# Evolution of Complex Networks via Edge Snapping

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***Abstract*—In this paper, we present a novel adaptive strategy for consensus and synchronization of complex networks. The strategy is inspired by bistable phenomena that are observed in a variety of mechanical systems. The novelty is that the adaptation involves the topology of the network itself rather than its coupling gains. In particular, we model the evolution of each coupling gain as a second order dynamical system that is subject to the action of a double-well potential. Through a new mechanism, termed as edge snapping, an unweighted network topology emerges at steady state. We assess the stability properties of the proposed scheme through analytical methods and numerical investigations. We conduct an extensive numerical study of the topological properties of the emerging network to elucidate the correlation between the initial conditions of the nodes' dynamics and the network structure.**

***Index Terms*—Adaptive control, consensus, networks, nonlinear systems, synchronization.**

## I. Introduction

SYNCHRONIZATION and consensus are common features in complex networked systems and are frequently encountered in nature and technology [1]–[9]. Synchronization has been the subject of considerable research effort in a diverse range of disciplines, from biology [10]–[13] to optoelectronics [14], [15] and cryptography [16]–[19]. In control theory, much ongoing research is focused on consensus problem [20]–[26]. Flocking, for example, is a typical consensus problem, in which a group of mobile agents has to align their velocity vectors and stabilize the interagent distances using decentralized nearest-neighbor interaction rules; see, for instance, [27]–[29]. Another common example of consensus is the rendezvous problem, in which a set of agents meets at a selected point of the space by using only information on the position of the nearest neighbors; see [30]–[32].

Typically, multiagent complex systems are modeled as networks of $N$ oscillators, often assumed to be identical. The individual dynamics of each oscillator is described by nonlinear ordinary differential equations of the form $\dot{x} = f(x)$, where $x \in \mathbb{R}^n$ is the state vector, $f : \mathbb{R}^n \to \mathbb{R}^n$ is a sufficiently smooth nonlinear vector field describing the system dynamics, the superimposed dot refers to the derivative with respect to the time variable $t$, and $n$ is a positive integer. The time evolution of the $i^{\mathrm{th}}$ dynamical system in the network is usually described by

$$\dot{x}_i = f(x_i) + \sigma \sum_{j\in\mathcal{N}_i} \left(h(x_j) - h(x_i)\right), \quad i = 1, 2, \ldots, N \quad (1)$$

where $x_i$ is the state of the $i^{\mathrm{th}}$ system, $\sigma$ is the constant coupling gain between neighboring nodes, $\mathcal{N}_i$ is the set of neighbors of node $i$, $h : \mathbb{R}^n \to \mathbb{R}^n$ is the output function, and the dependence on $t$ is omitted.

In general, when studying synchronization, the aim is to find conditions on the coupling gain $\sigma$ and the network topology that guarantee that the states of all network nodes converge to some common asymptotic evolution, that is,

$$\lim_{t\to\infty} (x_i(t) - x_j(t)) = 0 \quad (2)$$

for all $(i, j)$ such that $i \neq j$. Note that the classical consensus problems in control theory (see, for example, [20], [21], [23]–[26]) can be viewed as a particular case of synchronization, in which the agents are simple integrators whose dynamics is given by $f(x) = 0$.

The coupling gain is typically assumed to be common to all the network interconnections and equal to a constant value. Furthermore, the network topology is generally supposed to be fixed [2]–[4]. To overcome these limitations, adaptive gains strategies have been recently presented; see, for example, [33]–[40]. In these strategies, the network gains evolve according to integro-differential adaptation laws driven by global or local synchronization errors. For instance, a common adaptive gain for all the network nodes is studied in [33] and [34]. In [35]–[37], decentralized gain adaptation using different gains for each node [35], [36] or edge [36], [37] is proposed. In [38] and [39], gain adaptation is used to improve pinning control performance; while in [40], a hybrid adaptive mechanism is presented, where an edge is activated only if the estimated gain is above a certain threshold.

An interesting open problem is to find strategies to adapt not only the network gains but also the network structure to achieve synchronous behaviors. Indeed, many real-world applications show that the network topology is adaptively updated in response to changes of its nodes' dynamics. Examples can be found in nature (for example, in biological networks [41]–[43]) and in technology (for instance, wireless sensor networks that need to adapt to unexpected environmental changes [44]–[47]). More generally, the case of large scale extended systems with dynamic topologies was already discussed in the pioneering work by Šiljak in the late 1970s; see [22], [48]–[50].

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Recent works on synchronization over time-varying network topologies are relevant to the design of adaptation rules. In [51], a pinning control [52]–[57] technique based on a switching cyclical control of the network nodes is proposed. Master–slave synchronization under fast-switching periodic or stochastic linear feedback are presented in [58] and [59], respectively. Synchronization over time-varying stochastic networks through fast-switching is studied in [60]. Synchronization of networks with blinking interactions is studied in [61] and [62]; while in [63], the case of intermittent coupling with time scale ranging from very small to very large compared with the nodes' dynamics is investigated. Slowly time-varying topologies are also the focus of [64], where synchronization is adaptively achieved through the minimization of a local potential.

The aim of this paper is to introduce a novel network adaptation strategy based on the use of a continuous second order model for the gains' evolution. This model has a bistable steady state that leads to the formation of a self-emerging unweighted topology. Namely, edges are activated or deactivated according to local decentralized conditions that ensure the achievement of a common asymptotic evolution for the network nodes. We study the properties of the proposed strategy in networks of integrators (consensus problem) and in networks of nonlinear oscillators (synchronization). The strategy is effective in adaptively building network topologies that fulfill the synchronization objectives. We investigate the properties of the emerging topologies along with their correlation with the initial conditions of the networked system. Finally, we compare the model to the hybrid mechanism presented in [40]. Simulations are performed on a number of representative sample problems to illustrate and confirm the theoretical derivations.

## II. Edge Snapping

In recent literature on synchronization of adaptively coupled oscillators (see, for example, [35]–[37] and [40]), the network equations are written as

$$\dot{x}_i = f(x_i) + \sum_{j\in\mathcal{N}_i} \sigma_{ij}\left(h(x_j) - h(x_i)\right) \tag{3}$$

$$\dot{\sigma}_{ij} = g(e_{ij}) \tag{4}$$

where $\sigma_{ij}$ is the coupling gain associated to edge $(i,j)$, $\mathcal{N}_i$ is the set of neighbors of node $i$, and $g:\mathbb{R}^n\to\mathbb{R}$ is a real function of the error $e_{ij}$, defined as $e_{ij}=x_j-x_i$. The controlled evolution of the adaptive gains is based on first order dynamical systems, and the final topology in general results in a weighted Laplacian.

In this paper, we propose a smooth evolution of the adaptive gains that results into an unweighted self-emerging topology. We assume that nodes are initially able to communicate their mutual states over a *fundamental edge snapping topology*, described by $\mathcal{N}_i$ in (3). In analogy with [50], this is defined as the virtual network of interconnections used by the network nodes to decide whether to form physical links among themselves or not. Specifically, we suppose such fundamental connection topology to be an all-to-all network, over which the values of the physical gains $\sigma_{ij}$ (supposed to be initially zero) are updated. To obtain this behavior, we describe the evolution of the coupling gains $\sigma_{ij}$ between network nodes using second order dynamical systems. The model is a second order damped system subject to a bistable potential $V:\mathbb{R}\to\mathbb{R}$, driven by the force $g:\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}$, which is a function of the error $e_{ij}=x_j-x_i$:

$$\ddot{\sigma}_{ij} + d\dot{\sigma}_{ij} + \frac{\partial}{\partial\sigma_{ij}}V(\sigma_{ij}) = g(e_{ij}) \tag{5}$$

where $d$ is the damping. Network evolution is thus prominently dictated by the potential $V$.

Many mechanical systems, such as microelectromechanical devices [65]–[71] and nanomechanical structures [72]–[75], are characterized by bistability, that is, the coexistence of two different stable states. Switching between configurations is generally triggered by the external forcing. Examples of this *snap through* phenomenon can be found in [76]–[78].

The analogy with the evolution of the network topology is evident. At the onset of the evolution, all nodes are disconnected, that is, all the edges in the network are in the first equilibrium state. Then, on the basis of the error between nodes, representing the external forcing, the edges can leave the initial equilibrium point. Edge snapping is induced as the external forcing is strong enough to activate the corresponding link. In order to achieve a similar behavior, we select $V$ as a sufficiently smooth bistable potential with two local minima corresponding to the desired equilibria of the system: $\sigma_{ij}=0$ (non-active edge) and $\sigma_{ij}=1$ (active edge). Moreover, we set $g(e_{ij})=\mathcal{G}(\|e_{ij}\|)$, where $\mathcal{G}$ is selected from the set of functions of *class* $k_\infty$,[1] and $\|\cdot\|$ is the Euclidean norm.

In what follows, we study the stability of this model and the properties of the emerging topology. Initially, we focus on the simple case of a network of integrators (Section III), and later we analyze the more general case of network synchronization (Section IV).

## III. Consensus of $N$ Integrators

In the presence of the edge snapping mechanism described above, the dynamic equations of a network of integrators are

$$\dot{x}_i = u_i \tag{6}$$

where the communication protocol $u_i$ is defined as

$$u_i = \sum_{j=1}^{N} \sigma_{ij}^2 (x_j - x_i) \tag{7}$$

and the evolution of each $\sigma_{ij}$ is described by

$$\ddot{\sigma}_{ij} = -d\dot{\sigma}_{ij} - \frac{\partial}{\partial\sigma_{ij}}V(\sigma_{ij}) + \mathcal{G}\left(|x_j - x_i|\right). \tag{8}$$

In this case, we have $x_i\in\mathbb{R}$ and we consistently replace the error norm with the absolute value $|\cdot|$ in (8). Notice that we consider positive coupling gains in (7). In this way, none of the possible emerging topologies is unstable; see [79].

[1] A function $F: I\to\mathbb{R}$ is positive definite if $F(x)>0, \forall x\in I, x\neq 0$ and $F(0)=0$. A function $f:\mathbb{R}_{\geq 0}\to\mathbb{R}_{\geq 0}$ is of *class* $k$ if it is continuous, positive definite and strictly increasing. An unbounded function of *class* $k$ belongs to *class* $k_\infty$.

In order to assess the stability of this model, we recast the equation set into a more handleable form. To this aim, we define the average of the initial conditions of the integrators:

$$\alpha := \frac{\sum_{i=1}^{N} x_i(0)}{N}. \tag{9}$$

We note that

$$\bar{x}(t) := \frac{\sum_{i=1}^{N} x_i(t)}{N} = \alpha \quad \forall t \geq 0 \tag{10}$$

since $\dot{\bar{x}} = 0$. Thus, following [24], we define the *disagreement* vector $\delta = x - \alpha 1_N$, where $1_N$ is the $N$-dimensional unitary vector, and the *total disagreement* $\delta_{\text{tot}} = \delta^T \delta = \sum_{i=1}^{N} \delta_i^2$. Therefore, the governing equations of the network can be written as

$$\dot{\delta} = -L_\sigma \delta \tag{11}$$

$$\ddot{\sigma}_{ij} = -d\dot{\sigma}_{ij} - \frac{\partial}{\partial \sigma_{ij}} V(\sigma_{ij}) + \mathcal{G}(|\delta_j - \delta_i|). \tag{12}$$

Here, $L_\sigma = [l_{ij}]$ is the time-varying Laplacian matrix of the network defined as

$$l_{ij} = \begin{cases} \sum\limits_{\substack{j=1 \\ j \neq i}}^{N} \sigma_{ij}^2, & \text{if } i = j \\ -\sigma_{ij}^2, & \text{otherwise.} \end{cases} \tag{13}$$

### A. Stability Analysis

From (11), the evolution of the total disagreement is described by

$$\dot{\delta}_{\text{tot}} = -2\delta^T L_\sigma \delta = -\sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \sigma_{ij}^2 (\delta_j - \delta_i)^2 \leq 0. \tag{14}$$

Clearly, the total disagreement asymptotically approaches a constant value $0 \leq \delta_{\text{tot}}^{\infty} \leq \delta_{\text{tot}}(0)$. At steady state, we have $\dot{\delta}_{\text{tot}} = 0$, which implies that $\sigma_{ij} = 0, \forall (i,j)$ and/or $(\delta_j - \delta_i) = 0, \forall (i,j)$. Notice, nonetheless, that if for some $(i,j)$, $(\delta_j - \delta_i) \neq 0$, then (12) would imply that the corresponding $\sigma_{ij}$ is different than 0 and thus $\dot{\delta}_{tot}^{\infty} < 0$. Hence, we would get a contradiction. Therefore, $\delta_{\text{tot}}^{\infty} = 0$ and the network of integrators asymptotically reaches consensus on $\alpha$.

From (14), no claim can be made on the evolution of $\sigma_{ij}$. For the sake of clarity, we refer to one of many possible choices for the potential $V(\sigma_{ij})$, that is,

$$V(\sigma_{ij}) = b\sigma_{ij}^2 (\sigma_{ij} - 1)^2. \tag{15}$$

Nonetheless, the following consideration on stability can be easily extended to any smooth double-well potential, which is radially unbounded with power growth larger than 2. The shape of the double-well potential in (15) is depicted in Fig. 1. Notice that the height of the barrier separating the two equilibria can be properly tuned by varying the parameter $b$. With this choice of the potential, (12) can be viewed as a Duffing–Holmes oscillator, firstly introduced in [80] to model a buckled beam undergoing forced lateral vibrations. In our case, the forcing

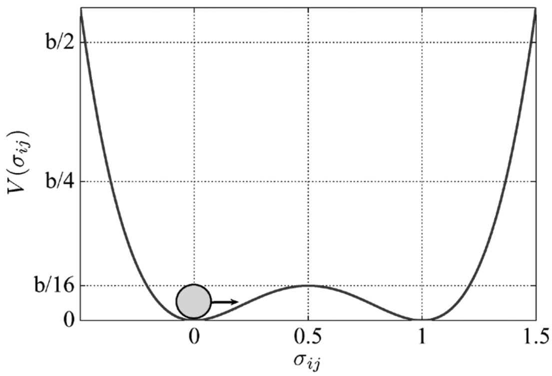


Fig. 1. Bistable potential driving the evolution of each $\sigma_{ij}$.

is not periodic as in [80], but it is bounded and asymptotically vanishes. Thus, considering that (12) is sufficiently smooth, we have $\ddot{\sigma}_{ij} \to -\infty^3$ as $\sigma_{ij} \to +\infty$. Hence, we conclude that each $\sigma_{ij}$ is upper bounded. Similar arguments can be used to show that $\sigma_{ij}$ is lower bounded. As a consequence, each $\sigma_{ij}$ converges to one of its equilibria, that is, $\sigma_{ij}^{\infty} \in \{0, 1/2, 1\}, \forall (i,j)$.

We first analyze the local stability of $\sigma_{ij} = 0$. From (14), we have that $|\delta_i| \leq \sqrt{\delta_{\text{tot}}(0)}$. Thus, we note that $\mathcal{G}(|\delta_j - \delta_i|) \leq \mathcal{G}(2\sqrt{\delta_{\text{tot}}(0)}) := \gamma < +\infty$ and that the initial condition for the total disagreement can be selected to have $\gamma$ arbitrary small. For local perturbations of $\sigma_{ij}$ and $\dot{\sigma}_{ij}$ from the origin and for sufficiently small $\gamma$, (12) can be approximated by

$$\ddot{\sigma}_{ij} + d\dot{\sigma}_{ij} + 2b\sigma_{ij} = \varepsilon \tag{16}$$

where $\varepsilon$ is bounded by $\gamma$ and asymptotically vanishes. Since $b$ and $d$ are positive quantities, the eigenvalues of the linear second order system in (16) have negative real part. Thus, for sufficiently small perturbations, $\sigma_{ij}$ remains bounded in the neighborhood of 0 and asymptotically goes to zero as $\varepsilon \to 0$. The same results can be obtained by linearizing (8) in the neighborhood of the equilibrium $\sigma_{ij} = 1$. The case of $\sigma_{ij} = 1/2$ is different as expected from Fig. 1. In fact, in the neighborhood of $\sigma_{ij} = 1/2$, $\dot{\sigma}_{ij} = 0$, system (12) can be approximated by

$$\ddot{\sigma}_{ij} + d\dot{\sigma}_{ij} - b(\sigma_{ij} - 1/2) = \varepsilon \tag{17}$$

which has one positive real eigenvalue that leads to an unstable equilibrium point. We conclude that almost all the solutions converge to $\delta = 0$ and $\sigma_{ij} = 0/1$. In other words, if we exclude the trivial case in which $\sigma_{ij}(0) = 1/2, \forall (i,j)$, and $\delta(0) = 0$, the network evolves to a final topology, whose structure depends on the initial conditions, the damping $d$, and the parameter $b$.

In our numerical experiments, we select $\mathcal{G}(|\delta_j - \delta_i|) = |\delta_j - \delta_i|$ and $d = 5$. As a first simulation case, we consider a network of 100 integrators, whose initial conditions are taken from a normal distribution with zero mean and standard deviation $std$ equal to 15. Moreover, we assume that the edge set is initially empty and the initial gains' growth rates are zero, that is, $\sigma_{ij}(0) = \dot{\sigma}_{ij}(0) = 0, \forall (i,j)$. By selecting $b = 16$, the integrators reach consensus on the average value $\alpha = 0.72$, and the gains $\sigma_{ij}$ asymptotically converge to one of the two stable equilibria; see Fig. 2. As a result of this evolution, a connected

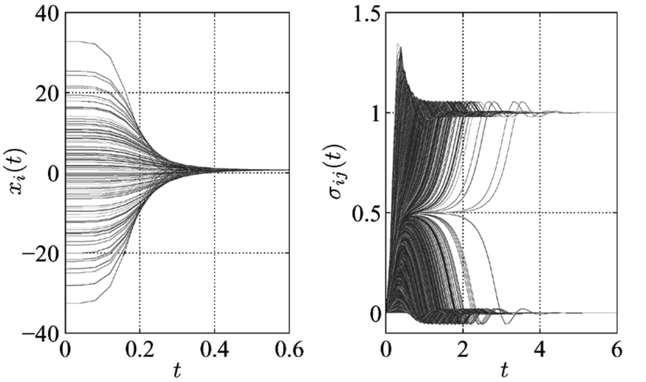


Fig. 2. Consensus of 100 integrators. Evolution of the states (left) and the coupling gains (right) when $b = 16$.

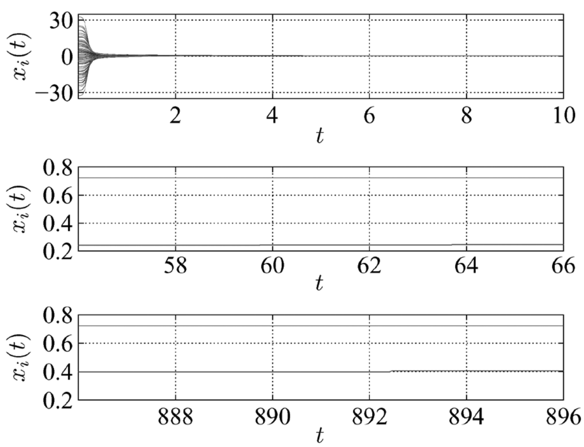


Fig. 3. Consensus of 100 integrators. Evolution of the states from time 0 to 10 (top), from time 56 to 66 (center) and from time 886 to 896 (bottom). Here $b \geq 180.36$. Note that consensus is slowly approached.

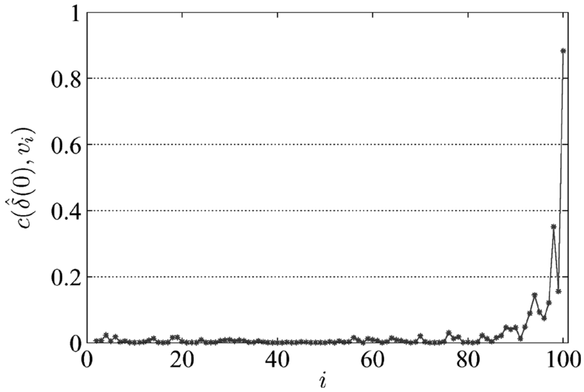


Fig. 4. Network of 100 integrators. Correlation of the $N$ eigenvectors of the emerging topology with the initial condition $\hat{\delta}(0)$ of the integrators.

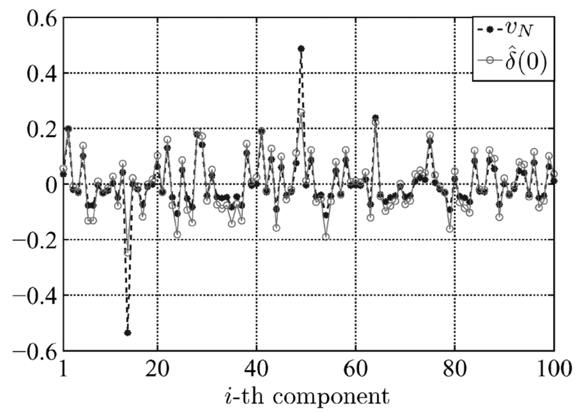


Fig. 5. Network of 100 integrators. Comparison of the components of the initial conditions $\hat{\delta}(0)$ and the maximum eigenvector $v_N$.

network emerges. By increasing the barrier $b$ between the two equilibria, the snapping dynamics becomes more difficult and less edges are activated. Above a critical value $\bar{b}$, that with our parameters and initial conditions is equal to 180.36, the final network becomes disconnected, and the settling time dramatically increases as illustrated in Fig. 3. It is worth emphasizing that, even though the emerging topology can be disconnected, consensus will be asymptotically reached. In fact, during the transient, the network remains connected with very low weights, which asymptotically vanish as consensus is achieved. Nonetheless, if the height of the barrier is properly tuned, the emerging topology is a connected network. Thus, edge snapping can be viewed as a distributed and adaptive way of building a topology.

In order to evaluate without any bias the properties of the self-emerging topology, we have considered the case in which each pair of nodes is able to communicate with each other and then decide whether activating or not an edge, see (7) where the summation goes from 1 to $N$. In other words, we have assumed an all-to-all fundamental edge snapping topology. Nevertheless, it is worth remarking that the stability analysis performed above requires only a connected fundamental edge snapping topology, that is, only a connected subset of edges can be activated. The aim of the following subsection is to investigate in greater detail the properties of this topology.

### B. Analysis of the Emerging Topology

The spectral properties of the Laplacian matrix have a fundamental role on the consensus and synchronization dynamics. Thus, to assess the properties of the emerging Laplacian, denoted with $\bar{L}_\sigma$, we investigate the relations between the properties of the Laplacian spectrum (in particular, the Laplacian eigenvectors) and the initial conditions. We denote by $0 = \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_N$ the eigenvalues of $\bar{L}_\sigma$ and with $v_1, \ldots, v_N$ the corresponding normalized eigenvectors. We refer to $v_N$ as the *maximum eigenvector*. We measure the correlation $c$ between the initial conditions and the eigenvectors of the Laplacian as the inner product between the normalized initial disagreement $\hat{\delta}(0) = \delta(0)/\|\delta(0)\|$ and each eigenvector:

$$c\left(\hat{\delta}(0), v_i\right) = \hat{\delta}(0)^T v_i. \tag{18}$$

This index is useful in investigating the relationship between the initial conditions of the nodes and the emerging topology. As an example, we calculate $c(\hat{\delta}(0), v_i)$ for the 100 nodes network previously analyzed, selecting $b = 180$. As illustrated in Fig. 4, only the last eigenvectors are significantly correlated to the initial conditions. In particular, the largest correlation index corresponds to the maximum eigenvector $v_N$, in fact $c(\hat{\delta}(0), v_N) = 0.88$. This correlation is also evident when plotting the components of the two vectors $v_N$ and $\hat{\delta}(0)$; see Fig. 5.

To give a statistical relevance to these considerations, we have performed 50 repetitions of these numerical experiments, generating the initial conditions from a normal distribution with zero

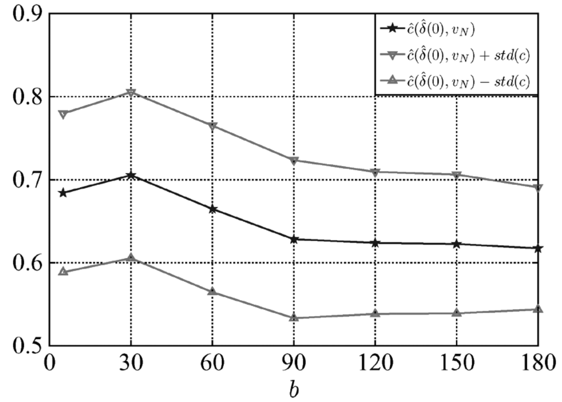


Fig. 6. Network of 100 integrators. Correlation between maximum eigenvector $v_N$ and initial conditions $\hat{\delta}(0)$ as a function of the height of the barrier $b$.

mean and $std = 15$. The average correlation $\hat{c}(\hat{\delta}(0), v_N)$ is equal to 0.62, which is sufficiently high. Furthermore, this correlation is always larger than 0.6 regardless of the barrier $b$ (see Fig. 6) and shows a peak at $b = 30$. In the analysis, we do not take into consideration the case of extremely low barriers, that would indeed results into highly connected networks. Similar evidence has been gathered for the influence of the damping $d$ on the correlation $c(\hat{\delta}(0), v_i)$, and data are not reported here for brevity.

Edge snapping dynamics tends to induce a topology in which the maximum eigenvector is strongly related to the initial conditions. Thus, it is a viable method to build topologies well-suited to rapidly synchronize integrators with prescribed initial conditions. Indeed, if a consensus protocol with initial conditions $x(0)$ is executed on the emerging topology, the time rate to consensus, that is, the exponential rate of convergence, will be approximately $\lambda_N$.

In addition, initial conditions play a key role in determining other topological features of the network, such as the *degree distribution* and the *betweenness centrality* of the nodes. The degree $k_i$ of a node $i$ is the number of incident edges, while the betweenness centrality is a measure of the importance of a node in the network [81], [82]. The betweenness centrality of a node $v$ is defined by

$$bc(v) = \sum_{\substack{i,j=1 \\ i\neq j\neq v}}^{N} \frac{s_{ij}(v)}{s_{ij}} \tag{19}$$

where $s_{ij}$ is the number of shortest paths connecting node $i$ to $j$ and $s_{ij}(v)$ is the number of shortest paths from $i$ to $j$ that vertex $v$ lies on. As illustrated in Figs. 7 and 8, nodes that are far from the mean value $\alpha$ are more likely to exhibit a higher final degree and betweenness centrality.

## IV. Synchronization of $N$ Nonlinear Oscillators

In this section, we explore the possibility of synchronizing a network of nonlinear oscillators via edge snapping. The governing equations of the network are

$$\dot{x}_i = f(x_i) - \rho \sum_{j=1}^{N} l_{ij} h(x_j) \tag{20}$$

$$\ddot{\sigma}_{ij} = -\, d\dot{\sigma}_{ij} - \frac{\partial}{\partial \sigma_{ij}} V(\sigma_{ij}) + \mathcal{G}\left(\|x_j - x_i\|\right) \tag{21}$$

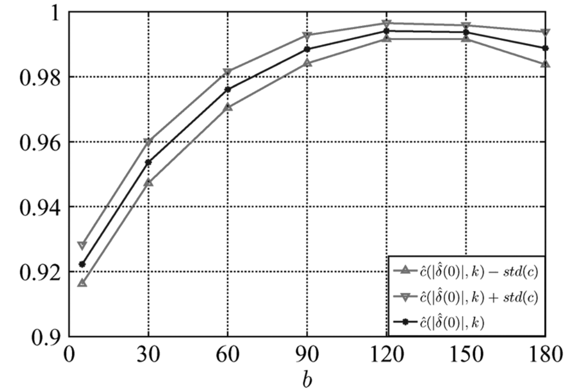


Fig. 7. Network of 100 integrators. Correlation between the degree distribution $k$ and initial conditions $\hat{\delta}(0)$ as a function of the height of the barrier $b$.

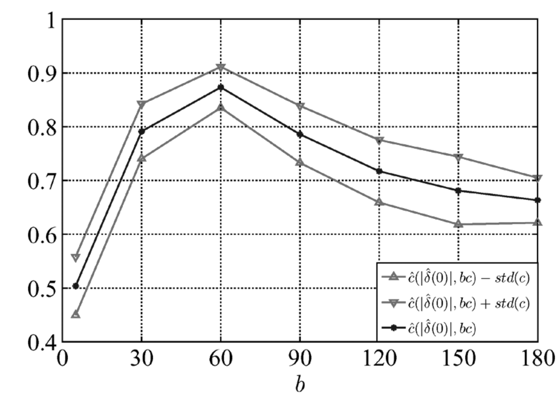


Fig. 8. Network of 100 integrators. Correlation between the betweenness centrality $bc$ and initial conditions $\hat{\delta}(0)$ as a function of the height of the barrier $b$.

where $l_{ij}$ is an element of the adaptive Laplacian matrix defined in (13), $\rho$ is an additional overall coupling gain, and for ease of illustration $\mathcal{G}(\|x_j - x_i\|)$ is selected to be $\|x_j - x_i\|^2$.

We denote with $x_s(t)$ the invariant synchronization manifold and with $\delta x_i = x_i - x_s$ the transverse dynamics of the $i^{\text{th}}$ oscillator and define $\delta x = (\delta x_1, \ldots, \delta x_N)^T \in \mathbb{R}^{n\times N}$. Similarly, we define $\delta\sigma = (\delta\sigma_{12}, \ldots, \delta\sigma_{1N}, \ldots, \delta\sigma_{N1}, \ldots, \delta\sigma_{NN})^T$ and $\delta\dot{\sigma} = (\delta\dot{\sigma}_{12}, \ldots, \delta\dot{\sigma}_{1N}, \ldots, \delta\dot{\sigma}_{N1}, \ldots, \delta\dot{\sigma}_{NN})^T$ as local perturbations from one of the locally stable equilibrium configurations of the coupling gains (that are $\sigma_{ij} = 0/1, \dot{\sigma}_{ij} = 0, \forall (i,j)$). Therefore, we have

$$\delta\dot{x} = \left[I_N \otimes Jf(x_s) - \rho \bar{L}_\sigma \otimes Jh(x_s)\right] \delta x \tag{22}$$

$$\delta\ddot{\sigma} = -\, d\delta\dot{\sigma} - 2b\delta\sigma \tag{23}$$

where $Jf(x_s)$ and $Jh(x_s)$ denote the Jacobian of $f$ and $h$ evaluated on the synchronization manifold respectively, $\bar{L}_\sigma$ is the constant Laplacian matrix corresponding to the considered equilibrium configuration, and $\otimes$ denotes the Kronecker product.

### A. Stability Analysis

Equations (22) and (23) are decoupled. Thus, their stability can be considered separately. Equation (23) has all eigenvalues with negative real part since $b$ and $d$ are positive constants. We

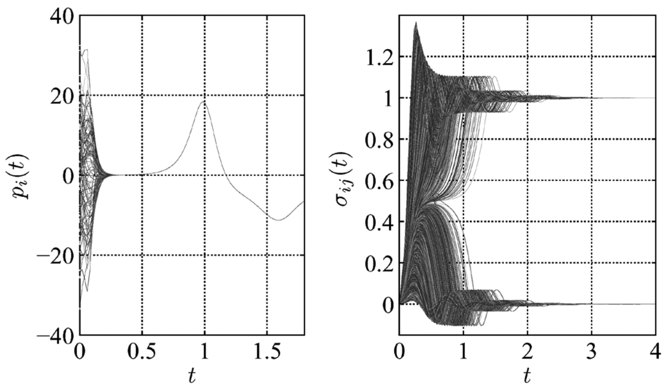


Fig. 9. Network of 100 Lorenz oscillators. Evolution of $p$ (left) and $\sigma$ (right).

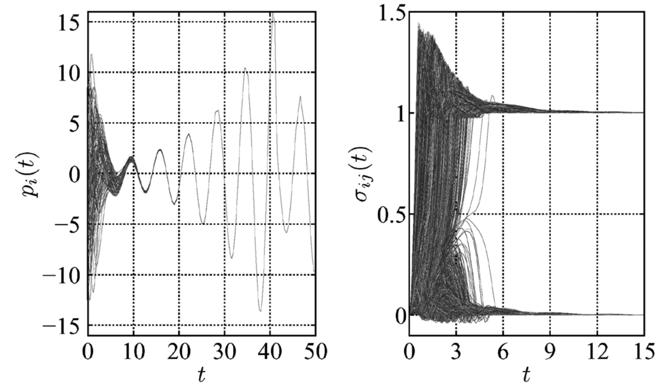


Fig. 10. Network of 100 Rössler oscillators. Evolution of $p$ (left) and $\sigma$ (right).

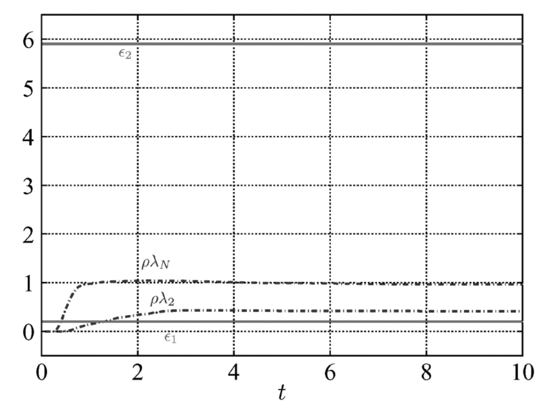


Fig. 11. Network of 100 Rössler oscillators. Evolution of $\tilde{\sigma}\lambda_2$ and $\tilde{\sigma}\lambda_N$ compared with the lower and upper bounds $\epsilon_1$ and $\epsilon_2$.

use the master stability function (MSF) approach, firstly introduced in [83], to study the transverse stability of the synchronization manifold. Clearly, it is possible to write $\delta x_i = \sum_{i=1}^{N} v_i \otimes \xi_i(t)$. The evolution of the coefficients $\xi_i$ is given by

$$\frac{d\xi_i}{dt} = \left[Jf(x_s) - \rho\lambda_i Jh(x_s)\right]\xi_i. \quad (24)$$

If we consider a generic $\nu$ instead of $\lambda_i$, we can extract $n$ Lyapunov exponents depending on $\nu$. The parametric behavior of the largest Lyapunov exponent is the MSF $\Lambda(\nu)$. Thus, for any given $\rho$, the only locally stable coupling configurations are those characterized by a Laplacian with a distribution of the eigenvalues such that the MSF is negative, that is, $\Lambda(\lambda_i) < 0, \forall i = 2, \ldots, N$ ($\lambda_1 = 0$ is not considered, since the corresponding Lyapunov exponents describe the evolution on the synchronization manifold). If we exclude the trivial case in which $\Lambda(0) \leq 0$, every uncoupled configuration ($\lambda_2 = 0$) is locally unstable. Note that the stability analysis can be slightly modified to encompass the case of an arbitrary $\mathcal{G}$ of class $k_\infty$. In this case, (24) is unidirectionally coupled to (23), and the line of argument in Section III can be adapted to ascertain its stability.

As an example, we consider a network of Lorenz oscillators that are coupled on all their state variables. In this case, the state has three components $x = (p, q, r)^T$ and $f(x) = (10(q-p), -pr + 28z - q, pq - (8/3)r)^T$. For $h(x) = x$, a network of Lorenz oscillators is characterized by an unbounded stability region. In fact, the master stability function is equal to $\Lambda(\nu) = \Lambda(0) - \nu \cong 0.9 - \nu$. In this case, the only eigenvalue that affects the local stability is $\lambda_2$. In our simulations, we select $d = 5$, $b = 180$, and we fix $\rho = 1$. Initial conditions for the states are taken from a normal distribution with zero mean and $std = 15$. As made clear by Fig. 9, the network rapidly escapes from the unstable disconnected configuration and settles in a locally stable synchronous configuration characterized by $\lambda_2 = 23.84$ and thus by $\Lambda(\lambda_2) = -22.94 < 0$.

As an additional illustration of the edge snapping method, we analyze a network of Rössler oscillators with state vector $x = (p, q, r)$. The dynamic equation of the Rössler oscillator is characterized by the vector field $f(x) = (-q - r, p - c_1 q, c_2 + (p - c_3)r)^T$. By choosing the parameters as in [51] ($c_1 = 0.165$, $c_2 = 0.2$ and $c_3 = 10$), the system exhibits chaotic behavior. We couple the chaotic oscillators only on the variables $p$ and $r$, that is, we select $h(x_i) = Hx_i$ with

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

In this case, the stability region is bounded. Thus, a coupling configuration is locally stable if and only if

$$0 < \epsilon_1 < \rho\lambda_2 \leq \ldots \leq \rho\lambda_N < \epsilon_2 < +\infty \quad (25)$$

with $\epsilon_1 = 0.2$ and $\epsilon_2 = 5.9$, as reported in [51]. For a given $\rho$, as the network grows, no coupling configuration satisfies the bounds in (25) simultaneously leading to unstable synchronization manifold. To prevent this behavior, we select $\rho = 1/N$. In this way, the bounds in (25) scale with the size of the network. Initial conditions for the states are taken randomly from a zero mean normal distribution with $std = 5$. By choosing $d = 5$ and $b = 50$, we can see from Fig. 10 that synchronization is achieved. Furthermore, we observe that the snapping dynamics adapts the network configuration in such a way that $\lambda_N$ and $\lambda_2$ satisfy (25), as illustrated in Fig. 11.

As for the consensus case, the above stability analysis is valid for any connected fundamental edge snapping topology. As a numerical example, we show the case where nodes exchange information on their states over a scale-free fundamental edge snapping topology with average degree $m = 5$ in a network of 100 Lorenz oscillators. We select $d = 5$, $b = 150$, $\rho = 1$ and

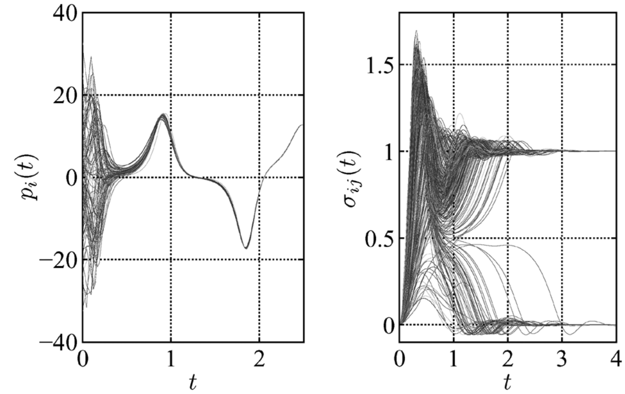


Fig. 12. Network of 100 Lorenz oscillators with scale-free fundamental edge snapping topology. Evolution of $p$ (left) and $\sigma$ (right).

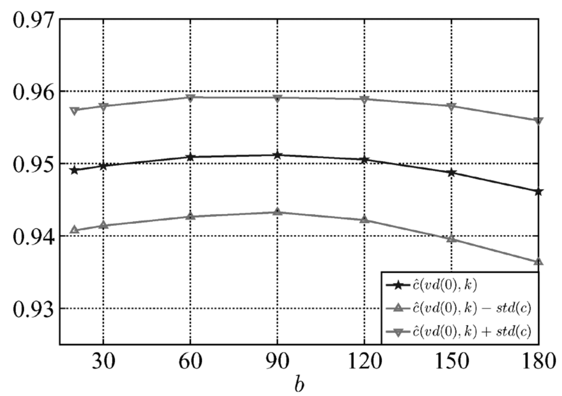


Fig. 13. Network of 100 Lorenz oscillators. Correlation between the degree distribution $k$ and initial distance from synchronization $vd(0)$ as a function of the height of the barrier $b$.

we take the initial conditions from a normal distribution with $std = 15$. As shown in Fig. 12, synchronization is asymptotically achieved.

## *B. Analysis of the Emerging Topology*

As for the consensus problem, we analyze the influence of the initial conditions on the properties of the emerging topology. We measure the initial node localization using the index *initial vertex distance from synchronization* $vd_i(0)$ to lump the vector structures of the network states. This index has been firstly introduced in [40] and is defined as

$$vd_i(0) = \sum_{j=1}^{N} \|e_{ij}(0)\| \tag{26}$$

where $e_{ij}(0) = x_j(0) - x_i(0)$. The correlation between this index and the topological features of the final topology, such as the eigenvectors, the degree distribution, and the betweenness centrality, is investigated in networks of Lorenz systems and Rössler chaotic oscillators.

As illustrated in Figs. 13 and 14, edge snapping dynamics leads to a final topology whose degree distribution is prominently dictated by the initial conditions of the nodes also for synchronization problems. Moreover, initial conditions are significantly correlated to the maximum eigenvector of the emerging

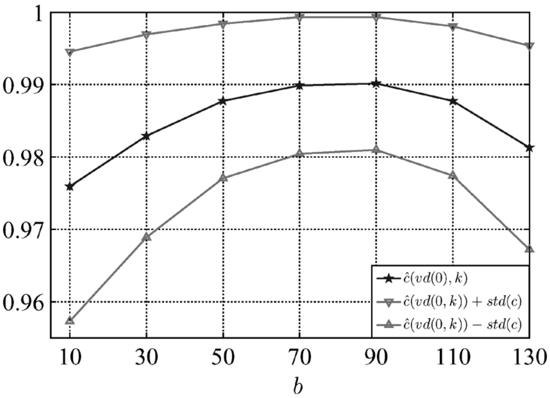


Fig. 14. Network of 100 Rössler oscillators coupled through $p$ and $r$. Correlation between the degree distribution $k$ and initial distance from synchronization $vd(0)$ as a function of the height of the barrier $b$.

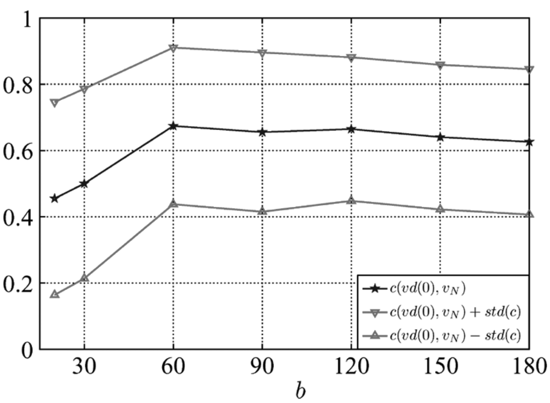


Fig. 15. Network of 100 Lorenz oscillators. Correlation between the maximum eigenvector $v_N$ and initial distance from synchronization $vd(0)$ as a function of the height of the barrier $b$.

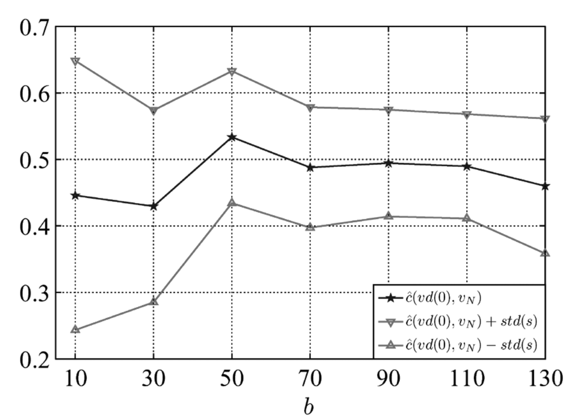


Fig. 16. Network of 100 Rössler oscillators coupled through $p$ and $r$. Correlation between the maximum eigenvector $v_N$ and initial distance from synchronization $vd(0)$ as a function of the height of the barrier $b$.

topology, as reported in Figs. 15 and 16. Nevertheless, in this case, the correlation is slightly lower as compared to the consensus problem. Similarly, the correlation with the betweenness centrality is lower, but still remarkable; see Figs. 17 and 18. As for consensus problems, correlations are significant for a broad range of $b$.

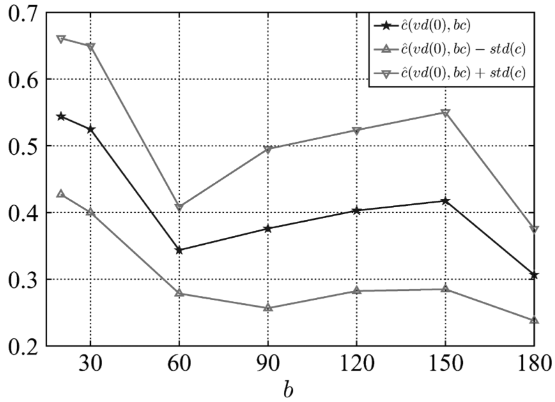

Fig. 17. Network of 100 Lorenz oscillators. Correlation between the betweenness centrality $bc$ and initial distance from synchronization $vd(0)$ as a function of the height of the barrier $b$.

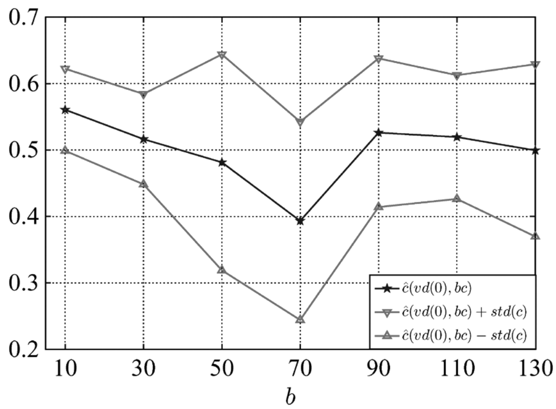

Fig. 18. Network of 100 Rössler oscillators coupled through $p$ and $r$. Correlation between the betweenness centrality $bc$ and initial distance from synchronization $vd(0)$ as a function of the height of the barrier $b$.

## V. Hybrid Adaptive Coupling

An alternative approach for network adaptation is based on a non-smooth link activation and deactivation. In this case, a link is activated, and consequently the corresponding gain switches instantaneously from 0 to 1, only when a certain condition is verified. In [40], an hybrid mechanism with an activation threshold on the coupling gain is proposed, that is,

$$\sigma_{ij}(t) := \begin{cases} \hat{\sigma}_{ij}, & \text{if } \Phi(\hat{\sigma}_{ij}, e_{ij}) > l \\ 0, & \text{otherwise} \end{cases} \tag{27}$$

where $\hat{\sigma}_{ij}$ is called *reference gain* and $\Phi$ *link activation function*. The evolution of the reference gain is governed by a first order law of the form

$$\dot{\hat{\sigma}}_{ij} = \mu \| e_{ij} \| \tag{28}$$

while $\Phi$ is chosen as

$$\Phi(e_{ij}, \hat{\sigma}_{ij}) = \hat{\sigma}_{ij}. \tag{29}$$

Thus, the link is instantaneously activated only when the corresponding estimated coupling gain is larger than the threshold and the gain continues to increase smoothly till the synchronization/consensus is achieved.

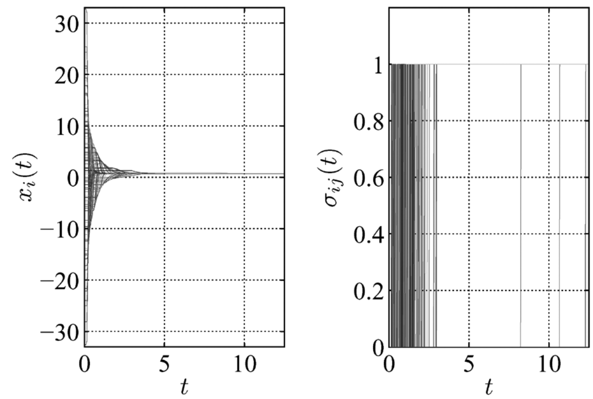

Fig. 19. Consensus of 100 integrators with hybrid coupling. Gain $\mu$ equal to 1 and threshold $l$ equal to 9. Initial conditions from a zero mean normal distribution with $std = 15$. Evolution of the states (left) and the coupling gains (right).

In order to evolve to a final unweighted topology, as in the edge snapping case, we modify (27) as follows:

$$\sigma_{ij}(t) := \begin{cases} 1, & \text{if } \Phi(\hat{\sigma}_{ij}, e_{ij}) > l \\ 0, & \text{otherwise.} \end{cases} \tag{30}$$

The mechanical analog of this hybrid strategy is the sticking phenomenon [84]. As an example, consider a particle located at the edge of a non-smooth surface that is forced by an external load while sticking to the surface through mechanical adhesions. As the external load overcomes the adhesion forces, the particle suddenly reaches a different location. In what follows, we compare edge snapping to this hybrid approach. An advantage of the edge snapping model is the smoothness of the gain evolution, which eases the stability analysis of the network evolution. As an example, the MSF approach is easily extended to analyze local stability of the synchronization manifold. On the other hand, the stability of the disconnected configurations in the consensus problem illustrated in Section III counts against. This problem never appears in the hybrid strategy, in which the final configuration is always connected. In fact, an increase of the threshold results in a delayed consensus and in a different, but still connected, network topology. Nonetheless, general claims on the stability properties of the hybrid strategy in the synchronization case are difficult to obtain. As for the transient dynamics, the hybrid strategy shows discontinuous nodes' dynamics. As one edge is activated, the corresponding nodes instantaneously modify their evolution and their trajectories converge towards each other, as depicted in Fig. 19.

We refer to the consensus problem to analyze the topology emerging from the hybrid approach, but similar results are obtained for synchronization and are not reported here for brevity. The final properties of the network are largely dictated by the initial conditions in the hybrid approach as well. In particular, the final degree distribution and the maximum eigenvector show a similar correlation to the initial conditions as compared to the edge snapping case; see Figs. 20 and 21. The correlation with the betweenness centrality is instead very sensitive to the threshold $l$. The higher the threshold is, the lower is the correlation, as illustrated in Fig. 22.

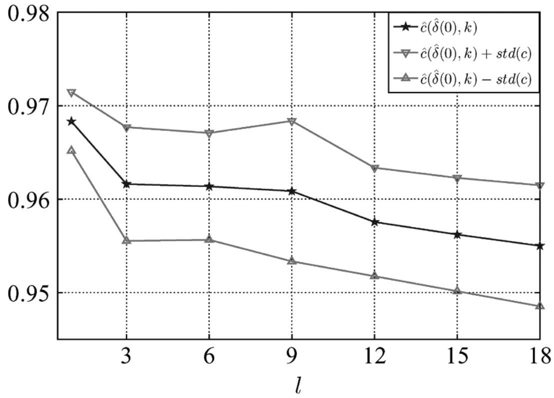


Fig. 20. Network of 100 integrators with hybrid coupling. Correlation between the degree distribution $k$ and initial conditions $\hat{\delta}(0)$ as a function of the threshold $l$.

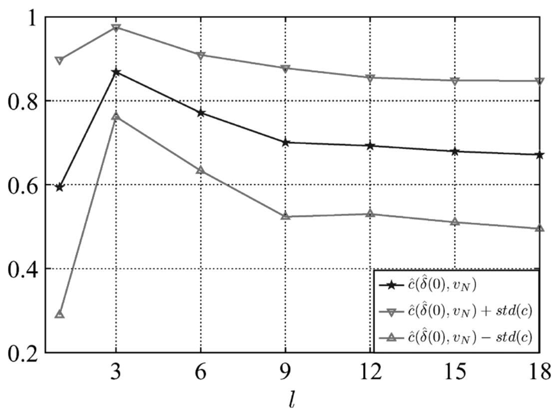


Fig. 21. Network of 100 integrators with hybrid coupling. Correlation between the maximum eigenvector $v_N$ and initial conditions $\hat{\delta}(0)$ as a function of the threshold $l$.

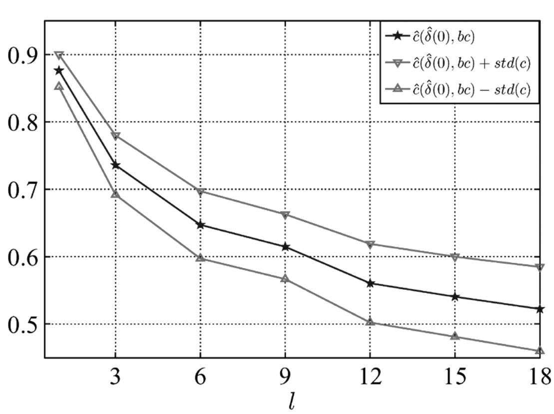


Fig. 22. Network of 100 integrators with hybrid coupling. Correlation between the betweenness centrality $bc$ and initial conditions $\hat{\delta}(0)$ as a function of the threshold $l$.

## VI. Partial Edge Snapping

As illustrated in the previous sections, edge snapping can be successfully used to drive the network evolution by using the states of all its nodes. The same mechanism can also be applied to control the evolution of a subset of the network edges in order to improve the properties of an existing static topology, see Fig. 23. In particular, edge snapping can be used to guide the evolution of a limited fraction of the edge set of the fundamental edge snapping topology, while blocking the rest of the network.

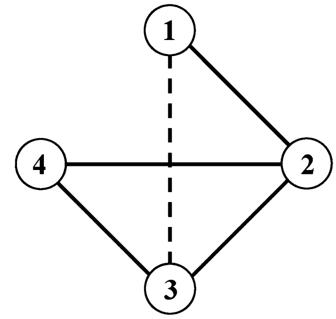


Fig. 23. Partial edge snapping. The solid line represents the edges of the existing static topology, while the dashed line represents the edges which are allowed to snap, that is, the edges of the fundamental edge snapping topology.

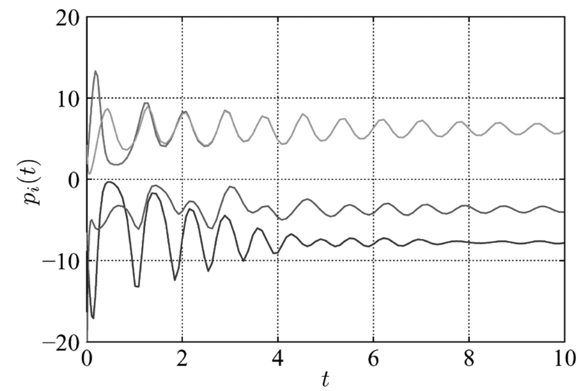


Fig. 24. Network of 4 Lorenz oscillators without snapping. Evolution of $p$.

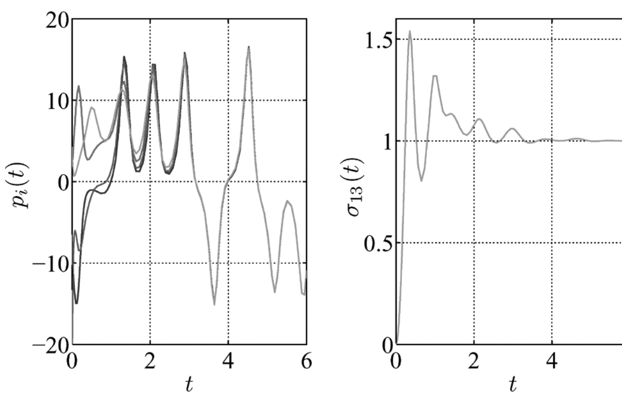


Fig. 25. Network of 4 Lorenz oscillators with partial edge snapping. Evolution of $p$ (left) and $\sigma$ (right).

As an example, we consider the network in Fig. 23, where each node represents a Lorenz oscillator. Initial conditions on the states taken from a normal distribution with $std = 15$. In this case, only the edge (1,3) is allowed to snap. In our simulations, we choose the overall coupling gain $\rho = 0.48$. Without snapping, the constant Laplacian matrix of the system is

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} \tag{31}$$

and thus $\lambda_2 = 1$. In this case $\Lambda(\lambda_2) = 0.9 - 0.48 = 0.42 > 0$, leading to the instability of the synchronization manifold. As shown in Fig. 24, synchronization is not achieved with this configuration. If we allow edge (1,3) to snap, we can see, as shown in Fig. 25, that edge (1,3) is activated. The addition of this new edge leads the network to a synchronous evolution. The final configuration of the network is locally asymptotically stable: in fact, in the emerging topology we have $\lambda_2 = 2$, and thus $\Lambda(\lambda_2) = 0.9 - 0.96 = -0.06 < 0$.

## VII. Conclusion

We presented a new viable strategy to adaptively control link activation in complex networks. This strategy is based on a simple mechanical analogy. The gain evolution, named *edge snapping*, mimics the damped motion of a particle in a one-dimensional space subject to a double-well potential.

We analyzed the stability properties of this strategy, both in the consensus and synchronization problems. In particular, we showed that consensus is achieved for every initial conditions and we studied local synchronization by extending the MSF approach to the case of adaptive couplings. The proposed strategy aims not only at synchronizing/leading to consensus the coupled dynamical systems, but also to adaptively determine the final network topology. We showed that the properties of this self-emerging topology are strongly tied to the initial conditions of the network. In particular, we found a remarkable correlation between the initial conditions of the network and the maximum eigenvector of the emerging topology. In the consensus case, this means that the final topology is well suited to rapidly lead to consensus prescribed initial conditions. Indeed, if a consensus protocol with initial conditions $x(0)$ is executed on the emerging topology, the time-rate to consensus will be approximately $\lambda_N$.

Furthermore, we showed that other topological features of the network, such as degree distribution and betweenness centrality, are prominently dictated by the initial conditions. The edge snapping strategy was compared with an hybrid approach, inspired by the one presented in [40]. This approach exhibits similar properties, but does not allow for immediate stability analyses.

Current research activities are devoted to establish qualitative procedures for *a priori* tailoring the network properties.

## Acknowledgment

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