The real and imaginary parts of this function are each realizable. We think of this complex forcing function as two forcing functions, a real one and an imaginary one, and as a consequence of linearity, the principle of superposition applies and thus the current response can be written as

$$i(t) = I_M \cos(\omega t + \phi) + jI_M \sin(\omega t + \phi) \tag{8.16}$$

where $I_M \cos(\omega t + \phi)$ is the response due to $V_M \cos \omega t$ and $jI_M \sin(\omega t + \phi)$ is the response due to $jV_M \sin \omega t$. This expression for the current containing both a real and an imaginary term can be written via Euler's equation as

$$i(t) = I_M e^{j(\omega t + \phi)} \tag{8.17}$$

Because of the preceding relationships, we find that rather than apply the forcing function $V_M \cos \omega t$ and calculate the response $I_M \cos(\omega t + \phi)$, we can apply the complex forcing function $V_M e^{j\omega t}$ and calculate the response $I_M e^{j(\omega t + \phi)}$, the real part of which is the desired response $I_M \cos(\omega t + \phi)$. Although this procedure may initially appear to be more complicated, it is not. It is through this technique that we will convert the differential equation to an algebraic equation that is much easier to solve.

EXAMPLE 8.4

Once again, let us determine the current in the RL circuit examined in Example 8.3. However, rather than apply $V_M \cos \omega t$, we will apply $V_M e^{j\omega t}$.

SOLUTION

The forced response will be of the form

$$i(t) = I_M e^{j(\omega t + \phi)}$$

where only I_M and ϕ are unknown. Substituting $v(t)$ and $i(t)$ into the differential equation for the circuit, we obtain

$$RI_M e^{j(\omega t + \phi)} + L\frac{d}{dt}\left(I_M e^{j(\omega t + \phi)}\right) = V_M e^{j\omega t}$$

Taking the indicated derivative, we obtain

$$RI_M e^{j(\omega t + \phi)} + j\omega L I_M e^{j(\omega t + \phi)} = V_M e^{j\omega t}$$

Dividing each term of the equation by the common factor $e^{j\omega t}$ yields

$$RI_M e^{j\phi} + j\omega L I_M e^{j\phi} = V_M$$

which is an algebraic equation with complex coefficients. This equation can be written as

$$I_M e^{j\phi} = \frac{V_M}{R + j\omega L}$$

[hint]

Summary of complex number relationships:

$x + jy = re^{j\theta}$

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\dfrac{y}{x}$

$x = r\cos\theta$

$y = r\sin\theta$

$\dfrac{1}{e^{j\theta}} = e^{-j\theta}$

Converting the right-hand side of the equation to exponential or polar form produces the equation

$$I_M e^{j\phi} = \frac{V_M}{\sqrt{R^2 + \omega^2 L^2}}\, e^{j[-\tan^{-1}(\omega L/R)]}$$

(A quick refresher on complex numbers is given in the Appendix for readers who need to sharpen their skills in this area.) The preceding form clearly indicates that the magnitude and phase of the resulting current are

$$I_M = \frac{V_M}{\sqrt{R^2 + \omega^2 L^2}}$$

and

$$\phi = -\tan^{-1}\frac{\omega L}{R}$$