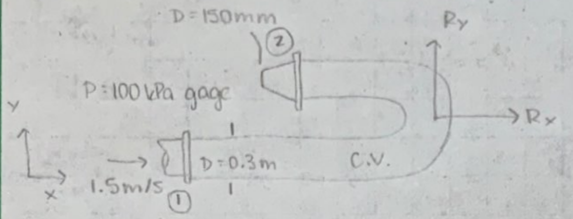
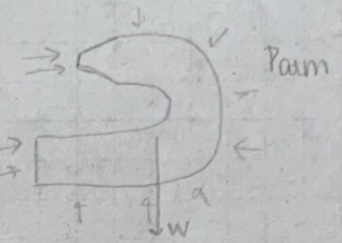
Veronica Loomis 620 HW.2

PROBlem 1

Determine forces in x & y direction
Water leaves nozzle as free jet
Interior volume $= 0.1\ m^3$
Assume: steady, incompressible flow. One dimensional





Neglect contributions from Patm & ignore τ_w

$\rho_1 = \rho_2$ (incompressible)

$P_1 = P_2$

$\iiint \frac{\partial(\rho v)}{\partial t}\, dV = 0$ (steady)

Start with continuity integral

$$\iint_{CS} (\rho \vec{v} \cdot d\vec{S}) = 0$$

$$\rho V_2 A_2 = \rho V_1 A_1$$

$$\rho (1.5 m/s)(0.3^2)\tfrac{1}{4} m^2 \pi = \rho V_1 (0.15^2)\tfrac{1}{4} m^2 \pi$$

$$V_1 = \frac{1.5(0.3)^2}{(0.15)^2}$$

$$V_1 = 6\ m/s$$

Balance integral momentum (x)

$$\iint_{CS} (\rho \vec{v} \cdot d\vec{S})\vec{v} = F_x = P_1 A_1 + P_2 A_2 + R_x$$

$$-V_1 \rho_1 V_1 A_1 + V_2 \rho_2 V_2 A_2 = P_1 A_1 + P_2 A_2 + R_x$$

$$\rho (V_2^2 A_2 - V_1^2 A_1) = P_1 A_1 + P_2 A_2 + R_x$$

$$R_x = \rho (V_2^2 A_2 - V_1^2 A_1) - P_1 A_1 - P_2 A_2$$

$$R_x = \rho \left((6)^2 (0.15)^2 \tfrac{\pi}{4} - (1.5)^2 (0.3)^2 \tfrac{\pi}{4}\right) - 100 kPa (0.3)^2 \tfrac{\pi}{4} - 100 kPa (0.15)^2 \tfrac{\pi}{4}$$

$$R_x = \rho (0.4771\ N\ m^3/kg) - 5301.4\ N$$

* $\rho_{water} = 997.77\ kg/m^3$

$$\boxed{R_x = -4825.36\ N}$$

Balance integral momentum (y)

$$0 = R_y - W$$

$$R_y = W = mg$$

$$R_y = m (9.81\ N/kg)$$

* $\rho_{water} = 997.77\ kg/m^3$

$\rho = m/V$ $(V = 0.1 m^3)$

$m = 99.777 kg$

$$\boxed{R_y = 978.81\ N}$$

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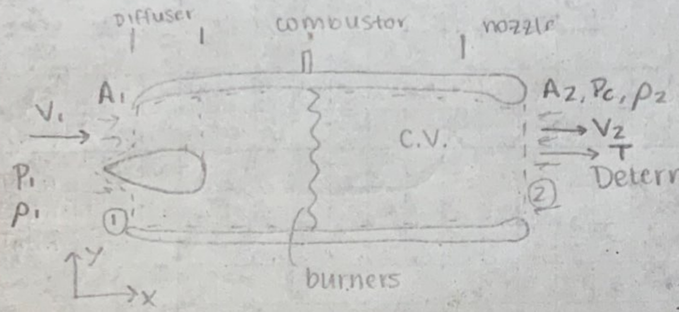
PROBLEM 2

Supersonic Ram Jet

Fuel burned in stream in combustion zone

Products of combustion expand to some P_c at nozzle exit

$P_c \neq P_{ambient}$



Moving at V_1

inlet area: A_1

exit vel: V_2

fuel burned per unit time: W_f

Determine Thrust developed by ramjet

neglect skin friction

integral momentum balance (x)

$$\iint_{CS} (\rho \vec{V} \cdot d\vec{s}) \vec{V}$$

$$V_2 \rho_2 V_2 A_2 - V_1(\rho_1 V_1 A_1 - W_f) = F_x$$

$$V_2 \rho_2 V_2 A_2 - V_1(\rho_1 V_1 A_1 - W_f) = P_1 A_1 - P_2 A_2 + T$$

$$\boxed{T = V_2 \dot{m}_2 - V_1(\dot{m}_1 - W_f) + P_2 A_2 - P_1 A_1}$$

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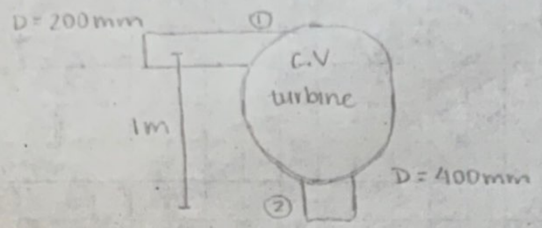
PROBLEM 3

Water flow rate: 220 L/sec
$P_1 = 170$ kPa gage
$P_2 = -20$ kPa gage

Determine power delivered to turbine from water

Incompressible, steady, 1-D flow at ① & ②
No friction, no heat transfer



$$\cancel{\frac{\dot{Q}}{\dot{m}}} + \frac{\dot{W}_s}{\dot{m}} + \cancel{\frac{\dot{W}_p}{\dot{m}}} + \cancel{\frac{\dot{W}_b}{\dot{m}}} + \cancel{\frac{\dot{W}_v}{\dot{m}}} + \left[h_1 + \frac{V_1^2}{2} + gz_1\right] = \left[h_2 + \frac{V_2^2}{2} + gz_2\right]$$

$\dot{m} = \rho v A$, $\iint_{cs} (\rho \vec{v} \cdot d\vec{S}) = 0$
Solve for V_1 & V_2 first
Continuity

$\dot{m} = 220$ L/sec $= 0.22\ m^3/s$ (ignore ρ)

$\dot{m} = V_1 A_1 = V_1 (0.2)^2 \pi/4 \; m^2 = 0.22\ m^3/s$

$V_1 = 7.003$ m/s

$\dot{m} = V_2 A_2 = V_2 (0.4)^2 \pi/4 \; m^2 = 0.22\ m^3/s$

$V_2 = 1.75$ m/s.

no heat transfer => isothermal => const internal energy

$$\frac{\dot{W}_s}{\dot{m}} = v(P_2 - P_1) + \left[\frac{V_2^2}{2} - \frac{V_1^2}{2}\right] + g[z_2 - z_1]$$

$$\frac{\dot{W}_s}{\dot{m}} = \frac{1}{997.77}\frac{m^3}{kg}\left(-20\,000 - 170\,000 \frac{kg}{m s^2}\right) + \left[\frac{(1.75)^2}{2} - \frac{(7)^2}{2}\right] m^2/s^2 + 9.81\,[1\,m]\; m^2/s^2$$

$\frac{\dot{W}_s}{\dot{m}} = -190.42 + (1.53125 - 24.5) + 9.81 \; m^2/s^2$

$\dot{W}_s/\dot{m} = -203.57875\ m^2/s^2$

$\dot{W}_s = \dot{m}(\dot{W}_s/\dot{m}) = (0.22\ m^3/s)(997.77\ \frac{kg}{m^3})(-203.57875\ m^2/s^2)$

$\dot{W}_s = -44\,687.45$

Power $\frac{kg\, m^2}{s^3}$

$\boxed{\text{Power} = -44.687\ kW}$

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