# HW 7

Saturday, April 8, 2023 12:40 PM

7.1

**7.1** Starting with Eq. (7.9), derive Eqs. (7.10) and (7.11).

$$\boxed{e_2 - e_1 = \frac{p_1 + p_2}{2}(v_1 - v_2)} \tag{7.9}$$

$e_2 - e_1 = \frac{1}{2}(P_1 + P_2)(v_1 - v_2)$

$e = c_v T \qquad v = \frac{RT}{P}$

$\frac{2R}{\gamma - 1}(T_2 - T_1) = P_1 + P_2\left(\frac{RT_1}{P_1} - \frac{RT_2}{P_2}\right)$

$$\boxed{\frac{T_2}{T_1} = \frac{p_2}{p_1}\left(\frac{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}{1 + \frac{\gamma+1}{\gamma-1}\frac{p_2}{p_1}}\right)} \tag{7.10}$$

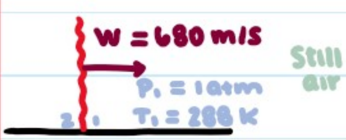
$$\boxed{\frac{\rho_2}{\rho_1} = \frac{1 + \frac{\gamma+1}{\gamma-1}\left(\frac{p_2}{p_1}\right)}{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}} \tag{7.11}$$

7.2

**7.2** Consider a normal shock wave moving with a velocity of 680 m/s into still air at standard atmospheric conditions ($p_1 = 1$ atm and $T_1 = 288$ K).

(*a*) Using the equations of Sec. 7.2, calculate $T_2$, $p_2$, and $u_p$ behind the shock wave.

(*b*) The normal shock tables, Table A.2, can be used to solve moving shock wave problems simply by noting that the tables pertain to flow velocities (hence Mach numbers) *relative to the wave*.




a) Find $T_2, P_2, u_p$

eq 7.14, solve for $\frac{P_2}{P_1}$

$W = a_1\sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{P_2}{P_1} - 1\right) + 1}$

$\frac{\gamma+1}{2\gamma}\left(\frac{P_2}{P_1} - 1\right) = \left(\frac{W}{\sqrt{\gamma R T_1}}\right)^2 - 1$

$\gamma = 1.4,\ T_1 = 288\text{ K},\ W = 680$

$\frac{2.4}{2.8}\left(\frac{P_2}{P_1} - 1\right) = \left(\frac{680}{\sqrt{1.4 \cdot 287 \cdot 288}}\right)^2 - 1$

$\frac{P_2}{P_1} = 4.495$

$P_2 = 4.495$ atm

Eq 7.10
$$\frac{T_2}{T_1} = \frac{P_2}{P_1}\left(\frac{\frac{\gamma+1}{\gamma-1} + \frac{P_2}{P_1}}{1 + \frac{\gamma+1}{\gamma-1}\frac{P_2}{P_1}}\right)$$

$\gamma = 1.4$, $\frac{P_2}{P_1} = 4.495$

$$\frac{T_2}{T_1} = 4.495\left(\frac{\frac{2.4}{0.4} + 4.495}{1 + \frac{2.4}{0.4}(4.495)}\right)$$

$$\frac{T_2}{T_1} = 1.687$$

$T_2 = 485.9$ K

$$u_p = \frac{a_1}{\gamma}\left(\frac{P_2}{P_1} - 1\right)\left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{P_2}{P_1} + \frac{\gamma-1}{\gamma+1}}\right)^{1/2}$$

$$u_p = \frac{\sqrt{1.4 \cdot 287 \cdot 288}}{1.4}(4.495 - 1)\left(\frac{\frac{2.8}{2.4}}{4.495 + \frac{0.4}{2.4}}\right)^{1/2}$$

$u_p = 424.8$ m/s

b) $T_2$, $P_2$, $u_p$ using Table A.2

Relative to SW, $u_1 = 680$ m/s

$a_1 = \sqrt{1.4 \cdot 287 \cdot 288} = 340.174$ m/s

$M_1 = \frac{u_1}{a_1} = 2.0$

↳ Table A.2

$\frac{P_2}{P_1} = 4.5$, $\frac{T_2}{T_1} = 1.687$, $M_2 = 0.5774$

$P_2 = 4.5$ atm

$T_2 = 485.9$ K

$a_2 = \sqrt{1.4 \cdot 287 \cdot 485.9}$

$a_2 = 441.85$ m/s

$u_2 = M_2 a_2 = 255.13$ m/s

$u_p = W - u_2 = 424.87$ m/s

7.3

**7.3** For the conditions of Prob. 7.2, calculate the total pressure and temperature of the gas behind the moving shock wave.

$P_{02}$, $T_{02}$

Moving SW → $T_{01} \neq T_{02}$

Use Table A.1, but get Mach #s relative to lab reference frame

$M_2 = \frac{u_P}{a_2}$

$M_2 = 0.96$

Table A.1

$\frac{P_{02}}{P_2} = 1.808$ $P_2 = 4.495$ atm

$T_2 = 485.9$ K

$\frac{T_{02}}{T_2} = 1.184$

$P_0 = 8.13$ atm

$T_0 = 575.3$ K