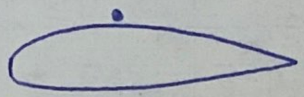
Veronica Loomis

Problem 3.1

$M = 0.7$, $P = 0.9$ atm, $T = 250K$



Assume

isentropic

one dim.

steady

ideal gas (air)

Calculate P_0, T_0, P^*, T^*, and a^*

Stagnation values => isentropic

Use table A-1 in textbook at $M = 0.7$

$$P_0/P = 0.1387 \times 10^1$$

$$P_0 = 1.387(0.9)$$

$$\boxed{P_0 = 1.2483 \text{ atm}}$$

$$T_0/T = 1.098$$

$$T_0 = 1.098(250K)$$

$$\boxed{T_0 = 274.5 \text{ K}}$$

Characteristic values => adiabatic, $M=1$

Use same table A-1 but now at $M=1$

$$P^* = \frac{P^*}{P^0} \times \frac{P_0}{P} \times P \rightarrow$$

1.2483 atm

$$P^* = \left(\frac{1}{1.893}\right)(1.387)(0.9) \text{ atm}$$

$$\boxed{P^* = 0.659 \text{ atm}}$$

$$T^* = \frac{T^*}{T_0} \times \frac{T_0}{T} \times T$$

$$T^* = \left(\frac{1}{1.2}\right)(1.098)(250) \text{ K}$$

$$\boxed{T^* = 228.75 \text{ K}}$$

$$a^* = \sqrt{\gamma R T^*}$$

$$a^* = \sqrt{(1.4)(287)(228.75)}$$

$$\boxed{a^* = 303.1695 \text{ m/s}}$$

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Problem 3.2

At a given point, (in supersonic wind tunnel)

Assume
isentropic
one dim.
steady
ideal gas

$P = 5\times10^4 \ N/m^2$, $T = 200K$

$P_0 = 1.5\times10^6 \ N/m^2$

Calculate local mach and total temperature

$$\frac{P_0}{P} = \frac{1.5\times10^6}{5\times10^4} = 30$$

Using table A-1 there is

$(P_0/P)_1 = 29.29$ and $(P_0/P)_2 = 31.59$ at $M_1 = 2.85$ and $(M)_2 = 2.9$

interpolating gives

$\boxed{M = 2.865}$

in this same row in A-1 we have

$(T_0/T)_1 = 2.624$ and $(2.682) = (T_0/T)_2$ at $M = 2.85$ and $M = 2.9$

Now that we know M, we can interpolate again

$T_0/T = 2.6414$

$T_0 = 2.6414(200)$

$\boxed{T_0 = 528.28 \ K}$

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Problem 3.4

Assume
ideal gas
isentropic after shock

Normal shock in air

Upstream: $M_1 = 3$, $P_1 = 1$ atm, $\rho_1 = 1.23$ kg/m^3

Calculate downstream $P_2, T_2, \rho_2, M_2, u_2, P_{02}$, and T_{02}

Use table A2 at $M = 3$

$P_2/P_1 = 10.33$
$P_2 = 10.33 (1 \text{ atm})$
$\boxed{P_2 = 10.33 \text{ atm}}$

$T_2/T_1 = 2.679$
$T_2 = 2.679 \left(\frac{P_1}{\rho_1 R}\right) = 2.679 \left(\frac{1 \text{ atm}}{1.23 \frac{kg}{m^3} (287)}\right) \times \frac{101325 N}{m^2}$
$\boxed{T_2 = 768.957 \text{ K}}$

$\rho_2/\rho_1 = 3.857$
$\rho_2 = 3.857 (1.23)$ kg/m^3
$\boxed{\rho_2 = 4.744 \text{ kg/m}^3}$

$\boxed{M_2 = 0.4752}$

$u_2 = M_2 a_2 = M_2 \sqrt{\gamma R T_2}$

$u_2 = 0.4752 \sqrt{(1.4)(287)(768.957)}$

$\boxed{u_2 = 264.139 \text{ m/s}}$

$P_{02}/P_1 = 12.06$
$P_{02} = 12.06 (1 \text{ atm})$
$\boxed{P_{02} = 12.06 \text{ atm}}$

For T_{02}, use table 1 at $M = 0.4752$
interpolate between $M = 0.46$, $M = 0.48$
$T_{02}/T_2 = 1.04504$
$T_{02} = 1.04504 (768.957)$ K
$\boxed{T_{02} = 803.59 \text{ K}}$

HW3

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Problem 3.6

Will append to PDF

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Problem 3.7

$M = 38$ $T_{atm} = 270\,K$

calculate T_o assuming $\gamma = 1.4$

Assume isentropic
Table A-1
$T_o/T = 289.8$

$T_o = 270(289.8)\,K$

$T_o = 78\,246\,K$

I don't think this is an accurate calculation.
I don't think we are allowed to assume this is isentropic since it is not reversible.
I think my answer is an overestimate

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Eqns 6.11, 12, 13, 17

Eqn 6.11
$$\frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho u \vec{V}) = \frac{-\partial P}{\partial x} + \rho f_x$$

turns into

$$\boxed{\frac{\partial(\rho u)}{\partial t} + u\frac{\partial(\rho u)}{\partial x} + v\frac{\partial(\rho u)}{\partial y} + w\frac{\partial(\rho u)}{\partial z} = \frac{-\partial P}{\partial x} + \rho f_x}$$

Eqn 6.12
$$\frac{\partial(\rho v)}{\partial t} + \nabla \cdot (\rho v \vec{V}) = \frac{-\partial P}{\partial y} + \rho f_y$$

turns into

$$\boxed{\frac{\partial(\rho v)}{\partial t} + u\frac{\partial(\rho v)}{\partial x} + v\frac{\partial(\rho v)}{\partial y} + w\frac{\partial(\rho v)}{\partial z} = \frac{-\partial P}{\partial y} + \rho f_y}$$

Eqn 6.13
$$\frac{\partial(\rho w)}{\partial t} + \nabla \cdot (\rho w \vec{V}) = \frac{-\partial P}{\partial z} + \rho f_z$$

$$\boxed{\frac{\partial(\rho w)}{\partial t} + u\frac{\partial(\rho w)}{\partial x} + v\frac{\partial(\rho w)}{\partial y} + w\frac{\partial(\rho w)}{\partial z} = \frac{-\partial P}{\partial z} + \rho f_z}$$

Eqn 6.17
$$\frac{\partial}{\partial t}\left[\rho\left(e + \frac{V^2}{2}\right)\right] + \nabla \cdot \left[\rho\left(e + \frac{V^2}{2}\right)\vec{V}\right] = -\nabla \cdot (p\vec{V}) + \rho\dot{q} + \rho(\vec{f} \cdot \vec{V})$$

$$\boxed{\begin{aligned}&\frac{\partial}{\partial t}\left[\rho\left(e + \frac{V^2}{2}\right)\right] + u\frac{\partial}{\partial x}\left[\rho\left(e + \frac{V^2}{2}\right)\right] + v\frac{\partial}{\partial y}\left[\rho\left(e + \frac{V^2}{2}\right)\right] + w\frac{\partial}{\partial z}\left[\rho\left(e + \frac{V^2}{2}\right)\right] \\ &= -\left[u\frac{\partial P}{\partial x} + v\frac{\partial P}{\partial y} + w\frac{\partial P}{\partial z}\right] + \rho\dot{q} \\ &+ \rho\left(f_x u + f_y v + f_z w\right)\end{aligned}}$$

HW3

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Problem 7

a) $A = 1,\ B = 1,\ C = 1,\ D = 1,\ E = 1,\ F = 1,\ G = 1$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + u + 1 = 0$$

$B^2 - 4AC = 1 - 4 = -3 < 0$

$\therefore$ This is an elliptic PDE

b) $A = 1,\ B = 0,\ C = -a^2,\ D = 0,\ E = 0,\ F = 0,\ G = 0$

$$\frac{\partial^2 u}{\partial x^2} - a^2 \frac{\partial^2 u}{\partial y^2} = 0$$

$B^2 - 4AC = +4a^2 > 0$

$\therefore$ This is a hyperbolic PDE

c) $A = 1,\ B = 0,\ C = 1,\ D = 1,\ E = 1,\ F = 1,\ G = 1$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + u + 1 = 0$$

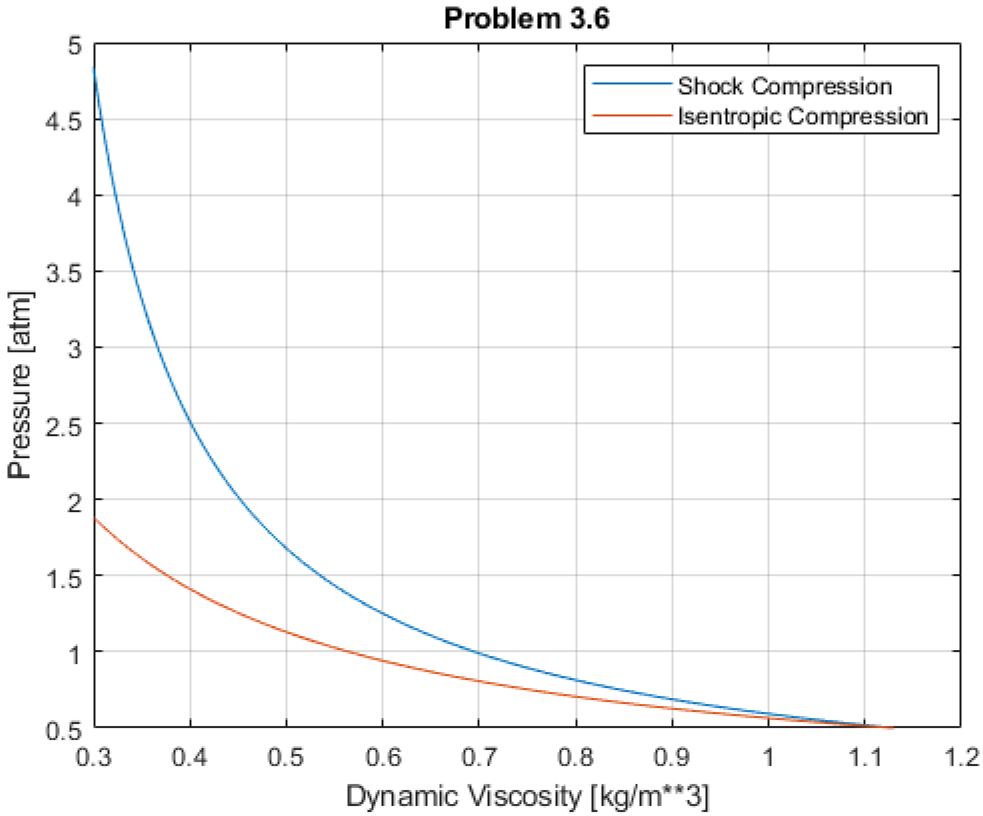
$B^2 - 4AC = 0 - 4 = -4 < 0$

$\therefore$ This is an elliptic PDE

620 HW 3

Problem 3.6 Consider the compression of air by means of a) shock compression b) isentropic compression Start from same (pressure)_1 and (dynamic viscosity)_1

```
gamma = 1.4;

% choosing my own P1
P1 = 0.5; % atm
% have nu be a vector
nu = linspace(1.13, 0.3, 1000);

P2_sh(1) = P1;
P2_isen(1) = P1;

for i=2:length(nu)
    % For shock compression
    P2_sh(i) = P1*((2.4/0.4)*nu(1)/nu(i) - 1) / ((2.4/0.4) - nu(1)/nu(i));

    % For an isentropic compression, P*nu is constant
    P2_isen(i) = P1*nu(1)/nu(i);
end

plot(nu, P2_sh)
hold on
plot(nu, P2_isen)
legend('Shock Compression','Isentropic Compression')
grid on
ylabel('Pressure [atm]')
xlabel('Dynamic Viscosity [kg/m**3]')
title('Problem 3.6')
```



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