

Veronica Loomis

Problem 1.1

At nose of missile, $P = 5.6 \text{ atm}$, $T = 850^\circ\text{R}$

Calculate density (ρ) and specific volume (v)

Note: $1 \text{ atm} = 2116 \text{ lbf/ft}^2$

Assuming a perfect gas:

$$P = \rho R T \quad [\text{Eqn 1.9}]$$

$$5.6 \text{ atm} \times 101325 \frac{\text{N}}{\text{m}^2 \text{ atm}} = \rho \left(287 \frac{\text{J}}{\text{kg K}} \right) \left(850^\circ\text{R} \times \frac{5}{9} \frac{\text{K}}{^\circ\text{R}} \right)$$

$$\rho = \frac{5.6(101325)}{287(850)(5/9)} \frac{\text{kg/s}^2\text{m}}{\text{m}^2/\text{s}^2} = 4.1867 \text{ kg/m}^3$$

$$v = 1/\rho = 0.2388 \text{ m}^3/\text{kg}$$

$$\rho = 4.1867 \text{ kg/m}^3$$

$$v = 0.2388 \text{ m}^3/\text{kg}$$

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Problem 1.3

For a calorically perfect gas, derive $C_p - C_v = R$

Repeat for a thermally perfect gas

Calorically Perfect Gas:

$$e = C_v T \quad \text{and} \quad h = C_p T$$

We know $h = e + Pv$

$$e + Pv = e + RT = C_p T$$

Rearrange

$$T = \frac{e}{C_p - R}$$

Substitute into $e = C_v T$

$$e = \frac{C_v e}{C_p - R}$$

$$C_p - R = C_v$$

$$C_p - C_v = R \quad \checkmark$$

Thermally Perfect

$$de = C_v dT \quad \text{and} \quad dh = C_p dT$$

Similar to calorically perfect,
 $dh = d/dT(e + Pv) = d/dT(e + RT)$
 $= de + R dT = C_p dT$

Replace

$$C_p dT - R dT = C_v dT$$

$$C_p - C_v = R \quad \checkmark$$

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Problem 1.4

Across a given portion of a shockwave:

$$\frac{P_2}{P_1} = 4.5$$

$$\frac{T_2}{T_1} = 1.687$$

1. ahead of SW
2. behind SW

Calculate change in entropy in (a) $[\text{ft} \cdot \text{lb} / \text{slug} \cdot ^\circ \text{R}]$ (b) $[\text{J} / \text{kg} \cdot \text{K}]$

Assuming calorically perfect gas

$$S_2 - S_1 = C_p \ln(T_2/T_1) - R \ln(P_2/P_1) \quad [\text{Eqn 1.36}]$$

(a)

If we assume $\gamma = 1.4$

$$C_v = \frac{R}{\gamma - 1} \quad [\text{Eqn 1.23}]$$

$$C_v = \frac{1716 \text{ ft} \cdot \text{lb} / \text{slug} \cdot ^\circ \text{R}}{0.4}$$

$$C_v = 4290 \text{ ft} \cdot \text{lb} / \text{slug} \cdot ^\circ \text{R}$$

$$C_p = C_v + R = 6006 \text{ ft} \cdot \text{lb} / \text{slug} \cdot ^\circ \text{R}$$

$$S_2 - S_1 = 6006 \ln(1.687) - 1716 \ln(4.5)$$

$$S_2 - S_1 = 559.85 \text{ ft} \cdot \text{lb} / \text{slug} \cdot ^\circ \text{R}$$

(b)

$$C_v = \frac{287}{0.4} = 717.5 \text{ J} / \text{kg} \cdot \text{K}$$

$$C_p = C_v + R = 1004.5 \text{ J} / \text{kg} \cdot \text{K}$$

$$S_2 - S_1 = 1004.5 \ln(1.687) - 287 \ln(4.5)$$

$$S_2 - S_1 = 93.63 \text{ J} / \text{kg} \cdot \text{K}$$

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Problem 1.5

Air flow through given duct is isentropic

At one point, $P_1 = 1800 \text{ lb/ft}^2$ $T_1 = 500^\circ\text{R}$

At second point, $T_2 = 400^\circ\text{R}$

Calculate P and ρ at second point

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1} \right)^{\gamma/(\gamma-1)} \quad [\text{Eqn 1.43}]$$

Assume $\gamma = 1.4$

$$P_2 = (1800) \left(400/500 \right)^{1.4/0.4}$$

$$P_2 = 824.3 \text{ lb/ft}^2$$

Assume ideal gas

$$(P = \rho RT)_2$$

$$\rho = \frac{824.3 \text{ lb/ft}^2}{(1716)(400) \frac{\text{ft} \cdot \text{lb}}{\text{slug} \cdot \text{R}}} = 0.0012 \text{ slug/ft}^3$$

$$\rho_2 = 0.0012 \text{ slug/ft}^3$$