# HW 6

Monday, March 27, 2023 2:44 PM

5.1

**5.1** A supersonic wind tunnel is designed to produce flow in the test section at Mach 2.4 at standard atmospheric conditions. Calculate:

(*a*) The exit-to-throat area ratio of the nozzle

(*b*) Reservoir pressure and temperature

Supersonic wind tunnel

$M = 2.4$

$T_e = 288\ K$

$P_e = 1\ atm$

Table A.1 with $M = 2.4$

a)

$$\frac{A_e}{A^*} = 2.403$$

$$\frac{P_0}{P_e} = 14.62$$

$$\frac{T_0}{T_e} = 2.152$$

b)

$P_0 = 14.62\ atm$

$T_0 = (2.152)(288)$

$T_0 = 619.78\ K$

5.4

**5.4** Consider the purely subsonic flow in a convergent-divergent duct. The inlet, throat, and exit area are 1 $m^2$, 0.7 $m^2$, and 0.85 $m^2$, respectively. If the inlet Mach number and pressure are 0.3 and $0.8 \times 10^5$ $N/m^2$, respectively, calculate:

(*a*) $M$ and $p$ at the throat

(*b*) $M$ and $p$ at the exit

Purely subsonic in Laval nozzle

$A_{inlet} = 1 m^2$, $A_t = 0.7 m^2$, $A_e = 0.85 m^2$

$M_{inlet} = 0.3$, $P_{inlet} = 0.8 \times 10^5 N/m^2$

Use A.1 to get $P_0$

$\frac{P_0}{P} = 1.064$ $\qquad \frac{A_{in}}{A^*} = 2.035$

$\therefore A^* = \left(\frac{1}{2.035}\right)(1 m^2) = 0.4914$

$A^* < A_t$

$P_0 = (1.064)(0.8 \times 10^5 N/m^2) = 0.8512 \times 10^5 N/m^2$

a) Throat M and P

$\frac{A_t}{A^*} = 1.425$

$M_t = 0.46$

$\frac{P_0}{P_t} = 1.156$

$P_t = \left(\frac{1}{1.156}\right)(0.8512 \times 10^5 N/m^2)$

$P_t = 0.736 \times 10^5 N/m^2$

b) Exit M and P

$\frac{A_e}{A^*} = 1.73$

$M_e = 0.36$

$\frac{P_0}{P_e} = 1.094$

$P_e = \left(\frac{1}{1.094}\right)(0.8512 \times 10^5 N/m^2)$

$P_e = 0.778 \times 10^5 N/m^2$

5.6

**5.6** The mass flow of a calorically perfect gas through a choked nozzle is given by

$$\dot{m} = \frac{p_o A^*}{\sqrt{T_o}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

Derive this relation.

$\dot{m} = \rho u A$

If choked,

$\dot{m} = \rho^* A^* a^*$

Substitute $a^* = \sqrt{\gamma R T^*}$ [3.20]

$\dot{m} = \rho^* A^* \sqrt{\gamma R T^*}$

We know $T^* = T_0\left(\frac{2}{\gamma+1}\right)$ [3.34]

$\dot{m} = \rho^* A^* \sqrt{\gamma R T_0 \left(\frac{2}{\gamma+1}\right)}$

Also $\rho^* = \rho_0 \left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)}$ [3.36]

$\dot{m} = \rho_0 A^* \sqrt{\gamma R T_0 \left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}}$

Ideal gas eq: $P_0 = \rho_0 R T_0$

$\dot{m} = \frac{P_0}{R T_0} A^* \sqrt{\gamma R T_0 \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1}}$

Rearrange

$\dot{m} = \frac{P_0}{\sqrt{T_0}} A^* \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$

5.8

**5.8** A blunt-nosed aerodynamic model is mounted in the test section of a supersonic wind tunnel. If the tunnel reservoir pressure and temperature are 10 atm and 800°R, respectively, and the exit-to-throat area ratio is 25, calculate the pressure and temperature at the nose of the model.

$P_0 = 10$ atm

$T_0 = 800$ °R

$\frac{A_e}{A^*} = 25$

calc P & T at nose

Table A.1 at $A_e/A^* = 25$

$M = 5$

$\frac{P_0}{P_e} = 529.1$

$P_e = 0.0189$ atm

Use A.2 at $M = 5$

$\frac{P_{02}}{P_{01}} = 0.06172$

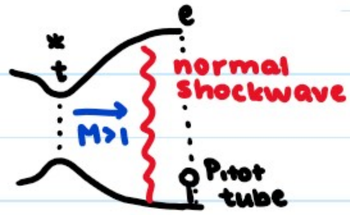
$P_{02}$ (nose) $= 0.617$

$T_{02} = T_{01} = 800$ °R

5.10

**5.10** Consider a supersonic nozzle with a Pitot tube mounted at the exit. The reservoir pressure and temperature are 10 atm and 500 K, respectively. The pressure measured by the Pitot tube is 0.6172 atm. The throat area is 0.3 m$^2$. Calculate:

(*a*) Exit Mach number $M_e$
(*b*) Exit area $A_e$
(*c*) Exit pressure and temperature $p_e$ and $T_e$
(*d*) mass flow through the nozzle




Supersonic

$P_{0,1} = 10$ atm

$T_0 = 500$ K

$P_{pitot,e} = 0.6172$ atm $= P_{0,2} \neq P_{0,1}$ Normal Shock!

$A_t = 0.3\,m^2$

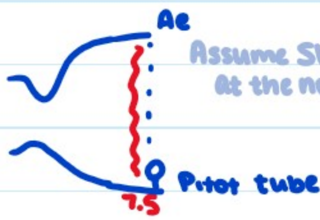
a) Me

$\frac{P_{02}}{P_{01}} = 0.06172$

Table A.2 → "Me" before shockwave = 5

$\boxed{\text{Me after sw} = 0.4152}$

b) Ae




Me = 5.0

Table A.1 → $\frac{Ae}{At} = 25$

$Ae = (25)(0.3\,m^2)$

$\boxed{Ae = 7.5\,m^2}$

c) Pe and Te

Me = 0.4152

Table A.1 → $\frac{P_0}{Pe} = 1.126$, $\frac{T_0}{Te} = 1.034$

$Pe = \left(\frac{1}{1.126}\right)(0.6172 \text{ atm})$

$\boxed{Pe = 0.548 \text{ atm}}$

$Te = \left(\frac{1}{1.034}\right)(500\,K)$

$\boxed{Te = 483.6\,K}$

d) $\dot{m}$

$\rho e = \frac{Pe}{R\,Te} = \frac{0.548 \text{ atm} \cdot 101325 \frac{N/m^2}{atm}}{287 \text{ J/kgK } (483.6K)}$

$\rho e = 0.4 \text{ kg/m}^3$

$Ae = 7.5\,m^2$

$u_e = M_e a_e = 0.4152\sqrt{1.4 \times 287 \times 483.6}$

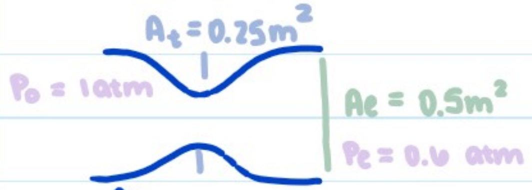
$u_e = 183.02$

$\dot{m} = \rho_e A_e u_e$

$\dot{m} = 549$ kg/s

5.11

**5.11** Consider a convergent-divergent duct with exit and throat areas of 0.5 m$^2$ and 0.25 m$^2$, respectively. The inlet reservoir pressure is 1 atm and the exit static pressure is 0.6 atm. For this pressure ratio, the flow will be supersonic in a portion of the nozzle, terminating with a normal shock inside the nozzle. Calculate the local area ratio ($A/A^*$) at which the shock is located inside the nozzle.




Find $\frac{A_e}{A^*}$ of normal shock

$$\left(\frac{P_e}{P_{01}}\right)\left(\frac{A_e}{A_t^*}\right) = \left(\frac{0.6}{1}\right)\left(\frac{0.5}{0.25}\right) = 1.2$$

$$M_e^2 = \frac{-1}{0.4} + \sqrt{\frac{1}{(0.4)^2} + \left(\frac{2}{0.4}\right)\left(\frac{2}{2.4}\right)^{2.4/0.4}\left(\frac{1}{1.2}\right)^2} \quad [5.28]$$

$M_e^2 = 0.223$

$M_e = 0.47186$

Table A.1 $\rightarrow$ $\frac{P_{02}}{P_e} = 1.165$

$$\frac{P_{02}}{P_{01}} = \left(\frac{P_{02}}{P_e}\right)\left(\frac{P_e}{P_{01}}\right) = (1.165)(0.6)$$

$\frac{P_{02}}{P_{01}} = 0.7$

Table A.2

$M_1 = 1.98$

Table A.1

$\frac{A}{A^*} = 1.66$