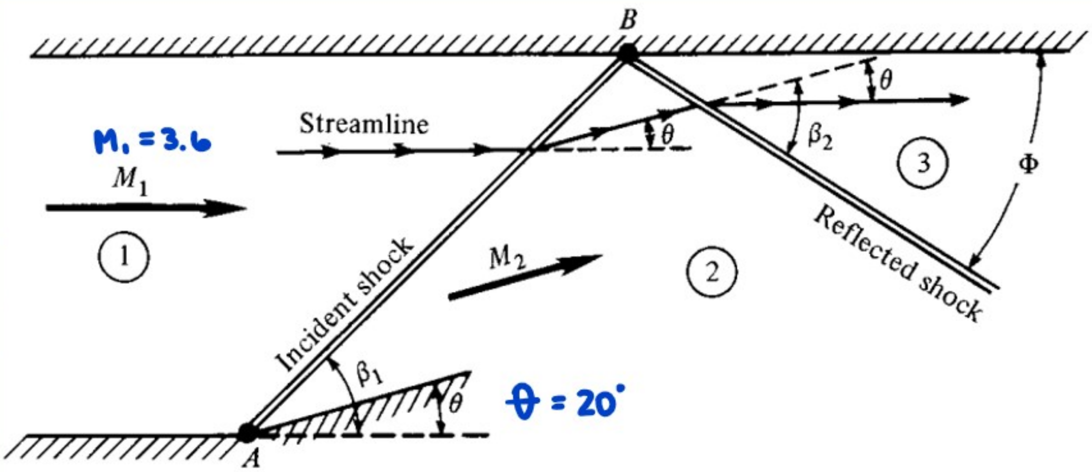
HW 5

Wednesday, March 22, 2023 2:42 PM

4.6

4.6 A supersonic stream at $M_1 = 3.6$ flows past a compression corner with a deflection angle of 20°. The incident shock wave is reflected from an opposite wall which is parallel to the upstream supersonic flow, as sketched in Fig. 4.14. Calculate the angle of the reflected shock relative to the straight wall.



Find Φ

$\theta = 20°$

$M_1 = 3.6, \quad \theta_1 = 20°$

Using relation, $\beta = 34°$

$\therefore M_{1n} = M_1 \sin\beta$

$M_{1n} = 2.013$

Using table 2 → $M_{2n} = 0.575$

$$M_2 = \frac{M_{2n}}{\sin(\beta-\theta)} = \frac{0.575}{\sin(14°)} = 2.3768$$

Now find β_2:

$M_2 = 2.3768, \ \theta = 20°$

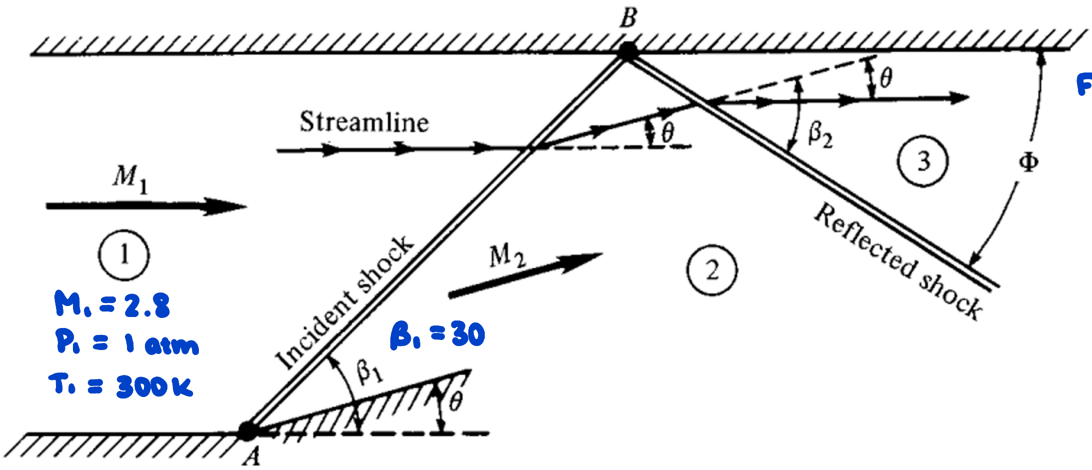
$\beta_2 \cong 45°$

$\Phi = \beta_2 - \theta$

$\Phi = 25°$

4.7

4.7 An incident shock wave with wave angle = 30° impinges on a straight wall. If the upstream flow properties are $M_1 = 2.8$, $p_1 = 1$ atm, and $T_1 = 300$ K, calculate the pressure, temperature, Mach number, and total pressure downstream of the reflected wave.



Find:
P_3
T_3
M_3
P_{03}

Find θ

$M_1 = 2.8$, $\beta_1 = 30°$

$\therefore$ $\theta \cong 11°$

$M_{1n} = 2.8 \sin(30)$

$M_{1n} = 1.4$

Use table A.2

$P_2/P_1 = 2.12$

$T_2/T_1 = 1.255$

$P_{02}/P_{01} = 0.9582$

$P_{02}/P_1 = 3.049$

$M_{2n} = 0.7397$

$M_2 = \dfrac{M_{2n}}{\sin(\beta_1 - \theta)} = 2.27$

Solve β_2

$M_2 = 2.27$, $\theta = 11°$

$\therefore \beta_2 = 35.7$

$M_{2n,new} = M_2 \sin(35.7)$

$M_{2n,new} = 1.32$

$P_3/P_2 = 1.866$

$T_3/T_2 = 1.204$

$P_{03}/P_{02} = 0.9758$

$M_{3n} = 0.776$

Putting it all together

$P_3 = \frac{P_3}{P_2}\frac{P_2}{P_1}P_1 = (1.866)(2.12)(1\ atm)$

$P_3 = 3.96\ atm$

$T_3 = (1.204)(1.255)(300\ K)$

$T_3 = 453.3\ K$

$P_{03} = \frac{P_{03}}{P_{02}}\frac{P_{02}}{P_1}P_1 = (0.9758)(3.049)(1\ atm)$

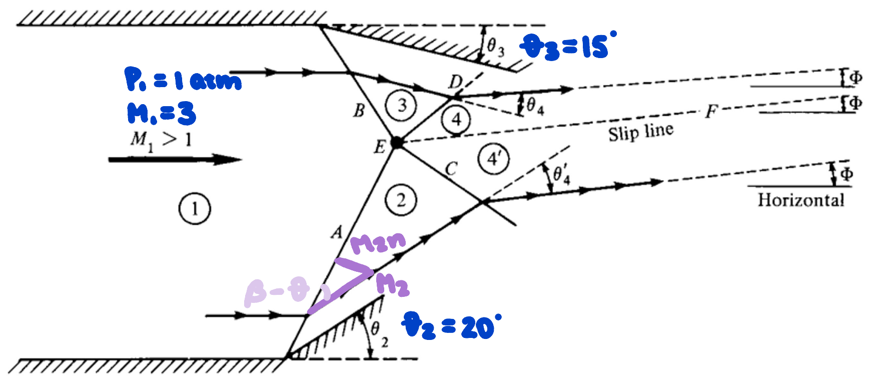
$P_{03} = 2.975\ atm$

$M_3 = \frac{M_{3n}}{\sin(\beta - \theta)}$

$M_3 = 1.857$

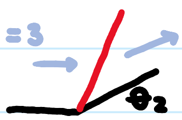
4.9

4.9 Consider the intersection of two shocks of opposite families, as sketched in Fig. 4.17. For $M_1 = 3$, $p_1 = 1$ atm, $\theta_2 = 20°$, and $\theta_3 = 15°$, calculate the pressure in regions 4 and 4′, and the flow direction Φ, behind the refracted shocks.



Find P_4 $(= P_{4'})$ and Φ

Starting with A:



Find β_A:

$M_1 = 3$, $\theta_2 = 20°$

$\beta_A = 37.76°$

$M_{1n} = 3 \sin(37.76)$

$M_{1n} = 1.837$

Use table A.2

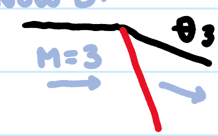
$\frac{P_2}{P_1} = 3.77$

$M_{2n} = 0.608$

$M_2 = \frac{M_{2n}}{\sin(\beta - \theta)}$

$M_2 = 1.99$

Now B:



Find β_B:

$M_1 = 3$, $\theta_3 = 15°$

$\beta_B \cong 32.24°$

$M_{1n} = 3 \sin(32.24)$

$M_{1n} = 1.6$

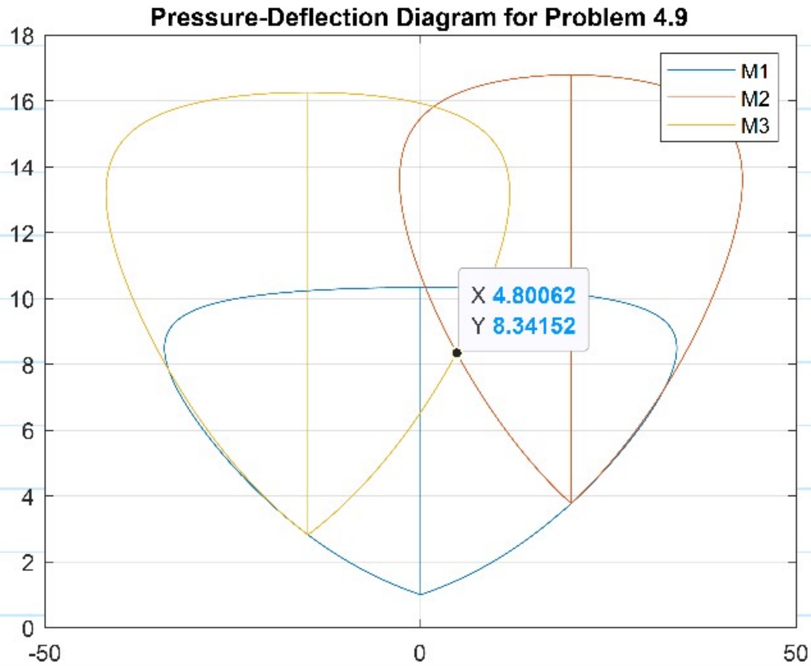
Table A2

$\frac{P_3}{P_1} = 2.82$

$M_{3n} = 0.6684$

$M_3 = \frac{M_{3n}}{\sin(\beta_B - \theta_3)}$

$M_3 = 2.255$



$\phi = 4.8^\circ$

$\theta_4' = 20 - 4.8 = 15.2^\circ$ $\qquad$ $\theta_4 = 4.8 + 15 = 19.8^\circ$

Using $M_2 = 1.99$ and $\theta_4' = 15.2^\circ$ $\qquad$ $M_3 = 2.255$ & $\theta_4 = 19.8$

$\beta_C = 45.86$ $\qquad$ $\beta_D = 46.559$

$M_{2n} = M_2 \sin(\beta_c)$ $\qquad$ $M_{3n} = 1.637$

$M_{2n} = 1.4281$ $\qquad$ Table A.2

Table A.2 $\qquad$ $\frac{P_4}{P_3} = 2.96$

$\frac{P_4'}{P_2} = 2.213$ $\qquad$ $P_4 = (2.96)(2.82)(1\,\text{atm})$

$P_4' = (2.213)(3.77)(1\,\text{atm})$ $\qquad$ $P_4 = 8.347$ atm

$P_4' = 8.34$ atm

To summarize, $P_4 = P_4' \cong 8.34$ atm

$\phi = 4.8^\circ$

4.10

4.10 Consider the flow past a 30° expansion corner, as sketched in Fig. 4.26. The upstream conditions are $M_1 = 2$, $p_1 = 3$ atm, and $T_1 = 400$ K. Calculate the following downstream conditions: M_2, p_2, T_2, T_{o_2}, and p_{o_2}.

Find:
$M_2, P_2, T_2, T_{02}, P_{02}$

$M_1 = 2$
$P_1 = 3$ atm
$T_1 = 400$ K

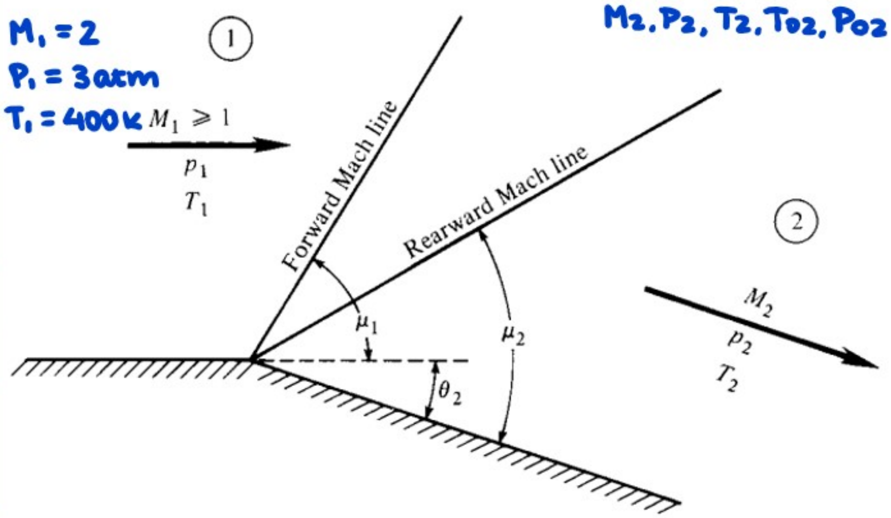


Table 5: $\nu(M_1) = 26.38°$

$\nu_2 = \theta + \nu_1 = 56.38°$

Table 5: $M_2 = 3.368$

Use isentropic relations at 1 for stag. values

$M_1 = 2 \rightarrow$ Table 1

$\frac{P_0}{P} = 7.824 \rightarrow P_0 = 23.472$ atm

$\frac{T_0}{T} = 1.8 \rightarrow T_0 = 720$ K

Now we want isen. at $M_2 = 3.368$

$\frac{P_0}{P_2} = 63.176$

$P_{01} = P_{02}$

$P_2 = \frac{23.472}{63.176}$

$P_2 = 0.3715$ atm

$\frac{T_0}{T_2} = 32.68$

$T_{01} = T_{02}$

$T_2 = \frac{720}{32.68}$

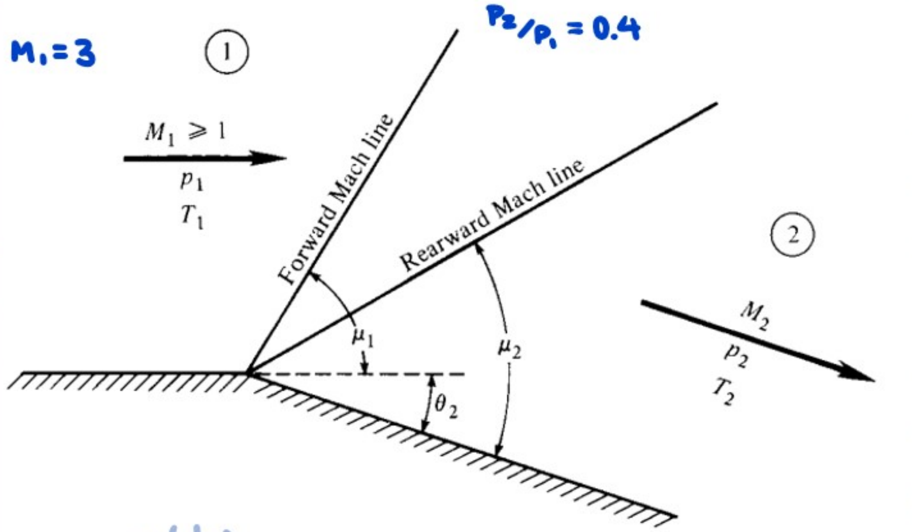
$T_2 = 22.03$ K

4.11

4.11 For a given Prandtl-Meyer expansion, the upstream Mach number is 3 and the pressure ratio across the wave is $p_2/p_1 = 0.4$. Calculate the angles of the forward and rearward Mach lines of the expansion fan relative to the free-stream direction.

$M_1 = 3$

$P_2/P_1 = 0.4$



$\mu_1 = \sin^{-1}\left(\frac{1}{M_1}\right)$

$\mu_1 = 19.47^\circ$

$\frac{P_0}{P_1}$ (at $M=3$) $= 36.73$

call $P_1 = 1$ atm

Then $P_0 = 36.73$ atm

If $P_2/P_1 = 0.4$

Then $P_2 = 0.4$ atm

$\frac{P_0}{P_2} = \frac{36.73}{0.4} = 113.857$

Find this in table 1

$M_2 = 3.79$

$\mu_2 = 15.3^\circ$

```
% Parametric Plot for Problem 4.9
% Function takes gamma, the 3 mach numbers from the 3 regions, and the
% index number of the region

% M was given
% M2 and M3 calculated in handwritten work
M = 3;
M2 = 1.99;
M3 = 2.255;

gamma = 1.4;

% send to function
[pressure1, theta1] = parametrics(gamma, M,  1);
[pressure2, theta2] = parametrics(gamma, M2, 2);
[pressure3, theta3] = parametrics(gamma, M3, 3);

% mirror the pressure diagram so we have a full diagram
p1 = [pressure1, pressure1];
th1 = [theta1, -theta1];
p2 = [pressure2, pressure2];
th2 = [theta2, -theta2];
p3 = [pressure3, pressure3];
th3 = [theta3, -theta3];

plot(th1, p1)
grid on
title('Pressure-Deflection Diagram for Problem 4.9')
hold on
plot(th2 +20, p2) % we are given theta2 so we can shift (left running)
plot(th3- 15, p3) % we are given theta3 so we can shift (right running)
legend('M1','M2','M3')


function [P2P1, theta] = parametrics(gamma, M, n)
% This function calculates the pressure ratio and theta for a given set of
% gamma, mach number, and index of flow region
    beta = linspace(asind(1/M), 90, 1000);

    % These are the pressure ratios calculated at each region after the
    % shockwave. They are applied to the P2P1 equation
    if n == 1
        a = 1;
    elseif n ==2
        a = 3.77;
    else
        a = 2.82;
    end

    for i = 1:length(beta)
        P2P1(i) = a * (1 + 2*gamma/(gamma+1) *(M*M*sind(beta(i))*sind(beta(i))
 - 1));
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        theta(i) = atand(2*cotd(beta(i))*(M*M*sind(beta(i))*sind(beta(i))
 -1) / (M*M*(gamma+cosd(2*beta(i)))+2));
    end
end
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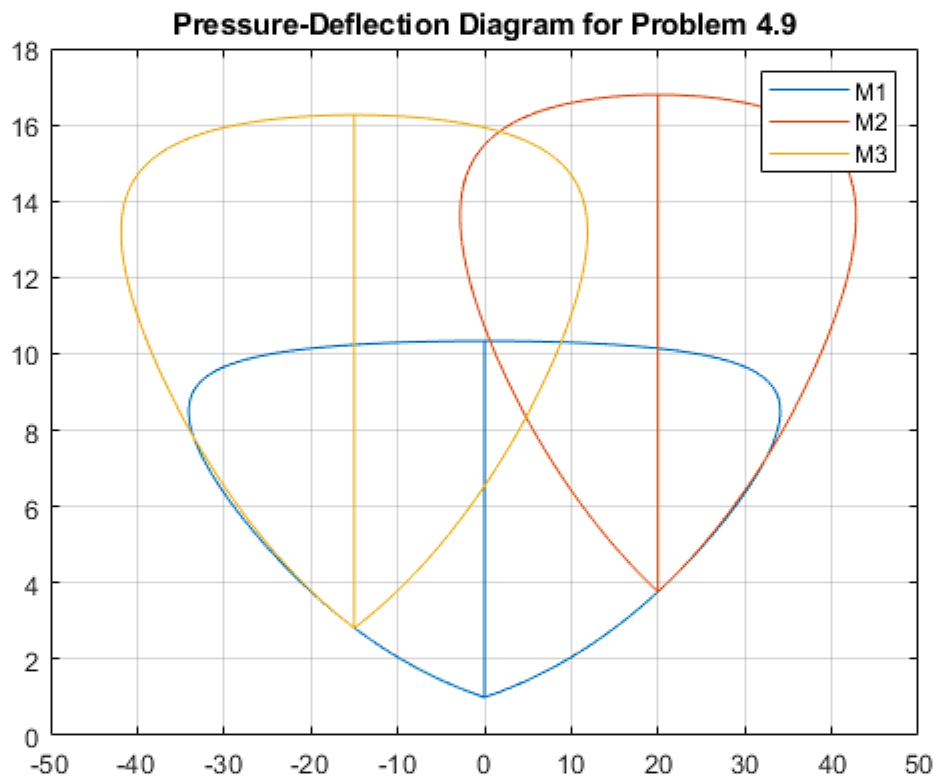

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