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2.6. Energy Equation

enthalpy

$h = e + Pv$

$$e_{tot} = e_{internal} + e_{potential} + e_{kinetic} + e_{other}$$

specific volume

$$e_{tot} = e + \frac{1}{2}V^2 + gz + Pv$$

pressure

energy per unit mass

system

flow work

$$e_{tot,F.S.} = h + \frac{1}{2}V^2 + gz$$

flowing stream

$$\dot{W} = \dot{W}_{shaft} + \dot{W}_{pressure} + \dot{W}_{body} + \dot{W}_{viscous\ stresses}$$
$$= \dot{W}_s + \dot{W}_p + \dot{W}_b + \dot{W}_v$$

integral - cons. of energy

positive/into system

②

$$\oiiint_{\mathcal{V}} \dot{q}\rho\, d\mathcal{V} - \oiint_S (p\, d\mathbf{S})\cdot\mathbf{V} + \oiiint_{\mathcal{V}} \rho\mathbf{f}\cdot\mathbf{V}\, d\mathcal{V} = \oiint_S \rho\left(e+\frac{V^2}{2}\right)\mathbf{V}\cdot d\mathbf{S} + \oiiint_{\mathcal{V}} \frac{\partial}{\partial t}\left[\rho\left(e+\frac{V^2}{2}\right)\right] d\mathcal{V}$$

$$\dot{Q} + \dot{W}_s + \dot{W}_p + \dot{W}_b + \dot{W}_v = \oiint_S \rho\left(h + \frac{v^2}{2} + gz\right)\vec{v}\cdot d\vec{s} + \underbrace{\oiiint_{V} \frac{\partial}{\partial t}\left[\rho\left(e + \frac{v^2}{2} + gz\right)\right] dV}_{\text{unsteady term}}$$

energy rate balance equation

intensive form - energy balance // steady // process ① → ②

single stream - $\dot{m}_{in} = \dot{m}_{out}$

$$\frac{\dot{Q}}{\dot{m}} + \frac{\dot{W}_s}{\dot{m}} + \frac{\dot{W}_p}{\dot{m}} + \frac{\dot{W}_b}{\dot{m}} + \frac{\dot{W}_v}{\dot{m}} + \left[h_1 + \frac{v_1^2}{2} + gz_1\right] = \left[h_2 + \frac{v_2^2}{2} + gz_2\right]$$

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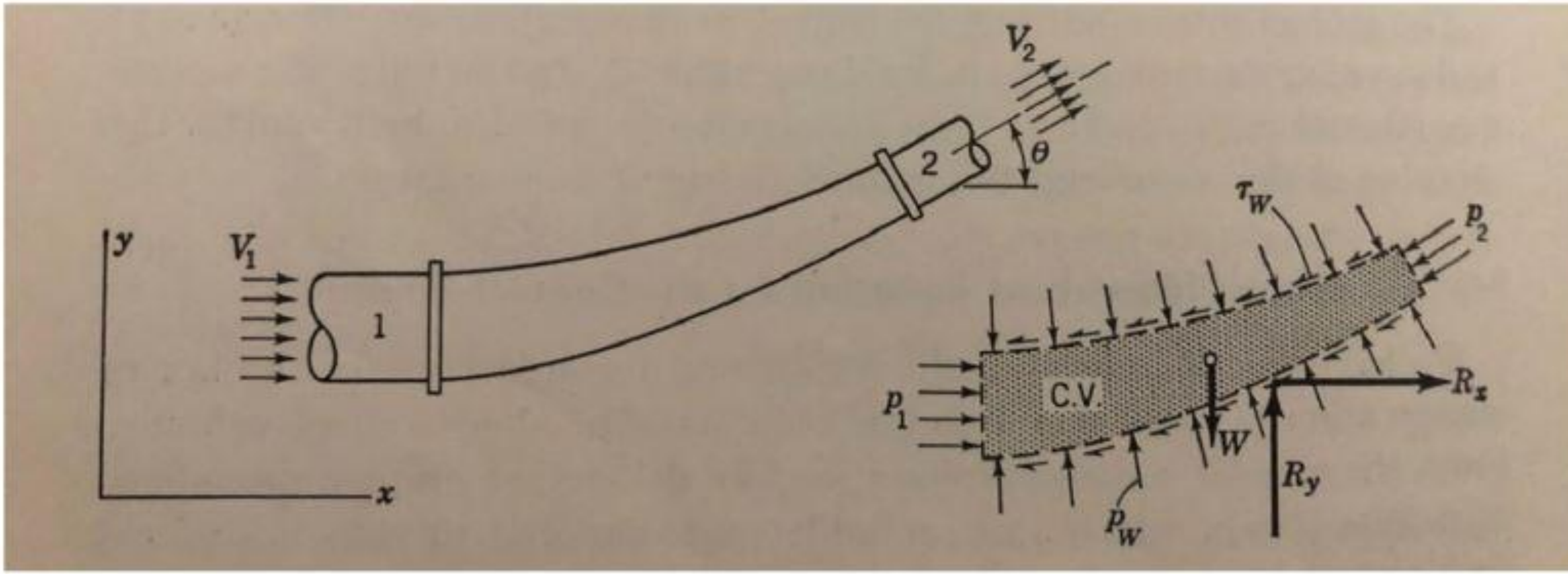
EXAMPLE PROBLEM. Evaluate the x-component force and y-component force resulting from internal, steady flow within the reducing elbow.



steady flow

0

$$\oiint_S (\rho \mathbf{V}\cdot d\mathbf{S})\mathbf{V} + \cancel{\iiint_{\mathcal{V}} \frac{\partial(\rho \mathbf{V})}{\partial t}\, d\mathcal{V}} = \underbrace{\iiint_{\mathcal{V}} \rho \mathbf{f}\, d\mathcal{V} - \oiint_S p\, d\mathbf{S} + \mathbf{F}_{viscous}}_{\text{net forces / C.V.}}$$

- neglect contributions Patm
- steady flow
- neglect τ_w terms
- internal flow

W = weight

$F_x = p_1 A_1 - p_2 A_2 \cos\theta + R_x$

$F_y = -p_2 A_2 \sin\theta - W + R_y$

$$\oiint_{c.s.} (\rho \vec{V} \cdot d\vec{s}) \vec{V}$$

consider x & y directions

balance integral momentum / x-direction

$$\oiint_{c.s.} (\rho \vec{V} \cdot d\vec{s}) \vec{V} = F_x = p_1 A_1 - p_2 A_2 \cos\theta + R_x$$

$$V_2 \cos\theta (\rho_2 V_2 A_2) - V_1 (\rho_1 V_1 A_1) = p_1 A_1 - p_2 A_2 \cos\theta + R_x$$

balance integral momentum / y-direction

$$V_2 \sin\theta (\rho_2 V_2 A_2) = F_y = -p_2 A_2 \sin\theta - W + R_y$$

continuity integral

$$\oiint_{c.s.} (\rho \vec{V} \cdot d\vec{S}) = 0 \qquad \rho_2 V_2 A_2 = \rho_1 V_1 A_1 = \dot{m}$$

mass flow rate

[single stream / steady flow]

EXAMPLE PROBLEM. Consider a pipe through which water is pumped to higher elevation. Inlet and outlet conditions are indicated. Determine the power requirement for the pump.

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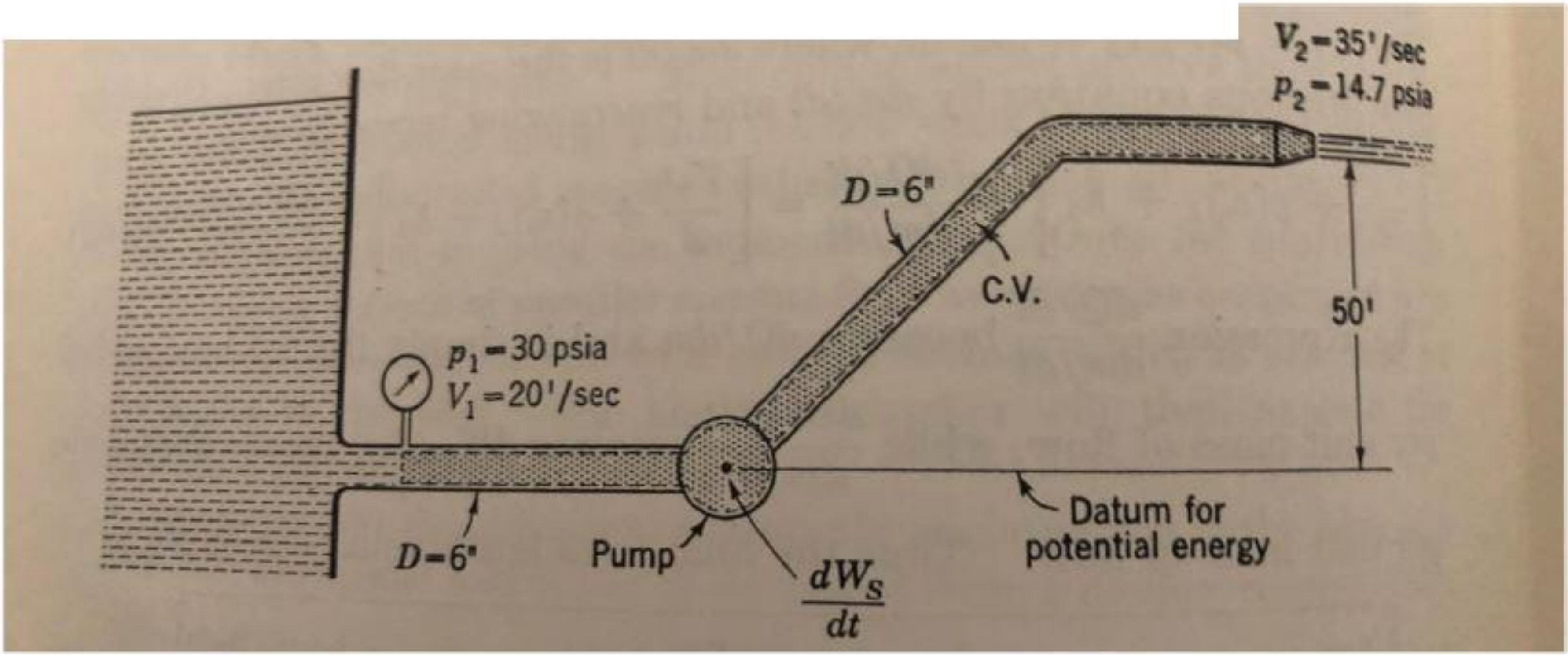

$$\cancel{\frac{Q}{m}}^{\,0} + \frac{W_s}{m} + \cancel{\frac{W_b}{m}}^{\,0} + \cancel{\frac{W_{\cdot}}{m}}^{\,0} + \cancel{\frac{W_{\cdot}}{m}}^{\,0} + \left[h_1 + \frac{V_1^2}{2} + g z_1\right] = \left[h_2 + \frac{V_2^2}{2} + g z_2\right]$$

assumptions

- steady flow
- single stream
- isothermal - constant temperature
- adiabatic - constant int. energy

$$-\frac{W_s}{m} = (h_2 - h_1) + \left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) + g(z_2 - z_1)$$

$$h = u + pv$$

$$h_2 - h_1 = \cancel{u_2}^{\,0} - \cancel{u_1}^{\,0} + P_2 v_2 - P_1 v_1$$

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$$\frac{W_s}{m} = [h_2 - h_1] + \left[\frac{V_2^2}{2} - \frac{V_1^2}{2}\right] + g[z_2 - z_1]$$

$$= [P_2 v_2 - P_1 v_1] + \left[\frac{V_2^2}{2} - \frac{V_1^2}{2}\right] + g[z_2 - z_1]$$

$v = \frac{1}{\rho}$ $\quad \rho = 62.4/g$

$= 62.4 \frac{lbm.}{ft^3} / 32.17 \frac{lbm.\ ft.}{lbf.\ sec^2} = \frac{lbf.}{ft^3}$

[if incompressible flow / const. ρ / const v

$v_1 = v_2$]

$$\frac{W_s}{m} = [P_2 v_2 - P_1 v_1] + \left[\frac{V_2^2}{2} - \frac{V_1^2}{2}\right] + g[z_2 - z_1]$$

$$= \left[\frac{\left[14.7 \frac{lbf}{in^2}\right]\left[144 \frac{in^2}{ft^2}\right]}{\left[62.4/32.17 \frac{lbm.\ ft.}{lbf.\ sec^2}\right] \frac{lbm.}{ft^3}} - \frac{\left[30.0 \frac{lbf.}{in^2}\right]\left[144 \frac{in^2}{ft^2}\right]}{\left[62.4 \frac{lbm.}{ft^3} / 32.17 \frac{lbm.\ ft.}{lbf\ sec^2}\right]}\right]$$

$$+ \left[\frac{(35\ ft./sec.)^2}{2} - \frac{(20\ ft./sec)^2}{2}\right] + 32.17 \frac{ft.}{sec^2}[50\ ft. - 0\ ft.]$$

Power $= 6740\ ft.\ lbf./sec.$