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Problem 4.1

Oblique shock wave with wave angle of 35°
Upstream of wave, $P_1 = 2000\ \text{lb}/\text{ft}^2$, $T_1 = 520°R$, and $V_1 = 3355$ ft/s
Calculate P_2, T_2, V_2, and flow deflection angle

$$M_1 = \frac{V_1}{a_1}$$

$a_1 = \sqrt{\gamma R T_1}$ assuming ideal real gas → air

$$a_1 = \sqrt{(1.4)(1716\ f^2/s^2R°)(520°R)}$$

$a_1 = 1117.7$ ft/s

$\therefore M_1 = 3355\ \text{ft/s} / 1117.7\ \text{ft/s}$

$M_1 = 3$

We are told $\beta = 35°$

So, $Mn_1 = M_1 \sin\beta = 3\sin(35°)$

$Mn_1 = 1.72$

Use this Mn_1 in table A2 and get

$P_2/P_1 = 3.285$, $T_2/T_1 = 1.473$, $Mn_2 = 0.6355$

Therefore:

$P_2 = (2000\ \text{lb}/\text{ft}^2)(3.285) = \boxed{6570\ \text{lb}/\text{ft}^2 = P_2}$

$T_2 = (520°R)(1.473) = \boxed{765.96\ °R = T_2}$

Using θ-β-M figure with $\beta = 35°$ and $M = 3$

$\boxed{\theta \cong 18°}$

$$M_2 = \frac{M_{2n}}{\sin(\beta-\theta)} = \frac{0.6355}{\sin(17°)} = 2.1736$$

$V_2 = M_2 a_2 = 2.1736\sqrt{1.4(1716)(765.96)}$

$\boxed{V_2 = 2948.53\ \text{ft/s}}$

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Problem 4.2

Wedge with half angle of 10° flying at Mach 2
Calculate ratio of TOTAL pressures across shockwave emanating from wedge LE

Using θ-β-M chart with $\theta = 10°$, $M = 2$

We get $\beta \cong 38°$

$M_{n1} = M_1 \sin\beta$

$M_{n1} = 2 \sin(38°) = 1.23$

Using table A2 with $M = 1.23$ & interpolating

$$\boxed{\frac{P_{02}}{P_{01}} = 0.99455}$$

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Problem 4.3

Calculate maximum surface pressure [N/m²] that can be achieved on forward face of wedge flying at Mach 3 at std sea level conditions ($P_1 = 1.01 \times 10^5$ N/m²) with attached shock wave

Using θ-β-M we want θ_{max} for $M = 3$ ($\theta_{max} \Rightarrow$ attached)

We get $\theta \cong 34^\circ$ at $\beta \cong 65^\circ$

$M_{n1} = M_1 \sin(65^\circ) = 3\sin(65^\circ) = 2.72$

Table A2 with $M = 2.72$ & interpolating

$P_2/P_1 = 8.4652$

$P_2 = (8.4652)(1.01 \times 10^5 \text{ N/m}^2)$

$$\boxed{P_2 = 854985.2 \text{ N/m}^2 = 8.55 \times 10^5 \text{ N/m}^2}$$

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Problem 4.4

In flow past compression corner, upstream $M_1 = 3.5$ and $P_1 = 1$ atm.
Downstream of corner, $P_2 = 5.48$ atm
Calculate deflection angle of corner

$P_2 / P_1 = 5.48$

Using table A2 this gives

$M = 2.2 = M_{n_1}$

$M_{n_1} = M_1 \sin(\beta)$

$\sin(\beta) = \frac{2.2}{3.5}$

$\beta = 38.9^\circ$

Using $\theta - \beta - M$ figure

$\boxed{\theta \cong 21^\circ}$

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Problem 4.5

Consider $20°$ half angle wedge at $M=3$ at std. sea level conditions

$P_1 = 2116\ lb/ft^2$, $T_1 = 519°R$

Calculate wave angle, surface pressure, temperature, and mach number

Use θ-β-M. with $\theta = 20°$, $M=3$ and get

$\boxed{\beta \cong 47°}$

$M_{n_1} = M_1 \sin(47°) = 2.194$

Use table A2 with $M=2.19$ & interpolating

$P_2/P_1 = 5.4292$

$T_2/T_1 = 1.8482$

$M_{2n} = 0.54848$

$\boxed{P_2 = (5.4292)(2116\ lb/ft^2) = 11488.187\ lb/ft^2}$

$\boxed{T_2 = (1.8482)(519°R) = 959.2\ °R}$

$M_2 = M_{n2} / \sin(\beta - \theta)$

$M_2 = 0.54848 / \sin(27°)$

$\boxed{M_2 = 1.208}$

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Problem 6

Consider viscous dissipation effects for supersonic flow approaching nosetip of RV at $M = 3.3$

Local static temp of flow = 350 K

Specific heat at const P = 1008 J/kg K

Recovery factor = 0.83

a) Determine magnitude of KE in flow [m^2/s^2]

$$KE = \frac{1}{2}V^2$$

$$V = Ma = M\sqrt{\gamma R T} \quad \text{(assuming ideal air)}$$

$$V = 3.3 \cdot \sqrt{1.4(287\,J/kgK)(350\,K)} = 1237.5\ m/s$$

$$\boxed{\therefore KE = 765730.35\ m^2/s^2 = 7.66\times10^5\ m^2/s^2}$$

b) Mag of KE divided by C_p in K

$$\frac{KE}{C_p} = \frac{765730.35\ m^2/s^2}{1008\ m^2/s^2K} = \boxed{759.65\ K = KE/C_p}$$

c) Determine stagnation temp of flow in K

$$\text{using} \quad C_pT + \frac{V^2}{2} = C_pT_0 \rightarrow T + \frac{V^2}{2C_p} = T_0$$

$$\therefore \underset{(b)}{759.65K} + \underset{(given)}{350K} = \boxed{1109.65K = T_0}$$

d) Mag of KE in flow that is recovered isentropically

$$\text{recovery factor } \alpha = \frac{T_{AD} - T}{T_0 - T} \quad \text{solve for } T_{AD}$$

$$T_{AD} = \alpha(T_0 - T) + T = 0.83(1109.65 - 350) + 350$$

$$T_{AD} = 980.5\ K$$

$$T_{recovered\ isen} = T_{AD} - T = 630.5K$$

$$KE_{rec} = (T_{AD} - T)\times C_p = \boxed{6.356\times10^5\ m^2/s^2 = KE_{rec}}$$

e) Mag of KE lost due to viscous dissipation

$$T_{lost} = T_0 - T_{AD} = 129.15K$$

$$KE_{lost} = (T_0 - T_{AD})C_p = \boxed{1.3\times10^5\ m^2/s^2 = KE_{lost}}$$

f) Mag KE divided by C_p recovered isentropically in K

(d) $KE_{rec} = 6.356 \times 10^5 \, m^2/s^2$

$$\boxed{\frac{KE_{rec}}{C_p} = 630.5K}$$

g) Mag KE divided by C_p lost to viscous dissipation in K

(e) $KE_{lost} = 1.3 \times 10^5 \, m^2/s^2$

$$\boxed{\frac{KE_{lost}}{C_p} = 129.15K}$$