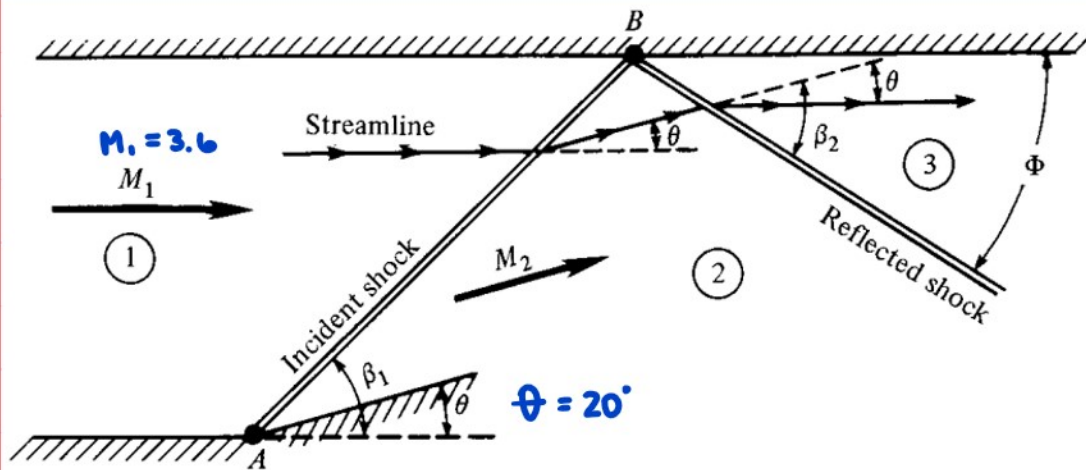


HW 5

Wednesday, March 22, 2023 2:42 PM

4.6

4.6 A supersonic stream at $M_1 = 3.6$ flows past a compression corner with a deflection angle of 20° . The incident shock wave is reflected from an opposite wall which is parallel to the upstream supersonic flow, as sketched in Fig. 4.14. Calculate the angle of the reflected shock relative to the straight wall.



Find Φ

$$M_1 = 3.6, \quad \theta = 20^\circ$$

Using relation, $\beta = 34^\circ$

$$\therefore M_{1n} = M_1 \sin \beta$$

$$M_{1n} = 2.013$$

Using table 2 $\rightarrow M_{2n} = 0.575$

$$M_2 = \frac{M_{2n}}{\sin(\beta - \theta)} = \frac{0.575}{\sin(14^\circ)} = 2.3768$$

Now find β_2 :

$$M_2 = 2.3768, \quad \theta = 20^\circ$$

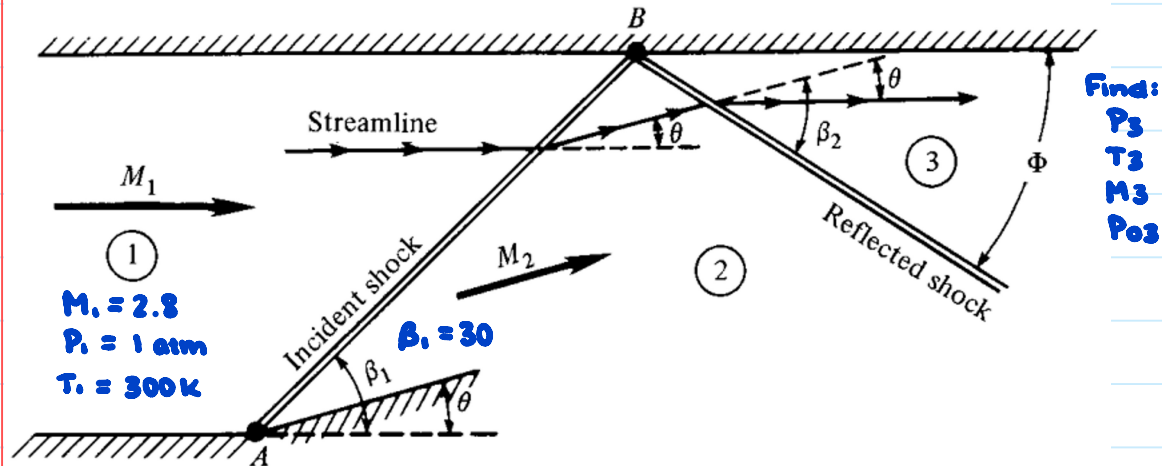
$$\beta_2 \approx 45^\circ$$

$$\Phi = \beta_2 - \theta$$

$$\Phi = 25^\circ$$

4.7

4.7 An incident shock wave with wave angle $= 30^\circ$ impinges on a straight wall. If the upstream flow properties are $M_1 = 2.8$, $p_1 = 1 \text{ atm}$, and $T_1 = 300 \text{ K}$, calculate the pressure, temperature, Mach number, and total pressure downstream of the reflected wave.



Find θ

$$M_1 = 2.8, \beta_1 = 30^\circ$$

$$\therefore \theta \cong 11^\circ$$

$$M_{1n} = 2.8 \sin(30)$$

$$M_{1n} = 1.4$$

Use table A.2

$$P_2/P_1 = 2.12$$

$$T_2/T_1 = 1.255$$

$$P_{02}/P_{01} = 0.9582$$

$$P_{02}/P_1 = 3.049$$

$$M_{2n} = 0.7397$$

$$M_2 = \frac{M_{2n}}{\sin(\beta_1 - \theta)} = 2.27$$

Solve β_2

$$M_2 = 2.27, \theta = 11^\circ$$

$$\therefore \beta_2 = 35.7$$

$$M_{2n, \text{new}} = M_2 \sin(35.7)$$

$$M_{2n, \text{new}} = 1.32$$

$$P_3/P_2 = 1.866$$

$$T_3/T_2 = 1.204$$

$$P_{03}/P_{02} = 0.9753$$

$$M_{3n} = 0.776$$

Putting it all together

$$P_3 = \frac{P_3}{P_2} \frac{P_2}{P_1} P_1 = (1.866)(2.12)(1 \text{ atm})$$

$$P_3 = 3.96 \text{ atm}$$

$$T_3 = (1.204)(1.255)(300 \text{ K})$$

$$T_3 = 453.3 \text{ K}$$

$$P_{03} = \frac{P_{03}}{P_{02}} \frac{P_{02}}{P_1} P_1 = (0.9758)(3.049)(1 \text{ atm})$$

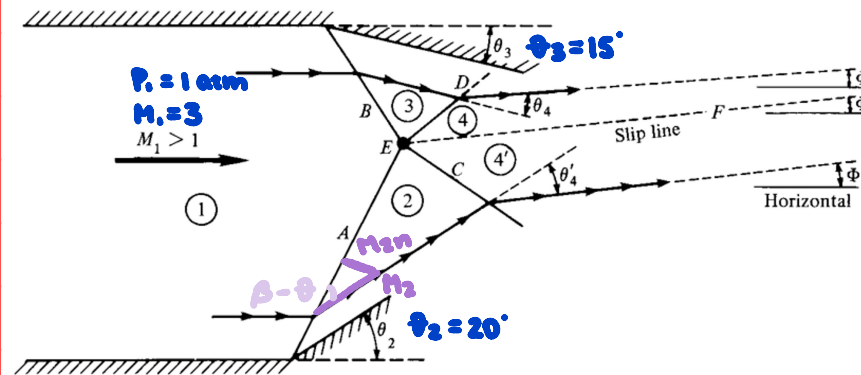
$$P_{03} = 2.975 \text{ atm}$$

$$M_3 = \frac{M_{3n}}{\sin(\beta - \theta)}$$

$$M_3 = 1.857$$

4.9

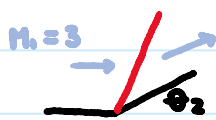
4.9 Consider the intersection of two shocks of opposite families, as sketched in Fig. 4.17. For $M_1 = 3$, $p_1 = 1$ atm, $\theta_2 = 20^\circ$, and $\theta_3 = 15^\circ$, calculate the pressure in regions 4 and 4', and the flow direction Φ , behind the refracted shocks.



Find $P_4 (=P_{4'})$

and Φ

Starting with A:



Find β_A :

$$M_1 = 3, \theta_2 = 20^\circ$$

$$\beta_A = 37.76^\circ$$

$$M_{2n} = 3 \sin(37.76)$$

$$M_{2n} = 1.837$$

Use table A.2

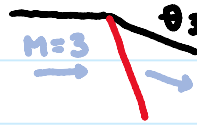
$$\frac{P_2}{P_1} = 3.77$$

$$M_{2n} = 0.608$$

$$M_2 = \frac{M_{2n}}{\sin(\beta - \theta)}$$

$$M_2 = 1.99$$

Now B:



Find β_B :

$$M_1 = 3, \theta_3 = 15^\circ$$

$$\beta_B = 32.24^\circ$$

$$M_{3n} = 3 \sin(32.24)$$

$$M_{3n} = 1.6$$

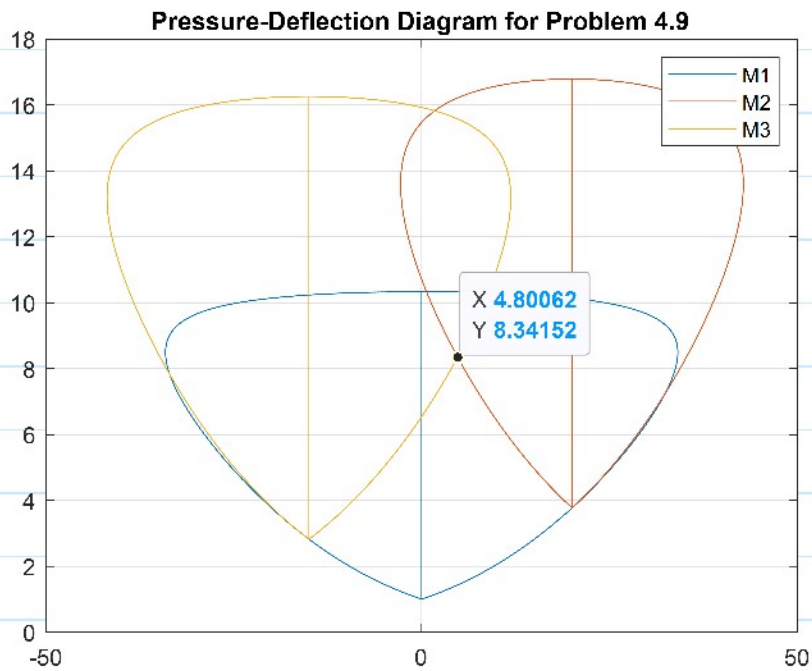
Table A2

$$\frac{P_3}{P_1} = 2.82$$

$$M_{3n} = 0.6684$$

$$M_3 = \frac{M_{3n}}{\sin(\beta - \theta)}$$

$$M_3 = 2.255$$



$$\Phi = 4.8^\circ$$

$$\theta_4' = 20 - 4.8 = 15.2^\circ$$

$$\theta_4 = 4.8 + 15 = 19.8^\circ$$

$$\text{Using } M_2 = 1.99 \text{ and } \theta_4' = 15.2^\circ$$

$$M_3 = 2.255 \text{ at } \theta_4 = 19.8^\circ$$

$$\beta_c = 45.86$$

$$\beta_D = 46.559$$

$$M_{2n} = M_2 \sin(\beta_c)$$

$$M_{3n} = 1.637$$

$$M_{2n} = 1.4281$$

Table A.2

Table A.2

$$\frac{P_4}{P_3} = 2.96$$

$$\frac{P_4'}{P_2} = 2.213$$

$$P_4 = (2.96)(2.82)(1 \text{ atm})$$

$$P_4' = (2.213)(3.77)(1 \text{ atm})$$

$$P_4 = 8.347 \text{ atm}$$

$$P_4' = 8.34 \text{ atm}$$

To summarize, $P_4 = P_4' \approx 8.34 \text{ atm}$

$$\Phi = 4.8^\circ$$

4.10

4.10 Consider the flow past a 30° expansion corner, as sketched in Fig. 4.26. The upstream conditions are $M_1 = 2$, $p_1 = 3 \text{ atm}$, and $T_1 = 400 \text{ K}$. Calculate the following downstream conditions: M_2 , p_2 , T_2 , T_{o2} , and p_{o2} .

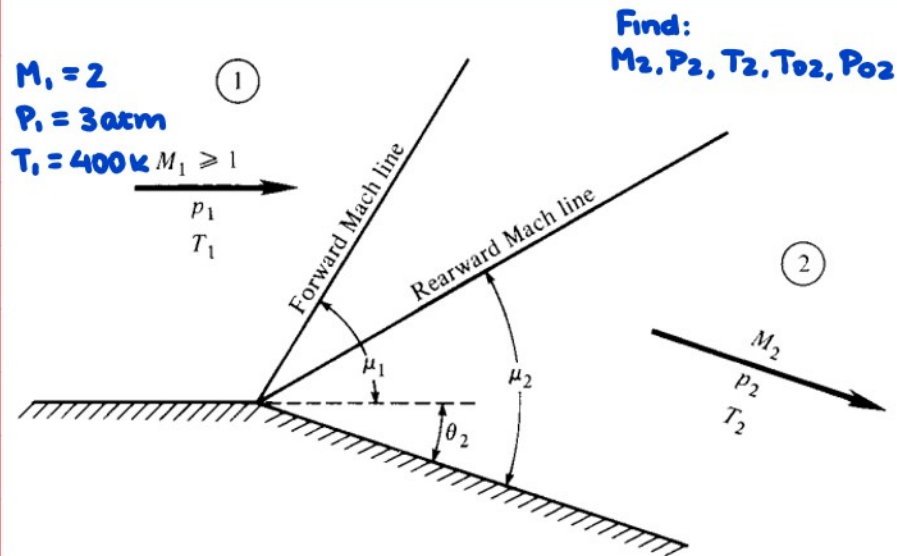


Table 5: $\nu(M_1) = 26.38^\circ$

$$\nu_2 = \theta + \nu_1 = 56.38^\circ$$

Table 5: $M_2 = 3.368$

Use isentropic relations at ① for stag. values

$M_1 = 2 \rightarrow$ Table 1

$$\frac{p_o}{p} = 7.824 \rightarrow p_o = 23.472 \text{ atm}$$

$$\frac{T_o}{T} = 1.8 \rightarrow T_o = 720 \text{ K}$$

Now we want isen. at $M_2 = 3.368$

$$\frac{p_o}{p_2} = 63.176$$

$$p_{o1} = p_{o2}$$

$$p_2 = \frac{23.472}{63.176}$$

$$p_2 = 0.3715 \text{ atm}$$

$$\frac{T_o}{T_2} = 32.68$$

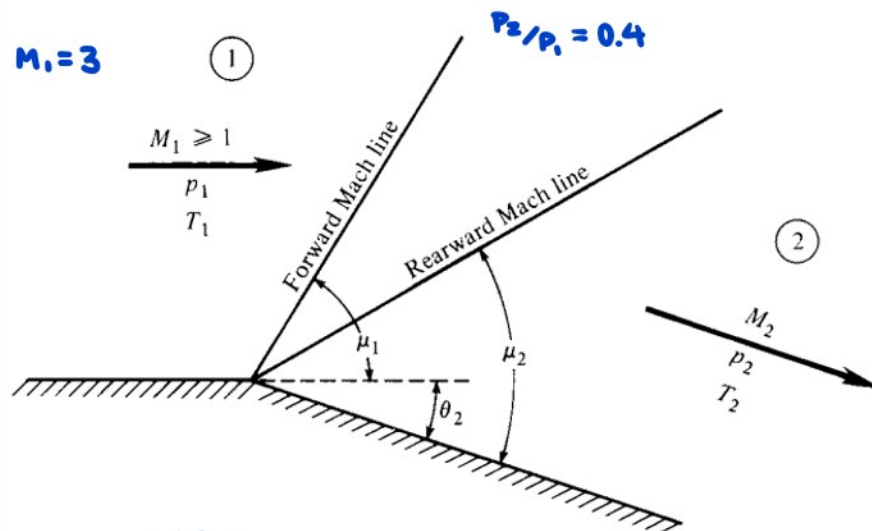
$$T_{o1} = T_{o2}$$

$$T_2 = \frac{720}{32.68}$$

$$T_2 = 22.03 \text{ K}$$

4.11

4.11 For a given Prandtl-Meyer expansion, the upstream Mach number is 3 and the pressure ratio across the wave is $p_2/p_1 = 0.4$. Calculate the angles of the forward and rearward Mach lines of the expansion fan relative to the free-stream direction.



$$\mu_1 = \sin^{-1}\left(\frac{1}{M_1}\right)$$

$$\mu_1 = 19.47^\circ$$

$$\frac{p_0}{p_1} (\text{at } M=3) = 36.73$$

$$\text{call } p_1 = 1 \text{ atm}$$

$$\text{Then } p_0 = 36.73 \text{ atm}$$

$$\text{If } p_2/p_1 = 0.4$$

$$\text{Then } p_2 = 0.4 \text{ atm}$$

$$\frac{p_0}{p_2} = \frac{36.73}{0.4} = 113.857$$

Find this in table 1

$$M_2 = 3.79$$

$$\mu_2 = 15.3^\circ$$