This Homework Must Be Uploaded onto CANVAS to Receive Credit.
Deadline: Shown in Syllabus

Name: ______________________

General Instructions

- Uploading Assignment: The entire homework assignment must be uploaded in the CANVAS dropbox in one file. Use the filename *xxHW_Lastname_revxx.doc* when uploading to CANVAS. Your homework must be written neatly or typed. If you want to write it out, you can scan it or take pictures of it with your phone. I must be able to read the uploaded file. Submitting all solutions in one file is required.
- Uploading spreadsheets or other programs: If you use spreadsheets or other programs, put in screenshots of your graphs or pertinent tables into your homework file submission. You do not have to upload your spreadsheets, videos, or programs unless specifically requested in the assignment sheet. When using computer programs, be sure to document in your homework submission the basic equations and example calculations with units showing how the program works.
- Re-submitting homework: If you submit your package and then resubmit an update before the deadline, the newest submission will be graded.
- Grading Rubric**:** The homework grading rubric is shown on CANVAS. The completeness of the entire homework package is also a component of the homework grade.

Required Homework Format (See Example at end of this Syllabus)

In the solution of problems, you are required to:

1. **Name:** Provide name of the student.
2. **Given**: State briefly and concisely (in your own words) the information provided.
3. **Find:** State the information that you have to find.
4. **Schematic**: Draw a schematic representation of the system and control volume if applicable.
5. **Assumptions:** List the simplifying assumptions that are appropriate to the problem and implied by the equations used.
6. **Basic Equations**: Outline the basic equations needed to do the analysis. Use the proper symbol from the book where applicable.
7. **Analysis:** Manipulate the basic equations to the point where it is appropriate to substitute numerical values. Substitute numerical values (using a consistent set of units) to obtain a numerical answer. Include appropriate units in calculations. If multiple repetitive calculations are done on a spreadsheet for example, show at least one example calculation in detail, including all units. The significant figures in the answer should be consistent with the given data. Check the answer and the assumptions made in effecting the solution to make sure they are reasonable.
8. **Answer**. Label the answer(s) with a box and an arrow from the right-hand margin.
9. **Comment**: Write a comment at the end of the homework that reflects on the limitations of the solution, the reasonableness of the solution, or something that you learned by doing the problem.

All nine formatting elements must be specifically shown in Each HW to receive full credit unless otherwise specified.

Assigned Problems:

- Textbook Problems

  2.6, 3.8, 4.24, 4.30

- Special Problems

SP01A Annotated Bibliography (Template in Appendix A)

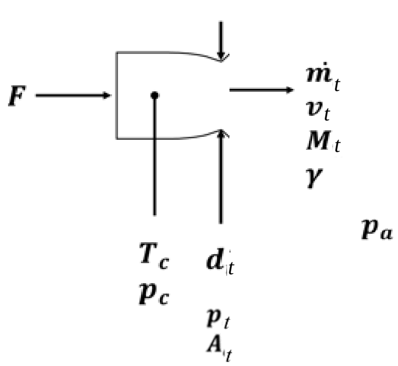
Complete a Two-Page Annotated Bibliography using the attached template. The paper is in the assignment drop box.

Berg, P., Loeblich, W., and Frederick, R., "Using CEQUEL for Thermochemistry Calculations in a Graduate Rocket Propulsion Course at UAH," 2023 AIAA SciTech, January 26, 2023. (Note this paper will have an AIAA Paper Number assigned later this month)

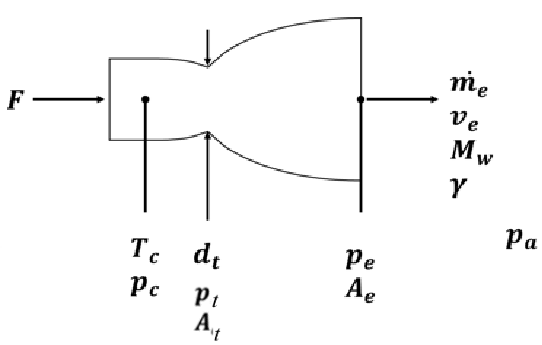
SP01B Basic Linear Momentum Control Volume Analysis of Rocket Motor (Ref. Material in Appendix B)

1. **Name:**
2. **Given:** Rocket motor with a converging nozzle, and then a second configuration adding a diverging nozzle.
3. **Find:**
   a. Starting with the complete linear momentum (Equation 3.37 in Appendix B) equation for a control volume, derive the thrust equation in a systematic manner for the rocket with the converging nozzle. Document each simplifying assumption, define the control volume, and use the symbols found in the schematic or consistent with the course textbook. Appendix B shows the basic starting equation with some explanations.
   b. Repeat the process in for the rocket with a converging-diverging nozzle; determine the force that the diverging section of the nozzle has on the circular ring of material around the rocket throat. You will need to draw a separate control volume for this analysis. For Special Problem SP01B assume that the cross-sectional area of the nozzle throat material is A_{ring}.
   c. Comment of the ratio of the thrust of the second rocket over the first for a typical supersonic converging-diverging nozzle. When is the diverging section adding to the overall thrust of the rocket motor?
4. **Schematic:**

Converging



Converging-Diverging

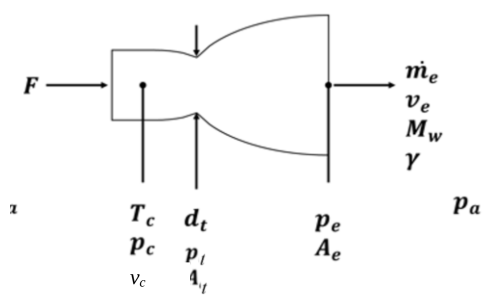


SP01C Conservation of Energy, Control Volume Analysis of Converging-Diverging Nozzle (Ref. Material in Appendix C)

1. **Name:**
2. **Given:** Rocket motor with a converging-diverging nozzle
3. **Find:**
   a. Starting with the conservation of energy equation for a control volume (see Appendix C), show a systematic derivation of the thrust equation that includes the velocity in the chamber entering the nozzle. State all the assumptions relevant to reducing and modifying the original equation and show the additional laws/equations that are inserted into the solution to arrive at Equation 4.23 in the textbook (with the added v_c term that is not included)

$$v_e^2 = \frac{2\gamma R_u T_c}{\mathfrak{M}(\gamma - 1)}\left[1 - (p_e/p_c)^{(\gamma-1)/\gamma}\right]$$

   b. Comment on the definitions and equations for thermally perfect gas and calorically perfect gas assumptions and why real rocket nozzles operation behave differently.
4. **Schematic:**



Two-Page Annotated Bibliography Template
(Select MAE 440 or MAE 540)
Summarize

Reference Document Examined:	List the complete citation of the reference here. Use the AIAA Journal reference format.
Reviewer:	Your Name
Source of Document:	List the source of the document (online, company, particular library, particular website, and any copyright information.
Date of Review:	Put in the date of your review
Electronic File Name:	**Put in the name of the electronic file**

Summary of Paper:

Type in your one-page summary, single space, here. This paragraph or set of paragraphs should at least complete the first page. You may include one picture (not to exceed ½ pages) in the summary.

B. Assess:

Important Facts from Document:

1. List five important facts you learned from the reference document you examined. Put them in the form of complete sentences.
2.

Key Figure from Document:

Put in one key figure from the paper with a caption

Important Relationships among Parameters Described in the Paper:

1. List 2 important relationships among parameters that are described in the paper
2. For example, when the pressure in the chamber goes up, the specific impulse increases;
3. For example, when a supplier goes out of business, the rocket community must turn to commercial industries that have a larger market to sustain the products.

C. Reflect

"Once you've summarized and assessed a source, you need to ask how it fits into your research. Was this source helpful to you? How can you use this source in a research project? Has it changed how you think about your topic?" Write this in your own words.

Appendix B – Linear Momentum Equation for a Control Volume[1]

3.4 The Linear Momentum Equation

In Newton's second law, Eq. (3.2), the property being differentiated is the linear momentum $m\mathbf{V}$. Therefore our dummy variable is $\mathbf{B} = m\mathbf{V}$ and $\beta = d\mathbf{B}/dm = \mathbf{V}$, and application of the Reynolds transport theorem gives the linear momentum relation for a deformable control volume:

$$\boxed{\frac{d}{dt}(m\mathbf{V})_{\text{syst}} = \sum \mathbf{F} = \frac{d}{dt}\left(\int_{CV} \mathbf{V}\rho \, d\mathcal{V}\right) + \int_{CS} \mathbf{V}\rho(\mathbf{V}_r \cdot \mathbf{n}) \, dA} \tag{3.35}$$

The following points concerning this relation should be strongly emphasized:

1. The term $\mathbf{V}$ is the fluid velocity relative to an *inertial* (nonaccelerating) coordinate system; otherwise Newton's second law must be modified to include noninertial relative acceleration terms (see the end of this section).
2. The term $\Sigma\,\mathbf{F}$ is the *vector* sum of all forces acting on the system material considered as a free body; that is, it includes surface forces on all fluids and solids cut by the control surface plus all body forces (gravity and electromagnetic) acting on the masses within the control volume.
3. The entire equation is a vector relation; both the integrals are vectors due to the term $\mathbf{V}$ in the integrands. The equation thus has three components. If we want only, say, the x component, the equation reduces to

$$\sum F_x = \frac{d}{dt}\left(\int_{CV} u\rho \, d\mathcal{V}\right) + \int_{CS} u\rho(\mathbf{V}_r \cdot \mathbf{n}) \, dA \tag{3.36}$$

and similarly, $\Sigma\, F_y$ and $\Sigma\, F_z$ would involve v and w, respectively. Failure to account for the vector nature of the linear momentum relation (3.35) is probably the greatest source of student error in control volume analyses.

For a fixed control volume, the relative velocity $\mathbf{V}_r \equiv \mathbf{V}$, and Eq. (3.35) becomes

$$\boxed{\sum \mathbf{F} = \frac{d}{dt}\left(\int_{CV} \mathbf{V}\rho \, d\mathcal{V}\right) + \int_{CS} \mathbf{V}\rho(\mathbf{V} \cdot \mathbf{n}) \, dA} \tag{3.37}$$

Again we stress that this is a vector relation and that $\mathbf{V}$ must be an inertial-frame velocity. Most of the momentum analyses in this text are concerned with Eq. (3.37).

[1] White, F.M., *Fluid Mechanics*, 7th Edition, ISBN 978-0-07-352934-9

One-Dimensional Momentum Flux

By analogy with the term *mass flow* used in Eq. (3.28), the surface integral in Eq. (3.37) is called the *momentum flux term.* If we denote momentum by **M**, then

$$\dot{\mathbf{M}}_{CS} = \int_{\text{sec}} \mathbf{V}\rho(\mathbf{V}\cdot\mathbf{n})\, dA \tag{3.38}$$

Because of the dot product, the result will be negative for inlet momentum flux and positive for outlet flux. If the cross section is one-dimensional, **V** and ρ are uniform over the area and the integrated result is

$$\dot{\mathbf{M}}_{\text{sec}i} = \mathbf{V}_i(\rho_i V_{ni} A_i) = \dot{m}_i \mathbf{V}_i \tag{3.39}$$

for outlet flux and $-\dot{m}_i\mathbf{V}_i$ for inlet flux. Thus if the control volume has only one-dimensional inlets and outlets, Eq. (3.37) reduces to

$$\boxed{\sum \mathbf{F} = \frac{d}{dt}\left(\int_{CV} \mathbf{V}\rho\, d\mathcal{V}\right) + \sum (\dot{m}_i\mathbf{V}_i)_{\text{out}} - \sum (\dot{m}_i\mathbf{V}_i)_{\text{in}}} \tag{3.40}$$

This is a commonly used approximation in engineering analyses. It is crucial to realize that we are dealing with vector sums. Equation (3.40) states that the net vector force on a fixed control volume equals the rate of change of vector momentum within the control volume plus the vector sum of outlet momentum fluxes minus the vector sum of inlet fluxes.

Appendix C – Conservation Of Energy for a Control Volume[2]

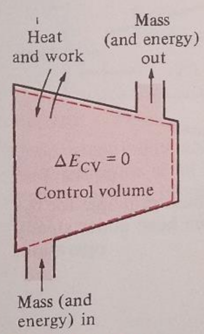


FIGURE 4-23

During a steady-flow process, the rate of energy flow into the control volume equals the rate of energy flow out of it.

Conservation of Energy

It was pointed out earlier that during a steady-flow process the total energy content of a control volume remains constant (E_{CV} = constant, as shown in Fig. 4-23). That is, the change in the total energy of the control volume during such a process is zero ($\Delta E_{CV} = 0$). Thus the amount of energy entering a control volume in all forms (heat, work, mass transfer) must be equal to the amount of energy leaving it for a steady-flow process.

Consider, for example, an ordinary electric hot-water heater under steady operation, as shown in Fig. 4-24. A cold-water stream with a mass flow rate $\dot{m}$ is continuously flowing into the water heater, and a hot-water stream of the same mass flow rate is continuously flowing out of it. The water heater (the control volume) is losing heat to the surrounding air at a rate of $\dot{Q}$, and the electric heating element is doing electrical work (heating) on the water at a rate of $\dot{W}$. On the basis of the conservation of

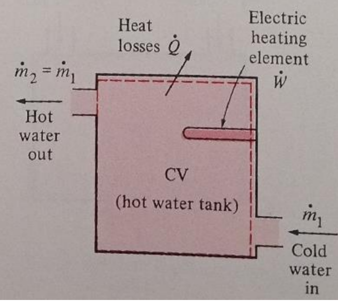


FIGURE 4-24

A water heater under steady operation.

energy principle, we can say that *the water stream will experience an increase in its total energy as it flows through the water heater, which is equal to the electric energy supplied to the water minus the heat losses.*

By this line of reasoning, the first law of thermodynamics or the **conservation of energy principle** for a general steady-flow sytem with multiple inlets and exits can verbally be expressed as

$$\begin{pmatrix}\text{Total energy}\\ \text{crossing boundary}\\ \text{as heat and work}\\ \text{per unit time}\end{pmatrix} = \begin{pmatrix}\text{Total energy}\\ \text{transported out of}\\ \text{CV with mass}\\ \text{per unit time}\end{pmatrix} - \begin{pmatrix}\text{Total energy}\\ \text{transported into}\\ \text{CV with mass}\\ \text{per unit time}\end{pmatrix}$$

or

$$\dot{Q} - \dot{W} = \Sigma \dot{m}_e \theta_e - \Sigma \dot{m}_i \theta_i \qquad (4\text{-}18)$$

where θ is the total energy of the flowing fluid, including the flow work, per unit mass (Fig. 4-25). It can also be expressed as

$$\dot{Q} - \dot{W} = \underbrace{\sum \dot{m}_e \left(h_e + \frac{\mathbf{V}_e^2}{2} + gz_e\right)}_{\text{for each exit}} - \underbrace{\sum \dot{m}_i \left(h_i + \frac{\mathbf{V}_i^2}{2} + gz_i\right)}_{\text{for each inlet}} \quad \text{(kW)} \qquad (4\text{-}19)$$

since $\theta = h + \text{ke} + \text{pe}$ (Eq. 4-13). Equation 4-19 is the general form of the first-law relation for steady-flow processes.

For single-stream (one-inlet, one-exit) systems the summations over the inlets and the exits drop out, and the inlet and exit states in this case are denoted by subscripts 1 and 2, respectively, for simplicity. The mass flow rate through the entire control volume remains constant ($\dot{m}_1 = \dot{m}_2$) and is denoted $\dot{m}$. Then the conservation of energy equation for *single-stream steady-flow systems* becomes

$$\dot{Q} - \dot{W} = \dot{m}\left[h_2 - h_1 + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2} + g(z_2 - z_1)\right] \quad \text{(kW)} \qquad (4\text{-}20)$$

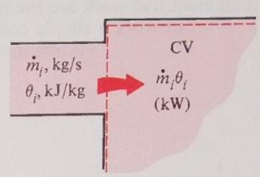


FIGURE 4-25

The product $\dot{m}_i\theta_i$ is the energy transported into the control volume by mass per unit time.

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[2] Cengel, Y.A and Boles, M.A., "Thermodynamics, - An Engineering Approach," ISBN 0-07-010356-9