This Homework Must Be Uploaded onto CANVAS to Receive Credit.
Deadline: Shown in Syllabus

Name: ______________________

General Instructions

- Uploading Assignment: The entire homework assignment must be uploaded in the CANVAS dropbox in one file. Use the filename *xxHW_Lastname_revxx.doc* when uploading to CANVAS. Your homework must be written neatly or typed. If you want to write it out, you can scan it or take pictures of it with your phone. I must be able to read the uploaded file. Submitting all solutions in one file is required.
- Uploading spreadsheets or other programs: If you use spreadsheets or other programs, put in screenshots of your graphs or pertinent tables into your homework file submission. You do not have to upload your spreadsheets, videos, or programs unless specifically requested in the assignment sheet. When using computer programs, be sure to document in your homework submission the basic equations and example calculations with units showing how the program works.
- Re-submitting homework: If you submit your package and then resubmit an update before the deadline, the newest submission will be graded.
- Grading Rubric**:** The homework grading rubric is shown on CANVAS. The completeness of the entire homework package is also a component of the homework grade.

Required Homework Format (See Example at end of this Syllabus)

In the solution of problems, you are required to:

1. **Name:** Provide name of the student.
2. **Given**: State briefly and concisely (in your own words) the information provided.
3. **Find:** State the information that you have to find.
4. **Schematic**: Draw a schematic representation of the system and control volume if applicable.
5. **Assumptions:** List the simplifying assumptions that are appropriate to the problem and implied by the equations used.
6. **Basic Equations**: Outline the basic equations needed to do the analysis. Use the proper symbol from the book where applicable.
7. **Analysis:** Manipulate the basic equations to the point where it is appropriate to substitute numerical values. Substitute numerical values (using a consistent set of units) to obtain a numerical answer. Include appropriate units in calculations. If multiple repetitive calculations are done on a spreadsheet for example, show at least one example calculation in detail, including all units. The significant figures in the answer should be consistent with the given data. Check the answer and the assumptions made in effecting the solution to make sure they are reasonable.
8. **Answer**. Label the answer(s) with a box and an arrow from the right-hand margin.
9. **Comment**: Write a comment at the end of the homework that reflects on the limitations of the solution, the reasonableness of the solution, or something that you learned by doing the problem.

All nine formatting elements must be specifically shown in Each HW to receive full credit unless otherwise specified.

Problem 11.4

1. **Name:** Robert Frederick

2. **Given:** 7-port hybrid rocket motor

LOX/HTPB: $I_{sp} = 350s,\ O/F = 2.5,\ \bar{F} = 500{,}000\ lb_f\ ,\ t_p = 100s,\ \rho_f = 0.036\frac{lb_m}{in^3},$
$optimal\ c^*(2.5) = 5000\frac{ft}{s},\ G_{ox,i} = 1\frac{lb_m}{in^2 s},\ r = 0.2G_{ox}^{0.68}\ in/s$

3. **Find:**

(i) Initial port diameter $D_{p,i}$

(ii) Total web thickness w assuming 7-port grain, no sliver burning, regression rate varies linearly with time.

(iii) Length of fuel grain and diameter of motor.

4. **Schematic:**

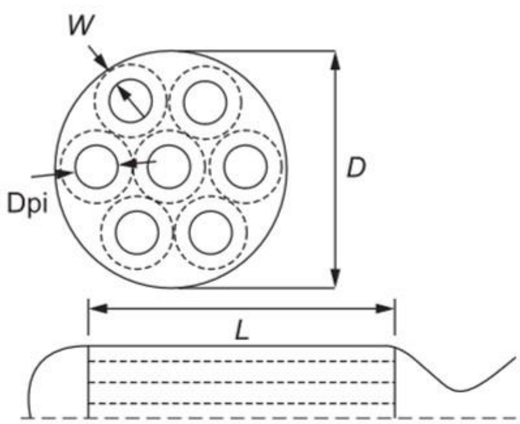


Figure 11.25 Diagram for Problem 11.4.

5. **Assume:**

1) Lumped parameter ballistics
2) No end burning
3) Burning rate is linear with time
4) 500,000 is initial thrust
5) $I_{sp} = 350s$ is initial value

6. **Basic Equations:**

$$c^* = \frac{P_c A_b}{\dot{m}_{ox}+\dot{m}_F} \qquad G_{ox} = \frac{\dot{m}_{ox}}{NA_p} \qquad \dot{m}_T = \dot{m}_{ox} + \dot{m}_F$$

$$r = aG_{ox}^n \qquad \dot{m}_{ox} = \frac{\dot{m}}{1+\frac{1}{O/F}} = \frac{\dot{m}\ O/F}{O/F+1}$$

$$\bar{r}_{avg} = \frac{r_i + r_f}{2} \qquad w = \bar{r}_{avg} t_b$$

7. **Analysis**:

i) Initial port diameter $D_{p,i}$

$$r = aG_{ox,i}^{n} = 0.2\left(1\frac{lb_m}{in^2 s}\right)^{0.68}$$

$$w = \left(\frac{r_i + r_f}{2}\right)t_b = \left[0.2 + 0.2\left(\frac{\dot{m}_{ox}/7}{\pi(R_o+w)^2}\right)^{0.68}\right]\frac{t_b}{2}$$

$$\dot{m}_{ox} = \frac{\bar{F}}{I_{sp}g\left(1+\frac{1}{O/F}\right)} = \frac{500{,}000\ lb_f}{(350s)\left(32.2\frac{ft}{s^2}\right)\left(1+\frac{1}{2.5}\right)}\left(\frac{32.2 lb_m\, ft}{lb_f\, s^2}\right) = 1020.41\frac{lb_m}{s}$$

$$R_o = \sqrt{\frac{\dot{m}_{ox}/N}{\pi G_{ox,i}}} = 6.81182in$$

$$\boldsymbol{D_{p,i}} = D_o = 2R_o = \boxed{\mathbf{13.6236}\boldsymbol{in}}$$

8i

ii) Total web thickness *w*

$$w = \left[0.2 + 0.2\left(\frac{\left(1020.41\frac{lbm}{s}\right)/7}{\pi(6.8118in+w)^2}\right)^{0.68}\right]\frac{100s}{2}$$

Solving iteratively for *w*, we get,

$$\boxed{\boldsymbol{w} = \mathbf{12.4351}\boldsymbol{in}}$$

8ii

iii) Length of fuel grain and diameter of motor

Find average port radius

$$R_{mid} = R_o + \frac{w}{2} = 6.81182in + \frac{12.4351in}{2} = 13.02932in$$

Using *O/F* ratio of 2

$$L = \frac{\dot{m}_{ox}/N}{2\pi\rho_f R_{mid}\, a\left(\frac{\dot{m}_{ox}}{N\pi R_{mid}^2}\right)^n O/F} = \frac{\left(1020.41\frac{lbm}{s}\right)/7}{2\pi\left(0.036\frac{lbm}{in^3}\right)(13.02937in)(0.2)\left(\frac{1020.41\frac{lbm}{s}}{7\pi(12.43in)^2}\right)^{0.68}(2.5)}$$

$$\Rightarrow \boxed{\boldsymbol{L} = \mathbf{238.9}\boldsymbol{in}}$$

8iii

$$\Rightarrow D = 6(R_o + w) = 6(6.8118in + 12.4351in)$$

$$\Rightarrow \boxed{\boldsymbol{D} = \mathbf{115.48}\boldsymbol{in}}$$

8iii

8. **Answers**:

(i) $\boldsymbol{D_{p,i}} = \mathbf{13.6236}\boldsymbol{in}$

(ii) $\boldsymbol{w} = \mathbf{12.4351}\boldsymbol{in}$

(iii) $\boldsymbol{L} = \mathbf{238.9}\boldsymbol{in}$; $\boldsymbol{D} = \mathbf{115.48}\boldsymbol{in}$

9. **Comments**:

Using the average burning rate for determining the web will overpredict the web distance for a 100*sec* burn. The burn rate really drops non-linearly with time, so the real average burn rate is lower that the first approximation.

Table of values are shown below.

1020.4

Area Ratio 6.243219

e 1.67835122 .

-0.001513

p

Dr. Robert Fredrick, Jr., The University of Alabama in Huntsville

Inputs		
F=	500,000	lb_f
Is=	350.0	s
OF=	2.5	--
t_b=	100	s
Outputs		
M_dot_ox=	1020.41	lb_m/s
D_i=	13.624	in
R_i=	6.81	in
Inputs		
a=	0.2	
n=	0.68	[-]
rho_p=	0.036	lbm/in^3
N=	7	-
Go=	1.00	lbm/s*in^2
Outputs		
R_0=	16.38	in
w=	9.57	in
D=	98.27	[-]
L=	229.155	in
R_{mid}=	11.595	in
t_{mid}	36.214	
Mox=	102,041	lbm
Mf=	40,252	lbm
O/F ave=	2.54	[-]

Time	R	OF	m_dot f all	Gox	r	m_dot_ox_t	M_dot_t	C* Curve Fit	Pc	Fvac(Cf=1.7)	Gamma	CF Calc	Fvac (var gamma)	r (11.4)	r_{ave}(11.4)	r_{ave} (SP)
(sec)	(in)	(-)	(lbm/s)	lbm/(s*in^2)	in/s	(lbm/s)	(lbm/s)	ft/s	psi	lbf	-	-	lbf	in/s	in/s	in/s
0.00	6.812	2.064	494.314	1.000	0.200	1020.408163	1514.7	5907.5	926.311	472,419	1.152819	1.7000	472,419	0.200	0.124351	0.099
5.00	7.727	2.160	472.385	0.777	0.168	1020.408163	1492.8	5899.8	911.7197	464,977	1.146272	1.7031886	465,849	0.1924	0.124351	0.099
10.00	8.514	2.237	456.170	0.640	0.148		1476.6	5886.8	899.8204	458,908	1.141893	1.7053474	460,352	0.1849	0.124351	0.099
15.00	9.213	2.301	443.396	0.547	0.133	1020.408163	1463.8	5871.8	889.7709	453,783	1.1388	1.7068851	455,621	0.1773	0.124351	0.099
20.00	9.846	2.357	432.910	0.479	0.121	1020.408163	1453.3	5856.4	881.0808	449,351	1.136545	1.7080131	451,469	0.1697	0.124351	0.099
25.00	10.429	2.406	424.047	0.427	0.112	1020.408163	1444.5	5841.2	873.4372	445,453	1.134872	1.7088539	447,773	0.1622	0.124351	0.099
30.00	10.970	2.451	416.395	0.386	0.105	1020.408163	1436.8	5826.6	866.6267	441,980	1.133621	1.7094846	444,446	0.1546	0.124351	0.099
35.00	11.477	2.491	409.676	0.352	0.098	1020.408163	1430.1	5812.5	860.4967	438,853	1.132687	1.7099564	441,424	0.1470	0.124351	0.099
36.21	11.595	2.500	408.168	0.345	0.097	1020.408163	1428.6	5809.2	859.1021	438,142	1.132501	1.7100509	440,733	0.1452	0.124351	0.099
40.00	11.955	2.528	403.699	0.325	0.093	1020.408163	1424.1	5799.2	854.934	436,016	1.131999	1.7103049	438,659	0.1395	0.124351	0.099
45.00	12.409	2.562	398.323	0.301	0.088	1020.408163	1418.7	5786.6	849.8522	433,425	1.131504	1.7105559	436,116	0.1319	0.124351	0.099
50.00	12.841	2.594	393.445	0.281	0.084	1020.408163	1413.9	5774.6	845.1836	431,044	1.131164	1.7107284	433,764	0.1244	0.124351	0.099
55.00	13.254	2.623	388.985	0.264	0.081	1020.408163	1409.4	5763.4	840.8741	428,846	1.13095	1.7108368	431,580	0.1168	0.124351	0.099
60.00	13.650	2.651	384.881	0.249	0.078	1020.408163	1405.3	5752.7	836.8799	426,809	1.130841	1.7108924	429,543	0.1092	0.124351	0.099
65.00	14.032	2.678	381.083	0.236	0.075	1020.408163	1401.5	5742.7	833.1647	424,914	1.130819	1.7109038	427,639	0.1017	0.124351	0.099
70.00	14.399	2.703	377.550	0.224	0.072	1020.408163	1398.0	5733.3	829.6984	423,146	1.130869	1.7108781	425,854	0.0941	0.124351	0.099
75.00	14.755	2.727	374.251	0.213	0.070	1020.408163	1394.7	5724.4	826.4555	421,492	1.130982	1.710821	424,175	0.0865	0.124351	0.099
80.00	15.099	2.749	371.159	0.204	0.068	1020.408163	1391.6	5716.0	823.4144	419,941	1.131147	1.7107372	422,594	0.0790	0.124351	0.099
85.00	15.432	2.771	368.249	0.195	0.066	1020.408163	1388.7	5708.1	820.5563	418,484	1.131357	1.7106304	421,101	0.0714	0.124351	0.099
90.00	15.757	2.792	365.503	0.187	0.064	1020.408163	1385.9	5700.6	817.8652	417,111	1.131606	1.7105039	419,688	0.0638	0.124351	0.099
100.00	16.379	2.831	360.440	0.173	0.061	1020.408163	1380.8	5687.0	812.9291	414,594	1.132202	1.710202	417,082	0.0487	0.124351	0.099

Problem 11.4

O/Fmid	2.5	w calc
w=	12.43505	12.43505
R_{mid}=	13.029	
L=	238.978	
D=	115.481	

Special Problem SP02B-A

1. **Name:** Robert Frederick

2. **Given:** Results from Problem 11.4, modify the time-dependent forms of the lumped parameter ballistics equations

3. **Find:**
(a) Total web distance burned
(b) Fuel grain length, assuming the optimum O/F ratio occurs at the mid-web radius
(c) Overall motor diameter
(d) Percentage difference of web distance and grain length using the integrated equation method and the results of 11.4
(e) Provide 5 plots
- Mixture ratio and oxidizer flux as f(t).
- c* vs f(O/F). Annotate motor start and stop points.
- $\dot{m}_F, \dot{m}_{ox}, \dot{m}_T$ vs time.
- P_c vs time with $\dot{m}_{ox}$ and $A_t = 300in^2$ and employing the c* data provided.
- F_{vac} vs time assuming $C_{Fv} = 1.7$ or using the thrust coefficient subroutine provided (assume an area ratio).

4. **Schematic:**

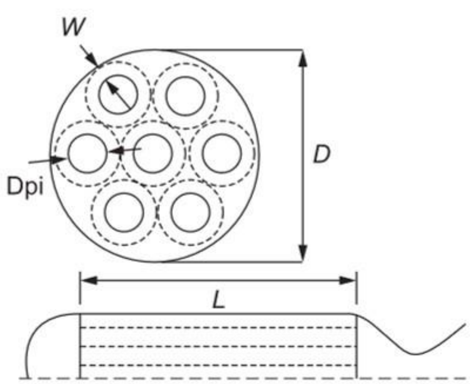


Figure 11.25 Diagram for Problem 11.4.

5. **Assume:**
1) Lumped parameter ballistics
2) No end burning
3) $r = aG_{ox}^n$

6. **Basic Equations:**
For single port

$$R(t) = \left\{2(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right\}^{\frac{1}{2n+1}}$$
$$\dot{m}_F(t) = 2a\rho_f L\pi^{1-n}\dot{m}_{ox}{}^n[R(t)]^{1-2n}$$

For multi-port with "N" ports and grain length "L"

$$R(t) = \left\{a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi N}\right)^n t + R_i^{2n+1}\right\}^{\frac{1}{2n+1}}$$

$$\dot{m}_F(t) = 2\pi N \rho_f L a \left(\frac{\dot{m}_{ox}}{\pi N}\right)^n [R(t)]^{1-2n}$$

$$\left.\frac{\dot{m}_{ox}}{\dot{m}_F}\right|(t) = \frac{1}{2\rho_f La} \left(\frac{\dot{m}_{ox}}{\pi N}\right)^{1-n} \left\{a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi N}\right)^n t + R_i^{2n+1}\right\}^{\frac{2n-1}{2n+1}}$$

$$r = aG_{ox}^n \qquad c^* = c^*(O/F) \qquad I_{sp} = \frac{F}{(\dot{m}_{ox}+\dot{m}_F)g} \qquad G_{ox} = \frac{\dot{m}_{ox}}{NA_p}$$

7. **Analysis**:

a) Total web distance burned

$$\dot{m}_{ox} = \frac{\dot{m}_T\,(O/F)}{1+(O/F)} = \frac{\bar{F}}{I_{sp}g}\left(\frac{(O/F)}{1+(O/F)}\right) = \frac{500{,}000\ lb_f}{(350s)\left(32.2\frac{ft}{s^2}\right)}\left(\frac{32.2 lb_m\, ft}{lb_f\, s^2}\right)\left(\frac{2.5}{1+2.5}\right) = 1020.41\,\frac{lb_m}{s}$$

$$D_{p,i} = \sqrt{\frac{4\dot{m}_{ox}}{\pi N G_{ox,i}}} = \sqrt{\frac{4*1020.41\frac{lb_m}{s}}{\pi*7*1\frac{lb_m}{in^2 s}}} = 13.6236in \implies R_o = 6.8118in$$

$$R(100s) = \left\{0.2(2*0.68+1)\left(\frac{1020.41\frac{lb_m}{s}}{7\pi}\right)^{0.68} 100s + (6.8118)^{2(0.68)+1}\right\}^{\frac{1}{2(0.68)+1}}$$

$$R(100s) = \{641.3 + 92.55\}^{0.4237} = 16.379in$$

$$\boldsymbol{w} = R(100s) - R_o = 16.379in - 6.8118in = \boxed{\boldsymbol{9.567in}}$$

8a

b) Fuel grain length

Radius at mid web (assume O/F =2.5)

$$R_{mid} = R_o + \frac{w}{2} = 6.81182in + \frac{9.567in}{2} = 11.595in$$

$$L = \frac{\dot{m}_{ox}/N}{2\pi\rho_f R_{mid}\, a\left(\frac{\dot{m}_{ox}}{N\pi R_{mid}^2}\right)^n O/F} = \frac{\left(1020.41\frac{lb_m}{s}\right)/7}{2\pi\left(0.036\frac{lb_m}{in^3}\right)(11.595in)(0.2)\left(\frac{1020.41\frac{lb_m}{s}}{7\pi(11.595in)^2}\right)^{0.68}(2.5)}$$

$$\implies \boxed{\boldsymbol{L = 229in}}$$

8b

c) Overall motor diameter

$$D = 6\big(R(100s)\big) = 6(16.379in)$$

$$\implies \boxed{\boldsymbol{D = 115.48in}}$$

8c

d) Percentage difference

$$\%diff\ Web = \frac{web_{SP} - web_{11.4}}{web_{SP}} * 100 = \frac{9.567-12.435}{9.567} * 100 \boxed{= \boldsymbol{-29.98\%}}$$

$$\%diff\ L = \frac{229-238.9}{229} * 100 \boxed{= \boldsymbol{-4.32\%}}$$

$$\%diff\ D = \frac{98.27-115.5}{98.27} * 100 \boxed{= \boldsymbol{-17.5\%}}$$

8d

e) Plots – Sample Calculations

$$O/F(t) = \frac{1}{2\rho_f La}\left(\frac{\dot{m}_{ox}}{\pi N}\right)^{1-n}\left\{a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi N}\right)^{n} t + R_i^{2n+1}\right\}^{\frac{2n-1}{2n+1}}$$

$$= \frac{1}{2\left(0.036\frac{lbm}{in^3}\right)(229in)(0.2)}\left(\frac{1020.41\frac{lbm}{s}}{7\pi}\right)^{1-0.68}\left\{0.2(2.36+1)\left(\frac{1020.41\frac{lbm}{s}}{7\pi}\right)^{0.68} t + (11.595in)^{2.36}\right\}^{0.1525}$$

$$O/F(t) = 1.035\{6.414t + 9.258\}^{0.1525}$$

$$O/F(0) = 1.035\{6.414 * 0 + 9.258\}^{0.1525} = 2.068$$

Check initial term

Mid-web timing

$$R(t) = \left\{a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi N}\right)^{n} t + R_i^{2n+1}\right\}^{\frac{1}{2n+1}}$$

$$\boldsymbol{t_{mid}} = \frac{R_{mid}^{2n+1} - R_i^{2n+1}}{a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi N}\right)^{n}} = \frac{11.595^{2.36} - 6.8118^{2.36}}{0.2(2.36)\left(\frac{1020.41\frac{lbm}{s}}{7\pi}\right)^{0.68}} = \boxed{\boldsymbol{36.21s}}$$

8e

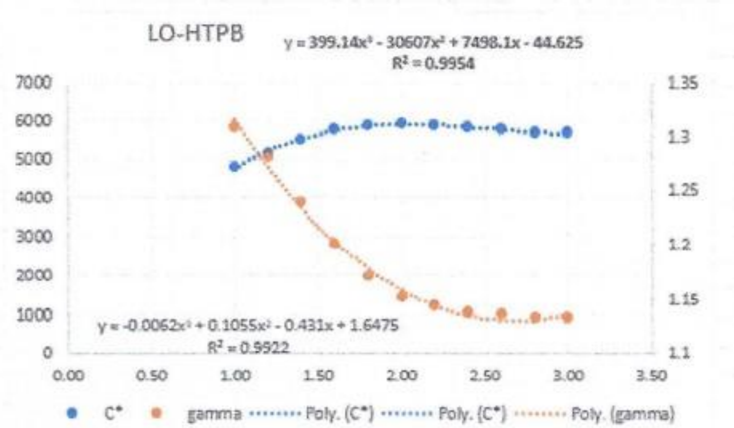
$$G_{ox} = \frac{\dot{m}_{ox}}{N\pi R(t)^2}, \quad \begin{cases} t = 0s & G_{ox} = 1\frac{lb_m}{in^2 s} \\ t = 36.21s & G_{ox} = 6.345\frac{lb_m}{in^2 s} \end{cases}$$

Equations for plots

$$c^* = \frac{P_c A_t}{\dot{m}_{ox} + \dot{m}_F}$$

$$P_c = \frac{c^*(O/F)(\dot{m}_{ox} + \dot{m}_F)}{A_t} = \frac{\left(\frac{ft}{s}\right)\left(\frac{lbm}{s}\right)}{in^2} * \frac{lb_f\, s^2}{32.2 lb_m\, ft} = \left[\frac{lb_f}{in^2}\right]$$

OF	C*	gamma
1.00	4825	1.308
1.20	5180	1.282
1.40	5543	1.239
1.60	5767	1.201
1.80	5882	1.171
2.00	5912	1.152
2.20	5885	1.143
2.40	5831	1.138
2.60	5768	1.135
2.80	5703	1.133
3.00	5693	1.132



Using 3rd-order fit of data provided,

$$c^* = 399.14(O/F)^3 - 3060.7(O/F)^2 + 7498.1(O/F) - 44.625$$

$$\gamma = -0.0062(O/F)^3 + 0.1055(O/F)^2 - 0.431(O/F) + 1.6475$$

Checkpoint at mid-web $c^*(2.5) = 6236.6 - 19129 + 18745 - 44.625 = 5807.813 ft/s$

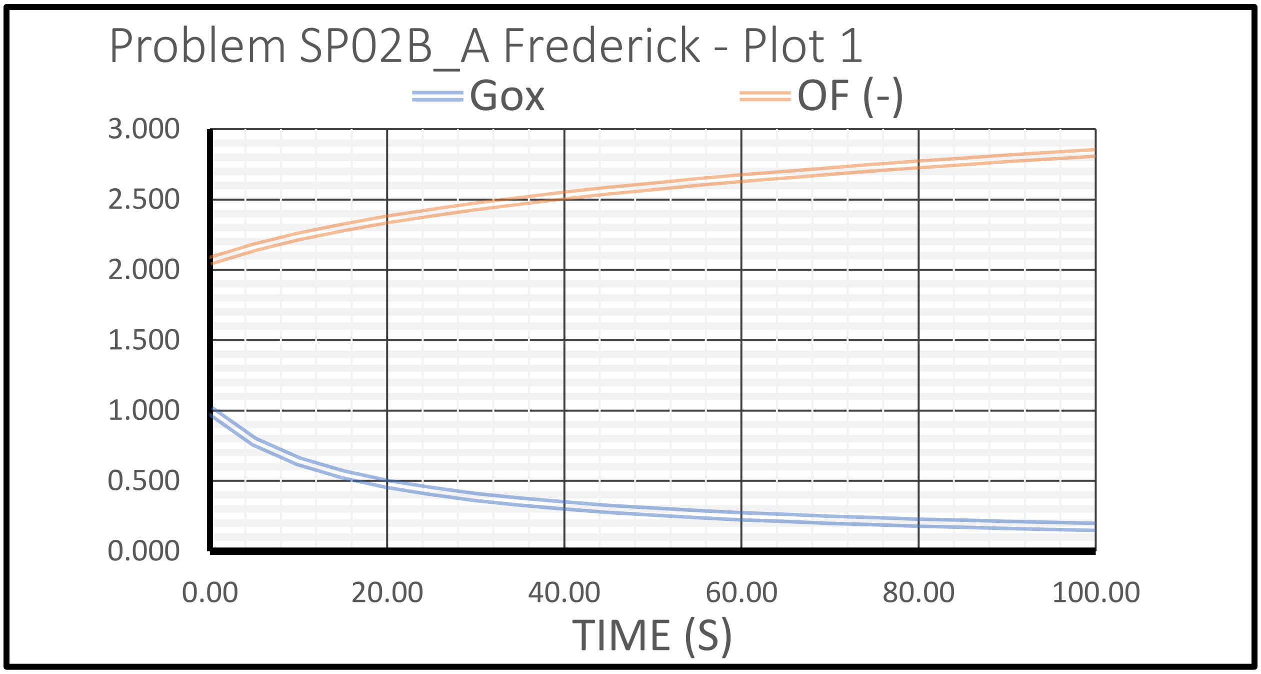
$$c^*(O/F = 2.064) = 5905.12 ft/s\ ,\ \dot{m}_T = 1514.7 lb_m/s$$

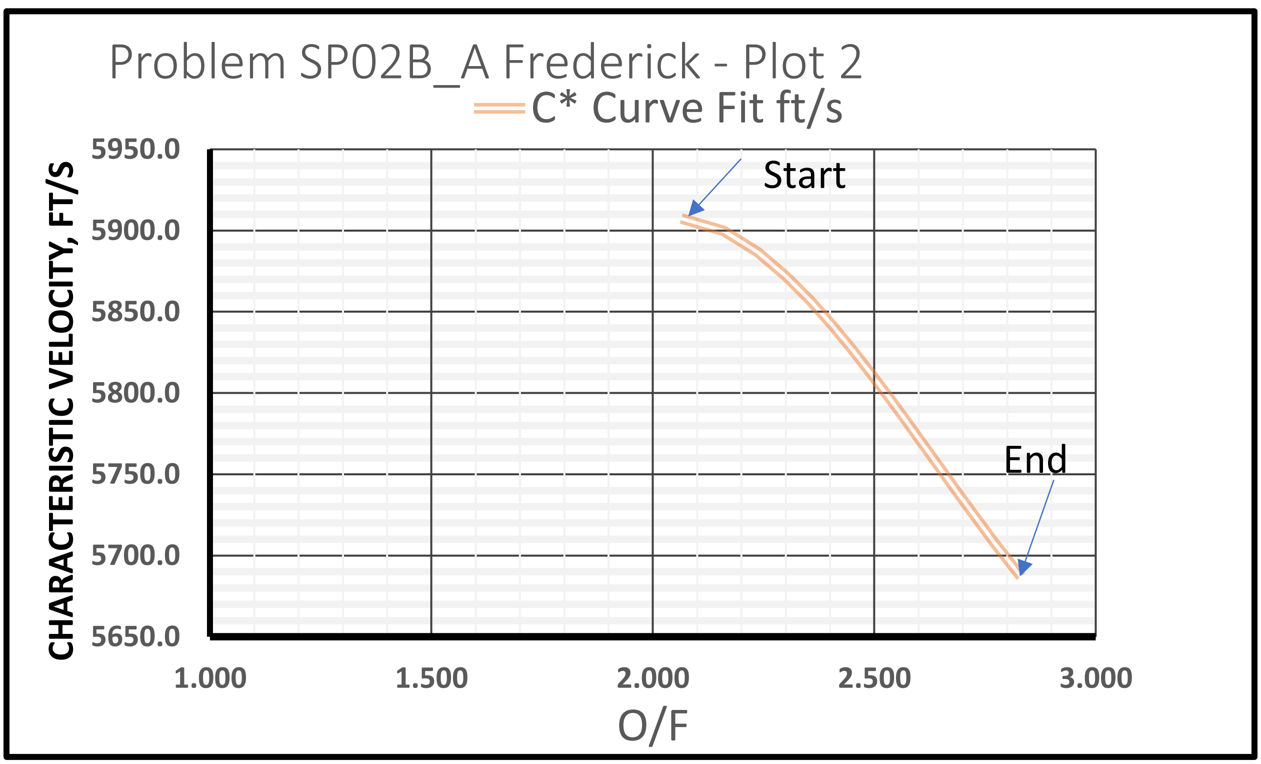
$$P_c = \frac{\left(5905\frac{ft}{s}\right)\left(1514.7\frac{lbm}{s}\right)}{300in^2} * \frac{lb_f\, s^2}{32.2 lb_m\, ft} = 925.8\frac{lb_f}{in^2}$$

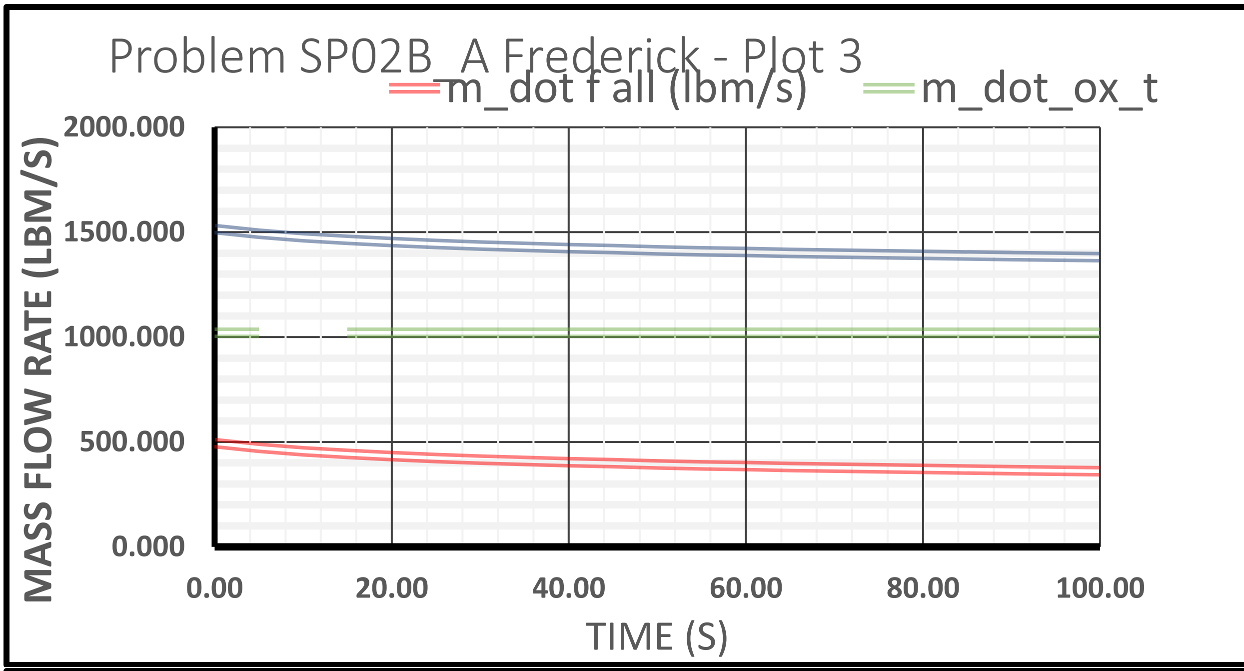
For thrust coefficient match $C_{Fvac} = 1.7$ @ $t = 0$

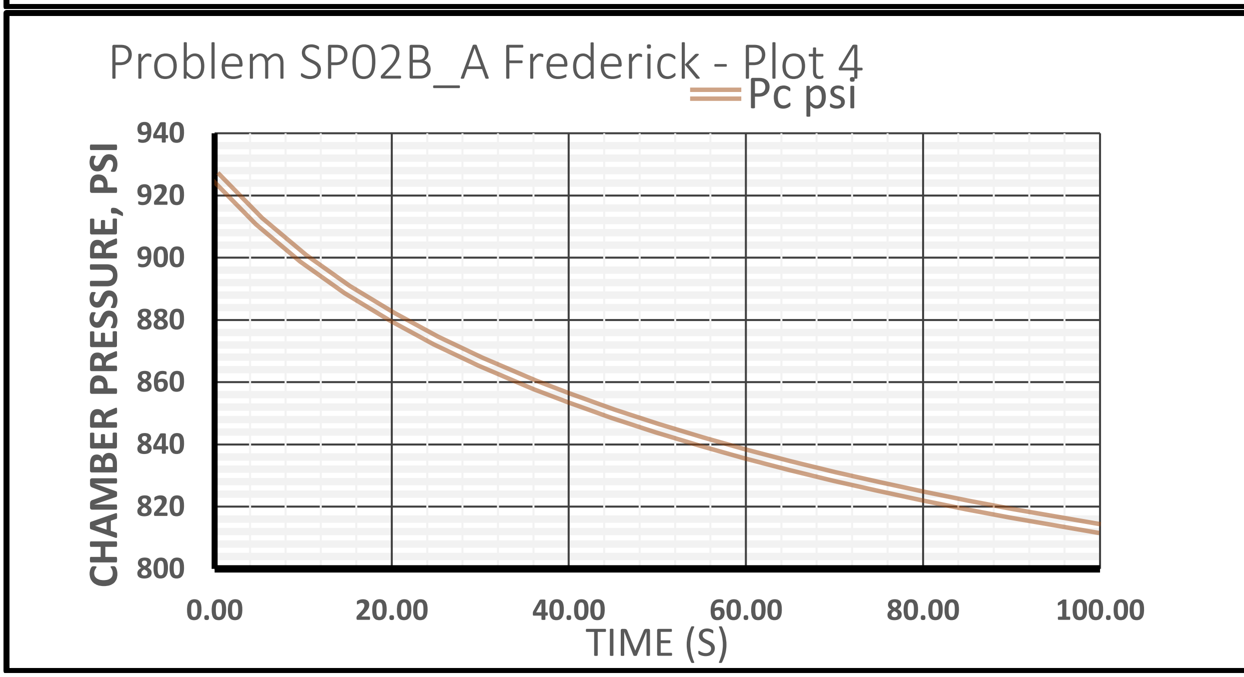
At $t = 0$, $\gamma = 1.15282$; using curve fit $C_{Fvac}(1.15282, \epsilon, 999{,}999) = 1.7$ iterate on ϵ to get $\epsilon = 6.24322$

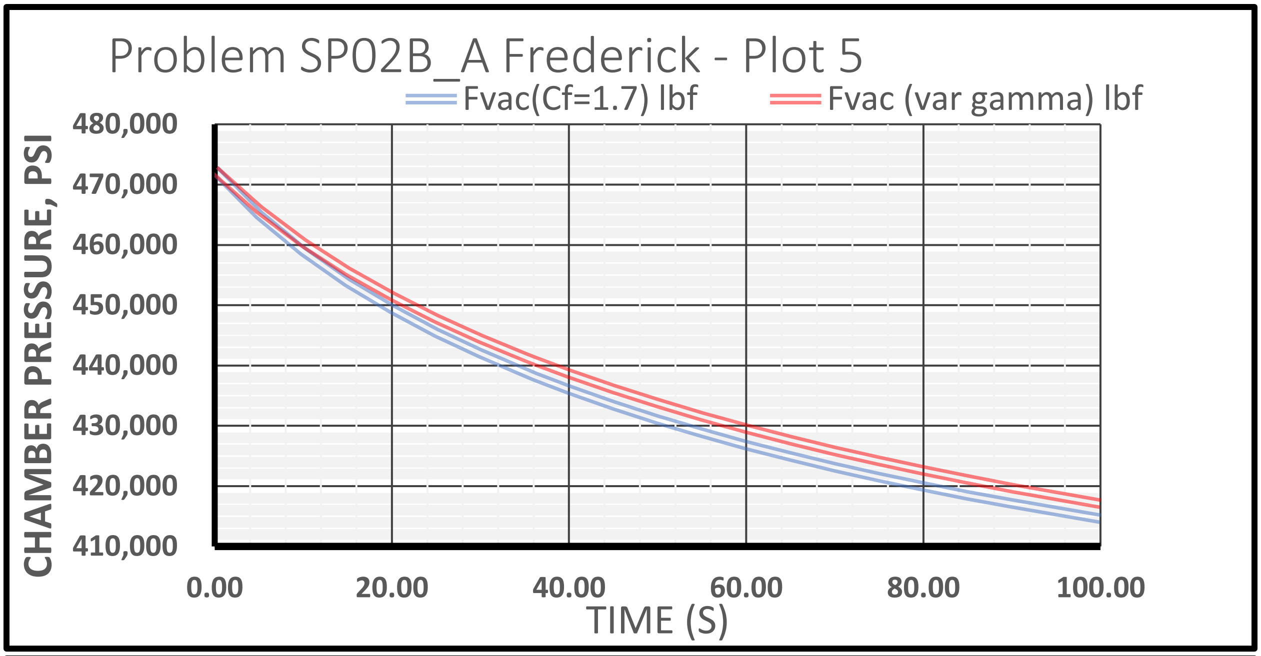
Use,
$C_{Fvac} = C_{Fvac}(\gamma)$
$C_{Fvac} = C_{Fvac}(O/F) = C_{Fvac}(\gamma, 6.224322{,}999{,}999)$

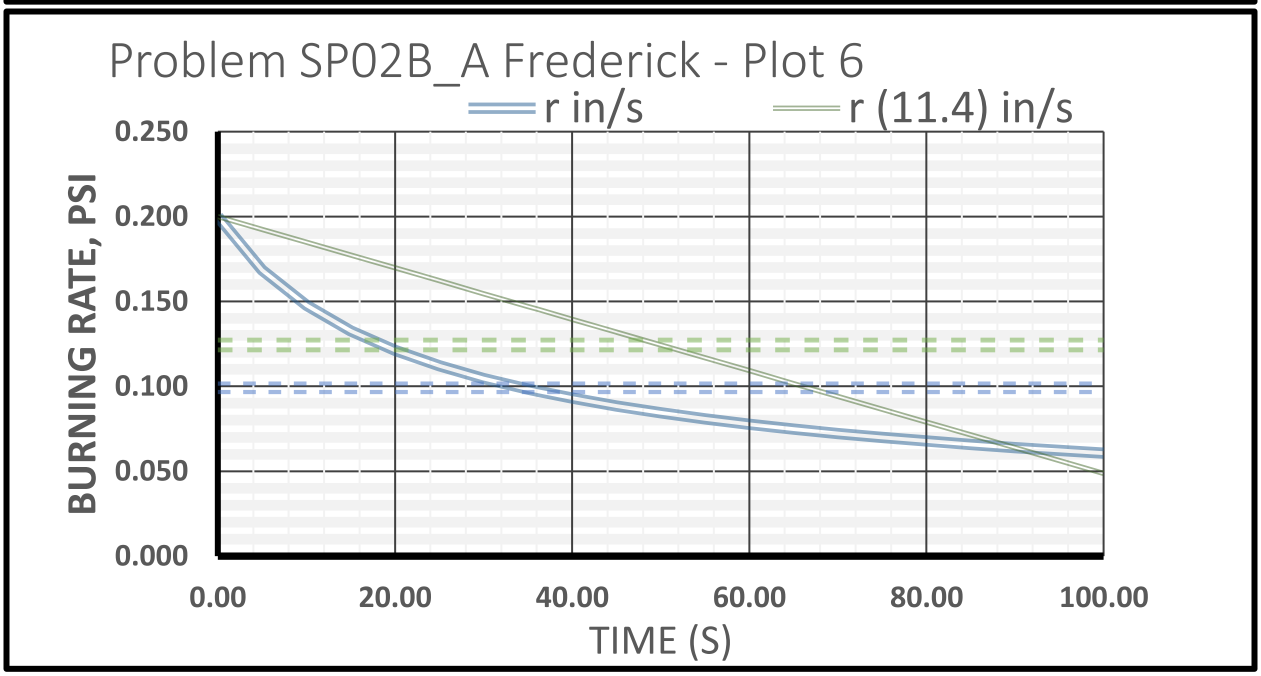





Problem SP02B_A Frederick - Plot 3
m_dot f all (lbm/s)
m_dot_ox_t
MASS FLOW RATE (LBM/S)
2000.000
1500.000
1000.000
500.000
0.000
0.00
20.00
40.00
60.00
80.00
100.00
TIME (S)


Problem SP02B_A Frederick - Plot 4
Pc psi
CHAMBER PRESSURE, PSI
940
920
900
880
860
840
820
800
0.00
20.00
40.00
60.00
80.00
100.00
TIME (S)





8. <u>Answers</u>: See above.

9. <u>Comments</u>: Small variations in γ with O/F ratio cause some changes in C_{Fvac}.

	Result	Result	% diff	Comment
	11-4	SP		
Dpi =	13.624	13.624	0%	Assume same intial oxidizer flow, number of ports and initial flux
w =	12.43505	9.57	-30%	the average burning rate assumtion leads to a higher buring rate
Mid Web O/F	2.5	2.5	0%	Assumed
L	238.9780007	229.1551886	-4%	The lower burning rate from SP leades to a smaller mid web diameter so a shorter length is needed to match the O/F
Dpi =	115.48	98.27	-18%	This is a consequeces of the high average burning rte
Fi=	500,000	472,419	-6%	This comes from the Isp and the C* of problem 11-4 resulting in an inrealistic thrust coefficient.