Name: Veronica Loomis

PROBLEM 11.3

Given:

Hybrid rocket (LOX/HTPB)

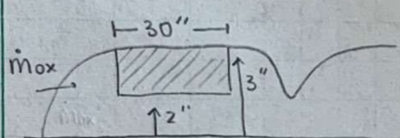
$c^* = -2520 + 6800\,(O/F) - 1320\,(O/F)^2 \qquad 2 < O/F < 3 \qquad [ft/s]$

$\rho_f = 0.0325\ lb/in^3$

$r = 0.16\, G_{ox}^{0.7}\ [in/s]$

Find: i) Optimal O/F for this propellant combination
ii) Oxidizer flow rate ($\dot{m}_{ox}$) which maximizes mid web performance
iii) Overall O/F shift for the firing, assuming fuel completely consumed

Schematic:



Assumptions:
Midweb => 2.5"
All of fuel is completely consumed

Basic Equations:

$O/F = \dot{m}_{ox}/\dot{m}_f$
$G_{ox} = \dot{m}_{ox}/(\pi R^2)$
$\dot{m}_f = r \rho_f A_b$
$A_b = 2\pi R L$

Analysis:
i) optimal O/F
derive c^* wrt O/F

call $\eta = O/F$

$$\frac{dc^*}{d\eta} = 6800 - 2(1320)\eta = 0$$

$$\eta = 2.575$$

i) Answer

$\boxed{\text{optimal } O/F = 2.575}$

ii) Oxidizer flow rate that maximizes performance @ midweb

Given $R_i = 2''$, $R_{end} = 3''$
so mid web $\rightarrow$ $R = 2.5''$

We know from (i) that $O/F = 2.575$
so
$\dot{m}_{ox} = 2.583\,\dot{m}_f$

$\dot{m}_f = r\rho_f A_b$
$= 0.16\left(\frac{\dot{m}_{ox}}{\pi R^2}\right)^n \rho_f\, 2\pi R L$

$= 0.16\left(\frac{\dot{m}_{ox}}{\pi(2.5)^2}\right)^{0.7} (0.0325)\, 2\pi(2.5)(30)$

$\dot{m}_f = 0.304876\,\dot{m}_{ox}^{0.7}$

Sub back in

$\dot{m}_{ox} = 2.575(0.304876)\,\dot{m}_{ox}^{0.7}$

ii) Answer

$\boxed{\dot{m}_{ox} = 0.44678 \text{ lb/s}}$

iii) Overall O/F shift

$\dot{m}_{ox} = 0.44678$ lb/s $\rightarrow$ max

min at either $R=2$, $R=3$

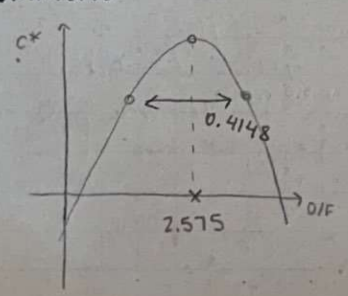
$\dot{m}_f @ R=2 \Rightarrow 0.16\left(\frac{0.44678}{\pi(2)^2}\right)^{0.7}(0.0325)\,2\pi(2)(30) = 0.18965 \rightarrow (O/F)_{min}$

$\dot{m}_f @ R=3 \Rightarrow 0.16\left(\frac{0.44678}{\pi(3)^2}\right)^{0.7}(0.0325)\,2\pi(3)(30) = 0.161256 \rightarrow (O/F)_{max}$

$(O/F)_{min} = 2.3558$, $(O/F)_{max} = 2.7706$ these are both $2 < O/F < 3$ ✓

$\boxed{\text{overall O/F shift} = 0.4148}$ iii) Answer

Comment:



Name: Veronica Loomis

Problem 11.7

Given:

Hybrid rocket, obeys $r = a G_{ox}^n$

$r = dR/dt$

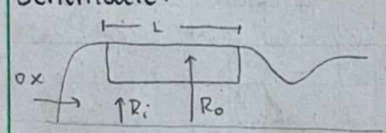
Find: i) Show $R(t) = \left[a(2n+1) \left(\frac{\dot{m}_{ox}}{\pi} \right)^n t + R_i^{2n+1} \right]^{1/(2n+1)}$

ii) Derive expressions for $\dot{m}_f$ and O/F

Is there a special value of n where fuel flow is const & no mixture ratio shifts?

iii) Plot $R(t)$, $\dot{m}_f(t)$, and $O/F(t)$ with the given conditions

Schematic:



Assumptions:

$\dot{m}_{ox}$ is constant

fuel regression rate is uniform

Basic Equations:

$$G_{ox} = \frac{\dot{m}_{ox}}{\pi R^2}$$

$$\dot{m}_f = r \rho_f A_b$$

$$A_b = 2 \pi R L$$

$$O/F = \dot{m}_{ox} / \dot{m}_f$$

Analysis:

i) $r = \frac{dR}{dt} = a\,G_{ox}^{n} = a\left(\frac{\dot{m}_{ox}}{\pi R^2}\right)^n$

$\int R^{2n}\, dR = \int a\,\dot{m}_{ox}^{n}\,\pi^{-n}\, dt$

$\frac{1}{2n+1}\left[R(t)\right]^{2n+1} = a\,\dot{m}_{ox}^{n}\,\pi^{-n} t + R_i^{2n+1}$

$$\boxed{R(t) = \left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right]^{1/2n+1}}$$ i) Answer

ii $\dot{m}_f$

$\dot{m}_f = r\rho_f A_b = a\left(\frac{\dot{m}_{ox}}{\pi R^2}\right)^n \rho_f\, 2\pi R L$

$\dot{m}_f = 2a\rho_f L\,\dot{m}_{ox}^{n}\,\pi^{1-n}\,R^{1-2n}$

Substitute R(t) from (i)

$$\boxed{\dot{m}_f = 2a\rho_f L\,\dot{m}_{ox}^{n}\,\pi^{1-n}\left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right]^{\frac{1-2n}{1+2n}}}$$ ii A) Answer

when $n = 1/2$, fuel flow is constant wrt time

O/F

$O/F = \dot{m}_{ox}/\dot{m}_f$

$$\boxed{O/F = \frac{\dot{m}_{ox}^{1-n}}{2a\rho_f L\,\pi^{1-n}}\left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right]^{\frac{2n-1}{2n+1}}}$$ ii B) Answer

when $n = 1/2$, there is no mixture ratio shift

iii) $L = 50''$, $R_i = 2''$, $R_o = 5''$, $\rho_f = 1\,g/cm^3 = 0.0361273\,lb/in^3$, $a = 0.1$, $n = 0.8$

G_{ox} initial $= 1.0\ lb/in^2 \cdot s$

We assumed $\dot{m}_{ox}$ was const so

$G_{ox} = \dot{m}_{ox}/\pi R$ @ initial values

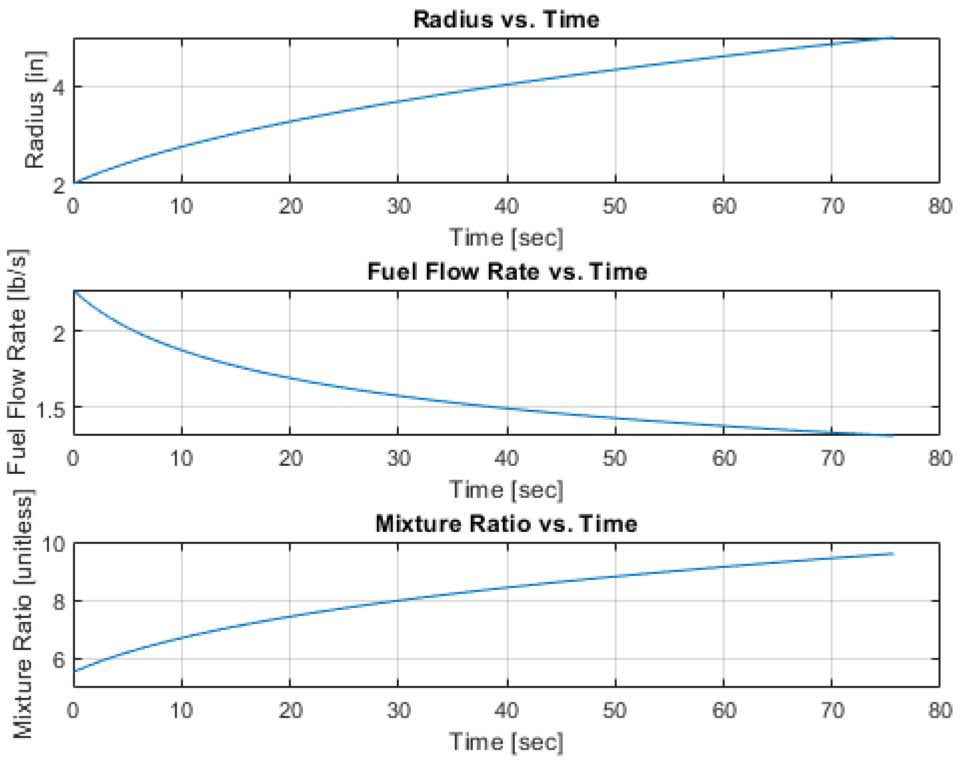
$\dot{m}_{ox} = G_{ox,init}\,\pi R_o$

Script & plots included next.

Veronica Loomis

HW02 - Book Problem 11.7 Part iii Using the values given Plot: R(t) vs. time mFdot(t) vs. time O/F(t) vs. time

```
L = 50;
Ri = 2;
Ro = 5;
rhoF = 0.0361273; % lb/in**3
a = 0.1;
n = 0.8;
Gox = 1.0; % lb/s-in**2
% since we are told mOxdot is constant,
% we can define it from the initial G and R values
mOxdot = Gox*pi*Ri^2;

allTime = linspace(0,100,1000);

i = 1;
testR = Ri;

while testR < 5
    R(i) = (((allTime(i)*a*(2*n+1))*mOxdot^n/(pi^n))+Ri^(2*n+1))^(1/
(2*n+1));
    mFdot(i) = 2*a*rhoF*L*mOxdot^n*pi^(1-n)*R(i)^(1-2*n);
    OF(i) = mOxdot/mFdot(i);
    testR = R(i);
    i = i+1;
end

t = allTime(1:i-1);

figure(1)
subplot(3,1,1)
plot(t,R)
title('Radius vs. Time')
grid on
xlabel('Time [sec]')
ylabel('Radius [in]')
subplot(3,1,2)
plot(t,mFdot)
title('Fuel Flow Rate vs. Time')
grid on
xlabel('Time [sec]')
ylabel('Fuel Flow Rate [lb/s]')
subplot(3,1,3)
plot(t,OF)
title('Mixture Ratio vs. Time')
grid on
xlabel('Time [sec]')
ylabel('Mixture Ratio [unitless]')
```



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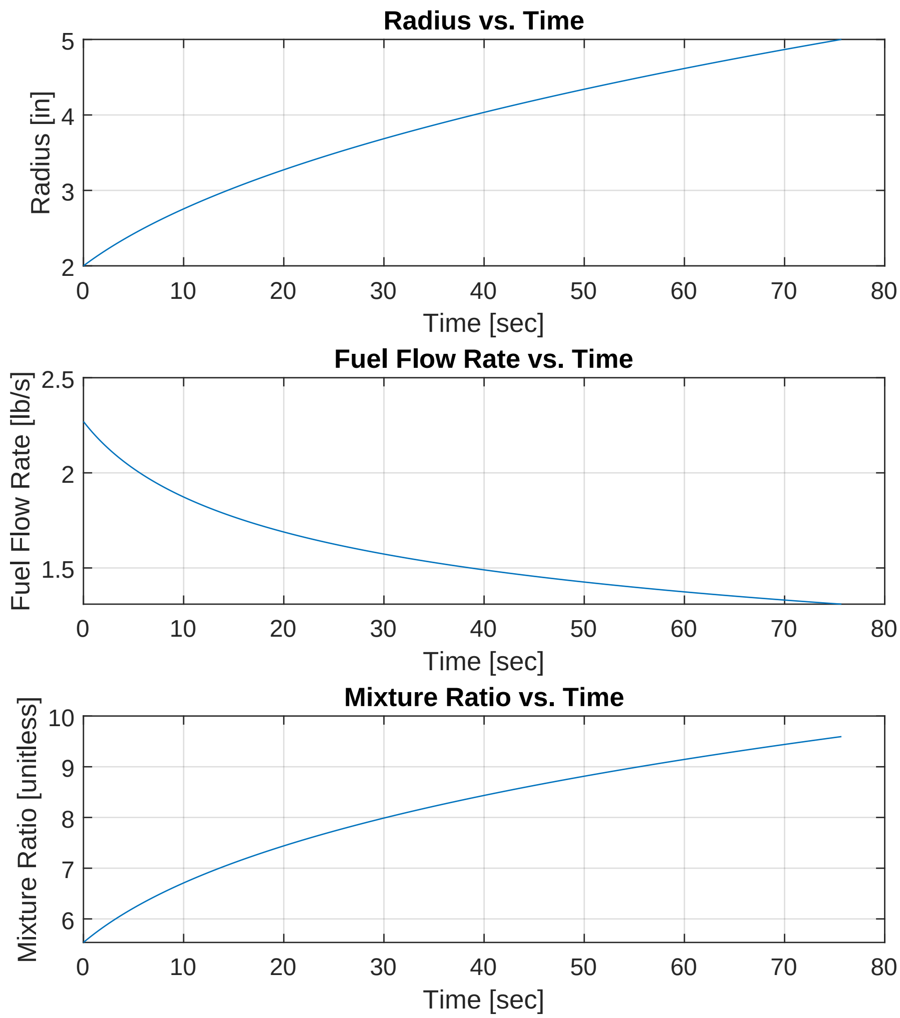


Radius vs. Time
Radius [in]
Time [sec]
Fuel Flow Rate vs. Time
Fuel Flow Rate [lb/s]
Time [sec]
Mixture Ratio vs. Time
Mixture Ratio [unitless]
Time [sec]

Name: Veronica Loomis

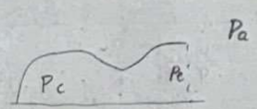
SP 02-HW SPA

Given: CF spreadsheet

Find: Include calculated separation line for $P_e/P_a = 1/3$
" for Normal Shock at exit

Schematic

Assumptions
Isentropic flow



Basic Equations:

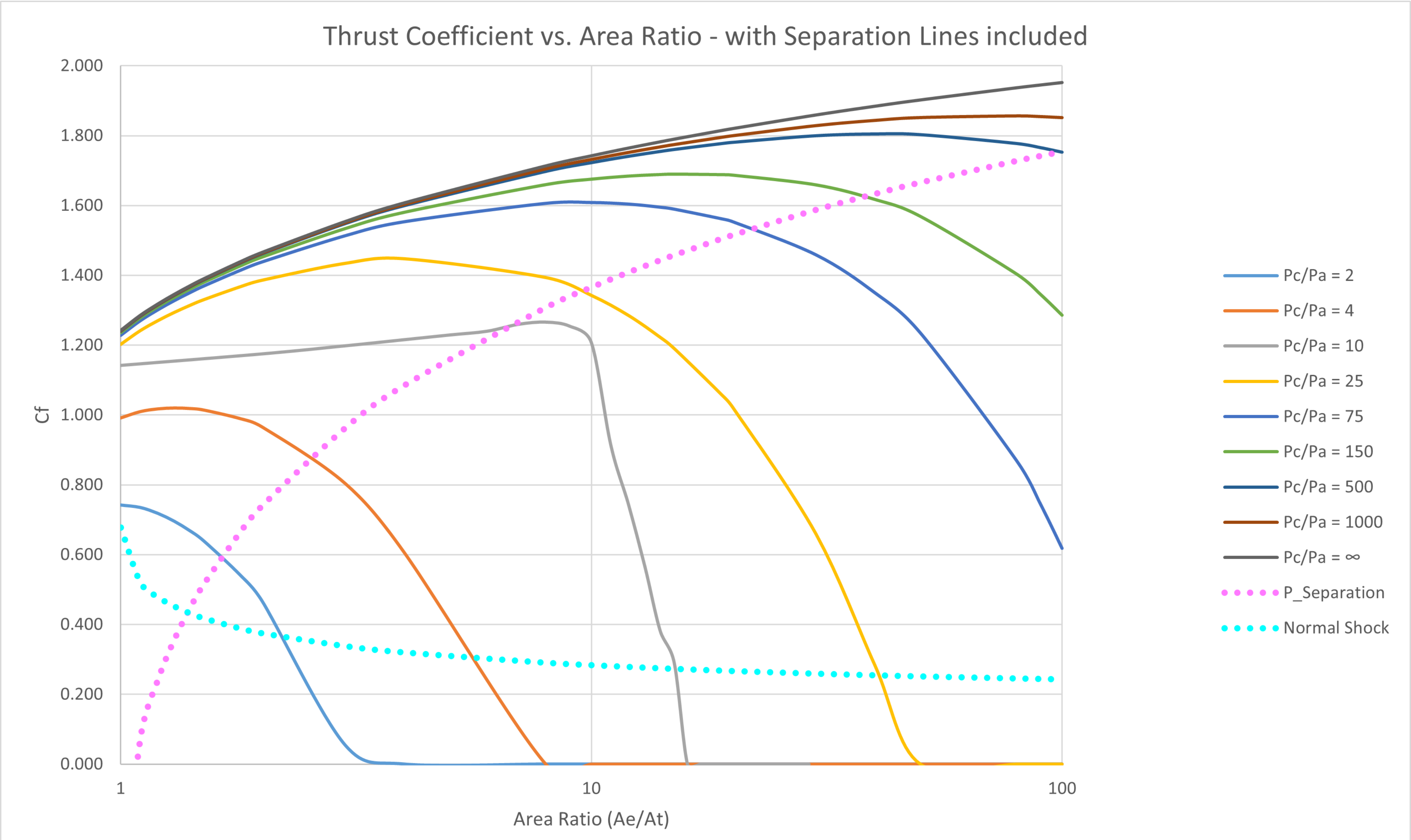
$$P_c/P_e\big|_{isentropic} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\gamma/(\gamma-1)}$$

$$P_e/P_a\big|_{sep\ 1/3} = 1/3$$

$$P_e/P_a\big|_{normal\ shock} = \left[\frac{2\gamma}{\gamma+1} M_e^2 - \frac{\gamma-1}{\gamma+1}\right]^{-1}$$

$$P_c/P_a = (P_c/P_e)(P_e/P_a)$$

Analysis, Answer, & Comparison are on the attached excel file.
The figure is the next page in this PDF.



Thrust Coefficient vs. Area Ratio - with Separation Lines included
2.000
1.800
1.600
1.400
1.200
1.000
0.800
0.600
0.400
0.200
0.000
Cf
1
10
100
Area Ratio (Ae/At)
Pc/Pa = 2
Pc/Pa = 4
Pc/Pa = 10
Pc/Pa = 25
Pc/Pa = 75
Pc/Pa = 150
Pc/Pa = 500
Pc/Pa = 1000
Pc/Pa = ∞
P_Separation
Normal Shock