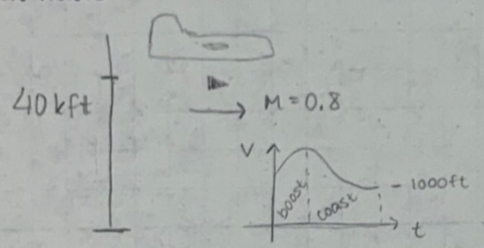
Name: Veronica Loomis

Problem 2.6

Given: Small interceptor launched horizontally
at $M = 0.8$
at $h = 40$ kft
$V = V_0(1 + 2\sin(\frac{\pi t}{2}))$ - missile vel history
$V_0 = V$ @ time of release
Missile is still able to intercept if $V \geq 1000$ ft/s
Neglect drag in BP
During coast:
$m_{missile} = 300$ lb $= 9.375$ slugs
average $C_D = 0.2$
$A_{ref} = 50\ in^2$

Find: (a) Range at the end of boost phase
(b) Total missile range

Schematic:

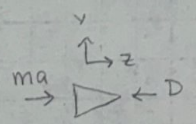


Assumptions:
Find ρ and a (sos) using atm table
↳ Purdue propulsion website

Basic Equations:

(a) $z = \int_0^{t_b} v(t)\,dt$

(b) $\sum F_z = ma = m\frac{dv}{dt} = -\frac{1}{2}\rho v^2 C_D A_{ref}$



Analysis:

(a) $V_0 = M \times a = 0.8 \times 968.076 = 774.46$ ft/s

$z = \int_0^{t_b} V_0(1 + 2\sin(\frac{\pi t}{2}))\,dt$

$z = 774.46\{t - \frac{4}{\pi}\cos(\frac{\pi t}{2})\}\Big|_0^1$

$z = 774.46(1 + \frac{4}{\pi})$

$z = 1760.533$ ft

(a) Answer

$\boxed{\text{Range at end of boost: } 1760.53 \text{ ft}}$

(b) $\sum F_z = ma = m\frac{dv}{dt} = -\frac{1}{2}\rho v^2 C_D A_{ref}$

$m\frac{dv}{dz}\frac{dz}{dt} = m\cancel{v}\frac{dv}{dz} = -\frac{1}{2}\rho v^{\cancel{2}} C_D A_{ref}$

$\int dz = \int_{V_0}^{V_f} \frac{-2m}{\rho v C_D A_{ref}}\,dv$

$z = \frac{-2m}{\rho C_D A_{ref}} \ln(v)\Big|_{V_i}^{1000}$

$V_i = V_0(1+2)$
$= 2323.38$

$z = \frac{-2(9.375)}{(0.000585)(0.2)(50/144)} \ln\left(\frac{1000}{2323.38}\right)$

$z = 389087.5$ ft

Answer

(b) $\boxed{\text{Total missile range: } 389087.5 \text{ ft or } 73.69 \text{ miles}}$

Comment: It makes sense that the BP range is MUCH smaller than coast range since $t_b = 1$

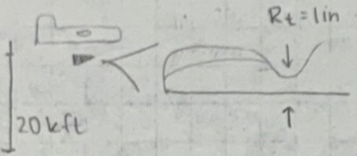
Name: Veronica Loomis

Problem 3.8

Given: Air launched missile
Operates at an altitude of 20 kft
Solid rocket propellant
$c^* = 5000$ ft/s
$\gamma = 1.2$
Nozzle expansion ratio limited to 30
$m_p = 250$ lb
$D_t = 2$ inch

Find: (a) Chamber pressure if $P_{sep} = \frac{1}{3} P_a$
(b) Assuming P_c from (a) is const. at 20 kft, find motor thrust & burn time
(c) Find sea level thrust & burning time using P_c from (a)

Schematic:



Assumptions:
$P_e \neq P_a$
$\varepsilon = 30$
Use Purdue atm data

Basic Equations:
(a) $P_{e,sep} = \frac{1}{3} P_a$ & tables
(b, c) $F = C_v A_t P_c$

Analysis:
(a) $P_{e,sep} = \frac{1}{3} P_a$ @ 20 kft → $P_a = 6.76$ psi (Purdue table)
$P_{e,sep} = 2.25$ psi $= 324.558$ lbf/ft^2
Since we know $\varepsilon = 30$
Use table to find
$\varepsilon = 30 \Rightarrow P_e/P_c = 0.0030119$
$\therefore P_c = P_e / 0.0030119$
$P_c = 107758.56$ lbf/ft^2

a) chamber pressure is
748.32 lbf/in^2

b) $F = (C_{F,vac} - (P_a/P_c)\varepsilon) A_t P_c$
From (a): $P_c = 748.32$ lbf/in^2
$A_t = \pi D_t^2 / 4 = \pi$ in^2
$\varepsilon = 30$
$P_a = 6.76$ lbf/in^2
Using Purdue table: $C_{F,vac}(@\gamma = 1.2, \varepsilon = 30) = 1.85928$

$\therefore F = [1.85928 - (0.009033569)30]\pi(748.32)$
$F = 3733.897$ lbf

@ 20 kft, the motor thrust is 3733.897 lbf
and the burn time is 16.5 seconds

Finding burn time:
$$t_b = \frac{m_p c^*}{A_t g_e P_c}$$
ft → in
$$t_b = \frac{(250)(5000)(12)}{\pi(32.2)(12)(748.32)}$$
$t_b = 16.51255$ sec

(c) redo (b) but now at sea level
use $P_c = 748.32\ lbf/in^2$

Pa at sea level: $14.696\ lbf/in^2$
Which gives $P_{e,sep} = 4.899\ lbf/in^2$
So $P_e/P_c = 0.0065462$
Gives a new $\varepsilon = 16.437$

$F = (C_{F vac} - (P_a/P_c)\varepsilon)A_t P_c$
$C_{F vac}(\gamma = 1.2, \varepsilon = 16.437) = 1.8$

$$F = \left[1.8 - \left(\frac{14.696}{748.32}\right)16.437\right]\pi(748.32)$$

$F = 3472.77\ lbf$

@ Sea level, the thrust is 3472.77 lbf
and burn time is still 16.5 seconds

Burn time

$$t_b = \frac{m_p c^*}{A_t g_e P_c}$$

Since none of these variables changed, we know it's the same tb we see in (b)

Answers:

a) chamber pressure is $748.32\ lbf/in^2$
b) at 20 kft, thrust is 3733.897 lbf burn time is 16.5 seconds
c) at sea level, thrust is 3472.77 lbf burn time is still 16.5 seconds

Comment:
Since this is using solid rocket propellant, it makes sense that the burn times stay the same.
It also makes sense that more thrust is needed at sea level since the air is denser

Scanned with CamScanner

Name: Veronica Loomis

Problem 4.24

Given: Hybrid Rocket

Maintain constant flow rate: $\dot{m}_{ox} = 10$ lb/s (lbm?)

Presume fuel flow given by: $\dot{m}_f = 3 - 0.6(t/t_b)$ $[lb_f/s]$

$t_b = 50$ sec

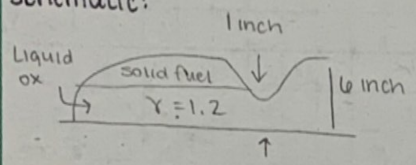
$c^* = 4800 + 800(t/t_b) - 800(t/t_b)^2$ [ft/s]

Find: (a) Initial thrust and Isp of the engine

(b) Expressions for thrust and Isp for any time t $(t \le t_b)$

(c) Maximum Isp delivered during firing. When does max occur?

Schematic:



Assumptions:

Ideal performance

Engine fired in vacuum

Use purdue tables for $\gamma = 1.2$

Basic Equations:

$I_{sp} = c^* C_F / g_e$

$F = C_F A_t P_c$

$\varepsilon = A_e / A_t$

Analysis:

(a) $\varepsilon = \pi(6)^2/\pi(1)^2 = 36$

using table $(\varepsilon = 36, \gamma = 1.2)$ we get

$C_{F vac} = 1.87535$

$P_c(t=0) = \dfrac{c^* \dot{m}}{A_t} = \dfrac{4800(3+10)}{\pi(1/12)^2} \times \dfrac{1}{32.2} = 88826.25\ lbf/ft^2$ or $616.8\ lbf/in^2$ $\left[\dfrac{ft/s \quad lbf/s}{ft}\right] \times \dfrac{lbf\, s^2}{32.2\ lbm\ ft}$

$F = [C_{F vac}] A_t P_c$

$F = 1.87535(\pi)(616.8)$

$F = 3633.9$ lbf at $t = 0$

$I_{sp} = \dfrac{c^* C_{F,vac}}{g_e} = \dfrac{(4800)(1.87535)}{32.2}$

$I_{sp} = 279.555$ sec

(b) $C_{Fvac} = 1.87535$

$A_t = \pi \text{ in}^2$

$$P_c = \frac{\left(4800 + 800\, t/t_b - 800(t/t_b)^2\right)\overbrace{\left(13 - 0.6(t/t_b)\right)}^{\dot{m}_{ox} + \dot{m}_f}}{\pi}$$

$$P_c = \frac{1}{\pi}\left(62400 - 7520(t/t_b) - 10880(t/t_b)^2 + 480(t/t_b)^3\right) \times \frac{1}{32.2}$$

$$F = C_{Fvac}(A_t)(P_c)$$

$$F = 0.05824\left[62400 - 7520(t/t_b) - 10880(t/t_b)^2 + 480(t/t_b)^3\right]$$

$$I_{sp} = c^* C_{Fvac} / g_c$$

$$I_{sp} = 0.05824\left[4800 + 800(t/t_b) - 800(t/t_b)^2\right] \quad \text{sec}$$

(c) max I_{sp} => max c^* since C_{Fvac} & g_c constant

$$\frac{dc^*}{dt} = 800 - 1600(t/t_b) = 0$$

$t/t_b = 1/2$ We're given $t_b = 50$

Max I_{sp} occurs at 25 seconds

$$I_{sp}(t=25) = 0.05824\left[4800 + 800(1/2) - 800(1/4)\right]$$

Max I_{sp} = 291.2 sec

Answers:

a) $F_{initial} = 3633.9$ lbf
$I_{sp,initial} = 280$ seconds

b) $F(t) = 3634.2 - 437.96(t/t_b) - 633.65(t/t_b)^2 + 27.955(t/t_b)^3$ lbf
$I_{sp}(t) = 279.552 + 46.592(t/t_b) - 46.592(t/t_b)^2$ sec

c) Max I_{sp} = 291.2 seconds
occurs at $t = 25$ seconds

Comment:

If you look at the answer for (b) you see that setting $t=0$ gives you the answers from (a)

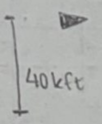
↳ They did not at first. I noticed that was wrong when checking my work

Name: Veronica Loomis
Problem 4:30

Given: Air launched missile @ $h = 40$ kft
$P_c = 500$ psi
$\gamma = 1.3$
$c^* = 4800$ ft/s
desired thrust = 2000 lb

Find: (a) Nozzle throat and exit diameter (D_t, D_e)
(b) Nozzle mass flow
(c) Isp of rocket engine
(d) Thrust coefficient for the nozzle
(e) Repeat a-d for operating at sea level

Schematic:



Assumptions:
Optimum expansion @ nozzle ($P_e = P_a$)
same nozzle in (e)
$\frac{1}{3}P_a = P_{e,sep}$
Use Purdue atm charts

Basic Equations:
$\dot{m} = P_c A_t / c^*$
$I_{sp} = c^* C_f / g_e$
$C_F = C_{F vac} - (P_a/P_c)\varepsilon$

Analysis:
(a) P_a at 40 kft (Purdue) $= 2.73$ lbf/in^2
$P_e = P_a = 2.73$ lb/in^2
$P_e/P_c = 0.00546$
Using Purdue table ($\gamma = 1.3$)
$\varepsilon = 14.92$, $C_{F,vac} = 1.7243871$
$F = C_F A_t P_c$
$A_t = F/(C_{F,vac} - (P_a/P_c)\varepsilon)(P_c)$
$A_t = 2000/(1.724 - (0.00546)(14.92))(500)$ [in^2]
$A_t = 2.435$ in^2
$\varepsilon = A_e/A_t$
$A_e = 14.92(2.435)$
$A_e = 36.334$ in^2

$D_t = 1.76$ in, $D_e = 6.8$ in

b) $\dot{m} = P_c A_t / c^*$

conv. to inches

$$\dot{m} = \left[500(2.435)/(4800)(\overset{ft \to in}{12})\right] \times 32 \times 12 \left[\frac{lbf}{in^2}\ in^2\ \frac{s}{in}\right]\ lbf \cdot s/in \times \frac{32.2\ lbm\ ft}{lbf\ s^2}$$

$\dot{m} = 8.1167\ lbm/s$

c) $I_{sp} = c^* C_F / g_e$

$I_{sp} = 4800 \times (1.724 - 0.00546(14.92))/32.2$

$I_{sp} = 244.85$ sec

d) $C_F = C_{F vac} - (P_a/P_c)\varepsilon$

$C_F = 1.724 - 0.00546(14.92)$

$C_F = 1.6425$

e) i) P_a @ SL $= 14.696\ lbf/in^2$

at low alt, $P_{e,sep} = 1/3 P_a = 4.89867$

$P_e/P_c = 0.009797$

Table ($\gamma = 1.3$)

$\varepsilon = 9.84$, $C_{F,vac} = 1.687$

$A_t = F/(C_{F,vac} - (P_a/P_c)\varepsilon)(P_c)$

$A_t = 2000/(1.687 - (0.02939)(9.84))(500)$

$A_t = 2.8616\ in^2$

$\varepsilon = A_e/A_t$

$A_e = (9.84)(2.8616)$

$A_e = 28.1585$

$D_t = 1.91$ in, $D_e = 5.9877$ in

ii) $\dot{m} = P_c A_t / c^*$

$\dot{m} = (500(2.8616)/(4800)(12))(32)(12)$

$\dot{m} = 9.539\ lbm/s$

iii) $I_{sp} = c^* C_F / g_e = 4800(1.687 - (0.02939)(9.84))/32.2$

$I_{sp} = 208.3677$ sec

iv) $C_F = 1.687 - (0.02939)(9.84)$

$C_F = 1.3978$

Answers: a) $D_t = 1.76$ in, $D_e = 6.8$ in

b) $\dot{m} = 8.1167$ lbm/s

c) $I_{sp} = 244.85$ sec

d) $C_F = 1.6425$

e) i) $D_t = 1.91$ in, $D_e = 5.9877$ in

ii) $\dot{m} = 9.539$ lbm/s

iii) $I_{sp} = 208.3677$ sec

iv) $C_F = 1.3978$

Comments:

In order to reach $F = 2000$ lb with $P_c = 500$ lbf/in^2

the throat at sea level > throat for altitude of 40kft

When looking at ambient pressures compared to P_c,

$$\varepsilon_{40kft} = 14.92 > \varepsilon_{SL} = 9.84$$

since as $P_e \downarrow$, $\varepsilon \uparrow$

This is why $D_{e,40kft} > D_{e,SL}$

Two-Page Annotated Bibliography
(MAE 640)
Summarize

Reference Document Examined:	Frederick, R. A., Berg, P., and Loeblich, W., "Using CEQUEL for Thermochemistry Calculations in a Graduate Rocket Propulsion Course at UAH," UAH Propulsion Research Center
Reviewer:	Veronica Loomis
Source of Document:	canvas.uah.edu
Date of Review:	January 18, 2023
Electronic File Name:	HW01_PaperReview

Summary of Paper:

This paper explains how to use code for a solid-fuel ramjet design project which utilizes a spreadsheet and python code. This code (Chemical Equilibrium in Excel – CEQUEL) includes calculations for thermochemical properties, internal ballistics, and flight performance characteristics. It demonstrates this in both a spreadsheet and the python-based implementation. This saves time by eliminating the need to look up data in tables or try and fit data on parametric curves.

CEQUEL is available for Microsoft Excel to perform thermochemistry calculations. The user can use it to perform calculations of things such as temperature, enthalpy, entropy, pressure, rocket area ratio, and so on. This is done using either the Equilibrium Wizard or the Isobaric Mixing Wizard. These contain libraries of over 100 reactants to test.

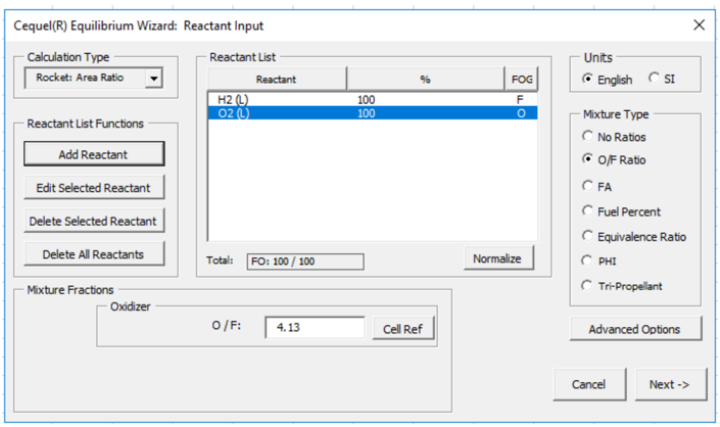


Figure 1: Equilibrium Wizard - Reactant Input Example

CEQUEL is also available via python. It is suggested that one is familiar with the Excel version first, so it is clear what potential options are able to be chosen and in what format. A way to test the usefulness of CEQUEL was by recreating a figure from the textbook which detailed the performance of rocket propellant combinations. The results were almost identical except for a few cases of ~10% differences.

One big project was the evaluation of three missions: all-solids using two center-perforated grains, a baseline mission to work out bugs, and a design mission where each team did studies on the fuel candidates provided and changed input parameters within the guidelines to find a maximum range. The results concluded that the advantages of all-solids is the compact size, max

Mach number, and lower time to target compared to other missions. It was also found that the advantage of a ramjet is that the range increased to almost double that of the baseline mission.

B. Assess:

Important Facts from Document:

1. CEQUEL is a very helpful tool when trying to calculate thermochemical properties.
2. CEQUEL is available in both Excel and Python and it is recommended that one is comfortable with the Excel version first.
3. When comparing data from both versions CEQUEL, the results are very close to identical with a few exceptions for scaling issues.
4. The advantages for all-solid propellant include its compact size, maximum Mach number, and time to target.
5. One of the advantages of ramjets is the increase in range during a mission.

Key Figure from Document:

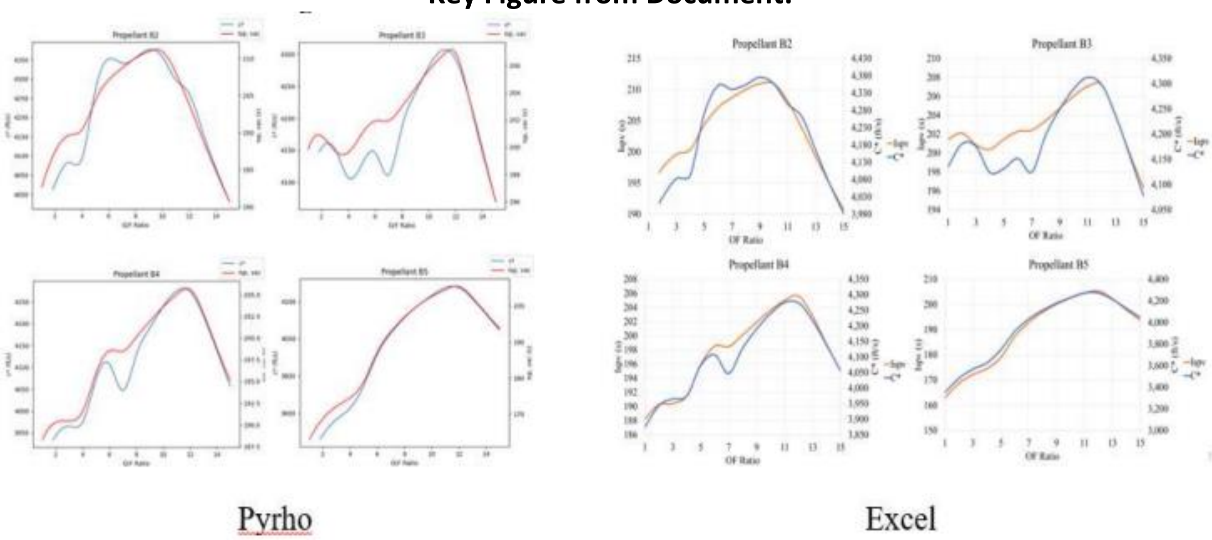


Figure 2: The results from the Python CEQUEL (left) compared to the Excel CEQUEL (right)

Important Relationships among Parameters Described in the Paper:

1. An increase in c* leads to an increase in specific impulse in a vacuum
2. The thrust of the rocket increases as the coefficient of thrust increases (which is when the nozzle area ratio decreases)

$$C_{f,i} = C_{fv,i} - \frac{P_a}{P_{c,i}}\varepsilon_i$$

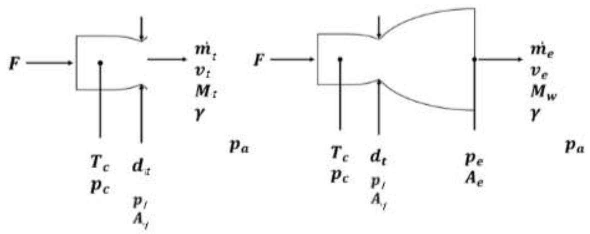
C. Reflect

This paper was very helpful in getting a basic understanding of CEQUEL and what it can be used for. It lays out the different programs it can be used in as well as demonstrations of problems it has worked through. This will be helpful in this class to do the more difficult and rigorous calculations.

HW1 SP01B

Monday, January 16, 2023 20:13

1. **Name:** Veronica Loomis
2. **Given:** Rocket motor with a converging nozzle, and then a second configuration adding a diverging nozzle.
3. **Find:**
   a. Starting with the complete linear momentum (Equation 3.37 in Appendix B) equation for a control volume, derive the thrust equation in a systematic manner for the rocket with the converging nozzle. Document each simplifying assumption, define the control volume, and use the symbols found in the schematic or consistent with the course textbook. Appendix B shows the basic starting equation with some explanations.
   b. Repeat the process in for the rocket with a converging-diverging nozzle; determine the force that the diverging section of the nozzle has on the circular ring of material around the rocket throat. You will need to draw a separate control volume for this analysis. For Special Problem SP01B assume that the cross-sectional area of the nozzle throat material is A_{ring}.
   c. Comment of the ratio of the thrust of the second rocket over the first for a typical supersonic converging-diverging nozzle. When is the diverging section adding to the overall thrust of the rocket motor?
4. **Schematic:**



Equations:

SP 01 B

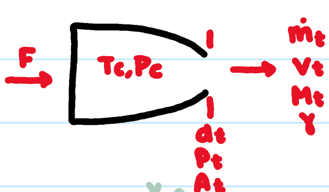
$$\sum F = \frac{d}{dt}\left(\int_{cv} \vec{V}\rho\, d\forall\right) + \int_{cs} \vec{V}\rho(\vec{v}\cdot\vec{n})\, dA \qquad [3.37]$$

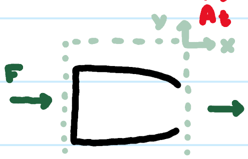
Assumptions: $(P_t \neq P_a)$

Analysis

Derive the thrust equation

a)





Forces: F, P

Momentum: $\dot{m} V_t$

$$\int_{CS} \vec{V}\rho(\vec{v}\cdot\vec{n})\,dA$$

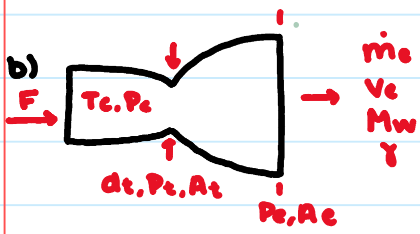
$$v(-v)\rho A_t = -\dot{m}_t V_t$$

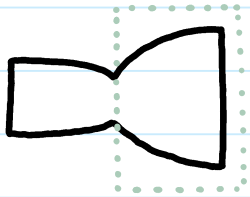
$$F + P_t A_t - P_a A + \theta A_{ring} = -\dot{m}_t V_t$$

Answer

$$\boxed{F = P_a A - P_t A_t - \theta A_{ring} - \dot{m}_t V_t}$$

b)





Forces: F, P $(P_t \neq P_a)$

Momentum: $\dot{m} V_t$

$$\int_{CS} \vec{V}\rho(\vec{v}\cdot\vec{n})\,dA$$

$$v(-v)\rho A_t = -\dot{m}_e V_e$$

$$F + P_e A_e - P_a A + \theta_1 A_{ring,L} + \theta_2 A_{ring,R} = -\dot{m}_e V_e$$

Answer

$$\boxed{F = P_a A - P_e A_e - \theta_1 A_{ring,L} - \theta_2 A_{ring,R} - \dot{m}_e V_e}$$

c)

Answer

$$\boxed{\frac{F_2}{F_1} = \frac{P_a A - P_e A_e - \theta_1 A_{ring,L} - \theta_2 A_{ring,R} - \dot{m}_e V_e}{P_a A - P_t A_t - \theta A_{ring} - \dot{m}_t V_t}}$$

Comment:

The second force is very similar to the first, but the shear stress on the throat and exit come more into play

HW1 SP01C

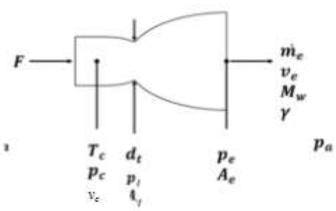
Wednesday, January 18, 2023 2:15 PM

1. **Name:** Veronica Loomis
2. **Given:** Rocket motor with a converging-diverging nozzle
3. **Find:**
   a. Starting with the conservation of energy equation for a control volume (see Appendix C), show a systematic derivation of the thrust equation that includes the velocity in the chamber entering the nozzle. State all the assumptions relevant to reducing and modifying the original equation and show the additional laws/equations that are inserted into the solution to arrive at Equation 4.23 in the textbook (with the added v_c term that is not included)

$$v_e^2 = \frac{2\gamma R_u T_c}{\mathfrak{M}(\gamma - 1)}[1 - (p_e/p_c)^{(\gamma-1)/\gamma}]$$

   b. Comment on the definitions and equations for thermally perfect gas and calorically perfect gas assumptions and why real rocket nozzles operation behave differently.

4. **Schematic:**



Equations:

Motor with C/D nozzle

$$\dot{Q} - \dot{W} = \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e\right) - \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i\right) \quad [kW]$$

$$\downarrow$$

$$V_e^2 = \frac{2\gamma R_u T_c}{M(\gamma-1)}\left[1 - (P_e/P_c)^{(\gamma-1)/\gamma}\right] + V_c$$

Assume:

Adiabatic: $\dot{Q} = 0$

No shaft work: $\dot{W} = 0$

assume very low M

$$0 = h_e + \frac{V_e^2}{2} + \cancel{gz_e} - h_i - \cancel{\frac{V_i^2}{2}} - \cancel{gz_i}$$

these are the same

$$V_e^2 = 2(h_i - h_e)$$

$$h_x = \frac{\gamma R T_x}{\gamma - 1}$$

$R = \frac{R_u}{M} \rightarrow$ script M

$$V_e^2 = 2\left[\frac{\gamma R T_i}{\gamma-1} - \frac{\gamma R T_e}{\gamma - 1}\right]$$

$$V_e^2 = \frac{2\gamma R}{\gamma - 1}(T_i - T_e)$$

$T_i = T_c$

$$V_e^2 = \frac{2\gamma R T_c}{\gamma - 1}\left(1 - \frac{T_e}{T_c}\right)$$

$$\boxed{V_e^2 = \frac{2\gamma R T_c}{\gamma - 1}\left[1 - \left(\frac{P_e}{P_c}\right)^{(\gamma-1)/\gamma}\right]}$$

Answer

b)

Answer

Thermally perfect: obeys $PV = nRT$

↳ can't always assume bc of gas particle interactions

Calorically perfect: C_v & C_p are constant w/in a temp range

↳ Generally invalid due to large temp changes involved

Comment:
It is convenient to assume thermally and calorically perfect since it makes the math easier, but oftentimes it is not truly the case