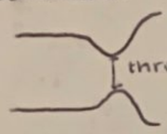
Name: Veronica Loomis

Problem 6.4

Given: convective heat transfer

Find: Prove convective heat transfer is max at throat

Schematic:



Assumptions:
negligible temp dependence of μ, C_p, and k

Basic Equations:

$$\dot{q} = h_g (T_\infty - T_w)$$

$$h_g = \frac{0.026}{D^{0.2}}\left(\frac{\mu^{0.2} C_p}{Pr^{0.6}}\right)\left(\rho_\infty V_\infty\right)^{0.8}\left(\frac{\rho_{am}}{\rho_\infty}\right)^{0.8}\left(\frac{\mu_{am}}{\mu_0}\right)^{0.2}$$

$$Pr = \frac{\mu_\infty C_{p\infty}}{k_\infty} \qquad V_\infty = \frac{\dot{m}}{\rho_\infty A} = \frac{4\dot{m}}{\rho_\infty \pi D^2}$$

Analysis:

max @ throat => $\dot{q}' = 0$, $\dot{q}'' > 0$ (derive wrt x)

$$\dot{q} = (T_\infty - T_w)\ \text{const}\ \frac{1}{D^{0.2}}\left(\frac{1}{D^2}\right)^{0.8} = \sim \times D^{-0.2} D^{-1.6} = \sim D^{-1.8}$$

$$\dot{q}' = \sim (-1.8) D^{-2.8} \frac{dD}{dx}$$

this is possible if $D \neq 0$, $\underbrace{dD/dx = 0}_{\text{true at throat}}$

Answer

$\dot{q}_{left} = \sim D^{-1.8}$ < $\dot{q}_{critical point}$ ∴ it is max at throat

Comment

This makes sense. Mass flow rate is max when flow is choked, so it makes sense that heat transfer is as well.

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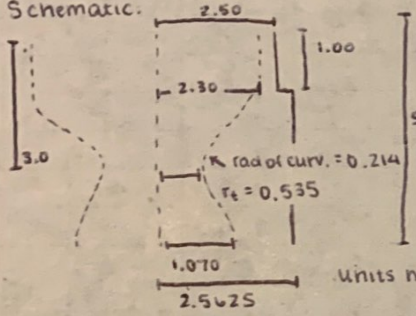
Problem 6.7

Given: Heavy walled steel nozzle
15 second test firing

Steel melting temp: 2500°F / 1370°C / 1643.15 K

Find: Temp profile at throat at $t = 2, 5, 10, 15$ sec. Will steel melt?

Schematic:



units not given.

5 1/2 in diam. BAR CRS

Assumptions:
- $\bar{h}_g = 10000\ W/m^2K$ for steel
- $k = 60.5\ W/mK$
- $\alpha = 17.7 \times 10^{-6}\ m^2/s$
- $P_c = 300$ psi $(lbf/in^2) = 2.068 \times 10^6\ N/m^2$
- Oxidizer: 85% aq H_2O_2 & HTPB as fuel
- $O/F = 7$
- $\gamma = 1.4$
- One dimensional
- constant k, c_p, α
- heat never conducted to back side

Basic Equations:

$$\frac{T(x,t) - T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \operatorname{Exp}\left[\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right] \operatorname{erfc}\left[\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right]$$

$$T_t = T_c \left[\frac{1}{1 + \frac{\gamma - 1}{2}}\right]$$

Analysis:

In CEQUEL, find T_c

Find T_t using $T_t = T_c \left[\frac{1}{1 + \frac{\gamma-1}{2}}\right]$

Substitute $T_t = T_\infty$, $T_i = 298$ K

Plot in MATLAB → Answer: If one uses the temperature given in solutions, the steel will melt.
If the temperature is calculated in CEQUEL, it will not melt.

Comment: The difference between given T_∞ and CEQUEL/converted T_∞ might have been a typo.
It may have also been a miscalculation since if I swap T_t and T_c in the equation above, my CEQUEL/calculated value becomes a lot closer to the one given in the solution.