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PROBLEM 11.3

Given:

Hybrid rocket (LOX/HTPB)

$c^* = -2520 + 6800\,(O/F) - 1320\,(O/F)^2$ $\quad 2 < O/F < 3 \quad$ [ft/s]

$\rho_f = 0.0325$ lb/in^3

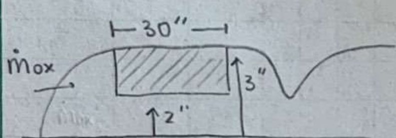
$r = 0.16\,G_{ox}^{0.7}$ [in/s]

Find: i) Optimal O/F for this propellant combination

ii) Oxidizer flow rate ($\dot{m}_{ox}$) which maximizes mid web performance

iii) Overall O/F shift for the firing, assuming fuel completely consumed

Schematic:



Assumptions:

Midweb => 2.5"

All of fuel is completely consumed

Basic Equations:

$O/F = \dot{m}_{ox} / \dot{m}_f$

$G_{ox} = \dot{m}_{ox} / (\pi R^2)$

$\dot{m}_f = r \rho_f A_b$

$A_b = 2\pi R L$

Analysis:

i) optimal O/F

derive c^* wrt O/F

call $\eta = O/F$

$$\frac{dc^*}{d\eta} = 6800 - 2(1320)\eta = 0$$

$$\eta = 2.575$$

i) Answer

$\boxed{\text{optimal } O/F = 2.575}$

ii) Oxidizer flow rate that maximizes performance @ midweb

Given $R_i = 2''$, $R_{end} = 3''$
so mid web → $R = 2.5''$

We know from (i) that $O/F = 2.575$
so
$\dot{m}_{ox} = 2.583\,\dot{m}_f$

$$\dot{m}_f = r\rho_f A_b$$
$$= 0.16\left(\frac{\dot{m}_{ox}}{\pi R^2}\right)^n \rho_f\, 2\pi R L$$
$$= 0.16\left(\frac{\dot{m}_{ox}}{\pi(2.5)^2}\right)^{0.7}(0.0325)\,2\pi(2.5)(30)$$
$$\dot{m}_f = 0.304876\,\dot{m}_{ox}^{0.7}$$

Sub back in

$$\dot{m}_{ox} = 2.575(0.304876)\,\dot{m}_{ox}^{0.7}$$

ii) Answer

$$\boxed{\dot{m}_{ox} = 0.44678 \text{ lb/s}}$$

iii) Overall O/F shift

$\dot{m}_{ox} = 0.44678$ lb/s → max

min at either $R = 2$, $R = 3$

$$\dot{m}_f \text{ @ } R=2 \Rightarrow 0.16\left(\frac{0.44678}{\pi(2)^2}\right)^{0.7}(0.0325)\,2\pi(2)(30) = 0.18965 \rightarrow (O/F)_{min}$$

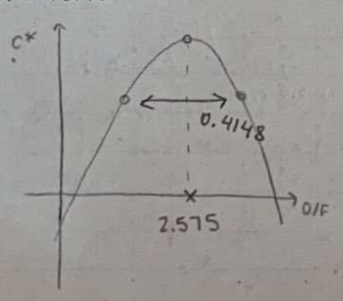
$$\dot{m}_f \text{ @ } R=3 \Rightarrow 0.16\left(\frac{0.44678}{\pi(3)^2}\right)^{0.7}(0.0325)\,2\pi(3)(30) = 0.161256 \rightarrow (O/F)_{max}$$

$(O/F)_{min} = 2.3558$, $(O/F)_{max} = 2.7706$ these are both $2 < O/F < 3$ ✓

$$\boxed{\text{overall } O/F \text{ shift} = 0.4148}$$

iii) Answer

Comment:



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Problem 11.7

Given:

Hybrid rocket, obeys $r = a G_{ox}^{n}$

$r = dR/dt$

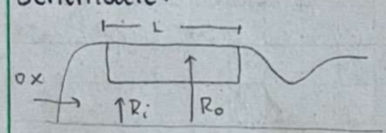
Find: i) Show $R(t) = \left[a(2n+1) \left(\frac{\dot{m}_{ox}}{\pi} \right)^n t + R_i^{2n+1} \right]^{1/(2n+1)}$

ii) Derive expressions for $\dot{m}_f$ and O/F

Is there a special value of n where fuel flow is const & no mixture ratio shifts?

iii) Plot $R(t)$, $\dot{m}_f(t)$, and $O/F(t)$ with the given conditions

Schematic:



Assumptions:

$\dot{m}_{ox}$ is constant

fuel regression rate is uniform

Basic Equations:

$$G_{ox} = \frac{\dot{m}_{ox}}{\pi R^2}$$

$$\dot{m}_f = r \rho_f A_b$$

$$A_b = 2\pi R L$$

$$O/F = \dot{m}_{ox} / \dot{m}_f$$

Analysis:

i) $r = \frac{dR}{dt} = a G_{ox}^n = a\left(\frac{\dot{m}_{ox}}{\pi R^2}\right)^n$

$$\int R^{2n}\, dR = \int a \dot{m}_{ox}^n \pi^{-n}\, dt$$

$$\frac{1}{2n+1}\left[R(t)\right]^{2n+1} = a \dot{m}_{ox}^n \pi^{-n} t + R_i^{2n+1}$$

$$\boxed{R(t) = \left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right]^{1/2n+1}}$$ i) Answer

ii $\dot{m}_f$

$$\dot{m}_f = r \rho_f A_b = a\left(\frac{\dot{m}_{ox}}{\pi R^2}\right)^n \rho_f\, 2\pi R L$$

$$\dot{m}_f = 2 a \rho_f L \dot{m}_{ox}^n \pi^{1-n} R^{1-2n}$$

Substitute R(t) from (i)

$$\dot{m}_f = 2 a \rho_f L \dot{m}_{ox}^n \pi^{1-n}\left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right]^{\frac{1-2n}{1+2n}}$$

when $n = 1/2$, fuel flow is constant wrt time

ii A) Answer

O/F

$O/F = \dot{m}_{ox} / \dot{m}_f$

ii B) Answer

$$O/F = \frac{\dot{m}_{ox}^{1-n}}{2 a \rho_f L \pi^{1-n}}\left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right]^{\frac{2n-1}{2n+1}}$$

when $n = 1/2$, there is no mixture ratio shift

iii) $L = 50''$, $R_i = 2''$, $R_o = 5''$, $\rho_f = 1\,g/cm^3 = 0.0361273\,lb/in^3$, $a = 0.1$, $n = 0.8$

G_{ox} initial $= 1.0\ lb/in^2 \cdot s$

we assumed $\dot{m}_{ox}$ was const so

$G_{ox} = \dot{m}_{ox} / \pi R$ @ initial values

$\dot{m}_{ox} = G_{ox,init}\, \pi R_o$

Script & plots included next.