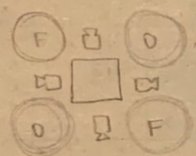
Name: Veronica Loomis
Problem 8.4

Given: N_2O_4 / MMH propellant combo
Equal volumes for both fuel & oxidizer tanks
Payload mass = 10 kg
I_{sp} = 300 sec
Required divert Δv = 1000 m/s
$\lambda = 0.5$

Find.: a) propellant mixture ratio
b) total propellant mass required
c) radius of spherical tanks
d) tank wall thickness if P_{burst} = 5 MPa and is titanium

Schematic:



Assumptions:
Each tank has same volume
Keep grav in Δv eq so units work
Thin wall tanks
$\rho_{ox} = 1440\ kg/m^3$ } googled
$\rho_f = 880\ kg/m^3$

Basic Equations:

$$r = m_{ox}/m_f, \quad \rho = m/V, \quad MR = \frac{m_o}{m_f}, \quad m_o = m_{PL} + m_p + m_i, \quad \lambda = \frac{m_p}{m_p + m_i}$$

$$V_{sph} = \frac{4}{3}\pi R^3, \quad \Delta v = I_{sp} \ln\left(\frac{m_o}{m_f}\right), \quad m_p = m_{PL}\left(\frac{MR-1}{MR-(MR-1)/\lambda}\right)$$

$$t_c = P_t R / \sigma_w, \quad \sigma_w = \min(F_{ty}/1.1, F_{tu}/\underbrace{1.25}_{\text{or } 1.4})$$

Analysis:

a) Propellant mixture ratio

$$\Delta v = g I_{sp} \ln\left(\frac{m_o}{m_f}\right) \longrightarrow \frac{m_o}{m_f} = e^{\Delta v / g I_{sp}}$$

$$\frac{m_o}{m_f} = e^{(1000\ m/s)/(9.81\ m/s^2)(300\ s)}$$

$$\boxed{\frac{m_o}{m_f} = 1.4 = MR}$$

b) Total propellant mass required

$$m_p = m_{PL}\left(\frac{MR-1}{MR-(MR-1)/\lambda}\right) = 10\left(\frac{0.4}{1.4 - \frac{0.4}{0.5}}\right) kg$$

$$\boxed{m_p = 6.67\ kg}$$

c) Radius of spherical tanks

know $m_p = 6.67$ kg

need $r = \frac{m_{ox}}{m_f}$

have $\rho_{ox} = \frac{m_{ox}}{V_{ox}}$ & $\rho_f = \frac{m_f}{V_f}$

told $V_f = V_{ox}$

$\therefore$ we can say $\frac{\rho_{ox}}{\rho_f} = \frac{m_{ox}}{m_f}$

$\rho_{ox}/\rho_f = 1440/880 = 1.636 = r$

$m_f = \frac{m_p}{1+r} = 2.53$ kg

$m_{ox} = r m_f = 4.14$ kg

$V_f = \frac{m_f}{\rho_f} = 0.002875\ m^3$

$V_{ox} = \frac{m_{ox}}{\rho_{ox}} = 0.002875\ m^3$

yes they are the same

$R = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.002875 m^3)}{4\pi}}$

$\boxed{R = 0.0882\ m}$

d) Tank wall thickness

$t_c = \frac{P_{t(max)} R}{\sigma_w}$ $P_t = 5$ MPa

$\sigma_w = \min\left(\frac{\text{yield}}{1.1}, \frac{\text{ult tens. strength}}{1.4}\right)$ look up for titanium

$\sigma_w = \min\left(\frac{241\ MPa}{1.1}, \frac{1400\ MPa}{1.4}\right) = 219.1$ MPa

$t_c = \frac{5(0.0882)}{219.1} \frac{MPa\ m}{MPa}$

$\boxed{t_c = 0.00201\ m = 2.01\ mm}$

Comments:

The wall thickness is allowed to be SUPER thin because of the material.

I also realized afterwards that (c) could just have been $V = \frac{m_p}{\rho_f + \rho_{ox}}$ ...
but i guess it was good to make sure $V_f = V_{ox}$ like we assumed.

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Problem 8.6

Given: LOX & Ethanol

$c^* = 5900\ ft/s$

$\gamma = 1.2$

$F_v = 30000\ lbf$

$\varepsilon = 50$

$P_c = 1200\ psi$

$M \leq 0.1$

$L^* = 40$ inches - char. chamber dim.

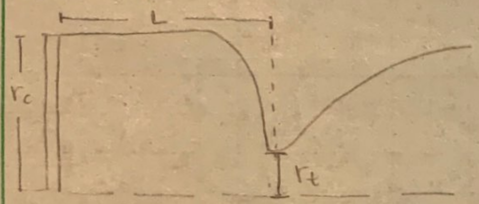
Find: a) Isp

b) r_t

c) r_c

d) L (chamber)

Schematic:



Assumptions:

Purdue tables

~~Analysis~~: Basic Equations:

$$I_{sp} = \frac{C_{fv} \cdot c^*}{g}, \quad F_v = C_{fv} P_c A_t, \quad A_c = (N_{ox} + N_{fuel}) / N_D$$

$$\varepsilon_c = A_c / A_t \qquad \frac{A_i}{A_t} = \frac{1}{M}\left[\frac{2 + (\gamma - 1)M^2}{\gamma + 1}\right]^{(\gamma+1)/2(\gamma-1)}$$

$$L^* = \frac{V_c}{A_t}$$

Analysis:

a) Isp

$$I_{sp} = \frac{C_{fv} \cdot c^*}{g}$$

Using $\gamma = 1.2$, $\varepsilon = 50$

$C_{fv} = 1.90236$

$$I_{sp} = \frac{(1.90236)(5900\ ft/s)}{32.2\ ft/s^2}$$

$$\boxed{I_{sp} = 348.57\ sec}$$

b) r_t

$$F_v = C_{Fv} P_c A_t$$

$$A_t = \frac{F_v}{C_{Fv} P_c} = \frac{30\,000 \text{ lbf}}{(1.90236)(1200 \text{ lbf/in}^2)}$$

$$A_t = 13.14 \text{ in}^2$$

$$r_t = \sqrt{A_t/\pi}$$

$$\boxed{r_t = 2.045 \text{ in}}$$

c) r_c $(M = 0.1)$

$$A_c = A_t \left(\frac{1}{0.1} \left[\frac{2 + (0.2)(0.1)^2}{2.2} \right]^{2.2/0.4} \right)$$

$$A_c = 78.22 \text{ in}^2$$

$$r_c = \sqrt{A_c/\pi}$$

$$\boxed{r_c = 5.0 \text{ in}}$$

d) L (chamber)

$$L^* = V_c / A_t$$

$$L^* = \frac{\pi r_c^2 L_c}{A_t}$$

$$L_c = \frac{L^* A_t}{\pi r_c^2} = \frac{40 \text{ in } (13.14 \text{ in}^2)}{\pi \, (5.0 \text{ in})^2}$$

$$\boxed{L_c = 6.69 \text{ in}}$$

Comment

$r_c > r_t$ → expected

L_c is close to r_c