



# MAE 640 Rocket Propulsion II

## Lecture 02L-A1 – Hybrid Rocket Engines

Items Included:

- [A] Announcements
- [B] Module Overview
- [C] 11.1 Introduction: General Arrangement & History
- [D] 11.2 HRE Combustion Fundamentals
- [E] 11.3 Lumped Parameter Ballistics
- [F] Initial Read of Homework Problems
- [G] Supplementary Material on Flow Separation and N.S.

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# [A] Announcements

## Resources

- General
  - **Technical Resources Module** on Canvas has Compressible Gas Dynamics Tables and Atmospheric Properties Tables.
  - **Thrust Coefficient** - Excel Spreadsheet for the thrust coefficient download is in Module 01 “Lectures” section and on 01HW Drop Box
  - **Questions** - Email me with your questions about HW01
- Homework 01
  - **Light My Fire 01** (Office Hours/Helps Session) Problems 2.8, 3.8, 4.24, and 4.30 are Discussed. I have posted the Video and Charts on CANVAS.

# Outline of Lecture 02-A1

## Lecture 03-A1 [This Package]

- [A] Announcements
- [B] Module Overview
- [C] 11.1 Introduction: General Arrangement & History
- [D] 11.2 HRE Combustion Fundamentals
- [E] 11.3 Lumped Parameter Ballistics
- [F] First Reading of HW02-A Problems

# [B] Module Learning Objectives

## Module 02

- Students will be able to correctly define fundamental concepts, advantages, and performance parameters of hybrid rockets engines.
- Students will be able to perform calculations of basic performance parameters and internal ballistics.
- Students will apply concepts using a computer program to describe time-dependent internal ballistics and thrust.

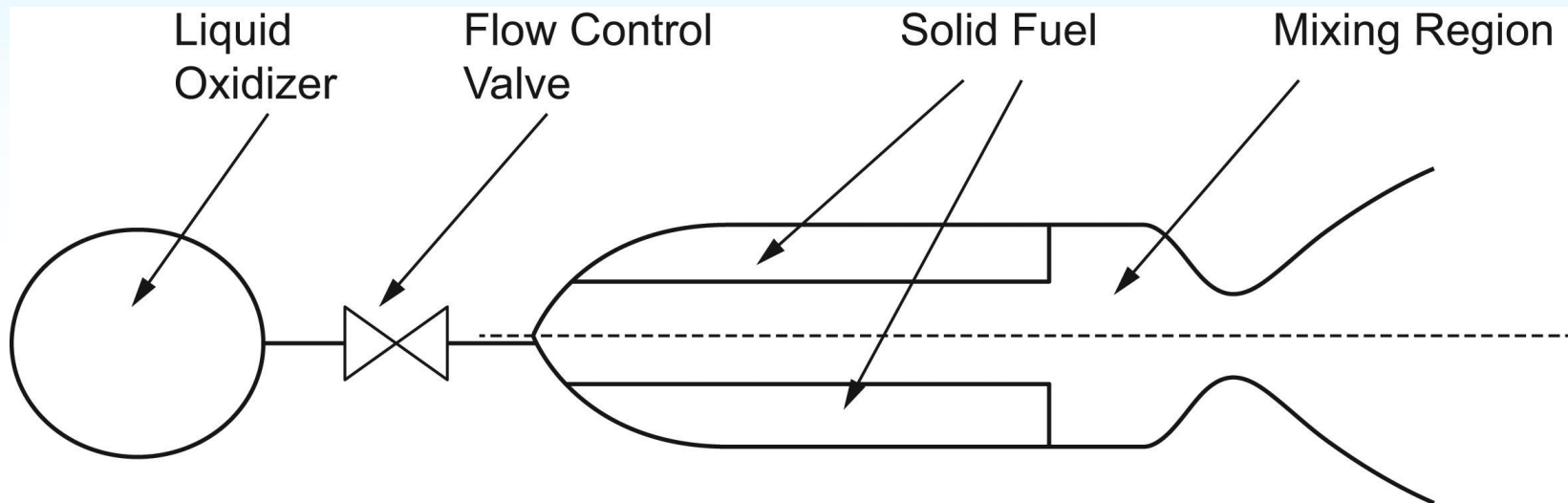
# [B] Module Components

## Module 02 – Hybrid Rocket Engines (Week 1)

- ☐ Attend or View Lecture 02-A1
- ☐ Study Textbook Chapter 11.1, 11.2, and 11.3
- ☐ Complete Quiz 2A (02QA)
- ☐ Participate or Review **LIGHT MY FIRE** 02A"
- ☐ Complete 02HW-A

# [C] 11. 1 Intro: General Arrangement & History

## Major Elements of a Hybrid Rocket Engine (HRE)



# [C] 11. 1 Intro: General Arrangement & History

## Some Advantages

- Non-Hazardous Fuels
- Separation of Fuel and Oxidizer
- Reduced Operation Cost
- Reduced Complexity Compared to Liquids
- Higher Specific Impulse than Solid Motors

## Some Disadvantages

- Low Fuel Regression
- Large Ports Result in Low Propellant Loading
- Low Propellant Loading Results in Low Mass Fraction
- Mixture Ratio Shifts
- Lower Specific Impulse than Liquid Engines

# [C] 11. 1 Intro: General Arrangement & History

First Hybrid Rocket was Flown on August 17, 1933



Mikhail Tikhonravov in 1925



*Soviet rocket pioneer M.K. Tikhonravov and the GIRD-9.*

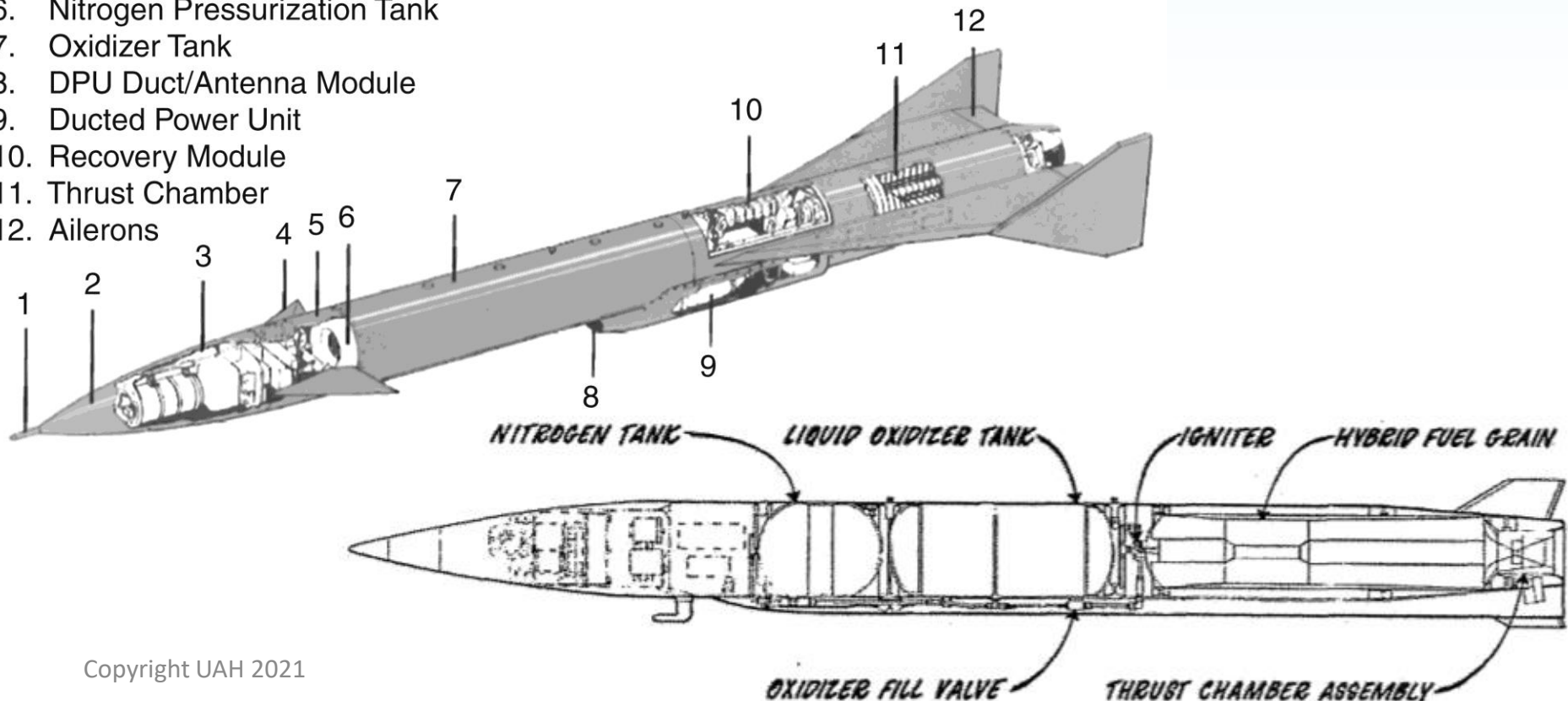


# [C] 11. 1 Intro: General Arrangement & History

## Firebolt High Altitude Supersonic Target (IRFNA and PMMA) Tested in the 1960's

### Firebolt Configuration

1. Pitot Tube
2. Radome
3. Equipment Compartment
4. Canards
5. Equipment Compartment
6. Nitrogen Pressurization Tank
7. Oxidizer Tank
8. DPU Duct/Antenna Module
9. Ducted Power Unit
10. Recovery Module
11. Thrust Chamber
12. Ailerons



# [C] 11. 1 Intro: General Arrangement & History

## AMROC 250,000 lbf Booster Tested in 1990's

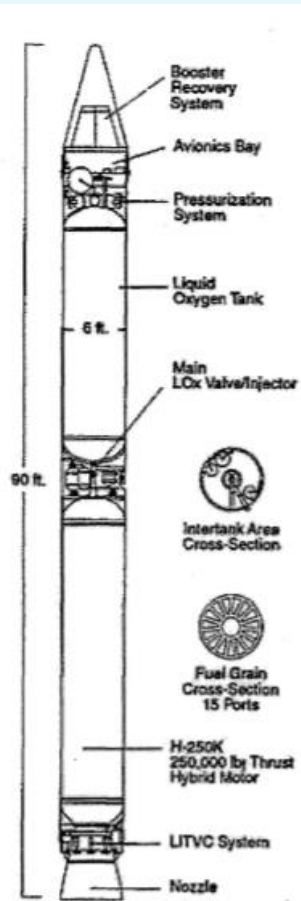
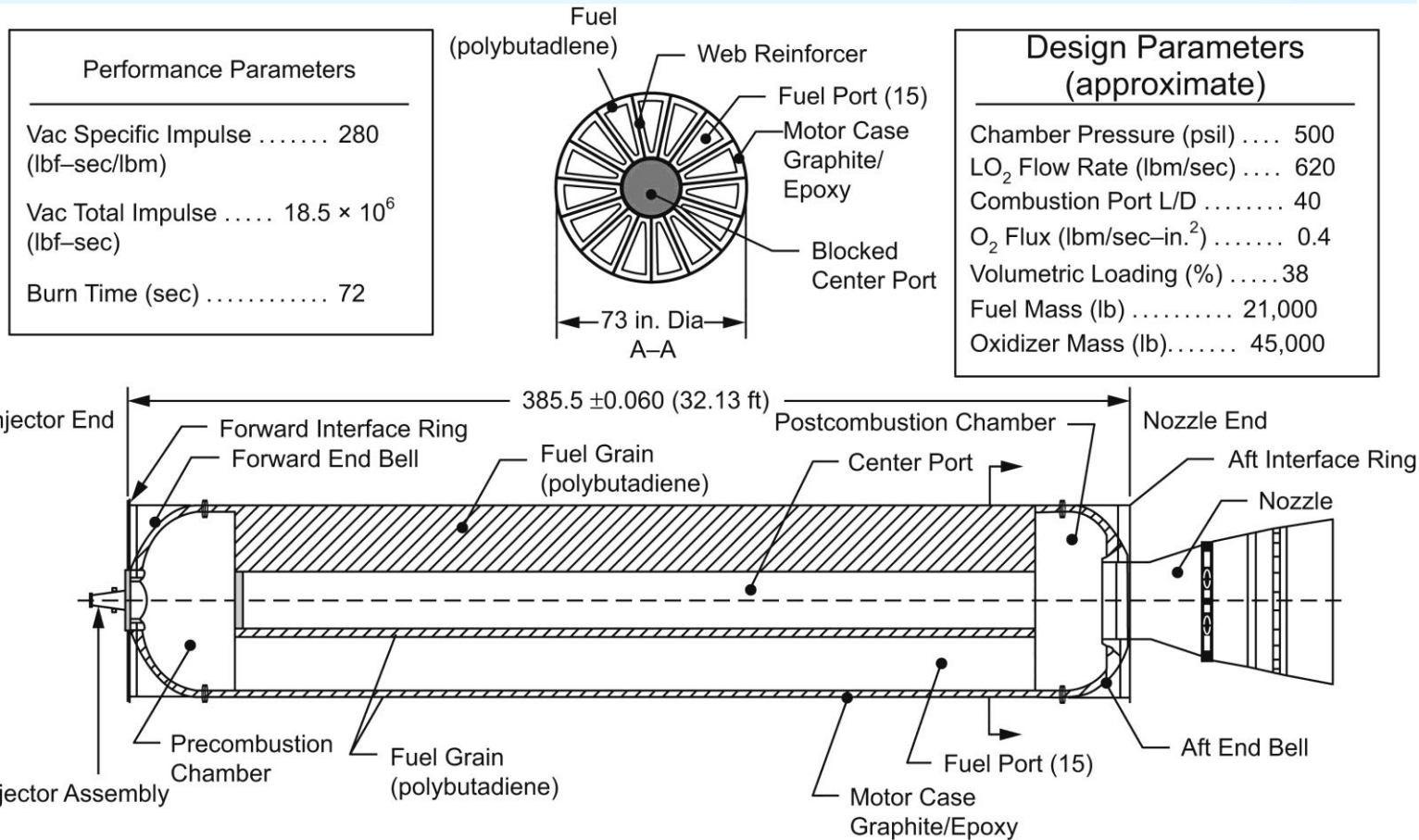


Figure 28 AMROC HyFLYER<sup>34</sup>



# [C] 11. 1 Intro: General Arrangement & History

## Virgin Galactic Spaceship 2 in 2010's



[https://youtu.be/K2kf1l8yx\\_4](https://youtu.be/K2kf1l8yx_4)

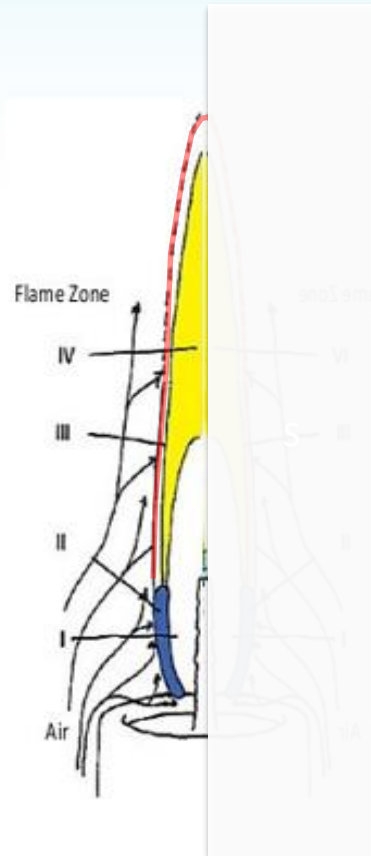
# [D] 11. 2 HRM Fundamentals

Candle



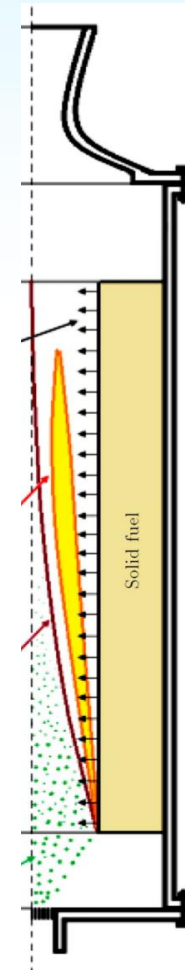
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Diffusion Flame

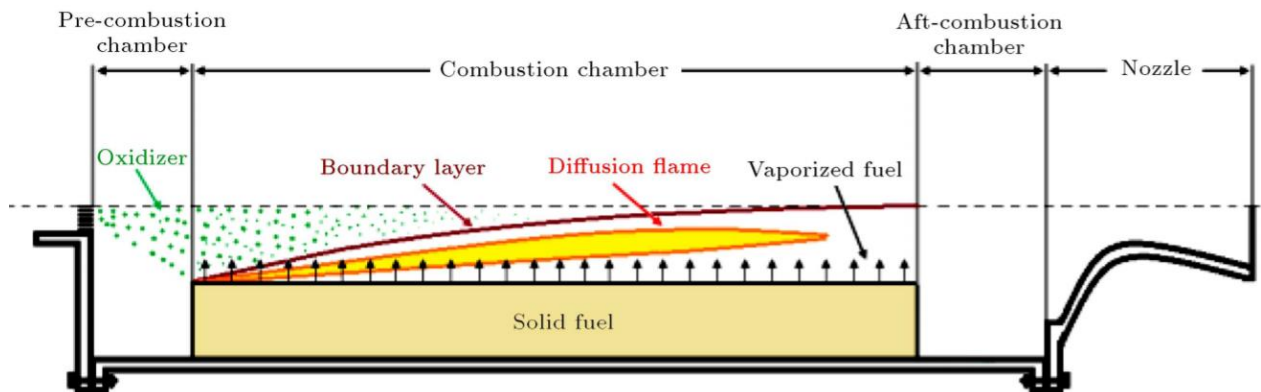
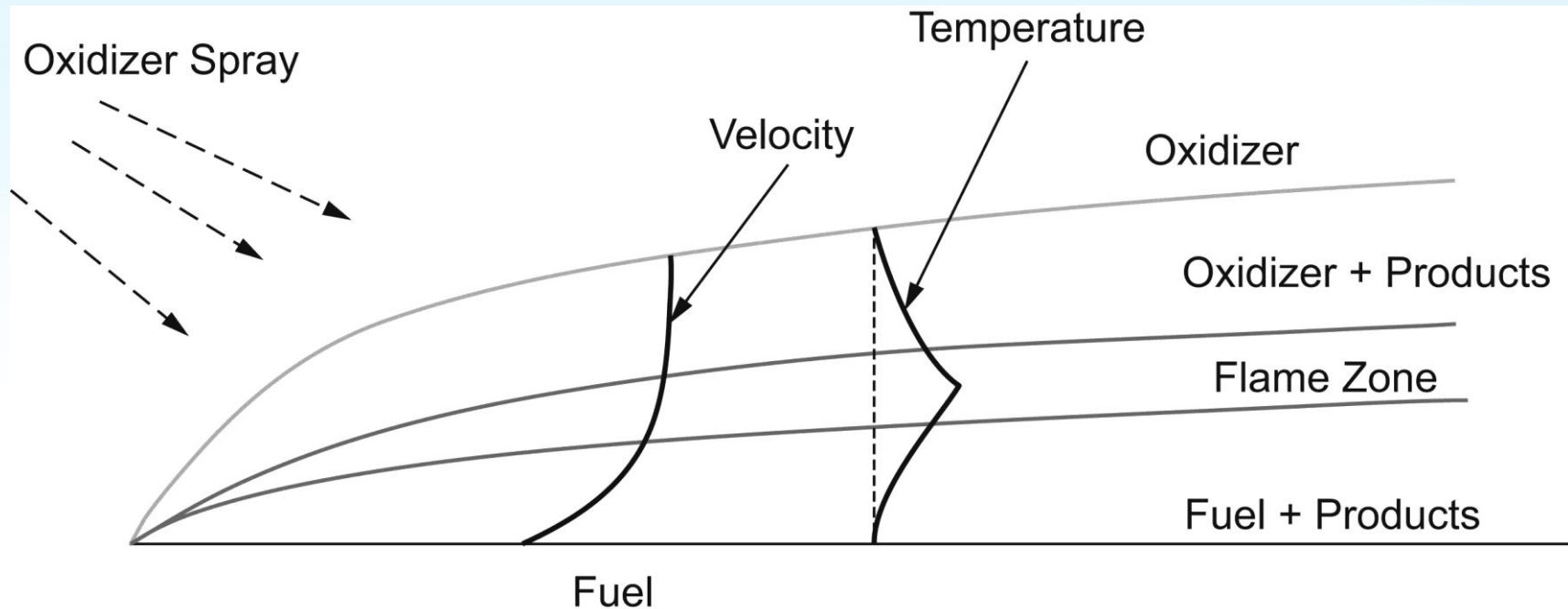


MAE 640

Hybrid Rocket  
Like a Candle  
Burning Inside Out

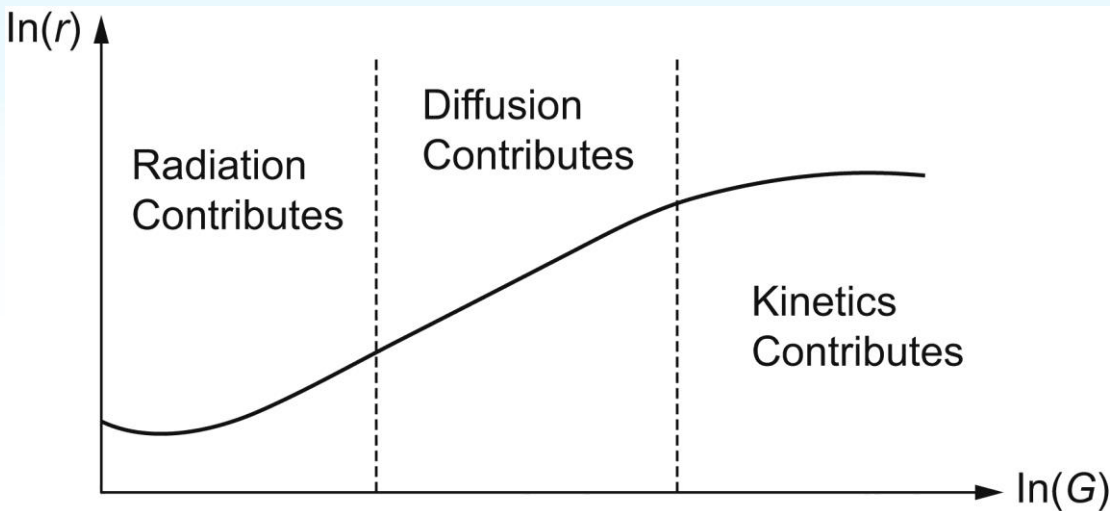


# [D] 11. 2 HRM Fundamentals



# [D] 11. 1 Intro: General Arrangement & History

Hybrid Rocket Regression Depends on Mass Flux

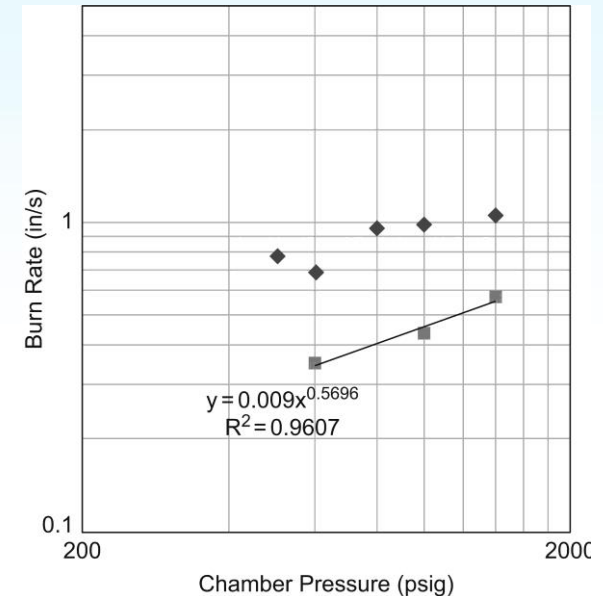


$$r = aG^n$$

$$r = aG_{ox}^n$$

$G_{ox}$  = Mass flow of Oxidizer/Area of Port

Solid Rocket Burning Rate Depends on Pressure



$$r_b = ap_c^n$$



# [E] 11. 3 HRE Lumped Parameter Ballistics

## Assumptions

- “Lumped” is Characterizing the Combustion Chamber with a Single Average Pressure that only Varies with Time.
- Regression Rate is Constant Along Each Port.
- In Reality there will be Pressure Drops Along the Ports and Some Non-uniform Regression Rates in the Fuel.

# [E] 11. 3 HRE Lumped Parameter Ballistics

$$\dot{m}_f = r\rho_f A_b$$

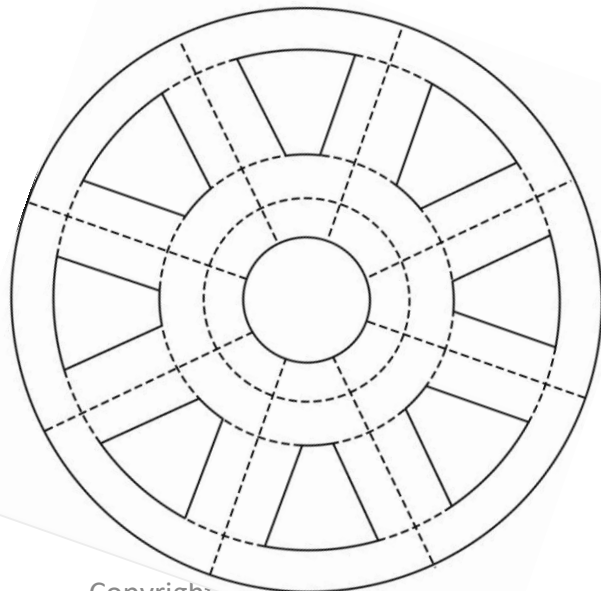
$$A_b = N \times \text{Per} \times L$$

$$r = aG_{\text{ox}}^n$$

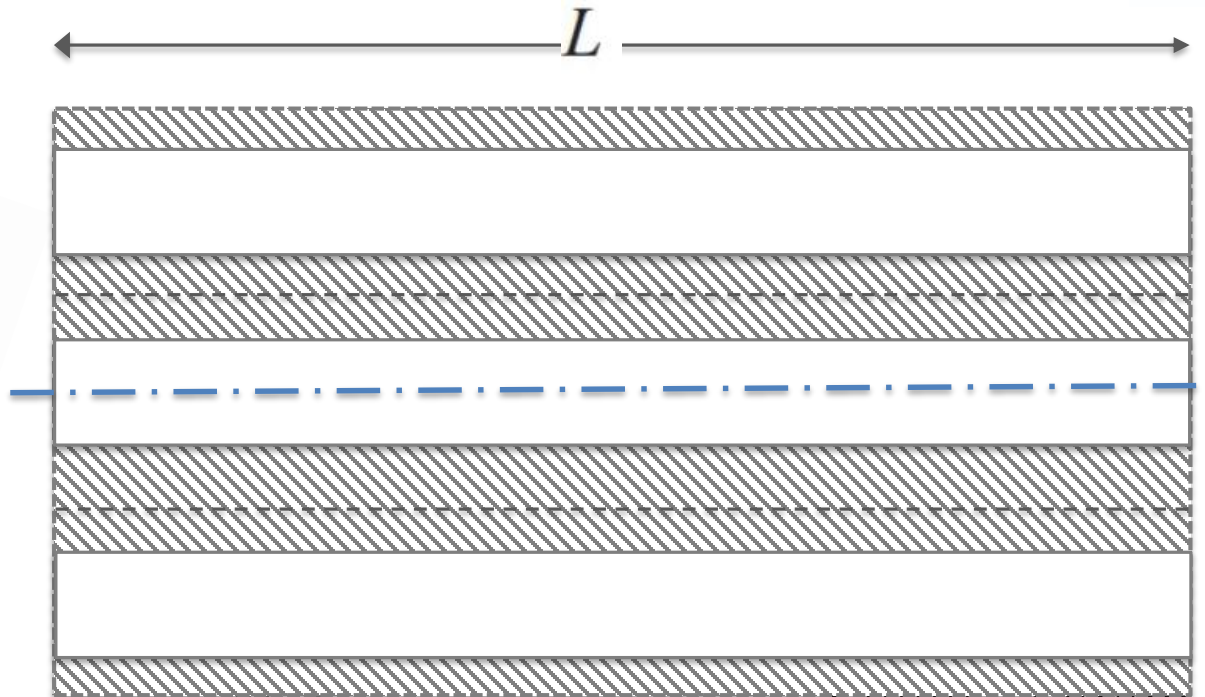


(11.2)

(11.3)



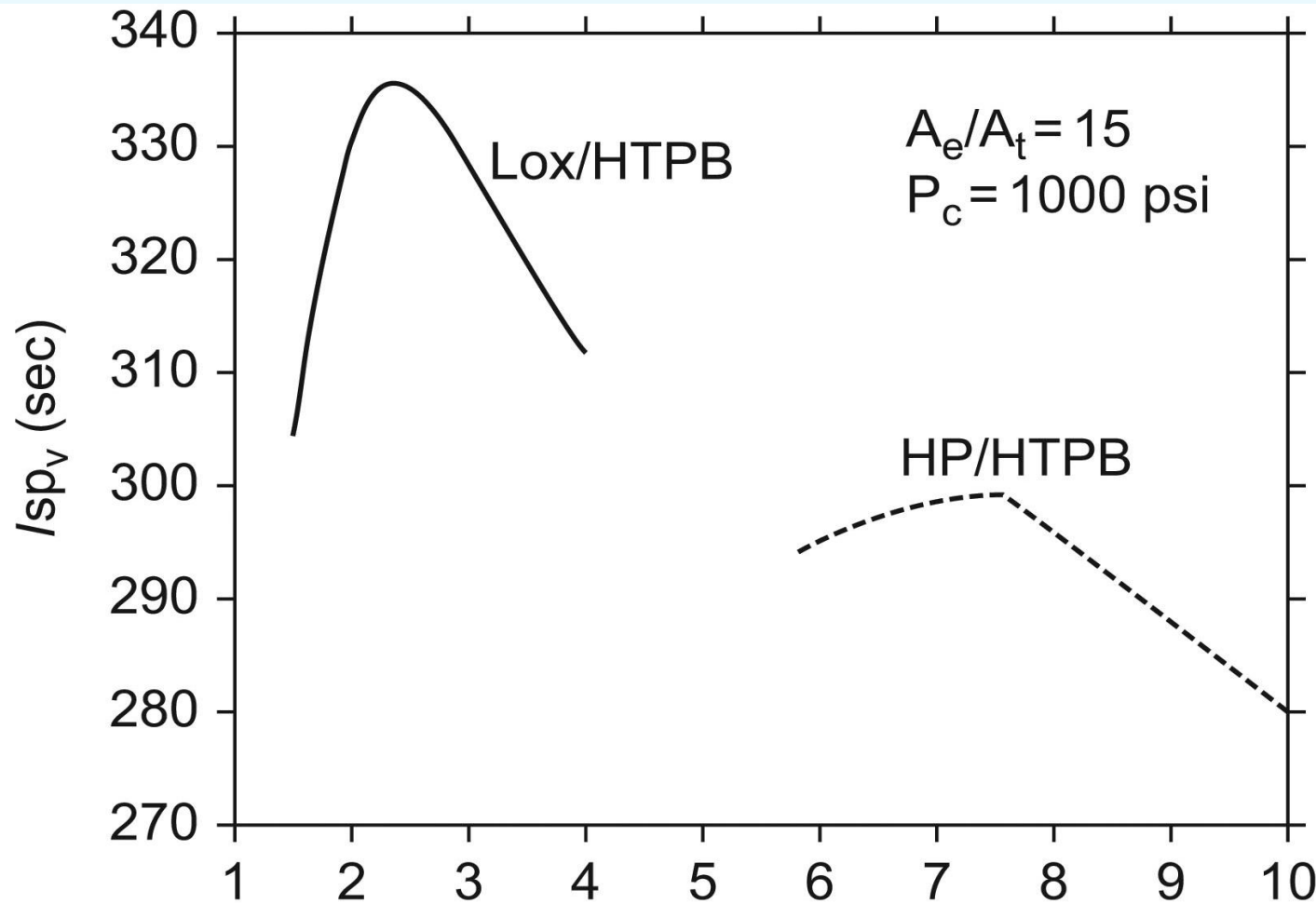
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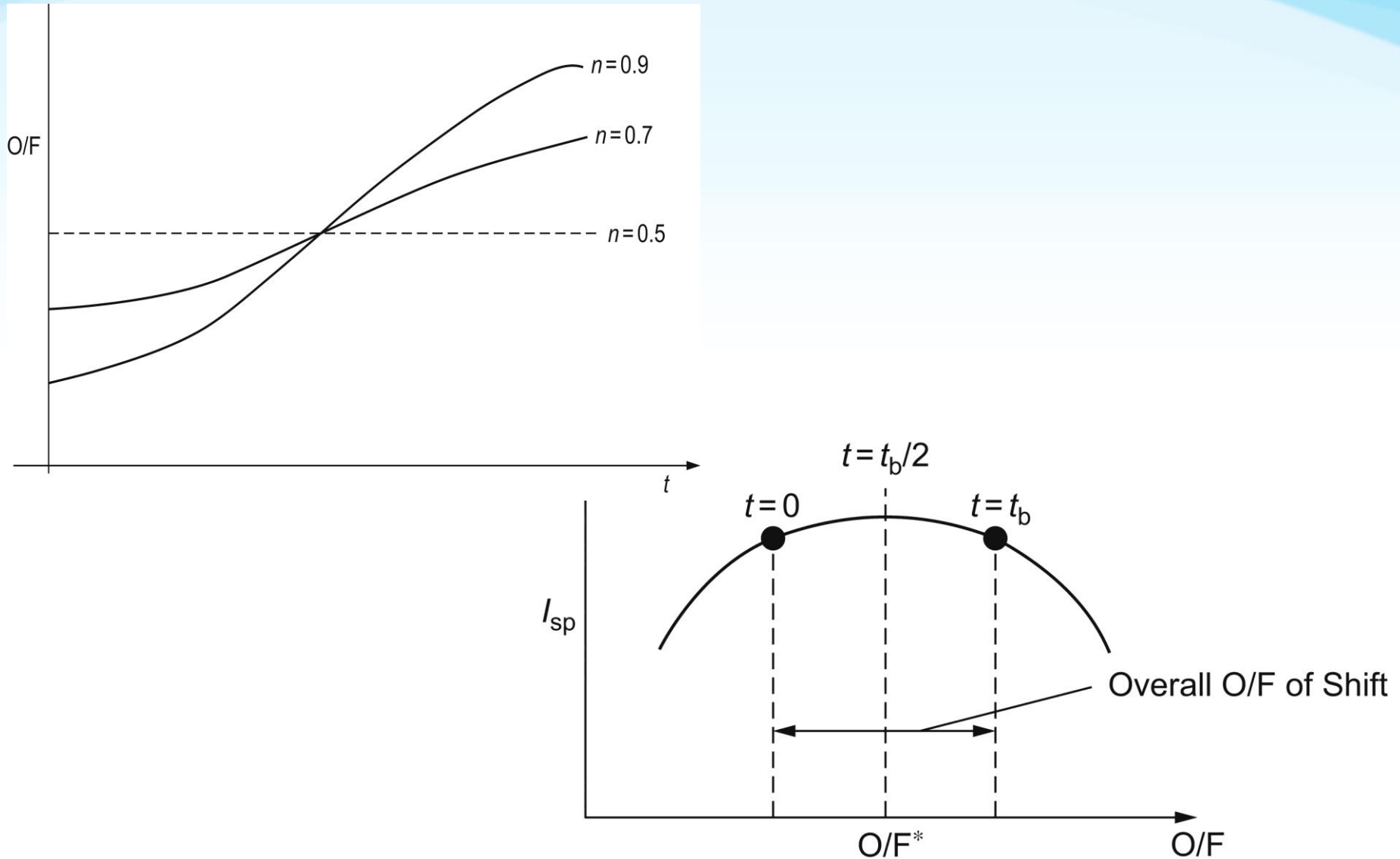


# [D] 11. 3 HRE Lumped Parameter Ballistics

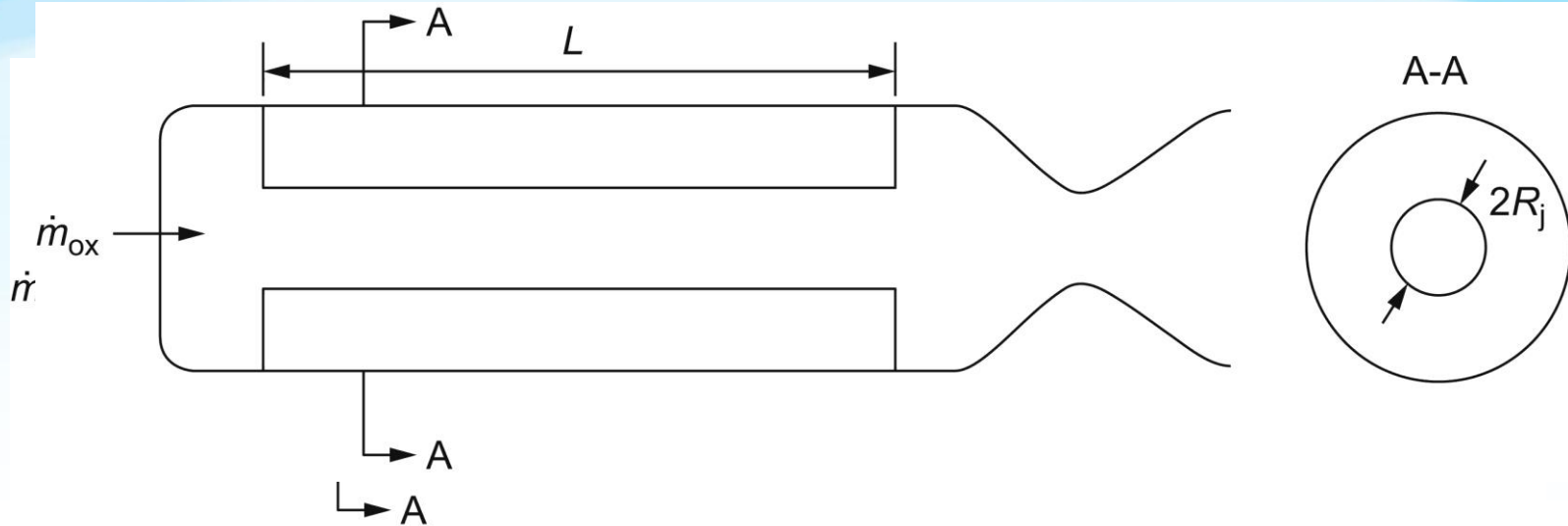
$$p_c = (\dot{m}_{ox} + \dot{m}_f)c^* / (gA_t) \quad (11.5)$$



# [D] 11. 3 HRE Lumped Parameter Ballistics



# [E] 11. 3 HRE Lumped Parameter Ballistics



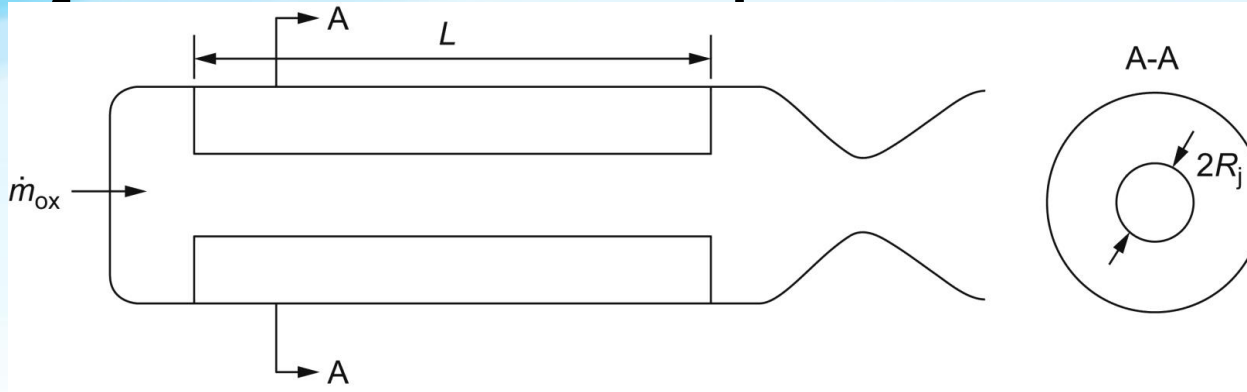
$$A_v = \pi R^2; A_b = 2\pi RL \quad (E1)$$

$$r = aG_{ox}^n = a\left(\frac{\dot{m}_{ox}}{\pi R^2}\right)^n \quad (E2)$$

$$\dot{m}_f = r\rho_f A_b = a\rho_f \left(\frac{\dot{m}_{ox}}{\pi R^2}\right)^n 2\pi RL = 2a\pi^{(1-n)} P_f L \dot{m}_{ox}^n R^{(1-2n)} \quad (E3)$$

Should be  $\rho_f$

# [E] 11. 3 HRE Lumped Parameter Ballistics



$$r = \frac{dR}{dt} = a \left( \frac{\dot{m}_{ox}}{\pi} \right)^n R^{-2n} \quad (E4)$$

Should be  $m\_dot\_ox$

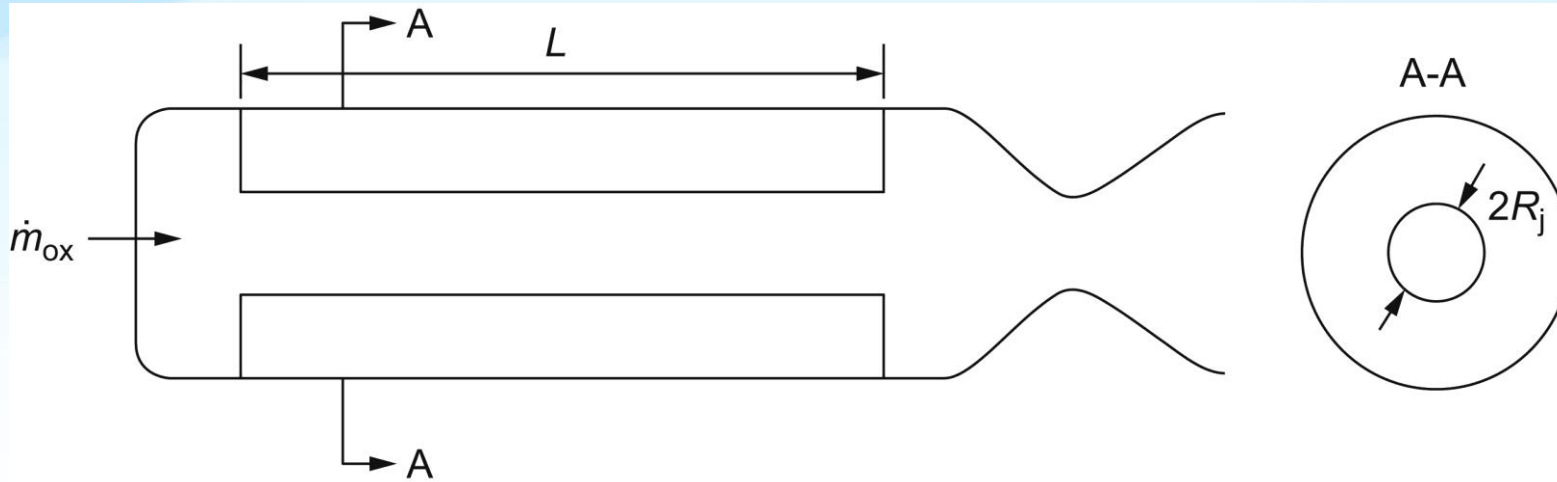
$$R^{2n} dR = a \left( \frac{\dot{m}_{ox}}{\pi} \right)^n dt \quad (E5)$$

$$R(t) = \left[ a(2n+1) \left( \frac{\dot{m}_{ox}}{\pi} \right)^n t + R_i^{2n+1} \right]^{\frac{1}{2n+1}} \quad (E6)$$

$$\dot{m}_f(t) = 2a\pi^{1-n} \rho_f L \dot{m}_{ox}^n \left[ a(2n+1) \left( \frac{\dot{m}_{ox}}{\pi} \right)^n t + R_i^{2n+1} \right]^{\frac{1-2n}{1+2n}} \quad (E7)$$

LLE

# [E] 11. 3 HRE Lumped Parameter Ballistics



$$\dot{m}_f(t) = 2a\pi^{1-n}\rho_f L \dot{m}_{ox}^n \left[ a(2n+1) \left( \frac{\dot{m}_{ox}}{\pi} \right)^n t + R_i^{2n+1} \right]^{\frac{1-2n}{1+2n}} \quad (E7)$$

Assuming Mass Flow Rate of Oxidizer is Constant

- Increasing the Length,  $L$ , Increases Fuel Flow Rate
- Fuel Flow Rate Varies With Time while Oxidizer Flow Rate is Constant (So the OF Ratio Shifts with Time)
- For  $n > 1/2$  the mass flow rate of fuel will decrease with time (Generally the case)
- For  $n = 1/2$  the O/F ration is constant with time
- Shifting OF causes shifting  $c^*$  (Characteristic Velocity )

# Summary

- Lecture 3A
  - Highlighted Course Resources
  - Discussed General Configuration of Hybrid Rocket Engines (HREs)
  - Described HRE Combustion Fundamentals
  - Started HRE Lumped Parameter Ballistics
- Lecture 2B Will Have First Read of HW02A



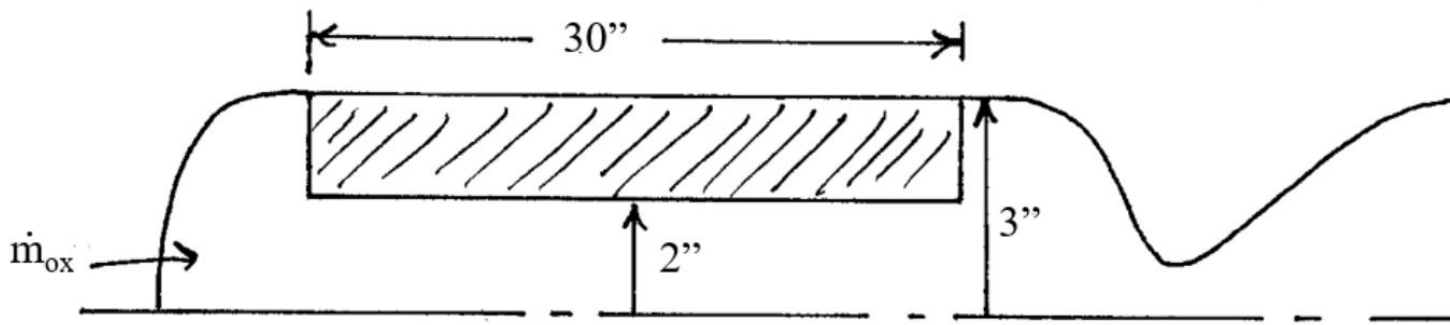
# [F] Initial Reading of Homework Problems

# HW02A – Problem 11.3

11.3 Consider the hybrid rocket test motor shown in Figure 11.24. This motor utilizes LOX/HTPB propellants. Data from the NASA thermochemistry code were curvefit for characteristic velocity:

$$c^* = -2520 + 6800(\text{O/F}) - 1320(\text{O/F})^2 \quad 2 < \text{O/F} < 3$$

with  $c^*$  in ft/s. In addition, the fuel density is known to be  $0.0325 \text{ lb/in}^3$  and the regression rate (in inches/s) obeys  $r = 0.16G_{\text{ox}}^{0.7}$ , where  $G_{\text{ox}}$  is the oxidizer massflux in  $\text{lb}/(\text{in}^2 \text{ s})$ . We desire to operate the engine at fixed oxidizer mass flow so we expect mixture ratio variations during the burn. For this reason, we wish to hit the optimum mixture ratio (max.  $c^*$ ) at the mid-web location. You may neglect the burning of the end faces of the fuel grain in your analysis. Under these assumptions, determine:

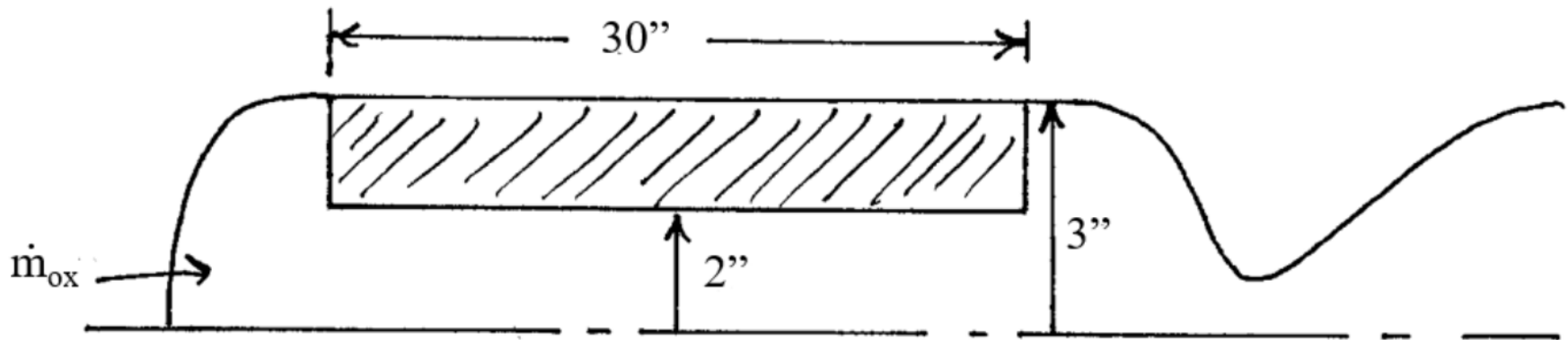




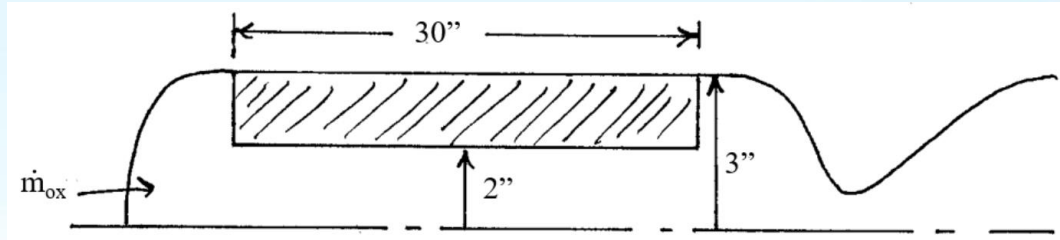
# HW02A – Problem 11.3

For this reason, we wish to hit optimum mixture ratio (max.  $c^*$ ) at the mid-web location. You may neglect the burning of the end faces of the fuel grain in your analysis. Under these assumptions, determine:

- The optimal O/F for this propellant combination.
- The oxidizer flowrate which maximizes performance at mid-web
- The overall O/F shift (max O/F – min O/F) for the firing assuming the fuel is completely consumed.



# HW02A – Problem 11.3

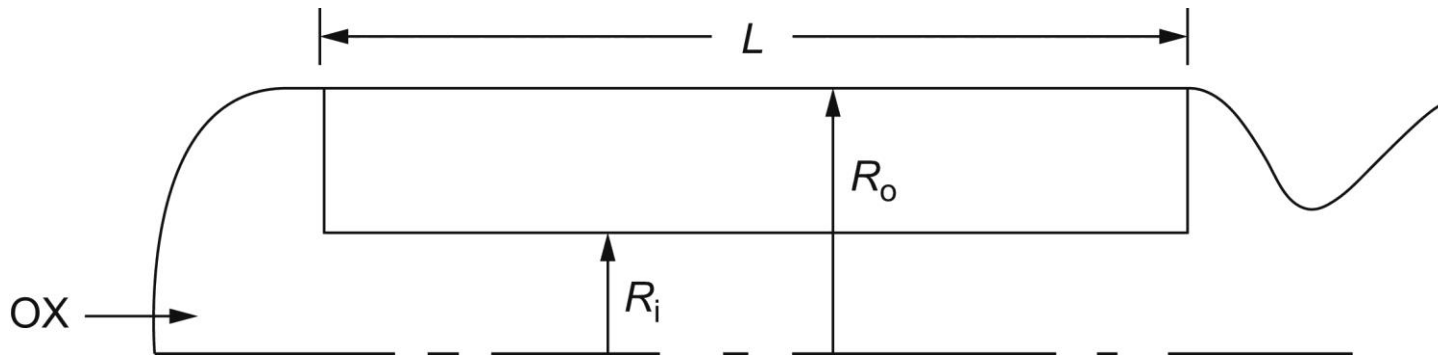


# HW02A – Problem 11.7

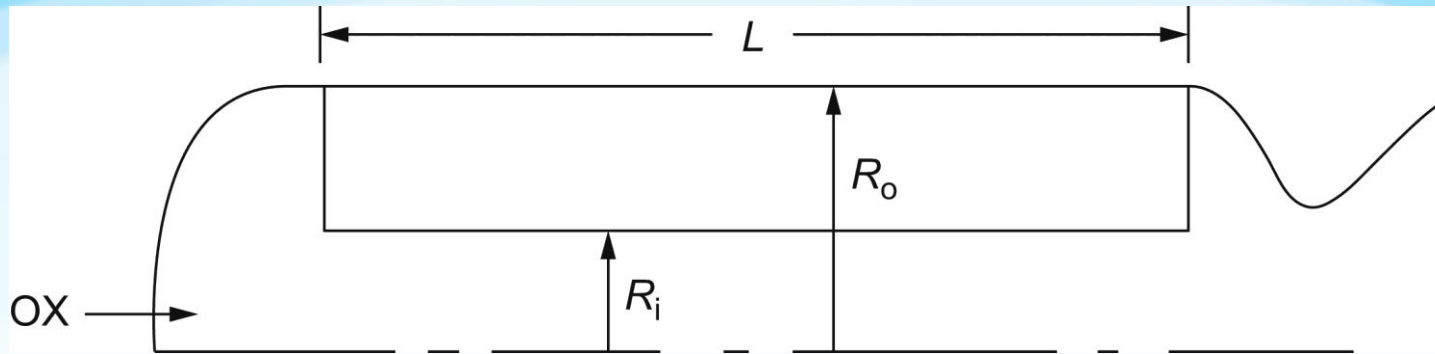
11.7 Consider a hybrid rocket with a simple tubular fuel grain as shown in Figure 11.26. Assume the fuel regression rate is uniform along the length of the grain and obeys

$$r = aG_{\text{ox}}^n \quad (1)$$

where  $G_{\text{ox}}$  is the oxidizer massflux in the fuel port. Assuming the oxidizer mass flow,  $\dot{m}_{\text{ox}}$ , is constant, one can actually solve for the port radius,  $R$ , as a function of time using Eq. 1 and the fact that  $r = dR/dt$ .



# HW02A – Problem 11.7

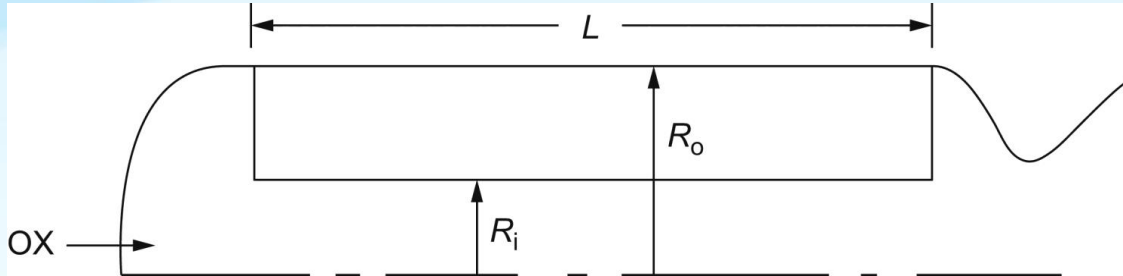


(i) Show that:

$$R(t) = \left[ a(2n + 1) \left( \frac{\dot{m}_{\text{ox}}}{\pi} \right)^n t + R_i^{2n+1} \right]^{\frac{1}{2n+1}}$$

- (ii) Using this result, derive expressions for the fuel flow,  $\dot{m}_f$ , and mixture ratio, O/F, as functions of time. Is there a special value of  $n$  which provides for constant fuel flow and no mixture ratio shifts?
- (iii) Suppose  $L = 50''$ ,  $R_i = 2''$ ,  $R_o = 5''$ ,  $\rho_f = 1 \text{ g/cc}$ , and  $r = 0.1 G_{\text{ox}}^{0.8}$  in inches/s with  $G_{\text{ox}} \sim \text{lb/in}^2 \text{ s}$ . Plot  $R(t)$ ,  $\dot{m}_f(t)$ , and O/F(t) assuming an initial  $G_{\text{ox}}$  of  $1.0 \text{ lb/in}^2 \text{ s}$ .

# HW02A – Problem 11.7



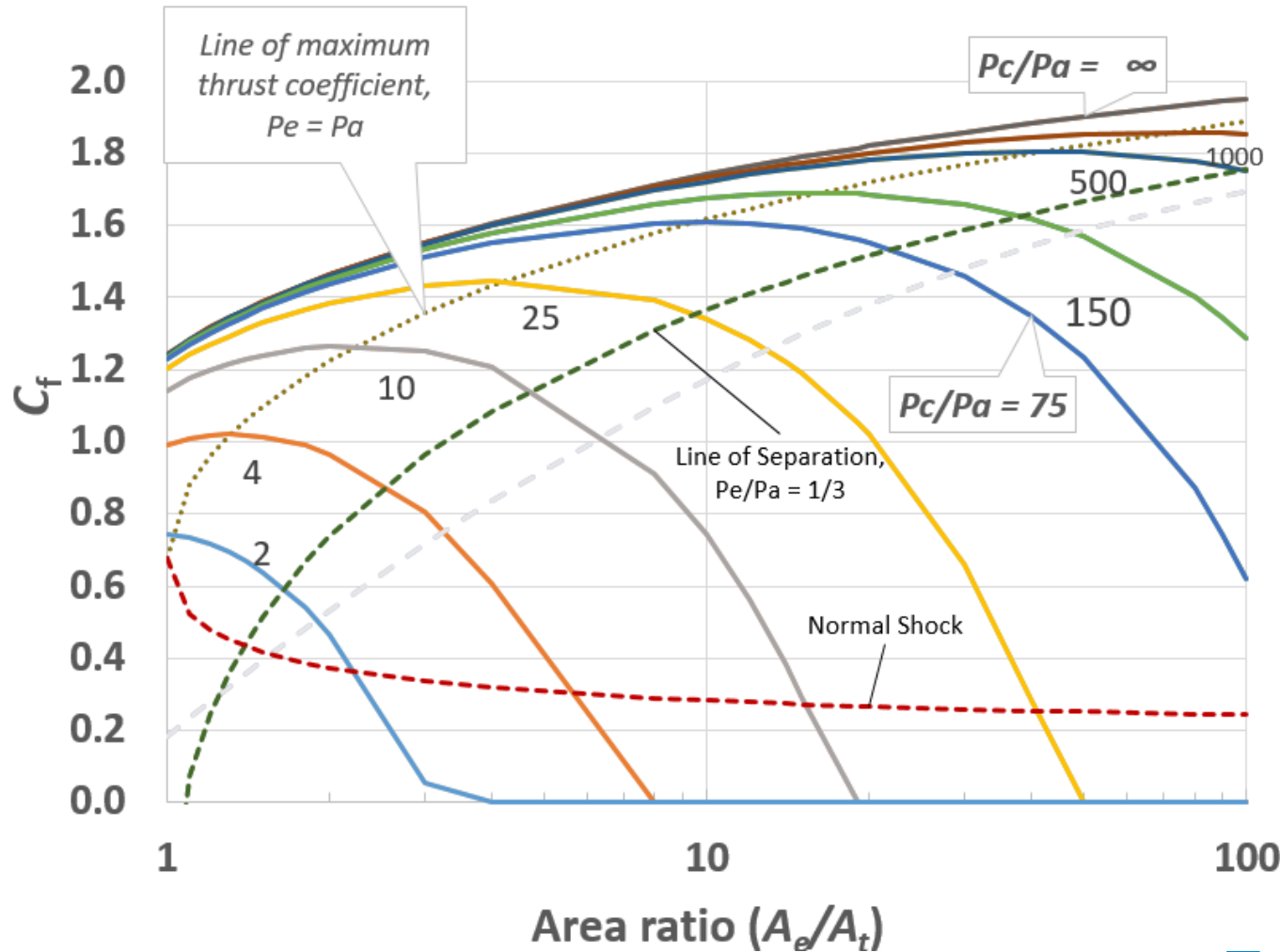
(i) Show that:

$$R(t) = \left[ a(2n + 1) \left( \frac{\dot{m}_{\text{ox}}}{\pi} \right)^n t + R_i^{2n+1} \right]^{\frac{1}{2n+1}}$$

# Special Problems

- Special Problem 02HW-SPA
  - Update the Thrust Coefficient Spreadsheet to include a calculated separation line for  $Pe/Pa = 1/3$ .
  - Update the Thrust Coefficient Spreadsheet to include a calculated separation line for a Normal Shock at the Exit
- Special Problem 02HW-SPB
  - Do an Annotated Bibliography on
    - Frederick, R., and Thomas, D., “Propulsion Research and Academic Programs at the University of Alabama in Huntsville,” 2023 AIAA SciTech, January 26, 2023.

# Special Problem 02HW-SPA

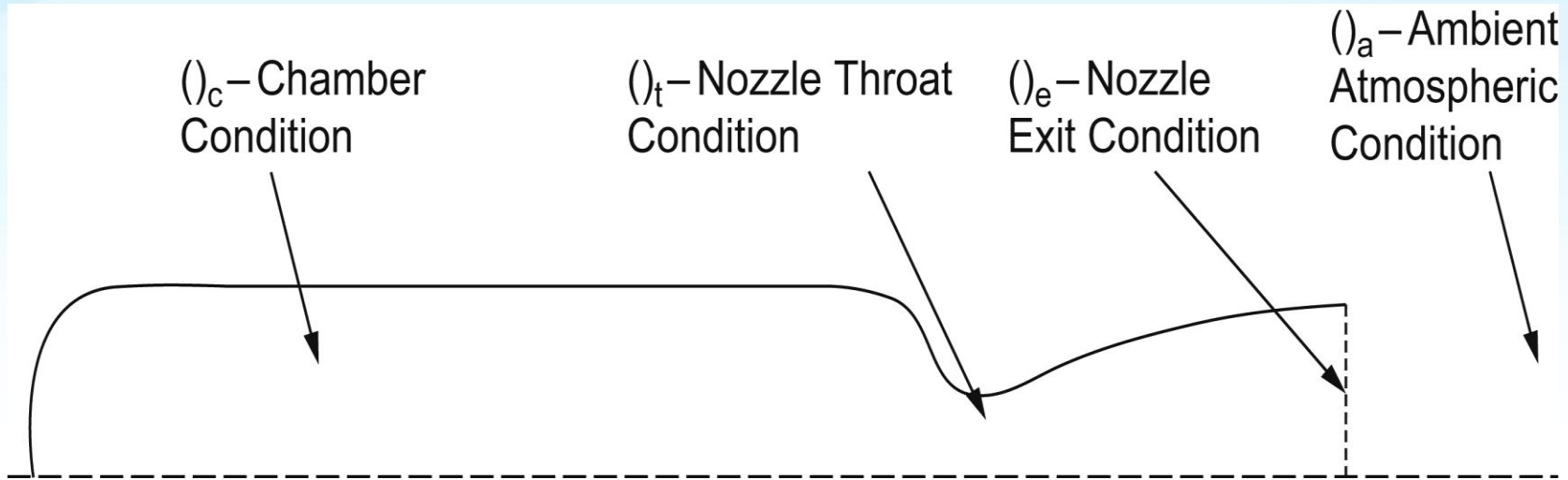




# [G] Supplementary Material on Flow Separation and Normal Shocks in Nozzles



# Nomenclature



## Isentropic Flow Equations

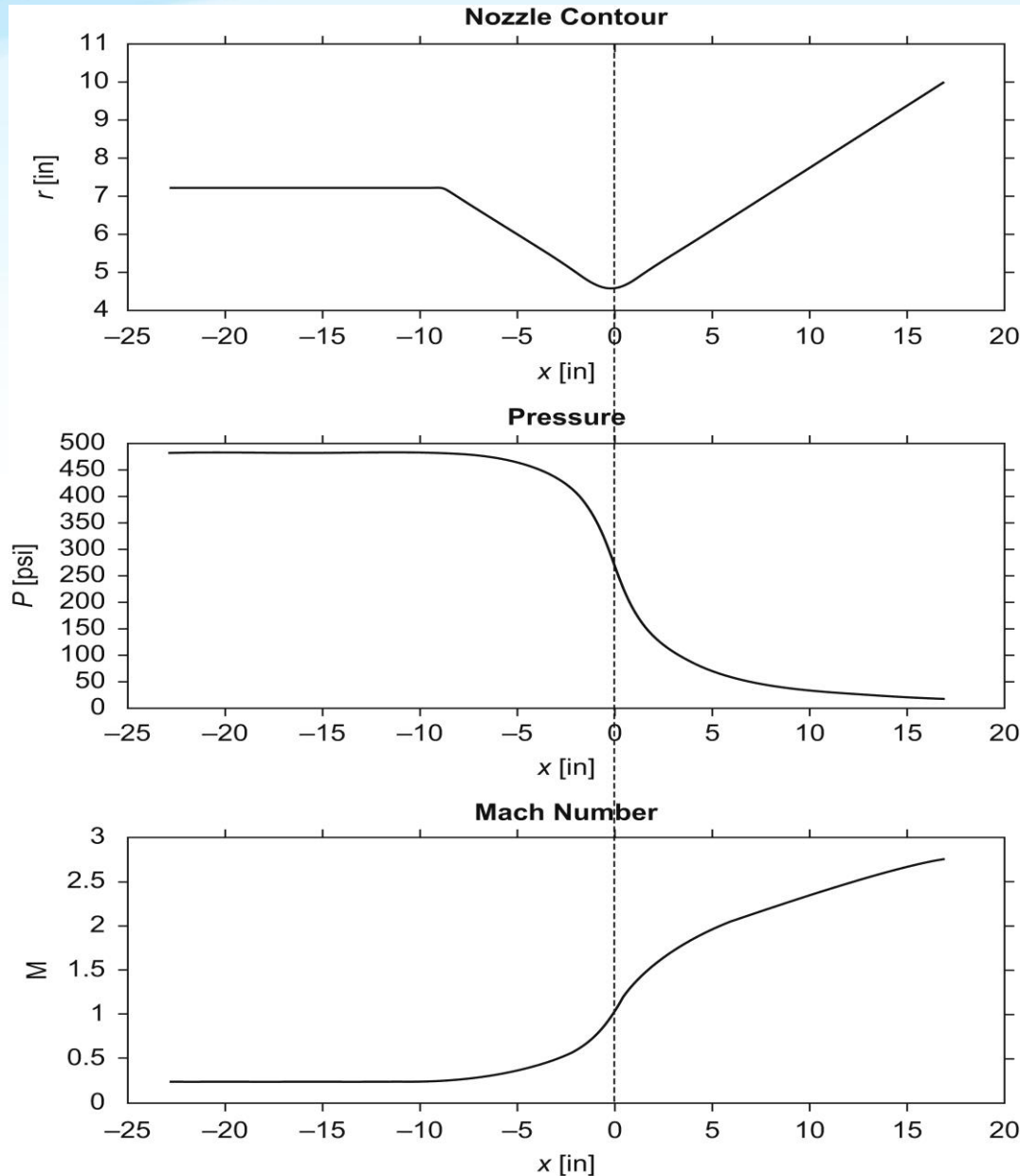
$$p_c/p = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

$$T_c/T = \left(1 + \frac{\gamma - 1}{2} M^2\right)$$

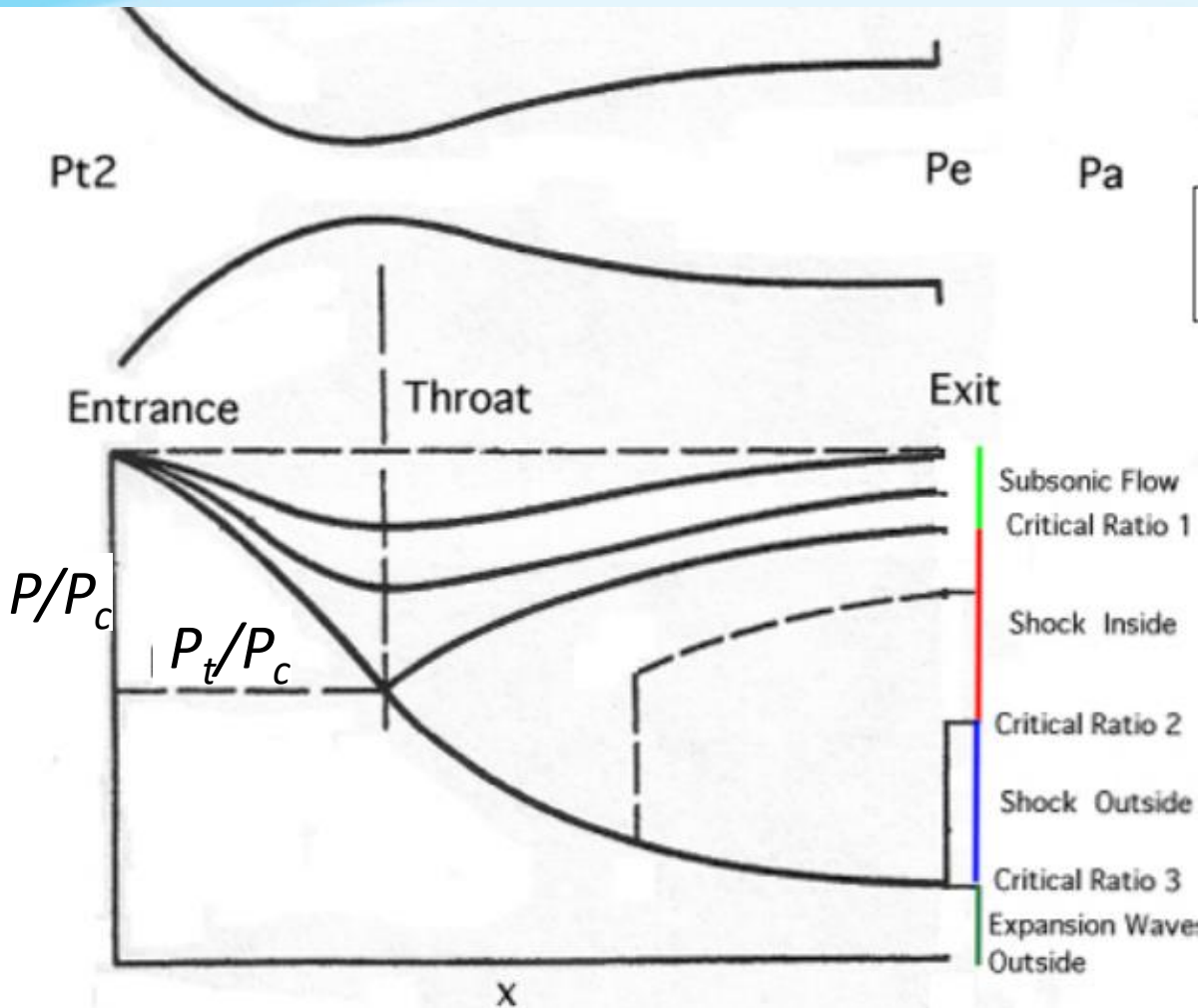
$$M = v/a = \frac{v}{\sqrt{\gamma RT}}$$

$$\frac{A}{A_t} = \frac{1}{M} \left\{ \frac{2 + (\gamma - 1)M^2}{(\gamma + 1)} \right\}^{\frac{\gamma+1}{2(\gamma-1)}}$$

# Review of Compressible Flow



# Nozzle Startup



Stagnation to static pressure

$$p_c/p = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

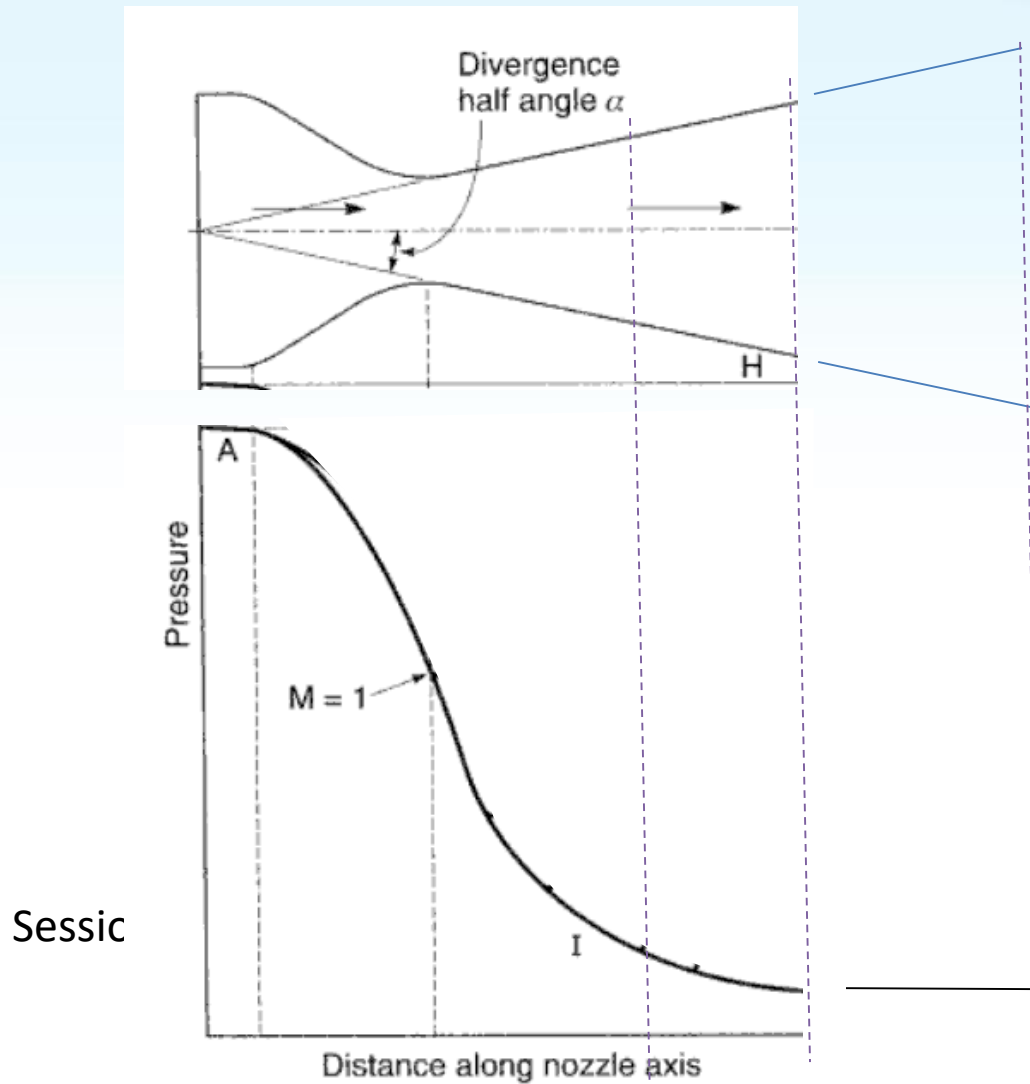
At the Throat

$$p_c/p_t = \left(1 + \frac{\gamma - 1}{2} 1^2\right)^{\gamma/(\gamma-1)} = \left[\frac{\gamma + 1}{2}\right]^{\gamma/(\gamma-1)}$$

For  $\gamma = 1.2$

$$\frac{P_t}{P_c} = 0.565$$

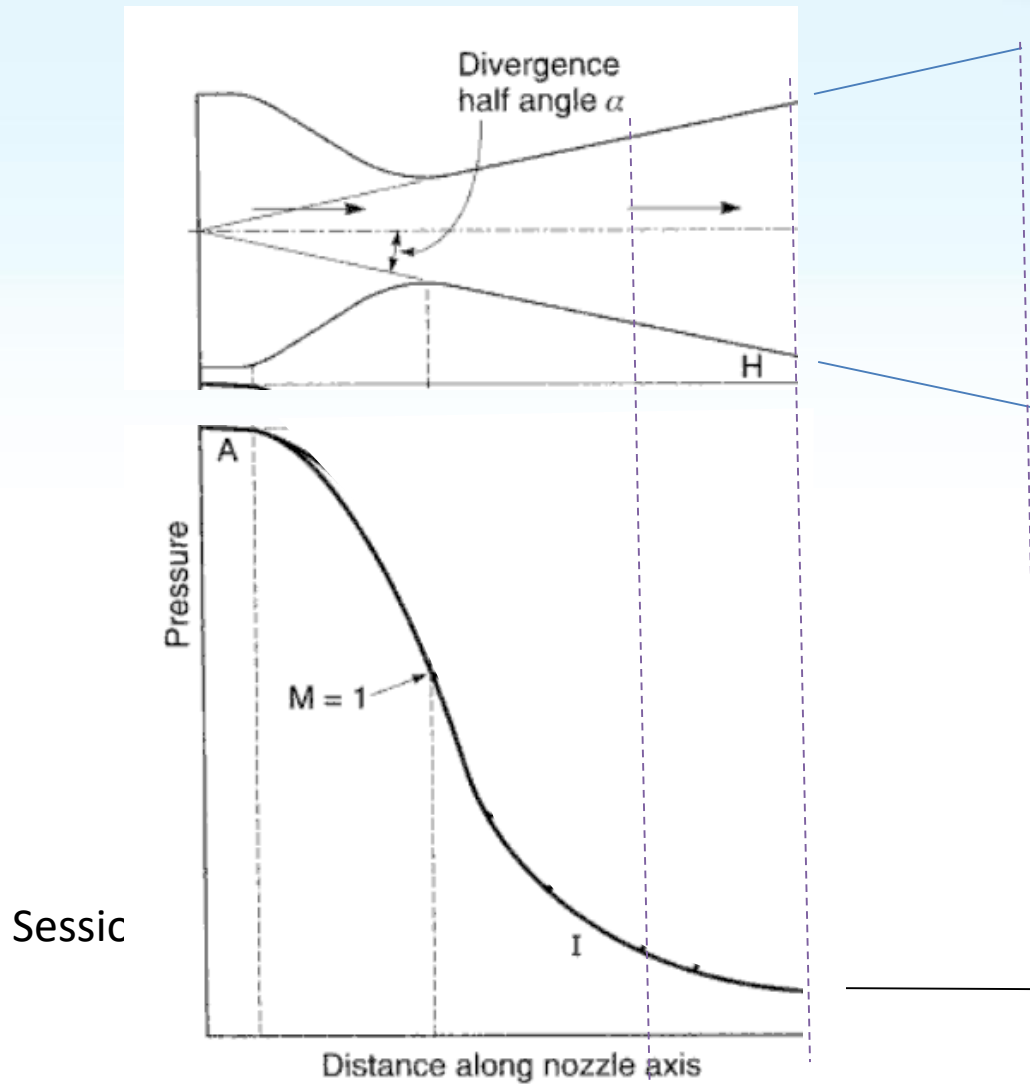
# Idea, Over, and Under Expansion



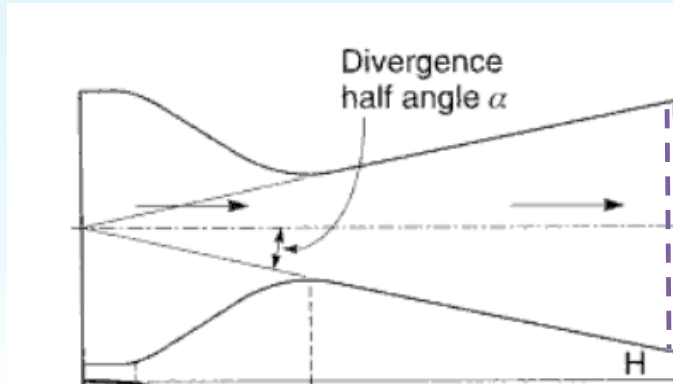
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$p_a$

# Idea, Over, and Under Expansion



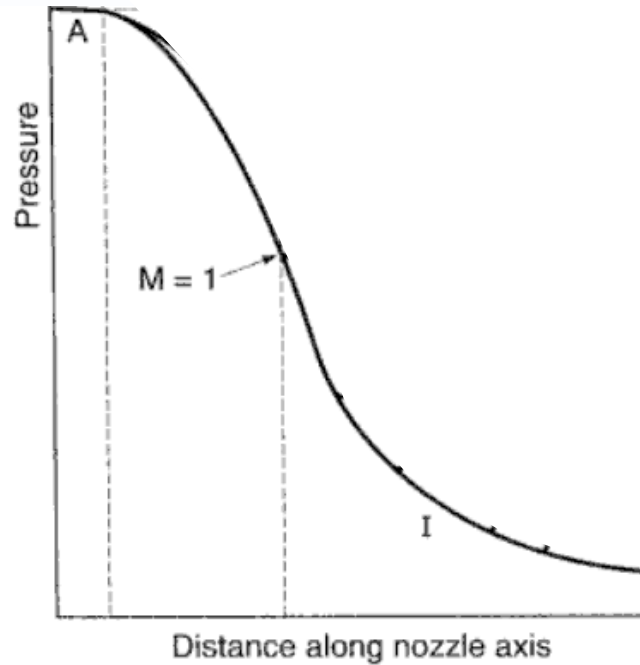
# Normal Shock at Exit



*Normal Shock*

*Pressure Drop Across Normal Shock*

$$\frac{p_3}{p_2} \Big|_{Normal Shock} = \frac{2k}{k+1} M_2^2 - \frac{k-1}{k+1}$$



$$\frac{p_2}{p_1} = \left[ 1 + \frac{k-1}{2} M_2^2 \right]^{\frac{-k}{k-1}}$$

Sessic

$p_3$

$p_2$

# Thrust Coefficient

$$C_F = \sqrt{\frac{2k^2}{k-1} \left( \frac{2}{k+1} \right)^{\frac{k+1}{k-1}} \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} \right] + \frac{p_2 - p_3}{p_1} \frac{A_2}{A_1}}$$

3) Solve ( $C_F$ )

Note on Subscripts:

1 = c (Chamber)

2 = e (Exit)

3 = a (ambient)

$$\frac{p_2}{p_1} = \left[ 1 + \frac{k-1}{2} M_2^2 \right]^{\frac{-k}{k-1}}$$

2) Solve ( $p_2/p_1$ )

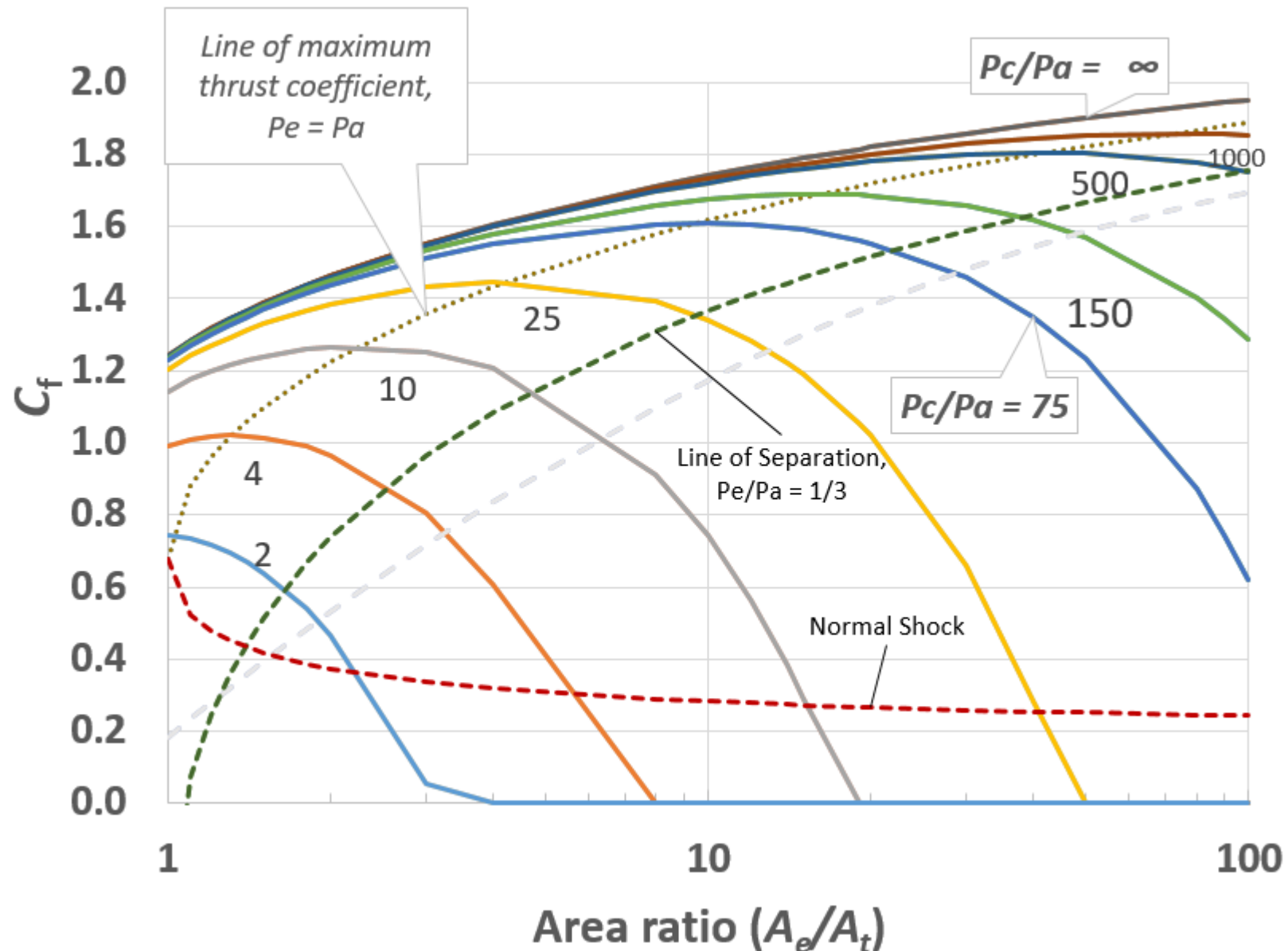
Session 04

Know ( $A_2/A_t$ ,  $\gamma$ )

1) Solve  $M_2$

$$\frac{A_2}{A_t} = \frac{1}{M_2} \left[ \frac{2}{\gamma+1} \left( 1 + \frac{\gamma-1}{2} M_2^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

# Notes/Comments/Questions





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