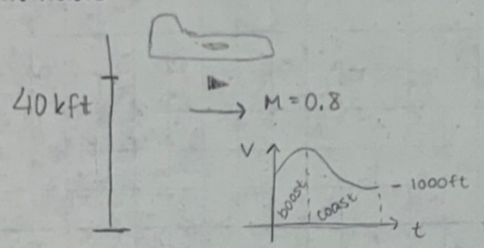
Name: Veronica Loomis

Problem 2.6

Given: Small interceptor launched horizontally
at $M = 0.8$
at $h = 40$ kft
$V = V_0(1 + 2\sin(\frac{\pi t}{2}))$ - missile vel history
$V_0 = V$ @ time of release
Missile is still able to intercept if $V \geq 1000$ ft/s
Neglect drag in BP
During coast:
$m_{missile} = 300$ lb $= 9.375$ slugs
average $C_D = 0.2$
$A_{ref} = 50\ in^2$

Find: (a) Range at the end of boost phase
(b) Total missile range

Schematic:

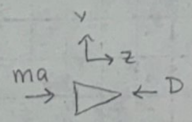


Assumptions:
Find ρ and a (sos) using atm table
↳ Purdue propulsion website

Basic Equations:

(a) $$z = \int_0^{t_b} v(t)\,dt$$

(b) $$\Sigma F_z = ma = m\frac{dv}{dt} = -\frac{1}{2}\rho v^2 C_D A_{ref}$$



Analysis:

(a) $V_0 = M \times a = 0.8 \times 968.076 = 774.46$ ft/s

$$z = \int_0^{t_b} V_0(1 + 2\sin(\tfrac{\pi t}{2}))\,dt$$

$$z = 774.46\left\{t - \frac{4}{\pi}\cos\left(\frac{\pi t}{2}\right)\right\}\Big|_0^1$$

$$z = 774.46\left(1 + \frac{4}{\pi}\right)$$

$$z = 1760.533\ \text{ft}$$

(a) Answer

$$\boxed{\text{Range at end of boost: } 1760.53\ \text{ft}}$$

(b) $$\Sigma F_z = ma = m\frac{dv}{dt} = -\frac{1}{2}\rho v^2 C_D A_{ref}$$

$$m\frac{dv}{dz}\frac{dz}{dt} = m\cancel{v}\frac{dv}{dz} = -\frac{1}{2}\rho v^{\cancel{2}} C_D A_{ref}$$

$$\int dz = \int_{V_0}^{V_f} \frac{-2m}{\rho v C_D A_{ref}}\,dv$$

$$z = \frac{-2m}{\rho C_D A_{ref}} \ln(v)\Big|_{V_i}^{1000}$$

$V_i = V_0(1+2)$
$= 2323.38$

$$z = \frac{-2(9.375)}{(0.000585)(0.2)(50/144)} \ln\left(\frac{1000}{2323.38}\right)$$

$$z = 389087.5\ \text{ft}$$

Answer

(b) $$\boxed{\text{Total missile range: } 389087.5\ \text{ft or } 73.69\ \text{miles}}$$

Comment: It makes sense that the BP range is MUCH smaller than coast range since $t_b = 1$

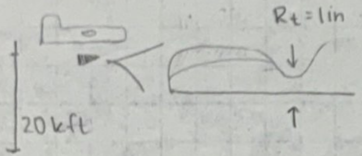
Name: Veronica Loomis

Problem 3.8

Given: Air launched missile
Operates at an altitude of 20 kft
Solid rocket propellant
$c^* = 5000$ ft/s
$\gamma = 1.2$
Nozzle expansion ratio limited to 30
$m_p = 250$ lb
$D_t = 2$ inch

Find: (a) Chamber pressure if $P_{sep} = \frac{1}{3} P_a$
(b) Assuming P_c from (a) is const. at 20 kft, find motor thrust & burn time
(c) Find sea level thrust & burning time using P_c from (a)

Schematic:



Assumptions:
$P_e \neq P_a$
$\varepsilon = 30$
Use Purdue atm data

Basic Equations:
(a) $P_{e,sep} = \frac{1}{3} P_a$ & tables
(b, c) $F = C_F A_t P_c$

Analysis:
(a) $P_{e,sep} = \frac{1}{3} P_a$ @ 20 kft → $P_a = 6.76$ psi (Purdue table)
$P_{e,sep} = 2.25$ psi $= 324.558$ lbf/ft^2
Since we know $\varepsilon = 30$
Use table to find
$\varepsilon = 30 \Rightarrow P_e/P_c = 0.0030119$
$\therefore P_c = P_e / 0.0030119$
$P_c = 107758.56$ lbf/ft^2

a) Chamber pressure is
748.32 lbf/in^2

b) $F = (C_{F,vac} - (P_a/P_c)\varepsilon) A_t P_c$
From (a): $P_c = 748.32$ lbf/in^2
$A_t = \pi D_t^2/4 = \pi$ in^2
$\varepsilon = 30$
$P_a = 6.76$ lbf/in^2
Using Purdue table: $C_{F,vac}(@\gamma = 1.2, \varepsilon = 30) = 1.85928$

$\therefore F = [1.85928 - (0.009033569)30]\pi(748.32)$
$F = 3733.897$ lbf

Finding burn time:
$$t_b = \frac{m_p c^*}{A_t g_e P_c}$$
ft → in
$$t_b = \frac{(250)(5000)(12)}{\pi(32.2)(12)(748.32)}$$
$t_b = 16.512655$ sec

@ 20 kft, the motor thrust is 3733.897 lbf
and the burn time is 16.5 seconds

(c) redo (b) but now at sea level
use $P_c = 748.32\ lbf/in^2$

Burn time

$$t_b = \frac{m_p c^*}{A_t g_e P_c}$$

Since none of these variables changed, we know it's the same t_b we see in (b)

P_a at sea level: $14.696\ lbf/in^2$
Which gives $P_{e,sep} = 4.899\ lbf/in^2$
So $P_e/P_c = 0.0065462$
Gives a new $\varepsilon = 16.437$

$$F = (C_{F vac} - (P_a/P_c)\varepsilon)A_t P_c$$

$$C_{F vac}(\gamma = 1.2, \varepsilon = 16.437) = 1.8$$

$$F = \left[1.8 - \left(\frac{14.696}{748.32}\right)16.437\right]\pi(748.32)$$

$$F = 3472.77\ lbf$$

@ Sea level, the thrust is 3472.77 lbf
and burn time is still 16.5 seconds

Answers:

a) chamber pressure is $748.32\ lbf/in^2$

b) at 20 kft, thrust is 3733.897 lbf
burn time is 16.5 seconds

c) at sea level, thrust is 3472.77 lbf
burn time is still 16.5 seconds

Comment:

Since this is using solid rocket propellant, it makes sense that the burn times stay the same.
It also makes sense that more thrust is needed at sea level since the air is denser

Name: Veronica Loomis

Problem 4.24

Given: Hybrid Rocket

Maintain constant flow rate: $\dot{m}_{ox} = 10\ lb/s$ ↗ lbm?

Presume fuel flow given by: $\dot{m}_f = 3 - 0.6(t/t_b)$ $[lb_f/s]$

$t_b = 50\ sec$

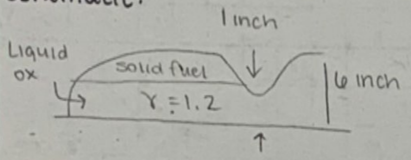
$c^* = 4800 + 800(t/t_b) - 800(t/t_b)^2$ [ft/s]

Find: (a) Initial thrust and Isp of the engine

(b) Expressions for thrust and Isp for any time t $(t \le t_b)$

(c) Maximum Isp delivered during firing. When does max occur?

Schematic:



Assumptions:

Ideal performance

Engine fired in vacuum

Use purdue tables for $\gamma = 1.2$

Basic Equations:

$I_{sp} = c^* C_F / g_e$

$F = C_F A_t P_c$

$\varepsilon = A_e/A_t$

Analysis:

(a) $\varepsilon = \pi(6)^2/\pi(1)^2 = 36$

using table $(\varepsilon = 36, \gamma = 1.2)$ we get

$C_{Fvac} = 1.87535$

$$P_c(t=0) = \frac{c^* \dot{m}}{A_t} = \frac{4800(3+10)}{\pi(1/12)^2} \times \frac{1}{32.2} = \begin{matrix} 88826.25\ lbf/ft^2 \\ \text{or } 616.8\ lbf/in^2 \end{matrix} \left[\frac{ft/s \quad lbf/s}{ft}\right] \times \frac{lbf\, s^2}{32.2\ lbm\ ft}$$

$F = [C_{Fvac}] A_t P_c$

$F = 1.87535(\pi)(616.8)$

$F = 3633.9\ lbf$ at $t = 0$

$$I_{sp} = \frac{c^* C_{F,vac}}{g_e} = \frac{(4800)(1.87535)}{32.2}$$

$I_{sp} = 279.555\ sec$

(b) $C_{Fvac} = 1.87535$

$A_t = \pi \text{ in}^2$

$$P_c = \frac{\left(4800 + 800\,t/t_b - 800(t/t_b)^2\right)\underbrace{(13 - 0.6(t/t_b))}_{\dot m_{ox} + \dot m_f}}{\pi}$$

$$P_c = \frac{1}{\pi}\left(62400 - 7520(t/t_b) - 10880(t/t_b)^2 + 480(t/t_b)^3\right) \times \frac{1}{32.2}$$

$F = C_{Fvac}(A_t)(P_c)$

$$F = 0.05824\left[62400 - 7520(t/t_b) - 10880(t/t_b)^2 + 480(t/t_b)^3\right]$$

$I_{sp} = c^* C_{Fvac} / g_c$

$$I_{sp} = 0.05824\left[4800 + 800(t/t_b) - 800(t/t_b)^2\right] \quad \text{sec}$$

(c) max I_{sp} => max c^* since C_{Fvac} & g_c constant

$$\frac{dc^*}{dt} = 800 - 1600(t/t_b) = 0$$

$t/t_b = 1/2$ We're given $t_b = 50$

Max I_{sp} occurs at 25 seconds

$$I_{sp}(t=25) = 0.05824\left[4800 + 800(1/2) - 800(1/4)\right]$$

Max I_{sp} = 291.2 sec

Answers:

a) $F_{initial} = 3633.9$ lbf
$I_{sp,initial} = 280$ seconds

b) $F(t) = 3634.2 - 437.96(t/t_b) - 633.65(t/t_b)^2 + 27.955(t/t_b)^3$ lbf
$I_{sp}(t) = 279.552 + 46.592(t/t_b) - 46.592(t/t_b)^2$ sec

c) Max I_{sp} = 291.2 seconds
occurs at $t = 25$ seconds

Comment:

If you look at the answer for (b) you see that setting $t=0$ gives you the answers from (a)

↳ They did not at first. I noticed that was wrong when checking my work

Scanned with CamScanner

Name: Veronica Loomis

Problem 4:30

Given: Air launched missile @ $h = 40$ kft

$P_c = 500$ psi

$\gamma = 1.3$

$c^* = 4800$ ft/s

desired thrust = 2000 lb

Find: (a) Nozzle throat and exit diameter (D_t, D_e)

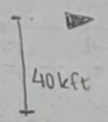
(b) Nozzle mass flow

(c) Isp of rocket engine

(d) Thrust coefficient for the nozzle

(e) Repeat a-d for operating at sea level

Schematic:



Assumptions:

optimum expansion @ nozzle ($P_e = P_a$)

same nozzle in (e)

$\frac{1}{3} P_a = P_{e,sep}$

Use Purdue atm charts

Basic Equations:

$\dot{m} = P_c A_t / c^*$

$I_{sp} = c^* C_f / g_e$

$C_F = C_{F vac} - (P_a/P_c)\varepsilon$

Analysis:

(a) P_a at 40 kft (Purdue) = 2.73 lbf/in^2

$P_e = P_a = 2.73$ lb/in^2

$P_e/P_c = 0.00546$

Using Purdue table ($\gamma = 1.3$)

$\varepsilon = 14.92$, $C_{F,vac} = 1.7243871$

$F = C_F A_t P_c$

$A_t = F/((C_{F,vac} - (P_a/P_c)\varepsilon)(P_c))$

$A_t = 2000/((1.724 - (0.00546)(14.92))(500))$ [in^2]

$A_t = 2.435$ in^2

$\varepsilon = A_e/A_t$

$A_e = 14.92(2.435)$

$A_e = 36.334$ in^2

$D_t = 1.76$ in, $D_e = 6.8$ in

b) $\dot{m} = P_c A_t / c^*$

conv. to inches

$\dot{m} = \left[500(2.435)/(4800)(12)\right] \times 32 \times 12 \left[\frac{lbf}{in^2} \; in^2 \; \frac{s}{in}\right] \; lbf \cdot s / in \times \frac{32.2 \; lbm \; ft}{lbf \; s^2}$

ft → in

$\dot{m} = 8.1167 \; lbm/s$

c) $I_{sp} = c^* C_F / g_e$

$I_{sp} = 4800 \times (1.724 - 0.00546(14.92))/32.2$

$I_{sp} = 244.85$ sec

d) $C_F = C_{F vac} - (P_a/P_c)\varepsilon$

$C_F = 1.724 - 0.00546(14.92)$

$C_F = 1.6425$

e) i) P_a @ SL $= 14.696 \; lbf/in^2$

at low alt, $P_{e,sep} = 1/3 P_a = 4.89867$

$P_e/P_c = 0.009797$

Table ($\gamma = 1.3$)

$\varepsilon = 9.84$, $C_{F,vac} = 1.687$

$A_t = F/(C_{F,vac} - (P_a/P_c)\varepsilon)(P_c)$

$A_t = 2000/(1.687 - (0.02939)(9.84))(500)$

$A_t = 2.8616 \; in^2$

$\varepsilon = A_e/A_t$

$A_e = (9.84)(2.8616)$

$A_e = 28.1585$

$D_t = 1.91$ in, $D_e = 5.9877$ in

ii) $\dot{m} = P_c A_t / c^*$

$\dot{m} = (500(2.8616)/(4800)(12))(32)(12)$

$\dot{m} = 9.539 \; lbm/s$

iii) $I_{sp} = c^* C_F / g_e = 4800(1.687 - (0.02939)(9.84))/32.2$

$I_{sp} = 208.3677$ sec

iv) $C_F = 1.687 - (0.02939)(9.84)$

$C_F = 1.3978$

Answers: a) $D_t = 1.76$ in, $D_e = 6.8$ in

b) $\dot{m} = 8.1167$ lbm/s

c) $I_{sp} = 244.85$ sec

d) $C_F = 1.6425$

e) i) $D_t = 1.91$ in, $D_e = 5.9877$ in

ii) $\dot{m} = 9.539$ lbm/s

iii) $I_{sp} = 208.3677$ sec

iv) $C_F = 1.3978$

Comments:

In order to reach $F = 2000$ lb with $P_c = 500$ lbf/in^2

the throat at sea level > throat for altitude of 40kft

When looking at ambient pressures compared to P_c,

$$\varepsilon_{40kft} = 14.92 > \varepsilon_{SL} = 9.84$$

since as $P_e \downarrow$, $\varepsilon \uparrow$

This is why $D_{e,40kft} > D_{e,SL}$