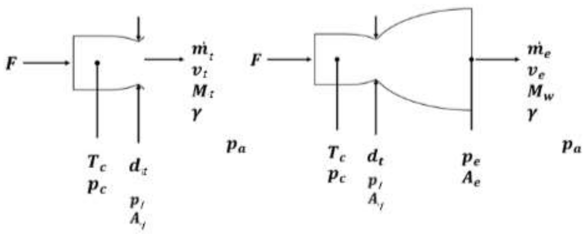
HW1 SP01B

Monday, January 16, 2023 20:13

1. **Name:** Veronica Loomis
2. **Given:** Rocket motor with a converging nozzle, and then a second configuration adding a diverging nozzle.
3. **Find:**
   a. Starting with the complete linear momentum (Equation 3.37 in Appendix B) equation for a control volume, derive the thrust equation in a systematic manner for the rocket with the converging nozzle. Document each simplifying assumption, define the control volume, and use the symbols found in the schematic or consistent with the course textbook. Appendix B shows the basic starting equation with some explanations.
   b. Repeat the process in for the rocket with a converging-diverging nozzle; determine the force that the diverging section of the nozzle has on the circular ring of material around the rocket throat. You will need to draw a separate control volume for this analysis. For Special Problem SP01B assume that the cross-sectional area of the nozzle throat material is A_{ring}.
   c. Comment of the ratio of the thrust of the second rocket over the first for a typical supersonic converging-diverging nozzle. When is the diverging section adding to the overall thrust of the rocket motor?
4. **Schematic:**

Converging Converging-Diverging



Equations:

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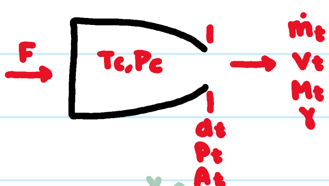
$$\sum F = \frac{d}{dt}\left(\int_{cv} \vec{V}\rho \, d\forall\right) + \int_{cs} \vec{V}\rho(\vec{v}\cdot\vec{n})\, dA \qquad [3.37]$$

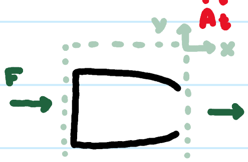
Assumptions: $(P_t \neq P_a)$

Analysis

Derive the thrust equation

a)





Forces : F, P

Momentum : $\dot{m}V_t$

$$\int_{cs} \vec{V}\rho(\vec{V}\cdot\vec{n})\,dA$$

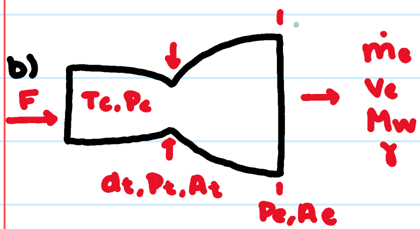
$$V(-V)\rho A_t = -\dot{m}_t V_t$$

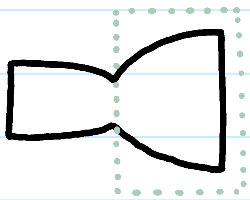
$$F + P_t A_t - P_a A + \theta A_{ring} = -\dot{m}_t V_t$$

Answer

$$\boxed{F = P_a A - P_t A_t - \theta A_{ring} - \dot{m}_t V_t}$$

b)





Forces: F, P ($P_t \neq P_a$)

Momentum: $\dot{m} V_t$

$$\int_{cs} \vec{V}\rho(\vec{V}\cdot\vec{n})\,dA$$

$$V(-V)\rho A_t = -\dot{m}_e V_e$$

$$F + P_e A_e - P_a A + \theta_1 A_{ring,L} + \theta_2 A_{ring,R} = -\dot{m}_e V_e$$

Answer

$$\boxed{F = P_a A - P_e A_e - \theta_1 A_{ring,L} - \theta_2 A_{ring,R} - \dot{m}_e V_e}$$

c)

Answer

$$\boxed{\frac{F_2}{F_1} = \frac{P_a A - P_e A_e - \theta_1 A_{ring,L} - \theta_2 A_{ring,R} - \dot{m}_e V_e}{P_a A - P_t A_t - \theta A_{ring} - \dot{m}_t V_t}}$$

Comment:

The second force is very similar to the first, but the shear stress on the throat and exit come more into play