Name: Veronica Loomis
Problem 11.4

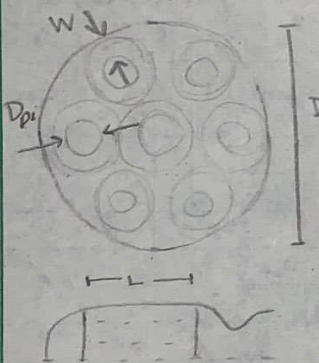
Given:

Hybrid Rocket with 7 port cluster
LOX / HTPB so Isp = 350 sec at O/F = 2.5
average thrust required: 500,000 lbf for t_b = 100 sec
HTPB fuel density: 0.036 lb/in^3
average c^* at optimal O/F: 5000 ft/s (at O/F = 2.5)

$G_{ox,initial}$ = 1.0 lb/in^{2}s
$r = 0.2\, G_{ox}^{0.68}$

Find:
a) Initial port diameter (D_{pi})
b) total web distance burned (w)
c) fuel grain length (L) and overall motor diameter (D)

Schematic:



Assumptions:
Characteristic velocity efficiency of 1.0
Initial Gox is for all 7 ports combined
$\dot{m}_{ox}$ is constant
Burning rate is linear
Given Isp and F are the initial values

Basic Equations:

$G_{ox} = (\dot{m}_{ox} / N A_p)$

$r = a\, G_{ox}^{n}$

$\dot{m}_{ox} = \dfrac{\dot{m}}{1 + 1/OF}$

$r_{av} = \dfrac{r_c + r_f}{2}$

$w = r_{av} t$

$c^* = P_c A_b / (\dot{m}_{ox} + \dot{m}_f)$

$\dot{m}_f = r \rho_f N\, Per(w) L$

Analysis

a) $r_i = a G_{ox,init}^n$

$r_i = 0.2(1)^{0.68} = 0.2 \text{ in/sec}$

$\dot{m}_{ox} = \dfrac{F}{I_{sp}\, g\,(1 + 1/OF)} = \dfrac{500\,000 \text{ lbf}}{(350 \text{ sec})(32.2 \text{ ft/s}^2)(1 + 1/2.5)} \times \dfrac{32.2 \text{ lbm ft}}{s^2 \text{ lbf}}$

$\dot{m}_{ox} = 1020.4 \text{ lbm/s}$

$G_{ox,init} = \dfrac{\dot{m}_{ox}}{7\pi R_i^2}$

$R_i^2 = \dfrac{\dot{m}_{ox}}{7\pi G_{ox,init}}$

$R_i = \sqrt{\dfrac{1020.4 \text{ lbm/s}}{7\pi (1.0 \text{ lb/in}^2\text{s})}}$

$R_i = 6.81179 \text{ in}$

$\boxed{D_{pi} = 2R = 13.62 \text{ in}}$

b) $w = \dfrac{r_i + r_f}{2} (t_b)$

$r_f = 0.2\left(\dfrac{\dot{m}_{ox}}{7\pi R_f^2}\right)^{0.68}$

$R_f = R_i + w$

$r_f = 0.2\left(\dfrac{1020.4}{7\pi(6.81179 + w)^2}\right)^{0.68}$

sub in

$$w = \left[\dfrac{0.2 + 0.2\left(\dfrac{1020.4}{7\pi(6.81179 + w)^2}\right)^{0.68}}{2}\right] \times 100 \quad [\text{in/s}] \times [\text{s}]$$

$$w = 10 + 10\left(\dfrac{46.4}{(6.81179 + w)^2}\right)^{0.68}$$

solve for w in MATLAB

$\boxed{w = 12.435 \text{ in}}$

c) $\dot{m}_f = r \rho_f N \, Per(w) L$

$$\dot{m}_f = 0.2 \left(\frac{\dot{m}_{ox}}{\pi R^2 (7)} \right)^{0.68} (0.036)(7)\, 2\pi (\,R_{mid}\,)\, L$$

Use R = mid burn radius

$R = 6.81179 + (12.435/2) = 13.03$ in

$\dot{m}_f = \frac{1}{7}\dot{m}_{ox} \, (1/O/F) = 1020.4$ lbm/s $(1/2.5)$

$\dot{m}_f = 408.16$ lbm/s

Rearrange & substitute

$$L = \frac{408.16 \text{ lbm/s}}{\underbrace{0.2 \left(\frac{1020.4}{7\pi (13.03)^2} \right)^{0.68}}_{\text{in/s}} (0.036 \text{ lb/in}^3)(7)(2\pi)(13.03 \text{ in})}$$

$\boxed{L = 238.98 \text{ in}}$

overall D

$D = (R_o + w)(2)(3)$

$D = (6.81179 + 12.435)(6)$ in

$\boxed{D = 115.48 \text{ in}}$

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SP 02 BA

Given:

$$R(t) = \left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right]^{1/2n+1}$$

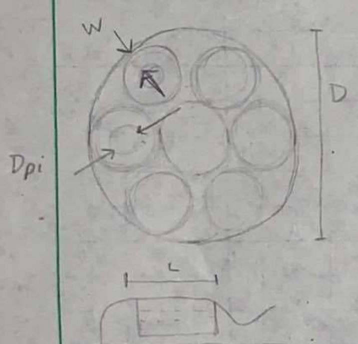
$$\dot{m}_f(t) = 2a\pi^{1-n} \rho_f L \dot{m}_{ox}^n \left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right]^{\frac{1-2n}{1+2n}}$$

Find:

Repeat 11.4 and find:

a) Total web dist burned
b) L assuming O/F is at mid web
c) overall motor diameter
d) % diff of web distance and L here vs in 11.4
e) 5 plots
  1) Mixture ratio & G_{ox} as f(t)
  2) C^* as f(O/F) and point out motor start & stop points
  3) $\dot{m}_f$, $\dot{m}_{ox}$, $\dot{m}_{prop}$ as f(t)
  4) P_c as f(t) with $A_t = 300 \text{ in}^2$ and using c^* from table
  5) F_{vac} as f(t) assuming $C_{fv} = 1.70$

Schematic:



Assumptions:
Same as 11.4
Use the 2 equations above

Basic Equations:

$G_{ox} = (\dot{m}_{ox} / N A_p)$

$r = a G_{ox}^n$

$\dot{m}_{ox} = \frac{\dot{m}}{1 + 1/OF}$

$r_{av} = \frac{r_c + r_f}{2}$

$w = r_{av} t$

$c^* = P_c A_t / (\dot{m}_{ox} + \dot{m}_f)$

$\dot{m}_f = r \rho_f N \, Per(w) L$

Analysis:

a) Total web dist. burned

$$\dot{m}_{ox} = \frac{F}{I_{sp} g}\left(\frac{O/F}{1+O/F}\right) = \frac{500\,000 \text{ lbf}}{(350\text{ s})(32.2\text{ ft/s}^2)}\left(\frac{2.5}{3.5}\right)\left(\frac{32.2 \text{ lbm ft}}{\text{lbf s}^2}\right)$$

$$\dot{m}_{ox} = 1020.4 \text{ lbm/s}$$

$$G_{ox,init} = \dot{m}_{ox}/(7\pi R_i^2)$$

$$R_i = \sqrt{\frac{1020.4 \text{ lbm/s}}{7\pi\,(1.0 \text{ lbm/in}^2\text{s})}}$$

$$R_i = 6.81179 \text{ in}$$

Now, we know $t_b = 100$s

$$R(100) = \left[\underbrace{0.2\,(1.36+1)\left(\frac{1020.4}{7\pi}\right)^{0.68}(100)}_{[\text{in/s}][\text{s}]} + (6.81179)^{(1.36+1)}\right]^{1/(1.36+1)}$$

$$R(100) = 16.379 \text{ in}$$

$$W = R(100) - R_i = 16.379 - 6.81179 \text{ in}$$

$$\boxed{W = 9.567 \text{ in}}$$

b) $R_{mid} = R_i + W/2 = 6.81179 + 4.7835$

$$R_{mid} = 11.595 \text{ in}$$

$$\dot{m}_f = 0.2\left(\frac{\dot{m}_{ox}}{7\pi R^2}\right)^{0.68}(0.036)\,7(2\pi)\,R_{mid}\,L$$

$$L = \frac{\dot{m}_{ox}\,(1/O/F) \text{ lbm/s}}{\underbrace{0.2\left(\frac{1020.4}{7\pi(11.595)^2}\right)^{0.68}}_{\text{in/s}}(0.036 \text{ lb/in}^3)\,7(2\pi)(11.595 \text{ in})}$$

$$\boxed{L = 229.15 \text{ in}}$$

c) overall diameter

$$D = (R_i + w)(3)(2)$$

$$D = (6.81179 + 9.567)(6)$$

$$\boxed{D = 98.273 \text{ in}}$$

d) % difference in w and L from 11.4 and here

W:

11.4 : $w = 12.435$ in

SP : $w = 9.567$ in

$$\%\text{ diff web} = \frac{SP - 11.4}{SP} = \frac{9.567 - 12.435}{9.567}$$

$$\boxed{\%\text{ diff web} = 29.978\%}$$

L:

11.4 : 238.98 in

SP : 229.15 in

$$\%\text{ diff L} = \frac{229.15 - 238.98}{229.15}$$

$$\boxed{\%\text{ diff L} = 4.2897\%}$$

e) Plots

$$O/F = \frac{\dot{m}_{ox}}{(7)2a\pi^{1-n}\rho_f L \dot{m}_{ox}^{n}\left[a(2n+1)\left(\frac{\dot{m}_{ox}}{7\pi}\right)^{n} t + R_i^{2n+1}\right]^{1-2n/1+2n}}$$

c^* as $f(O/F)$ → plot Mixture ratio & c^* table & curve fit

This is done in excel

$$c^* = 359.8(O/F)^3 - 2848.3(O/F)^2 + 7136.2(O/F) + 148.72$$

at $O/F = 2.5$, $c^* = 5809.22$ ft/s

at $R_o = 6.81179$ in, $O/F = 2.1233$ which gives $c^* = 5903.984$ ft/s

$$P_c = \frac{c^*(\dot{m}_{ox} + \dot{m}_f)}{A_t} = \frac{(5903.984 \text{ ft/s})(\dot{m}_{ox}\,[\text{lbm/s}] + \dot{m}_f\,[\text{lbm/s}])}{(300 \text{ in}^2)(1/144)\,\text{ft}^2/\text{in}^2 \times 32.2 \text{ lbm ft/lbf s}^2}$$

Assuming same A_t for thrust in vacuum

Problem 11.4-b

Iteratively solving for w

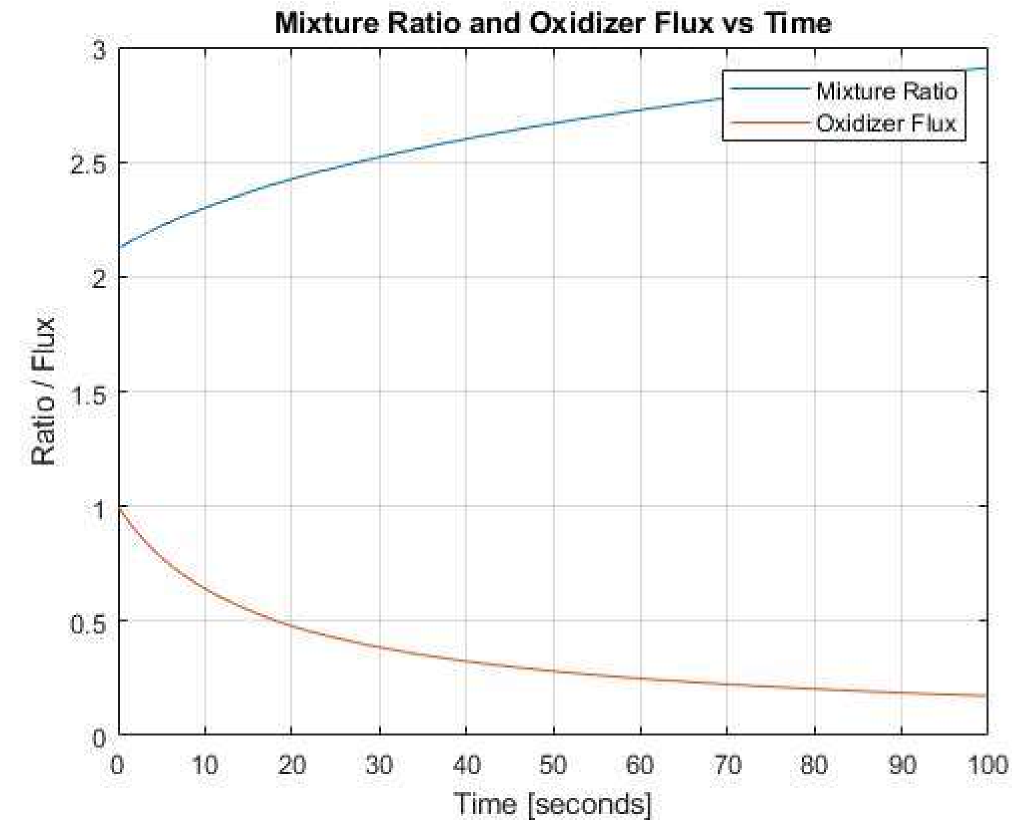
```
wGuess = 5;
tol = 1;

while tol > 0.001
    w = 10 + 10*(46.4/(6.81179+wGuess)^2)^0.68;
    tol = abs(wGuess-w);
    wGuess = w;
end

fprintf('The web distance is %0.3f inches\n\n', wGuess)
```

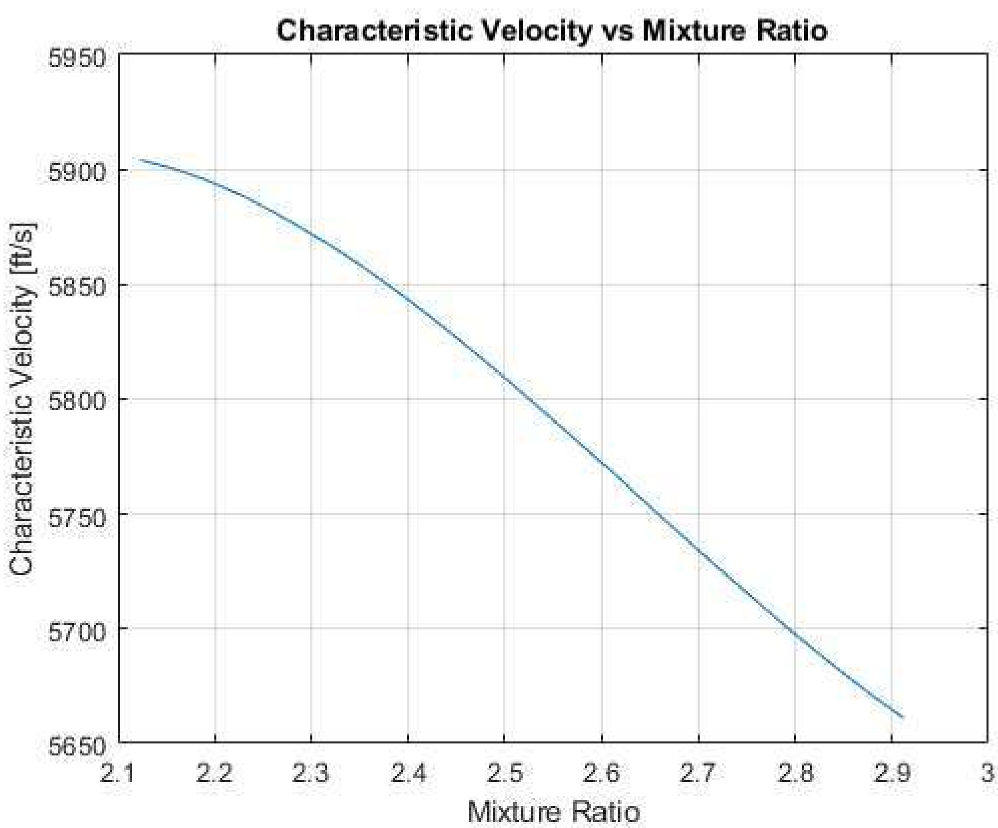
The web distance is 12.435 inches

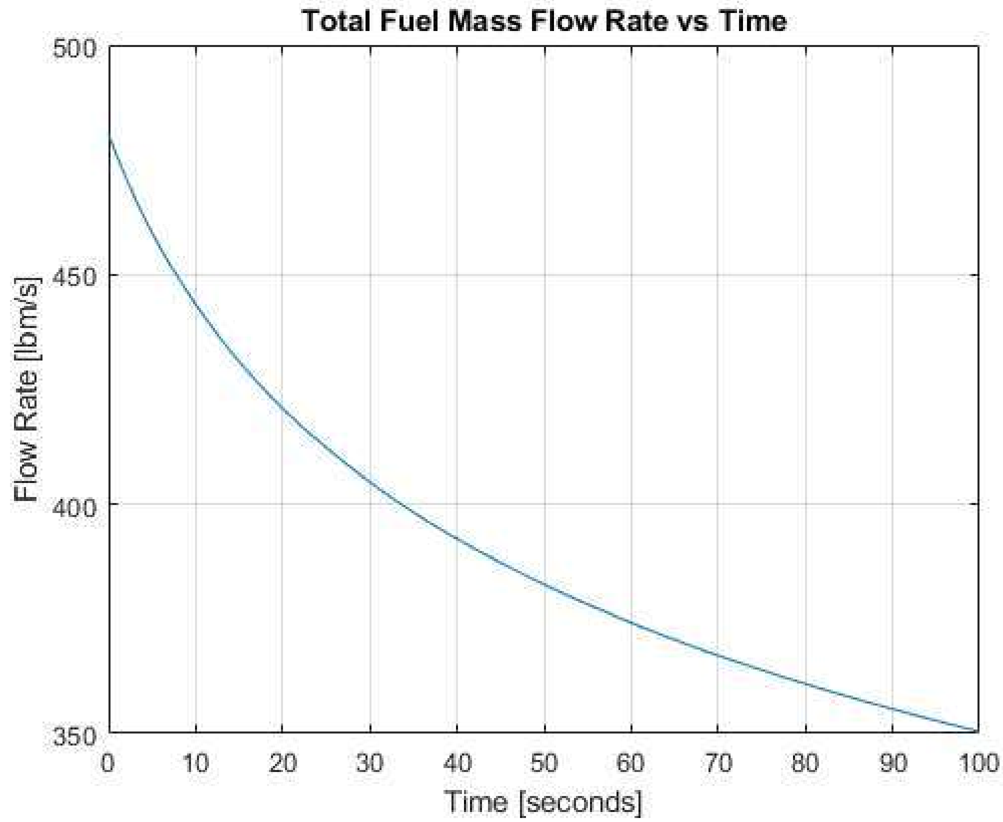
Special Problem Plots

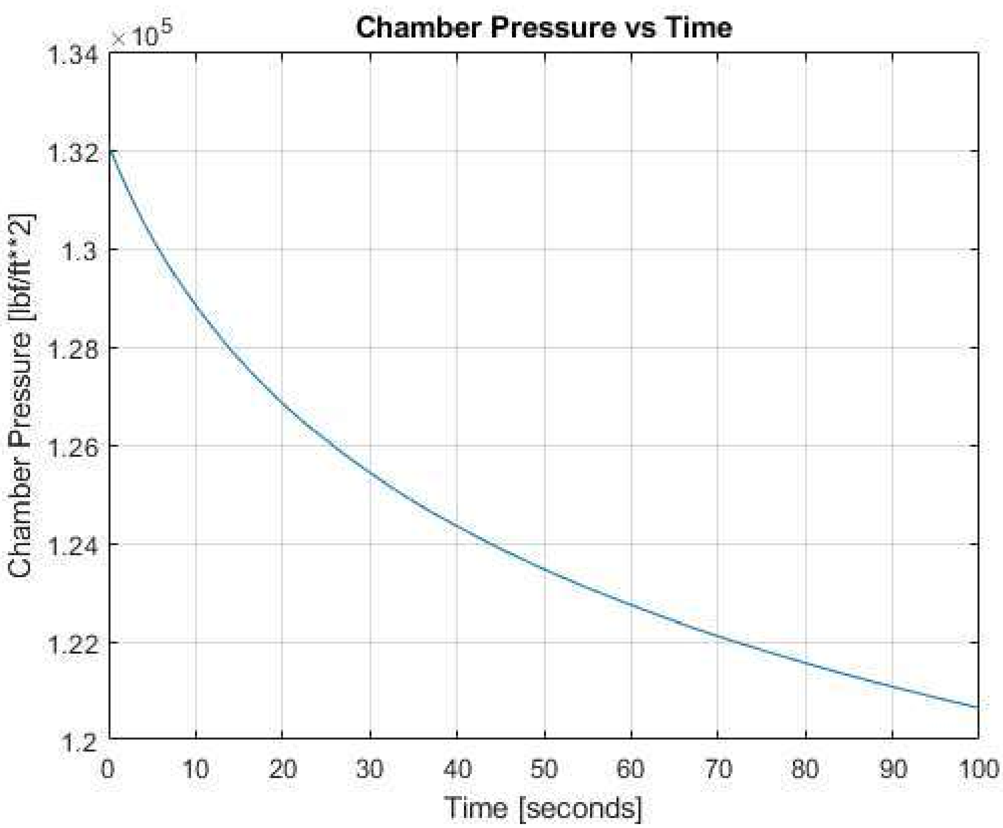
```
mox = 1020.4; % lbm/s
rhof = 0.035; % lb / in**3
a = 0.2;
n = 0.68;
L = 229.15; % in
Rinit = 6.81179; % in
cfv = 1.7;

t = linspace(0,100,1000);

% calculated in Excel
cStarConst = 5903.984; % ft/s
At = 300/144;      % ft**2

for i=1:1000
    R(i) = (a*(2*n+1)*((mox/(7*pi))^n)*t(i) + Rinit^(2*n+1))^(1/(1+2*n));
    Gox(i) = mox / (7*pi*R(i)^2);
    mf(i) = a*((Gox(i))^n)*rhof*7*2*pi*R(i)*L;
    OF(i) = mox / mf(i);

    cStar(i) = 359.8*(OF(i))^3 - 2848.3*(OF(i))^2 + 7136.2*(OF(i)) + 148.72;
    Pc(i) = cStarConst*(mox+mf(i))/(At*32.2);

    tVac(i) = cfv*Pc(i)*At;
end

% Plots
figure(1)
plot(t,OF)
title('Mixture Ratio and Oxidizer Flux vs Time')
xlabel('Time [seconds]')
ylabel('Ratio / Flux')
```

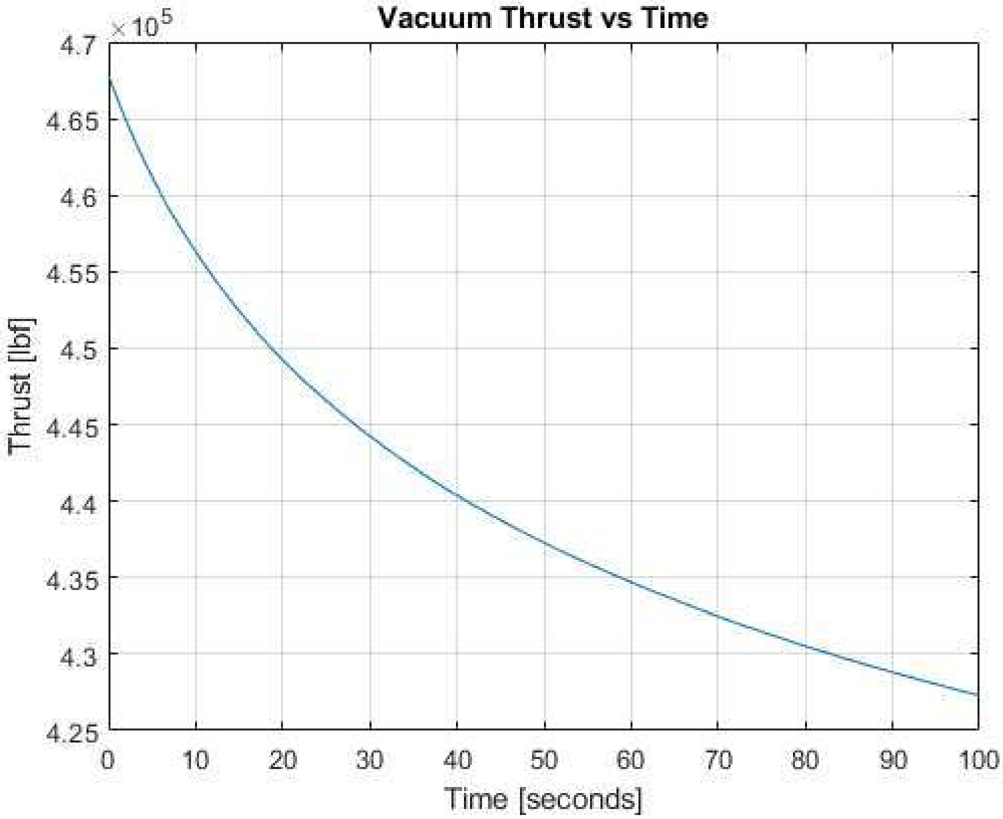
```
hold on
grid on
plot(t,Gox)
legend('Mixture Ratio','Oxidizer Flux')

figure(2)
plot(OF, cStar)
grid on
title('Characteristic Velocity vs Mixture Ratio')
xlabel('Mixture Ratio')
ylabel('Characteristic Velocity [ft/s]')

figure(3)
plot(t, mf)
grid on
title('Total Fuel Mass Flow Rate vs Time')
xlabel('Time [seconds]')
ylabel('Flow Rate [lbm/s]')

figure(4)
plot(t, Pc)
grid on
title('Chamber Pressure vs Time')
xlabel('Time [seconds]')
ylabel('Chamber Pressure [lbf/ft**2]')

figure(5)
plot(t, tVac)
grid on
title('Vacuum Thrust vs Time')
xlabel('Time [seconds]')
ylabel('Thrust [lbf]')
```


Mixture Ratio and Oxidizer Flux vs Time
Mixture Ratio
Oxidizer Flux
Ratio / Flux
Time [seconds]
0
0.5
1
1.5
2
2.5
3
0
10
20
30
40
50
60
70
80
90
100


Characteristic Velocity vs Mixture Ratio
Characteristic Velocity [ft/s]
Mixture Ratio
5650
5700
5750
5800
5850
5900
5950
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
3

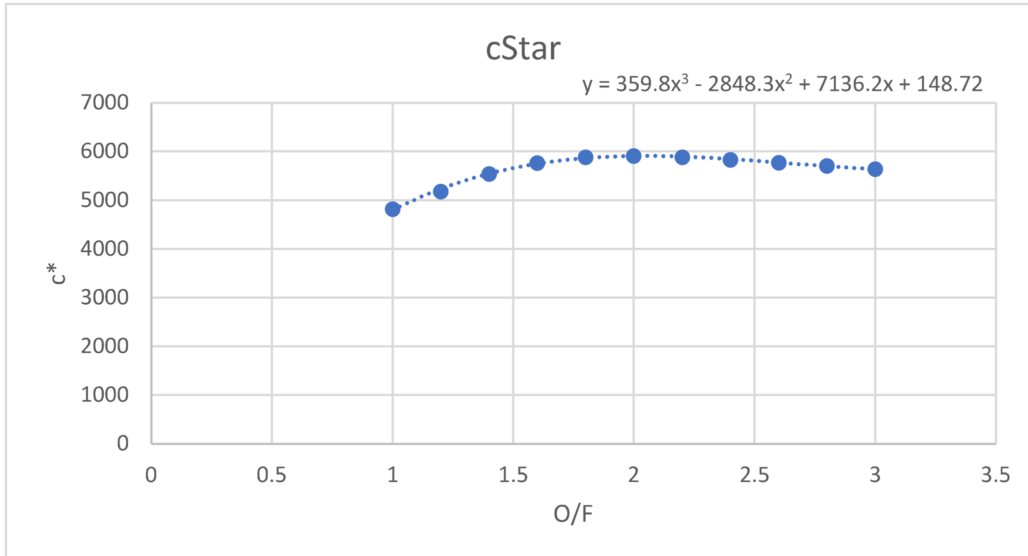

Total Fuel Mass Flow Rate vs Time
Flow Rate [lbm/s]
Time [seconds]
500
450
400
350
0
10
20
30
40
50
60
70
80
90
100


Chamber Pressure vs Time
×10^5
Chamber Pressure [lbf/ft**2]
Time [seconds]
1.34
1.32
1.3
1.28
1.26
1.24
1.22
1.2
0
10
20
30
40
50
60
70
80
90
100

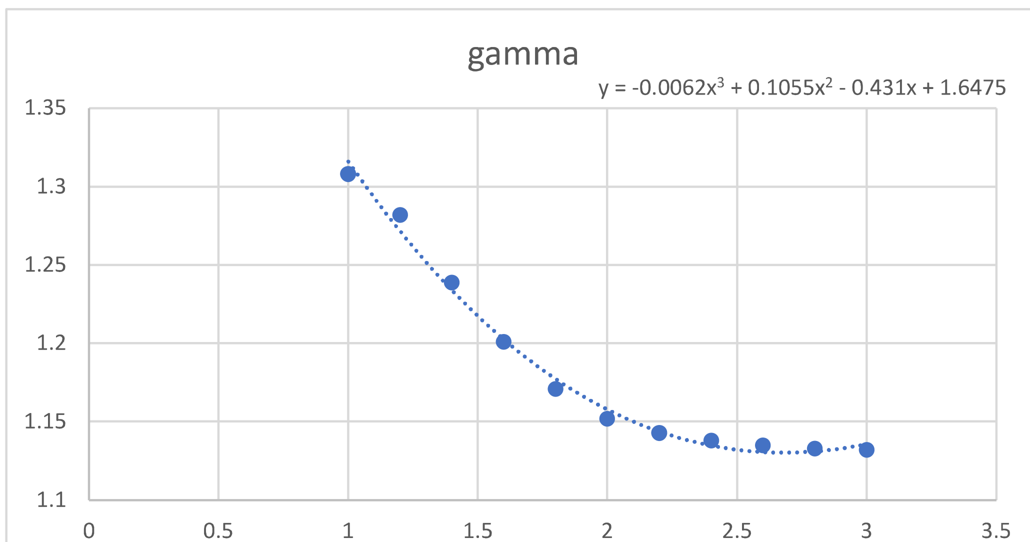


Published with MATLAB® R2022b

OF	cStar	gamma
1	4825	1.308
1.2	5180	1.282
1.4	5543	1.239
1.6	5767	1.201
1.8	5882	1.171
2	5912	1.152
2.2	5885	1.143
2.4	5831	1.138
2.6	5768	1.135
2.8	5703	1.133
3	5639	1.132



	cSTAR	
359.8		2.1233
-2848.3		
7136.2		
148.72		5903.984



	gamma	
-0.0062		2.1233
0.1055		
-0.431		
1.6475		1.148644

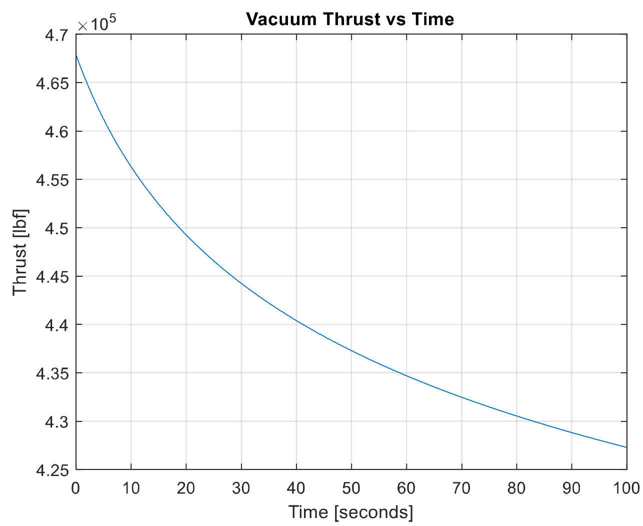
Homework #02B – MAE 640 Rev 2023-01

Name: ____Veronica____________________

Homework 02HWB Summary One-Page Cover Sheet

Problem 11:4 and SP02B-A Summary of Results

Result	Prob. 11-4	SP02B_A	Comment on Reasons for Any Differences
Initial Port Diameter, $2R_{pi}$ *(in)*	13.62	13.62	
Web Thickness, *w (in)*	12.435	9.567	When assumed to be linear with time, the web thickness is larger since this assumes it is constantly burning at the same rate
Mid-web O/F ratio	2.5	2.5	
Grain Length, *L (in)*	238.98	229.15	This is for the same reason as the web thickness. When it is assumed that it is linear with time, more grain is burned
Motor Diameter, *D (in)*	115.48	98.273	Since the web thickness changed between the two, the motor diameter also changed
Initial Thrust, *F*, (lb*f)*	500,000	467,850	Assuming a cfv of 1.7 brings the initial thrust down



Graph of Thrust as a Function of time from SP02B-A

Copy of the "Reflect" Section of Your Literature Review (McFarlane)

This paper highlights the hybrid rocket and dives into a test ran on a motor being developed. Hybrid rockets are not used in regular launches, so this method still requires a lot of testing and research. It offers many advantages when compared to solid and liquid rockets. This is good to know as an aerospace engineer that there are always different ways of doing something that has always been done the same one or two ways in the past.

Two-Page Annotated Bibliography Template

Summarize

Reference Document Examined:	McFarlane, J.S., "Design and Testing of AMROCS 250,000 pound Thrust Hybrid Motor," AIAA Paper 1993-2551, 1993.
Reviewer:	Veronica Loomis
Source of Document:	Canvas
Date of Review:	January 31, 2023
Electronic File Name:	1993_McFarlane_AMROC_250klbf_Thrust_Hybrid_Motor.pdf

Summary of Paper:

The American Rocket Company (AMROC) was developing a 250,000 pound thrust hybrid rocket motor that would be used on future space launch vehicles. A hybrid rocket motor was chosen since it offers significant advantages over both solid and liquid propulsion systems in terms of safety, cost, and operational flexibility. However, hybrid propulsion is less developed then either solid or liquid and so it is the goal of AMROC to bring this technology to commercial flight status. Specifically, AMROC was working on an orbital launch vehicle for delivering payloads to low Earth orbits.

Most of the fabrication took place in Camarillo, CA until the size grew too big and it was moved to Vendenberg AFB after a Joint Operating Agreement with the US Air Force. The motor was tested in a series of four firings in 1992. The first test on January 22 went smoothly with no chamber pressure overshoot. Three chamber pressure spikes were seen during the test and the motor dampened oscillations to provide stability. The second test on February 17 behaved similarly. The third test on March 11 aborted its first ignition attempt due to a faulty transducer downstream of the ignition valve. It was replaced and a successful test was ran showing similar results to the first two tests. The final test occurred on March 23 but was ended early when the motor case failed. This was caused by combustion gasses leaking through the insulation and exiting through the case cylinder. This weakened the material at the interface which caused failure.

B. Assess:

Important Facts from Document:

1. Hybrid propulsion are less developed than either solids or liquids, but offer advantages in terms of safety, cost, and operational flexibility.
2. The hybrid combustion process is independent of chamber pressure.
3. There were 4 tests ran but only 3 were successful.
4. There were plans to produce a LEO booster known as the Aquila launch vehicle.
5. This privately funded project got to test at the US Air Force Vandenberg AFB.

Key Figure from Document:

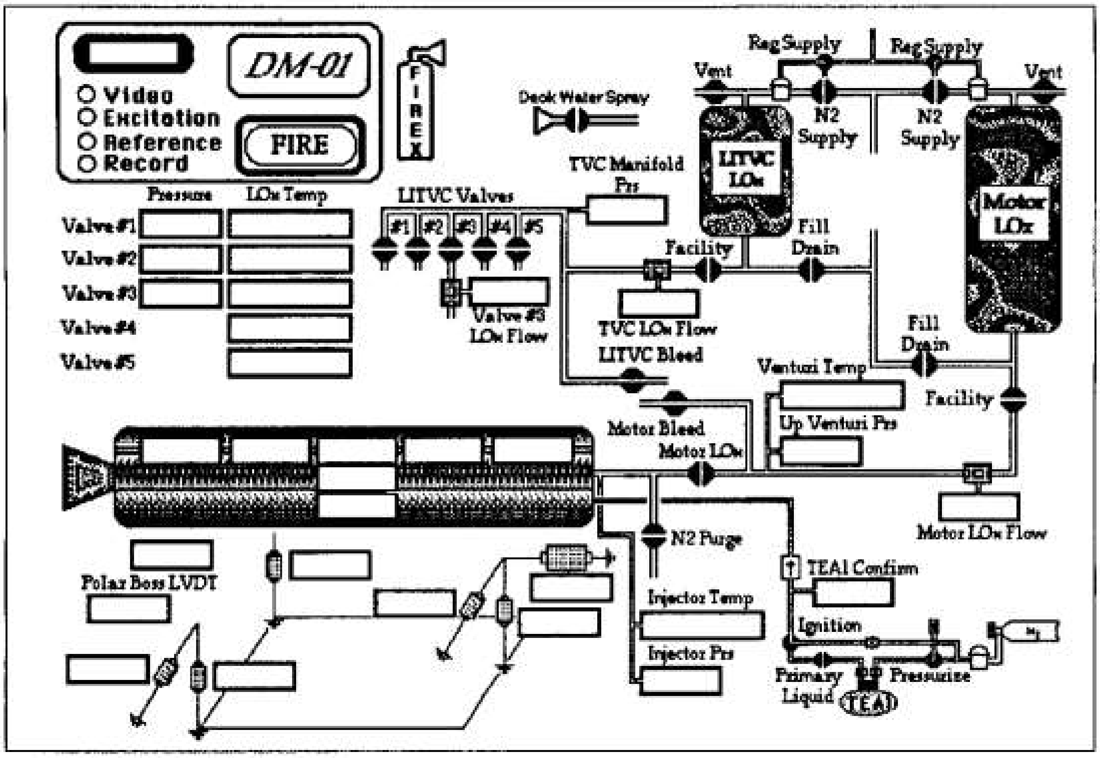


Figure 1: DM-01 Process Flow Diagram

Important Relationships among Parameters Described in the Paper:

1. When combustion gasses leak through the insulation, the structure and material of the motor can fail.
2. Good dynamic stability means that the severity of oscillations does not increase

C. Reflect

This paper highlights the hybrid rocket and dives into a test ran on a motor being developed. Hybrid rockets are not used in regular launches, so this method still requires a lot of testing and research. It offers many advantages when compared to solid and liquid rockets. This is good to know as an aerospace engineer that there are always different ways of doing something that has always been done the same one or two ways in the past.