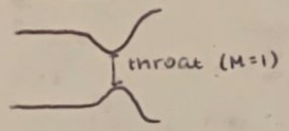
Name: Veronica Loomis

Problem 6.4

Given: convective heat transfer

Find: Prove convective heat transfer is max at throat

Schematic:



Assumptions:

negligible temp dependence of μ, c_p, and k

Basic Equations:

$$\dot{q} = h_g (T_\infty - T_w)$$

$$h_g = \frac{0.026}{D^{0.2}} \left(\frac{\mu^{0.2} c_p}{Pr^{0.6}} \right) (\rho_\infty V_\infty)^{0.8} \left(\frac{\rho_{am}}{\rho_\infty} \right)^{0.8} \left(\frac{\mu_{am}}{\mu_0} \right)^{0.2}$$

$$Pr = \frac{\mu_\infty c_{p\infty}}{k_\infty} \qquad V_\infty = \frac{\dot{m}}{\rho_\infty A} = \frac{4\dot{m}}{\rho_\infty \pi D^2}$$

Analysis:

max @ throat $\Rightarrow$ $\dot{q}' = 0$, $\dot{q}'' > 0$ (derive wrt x)

$$\dot{q} = (T_\infty - T_w)\ \text{const}\ \frac{1}{D^{0.2}} \left(\frac{1}{D^2} \right)^{0.8} = \sim \times D^{-0.2} D^{-1.6} = \sim D^{-1.8}$$

$$\dot{q}' = \sim (-1.8) D^{-2.8} \frac{dD}{dx}$$

this is possible if $D \neq 0$, $\underbrace{dD/dx = 0}_{\text{true at throat}}$

Answer

$\dot{q}_{left} = \sim D^{-1.8}$ $<$ $\dot{q}_{critical point}$ $\therefore$ it is max at throat

Comment

This makes sense. Mass flow rate is max when flow is choked, so it makes sense that heat transfer is as well.

Name: Veronica Loomis

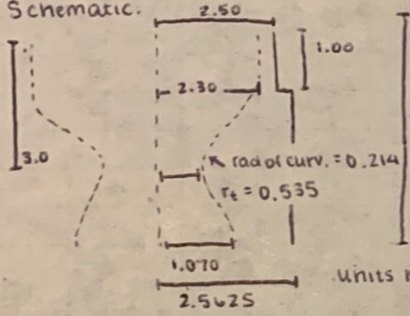
Problem 6.7

Given: Heavy walled steel nozzle
15 second test firing

Steel melting temp: 2500°F / 1370°C / 1643.15 K

Find: Temp profile at throat at t = 2, 5, 10, 15 sec. Will steel melt?

Schematic:



units not given.

5 1/2 in diam. BAR CRS

Assumptions:
$\bar{h}_g$ = 10000 W/m²K for steel
k = 60.5 W/mK
α = 17.7×10^{-6} m²/s
P_c = 300 psi (lbf/in²) = 2.068×10^{6} N/m²
Oxidizer: 85% aq H_2O_2 & HTPB as fuel
O/F = 7
$\gamma = 1.4$
One dimensional
constant k, c_p, α
heat never conducted to back side

Basic Equations:

$$\frac{T(x,t) - T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \operatorname{Exp}\left[\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right] \operatorname{erfc}\left[\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right]$$

$$T_t = T_c\left[\frac{1}{1+\frac{\gamma-1}{2}}\right]$$

Analysis:

In CEQUEL, find T_c

Find T_t using $T_t = T_c\left[\frac{1}{1+\frac{\gamma-1}{2}}\right]$

Substitute $T_t = T_\infty$, T_i = 298 K

Plot in MATLAB → Answer: If one uses the temperature given in solutions, the steel will melt.
If the temperature is calculated in CEQUEL, it will not melt.

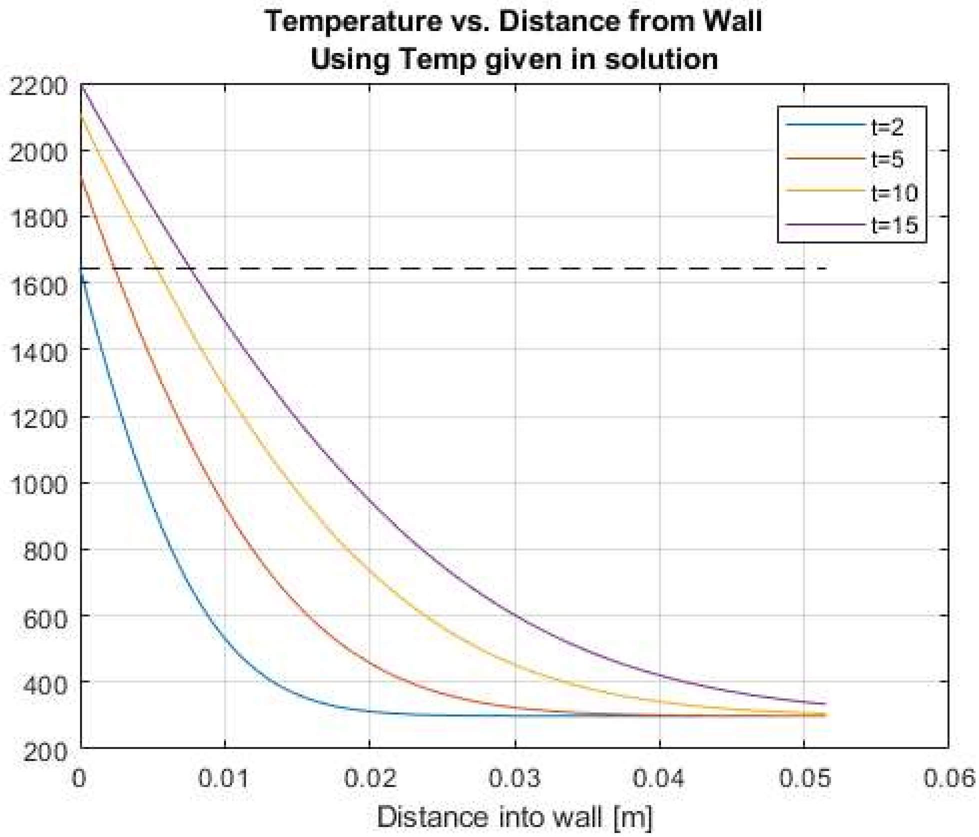
Comment: The difference between given T_∞ and CEQUEL/converted T_∞ might have been a typo.
It may have also been a miscalculation since if I swap T_t and T_c in the equation above, my CEQUEL/calculated value becomes a lot closer to the one given in the solution.

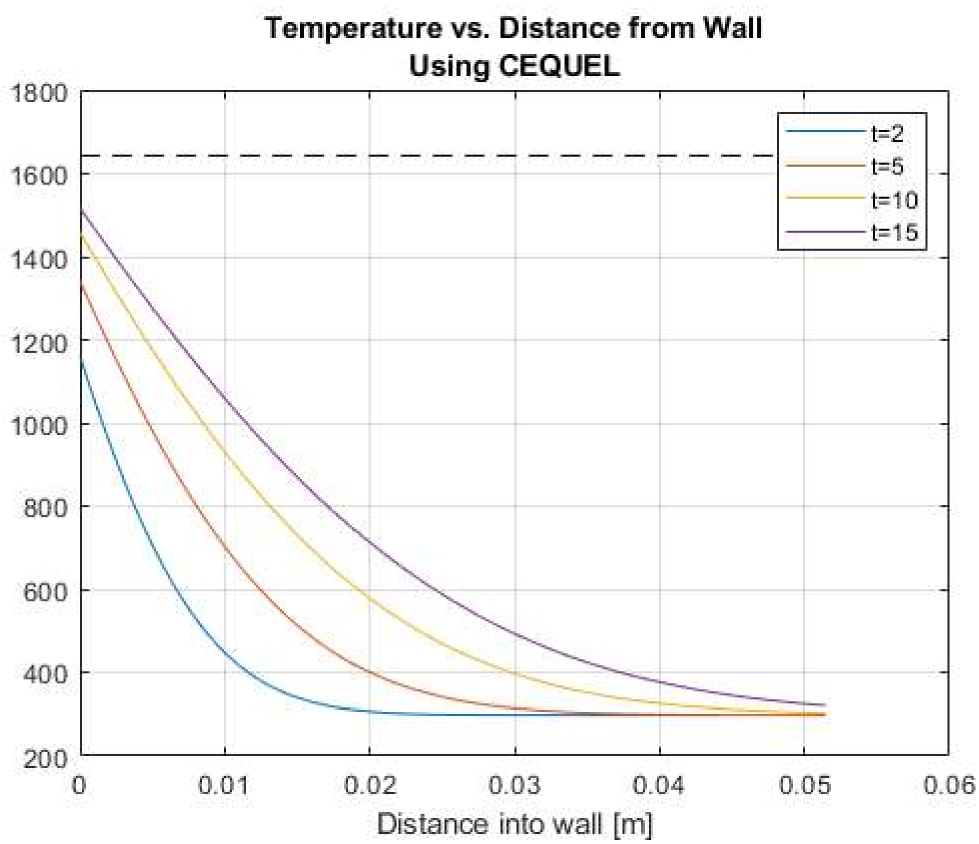
Scanned with CamScanner

OF	7		OF	7	
Pc (psi)	300		Pc (psi)	300	
Pc (bar)	20.6843		Pc (bar)	20.6843	
Area Ratio	4		Area Ratio	4	
OF	#NAME?	DIMENSIONLESS	OF	7	DIMENSIONLESS
FPCT	#NAME?	DIMENSIONLESS	FPCT	12.5	DIMENSIONLESS
ERATIO	#NAME?	DIMENSIONLESS	ERATIO	0.984183	DIMENSIONLESS
Phi	#NAME?	DIMENSIONLESS	Phi	0.968289	DIMENSIONLESS
T	#NAME?	DEG K	T	2180.01	DEG K
			T @ throat	1816.675	

Problem 6.7 in textbook

```
% Given
h = 10000; % W/Km**2
k = 60.5;  % W/Km
alpha = 17.7e-6; % m**2/s
Pc = 300; % psi (2.068e6 N/m**2)
% oxidizer = 85% aqueous H2O2
% fuel = HTPB
OF = 7;

t = [2,5,10,15];
xDist = (2.5625- 0.535)/39.37; % convert in to m
x = linspace(0,xDist,100); % range of x values we are interested in

Ti = 298; % K - initial wall temperature
TinfGiven = 2667.2; % K - hot gas temperature
TinfCEQ = 1816.675; % K - hot gas temperature
                    % found from CEQUEL and converting to throat temp

% Equation
% iterate over x, plug in t values that we want for plot purposes
for i=1:100
    for j = 1:4
        T_given(i,j) = Ti + (TinfGiven - Ti)*(erfc(x(i)/(2*sqrt(alpha*t(j))))
 - exp((h*x(i)/k) + (h^2*alpha*t(j))/(k^2)).*erfc((x(i)/
(2*sqrt(alpha*t(j))))+(h*sqrt(alpha*t(j))/k)));
        T_CEQ(i,j) = Ti + (TinfCEQ - Ti)*(erfc(x(i)/(2*sqrt(alpha*t(j))))
 - exp((h*x(i)/k) + (h^2*alpha*t(j))/(k^2)).*erfc((x(i)/
(2*sqrt(alpha*t(j))))+(h*sqrt(alpha*t(j))/k)));
    end
end

figure(1)
plot(x,T_given)
hold on
% Plot steel melting temperature
plot(x,ones(1,length(x))*1643.15,'k--')
grid on
legend('t=2','t=5','t=10','t=15')
title({'Temperature vs. Distance from Wall','Using Temp given in solution'})
xlabel('Distance into wall [m]')

figure(2)
plot(x,T_CEQ)
hold on
% Plot steel melting temperature
plot(x,ones(1,length(x))*1643.15,'k--')
grid on
legend('t=2','t=5','t=10','t=15')
title({'Temperature vs. Distance from Wall','Using CEQUEL'})
xlabel('Distance into wall [m]')
```


Temperature vs. Distance from Wall
Using Temp given in solution
t=2
t=5
t=10
t=15
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
0.01
0.02
0.03
0.04
0.05
0.06
Distance into wall [m]


Temperature vs. Distance from Wall
Using CEQUEL
t=2
t=5
t=10
t=15
1800
1600
1400
1200
1000
800
600
400
200
0
0.01
0.02
0.03
0.04
0.05
0.06
Distance into wall [m]

Published with MATLAB® R2022b

SP04A

Name Veronica Loomis

Given: O/F = 7.934
Pc = 50 atm
Equilibrium Flow
Nozzle Area Ratios: 0, 1, 2, 5, 10, 20

Find: Plots of values as a function of the nozzle area ratio

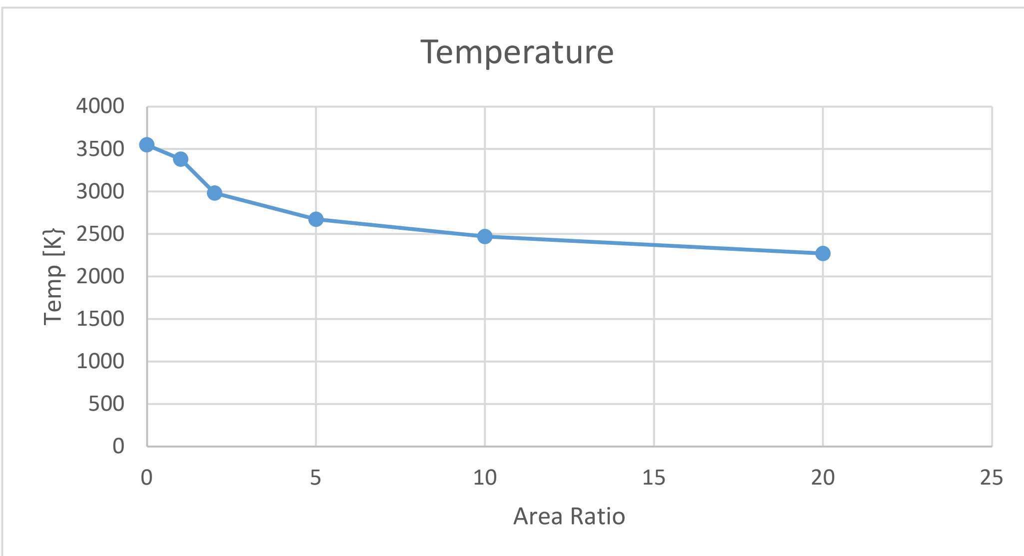
Analysis: CEQUEL

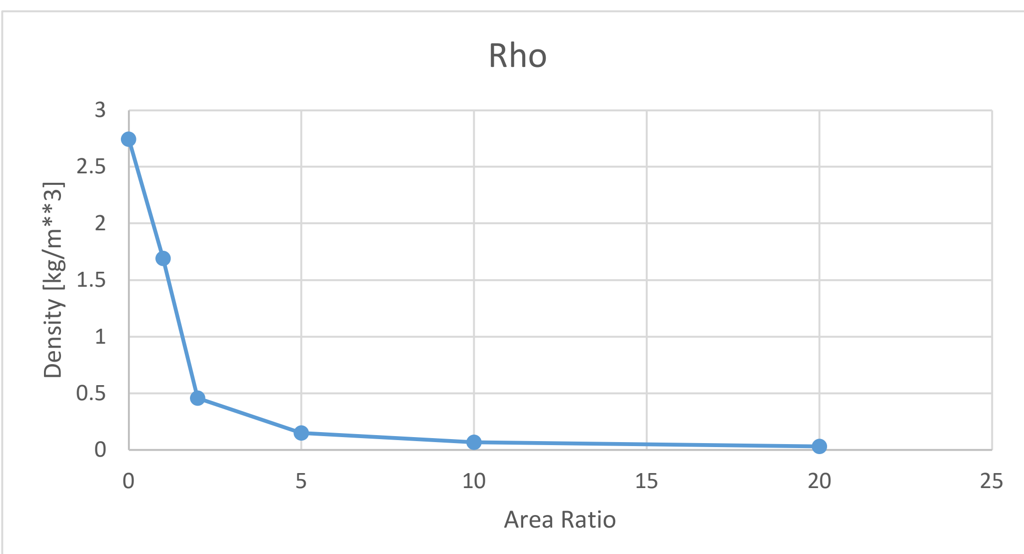
O/F	7.934
Pc (atm)	50
Pc (bar)	50.6625

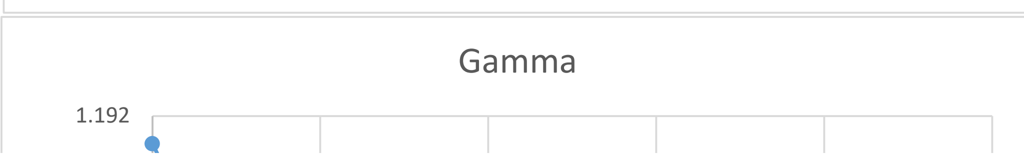
Area Ratios	0	1	2	5	10	20	
OF	7.934	7.934	7.934	7.934	7.934	7.934	DIMENSIONLESS
FPCT	11.19319	11.19319	11.19319	11.19319	11.19319	11.19319	DIMENSIONLESS
ERATIO	1.000338	1.000338	1.000338	1.000338	1.000338	1.000338	DIMENSIONLESS
Phi	1.000338	1.000338	1.000338	1.000338	1.000338	1.000338	DIMENSIONLESS
AR	0	1	2	5	10	20	DIMENSIONLESS
P	50.6625	29.35713	6.780241	1.920466	0.791986	0.334503	BAR
T	3548.422	3381.747	2980.844	2673.461	2468.599	2270.965	DEG K
RHO	2.744301	1.691015	0.458145	0.148395	0.067298	0.031285	KG/M^3
H	-860.601	-1837.49	-4187.76	-5934.71	-7028.45	-7996.04	KJ/KG
U	-2706.7	-3573.56	-5667.7	-7228.86	-8205.28	-9065.27	KJ/KG
GFE	-57163.8	-55496.1	-51485.2	-48354.8	-46198	-44029.7	KJ/KG
S	15.86712	15.86712	15.86712	15.86712	15.86712	15.86712	KJ/(KG K)
Z	1	1	1	1	1	1	DIMENSIONLESS
MW	15.98148	16.19614	16.74688	17.17609	17.44107	17.65943	MOL WT
CP	10.39491	10.16045	9.115419	7.783356	6.67492	5.540151	KJ/(KG K)
CPG	3.238026	3.215358	3.150319	3.085956	3.033401	2.973607	KJ/(KG K)
GammaG	1.191429	1.189994	1.187081	1.186048	1.186461	1.188121	DIMENSIONLESS
Gamma	1.128506	1.12541	1.119764	1.119001	1.121974	1.12928	DIMENSIONLESS
C	1443.376	1397.779	1287.313	1203.395	1149.075	1098.844	M/S
MW_MIX	15.98148	16.19614	16.74688	17.17609	17.44107	17.65943	MOL WT
Viscosity	1.110518	1.071734	0.975276	0.897615	0.843612	0.789487	milliPOISE
Specific_Heat	10.39516	10.16055	9.114248	7.782729	6.674583	5.54	KJ/(KG K)
Conductivity_	22.1023	20.77815	16.62412	12.60708	9.756744	7.160257	milliW/(CM K)
Prandtl_Eq	0.522299	0.52408	0.5347	0.554125	0.577114	0.610839	DIMENSIONLESS
Specific_Heat	3.238087	3.215396	3.150436	3.085993	3.033415	2.973612	KJ/(KG K)
Conductivity_	4.909735	4.663019	4.058006	3.582104	3.259603	2.945546	milliWATTS/(CM K)
Prandtl_FR	0.732413	0.739017	0.757157	0.773298	0.785073	0.79701	DIMENSIONLESS
PINJ_P	-999.999	-999.999	-999.999	-999.999	-999.999	-999.999	N/A
PC_P	1	1.725731	7.472079	26.38032	63.96894	151.456	DIMENSIONLESS
MACH	0	0.999998	2.003862	2.647198	3.056562	3.437869	DIMENSIONLESS
AR	0	1	2	5	10	20	DIMENSIONLESS
CSTAR	0	2143	2143	2143	2143	2143	M/S
CF		0.652133	1.203513	1.486255	1.638628	1.762479	DIMENSIONLESS

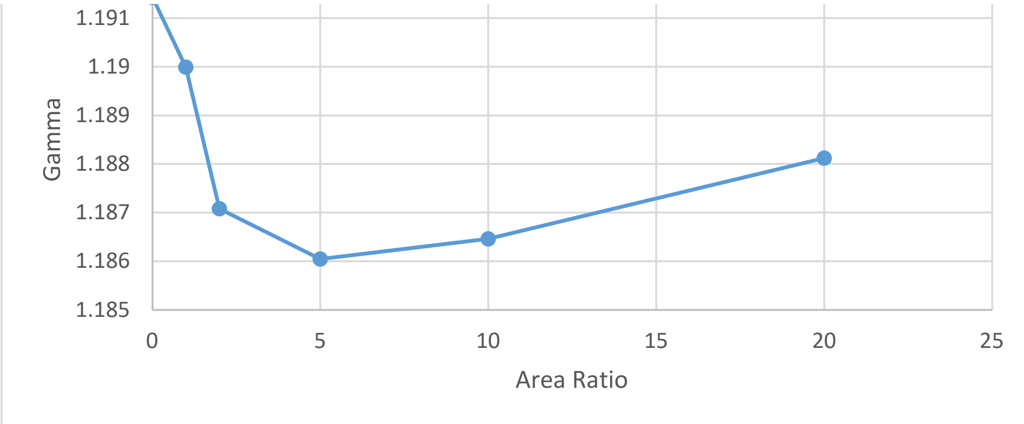
ISPV		2639.796	3153.305	3591.874	3847.288	4060.72	M/S
ISP		1397.776	2579.598	3185.626	3512.22	3777.682	M/S
ISPVRHO		0	0	0	0	0	KG/(M^2 S)
AR	0	1	2	5	10	20	DIMENSIONLESS
H	0.001984	0.001678	0.000978	0.000524	0.000292	0.000137	MASS
H2	0.014558	0.013224	0.0097	0.00679	0.004873	0.003184	MASS
H2O	0.797388	0.819032	0.87361	0.915653	0.941706	0.963468	MASS
H2O2	6.93E-05	4.37E-05	1.18E-05	0	0	0	MASS
HO2	0.000375	0.000252	8.02E-05	2.58E-05	1.03E-05	0	MASS
O	0.014359	0.011811	0.006307	0.003074	0.001579	0.000672	MASS
O2	0.067528	0.063332	0.050333	0.037525	0.028067	0.019038	MASS
OH	0.103738	0.090627	0.058981	0.036405	0.023471	0.013497	MASS

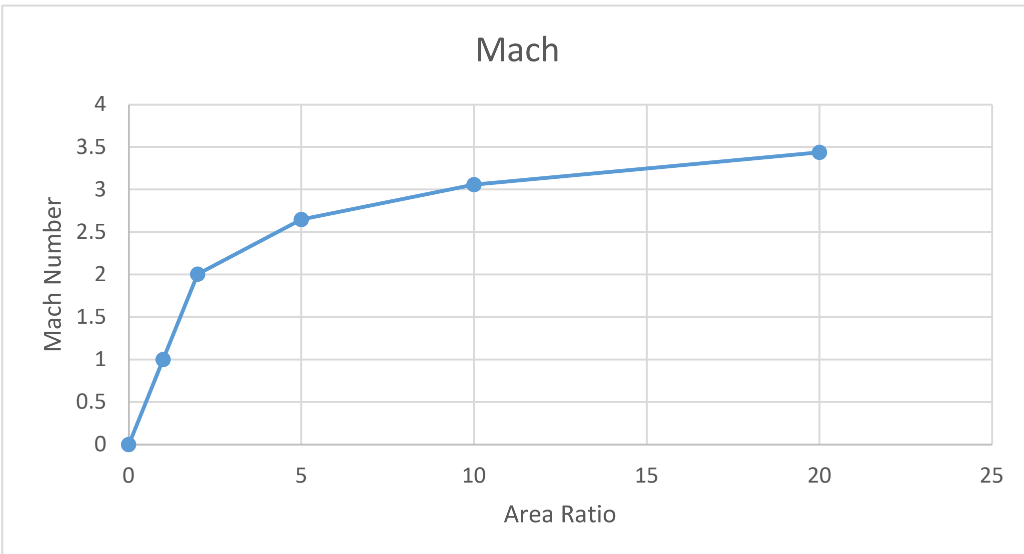
Answer:

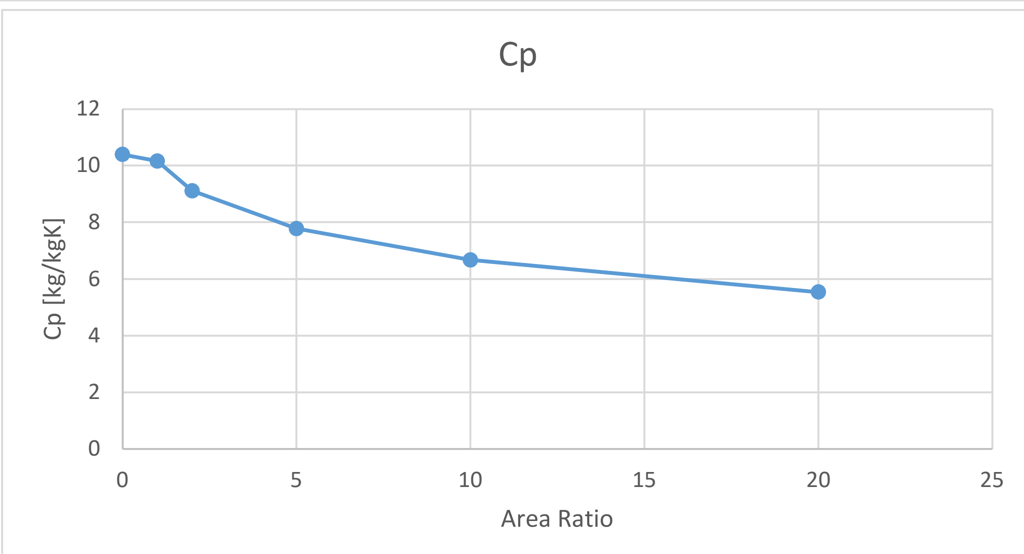


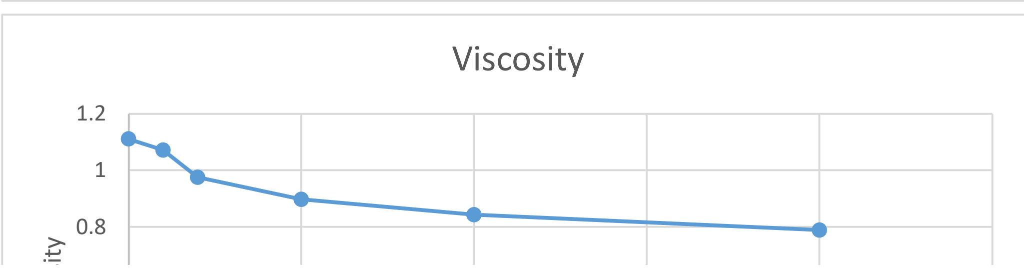


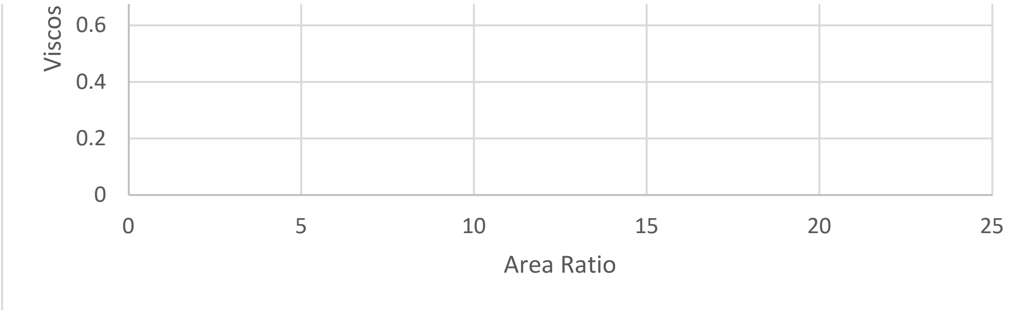



1.191
1.19
1.189
1.188
1.187
1.186
1.185
Gamma
0
5
10
15
20
25
Area Ratio


Mach
4
3.5
3
2.5
2
1.5
1
0.5
0
Mach Number
0
5
10
15
20
25
Area Ratio


Cp
12
10
8
6
4
2
0
Cp [kg/kgK]
0
5
10
15
20
25
Area Ratio


Viscosity
1.2
1
0.8



Comment: The tempeartures are lower for liquid than they are for gaseous propellants This makes sense since gases usually exist at higher temperatures, so not as much work has to be done compared to converting them from liquid to gas.

Two-Page Annotated Bibliography Template

Summarize

Reference Document Examined:	Bartz, D.R., "Turbulent boundary layer heat transfer from fast accelerating flow of rocket exhaust gases and heated air." NASA CR-62615, December 1, 1963
Reviewer:	Veronica Loomis
Source of Document:	Canvas
Date of Review:	March 1, 2023
Electronic File Name:	ref_Bartz.pdf

Summary of Paper:

It is important to know heat transfer and boundary layer development within the combustion chamber and nozzle of an engine. Many advancements are being made that are increasing the pressures and temperatures of the gases within the engine, which makes it more crucial to understand the fluxes that occur within. This problem is by no means solved yet, however there is a lot of analysis and experimentation that is ongoing to try and get a better grasp of it. There are many components that go into this analysis with their own constraints, so it is important to gather those and understand what the expectations are.

B. Assess:

Important Facts from Document:

1. Recent advancements have seen engines with chamber pressures reaching 1000 lb/in^2 and it's not unlikely that these will double in the future.
2. More energetic propellants being developed are driving up gas temperatures to roughly 8000^{o}F.
3. It is becoming increasingly more important to know about heat transfer and boundary-layer development in combustion chambers and nozzles.
4. It is difficult to characterize these since free stream flow cannot be successfully described in terms of steady, average, 1-D flow variables.
5. There are three unique characteristics to the thrust chamber: rapid establishment of steady flow, high heat fluxes, and a sharp axial gradient of heat flux.

Key Figure from Document:

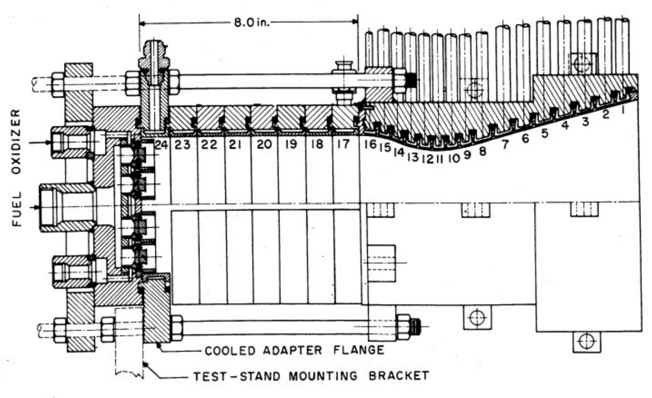


Figure 28: Since fluxes are high, the thrust chamber is divided into axially short segment lengths which are individually cooled.

Important Relationships among Parameters Described in the Paper:

1. Overdesigned wall protection led to excessive pressure drops and weight, or a shift towards lower performance.
2. A sharp axial gradient of heat flux creates a requirement for very localized measurements.

C. Reflect

This paper was interesting to read since it's a very real example of how this field of study is still ongoing and there is plenty more research to be done into it. Technology is always advancing and it is very important for the math and designs to stay up to date as well.

Name: Veronica Loomis

SP04C

Given: $\overline{T_l} = 100\ °F$

$k_{water} = 1.07 \times 10^{-4}$ Btu/s ft F $= 0.3852$ Btu/hr ft F

$T_{gas} = 4500\ °F$

$\mu_{water} = 2.5 \times 10^{-5}$ lbf · s/ft^2

$\bar{c}_{water} = 1.3$ Btu/lb F

cooling dim: 1/4 × 1/2 in

$\dot{m}_{water} = 0.585$ lbm/s

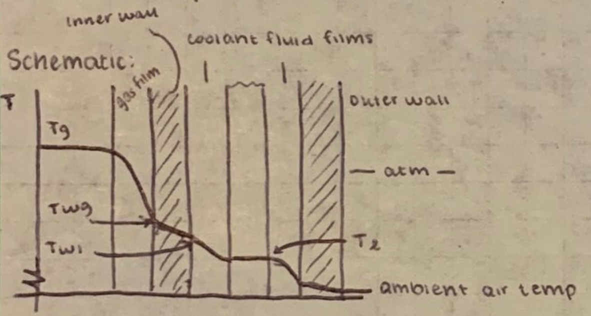
$t_{w,inner} = 1/8$ in

$\bar{q}_{abs} = 1.3$ Btu/in^{2}s (also examine 1.0)

$k_{wall} = 26$ Btu/hr ft F

Find
a) Film coef. of coolant [h]
b) Wall temp on coolant side
c) Wall temp on gas side
d) Compare heat transfer coef equation here w/ one in textbook

Schematic:



radial dist from thrust chamber center line

Assumptions:
No losses

Basic Equations:

$$h_{liq} = 0.023\ \bar{c}_p \frac{\dot{m}_{water}}{A} \left(\frac{D v \rho}{\mu g_c}\right)^{-0.2} \left(\frac{\mu g_c \bar{c}_w}{k_w}\right)^{-2/3}$$ [HW4 PDF]

100k up

$$q_{conv} = h_{liq}(T_{wl} - T_\infty)$$

$$q_{cond} = -k(dt/dx)$$

Analysis:

a) Film coef. of fluid [h_{liq}]

$$h_l = 0.023\ \bar{C}_p\ \frac{\dot{m}_w}{A}\left(\frac{Dv\rho}{\mu g_c}\right)^{-0.2}\left(\frac{\mu g_c \bar{C}_p}{k_w}\right)^{-2/3}$$

$\bar{C}_p$ = 1.3 Btu/lbm°F
$\dot{m}_w$ = 0.585 lbm/s
A = 1/8 in^2
D = $\sqrt{4A/\pi}$ = 0.3989 in
ρ_w = 62.4 lbm/ft^3
v = $\dot{m}_w/\rho_w A$ = 0.585/(62.4 × 1/8) = 0.075 $\frac{\text{lbm}\ \text{ft}^3}{\text{s}\cdot\text{lbm}\ \text{in}^2} \times \frac{144\ \text{in}^2}{\text{ft}^2}$ = 10.8 ft/s
μ = 2.5×10^{-5} lbfs/ft^2
k_w = 1.07×10^{-4} Btu/sft°F

$$h_{liq} = 0.023 \times \frac{1.3\ \text{Btu}}{\text{lbm°F}} \times \frac{0.585\ \text{lbm}}{1/8\ \text{in}^2\text{s}} \times \left[\frac{0.3989\ \cancel{\text{in}} \times 10.8\ \cancel{\text{ft}} \times 62.4\ \cancel{\text{lbm}}\ \cancel{\text{ft}^2}\ \cancel{\text{lbf}\cdot\text{s}^2} \times \cancel{\text{ft}}}{\cancel{\text{sec}}\ \cancel{\text{ft}^3} \times 2.5\times10^{-5}\ \cancel{\text{lbf}\cdot\text{sec}} \times 32.2\ \cancel{\text{lbm ft}} \times 12\ \cancel{\text{in}}}\right]^{-0.2}$$

$$\times \left[\frac{2.5\times10^{-5}\ \text{lbf s} \times 32.2\ \text{lbm ft} \times 1.3\ \text{Btu}\ \ \text{s ft °F}}{\text{ft}^2\ \text{lbf s}^2\ \text{lbm °F} \times 1.07\times10^{-4}\ \text{Btu}}\right]^{-2/3}$$

$$h_{liq} = 0.139932\ \text{Btu/in}^2\text{s°F} \times (27828.85)^{-0.2} \times (9.78037)^{-2/3}$$

$$h_{liq} = 0.139932\ (0.1291519)(0.2186569)\ \text{Btu/in}^2\text{s°F}$$

$$\boxed{h_{liq} = 0.00395167\ \text{Btu/in}^2\text{s°F}}$$

b) Wall temp - coolant side (convec)

$q = h_l(T_w - T_\infty)$
q = 1.3 Btu/in^{2}s
h_l = 0.00395 Btu/in^{2}s°F
$T_\infty = \bar{T}_l$ = 100°F

$$T_{wall} = \frac{q}{h_l} + T_\infty$$

$$T_{wall} = \frac{1.3\ \text{Btu}\ \text{in}^2\text{s°F}}{\text{in}^2\text{s}\ 0.00395\ \text{Btu}} + 100°\text{F}$$

$$\boxed{T_{wall} = 429.114\ °\text{F}}$$

c) wall temp - gas side (conduc.)

$$q = -k\left(\frac{dT}{dx}\right) = \frac{k}{t_w}\left(T_{wg} - T_{wl}\right)$$

$q = 1.3$ Btu/in^{2}s
$k_{wall} = 26$ Btu/hr ft °F
$t_w = 1/8$ in
$T_{wl} = 429.114$ °F

$$T_{wg} = \frac{q\,t_w}{k} + T_{wl}$$

$$T_{wg} = \frac{1.3\ \text{Btu}\ \ \frac{1}{8}\ \cancel{\text{in}}\ \ \cancel{\text{hr}}\ \cancel{\text{ft}}\ °F\ \ 12\ \cancel{\text{in}}\ \ 3600\ \cancel{\text{sec}}}{\cancel{\text{in}^2 \text{s}}\ \ 26\ \cancel{\text{Btu}}\ \ \cancel{\text{ft}}\ \ \cancel{\text{hr}}} + 429.114\ °F$$

$$T_{wg} = 270\ °F + 429.114\ °F$$

$$\boxed{T_{wg} = 699.114\ °F}$$