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Problem 11.4

Given: Hybrid Rocket with 7 port cluster

LOX / HTPB so $I_{sp} = 350$ sec at $O/F = 2.5$

average thrust required: 500,000 lbf for $t_b = 100$ sec

HTPB fuel density: 0.036 lb/in^3

average c^* at optimal O/F: 5000 ft/s (at $O/F = 2.5$)

$G_{ox,initial} = 1.0$ lb/in^{2}s

$r = 0.2\, G_{ox}^{0.68}$

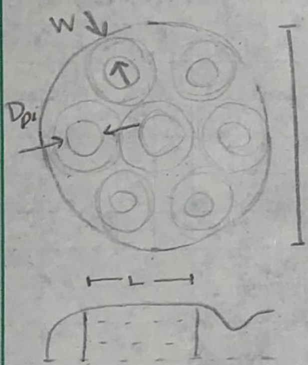
Find:

a) Initial port diameter (D_{pi})

b) total web distance burned (w)

c) fuel grain length (L) and overall motor diameter (D)

Schematic:



Assumptions:

Characteristic velocity efficiency of 1.0

Initial G_{ox} is for all 7 ports combined

$\dot{m}_{ox}$ is constant

Burning rate is linear

Given I_{sp} and F are the initial values

Basic Equations:

$$G_{ox} = (\dot{m}_{ox} / N A_p)$$

$$r = a\, G_{ox}^{n}$$

$$\dot{m}_{ox} = \frac{\dot{m}}{1 + 1/OF}$$

$$r_{av} = \frac{r_c + r_f}{2}$$

$$w = r_{av} t$$

$$c^* = P_c A_b / (\dot{m}_{ox} + \dot{m}_f)$$

$$\dot{m}_f = r \rho_f N\, Per(w) L$$

Analysis

a) $r_i = a\,G_{ox,init}^{\,n}$

$r_i = 0.2\,(1)^{0.68} = 0.2 \text{ in/sec}$

$\dot{m}_{ox} = \dfrac{F}{I_{sp}\, g\,(1+1/_{OF})} = \dfrac{500\,000 \text{ lbf}}{(350 \text{ sec})(32.2 \text{ ft/s}^2)(1+1/2.5)} \times \dfrac{32.2 \text{ lbm ft}}{s^2 \text{ lbf}}$

$\dot{m}_{ox} = 1020.4 \text{ lbm/s}$

$G_{ox,init} = \dfrac{\dot{m}_{ox}}{7\pi R_i^2}$

$R_i^2 = \dfrac{\dot{m}_{ox}}{7\pi G_{ox,init}}$

$R_i = \sqrt{\dfrac{1020.4 \text{ lbm/s}}{7\pi\,(1.0 \text{ lb/in}^2\text{s})}}$

$R_i = 6.81179 \text{ in}$

$\boxed{D_{pi} = 2R = 13.62 \text{ in}}$

b) $w = \dfrac{r_i + r_f}{2}\,(t_b)$

$r_f = 0.2\left(\dfrac{\dot{m}_{ox}}{7\pi R_f^2}\right)^{0.68}$

$R_f = R_i + w$

$r_f = 0.2\left(\dfrac{1020.4}{7\pi(6.81179+w)^2}\right)^{0.68}$

sub in

$w = \left[\dfrac{0.2 + 0.2\left(\dfrac{1020.4}{7\pi(6.81179+w)^2}\right)^{0.68}}{2}\right] \times 100 \quad [\text{in/s}] \times [\text{s}]$

$w = 10 + 10\left(\dfrac{46.4}{(6.81179+w)^2}\right)^{0.68}$

solve for w in MATLAB

$\boxed{w = 12.435 \text{ in}}$

c) $\dot{m}_f = r \rho_f N \, Per(w) L$

$$\dot{m}_f = 0.2\left(\frac{\dot{m}_{ox}}{\pi R^2 (7)}\right)^{0.68} (0.036)(7)\, 2\pi (R_{mid})\, L$$

Use R = mid burn radius

$$R = 6.81179 + (12.435/2) = 13.03 \text{ in}$$

$$\dot{m}_f = \tfrac{1}{7}\dot{m}_{ox}\,(1/O/F) = 1020.4 \text{ lbm/s}\,(1/2.5)$$

$$\dot{m}_f = 408.16 \text{ lbm/s}$$

Rearrange & substitute

$$L = \frac{408.16 \text{ lbm/s}}{\underbrace{0.2\left(\frac{1020.4}{7\pi(13.03)^2}\right)^{0.68}}_{\text{in/s}} (0.036 \text{ lb/in}^3)(7)(2\pi)(13.03 \text{ in})}$$

$$\boxed{L = 238.98 \text{ in}}$$

overall D

$$D = (R_o + w)(2)(3)$$

$$D = (6.81179 + 12.435)(6) \text{ in}$$

$$\boxed{D = 115.48 \text{ in}}$$

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Given:

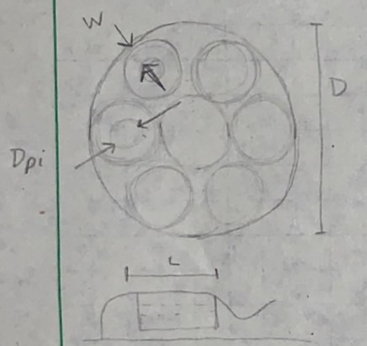
$$R(t) = \left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right]^{1/2n+1}$$

$$\dot{m}_f(t) = 2a\pi^{1-n} \rho_f L \dot{m}_{ox}^n \left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right]^{\frac{1-2n}{1+2n}}$$

Find:

Repeat 11.4 and find:
a) Total web dist burned
b) L assuming O/F is at mid web
c) overall motor diameter
d) % diff of web distance and L here vs in 11.4
e) 5 plots
1) Mixture ratio & G_{ox} as f(t)
2) C^* as f(O/F) and point out motor start & stop points
3) $\dot{m}_f$, $\dot{m}_{ox}$, $\dot{m}_{prop}$ as f(t)
4) Pc as f(t) with $A_t = 300\ in^2$ and using c^* from table
5) F_{vac} as f(t) assuming $C_{fv} = 1.70$

Schematic:



Assumptions:
Same as 11.4
Use the 2 equations above

Basic Equations:

$G_{ox} = (\dot{m}_{ox}/NA_p)$

$r = a G_{ox}^n$

$\dot{m}_{ox} = \frac{\dot{m}}{1 + 1/OF}$

$r_{av} = \frac{r_c + r_f}{2}$

$W = r_{av} t$

$c^* = P_c A_b / (\dot{m}_{ox} + \dot{m}_f)$

$\dot{m}_f = r \rho_f N\ Per(W) L$

Analysis:

a) Total web dist. burned

$$\dot{m}_{ox} = \frac{F}{I_{sp} g}\left(\frac{O/F}{1+O/F}\right) = \frac{500\,000 \text{ lbf}}{(350 \text{ s})(32.2 \text{ ft/s}^2)}\left(\frac{2.5}{3.5}\right)\left(\frac{32.2 \text{ lbm ft}}{\text{lbf s}^2}\right)$$

$$\dot{m}_{ox} = 1020.4 \text{ lbm/s}$$

$$G_{ox,init} = \dot{m}_{ox} / (7\pi R_i^2)$$

$$R_i = \sqrt{\frac{1020.4 \text{ lbm/s}}{7\pi (1.0 \text{ lbm/in}^2\text{s})}}$$

$$R_i = 6.81179 \text{ in}$$

Now, we know $t_b = 100\text{s}$

$$R(100) = \left[0.2\,(1.36+1)\underbrace{\left(\frac{1020.4}{7\pi}\right)^{0.68}}_{[\text{in/s}][\text{s}]}(100) + (6.81179)^{(1.36+1)}\right]^{1/(1.36+1)}$$

$$R(100) = 16.379 \text{ in}$$

$$W = R(100) - R_i = 16.379 - 6.81179 \text{ in}$$

$$\boxed{W = 9.567 \text{ in}}$$

b) $R_{mid} = R_i + W/2 = 6.81179 + 4.7835$

$$R_{mid} = 11.595 \text{ in}$$

$$\dot{m}_f = 0.2\left(\frac{\dot{m}_{ox}}{7\pi R^2}\right)^{0.68}(0.036)\,7(2\pi)\,R_{mid}\,L$$

$$L = \frac{\dot{m}_{ox}\left(1/O/F\right) \text{ lbm/s}}{0.2\underbrace{\left(\frac{1020.4}{7\pi(11.595)^2}\right)^{0.68}}_{\text{in/s}}(0.036 \text{ lb/in}^3)\,7(2\pi)(11.595 \text{ in})}$$

$$\boxed{L = 229.15 \text{ in}}$$

c) overall diameter

$$D = (R_i + w)(3)(2)$$

$$D = (6.81179 + 9.567)(6)$$

$$\boxed{D = 98.273 \text{ in}}$$

d) % difference in w and L from 11.4 and here

W:

11.4 : $w = 12.435$ in

SP : $w = 9.567$ in

$$\%\text{ diff web} = \frac{SP - 11.4}{SP} = \frac{9.567 - 12.435}{9.567}$$

$$\boxed{\%\text{ diff web} = 29.978\%}$$

L:

11.4 : 238.98 in

SP : 229.15 in

$$\%\text{ diff } L = \frac{229.15 - 238.98}{229.15}$$

$$\boxed{\%\text{ diff } L = 4.2897\%}$$

e) Plots

$$O/F = \frac{\dot{m}_{ox}}{(7)2a\pi^{1-n}\rho_f L \dot{m}_{ox}^{n}\left[a(2n+1)\left(\frac{\dot{m}_{ox}}{7\pi}\right)^n t + R_i^{2n+1}\right]^{1-2n/1+2n}}$$

c^* as $f(O/F)$ → plot Mixture ratio & c^* table & curve fit

This is done in excel

$$c^* = 359.8(O/F)^3 - 2848.3(O/F)^2 + 7136.2(O/F) + 148.72$$

at $O/F = 2.5$, $c^* = 5809.22$ ft/s

at $R_o = 6.81179$ in, $O/F = 2.1233$ which gives $c^* = 5903.984$ ft/s

$$P_c = \frac{c^*(\dot{m}_{ox} + \dot{m}_f)}{A_t} = \frac{(5903.984 \text{ ft/s})(\dot{m}_{ox}[\text{lbm/s}] + \dot{m}_f[\text{lbm/s}])}{(300 \text{ in}^2)(1/144)\,\text{ft}^2/\text{in}^2 \times 32.2 \text{ lbm ft/lbf s}^2}$$

Assuming same A_t for thrust in vacuum