MAE 640 2023 Exam 01 Solution

MAE 640 Module 01 Problem - 2023

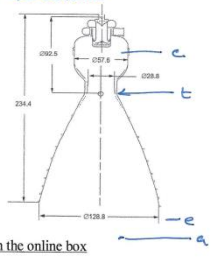
Module 01 Problem

1. NAME: DR. ROBERT FREDERICK

2. **Given:**

The TRW Ultra Low Cost Engine (ULCE) shown in figure (dimensions are in inches) operates nominally at a chamber pressure of 700 psi and uses LOX/LH$_2$ propellant with a c^*=7800 ft/s and a mixture ratio (O/F = 6.0), and a γ = 1.2.

4. Schematic



3. **Find:**

(a) Total engine propellant flow rate in lb_m/s
(b) Sea level thrust in lb_f
(c) Vacuum thrust lb_f
(d) Sea level and vacuum I_{sp}
(e) If the engine is tested at sea level, determine the area ratio in the exit cone where the flow is likely to separate assuming p_{sep}/p_a = 0.33333
(f) Enter the area ratio that you calculated for part (e) in the online box

5. ASSUME

(a) STEADY STATE (b) ATM = 14.7 psi (c) ISENTROPIC, 1-D FLOW

6. BASIC EQUATIONS

$$c^* = \frac{P_c A_t}{\dot{m}} \qquad F = C_F A_t P_c \qquad I_s = \frac{C_F c^*}{g_c}$$

$$\dot{m} = \frac{P_c A_t}{c^*} \qquad A_t = \frac{\pi D_t^2}{4} = \frac{\pi (28.8\,in)^2}{4} = 651.44\,in^2$$

$$\dot{m} = \frac{700\,lbf/in^2 (651.44\,in^2)}{7800\,ft/s}\left(\frac{32.2\,lbm\,ft}{lbf \cdot s^2}\right) = \boxed{1882.5\ \frac{lbm}{s}} \quad (a)$$

FROM GRAPH

$\epsilon = (128.8/28.8)^2 = 20$, $\gamma = 1.2$ (Figure 4.6), $P_c/P_{atm} = \frac{700}{14.7} = 48$

$C_{F_{VAC}} = 1.82$, $C_{F_{SL}} = 1.40$ READING FROM FIGURE 4.6

PURDUE TABLE $\rightarrow C_{F_{VAC}} = 1.8205$, $C_{F_{SL}} = 1.8205 - (14.7/700)(20) = 1.400$ (b)

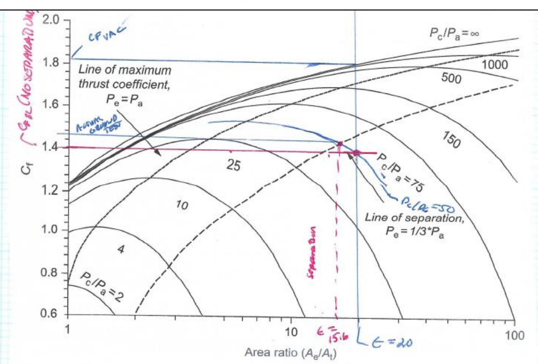
$$F_{VAC} = 1.8205 (651.4\,in^2)(700\,lbf/in^2) = \boxed{830{,}165\ lbf} \quad (c)$$

$$F_{SL} = F_{VAC}\left(\frac{1.400}{1.8205}\right) = \boxed{638{,}411\ lbf}$$

(d)

$$I_s = \frac{C_F c^*}{g}\ ; \ I_{s_{VAC}} = \frac{1.8205 (7800\,ft/s)}{32.2\,ft/s} = \boxed{441\ sec} \quad (d)$$

$$I_{s_{SL}} = \left(\frac{1.400}{1.8205}\right)(440s) = \boxed{339\ sec}$$



$C_{F_{SL}} = 1.467$ $\quad F_{SL_{sep}} = 1.467(651.44)(700) = 668{,}963\,lbf$

ABOUT 5% HIGHER

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(e) $P_{SEP} = 0.3333\,(14.7\,psi) = 4.9\,psi$

$$\frac{P_{sep}}{P_c} = \frac{4.9\,psi}{700\,psi} = 0.007 \Rightarrow \frac{P_e}{P_c} = 0.007$$ FOR ISENTROPIC AREA RATIO OF SEPARATION

PURDUE C_F TABLES $\gamma = 1.2$.

ϵ	P_e/P_c
15	.00736
	.007
16	.00677

$\frac{3.6}{5.9}$.

SO $\boxed{\epsilon_{SEP} \approx 15.6}$ $\leftarrow$ 3(e)

CHECKS ON SPREADSHEET

9. COMMENT

- THE OPERATION CAN BE VISUALIZED OF A C_F PLOT AND CROSS-CHECKED
- THE SEA-LEVEL THRUST WOULD BE OFF IF THE FLOW SEPARATED AS PREDICTED, AND WOULD ACTUALLY BE HIGHER

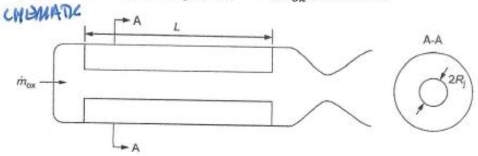
MAE 640 Module 02 Problem -2023

Module 02 Problem ..

1. NAME: ROBERT FREDERICK, UAH

2. **Given:** Consider a single port hybrid rocket motor shown in the figure below. The propellant density (ρ_p), the initial radius of the bore (R_i), the length of the grain (L), the mass flow rate of the oxidizer($\dot{m}_{ox}$), a characteristic velocity $c^* = \alpha + \beta \cdot (O/F) + \gamma \cdot (O/F)^2$ [ft/s], the area of the throat (A_t), and burning rate equation, $r = aG_{ox}^n$ are known.

4. SCHEMATIC



3. **Find:**

(a) Develop a *symbolic equation* for the initial chamber pressure in terms of the oxidizer flow rate assuming the initial oxidizer-to-fuel ration is $(O/F)_0$ and uses all of the given information. (Show steps of development).

(b) Develop *symbolic equation* for how the O/F ratio varies with time and uses all of the given information. (Show steps of development).

(c) If $\alpha = 2515$; $\beta = 3075$; and $\gamma = -690$, calculate the value O/F ratio of the maximum c^* and the value of the maximum c^*

(d) Enter the O/F_{max} in the box below during this exam.

6. ASSUME

(a) LUMPED PARAMETER BALLISTICS
(b)

(b)

7/8. BASIC EQUATIONS/ANALYSIS

$$r = a(G_{ox})^n \qquad G_{ox} = \frac{\dot{m}}{A_p} \qquad c^* = \frac{P_c A_t}{(\dot{m}_{ox} + \dot{m}_p)} \ ; \ \dot{m}_f = \rho_f A_p r$$

$$P_c = \frac{c^*(\dot{m}_{ox} + \dot{m}_f)}{A_t}$$

$$\dot{m}_f = \rho_f A_b r = \rho_f (\pi R L)\left(a\left(\frac{\dot{m}_{ox}}{\pi R^2}\right)^n\right) 2$$

$$\boxed{P_c = \frac{(\alpha + \beta(O/F) + \gamma(O/F)^2)\left(\dot{m}_{ox} + 2\rho_f(\pi R L)\left(a\left(\frac{\dot{m}_{ox}}{\pi R^2}\right)^n\right)\right)}{A_t}} \ (a)$$

$$O/F \quad \frac{\dot{m}_{ox}}{\dot{m}_f} = \frac{\dot{m}_{ox}}{2a\pi^{1-n}\rho_f L \dot{m}_{ox}^n\left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right]^{\frac{1-2n}{1+2n}}}$$

SIMPLIFYING

$$\boxed{O/F(t) = \frac{1}{2}\dot{m}_{ox}^{1-n} a^{-1} \pi^{n-1} L^{-1} \left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right]^{\frac{2n-1}{1+2n}}} \ (b)$$

$$c^* = 2515 + 3075(O/F) - 690(O/F)^2$$

$$\frac{dc^*}{d(O/F)} = 0 = 3075 - 2(690)(O/F)$$

$$O/F_{max} = \frac{3075}{2(690)} = \boxed{2.23} \leftarrow O/F|_{MAX}$$

$$c^*_{MAX} = 2515 + 3075(2.23) - 690(2.23)^2 = \boxed{5941 \text{ ft/s}} \leftarrow$$

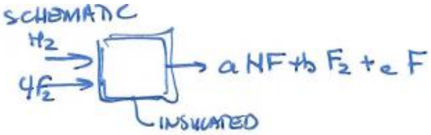
COMMENT:

C* WILL SHIFT WITH RADIUS (TIME) AS THE ENGINE OPERATES

MAE 640 Module 03 Problem - 2021

(a) The O/F ratio of the reactants assuming difluorine F_2 is the oxidizer and H_2 is the fuel.
(b) A mathematical *equation* for "*c*" as only a function of "*b*" by balancing the atoms in the reaction equation.
(c) A *mathematical equation* for the equilibrium constant K_p for the dissociation reaction shown. Express K_p as a function of pressure (P) and "*b*".
(d) A *mathematical equation* for the enthalpy balance of the products and the reactants for an adiabatic combustion, assuming that the specific heats all of the products do not change with temperature (i.e. $C_{p,HF}$ = const.; $C_{p,F2}$ = cont.; and $C_{p,F}$ =const.). The equation should allow the calculation of enthalpy as a function of product temperature.
(e) A *mathematical equation* for the enthalpy balance of the products and the reactants for an adiabatic combustion, assuming that the specific heats all of the products do change with temperature with a third-order polynomial curve fit as shown in the book. The equation should allow the calculation of enthalpy as a function of product temperature. Complete any mathematical integrations. Include symbols for any relevant heats of formation, and complete any integrals. (Do not solve)
(f) If the pressure were doubled, would there be more or less dissociation of F_2? Would the calculated product temperature increase or decrease?
(g) Enter the O/F ratio that you calculated in part (a) in the online box below

4. SCHEMATIC



5. ASSUME
1. CONSTANT PRESSURE COMBUSTION
2. DISSOCIATION REACTION GIVEN
3. STEADY STATE
4. ADIABATIC COMBUSTION
5. CONSTANT SPECIFIC HEAT VALUES FOR THE PRODUCTS AND REACTANTS

6. BASIC EQUATIONS/ANALYSIS

$$O/F = \frac{m_{ox}}{m_f} = \frac{4\ \text{moles}_{F_2}}{1\ \text{mole}_{H_2}} \frac{(38\ g/\text{mole})}{(2\ g/\text{mole})} = \boxed{76}$$

$$H_2 + 4F_2 \rightarrow a\,HF + b\,F_2 + e\,F$$

H: $2 = a$ $\qquad a = 2$

F $\quad 8 = a + 2b + e$

$\quad 8 = 2 + 2b + e \qquad \boxed{c = 6 - 2b}$

$F_2 \rightleftharpoons 2F$

$$K_P = \frac{(X_F)^2}{(X_{F_2})^1} P^{2-1} = \frac{(X_F)^2}{X_{F_2}} P$$

$$X_F = \frac{c}{n}$$

$$X_{F_2} = \frac{b}{n}$$

$$n = a + b + c = 2 + b + (6-2b) = 8 - b$$

$$K_P = \frac{\left(\frac{6-2b}{8-b}\right)^2}{\left(\frac{b}{8-b}\right)} P = \boxed{\frac{\underbrace{(6-2b)^2}_{4(3-b)^2}}{b(8-b)} P = K_p} \leftarrow (c)$$

ENTHALPY

$$H_P - H_R = a\bar{h}_{HF} + b\bar{h}_F + c\bar{h}_F - 1\cdot\bar{h}_{H_2} - 4\bar{h}_{F_2} = 0$$

$$\bar{h} = \int_{T_0}^{T_2} c_p\,dT + h_f^o \quad \Big| \text{ IF } C_P \text{ CONSTANT } = \left[C_P(T_2 - T_0) + h_f^o\right]_i$$

$$a\left[C_P(T_2-T_0)+h_f^o\right]_{HF} + b\left[C_P(T_2-T_0)+h_f^o\right] + c\left[C_P(T_2-T_0)+h_f^o\right]$$
$$- 1.0\left[C_P(\cancel{T_2}-T_0)+\cancel{h_f^o}\right]_{H_2} - 4\left[\cancel{C_P(T_2-T_0)}\ \cancel{h_F}\right]_{F_2}$$

$$\boxed{0 = 2\left[C_P(T_2-T_0)+h_f^o\right]_{HF} + b\left[C_P(T_2-T_0)+\cancel{h_f^o}\right]_{F_2} + (6-2b)\left[C_P(T_2-T_0)+h_f^o\right]_F}$$

olynomial $Cp = a' + b'\left(\frac{T}{1000}\right) + c'\left(\frac{T}{1000}\right)^2 + d'\left(\frac{T}{10000}\right)^3$

$$H_2 = 0 = 2\left[a'T + \frac{b'T}{2}\left(\frac{T}{1000}\right) \quad + \frac{c'T}{3}\left(\frac{T}{1000}\right)^2 \ \frac{d'T}{4}\left(\frac{T}{1000}\right)^3\right]_{T^0}^{T_2} + \Delta h_f^o\Big|_{HF}$$

$$+(b)\left[a'T + \frac{b'T}{2}\left(\frac{T}{1000}\right) \quad + \frac{c'T}{3}\left(\frac{T}{1000}\right)^2 - \frac{d'T}{4}\left(\frac{T}{1000}\right)^3 + \Delta h_f^o\Big|_{HF}\right]_{T^0}^{T_2}\Bigg|_{F_2}$$

$$+(6-2b)\left[a'T + \frac{bT'}{2}\left(\frac{T}{1000}\right) \quad + \frac{c'T}{3}\left(\frac{T}{1000}\right)^2 + \frac{d'T}{4}\left(\frac{T}{1000}\right)^3\right]_{T^0}^{T_2}\Bigg|_F$$

IF PRESSURE DOUBLED LESS DISSOCIATION
HIGHER TEMPERATURE (LESS F)

COMMENT! IN REALITY WOULD USE $c_p = f(T)$ AND CURVE FITS FOR REACTION RATE EQUATIONS.

Pressure	100	200

OF	75.35	75.35
FPCT	1.309758	1.309758
ERATIO	0.250149	0.250149
Phi	0.250149	0.250149
P	99.99996	199.9999
T	2847.332	2975.286
MW	22.10102	22.35648
AR	0	0
F	56%	55%
F2	15%	16%
HF	29%	29%