Module 04 Heat Transfer Problem

(You may screen shot or paint this page and insert in in your exam package to save time)

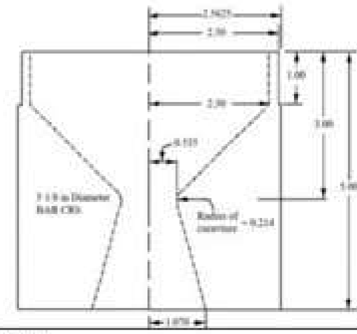
1. **Name:** ______________________
2. **Given:**

A small test rocket uses heat sink cooling of the nozzle throat by making this part out of a thick metal. The baseline nozzle material is **steel** and the alternative material is **copper**.
Assume 1-D transient conduction in the metal and that the thickness of the nozzle material is much greater than the heat-affected zone.

For the $(T_\infty - T_i)$ term use:

- Temperature of Throat = $2667.2K = T_\infty = T_r$
- Initial Temperature = $298K = T_i$

4. **Schematic**



Material Properties

Steel	Copper
$h = 10{,}000\ W/(m^2{\cdot}K)$	$h = 15{,}100\ W/(m^2{\cdot}K)$
$k = 60.5\ W/(m{\cdot}K)$	$k = 380\ W/(m{\cdot}K)$
$\alpha = 17.7 \times 10^{-6}\ m^2/s$	$\alpha = 7.75 \times 10^{-6}\ m^2/s$
$T_{melt} = 1643K$	$T_{melt} = 1185\ K$

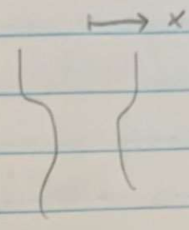
5. **Find:**
   a) A plot of the steel thermal profiles (x from 0.0 to 3.0) at 2, 5, 10 and 15 seconds.
   b) Determine the time when steel reaches melting point.
   c) A plot of the copper thermal profiles (x from 0.0 to 3.0) at 2, 5, 10 and 20 seconds.
   d) Determine the time when copper reaches melting point.
   e) Compare the thermal properties of each material and describe the parameters help extend the possible test and why they effect the answer.
6. **Assume:**

7. **Analysis**
   - Show equations used in symbolic form.
   - You do not have to show sample calculations with units.
   - You can use the software developed on your homework assignment to make the calculations.
   - Put a screen shot of tabular data and graphs in Exam Upload
   - Upload any software used to make your calculations

Write your answer to question (b) in the box below before the end of the exam period.

Mod 04 Problem

Steel & copper



Assume:

1-D heat trans.

conduction in x dir

k, α const

Analysis:

Use MATLAB for plots

$$\frac{T(x,t) - T_i}{T_\infty - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left[\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right] \text{erfc}\left[\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right]$$

2667.2 $\rightarrow T_\infty - T_i$

$x = (0,3)$, $t = 2, 5, 10, 15$

↳ Assume inches. I convert to m

Steel: $h = 10\,000$, $k = 60.5$, $\alpha = 17.7 \times 10^{-6}$, $T_{melt} = 1643K$

b) t when melting point reached

used MATLAB plots

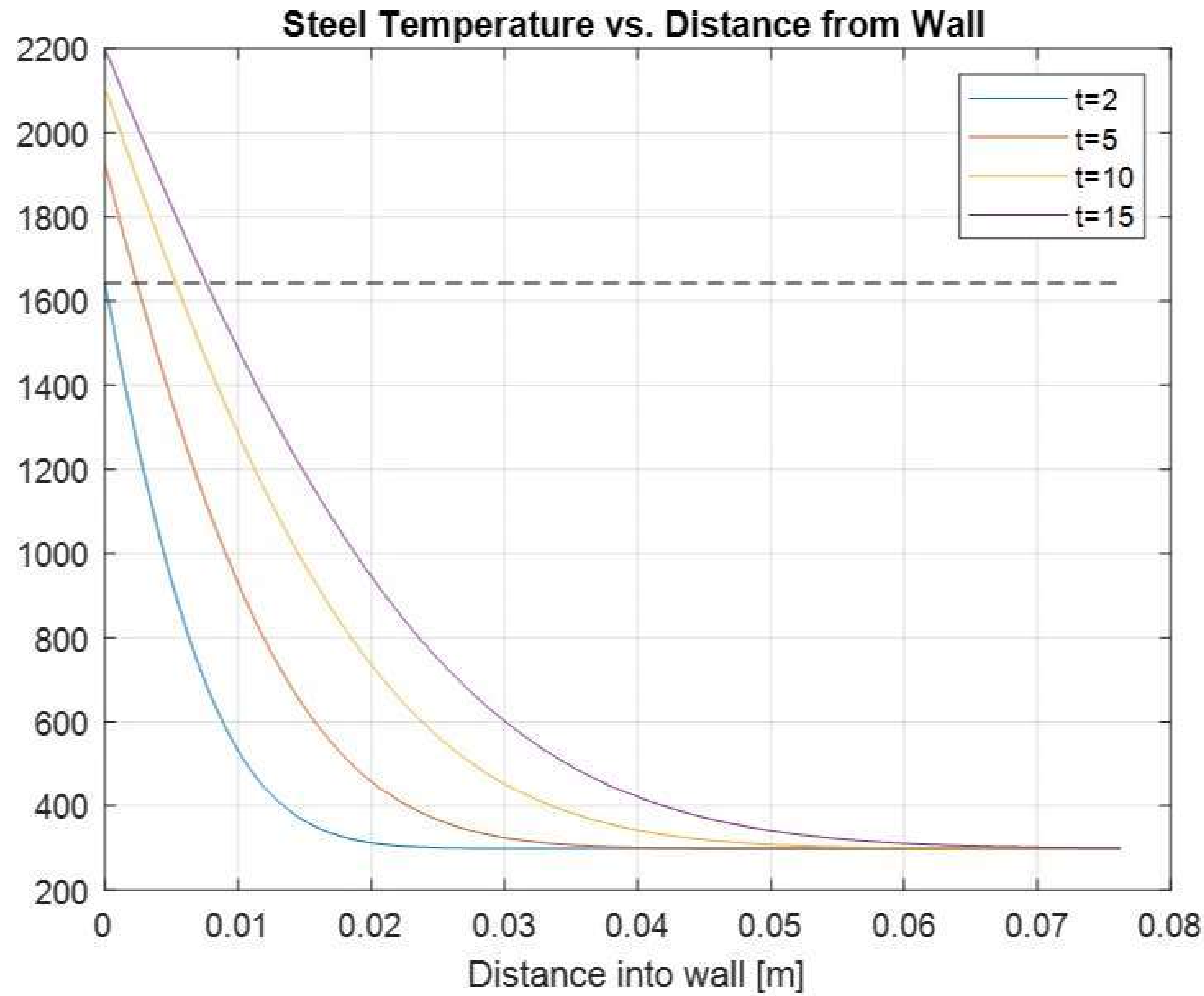
this happens at $t = 2$ when $x = 0$

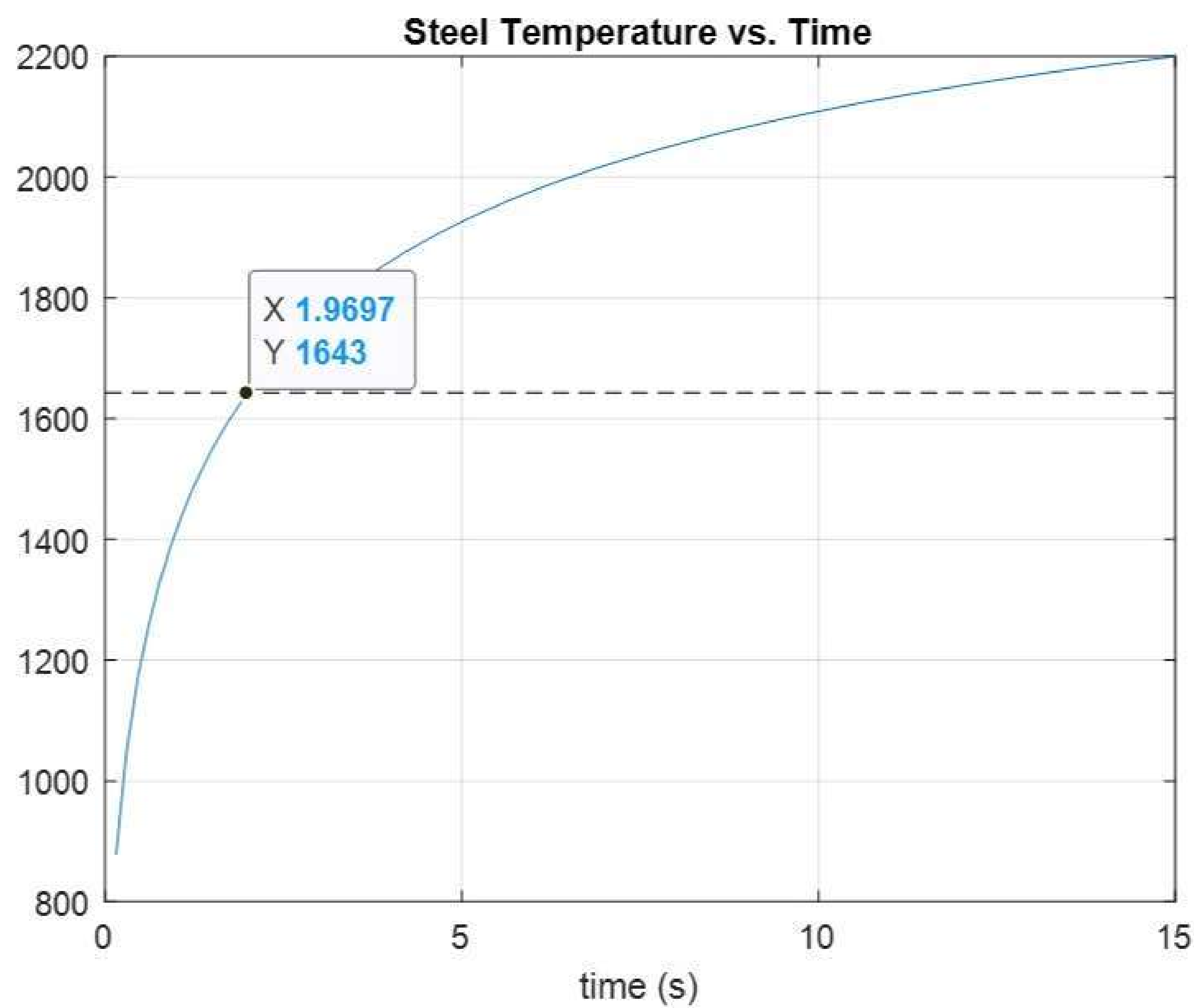
c) Plot copper profile

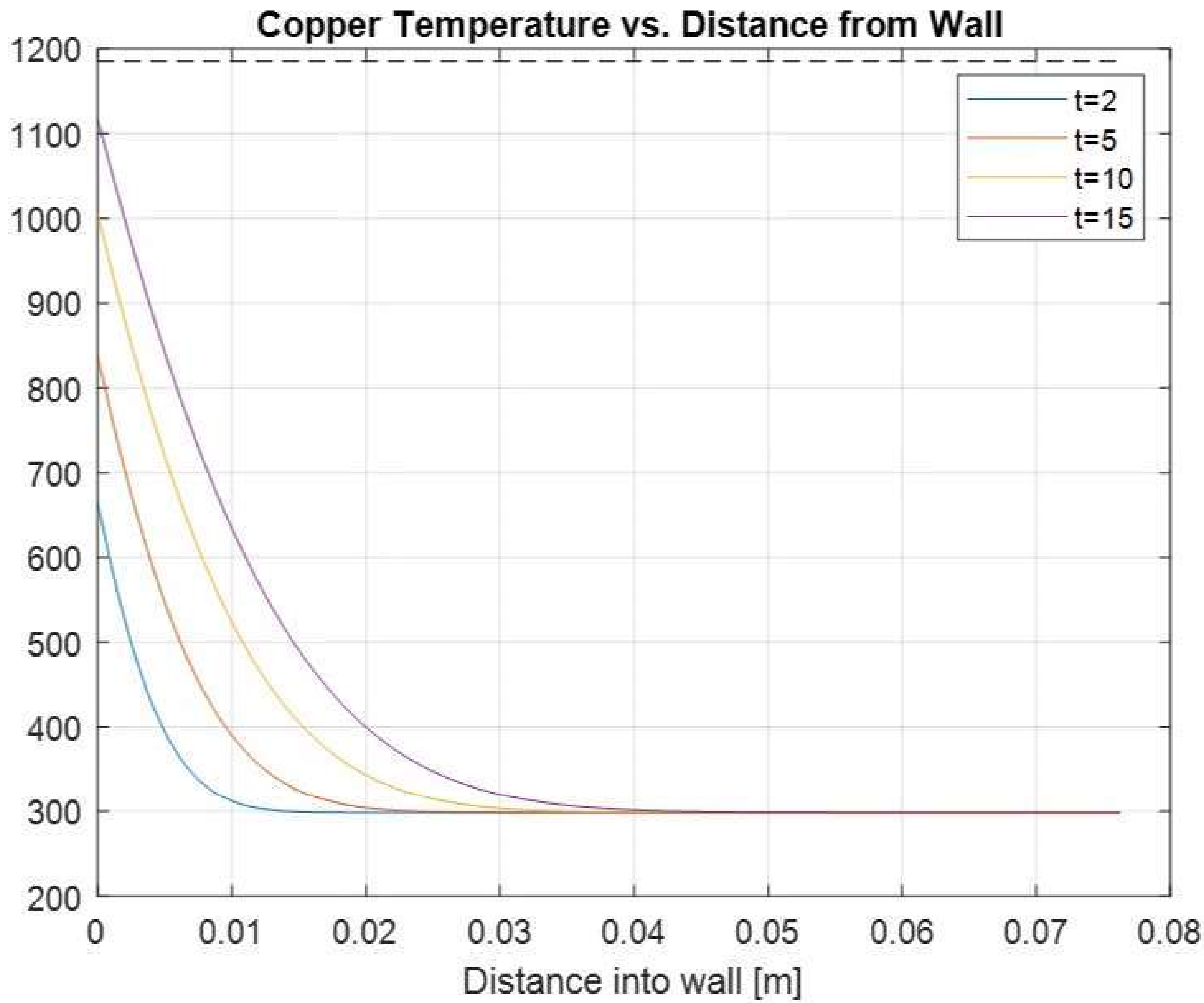
d) t when copper melts

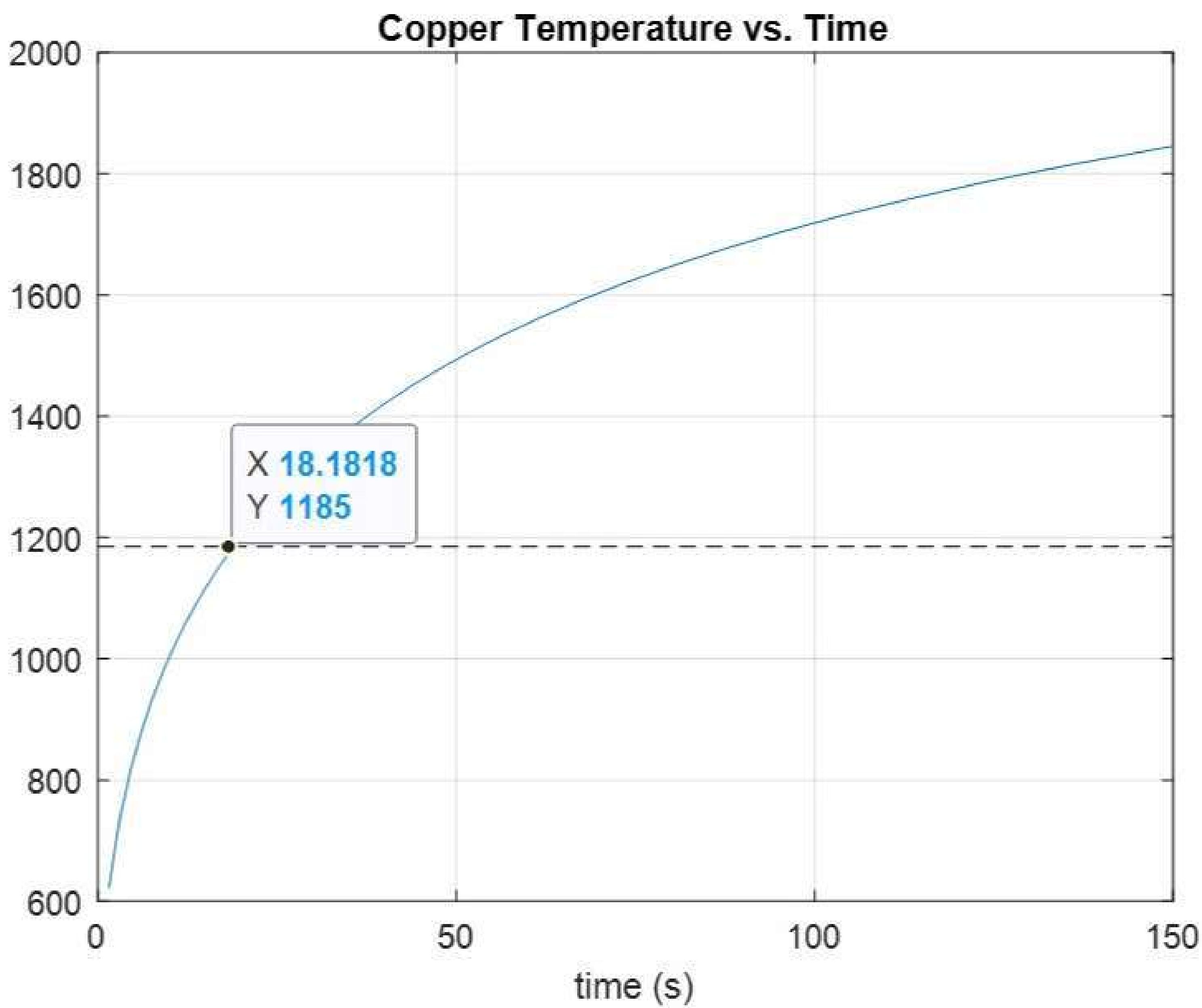
Copper ex: will melt at 18 sec at $x = 0$

e) copper is more conductive (higher k) which means heat is drawn away faster than in steel


Steel Temperature vs. Distance from Wall
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
Distance into wall [m]
t=2
t=5
t=10
t=15


Steel Temperature vs. Time
2200
2000
1800
1600
1400
1200
1000
800
0
5
10
15
time (s)
X 1.9697
Y 1643


Copper Temperature vs. Distance from Wall
t=2
t=5
t=10
t=15
1200
1100
1000
900
800
700
600
500
400
300
200
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
Distance into wall [m]



Copper Temperature vs. Time
2000
1800
1600
1400
1200
1000
800
600
0
50
100
150
time (s)
X 18.1818
Y 1185

Table of Contents

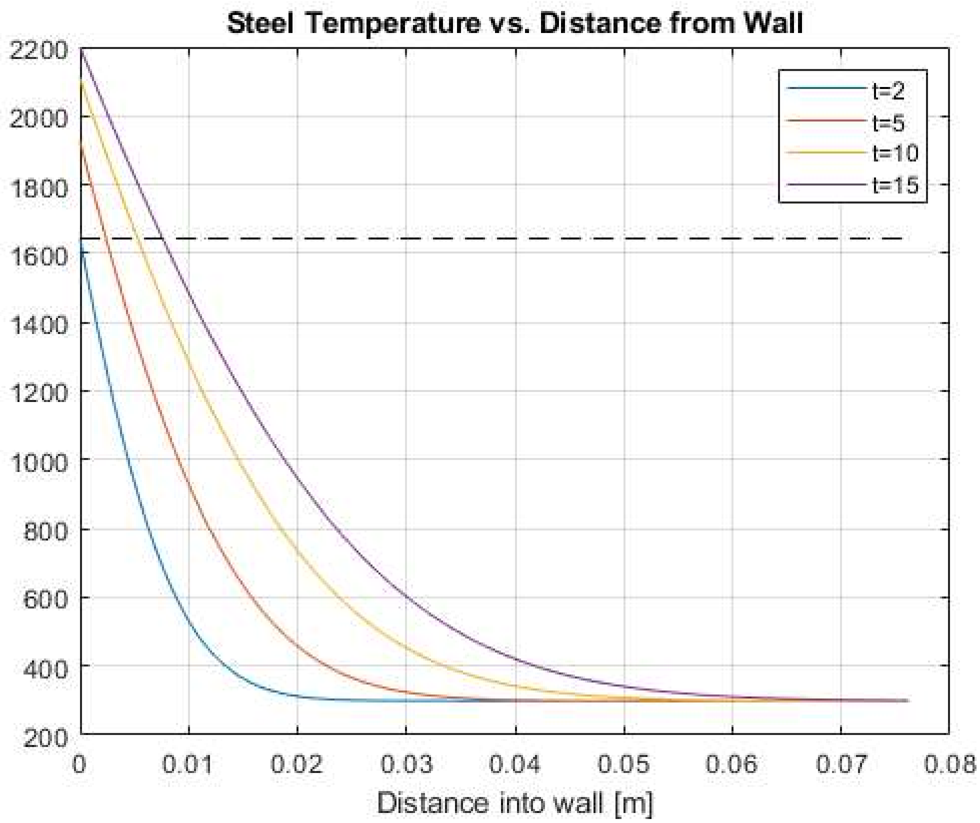
mod 04 steel

```
clear
clc
% Given
h = 10000; % W/Km**2
k = 60.5;  % W/Km
alpha = 17.7e-6; % m**2/s
TDiff = 2667.2; % K

t = [2,5,10,15];
x = linspace(0,3/39.37,100); % convert in to m

Ti = 298; % K - initial wall temperature
% Equation
% iterate over x, plug in t values that we want for plot purposes
for i=1:100
    for j = 1:4
        T_given1(i,j) = Ti + (TDiff-Ti)*(erfc(x(i)/(2*sqrt(alpha*t(j))))
 - exp((h*x(i)/k) + (h^2*alpha*t(j))/(k^2)).*erfc((x(i)/
(2*sqrt(alpha*t(j))))+(h*sqrt(alpha*t(j))/k)));
    end
end

figure(1)
plot(x,T_given1)
hold on
% Plot steel melting temperature
plot(x,ones(1,length(x))*1643,'k--')
grid on
legend('t=2','t=5','t=10','t=15')
title('Steel Temperature vs. Distance from Wall')
xlabel('Distance into wall [m]')
hold off
```



mode 04 steel - time

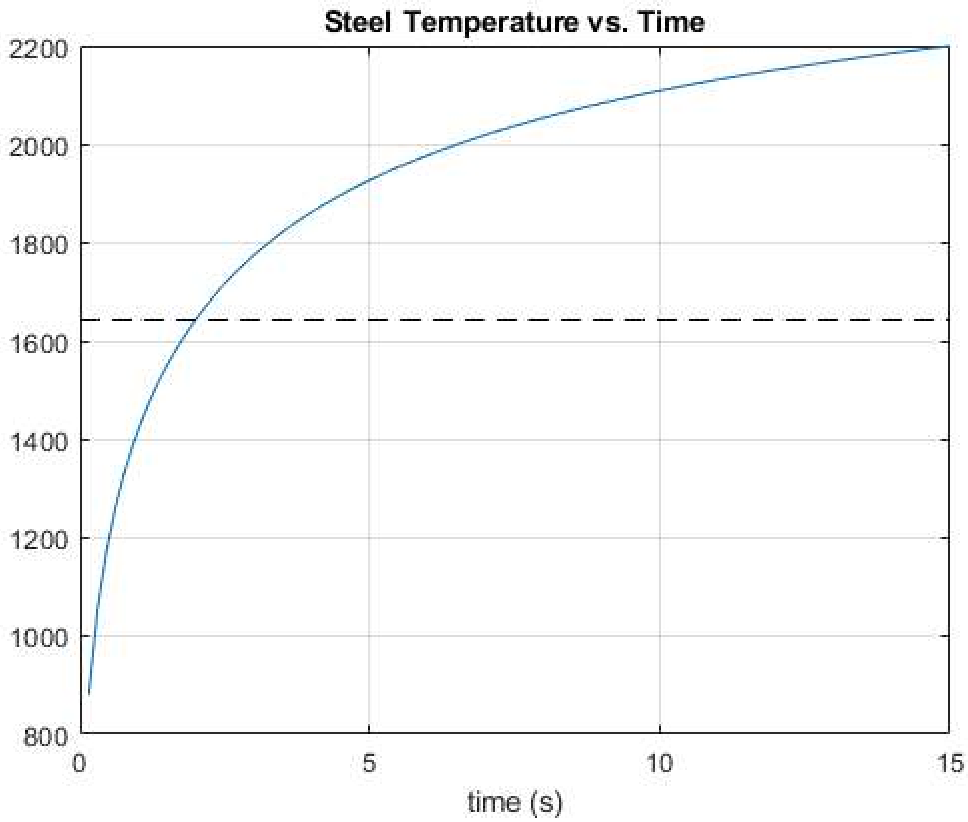
```
clear
clc
% Given
h = 10000; % W/Km**2
k = 60.5;  % W/Km
alpha = 17.7e-6; % m**2/s
TDiff = 2667.2; % K

t = linspace(0,15,100);
x = 0; % convert in to m

Ti = 298; % K - initial wall temperature
% Equation
% iterate over x, plug in t values that we want for plot purposes
for j=1:100
        T_given2(j) = Ti + (TDiff-Ti)*(erfc(x/(2*sqrt(alpha*t(j))))
 - exp((h*x/k) + (h^2*alpha*t(j))/(k^2)).*erfc((x/
(2*sqrt(alpha*t(j))))+(h*sqrt(alpha*t(j))/k)));
end

figure(2)
plot(t,T_given2)
hold on
```

```
% Plot steel melting temperature
plot(t,ones(1,length(t))*1643,'k--')
grid on
title('Steel Temperature vs. Time')
xlabel('time (s)')
hold off
```



mod 04 copper

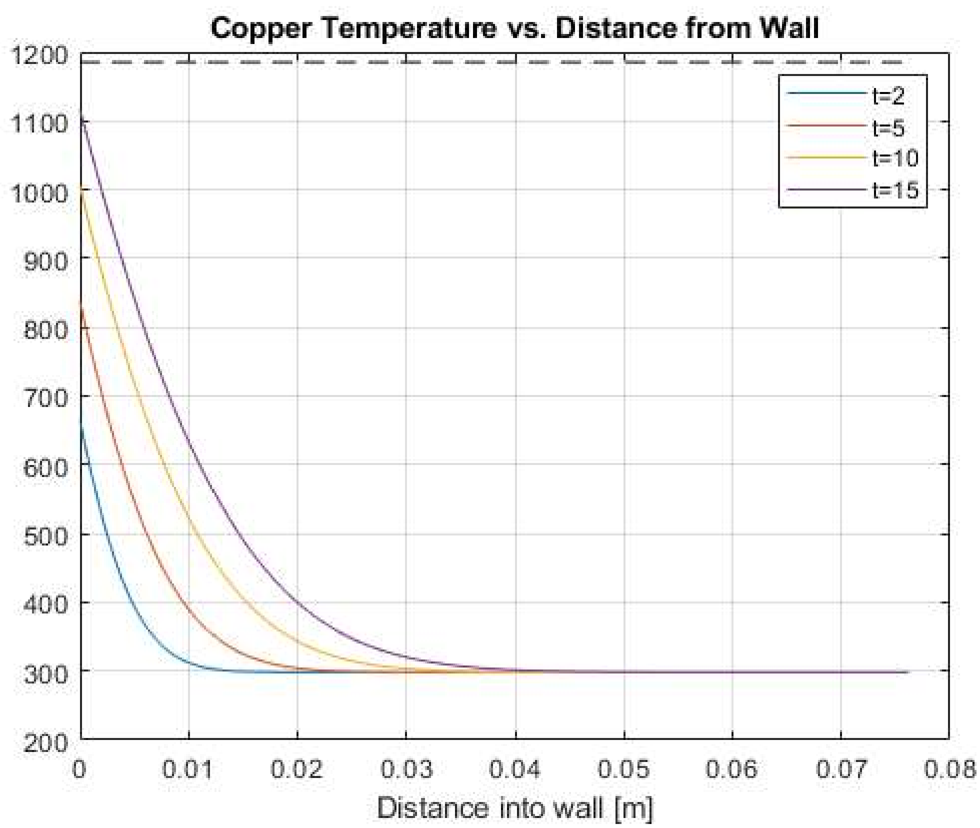
```
clear
clc
% Given
h = 15100; % W/Km**2
k = 380;   % W/Km
alpha = 7.75e-6; % m**2/s
TDiff = 2667.2; % K

t = [2,5,10,15];
x = linspace(0,3/39.37,100); % convert in to m

Ti = 298; % K - initial wall temperature
% Equation
% iterate over x, plug in t values that we want for plot purposes
for i=1:100
    for j = 1:4
```

```
        T_given3(i,j) = Ti + (TDiff-Ti)*(erfc(x(i)/(2*sqrt(alpha*t(j))))
 - exp((h*x(i)/k) + (h^2*alpha*t(j))/(k^2)).*erfc((x(i)/
(2*sqrt(alpha*t(j))))+(h*sqrt(alpha*t(j))/k)));
    end
end

figure(3)
plot(x,T_given3)
hold on
% Plot steel melting temperature
plot(x,ones(1,length(x))*1185,'k--')
grid on
legend('t=2','t=5','t=10','t=15')
title('Copper Temperature vs. Distance from Wall')
xlabel('Distance into wall [m]')
hold off
```



copper time

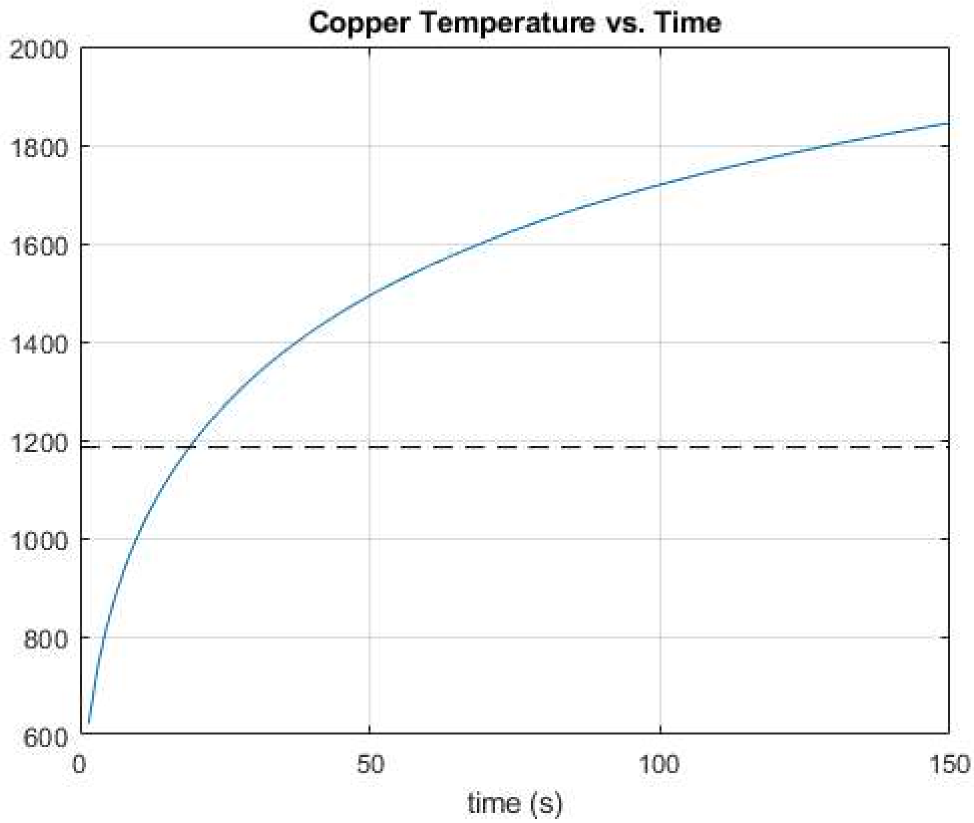
```
clear
clc
% Given
h = 15100; % W/Km**2
k = 380;  % W/Km
alpha = 7.75e-6; % m**2/s
TDiff = 2667.2; % K
```

```
t = linspace(0,150,100);
x = 0; % convert in to m

Ti = 298; % K - initial wall temperature
% Equation

for j=1:100
        T_given4(j) = Ti + (TDiff-Ti)*(erfc(x/(2*sqrt(alpha*t(j))))
 - exp((h*x/k) + (h^2*alpha*t(j))/(k^2)).*erfc((x/
(2*sqrt(alpha*t(j))))+(h*sqrt(alpha*t(j))/k)));
end

figure(4)
plot(t,T_given4)
hold on
% Plot steel melting temperature
plot(t,ones(1,length(t))*1185,'k--')
grid on
title('Copper Temperature vs. Time')
xlabel('time (s)')
```



Published with MATLAB® R2022b

Module 05 Liquid Rocket Problem

(You may screen shot or print this page and insert in in your exam package to save time)

1. **Name:** __________
2. **Given:** A Liquid Oxygen/Liquid Hydrogen thrust chamber that burns for 2 minutes at a total flow rate of 347 kg/s with a mixture ratio of 6.0. The propellants are stored at their boiling temperatures and atmospheric pressure.
3. **Find:**

a. The mass of the fuel for the 2 minute burn (without contingencies)
b. The mass of the oxidizer for the 2 minute burn (without contingencies)
c. Volume of hydrogen for the 2 minute burn (without contingencies)
d. Total length (in meters) of a cylindrical tank with hemispherical ends for the hydrogen if the inner diameter is 1.6 meters

Write your answer for (d) in the box below.

Mod 05

Assume - LOX/LH 2 min

$\dot{m} = 347$ kg/s $r = 6.0$ Patm

a) mass for 2 minute burn (Fuel)

$M_{total} = 347 \times 120\,sec = 41640$ kg total ✓

$\dot{m}_f = \dfrac{\dot{m}_{tot}}{1+r} = \dfrac{347}{7} = 49.57$

$m_f = \dot{m}_f (120)$

$\boxed{m_f = 5948.57 \text{ kg}}$

b) $\dot{m}_{ox} = r \times \dot{m}_f = (49.57)(6) = 297.42$

$m_{ox} = \dot{m}_{ox}\ (120)$

$\boxed{m_{ox} = 35690.4 \text{ kg}}$

c) vol H for 2 min burn

$\rho = m/V$

$V = m/\rho = \underset{m_f}{5948.57} / \underset{\text{google}}{0.08375 \text{ kg/m}^3}$

$\boxed{V_H = 71027.7 \text{ m}^3}$

d) length of cyl tank for H if inner diam = 1.6

$t_r = \rho_c A_c l_c \dot{m}$

$l_c = \dfrac{t_r}{\rho_c A_c \dot{m}} = \dfrac{120 \text{ sec}}{0.08375 \frac{kg}{m^3} \cdot \frac{1.6^2 \pi}{4} m^2 \cdot 49.57 \text{ kg/s}}$

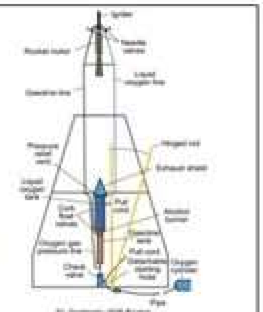
$\boxed{l_c = 45.16 \text{ m}}$

Module 06 Liquid Rocket Propellants

(You may screen shot or print this page and insert in in your exam package to save time)

1. **Name:** _______________

2. **Given:** On March 16, 1926, Robert Goddard set up his rocket, which he later called Nell, fueled with gasoline and liquid oxygen, on a farm in Auburn, Massachusetts. Assume a chamber pressure of 250 psi, and an area ratio of 5.0. It burned for 20 seconds.



3. **Find**:
   a) Using CEQUEL, make a table and a plot of c^* and the vacuum specific impulse as a function of mixture ratio. (Mixture ratios from 1 to 10)
   b) What is a mixture ratio to achieve high performance?
   c) Estimate the Nozzle throat dimeter using the given data and the results plus your engineering judgement on whatever data might be missing

- Outline problem in Homework Format
- Insert Screen Shots of tables and requested graphs into the exam submission file.
- Upload computer programs used to make the calculations in the exam upload site

Mod 06

gas & LOX

Assume- $P_c = 250$ psi, ARatio = 5.0 time = 20 sec

Analysis - a) on CEQUEL

b) highest performance ⇒ high Isp

this occurs at mixture ratio of $\boxed{3}$

c) $\varepsilon = 5.0$ e/t $P_c = 250$

$$A_t = \frac{F_v\, c^*}{Isp_v P_c\, g_o} = \quad \text{est } F_v = 30\,000$$

$$A_t = \frac{30\,000 \times 4258}{220.9 \times 250 \times 32.2}$$

$$\boxed{A_t \approx 71.83 \text{ in}^2}$$

Pc	250	psi
area ratio	5	
time	20	sec
ox	LOX	
f	gas	

mixture ratio	c*	Isp Vac
1	4258	220.916
2	5726	291.304
3	5677	296.292
4	5400	282.054
5	5172	269.821
6	4982	259.272
7	4816	249.625
8	4663	240.471
9	4519	231.759
10	4379	223.557

Assumed vacuum thrust		Throat Area
30000		71.8295