Name: Veronica Loomis

PROBLEM 11.3

Given:

Hybrid rocket (LOX/HTPB)

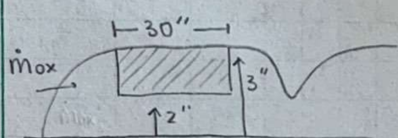
$c^* = -2520 + 6800\,(O/F) - 1320\,(O/F)^2$ $2 < O/F < 3$ [ft/s]

$\rho_f = 0.0325\ \text{lb/in}^3$

$r = 0.16\, G_{ox}^{0.7}$ [in/s]

Find: i) Optimal O/F for this propellant combination
ii) Oxidizer flow rate ($\dot{m}_{ox}$) which maximizes mid web performance
iii) Overall O/F shift for the firing, assuming fuel completely consumed

Schematic:



Assumptions:
Midweb => 2.5"
All of fuel is completely consumed

Basic Equations:

$O/F = \dot{m}_{ox}/\dot{m}_f$
$G_{ox} = \dot{m}_{ox}/(\pi R^2)$
$\dot{m}_f = r \rho_f A_b$
$A_b = 2\pi R L$

Analysis:
i) optimal O/F
derive c^* wrt O/F

call $\eta = O/F$

$$\frac{dc^*}{d\eta} = 6800 - 2(1320)\eta = 0$$

$$\eta = 2.575$$

i) Answer

$\boxed{\text{optimal } O/F = 2.575}$

ii) Oxidizer flow rate that maximizes performance @ midweb

Given $R_i = 2''$, $R_{end} = 3''$
so mid web → $R = 2.5''$

We know from (i) that $O/F = 2.575$
so
$\dot{m}_{ox} = 2.583\,\dot{m}_f$

$\dot{m}_f = r \rho_f A_b$
$= 0.16 \left(\frac{\dot{m}_{ox}}{\pi R^2}\right)^n \rho_f\, 2\pi R L$

$= 0.16 \left(\frac{\dot{m}_{ox}}{\pi (2.5)^2}\right)^{0.7} (0.0325)\, 2\pi(2.5)(30)$

$\dot{m}_f = 0.304876\,\dot{m}_{ox}^{0.7}$

sub back in

$\dot{m}_{ox} = 2.575(0.304876)\,\dot{m}_{ox}^{0.7}$

ii) Answer

$\boxed{\dot{m}_{ox} = 0.44678 \text{ lb/s}}$

iii) overall O/F shift

$\dot{m}_{ox} = 0.44678$ lb/s → max

min at either $R = 2$, $R = 3$

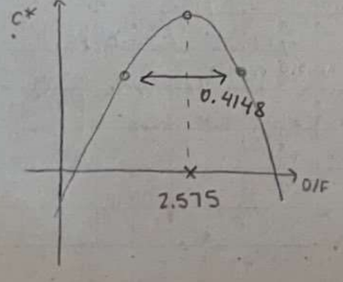
$\dot{m}_f$ @ $R = 2 \Rightarrow 0.16 \left(\frac{0.44678}{\pi (2)^2}\right)^{0.7} (0.0325)\, 2\pi(2)(30) = 0.18965 \rightarrow (O/F)_{min}$

$\dot{m}_f$ @ $R = 3 \Rightarrow 0.16 \left(\frac{0.44678}{\pi (3)^2}\right)^{0.7} (0.0325)\, 2\pi(3)(30) = 0.161256 \rightarrow (O/F)_{max}$

$(O/F)_{min} = 2.3558$, $(O/F)_{max} = 2.7706$ these are both $2 < O/F < 3$ ✓

$\boxed{\text{overall O/F shift} = 0.4148}$ iii) Answer

Comment:



Name: Veronica Loomis

Problem 11.7

Given:

Hybrid rocket, obeys $r = a G_{ox}^n$

$r = dR/dt$

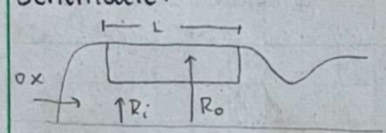
Find: i) Show $R(t) = \left[a(2n+1) \left(\frac{\dot{m}_{ox}}{\pi} \right)^n t + R_i^{2n+1} \right]^{1/(2n+1)}$

ii) Derive expressions for $\dot{m}_f$ and O/F

Is there a special value of n where fuel flow is const & no mixture ratio shifts?

iii) Plot $R(t)$, $\dot{m}_f(t)$, and $O/F(t)$ with the given conditions

Schematic:



Assumptions:

$\dot{m}_{ox}$ is constant

fuel regression rate is uniform

Basic Equations:

$$G_{ox} = \frac{\dot{m}_{ox}}{\pi R^2}$$

$$\dot{m}_f = r \rho_f A_b$$

$$A_b = 2\pi R L$$

$$O/F = \dot{m}_{ox} / \dot{m}_f$$

Analysis:

i) $r = \frac{dR}{dt} = a G_{ox}^{n} = a\left(\frac{\dot{m}_{ox}}{\pi R^2}\right)^n$

$\int R^{2n}\, dR = \int a \dot{m}_{ox}^{n} \pi^{-n}\, dt$

$\frac{1}{2n+1}\left[R(t)\right]^{2n+1} = a \dot{m}_{ox}^{n} \pi^{-n} t + R_i^{2n+1}$

$$\boxed{R(t) = \left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right]^{1/2n+1}}$$ i) Answer

ii $\dot{m}_f$

$\dot{m}_f = r \rho_f A_b = a\left(\frac{\dot{m}_{ox}}{\pi R^2}\right)^n \rho_f 2\pi R L$

$\dot{m}_f = 2 a \rho_f L \dot{m}_{ox}^{n} \pi^{1-n} R^{1-2n}$

Substitute R(t) from (i)

$$\dot{m}_f = 2a\rho_f L \dot{m}_{ox}^{n} \pi^{1-n}\left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right]^{\frac{1-2n}{1+2n}}$$ ii A) Answer

when $n = 1/2$, fuel flow is constant wrt time

O/F

$O/F = \dot{m}_{ox} / \dot{m}_f$

$$O/F = \frac{\dot{m}_{ox}^{1-n}}{2a\rho_f L \pi^{1-n}}\left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right]^{\frac{2n-1}{2n+1}}$$ ii B) Answer

when $n = 1/2$, there is no mixture ratio shift

iii) $L = 50''$, $R_i = 2''$, $R_o = 5''$, $\rho_f = 1 g/cm^3 = 0.0361273\ lb/in^3$, $a = 0.1$, $n = 0.8$

G_{ox} initial $= 1.0\ lb/in^2 \cdot s$

we assumed $\dot{m}_{ox}$ was const so

$G_{ox} = \dot{m}_{ox}/\pi R$ @ initial values

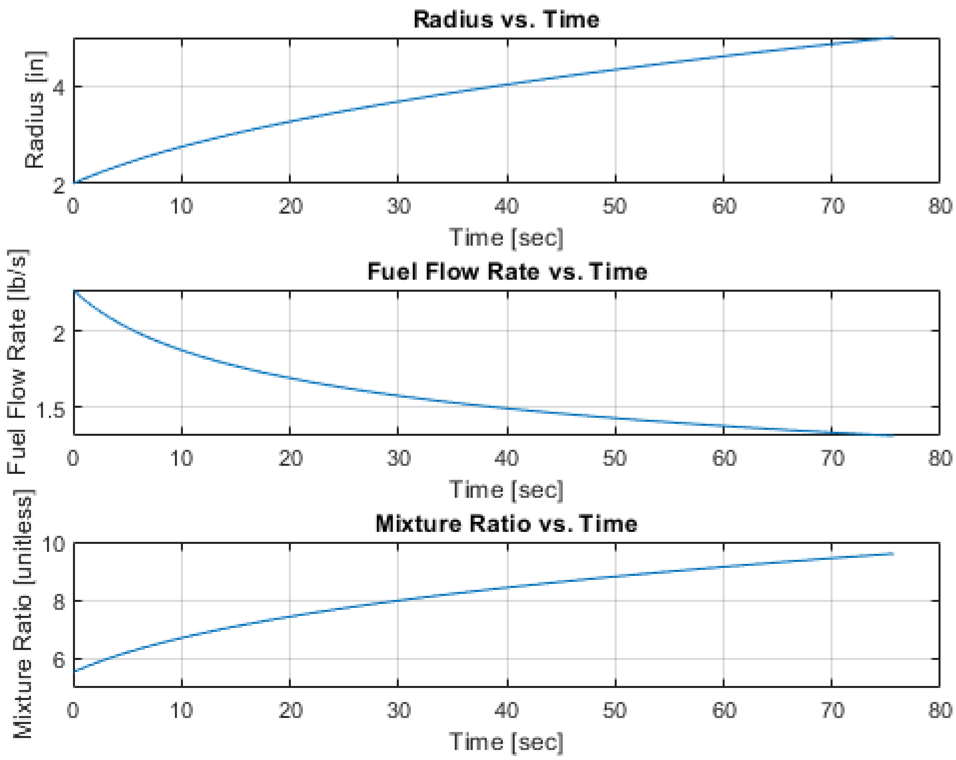
$\dot{m}_{ox} = G_{ox,init}\, \pi R_o$

Script & plots included next.

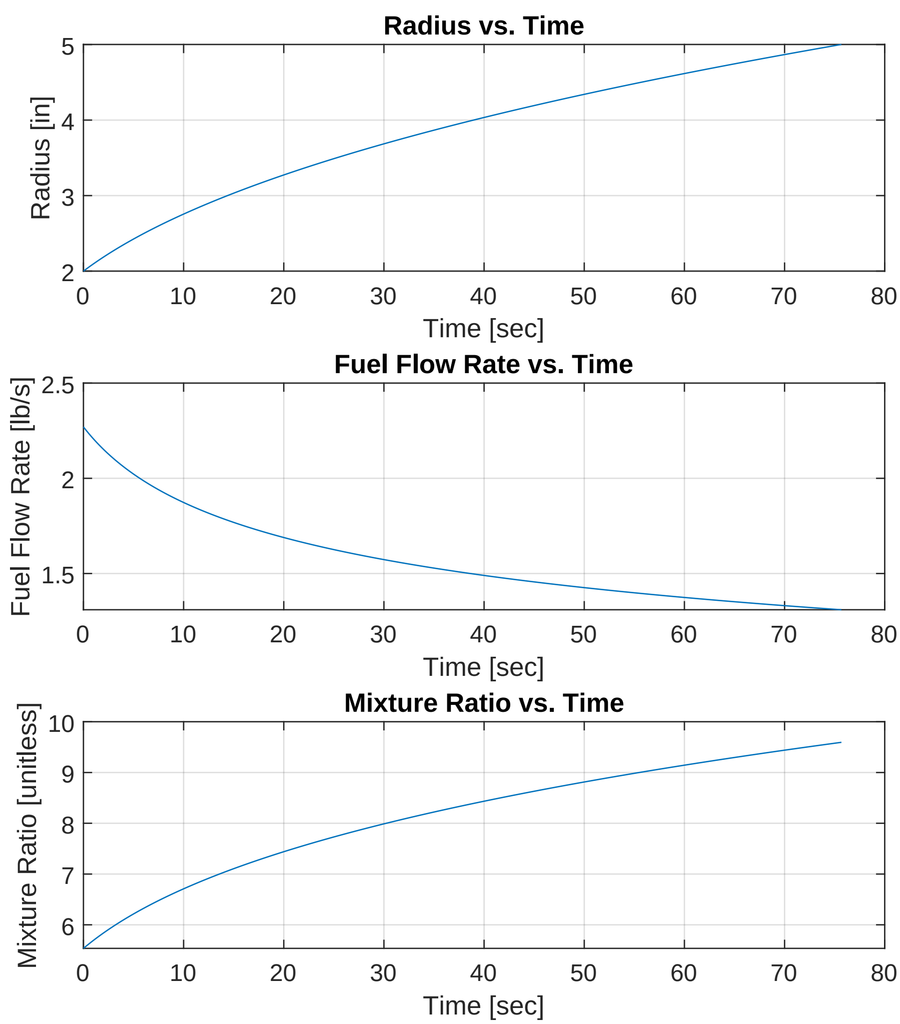
Veronica Loomis

HW02 - Book Problem 11.7 Part iii Using the values given Plot: R(t) vs. time mFdot(t) vs. time O/F(t) vs. time

```
L = 50;
Ri = 2;
Ro = 5;
rhoF = 0.0361273; % lb/in**3
a = 0.1;
n = 0.8;
Gox = 1.0; % lb/s-in**2
% since we are told mOxdot is constant,
% we can define it from the initial G and R values
mOxdot = Gox*pi*Ri^2;

allTime = linspace(0,100,1000);

i = 1;
testR = Ri;

while testR < 5
    R(i) = (((allTime(i)*a*(2*n+1))*mOxdot^n/(pi^n))+Ri^(2*n+1))^(1/
(2*n+1));
    mFdot(i) = 2*a*rhoF*L*mOxdot^n*pi^(1-n)*R(i)^(1-2*n);
    OF(i) = mOxdot/mFdot(i);
    testR = R(i);
    i = i+1;
end

t = allTime(1:i-1);

figure(1)
subplot(3,1,1)
plot(t,R)
title('Radius vs. Time')
grid on
xlabel('Time [sec]')
ylabel('Radius [in]')
subplot(3,1,2)
plot(t,mFdot)
title('Fuel Flow Rate vs. Time')
grid on
xlabel('Time [sec]')
ylabel('Fuel Flow Rate [lb/s]')
subplot(3,1,3)
plot(t,OF)
title('Mixture Ratio vs. Time')
grid on
xlabel('Time [sec]')
ylabel('Mixture Ratio [unitless]')
```



Published with MATLAB® R2019b


Radius vs. Time
Radius [in]
5
4
3
2
0
10
20
30
40
50
60
70
80
Time [sec]
Fuel Flow Rate vs. Time
Fuel Flow Rate [lb/s]
2.5
2
1.5
0
10
20
30
40
50
60
70
80
Time [sec]
Mixture Ratio vs. Time
Mixture Ratio [unitless]
10
9
8
7
6
0
10
20
30
40
50
60
70
80
Time [sec]

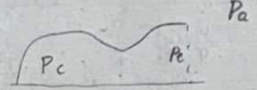
Name: Veronica Loomis

SP 02-HW SPA

Given: CF spreadsheet

Find: Include calculated separation line for $P_e/P_a = 1/3$
" for Normal Shock at exit

Schematic



Assumptions
Isentropic flow

Basic Equations:

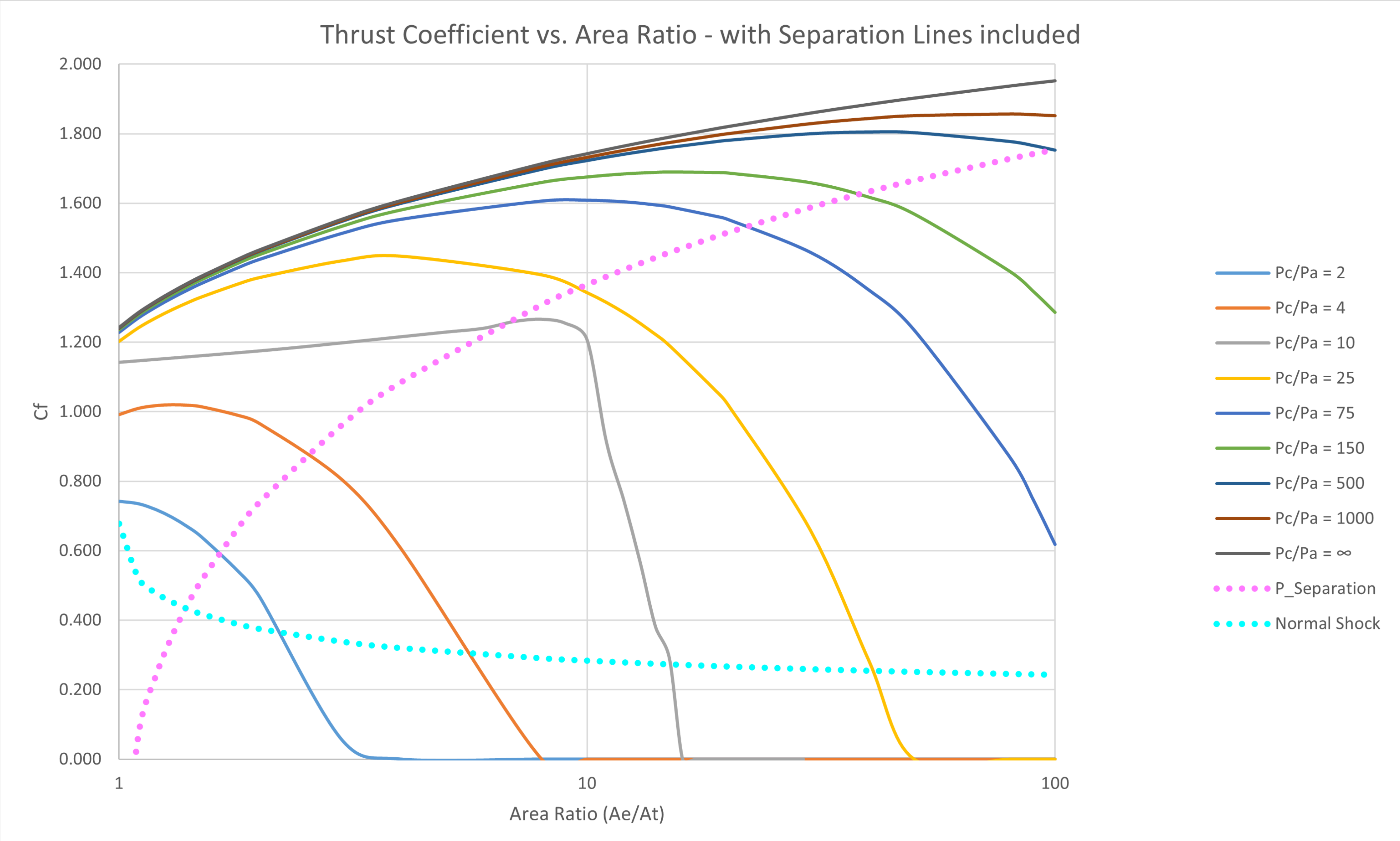
$$P_c/P_e|_{isentropic} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\gamma/(\gamma-1)}$$

$$P_e/P_a|_{sep\ 1/3} = 1/3$$

$$P_e/P_a|_{normal\ shock} = \left[\frac{2\gamma}{\gamma+1} M_e^2 - \frac{\gamma-1}{\gamma+1}\right]^{-1}$$

$$P_c/P_a = (P_c/P_e)(P_e/P_a)$$

Analysis, Answer, & Comparison are on the attached excel file.
The figure is the next page in this PDF.


Thrust Coefficient vs. Area Ratio - with Separation Lines included
Cf
2.000
1.800
1.600
1.400
1.200
1.000
0.800
0.600
0.400
0.200
0.000
1
10
100
Area Ratio (Ae/At)
Pc/Pa = 2
Pc/Pa = 4
Pc/Pa = 10
Pc/Pa = 25
Pc/Pa = 75
Pc/Pa = 150
Pc/Pa = 500
Pc/Pa = 1000
Pc/Pa = ∞
P_Separation
Normal Shock

Two-Page Annotated Bibliography Template
(MAE 640)
Summarize

Reference Document Examined:	Frederick, R., and Thomas, D., "Propulsion Research and Academic Programs at the University of Alabama in Huntsville," 2023 AIAA SciTech, January 26, 2023.
Reviewer:	Veronica Loomis
Source of Document:	canvas.uah.edu
Date of Review:	January 25, 2023
Electronic File Name:	HW01_PaperReview

Summary of Paper:

The Propulsion Research Center at the University of Alabama in Huntsville provides an important environment for connecting academic research with real-world needs within the propulsion community. UAH ranks very high on the list of schools with NASA and DoD funding and research, and it achieved the highest research activity rating on the Carnegie Classification of Institutes of Higher Education. One detail showing the growth of the propulsion program at UAH is the fact that the program went from 15 to 150 students from the years 1991-2022. The University has BSME and BSAE as two different majors, and as of 2018 there have been more AE undergraduates enrolled than there are for ME.

UAH is deeply rooted in rocket propulsion since Dr. Werner Von Braun (among others) facilitated state funding to expand UAH to attract and teach people in order to build up the US Space program. Enrollment for the core propulsion classes at UAH has steadily increased over the past few years and dates as far back as 1959 to educate those in the NASA Apollo program.

Research expenditures have increased dramatically since the inception of the Propulsion Research Center (a growth of 176% over the past 5 years). The total research expenditures over the past 31 years is $54 million ($74 million adjusted for inflation). This averages to roughly $240,000 per advanced degree.

B. Assess:

Important Facts from Document:

1. The Propulsion Research Center is a large draw for aerospace students at UAH
2. UAH is as popular as it is thanks to the US Space program and funding from those who wanted those in the program to be well educated.
3. UAH ranks highly when it comes to funding, aerospace, computer science, and overall research activity.
4. The PRC is large and has many branches of technical topics that are all "monitored" by a faculty member.
5. Research is expensive.

Key Figure from Document:

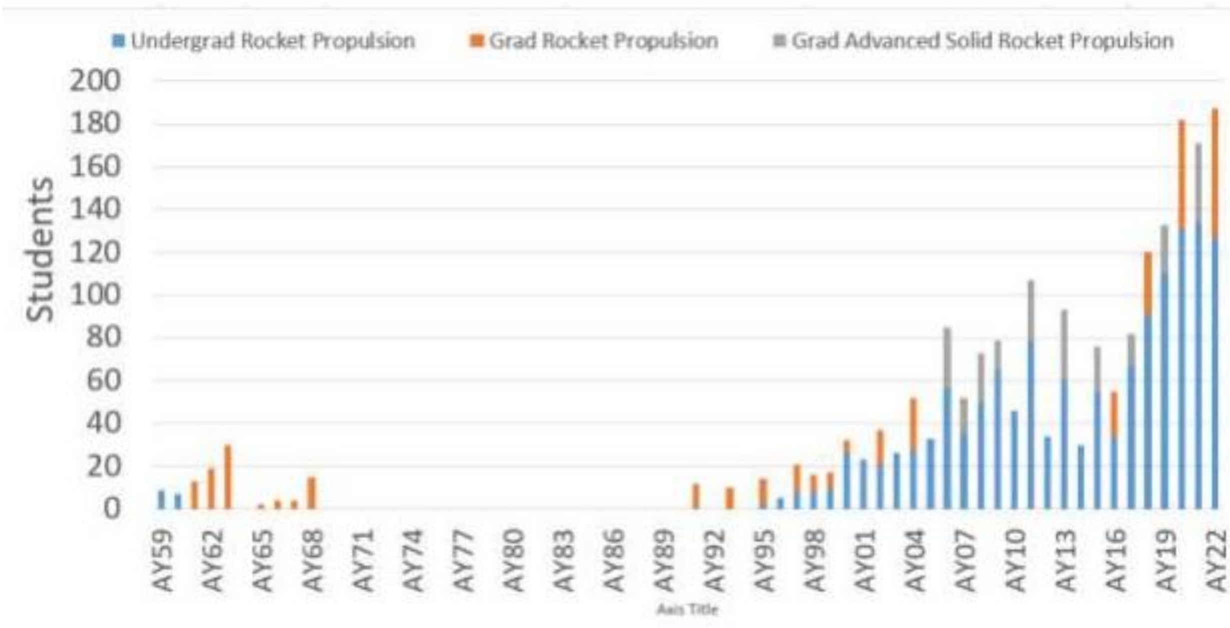


Figure 1: Enrollment Trends for Propulsion Classes at UAH

Important Relationships among Parameters Described in the Paper:

1. If you build it, they will come.
    a. UAH, and more specifically the PRC, is successful because many aerospace students are interested in propulsion research and want to enroll in a university with a rich history within that field.
2. Since more students are becoming more interested in this field and since the demand in the industry calls for it, more specialized classes are being added to further propulsion knowledge in fields like nuclear, fusion, and electric propulsion.

C. Reflect

This paper made me proud to be a UAH student who is taking a propulsion class. The PRC is a large organization that not only provides a bridge between academic research and real-world knowledge, but also branches into so many smaller technical topics that can be studied.