MAE 640 Rocket Propulsion II

Lecture 02L-B-1 – Hybrid Rocket Engines

Items Included:

[A] Announcements
[B] Module Overview
[C] Chapter 11.3 Lumped Parameter Ballistics
[D] Chapter 11.6 HRP Propellants
[E] Chapter 11.7 Design, UAH Designs, AIAA Design Example
[F] Compile Questions on 02HW-A for Help Session (2:00 Today)

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[A] Announcements

Resources

- General
  - **Thrust Coefficient -** Excel Spreadsheet for the starting the thrust coefficient download is in Module 01 “Lectures” section and on 02HW Drop Box
  - **Questions** - Email me with your questions about HW02
  - **January 30 Lecture** will be pre-recorded (will be on travel)
  - 01HW Grades are now posted.
- Homework 02
  - ***Light My Fire 02-A*** **(Office Hours/Helps Session)** (Today, Monday, 2:00 - 3:00 Online from my Office (TH S226) Homework Problems are discussed. I will post the Video and Charts on CANVAS.

[B] Module Overview

Module 02 Objectives

- Students will be able to correctly define and describe fundamental concepts, advantages, and performance parameters of hybrid rockets engines.
- Students will be able to perform calculations of basic performance parameters and internal ballistics.
- Students will apply concepts using a computer program to describe time-dependent internal ballistics and thrust.

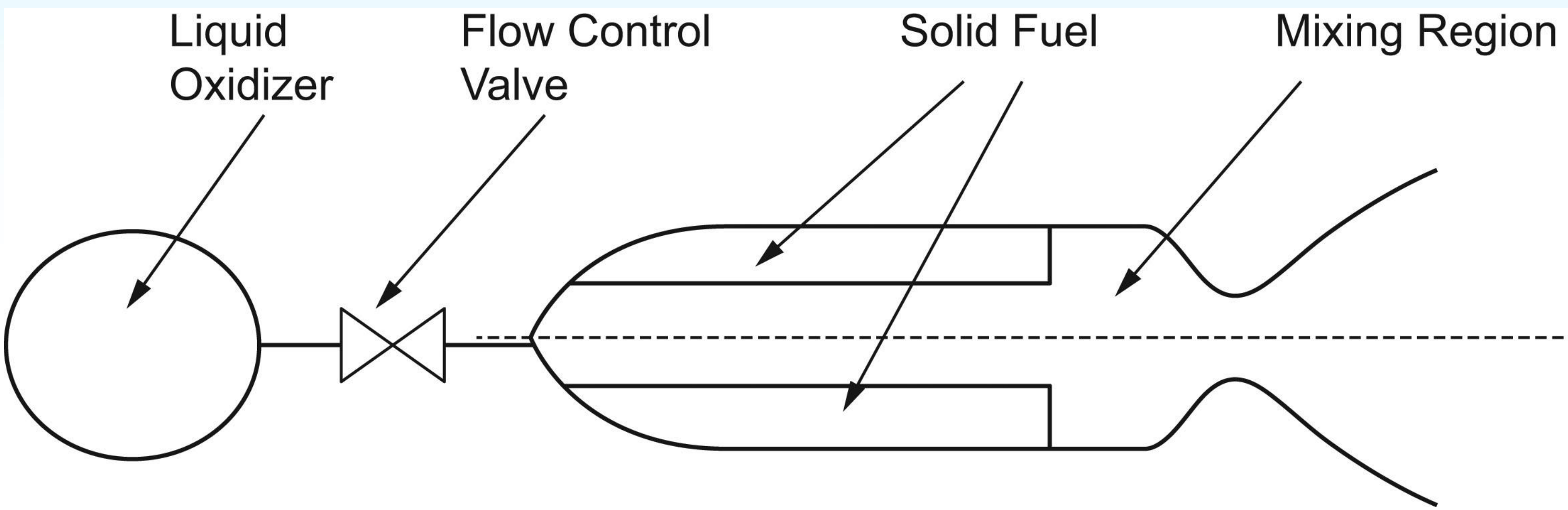
[B] Module Overview

Module 02 – Hybrid Rocket Engines (Week 2)

- ☐ Attend or View Lecture 02-B-1 and 02-B-2
- ☐ Study Textbook Chapter 11.4 to 11.7
- ☐ Complete Quiz 2B (02QA)
- ☐ Participate or Review LIGHT MY FIRE 02B
- ☐ Complete 02HW-B

[B] Module Overview

Major Elements of a Hybrid Rocket Engine (HRE)



[B] Module Overview

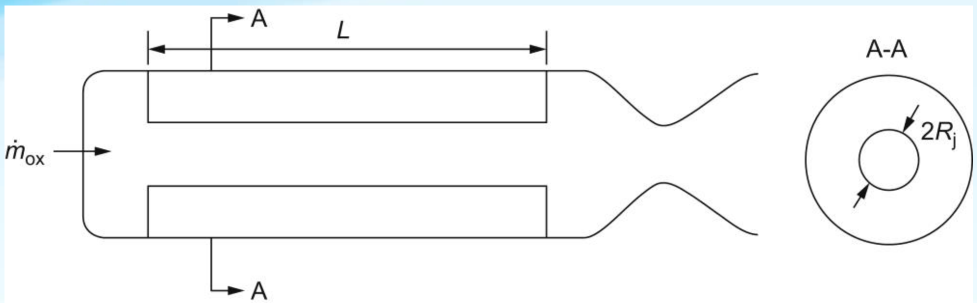
Hybrid Rocket Motor Firing at UAH Propulsion Research Center

[C] 11. 3 HRE Lumped Parameter Ballistics

Assumptions

- "Lumped" is Characterizing the Combustion Chamber with a Single Average Pressure that only Varies with Time.
- Regression Rate is Constant Along Each Port.
- In Reality there will be Pressure Drops Along the Ports and Some Non-uniform Regression Rates in the Fuel.
- The Oxidizer-to-Fuel Ratio Does Shift With Time causing shifts in the c^* *and* γ With Time

[C] 11. 3 HRE Lumped Parameter Ballistics



$$A_p = \pi R^2; A_b = 2\pi R L \tag{E1}$$

$$r = aG_{ox}^n = a\left(\frac{\dot{m}_{ox}}{\pi R^2}\right)^n \tag{E2}$$

%&=qMnt9V5

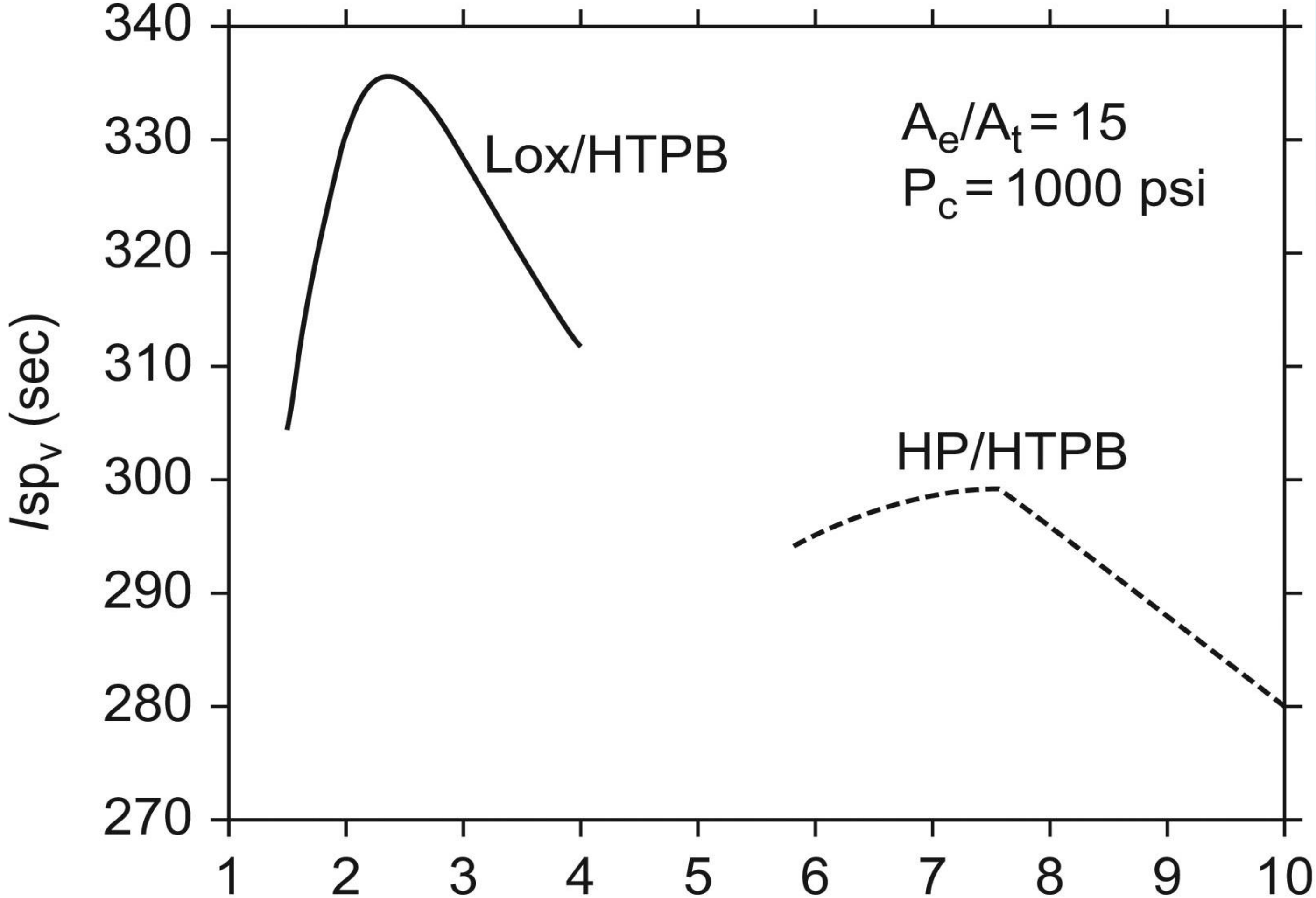
$$\dot{m}_f = r\rho_f A_b = a\rho_f\left(\frac{\dot{m}_{ox}}{\pi R^2}\right)^n 2\pi R L = 2a\pi^{(1-n)} P_f L \dot{m}_{ox}{}^n R^{(1-2n)} \tag{E3}$$

$$r = \frac{dR}{dt} = a\left(\frac{\dot{m}_{ox}}{\pi}\right)^n R^{-2n} \tag{E4}$$

$$R(t) = \left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + Ri^{2n+1}\right]^{\frac{1}{2n+1}} \tag{E6}$$

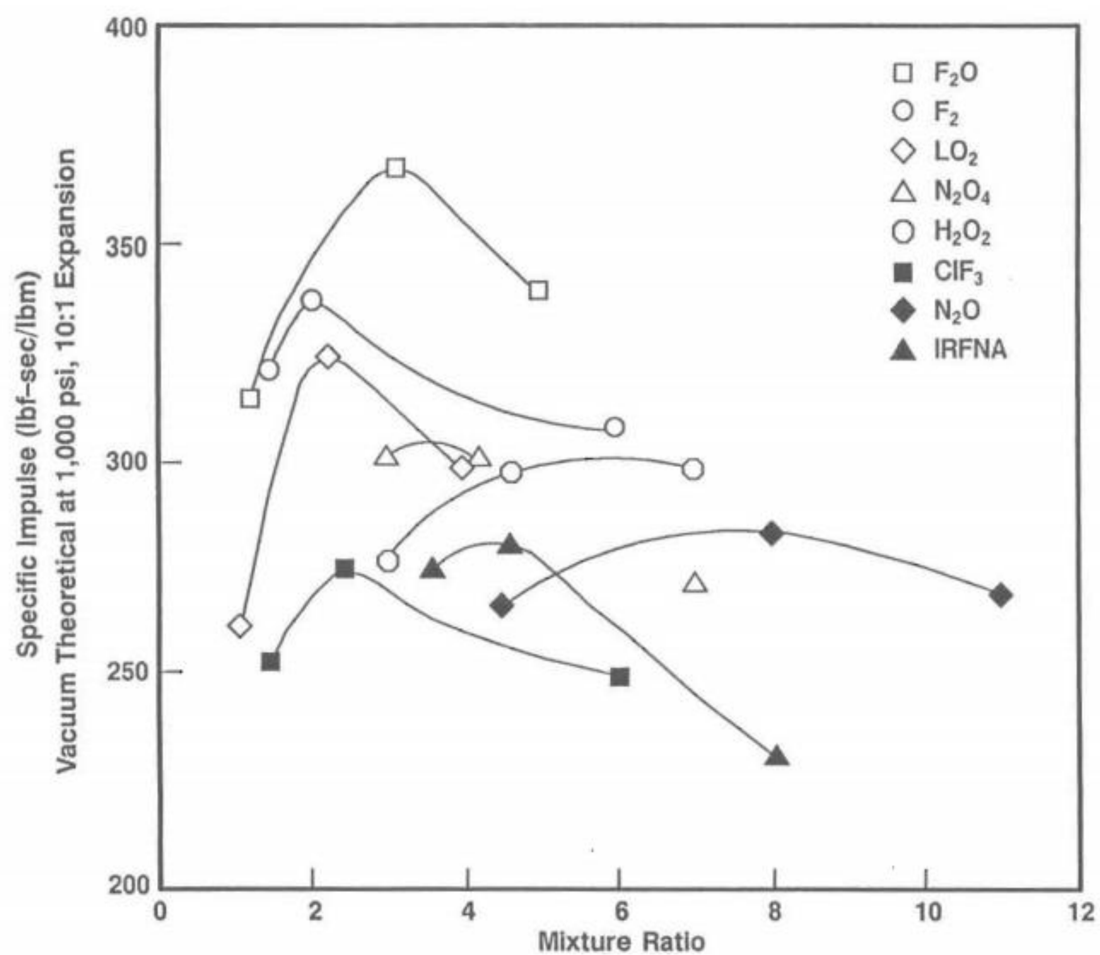
[D] 11. 6 HRE Propellants

$$p_c = (\dot{m}_{ox} + \dot{m}_f)c * /(gA_t) \tag{11.5}$$



[D] 11. 6 HRE Propellants

Performance of Several Oxidizers as a function of O/F ratio
HTPB Fuel, 1000 psi chamber, and 10:1 expansion to seal level



$$I_{sp} = \frac{F}{\dot{m}g} = \frac{c_f p_c A_t}{g p_c A_t / c^*} = c_f c^* / g$$

LO2

Mixture Ratio	C* (ft/sec)	Gamma
1.0	4,825	1.308
1.2	5,180	1.282
1.4	5,543	1.239
1.6	5,767	1.201
1.8	5,882	1.171
2.0	5,912	1.152
2.2	5,885	1.143
2.4	5,831	1.138
2.6	5,768	1.135
2.8	5,703	1.133
3.0	5,639	1.132

[D] 11. 6 HRE Oxidizers

- **LOX- Oxygen (O_2)**
  - Gaseous is popular for Lab. Studies
  - Liquid Oxygen (LOX) more applicable to flight because of higher density
  - Not Storable
- **Nitrous Oxide (N_2O) (Also a Monopropellant)**
  - Self Pressurizing
  - “safe” and “deadly
- **Inhibited Red Fuming Nitric Acid (IRFNA) and WFNA**
  - Good density and storability
  - Toxic and Corrosive
- **HP -Hydrogen Peroxide (H_2O_2) (Also a Monopropellant)**
  - 90-98% concentrations
  - Can detonate at certain conditions

[D] 11. 6 Some HRE Fuels

- **Poly(methyl methacrylate) (PMMA)**
  - Cast Plexiglas $(C_5O_2H_8)_n$
  - Used for Baseline Laboratory Studies
- **Hydroxyterminated Polybutadiene (HTPB) and CTPB and PBAN**
  - Rubber-like Materials have good mechanical properties
  - Produce High-Flame Temperatures
  - Low Regression Rates
- **Diclopolybutaduene (DCPD)**
  - More like a hard plastic
  - Also Polyethylene and Polystyrene
- **Paraffin-Based Fuels**
  - Enhance Regression Rate
  - Have Liquid Layer on Surface that Sheds droplets into flow
  - Structural Considerations
- **Metal Additives**
  - Aluminum, Boron, others
  - Increase theoretical *Isp* and fuel density
  - Considerations of Combustion Efficiency and Slag
- **Rapid Prototype Materials**

[E] 11. 7 HRE Design

System Requirements
Δv, F_{avg}, F_i, t_b, $g_{\max}$, $q_{\max}$

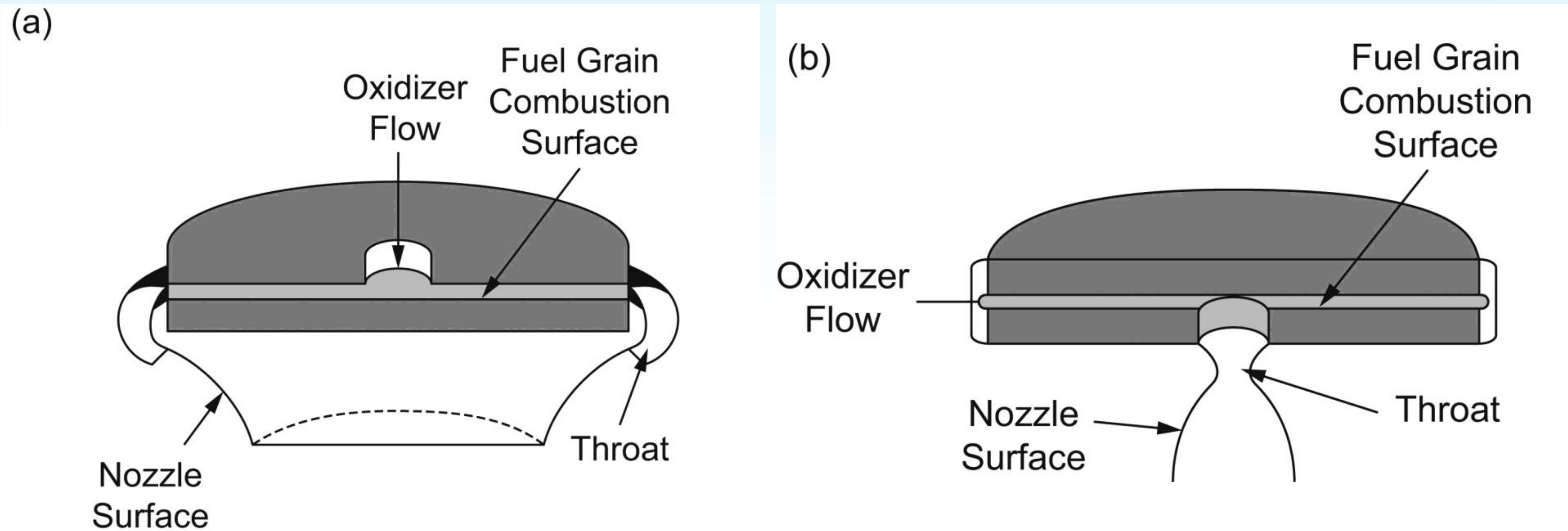
Propellant Selection
ρ_f, ρ_{ox}, a, n, P_c, Isp, c^*

Fuel Section Design
Topology, # Ports, L

System Sizing
m_f, m_{ox}, V_f, V_{ox}, A_t, L_f

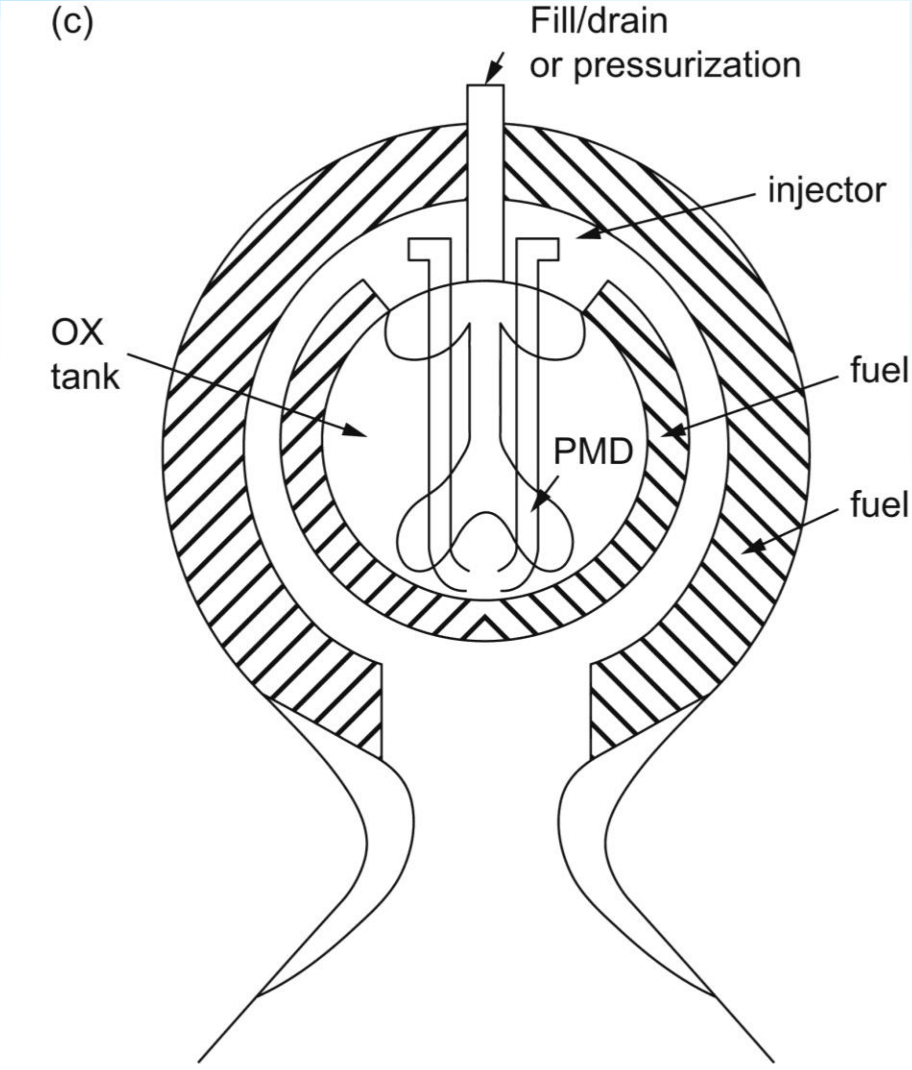
[E] 11. 7 HRE Design

"Pancake" Motors with Outward or Inward-Flowing Oxidizer



- Radially Outward (a) or Radially Inward (b)
- Desirable for Length-Limited Systems
- (a) Shows Aerospike Nozzle
- Mass Flux Decreases with increasing distance from Center of Motor

[E] 11. 7 HRE Design



- Spherical Oxidizer Tank is in the center of the System
- Outer Wall of Tank Lined With Fuel
- Second Concentric Fuel Layer Lining Inside of the Case
- Improved packaging
- Lower Inert Weight

[E] UAH HRE Design

Hybrid Rocket Upper Stage Performance

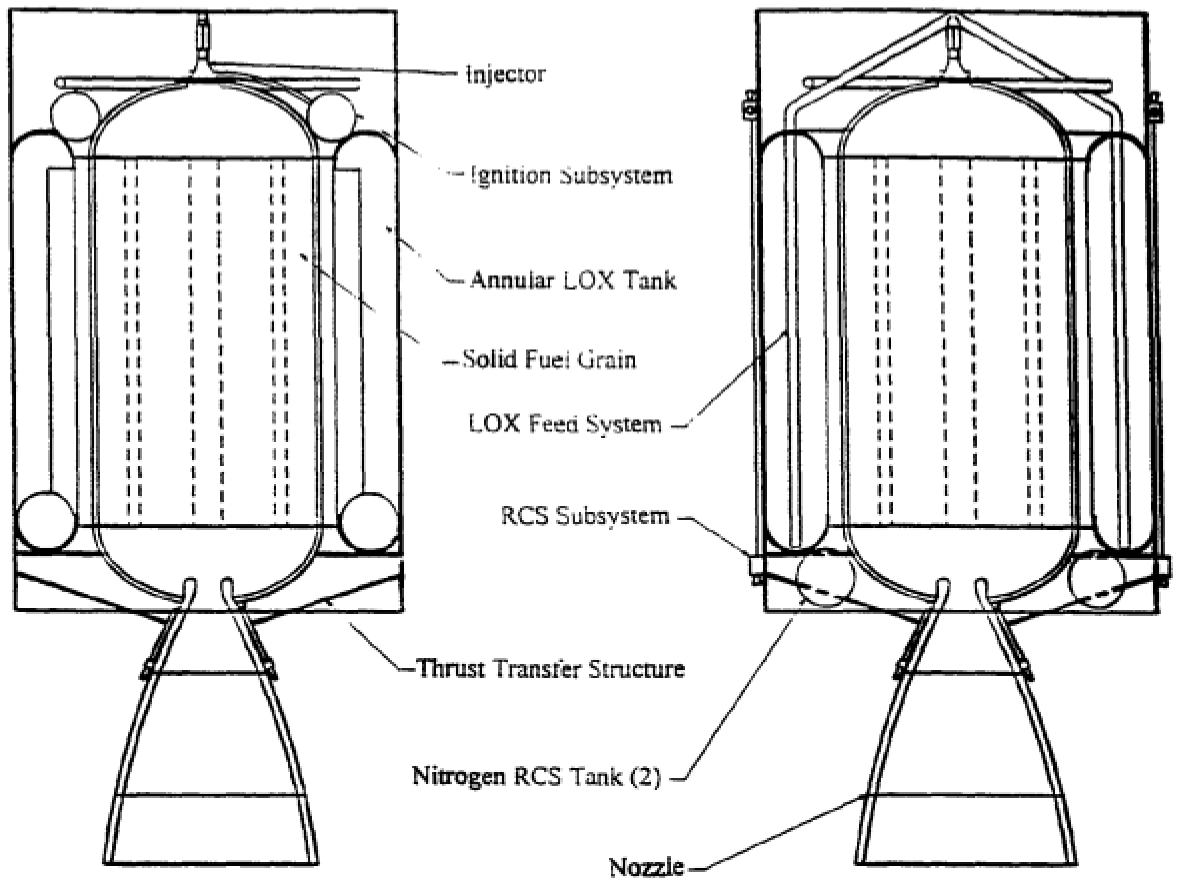


Figure 3. Side View of HRPUS Demonstrator

Figure 4. Side View of HRPUS Demonstrator Rotated 90 Degrees

LaSarge, P.A., Ford, S.I., and Frederick, R.A., "Conceptual Design of Hybrid Rocket Powered Upper Stage (HRPUS) Demonstrator," AIAA Paper 96-2841, July 1996.

[E] UAH HRE Design

Hybrid Rocket Upper Stage Performance

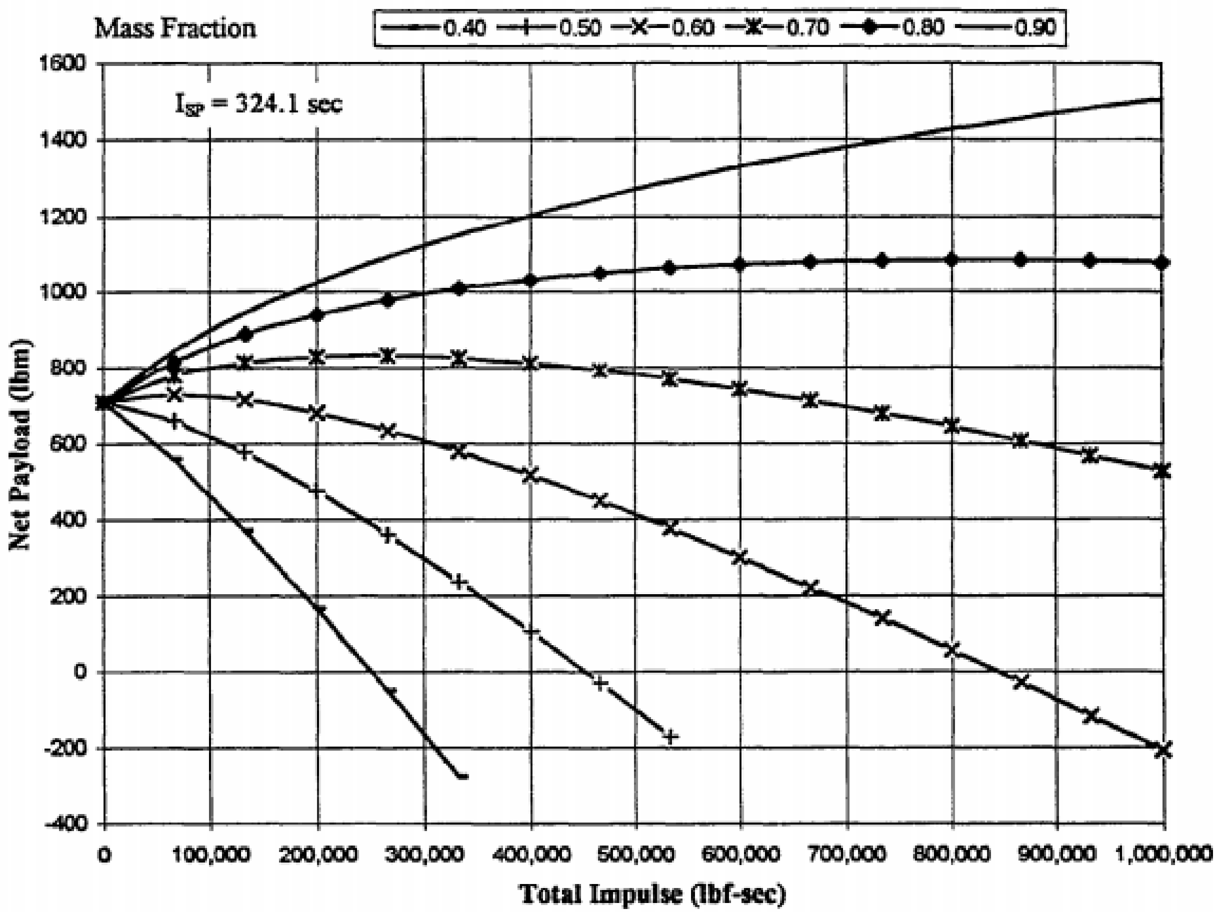


Figure 1. Plot Indicating Relationship Between Specific Impulse, Mass Fraction, Total Impulse, and Net Payload

LaSarge, P.A., Ford, S.I., and Frederick, R.A., "Conceptual Design of Hybrid Rocket Powered Upper Stage (HRPUS) Demonstrator," AIAA Paper 96-2841, July 1996.

The HRPUS was designed as a fourth stage vehicle that was to mate with the 48 inch third stage interface and payload shroud interface.[1] This interface was to be both physical as well as functional. A minimum delivered net payload was set at 200 pounds to a 100 nautical mile orbit at an inclination of 28.5°. NASA set a target cost of \$15 million for one demonstrator vehicle to be ready for a 1999 launch date. A low technology risk and a high degree of manufacturability was also a criteria for the vehicle. The HRPUS was to be 3-axis stabilized and able to withstand a maximum axial load of 7.0 g's and a maximum lateral load of 2.5 g's. The maximum dynamic pressure was not to exceed 2,700 pounds per square foot.

- Payload Performance
- 4th Stage Hybrid Rocket
- Function of Total Impulse
- Stage Mass Fractions

[E] UAH HRE Design

Hybrid Rocket Upper Stage Performance

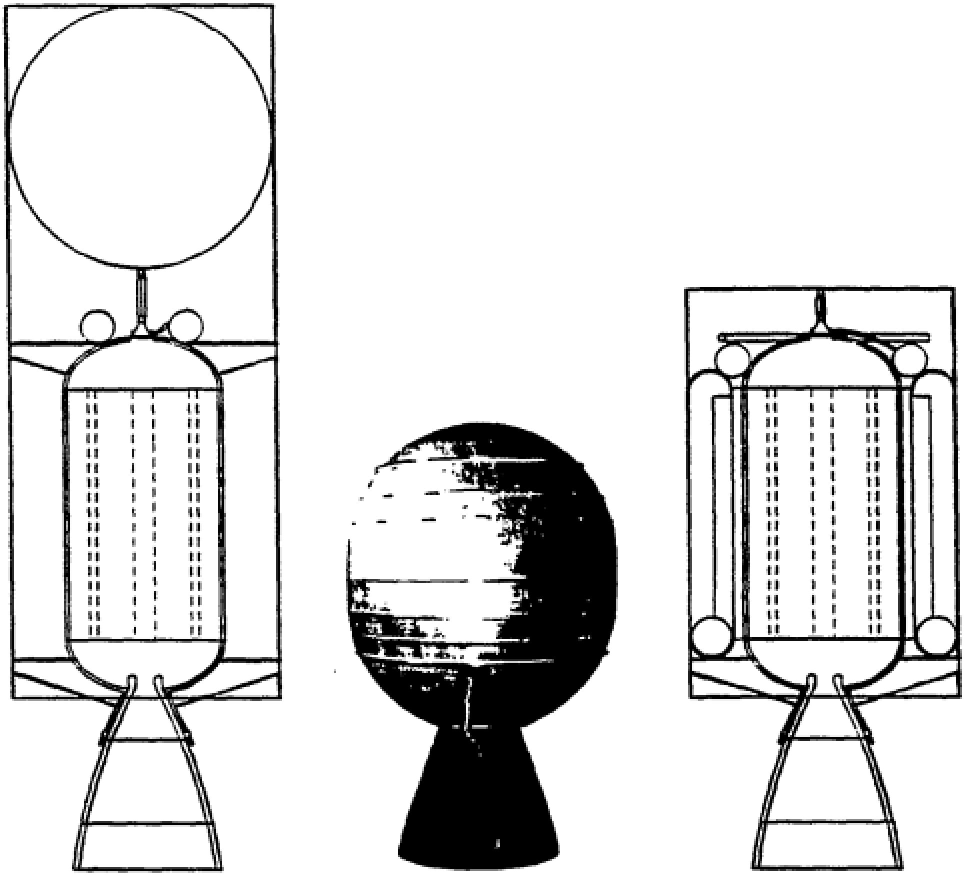


Figure 7. Scale Drawings Showing Size Comparison Between Stacked Configuration (l), STAR-48 (m), and Annular Configuration (r)

Comparisons

- Convectional Layout
- Star-48 Solid Propellant
- Compact Hybrid Reconfiguration

Table 2. Comparison of Configuration Performance Parameters

Parameter	Stacked	STAR -48	Annular
Overall Length (in)	155	80	103
Total Stage Weight (lbm)	4,987	4,665*	4,628
Stage Mass Fraction	0.60	0.94**	0.66
Net Payload (lbm)	59	800	418

*Indicates actual motor weight only.
**Indicates motor mass fraction.

LaSarge, P.A., Ford, S.I., and Frederick, R.A., "Conceptual Design of Hybrid Rocket Powered Upper Stage (HRPUS) Demonstrator," AIAA Paper 96-2841, July 1996.

[B] UAH HRE Design

Hybrid Rocket Upper Stage Performance

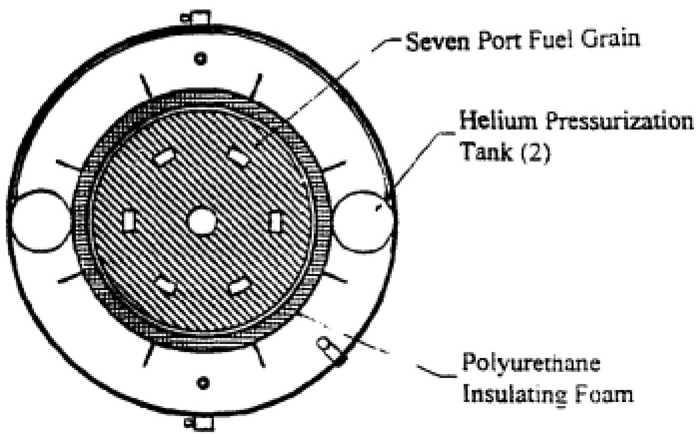


Figure 5. Cross-sectional View of HRPUS Demonstrator

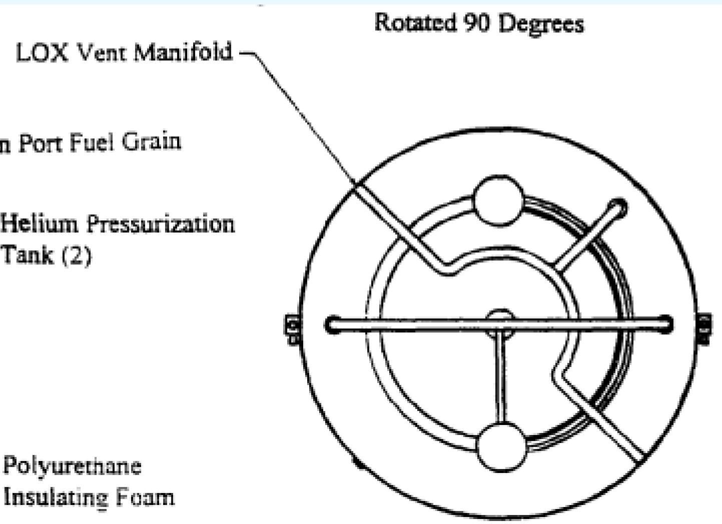


Figure 6. Top View of HRPUS Demonstrator

LaSarge, P.A., Ford, S.I., and Frederick, R.A., "Conceptual Design of Hybrid Rocket Powered Upper Stage (HRPUS) Demonstrator," AIAA Paper 96-2841, July 1996.

[E] UAH HRE Design

Hybrid Rocket Upper Stage Performance

Table 1. Evaluation of HRPUS Performance

Evaluation Criteria	Requirement	Result	Meets Requirements
Total Program Cost	$15M	$20.8M	No
Net Payload (lbm)	200	418	Yes
Performance			
Maximum Acceleration (g's)	7	5.66	Yes
Maximum Q (lb/ft^2)	2,600	2,739	No
Manufacturability	Med. - High	High	Yes
Major Risk Factors			
Cost Risk			
Probability to Exceed $15M	Low (<=15%)	High	No
Technology Risk	*Low	Low	Yes
Schedule Risk			
Launch Date (31 Dec. 99)	Low	Low	Yes

LaSarge, P.A., Ford, S.I., and Frederick, R.A., "Conceptual Design of Hybrid Rocket Powered Upper Stage (HRPUS) Demonstrator," AIAA Paper 96-2841, July 1996.

[E] UAH Porous Fuel HRE Design

In this configuration, the oxidizer is flowed through a porous grain or through small ports to the surface where combustion occurs. This design is in contrast to conventional hybrids where combustion occurs in ports cast into the grain.

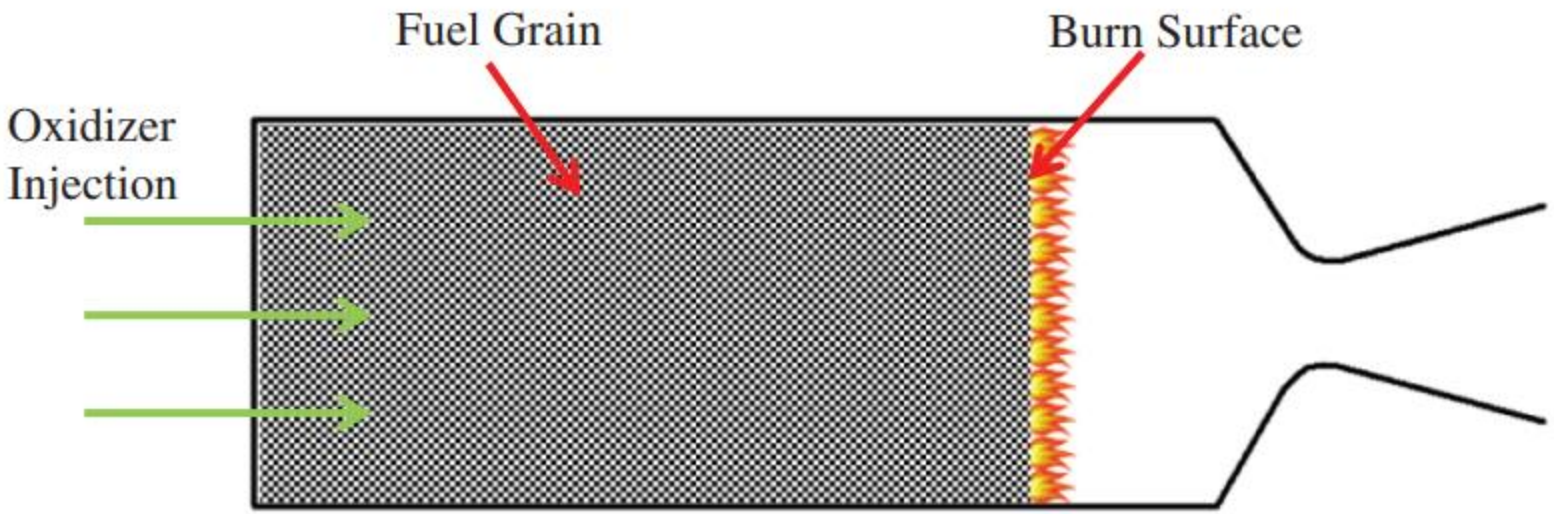
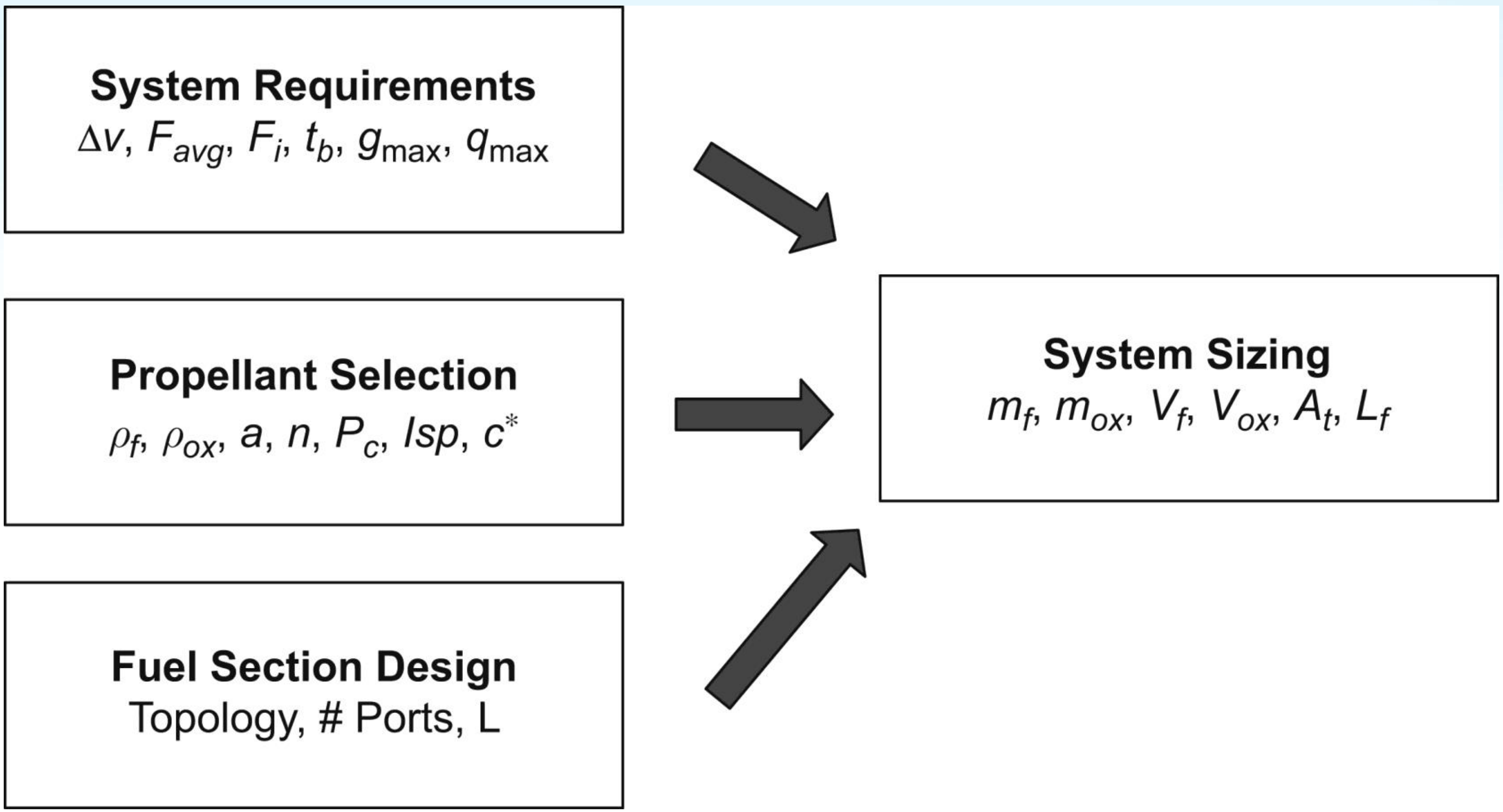


Fig. 1 Axial-injection, end-burning hybrid [2].

Hitt, M.A., and Frederick, R.A., Jr., "Regression Rate Model Predictions of an Axial-Injection End-Burning Hybrid Motor," ALAA Journal of Propulsion and Power, Vol.34, No. 5 (2018), pp. 1116-1123, https://doi.org/10.2514/1.B36839

[E] 11. 7 HRE Design



[E] Hybrid Rocket "Short Course"

This material was assembled by the AIAA Hybrid Rocket Technical Committee. The following individuals provided input and critical review:

T. A. Boardman	Thiokol Corporation
S. E. Claflin	Rockwell Corporation/Rocketdyne Division
N. S. Cohen	Cohen Professional Services
R. A. Frederick	University of Alabama at Huntsville
S. D. Heister	Purdue University
C. W. Shaeffer	United Technologies Corporation/Chemical Systems Division

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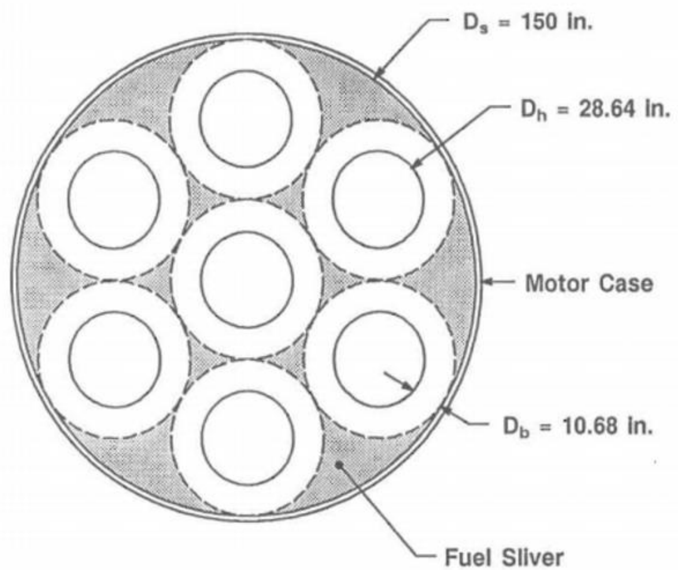
AIAA HRTC SHORT COURSE ii

BOOSTER DESIGN EXAMPLE

Circular Port Hybrid Grain Configuration



[E] AIAA Nomenclature

α	thermal diffusivity
β	polynomial correlation coefficient
γ	polynomial correlation coefficient
δ	boundary layer displacement thickness
ϵ	emissivity
μ	viscosity (absolute)
ρ	density
σ	Stefan Boltzmann constant
τ	shear stress
υ	viscosity (kinematic)
a	solid propellant burn rate coefficient, hybrid flux coefficient
A	area
B	blowing coefficient
C^*	characteristic velocity
C_f	skin friction coefficient
C_h	Stanton number
Cp	specific heat (constant pressure)
D_p	port diameter
G	flux
g_c	gravitational constant
h	film cooling heat transfer coefficient
H	enthalpy
h_v	heat of vaporization
k	thermal conductivity
L	arbitrary length scale
L_g	grain length
m	mass flow rate
P	chamber pressure
Pr	Prandtl number
$\dot{Q}$	heating rate
$\dot{r}$	burn rate, regression rate
Re	Reynolds number
T	temperature
u	velocity (x-direction)
U	freestream velocity
v	velocity (y-direction)

Subscripts

c	chamber
e	boundary layer edge
f	fuel
F	flame
g	gas
o	oxidizer
p	propellant
r	radiation
s	surface
t	throat

Supercripts

l	polynomial correlation exponent
m	polynomial correlation exponent
n	solid propellant pressure exponent, hybrid flux exponent

DETERMINE DESIGN CHARACTERISTICS OF A SPACE–SHUTTLE–CLASS HYBRID ROCKET BOOSTER

DESIGN EXAMPLE

Design Requirements	
Fuel	HTPB
Oxidizer	LO_2
Required booster initial thrust (vacuum)	3.1×10^6 lbf
Burn time	120 sec
Fuel grain outside diameter	150 in.
Initial chamber pressure	700 psia
Initial mixture ratio	2.0
Initial expansion ratio	7.72

REGRESSION RATE EQUATION CAN BE INTEGRATED TO YIELD EXPRESSIONS AS FUNCTIONS OF BURN TIME

N = NUMBER OF CIRCULAR COMBUSTION PORTS IN FUEL GRAIN

- Combustion port radius

$$r(t) = \left\{ a(2n+1) \left[\frac{\dot{m}_{O_T}}{\pi N}\right]^n t + R_i^{2n+1} \right\}^{\frac{1}{2n+1}}$$

- Fuel flow rate

$$\dot{m}_f(t) = 2\pi N \varrho_f L a \left[\frac{\dot{m}_{O_T}}{\pi N}\right]^n \left\{ a(2n+1) \left[\frac{\dot{m}_{O_T}}{\pi N}\right]^n t + R_i^{2n+1} \right\}^{\frac{1-2n}{1+2n}}$$

- Mixture ratio

$$\frac{\dot{m}_o(t)}{\dot{m}_f(t)} = \frac{1}{2\varrho_f L a} \left[\frac{\dot{m}_{O_T}}{\pi N}\right]^{1-n} \left\{ a(2n+1) \left[\frac{\dot{m}_{O_T}}{\pi N}\right]^n t + R_i^{2n+1} \right\}^{\frac{2n-1}{2n+1}}$$

- For constant oxidizer flow and n >1/2
  - Fuel flow rate always decreases with time
  - Mixture ratio always increases with time
- For n = 1/2, no mixture ratio shift occurs during operation—regression rate decrease is exactly balanced by fuel surface area increase

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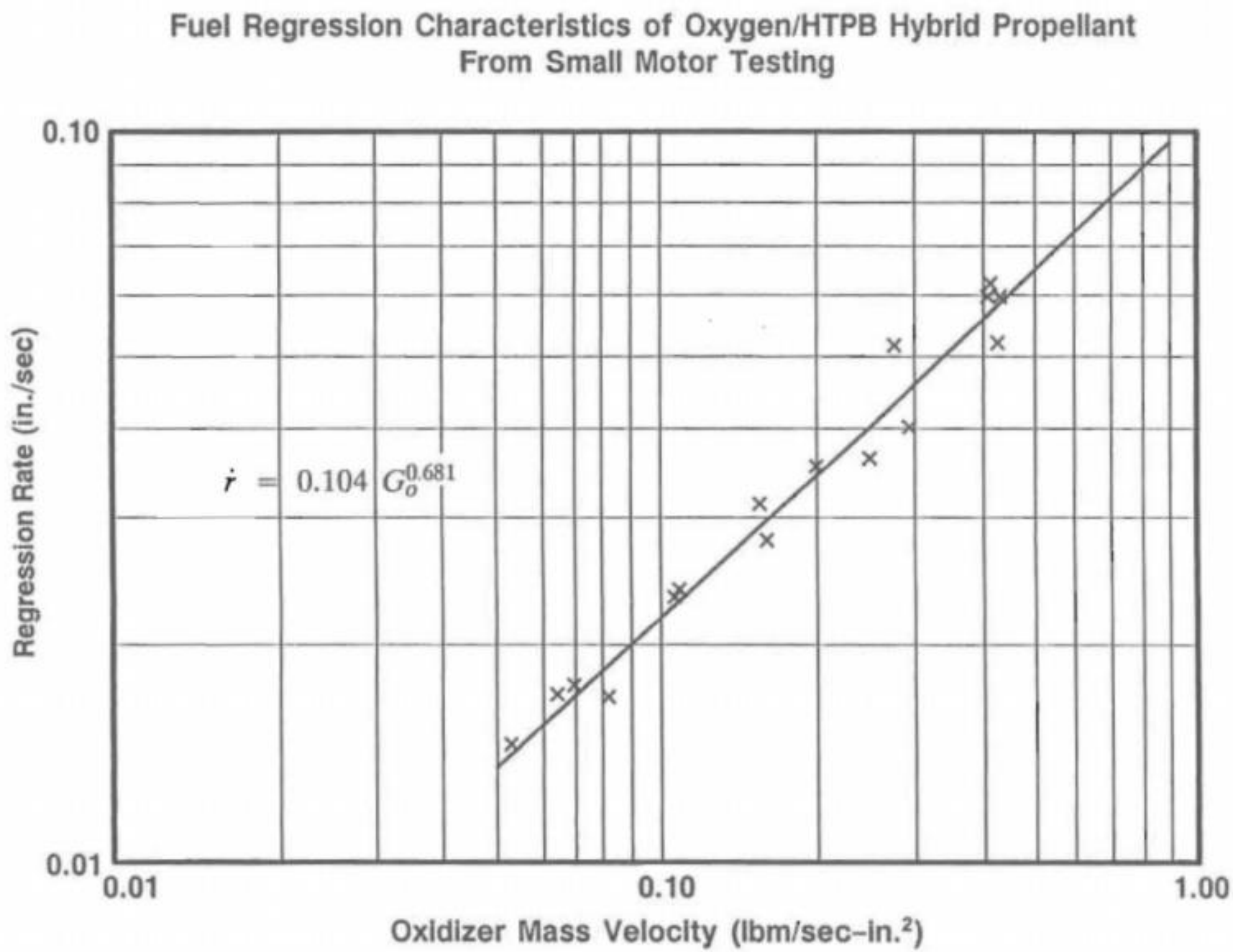
TYPICAL EXPERIMENTALLY DERIVED FUEL REGRESSION RATE CORRELATION

DESIGN DATA



THEORETICAL CHARACTERISTIC VELOCITY (C*) AND RATIO OF SPECIFIC HEATS FOR LO_2/HTPB FUEL

DESIGN DATA

Mixture Ratio	C* (ft/sec)	Gamma
1.0	4,825	1.308
1.2	5,180	1.282
1.4	5,543	1.239
1.6	5,767	1.201
1.8	5,882	1.171
2.0	5,912	1.152
2.2	5,885	1.143
2.4	5,831	1.138
2.6	5,768	1.135
2.8	5,703	1.133
3.0	5,639	1.132

CALCULATE REQUIRED THROAT AREA AND PROPELLANT MASS FLOW RATES TO PRODUCE DESIRED INITIAL THRUST

- Throat area

$$A_t = \frac{F_v}{C_{f_v} P_c} = \frac{3.1x10^6 \; lbf}{(1.735)(700 \; lbf/in.^2)} = 2,552.5 \; in.^2 \rightarrow D_t = 57.01 \; in.$$

- Total propellant flow rate

$$\dot{m}_T = \frac{g_c P_c A_t}{\eta C^*} = \frac{\left(32.174 \frac{lbm\text{–}ft}{lbf\text{–}sec^2}\right)\left(700 \frac{lbf}{in.^2}\right)(2,552.5 \; in.^2)}{(0.95)\;(5,912 \; ft/\sec^2)} = 10,236 \; lbm/sec$$

- Initial fuel and oxidizer flow rates

$$\dot{m}_T = \dot{m}_{o_T} + \dot{m}_{f_T} = \dot{m}_{f_T}\left[\frac{O}{F} + 1\right]$$

$$\dot{m}_{f_T} = \frac{10,236 \; lbm/sec}{3} = 3,412 \; lbm/sec$$

$$\dot{m}_{o_T} = 10,236 - 3,412 = 6,824 \; lbm/sec$$

ARBITRARILY SELECT SEVEN CIRCULAR FUEL PORTS FOR INITIAL DESIGN

- Use expression for R(t) to determine fuel distance burned in 120 sec (fuel web)

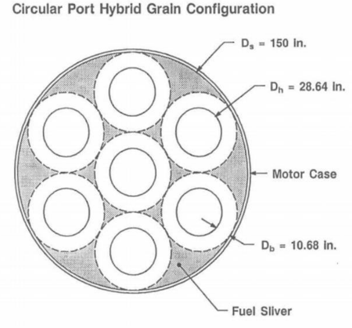
$$D_b = R\,(t,\ R_i)\big|_{t-120} - R_i$$

- Motor diameter constraint (150 in.) is satisfied by the following relationship

$$150\ in. = 6\,R_i + \overset{6}{D_b}$$

- The above equations with two unknowns (R_i and D_b) are solved to yield

Circular Port Hybrid Grain Configuration



$$R_i = 14.32\ in.$$

$$D_b = 10.68\ in.$$

DETERMINE THE FUEL GRAIN OVERALL LENGTH

- Use combustion port geometry to determine oxidizer mass flux

$$G_o = \frac{\dot{m}_{o_T}}{N\ A_p} = \frac{6,824\ lbm/sec}{(7)(\pi)(14.32\ in.)^2} = 1.51\ lbm/in.^2-sec$$

- Use oxidizer mass flux to determine initial fuel regression rate

$$\dot{r}_i = 0.104\ G_{o_i}^{0.681} = 0.104\ (1.51\ lbm/sec)^{0.681} = 0.138\ in./sec$$

- Use initial fuel regression rate to determine required fuel grain length

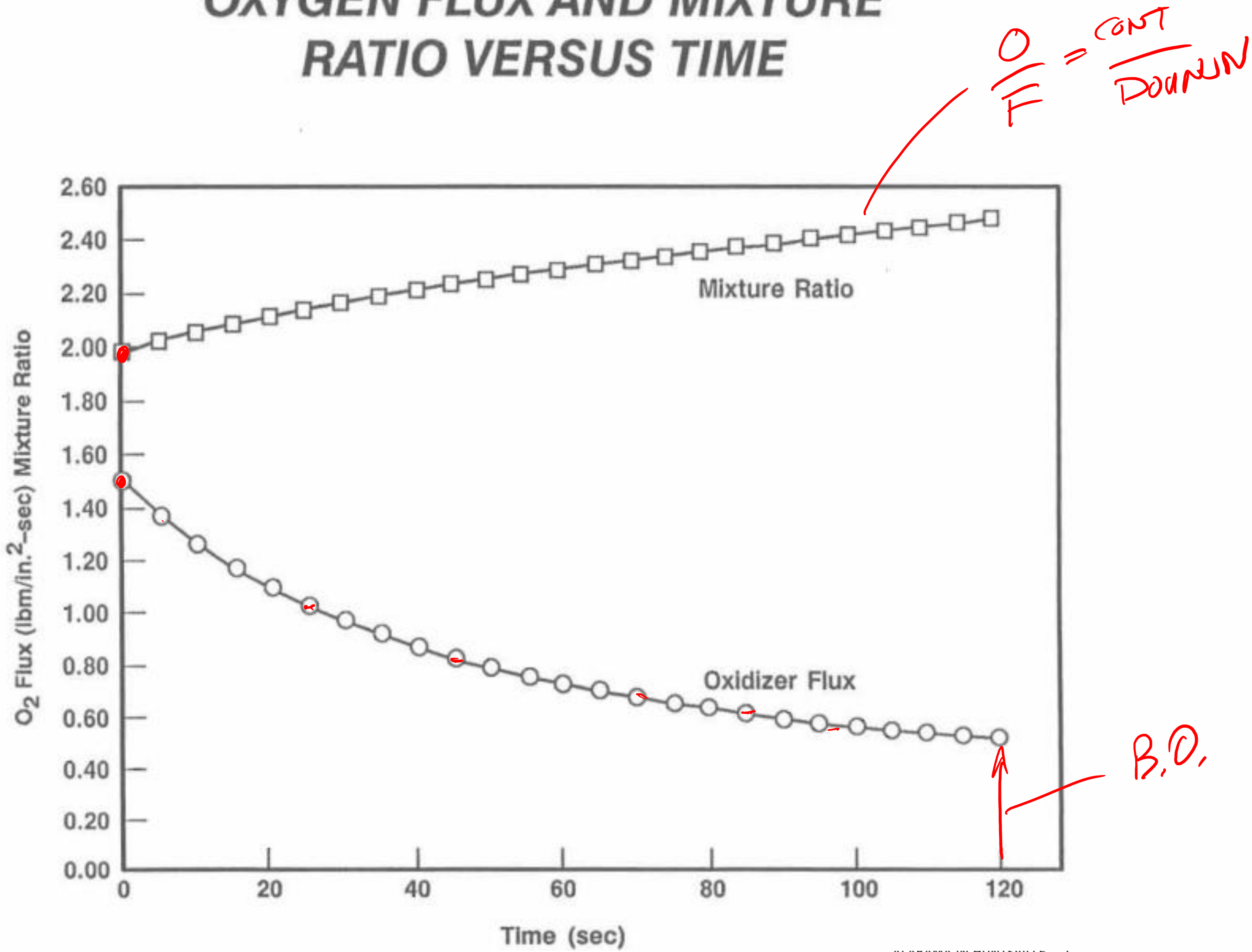
$$L = \frac{\dot{m}_{f_T}/N}{2\pi\ R_i\ \varrho_f\ \dot{r}_i} = \frac{\frac{3,412\ lbm/sec}{7}}{(\pi)\ (28.64\ in.)\ (0.033\ lbm/in.)^3\ (0.138\ in./sec)} = 1,189.6\ in.$$

INITIAL SPECIFIC IMPULSE CAN BE EASILY DETERMINED

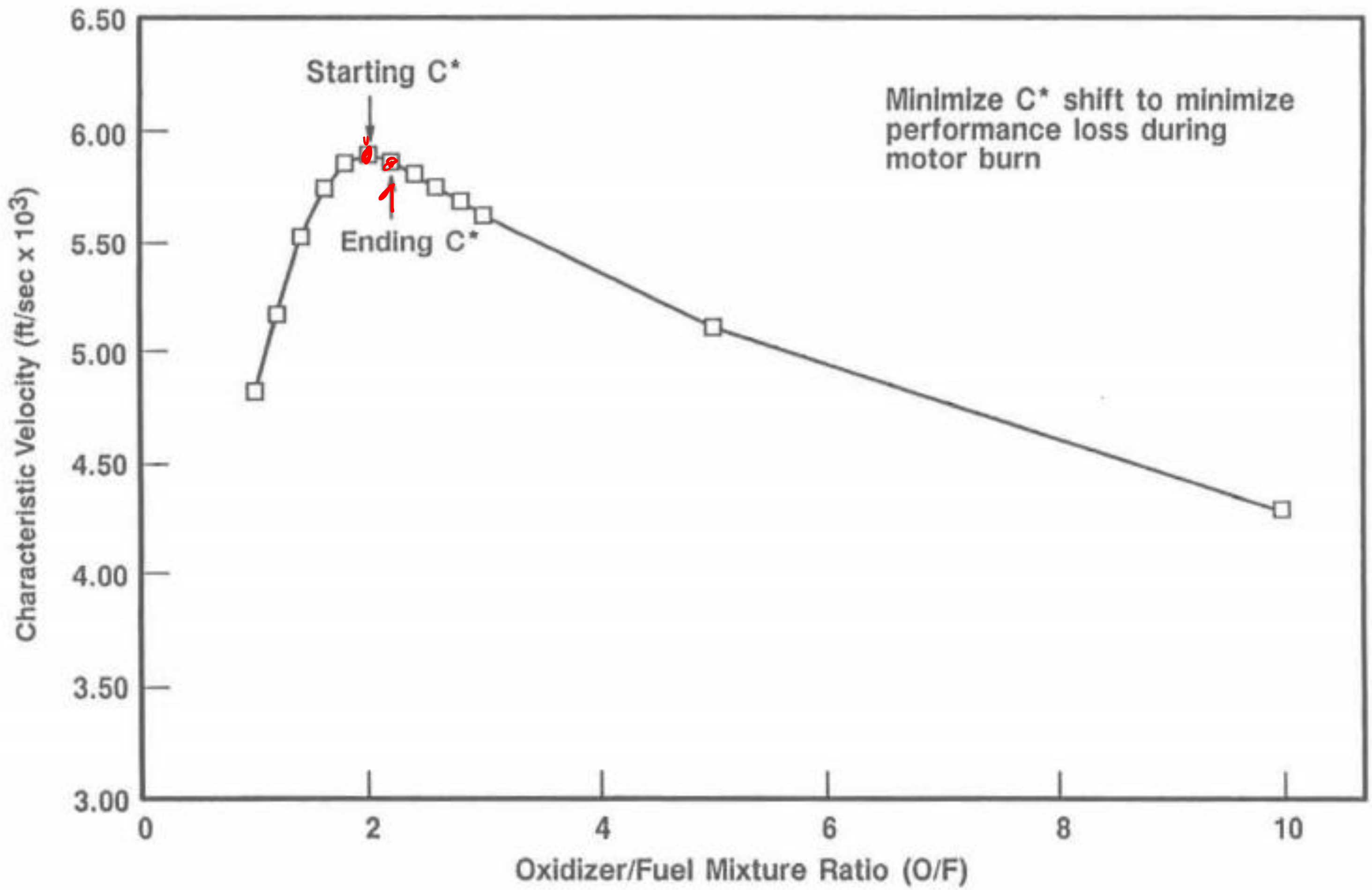
- Vacuum delivered specific impulse

$$I_{sp_v} = \frac{C_{f_v} \eta C^*}{g_c} = \frac{(1.735)\ (0.95)\ (5,912\ ft/sec)}{32.174\ lbm-ft/lbf-sec^2} = 302.87\ lbf-sec/lbm$$

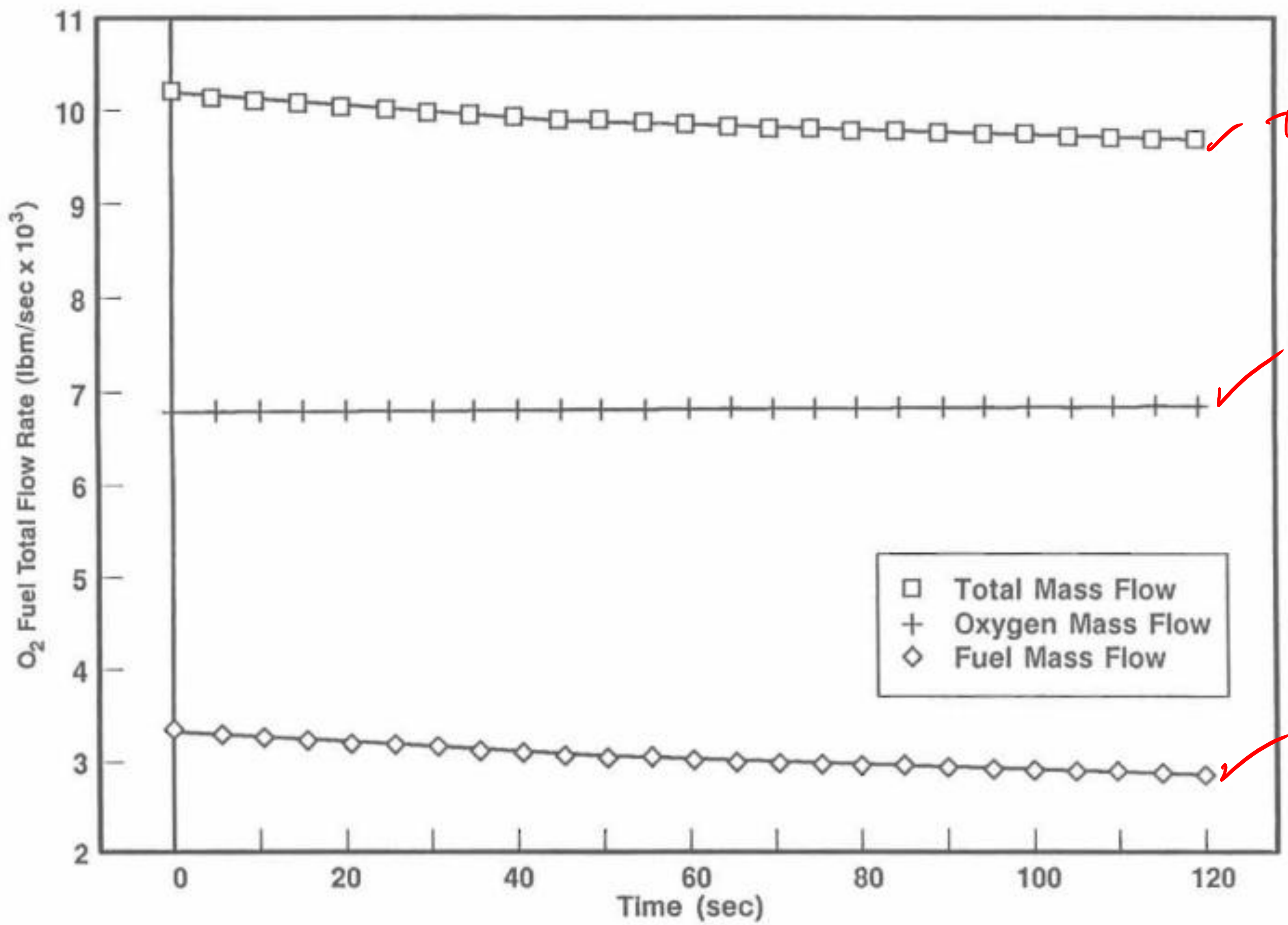
OXYGEN FLUX AND MIXTURE RATIO VERSUS TIME



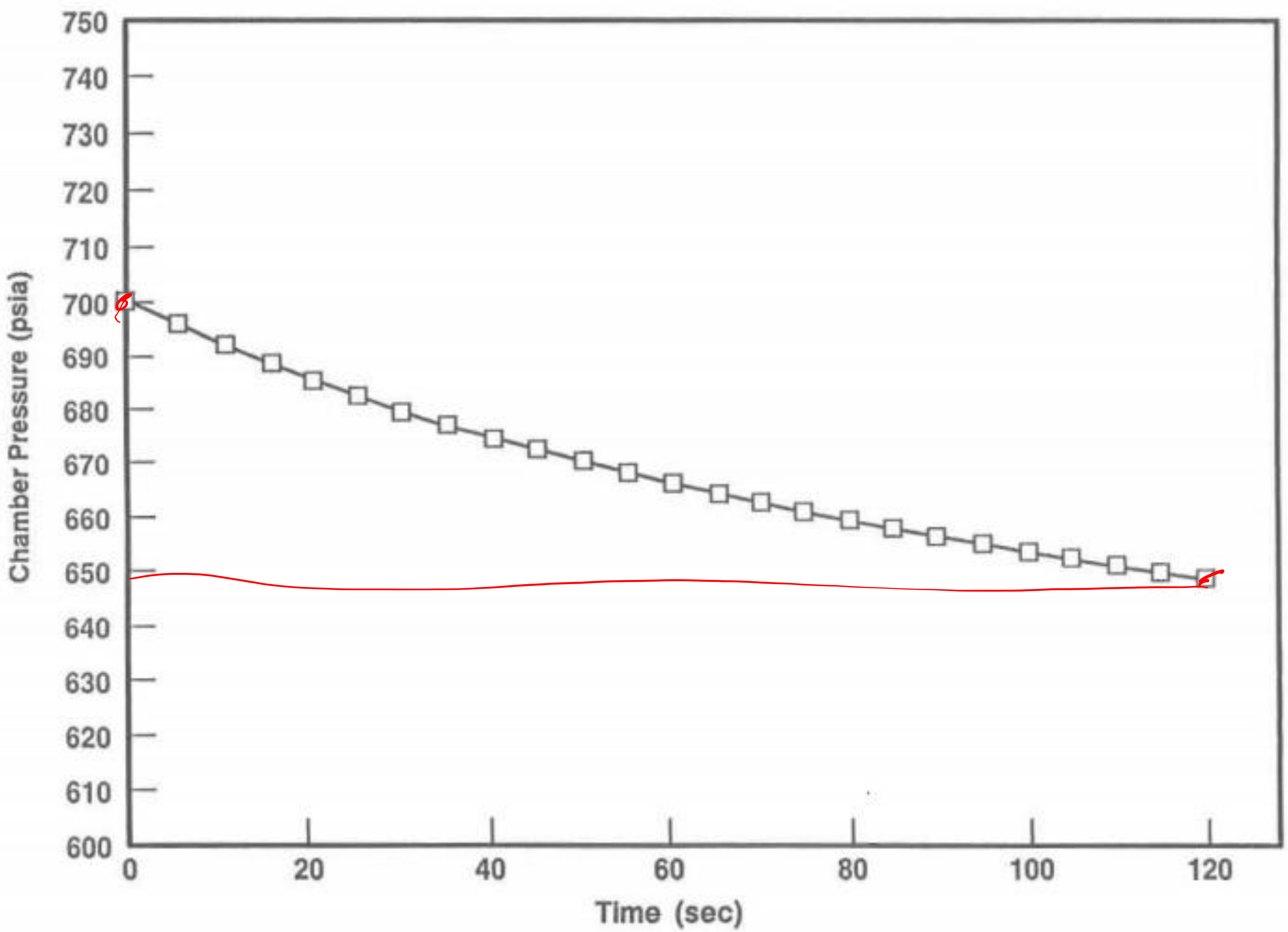
THEORETICAL CSTAR VERSUS MIXTURE RATIO FOR LO_2/HTPB PROPELLANT



PROPELLANT FLOW RATE VERSUS TIME



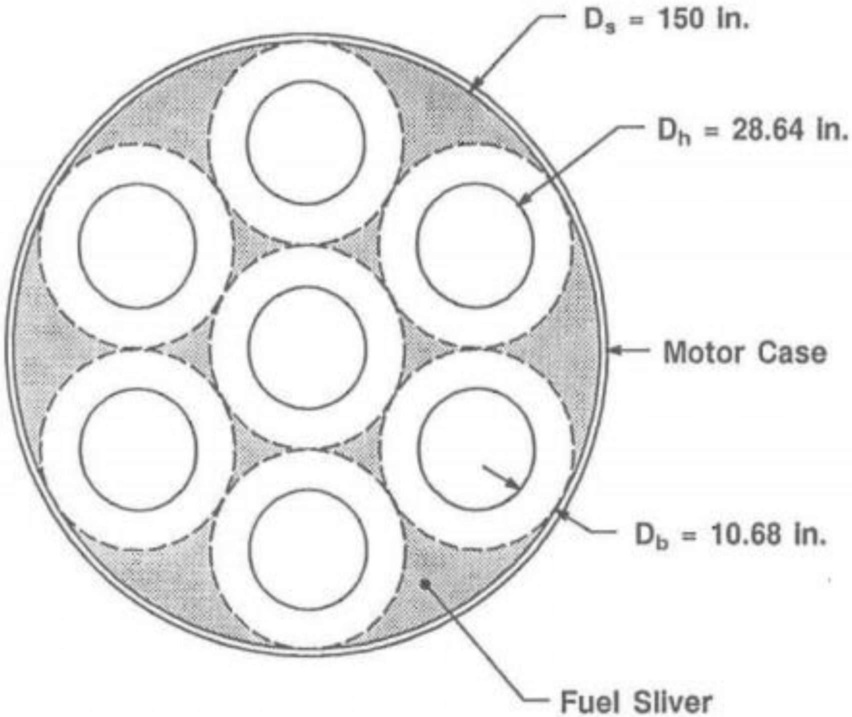
CHAMBER PRESSURE VERSUS TIME



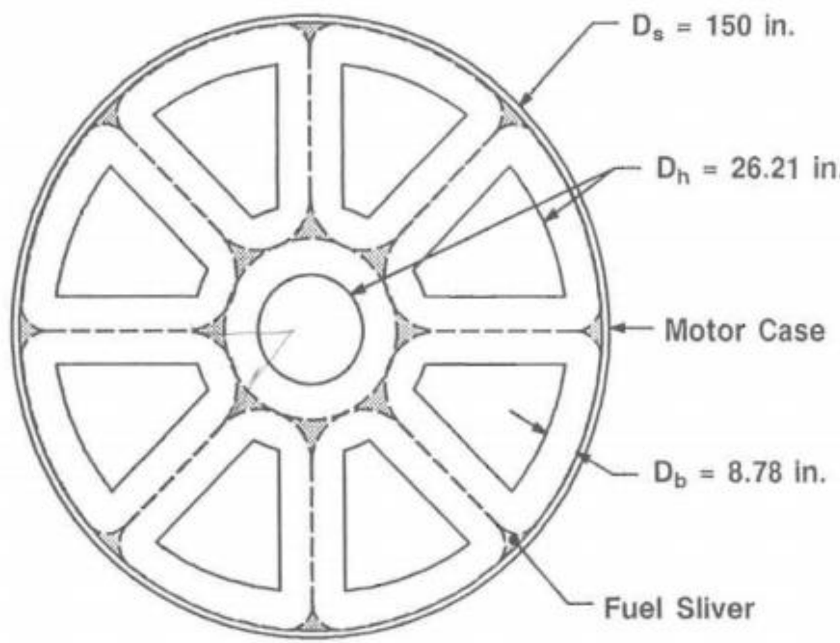
CIRCULAR PORT AND QUADRILATERAL PORT DESIGNS

FUEL SLIVER FRACTIONS ARE HIGH WITH CIRCULAR PORT DESIGNS AND CAN BE REDUCED BY USING A QUADRILATERAL PORT GRAIN DESIGN

Circular Port Hybrid Grain Configuration



Quadrilateral Port Hybrid Grain Configuration



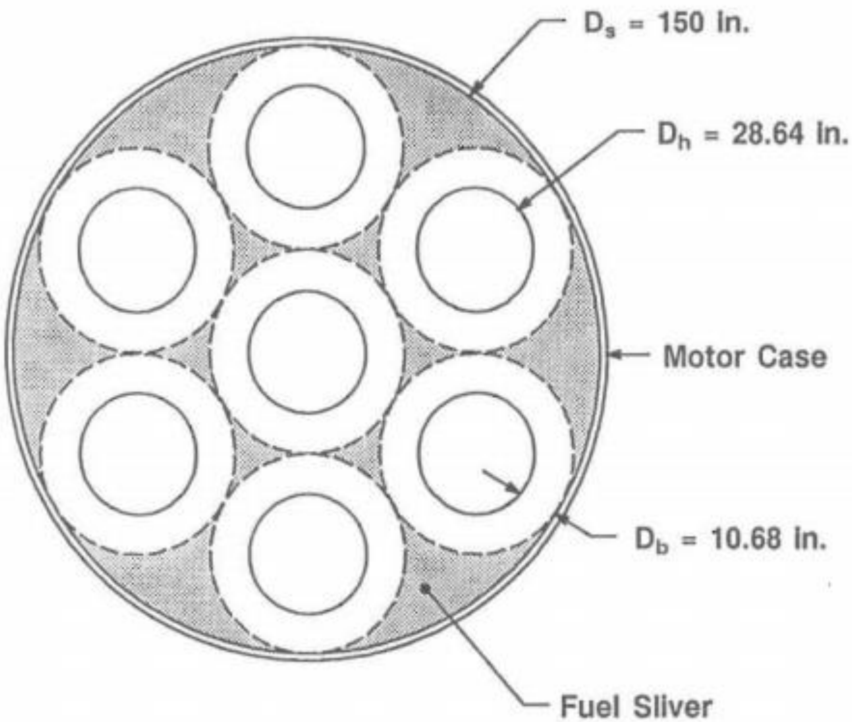
COMPARISON OF CIRCULAR PORT AND QUADRILATERAL PORT GRAIN DESIGNS

DESIGN PARAMETERS	CIRCULAR PORT	QUADRILATERAL PORT
Oxidizer Flow Rate (lbm/sec)	6,824	6,824
Initial Fuel Flow Rate (lbm/sec)	3,412	3,412
Burn Time (sec)	120	120
Grain Diameter (in.)	150	150
Number of Combustion Ports	7	9
Oxidizer Flux (lbm/sec/in.^{2})	1.51	1.07
Fuel Regression Rate (in./sec)	0.138	0.109
Distance Burned (in.)	10.68	8.78
Grain Length (in.)	1,189.6	976.1
Combustion Port L/D	41.5	37.2
Loaded Fuel Mass (lbm)	516,664	364,170
Fuel Consumed (lbm)	362,577	348,584
Theoretical Sliver Fraction (%)	29.8	4.28

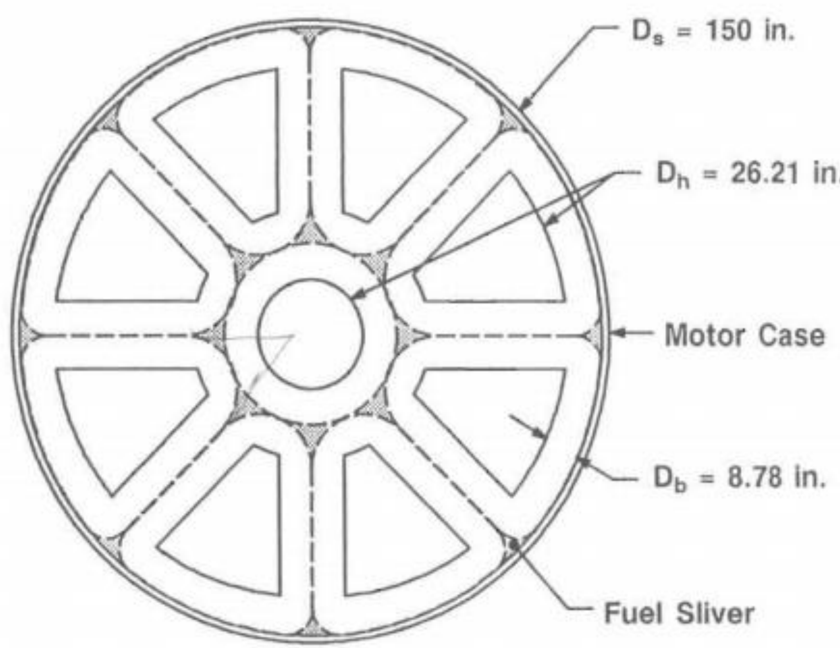
CIRCULAR PORT AND QUADRILATERAL PORT DESIGNS

FUEL SLIVER FRACTIONS ARE HIGH WITH CIRCULAR PORT DESIGNS AND CAN BE REDUCED BY USING A QUADRILATERAL PORT GRAIN DESIGN

Circular Port Hybrid Grain Configuration



Quadrilateral Port Hybrid Grain Configuration



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REGRESSION RATE EQUATION CAN BE INTEGRATED TO YIELD EXPRESSIONS AS FUNCTIONS OF BURN TIME

N = NUMBER OF CIRCULAR COMBUSTION PORTS IN FUEL GRAIN

- Combustion port radius

$$r(t) = \left\{ a(2n+1) \left[\frac{\dot{m}_{O_T}}{\pi N}\right]^n t + R_i^{2n+1} \right\}^{\frac{1}{2n+1}}$$

- Fuel flow rate

$$\dot{m}_f(t) = 2\pi N \varrho_f L a \left[\frac{\dot{m}_{O_T}}{\pi N}\right]^n \left\{ a(2n+1) \left[\frac{\dot{m}_{O_T}}{\pi N}\right]^n t + R_i^{2n+1} \right\}^{\frac{1-2n}{1+2n}}$$

- Mixture ratio

$$\frac{\dot{m}_O(t)}{\dot{m}_f(t)} = \frac{1}{2\varrho_f L a} \left[\frac{\dot{m}_{O_T}}{\pi N}\right]^{1-n} \left\{ a(2n+1) \left[\frac{\dot{m}_{O_T}}{\pi N}\right]^n t + R_i^{2n+1} \right\}^{\frac{2n-1}{2n+1}}$$

- For constant oxidizer flow and $n > 1/2$
  - Fuel flow rate always decreases with time
  - Mixture ratio always increases with time
- For $n = 1/2$, no mixture ratio shift occurs during operation—regression rate decrease is exactly balanced by fuel surface area increase

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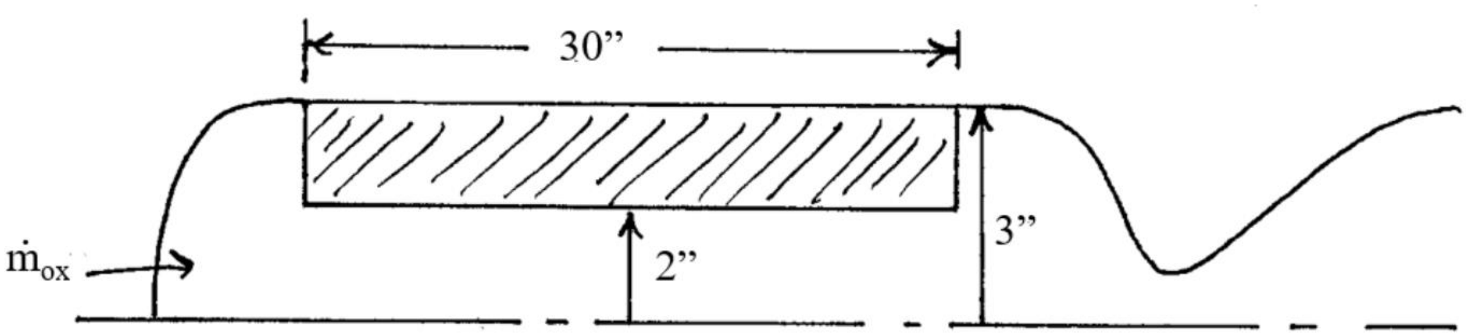
[F] Initial Reading of Homework Problems

HW02A – Problem 11.3

11.3 Consider the hybrid rocket test motor shown in Figure 11.24. This motor utilizes LOX/HTPB propellants. Data from the NASA thermochemistry code were curvefit for characteristic velocity:

$$c* = -2520 + 6800(\mathrm{O/F}) - 1320(\mathrm{O/F})^2 \quad 2 < \mathrm{O/F} < 3$$

with c^* in ft/s. In addition, the fuel density is known to be 0.0325 lb/in^3 and the regression rate (in inches/s) obeys $r = 0.16 G_{\mathrm{ox}}^{0.7}$, where G_{ox} is the oxidizer massflux in lb/(in^2 s). We desire to operate the engine at fixed oxidizer mass flow so we expect mixture ratio variations during the burn. For this reason, we wish to hit the optimum mixture ratio (max. c^*) at the mid-web location. You may neglect the burning of the end faces of the fuel grain in your analysis. Under these assumptions, determine:



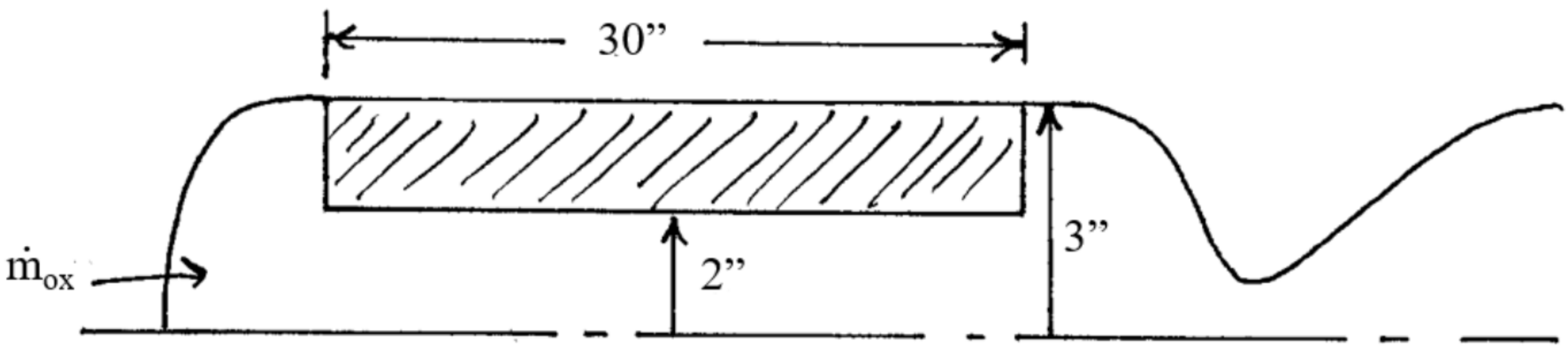
HW02A – Problem 11.3

For this reason, we wish to hit optimum mixture ratio (max. c*) at the mid-web location. You may neglect the burning of the end faces of the fuel grain in your analysis. Under these assumptions, determine:

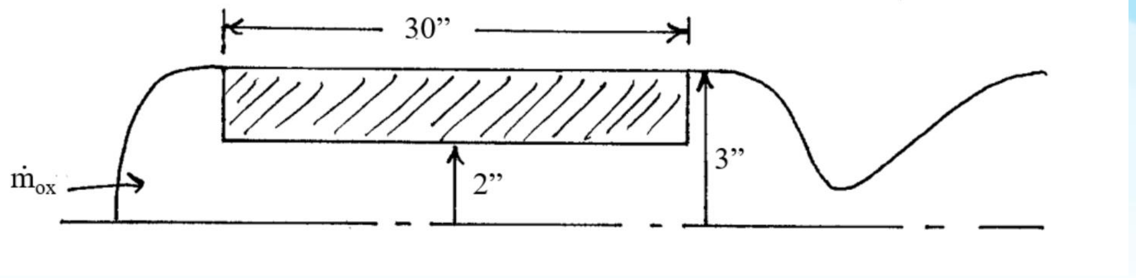
i) The optimal O/F for this propellant combination.

ii) The oxidizer flowrate which maximizes performance at mid-web

iii) The overall O/F shift (max O/F – min O/F) for the firing assuming the fuel is completely consumed.



HW02A – Problem 11.3

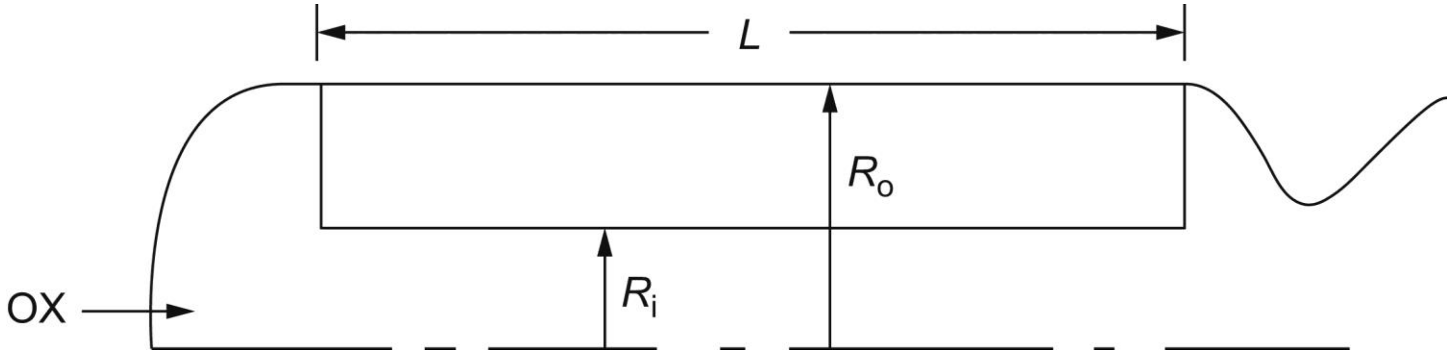


HW02A – Problem 11.7

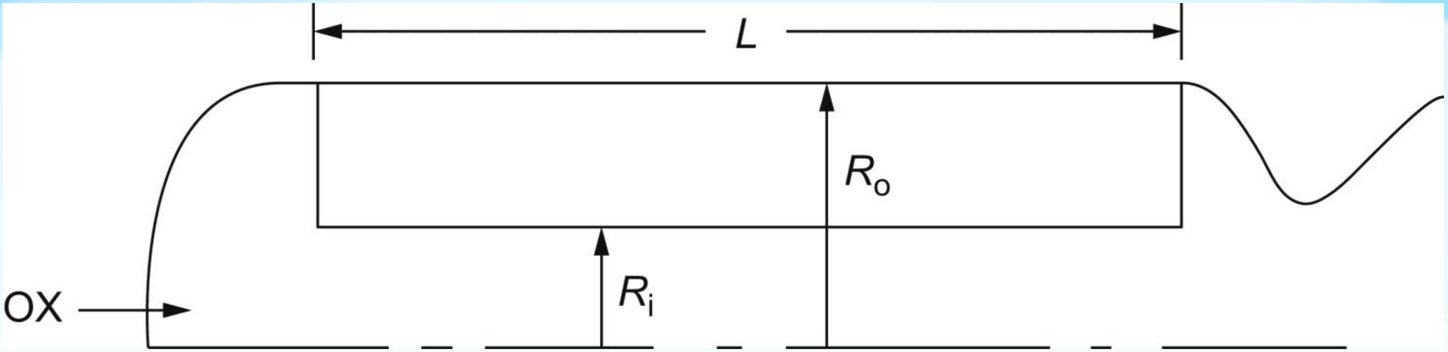
11.7 Consider a hybrid rocket with a simple tubular fuel grain as shown in Figure 11.26. Assume the fuel regression rate is uniform along the length of the grain and obeys

$$r = aG_{\mathrm{ox}}^{n} \tag{1}$$

where G_{ox} is the oxidizer massflux in the fuel port. Assuming the oxidizer mass flow, $\dot{m}_{\mathrm{ox}}$, is constant, one can actually solve for the port radius, R, as a function of time using Eq. 1 and the fact that $r = \mathrm{d}R/\mathrm{d}t$.



HW02A – Problem 11.7



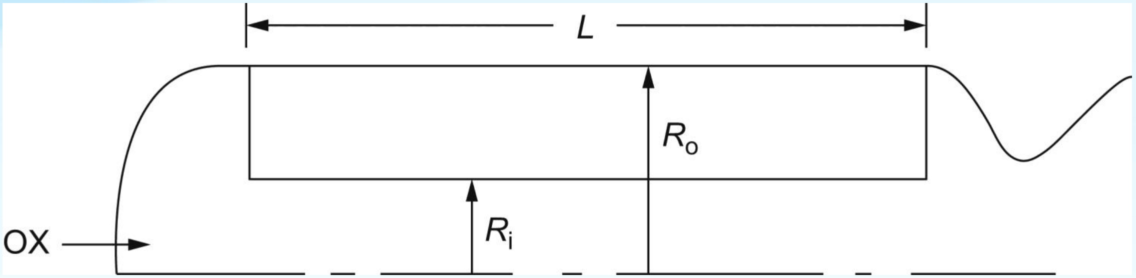
(i) Show that:

$$R(t) = \left[a(2n+1)\left(\frac{\dot{m}_{ox}}{\pi}\right)^n t + R_i^{2n+1}\right]^{\frac{1}{2n+1}}$$

(ii) Using this result, derive expressions for the fuel flow, $\dot{m}_f$, and mixture ratio, O/F, as functions of time. Is there a special value of n which provides for constant fuel flow and no mixture ratio shifts?

(iii) Suppose $L = 50''$, $R_i = 2''$, $R_o = 5''$, $\rho_f = 1$ g/cc, and $r = 0.1\ G_{ox}^{0.8}$ in inches/s with $G_{ox} \sim \mathrm{lb/in^2 s}$. Plot $R(t)$, $\dot{m}_f(t)$, and O/F(t) assuming an initial G_{ox} of 1.0 $\mathrm{lb/in^2}$ s.

HW02A – Problem 11.7

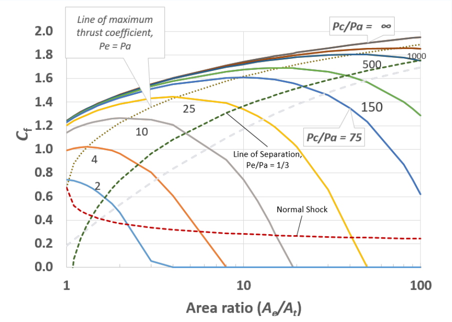


(i) Show that:

$$R(t) = \left[a(2n+1)\left(\frac{\dot{m}_{\text{ox}}}{\pi}\right)^n t + R_i^{2n+1}\right]^{\frac{1}{2n+1}}$$

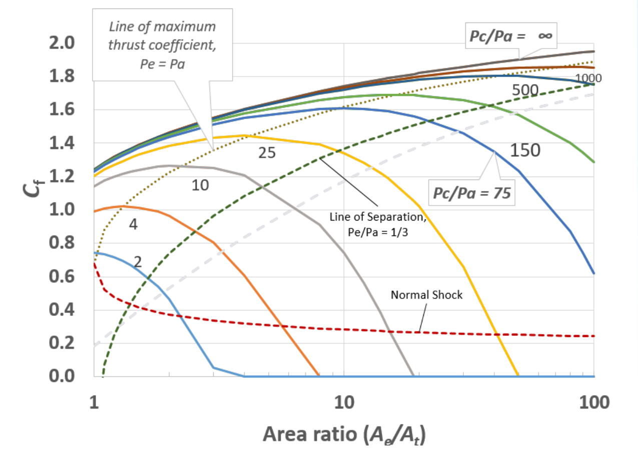
Special Problems

- Special Problem 02HW-SPA
  - Update the Thrust Coefficient Spreadsheet to include a calculated separation line for *Pe/Pa* = 1/3.
  - Update the Thrust Coefficient Spreadsheet to include a calculated separation line for *a* Normal Shock at the Exit
- Special Problem 02HW-SPB
  - Do an Annotated Bibliography on
    - Frederick, R., and Thomas, D., "Propulsion Research and Academic Programs at the University of Alabama in Huntsville," 2023 AIAA SciTech, January 26, 2023.



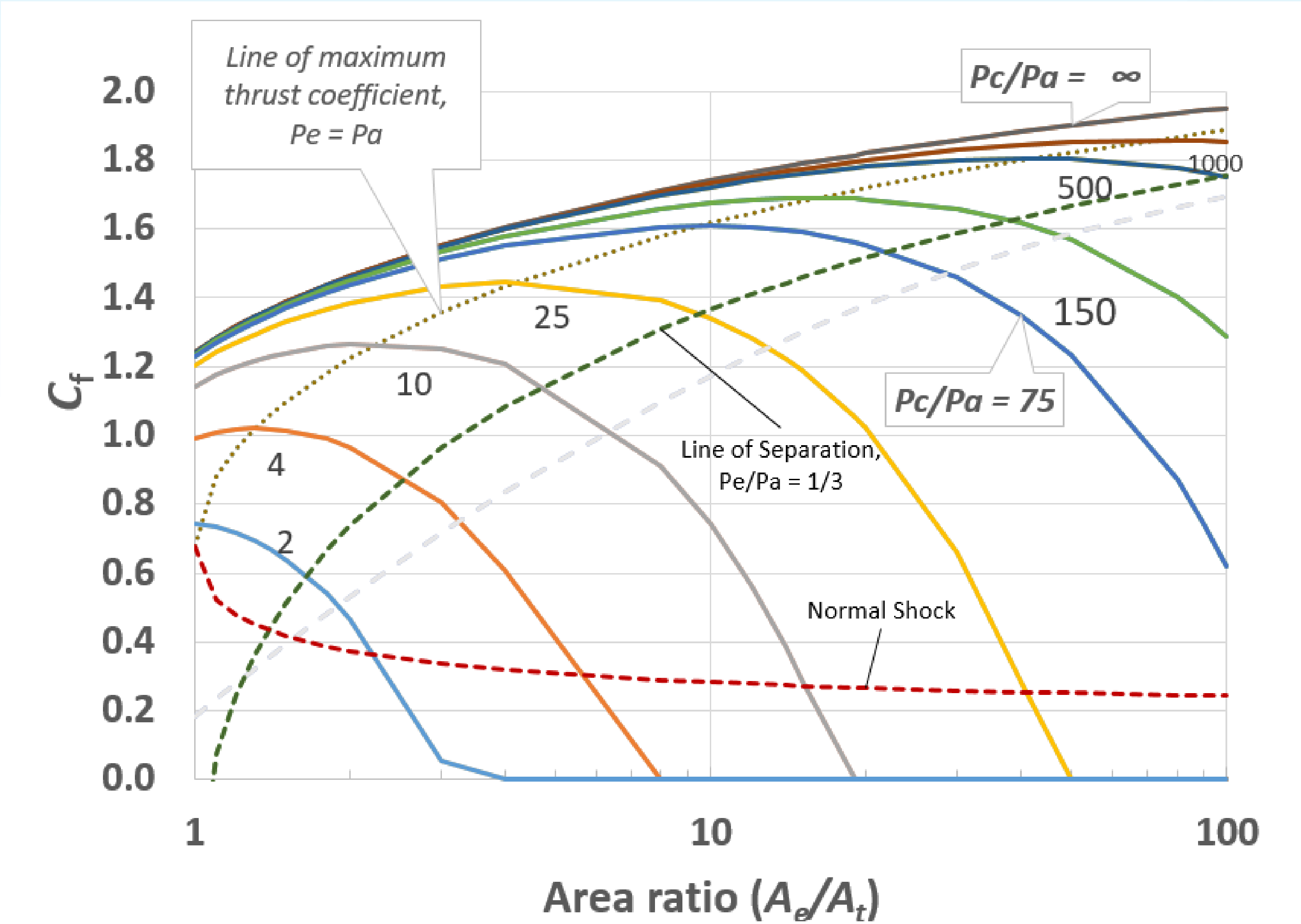
Special Problems

- Special Problem 02HW-SPA



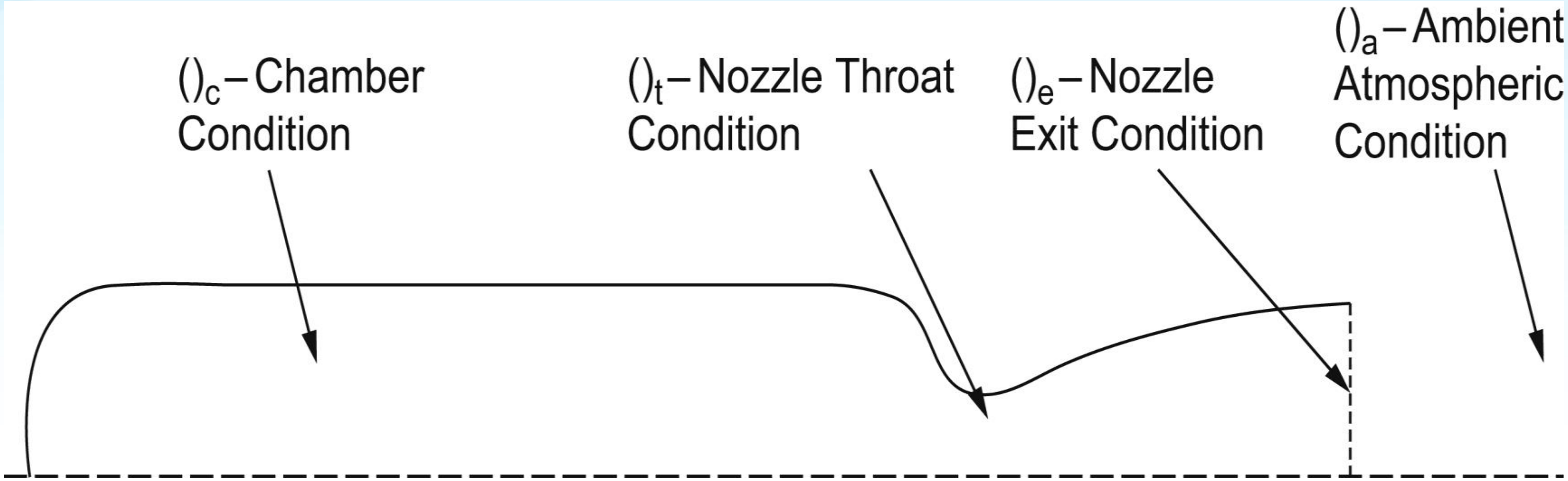
- Special Problem 02HW-SPB

Special Problem 02HW-SPA



Supplementary Material on Flow Separation and Normal Shocks in Nozzles

Nomenclature



Isentropic Flow Equations

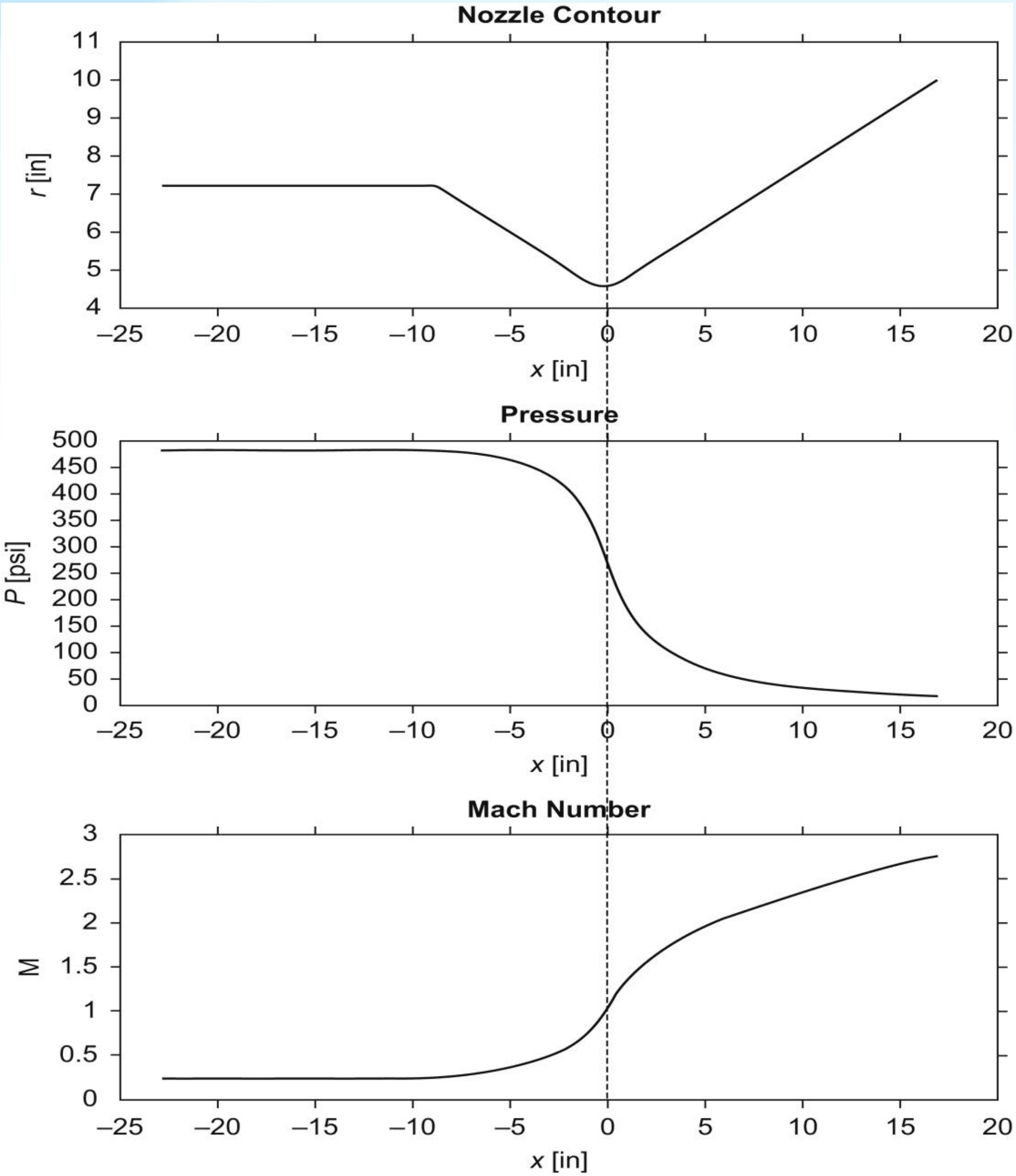
$$p_c/p = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

$$T_c/T = \left(1 + \frac{\gamma - 1}{2} M^2\right)$$

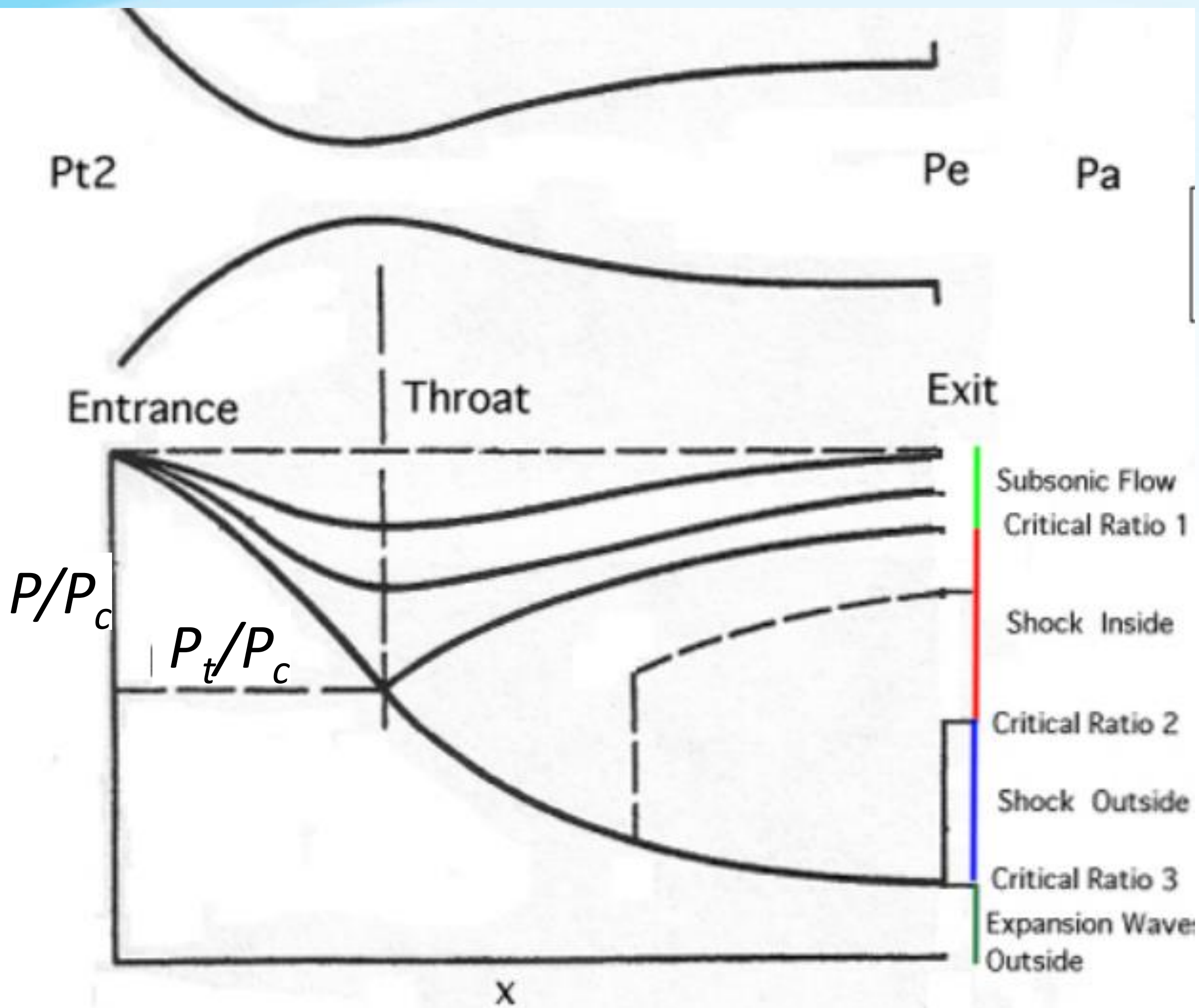
$$M = v/a = \frac{v}{\sqrt{\gamma R T}}$$

$$\frac{A}{A_t} = \frac{1}{M}\left\{\frac{2 + (\gamma - 1)M^2}{(\gamma + 1)}\right\}^{\frac{\gamma+1}{2(\gamma-1)}}$$

Review of Compressible Flow



Nozzle Startup



Stagnation to static pressure

$$p_c/p = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

At the Throat

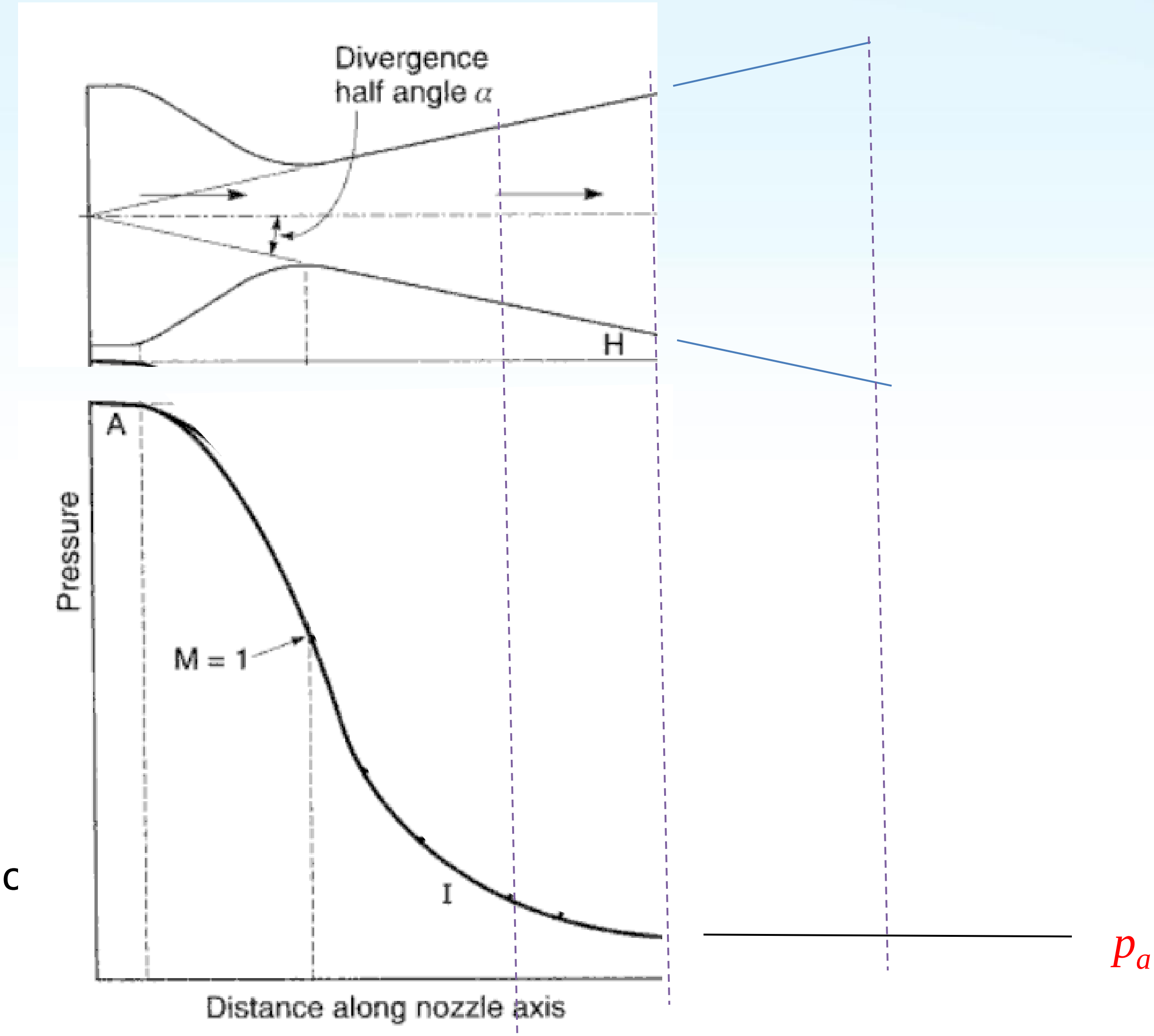
$$p_c/p_t = \left(1 + \frac{\gamma - 1}{2} 1^2\right)^{\gamma/(\gamma-1)} =$$

$$= \left[\frac{\gamma + 1}{2}\right]^{\gamma/(\gamma-1)}$$

For $\gamma = 1.2$

$$\frac{P_t}{Pc} = 0.565$$

Idea, Over, and Under Expansion



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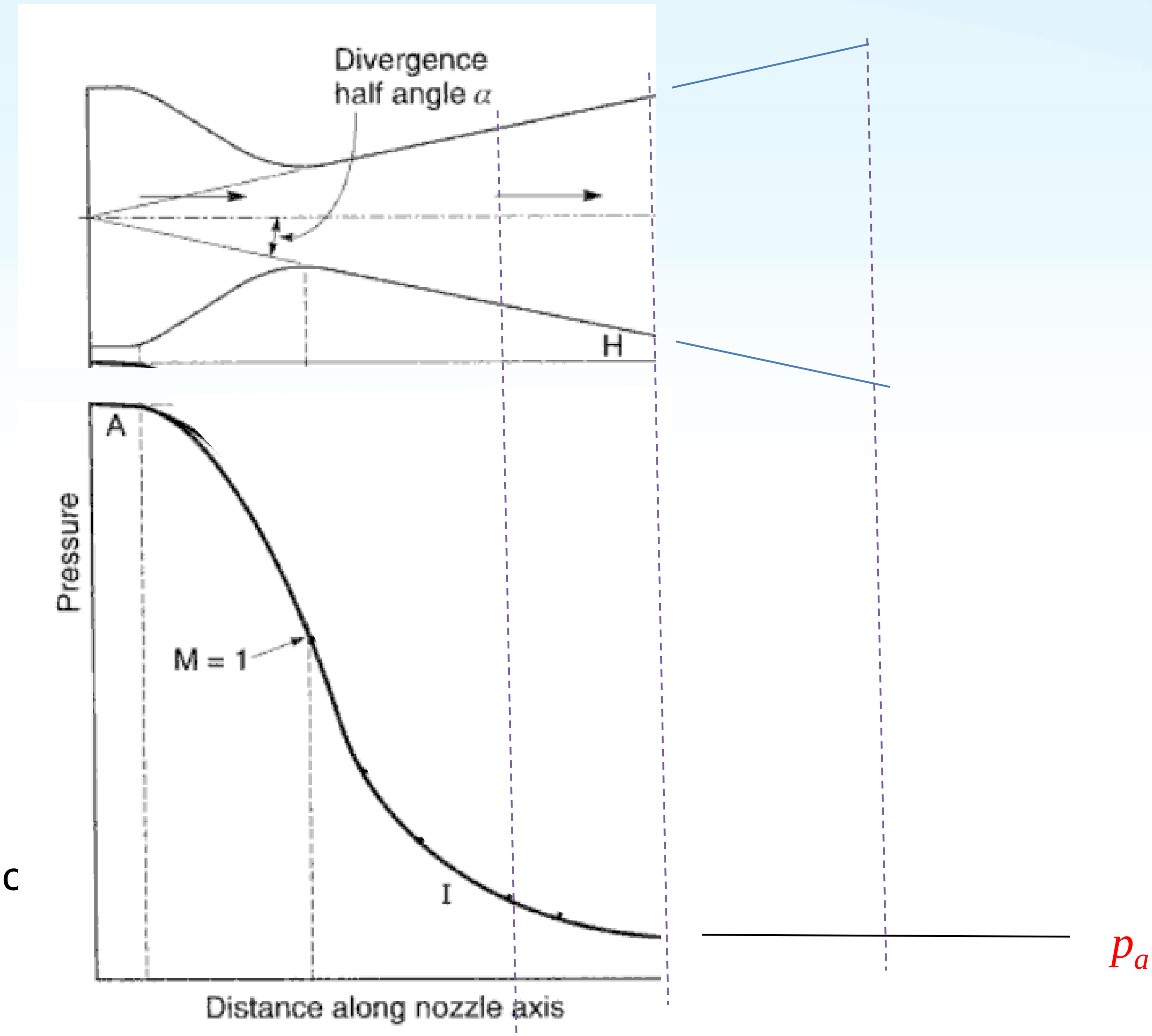
Idea, Over, and Under Expansion



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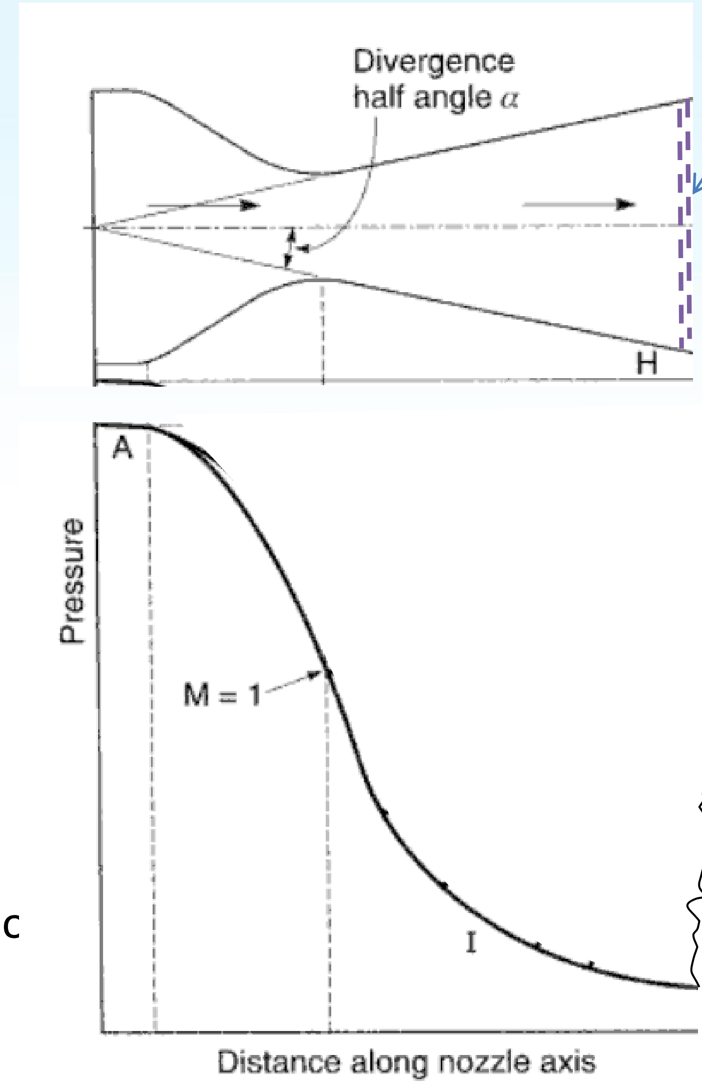
Normal Shock at Exit



Normal Shock

Pressure Drop Across Norma Shock

$$\left.\frac{p_3}{p_2}\right|_{NormalShok} = \frac{2k}{k+1} M_2^2 - \frac{k-1}{k+1}$$

$$\frac{p_2}{p_1} = \left[1 + \frac{k-1}{2} M_2^2\right]^{\frac{-k}{k-1}}$$

p_3

p_2

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Thrust Coefficient

$$C_F = \sqrt{\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]} + \frac{p_2 - p_3}{p_1}\frac{A_2}{A_1}$$

3) Solve (C_F)

Note on Subscripts:

1 = c (Chamber)
2 = e (Exit)
3 = a (ambient)

$$\frac{p_2}{p_1} = \left[1 + \frac{k-1}{2}M_2^2\right]^{\frac{-k}{k-1}}$$

2) Solve (p_2/p_1)

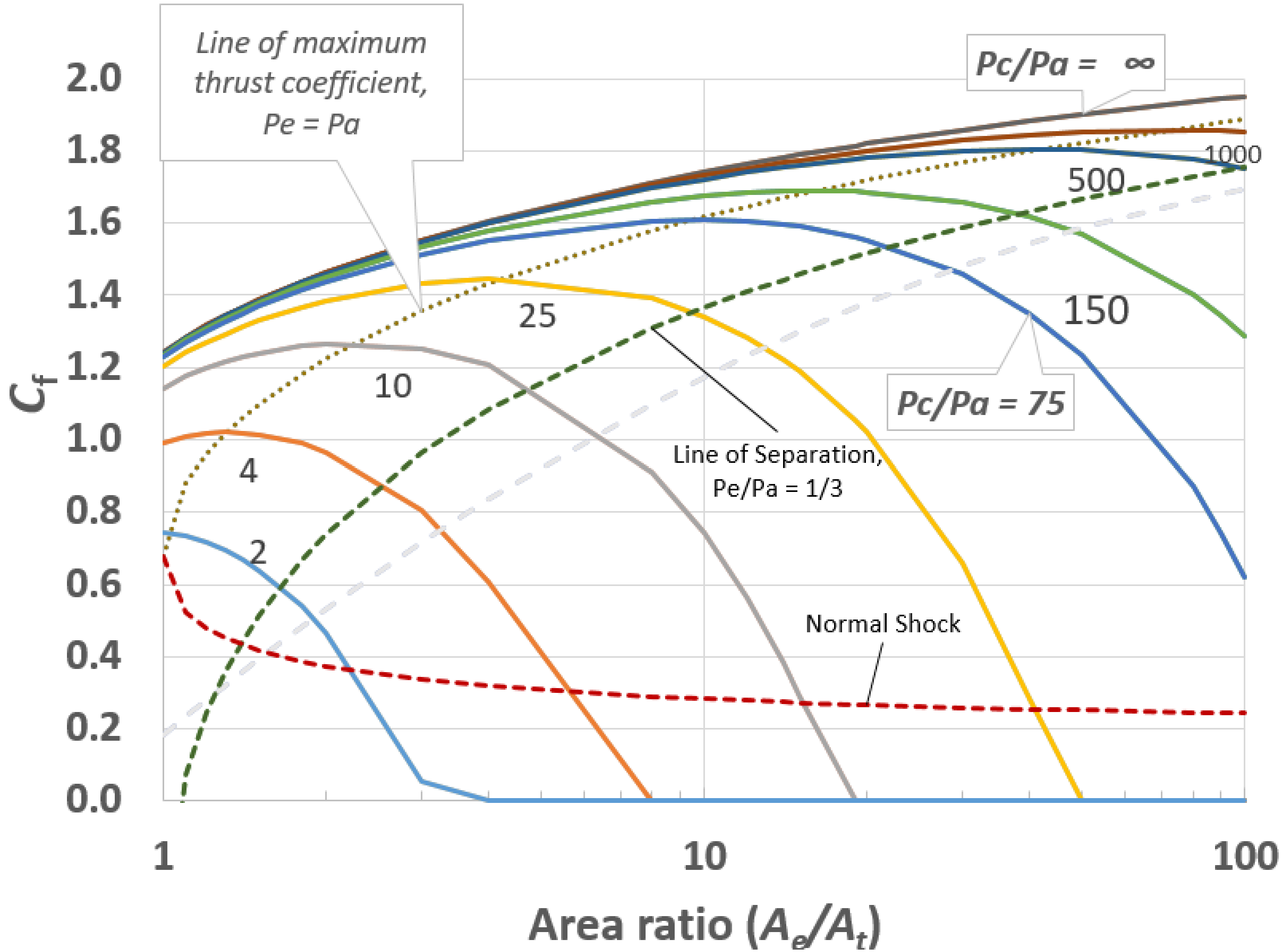
Session 04

Know $(A_2/A_t\,,\,\gamma)$

1) Solve $M2$

$$\frac{A_2}{A_t} = \frac{1}{M_2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_2^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Notes/Comments/Questions



Notes/Comments/Questions

Notes/Comments/Questions