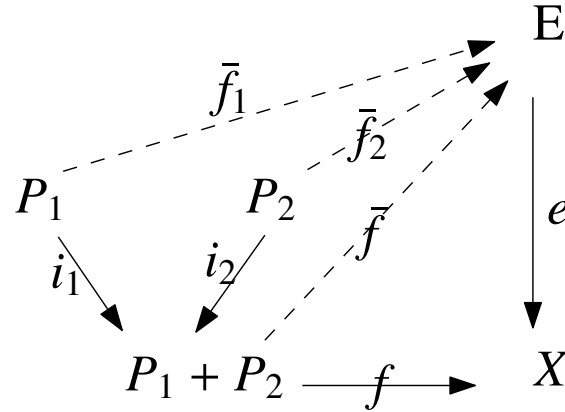


Exercise 7

Let P_1 and P_2 be projectives, $e : E \rightarrow X$ any epimorphism and $f : P_1 + P_2 \rightarrow X$. We need to show that f factorises through e .

We start with the solid lines in the following diagram.



Because P_1 is a projection, the function $f \circ i_1$ factorises over E (call the function \bar{f}_1). The same holds for P_2 . Now we can use the universal property of the coproduct and construct \bar{f} such that $\bar{f} \circ i_1 = \bar{f}_1$ and $\bar{f} \circ i_2 = \bar{f}_2$.

Now all paths starting from P_1 and P_2 commute. Using the universal property of the coproduct again, we see that $e \circ \bar{f}$ and f both go from $P_1 + P_2$ to X and must therefore be the same (unique) morphism that makes the diagram commute and we are done since we have found $e \circ \bar{f} = f$.

Exercise 15

Let $f, g : X \rightarrow Y$ be two morphisms in **Top**. I claim the coequaliser of f and g is the projection π to Y/\sim where \sim is the smallest equivalence relation containing $\{(f(x), g(x)) \mid x \in X\}$.

This means, that Y/\sim is the space of equivalence classes of \sim and the open sets are exactly those sets S such that $\pi^{-1}(S)$ is open in Y , where π denotes the projection $x \mapsto [x]_\sim$. It is simple to check that this indeed constitutes a valid topology.

Now for the universal property: Assume there is a morphism $c : Y \rightarrow Z$ such that $c \circ f = c \circ g$. This means that c respects \sim , namely $y_1 \sim y_2 \Rightarrow c(y_1) = c(y_2)$. Therefore $c^* : Y/\sim \rightarrow Z$ given by $[x]_\sim \mapsto c(x)$ is a well-defined function such that $c^* \circ \pi = c$. It is also continuous since, if $O \subset Z$ is open, then so is $c^{*-1}(O) \subset Y$ because this just means that $\pi^{-1}(c^{*-1}(O)) = c^{-1}(O)$ must be open which is true because c is continuous. It is obvious, that we could not have defined c^* differently, so the factorisation over Y/\sim is unique as we want it to be.