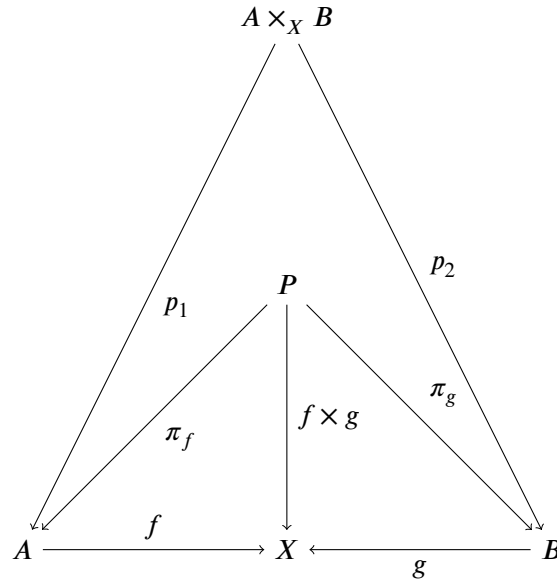


## Exercise 1

Let  $f \times g : P \rightarrow X$  be the product of  $f$  and  $g$  in the slice category (note: This is not the normal product!), with projections  $\pi_f : P \rightarrow A$  and  $\pi_g : P \rightarrow B$  such that  $f \circ \pi_f = g \circ \pi_g = f \times g$  (this is what it means to be a morphism in the slice category).

Because of this equality we immediately know that  $\pi_f$  and  $\pi_g$  factorise uniquely over  $A \times_X B$  via a morphism  $i_1 : P \rightarrow A \times_X B$  (this uses the property of the pullback).

On the other hand  $p_1$  and  $p_2$  are morphisms from  $f \circ p_1 = g \circ p_2$  to  $f$  and  $g$  in the slice category respectively, therefore they factorise uniquely over the product  $f \times g$  via some unique morphism  $i_2 : A \times_X B \rightarrow P$ . By uniqueness, we have that  $i_1$  and  $i_2$  must be inverse to one another and therefore pullbacks are exactly products in the slice category.



## Exercise 7

Let  $L$  be the limit of  $D$  with (with morphisms  $l_i : L \rightarrow C_i$ ).

Let  $L'$  be the limit of  $\text{Hom}(C, -) \circ D$  (with morphisms  $l'_i : L' \rightarrow \text{Hom}(C, C_i)$ ).

We have to show that  $\text{Hom}(C, L) \cong L'$ .

The unique morphism from  $\text{Hom}(C, L)$  to  $L'$  exists since  $L'$  is a limit and mapping a cone via a functor yields a cone. The other direction is the interesting one:

Let  $x \in L'$  be an element of  $L'$  (note that we are in **Sets**). The morphisms  $l'_i$  map this  $x$  to morphisms  $l'_i(x) : C \rightarrow C_i$ . Because  $L$  was a limit over  $D$ , we know that there is a factorisation through  $L$ , i.e. there exists a unique  $\xi : C \rightarrow L$  such that  $l'_i(x) = l_i \circ \xi$ . In other words, we identified the unique element of  $\text{Hom}(C, L)$  that behaves under  $l_i$  in the same way that  $x$  behaves under  $l'_i$ . Since  $x \in L'$  was chosen arbitrarily, we conclude that there is a unique map from  $L' \rightarrow \text{Hom}(C, L)$  that makes everything commute. This

completes the construction of the isomorphism and  $\text{Hom}(C, L)$  is therefore also a limit of  $\text{Hom}(C, -) \circ D$ .