Excercise 3

Let $(G, +)$ be an abelian group. We have to show, that $+ : G \times G \to G$ and $\cdot^{-1} : G \to G$ are group homomorphisms.

This means that:

$$(g_1 + h_1) + (g_2 + h_2) = (g_1 + g_2) + (h_1 + h_2)$$
$$(g_1 + g_2)^{-1} = g_1^{-1} + g_2^{-1}$$

The former is clear since $+$ is commutative, the latter is also clear since $(g_1 + g_2)$ is the inverse to both sides (again using commutativity).

The remaining properties of $+$, i and u are simple to check since they are already implicit in the fact that $(G, +)$ is a group.

Excercise 4

Let M be a monoid in the category of groups, i.e. let $(M, \cdot)$ be a group $\star : M \times M \to M$ an associative group homomorphism and e the neutral (generalised) element of M. Since $\star$ is a group homomorphism we have (mixing infix and prefix notation when it makes things look easier):

$$(g_1 \cdot h_1) \star (g_2 \cdot h_2) = \star(g_1 \cdot h_1, g_2 \cdot h_2) = \star((g_1, g_2) \cdot (h_1, h_2)) = \star(g_1, g_2) \cdot \star(h_1, h_2) = (g_1 \star g_2) \cdot (h_1 \star h_2)$$

By Proposition 4.5 (Eckmann-Hilton) we now know that $\star = \cdot$ and that this operation is commutative.

Therefore M was an abelian group and the operation $\cdot^{-1}$ is a group homomorphism. M can therefore be represented as an internal group (since exactly abelian groups are internal groups in **Grp**).

Excercise 7

This is simple:

$f \sim f'$ implies $g \circ f \circ id \sim g \circ f' \circ id$.
$g \sim g'$ implies $id \circ g \circ f' \sim id \circ g' \circ f'$.
Using transitivity we get $g \circ f \sim g' \circ f'$.

This proofs the claim.