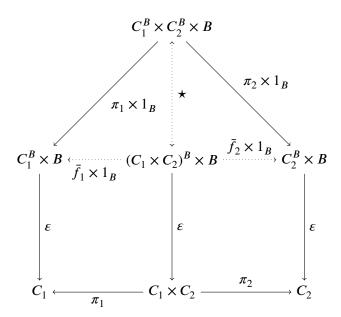
Excercise 2

a) I show that $(C_1 \times C_2)^B \cong C_1^B \times C_2^B$. Consider the following diagram:



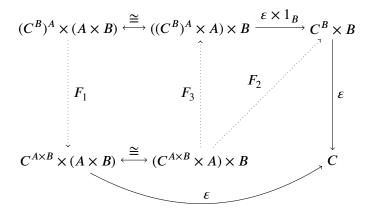
The solid lines are the evaluation functions for the exponentials (note: the three ε are all different) and the canonical projections for products.

By the universal property of the exponential, there is exactly one \bar{f}_1 and \bar{f}_2 such that $\bar{f}_1 \times 1_B$ and $\bar{f}_2 \times 1_B$ make the diagram commute at the dashed lines (f_1 is $\pi_1 \circ \varepsilon$ here). Now, by the universal property of the product it is exactly $\bar{f}_1 \times \bar{f}_2 \times 1_B$ in the upwards direction at \star that makes the diagram commute (requiring identity on the B part).

For the other direction note that by the universal property of the product there is exactly one morphism from $C_1^B \times C_2^B \times B$ to $C_1 \times C_2$ that makes the diagram commute and this gives us (by the universal property of the exponential) the unique morphism $f: C_1^B \times C_2^B \to (C_1 \times C_2)^B$ such that $f \times 1_B$ in the downward direction of \star that makes the diagram commute.

Together we have found two unique morphisms from at \star that don't touch the *B* part. Those must therefore be isomorphisms.

b) Consider the following diagram:



It uses the cannonical isomorphisms (using associativity of products) and the respective evaluation functions of exponentials.

We can construct the following morphisms uniquely:

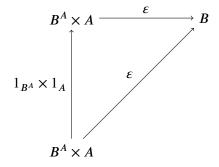
- f_1 such that $F_1 = f_1 \times 1_{A \times B}$ makes the diagram commute (property of exponentials).
- f_2 such that $F_2 = f_2 \times 1_B$ makes the diagram commute (property of exponentials).
- f_3 such that $F_3 = f_3 \times 1_A \times 1_B$ makes the diagram commute (property of exponentials).

We have now constructed $f_1:(C^B)^A\to C^{A\times B}$ and f_3 in the other direction uniquely. These two functions must therefore be isomorphisms.

Excercise 3

We know that the transpose of a function is unique, i.e. if we find a function fullfilling the property of the transpose, we know that it must be the transpose. That being said:

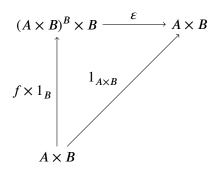
- The transpose of ε is 1_{B^A} , since the following commutes:



• The transpose of $1_{A \times B}$ in **Sets** is:

$$f: a \mapsto (f_a: b \mapsto (a, b))$$

because this makes the diagram commute:



• The transpose of $\varepsilon \circ \tau$ in **Sets** is:

$$f: a \mapsto (i_a: g \mapsto g(a))$$

(i.e. f is the function that maps a to the function i_a that inserts a into whatever function it gets as its paramter) because this makes the diagram commute:

