MEMORIAL UNIVERSITY

Computational Exploration of Reaction-Diffusion Models

Name: Vasu Manocha **Student ID: 202172276**

1 Introduction

Reaction-diffusion models play a crucial role in understanding the complex dynamics of two interacting species. These mathematical models capture the evolution of spatially distributed populations and provide insights into the patterns and behaviors that emerge over time. Let's delve into the specifics of each model:

1.1 Chemotaxis Model

The Chemotaxis Model involves the following parameters:

- u and v: Concentrations of the two interacting species.
- k: Chemotactic sensitivity, influencing the movement of the species towards or away from each other based on chemical signals.

The equations for the Chemotaxis Model are:

$$\frac{\partial u}{\partial t} = ku(1-u)$$
$$\frac{\partial v}{\partial t} = -v^2 + u$$

1.1.1 Fixed Points, Stability, and Eigenvalues

For the Chemotaxis Model:

$$f(u,v) = ku(1-u)$$
$$g(u,v) = -v^2 + u$$

Jacobian matrix:

$$J(u,v) = \begin{bmatrix} k(1-2u) & 0 \\ 1 & -2v \end{bmatrix}$$

Fixed points (u^*, v^*)**:**

$$
\begin{aligned}
&1. \quad (0,0)\\
&2. \quad (1,-1)\\
&3. \quad (1,1)
\end{aligned}
$$

Eigenvalues for each fixed point:

1. $(0,0)$: Calculate eigenvalues for this point using Jacobian
2. $(1,-1)$: Calculate eigenvalues for this point using Jacobian
3. $(1,1)$: Calculate eigenvalues for this point using Jacobian

Fixed Point 1: $(0,0)$ Jacobian matrix at Fixed Point 1:

$$J(0,0) = \begin{bmatrix} k & 0 \\ 1 & 0 \end{bmatrix}$$

Fixed Point 1: $(0,0)$ Jacobian matrix at Fixed Point 1:

$$J(0,0) = \begin{bmatrix} k & 0 \\ 1 & 0 \end{bmatrix}$$

Eigenvalues at Fixed Point 1: Calculate the eigenvalues for this point. The characteristic equation is given by $\det(J - \lambda I) = 0$:

$$\det\left(\begin{bmatrix} k-\lambda & 0 \\ 1 & -\lambda \end{bmatrix}\right) = (k-\lambda)(-\lambda) = 0$$

This leads to the eigenvalues $\lambda_1 = k$ and $\lambda_2 = 0$.

Stability:

- When $k > 0$, $\lambda_1 = k$ is unstable, and $\lambda_2 = 0$ is marginally stable.
- When $k < 0$, both eigenvalues are unstable.

Fixed Point 2: $(1,-1)$ Jacobian matrix at Fixed Point 2:

$$J(1,-1) = \begin{bmatrix} -k & 0 \\ 1 & 2 \end{bmatrix}$$

Eigenvalues at Fixed Point 2: Calculate the eigenvalues for this point. The characteristic equation is given by $\det(J - \lambda I) = 0$:

$$\det\left(\begin{bmatrix} -k-\lambda & 0 \\ 1 & 2-\lambda \end{bmatrix}\right) = (-k-\lambda)(2-\lambda) = 0$$

This leads to the eigenvalues $\lambda_1 = -k$ and $\lambda_2 = 2$.

Stability:

- When $k > 0$, $\lambda_1 = -k$ is stable, and $\lambda_2 = 2$ is unstable.
- When $k < 0$, $\lambda_1 = -k$ is unstable, and $\lambda_2 = 2$ is also unstable.

Fixed Point 3: $(1, 1)$ Jacobian matrix at Fixed Point 3:

$$J(1,1) = \begin{bmatrix} -k & 0 \\ 1 & -2 \end{bmatrix}$$

Eigenvalues at Fixed Point 3: Calculate the eigenvalues for this point. The characteristic equation is given by $\det(J - \lambda I) = 0$:

$$\det\left(\begin{bmatrix} -k-\lambda & 0 \\ 1 & -2-\lambda \end{bmatrix}\right) = (-k-\lambda)(-2-\lambda) = 0$$

This leads to the eigenvalues $\lambda_1 = -k$ and $\lambda_2 = -2$.

Stability:

- When $k > 0$, $\lambda_1 = -k$ is stable, and $\lambda_2 = -2$ is stable.
- When $k < 0$, $\lambda_1 = -k$ is unstable, and $\lambda_2 = -2$ is also stable.

1.2 Brusselator Model

The Brusselator Model introduces additional parameters:

- u and v: Concentrations of the two interacting species.
- A: Rate of chemical production in the system.
- B: Rate of chemical removal or consumption.

The equations for the Brusselator Model are:

$$\frac{\partial u}{\partial t} = A - (B+1)u + u^2 v$$
$$\frac{\partial v}{\partial t} = Bu - u^2 v$$

These parameters collectively influence the chemical reactions and diffusion processes, giving rise to diverse spatiotemporal patterns.

1.2.1 Fixed Points, Stability, and Eigenvalues

For the Brusselator Model:

$$f(u,v) = A - (B+1)u + u^2 v$$
$$g(u,v) = Bu - u^2 v$$

Jacobian matrix:

$$J(u,v) = \begin{bmatrix} -(B+1) + 2uv & u^2 \\ B - 2uv & -u^2 \end{bmatrix}$$

Fixed points (u^*, v^*)**:**

$$1. \quad \left(A, \frac{B}{A}\right)$$

Jacobian matrix at Fixed Point:

$$J(A, B/A) = \begin{bmatrix} -(B+1) + 2Av & A^2 \\ B - 2Av & -A^2 \end{bmatrix}$$

Eigenvalues for each fixed point: The Brusselator has a fixed point at $\{U\} = A$ and $\{V = \frac{B}{A}$. The fixed point becomes unstable when $B > 1 + A^2$.

1.3 Simulation and Results

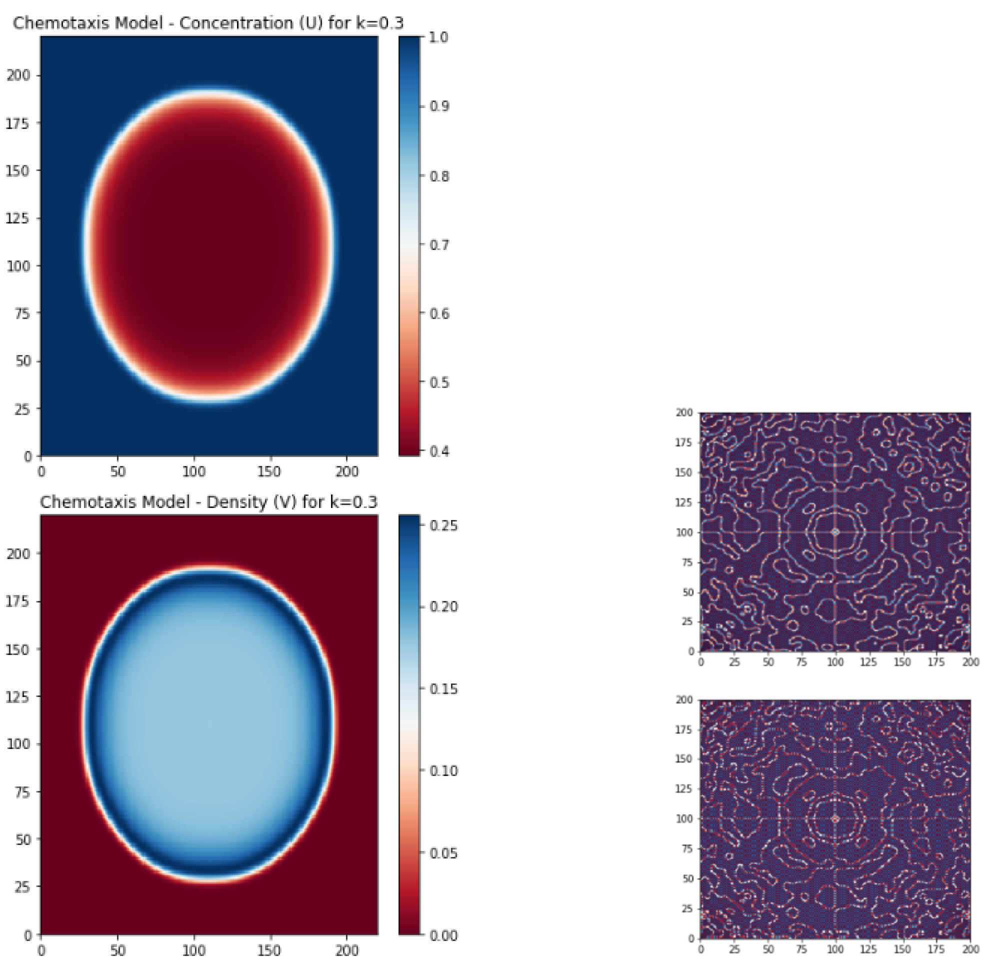

Figure 1: Simulation results for the Chemotaxis Model.

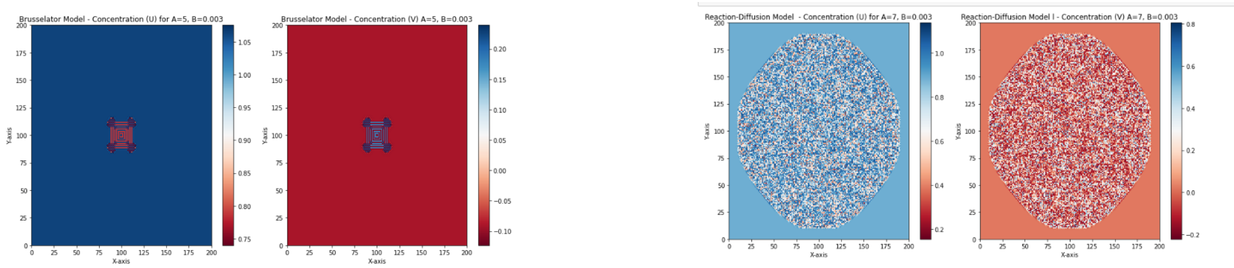

Figure 2: Simulation results for the Brusselator Model.

Parameter Values:

- Chemotaxis Model: $k = 0.03$
- Chemotaxis Model: $k = 10$

Parameter Values:

- Brusselator Model: $A = 5, B = 0.003$
- Brusselator Model: $A = 7, B = 0.003$

Observations for Chemotaxis Model: For various values of k, the simulation results show that the pattern remains consistent, suggesting robustness in the model.

Observations for Brusselator Model: For certain parameter values, specifically when $B = 0.003$ and as A increases, the model exhibits unexpected behavior at $A = 7$. A bifurcation in the pattern is observed at this point, indicating a critical transition. Further research is required to comprehensively understand and analyze the implications of this bifurcation.

2 Conclusion

Further research is required to comprehend unusual behaviors observed at specific parameter values. As the models exhibit intricate dynamics, additional investigation is needed to explore the underlying reasons for these observed phenomena. The complexity of reaction-diffusion systems suggests a rich area for extended study and analysis.