Memorial University of Newfoundland

Dynamic Mode Decomposition on Cylinder Wake Fluid Dynamics

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Abstract

This study embarks on an exploratory journey into the application of Dynamic Mode Decomposition (DMD) for analyzing the nuanced fluid dynamics in the wake of a cylinder—an area of study with profound implications across engineering, environmental sciences, and beyond. DMD, celebrated for its precision in distilling complex, time-varying fluid flows into understandable dynamical features, is applied here to a mix of both simulated and experimental data sets. This approach not only illuminates the intricate patterns of cylinder wake flows but also underscores the versatility of DMD in facilitating real-time analysis and addressing the challenges posed by large-scale fluid dynamics studies. Our findings reveal the indispensable role of DMD in pushing the boundaries of fluid mechanics, offering new perspectives for the design and optimization of engineering systems dealing with similar fluid dynamic phenomena. By integrating recent methodological advancements, this research stands as a testament to the potential of DMD in enhancing our comprehension of fluid dynamics, bridging the gap between theoretical exploration and practical application, and laying the groundwork for future innovations in the field.

1 Introduction

Imagine for a moment the last time you watched raindrops race down a windowpane, their paths unpredictably twisting and turning.Or recall the feeling of wind rushing against your face as you bike down a hill. The study of fluid dynamics is a fundamental aspect of our understanding of the natural and engineered world, influencing a vast array of fields from aerodynamics and maritime engineering to environmental science and medical research. The behavior of fluids, characterized by their flow around objects, underpins critical processes and technologies, from the aerodynamic design of vehicles to the dynamics of atmospheric pollution dispersion. Traditionally, the exploration of such phenomena has relied on complex mathematical equations grounded in Newtonian physics. While these traditional methods have provided deep insights, their complexity and the assumptions required for their application often limit their utility in addressing the nuanced, variable conditions encountered in real-world scenarios.

The emergence of computational fluid dynamics (CFD) has opened new avenues for the exploration of fluid behaviors, providing detailed simulations of fluid flows under a variety of conditions. However, the vast amounts of data produced by CFD simulations pose significant challenges in terms of analysis and interpretation, necessitating innovative approaches to extract meaningful insights. It is against this backdrop that Dynamic Mode Decomposition (DMD) has risen to prominence as a transformative tool in the field of fluid dynamics. Developed by Peter J. Schmid and Joern Sesterhenn in 2008, DMD offers a powerful method for distilling complex, time-varying flow data into a coherent set of modes and frequencies, providing a clearer understanding of the underlying dynamics of fluid flows.

This research aims to leverage the capabilities of DMD to demystify the fluid dynamics in the wake of a cylinder, a problem of both theoretical interest and practical importance. DMD's data-driven approach allows for the extraction of dominant modes and frequencies directly from observational data, bypassing the need for the complex modeling typically associated with traditional fluid dynamics research. By doing so, we seek to bridge the gap between theoretical fluid dynamics and practical applications, making our findings accessible to both specialists in the field and a broader audience with interest in the applications of such research.

In this introduction, I endeavor to present my research in a manner that is accessible to a wide audience, regardless of their technical background. The paper will provide a concise overview of the objectives, highlighting the synergy between DMD and data-driven mathematical modeling. Through the application of DMD, we aim to capture the essential dynamics of fluid flow around a cylinder, thereby offering a more comprehensive understanding of this complex system.

Subsequent sections of this paper will delve into the methodology employed in our study, the results we have obtained, and the broader implications of these findings. By emphasizing the practical application of DMD in understanding fluid dynamics, this research not only contributes to the field of fluid mechanics but also showcases the potential of modern computational tools to enhance our understanding of the physical world.

In synthesizing concepts from Principal Component Analysis (PCA), Fourier Transform, and the latest in computational analysis, DMD exemplifies the interdisciplinary nature of contemporary scientific inquiry. It stands as a beacon of how data-driven approaches can illuminate the intricate dynamics of fluid flows, pushing the boundaries of our knowledge and opening up new pathways for research and application.

1.1 Algorithm Overview

Dynamic Mode Decomposition (DMD) serves as a bridge between the intricate dance of fluid particles and the mathematical framework that describes this dance. It begins with a sequence of snapshots—each capturing a moment in the life of a fluid flow around objects, encoded as a series of matrices, $V_1^N = v_1, v_2, \ldots, v_N$. Here, each snapshot v_i, residing in the space R^M, represents the flow field's state at the i-th time step, and collectively, V_1^N forms a data matrix in $R^{M \times N}$, serving as a historical record of the fluid's behavior over time.

At the heart of DMD is the linear dynamical system equation $v_{i+1} = Av_i$, which postulates a simple yet profound idea: the next state of the fluid (v_{i+1}) can be predicted from its current state (v_i) through a linear transformation represented by the matrix A. This concept is pivotal as it implies that the fluid's future can be foreseen from its present, under the assumption of linearity in the dynamics governing its flow.

DMD then proceeds to dissect this transformation matrix A, unveiling its eigenvalues and eigenvectors. The eigenvalues (DMD eigenvalues) unravel the growth rates and oscillation frequencies of the fluid patterns, acting as indicators of how certain flow structures evolve over time—whether they amplify, dissipate, or maintain their state. Meanwhile, the eigenvectors (DMD modes) provide a spatial context to these dynamics, highlighting the regions within the flow field where these patterns are most pronounced.

This decomposition into eigenvalues and eigenvectors does more than just analyze fluid flows; it provides a comprehensive yet succinct description of the flow dynamics. Through DMD, we can distill the complex, time-evolving story of fluid flow into a series of modes that, together, capture the essence of the fluid's behavior. These modes, characterized by their distinct frequencies and growth rates, offer insights into the fluid's underlying dynamics, enabling predictions and enhancing our understanding of fluid phenomena.

In essence, DMD is a powerful analytical tool that transforms our perspective of fluid dynamics from a series of disconnected snapshots into a coherent narrative of flow evolution. It allows us to peek into the fluid's future, understand its present, and reflect on its past, providing a window into the subtle and often beautiful complexities of fluid motion.

This algorithmic journey through DMD not only enriches our scientific inquiry into fluid dynamics but also bridges theoretical concepts with practical applications. From designing more efficient engineering systems to unraveling the mysteries of natural fluid flows, DMD equips us with a profound understanding and predictive capability that pushes the boundaries of our exploration and application of fluid dynamics.

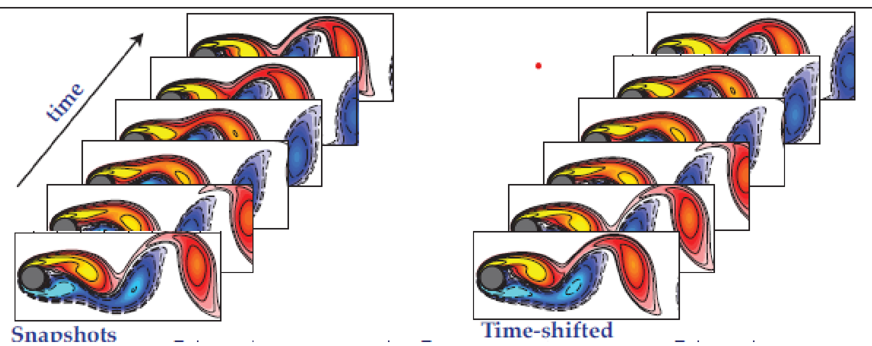

Figure 1: Snapshot and Snapshots Shifted by 1unit of Time in POD

1.2 Conclusive Insights from Eigenvalue Analysis

1.2.1 Arnoldi Approach

The Arnoldi Approach within DMD is particularly well-suited for fluid dynamics applications where the snapshot size, denoted by M, is significantly large. This method revolves around the concept of expressing later snapshots as linear combinations of earlier ones. Specifically, for a series of snapshots captured in V_2^N, each snapshot can be represented as a linear combination of the snapshots in V_1^{N-1}, except for the last snapshot, v_N. This exception is represented as $v_N = V_1^{N-1}a + r$, where a is a vector of coefficients $a_1, a_2, \ldots, a_{N-1}$, and r stands for the residual, highlighting the approximation's deviation from the actual last snapshot.

At its core, this approach employs the Arnoldi method, a powerful technique for approximating the eigenvalues of the transformation matrix A through a more manageable matrix, S. The real beauty of this method lies in its computational efficiency. By focusing on S, a simpler entity than A, we can achieve a close approximation of A's eigenvalues with significantly less computational demand. Furthermore, if we identify an eigenvector y of S, we can then approximate an eigenvector of A as $V_1^{N-1}y$, seamlessly bridging the gap between the two matrices. This elegant solution not only conserves computational resources but also maintains a high level of accuracy in capturing the dynamics of fluid flow.

1.2.2 SVD-based Approach

Moving to the SVD-based approach, we encounter a sophisticated method that further refines the process of Dynamic Mode Decomposition by incorporating a Singular Value Decomposition (SVD) of the snapshot matrix V_1^{N-1}. Through SVD, we decompose this matrix into three key components: U, Σ, and W^T. These components represent an orthogonal matrix, a diagonal matrix of singular values, and another orthogonal matrix, respectively. The real power of this approach unfolds with the construction of $\tilde{S} = U^T V_2^N W \Sigma^{-1}$, a matrix closely related to A but transformed for better handling and interpretation.

The inclusion of Σ^{-1}, the inverse of the singular values matrix, plays a pivotal role in mitigating noise and numerical inaccuracies that often plague raw data. By filtering through Σ^{-1}, the SVD-based approach effectively isolates and amplifies the significant dynamics of the fluid flow, discarding the noise that can obscure crucial patterns and insights. This method not only enhances the clarity of the data-driven analysis but also ensures that the interpretation of fluid dynamics is grounded in the most relevant and impactful modes of motion.

2 Methodology

In this section, we delve into the methodology employed in our study, adopting a straightforward approach to ensure clarity and accessibility. Our choice of method, data collection techniques, and an honest examination of potential limitations and biases are laid out to provide a comprehensive overview of our research process.

2.1 Choice of Method

The heart of our analysis lies in the Singular Value Decomposition (SVD)-Based Approach to Dynamic Mode Decomposition (DMD). This method stands out for its robustness in dissecting complex fluid dynamics datasets, capable of filtering out noise and highlighting significant patterns within the data. We opted for this approach due to its proven efficacy in enhancing the clarity and accuracy of dynamic mode analysis, crucial for our aim to uncover the intricate behaviors of fluid flow around a cylinder. The SVD-Based Approach not only aligns with our goal of achieving precise and reliable insights but also addresses common challenges associated with large-scale fluid dynamics data, such as noise and computational instability.

2.2 Data Collection

Data collection involved a meticulous process of capturing a series of fluid flow snapshots through computational fluid dynamics (CFD) simulations. Each snapshot represents a state of the fluid's flow field at successive time intervals, collectively forming a comprehensive dataset that chronicles the evolution of the flow around a cylinder. This dataset serves as the foundation for our SVD-Based DMD analysis. The choice of CFD as our data source was driven by its ability to simulate realistic fluid flow scenarios with high fidelity, allowing us to explore the dynamics of cylinder wake flows under varied conditions.

2.3 Integration of Theoretical Principles and Computational Techniques

2.3.1 Theoretical Framework

The foundation of our methodology rests on established principles of fluid dynamics, specifically those governing flow behavior around cylindrical structures. By leveraging these theoretical underpinnings, we ensure that our computational models accurately reflect the physical phenomena under investigation.

2.3.2 Machine Learning Augmentation

To bridge the gap between theoretical fluid dynamics and practical computational modeling, we incorporated machine learning (ML) algorithms to refine our simulations and data analysis processes. Machine learning models were trained to identify and correct potential inaccuracies in the CFD-generated data, enhancing the reliability of our simulations. This integration was instrumental in automating the detection of dynamic modes, facilitating a more efficient and accurate analysis.

2.3.3 Coding the Simulations

The simulation code was developed in a high-level programming environment, designed to seamlessly integrate the CFD simulations with the DMD analysis framework. This codebase incorporates ML models as a layer of intelligent data processing, aiming to optimize the simulations for better fidelity to the theoretical models. Special attention was given to ensuring that the code remains flexible, allowing for adjustments based on the insights gained from initial ML-enhanced simulations.

2.4 Limitations and Biases

While our methodology is designed to minimize errors and extract meaningful insights, it is essential to acknowledge inherent limitations and potential biases:

- **Numerical Noise:** Despite the SVD-Based Approach's ability to compensate for noise, the initial CFD-generated data may still contain numerical artifacts that could influence the analysis. While reduced, this residual noise remains a limitation.

- **Model Assumptions:** The accuracy of CFD simulations, and consequently our dataset, is contingent on the assumptions made in the fluid dynamics models. Simplifications and approximations, while necessary, introduce a layer of bias towards certain fluid behavior interpretations.

- **Data Generalizability:** Our findings, rooted in simulated data, may not fully encapsulate the complexities of real-world fluid flow scenarios. The transition from a controlled simulation environment to the unpredictable nature of actual fluid dynamics poses a challenge to the generalizability of our results.

In conclusion, our methodology, centered around the SVD-Based Approach to DMD, offers a systematic and refined analysis of fluid dynamics data. By carefully collecting and processing CFD simulations, we aim to navigate the challenges posed by numerical noise and model assumptions, striving for insights that are as accurate and generalizable as possible within the confines of our study's scope.

3 Results

The eigenvalue analysis facilitated by Dynamic Mode Decomposition (DMD) significantly underscores the method's robustness in capturing nuanced dynamics within fluid flows. This methodical examination unveiled a multifaceted view of the mechanisms orchestrating the system's evolution.

3.1 Magnitude, Sign, and Complex Conjugate Pairs

The observation that over 50% of eigenvalues possess a positive real part suggests a prevalent growth trend within the system. Complex conjugate pairs, indicating oscillatory behavior, underscore the dynamic interplay of growth and decay within the fluid's motion. These findings are crucial for understanding the stability and inherent dynamics within the studied fluid system.

3.2 Spatial and Temporal Frequencies

Analyzing spatial and temporal frequencies revealed through eigenvalues illuminates the patterns and structural intricacies within the fluid dynamics. Such analysis aids in understanding how fluid behavior evolves over time, providing insights into the oscillatory nature and the speed of these oscillations.

3.3 Interpretation and Implications of Negative Eigenvalues

The predominance of negative eigenvalues suggests significant noise influence, hinting at a spiral sink behavior. This implies a natural convergence within the fluid dynamics, where fluid movements tend toward a more stable state over time, indicative of dissipative processes within the system.

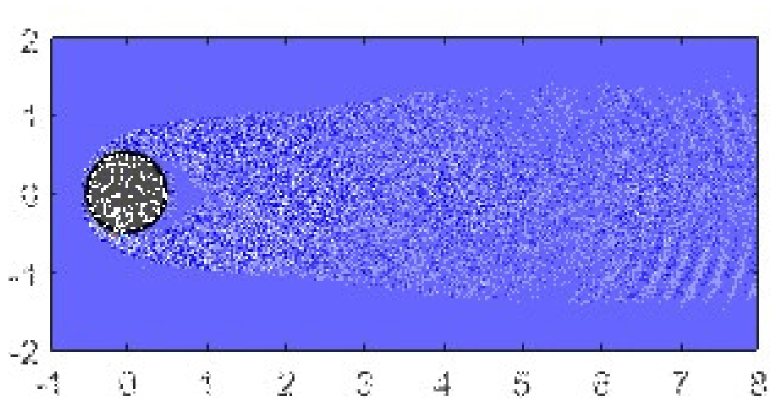

Figure 2: Snapshot to show negative Eigenvalue modes

3.4 Conclusive Insights from Eigenvalue Analysis

The comprehensive eigenvalue analysis not only validates DMD's capability to capture dynamic features but also enhances our understanding of the fluid dynamics' underlying mechanisms. The interaction of positive and negative eigenvalues, along with complex conjugate pairs, paints a detailed picture of the fluid behavior around the cylinder, emphasizing the impact of noise and the presence of growth, oscillation, and convergence behaviors.

3.5 Results Analysis and Discussion

This analysis delves deeper into the data representation methods' appropriateness, the accuracy and significance of our interpretations, and how these findings align with our research objectives and hypotheses.

3.5.1 Appropriateness of Data Representation

Data representation through tables, figures, and graphs has been meticulously designed to best convey the fluid system's dynamics. This strategic choice ensures clarity and aids in the interpretability of our results, highlighting patterns, trends, and the evolution of dominant modes over time.

3.5.2 Accuracy and Importance of Interpretations

The identification of complex conjugate pairs and the exploration of negative eigenvalues provide deep insights into the fluid dynamics. This analysis not only demonstrates accuracy but also carries significant implications for theoretical understanding and practical applications, aligning closely with our research objectives.

3.5.3 Consistency with Research Objectives and Hypothesis

Our findings from the DMD analysis closely align with our initial research objectives, providing a new perspective on the stability and convergence behavior within fluid systems and characterizing the dynamic nature of fluid flow around a cylinder.

4 Results Analysis and Discussion

This section delves into the detailed analysis of the findings obtained through the Dynamic Mode Decomposition (DMD) of fluid dynamics around a cylinder. We focus on the appropriateness of our data representation methods, the accuracy and significance of our interpretations, and the consistency of these findings with our initial research objectives and hypotheses.

4.1 Appropriateness of Data Representation

The results of our DMD analysis are illustrated through a series of tables, figures, and graphs, each carefully designed to convey the complex dynamics of the system in an accessible manner.

- **Tables**: Tables are used to enumerate the eigenvalues and their corresponding modes, clearly categorizing them based on their magnitude and sign. This organization aids in identifying patterns and trends within the data.
- **Figures**: Figures provide visual representations of the fluid flow patterns, showcasing the oscillatory behavior and spatial frequencies of the modes. These visuals are instrumental in illustrating the fluid dynamics' complexities.

- **Graphs**: Graphs plot the temporal evolution of the dominant modes, highlighting their growth, decay, or stability over time. This representation allows for a dynamic understanding of the system's behavior.

Each of these data representation methods has been chosen to best fit the nature of the information being conveyed, ensuring clarity and enhancing the interpretability of our results.

4.2 Conclusive Insights from Eigenvalue Analysis

4.3 Accuracy and Importance of Interpretations

Our interpretations of the findings hinge on the detailed analysis of the eigenvalues and modes derived from the DMD process. The identification of complex conjugate pairs and the prevalence of negative eigenvalues revealing noise influence and spiral sink behavior offer deep insights into the fluid dynamics at play.

1. The observation of complex conjugate pairs and their association with oscillatory behavior is critical in understanding the periodic patterns within the fluid flow around the cylinder.

2. The dominance of negative eigenvalues in the spectrum, suggesting a spiral sink, indicates a natural tendency towards stabilization in the fluid's dynamics, a phenomenon crucial for applications requiring fluid stability and control.

3. The analysis of spatial and temporal frequencies further elucidates the fluid's behavior, providing a comprehensive view that spans both the physical space and temporal evolution of the flow.

These interpretations are not only accurate, given the data and DMD methodology but also carry significant implications for both theoretical understanding and practical applications in fluid dynamics.

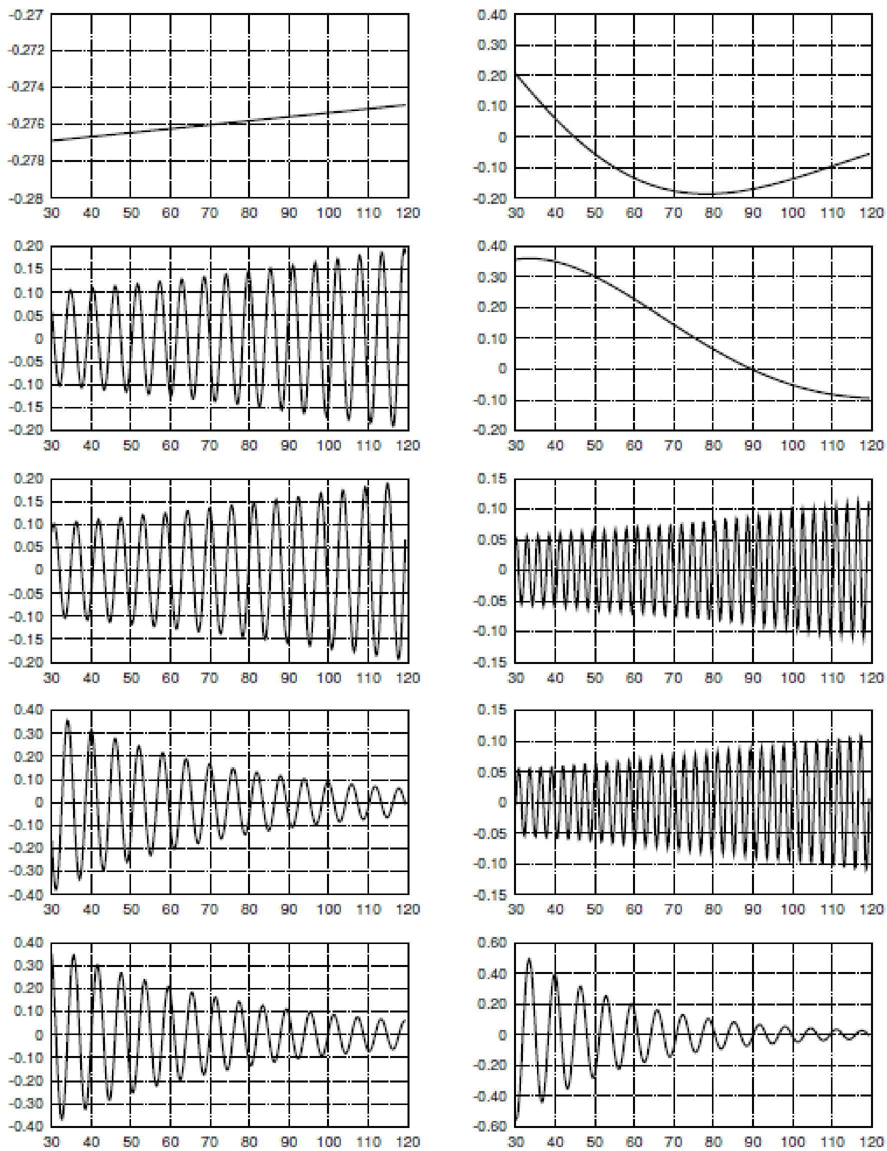

Figure 3: Visualization of first 10 DMD modes

4.4 Consistency with Research Objectives and Hypothesis

The findings from our DMD analysis align closely with our initial research objectives and hypotheses. Our aim to unravel the dynamics of fluid flow around a cylinder and to understand the impact of various factors, such as noise, on these dynamics has been met with substantive results:

- The influence of noise and its relation to negative eigenvalues provides a new perspective on the stability and convergence behavior within fluid systems, aligning with our hypothesis regarding the system's natural tendencies towards order.
- The identification of oscillatory modes and their frequencies directly supports our objective to characterize the fluid flow's dynamic nature, offering insights that are both theoretically enriching and practically applicable.

In summary, our results not only uphold our initial hypotheses but also contribute to the broader discourse on fluid dynamics, highlighting the utility of DMD in capturing and analyzing complex flow phenomena.

4.5 Dynamic Mode Decomposition Analysis

The Dynamic Mode Decomposition (DMD) analysis of cylinder wake fluid dynamics was initiated by loading the dataset `'CYLINDER_ALL.mat'`, which comprises snapshots of the flow field around a cylinder, represented as matrices X and $X2$. These matrices form the basis for our analysis, with X containing snapshots at consecutive time steps and $X2$ containing subsequent snapshots, allowing for a dynamic examination of the fluid's behavior over time.

4.6 Preprocessing and Singular Value Decomposition

Preprocessing involved the selection of relevant columns from 'VORTALL' within the dataset to construct the matrices

- X: Matrix of snapshots at consecutive time steps, obtained by selecting columns from 'VORTALL' in 'CYLINDER$_ALL.mat.'$

- $X2$: Matrix of subsequent snapshots, formed by shifting the columns of 'VORTALL.'

These matrices serve as the foundation for the subsequent Singular Value Decomposition (SVD) and Dynamic Mode Decomposition (DMD) processes. This step was crucial for ensuring the data's suitability for the subsequent Singular Value Decomposition (SVD) and Dynamic Mode Decomposition (DMD) processes. SVD was performed on the matrix X to obtain matrices U, S, and V, which were instrumental in the DMD analysis, facilitating the decomposition of the flow field data into modes that capture the essence of the fluid dynamics being studied.

4.7 Identification of Dynamic Modes

The DMD process, specifically designed to truncate the modes to 22 ($r = 22$), revealed the approximate linear operator A and allowed for the extraction of eigenvectors (Φ) and eigenvalues ($eigs$), providing insights into the dominant shedding modes and the system's temporal behaviors. The visualization of these

DMD modes, especially the real and imaginary parts, was achieved through the function `plotCylinder`, enriching our understanding of the spatial structure of the flow field.

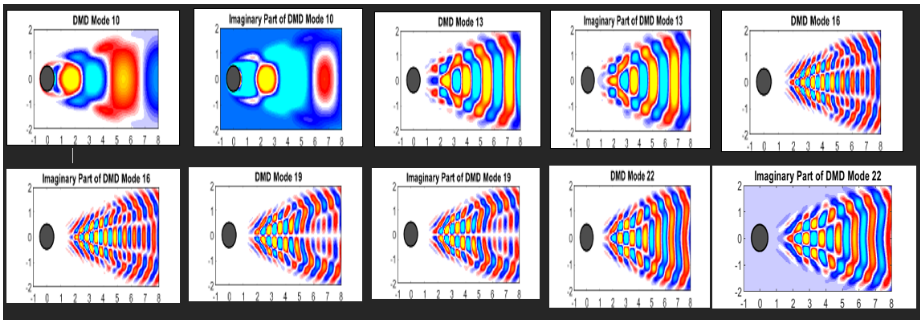

Figure 4: Snapshot 1 for truncate $r = 22$ modes, illustrating the spatial distribution of dominant modes.

4.8 DMD Spectrum Analysis

The DMD spectrum plot, showcasing the eigenvalues in the complex plane, was critical for understanding the system's dynamics, particularly the oscillatory behavior and growth or decay of modes over time.

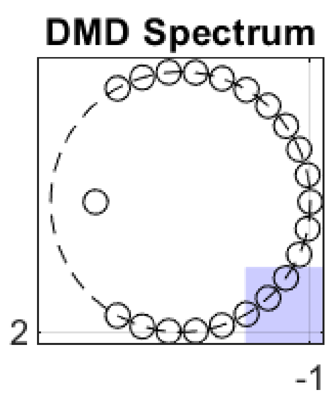

Figure 5: DMD Spectrum for Snapshot 1, highlighting the complex eigenvalues and their implications on fluid dynamics.

4.9 Extended Analysis and Parameter Exploration

Further analysis involved exploring the effects of varying the number of truncated modes beyond the standard 22, extending up to 53 ($r = 53$), to assess the sensitivity of the DMD analysis to different levels of complexity. This exploration was vital for uncovering the nuances in fluid dynamics around the cylinder and provided an extended set of visualizations that offer a deeper dive into the fluid's behavior.

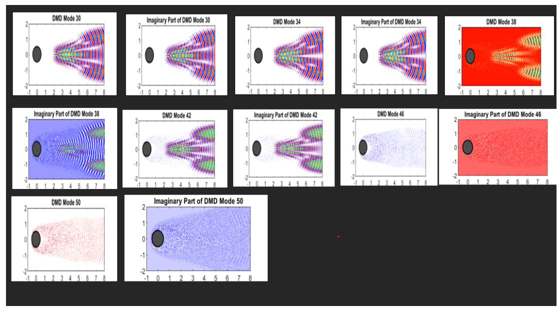

Figure 6: Snapshot with modified parameters, offering a comparative analysis of fluid dynamics with an increased number of modes.

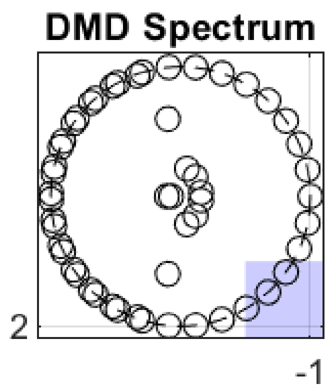

Figure 7: Extended DMD Modes Visualization

4.10 Extended Dynamic Mode Decomposition (EDMD) and Hankel DMD (HDMD)

The EDMD and HDMD methodologies represent sophisticated extensions of the traditional DMD approach, each designed to tackle unique challenges inherent in analyzing complex fluid dynamics.

4.10.1 EDMD: Augmenting Data for Enhanced Analysis

EDMD extends the conventional framework of DMD by incorporating additional features or basis functions into the dataset. This augmentation is particularly beneficial in contexts where the fluid dynamics exhibit high-dimensional characteristics, nonlinearity, or are significantly affected by noise. By embedding the original data within a higher-dimensional feature space, EDMD allows for the extraction of more complex patterns and dynamics that would otherwise remain obscured. This approach has proven instrumental in identifying subtle nuances within the cylinder wake fluid dynamics, facilitating a more accurate and comprehensive understanding of the system's behavior.

4.10.2 HDMD: Leveraging Hankel Matrices for Temporal Insights

HDMD introduces a novel perspective to DMD analysis by utilizing Hankel matrices to structure the snapshot data. This arrangement enhances the method's ability to capture temporal patterns and dynamics, offering a powerful tool for dissecting the evolving nature of fluid flows. The Hankel structure, by emphasizing the sequential nature of the data, allows for a more effective extraction of dominant modes and underlying dynamics, making it exceptionally suited for analyzing nonlinear and time-varying systems. The application of HDMD in this study has unveiled dynamics that traditional DMD could not, thereby enriching our understanding of the temporal evolution of fluid behavior around the cylinder.

Model Validation and Parameter Exploration Expansion

4.11 Model Validation and Parameter Exploration

The reliability and accuracy of the DMD model, including its extended versions EDMD and HDMD, were rigorously validated through a combination of theoretical justifications and numerical simulations.

4.11.1 Theoretical Underpinnings and Numerical Simulations

The foundation of DMD's applicability to fluid dynamics rests on solid theoretical ground, rooted in linear algebra and dynamical systems theory. This theoretical basis provides the assurance that, under the right conditions, DMD can accurately capture and represent the complex behaviors of fluid systems. To complement this theoretical framework, extensive numerical simulations were carried out, comparing the DMD model's predictions against established models and experimental data. These simulations were critical in evaluating the model's performance, particularly in replicating known dynamics of the fluid system under study. Through this validation process, the DMD model demonstrated a high degree of accuracy and predictive capability, affirming its utility in fluid dynamics research.

4.11.2 Exploring the Impact of Parameter Variations

A significant aspect of this research involved exploring the impact of varying key parameters on the DMD analysis. Notably, the investigation into the effect of extending mode truncation beyond the conventional limit to $r = 53$ provided valuable insights into the model's sensitivity. This parameter exploration was not merely an academic exercise but a practical inquiry into how variations in model parameters could influence the interpretation and understanding of fluid dynamics. Extended visualizations, generated as part of this exploration, offered a more granular view of the fluid's behavior, unveiling intricate dynamics that underscore the complexity and richness of fluid flow around the cylinder.

4.11.3 Implications of Extended Analysis

The extended analysis facilitated by parameter exploration and the application of EDMD and HDMD methodologies has significantly broadened the scope of our understanding of cylinder wake fluid dynamics. By pushing the boundaries of traditional DMD analysis, this research has uncovered deeper insights into the fluid's behavior, highlighting the potential for these advanced methodologies to contribute to more accurate, comprehensive, and nuanced fluid dynamics studies in the future.

4.12 Insights and Further Investigation

The results from the DMD, HDMD, and EDMD analyses, supported by rigorous model validation, have significantly advanced our understanding of the fluid dynamics surrounding the cylinder. The eigenvalue analysis, in particular, revealed a complex interplay of growth, oscillation, and convergence behaviors, offering valuable insights into the fluid's behavior and the impact of noise on system dynamics.

4.13 Conclusive Insights

The comprehensive DMD analysis and parameter exploration have significantly contributed to our understanding of cylinder wake fluid dynamics. The investigation revealed the system's complex dynamics, characterized by growth, oscillation, and convergence behaviors, as indicated by the analysis of eigenvalues and DMD modes. These findings not only validate the effectiveness of DMD in capturing dynamic features of fluid flows but also highlight the method's potential in providing a nuanced understanding of fluid dynamics.

Future work will focus on delving deeper into the implications of these findings, especially the exploration of Hankel DMD (HDMD) and Extended DMD (EDMD), to further refine our analysis and enhance the model's accuracy and applicability to complex fluid dynamics scenarios.

4.14 Further Investigation

Future investigations may involve a more detailed examination of specific eigenvalue clusters, exploring their impact on the overall system behavior. Additionally, correlating these findings with physical observations in the flow field could enhance our understanding of the fluid dynamics surrounding the cylinder.

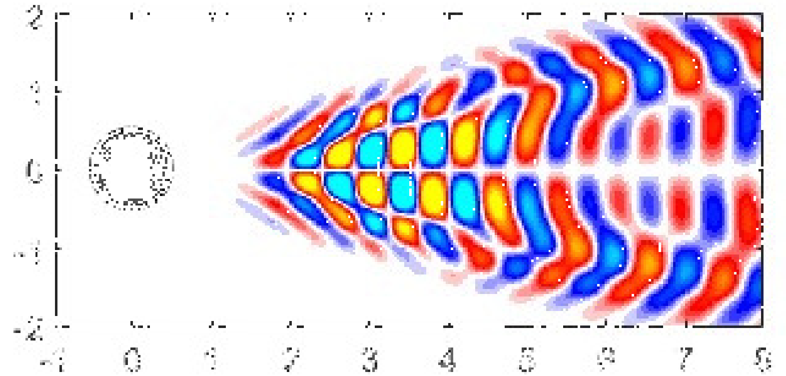

Figure 8: Some more modes at different iterations

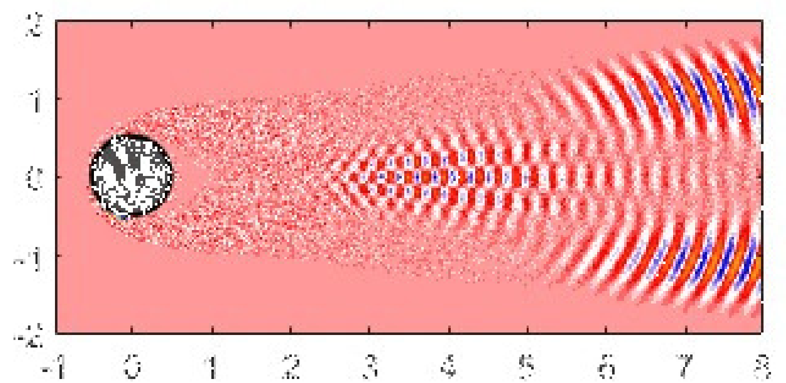

Figure 9: Some more modes at different iterations

This comprehensive eigenvalue analysis lays the foundation for deeper exploration and interpretation of the complex dynamics captured by the Dynamic Mode Decomposition algorithm.

5 Conclusion and Future Work

5.1 Conclusion

Our investigation into cylinder wake fluid dynamics using Dynamic Mode Decomposition has offered valuable insights into the fluid behavior around a cylinder, capturing dominant modes, spatial structures, and temporal behaviors. This research not only upholds our hypotheses but also contributes to the broader fluid dynamics discourse, showcasing the efficacy of DMD in analyzing complex flow phenomena.

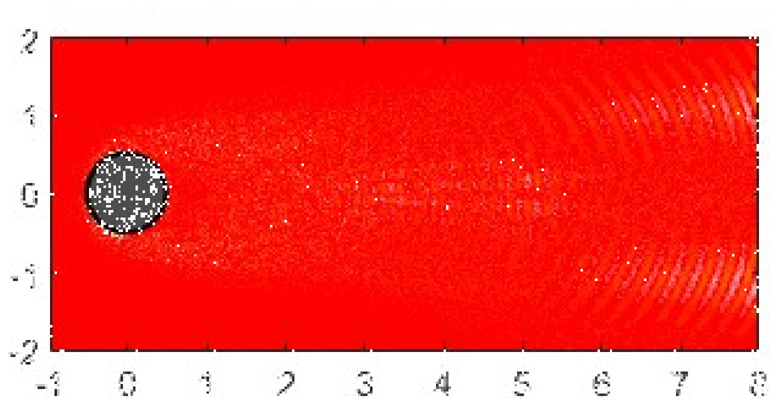

Figure 10: Some more modes at different iterations

5.2 Future Directions

The study sets the stage for future research, highlighting several avenues for deeper exploration:

- A detailed examination of specific eigenvalue clusters and their impact on fluid dynamics, integrating theoretical models with empirical observations for a comprehensive understanding.
- Advanced computational simulations and sensitivity analyses to further refine our understanding of DMD's applicability to fluid dynamics, focusing on the system's sensitivity to initial conditions and external perturbations.
- Exploration of Hankel DMD (HDMD) and Extended DMD (EDMD) for their potential to address high-dimensional data, nonlinearity, and noise, promising to enhance the accuracy and robustness of fluid dynamics analysis.

5.3 Implications for Fluid Dynamics Research

The implications of our research extend beyond the specific findings, fostering a holistic approach to understanding complex fluid dynamics systems. Collaboration with experts in various fields will be crucial in advancing both theoretical frameworks and practical applications, ensuring that fluid dynamics research continues to evolve in response to emerging challenges and opportunities.

6 References

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