Dynamic Mode Decomposition in Cylinder Wake Fluid Dynamics

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Outline

Objectives

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- Utilizing data-driven mathematical models: Implementing mathematical models based on real-world data to understand fluid dynamics more effectively.
- Enhancing understanding of fluid dynamics: Using DMD to extract meaningful insights from complex flow patterns, contributing to our understanding of fluid behavior.
- Making the research accessible to a broad audience: Presenting findings in a clear and understandable manner, ensuring that the research is beneficial to experts and non-experts alike.

Problem Statement

Aim

- This research aims to leverage the Dynamic Mode Decomposition (DMD) algorithm to analyze fluid dynamics in the wake of a cylinder.
- By extracting dominant modes and frequencies directly from observational data, it provides insights into the temporal behaviors of the fluid flow.

Significance

- Capture essential dynamics of fluid flow for comprehensive understanding

Theoretical Background

Introduction to Dynamic Mode Decomposition (DMD)

- DMD is a Dynamical System of coupled Spatial Temporal modes
- It involves fetching real-world data and running simulations using specific DMD algorithms.
- DMD differs from other dimensionality reduction techniques, such as Principal Component Analysis (PCA), and Fourier Transform .

Application

- Widely used in various fields like, robotics and neuroscience,including fluid mechanics, for extracting essential dynamics from time-series data

Singular Value Decomposition (SVD)

Definition

- SVD is a matrix factorization method that decomposes a matrix V into three other matrices: $V \approx U \Sigma W^T$, where
  - U is an $M \times M$ orthonormal matrix containing the left singular vectors,
  - Σ is an $M \times N$ diagonal matrix containing the singular values,
  - W^T is an $N \times N$ orthonormal matrix containing the right singular vectors.

Standard DMD

Dataset

- Snapshot sequence $V_1^{N-1} = \{v_1, v_2, ..., v_N - 1\}$, where $v_i \in \mathbb{R}^M$ is the i-th snapshot of the flow field, and $V_1^N \in \mathbb{R}^{M \times N}$ is a data matrix.
- Here, $v_1, v_2, ..., v_N$ are column vectors with millions of values. However, in DMD, we only consider the values with the most significance, which actually change the dynamics of the fluid in the long run.
- $V_1^N = \{v_1, v_2, ..., v_N\}$,where V_1^N is the Snapshot sequence one step ahead in time

"DMD is like a curious little explorer, unraveling the mysteries of fluid dynamics one snapshot at a time."

DMD

Linear Dynamical System

- $V_1^N = AV_1^{N-1}$, where v_{i+1} depicts time-shifted approximations.
- Here, V_1^N represents the N-th snapshot of the flow field, and A is the linear operator representing the dynamics of the system.

Methods of Solving DMD

Arnoldi Method

- DMD is an Arnoldi method, which approximates the eigenvalues of A

SVD Method

- Instead of computing the companion matrix V_1^{N-1}, the SVD-based approach yields the matrix $\tilde{V}$, related to A via a similarity transform.
- The SVD of $V_1^{N-1} = U\Sigma W^T$ is computed, and $\tilde{V} = U^T V_2^N W \Sigma^{-1}$, where Σ^{-1} compensates for noise and numerical issues.

SVD-Based Approach is better

SVD-Based Approach

- This approach compensates for noise and numerical issues, providing more accurate results.

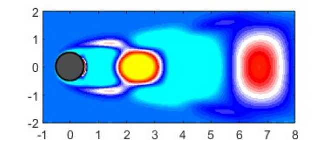

Figure: Approximation of Arnoldi Method

Eigenvalues and Predictions

Eigenvalue Analysis

- Comprehensive analysis reveals intriguing insights into the dynamics of the system

Magnitude and Sign

- Magnitude of eigenvalues indicates system's dynamics
- Presence of complex conjugate pairs suggests oscillatory behavior

Spatial and Temporal Frequency

- Imaginary part of eigenvalues represents spatial frequency
- Temporal frequency provides insights into speed and frequency of oscillations

Eigenvalues and Predictions

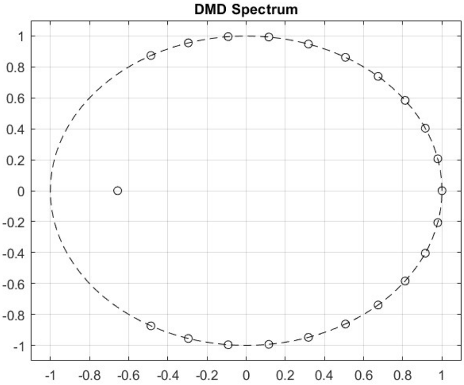

Figure: DMD Spectrum

Plots

Simulation

- We conducted simulations of the standard DMD with our data using the SVD approach.

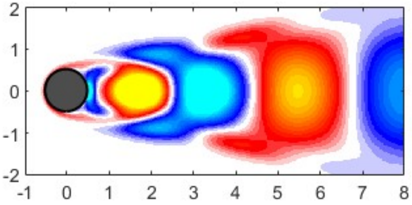

Figure: 1st DMD plot

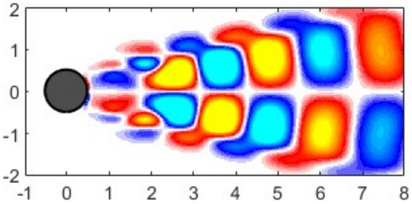

Figure: 2nd DMD plot

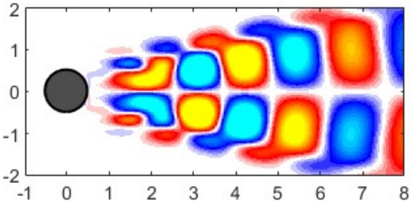

Figure: 3rd DMD plot

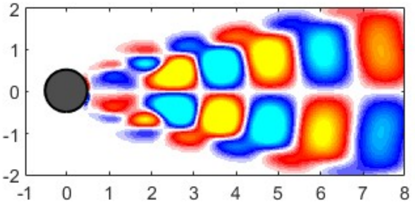

Figure: 4th DMD plot

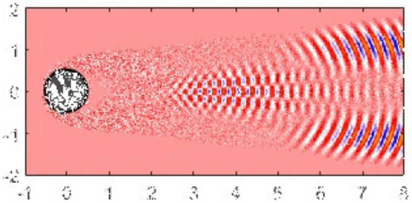

Figure: 5th DMD plot

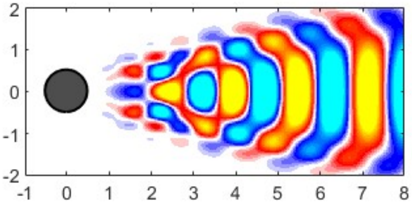

Figure: 6th DMD plot

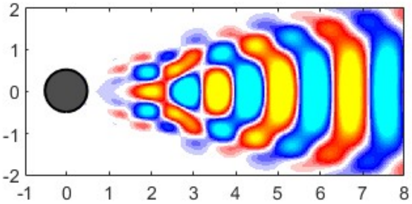

Figure: 7th DMD plot

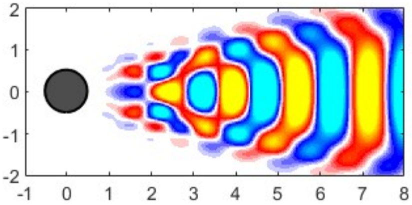

Figure: 8th DMD plot

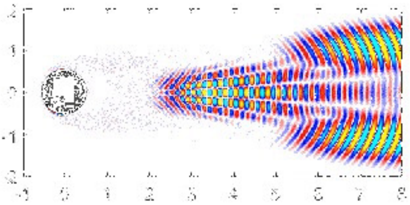

Figure: 47th DMD plot

Animation OF dmd Plots

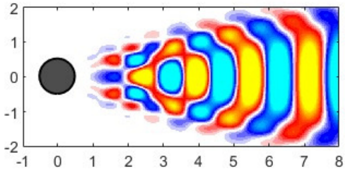

Figure: simulation with 46 iterations

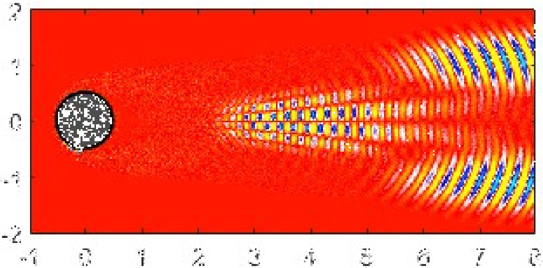

Figure: Plot for Negative Eigenvalue

Conclusion and Future Directions

Conclusion

- Valuable insights into cylinder wake fluid dynamics using DMD

Future Directions

- Explore different types of DMD algorithms, such as Exact DMD, Projected DMD, Hankel DMD, and Extended DMD, to predict data and compare with the SVD-based approach.
- Understanding the limitations and advantages of various DMD algorithms and optimizing them can lead to improved prediction accuracy and computational efficiency in analyzing fluid dynamics and other complex systems.

Questions?

Thank You!

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