Learning Chaos: Visualizing Sensitivity in the Logistic Map

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Outline

Why Study Chaos?

Key Ideas

- Simple deterministic rules can lead to unpredictable, chaotic behavior.
- Chaos theory explains real-world phenomena: weather, finance, biology.
- The logistic map models population growth, but also period-doubling and chaos.
- It's a simple gateway into understanding how complexity emerges from simplicity.

Model Definition

Logistic Map Formula

$$x_{n+1} = rx_n(1 - x_n)$$

Terms

- x_n: population at step n
- r: growth rate parameter
- Behavior depends heavily on r

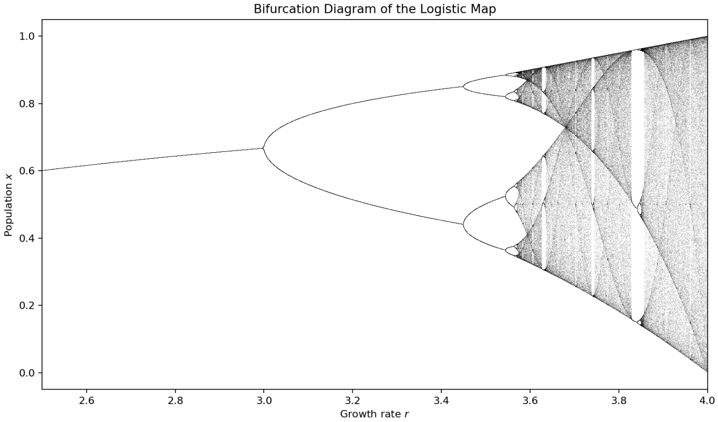
Computational Methods

Implementation Overview

- Implemented map in Python with NumPy and Matplotlib
- Generated:
  - Bifurcation diagrams (full and zoomed)
  - Sensitivity plots for close initial conditions
  - Feigenbaum constant estimates
- Used Jupyter for reproducible analysis

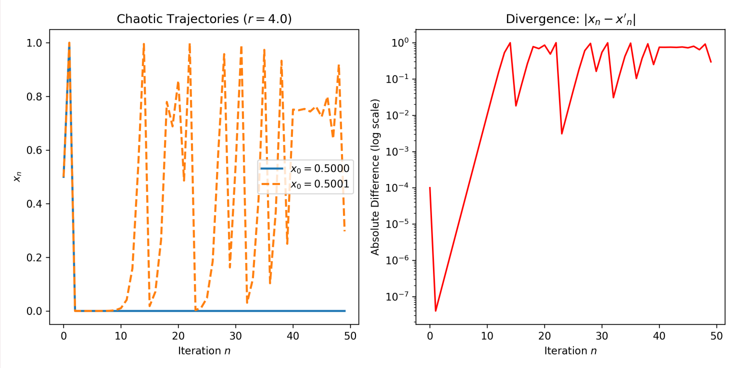
Bifurcation Diagram

Diagram



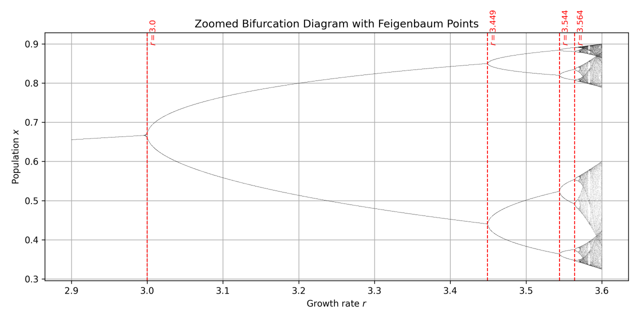
Sensitivity to Initial Conditions

Visualizing Divergence



Feigenbaum Constant Estimation

Zoomed Bifurcation with Annotations



Observation

- Estimated $\delta_2 \approx 4.7263$, $\delta_3 \approx 4.7500$
- Close to theoretical value: $\delta \approx 4.669$

Summary

Key Points

- Visualized how chaos emerges in the logistic map
- Demonstrated sensitive dependence and period-doubling
- Estimated a universal constant with simple code

Next Steps

Future Work

- Automate bifurcation detection
- Compute Lyapunov exponents
- Explore other chaotic maps (tent, sine)

Thank You

Questions?

Thank you for your attention.