

Econ 7330, Problem Set 3

October 11, 2019

This is due at the beginning of section Friday, 10/18. Feel free to work in groups, but write your solutions individually.

1. Let continuous random variable T represent the lifetime of any object. T could represent the lifetime of an individual, or the length of an unemployment spell, or the amount of time buyers and sellers have to wait before meeting with each other and getting the opportunity to make a transaction. Let f and F represent the p.d.f. and c.d.f. of T respectively.
 - a) The function $S(t) = P(T > t)$ is known as the *survival function*. What is $S(t)$ in terms of f and/or F ? Explain, in words, why this is called a survival function.
 - b) A very useful concept in applied microeconomics is the *hazard function* $\lambda(t) = \lim_{\delta \rightarrow 0} \frac{P(t \leq T < t + \delta | T \geq t)}{\delta}$. Show that $\lambda(t) = \frac{f(t)}{S(t)}$.
 - c) Show that $\lambda(t) = -\frac{S'(t)}{S(t)}$.
 - d) Show that for any continuous and differentiable function f , $\frac{f'(x)}{f(x)} = \frac{d}{dx} \log f(x)$. Explain in words what $\frac{f'(x)}{f(x)}$ represents. This is an important concept that you will encounter repeatedly in macroeconomics (and microeconomics).
 - e) Show that $\lambda(t) = -\frac{d}{dt} \log(S(t))$.
 - f) Explain in words what this function represents. (Think hard about this. It may help to use the definition of λ from part (b), or it may help to use insights from the remainder of the problem.)
 - g) If $T \sim U(0, \alpha)$, derive the hazard function.
2. Here you will play with Chebyshev's inequality a little bit.
 - a) Compute the upper bound for $P(|X - \mu_X| \geq k\sigma_X)$ for any random variable X .
 - b) Compute $P(|X - \mu_X| \geq k\sigma_X)$ for $X \sim U(0, 1)$. (Hint: What is μ_X ? What is σ_X ?)

- c) Order your answers from (a) and (b) for $k = 1$ and comment briefly.
3. Consider the joint pdf $f_{XY}(x, y) = \alpha x + \beta y$ for $x \in [0, 1]$, $y \in [0, 1]$.
- What is the marginal pdf of X ? Of Y ? Of $X|Y$?
 - What restriction on α and β must hold in order for this to be a joint pdf?
 - What is $P\left(X < \frac{\beta}{\alpha}Y\right)$?
 - What is $P\left(X > \sqrt{Y}\right)$?
 - What is $P\left(\sqrt{X - Y} > \frac{2}{3}\right)$?
4. Approximately 36,750 people died in car accidents in the US in 2018, and the 2018 population of the US was approximately 327.2 million. Suppose all cars have one driver and no passengers. Let D be the event that a person drives drunk, and let X be the event that a person dies in a car accident.
- In terms of the events and their complements, write out the treatment effect of drunk driving on death.
 - Your friend at the coroner's office tells you that 45% of dead drivers have alcohol in their blood. What other single piece of information would you need to calculate the effect of driving drunk on death? How could you obtain that information (2 sentences max)? Using your intuition and 15 minutes searching the internet, find a plausible value for it and compute the treatment effect.