Econ 7330, Problem Set 4

November 5, 2019

This is due by on Friday, 11/8 at the beginning of section. Feel free to work in groups, but write your solutions individually.

1. Let $X_1 ... X_n$ be a random sample with pdf $f(x) = \frac{\theta}{x^2}$ for $0 < \theta \leq x < \infty$.
    a) Derive the method of moments estimator for θ.
    b) Derive the maximum likelihood estimator for θ.
2. $f(x)$ is a differentiable function of x for which $f'' < 0$.
    a) Let $g()$ be a monotonically increasing, differentiable function. Using calculus, prove that $\arg\max g(f(x)) = \arg\max f(x)$.
    b) Why does $g()$ have to be monotonically increasing? What if it is monotonically decreasing? Where does the proof go wrong?
    c) Now, suppose $f(x)$ is not necessarily differentiable, but $\arg\max f(x)$ exists, and suppose that $g()$ is a monotincally increasing, non-differentiable function. Prove that $\arg\max g(f(x)) = \arg\max f(x)$.
    d) Given a random sample $X_1 ... X_n$ from a population with pdf $f(x)$, argue that maximizing the likelihood function $L(\theta|x)$ is equivalent to maximizing the log likelihood function $\log L(\theta|x)$.
3. Let $X_1 ... X_n$ be iid $N(\mu, 1)$.
    a) Derive the method of moments estimator for μ.
    b) Derive the maximum likelihood estimator for μ.
    c) Now suppose that $\mu \geq 0$ by assumption. Derive the maximum likelihood estimator for μ. (Hint: How does the likelihood function behave the in the restricted domain $\mu \geq 0$?)