Econ 7330, Problem Set 2

September 11, 2019

This is due at 9:30AM on Tuesday, 9/24. Feel free to work in groups, but write your solutions individually.

1. Consider the sample space $S = \{x | 2x \in \mathbb{Z}_{++}\}$ (S is the space of all numbers x such that $2x$ is a positive integer).
   (a) Describe in words an experiment that this sample space would correspond to.
   (b) What are the 4 smallest outcomes in S?
   (c) Consider the events $A = \{x | x \in \mathbb{Z}_{++}\}$ and $B = \{x | x/2 \in \mathbb{Z}_{++}\}$. Is $A \subset B$? Is $B \subset A$? What is $A \cup B$? What is $A \cap B$? What is A^c? Are A and B disjoint?
   (d) Consider the collection of events $C_i = \{i, i + 1/2\}$ for $i = 1, 2, ...$ Are the C_i pairwise disjoint? If so, do they form a partition of S?
2. Consider the sample space $S = \{x | x \in \mathbb{Z}\}$. Construct three examples of σ-algebras on S.
3. The king and queen are growing older and want their daughter to marry. There are N local candidates lined up (randomly) outside the palace. As the princess' learned adviser, you can objectively rank all of the candidates that you see (but only the ones that you see). Each one enters the palace one at a time, and the princess decides whether to not to marry the candidate. If she says yes, the wedding is held, and the remaining candidates are never seen. If she says no, the candidate is fed to a lion, and the next candidate enters the castle. The princess only marries the n^{th} candidate if they are better than all previous candidates (condition A).
   (a) Suppose the princess marries the m^{th} candidate. What is the probability that she marries the best of all N candidates?
   (b) Suppose the princess cannot marry the first candidate, but she then commits to marrying the next candidate who satisfies condition A. What is the probability that she never marries?
   (c) Now suppose the princess cannot marry the first candidate, but she then commits to marrying the next candidate who is better than at least one of the ones that she has seen before (condition B). What is the probability that she never marries?
   (d) Explain in words (no computation, 5 sentences max). Do you think that the princess is being too picky or not picky enough when using condition A? How about condition B? What makes a stopping rule "good"? What do you think a "better" stopping rule would be?

4. There are two events A and B with $P(A) > 0$ and $P(B) > 0$. You are to prove that they cannot be both mutually exclusive (i.e., $P(A|B) = 0$, $P(B|A) = 0$) and independent.

   (a) Write down the two logical statements that are necessary and sufficient for this proof. (Hint: they should be of the form "if x then y")

   (b) Prove both statements.

5. Let X be a continuous random variable with pdf f and cdf F. Define the function

$$g(x) = \begin{cases} \frac{f(x)}{1-F(k)} & x \geq k \\ 0 & x < k \end{cases} \tag{1}$$

   where k is a constant. Assume that $F(k) < 1$.

   (a) *2 pts.* Verify that g is a pdf.

   (b) *2 pts.* Suppose that x is income and k is the poverty line. In one sentence, what does $g(\cdot)$ represent?