Module 9: Lecture 26
Miss-Distance Analysis of RTPN

Keywords. RTPN, Miss-distance, Capture region

10.4.2 Miss distance analysis for RTPN

Starting with the equation,

$$V_\theta^2 - (N-1)V_R^2 = k = V_{\theta_0}^2 - (N-1)V_{R0}^2$$
$$\Rightarrow \quad R\ddot{R} - (N-1)\dot{R}^2 = k \tag{10.57}$$

As before

$$\ddot{R} = \dot{V}_R = \frac{dV_R}{dt} = \frac{dV_R}{dR}\frac{dR}{dt} = V_R\frac{dV_R}{dR} \tag{10.58}$$

Substituting the above in (10.57), we get

$$RV_R\frac{dV_R}{dR} - (N-1)V_R^2 = k \tag{10.59}$$

Letting $\hat{k} = N-1$ and separating the variables, we get

$$\frac{dR}{R} = \frac{V_R}{k+(N-1)V_R^2}dV_R = \frac{V_R}{k+\hat{k}V_R^2}dV_R \tag{10.60}$$

Assuming that $N \neq 1 \Rightarrow k \neq 0$, the above equation may be integrated to yield,

$$\ln R + c_1 = \frac{1}{2\hat{k}}\ln\left(k+\hat{k}V_R^2\right) \tag{10.61}$$

where,

$$c_1 = \frac{1}{2\hat{k}}\ln\left(k+\hat{k}V_{R0}^2\right) - \ln R_0 \tag{10.62}$$

Substituting, we get,

$$\ln\left(\frac{R}{R_0}\right) = \frac{1}{2\hat{k}}\ln\left(\frac{k+\hat{k}V_R^2}{k+\hat{k}V_{R0}^2}\right)$$
$$\Rightarrow \quad \left(\frac{R}{R_0}\right)^{2\hat{k}} = \frac{k+\hat{k}V_R^2}{k+\hat{k}V_{R0}^2} = \frac{k+\hat{k}V_R^2}{V_{\theta 0}^2}$$
$$\Rightarrow \quad V_{\theta 0}^2 + \hat{k}\left(V_R^2 - V_{R0}^2\right) = \left(\frac{R}{R_0}\right)^{2\hat{k}} V_{\theta 0}^2$$
$$\Rightarrow \quad \hat{k}\left(V_R^2 - V_{R0}^2\right) = \left[\left(\frac{R}{R_0}\right)^{2\hat{k}} - 1\right] V_{\theta 0}^2 \tag{10.63}$$

When $N = 1 \Rightarrow \hat{k} = 0$ (10.60) can be integrated to yield,

$$\ln R + c_2 = \frac{1}{2k} V_R^2 \tag{10.64}$$

with

$$c_2 = \frac{1}{2k} V_{R0}^2 - \ln R_0, \quad k = V_{\theta_0}^2 - (N-1) V_{R0}^2 = V_{\theta_0}^2 \tag{10.65}$$

Substituting which, we get,

$$\begin{aligned} \ln\left(\frac{R}{R_0}\right) &= \frac{1}{2k}\left(V_R^2 - V_{R0}^2\right) \\ \Rightarrow \quad \left(V_R^2 - V_{R0}^2\right) &= 2V_{\theta 0}^2 \ln\left(\frac{R}{R_0}\right) \end{aligned} \tag{10.66}$$

Now, putting the above results together in the following two cases:

Case 1: $N \neq 1 \Rightarrow \hat{k} \neq 0$.

$$\hat{k}\left(V_R^2 - V_{R0}^2\right) = \left[\left(\frac{R}{R_0}\right)^{2\hat{k}} - 1\right] V_{\theta 0}^2 \tag{10.67}$$

Now, at $R = R_m$, we have $V_R = 0$. Then,

$$-\hat{k} V_{R0}^2 = \left[\left(\frac{R_m}{R_0}\right)^{2\hat{k}} - 1\right] V_{\theta 0}^2 \tag{10.68}$$

$$\Rightarrow \quad \left(\frac{R_m}{R_0}\right)^{2\hat{k}} = 1 - \frac{\hat{k} V_{R0}^2}{V_{\theta 0}^2} \tag{10.69}$$

So, given the initial condition, this equation can be used to compute what the miss-distance would be when the initial condition is in the negative V_{R0} region. Likewise, given R_m/R_0, this equation can also be used to define the region for which this miss-distance is achieved as follows:

Since $R_m/R_0 < 1$, for $N > 1$ or $\hat{k} > 0$, we have $(R_m/R_0)^{2\hat{k}} - 1 < 0$.

Similarly, for $N < 1$ or $\hat{k} < 0$, we have $(R_m/R_0)^{2\hat{k}} - 1 > 0$.

Thus, in both cases the boundary of the capture region is given by (10.68) or (10.69).

The capture region itself will be given by,

$$
\begin{aligned}
|V_{\theta 0}| &< \left[\frac{\hat{k}}{1 - \left(\frac{R_m}{R_0}\right)^{2\hat{k}}} \right]^{1/2} (-V_{R0}), \quad V_{R0} < 0 \\
\Rightarrow \quad |V_{\theta 0}| &< \frac{\sqrt{N-1}}{\sqrt{1 - \left(\frac{R_m}{R_0}\right)^{2(N-1)}}} (-V_{R0}), \quad V_{R0} < 0
\end{aligned}
\tag{10.70}
$$

Case 2: $N = 1 \Rightarrow \hat{k} = 0$.

We have

$$
\left(V_R^2 - V_{R0}^2\right) = 2V_{\theta 0}^2 \ln\left(\frac{R}{R_0}\right) \tag{10.71}
$$

When miss-distance occurs, $R = R_m$ and $V_R = 0$, and so,

$$
-V_{R0}^2 = 2V_{\theta 0}^2 \ln\left(\frac{R}{R_0}\right) \tag{10.72}
$$

which gives the boundary of the capture region, which itself is given by,

$$
|V_{\theta 0}| < \frac{1}{\sqrt{-2\ln\left(\frac{R_m}{R_0}\right)}} (-V_{R0}), \quad V_{R0} < 0 \tag{10.73}
$$

The capture region is shown in Figure 10.18.

To compare with the capture region of the original TPN, substitute

$$
N = -\frac{c}{V_{R0}} \tag{10.74}
$$

then

$$
a_M = -\left(-\frac{c}{V_{R0}}\right) V_R \dot{\theta} \tag{10.75}
$$

When $N \neq 1 \Rightarrow c \neq -V_{R0}$, we can use (10.69) to obtain,

$$
\begin{aligned}
\left(\frac{R_m}{R_0}\right)^{2(-c/V_{R0}-1)} &= 1 - \frac{(-c/V_{R0} - 1)V_{R0}^2}{V_{\theta 0}^2} \\
\Rightarrow \quad \left(\frac{R_m}{R_0}\right)^{2(-c/V_{R0}-1)} &= 1 - \frac{\left(-cV_{R0} - V_{R0}^2\right)}{V_{\theta 0}^2}
\end{aligned}
\tag{10.76}
$$

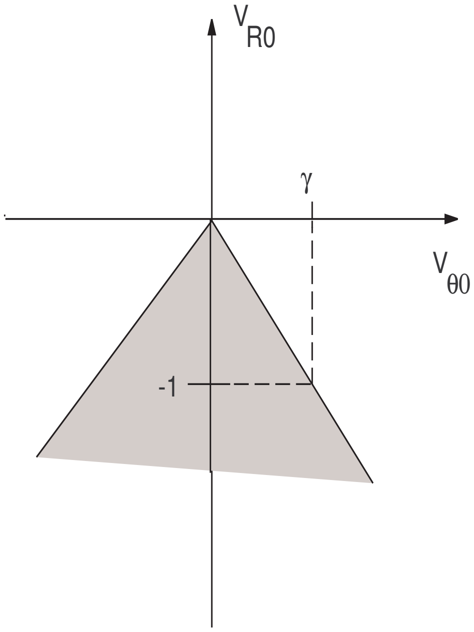


Figure 10.18: *The capture region for RTPN with non-zero miss-distance; When* $N \neq 1,\ \gamma = \sqrt{(N-1)/[1-(R_m/R_0)^{2(N-1)}]}$*; When* $N = 1,\ \gamma = \sqrt{1/[-2\ln(R_m/R_0)]}$

Let

$$p = 1 - \left(\frac{R_m}{R_0}\right)^{2(-c/V_{R0}-1)} \tag{10.77}$$

Then we have,

$$pV_{\theta 0}^2 + V_{R0}^2 + cV_{R0} = 0 \tag{10.78}$$

$$\Rightarrow \quad \left(V_{R0} + \frac{c}{2}\right)^2 + pV_{\theta 0}^2 = \left(\frac{c}{2}\right)^2 \tag{10.79}$$

The above equation forms the boundary of the capture region. The capture region itself is given by,

$$\left(V_{R0} + \frac{c}{2}\right)^2 + pV_{\theta 0}^2 < \left(\frac{c}{2}\right)^2 \tag{10.80}$$

for $V_{R0} \neq -c$. For $V_{R0} = -c$, we have,

$$|V_{\theta 0}| < \frac{c}{\sqrt{-2\ln\left(\frac{R_m}{R0}\right)}} \tag{10.81}$$

The capture region is shown in Figure 10.19.

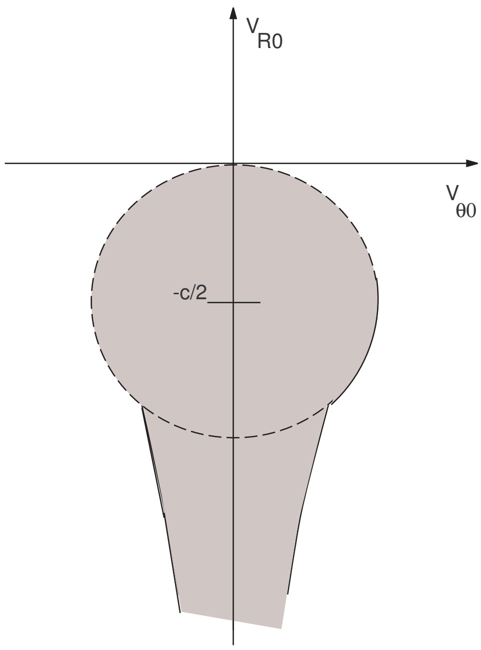


Figure 10.19: *The capture region for modified RTPN with non-zero miss-distance*

10.5 Concluding Remarks

The results discussed here covers many of the important results available in the literature on the analytical evaluation of TPN guidance law in its original form.

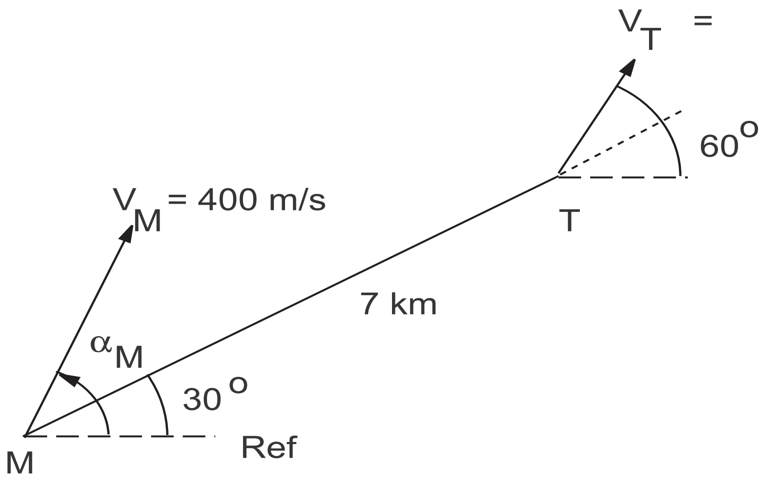
A realistic version of TPN is considered in which the time-varying nature of the closing velocity is taken into account. This shows some drastic performance reduction in the capturability of the TPN laws. These are perhaps the most convincing arguments against using TPN as a guidance law.

Assignment

1. **TPN.** Consider an initial geometry as given below. Assume a value of V_T lying between 0.5 and 0.7 of the value of V_M.

   (a) The guidance law is given by $a_M = c\dot{\theta}$ with $c = -3V_{R0}$.

   – Find the values of α_{M0} for which capture is possible.



- Plot the corresponding initial conditions in the $(V_{\theta 0}, V_{R0})$-space.
- Compute the interception point for each such point in the capture region and plot α_{M0} vs. t_f.

(b) Simulate the missile target engagement for the above problem and for any two initial values of α_{M0} (of which one is inside the capture region and the other outside). Plot the following:

- Missile-target trajectories
- Missile latax against time
- Trajectory in (V_θ, V_R)-space

(c) Same as (b) but assume that the target executes a maneuver with a_T = 30 m/s^2 in the clockwise direction.

(d) Compute and plot the capture region in the $(V_{\theta 0}, V_{R0})$-space for the case when capture is defined for a miss-distance less than 20 percent of the initial separation, and $a_M = c\dot{\theta}$ with $c = 300$. Note that only V_M and V_T are as given above. All other parameters are free.

2. **RTPN.**

Do exactly the same as before, but with $a_M = NV_R\dot{\theta}$, with $N = 3$.

For (d) assume $a_M = -(c/V_{R0})V_R\dot{\theta}$ with $c = 300$.

References

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2. *P.-J. Yuan and J.-S. Chern*: Solutions of true proportional navigation for maneuvering and non-maneuvering targets, *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 1, January-February 1992, pp. 268-271.