# Module 6: Lecture 18
# The Capture Region; Implementation

**Keywords.** Deviated pursuit; Capture region; Time of interception, Latax history, Implementation

### 7.2.3 The capture region

Based on the above discussion we can now identify the *capture region* of the deviated pursuit guidance law. Note that even in this case, if the initial geometry does not satisfy the collision triangle condition (i.e., the initial point is not on the negative $V_R$ axis) the capture is possible if and only if $V_T < V_M$. In Figure 7.11(a) we show the capture region for the deviated pursuit guidance law for a fixed $\delta > 0$. The figure shows that for a fixed $\delta$ the capture region for the deviated pursuit guidance law is of the same size as the pure pursuit guidance law. But the capture circle is now rotated by an angle $\delta$ clockwise. Another point to note is that a portion of the positive $V_R$ region has also become a part of the capture region. In this respect the deviated pursuit guidance law performs better than the pure pursuit guidance law.

Since the angle $\delta$ is a guidance parameter that can be selected by the designer of the guidance algorithm, we can see that by selecting the value of $\delta$ differently we can demarcate different regions as capture region. If we allow $\delta$ to vary between $-\pi/2$ and $+\pi/2$ the total capture region will be the union of all the individual capture regions for each $\delta$ and is shown in Figure 7.11(b). Note that for $\delta < 0$ the capture region is obtained by rotating the capture region for the pure pursuit guidance law ($\delta = 0^o$) in the anti-clockwise direction. Thus, if we consider $\delta$ to be a freely selected guidance parameter then the capture region expands considerably.

*Can you guess why I did not consider $\delta > \pi/2$ or $\delta < -\pi/2$ to expand the capture region even further?*

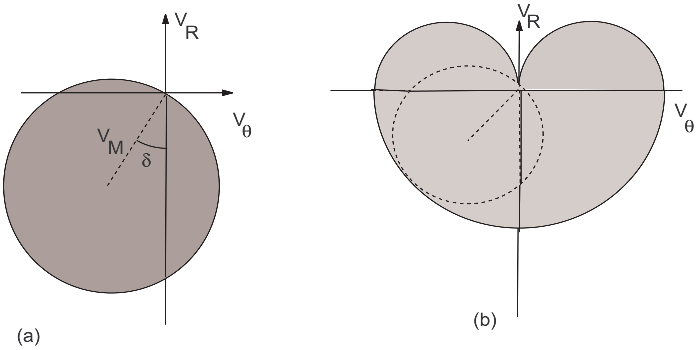



Figure 7.11: *Capture region for the deviated pursuit guidance law* (a) *Fixed* $\delta$ (b) $-\pi/2 < \delta < \pi/2$

### 7.2.4 Time of interception

To obtain the interception time when the initial state is within the capture region, let us consider (7.28).

$$\begin{aligned} & (V_R + V_M\cos\delta)^2 + (V_\theta + V_M\sin\delta)^2 = V_T^2 \\ \Rightarrow\ & V_R^2 + V_M^2 + 2V_MV_R\cos\delta + V_\theta^2 + 2V_MV_\theta\sin\delta = V_T^2 \\ \Rightarrow\ & V_R^2 + 2V_MV_R\cos\delta + V_\theta(V_\theta + V_M\sin\delta) + V_MV_\theta\sin\delta = V_T^2 - V_M^2 \\ \Rightarrow\ & \dot{R}^2 + R\ddot{R} + 2V_M\cos\delta\dot{R} + V_MV_\theta\sin\delta = V_T^2 - V_M^2 \end{aligned} \tag{7.39}$$

Note that in the above equation we used (7.31). From (7.32),

$$\begin{aligned} R\dot{V}_\theta &= -V_\theta V_R - V_\theta V_M\cos\delta \\ &\Rightarrow\ V_\theta V_M = \frac{R\dot{V}_\theta + V_\theta V_R}{-\cos\delta} \\ &\Rightarrow\ V_\theta V_M\sin\delta = \left(R\dot{V}_\theta + V_\theta V_R\right)(-\tan\delta) = \frac{d}{dt}(RV_\theta)(-\tan\delta) \end{aligned}$$

Substituting the above in (7.39), we obtain,

$$\frac{d}{dt}(RV_R) + 2V_M\cos\delta\frac{d}{dt}(R) - \tan\delta\frac{d}{dt}(RV_\theta) = V_T^2 - V_M^2$$

which, on integration, yields

$$R(V_R + 2V_M\cos\delta - V_\theta\tan\delta) = \left(V_T^2 - V_M^2\right)t + c \tag{7.40}$$

where,

$$c = R_0\left(V_{R0} + 2V_M\cos\delta - V_{\theta 0}\tan\delta\right)$$

If interception occurs, then at $t = t_f$ we have $R = 0$, which yields

$$t_f = \frac{-c}{V_T^2 - V_M^2} = \frac{R_0\,(V_{R0} + 2V_M \cos\delta - V_{\theta 0} \tan\delta)}{V_M^2 - V_T^2} \tag{7.41}$$

### 7.2.5 Lateral acceleration history

Exactly as in pure pursuit case, it is possible to derive similar equations for the deviated pursuit case. We omit the details which can be found in Locke (1955). It turns out that if interception occurs the terminal value of $a_M$ is given by,

$$\begin{aligned} 1 < \nu < 2 \quad & a_M \rightarrow \text{A finite value} \\ \nu \geq 2 \quad & a_M \rightarrow \infty \end{aligned} \tag{7.42}$$

## 7.3 Implementation

Here, we start with an assumption that the missile is initially on a pursuit course. but what happens when the missile points in a direction different from the pursuit geometry (pure or deviated) initially? We can examine two alternatives:

1. Missile applies the maximum lateral acceleration till it is on a pursuit course and then applies the pursuit lateral acceleration.
2. If there is no bound on the missile lateral acceleration, then the missile can turn instantaneously and then apply the pursuit acceleration.

However, note that both these alternatives are open-loop in nature and requires an inordinate amount of computations to make them feasible. Thus, errors in measurements, and mismatch between the missile flight angle and the LOS angle, will lead to large miss-distances.Moreover, even if a closed-loop implementation is devised, based upon continuous measurements of the states, latax oscillations will be caused by the high demand on latax. This is bound to occur due to the dynamics of the system.

A practical implementation of the pure pursuit guidance law would be to use a feedback law as follows:

$$a_M = -K(\alpha_M - \theta) \tag{7.43}$$

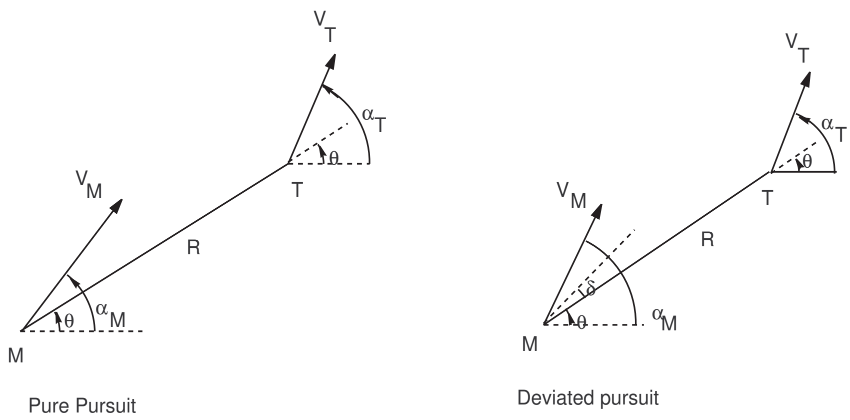



Figure 7.12: Pursuit guidance law: Implementation geometry

with $K > 0$. This is with reference to the Figure 7.12. However, this is not enough. For pursuit to be effective, we must have,

$$\dot{\alpha}_M = \dot{\theta} \tag{7.44}$$

Since

$$\dot{\alpha}_M = \frac{a_M}{V_M} \tag{7.45}$$

From which we get,

$$a_M = V_M \dot{\theta} \tag{7.46}$$

Putting (7.43) and (7.46) together, an implementable pure pursuit guidance law would have a form,

$$a_M = V_M \dot{\theta} - K(\alpha_M - \theta) \tag{7.47}$$

The first term equalizes the missile flight path angle rate with the LOS rate and the second term generates a latax proportional to the angular difference between the flight path and LOS.

We will have occasion to refer to this guidance law again when we discuss the proportional navigation guidance law.

Similarly, the deviated pursuit guidance law may be implemented as,

$$a_M = V_M \dot{\theta} - K(\alpha_M - \theta - \delta) \tag{7.48}$$

In both these cases, the choice of $K$ is important.

## 7.4 Concluding Remarks

In this chapter we derived some analytical results about the pursuit guidance law for both pure and deviated versions. In both cases capture is shown to be guaranteed when the missile velocity is higher than the target velocity. It was shown that the capture region for pure pursuit and deviated pursuit is a circle of radius equal to the missile velocity $V_M$. However, if the angle of deviation $\delta$ is considered as a flexible guidance parameter for deviated pursuit, then the deviated pursuit guidance law has a much larger capture region.

The latax history showed that the terminal latax is very high when the missile has more than double the target velocity, but is bounded by a finite value when the missile velocity is more than the target velocity but less than twice the target velocity.

*Questions*

1. Show that the trajectory in the relative velocity space when the missile uses a deviated pursuit guidance law right from the beginning of the engagement, is a circle. Determine the direction of movement of a point on this trajectory.

2. Obtain the capture region of the deviated pursuit guidance law in the relative velocity space.

3. **PART A: Deviated Pursuit,** $\delta = 25^o$**;** $\alpha_T \in [0^o,\ 180^o]$

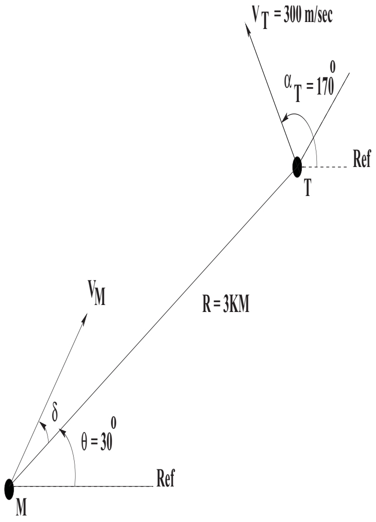
   A1. Integrate the equations of motion and plot $R$, $\theta$, $V_\theta$, $V_R$, and $a_M$ against time for $\nu = 0.9, 1.5, 3$. Note that $\nu = V_M/V_T$.

   A2. Also plot the missile and target trajectories for each case.

   A3. Check if interception occurs. If yes, then find $t_f$. If no then find $t_{\mathbf{miss}}$ and $R_{\mathbf{miss}}$.

   A4. Plot $(V_\theta, V_R)$ in $(V_\theta, V_R)$-space.

   A5. Interpret the simulation results in the light of the analytical results discussed in the class.




**PART B: Implementation**

B1. Use the implementation of deviated pursuit, with $\delta = 8^o$, as

$$a_M = V_M \dot{\theta} - k(\alpha_M - \theta - \delta)$$

when, in the above figure, the initial missile velocity vector lags the LOS by $10^o$.

Plot $R$, $\theta$, $V_\theta$, $V_R$, $\alpha_M$ and $a_M$ against time for $\nu = 0.9, 1.5, 3$; $k = 1, 5, 10$.

Plot $(V_\theta, V_R)$ in $(V_\theta, V_R)$-space.

Plot the missile and target trajectories for each case.

**PART C: Comparisons**

Compare the results obtained for pure pursuit in the last section with the results obtained for deviated pursuit.

# *References*


1. *C.-F. Lin: Modern Navigation, Guidance, and Control Processing*, Prentice Hall, Englewood Cliffs, NJ, 1991.

2. A.S. LOCKE: *Guidance*, D. Van Nostrand Co., 1955.