VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ
Fakulta informačních technologií

ELEKTRONIKA PRO INFORMAČNÍ TECHNOLOGIE

2016/2017

SEMESTRÁLNÍ PROJEKT

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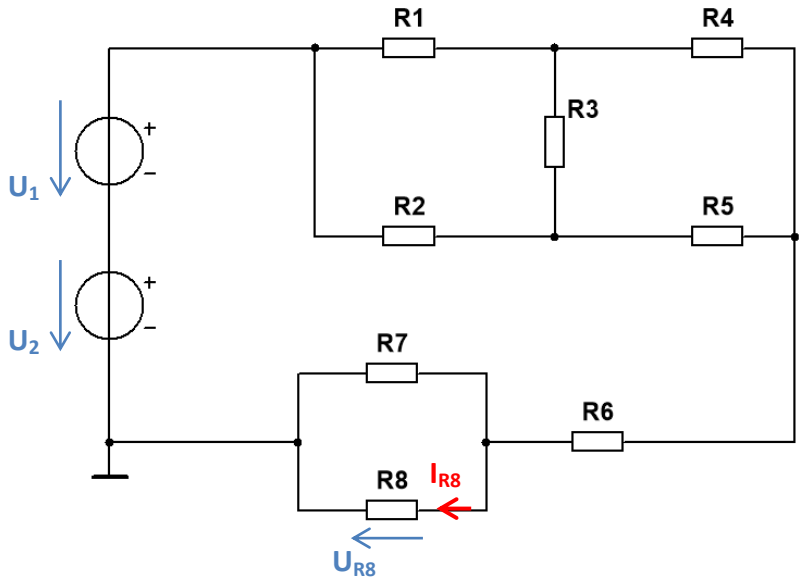
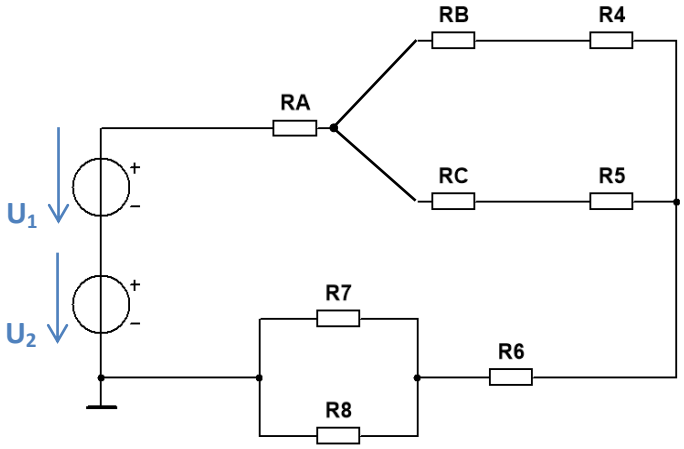
1. H

Stanovte napětí U_{R8} a proud I_{R8}. Použijte metodu postupného zjednodušování obvodu.

$U_1 = 135V$	$R_1 = 680\Omega$	$R_3 = 260\Omega$	$R_5 = 575\Omega$	$R_7 = 355\Omega$
$U_2 = 80V$	$R_2 = 600\Omega$	$R_4 = 310\Omega$	$R_6 = 870\Omega$	$R_8 = 265\Omega$

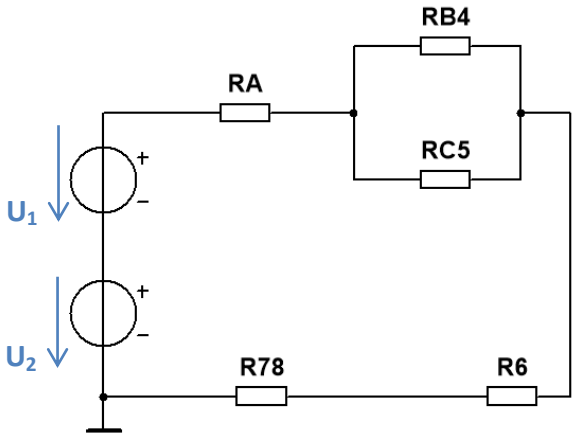




$$R_A = \frac{R_1 \times R_2}{R_1 + R_2 + R_3} = \frac{680\Omega \times 600\Omega}{680\Omega + 600\Omega + 260\Omega} = 264.9351\Omega$$

$$R_B = \frac{R_1 \times R_3}{R_1 + R_2 + R_3} = \frac{680\Omega \times 260\Omega}{680\Omega + 600\Omega + 260\Omega} = 114.8052\Omega$$

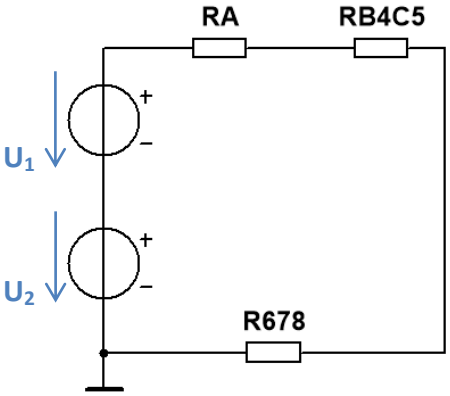
$$R_C = \frac{R_2 \times R_3}{R_1 + R_2 + R_3} = \frac{600\Omega \times 260\Omega}{680\Omega + 600\Omega + 260\Omega} = 101.2988\Omega$$



$$R_{B4} = R_B + R_4 = 114.8052\Omega + 310\Omega = 424.8052\Omega$$

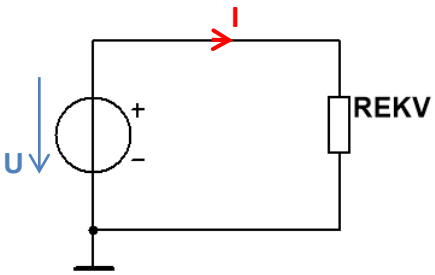
$$R_{C5} = R_C + R_5 = 101.2988\Omega + 575\Omega = 676.2988\Omega$$

$$R_{78} = \frac{R_7 \times R_8}{R_7 + R_8} = \frac{355\Omega \times 265\Omega}{355\Omega + 265\Omega} = 151.7339\Omega$$



$$R_{B4C5} = \frac{R_{B4} \times R_{C5}}{R_{B4} + R_{C5}} = \frac{424.8052\Omega \times 676.2988\Omega}{424.8052\Omega + 676.2988\Omega} = 260.9156\Omega$$

$$R_{678} = R_6 + R_{78} = 870\Omega + 151.7339\Omega = 1021.7339\Omega$$



$$U = U_1 + U_2 = 135V + 80V = 215V$$

$$R_{EKV} = R_A + R_{B4C5} + R_{678} = 264.9351\Omega + 260.9156\Omega + 1021.7339\Omega = \\ = 1547.5846\Omega$$

$$I = \frac{U}{R_{EKV}} = \frac{215V}{1547.5846\Omega} = 0.1389A$$

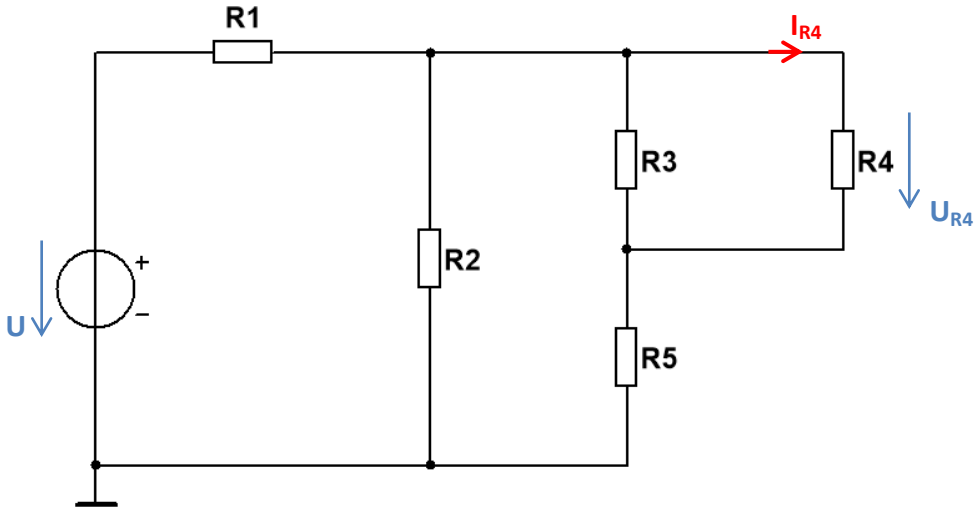
$$U_{R_{78}} = R_{78} \times I = 151.7339\Omega \times 0.1389A = 21.0798V \qquad U_{R_{78}} = U_{R_8}$$

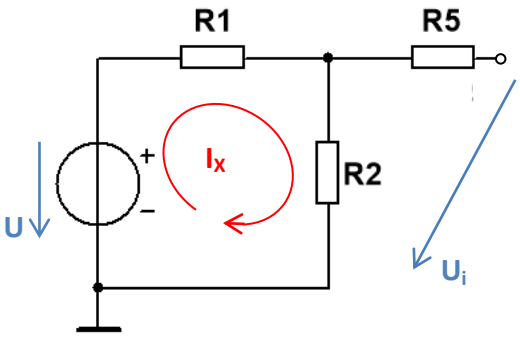
$$I_{R_8} = \frac{U_{R_{78}}}{R_8} = \frac{21.0798V}{265\Omega} = 79.5464mA$$

2. H

Stanovte napětí U_{R4} a proud I_{R4}. Použijte metodu Théveninovy věty.

U = 220V $R_2 = 580\Omega$ $R_4 = 560\Omega$
$R_1 = 360\Omega$ $R_3 = 205\Omega$ $R_5 = 350\Omega$

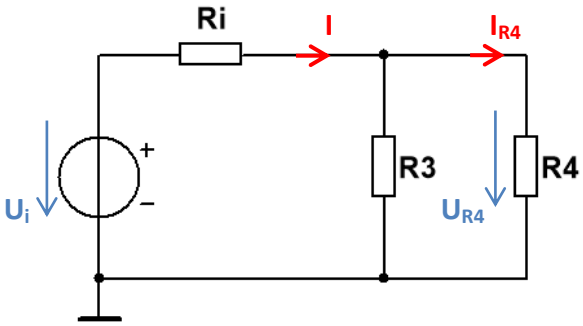




$$R_i = \frac{R_1 \times R_2}{R_1 + R_2} + R_5 = \frac{360\Omega \times 580\Omega}{360\Omega + 580\Omega} + 350\Omega = 572.1277\Omega$$

$$I_X = \frac{U}{R_1 + R_2} = \frac{220V}{360\Omega + 580\Omega} = 0.23404A$$

$$U_i = I_X \times R_i = 0.23404A \times 572.1277\Omega = 135.7447V$$



$$R_{34} = \frac{R_3 \times R_4}{R_3 + R_4} = \frac{205\Omega \times 560\Omega}{205\Omega + 560\Omega} = 150.0654\Omega$$

$$I = \frac{U_i}{R_i + R_{34}} = \frac{135.7447V}{572.1277\Omega + 150.0654\Omega} = 0.18796A$$

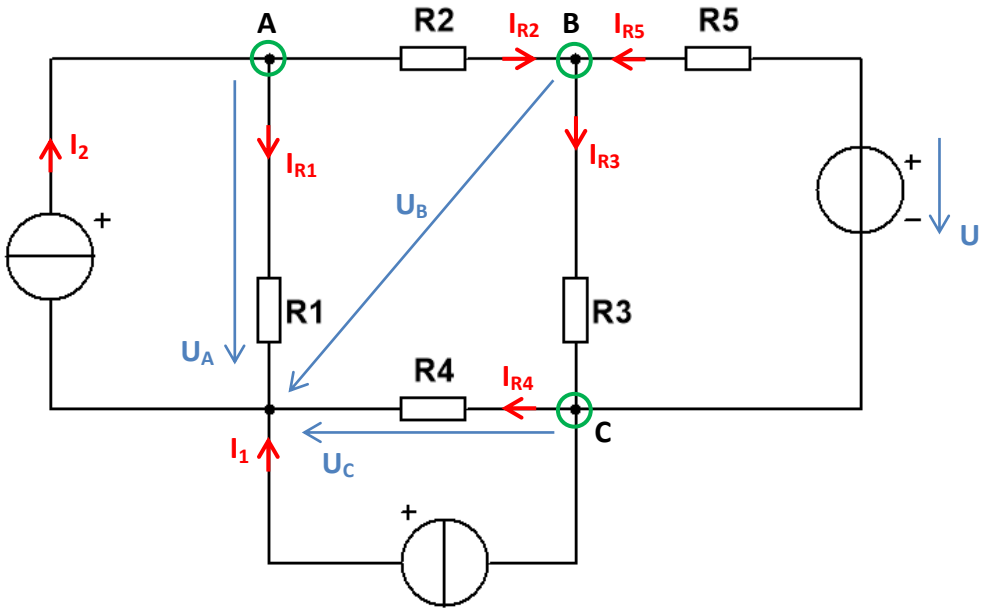
$$U_{R_4} = R_{34} \times I = 150.0654\Omega \times 0.18796A = 28.2066V$$

$$I_{R_4} = \frac{U_{R_4}}{R_4} = \frac{28.2066V}{560\Omega} = 50.3688mA$$

3. E

Stanovte napětí U_{R4} a proud I_{R4}. Použijte metodu uzlových napětí (U_A, U_B, U_C).

U = 135V	I_2 = 0.65A	R_2 = 42Ω	R_4 = 42Ω
I_1 = 0.55A	R_1 = 52Ω	R_3 = 52Ω	R_5 = 21Ω



Podle I. kirchhoffova zákona:

uzel A: $I_2 - I_{R_1} - I_{R_2} = 0$

uzel B: $I_{R_2} + I_{R_5} - I_{R_3} = 0$

uzel C: $I_{R_3} - I_{R_4} - I_{R_5} - I_1 = 0$

Rovnice pro jednotlivé proudy:

$$I_{R_1} = \frac{U_A}{R_1}$$

$$-U_A + R_2 \times I_{R_2} + U_B = 0 \Rightarrow I_{R_2} = \frac{U_A - U_B}{R_2}$$

$$-U_B + R_3 \times I_{R_3} + U_C = 0 \Rightarrow I_{R_3} = \frac{U_B - U_C}{R_3}$$

$$I_{R_4} = \frac{U_C}{R_4}$$

$$-U_C - U + R_5 \times I_{R_5} + U_B = 0 \Rightarrow I_{R_5} = \frac{U + U_C - U_B}{R_5}$$

$$I_2 - \frac{U_A}{R_1} - \frac{U_A - U_B}{R_2} = 0$$

$$\frac{U_A - U_B}{R_2} + \frac{U + U_C - U_B}{R_5} - \frac{U_B - U_C}{R_3} = 0$$

$$\frac{U_B - U_C}{R_3} - \frac{U_C}{R_4} - \frac{U + U_C - U_B}{R_5} - I_1 = 0$$

$$0.65 - \frac{U_A}{52} - \frac{U_A - U_B}{42} = 0 \qquad /\times 1092$$

$$\frac{U_A - U_B}{42} + \frac{135 + U_C - U_B}{21} - \frac{U_B - U_C}{52} = 0 \quad /\times\ 1092$$

$$\frac{U_B - U_C}{52} - \frac{U_C}{42} - \frac{135 + U_C - U_B}{21} - 0.55 = 0 \ /\times 1092$$

$$47U_A - 26U_B = 709.8$$

$$26U_A - 99U_B + 73U_C = -7020$$

$$-73U_B + 99U_C = -7620.6$$

$$\begin{matrix} U_A & U_B & U_C & \\ \end{matrix}$$
$$\left(\begin{array}{ccc|c} 47 & -26 & 0 & 709.8 \\ 26 & -99 & 73 & -7020 \\ 0 & -73 & 99 & -7620.6 \end{array}\right)$$

$$\begin{pmatrix} 47 & -26 & 0 \\ 26 & -99 & 73 \\ 0 & -73 & 99 \end{pmatrix} \begin{matrix} 47 & -26 \\ 26 & -99 \\ 0 & -73 \end{matrix} \qquad D = -143260$$

$$\begin{pmatrix} 47 & -26 & 709.8 \\ 26 & -99 & -7020 \\ 0 & -73 & -7620.6 \end{pmatrix} \begin{matrix} 47 & -26 \\ 26 & -99 \\ 0 & -73 \end{matrix} \qquad D_{U_C} = 4874305.8$$

$$U_C = \frac{D_{U_C}}{D} = \frac{4874305.8}{-143260} = -34.0242V$$

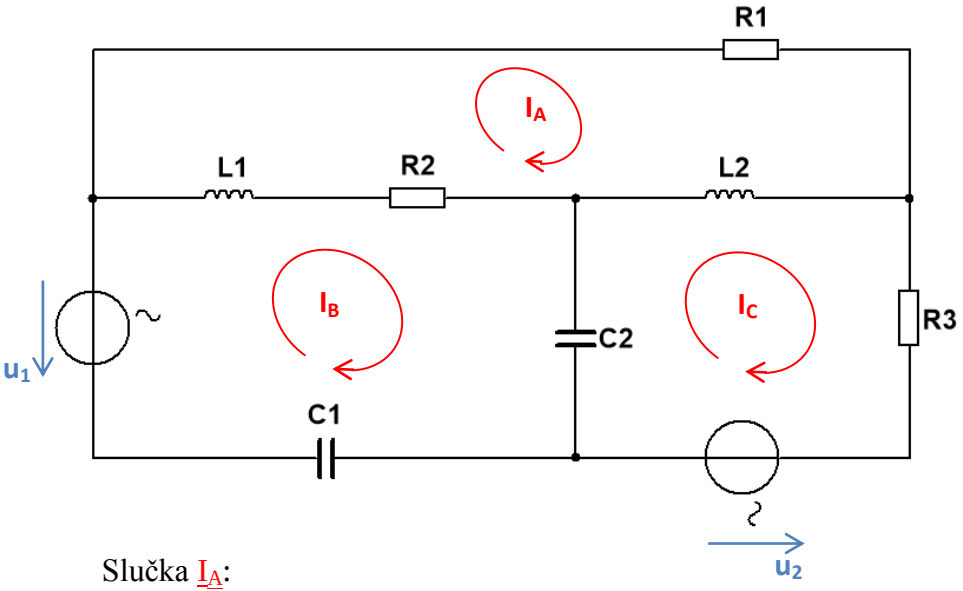
$$U_C = U_{R_4} = 34.0242V$$

$$I_{R_4} = \frac{U_{R_4}}{R_4} = \frac{34.0242V}{42\Omega} = 810.0998mA$$

4. H

Pro napájecí napětí platí: $u_1 = U_1 \cdot \sin(2\pi f\, t)$, $u_2 = U_2 \cdot \sin(2\pi f\, t)$. Ve vztahu pro napětí $u_{C1} = U_{C1} \cdot \sin(2\pi f\, t + \phi_{C1})$ určete $|U_{C1}|$ a ϕ_{C1}. Použijte metodu smyčkových proudů. Pozn: Pomocné "směry ˇšipek napájecích zdrojů platí pro speciální ˇčasový okamžik ($t = \pi\, 2\omega$)."

$U_1 = 65V$	$R_1 = 10\Omega$	$R_3 = 12\Omega$	$L_2 = 75mH$	$C_2 = 70\mu F$
$U_2 = 60V$	$R_2 = 10\Omega$	$L_1 = 160mH$	$C_1 = 155\mu F$	$f = 95Hz$



Slučka I_A:

$$R_1 I_A + X_{L_2}(I_A - I_C) + R_2(I_A - I_B) + X_{L_1}(I_A - I_B) = 0$$

$$R_1 I_A + j\omega L_2 I_A - j\omega L_2 I_C + R_2 I_A - R_2 I_B + j\omega L_1 I_A - j\omega L_1 I_B = 0$$

$$I_A(R_1 + R_2 + j\omega L_2 + j\omega L_1) - I_B(R_2 + j\omega L_1) - I_C j\omega L_2 = 0$$

Slučka I_B:

$$-X_{L_1}(I_A - I_B) - R_2(I_A - I_B) + X_{C_2}(I_B - I_C) + X_{C_1} I_B - U_1 = 0$$

$$j\omega L_1 I_B - j\omega L_1 I_A - R_2 I_A + R_2 I_B + \frac{1}{j\omega C_2} I_B - \frac{1}{j\omega C_2} I_C + \frac{1}{j\omega C_1} I_B - U_1 = 0$$

$$-I_A(j\omega L_1 + R_2) + I_B\left(j\omega L_1 + R_2 + \frac{1}{j\omega C_2} + \frac{1}{j\omega C_1}\right) - I_C \frac{1}{j\omega C_2} = U_1$$

Slučka I_C:

$$R_3 I_C - U_2 - X_{C_2}(I_B - I_C) - X_{L_2}(I_A - I_C) = 0$$

$$R_3 I_C - U_2 - \frac{1}{j\omega C_2} I_B + \frac{1}{j\omega C_2} I_C - j\omega L_2 I_A + j\omega L_2 I_C = 0$$

$$-I_A j\omega L_2 - I_B \frac{1}{j\omega C_2} + I_C\left(R_3 + \frac{1}{j\omega C_2} + j\omega L_2\right) = U_2$$

Sestava 3 rovnic I_A, I_B, I_C: $\omega = 2\pi f = 2\pi \times 95Hz = 190\pi\ rad/s$

$$I_A(R_1 + R_2 + j\omega L_2 + j\omega L_1) - I_B(R_2 + j\omega L_1) - I_C j\omega L_2 = 0$$

$$-I_A(j\omega L_1 + R_2) + I_B\left(j\omega L_1 + R_2 + \frac{1}{j\omega C_2} + \frac{1}{j\omega C_1}\right) - I_C\frac{1}{j\omega C_2} = U_1$$

$$-I_A j\omega L_2 - I_B\frac{1}{j\omega C_2} + I_C\left(R_3 + \frac{1}{j\omega C_2} + j\omega L_2\right) = U_2$$

$$I_A(10 + 10 + 190\pi \times 160m\,j + 190\pi \times 75m\,j) - I_B(10 + 190\pi \times 160m\,j) - I_C 190\pi \times 75m\,j = 0$$

$$-I_A(190\pi \times 160m\,j + 10) + I_B\left(190\pi \times 160m\,j + 10 + \frac{1}{190\pi \times 155\mu\,j} + \frac{1}{190\pi \times 70\mu\,j}\right) - I_C\frac{1}{190\pi \times 70\mu\,j} = 65$$

$$-I_A 190\pi \times 75m\,j - I_B\frac{1}{190\pi \times 70\mu\,j} + I_C\left(12 + \frac{1}{190\pi \times 70\mu\,j} + 190\pi \times 75m\,j\right) = 60$$

$$I_A(20 + 140.2721j) - I_B(10 + 95.5044j) - I_C 44.7677j = 0$$

$$-I_A(10 + 95.5044j) + I_B(10 + 60.7629j) + I_C 23.9331j = 65$$

$$-I_A 44.7677j + I_B 23.9331j + I_C(12 + 20.8346j) = 60$$

$$\begin{matrix} I_A & I_B & I_C & \\ \end{matrix}$$
$$\left(\begin{array}{ccc|c} 20 + 140.2721j & -10 - 95.5044j & -44.7677j & 0 \\ -10 - 95.5044j & 10 + 60.7629j & 23.9331j & 65 \\ -44.7677j & 23.9331j & 12 + 20.8346j & 60 \end{array}\right)$$

$$\left(\begin{array}{ccc} 20 + 140.2721j & -10 - 95.5044j & -44.7677j \\ -10 - 95.5044j & 10 + 60.7629j & 23.9331j \\ -44.7677j & 23.9331j & 12 + 20.8346j \end{array}\right)\begin{array}{cc} 20 + 140.2721j & -10 - 95.5044j \\ -10 - 95.5044j & 10 + 60.7629j \\ -44.7677j & 23.9331j \end{array}$$

$$D = 3693.1225 + 20504.2205j$$

$$\left(\begin{array}{ccc} 20 + 140.2721j & 0 & -44.7677j \\ -10 - 95.5044j & 65 & 23.9331j \\ -44.7677j & 60 & 12 + 20.8346j \end{array}\right)\begin{array}{cc} 20 + 140.2721j & 0 \\ -10 - 95.5044j & 65 \\ -44.7677j & 60 \end{array}$$

$$D_{I_B} = -99195.7664 + 134638.118j$$

$$I_B = \frac{D_{I_B}}{D} = \frac{-99195.7664 + 134638.118j}{3693.1225 + 20504.2205j} = 5.516 + 5.8313j$$

$$U_{C_1} = X_{C_1} \times I_B = \frac{1}{j\omega C_1} \times I_B = \frac{1}{190\pi \times 155 \times 10^{-6}j} \times 5.516 + 5.8313j = 63.0281 - 59.6195j$$

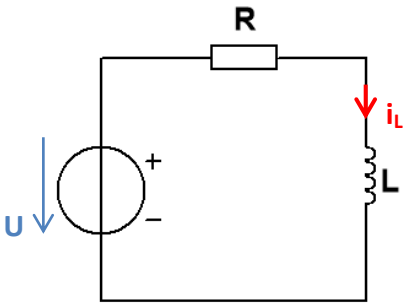
$$\left|U_{C_1}\right| = \sqrt{Re^2 + Im^2} = \sqrt{(63.0281)^2 + (-59.6195)^2} = 86.7584V$$

$$\varphi_{C_1} = arctan\left(\frac{Im(U_{C_1})}{Re(U_{C_1})}\right) = arctan\left(\frac{-59.6195}{63.0281}\right) = -43.4081° = -0.7576rad$$

5. H

Sestavte diferenciální rovnici popisující chovaní obvodu na obrázku, dále ji upravte dosazením hodnot parametrů. Vypočítejte analytické řešení $i_L = f(t)$. Proveďte kontrolu výpočtu dosazením do sestavené diferenciální rovnice.

U = 5V R = 40Ω
L = 50H $i_L(0) = 2A$



Platí: $i_L' = \frac{1}{L} u_L$

$$-U + R \times i_L + u_L = 0 \;\Rightarrow\; u_L = U - R \times i_L$$

Dosadíme u_L do předchozí rovnice:

$$i_L' = \frac{U - R \times i_L}{L}$$

$$i_L' = \frac{5 - 40 \times i_L}{50}$$

$$50 i_L' + 40 i_L = 5$$

$$50\lambda + 40 = 0 \;\Rightarrow\; \lambda = -0.8$$

$$i_L(t) = c(t) e^{\lambda t}$$

$$i_L(t) = c(t) e^{-0.8t}$$
$$i_L' = c'(t) e^{-0.8t} + c(t) e^{-0.8t} \times -0.8$$

$$50(c'(t) e^{-0.8t} + c(t) e^{-0.8t} \times -0.8) + 40 c(t) e^{-0.8t} = 5$$
$$50 c'(t) e^{-0.8t} - 40 c(t) e^{-0.8t} + 40 c(t) e^{-0.8t} = 5$$
$$50 c'(t) e^{-0.8t} = 5$$
$$c'(t) e^{-0.8t} = 0.1$$
$$c'(t) = 0.1 e^{0.8t}$$

$$\int c'(t)\, dt = \int 0.1 e^{0.8t}\, dt$$
$$c(t) + K_1 = \frac{0.1}{0.8} e^{0.8t} + K_2$$
$$c(t) = \frac{1}{8} e^{0.8t} + (K_2 - K_1)$$
$$c(t) = \frac{1}{8} e^{0.8t} + K$$

$$i_L(t) = c(t)e^{-0.8t}$$
$$i_L(t) = \left(\frac{1}{8}e^{0.8t} + K\right)e^{-0.8t}$$
$$i_L(t) = \frac{1}{8} + Ke^{-0.8t}$$

$$i_L(0) = 2$$

$$\frac{1}{8} + Ke^{0} = 2$$
$$K = 2 - \frac{1}{8} = \frac{15}{8}$$

$$i_L(t) = \frac{1}{8} + \frac{15}{8}e^{-0.8t} = \frac{1}{8}(1 + 15e^{-0.8t})$$

Kontrola výpočtu:

dif. rovnice: $50i_L' + 40i_L = 5$ $\qquad$ $i_L(0) = 2$

$$i_L(t) = \frac{1}{8} + \frac{15}{8}e^{-0.8t}$$
$$i_L'(t) = -0.8 \times \frac{15}{8}e^{-0.8t} = -\frac{3}{2}e^{-0.8t}$$

$$50 \times -\frac{3}{2}e^{-0.8t} + 40\left(\frac{1}{8} + \frac{15}{8}e^{-0.8t}\right) = 5$$
$$-75e^{-0.8t} + 5 + 75e^{-0.8t} = 5$$
$$5 = 5$$

1	2	3	4	5
H	H	E	H	H
U_{R8} = 21.0798V I_{R8} = 79.5464mA	U_{R4} = 28.2066V I_{R4} = 50.3688mA	U_{R4} = 34.0242V I_{R4} = 810.0998mA	$\lvert U_{C1} \rvert$ = 86.7584V ϕ_{C1} = -43.4081°/ -0.7576rad	$i_L(t) = \frac{1}{8}(1 + 15e^{-0.8t})$