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```
y = 1;
K = 0;
while (K <> n) do
{ y := y * n; K := K + 1; } ] c
```

* Assume $n = 4$

$$\frac{(m,e) \Rightarrow 0}{(m, \text{while } e \text{ do } c) \Rightarrow m} \qquad \frac{(m,e) \Rightarrow n \text{ nonzero } (m,c) \Rightarrow m' \quad (m', \text{while } e \text{ do } c) \Rightarrow m''}{(m, \text{while } e \text{ do } c) \Rightarrow m''}$$

$(m, K=0) \Rightarrow (K=0, y=1) \Rightarrow m'$

$(m', e) \Rightarrow (m', c) \Rightarrow (y=4, K=1)\ m''$

$(m'', e) \Rightarrow (m'', c) \Rightarrow (y=16, K=2)\ m'''$

$(m''', e) \Rightarrow (m''', c) \Rightarrow (y=64, K=3)\ m^4$

$(m^4, e) \Rightarrow (m^4, c) \Rightarrow (y=256, K=4)\ m^5$

$(m^5, e) \Rightarrow (m^5)$

Assume P $\quad \boxed{P = n > 0}$

```
y = 1;
K = 0;
```

* Assume $n = 4$

Assert Q $\quad \boxed{Q = y = n^K}$

```
while (K <> n) do
{ y = y * n
  K = K + 1
}
```

$\boxed{LI = K = n}$

$y = 1 * 4$	$y = 4$	n^K
$K = 0 + 1$	$K = 1$	$4^1 = 4$
$y = 4 * 4$	$y = 16$	$4^2 = 16$
$K = 1 + 1$	$K = 2$	$4^3 = 64$
$y = 16 * 4$	$y = 64$	$4^4 = 256$
$K = 2 + 1$	$K = 3$	
$y = 64 * 4$	$y = 256$	
$K = 3 + 1$	$K = 4$	

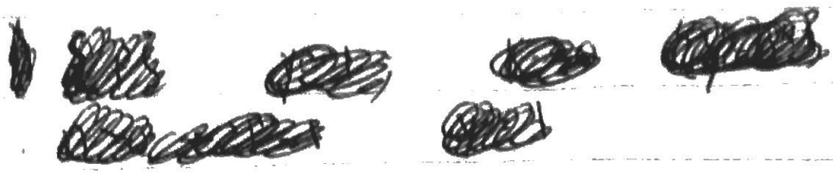
Assert R $\quad \boxed{R = y = n^n}$

n^n

$4^4 = 256$

Rules:

1) $\{P\}\ S\ \{Q\}$
2) $\{(Q \wedge \square)\}\ S1\ \{Q\}$
3) $\{Q \wedge L1\} \rightarrow \{R\}$



$\{P\}\ y := 1,\ K := 0\ \{y = n^K\}\ (n > 0 \rightarrow 1 = 1)\ y = n^1 \Rightarrow y = 1$ ✓

$\{(y = n^K) \wedge (K <> n)\}\ y := y * n;\ K := K + 1\ \{y = n^K\}\ y * n = n^{(K+1)}$

$y = n^K$ and $K = n$ | $y *$

$y = n^{(K+1)}$ ✓

$\{(y = n^K) \wedge (K = n)\} \rightarrow y = n^n$! $y = n^K$; $K = n \rightarrow y = n^n = y = n^n$

Substitute K by n

intro:	$\frac{E[x:t] \vdash$
	$E \vdash$ fun
elim:	$\frac{E \vdash e1 : t$
	$E \vdash e$

3) fun f → fun x → fun y → (f x) y ;;

$[f : t_1] \vdash$ fun x → fun y → (f x) y : t_2 INTRO

$[f : t_1]\ [x : t_3] \vdash$ fun y → (f x) y : t_2 INTRO

$[f : t_1]\ [x : t_3]\ [y : t_4] \vdash$ (f x) y : t_2 INTRO

$[f : t_1]\ [x : t_3]\ [y : t_4] \vdash f\ x : t_5 \rightarrow t_2$ $\quad x : t_6$ ELIM

$[f : t_1]\ [x : t_3]\ [y : t_4] \vdash f : t_6 \rightarrow (t_5 \rightarrow t_2)$ ELIM

$f : t_1 = t_6 \rightarrow (t_5 \rightarrow t_2)$

$x : t_3 = t_6$ $\qquad t_6 : a'$

$y : t_4 = t_5$ $\qquad t_5 : b'$, $t_2 : c'$

$(t_6 \rightarrow t_5 \rightarrow t_2) \rightarrow t_6 \rightarrow t_5 \rightarrow t_2$

$('a \rightarrow 'b \rightarrow 'c) \rightarrow 'a \rightarrow 'b \rightarrow 'c$