

# A Brief Table of Integrals

## Basic Forms

1.  $\int k \, dx = kx + C, \quad k \text{ any number}$
2.  $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
3.  $\int \frac{dx}{x} = \ln|x| + C$
4.  $\int e^x \, dx = e^x + C$
5.  $\int a^x \, dx = \frac{a^x}{\ln a} + C \quad (a > 0, a \neq 1)$
6.  $\int \sin x \, dx = -\cos x + C$
7.  $\int \cos x \, dx = \sin x + C$
8.  $\int \sec^2 x \, dx = \tan x + C$
9.  $\int \csc^2 x \, dx = -\cot x + C$
10.  $\int \sec x \tan x \, dx = \sec x + C$
11.  $\int \csc x \cot x \, dx = -\csc x + C$
12.  $\int \tan x \, dx = \ln|\sec x| + C$
13.  $\int \cot x \, dx = \ln|\sin x| + C$
14.  $\int \sinh x \, dx = \cosh x + C$
15.  $\int \cosh x \, dx = \sinh x + C$
16.  $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$
17.  $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$
18.  $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C$
19.  $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1} \frac{x}{a} + C \quad (a > 0)$
20.  $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \cosh^{-1} \frac{x}{a} + C \quad (x > a > 0)$

## Forms Involving $ax + b$

21.  $\int (ax + b)^n \, dx = \frac{(ax + b)^{n+1}}{a(n+1)} + C, \quad n \neq -1$
22.  $\int x(ax + b)^n \, dx = \frac{(ax + b)^{n+1}}{a^2} \left[ \frac{ax + b}{n+2} - \frac{b}{n+1} \right] + C, \quad n \neq -1, -2$
23.  $\int (ax + b)^{-1} \, dx = \frac{1}{a} \ln|ax + b| + C$
24.  $\int x(ax + b)^{-1} \, dx = \frac{x}{a} - \frac{b}{a^2} \ln|ax + b| + C$
25.  $\int x(ax + b)^{-2} \, dx = \frac{1}{a^2} \left[ \ln|ax + b| + \frac{b}{ax + b} \right] + C$
26.  $\int \frac{dx}{x(ax + b)} = \frac{1}{b} \ln \left| \frac{x}{ax + b} \right| + C$
27.  $\int (\sqrt{ax + b})^n \, dx = \frac{2}{a} \frac{(\sqrt{ax + b})^{n+2}}{n+2} + C, \quad n \neq -2$
28.  $\int \frac{\sqrt{ax + b}}{x} \, dx = 2\sqrt{ax + b} + b \int \frac{dx}{x\sqrt{ax + b}}$

$$29. (a) \int \frac{dx}{x\sqrt{ax+b}} = \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax+b} - \sqrt{b}}{\sqrt{ax+b} + \sqrt{b}} \right| + C$$

$$(b) \int \frac{dx}{x\sqrt{ax-b}} = \frac{2}{\sqrt{b}} \tan^{-1} \sqrt{\frac{ax-b}{b}} + C$$

$$30. \int \frac{\sqrt{ax+b}}{x^2} dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2} \int \frac{dx}{x\sqrt{ax+b}} + C$$

$$31. \int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b} \int \frac{dx}{x\sqrt{ax+b}} + C$$

**Forms Involving  $a^2 + x^2$**

$$32. \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$33. \int \frac{dx}{(a^2 + x^2)^2} = \frac{x}{2a^2(a^2 + x^2)} + \frac{1}{2a^3} \tan^{-1} \frac{x}{a} + C$$

$$34. \int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1} \frac{x}{a} + C = \ln(x + \sqrt{a^2 + x^2}) + C$$

$$35. \int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \ln(x + \sqrt{a^2 + x^2}) + C$$

$$36. \int x^2 \sqrt{a^2 + x^2} dx = \frac{x}{8} (a^2 + 2x^2) \sqrt{a^2 + x^2} - \frac{a^4}{8} \ln(x + \sqrt{a^2 + x^2}) + C$$

$$37. \int \frac{\sqrt{a^2 + x^2}}{x} dx = \sqrt{a^2 + x^2} - a \ln \left| \frac{a + \sqrt{a^2 + x^2}}{x} \right| + C$$

$$38. \int \frac{\sqrt{a^2 + x^2}}{x^2} dx = \ln(x + \sqrt{a^2 + x^2}) - \frac{\sqrt{a^2 + x^2}}{x} + C$$

$$39. \int \frac{x^2}{\sqrt{a^2 + x^2}} dx = -\frac{a^2}{2} \ln(x + \sqrt{a^2 + x^2}) + \frac{x\sqrt{a^2 + x^2}}{2} + C$$

$$40. \int \frac{dx}{x\sqrt{a^2 + x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 + x^2}}{x} \right| + C$$

$$41. \int \frac{dx}{x^2\sqrt{a^2 + x^2}} = -\frac{\sqrt{a^2 + x^2}}{a^2x} + C$$

**Forms Involving  $a^2 - x^2$**

$$42. \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right| + C$$

$$43. \int \frac{dx}{(a^2 - x^2)^2} = \frac{x}{2a^2(a^2 - x^2)} + \frac{1}{4a^3} \ln \left| \frac{x+a}{x-a} \right| + C$$

$$44. \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

$$45. \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$46. \int x^2 \sqrt{a^2 - x^2} dx = \frac{a^4}{8} \sin^{-1} \frac{x}{a} - \frac{1}{8} x \sqrt{a^2 - x^2} (a^2 - 2x^2) + C$$

$$47. \int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

$$48. \int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\sin^{-1} \frac{x}{a} - \frac{\sqrt{a^2 - x^2}}{x} + C$$

$$49. \int \frac{x^2}{\sqrt{a^2 - x^2}} dx = \frac{a^2}{2} \sin^{-1} \frac{x}{a} - \frac{1}{2} x \sqrt{a^2 - x^2} + C$$

$$50. \int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

$$51. \int \frac{dx}{x^2\sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2x} + C$$

**Forms Involving  $x^2 - a^2$**

$$52. \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln|x + \sqrt{x^2 - a^2}| + C$$

$$53. \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln|x + \sqrt{x^2 - a^2}| + C$$

$$107. \int x^n \cos^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \cos^{-1} ax + \frac{a}{n+1} \int \frac{x^{n+1} dx}{\sqrt{1-a^2x^2}}, \quad n \neq -1$$

$$108. \int x^n \tan^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \tan^{-1} ax - \frac{a}{n+1} \int \frac{x^{n+1} dx}{1+a^2x^2}, \quad n \neq -1$$

### Exponential and Logarithmic Forms

$$109. \int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$110. \int b^{ax} dx = \frac{1}{a \ln b} b^{ax} + C, \quad b > 0, b \neq 1$$

$$111. \int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1) + C$$

$$112. \int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx$$

$$113. \int x^n b^{ax} dx = \frac{x^n b^{ax}}{a \ln b} - \frac{n}{a \ln b} \int x^{n-1} b^{ax} dx, \quad b > 0, b \neq 1$$

$$114. \int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + C$$

$$115. \int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + C$$

$$116. \int \ln ax \, dx = x \ln ax - x + C$$

$$117. \int x^n (\ln ax)^m dx = \frac{x^{n+1} (\ln ax)^m}{n+1} - \frac{m}{n+1} \int x^n (\ln ax)^{m-1} dx, \quad n \neq -1$$

$$118. \int x^{-1} (\ln ax)^m dx = \frac{(\ln ax)^{m+1}}{m+1} + C, \quad m \neq -1$$

$$119. \int \frac{dx}{x \ln ax} = \ln |\ln ax| + C$$

### Forms Involving $\sqrt{2ax - x^2}$ , $a > 0$

$$120. \int \frac{dx}{\sqrt{2ax - x^2}} = \sin^{-1} \left( \frac{x-a}{a} \right) + C$$

$$121. \int \sqrt{2ax - x^2} \, dx = \frac{x-a}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x-a}{a} \right) + C$$

$$122. \int (\sqrt{2ax - x^2})^n dx = \frac{(x-a)(\sqrt{2ax - x^2})^n}{n+1} + \frac{na^2}{n+1} \int (\sqrt{2ax - x^2})^{n-2} dx$$

$$123. \int \frac{dx}{(\sqrt{2ax - x^2})^n} = \frac{(x-a)(\sqrt{2ax - x^2})^{2-n}}{(n-2)a^2} + \frac{n-3}{(n-2)a^2} \int \frac{dx}{(\sqrt{2ax - x^2})^{n-2}}$$

$$124. \int x \sqrt{2ax - x^2} \, dx = \frac{(x+a)(2x-3a)\sqrt{2ax - x^2}}{6} + \frac{a^3}{2} \sin^{-1} \left( \frac{x-a}{a} \right) + C$$

$$125. \int \frac{\sqrt{2ax - x^2}}{x} dx = \sqrt{2ax - x^2} + a \sin^{-1} \left( \frac{x-a}{a} \right) + C$$

$$126. \int \frac{\sqrt{2ax - x^2}}{x^2} dx = -2 \sqrt{\frac{2a-x}{x}} - \sin^{-1} \left( \frac{x-a}{a} \right) + C$$

$$127. \int \frac{x \, dx}{\sqrt{2ax - x^2}} = a \sin^{-1} \left( \frac{x-a}{a} \right) - \sqrt{2ax - x^2} + C$$

$$128. \int \frac{dx}{x \sqrt{2ax - x^2}} = -\frac{1}{a} \sqrt{\frac{2a-x}{x}} + C$$

### Hyperbolic Forms

$$129. \int \sinh ax \, dx = \frac{1}{a} \cosh ax + C$$

$$130. \int \cosh ax \, dx = \frac{1}{a} \sinh ax + C$$

$$131. \int \sinh^2 ax \, dx = \frac{\sinh 2ax}{4a} - \frac{x}{2} + C$$

$$132. \int \cosh^2 ax \, dx = \frac{\sinh 2ax}{4a} + \frac{x}{2} + C$$

$$133. \int \sinh^n ax \, dx = \frac{\sinh^{n-1} ax \cosh ax}{na} - \frac{n-1}{n} \int \sinh^{n-2} ax \, dx, \quad n \neq 0$$

$$134. \int \cosh^n ax \, dx = \frac{\cosh^{n-1} ax \sinh ax}{na} + \frac{n-1}{n} \int \cosh^{n-2} ax \, dx, \quad n \neq 0$$

$$135. \int x \sinh ax \, dx = \frac{x}{a} \cosh ax - \frac{1}{a^2} \sinh ax + C$$

$$136. \int x \cosh ax \, dx = \frac{x}{a} \sinh ax - \frac{1}{a^2} \cosh ax + C$$

$$137. \int x^n \sinh ax \, dx = \frac{x^n}{a} \cosh ax - \frac{n}{a} \int x^{n-1} \cosh ax \, dx$$

$$138. \int x^n \cosh ax \, dx = \frac{x^n}{a} \sinh ax - \frac{n}{a} \int x^{n-1} \sinh ax \, dx$$

$$139. \int \tanh ax \, dx = \frac{1}{a} \ln (\cosh ax) + C$$

$$140. \int \coth ax \, dx = \frac{1}{a} \ln |\sinh ax| + C$$

$$141. \int \tanh^2 ax \, dx = x - \frac{1}{a} \tanh ax + C$$

$$142. \int \coth^2 ax \, dx = x - \frac{1}{a} \coth ax + C$$

$$143. \int \tanh^n ax \, dx = -\frac{\tanh^{n-1} ax}{(n-1)a} + \int \tanh^{n-2} ax \, dx, \quad n \neq 1$$

$$144. \int \coth^n ax \, dx = -\frac{\coth^{n-1} ax}{(n-1)a} + \int \coth^{n-2} ax \, dx, \quad n \neq 1$$

$$145. \int \operatorname{sech} ax \, dx = \frac{1}{a} \sin^{-1} (\tanh ax) + C$$

$$146. \int \operatorname{csch} ax \, dx = \frac{1}{a} \ln \left| \tanh \frac{ax}{2} \right| + C$$

$$147. \int \operatorname{sech}^2 ax \, dx = \frac{1}{a} \tanh ax + C$$

$$148. \int \operatorname{csch}^2 ax \, dx = -\frac{1}{a} \coth ax + C$$

$$149. \int \operatorname{sech}^n ax \, dx = \frac{\operatorname{sech}^{n-2} ax \tanh ax}{(n-1)a} + \frac{n-2}{n-1} \int \operatorname{sech}^{n-2} ax \, dx, \quad n \neq 1$$

$$150. \int \operatorname{csch}^n ax \, dx = -\frac{\operatorname{csch}^{n-2} ax \coth ax}{(n-1)a} - \frac{n-2}{n-1} \int \operatorname{csch}^{n-2} ax \, dx, \quad n \neq 1$$

$$151. \int \operatorname{sech}^n ax \tanh ax \, dx = -\frac{\operatorname{sech}^n ax}{na} + C, \quad n \neq 0$$

$$152. \int \operatorname{csch}^n ax \coth ax \, dx = -\frac{\operatorname{csch}^n ax}{na} + C, \quad n \neq 0$$

$$153. \int e^{ax} \sinh bx \, dx = \frac{e^{ax}}{2} \left[ \frac{e^{bx}}{a+b} - \frac{e^{-bx}}{a-b} \right] + C, \quad a^2 \neq b^2$$

$$154. \int e^{ax} \cosh bx \, dx = \frac{e^{ax}}{2} \left[ \frac{e^{bx}}{a+b} + \frac{e^{-bx}}{a-b} \right] + C, \quad a^2 \neq b^2$$

### Some Definite Integrals

$$155. \int_0^\infty x^{n-1} e^{-x} \, dx = \Gamma(n) = (n-1)!, \quad n > 0$$

$$156. \int_0^\infty e^{-ax^2} \, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}, \quad a > 0$$

$$157. \int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \cos^n x \, dx = \begin{cases} \frac{1 \cdot 3 \cdot 5 \cdots (n-1)}{2 \cdot 4 \cdot 6 \cdots n} \cdot \frac{\pi}{2}, & \text{if } n \text{ is an even integer } \geq 2 \\ \frac{2 \cdot 4 \cdot 6 \cdots (n-1)}{3 \cdot 5 \cdot 7 \cdots n}, & \text{if } n \text{ is an odd integer } \geq 3 \end{cases}$$

## Basic Algebra Formulas

### Arithmetic Operations

$$a(b + c) = ab + ac, \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}, \quad \frac{a/b}{c/d} = \frac{a}{b} \cdot \frac{d}{c}$$

### Laws of Signs

$$-(-a) = a, \quad \frac{-a}{b} = -\frac{a}{b} = \frac{a}{-b}$$

**Zero** Division by zero is not defined.

$$\text{If } a \neq 0: \quad \frac{0}{a} = 0, \quad a^0 = 1, \quad 0^a = 0$$

$$\text{For any number } a: \quad a \cdot 0 = 0 \cdot a = 0$$

### Laws of Exponents

$$a^m a^n = a^{m+n}, \quad (ab)^m = a^m b^m, \quad (a^m)^n = a^{mn}, \quad a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

If  $a \neq 0$ , then

$$\frac{a^m}{a^n} = a^{m-n}, \quad a^0 = 1, \quad a^{-m} = \frac{1}{a^m}.$$

**The Binomial Theorem** For any positive integer  $n$ ,

$$\begin{aligned} (a + b)^n &= a^n + na^{n-1}b + \frac{n(n-1)}{1 \cdot 2} a^{n-2}b^2 \\ &\quad + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} a^{n-3}b^3 + \cdots + nab^{n-1} + b^n. \end{aligned}$$

For instance,

$$\begin{aligned} (a + b)^2 &= a^2 + 2ab + b^2, & (a - b)^2 &= a^2 - 2ab + b^2 \\ (a + b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3, & (a - b)^3 &= a^3 - 3a^2b + 3ab^2 - b^3. \end{aligned}$$

### Factoring the Difference of Like Integer Powers, $n > 1$

$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + ab^{n-2} + b^{n-1})$$

For instance,

$$\begin{aligned} a^2 - b^2 &= (a - b)(a + b), \\ a^3 - b^3 &= (a - b)(a^2 + ab + b^2), \\ a^4 - b^4 &= (a - b)(a^3 + a^2b + ab^2 + b^3). \end{aligned}$$

**Completing the Square** If  $a \neq 0$ , then

$$ax^2 + bx + c = au^2 + C \quad \left( u = x + (b/2a), C = c - \frac{b^2}{4a} \right)$$

### The Quadratic Formula

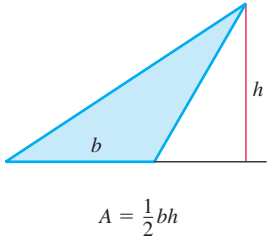
If  $a \neq 0$  and  $ax^2 + bx + c = 0$ , then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

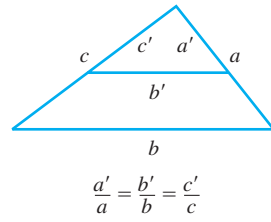
## Geometry Formulas

$A$  = area,  $B$  = area of base,  $C$  = circumference,  $S$  = surface area,  $V$  = volume

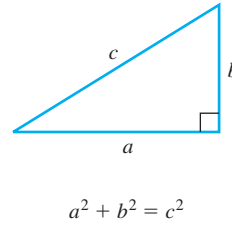
### Triangle



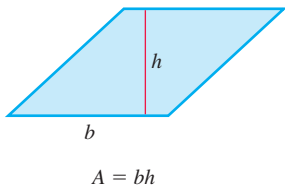
### Similar Triangles



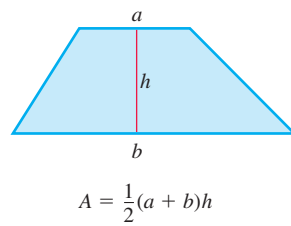
### Pythagorean Theorem



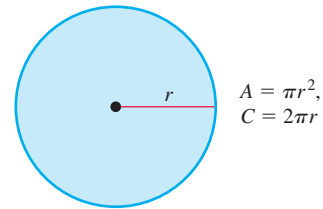
### Parallelogram



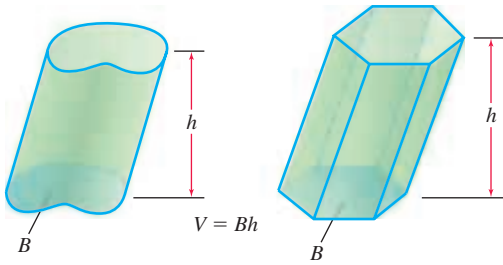
### Trapezoid



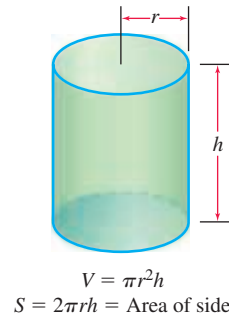
### Circle



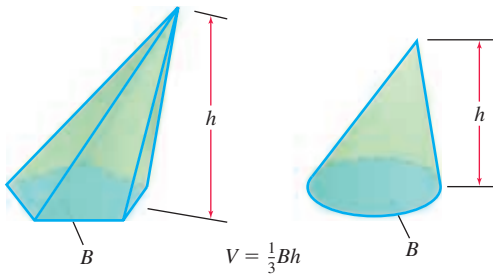
### Any Cylinder or Prism with Parallel Bases



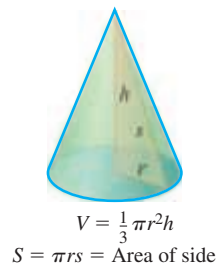
### Right Circular Cylinder



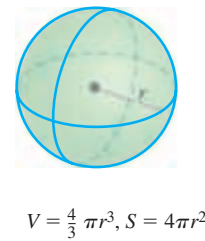
### Any Cone or Pyramid



### Right Circular Cone

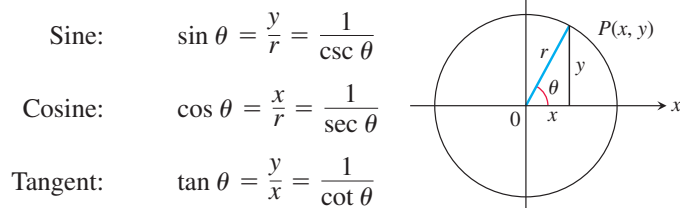


### Sphere



# Trigonometry Formulas

## Definitions and Fundamental Identities



## Identities

$$\begin{aligned}\sin(-\theta) &= -\sin \theta, & \cos(-\theta) &= \cos \theta \\ \sin^2 \theta + \cos^2 \theta &= 1, & \sec^2 \theta &= 1 + \tan^2 \theta, & \csc^2 \theta &= 1 + \cot^2 \theta \\ \sin 2\theta &= 2 \sin \theta \cos \theta, & \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ \cos^2 \theta &= \frac{1 + \cos 2\theta}{2}, & \sin^2 \theta &= \frac{1 - \cos 2\theta}{2} \\ \sin(A + B) &= \sin A \cos B + \cos A \sin B \\ \sin(A - B) &= \sin A \cos B - \cos A \sin B \\ \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \cos(A - B) &= \cos A \cos B + \sin A \sin B\end{aligned}$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\sin\left(A - \frac{\pi}{2}\right) = -\cos A, \quad \cos\left(A - \frac{\pi}{2}\right) = \sin A$$

$$\sin\left(A + \frac{\pi}{2}\right) = \cos A, \quad \cos\left(A + \frac{\pi}{2}\right) = -\sin A$$

$$\sin A \sin B = \frac{1}{2} \cos(A - B) - \frac{1}{2} \cos(A + B)$$

$$\cos A \cos B = \frac{1}{2} \cos(A - B) + \frac{1}{2} \cos(A + B)$$

$$\sin A \cos B = \frac{1}{2} \sin(A - B) + \frac{1}{2} \sin(A + B)$$

$$\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

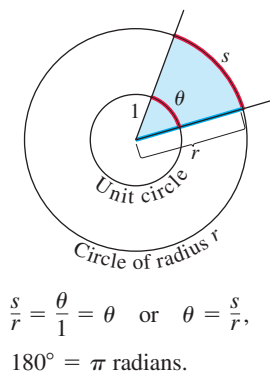
$$\sin A - \sin B = 2 \cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

$$\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\cos A - \cos B = -2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

## Trigonometric Functions

### Radian Measure



| Degrees | Radians |
|---------|---------|
|         |         |
|         |         |

The angles of two common triangles, in degrees and radians.

