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Integration (M-IV)

* $\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + c$

$\int \dfrac{1}{x} \, dx = \ln|x| + c$

$\int a^x \, dx = \dfrac{a^x}{\ln a} + c$

$\int \sin x \, dx = -\cos x + c$

$\int \sec^2 dx = \tan x + c$

$\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

$\int \sec x . \tan x \, dx = \sec x + c$

$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$

* $\int \dfrac{1}{a^2 + x^2} \, dx = \dfrac{1}{a} \tan^{-1}\left(x/a\right) + c$

$\int \dfrac{1}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(x/a\right) + c$

$\int \dfrac{dx}{|x|\sqrt{x^2 - a^2}} = \dfrac{1}{a} \sec^{-1}\left(x/a\right) + c$

$\int -\dfrac{1}{\sqrt{a^2 - x^2}} = \cos^{-1}\left(x/a\right) + c$

$\int \dfrac{-1}{a^2 + x^2} \, dx = \dfrac{1}{a} \cot^{-1}\left(x/a\right) + c$

$\int -\dfrac{1}{x\sqrt{x^2 - a^2}} \, dx = \dfrac{1}{a} \operatorname{cosec}^{-1}\left(x/a\right) + c$

* $\int \cot x \, dx = \log|\sin x| + c$

$\int \tan x \, dx = -\log|\cos x| + c$

$\int \sec x \, dx = \log|\sec x + \tan x| + c$
$= \log\left|\left(\tan\left(\pi/4 + x/2\right)\right)\right| + c$

$\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + c$
$= \log\left|\left(\tan\left(x/2\right)\right)\right| + c$
$= -\log|(\operatorname{cosec} x + \cot x)| + c$

* $\int \dfrac{dx}{\sqrt{x^2 + a^2}} = \ln\left|x + \sqrt{x^2 + a^2}\right| + c$

$\int \dfrac{dx}{\sqrt{x^2 - a^2}} = \ln\left|x + \sqrt{x^2 - a^2}\right| + c$

$\int \dfrac{dx}{a^2 - x^2} = \dfrac{1}{2a} \ln\left|\dfrac{a+x}{a-x}\right| + c$

$\int \dfrac{dx}{x^2 - a^2} = \dfrac{1}{2a} \ln\left|\dfrac{x-a}{x+a}\right| + c$

$$\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left|x+\sqrt{x^2+a^2}\right|+c \qquad \int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right|+c$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+c \qquad \int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+c$$

$$\int \sqrt{a^2-x^2}\,.dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c$$

$$\int \sqrt{a^2+x^2}\,.dx = \frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\ln\left(\frac{x+\sqrt{x^2+a^2}}{a}\right)+c$$

$$\int \sqrt{x^2-a^2}\,.dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\ln\left(\frac{x+\sqrt{x^2-a^2}}{a}\right)+c$$

$$\int e^{ax}\sin bx\,.dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)+c$$

$$\int e^{ax}\cos bx\,.dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)+c$$

Theorem of Integration.

$$\int f(ax+b)\,.dx = \frac{g(ax+b)}{a}+c$$

Integration by substitution.

$$f(x)=t$$

$$f'(x)\,.dx = dt$$

Integration by part:

$$\int [f(x)\,g(x)]\,.dx = f(x)\int g(x)\,.dx - \int\left(\frac{d}{dx}(f(x))\int (g(x))\,.dx\right).dx$$

$$= f(x)\int g(x)\,.dx - \int f'(x)\left(\int g(x)\,.dx\right).dx$$

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ILATE Rule for integration by parts :-

Inverse Trigno $\uparrow$ 1st
Log
Algebra
Trigno
Exponential

Integration of different types :-

① # $\sqrt{a^2-x^2}$ — $x = a\sin\theta$ or $a\cos\theta$
$\sqrt{a^2+x^2}$ — $x = a\tan\theta$ or $a\cot\theta$
$\sqrt{x^2-a^2}$ — $x = a\sec\theta$ or $a\operatorname{cosec}\theta$
$\sqrt{\dfrac{a^2-x^2}{a^2+x^2}}$ — $x^2 = a^2\cos 2\theta$

② # $\int \dfrac{dx}{\text{Quad.} / \sqrt{\text{Quad.}}}$ — means "or" (not division)

★ Make prefect square then integrate

$\int \dfrac{\text{linear}}{\text{Quad}/\sqrt{\text{Quad}}}\, dx$

★ N^r = linear = $A.\dfrac{d(\text{Quad.})}{dx} + B$

③ # $\int \sin^m x \cos^n x \, dx$

m = odd — $t = \cos x$
n = odd — $t = \sin x$

m+n = −ve even integer
$\boxed{t = \tan x}$

⑥ $\int e^x\left(f(x)+f'(x)\right).dx = e^x f(x)+c$

$\int \left(f(x)+x.f'(x)\right).dx = x.f(x)+c$

④ # $\int \dfrac{dx}{(ax+b)^m(cx+d)^n}$ — $\boxed{m+n=2 \quad t = \dfrac{ax+b}{cx+d}}$

⑦ Proper function ($D_{\text{Numerator}} < D_{\text{Denominator}}$)

$\dfrac{\text{(...)}}{x.x(x-2)(x-2)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-2} + \dfrac{D}{(x-2)^2}$

$\dfrac{\text{(...)}}{(x-1)(x^2+4)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+4}$

⑤ $\int \ln x.\, dx = x.(\ln(x)-1)+c$

$\int e^x.f(x).dx = e^x\left(f(x)-f'(x)+f''(x)- \ldots\ldots \text{vanishes}\right)+c$

(8) $\int \frac{a\sin x + b\cos x + c}{d\sin x + e\cos x + f}\,dx$

$\boxed{N^R = A(D^R) + B(D^R)' + C}$

(9) Trigo Twins

$N^R \rightarrow \cos x \pm \sin x$ / $b + \sin x$ / $\sqrt{b + \sin(2x)}$

assume $\boxed{t = \sin x \pm \cos x}$

(10) Algebric Twins :-

$\int \frac{x^2+1}{x^4+Kx^2+1}\,dx$ / $\int \frac{x^2-1}{x^4+Kx^2+1}\,dx$

Divide N^R & D^R by x^2 and put $\boxed{x + \frac{1}{x} = t}$

$\int \frac{dx}{a+b\cos^2 x}$ / $a + b\sin^2 x$ / $b\cos^2 x + b\sin x.\cos x + c\sin^2 x$

$\boxed{\text{Divide } N^R \text{ \& } D^R \text{ by } \cos^2 x \text{ and enjoy}}$

$\int \frac{dx}{a+b\cos x}$ / $a+b\sin x$ / $a\cos x + b\sin x + c$

$\sin x = \frac{2\tan(x/2)}{1+\tan^2(x/2)}$; $\cos x = \frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}$

(11) # $\int \frac{dx}{L\sqrt{Q}}$ $Q = px^2 + qx + r$
$L = ax + b$

put $\boxed{ax + b = \frac{1}{t}}$

$\int \frac{dx}{(ax^2+b)\sqrt{px^2+q}}$ $\boxed{\text{Put } \boxed{x = 1/t}}$

(12) $\int \frac{dx}{(ax+b)\sqrt{px+q}}$ or $\int \frac{1}{(ax^2+bx+c)\sqrt{px+q}}$

Put $\boxed{px + q = t^2}$

(13) # $\int \sqrt{\frac{x-\alpha}{\beta-x}}\,dx$ or

$\int \sqrt{(x-\alpha)(\beta-x)}\,dx$

$\boxed{x = \alpha\cos^2\theta + \beta\sin^2\theta}$

$\int \sqrt{\frac{x-\alpha}{x-\beta}}$ or $\int \sqrt{(x-\alpha)(x-\beta)}$

$\boxed{x = \alpha\sec^2\theta - \beta\tan^2\theta}$

$\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$

$\boxed{x - \alpha = t^2 \text{ or } x - \beta = t^2}$

Reduction formula:

(1) $\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - I_{n-2}$

(2) $\int \cot^n x\,dx = \frac{-\cot^{n-1}x}{n-1} - I_{n-2}$

(3) $\int \sec^n x\,dx = \frac{\tan x \sec^{n-2}x}{n-1} + \frac{n-2}{n-1} I_{n-2}$

(4) $\int \operatorname{cosec}^n x\,dx = \frac{\cot x \operatorname{cosec}^{n-2}x}{n-1} + \frac{n-2}{n-1} I_{n-2}$