Integral Invariants in Recognition of Handwritten Symbols

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Introduction

It was shown in previous work that truncated Legendre-Sobolev expansions of coordinate functions of stroke curves can be applied to recognition of handwritten mathematical symbols. Test results confirm that this technique is indeed effective and allows to achieve 97.5% recognition rate [1]. In this poster, we allow the symbols to be subject to a rotation or, in general, any affine transformation, which commonly occurs in practice. We ask to which extent these transformations affect the classification rates and present a new algorithm for classifying symbols in the presence of such transformations, based on the theory of integral invariants of parametric curves [2]. The proposed algorithm is on-line, in the sense that most computation is done while the symbol is written, and therefore does not cause delays.

Integral Invariants

Integral invariants provide an elegant approach to planar and spatial curves classification. This method helps to eliminate dependence on a character's orientation by assigning an invariant function to the character. Such function is invariant under an affine transformation of the curve, which allows to obtain affine-invariant classification methods.
The first integral invariant has the following form:

$$I_1(T) = \int_0^T X(t)Y'(t)dt - \frac{1}{2}X(T)Y(T)$$

and can be geometrically represented as the area between the curve and its secant.

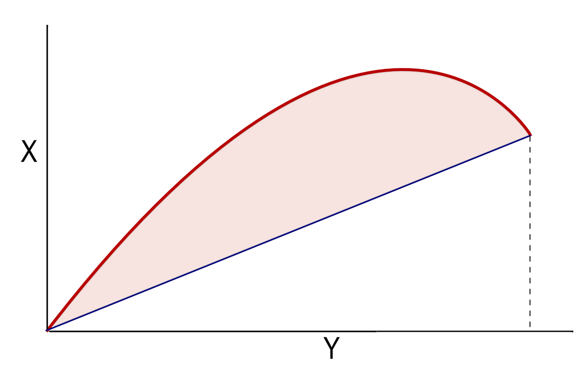

Figure 1: Geometric representation of integral invariant of the first order.

Along with the first integral invariant, we use integral invariants of orders 0 and 2, since they improve recognition rate, while only slightly increasing complexity. These integral invariants can be respectively written as follows:

$$I_0(T) = \sqrt{X^2(T) + Y^2(T)} = r(T), \text{ and}$$

$$I_2(T) = \int_0^T X(t)Y(t)Y'(t)dt - \frac{1}{2}\int_0^T (X(t))^2 Y'(t)dt - \frac{1}{6}X^2(T)Y^2(T)$$

Such integral invariant representation of a character curve is then approximated with Legendre-Sobolev polynomials. Coefficients of the truncated Legendre-Sobolev expansion, similar to our previous method, are used to construct a point for every given character and classify the character, based on the distance from the point to corresponding convex hulls.
Our experiments show that the third invariant gives minor increase in the recognition rate (about 1%) while significantly undermining the computational simplicity of the algorithm. Also, considering the nature of our algorithm, the accuracy of integral invariants classification is not crucially important. Integral invariants are used only for the purpose of selecting top N candidate classes, which are subsequently analyzed. Therefore, the function, obtained with invariants of orders 0,1 and 2, was chosen as sufficient enough to introduce the algorithm, independent of a character slant. The N stands for the number of the closest classes to a character and may be determined empirically to ensure, that the correct class is within the N. In our case, the first 9 classes contain the correct class in 99%.

Integral Invariants vs Legendre-Sobolev

According to our empirical results, classification with Legendre-Sobolev expansion of integral invariants give 85% recognition rate. This result does not depend on the angle to which we allow to rotate test characters. On the other hand recognition rate of Legendre-Sobolev expansion of coordinate functions is decreasing with the increase of the rotational angle and matches the recognition rate, obtained with integral invariants, when we allow the characters in the testing set to have a random angle in the approximate interval of (-23,23) degrees.

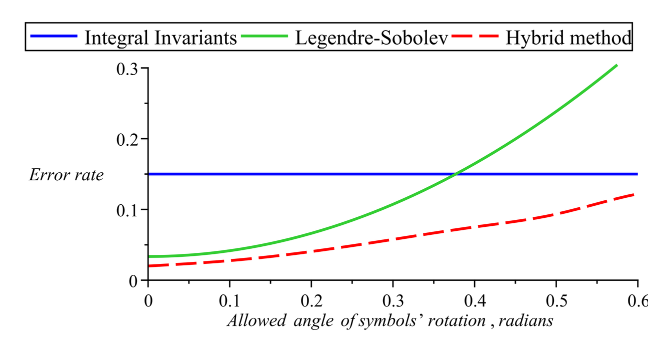

Figure 2: Integral Invariants vs Legendre-Sobolev.

Preliminary results show that with the new algorithm, it is possible to achieve recognition rate, which is marked as red dashed line on Fig. 2.

Algorithm independent on symbol's orientation

The test symbol is first represented as an invariant function, using integral invariants of orders 0,1 and 2. This function is then approximated with truncated Legendre-Sobolev series, and coefficients of the series are used to select top N closest candidate classes. We choose N as big as nesessary to achieve high probability of the test class being within the selected classes. The optimal rotation angle is selected as a solution of minimization problem of distance between the symbol and the candidate classes with respect to the angle, to which the character is rotated:

$$\alpha = -\arctan\left(\frac{\sum_i C_y^i c_x^i - \sum_i C_x^i c_y^i}{\sum_i C_x^i c_x^i + \sum_i C_y^i c_y^i}\right)$$

where C_x^i, C_y^i are coefficients of Legendre-Sobolev approximation of training symbols, and c_x^i, c_y^i are coefficients of the test symbol.
Experiments in progress.

References

[1] O. Golubitsky and S.M. Watt. Distance-Based Classification of Handwritten Symbols. Technical Report, ORCCA, University of Western Ontario. http://www.orcca.on.ca/TechReports/TechReports/2009/TR-09-03.pdf

[2] S. Feng, I.A. Kogan and H. Krim. Classification of curves in 2D and 3D via affine integral signatures. http://arxiv.org/abs/0806.1984