Inferences: Integrals and Derivatives

Graham Gal

Isenberg School of Management, University of Massachusetts, Amherst, MA 01003 USA

Abstract

Orbst's [1] paper on ontological interoperability presents a continuum from descriptions with very week semantics to those with strong semantics. Those ontologies represented in terms of axioms in first order logic have the strongest semantics and therefore the highest level of interoperability. These representations also allow for inferences about the system described by the axioms. The mathematicians of the early 20th century were focused on the possibility of determining which inferences were possible from the axioms. The problem for otologists working in describing services and business models is reversed. We know what inferences must be possible within the model, and it is therefore incumbent on the developers of a particular ontology to show whether these inferences are possible. For example, it is critical for a set of axioms that purport to describe the REA ontology to make inferences about not only accounting artifacts, but also other auditing conclusions. This paper attempts to describe the types of inferences that are required of any ontology.

Keywords

Inferences, Derivatives, Integrals, Attribute Bundles, Naming versus Defining

1. Introduction

The development of a set of axioms which describe a particular domain establishes the essential ontological categories and relationship within the domain. Conclusions or inferences from these axioms is also a critical aspect of understanding and validating the particular domain ontology. The results of these inferences (or formulae) are obtained from initial formulae through a series of symbolic manipulations [2]. Within mathematics, Hilbert [3]1 gave the study of mathematical proofs and symbolic logic (the symbolic manipulations) the name metamathematics. Hilbert argued that the formulae used to represent the axioms and theorems of a particular system also describe the chain of deductions in the system, i.e. the conclusions obtainable within the system. This can also be viewed in reverse as any theorem which can be deduced from the formulae is also an essential component of the system's axioms. Thus, a theorem that results from the application of rules of inference is equivalent to the axioms contained in the original system. The goal of metamathematics was to provide an understanding of whole classes of theorems which could be proved [2]. This goal was to determine a priori the potential for a particular theorem to be derivable from the given axioms. Another implication of the equivalence of the axioms to the derivable theorems or formulae is that if a demonstrability false formula can be obtained, then it can also be concluded that the axioms are incorrect or inconsistent. For example, if the following axioms are presented:

Proceedings of the 16th International Workshop on Value Modelling and Business Ontologies (VMBO 2022), held in conjunction with the 34th International Conference on Advanced Information Systems Engineering (CAiSE 2022), June 06–10, 2022, Leuven, Belgium

Email: gfgal@isenberg.umass.edu
ORCID: 0000-0001-6526-9367

[1] Hilbert original work [4]

Mammals give live birth

Humans are mammals

Men are Humans

Women are Humans

Then it can be concluded that Men can give live birth. This is a conclusion that is not true in our world, so therefore these axioms, and their ontological validity are in question. This would be the same as if the axioms of Principia Mathematica[5] could be used to show that the sum of two even numbers would result in an odd number.

Another issue that can arise when trying to infer theorems within a particular system is that some propositions may be undecidable. This conundrum is the result of Gödel's [6]2 proof that concluded that within certain systems described by what deemed to be fundamental axioms, such as those described in Principia Mathematica [5], there are theorems that would exist that were undecidable, i.e. it would not be possible to prove that they were either true or false. In business ontologies the problem faced by mathematicians, trying to determine whether certain theorems are contained, derivable, is actually reversed. In the business domain certain formulae, using axioms which are purported to describe the fundamentals of a business ontology, must be derivable using standard rules of inference. For example, accounting statements are fundamental descriptions of a state of a business. Therefore, if it is suggested that a set of axioms present a model of a business enterprise, then through rules of inference accounting statements must be obtainable. The demonstration that the axioms presented by Gal [7] can be used to derive the accounting statements is critical to accepting the ontology described by the axioms. That the paper goes beyond the development of accounting statements to include inferences related to auditing conclusions is a further indication of the completeness of the representations contained in the axioms. Certainly, Gödel's proof suggests that there are propositions which will not be derivable, and this possibility cannot be ignored. The purpose of this paper is to explore some issues related to inferences that go beyond the demonstration of accounting and auditing propositions and to suggest some concerns. The following sections will explore some of the potential issues.

The next section will look types of inferences related to integrals, i.e. different ways to integrate a set of events to make conclusions about domain. The third section examines propositions that use derivatives to make inferences about the possibility of future events. Section four looks at some of the implications for using these different approaches to inferences about the nature of organizations. These sections will use views of events corresponding to a soccer match from the perspective to three individuals as an example to demonstrate certain issues. Person A considers the events from the perspective of a fan. Person B views the events from the perspective of the Head Referee. Finally, Person C is a player.

2. Inferences Related to Integrating Events

Bubenko's [8] temporal data cube is a conceptualization of an approach to information modeling. While, its value in modeling data for an organization is established, it is also possible to use this model to conceptualize the set of all possible events. Figure 1shows this conceptualization as a Universal Relation (one with all attributes in the intension) and that extends from time 0 (t_0). At each point in time a new layer is added to the front of the cube with values for the attributes that describe this event at the new time (t_α). A corporate database includes the subset of event instances considered relevant for management and the attributes management considers most appropriate to describe these events.[3]

[2] Gödel's original work published in 1931 and is translated in this reference.

[3] Denna et al. [9] argues that the decision about the events and attributes to be included is based on managements' need to plan, control, and evaluate the organization.

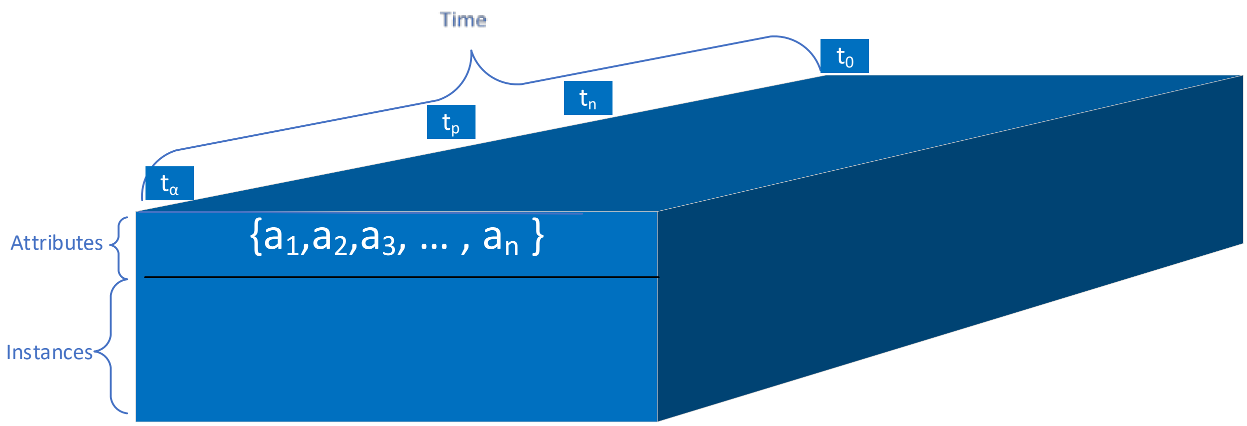

Figure 1: Universal Relation (From Gal [6])

From the larger event data cube, the three individuals can select events and the attributes which they consider most salient to their view of the soccer match. However, this idiosyncratic bundling of events implies a difficulty is defining any particular bundle as the "soccer match". For instance, when asked about the match the fan, Person A, responds that it was important to leave their house early in the morning so his mates could get to the stadium before it got crowded. However, when asked about the match the Head Referee speaks about the meeting the day before when all the officials and the League's commissioner discuss the previous match between the two teams when fisticuffs broke out after the final whistle. Finally, the player, Person C, indicates that when he met with his agent two days ago they agreed that this match is important as they go into contract negotiations. Each of these individuals sees the match as "beginning" at a different point in time and including different attributes (and their values). Each person may also have a different "ending" point in time. This means that the bundle of events corresponding to their match will be different. Grupp [10] discusses this issue and the problem of bundling when attempting to construct a theory of objects. These different bundles of the match indicate different definitions of what are the salient events to consider when making inferences about that match. The distinction between definitions results in assigning or naming different bundles as the match is critical if the goal is to develop an ontology of a soccer match. This is an example of the problem with naming versus defining objects [11]. Conceptually this implies that each person will use different integrals when making inferences concerning the events (and attributes) in their individual bundles. So, when the fan concludes that it was a good game, they are making an inference based on the integration of the events in their bundle. The referee's inference that the game was good was based on the lack of any fisticuff events in their bundle. Finally, the player's inference that is was a bad game is based on his event bundle lacking personal scoring events. Each of these inferences can also be influenced by each observer's comparison to historical comparison (or inferences) to events which comprised past "matches" [12].

There are well-established inferences that integrate events to form views of a corporate data cube. As mentioned in the introduction, Gal [6] derives certain conclusions using axioms that are integrals (aggregations) of the corporate data cube. That these conclusions are defined in the accounting literature makes it easier to validate their correctness [13]. Another set of inferences that are necessary for an ontological representation of an organization includes those about future events. These are the set of inferences based on derivatives of functions which use different events.

3. Inferences Related to Derivatives

When someone faces a question about an expectation of some future event(s) they must create a function based on the events (and attributes) that have occurred, which they view as relevant for forecasting the new event. For example, Barbour [14, p. 10] says that astronomers observe events related to changes the shape of the universe and deduce (infer) that it is expanding. In a similar fashion the attendees at the soccer match use observations of the events relevant to their forecasts and use them to make inferences about future events. For example, the fan, observer A, makes the following forecast, "I was sure that X was going to score." In contrast the referee, observer B, forecasts that, "X was going to shoot wide." While, the player, observer C, forecasts "the goalie was going to make the save." So, each observer has the potential to see the same events, and yet they all make different forecasts. As with the observers' conclusions about the integrals of the events, the data cube must include the events and attributes of these events which allow observers to make their idiosyncratic forecasts. To be clear at some point in the future there will be an event where the player either scores or doesn't. Then this event can be used by the various observers in their integrals or aggregations. As the fan makes the inference that the player who did not score is worthless, and the teammate concludes that he should not have passed him the ball. Any ontology which seeks to model soccer matches, business organizations, airline travel, services, etc. must support the forecasts (inferences) of the observers. The next section looks at some issues that need to be considered.

4. Some Concluding Issues

The different inferences made by the three observers raise a few questions. Perhaps the most relevant question is whether it could be concluded that they were all at the same game. Is the only criterion that matters their similar spatiotemporal location attributes? Is, the lack of any agreed upon view of the match irrelevant to the conclusion (inference) that they all attended the same match. Clearly, they did not occupy the exact same spatiotemporal coordinates, so any observer must also make an inference that they were at the same match. Again, we are faced with the issue of naming versus defining. The question may be phrased as whether the three observers' spatiotemporal coordinates were similar enough to allow an observer to make such a conclusion. Must all observers come to the same conclusion or are they free to name their observations, "attended or did not attend the match?"

There is also an issue with the events related to forecasts of future events. If a pen were held 1meter above the floor and then let go, it would be a rare observer indeed to forecast that the pen would not hit the floor. So, why did each observer make a different forecast about the scoring play? What is the difference between scoring a goal and dropping the pen? Is it simply that there are more events needed to forecast goal scoring as opposed dropping the pen? As the player gets closer to the goal is the inference of the two events of similar certainty? In either case the problem for the otologist is the inclusion of sufficient detail to allow different forecasts to be done with the same certainty.

Finally, a comment about what events are not. Events only have values for time and their other attributes. There are no events that are late, early, fast, slow, etc. These are all inferences about an event. Guarino [15] discusses the issue of identifiers for future events. If they are to be the result of a forecast, then they are put in the derivative of a function. They are when they are. The future events will be integrated into the data cube as they occur. A future event cannot be late. Someone may wish the event had happened at an earlier time. The wishing that the event had already taken place is itself an event, but it has no bearing on the existence of an event that has yet to occur.

Developers of ontologies must be aware of the inferences that are central to the domain of interest. Ontologies that seek to describe organizations, must allow for inferences of the types discussed in this paper. If, for example, an ontology of soccer matches, does not allow for a user to infer what the final score was, then it has a fundamental flaw. However, it must also allow for conclusions about the quality of a match; as idiosyncratic as they may be. To evaluate the completeness of veracity of a particular ontology based solely on the axioms describing without also considering the inferences is to avoid a central requirement of an ontology. This paper has suggested two classes or sets of inferences, those that deal with integrals of events and those that deal with the forecasting of changes to objects represented by the domain.

5. References

[1] L. Obrst, "Ontologies for Semantically Interoperable Systems," in *Conference on Information and Knowledge Management*, New Orleans, LA, USA, 2003, pp. 366–369.

[2] R. B. Braithwaite, "Introduction," in *On Formally Undecidable Propositions of Principia Mathematica and Related Systems*, New York, NY, USA: Dover Publications, 1992, pp. 1–32.

[3] D. Hilbert, "The Foundations of Geometry," 1950.

[4] D. Hilbert, *Grundlagen der Geometrie*. 1899.

[5] A. N. Whitehead and B. Russell, *Principia Mathematica*, 2nd ed. Cambridge, UK: Cambridge University Press, 1910.

[6] K. Gödel, *On Formally Undecidable Propositions on Principia Mathematica and Related Systems*. New York, NY: Dover Publications, 1992.

[7] G. Gal, "The Metaphysics of Internal Controls." 2022.

[8] J. A. Bubenko, *The Temporal Dimension in Information Modelling*. Amsterdam: North-Holland, 1977.

[9] E. Denna, J. O. Cherrington, D. P. Andros, and A. S. Hollander, *Event-Driven Business Solutions:Today's Revolution in Business and Information Technology*. Homewood, IL: Irwin, 1993.

[10] J. Grupp, "Compresence is a Bundle: A problem for the Bundle Theory of Objects," *Metaphys. Int. J. Ontol. Metaphys.*, vol. 5, no. 2, pp. 63–72, 2004.

[11] S. Kripke, *Naming and Necessity*. Princeton University Press, 1970.

[12] R. Schank, "Memory Organization Packets," in *Dynamic Memory Revisited*, Cambridge, UK: Cambridge University Press, 1999, pp. 123–136.

[13] Financial Accounting Standards Board, "Statement of Financial Accounting Concepts No. 6 - Elements of Financial Statements," Financial Accounting Standards Board, Norwalk, CT, Accounting Standard Concept Statement No 6, 1985.

[14] J. Barbour, "Time and its Arrows," in *The Janus Point*, Basic Books, 2020, pp. 1–22.

[15] N. Guarino, "On the Semantics of Ongoing and Future Occurrence Identifiers," in *Conceptual Modeling*, vol. 10650, H. C. Mayr, G. Guizzardi, H. Ma, and O. Pastor, Eds. Cham: Springer International Publishing, 2017, pp. 477–490. doi: 10.1007/978-3-319-69904-2_36.