3.42 Twins. About 30% of human twins are identical, and the rest are fraternal. Identical twins are necessarily the same sex – half are males and the other half are females. One-quarter of fraternal twins are both male, one-quarter both female, and one-half are mixes: one male, one female. You have just become a parent of twins and are told they are both girls. Given this information, what is the probability that they are identical?

Given:

$P(I) = .3$

$P(Fr) = .7 = P(I^c)$

$P(FF \mid I) = .5$

$P(MM \mid I) = .5$

$P(MM \mid I^c) = .25$

$P(FF \mid I^c) = .25$

$P(MF$ order doesn't matter $\mid I^c) = .5$

$P(I \mid FF) = ?$

BAYES' THEOREM: INVERTING PROBABILITIES

Consider the following conditional probability for variable 1 and variable 2:

$$P(\text{outcome } A_1 \text{ of variable 1} \mid \text{outcome } B \text{ of variable 2})$$

Bayes' Theorem states that this conditional probability can be identified as the following fraction:

$$\frac{P(B|A_1)P(A_1)}{P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \cdots + P(B|A_k)P(A_k)}$$

where A_2, A_3, ..., and A_k represent all other possible outcomes of the first variable.

$$P(I \mid FF) = \frac{P(I) \times P(FF \mid I)}{P(FF \mid I) \times P(I) + P(FF \mid I^c) \times P(I^c)}$$

$$= \frac{.3 \times .5}{.5 \times .3 + .25 \times .7}$$

$$= \frac{.3 \times .5}{.325}$$

$$= .461$$