Applied AI & ML Industry Projects Lab (AAM-IPL)
Transforming Learning into Industry Solutions

Week-3 – Linear Regression

Linear Regression and Boston Housing Price Prediction Project Overview

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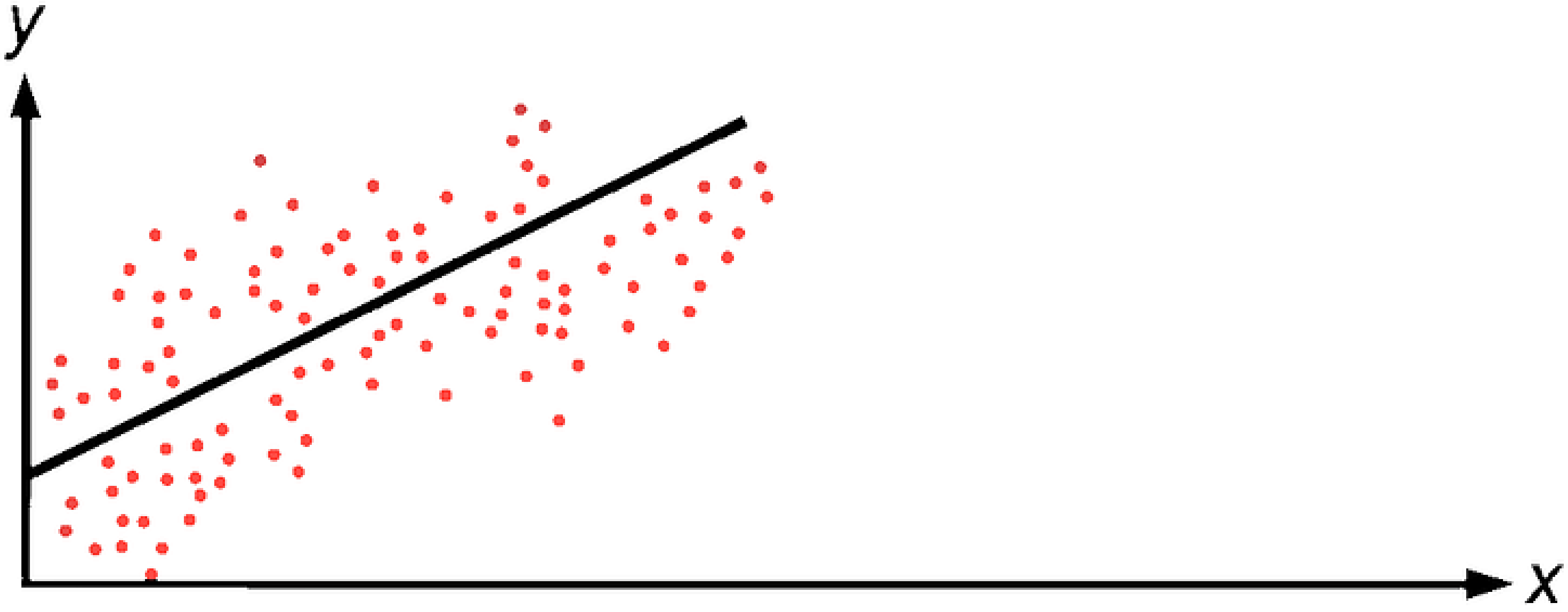
Lecture Outline

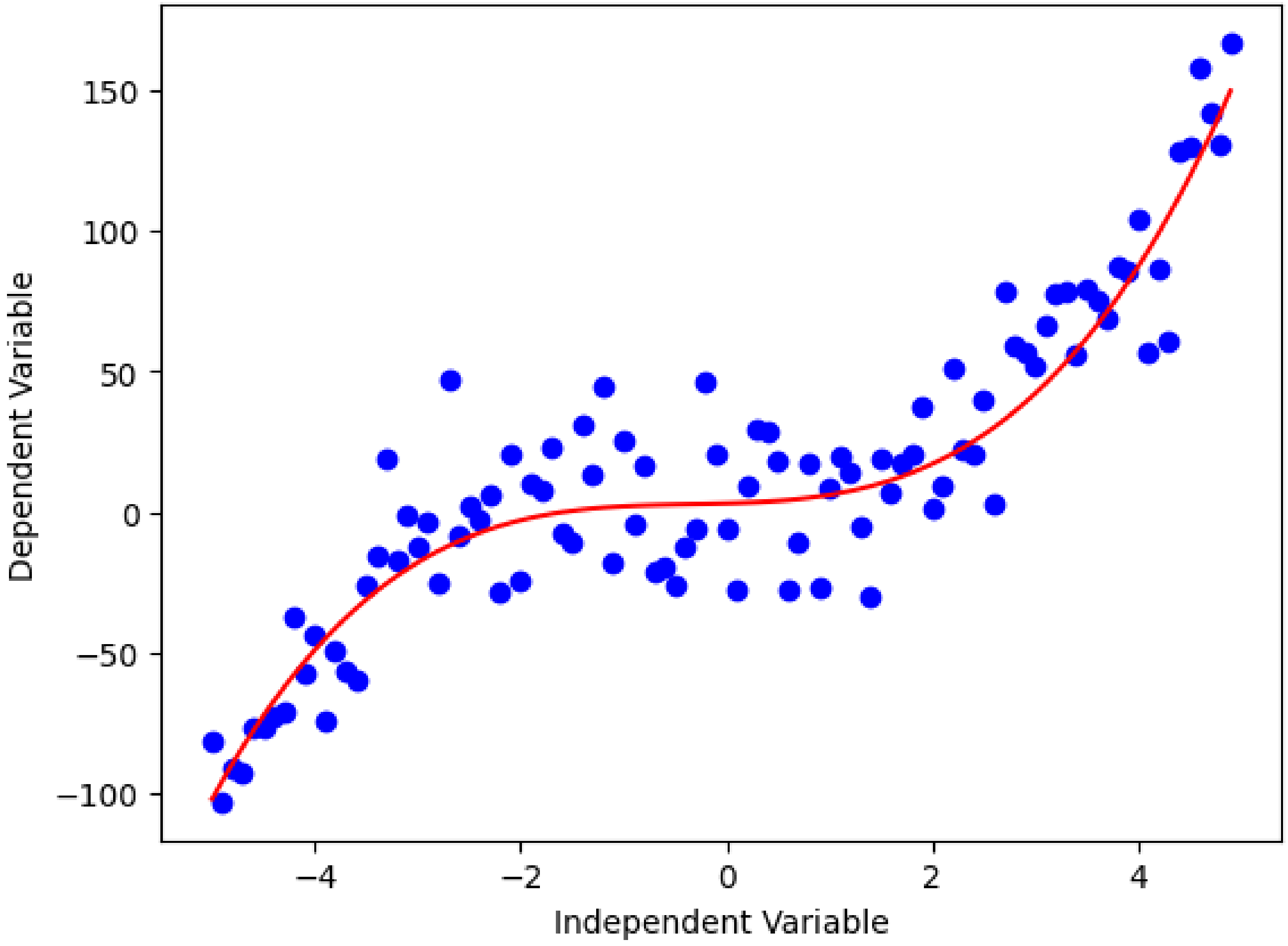
- Data Sets and Distribution
- Data Points and Line of Regression
- What is Linear Regression?
- Dependent and Independent Variables
- Linear Regression Visualisation
- Mathematical Model and Training
- Computing Regression Coefficients
- Boston Housing Price Prediction Project - Overview and Implementation Steps
- Demo
- Q&A

Data Sets and Distribution

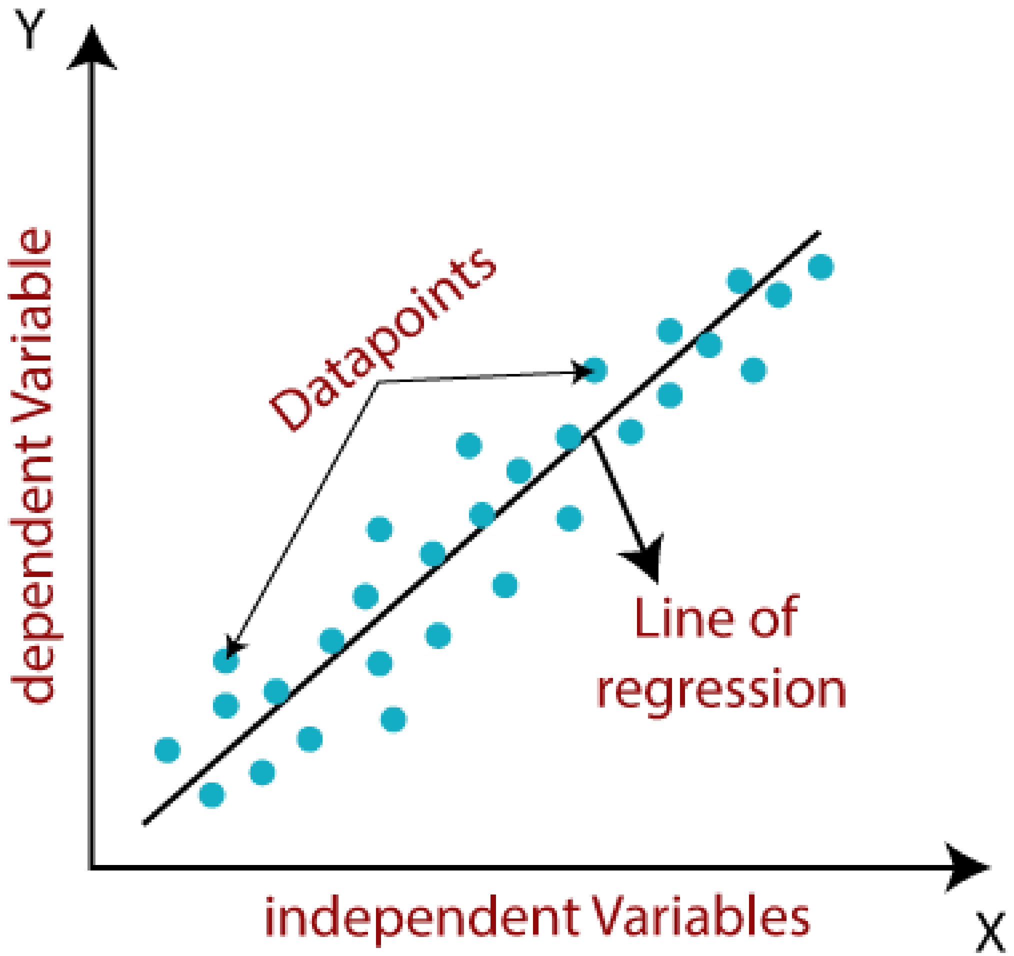


- Y is plotted based on X
- y is dependent or related on $x \Rightarrow y = f(x)$

Data Sets and Distribution



Data Points and Line of Regression



What is Linear Regression?

- A statistical method
- Models relationships between dependent and independent variables for estimating the dependent variable
- Dependent variable is called **target variable** and independent variables are called regressors
- Examples
    - Predicting stock prices, weather forecasting, housing prices, etc.

Linear Regression

- Types of regressions
  - Simple – univariate regression
    - One dependent variable and one independent variable
  - Multiple – multi-variate regression
    - One dependent variable and multiple independent variable

Dependent and Independent Variables

Regressor $\overline{x}$ or Independent Variable of 13-dimensions

	crim	zn	indus	chas	nox	rm	age	dis	rad	tax	ptratio	b	lstat	medv
0	0.00632	18.0	2.31	0	0.538	6.575	65.2	4.0900	1	296	15.3	396.90	4.98	24.0
1	0.02731	0.0	7.07	0	0.469	6.421	78.9	4.9671	2	242	17.8	396.90	9.14	21.6
2	0.02729	0.0	7.07	0	0.469	7.185	61.1	4.9671	2	242	17.8	392.83	4.03	34.7
3	0.03237	0.0	2.18	0	0.458	6.998	45.8	6.0622	3	222	18.7	394.63	2.94	33.4
4	0.06905	0.0	2.18	0	0.458	7.147	54.2	6.0622	3	222	18.7	396.90	5.33	36.2
...														

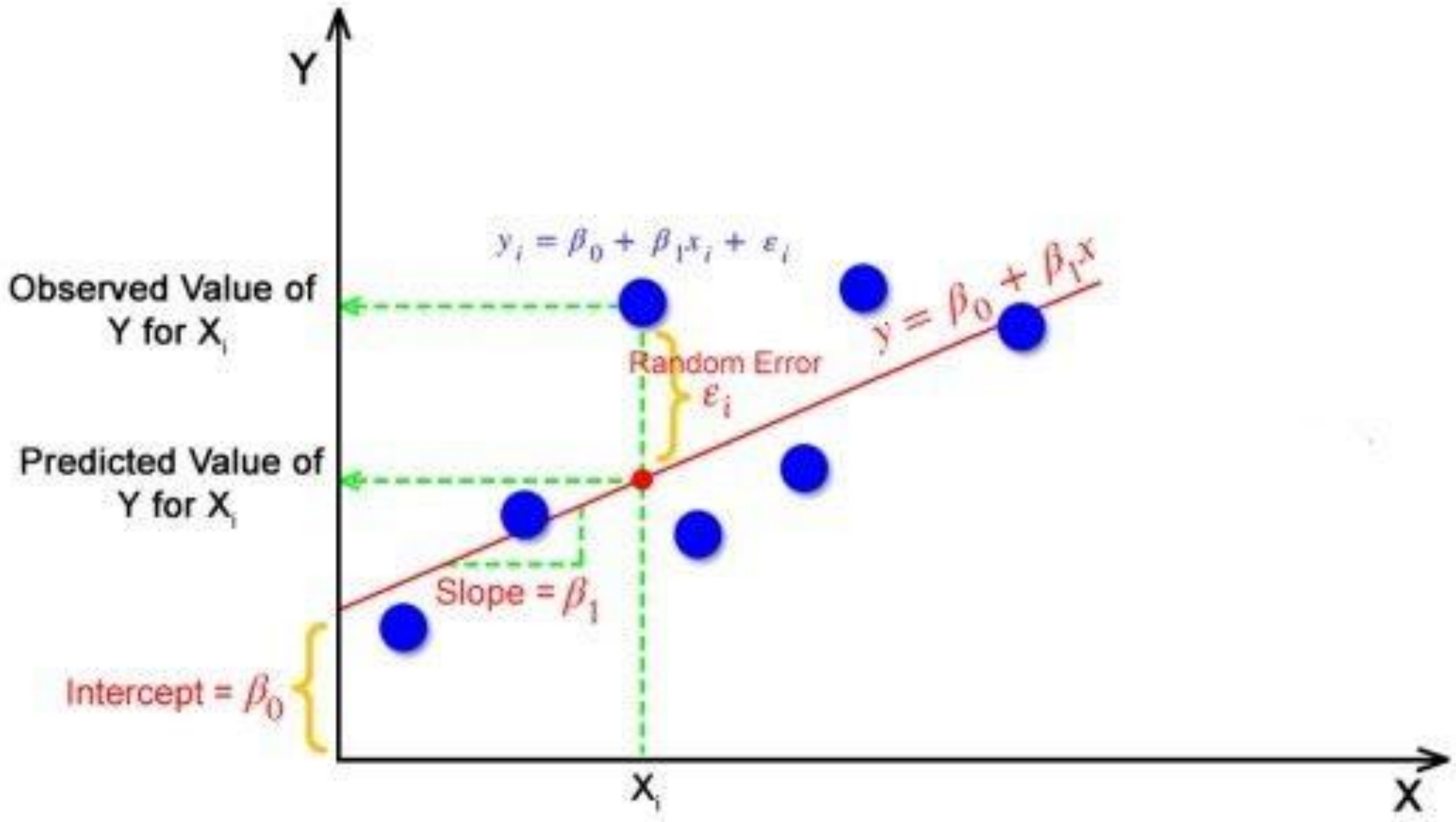
[506 rows x 14 columns]

Response or Target or Dependent Variable

CRIM Per capita crime rate by town
ZN Proportion of residential land zoned for lots over 25,000 sq. ft.
INDUS Proportion of non-retail business acres per town
CHAS Charles River dummy variable (1 if tract bounds river; 0 otherwise)
NOX Nitric oxide concentration (parts per 10 million)
RM Average number of rooms per dwelling
AGE Proportion of owner-occupied units built prior to 1940
DIS Weighted distances to five Boston employment centres
RAD Index of accessibility to radial highways
TAX Full-value property tax rate per $10,000
PTRATIO Pupil-teacher ratio by town
B 1000(Bk - 0.63)^2 where Bk is the proportion of Black residents by town
LSTAT Percentage of lower-status population
MEDV Median value of owner-occupied homes in $1000s (target variable)

$$\overline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{13} \end{bmatrix}$$

Linear Regression Visualisation



Mathematical Model

- In Boston housing data set, housing price is captured as a function of 13 other independent features/variables
  - Number of rooms, per capita crime in the town, distance to Boston employment center, Charles river bank side, etc.
- Price of any given house $k, y(k)$ in Boston with above independent feature values $\bar{x}_i$ is:

$$y(k) = h_0 \times 1 + h_1 x_1(k) + h_2 x_2(k) + h_3 x_3(k) + \cdots + h_{13} x_{13}(k) + \epsilon(k)$$

Bias

Regression Coefficients - Weights

Model Error

Regressors

Mathematical Model

$$y(k) = h_0 \times 1 + h_1 x_1(k) + h_2 x_2(k) + h_3 x_3(k) + \cdots + h_n x_n(k) + \epsilon(k)$$

$$y(k) = \underbrace{\begin{bmatrix} 1 & x_1(k) & x_2(k) & x_3(k) & \dots & x_n(k) \end{bmatrix}}_{\bar{x}^T} \underbrace{\begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ \vdots \\ h_n \end{bmatrix}}_{\bar{h}} + \underbrace{\begin{bmatrix} \epsilon_0 \\ \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_{n+1} \end{bmatrix}}_{\bar{\epsilon}}$$

$$y(k) = \bar{x}^T(k) \times \bar{h} + \epsilon(k)$$

Linear Regression

Training

- Finding the values of regression coefficients using training data set
- Training data set can be mathematically represented as pairs of target and regressor vectors
  - $\left(y(k), \bar{x}(k)\right), k = 1\ to\ M$

$$\begin{bmatrix} y(1) \\ y(2) \\ y(3) \\ \vdots \\ y(M) \end{bmatrix} = \begin{bmatrix} \bar{x}^T(1) \\ \bar{x}^T(2) \\ \bar{x}^T(3) \\ \vdots \\ \bar{x}^T(M) \end{bmatrix} \bar{h} + \begin{bmatrix} \epsilon(1) \\ \epsilon(2) \\ \epsilon(3) \\ \vdots \\ \epsilon(M) \end{bmatrix}$$

$\bar{y}\ - M \times 1$ | $X\ - M \times (n+1)$ | $\bar{\epsilon}\ - M \times 1$

$$\bar{y} = X\bar{h} + \bar{\epsilon}$$

Computing Regression Coefficients

$$\bar{y} = X\bar{h} + \bar{\epsilon}$$

$$\bar{\epsilon} = \bar{y} - X\bar{h}$$

- Start with random values for $\bar{h}$
- Goal is to reduce $\bar{\epsilon}$ to zero/minimum across the training dataset

$$\Rightarrow min\ \bar{\epsilon} = min\|\bar{\epsilon}\|^2 = min\left\|\bar{y} - X\bar{h}\right\|^2$$

- Least Squares Problem/Solution
- Solved using matrix algebra leading to $\bar{h}$
- $\bar{h} = (X^T X)^{-1} X^T \bar{y}$
- $(X^T X)^{-1} X^T$ - Pseudo Inverse of X
  - $\Rightarrow (X^T X)^{-1} X^T X = I$

Why Do We Square Error For Minimisation?

- Magnify error to penalise wrong predictions
- Avoiding cancellation of negative and positive errors
- Squaring makes it differentiable – critical for gradient descent algo
- Least square solution has a closed form formula using matrix algebra

Boston Housing Data Set and House Price Prediction

- **Objective - To build a model to predict the MEDV price of a house**
- The Boston Housing dataset is a classic dataset used for regression tasks, particularly in the domain of housing price prediction
- Contains information collected by the U.S. Census Service concerning housing in the Boston suburbs
- The dataset has been widely used to illustrate the workings of machine learning algorithms, particularly linear regression
- Dataset Overview
    - **Number of Samples**: 506
    - **Number of Features Per Sample**: 13
    - **Target Variable**: MEDV (Median value of owner-occupied homes in $1000s)
- Features
    - CRIM: Per capita crime rate by town
    - ZN: Proportion of residential land zoned for lots over 25,000 sq. ft.
    - INDUS: Proportion of non-retail business acres per town
    - CHAS: Charles River dummy variable (1 if tract bounds river; 0 otherwise)
    - NOX: Nitric oxide concentration (parts per 10 million)
    - RM: Average number of rooms per dwelling
    - AGE: Proportion of owner-occupied units built prior to 1940
    - DIS: Weighted distances to five Boston employment centres
    - RAD: Index of accessibility to radial highways
    - TAX: Full-value property tax rate per $10,000
    - PTRATIO: Pupil-teacher ratio by town
    - B: $1000(Bk - 0.63)^2$ where Bk is the proportion of Black residents by town
    - LSTAT: Percentage of lower-status population

Implementation Steps

- Perform Exploratory Data Analysis (EDA)
  - Print any missing values in the provided dataset
  - Print total data samples/points in the data set
  - Print first 5 rows of the data in the set
  - Plot histograms of all features with continuous values
  - Plot correlation heatmap of features
- Build a Linear Regression Model
  - Standardize the dataset and train the model (test size – 20%)
  - Predict the target variable (MEDV) using the independent features
  - Plot Actual Vs Predicted Home Prices
- Evaluate the Model - Use evaluation metrics to assess model's performance
  - R^2 Score
  - Mean Squared Error (MSE)
  - Root MSE
  - Mean Absolute Error (MAE)
  - Mean Absolute Percentage Error (MAPE)
- Print the Regression Coefficients of the Model
- Generate the PDF of Code and Output

Model Evaluation Metrics

- R^2 Score (Coefficient of Determination)
  - Measures the proportion of variance in the dependent variable that is predictable from the independent variables. An R2R 2 value of 1 indicates a perfect fit, while 0 indicates no predictive power.

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2}$$

- Mean Squared Error (MSE)
  - Measures the average of the squared differences between actual and predicted values. It's sensitive to outliers due to squaring.

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

- Root Mean Squared Error (RMSE)
  - It's the square root of MSE, providing an error value in the same units as the dependent variable.

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2}$$

Model Evaluation Metrics

- Mean Absolute Error (MAE)
  - Measures the average of the absolute differences between actual and predicted values. It's less sensitive to outliers compared to MSE.

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$$

- Mean Absolute Percentage Error (MAPE)
  - Measures the average of the absolute percentage differences between actual and predicted values.

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right| \times 100$$

Project Implementation Demo


Q & A

Interested in building a Gen AI application?

Reach out to venkat@brillium.in

THANK YOU!

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