



## EE932 Assignment-2 Solution

### eMasters in Communication Systems, IITK

#### EE932: Introduction to Reinforcement Learning

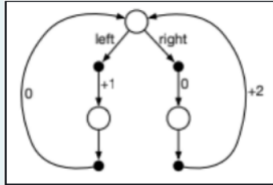
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#### Question 8:

Consider the continuing MDP shown. The only decision to be made is in the top state, where two actions are available, left and right. In the other two states, only one action is available, and hence there is nothing to decide. The numbers show the rewards that are received deterministically after each action. There are exactly two deterministic policies,  $\pi_{left}$  and  $\pi_{right}$ . What policies are optimal for the three cases given below? Show your calculations and upload an image. Case 1:  $\gamma = 0$ , Case 2:  $\gamma = 0.9$ , Case 3:  $\gamma = 0.5$ .



#### Solution:

Total rewards (Returns) for Continuing Task

$$G_t = \sum_{i=0}^{\infty} \gamma^i R_{t+1+i} = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots$$

#### Case $\gamma = 0$ :

For this case, the delayed/future rewards do not count, so the total reward from top state

$$\therefore G_{t-\pi_{left}} = 1 \text{ and } G_{t-\pi_{right}} = 0 \Rightarrow \pi_{left} \text{ is optimal}$$

#### Case $\gamma = 0.9$

For this case, delayed/future rewards would have substantial weightage as  $\gamma = 0.9$

The total rewards from the top state,

$$\begin{aligned} G_{t-\pi_{left}} &= 1 + 0 * 0.9 + 1 * 0.9^2 + 0 * 0.9^3 + 1 * 0.9^4 + \dots \\ &= 1 + 0.9^2 + 0.9^4 + \dots \\ &= \frac{1}{(1-0.9^2)} = 5.2632 \end{aligned}$$

$$\begin{aligned} G_{t-\pi_{right}} &= 0 + 2 * 0.9 + 0 * 0.9^2 + 2 * 0.9^3 + 0 * 0.9^4 + 2 * 0.9^5 \dots \\ &= 2 * 0.9 + 2 * 0.9^3 + 2 * 0.9^5 + \dots \\ &= 2 * 0.9(1 + 0.9^2 + 0.9^4 + \dots) \\ &= 2 * 0.9 * \frac{1}{(1-0.9^2)} = 9.4737 \end{aligned}$$

$$\therefore \pi_{right} \text{ is optimal}$$

#### Case $\gamma = 0.5$

For this case, delayed/future rewards would have substantial weightage as  $\gamma = 0.5$

The total rewards from the top state,

$$\begin{aligned} G_{t-\pi_{left}} &= 1 + 0 * 0.5 + 1 * 0.5^2 + 0 * 0.5^3 + 1 * 0.5^4 + \dots \\ &= 1 + 0.5^2 + 0.5^4 + \dots \\ &= \frac{1}{(1-0.5^2)} = 1.3333 \end{aligned}$$



$$\begin{aligned}G_{t-\pi_{right}} &= 0 + 2 * 0.5 + 0 * 0.5^2 + 2 * 0.5^3 + 0 * 0.5^4 + 2 * 0.5^5 \dots \\&= 2 * 0.5 + 2 * 0.5^3 + 2 * 0.5^5 + \dots \\&= 2 * 0.5(1 + 0.5^2 + 0.5^4 + \dots) \\&= 2 * 0.5 * \frac{1}{(1 - 0.5^2)} = 1 * \frac{1}{(1 - 0.5^2)} = 1.3333\end{aligned}$$

$\therefore$  Both  $\pi_{left}$  and  $\pi_{right}$  policies are optimal

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