Geometric Programs

- can be converted into convex form

<u>Def:</u> (1) monomial: $h(x) = d\, x_1^{a^{(1)}} x_2^{a^{(2)}} \cdots x_n^{a^{(n)}}$

where $d \geq 0$ $a^{(i)} \in \mathbb{R}$

$h: \mathbb{R}_{++}^n \to \mathbb{R}_{++}$

Eg $4x_1^2 x_2^{-1/2} x_3^{-3}$ or x_1/x_2

but $-x_1$ not a monomial

note $h(x)$ monomial $\Leftrightarrow$ $\frac{1}{h(x)}$ monomial

$$\frac{1}{h(x)} = \left(\frac{1}{d}\right) x_1^{-a^{(1)}} x_2^{-a^{(2)}} \cdots x_n^{-a^{(n)}}$$

(2) Posynomial = sum of monomials

$$g(x) = \sum_{k=1}^{K} d_k\, x_1^{a_k^{(1)}} x_2^{a_k^{(2)}} \cdots x_n^{a_k^{(n)}}$$

Eg $x_1^2 + x_2$ $x_1 - x_2$

$x_1 x_2 + x_2/x_3$ not a posynomial

$\frac{1}{x_1 + x_2}$

(GP) $\min \ g_0(\underline{x})$

posynomial $\leftarrow$ $g_i(x) \preceq 1 \qquad i=1 \ldots m$

monomial $\leftarrow$ $h_\ell(x) = 1 \qquad \ell = 1 \ldots p$

Note: monomials / posynomials not convex functions

convex form? $\quad y_i = \log(x_i) \qquad$ or $\quad x_i = e^{y_i}$

$$h_\ell(\underline{x}) = d_\ell \, x_1^{a_\ell^{(1)}} x_2^{a_\ell^{(2)}} \cdots x_n^{a_\ell^{(n)}}$$

$$= d_\ell \left(e^{a_\ell^{(1)} y_1 + a_\ell^{(2)} y_2 + \cdots + a_\ell^{(n)} y_n} \right)$$

$$= \exp(\underline{a}_\ell^T y + b_\ell)$$

$$a_\ell = \begin{bmatrix} a_\ell^{(1)} \\ \vdots \\ a_\ell^{(n)} \end{bmatrix}$$

$$b_\ell = \log d_\ell$$

Therefore:

$h(x) = 1 \iff a_\ell^T y + b_\ell = 0 \qquad$ affine eq.

likewise $$g_i(x) = \sum_{k=1}^{K} d_{ik}\, x_1^{a_{ik}^{(1)}} x_2^{a_{ik}^{(2)}} \cdots x_n^{a_{ik}^{(n)}}$$

$$= \sum_{k=1}^{K} \exp(\underline{a}_{ik}^T y + \underline{b}_{ik})$$

$$g_i(x) \le 1 \iff \log(g_i(\underline{x})) \le 0$$

$$\iff \log\left(\sum_{k=1}^{K} \exp(a_{ik}^T y + b_{ik})\right) \le 0$$

convex log-sum-exp

Summary
$$\min \log\left(\sum_{k=1}^{K} \exp(a_{0k}^T y + b_{0k})\right)$$

$$\log\left(\sum_{k=1}^{K} \exp(a_{ik}^T y + b_{ik})\right) \le 0 \quad i = 1 \ldots m$$

$$a_l^T y + b_l = 0 \quad l = 1 \ldots p$$

Manipulating GPs

(1) $\max h(x) \equiv \min \frac{1}{h(x)}$

(2) posynomial $\leq$ monomial

$$g_i(x) \leq h_i(x) \iff \frac{g_i(x)}{h_i(x)} \leq 1$$

posynomial

(3) product of posynomials = posynomial

(4)
$$\min \frac{g_1(x)}{h(x) - g_2(x)} \equiv \min t$$

$$\frac{g_1(x)}{h(\underline{x}) - g_2(x)} \leq t$$

$$\frac{g_1(x) + t g_2(x)}{t h(x)} \leq 1$$

here $\frac{g_1(x)}{t h(x)}$ & $\frac{g_2(x)}{h(x)}$ are posynomials