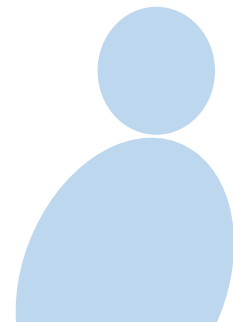


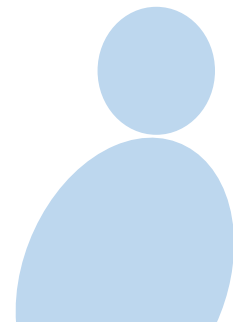
eMasters in Communication Systems

**Prof. Aditya
Jagannatham**



Elective Module:

**Detection for Wireless
Communication**

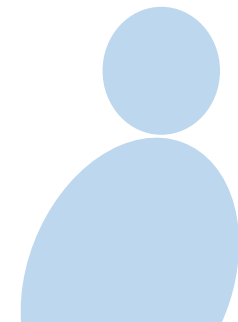


Chapter 5

Bayesian Detection

Min P_e Detector

min probability
of Error Decision rule.



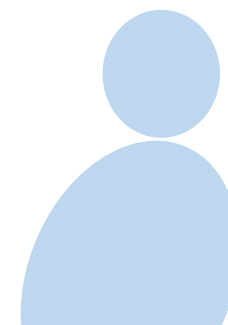
Bayesian Detection

- Consider a **Bayesian Binary**

Hypothesis Testing:

$$0 \leq \pi_0, \pi_1 \leq 1$$

$$\begin{aligned} P(\mathcal{H}_0) &= \pi_0 \\ P(\mathcal{H}_1) &= \pi_1 \end{aligned} \quad \left. \vphantom{\begin{aligned} P(\mathcal{H}_0) &= \pi_0 \\ P(\mathcal{H}_1) &= \pi_1 \end{aligned}} \right\} \begin{aligned} &\pi_0 + \pi_1 = 1 \\ &\text{Prior probabilities of hypotheses -} \end{aligned}$$

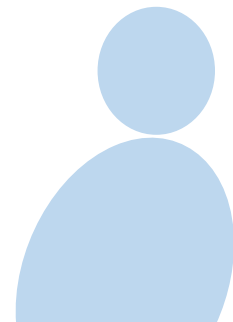


Bayesian Detection

- Consider a **Bayesian Binary Hypothesis Testing**:

$$P(\mathcal{H}_0) = \pi_0$$

$$P(\mathcal{H}_1) = \pi_1$$

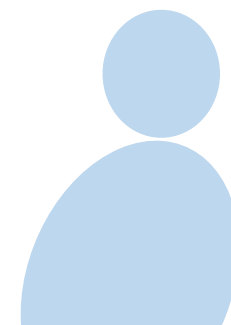


Bayesian Detection

- **Decision region** for \mathcal{H}_1 is $R_1 \subset \mathbb{R}^N$
if $\bar{y} \in R_1$ Then choose \mathcal{H}_1 ,
if $\bar{y} \in R_0$ Then choose \mathcal{H}_0 .
- **Decision region** for \mathcal{H}_0 is $R_0 = \mathbb{R}^N - R_1$

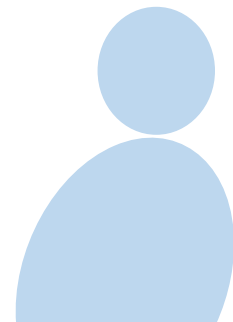
$$R_1 \cup R_0 = \mathbb{R}^N.$$

$$R_1 \cap R_0 = \emptyset.$$



Bayesian Detection

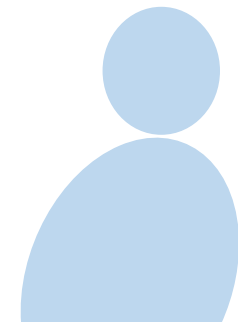
- **Decision region** for \mathcal{H}_1 be $\bar{\mathbf{y}} \in R_1$
- **Decision region** for \mathcal{H}_0 is $\bar{\mathbf{y}} \in R_0 = \mathcal{R}^N - R_1$



Bayesian Detection

- The probability of error P_e is given as

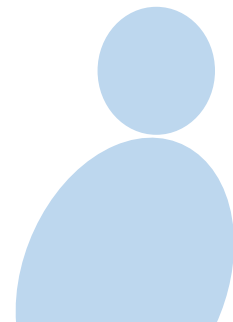
$$P_e = \underbrace{P_r(\mathcal{H}_0)}_{P_{FA}} \underbrace{Pr(\mathcal{H}_1 | \mathcal{H}_0)}_{\text{Probability of False Alarm}} + \underbrace{P_r(\mathcal{H}_1)}_{P_{MD}} \underbrace{Pr(\mathcal{H}_0 | \mathcal{H}_1)}_{\text{Probability of miss detection}}$$
$$= \pi_0 \cdot \underbrace{Pr(\mathcal{H}_1 | \mathcal{H}_0)}_{\bar{y} \in R_1; \mathcal{H}_0} + \pi_1 \cdot \underbrace{Pr(\mathcal{H}_0 | \mathcal{H}_1)}_{\bar{y} \in R_0; \mathcal{H}_1}$$



Bayesian Detection

- The probability of error P_e is given as

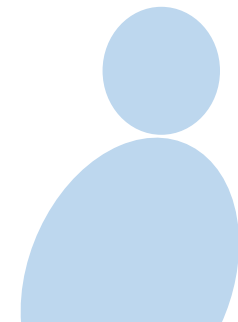
$$\begin{aligned} P_e &= P(\mathcal{H}_0|\mathcal{H}_1)P(\mathcal{H}_1) + P(\mathcal{H}_1|\mathcal{H}_0) \Pr(\mathcal{H}_0) \\ &= P(\mathcal{H}_0|\mathcal{H}_1)\pi_1 + P(\mathcal{H}_1|\mathcal{H}_0)\pi_0 \end{aligned}$$



Bayesian Detection

- P_e can be expressed as

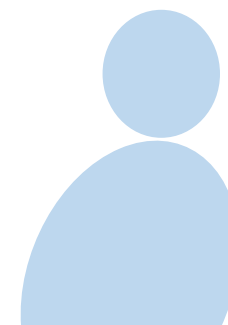
$$P_e = \pi_1 \int_{R_0} p(\bar{y} | \mathcal{H}_1) d\bar{y} + \pi_0 \int_{R_1} p(\bar{y} | \mathcal{H}_0) d\bar{y}$$
$$= \pi_1 \int_{R_0} p(\bar{y} | \mathcal{H}_1) d\bar{y} + \pi_0 \cdot \left(1 - \int_{R_0} p(\bar{y} | \mathcal{H}_0) d\bar{y} \right)$$



Bayesian Detection

$$\int_{R_1} p(\bar{y}|\mathcal{H}_0) d\bar{y} + \int_{R_0} p(\bar{y}|\mathcal{H}_0) d\bar{y} = 1$$

$$P_e = \underbrace{\int_{R_0} (\pi_1 p(\bar{y}|\mathcal{H}_1) - \pi_0 p(\bar{y}|\mathcal{H}_0)) d\bar{y}}_{\text{minimize}} + \underbrace{\pi_0}_{\text{constant}}.$$

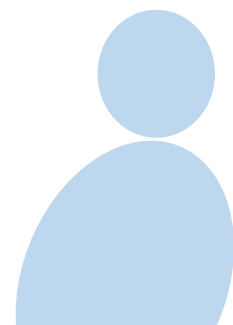


Bayesian Detection

To minimize $\int_{R_0} (\pi_1 P(\bar{y}|\mathcal{H}_1) - \pi_0 P(\bar{y}|\mathcal{H}_0)) d\bar{y}$

include all \bar{y} in R_0 such that

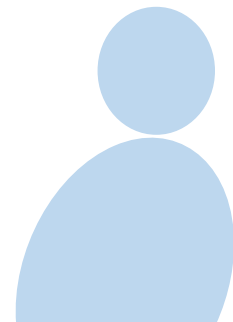
$$\pi_1 P(\bar{y}|\mathcal{H}_1) - \pi_0 P(\bar{y}|\mathcal{H}_0) \leq 0$$



Bayesian Detection

- P_e can be expressed as

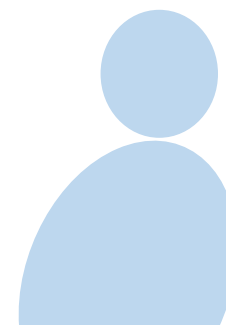
$$\pi_1 \int_{R_0} p(\bar{\mathbf{y}}|\mathcal{H}_1) d\bar{\mathbf{y}} + \pi_0 \int_{R_1} p(\bar{\mathbf{y}}|\mathcal{H}_0) d\bar{\mathbf{y}}$$



Bayesian Detection

$$\pi_1 \int_{R_0} p(\bar{\mathbf{y}}|\mathcal{H}_1) d\bar{\mathbf{y}} + \pi_0 \left(1 - \int_{R_0} p(\bar{\mathbf{y}}|\mathcal{H}_0) d\bar{\mathbf{y}} \right)$$

$$= \int_{R_0} (\pi_1 p(\bar{\mathbf{y}}|\mathcal{H}_1) - \pi_0 p(\bar{\mathbf{y}}|\mathcal{H}_0)) d\bar{\mathbf{y}} + \pi_0$$



Bayesian Detection

- To minimize, choose R_0 as

\bar{y} such that

$$\pi_1 p(\bar{y} | \mathcal{H}_1) - \pi_0 p(\bar{y} | \mathcal{H}_0) \leq 0$$
$$\Rightarrow \frac{p(\bar{y} | \mathcal{H}_0)}{p(\bar{y} | \mathcal{H}_1)} \geq \left(\frac{\pi_1}{\pi_0} \right) \quad \tilde{\gamma}$$

Reduces to Likelihood Ratio Test (LRT)

$$\tilde{\gamma} = \frac{\pi_1}{\pi_0}$$

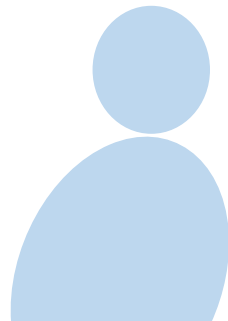
Bayesian Detection

- To minimize, choose R_0 as

$$\pi_1 p(\bar{\mathbf{y}}|\mathcal{H}_1) - \pi_0 p(\bar{\mathbf{y}}|\mathcal{H}_0) \leq 0$$

$$\Rightarrow \pi_0 p(\bar{\mathbf{y}}|\mathcal{H}_0) \geq \pi_1 p(\bar{\mathbf{y}}|\mathcal{H}_1)$$

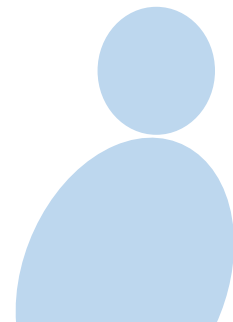
$$\Rightarrow \frac{p(\bar{\mathbf{y}}|\mathcal{H}_0)}{p(\bar{\mathbf{y}}|\mathcal{H}_1)} \geq \frac{\pi_1}{\pi_0}$$



Bayesian Detection

- It reduces to LRT with

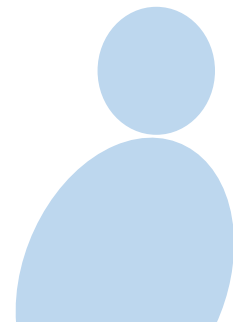
$$\tilde{\gamma} = \frac{\pi_1}{\pi_0} = \frac{\Pr(\mathcal{H}_1)}{\Pr(\mathcal{H}_0)}$$



Bayesian Detection

- It reduces to LRT with

$$\tilde{\gamma} = \frac{\pi_1}{\pi_0} = \frac{\Pr(\mathcal{H}_1)}{\Pr(\mathcal{H}_0)}$$



MAP Decision Rule

- Choose \mathcal{H}_0 if

$$\pi_0 p(\bar{y}|\mathcal{H}_0) \geq$$

$$\pi_1 p(\bar{y}|\mathcal{H}_1).$$

$$\Rightarrow \frac{\pi_0 p(\bar{y}|\mathcal{H}_0)}{\pi_0 p(\bar{y}|\mathcal{H}_0) + \pi_1 p(\bar{y}|\mathcal{H}_1)}$$

$$\Rightarrow \frac{\pi_1 p(\bar{y}|\mathcal{H}_1)}{\pi_0 p(\bar{y}|\mathcal{H}_0) + \pi_1 p(\bar{y}|\mathcal{H}_1)}.$$

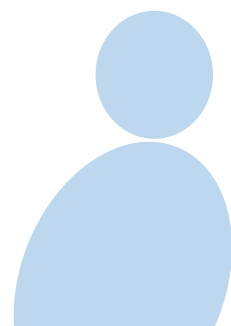
$$\Rightarrow \frac{Pr(\mathcal{H}_0) p(\bar{y}|\mathcal{H}_0)}{Pr(\mathcal{H}_0) p(\bar{y}|\mathcal{H}_0) + Pr(\mathcal{H}_1) p(\bar{y}|\mathcal{H}_1)}$$

$$\geq \frac{Pr(\mathcal{H}_1) p(\bar{y}|\mathcal{H}_1)}{Pr(\mathcal{H}_0) p(\bar{y}|\mathcal{H}_0) + Pr(\mathcal{H}_1) p(\bar{y}|\mathcal{H}_1)}$$

Bayes Rule:
 $Pr(\mathcal{H}_0|\bar{y})$

$Pr(\mathcal{H}_1|\bar{y})$

A posteriori probabilities.



MAP Decision Rule

- Chose \mathcal{H}_0 if

$$\Rightarrow \frac{p(\bar{y}|\mathcal{H}_0)}{p(\bar{y}|\mathcal{H}_1)} \geq \frac{\pi_1}{\pi_0}$$

$$\tilde{\gamma} = \frac{\pi_1}{\pi_0}$$

$$\pi_1 = \Pr(\mathcal{H}_1)$$

$$\pi_0 = \Pr(\mathcal{H}_0)$$

$$\Rightarrow \frac{\Pr(\mathcal{H}_0) p(\bar{y}|\mathcal{H}_0)}{\Pr(\mathcal{H}_0) p(\bar{y}|\mathcal{H}_0)} \geq \frac{\Pr(\mathcal{H}_1) p(\bar{y}|\mathcal{H}_1)}{\Pr(\mathcal{H}_1) p(\bar{y}|\mathcal{H}_1)}$$

$$\Rightarrow \frac{\Pr(\mathcal{H}_0) p(\bar{y}|\mathcal{H}_0) + \Pr(\mathcal{H}_1) p(\bar{y}|\mathcal{H}_1)}{\Pr(\mathcal{H}_1) p(\bar{y}|\mathcal{H}_1)}$$

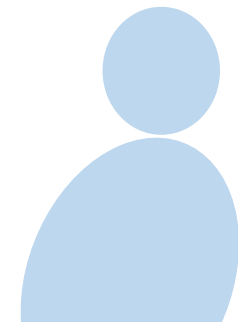
$$\geq \frac{\Pr(\mathcal{H}_0) p(\bar{y}|\mathcal{H}_0) + \Pr(\mathcal{H}_1) p(\bar{y}|\mathcal{H}_1)}{\Pr(\mathcal{H}_0) p(\bar{y}|\mathcal{H}_0) + \Pr(\mathcal{H}_1) p(\bar{y}|\mathcal{H}_1)}$$

MAP Decision Rule

- Choose \mathcal{H}_0 if

$$\left(\Pr(\mathcal{H}_0 | \bar{y}) \geq \Pr(\mathcal{H}_1 | \bar{y}) \right)$$

↙ choose hypothesis with maximum
aposteriori probability (MAP).
MAP decision rule!



MAP Decision Rule

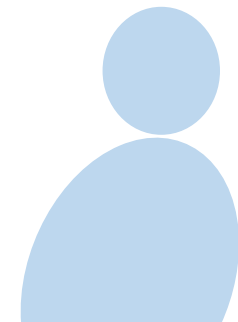
Aposteriori Probabilities.
 $\Pr(\mathcal{H}_0 | \bar{y}) \geq \Pr(\mathcal{H}_1 | \bar{y})$

- Choose \mathcal{H}_0 if

$$\Rightarrow \Pr(\mathcal{H}_0 | \bar{y}) \geq \Pr(\mathcal{H}_1 | \bar{y})$$

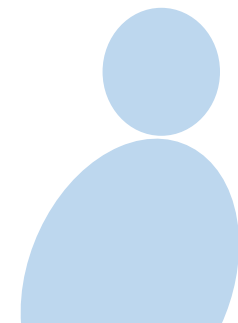
choose \mathcal{H}_1 if
 $\Pr(\mathcal{H}_0 | \bar{y}) < \Pr(\mathcal{H}_1 | \bar{y})$

- i.e. choose hypothesis with the **Maximum**
Aposteriori Probability (MAP)



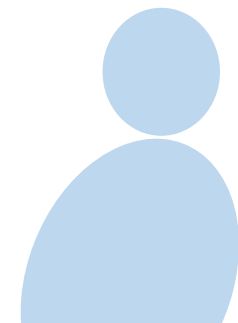
MAP Decision Rule

- This is termed as the **Maximum Aposteriori Probability (MAP) rule**
- Therefore, MAP rule **minimizes the**
probability of error P_e



MAP Decision Rule

- This is termed as the **Maximum A posteriori Probability (MAP) rule**
- Therefore, MAP rule **minimizes the probability of error**

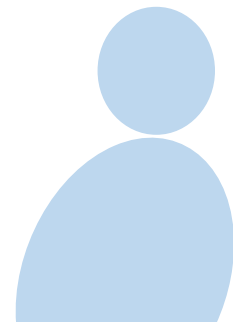


MAP Decision Rule

- Finally, when $\pi_0 = \pi_1 = \frac{1}{2}$

$$\tilde{\gamma} = \frac{\pi_1}{\pi_0} = \frac{1/2}{1/2} = 1 \Rightarrow ML$$

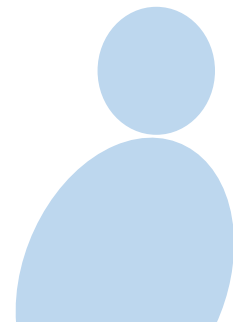
- MAP reduces to ML Decision rule!



MAP Decision Rule

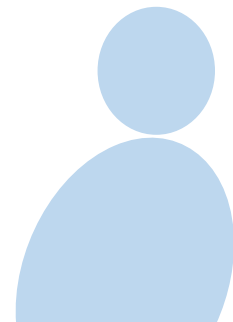
- Finally, when $\pi_0 = \pi_1 = \frac{1}{2}$
$$\tilde{\gamma} = \frac{\pi_1}{\pi_0} = 1$$

- MAP reduces to ML!



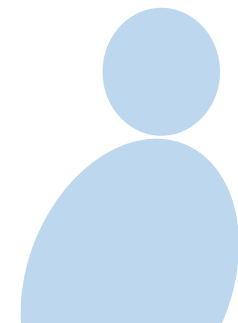
MAP Decision Rule

- When prior probabilities are equal, MAP becomes the Maximum Likelihood (ML) rule.



MAP Decision Rule

- When prior probabilities are equal, MAP becomes the **ML decision rule**



Bayesian Detection

$$\begin{aligned} E\{\bar{v}\} &= 0 \\ E\{\bar{v}\bar{v}^T\} &= \sigma^2 I \end{aligned}$$

- Consider now the signal detection problem

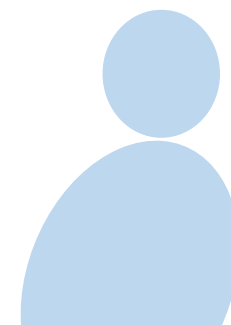
\mathcal{H}_0 :

$$\bar{y} = \bar{v}$$

← noise

\mathcal{H}_1 :

$$\bar{y} = \bar{s} + \bar{v}$$

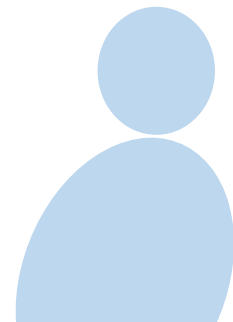


Bayesian Detection

- Consider now the signal detection problem

$$\mathcal{H}_0: \bar{y} = \bar{v}$$

$$\mathcal{H}_1: \bar{y} = \bar{s} + \bar{v}$$



LRT

$$\bar{s}^T \bar{y} = \sum_{i=1}^{N-1} y(i)s(i)$$

- Choose \mathcal{H}_0 if

$$\bar{s}^T \bar{y} = \sum_{i=1}^N y(i)s(i) \leq \frac{\|\bar{s}\|^2 - 2\sigma^2 \ln \tilde{\gamma}}{2} = \gamma$$

$$\tilde{\gamma} = \frac{\pi_1}{\pi_0}$$

$$\gamma = \frac{\|\bar{s}\|^2 - 2\sigma^2 \ln \frac{\pi_1}{\pi_0}}{2}$$

MAP rule
min P_e .

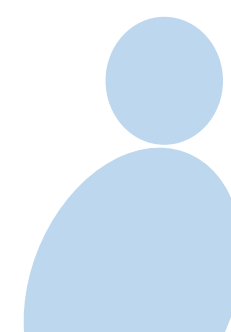
$$\text{choose } \mathcal{H}_0 \text{ if } \bar{s}^T \bar{y} \leq \frac{\|\bar{s}\|^2 - 2\sigma^2 \ln \frac{\pi_1}{\pi_0}}{2}$$

LRT

- Choose \mathcal{H}_0 if

$$\sum_{i=1}^N y(i)s(i) \leq \frac{\|\bar{\mathbf{s}}\|^2 - 2\sigma^2 \ln \tilde{\gamma}}{2} = \gamma$$

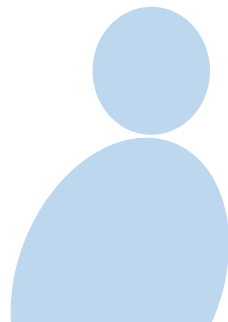
$$\gamma = \frac{\|\bar{\mathbf{s}}\|^2}{2} - \sigma^2 \ln \left(\frac{\pi_1}{\pi_0} \right)$$



LRT

- Choose \mathcal{H}_0 if

$$\sum_{i=1}^N y(i)s(i) \leq \frac{\|\bar{\mathbf{s}}\|^2}{2} - \sigma^2 \ln \frac{\pi_1}{\pi_0}$$



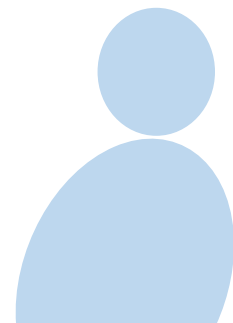
LRT

- Choose \mathcal{H}_1 if

$$\bar{S}^T \bar{y} = \sum_i y(i) S(i) > \frac{\|\bar{S}\|^2 - 2\sigma^2 \ln \tilde{\gamma}}{2}$$

$$\tilde{\gamma} = \frac{\bar{\pi}_1}{\bar{\pi}_0}$$

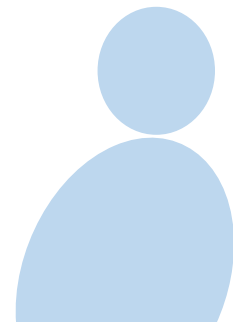
$$\bar{S}^T \bar{y} > \frac{\|\bar{S}\|^2}{2} - \sigma^2 \ln \left(\frac{\bar{\pi}_1}{\bar{\pi}_0} \right).$$



LRT

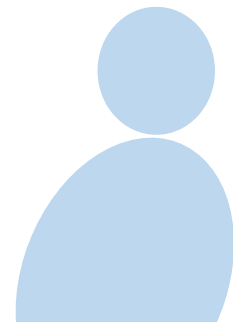
- Choose \mathcal{H}_1 if

$$\bar{\mathbf{s}}^T \bar{\mathbf{y}} > \frac{\|\bar{\mathbf{s}}\|^2}{2} - \sigma^2 \ln \frac{\pi_1}{\pi_0}$$



Performance ML *What is min P_e ?*

- The min P_e can now be determined as follows

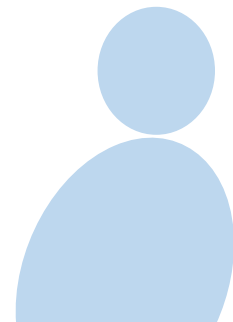


Performance ML

Probability of False Alarm

- Recall, P_{FA} is

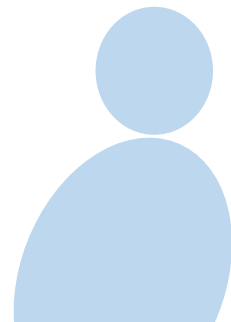
$$P_{FA} = Q\left(\frac{\gamma}{\sigma \|\bar{\mathbf{s}}\|}\right) = Q\left(\frac{\frac{\|\bar{\mathbf{s}}\|^2}{2} - \sigma^2 \ln\left(\frac{\pi_1}{\pi_0}\right)}{\sigma \|\bar{\mathbf{s}}\|}\right)$$
$$= Q\left(\frac{\|\bar{\mathbf{s}}\|^2 - 2\sigma^2 \ln\left(\frac{\pi_1}{\pi_0}\right)}{2\sigma \|\bar{\mathbf{s}}\|}\right)$$



Performance ML

- Recall, P_{FA} is

$$P_{FA} = Q\left(\frac{\gamma}{\sigma \|\bar{\mathbf{s}}\|}\right) = Q\left(\frac{\|\bar{\mathbf{s}}\|^2 - 2\sigma^2 \ln \frac{\pi_1}{\pi_0}}{2\sigma \|\bar{\mathbf{s}}\|}\right)$$



Performance ML

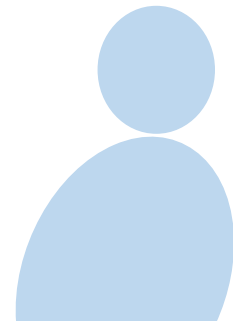
Probability of
Detection.

- Recall, P_D is

$$P_D = Q\left(\frac{\gamma - \|\bar{\mathbf{s}}\|^2}{\sigma \|\bar{\mathbf{s}}\|}\right) = Q\left(\frac{\frac{\|\bar{\mathbf{z}}\|^2}{2} - \sigma^2 \ln\left(\frac{\pi_1}{\pi_0}\right) - \|\bar{\mathbf{z}}\|^2}{\sigma \|\bar{\mathbf{z}}\|}\right)$$

$$= Q\left(\frac{-\frac{\|\bar{\mathbf{z}}\|^2}{2} - \sigma^2 \ln\left(\frac{\pi_1}{\pi_0}\right)}{\sigma \|\bar{\mathbf{z}}\|}\right)$$

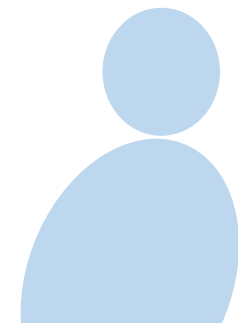
$$= Q\left(\frac{-\|\bar{\mathbf{z}}\|^2 - 2\sigma^2 \ln\left(\frac{\pi_1}{\pi_0}\right)}{\sigma \|\bar{\mathbf{z}}\|}\right)$$



Performance ML

- Recall, P_D is

$$P_D = Q\left(\frac{\gamma - \|\bar{\mathbf{s}}\|^2}{\sigma \|\bar{\mathbf{s}}\|}\right) = Q\left(\frac{-\|\bar{\mathbf{s}}\|^2 - 2\sigma^2 \ln \frac{\pi_1}{\pi_0}}{2\sigma \|\bar{\mathbf{s}}\|}\right)$$



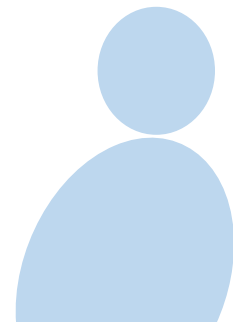
Performance ML

$$1 - Q(-x) = Q(x)$$

$$P_{MD} = 1 - P_D = 1 - Q\left(-\frac{\|\bar{z}\|^2 + 2\sigma^2 \ln\left(\frac{\pi_1}{\pi_0}\right)}{2\sigma\|\bar{z}\|}\right)$$

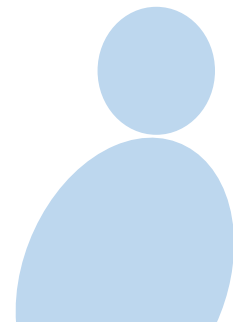
$$= Q\left(\frac{\|\bar{z}\|^2 + 2\sigma^2 \ln\left(\frac{\pi_1}{\pi_0}\right)}{2\sigma\|\bar{z}\|}\right)$$

↑
Probability of
mis detection



Performance ML

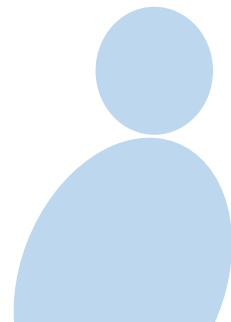
$$\begin{aligned} P_{MD} &= 1 - P_D = 1 - Q \left(\frac{-\|\bar{\mathbf{s}}\|^2 - 2\sigma^2 \ln \frac{\pi_1}{\pi_0}}{2\sigma\|\bar{\mathbf{s}}\|} \right) \\ &= Q \left(\frac{\|\bar{\mathbf{s}}\|^2 + 2\sigma^2 \ln \frac{\pi_1}{\pi_0}}{2\sigma\|\bar{\mathbf{s}}\|} \right) \end{aligned}$$



Performance ML

- Therefore, P_e is

$$\pi_0 P_{FA} + \pi_1 P_{MD}$$
$$= \pi_0 Q\left(\frac{\|\bar{z}\|^2 - 2\sigma^2 \ln\left(\frac{\pi_1}{\pi_0}\right)}{2\sigma\|\bar{z}\|}\right) + \pi_1 Q\left(\frac{\|\bar{z}\|^2 + 2\sigma^2 \ln\left(\frac{\pi_1}{\pi_0}\right)}{2\sigma\|\bar{z}\|}\right)$$



Performance ML

- Therefore, P_e is

$$\pi_0 P_{FA} + \pi_1 P_{MD}$$

Probability of Error

$$P_{FA} = \Pr(H_0) \Pr(H_1 | H_0) + \Pr(H_1) \Pr(H_0 | H_1)$$

P_{MD}

$$= \pi_0 Q\left(\frac{\|\bar{\mathbf{s}}\|^2 - 2\sigma^2 \ln \frac{\pi_1}{\pi_0}}{2\sigma \|\bar{\mathbf{s}}\|}\right) + \pi_1 Q\left(\frac{\|\bar{\mathbf{s}}\|^2 + 2\sigma^2 \ln \frac{\pi_1}{\pi_0}}{2\sigma \|\bar{\mathbf{s}}\|}\right)$$

$$= \pi_0 Q\left(\frac{\|\bar{\mathbf{s}}\|}{2\sigma} - \frac{\sigma}{\|\bar{\mathbf{s}}\|} \ln\left(\frac{\pi_1}{\pi_0}\right)\right) + \pi_1 Q\left(\frac{\|\bar{\mathbf{s}}\|}{2\sigma} + \frac{\sigma}{\|\bar{\mathbf{s}}\|} \ln\left(\frac{\pi_1}{\pi_0}\right)\right)$$

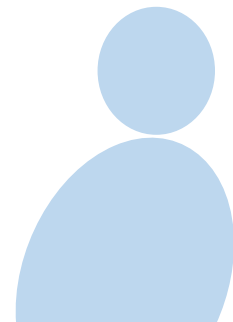
Performance ML

$$\frac{\|\tilde{z}\|}{\sigma} = \sqrt{\frac{\|\tilde{z}\|^2}{\sigma^2}} = \sqrt{\text{SNR}}.$$

- This can be simplified as

$$\text{SNR} = \frac{\|\tilde{z}\|^2}{\sigma^2}$$

$$P_e = \pi_0 Q\left(\frac{1}{2}\sqrt{\text{SNR}} - \frac{1}{\sqrt{\text{SNR}}} \ln\left(\frac{\pi_1}{\pi_0}\right)\right) + \pi_1 Q\left(\frac{1}{2}\sqrt{\text{SNR}} + \frac{1}{\sqrt{\text{SNR}}} \ln\left(\frac{\pi_1}{\pi_0}\right)\right)$$

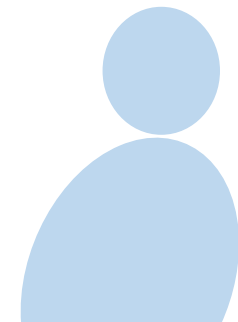


Performance ML

- This can be simplified as

Minimum Prob
of Error

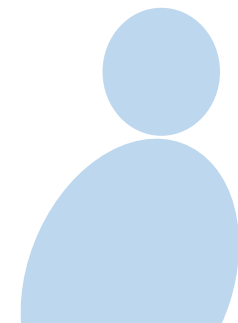
$$\pi_0 Q \left(\frac{1}{2} \sqrt{SNR} - \frac{1}{\sqrt{SNR}} \ln \frac{\pi_1}{\pi_0} \right) \\ + \pi_1 Q \left(\frac{1}{2} \sqrt{SNR} + \frac{1}{\sqrt{SNR}} \ln \frac{\pi_1}{\pi_0} \right)$$



Simple example $= 10$

$$\bar{\pi}_0 = 1 - \bar{\pi}_1 = 0.2$$

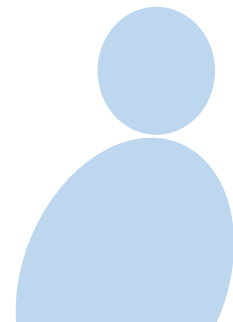
- $SNR = 10 \text{ dB}$. $\pi_1 = 0.8$
- **Minimum probability of error?**



Simple example

- We have

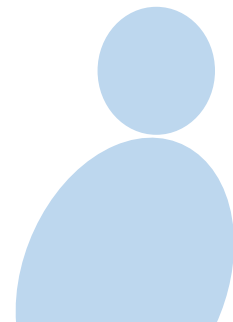
$$SNR = 10 \text{ dB} = 10$$



Simple example

- We have

$$SNR = \frac{\|\bar{s}\|^2}{\sigma^2} = 10$$



Simple example

$$\frac{\pi_1}{\pi_0} = \frac{0.8}{0.2} = 4$$

- Min P_e is

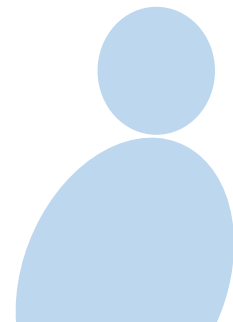
$$P_e = \pi_1 \times Q\left(\frac{1}{\sqrt{SNR}} \ln\left(\frac{\pi_1}{\pi_0}\right) + \frac{1}{2}\sqrt{SNR}\right) + \pi_0 \times Q\left(\frac{1}{2}\sqrt{SNR} - \frac{1}{\sqrt{SNR}} \ln\left(\frac{\pi_1}{\pi_0}\right)\right)$$

$$= 0.8 Q\left(\frac{1}{2}\sqrt{10} + \frac{1}{\sqrt{10}} \ln 0.4\right) + 0.2 Q\left(\frac{1}{2}\sqrt{10} - \frac{1}{\sqrt{10}} \ln 0.4\right)$$

$$= 0.8 Q(2.01) + 0.2 Q(1.14)$$

$$= 0.0408$$

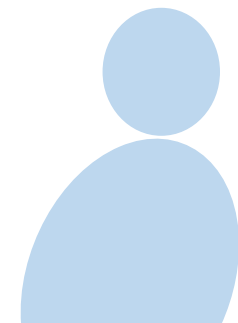
Min P_e .



Simple example

- Min P_e is

$$\begin{aligned} & \pi_1 \times Q\left(\frac{1}{\sqrt{SNR}} \ln\left(\frac{\pi_1}{\pi_0}\right) + \frac{1}{2} \sqrt{SNR}\right) + \pi_0 \times Q\left(\frac{1}{2} \sqrt{SNR} - \frac{1}{\sqrt{SNR}} \ln\left(\frac{\pi_1}{\pi_0}\right)\right) \\ &= 0.8 \times Q\left(\frac{1}{\sqrt{10}} \ln(4) + \frac{1}{2} \sqrt{10}\right) + 0.2 \times Q\left(\frac{1}{2} \sqrt{10} - \frac{1}{\sqrt{10}} \ln(4)\right) \\ &= 0.8 \times Q(2.01) + 0.2 \times Q(1.14) = 0.0408 \end{aligned}$$



Instructors may use this white area (14.5 cm / 25.4 cm) for the text.
Three options provided below for the font size.

Font: Avenir (Book), Size: 32, Colour: Dark Grey

Font: Avenir (Book), Size: 28, Colour: Dark Grey

Font: Avenir (Book), Size: 24, Colour: Dark Grey

Do not use the space below.

