Assignment - 5

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Que 1:- Express the following as an SOCP

$$\min c^T x \quad (1) \quad \text{st } x^T x \le yz$$
$$y^2 + z^2 \le 1$$
$$y \ge 0, z \ge 0$$
$$x \in \mathbb{R}^n, y, z \in \mathbb{R}$$

Soln:- General SOCP form

$$\min_x f^T x$$

Given $\Rightarrow$ $x^T x \le yz$

$$\|x\|^2 \le yz$$
$$4\|x\|^2 \le 4yz$$
$$4\|x\|^2 \le (y+z)^2 - (y-z)^2$$
$$4\|x\|^2 + (y-z)^2 \le (y+z)^2$$

$$\Rightarrow \left\| \begin{bmatrix} 2\|x\| \\ y-z \end{bmatrix} \right\| \le y+z$$

$$\bar{x}^T \bar{x} \le yz$$

$$\Rightarrow \left\| \begin{bmatrix} 2\bar{x} \\ y-z \end{bmatrix} \right\| \le y+z \qquad y\ z \ge 0$$

Que 2:- formulate the following as SOCP

(a) $$\max \left(\sum_{i=1}^{m} 1/(a_i^T x - b_i) \right)^{-1} \quad -(1)$$

$$\text{st. } a_i^T x - b_i \ge 0$$

$$= \min \sum_{i=1}^{m} \frac{1}{a^T x - b_i}$$

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$$\min \sum_{i=1}^{n} t_i$$

for all $t_i \geq 0$ $(i=1,\dots,n)$

$t_i(a_i^T x - b_i) \geq 1$

epigraph form

$$\frac{1}{a_i^T x - b_i} \leq t_i$$

$\Rightarrow 1 \leq (a_i^T x - b_i) t_i$

~~b min~~

Ques 3:- Solve the least norm problem

$$\min \|x\|_2 \quad \text{st } Ax = b$$

where $A \in R^{m \times n}$ with $m < n$ & $b \in R(A)$

Soln:- $\min \|\bar{x}\|_2^2$

$= \bar{x}^T x \quad \text{st } Ax = b$

Using lagrangian duality

$F(\bar{x}, \lambda) = \bar{x}^T \bar{x} + \bar{\lambda}^T (A\bar{x} - \bar{b})$

$F = \bar{x}^T \bar{x} + \bar{\lambda}^T (A\bar{x} - \bar{b})$

$\nabla F = 2\bar{x} + A^T \bar{\lambda} - 0 = 0$ (// setting gradient as 0)

$\Rightarrow \bar{x} = -\frac{1}{2} A^T \bar{\lambda}$ — (1)

finding $\bar{\lambda}$ using constraint

$A\bar{x} = \bar{b}$

$\Rightarrow A(-\frac{1}{2} A^T \lambda) = \bar{b}$

$\Rightarrow -\frac{1}{2}(AA^T)\bar{\lambda} = b$

$\Rightarrow \bar{\lambda} = -2(AA^T)^{-1} b$ — (2)

Substituting of (6) in (1)

$$\bar{x} = -\frac{1}{2} A^T \bar{\lambda}$$

$$= -\frac{1}{2} A^T \left(-2 (AA^T)^{-1} \bar{b} \right)$$

$$\hat{x} = A^T (AA^T)^{-1} \bar{b}$$ // least norm soluⁿ