You

What happens to the Gaussian discriminant analysis classifier when the prior probabilities of C_0 is p_0 and C_1 is p_1?

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In Gaussian Discriminant Analysis (GDA), the prior probabilities of classes C_0 and C_1 influence the classification process. GDA assumes that the data from each class are normally distributed, and it models the distribution of each class with a Gaussian (normal) distribution. The classifier computes the probability of a sample belonging to each class given its features, and then assigns the sample to the class with the highest probability.

When the prior probabilities of classes C_0 and C_1 are denoted as p_0 and p_1 respectively, the classifier adjusts the decision boundary to reflect these probabilities. Mathematically, it means incorporating the prior probabilities into the computation of class conditional probabilities.

To be specific, let's denote $p(x|C_i)$ as the probability density function (PDF) of the features x given class C_i. In GDA, this PDF is modeled using Gaussian distributions. Then, the posterior probability of class C_i given features x can be computed using Bayes' theorem:

$$P(C_i|x) = \frac{p(x|C_i) \times P(C_i)}{p(x)}$$

Where:

$P(C_i|x)$ is the posterior probability of class C_i given features x.
$p(x|C_i)$ is the likelihood of observing features x given class C_i.
$P(C_i)$ is the prior probability of class C_i.
$p(x)$ is the evidence, the probability of observing features x.

In Gaussian Discriminant Analysis:

$p(x|C_i)$ is modeled as a Gaussian distribution for each class.
$P(C_i)$ is the prior probability of class C_i.
$p(x)$ can be computed as $p(x) = \sum_{i=1}^{n} P(C_i) \times p(x|C_i)$, where n is the number of classes.

The decision boundary is determined by comparing the posterior probabilities for each class and assigning the sample to the class with the highest posterior probability.

In summary, when the prior probabilities of classes C_0 and C_1 are p_0 and p_1 respectively, they are incorporated into the computation of posterior probabilities, affecting the decision boundary of the classifier. This adjustment allows the classifier to account for class imbalances and varying prior knowledge about the classes.