EE910: Digital Communication Systems-I

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Lecture #4A: Continuous-Phase Frequency-Shift Keying (CPFSK)

Continuous-Phase Frequency-Shift Keying (CPFSK)

- We consider a class of digital modulation methods in which the phase of the signal is constrained to be continuous.
- This constraint results in a phase or frequency modulator that has memory.
- To represent a CPFSK signal, we begin with a PAM signal

$$d(t) = \sum_{n} I_n g(t - nT) \tag{1}$$

where $\{I_n\}$ denotes the sequence of amplitudes obtained by mapping k-bit blocks of binary digits from the information sequence $\{a_n\}$ into the amplitude levels $\pm 1, \pm 3, \cdots, \pm(M-1)$ and $g(t)$ is a rectangular pulse of amplitude $1/2T$ and duration T seconds.

Continuous-Phase Frequency-Shift Keying (CPFSK)

- The signal $d(t)$ is used to frequency-modulate the carrier. Consequently, the equivalent lowpass waveform $\upsilon(t)$ is expressed as

$$\upsilon(t) = \sqrt{\frac{2\varepsilon}{T}} e^{j\left[4\pi T f_d \int_{-\infty}^{t} d(\tau) d\tau + \phi_0\right]} \tag{2}$$

where f_d is the peak frequency deviation and ϕ_0 is the initial phase of the carrier.

- The carrier-modulated signal corresponding to Equation (2) may be expressed as

$$s(t) = \sqrt{\frac{2\varepsilon}{T}} cos\left[2\pi f_c t + \phi(t; \mathbf{I}) + \phi_0\right] \tag{3}$$

where $\phi(t; \mathbf{I})$ represents the time-varying phase of the carrier.

Continuous-Phase Frequency-Shift Keying (CPFSK)

- We have

$$\begin{aligned}\phi(t;\mathbf{I}) &= 4\pi T f_d \int_{-\infty}^{t} d(\tau)d\tau \\ &= 4\pi T f_d \int_{-\infty}^{t} \left[\sum_n I_n g(\tau - nT)\right] d\tau\end{aligned} \tag{4}$$

- Although $d(t)$ contains discontinuities, the integral of $d(t)$ is continuous. Hence, we have a continuous-phase signal.
- The phase of the carrier in the interval $nT \leq t \leq (n+1)T$ is determined by integrating Equation (4).

Continuous-Phase Frequency-Shift Keying (CPFSK)

- Thus,

$$\begin{aligned}\phi(t;I) &= 2\pi f_d T \sum_{k=-\infty}^{n-1} I_k + 2\pi h I_n q(t-nT) \\ &= \theta_n + 2\pi h I_n q(t-nT)\end{aligned} \tag{5}$$

where h, θ_n , and $q(t)$ are defined as

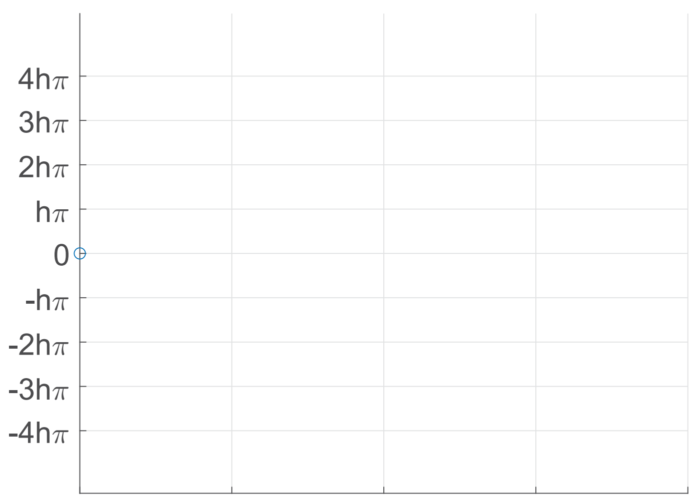
$$\begin{aligned} h &= 2f_d T \\ \theta_n &= \pi h \sum_{k=-\infty}^{n-1} I_k \\ q(t) &= \begin{cases} 0 & t < 0 \\ \frac{t}{2T} & 0 \leq t \leq T \\ \frac{1}{2} & t > T \end{cases}\end{aligned} \tag{6}$$

- θ_n represents the accumulation (memory) of all symbols up to time $(n-1)T$.
- The parameter h is called the modulation index.

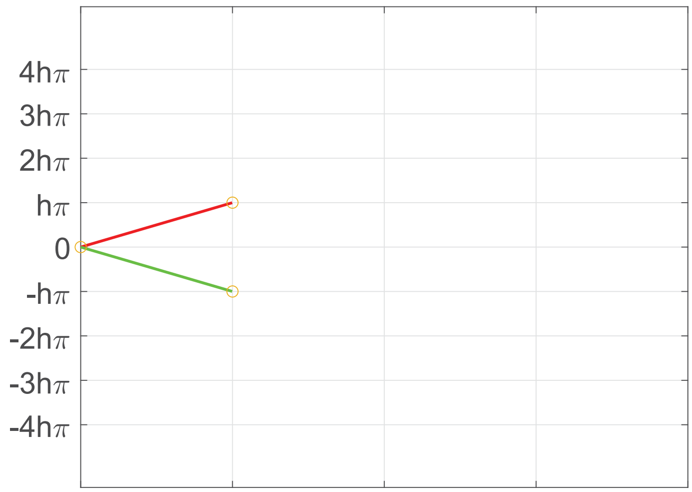
Continuous-Phase Modulation (CPM)

- One can sketch the set of phase trajectories $\phi(t; I)$ generated by all possible values of the information sequence $\{I_n\}$.
- Consider the case of CPFSK with binary symbols $I_n = \pm 1$, the set of phase trajectories beginning at time $t = 0$:

Continuous-Phase Modulation (CPM)



Continuous-Phase Modulation (CPM)



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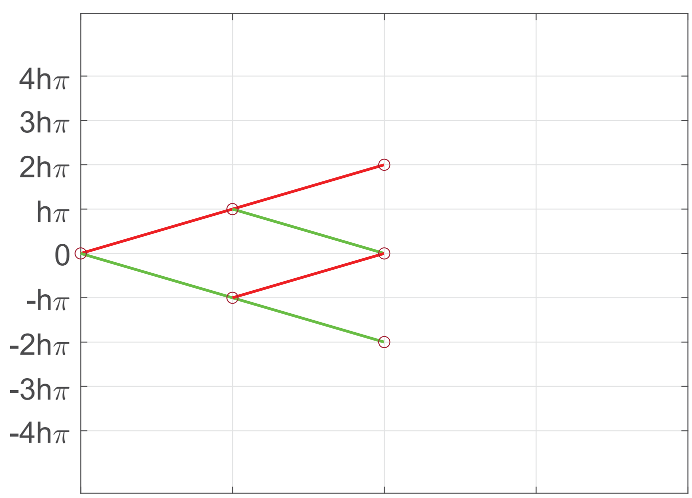
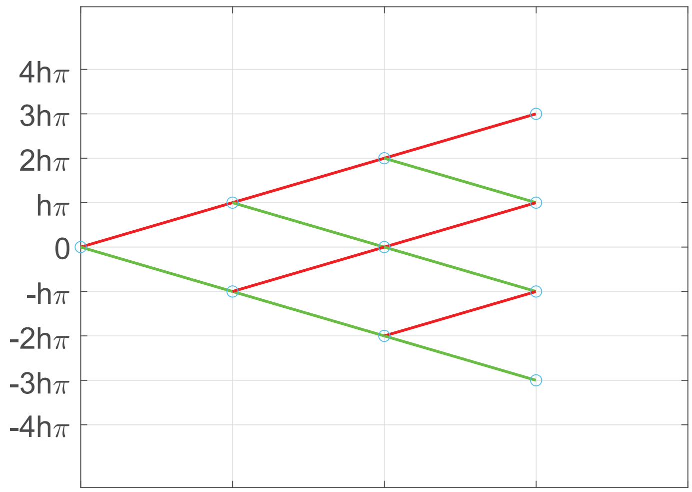
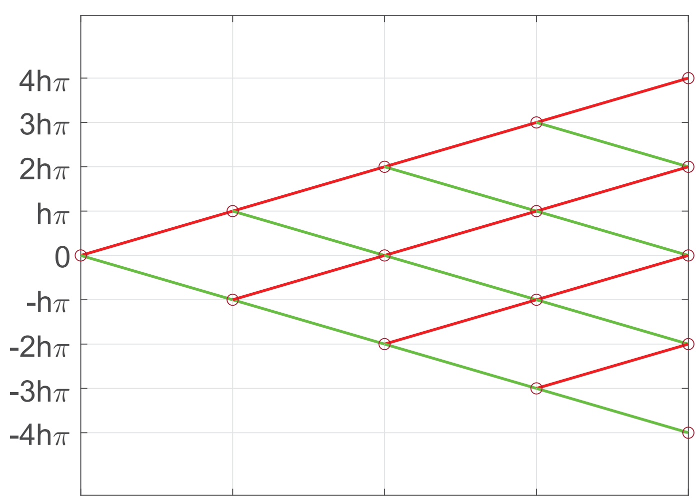


Continuous-Phase Modulation (CPM)



Continuous-Phase Modulation (CPM)



Minimum-Shift Keying (MSK)

- MSK is a special form of binary CPFSK (and, therefore, CPM) in which the modulation index $h = \frac{1}{2}$ and $g(t)$ is a rectangular pulse of duration T.
- The phase of the carrier in the interval $nT \leq t \leq (n+1)T$is

$$\begin{aligned}\phi(t; I) &= \frac{1}{2}\pi \sum_{k=-\infty}^{n-1} I_k + \pi I_n q(t - nT) \\ &= \theta_n + \frac{1}{2}\pi I_n(\frac{t - nT}{T}), \qquad nT \leq t \leq (n+1)T\end{aligned} \tag{7}$$

and the modulated carrier signal is

$$\begin{aligned}s(t) &= Acos\left[2\pi f_c t + \theta_n + \frac{1}{2}\pi I_n(\frac{t-nT}{T})\right] \\ &= Acos\left[2\pi(f_c + \frac{1}{4T}I_n)t - \frac{1}{2}n\pi I_n + \theta_n\right], \qquad nT \leq t \leq (n+1)T\end{aligned} \tag{8}$$

Minimum-Shift Keying (MSK)

- Equation (8) indicates that the binary CPFSK signal can be expressed as a sinusoid having one of two possible frequencies in the interval $nT \leq t \leq (n+1)T$.
- If we define these frequencies as

$$\begin{aligned}f_1 &= f_c - \frac{1}{4T} \\ f_2 &= f_c + \frac{1}{4T}\end{aligned} \tag{9}$$

then the binary CPFSK signal given by Equation (8) may be written in the form

$$s_i(t) = Acos\left[2\pi f_i t + \theta_n + \frac{1}{2}n\pi(-1)^{i-1}\right], \qquad i = 1, 2 \tag{10}$$

which represents an FSK signal with frequency separation of $\triangle f = f_2 - f_1 = 1/2T$.

Minimum-Shift Keying (MSK)

- Recall that $\Delta f = 1/2T$ is the minimum frequency separation needed to ensure orthogonality of signals $s_1(t)$ and $s_2(t)$ over a signalling interval of length T.
- This is why binary CPFSK with $h = \frac{1}{2}$ is called minimum shift keying (MSK).

Minimum Shift Keying

- In an MSK signal, the initial state for the phase is either 0 or π rad. Determine the terminal phase state for the following four input pairs of input data:
  1. 00
  2. 01
  3. 10
  4. 11
- We assume that the input bits 0,1 are mapped to the symbols -1 and 1 respectively. The terminal phase of an MSK signal at time instant n is given by

$$\theta(n; \mathbf{a}) = \frac{\pi}{2} \sum_{k=0}^{k} a_k + \theta_0$$

where θ_0 is the initial phase and a_k is ± 1 depending on the input bit at the time instant k.

Minimum Shift Keying

- The following table shows $\theta(n; \mathbf{a})$ for two different values of $\theta_0(0, \pi)$, and the four input pairs of data: $\{00, 01, 10, 11\}$.

θ_0	b_0	b_1	a_0	a_1	$\theta(n; \mathbf{a})$
0	0	0	-1	-1	$-\pi$
0	0	1	-1	1	0
0	1	0	1	-1	0
0	1	1	1	1	π
π	0	0	-1	-1	0
π	0	1	-1	1	π
π	1	0	1	-1	π
π	1	1	1	1	2π