

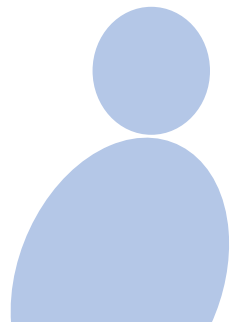
Online Example

Online Estimation

- Consider the problem

$$\bar{\mathbf{y}} = \begin{bmatrix} -2 \\ -3 \\ 1 \\ -2 \end{bmatrix}, \mathbf{X} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -1 & -1 \\ -1 & 1 \end{bmatrix}$$

Pilot matrix

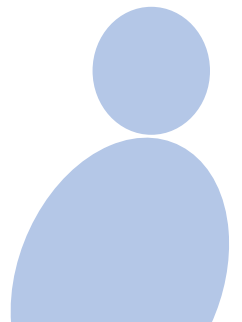


Online Example

- Let $\sigma^2 = 4$. Error covariance is

Error covariance
at $N=4$.

$$\begin{aligned} \mathbf{P}(N) &= \sigma^2 (\mathbf{X}^T \mathbf{X})^{-1} = \mathbf{I} \\ &= 4 \left(4\mathbf{I} \right)^{-1} = 4 \times \frac{1}{4} \mathbf{I} \\ &= \mathbf{I} = \mathbf{P}(4). \end{aligned}$$



Online Example

- Consider now a new input-output

$$y(5) = -2, \bar{\mathbf{x}}(5) = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

output *input*



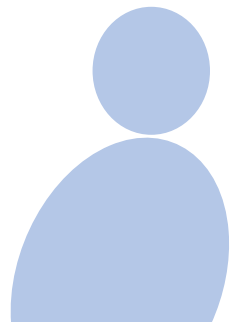
Online Example

Gain $N+1=5$

- The estimate $\hat{\mathbf{h}}(5)$ can be evaluated as follows

$$\bar{\mathbf{k}}(5) = \frac{\mathbf{P}(N)\bar{\mathbf{x}}(N+1)}{\sigma^2 + \bar{\mathbf{x}}^T(N+1)\mathbf{P}(N)\bar{\mathbf{x}}(N+1)}$$

$$= \frac{\mathbf{I} \times \begin{bmatrix} -2 \\ 2 \end{bmatrix}}{4 + \begin{bmatrix} -2 & 2 \end{bmatrix} \mathbf{I} \times \begin{bmatrix} -2 \\ 2 \end{bmatrix}} = \frac{1}{12} \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$



Error Covariance Update

- The error covariance update for time $N + 1$ is

$$P(N+1) = (I - \bar{K}(N+1)\bar{x}^T(N+1))P(N)$$

$$P(5) = (I - \bar{K}(5)\bar{x}^T(5))P(4).$$



Error Covariance Update

- The error covariance update for time $N + 1$ is

$$\mathbf{P}(N + 1) = \left(\mathbf{I} - \bar{\mathbf{k}}(N + 1)\bar{\mathbf{x}}^T(N + 1) \right) \mathbf{P}(N)$$



Error Covariance Update

- The error covariance for time $N + 1$ can be evaluated as

$$\mathbf{P}(N + 1) = \left(\mathbf{I} - \bar{\mathbf{k}}(N + 1) \bar{\mathbf{x}}^T(N + 1) \right) \mathbf{P}(N)$$

$$\begin{aligned} &= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{1}{12} \begin{bmatrix} -2 \\ 2 \end{bmatrix} \begin{bmatrix} -2 & 2 \end{bmatrix} \right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{Error covariance } N+1=5 \\ &= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix} \right) = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} = \mathbf{P}(5). \end{aligned}$$

Error Covariance Update

- The error covariance for time $N + 1$ can be evaluated as

$$\begin{aligned} \mathbf{P}(N + 1) &= \left(\mathbf{I} - \bar{\mathbf{k}}(N + 1) \bar{\mathbf{x}}^T(N + 1) \right) \mathbf{P}(N) \\ &= \left(\mathbf{I} - \frac{1}{12} \begin{bmatrix} -2 \\ 2 \end{bmatrix} \begin{bmatrix} -2 & 2 \end{bmatrix} \right) \mathbf{I} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} \end{aligned}$$

Handwritten notes in red:
 $\uparrow h(5) = \text{Estimate at } N+1=5$
 $\uparrow p(5) = \text{Error cov at } N+1=5$

