eMasters in Communication Systems

Prof. Aditya Jagannatham

Core Module:

Wireless Communication

Chapter 4

OFDM Technology

ORTHOGONAL
FREQUENCY
DIVISION
MULTIPLEXING.

OFDM

One of the most widely used technologies.

- Orthogonal Frequency Division Multiplexing (OFDM) is one of the most **extensively used** wireless technologies

MIMO + OFDM = MIMO−OFDM :
4G. 5G.

- OFDM is used in 4G LTE (**Long Term Evolution**), 5G NR (***New Radio***)
- Wi-Fi – **802.11n, 802.11 ac, 802.11ax**…

OFDM

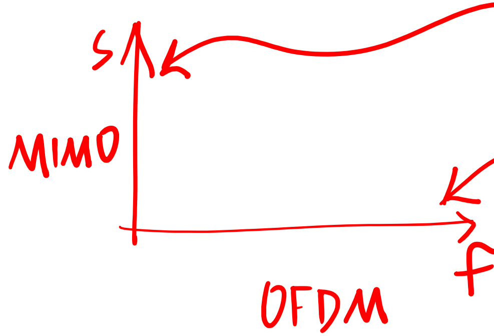
Transmission over
Very Large Bandwidth.

- OFDM is widely employed in most of the **modern cellular** and **Wi-Fi systems.**
- OFDM enables **ultrahigh** Data Rates. .

Space + Frequency
Multiplexing
⇒ Extraordinarily
high data
rates!!!

space Dimension

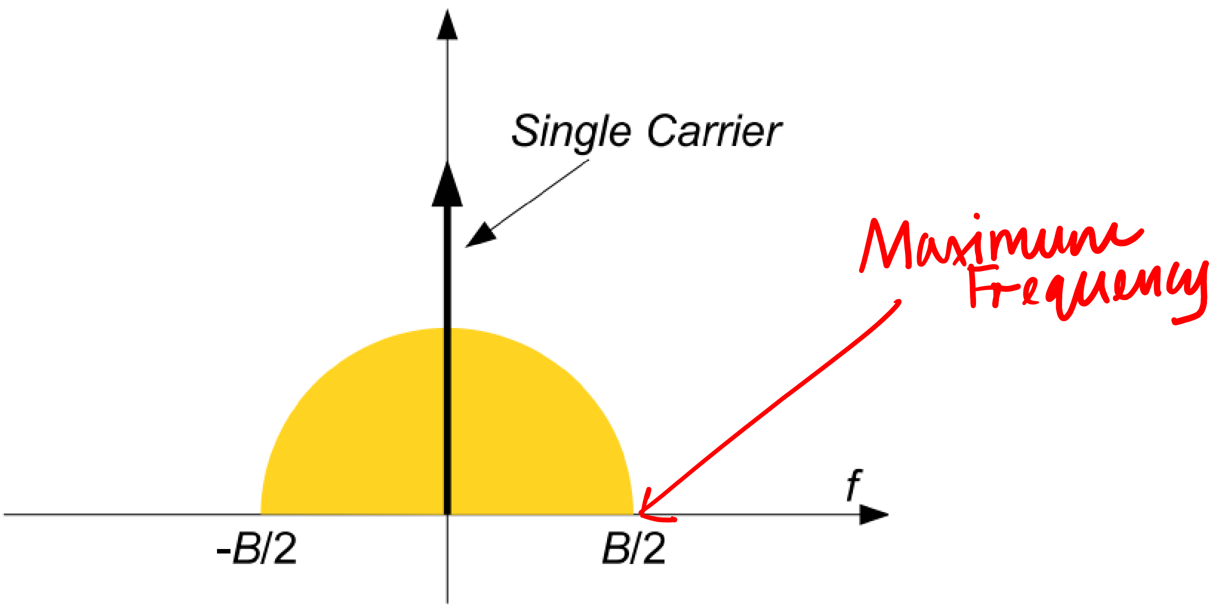
Frequency
dimension



Single Carrier Modulation

- Consider bandwidth $\frac{B}{2}$ and a ***single*** Carrier.

- Symbol duration = $\frac{1}{B}$



Single Carrier Modulation

$B = 10 \times 10^{6}\,Hz$

- Consider single carrier system with **bandwidth** $B = 10\ MHz$.

Symbol Duration

- The corresponding ***symbol duration*** is

$$\frac{1}{B} = \frac{1}{10 \times 10^{6}} = 0.1 \times 10^{-6} = 0.1\,\mu s.$$

Single Carrier Modulation

0.1 μs.

Larger Bandwidths - $\Rightarrow$ Smaller symbol Duration

- The symbol duration above is ***extremely small***.
- What happens when the **symbol duration** is extremely small?

Bandwidth $\uparrow$ $\rightarrow$ Symbol Duration $\downarrow$

Multipath Propagation

- Recall, the wireless channel is characterized by MULTIPATH PROPAGATION.

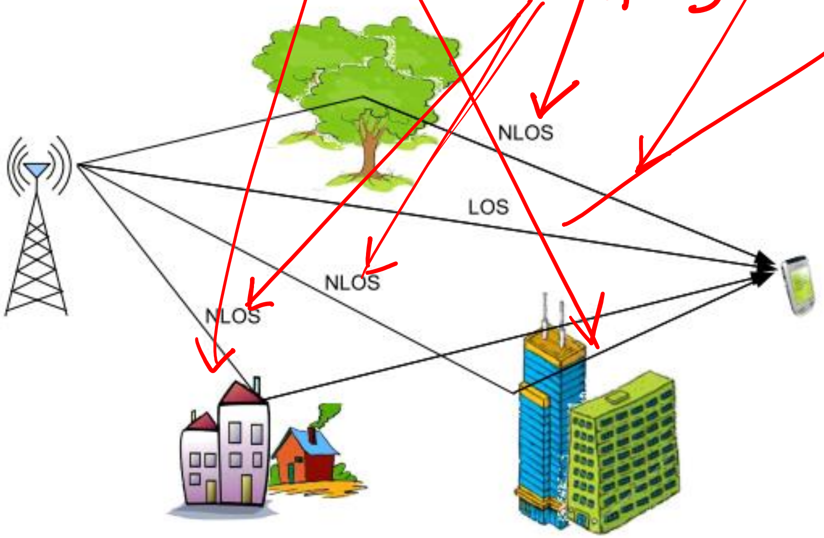
$$\overbrace{\tau_0, \tau_1, \tau_2, \ldots, \tau_{L-1}}^{\text{Delays}}$$

- Different copies arriving with DIFFERENT DELAYS!.

SCATTERERS.

Non Line of Sight

LINE OF SIGHT



Multipath Propagation

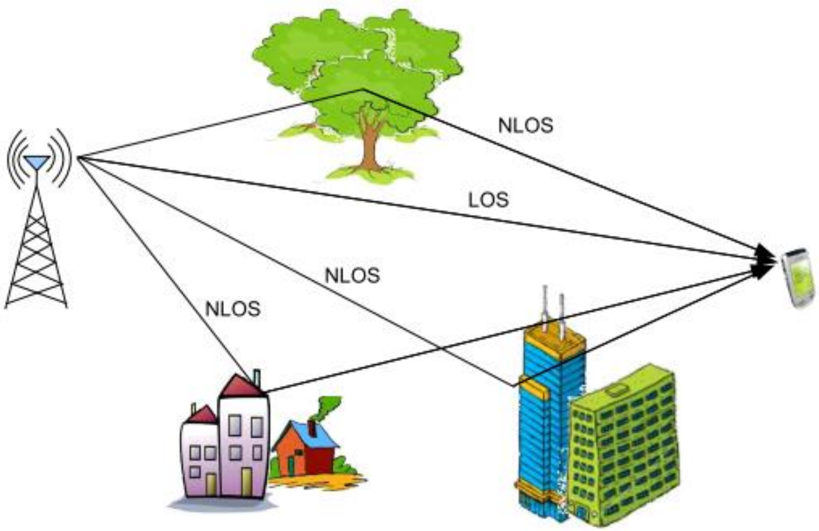
$\uparrow_{\tau_0} \quad \uparrow_{\tau_1} \quad \tau_2 \uparrow \cdots \uparrow_{\tau_{L-1}}$

components are spread over Time.

$\delta(t-\tau_i) \Rightarrow$ Delay of τ_i

- As a result the multipath components are **spread over time** $\approx$ 2-3 μs.
- This is termed as the DELAY SPREAD OF WIRELESS CHANNEL.

Large BW $\Rightarrow$ Delay Spread >> Symbol Time!!



Multipath Propagation

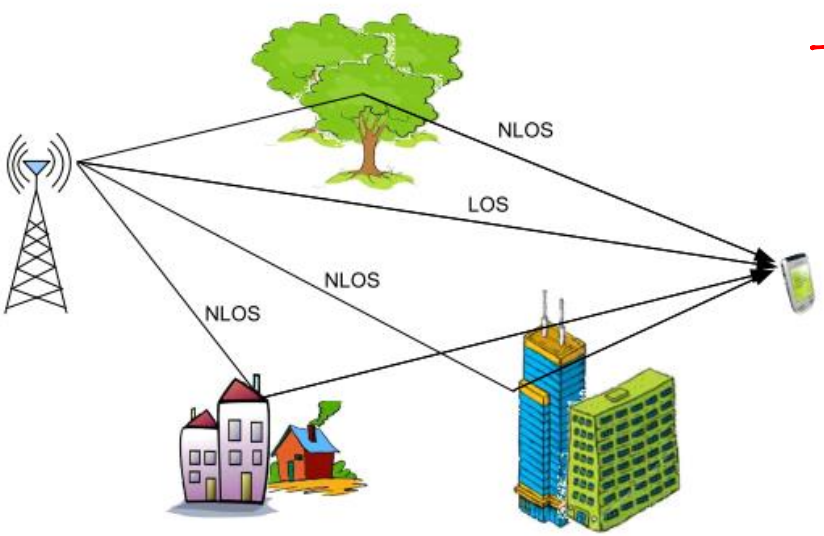
+ Delay spread << symbol Time.

- The signal copies with *different delays* superpose at the receiver
- This leads to Inter symbol interference (ISI).

⇒ Significant Distortion!!

⇒ BER VERY HIGH

⇒ INFORMATION LOST!!

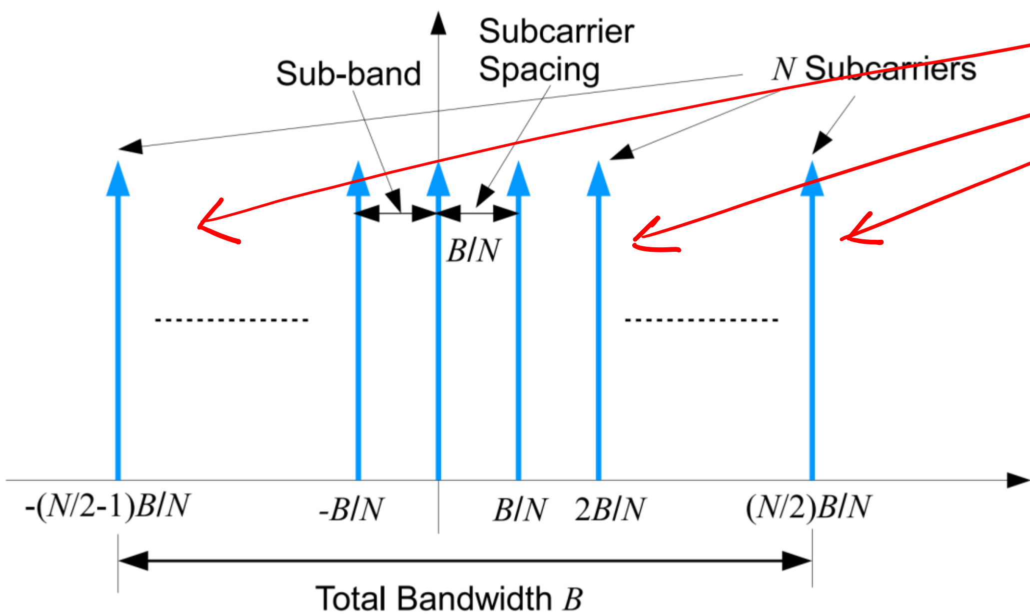


Multicarrier Modulation

ELIMINATE ISI? — PRECISELY WHAT OFDM ACHIEVES.

- How to ***avoid ISI***?
- Instead of using ~~one~~ Single carrier carrier, use N SUB CARRIERS

⇒ MULTIPLE CARRIES ⇒ MULTI CARRIER MODULATION.



MULTIPLE CARRIERS.

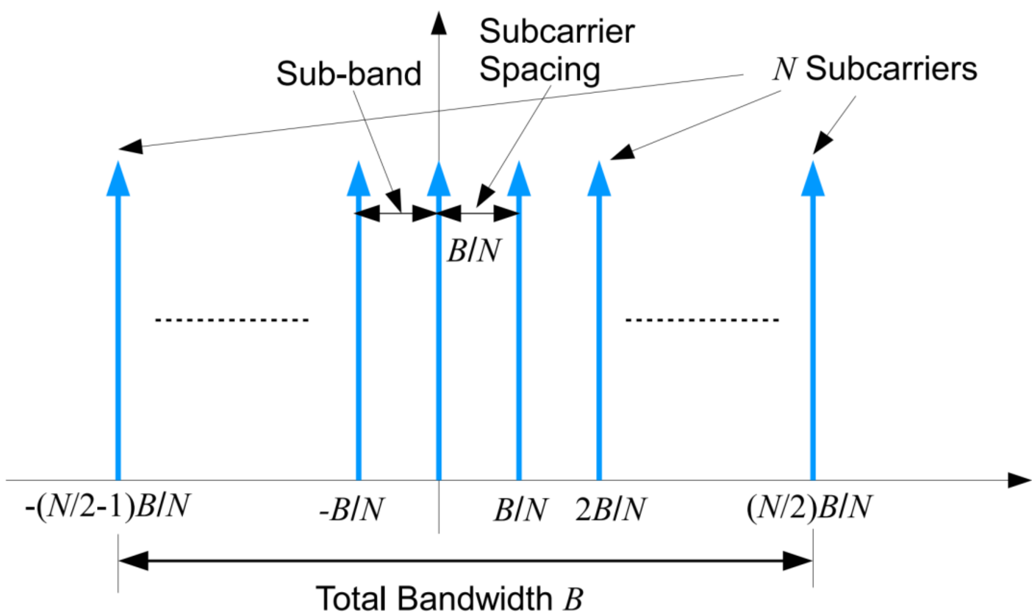
Multicarrier Modulation

multicarrier modulation

- This is also known as ***multicarrier modulation***. (MCM)
- Divide the bandwidth into N SUBBANDS.
- Width of each subband is $\frac{B}{N}$.

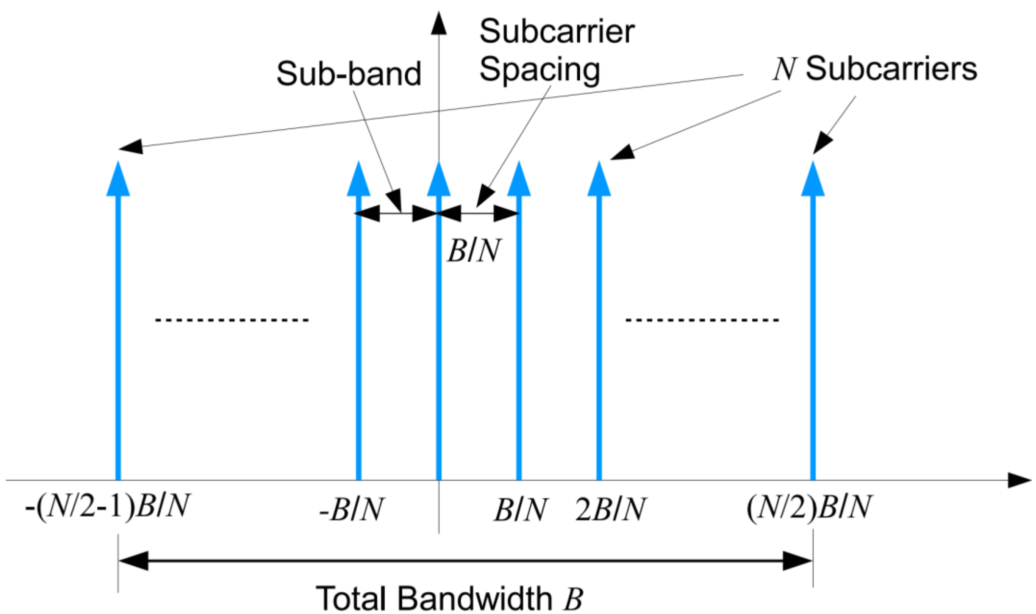
BW OF each subband.

B = Total BW.



Multicarrier Modulation - Example

- $B = 10\ MHz$ $= 10 \times 10^6\ Hz$
- $N = 1000$ subcarriers
- What is the **bandwidth** of each subcarrier? $= \frac{B}{N}$



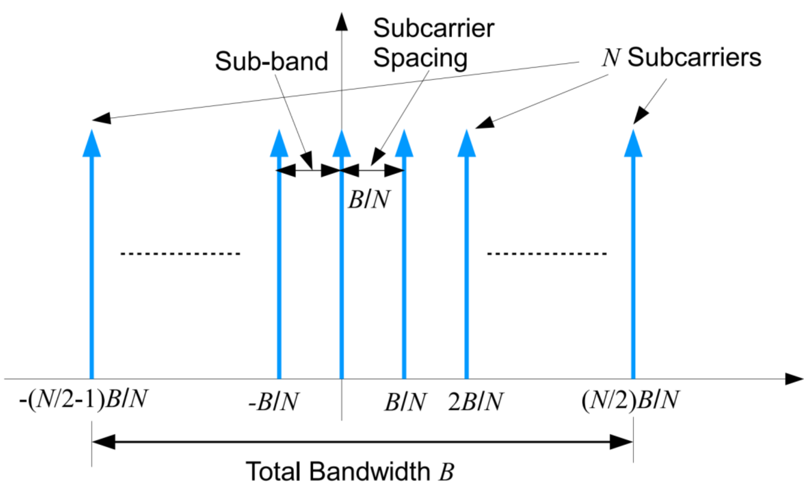
Multicarrier Modulation - Example

- $B = 10\ MHz$
- $N = 1000$
- What is the width of each **subcarrier**?

BW of each subcarrier

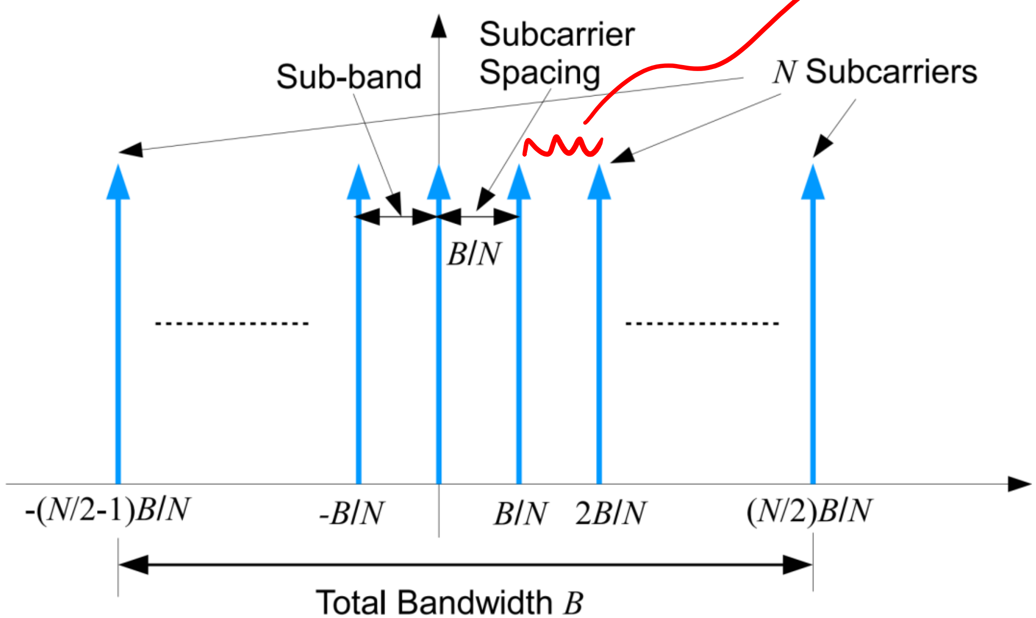
$$\frac{B}{N} = \frac{10 \times 10^6\ Hz}{1000} = 10 \times 10^3\ Hz$$

$= 10\ kHz$



Multicarrier Modulation - Example

$$\frac{B}{N} = \frac{10\ MHz}{1000} = \underline{\underline{10\ kHz}}.$$

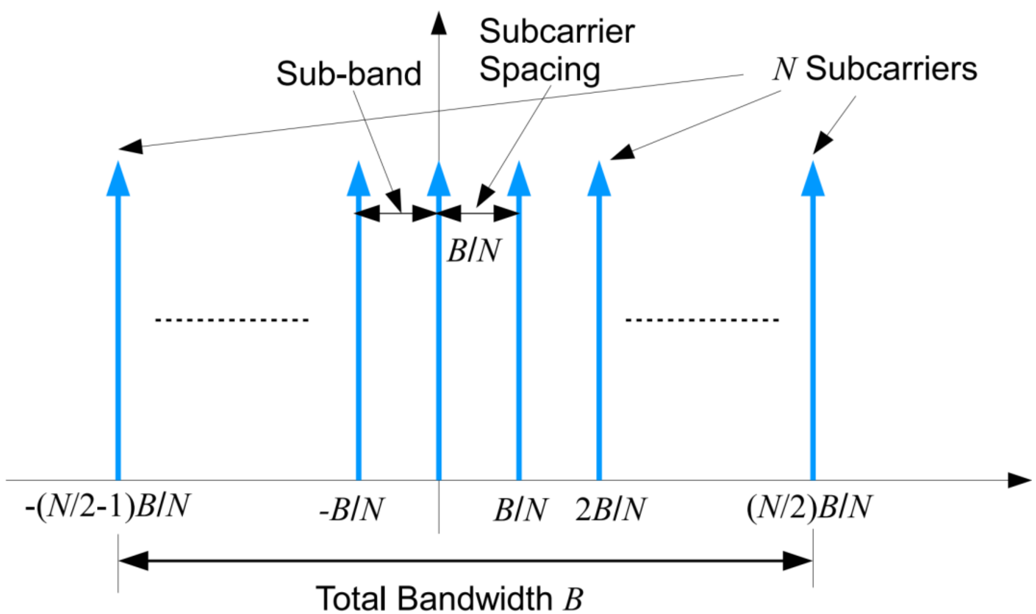


Carrier spacing
= BW of each subcarrier
$= \frac{B}{N}. = \underline{10\ kHz}.$

Multicarrier Modulation - Example

- What is the ***subcarrier spacing*** $= \frac{B}{N}$?

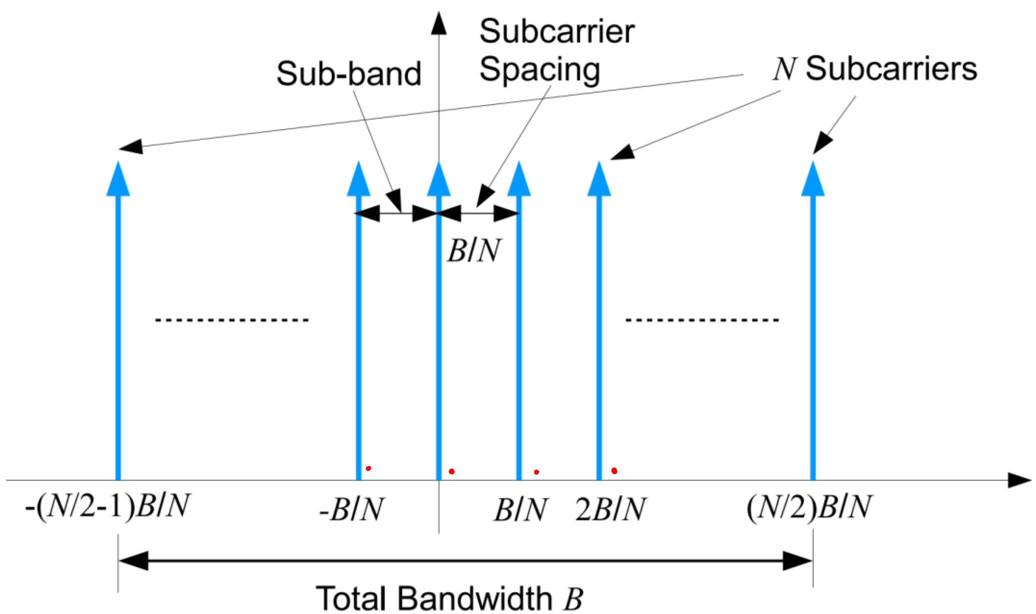
$\frac{B}{N}$.



Multicarrier Modulation

- Subcarriers are placed at integer multiples of $\frac{B}{N}$.

$\ldots, -\frac{2B}{N}, -\frac{B}{N}, 0, \frac{B}{N}, \frac{2B}{N}, \frac{3B}{N}, \ldots$



Multicarrier Modulation

- $\frac{B}{N} = f_0$
- Subcarriers are placed at

Number of subcarriers $= N$.

integer multiples of f_0.

$f_0 = \frac{B}{N}$.

$\ldots\ldots, -2f_0, -f_0, 0, f_0, 2f_0, 3f_0, \ldots$

Multicarrier Modulation

- $\frac{B}{N} = f_0$
- Subcarriers are placed at

$$\ldots, -2f_0, -f_0, 0, f_0, 2f_0, \ldots$$

Multicarrier Modulation

k^{th} subcarrier

- kth **Subcarrier** is placed at kf_0

$$e^{j2\pi k f_0 t}$$

Carrier Freq. F_c $\Rightarrow$ $e^{j2\pi F_c t}$

Multicarrier Modulation

On each subcarrier modulate symbol.

- **Modulate** the symbol X_k on the kth subcarrier

$$\underbrace{X_k \cdot e^{j2\pi k F_0 t}}_{\text{Symbol } X_k \text{ modulated on } k^{th} \text{ subcarrier.}}$$

Multicarrier Modulation

- Take the ***sum of the signals*** across all the subcarriers.
- The **transmit signal** is given as

$$x(t) = \underbrace{\overbrace{\sum_{k} X_k e^{j2\pi k F_0 t}}^{\text{Sum across all subcarriers.}}}_{\text{Transmit Signal.}}$$

$\Rightarrow$ Multicarrier Modulated Signal.

Symbol on subcarrier k.

Subcarrier k.

$$x(t) = \sum_{k} X_k e^{j2\pi k f_0 t}$$

Subcarrier index

Symbol.
LOADED on subcarrier k.

Multicarrier Demodulation

At Receiver

- Consider a ***noiseless scenario***

$$\underbrace{y(t) = x(t) = \sum_{k} X_k e^{j2\pi k f_0 t}}_{\text{ignoring noise for simplicity!!!}} = y(t)$$

Multicarrier Demodulation

EXISTS FOR COTINUOS - PERIODIC SIGNALS -

- Note that $\sum_k X_k e^{j2\pi k f_0 t}$ is the FOURIER SERIES expansion of $x(t)$.

- How to extract the coefficient X_l?

Multicarrier Demodulation

- The coefficient X_l can be **extracted** as follows

$$X_l = F_0 \int_{-\frac{1}{2F_0}}^{\frac{1}{2F_0}} y(t) e^{-j2\pi l F_0 t} dt$$

$\ldots, -2F_0, -F_0, 0, F_0, 2F_0, 3F_0$

kF_0 Harmonic $\Rightarrow$ Multiple of F_0.

$T = \frac{1}{F_0}$ = Fundamental Period.

$$= \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} y(t) e^{-j2\pi l F_0 t} dt$$

Multicarrier Demodulation

- The coefficient $X(l)$ can be **extracted** as follows

$f_0 = \frac{B}{N}$

integrating over one period $= T = \frac{1}{f_0}$.

$$X_l = f_0 \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} y(t)e^{-j2\pi l f_0 t}\,dt$$

$T = \frac{1}{f_0} = \frac{N}{B}$.

OFDM Symbol Duration.

Multicarrier Demodulation

OFDM Symbol Time

$$= \frac{N}{B} = N \times \frac{1}{B}.$$

$N \sim 10^3$

$\Rightarrow \frac{N}{B} >>$ Delay spread.

$\Rightarrow$ NO ISI.

Thus OFDM avoids intersymbol interference.

Multicarrier Demodulation

- To extract the lth symbol X_l.

$$\underbrace{f_0\int_0^{\frac{1}{f_0}} \overbrace{\left(\sum_k X_k e^{j2\pi k f_0 t}\right)}^{y(t)} e^{-j2\pi l f_0 t}\,dt}$$

$$= \underbrace{f_0 \cdot \int_0^{\frac{1}{f_0}} y(t)\, e^{-j2\pi l f_0 t}\,dt}_{\text{Demodulation}}$$

Coherent Demod.

Multicarrier Demodulation

$$F_0 \int_0^{\frac{1}{F_0}} y(t)\, e^{-j2\pi \ell F_0 t}\, dt$$

$$= F_0 \int_0^{\frac{1}{F_0}} \left(\sum_k X_k\, e^{j2\pi k F_0 t} \right) e^{-j2\pi \ell F_0 t}\, dt$$

$$= \sum_k X_k \underbrace{\left(F_0 \int_0^{\frac{1}{F_0}} e^{j2\pi (k-\ell) F_0 t}\, dt \right)}_{= \begin{cases} 0 & \text{if } k \neq \ell \\ 1 & \text{if } k = \ell \end{cases}}$$

$$= \sum_k X_k\, \delta(k-\ell)$$

$$= X_\ell$$

Multicarrier Demodulation

$$f_0 \int_0^{\frac{1}{f_0}} e^{j2\pi(k-\ell)f_0 t}\, dt = \begin{cases} 0 & \text{if } k \neq \ell \\ 1 & \text{if } k = \ell \end{cases}$$

$\Rightarrow$ Subcarriers are ORTHOGONAL!

inner product of 2 different subcarriers $= 0$.

$$f_0 \int_0^{\frac{1}{f_0}} \left(\sum_k X_k e^{j2\pi k f_0 t}\right) e^{-j2\pi l f_0 t}\, dt$$

$$= f_0 \int_0^{\frac{1}{f_0}} \left(\sum_k X_k e^{j2\pi (k-l) f_0 t}\right) dt$$

ABLE TO RECOVER X_ℓ.

$$= \sum_k X_k \left(\overbrace{f_0 \int_0^{\frac{1}{f_0}} e^{j2\pi (k-l) f_0 t}\, dt}^{\delta(k-\ell) = \begin{cases} 1 & \text{if } k=\ell \\ 0 & \text{if } k \neq \ell \end{cases}} \right) = \sum_k X_k \delta(k-\ell) = X_\ell.$$

$$\underbrace{f_0 \int_0^{\frac{1}{f_0}} e^{j2\pi (k-l) f_0 t}\, dt}_{} = \begin{cases} 1 & k = l \\ 0 & k \neq l \end{cases}$$

Multicarrier Demodulation

- $e^{j2\pi k f_0 t}, e^{j2\pi l f_0 t}$ are ORTHOGONAL
- This explains the name **OFDM** ← Orthogonal. Frequency Division Multiplexing.
- **Orthogonal** $\Rightarrow$ Subcarriers are ORTHOGONAL

  \+
- **Frequency Division** $\Rightarrow$ Dividing BW into multiple subbands.

  \+
- **Multiplexing** $\Rightarrow$ Parallel Transmission of multiple symbols over multiple subcarriers. in same band.

Multicarrier Demodulation

OFDM
+ MIMO.
Wide Bandwidth
channel.

LTE ~ 100 Mbps!

- **<u>Orthogonal</u>**: Subcarriers are **ORTHOGONAL**.
- **<u>Frequency Division</u>**: Dividing the band into ***multiple subbands***.
- **<u>Multiplexing</u>**: Simultaneous transmission of **multiple symbols** over same channel.

OFDM :- Enables Transmission
over Large BW
without ISI ⇒ Extremely
High Data Rates.

OFDM Generation

'Sampling'

- Is there a **simple technique** to generate the OFDM signal?
- **Answer is YES**!!!!!

$\sum_{k} X_k e^{j2\pi k F_0 t} \Rightarrow$

N ~ 1000.

1000s of subcarriers!

Difficult to generate!

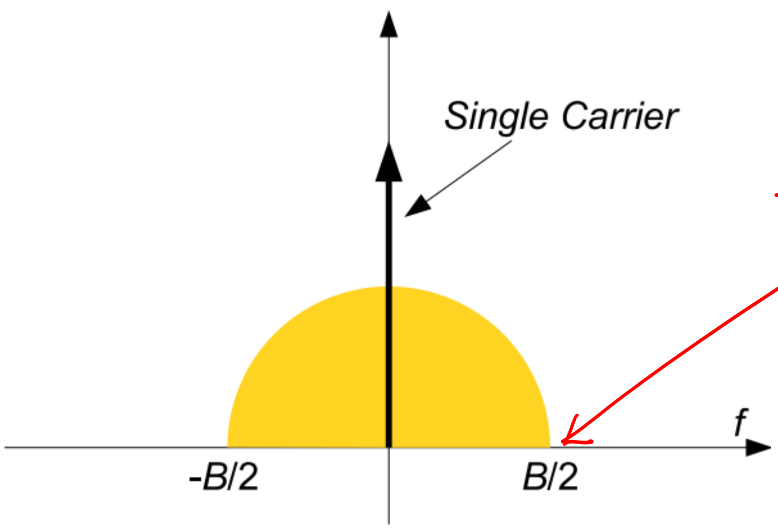
OFDM Generation

- Signal **bandlimited** to $\frac{B}{2}$.

NYQUIST CRITERION.

$$f_s = 2 f_{max} = 2 \times \frac{B}{2} = B.$$

- What is the minimum **sampling frequency**?



maximum Frequency.

$$f_{max} = \frac{B}{2}$$

OFDM Generation

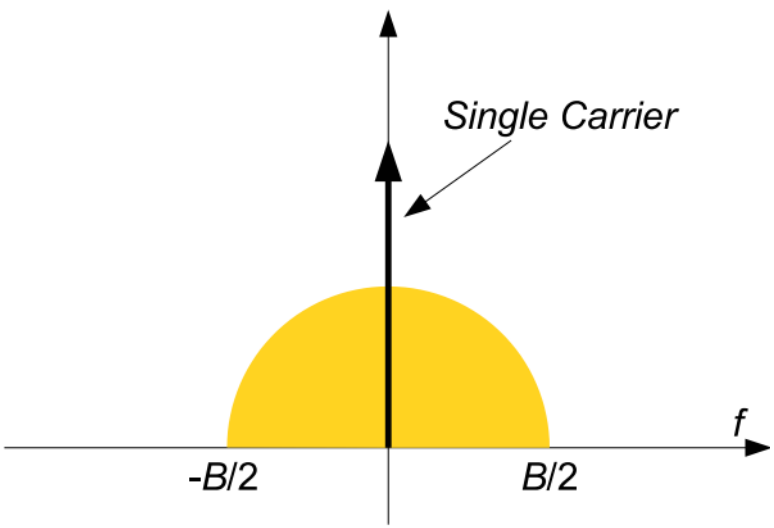
SAMPLING FREQ.

$\frac{B}{2} = F_{max}$.
$= $ maximum Frequency component in signal.

- $f_s = 2 \times \frac{B}{2} =$ ___B___: This is termed ___NYQUIST___ criterion!

Sampling interval.

- **Sampling duration** $T_s =$ ___$\frac{1}{F_s} = \frac{1}{B}$___



OFDM Generation

- lth sample will be at $t = \frac{l}{B}$

$l \times T_s = \frac{l}{B}.$

$$x(l) = \sum_k X_k e^{j2\pi k F_0 \frac{l}{B}} = \sum_k X_k e^{j2\pi k \frac{\cancel{B}}{N} \frac{l}{\cancel{B}}}.$$

$$= \underbrace{\frac{1}{N}}_{\text{SCALING FACTOR.}} \underbrace{\sum_k X_k e^{j2\pi \frac{kl}{N}}}_{\text{IDFT}}$$

IDFT

inverse Discrete Fourier Transform!

SCALING FACTOR.

Which can be Efficiently implented using IFFT!

OFDM Generation

- lth sample will be at $t = \frac{l}{B}$ $= l T_s = l \times \frac{1}{B}$

$$x(l) = \sum_k X(k) e^{j2\pi k \overbrace{\frac{\cancel{B}}{N}}^{F_0} \frac{l}{\cancel{B}}}$$

$t = \frac{l}{B}$

Simple scaling

$$\underbrace{x(l) = \frac{1}{N} \sum_k X(k) e^{j2\pi \frac{kl}{N}}}_{IDFT} \equiv IFFT$$

OFDM Generation

Inverse Discrete Fourier Transform

- What is the advantage of IDFT?
  - We can implement is very efficiently using **IFFT**.

Inverse Fast Fourier Transform

- What is this telling us?

OFDM Generation

Widely implemented!

DSP chipsets.

- **Samples** of the OFDM signal can be generated very efficiently using the IFFT algorithm!!

$N \log_2 N \sim N$.

$$\underbrace{X_0, X_1, \ldots, X_{N-1}}_{\text{Freq Domain}}$$

Load Subcarriers.

$$\downarrow IFFT$$

Time Domain Samples.

$$x(0), x(1), \ldots \ldots, x(N-1)$$

$$\underbrace{\quad}_{\text{Time Domain Samples}}$$

OFDM Generation

Modulati Loading. $(A^{-1})^{-1} = A$.

Tx IFFT — Rx FFT

Demod. unload subcarriers.

- ***Loading the subcarriers.*** – IFFT performed at transmitter.

Fast Fourier Transform

- ***What about at receiver for decoding??***
  - ___Perform inverse IFFT = FFT___!!!

inverse IFFT ≡ FFT

ISI Channel Model

Only Fading
No ISI.

$y(k) = h\,x(k) + n(k)$

intersymbol interference.

The **ISI channel model** is given as

Model for ISI channel.

current symbol

L-1 Previous symbols!

$$y(k) = h(0)x(k) + h(1)x(k-1) + \cdots + h(L-1)x(k-L+1) + v(k)$$

$h(0), h(1), \ldots, h(L-1) \equiv$ channel Taps.

$$= \sum_{l=0}^{L-1} h(l)\,x(k-l) + v(k)$$

ISI Channel Model

$$\underbrace{h(0), h(1), \ldots, h(L-1)}_{\text{channel taps}}$$

$$y(k) = h * x + v(k)$$

Linear convolution

Thus the channel performs

____LINEAR.____ convolution

Cyclic Prefix

IFFT

CP.

$N + \tilde{L}$

Prior to transmission we add the CYCLIC PREFIX (CP.) to an OFDM block

cyclic Fashion

Prefix at Head.

$\tilde{L}$ samples From end.

$$\underbrace{x(N-\tilde{L}),\dots,x(N-2),x(N-1)}_{\textbf{Cyclic Prefix}},\underbrace{x(0),x(1),x(2),\dots,x(N-1)}_{\textbf{Original Samples} = N}$$

Generated By IFFT.

Total # Samples $= N + \tilde{L}$

Cyclic Prefix

$\tilde{L} \approx 10-15\%$ of N
25% N.
$\tilde{L} << N$
NOT TO SCALE!

$$\overbrace{x(N-\tilde{L}),\dots,x(N-2),x(N-1)}^{\text{CP.}},\overbrace{\underline{x(0)},x(1),x(2),\dots,x(N-1)}^{\text{Original.}}$$

- Let us once again examine output corresponding to $x(0)$

$$y(0)=h(0)x(0)+h(1)x(N-1)+\cdots+h(L-1)x(N-L+1)$$
$$+\cdots+0\times x(2)+0\times x(1)+v(0)$$

$$y(1)=h(0)x(1)+h(1)x(0)+h(2)x(N-1)+\cdots+h(L-1)x(N-L+2)$$
$$+\cdots+0\times x(3)+0\times x(2)+v(1)$$

$$y(2)=h(0)x(2)+h(1)x(1)+h(2)x(0)+\cdots+h(L-1)x(N-L+3)$$
$$+\cdots+0\times x(4)+0\times x(3)+v(2)$$

Cyclic Prefix

$$x(N-\tilde{L}), \ldots, x(N-2), x(N-1), x(0), x(1), x(2), \ldots, x(N-1)$$

- Let us once again examine output corresponding to $x(0)$

$$y(0) = h(0)x(0) + h(1)x(N-1) + \cdots + h(L-1)x(N-L+1)$$
$$+ \cdots + 0 \times x(2) + 0 \times x(1) + v(0)$$

$$y(1) = h(0)x(1) + h(1)x(0) + \cdots + h(L-1)x(N-L+2)$$
$$+ \cdots + 0 \times x(3) + 0 \times x(2) + v(1)$$

$$y(2) = h(0)x(2) + h(1)x(1) + \cdots + h(L-1)x(N-L+3)$$
$$+ \cdots + 0 \times x(4) + 0 \times x(3) + v(2)$$

OFDM samples are shifting in a circular fashion! $\Rightarrow$ CIRCULAR CONVOLUTION!

Cyclic Prefix

- Because of the addition of **cyclic prefix,** ____LINEAR.____ ***convolution*** becomes a ____CIRCULAR.____ ***convolution***

Circular convolution Because of CP!

- **What is the output of the OFDM channel?**

$$y(k) = h \circledast x + v$$

Cyclic Prefix

$*$ = Linear convolution

$\circledast$ = circular convolution

- What happens when we take the **FFT**?

Output

channel

OFDM signal.

Noise

$$y(l) = h \circledast x + v(l)$$

$$\downarrow FFT$$

at Receiver

Symbol on subcarrier k.

Noise for subcarrier k.

$$Y_k = H_k \times X_k + V_k$$

Output for subcarrier k.

Product in Freq Domain!

channel coeff for subcarrier k.

OFDM Model

No ISI on any subcarrier!!

$k = 0, 1, \ldots, N-1$ SUBCARRIERS.

- Therefore we have?

$Y_k = H_k X_k + V_k$.

$$Y_0 = H_0 \times X_0 + V_0$$
$$Y_1 = H_1 \times X_1 + V_1$$
$$\vdots$$
$$Y_{N-1} = H_{N-1} \times X_{N-1} + V_{N-1}$$

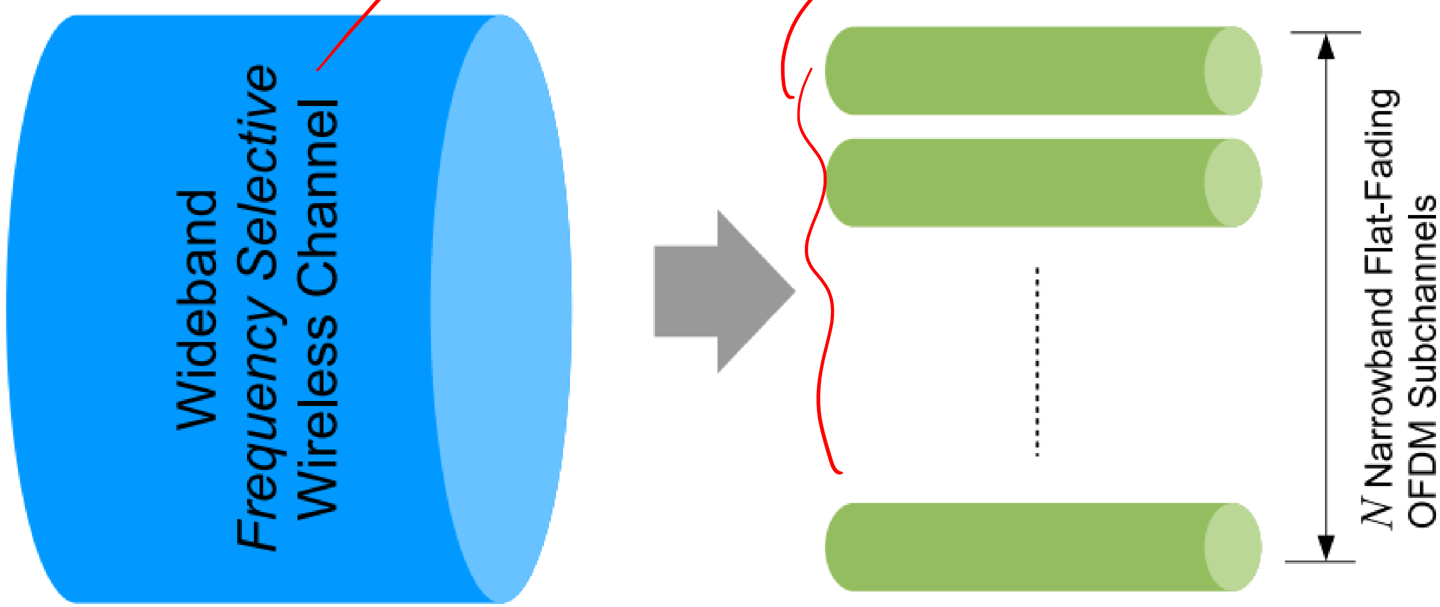
N parallel channels.

One for EACH SUBCARRIER.

OFDM

WIDEBAND CHANNEL

N PARALLEL FLAT FADING SUBCARRIER CHANNELS.



OFDM

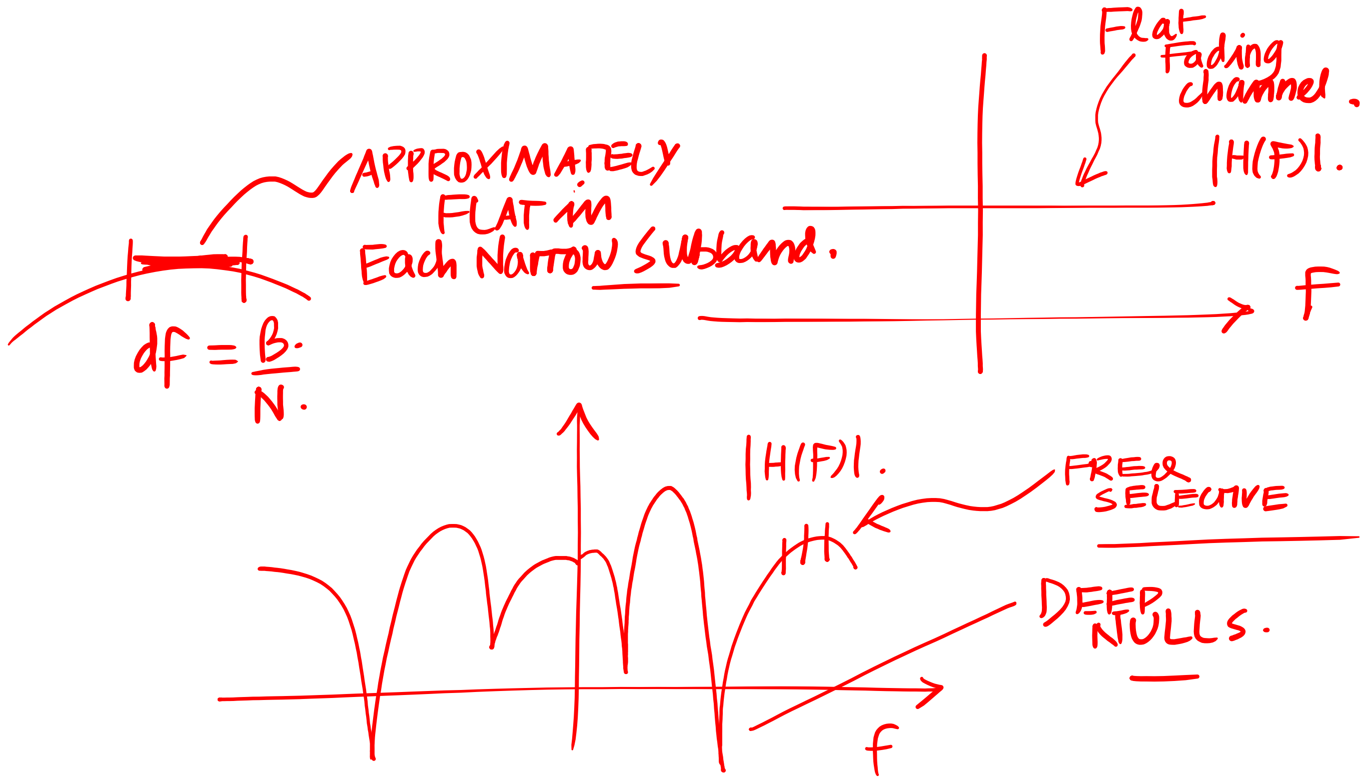
N Parrallel
ISI Free
Freq Domain channels !!!

- What is **OFDM achieving**?
- OFDM is converting a ***time-domain ISI channel*** into N ***parallel subcarrier channels***, each of which is **ISI free**.

ISI ⇒ FREQ SELECTIVE
⇒ DISPERSIVE

FLAT FADING ⇒ FREQ FLAT
NO ISI !

OFDM



APPROXIMATELY FLAT in Each Narrow Subband.
df = B/N.
Flat Fading channel.
|H(F)|.
F
|H(F)|.
FREQ SELECTIVE
DEEP NULLS.
f

OFDM System Model

Subcarrier channel coeffs.

- Channel coefficients H_k are given as

$$\overbrace{h(0), h(1), \ldots, h(L-1)}^{L}, \overbrace{0, 0, \ldots, 0}^{N-L \text{ zeros.}}$$

$$H_k = \sum_{l=0}^{L-1} h(l) e^{-j2\pi \frac{kl}{N}}$$

$\downarrow$ N pt zero padded. FFT

$$H_0, H_1, H_2, \ldots, H_{N-1}$$

COEFFICIENT FOR

k^{th} SUBCARRIER.

OFDM System Model

Frequency Domain ~~Subcarrier~~ Outputs.

- Outputs Y_k are given as

Time Domain.

$y(0), y(1), \ldots, y(N-1)$

$\downarrow$ FFT

$Y_0, Y_1, \ldots, Y_{N-1}$

FREQ DOMAIN

Cyclic Prefix

$$x(N-\tilde{L}),\ldots,x(N-2),x(N-1),x(0),x(1),x(2),\ldots,x(N-1)$$

START LOOKING AT OUTPUTS FROM $y(0)$ FOR $x(0)$

- Note at the receiver we **remove the outputs** corresponding to the CP
- This is termed CP. REMOVAL

Remove outputs corresponding to CP.

- We only consider outputs starting from $y(0)$

$$y(0)=h(0)x(0)+h(1)x(N-1)+\cdots+h(L-1)x(N-L+1)$$
$$+\cdots+0\times x(2)+0\times x(1)+v(0)$$

Instructors may use this white area (14.5 cm / 25.4 cm) for the text. Three options provided below for the font size.

Font: Avenir (Book), Size: 32, Colour: Dark Grey

Font: Avenir (Book), Size: 28, Colour: Dark Grey

Font: Avenir (Book), Size: 24, Colour: Dark Grey

Do not use the space below.

Instructors may use this white area (14.5 cm / 25.4 cm) for the text.
Three options provided below for the font size.

Font: Avenir (Book), Size: 32, Colour: Dark Grey

Font: Avenir (Book), Size: 28, Colour: Dark Grey

Font: Avenir (Book), Size: 24, Colour: Dark Grey

Do not use the space below.