eMasters in Communication Systems

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Elective Module:

Advanced ML Techniques

Chapter 8

Linear Discriminant Analysis

Discriminant Functions

- Consider a **classifier** built using functions

$g_i(\bar{x}), \quad i = 1, 2, \ldots, L$

$$\left.\begin{array}{l} g_1(\bar{x}) \\ g_2(\bar{x}) \\ \vdots \\ g_L(\bar{x}) \end{array}\right\} L \text{ Discriminant Functions.}$$

Discriminant Functions

- Consider a **classifier** built using functions

$$g_i(\bar{\mathbf{x}}), i = 1,2, \dots, L$$

Discriminant Functions

Feature vector

- The **input vector** $\bar{\mathbf{x}}$ is assigned to class l if

$$l = \arg\max_i g_i(\bar{x})$$

Assign class L, For which discriminant function is maximum

Discriminant Functions

- The **input vector** $\bar{\mathbf{x}}$ is assigned to class l if

$$g_l(\bar{\mathbf{x}}) = \max_{1 \le i \le L} g_i(\bar{\mathbf{x}})$$

Discriminant Functions

- These $g_i(\bar{\mathbf{x}})$ are termed

Discriminant Functions.

Discriminant Functions

- These $g_i(\bar{\mathbf{x}})$ are termed **discriminant functions**

Gaussian Density

Gaussian Random variables.

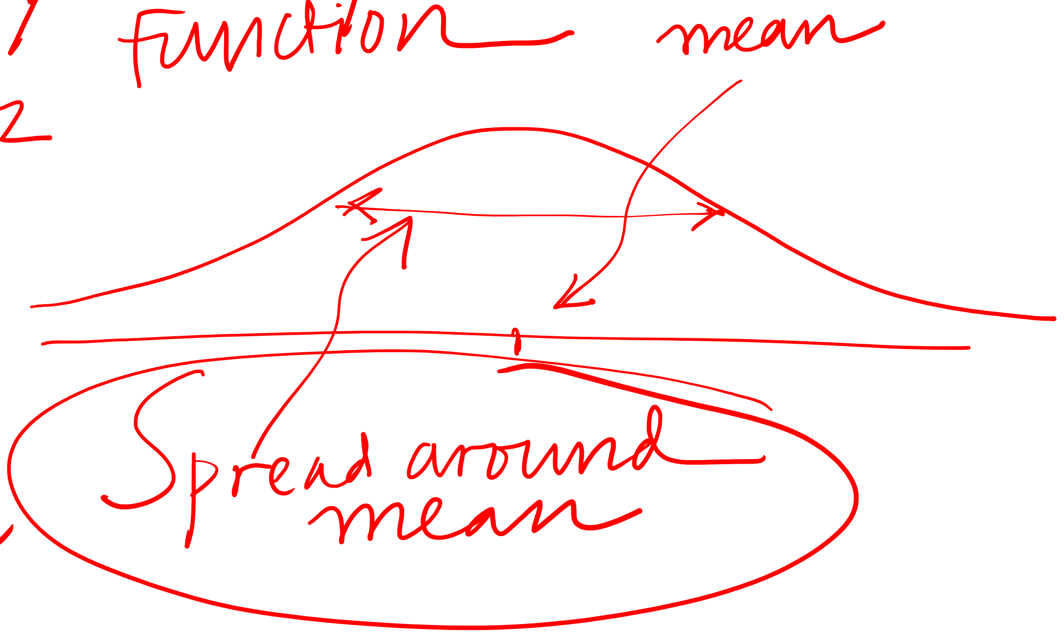
- Recall, the expression for the **Gaussian PDF** is

Probability density function

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

$$\mu = E\{X\} = \text{mean}$$

$$\sigma^2 = E\{(X-\mu)^2\}$$



Gaussian Density

- Recall, the expression for the ***Gaussian PDF*** is

mean

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Variance

Gaussian Density

- The ***mean*** and ***variance*** of the Gaussian RV are

$$E\{X\} = \mu$$

$$E\{(X-\mu)^2\} = \sigma^2$$

Gaussian Density

- The ***mean*** and ***variance*** of the Gaussian RV are

$$E\{X\} = \mu$$
$$E\{(X-\mu)^2\} = \sigma^2$$

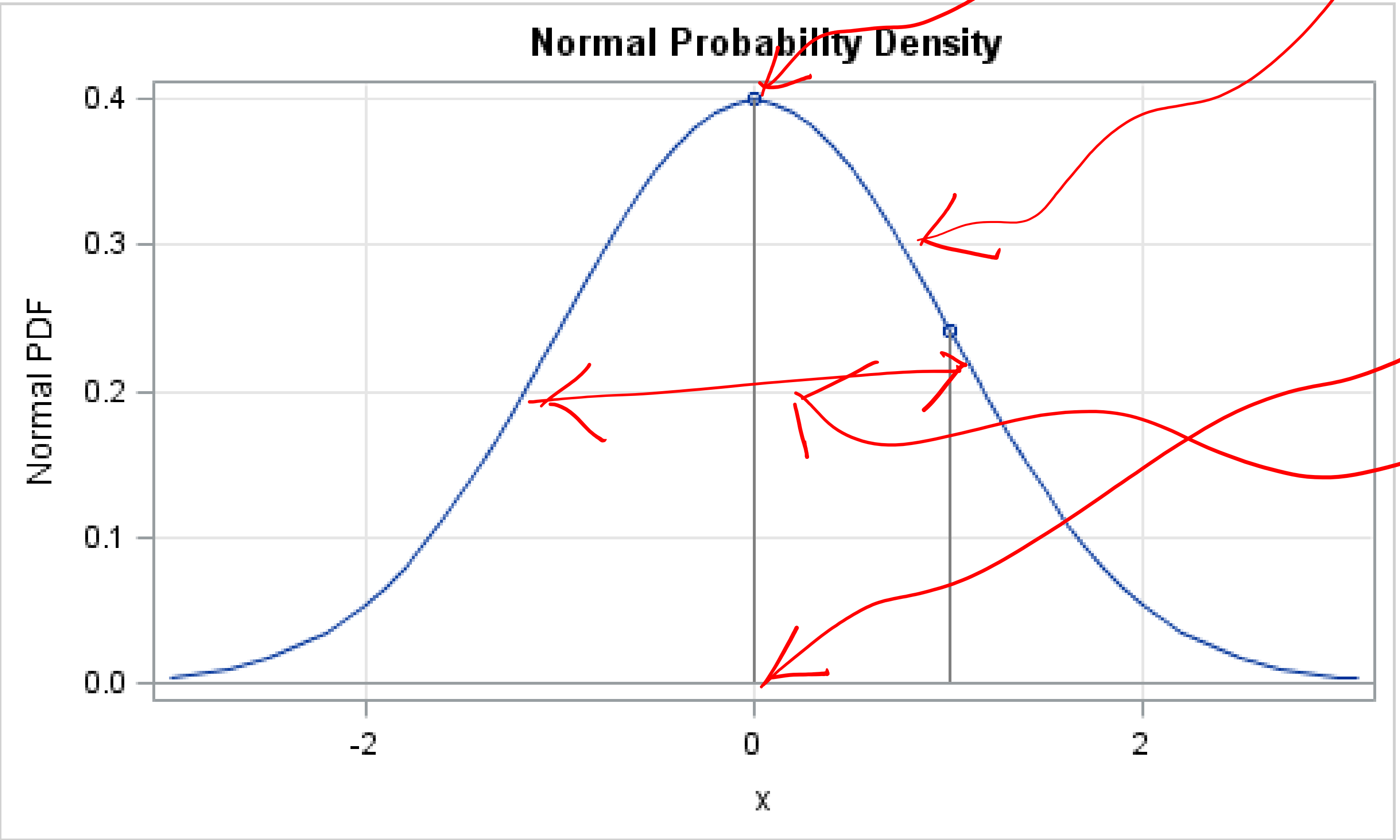
Gaussian Density

Peak

Bell Shaped curve

mean = 0

Spread is characterized by variance.



Multivariate Gaussian Density

- Recall, the PDF of a **Gaussian random vector** is given as

Multivariate Gaussian

$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$$

Components

Features.

Feature vector

$$f_{\bar{X}}(\bar{x}) = \frac{1}{\sqrt{(2\pi)^n |R|}} e^{-\frac{1}{2}(\bar{x}-\bar{\mu})^T R^{-1} (\bar{x}-\bar{\mu})}.$$

R = covariance matrix.

$\bar{\mu}$ = mean vector

Multivariate Gaussian Density

- Recall, the PDF of a ***Gaussian random vector*** is given as

$$f_{\bar{\mathbf{X}}}(\bar{\mathbf{x}}) = \frac{1}{\sqrt{(2\pi)^n |\mathbf{R}|}} e^{-\frac{1}{2}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}})^T \mathbf{R}^{-1} (\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}})}$$

$|R|$ = Determinant of R

Multivariate Gaussian Density

- The ***mean*** and ***covariance matrix*** are defined as

$$E\{\bar{x}\} = \bar{\mu}$$

$$E\left\{\begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}\right\} = \begin{bmatrix} E\{x_1\} \\ \vdots \\ E\{x_N\} \end{bmatrix} = \bar{\mu}$$

$$E\{(\bar{x}-\bar{\mu})(\bar{x}-\bar{\mu})^T\} = R$$

covariance matrix

Multivariate Gaussian Density

- The ***mean*** and ***covariance matrix*** are defined as

$$E\{\bar{\mathbf{x}}\} = \bar{\boldsymbol{\mu}}$$

mean

$$E\{(\bar{\mathbf{x}} - \bar{\boldsymbol{\mu}})(\bar{\mathbf{x}} - \bar{\boldsymbol{\mu}})^T\} = \mathbf{R}$$

covariance matrix

Multivariate Gaussian PDF - Example

- Find **multivariate Gaussian** PDF

$N=2$

$$\bar{\boldsymbol{\mu}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \mathbf{R} = \begin{bmatrix} 7 & 2 \\ 2 & 1 \end{bmatrix}$$

Probability density function

Multivariate Gaussian PDF - Example

$$|\mathbf{R}| = \left| \begin{bmatrix} 7 & 2 \\ 2 & 1 \end{bmatrix} \right| = 7 \times 1 - 2 \times 2$$

$$= 7 - 4 = 3$$

Multivariate Gaussian PDF - Example

$$|\mathbf{R}| = \left|\begin{bmatrix} 7 & 2 \\ 2 & 1 \end{bmatrix}\right| = 7 - 4 = 3$$

Multivariate Gaussian PDF - Example

$$\mathbf{R} = \begin{bmatrix} 7 & 2 \\ 2 & 1 \end{bmatrix}$$

inverse of R

$$\mathbf{R}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & -2 \\ -2 & 7 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Multivariate Gaussian PDF - Example

$$\mathbf{R} = \begin{bmatrix} 7 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\mathbf{R}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & -2 \\ -2 & 7 \end{bmatrix}$$

Multivariate Gaussian PDF - Example

- The **multivariate Gaussian** PDF is given as follows

$$\frac{1}{\sqrt{(2\pi)^n|\mathbf{R}|}}e^{-\frac{1}{2}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}})^T\mathbf{R}^{-1}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}})}$$

$$= \frac{1}{\sqrt{(2\pi)^n|R|}}e^{-\frac{1}{2}\begin{bmatrix} x_1-1 & x_2-2\end{bmatrix}R^{-1}\begin{bmatrix} x_1-1 \\ x_2-2\end{bmatrix}}$$

Multivariate Gaussian PDF - Example

- The **multivariate Gaussian** PDF is given as follows

$$\frac{1}{\sqrt{(2\pi)^n|\mathbf{R}|}}e^{-\frac{1}{2}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}})^T\mathbf{R}^{-1}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}})}$$

$$=\frac{1}{\sqrt{(2\pi)^2\times 3}}e^{-\frac{1}{6}\begin{bmatrix}x_1-1 & x_2-2\end{bmatrix}\begin{bmatrix}1 & -2\\ -2 & 7\end{bmatrix}\begin{bmatrix}x_1-1\\ x_2-2\end{bmatrix}}$$

Multivariate Gaussian PDF - Example

$$(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}})^T\mathbf{R}^{-1}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}})$$

$$=\frac{1}{3}\begin{bmatrix} x_1-1 & x_2-2\end{bmatrix}\begin{bmatrix}1 & -2\\ -2 & 7\end{bmatrix}\begin{bmatrix}x_1-1\\ x_2-2\end{bmatrix}$$

$$\begin{array}{r} -8\\ +1\\ +28\\ \hline 21. \end{array}$$

$$=\frac{1}{3}\left((x_1-1)^2+7(x_2-2)^2-2\times 2\times(x_1-1)(x_2-2)\right)$$

$$=\frac{1}{3}\left(x_1^2+7x_2^2+6x_1-24x_2-4x_1x_2+21\right)$$

Multivariate Gaussian PDF - Example

$$\begin{bmatrix} x_1 - 1 & x_2 - 2 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} x_1 - 1 \\ x_2 - 2 \end{bmatrix}$$

$$= (x_1 - 1)^2 + 7(x_2 - 2)^2 - 2 \times 2(x_1 - 1)(x_2 - 2)$$

$$= x_1^2 + 7x_2^2 + 6x_1 - 24x_2 - 4x_1x_2 + 21$$

Multivariate Gaussian PDF - Example

- The **multivariate Gaussian** PDF is derived as

$$\frac{1}{\sqrt{(2\pi)^n|\mathbf{R}|}}e^{-\frac{1}{2}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}})^T\mathbf{R}^{-1}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}})}$$

$$=\frac{1}{\sqrt{12\pi^2}}e^{-\frac{1}{6}\left(x_1^2+7x_2^2+6x_1-24x_2-4x_1x_2+21\right)}.$$

Multivariate Gaussian PDF for given example.

Multivariate Gaussian PDF - Example

- The **multivariate Gaussian** PDF is derived as

$$\frac{1}{\sqrt{(2\pi)^n|\mathbf{R}|}}e^{-\frac{1}{2}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}})^T\mathbf{R}^{-1}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}})}$$

$$=\frac{1}{\sqrt{(2\pi)^2\times 3}}e^{-\frac{1}{6}\left(x_1^2+7x_2^2+6x_1-24x_2-4x_1x_2+21\right)}$$

Gaussian Discriminant Analysis

- Consider the input vectors $\bar{\mathbf{x}}$ drawn from ***two Gaussian classes***

$\mathcal{C}_0$: $\bar{\mu}_0, R$

$\mathcal{C}_1$: $\bar{\mu}_1, R$

Gaussian Discriminant Analysis

- Consider the input vectors $\bar{\mathbf{x}}$ drawn from ***two Gaussian classes***

$\mathcal{C}_0$: **Mean** $\bar{\boldsymbol{\mu}}_0$ and **covariance** $\mathbf{R}$

$\mathcal{C}_1$: **Mean** $\bar{\boldsymbol{\mu}}_1$ and **covariance** $\mathbf{R}$

class 0 class 1

Gaussian Discriminant Analysis

- Also termed **Gaussian Discriminant Analysis**

Gaussian
Discriminant
Analysis.

Gaussian Discriminant Analysis

- Thus, the ***likelihoods*** of the two classes are

$$p(\bar{\mathbf{x}}; \mathcal{C}_0) = \frac{1}{\sqrt{(2\pi)^n |R|}} e^{-\frac{1}{2}(\bar{x}-\bar{\mu}_0)^T R^{-1}(\bar{x}-\bar{\mu}_0)}$$

$$p(\bar{\mathbf{x}}; \mathcal{C}_1) = \frac{1}{\sqrt{(2\pi)^n |R|}} e^{-\frac{1}{2}(\bar{x}-\bar{\mu}_1)^T R^{-1}(\bar{x}-\bar{\mu}_1)}$$

Likelihood of $\mathcal{C}_0$.

Likelihood for class $\mathcal{C}_1$.

Gaussian Discriminant Analysis

- Thus, the ***likelihoods*** of the two classes are

$$p(\bar{\mathbf{x}}; \mathcal{C}_0) = \frac{1}{\sqrt{(2\pi)^n |\mathbf{R}|}} e^{-\frac{1}{2}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}}_0)^T \mathbf{R}^{-1} (\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}}_0)}$$

$$p(\bar{\mathbf{x}}; \mathcal{C}_1) = \frac{1}{\sqrt{(2\pi)^n |\mathbf{R}|}} e^{-\frac{1}{2}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}}_1)^T \mathbf{R}^{-1} (\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}}_1)}$$

Maximum Likelihood rule

- Choose the class that ***maximizes the likelihood***

Maximum Likelihood

ML rule

ML classifier

Maximum Likelihood rule

likelihood of C_0.

likelihood of C_1.

- Therefore, **choose C_0** if

$$p(\bar{\mathbf{x}}; C_0) \geq p(\bar{\mathbf{x}}; C_1)$$

$$\Rightarrow \cancel{\frac{1}{\sqrt{(2\pi)^n|R|}}} \cdot e^{-\frac{1}{2}(\bar{x}-\bar{\mu}_0)^T R^{-1}(\bar{x}-\bar{\mu}_0)} \geq \cancel{\frac{1}{\sqrt{(2\pi)^n|R|}}} \cdot e^{-\frac{1}{2}(\bar{x}-\bar{\mu}_1)^T R^{-1}(\bar{x}-\bar{\mu}_1)}$$

$$\Rightarrow (\bar{x}-\bar{\mu}_0)^T R^{-1}(\bar{x}-\bar{\mu}_0) \leq (\bar{x}-\bar{\mu}_1)^T R^{-1}(\bar{x}-\bar{\mu}_1)$$

Maximum Likelihood rule

- Therefore, **choose** $\mathcal{C}_0$ if

$$p(\bar{\mathbf{x}}; \mathcal{C}_0) \geq p(\bar{\mathbf{x}}; \mathcal{C}_1)$$

$$\Rightarrow \frac{1}{\sqrt{(2\pi)^n |\mathbf{R}|}} e^{-\frac{1}{2}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}}_0)^T \mathbf{R}^{-1}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}}_0)} \geq \frac{1}{\sqrt{(2\pi)^n |\mathbf{R}|}} e^{-\frac{1}{2}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}}_1)^T \mathbf{R}^{-1}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}}_1)}$$

$$\Rightarrow (\bar{\mathbf{x}} - \bar{\boldsymbol{\mu}}_0)^T \mathbf{R}^{-1} (\bar{\mathbf{x}} - \bar{\boldsymbol{\mu}}_0) \leq (\bar{\mathbf{x}} - \bar{\boldsymbol{\mu}}_1)^T \mathbf{R}^{-1} (\bar{\mathbf{x}} - \bar{\boldsymbol{\mu}}_1)$$

Because of -ve sign
in Exponent

Maximum Likelihood rule

- This ***discriminant function*** can be simplified as

Choose $\mathcal{C}_0$: $\bar{h}^T(\bar{x}-\tilde{\mu}) \geq 0$

Choose $\mathcal{C}_1$: $\bar{h}^T(\bar{x}-\hat{\mu}) < 0$

Maximum Likelihood rule

- This ***discriminant function*** can be simplified as

$g(\bar{x})$,

Choose C_0: $\bar{\mathbf{h}}^T(\bar{\mathbf{x}} - \tilde{\boldsymbol{\mu}}) \geq 0$

Choose C_1: $\bar{\mathbf{h}}^T(\bar{\mathbf{x}} - \tilde{\boldsymbol{\mu}}) < 0$

Maximum Likelihood rule

- where

$$\tilde{\mu} = \frac{\bar{\mu}_0 + \bar{\mu}_1}{2}$$

Midpoint of Both classes

$$\bar{h} = R^{-1}(\bar{\mu}_0 - \bar{\mu}_1).$$

Linear classifier

$$C_0: (\bar{\mu}_0 - \bar{\mu}_1)^T R^{-1}\left(\bar{x} - \frac{\bar{\mu}_0 + \bar{\mu}_1}{2}\right) \geq 0$$

$$C_1: (\bar{\mu}_0 - \bar{\mu}_1)^T R^{-1}\left(\bar{x} - \frac{\bar{\mu}_0 + \bar{\mu}_1}{2}\right) < 0$$

Maximum Likelihood rule

- where

$$\tilde{\boldsymbol{\mu}} = \frac{1}{2}(\bar{\boldsymbol{\mu}}_0 + \bar{\boldsymbol{\mu}}_1)$$

$$\bar{\mathbf{h}} = \mathbf{R}^{-1}(\bar{\boldsymbol{\mu}}_0 - \bar{\boldsymbol{\mu}}_1)$$

Linear classifier

- Thus, the classifier is ***linear***
- It is characterized by the ***hyperplane***

$$\bar{h}^T(\bar{x}-\hat{\mu}) \geq 0 \rightarrow \text{hyperplane}$$

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n \geq b$$

$2x_1 + 3x_2 \geq -7$: Line 2D

$4x_1 - 8x_2 + 17x_3 \geq -2$: Plane 3D.

n Dimensions: hyperplane

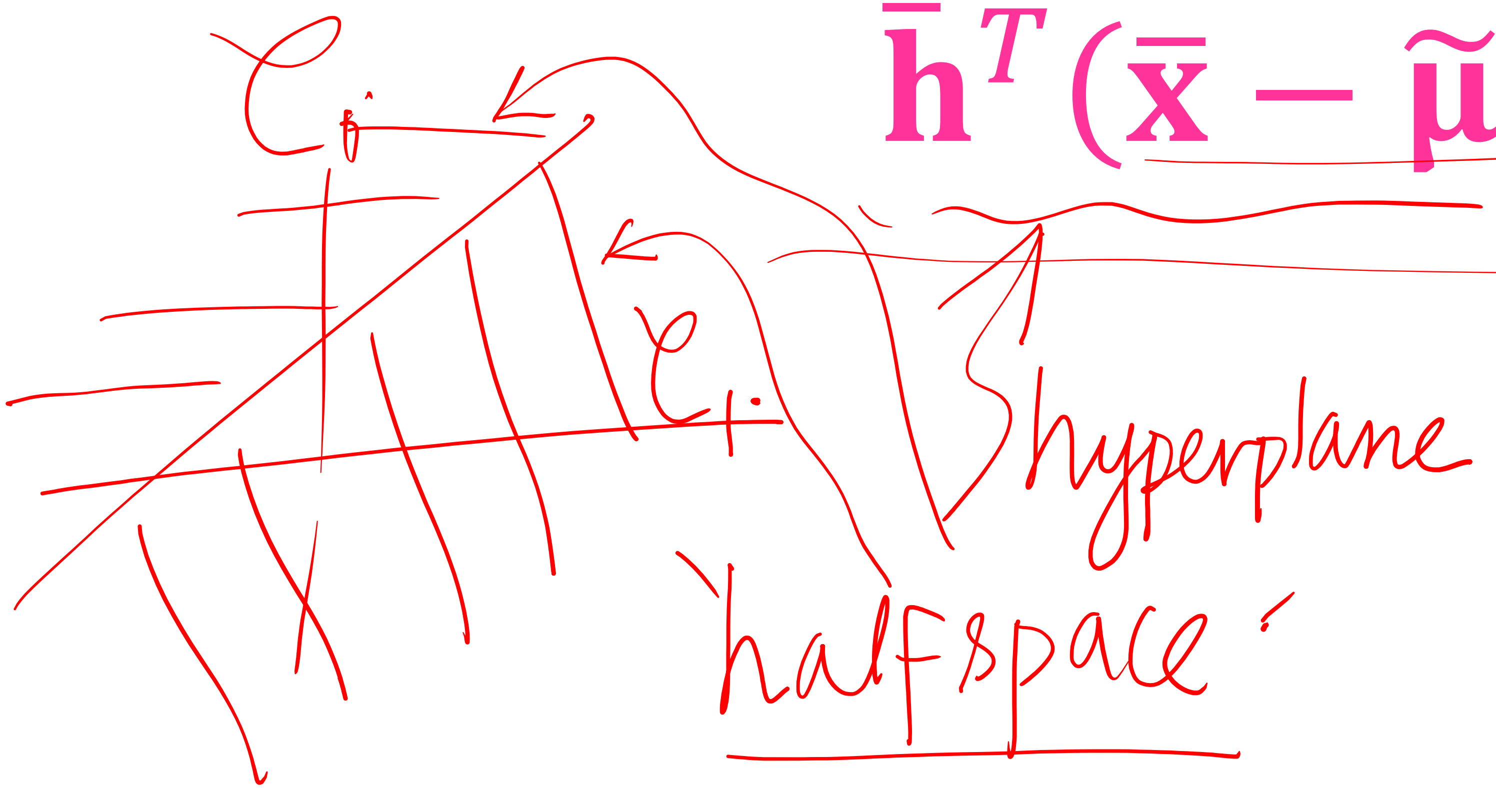
Linear classifier

Discriminant Functions are Linear $\Rightarrow$ Linear Discriminant Analysis

- Thus, the classifier is ***linear***
- It is characterized by the ***hyperplane***

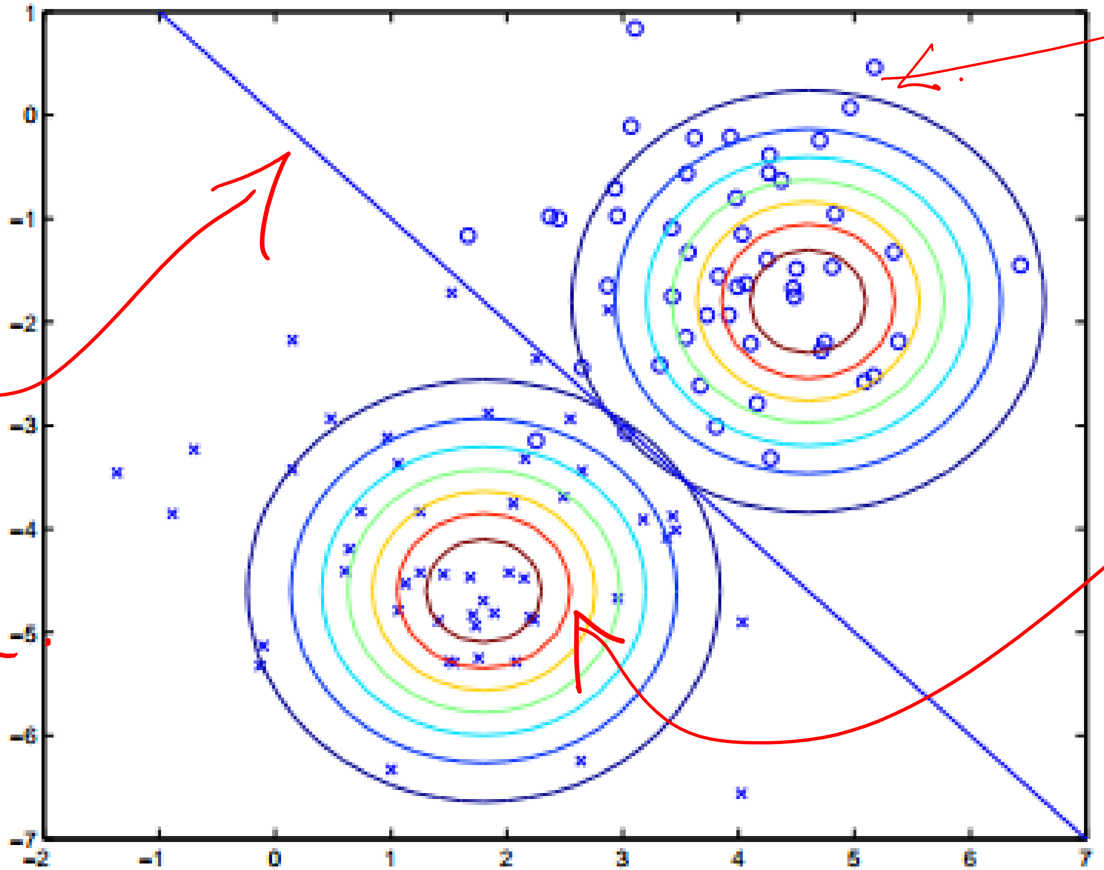
$$\bar{\mathbf{h}}^T(\bar{\mathbf{x}} - \tilde{\boldsymbol{\mu}})$$: Linear

LDA.



Gaussian Discriminant Classifier

Gaussian PDF C_0.



Gaussian PDF C_1.

Linear Discriminant.

Special case LDA

- Consider the ***special case*** $\mathbf{R} = \sigma^2\mathbf{I}$

$R \propto I$

- It follows that

$\bar{\mathbf{h}} = R^{-1}(\bar{\mu}_0 - \bar{\mu}_1)$

$= \frac{1}{\sigma^2} \cdot I(\bar{\mu}_0 - \bar{\mu}_1)$

$\equiv (\bar{\mu}_0 - \bar{\mu}_1)$

Special case

- Consider the ***special case*** $\mathbf{R} = \sigma^2\mathbf{I}$
- It follows that

$$\bar{\mathbf{h}} = \frac{1}{\sigma^2}\mathbf{I}(\bar{\boldsymbol{\mu}}_0 - \bar{\boldsymbol{\mu}}_1) \sim (\bar{\boldsymbol{\mu}}_0 - \bar{\boldsymbol{\mu}}_1)$$

Special case

- The **hyperplane** reduces to

Choose c_0: $\bar{h}^T(\bar{x}-\hat{\bar{\mu}}) \geq 0$ $\qquad \frac{\bar{\mu}_0+\bar{\mu}_1}{2}$

$$\Rightarrow (\bar{\mu}_0-\bar{\mu}_1)^T(\bar{x}-\hat{\bar{\mu}}) \geq 0$$

Special case

- The **hyperplane** reduces to

$$(\bar{\boldsymbol{\mu}}_0 - \bar{\boldsymbol{\mu}}_1)^T \left(\bar{\mathbf{x}} - \frac{1}{2}(\bar{\boldsymbol{\mu}}_0 + \bar{\boldsymbol{\mu}}_1)\right) \geq \mathbf{0}$$

Special case

$$(\bar{\mu}_0 - \bar{\mu}_1)^T\left(\bar{x} - \left(\frac{\bar{\mu}_0 + \bar{\mu}_1}{2}\right)\right)$$

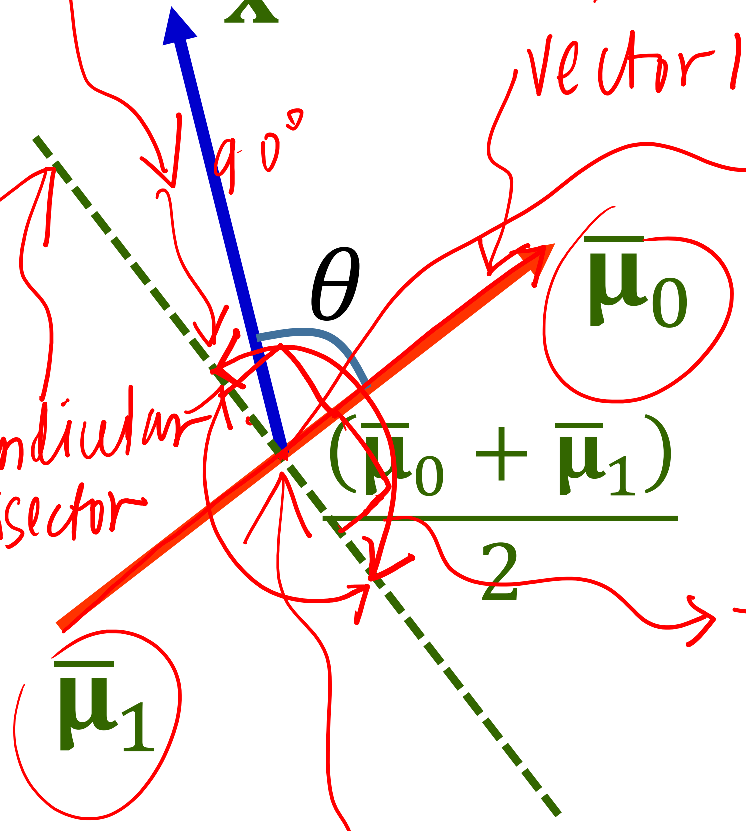
Vector from $\bar{\mu}_0$ to $\bar{\mu}_1$

Vector 2

Vector 1

Vector from $\frac{\bar{\mu}_0 + \bar{\mu}_1}{2}$ to $\bar{x}$

Dot product Between vectors.



Perpendicular Bisector

-90°

$\bar{a} \cdot \bar{b} \geq 0$

$\Rightarrow \|\bar{a}\|\|\bar{b}\| \cos\theta \geq 0$

$\Leftrightarrow \cos\theta \geq 0$

$\Leftrightarrow -90^\circ \leq \theta \leq 90^\circ$

$\frac{\bar{\mu}_0 + \bar{\mu}_1}{2}$ = Mid point $\bar{\mu}_0, \bar{\mu}_1$

Special case

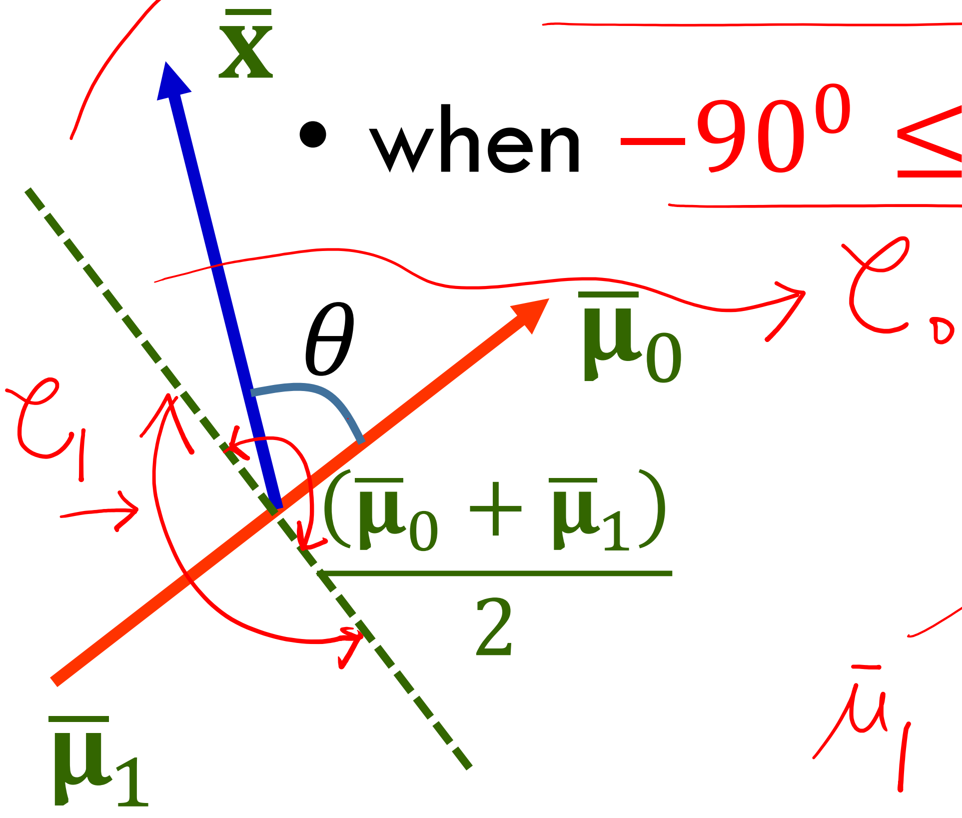
- It can be seen that

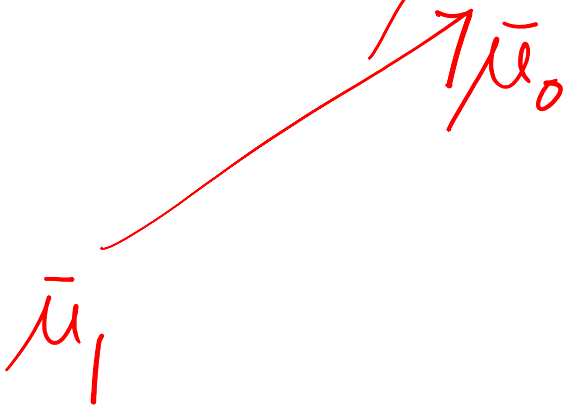
$$(\bar{\boldsymbol{\mu}}_0 - \bar{\boldsymbol{\mu}}_1)^T \left(\bar{\mathbf{x}} - \frac{1}{2}(\bar{\boldsymbol{\mu}}_0 + \bar{\boldsymbol{\mu}}_1) \right) \geq 0$$

Linear discriminant is perpendicular Bisector of means.

$\bar{\mu}_0 - \bar{\mu}_1$ vector from midpoint of $\bar{\mu}_0, \bar{\mu}_1$ to $\bar{x}$

- when $-90^0 \leq \theta \leq 90^0$





Special case

- Thus, the hyperplane is the ***perpendicular bisector*** of $\bar{\boldsymbol{\mu}}_0$ and $\bar{\boldsymbol{\mu}}_1$

Example

What is the Linear Discriminant classifier

- Consider the **Gaussian classification** problem
- The two classes C_0, C_1 are distributed as

$$C_0 \sim N\left(\begin{bmatrix}-1\\2\end{bmatrix}, \begin{bmatrix}\frac{1}{2} & 0\\0 & \frac{1}{4}\end{bmatrix}\right), C_1 \sim N\left(\begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}\frac{1}{2} & 0\\0 & \frac{1}{4}\end{bmatrix}\right)$$

R

$\bar{\mu}_0$ $\bar{\mu}_1$

Example

- Calculate $\bar{\mathbf{h}}$

$$R = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{4} \end{bmatrix} \quad R^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\bar{\mathbf{h}} = \mathbf{R}^{-1}(\bar{\boldsymbol{\mu}}_0 - \bar{\boldsymbol{\mu}}_1)$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$\bar{h} = \begin{bmatrix} -6 \\ 4 \end{bmatrix}$$

Example

- Calculate $\bar{\mathbf{h}}$

$$\bar{\mathbf{h}} = \mathbf{R}^{-1}(\bar{\boldsymbol{\mu}}_0 - \bar{\boldsymbol{\mu}}_1)$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ 4 \end{bmatrix}$$

Example

- Further,

$$\tilde{\boldsymbol{\mu}} = \frac{1}{2}(\bar{\boldsymbol{\mu}}_0 + \bar{\boldsymbol{\mu}}_1)$$

$$= \frac{1}{2}\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1/2 \\ 3/2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Example

- Further,

$$\tilde{\boldsymbol{\mu}} = \frac{1}{2}(\bar{\boldsymbol{\mu}}_0 + \bar{\boldsymbol{\mu}}_1)$$

$$= \frac{1}{2}\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Example

- The classifier chooses C_0 if

$$\bar{\mathbf{h}}^T(\bar{\mathbf{x}} - \tilde{\boldsymbol{\mu}}) \geq 0$$

Linear Discriminant

Choose C_0 if $\begin{bmatrix}-6 & 4\end{bmatrix}\left(\begin{bmatrix}x_1\\x_2\end{bmatrix} - \frac{1}{2}\begin{bmatrix}1\\3\end{bmatrix}\right) \geq 0$

$$-6x_1 + 4x_2 + 3 - 6 \geq 0$$

$$\Rightarrow -6x_1 + 4x_2 \geq 3$$

$$\left.\begin{array}{l} \Rightarrow 6x_1 - 4x_2 \leq -3 \\ C_1 \text{ if } 6x_1 - 4x_2 > -3 \end{array}\right\}$$

Example

- The classifier chooses C_0 if

$$\bar{\mathbf{h}}^T(\bar{\mathbf{x}} - \tilde{\boldsymbol{\mu}}) \geq 0$$

$$\Rightarrow \begin{bmatrix} -6 & 4 \end{bmatrix} \left(\bar{\mathbf{x}} - \frac{1}{2}\begin{bmatrix} 1 \\ 3 \end{bmatrix} \right) \geq \mathbf{0}$$

$$\Rightarrow -6x_1 + 4x_2 \geq 3$$

$$\Rightarrow 6x_1 - 4x_2 \leq -3$$

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Font: Avenir (Book), Size: 28, Colour: Dark Grey

Font: Avenir (Book), Size: 24, Colour: Dark Grey

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