Course Introduction

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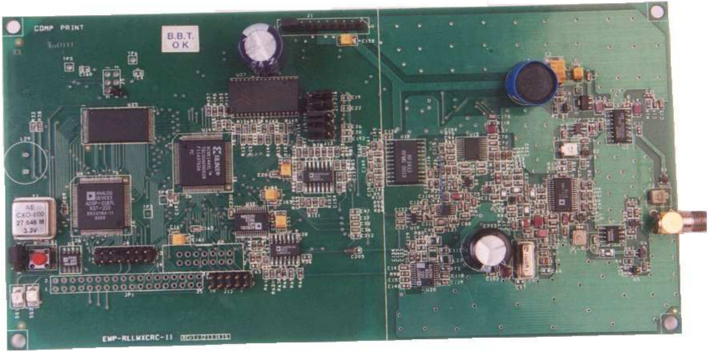
Applied Linear Algebra for Wireless Communications

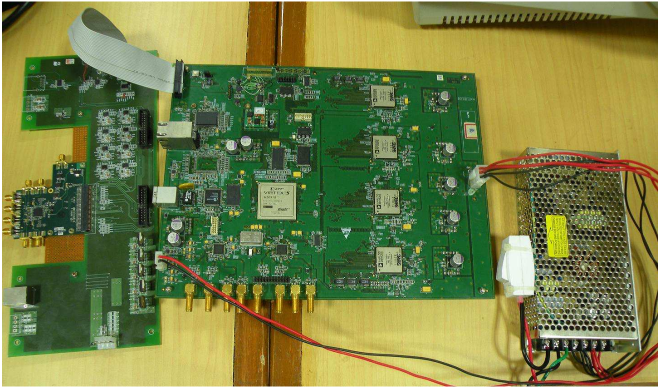
Agenda for today's class

- Motivation for learning applied linear algebra
- Motivate its application in 4G/5G wireless systems

3G wireless systems – single antenna

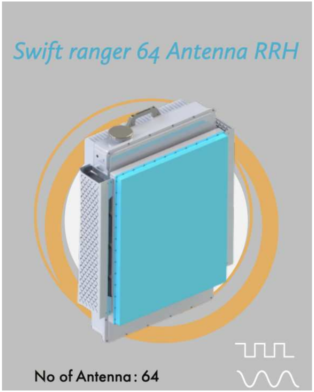


4G wireless systems – two/four antennas



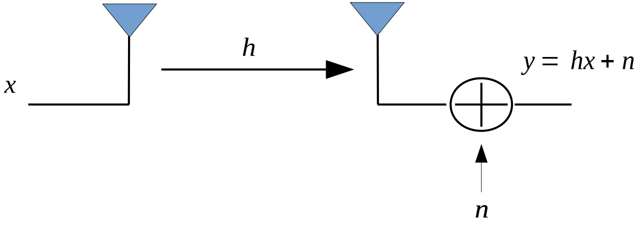
- Designed this system in 2010

5G base station at IIT Kanpur – 64 antennas



- Designed this system in 2020

Single antenna wireless system



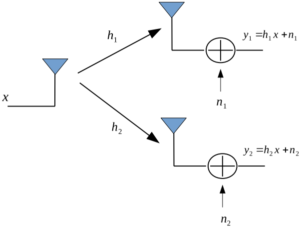
- Consider 1 transmit antenna and 1 receive antennas
- System is mathematically represented as

$$y = hx + n$$

- x is transmit symbol, n is receiver thermal noise
- h is wireless channel between the transmitter and receiver

Single transmit and two receive antenna systems (1)

- Consider a system with 1 transmit antenna and 2 receive antennas



- System is mathematically represented as

$$
\begin{aligned}
y_1 &= h_1 x + n_1 \\
y_2 &= h_2 x + n_2 \\
\mathbf{y} &= \mathbf{h} x + \mathbf{n}
\end{aligned}
$$

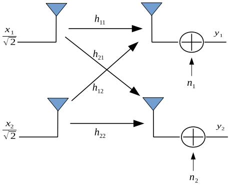
Single transmit and two receive antenna systems (2)

- System is mathematically represented as

$$\mathbf{y} = \mathbf{h}x + \mathbf{n}$$

- Here $\mathbf{y} = [y_1, y_2]^T$, $\mathbf{h} = [h_1, h_2]^T$ and $\mathbf{n} = [n_1, n_2]^T$
- Design a receiver to detect transmit signal x from receive vector $\mathbf{y}$
  - Cauchy-Schwartz inequality helps us in doing that

Multiple transmit and receive antenna system (1)



- Transmit $\frac{x_1}{\sqrt{2}}$ from first antenna and $\frac{x_2}{\sqrt{2}}$ from second antenna
- Received signal is

$$\begin{aligned} y_1 &= \frac{h_{11}x_1}{\sqrt{2}} + \frac{h_{12}x_2}{\sqrt{2}} + n_1 \\ y_2 &= \frac{h_{21}x_1}{\sqrt{2}} + \frac{h_{22}x_2}{\sqrt{2}} + n_2 \\ \mathbf{y} &= \mathbf{Hx} + \mathbf{n} \end{aligned}$$

Multiple transmit and receive antennas (3)

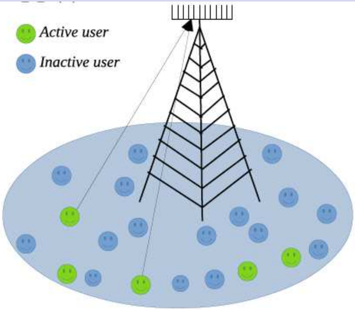
- $\mathbf{x} = [x_1/\sqrt{2}\ x_2/\sqrt{2}]^T$ is transmit vector
- $\mathbf{y} = [y_1\ y_2]^T$ is receiver vector
- $\mathbf{n} = [n_1\ n_2]^T$ is the receiver noise vector

  $\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$ is channel matrix
- Two symbols in transmit vector $\mathbf{x}$ interfere with each other at the receiver
  - Need to design a receiver to recover $\mathbf{x}$ from $\mathbf{y}$
- Most common receiver is zero forcing $\mathbf{W} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T$

$$\begin{aligned} \mathbf{W}\mathbf{y} = \tilde{\mathbf{y}} &= \mathbf{W}\mathbf{H}\mathbf{x} + \mathbf{W}\mathbf{n} \\ &= (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{H}\mathbf{x} + \underbrace{\mathbf{W}\mathbf{n}}_{\tilde{\mathbf{n}}} \\ &= \mathbf{x} + \tilde{\mathbf{n}} \end{aligned}$$

- Above operation is called left-inverse of a matrix

5G machine type communications (MTC) (1)



- Consider M single-antenna mMTC devices and N-antenna base-station (BS)
- Only few mMTC active devices transmit data which BS need to process
- BS does not know which devices are active
- All active M mMTC devices transmit simultaneously

5G machine type communications (MTC) (2)

- Received signal assuming all devices are active

$$
\begin{aligned}
y_1 &= h_{11}x_1 + h_{12}x_2 + \cdots + h_{1M}x_M + n_1 \\
y_2 &= h_{21}x_1 + h_{22}x_2 + \cdots + h_{2M}x_M + n_2 \\
\vdots &= \vdots \\
y_N &= h_{N1}x_1 + h_{N2}x_2 + \cdots + h_{NM}x_M + n_N \\
\mathbf{y} &= \mathbf{H}\mathbf{x} + \mathbf{n}
\end{aligned}
$$

- Transmit signal $\mathbf{x} = [x_1, \cdots, x_M]^T$, receive signal $\mathbf{y} = [y_1, \cdots, y_N]^T$, and receiver noise $\mathbf{n} = [n_1, \cdots, n_N]^T$
- Transmit vector $\mathbf{x}$ contains only few non-zero values
  $\mathbf{x} = [1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0 \cdots, 0]^T$
- Transmit signal is sparse

Other applications

- Linear algebra is used in the subjects which you will learn in emasters
- Wireless communications
- Machine learning for wireless
- Convex optimization for signal processing
- Coding theory

Books

- Introduction to Linear Algebra, 5th edition
    - Gilbert Strang, WELLESLEY CAMBRIDGE PRESS