Optimization

$$\min f_0(x) \leftarrow \text{objective}$$

$$\text{s.t.} \quad f_i(x) \le 0 \qquad i = 1, 2, \ldots m$$

$x \in \mathbb{R}^n$

$x_{n \times 1}$

constraint

$$\text{or} \quad f_j(x) = 0$$

Eg $n = 1$

$$\min (x-1)^2 \qquad x^* = 1$$

$$x \geq 0$$

Eg $n = 2$

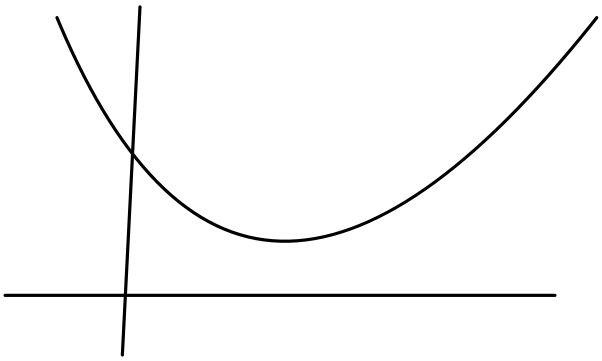
$$\min_{x \geq 0} \|\underline{x} - \underline{a}\|^2 \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad a = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

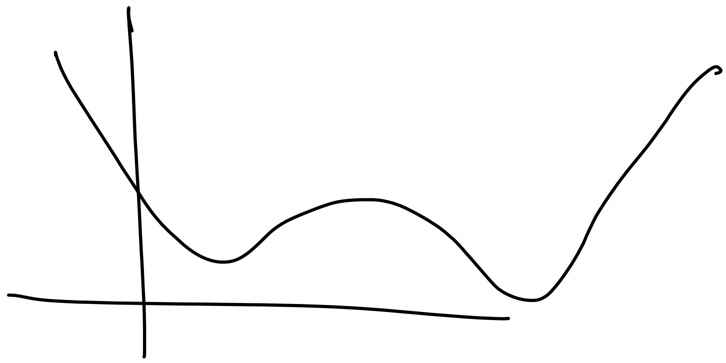
$$= \min (x_1 - 1)^2 + (x_2 - 2)^2$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

$$\underline{x} = \begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Convex Optimization





- easy to optimize over
- mathematical properties
- guaranteed to converge (solution → optimum)

  predictable algorithms

Eg. Least Squares problem

$$x^* = \arg\min \|A\underline{x} - \underline{b}\|_2^2$$

$$(Ax-b)^T(Ax-b)$$

$x \in \mathbb{R}^n$
$b \in \mathbb{R}^m$
$A \in \mathbb{R}^{m\times n}$
$(Ax) \in \mathbb{R}^m$
$Ax - b \in \mathbb{R}^m$

$$\underbrace{[\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot]}_{m \text{ entries}} \quad \left.\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}\right\} m \text{ entries}$$

Special case: $n = m$ and A^{-1} exists

$x^* = A^{-1}b$ so that $\|Ax^* - b\| = 0$

$n \neq m$ then $x^* = (A^TA)^{-1}A^Tb$ when $(A^TA)^{-1}$ exists
closed form (formula)

- well studied

MATLAB : A\b
lscov(A, b)

try these for simple cases

Eg Linear Programming

$$\min c^T x \qquad x \in \mathbb{R}^n$$

$$Ax \leq b \quad \text{---} \quad b \in \mathbb{R}^m$$

$$A = \begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ & \vdots & \\ - & \vdots & - \end{bmatrix}$$

$a_i^T x \leq b_i \quad i = 1, 2 \dots m$

$$a_1^T x \leq b_1$$
$$a_2^T x \leq b_2$$
$$\vdots \qquad \vdots$$
$$a_m^T x \leq b_m$$

$[A]_{ij} = a_{ij}$

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad i = 1, 2, \dots, m$$

Complexity ?

\# floating point operations (eg. $\times, \div, +, -$)

$\sim O(mn^2)$ $\quad x \in \mathbb{R}^n$ $\quad m > n$

Roughly : convex $\sim O(n^3)$

(size n)

= max (# variables , # constraints)

\# operations $\leq$ (const) $\times n^3$

$\downarrow$

does not depend on

n

problem structure / condition number

vs. General optimization problems $O(e^n)$

$e^n \gg n^3$ or n^5 ...

for large n

e.g. $e^{10} \sim 22026$

$n^3 \sim 1000$