EE910: Digital Communication Systems-I

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Lecture #8A: Maximum Likelihood Sequence Estimation: Viterbi Algorithm

Maximum Likelihood Sequence Estimation

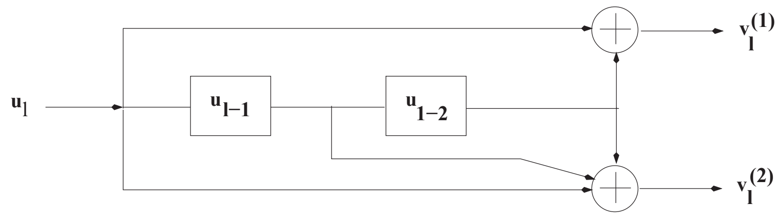
- Any system with memory can be represented by its state diagram or trellis diagram.
- Viterbi algorithm is an computational efficient way to find maximum likelihood sequence estimate.
- We will take an example of a simple error correcting code having four states to illustrate how maximum likelihood sequence estimation works.

Viterbi Decoding

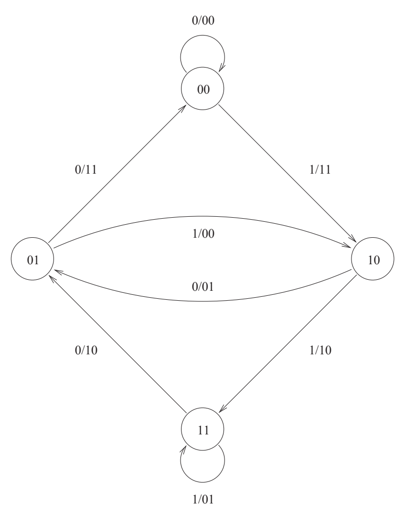
- We will consider an example where each information bit is encoded into two coded bits using the error correcting code that has memory two.
- We will assume that the data is transmitted over a binary symmetric channel.

Viterbi Decoding

Example:



Viterbi Decoding



Viterbi decoding

On BSC:

- Let the information sequence of length L

$$\mathbf{u} = (u_0, u_1, \cdots, u_l, \cdots, u_{L-1})$$

is encoded into code sequence of length $N \triangleq L$

$$\mathbf{v} = (\mathbf{v}_0, \mathbf{v}_1, \cdots, \mathbf{v}_l, \cdots, \mathbf{v}_{L-1})$$

- If the code sequence $\mathbf{v}$ is transmitted over a channel, let the received sequence is,

$$\mathbf{r} = (\mathbf{r}_0, \mathbf{r}_1, \cdots, \mathbf{r}_l, \cdots, \mathbf{r}_{L-1}),$$

where the l^{th} received block is

$$\mathbf{r}_l = (r_l^{(1)}, r_l^{(2)}).$$

Viterbi decoding

On BSC:

- A maximum likelihood decoder finds a path through the trellis that maximizes the path conditional probability

$$P(\mathbf{r}|\mathbf{v}) = \prod_{l=0}^{L} P(\mathbf{r}_l|\mathbf{v}_l)$$

where the branch conditional probability

$$P(\mathbf{r}_l|\mathbf{v}_l) = \prod_{l=1}^{2} P(r_l^{(i)}|v_l^{(i)})$$

- The bit conditional probabilities $P(r_l^{(i)}|v_l^{(i)})$ are the channel transition probabilities.

Viterbi decoding

On BSC:

- Maximizing $P(\mathbf{r}|\mathbf{v})$ is equivalent to maximizing

$$M(\mathbf{r}|\mathbf{v}) \triangleq \log P(\mathbf{r}|\mathbf{v})$$

- $M(\mathbf{r}|\mathbf{v})$ is called the path metric.

Viterbi decoding

- We have

$$\begin{aligned} M(\mathbf{r}|\mathbf{v}) &= \sum_{l=0}^{L} \log P(\mathbf{r}_l|\mathbf{v}_l) \\ &= \sum_{l=0}^{L} M(\mathbf{r}_l|\mathbf{v}_l), \qquad \text{(branch metrics)} \end{aligned}$$

$$\begin{aligned} M(\mathbf{r}_l|\mathbf{v}_l) &= \sum_{i=1}^{2} \log P(r_l^{(i)}|v_l^{(i)}) \\ &= \sum_{i=1}^{2} M(r_l|v_l), \qquad \text{(bit metrics)} \end{aligned}$$

Viterbi decoding

- The partial path metric for the first j branches of a path $\mathbf{v}$ is given by

$$M([\mathbf{r}|\mathbf{v}]_j) = \sum_{l=0}^{j-1} M(\mathbf{r}_l|\mathbf{v}_l)$$

- For BSC, the maximum likelihood decoder decodes the received sequence $\mathbf{r}$ into code sequence $\mathbf{v}$ that minimizes the Hamming distance $d(\mathbf{r}, \mathbf{v})$
- The Viterbi algorithm is a computationally efficient method of finding the path through the trellis with the best metric.

Viterbi decoding

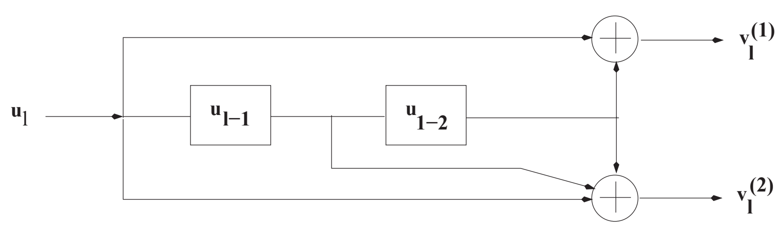
- The Viterbi decoder proceeds through the trellis level by level in search of the path with the best metric.
- At each level, the decoder compares the metric of all partial paths entering each state.
- The decoder stores the partial path entering each state with the best metric (survivor path) and eliminates all other partial paths.
- For $m \leq l \leq L$, there are total 2^m survivors.
- To bring the code to zero state is known as termination process.
- The number of survivors decrease during the termination process, until there is only one survivor left.
- The surviving path is the maximum likelihood path.

Viterbi decoding

- Step 1: Starting at level $l = m$ in the trellis, compute the partial metric for the single path entering each m^{th} level state. Store the survivor path and its metric for each state.
- Step 2: Increase time l by one. Compute the partial metric for all the paths entering at the $(l+1)^{th}$ level state by adding the branch metric entering that state to the metric of the connecting survivor path at the previous l^{th} level state. Store the survivor path and its metric for each state.
- Step 3: Repeat Step 2 until you are at the end of the trellis.

Viterbi Decoding

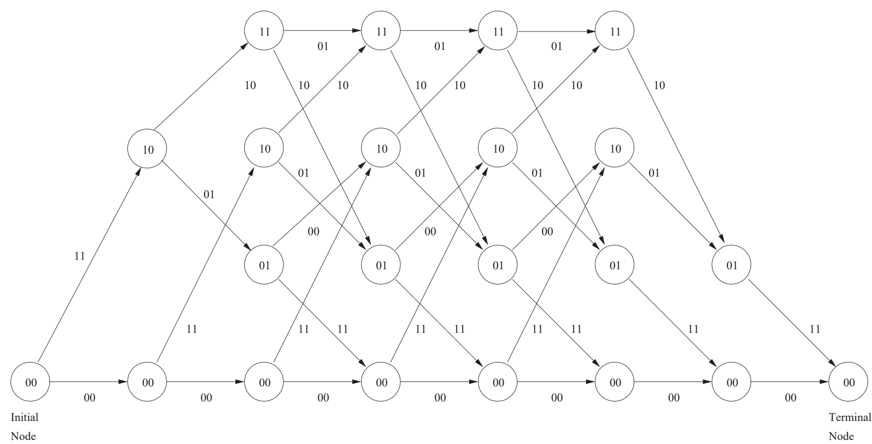
Example:



- This $(2,1,2)$ convolutional code with $L = 7$(including termination bits) is used on a BSC. The received sequence is

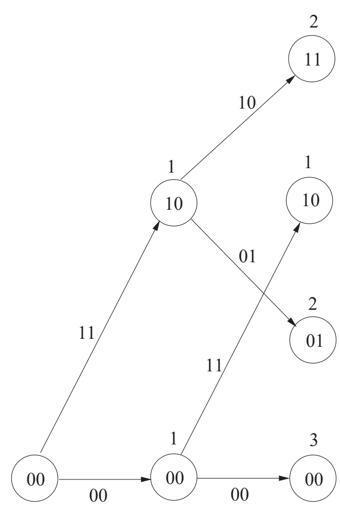
$$\mathbf{r} = (01,\ 11,\ 10,\ 10,\ 00,\ 11,\ 10)$$

Viterbi Decoding



Trellis diagram of $(2, 1, 2)$ convolutional code.

Viterbi Decoding



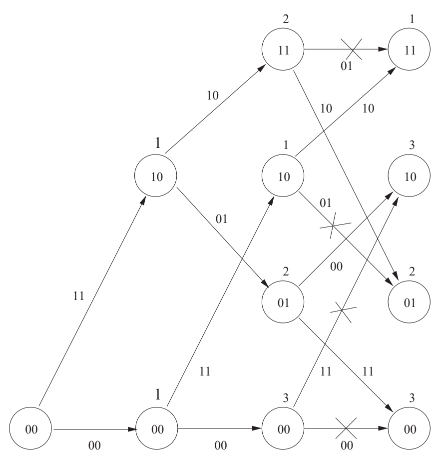
Level 2

Viterbi Decoding

$$\mathbf{r} = (01,\ 11\ )$$

$$\begin{array}{ll}
\mathbf{v}_1 = (00,\ 00\ ) & d(\mathbf{v}_1, \mathbf{r}) = 3 \\
\mathbf{v}_2 = (00,\ 11\ ) & d(\mathbf{v}_2, \mathbf{r}) = 1 \\
\mathbf{v}_3 = (11,\ 01\ ) & d(\mathbf{v}_3, \mathbf{r}) = 2 \\
\mathbf{v}_4 = (11,\ 10\ ) & d(\mathbf{v}_4, \mathbf{r}) = 2
\end{array}$$

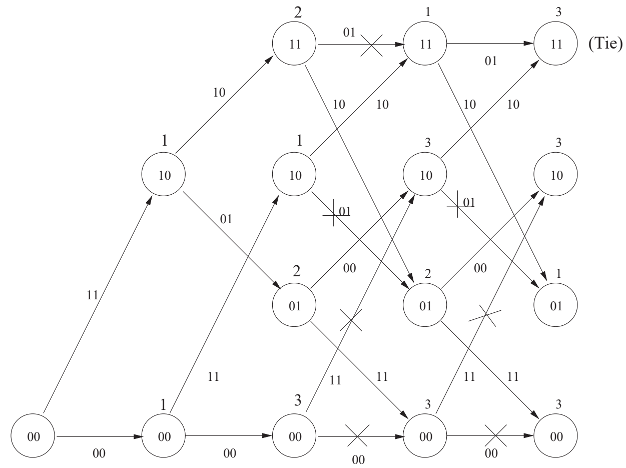
Viterbi Decoding



$$\mathbf{r} = (01,\ \ 11\ \ 10)$$

Level 3

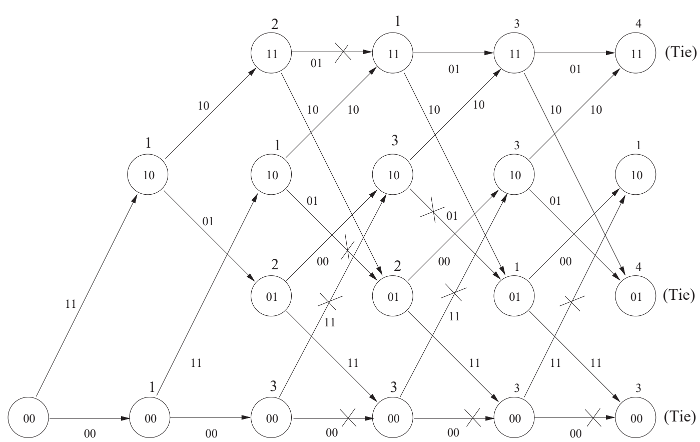
Viterbi Decoding



$\mathbf{r} = (01,\;\; 11 \;\; 10 \;\; 10)$

Level 4

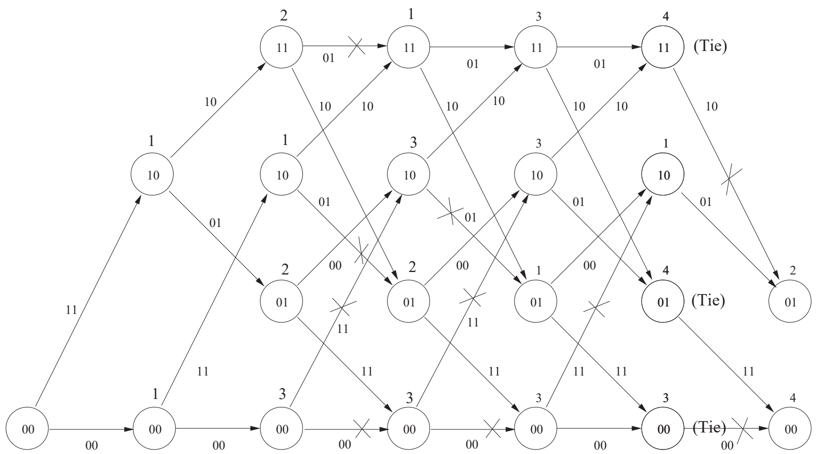
Viterbi Decoding



$\mathbf{r} = (01,\;\; 11 \;\; 10 \;\; 10 \;\; 00)$

Level 5

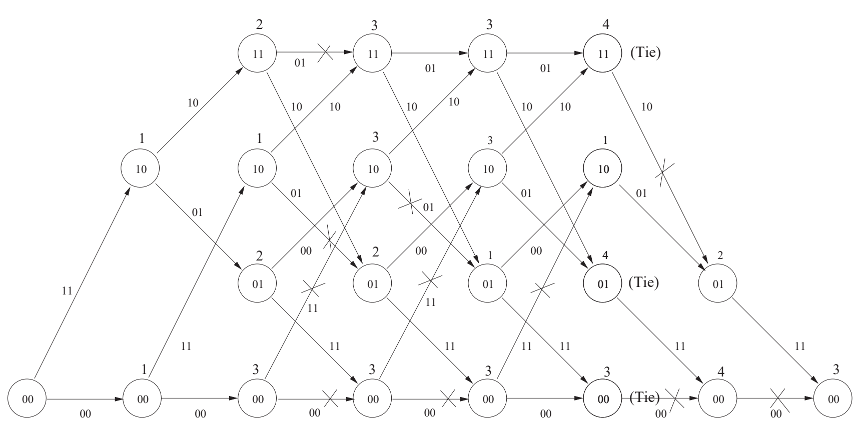
Viterbi Decoding



$\mathbf{r} = (01, \;\; 11 \;\; 10 \;\; 10 \;\; 00 \;\; 11)$

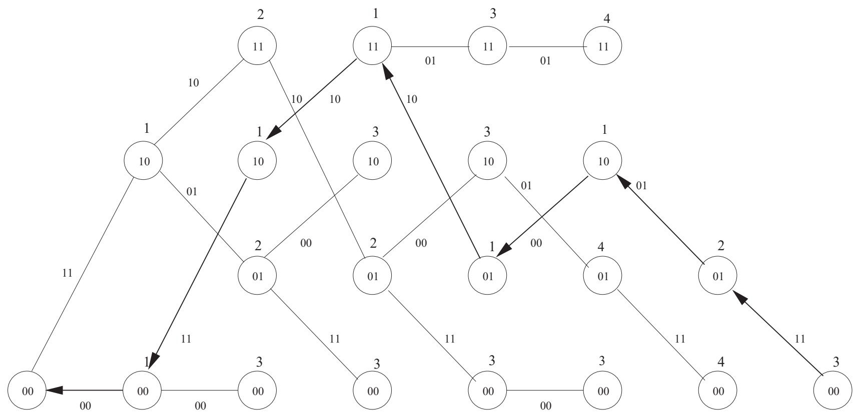
Level 6

Viterbi Decoding



$\mathbf{r} = (01, \;\; 11 \;\; 10 \;\; 10 \;\; 00 \;\; 11 \;\; 10)$

Level 7



$\hat{\mathbf{v}} = (00, \; 11 \; 10 \; 10 \; 00 \; 01 \; 11)$

Level 7