Live Interaction #4:

27th October 2023

E-masters Communication Systems

Detection for Wireless

- Probability of error for M –ary QAM:
- P_e for interior point:

$$2Q\left(\frac{A}{\sigma}\right)$$

- P_e for edge point:

$$Q\left(\frac{A}{\sigma}\right)$$

- Overall P_e

$$P_{int} \times 2Q\left(\frac{A}{\sigma}\right) + P_{edge} \times Q\left(\frac{A}{\sigma}\right)$$

$$= \frac{(M-2)}{M} \times 2Q\left(\frac{A}{\sigma}\right) + \frac{2}{M} \times Q\left(\frac{A}{\sigma}\right)$$

$$= 2\left(1 - \frac{1}{M}\right) Q\left(\frac{A}{\sigma}\right)$$

- Average symbol power:

$$E_s = \frac{A^2}{3}(M^2 - 1)$$

$$P_e = 2\left(1 - \frac{1}{M}\right) Q\left(\frac{\sqrt{\frac{3E_s}{M^2-1}}}{\sqrt{\frac{N_0}{2}}}\right)$$

$$P_e = 2\left(1-\frac{1}{M}\right)Q\left(\sqrt{\frac{6E_s}{(M^2-1)N_0}}\right)$$

- Probability of error for M −ary QAM
- In-phase PAM, Quadrature PAM: $\sqrt{M}$

$$2\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3E_s}{(M-1)N_0}}\right)$$

- Probability of symbol error:

$$2\times 2\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3E_s}{(M-1)N_0}}\right)$$

$$=4\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3E_s}{(M-1)N_0}}\right)$$

- $SNR = 22\ dB$. What is the P_e for 16 QAM?

$$SNR = \frac{E_s}{N_0} = 22\ dB = 10^{2.2}$$

$$P_e = 4\left(1-\frac{1}{4}\right)Q\left(\sqrt{\frac{3\times 10^{2.2}}{15}}\right)$$

$$=3Q\left(\sqrt{\frac{3\times 10^{2.2}}{15}}\right) = 2.7\times 10^{-8}$$

- Min P_e decision rule:
- $\Pr(\mathcal{H}_0) = \pi_0$, $\Pr(\mathcal{H}_1) = \pi_1$

- Choose $\mathcal{H}_0$ if

$$\frac{p(\bar{\mathbf{y}}|\mathcal{H}_0)}{p(\bar{\mathbf{y}}|\mathcal{H}_1)} \geq \frac{\pi_1}{\pi_0} = \tilde{\gamma}$$

$$\Rightarrow \qquad \underbrace{p(\mathcal{H}_0|\bar{\mathbf{y}}) \geq p(\mathcal{H}_1|\bar{\mathbf{y}})}_{\text{MAP:Maximum Aposteriori Probability}}$$

- **MAP Decision rule**:
- **MAP reduces to ML when** $\pi_1 = \pi_0 = \frac{1}{2}$
- Consider our signal detection problem:

$$\mathcal{H}_0 : \bar{\mathbf{y}} = \bar{\mathbf{v}}$$

$$\mathcal{H}_1 : \bar{\mathbf{y}} = \bar{\mathbf{s}} + \bar{\mathbf{v}}$$

$$\tilde{\gamma} = \frac{\pi_1}{\pi_0}$$

$$\gamma = \frac{\|\bar{\mathbf{s}}\|^2 - 2\sigma^2 \ln \frac{\pi_1}{\pi_0}}{2}$$

$$\underbrace{P_e^{MAP} = \pi_0 Q\left(\frac{1}{2}\sqrt{SNR} - \frac{1}{\sqrt{SNR}} \ln \frac{\pi_1}{\pi_0}\right) + \pi_1 Q\left(\frac{1}{2}\sqrt{SNR} + \frac{1}{\sqrt{SNR}} \ln \frac{\pi_1}{\pi_0}\right)}_{P_e \text{ for MAP rule}}$$

- Probability of error for ML decoder:

$$P_e^{ML} = Q\left(\frac{1}{2}\sqrt{SNR}\right)$$

- $SNR = 15\, dB = 10^{1.5}$. Calculate P_e^{MAP}, P_e^{ML}.
- $\pi_0 = 0.90$

$$P_e^{MAP} = 0.90 \times Q\left(\frac{1}{2}\sqrt{10^{1.5}} + \frac{1}{\sqrt{10^{1.5}}} \ln 9\right) + 0.10 \times Q\left(\frac{1}{2}\sqrt{10^{1.5}} - \frac{1}{\sqrt{10^{1.5}}} \ln 9\right) = 0.0014$$

$$P_e^{ML} = Q\left(\frac{1}{2}\sqrt{10^{1.5}}\right) = 0.0025$$