EE908 Assignment 1 31 March 2024

1. Solve these problems and submit by 7th April (Sunday) 9am before the discussion session.
2. There is no penalty for submitting incorrect attempts
3. However, plagiarism will result in serious penalties, such as an F grade.

1. Inner product and norms
   - $\boxed{1}$ (a) $\|\mathbf{u}+\mathbf{v}\|_2^2 = \|\mathbf{u}\|_2^2 + \|\mathbf{v}\|_2^2$ if and only if $\mathbf{u}^T\mathbf{v} = 0$.
   - $\boxed{1}$ (b) $2\langle \mathbf{a}, \mathbf{b}\rangle + 2\langle \mathbf{x}, \mathbf{y}\rangle = \langle \mathbf{a}+\mathbf{x}, \mathbf{b}+\mathbf{y}\rangle + \langle \mathbf{a}-\mathbf{x}, \mathbf{b}-\mathbf{y}\rangle$
   - $\boxed{1}$ (c) $\|\mathbf{x}\|_1 \le \sqrt{n}\,\|\mathbf{x}\|_2$
   - $\boxed{1}$ (d) $\|\mathbf{x}\|_1 \ge \|\mathbf{x}\|_2 \ge \|\mathbf{x}\|_\infty$

$\boxed{2}$ 2. Prove the triangle inequality for matrices, $\|\mathbf{A}+\mathbf{B}\|_F \le \|\mathbf{A}\|_F + \|\mathbf{B}\|_F$.

3. The following are some useful inequalities for ℓ_2 norm. Prove them:
   - $\boxed{1}$ (a) $2\langle \mathbf{x}, \mathbf{y}\rangle \le \|\mathbf{x}\|^2 + \|\mathbf{y}\|^2$
   - $\boxed{1}$ (b) $2\langle \mathbf{x}, \mathbf{y}\rangle \le \epsilon\,\|\mathbf{x}\|^2 + \frac{1}{\epsilon}\,\|\mathbf{y}\|^2$ for any $\epsilon > 0$
   - $\boxed{1}$ (c) $\|\mathbf{x}+\mathbf{y}\|^2 \le (1+\epsilon)\,\|\mathbf{x}\|^2 + (1+1/\epsilon)\,\|\mathbf{y}\|^2$ for any $\epsilon > 0$
   - $\boxed{1}$ (d) $\|\mathbf{x}_1 + \mathbf{x}_2 + \ldots + \mathbf{x}_n\|^2 \le n\,\|\mathbf{x}_1\|^2 + n\,\|\mathbf{x}_2\|^2 + \ldots + n\,\|\mathbf{x}_n\|^2$