1. Three urns contain 6 red, 4 black; 4 red, 6 black, and 5 red, 5 black balls respectively. One of the urns is selected at random and a ball is drawn from it. Given the ball drawn is red, the probability that it is drawn from the first urn can be found using Bayes principle as follows

$$P(U_1|R) = \frac{P(R|U_1)P(U_1)}{P(R)} = \frac{P(R|U_1)P(U_1)}{P(R|U_1)P(U_1) + P(R|U_2)P(U_2) + P(R|U_3)P(U_3)}$$

$$= \frac{\frac{6}{10} \times \frac{1}{3}}{\frac{6}{10} \times \frac{1}{3} + \frac{4}{10} \times \frac{1}{3} + \frac{5}{10} \times \frac{1}{3}} = \frac{\frac{6}{10}}{\frac{15}{10}} = \frac{6}{15} = \frac{2}{5}$$

   Ans a
2. The Naïve Bayes assumption can be verbally expressed as The features are conditionally independent given the label
   Ans b
3. The probability $p(y = 1)$ can be evaluated as

$$1 - p(y = 0) = 1 - \frac{\sum_{i=1}^{M} 1(y(i) = 1)}{N}$$

   Ans d
4. Given a new observation $\bar{\mathbf{x}} = \bar{\mathbf{v}}$, it can be labeled as belonging to the class $y = 1$ if

$$\frac{\prod_{j=1}^{N} p\left(x_j = v_j | y = 1\right) \times p(y = 1)}{p(\bar{\mathbf{x}} = \bar{\mathbf{v}})} > \frac{\prod_{j=1}^{N} p\left(x_j = v_j | y = 0\right) \times p(y = 0)}{p(\bar{\mathbf{x}} = \bar{\mathbf{v}})}$$

$$\Rightarrow \prod_{j=1}^{N} p\left(x_j = v_j | y = 1\right) \times p(y = 1) > \prod_{j=1}^{N} p\left(x_j = v_j | y = 0\right) \times p(y = 0)$$

   Ans b
5. Given the data below

SNo.	Weather condition	Road condition	Traffic condition	Engine problem	Accident
1	Rain	bad	high	no	yes
2	snow	average	normal	yes	yes
3	clear	bad	light	no	no
4	clear	good	light	yes	yes
5	snow	good	normal	no	no
6	rain	average	light	no	no
7	rain	good	normal	no	no
8	snow	bad	high	no	yes
9	clear	good	high	yes	no
10	clear	bad	high	yes	yes

   Q_1 for accident occurring with rainy weather over a bad road with high traffic is traffic and no engine problem

$$p(yes) \times p(Rain|yes) \times p(bad|yes) \times p(high|yes) \times p(no|yes)$$

$$= \frac{1}{2} \times \frac{1}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{2}{5} = \frac{18}{1250}$$

   Ans b
6. Unsupervised learning Requires data, but NO labels
   Ans a
7. The cluster assignment indicators $\alpha_i(j)$ for K-means satisfy

$$\sum_{i=1}^{K} \alpha_i(j) = 1, \alpha_i(j) \in \{0,1\}$$

   Ans c
8. The K-means algorithm is imported in PYTHON as

from sklearn.cluster import KMeans

Ans d

9. Given

$$\bar{\mathbf{x}}(1) = \begin{bmatrix} -2 \\ -4 \end{bmatrix}, \bar{\mathbf{x}}(2) = \begin{bmatrix} -4 \\ -2 \end{bmatrix} \in \mathcal{C}_0$$

$$\bar{\mathbf{x}}(3) = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \bar{\mathbf{x}}(4) = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \in \mathcal{C}_1$$

The centroids are given as

$\bar{\boldsymbol{\mu}}_0 = \frac{\bar{\mathbf{x}}(1)+\bar{\mathbf{x}}(2)}{2} = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$, $\bar{\boldsymbol{\mu}}_1 = \frac{\bar{\mathbf{x}}(3)+\bar{\mathbf{x}}(4)}{2} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

Ans b

10. Given the data $\bar{\mathbf{x}}(1) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, and centroids below

$$\bar{\boldsymbol{\mu}}_0 = \begin{bmatrix} -2 \\ -1 \end{bmatrix}, \bar{\boldsymbol{\mu}}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Distance to centroid 0= $\sqrt{4^2 + 2^2} = \sqrt{20}$

Distance to centroid 1 = $\sqrt{1^2 + 1^2} = \sqrt{2}$

Hence, it is assigned to **cluster 1** as distance is minimum. Therefore, it follows that $\alpha_0(1) = 0, \alpha_1(1) = 1$

Ans b