# Convex Optimization Problems

Std. form: $x^* = \arg\min_x f_0(x)$ (convex)

$$f_i(x) \le 0 \qquad i = 1, 2, \ldots, m$$

affine: $h_j(x) = a_j^T x - b_j = 0 \qquad j = 1, \ldots, p$

Note:
- seemingly non-convex problems may be converted into convex optimization problems
- software may only recognize the standard form

Eg

| | | | |
|---|---|---|---|
| | $\min \; x_1^2 + x_2^2$ | | $\min \; x_1^2 + x_2^2$ |
| non-convex constraint | $\frac{x_1}{1+x_2^2} \le 0$ | $\Leftrightarrow$ | $x_1 \le 0$ |
| non-affine equality | $(x_1 + x_2)^2 = 0$ | $\Leftrightarrow$ | $x_1 + x_2 = 0$ |

in standard form

Software will throw error: not convex!

- many problems thought to be non-convex for many years, turned out to be convex

Eg $\min f_0(x)$ $\Leftrightarrow$ $\min \alpha f_0(x)$

$f_i(x) \leq 0$ ↓ equivalent $\beta_i \cdot f_i(x) \leq 0$

$a_i^T x - b_i = 0$ obtain solution of one from the other $\gamma_i (a_i^T x - b_i) = 0$

here:

$\alpha, \beta > 0$

$\gamma_i \neq 0$

- same $x^*$
- but objective value will be different