Generalized LFP

- not a convex problem

$$\min \left\{ \max_i \frac{c_i^T x + d_i}{e_i^T x + f_i} \right\}$$

$Gx \leq h$

$Ax = b$

Charnes-Cooper transform not applicable

epigraph

$$\min_{x,t} \; t$$

dom: $e_i^T x + f_i > 0$

$$\frac{c_i^T x + d_i}{e_i^T x + f_i} \leq t \quad \forall i$$

How to solve?

$Gx \leq h$

$Ax = b$

Observe: $g_i(\underline{x}) = \dfrac{c_i^T x + d_i}{e_i^T x + f_i}$ neither convex nor concave

but quasiconvex.

given α, $C_\alpha^i = \{ \underline{x} \mid g_i(\underline{x}) \leq \alpha \}$ convex set.

$$= \{ x \mid c_i^T x + d_i \leq \alpha (e_i^T x + f_i) \}$$

$$= \{ x \mid \underbrace{(c_i - \alpha e_i)^T x + (d_i - \alpha f_i) \leq 0}_{\text{half space (given } \alpha)} \}$$

$\Rightarrow$ line search (e.g. bisection) on α

consider feasibility problem (convex)

$$\begin{aligned} &\text{find}_x \ x && (P_\alpha) \\ \text{s.t. } & g_i(\underline{x}) \leq \alpha && \leftarrow \text{convert to convex for fixed } \alpha \\ & Gx \leq h \\ & Ax = b \end{aligned}$$

for $\alpha \to \infty$ constraint easily satisfied

for $\alpha \to -\infty$ constraint not satisfied

find smallest α s.t. constraint satisfied suppose (we know apriori) $l \preceq \alpha \preceq u$

for each α solve P_α

Bisection

given $l \leq \alpha \leq u$

set $\alpha = \frac{l+u}{2}$

solve P_α

$\rightarrow$ infeasible : $u = \alpha$

$\rightarrow$ feasible $l = \alpha$

until $(u-l) \leq \epsilon$

takes $\sim O(\log 1/\epsilon)$ iterations

solve using a series of convex problems

Summary: G-LFP

P_α : find x

$$(c_i - \alpha e_i)^T x + (d_i - \alpha f_i) \leq 0 \quad \forall i$$

$$Gx \leq h$$

$$Ax = b$$

$$e_i^T x + f_i > 0 \quad \forall i$$

(convex) $\rightarrow$ CVX

quasi-convex

$\rightarrow$ series of convex

$\log(1/\epsilon)$