$$\langle u, v \rangle \leq \|u\| \|v\|$$

Desire: $\|\underline{u} + t\underline{v}\| \geq 0$

so $(\underline{u} + t\underline{v})^T (\underline{u} + t\underline{v}) \geq 0$

$$\underline{u}^T u + 2t(u^T v) + t^2 (v^T v)$$

$u^T v = v^T u$ $\quad \forall\, t \in \mathbb{R}$

$$\min_t (\ ) \geq 0$$

$$t = -\frac{u^T v}{\|u\|_2^2}$$

$$\Rightarrow \quad (u^T v)^2 \leq \|u\|^2 \|v\|^2$$

Similarly: $tr(A^T B) \leq \|A\|_F \|B\|_F$