EE 900 - Mid Semester Exam (Quiz-2)

Name : Venkateswar Reddy

Roll No : 23156022

Course : EE 900

Date : 13-08-2023

Program : eMasters - Comm. Systems

Q1 Solution:

(i) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \& \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$

- The smallest subspace spanned in the set of all vectors of the form $(c_1, -c_2, 0)$ where c_1 and c_2 are real numbers.
- This is a one-dimensional subspace in $\mathbb{R}^3$

(ii) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$\uparrow A$ $\uparrow B$

- $\text{span}\{A, B\} = \left\{ \begin{bmatrix} c_1 & c_1 \\ 0 & c_1 \end{bmatrix} \mid c_1, c_2 \in \mathbb{R} \right\}$
- The smalled subspau spanned is set of all matrices of form $\begin{bmatrix} c_1 & c_1 \\ 0 & c_1 \end{bmatrix}$

Q2 Solution:

$$A = \begin{bmatrix} 2 & 4 & 6 \\ -4 & -2 & -2 \\ 3 & 6 & 12 \end{bmatrix}$$

Applying Gauss-Jordan and Gauss-elimination procedure to obtain $A \to R$

$$\xrightarrow{R_2 + 2R_1} \begin{bmatrix} 2 & 4 & 6 \\ 0 & 6 & 10 \\ 3 & 6 & 12 \end{bmatrix} \xrightarrow{2r_3 - 3r_1} \begin{bmatrix} \textcircled{2} & 4 & 6 \\ 0 & \textcircled{6} & 10 \\ 0 & 0 & \textcircled{6} \end{bmatrix} \leftarrow U$$

$6R_1 - 4R_2$ [illegible] r_2

$$\xrightarrow{6R_1 - 4R_2} \begin{bmatrix} 12 & 0 & -4 \\ 0 & 6 & 10 \\ 0 & 0 & 6 \end{bmatrix} \xrightarrow[6R_2 - 10R_3]{} \begin{bmatrix} 12 & 0 & -4 \\ 0 & 36 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\xrightarrow{6R_1 + 4R_3} \begin{bmatrix} 72 & 0 & 0 \\ 0 & 36 & 0 \\ 0 & 0 & 6 \end{bmatrix} \xrightarrow[R_3/6]{R_1/72,\ R_2/36} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\uparrow$ rref(A)

(8) -

$$B = \begin{bmatrix} 4 & 12 & 16 \\ 2 & -3 & 8 \\ 4 & 12 & 5 \end{bmatrix}$$

Applying Gaussian elimination & Gauss-Jordan Procedure

$$\xrightarrow{2R_2 - R_1} \begin{bmatrix} 4 & 12 & 16 \\ 0 & -18 & 0 \\ 4 & 12 & 5 \end{bmatrix} \xrightarrow{R_3 - R_1} \begin{bmatrix} 4 & 12 & 16 \\ 0 & -18 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

further reduing to rref

$$\xrightarrow{\cdot 18R_1 + 12R_2} \begin{bmatrix} 18 & 0 & 288 \\ 0 & -18 & 0 \\ 0 & 0 & 11 \end{bmatrix} \xrightarrow{}$$

$$\xrightarrow{11R_1 - 288R_3} \begin{bmatrix} 198 & 0 & 0 \\ 0 & -18 & 0 \\ 0 & 0 & 11 \end{bmatrix} \xrightarrow{\substack{R_1/198 \\ R_2/-18 \\ R_3/11}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q3 Solution

Writing augumented matrix for the equations

$$\begin{bmatrix} 1 & 1 & b_1 \\ 2 & 3 & b_2 \\ 0.5 & 1 & b_3 \\ 5 & 10 & b_4 \end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix} 1 & 1 & b_1 \\ 0 & 1 & b_2 - 2b_1 \\ 0.5 & 1 & b_3 \\ 5 & 10 & b_4 \end{bmatrix}$$

$2R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & b_1 \\ 0 & 1 & b_2 - 2b_1 \\ 0 & 1 & 2b_3 - b_1 \\ 5 & 10 & b_4 \end{bmatrix}$$

$R_4 - 5R_1$

$$\begin{bmatrix} 1 & 1 & b_1 \\ 0 & 1 & b_2 - 2b_1 \\ 0 & 1 & 2b_3 - b_1 \\ 0 & 5 & b_4 - 5b_1 \end{bmatrix}$$

↑p ↑p

- 2 Pivot
- 2 free variable

$x_1 + x_2 = 0$

$x_2 = b_2 - 2$

Solvable when

$b_2 - 2b_1 = 0$ & $b_1 = 0$

$2b_3 - b_1 = 0$

$b_4 - 5b_1 = 0$

Q4 Solution:

(a) $x = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ (b) $y = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

$p = x\hat{x}$ $\quad e = b - p$

$p = aa^T b$ $\quad \hat{x} = y x^{-1}$

$= yy^T$

$\therefore p = \dfrac{aa^T b}{a^T a} = \dfrac{(1,-1,-1).(1,-1,-1)\, b}{(1,-1,-1)(1,-1,-1)}$

$= \dfrac{(1+1+1)}{(1+1+1)}(1,3,2) = (1,3,2)$

$e = b - p = (1,3,2) - (1,3,2)$

$= (0,0,0)$

The dot product of e to y is

$(0,0,0)(1,-1,-1) = (0,0,0)$

$\therefore e \perp y$

Q5 solution

$$\det A = ad - bc = 0 - 20 = -20$$

$$\det A^{-1} = \frac{1}{\det A} = \frac{1}{20}$$

$$\det A^3 = \det A \cdot \det A \cdot \det A = (-20)(-20)(-20)$$

$$= -800$$