Live Interaction #8:

25th November 2023

E-masters Communication Systems

Estimation for Wireless

- **Online estimation**:
- Continuous update of the estimate
- Multi-antenna channel estimation model:

$$\underbrace{\begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(N) \end{bmatrix}}_{\bar{\mathbf{y}}} = \underbrace{\begin{bmatrix} \bar{\mathbf{x}}^T(1) \\ \bar{\mathbf{x}}^T(2) \\ \vdots \\ \bar{\mathbf{x}}^T(N) \end{bmatrix}}_{\mathbf{X}} \bar{\mathbf{h}} + \bar{\mathbf{v}}$$

- ML estimate is given as

$$\hat{\mathbf{h}}(N) = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\bar{\mathbf{y}}$$

$$\mathbf{P}(N) = \sigma^2(\mathbf{X}^T\mathbf{X})^{-1}$$

- At time $N+1$ we have a new pilot output

$$y(N+1) = \bar{\mathbf{x}}^T(N+1)\bar{\mathbf{h}} + v(N+1)$$

- How to update $\hat{\mathbf{h}}(N)$ to obtain $\hat{\mathbf{h}}(N+1)$?

$$\underbrace{\bar{\mathbf{k}}(N+1) = \frac{\mathbf{P}(N)\bar{\mathbf{x}}(N+1)}{\sigma^2 + \bar{\mathbf{x}}^T(N+1)\mathbf{P}(N)\bar{\mathbf{x}}(N+1)}}_{\text{Gain}}$$

$$\underbrace{e(N+1) = y(N+1) - \bar{\mathbf{x}}^T(N+1)\hat{\mathbf{h}}(N)}_{\text{Prediction error}}$$

$$\underbrace{\hat{\mathbf{h}}(N+1) = \hat{\mathbf{h}}(N) + \bar{\mathbf{k}}(N+1)e(N+1)}_{\text{Update equation}}$$

$$\mathbf{P}(N+1) = \left(\mathbf{I} - \bar{\mathbf{k}}(N+1)\bar{\mathbf{x}}^T(N+1)\right)\mathbf{P}(N)$$

- **Advantage**: **Very very low complexity!**
- Example:
- $N = 4, \sigma^2 = 4$

$$\mathbf{X} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & -1 \\ -1 & -1 \end{bmatrix}, \bar{\mathbf{y}} = \begin{bmatrix} -2 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

$$\mathbf{X}^T\mathbf{X} = 4\mathbf{I}$$

$$\hat{\mathbf{h}}(4) = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\bar{\mathbf{y}}$$

$$= \frac{1}{4}\begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -2 \\ -6 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -1 \\ -3 \end{bmatrix}$$

$$P(4) = \sigma^2(\mathbf{X}^T\mathbf{X})^{-1} = \mathbf{I}$$

- At time $N + 1 = 5$,

$$y(5) = -2, \bar{\mathbf{x}}(5) = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$\bar{\mathbf{k}}(5) = \frac{\mathbf{P}(4)\bar{\mathbf{x}}(5)}{\sigma^2 + \bar{\mathbf{x}}^T(5)\mathbf{P}(4)\bar{\mathbf{x}}(5)}$$

$$= \frac{\begin{bmatrix}-2\\-2\end{bmatrix}}{4 + \begin{bmatrix}-2 & -2\end{bmatrix}\begin{bmatrix}-2\\-2\end{bmatrix}} = \frac{\begin{bmatrix}-2\\-2\end{bmatrix}}{4+8} = \frac{1}{12}\begin{bmatrix}-2\\-2\end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix}-1\\-1\end{bmatrix}$$

$$e(5) = y(5) - \bar{\mathbf{x}}^T(5)\hat{\mathbf{h}}(4)$$

$$= -2 - \begin{bmatrix}-2 & -2\end{bmatrix}\frac{1}{2}\begin{bmatrix}-1\\-3\end{bmatrix}$$

$$= -2 - 4 = -6$$

$$\hat{\mathbf{h}}(N+1) = \hat{\mathbf{h}}(N) + \bar{\mathbf{k}}(N+1)e(N+1)$$

$$= \frac{1}{2}\begin{bmatrix}-1\\-3\end{bmatrix} + \frac{1}{6}\begin{bmatrix}-1\\-1\end{bmatrix}(-6)$$

$$= \frac{1}{2}\begin{bmatrix}-1\\-3\end{bmatrix} + \begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}\frac{1}{2}\\-\frac{1}{2}\end{bmatrix}$$

$$\mathbf{P}(5) = \left(\mathbf{I} - \bar{\mathbf{k}}(5)\bar{\mathbf{x}}^T(5)\right)\mathbf{P}(4)$$

$$= \mathbf{I} - \frac{1}{6}\begin{bmatrix}-1\\-1\end{bmatrix}\begin{bmatrix}-2 & -2\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} - \begin{bmatrix}\frac{1}{3} & \frac{1}{3}\\\frac{1}{3} & \frac{1}{3}\end{bmatrix} = \begin{bmatrix}\frac{2}{3} & -\frac{1}{3}\\-\frac{1}{3} & \frac{2}{3}\end{bmatrix}$$

- **MIMO Receiver:**
- **ZF Receiver:**

$$\bar{\mathbf{y}} = \mathbf{H}\bar{\mathbf{x}} + \bar{\mathbf{v}}$$

$$\underbrace{\hat{\mathbf{x}} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\bar{\mathbf{y}}}_{\text{ZF receiver}}$$

- BLUE: Best Linear Unbiased Estimate.
- Assignment 8 Deadline – 25th November Saturday 11:59 PM.
- Assignment 7, 8 Discussion – 26th November Sunday 12:30-1:00 PM.
- Quiz #4 – 26th November Sunday 1:15 – 2:00 PM.

Final Exam:

- Total questions in Final: 40
- Multiple choice questions with four given options and only one correct option.
- NO negative marking
- Closed-book exam
- Duration is 3 hrs
- One mark per question
- Question Paper PATTERN
- 8 questions: Recall type (Purely formula), one from each week
- 16 questions: Seen, Directly from assignments, two from each week
- 16 questions: Unseen, Roughly 2 from every week based on assignment questions
- Weightage:

	Proposed Weightage

Assignments (Theory)	20%
Quizzes	30%
Final	40%
Attendance Minimum 80% attendance	10%

- **Best 3 out of four quizzes**
- **Best 6 out of 8 assignments**