EE908 Assignment 2 5 April 2024

1. Solve these problems and submit by 14th April (Sunday) 9am before the discussion session.
2. There is no penalty for submitting incorrect attempts
3. However, plagiarism will result in serious penalties, such as an F grade.

$\boxed{2}$ 1. Let $\lambda_i(\mathbf{A})$ denote an eigenvalues of a symmetric matrix $\mathbf{A}$. Find the following in terms of $\lambda_i(\mathbf{A})$

   (a) $\text{Tr}(\mathbf{A}^3)$

   (b) $\lambda_i(\mathbf{A}^{-2})$

   (c) $\lambda_i(\mathbf{A} - \mathbf{I})$

   (d) $\lambda_i(\mathbf{I} + 2\mathbf{A})$

$\boxed{2}$ 2. Prove the following results for $\mathbf{A} \succ 0$:

   (a) $\mathbf{A}^{-1} \succ 0$

   (b) $[\mathbf{A}]_{ii} > 0$ for all i, where $[\mathbf{A}]_{ii}$ denotes the i-th diagonal entry of $\mathbf{A}$.

$\boxed{2}$ 3. A matrix $\mathbf{A}$ is idempotent if $\mathbf{A}^2 = \mathbf{A}$. Show that the only possible eigenvalues of an idempotent matrix are $\lambda = 0$ and $\lambda = 1$.

$\boxed{2}$ 4. Given an $m \times n$ matrix $\mathbf{A}$ with SVD $\mathbf{A} = \mathbf{A} = \sum_{i=1}^{r} \sigma_i \mathbf{v}_i \mathbf{u}_i^T$, show that $\|\mathbf{A}\|_F^2 := \text{tr}(\mathbf{A}^T\mathbf{A}) = \sum_{i=1}^{r} \sigma_i^2$.

$\boxed{2}$ 5. The ℓ_2 norm of a matrix $\mathbf{A}$ is defined as

$$\|\mathbf{A}\|_2 = \max_{\|\mathbf{x}\|_2 = 1} \|\mathbf{A}\mathbf{x}\|_2$$

   Derive an expression for $\|\mathbf{A}\|_2$ in terms of $\{\sigma_i\}_{i=1}^{r}$.