eMasters in Communication Systems

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Elective Module:

Detection for Wireless Communication

Chapter 4

Detection with Multiple Symbols

Multiple Symbol Constellation

- Consider the **BHT problem**

$\mathcal{H}_0$: $\bar{y} = \bar{s}_0 + \bar{v}$ Signal for $\mathcal{H}_0$

$\mathcal{H}_1$: $\bar{y} = \bar{s}_1 + \bar{v}$ Signal for $\mathcal{H}_1$

Multiple Symbol Constellation

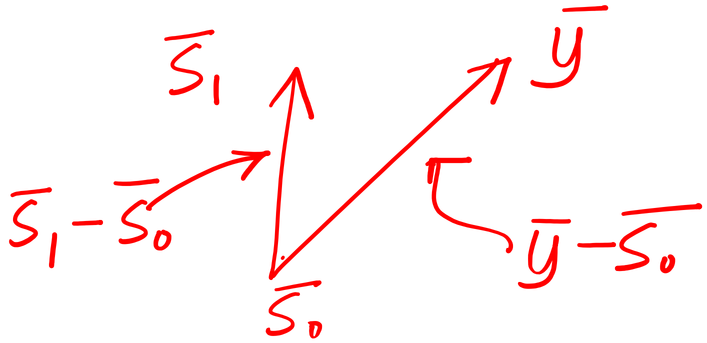
- Consider the BHT problem

$$\mathcal{H}_0 : \bar{\mathbf{y}} = \bar{\mathbf{s}}_0 + \bar{\mathbf{v}}$$

$$\mathcal{H}_1 : \bar{\mathbf{y}} = \bar{\mathbf{s}}_1 + \bar{\mathbf{v}}$$

Multiple Symbol Constellation

- It was seen that the **optimal detector** is



- Choose $\mathcal{H}_0$ if

$$\bar{s}^T \tilde{y} \lessgtr \gamma$$

$$(\bar{s}_1-\bar{s}_0)^T(\bar{y}-\bar{s}_0) \lessgtr \gamma$$

Multiple Symbol Constellation

- It was seen that the **optimal detector** is
- Choose $\mathcal{H}_0$ if

$$\bar{\mathbf{s}}^T \tilde{\mathbf{y}} \leq \gamma$$

$$(\bar{\mathbf{s}}_1 - \bar{\mathbf{s}}_0)^T \tilde{\mathbf{y}} \leq \gamma$$

For ML

$$\gamma = \frac{\|\bar{s}\|^2}{2} = \frac{\|\bar{s}_1 - \bar{s}_0\|^2}{2}$$

Multiple Symbol Constellation

- For ML, this can be simplified as follows
- Choose $\mathcal{H}_0$ if

$$(\bar{s}_1 - \bar{s}_0)^T \overbrace{(\bar{y} - \bar{s}_0)}^{\tilde{y}} \lessgtr \frac{\|\bar{s}_1 - \bar{s}_0\|^2}{2}$$

Multiple Symbol Constellation

- For ML, this can be simplified as follows
- Choose $\mathcal{H}_0$ if

$\bar{y} - \bar{s}_0$

$$(\bar{\mathbf{s}}_1 - \bar{\mathbf{s}}_0)^T \tilde{\mathbf{y}} \leq \frac{\|\bar{\mathbf{s}}_1 - \bar{\mathbf{s}}_0\|^2}{2}$$

Multiple Symbol Constellation $\|\bar{u}\|^2 = \bar{u}^T\bar{u}$

- This can be further simplified as

choose $\mathcal{H}_0$

$$(\bar{s}_1 - \bar{s}_0)^T(\bar{y} - \bar{s}_0) \leq \frac{\|\bar{s}_1 - \bar{s}_0\|^2}{2}$$

$$\Rightarrow 2(\bar{s}_1^T - \bar{s}_0^T)(\bar{y} - \bar{s}_0) \leq (\bar{s}_1 - \bar{s}_0)^T(\bar{s}_1 - \bar{s}_0)$$

$$\Rightarrow 2\bar{s}_1^T\bar{y} - 2\bar{s}_0^T y - \cancel{2\bar{s}_1^T\bar{s}_0} + 2\|\bar{s}_0\|^2 \leq \|\bar{s}_1\|^2 + \|\bar{s}_0\|^2 - \cancel{2\bar{s}_1^T\bar{s}_0}$$

Multiple Symbol Constellation

Choose $\mathcal{H}_0$ if

$$\Rightarrow \quad \|\bar{s}_0\|^2 - 2\bar{s}_0^T\bar{y} \leq \|\bar{s}_1\|^2 - 2\bar{s}_1^T\bar{y}$$

$$\Rightarrow \quad \|\bar{s}_0\|^2 + \|\bar{y}\|^2 - 2\bar{s}_0^T\bar{y} \leq \|\bar{s}_1\|^2 + \|\bar{y}\|^2 - 2\bar{s}_1^T\bar{y}$$

$$\Rightarrow \quad \|\bar{y} - \bar{s}_0\|^2 \leq \|\bar{y} - \bar{s}_1\|^2$$

$$\Rightarrow \quad \|\bar{y} - \bar{s}_0\| \leq \|\bar{y} - \bar{s}_1\|.$$

Multiple Symbol Constellation

choose $\mathcal{H}_0$ if

$$\|\bar{y}-\bar{s}_0\| \leq \|\bar{y}-\bar{s}_1\|$$

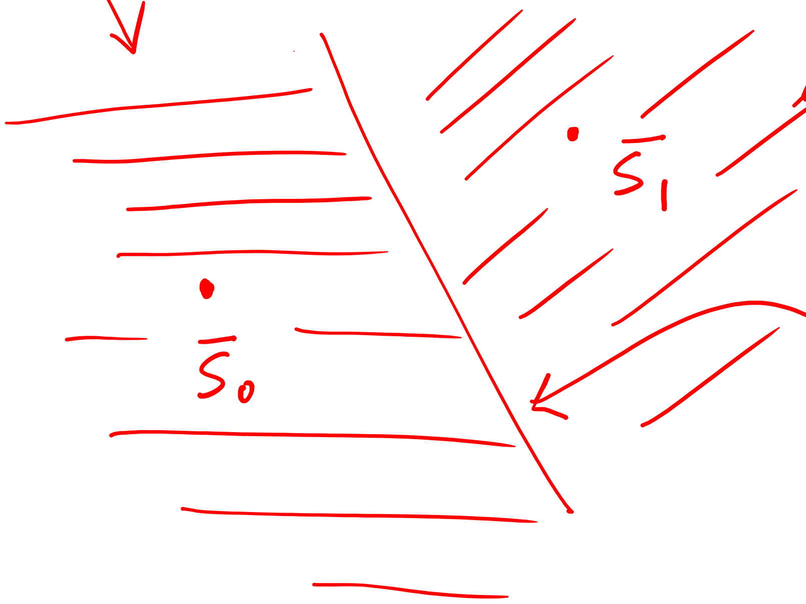
Minimum distance <u>decoder</u>

decision region $\mathcal{H}_0$.

Set of all points closer to $\bar{s}_0$

set of all points closer to $\bar{s}_1$.

$\perp$ bisector of $\bar{s}_1-\bar{s}_0$



decision region for $\mathcal{H}_1$.

Multiple Symbol Constellation

- This can be simplified as follows
- Choose $\mathcal{H}_0$ if

$$(\bar{\mathbf{s}}_1 - \bar{\mathbf{s}}_0)^T \tilde{\mathbf{y}} \leq \gamma$$

$$\Rightarrow (\bar{\mathbf{s}}_1 - \bar{\mathbf{s}}_0)^T (\bar{\mathbf{y}} - \bar{\mathbf{s}}_0) \leq \gamma$$

Multiple Symbol Constellation

- For ML detection
- Choose $\mathcal{H}_0$ if

$\bar{y} - \bar{s}_0$

$$(\bar{\mathbf{s}}_1 - \bar{\mathbf{s}}_0)^T \tilde{\mathbf{y}} \leq \frac{\|\bar{\mathbf{s}}_1 - \bar{\mathbf{s}}_0\|^2}{2}$$

$$\Rightarrow 2(\bar{\mathbf{s}}_1 - \bar{\mathbf{s}}_0)^T (\bar{\mathbf{y}} - \bar{\mathbf{s}}_0) \leq \|\bar{\mathbf{s}}_1 - \bar{\mathbf{s}}_0\|^2$$

$$2(\bar{\mathbf{s}}_1 - \bar{\mathbf{s}}_0)^T(\bar{\mathbf{y}} - \bar{\mathbf{s}}_\mathbf{0}) \le \|\bar{\mathbf{s}}_1 - \bar{\mathbf{s}}_0\|^2$$

$$\Rightarrow \mathbf{2}\bar{\mathbf{s}}_1^T\bar{\mathbf{y}} - \mathbf{2}\bar{\mathbf{s}}_0^T\bar{\mathbf{y}} - \mathbf{2}\bar{\mathbf{s}}_1^T\bar{\mathbf{s}}_\mathbf{0} + \mathbf{2}\|\bar{\mathbf{s}}_0\|^2 \le \|\bar{\mathbf{s}}_0\|^2 + \|\bar{\mathbf{s}}_1\|^2 - 2\bar{\mathbf{s}}_1^T\bar{\mathbf{s}}_\mathbf{0}$$

$$\Rightarrow \|\bar{\mathbf{s}}_0\|^2 - 2\bar{\mathbf{s}}_0^T\bar{\mathbf{y}} \le \|\bar{\mathbf{s}}_1\|^2 - \mathbf{2}\bar{\mathbf{s}}_1^T\bar{\mathbf{y}}$$

$$\Rightarrow \|\bar{\mathbf{s}}_0\|^2 - 2\bar{\mathbf{s}}_0^T\bar{\mathbf{y}} + \|\bar{\mathbf{y}}\|^2 \le \|\bar{\mathbf{s}}_1\|^2 - 2\bar{\mathbf{s}}_1^T\bar{\mathbf{y}} + \|\bar{\mathbf{y}}\|^2$$

Multiple Symbol Constellation

- This reduces to
- Choose $\mathcal{H}_0$ if

Nearest neighbor decision rule.
Nearest neighbor decoder

$$\|\bar{\mathbf{y}} - \bar{\mathbf{s}}_0\|^2 \le \|\bar{\mathbf{y}} - \bar{\mathbf{s}}_1\|^2$$

$$\Rightarrow \|\bar{\mathbf{y}} - \bar{\mathbf{s}}_0\| \le \|\bar{\mathbf{y}} - \bar{\mathbf{s}}_1\|$$

choose $\mathcal{H}_1$ if

$$\|\bar{y} - \bar{s}_1\| < \|\bar{y} - \bar{s}_0\|.$$

Multiple Symbol Constellation

- Choose $\mathcal{H}_0$ if $\|\bar{y}-\bar{s}_0\| \leqslant \|\bar{y}-\bar{s}_1\|$

choose $\mathcal{H}_1$ if $\|\bar{y}-\bar{s}_1\| \leqslant \|\bar{y}-\bar{s}_0\|$.

Nearest neighbor decoder

Multiple Symbol Constellation

- Choose $\mathcal{H}_0$ if

$$\|\bar{\mathbf{y}} - \bar{\mathbf{s}}_0\| \leq \|\bar{\mathbf{y}} - \bar{\mathbf{s}}_1\|$$

- Choose closest signal! NN Decoder
  - This is **nearest neighbor** decoder

Multiple Symbol Constellation

- Consider now a multiple hypothesis testing problem

M-ary hypothesis testing problem.

$\bar{s}_0, \bar{s}_1, \ldots, \bar{s}_{M-1}$: M signals.

$$\mathcal{H}_0 : \bar{\mathbf{y}} = \bar{\mathbf{s}}_0 + \bar{\mathbf{v}}$$

$$\mathcal{H}_1 : \bar{y} = \bar{s}_1 + \bar{v}$$

$$\vdots$$

$$\mathcal{H}_{M-1} : \bar{y} = \bar{s}_{M-1} + \bar{v}$$

Multiple Symbol Constellation

- Consider now a multiple hypothesis testing problem

$$\mathcal{H}_0 : \bar{\mathbf{y}} = \bar{\mathbf{s}}_0 + \bar{\mathbf{v}}$$
$$\mathcal{H}_1 : \bar{\mathbf{y}} = \bar{\mathbf{s}}_1 + \bar{\mathbf{v}}$$
$$\vdots$$
$$\mathcal{H}_{M-1} : \bar{\mathbf{y}} = \bar{\mathbf{s}}_{M-1} + \bar{\mathbf{v}}$$

Multiple Symbol Constellation

- Nearest neighbour decoder reduces to

choose $\mathcal{H}_i$ corresponding to signal $\bar{s}_i$ which is closest

- Choose $\mathcal{H}_i$ such that

$$i = \arg\min_j \|\bar{y} - \bar{s}_j\|.$$

Multiple Symbol Constellation

- Nearest neighbour decoder reduces to
- Choose $\mathcal{H}_i$ such that

$$\arg\min_i \underline{\|\bar{\mathbf{y}} - \bar{\mathbf{s}}_i\|}$$

Figure of Decision Region

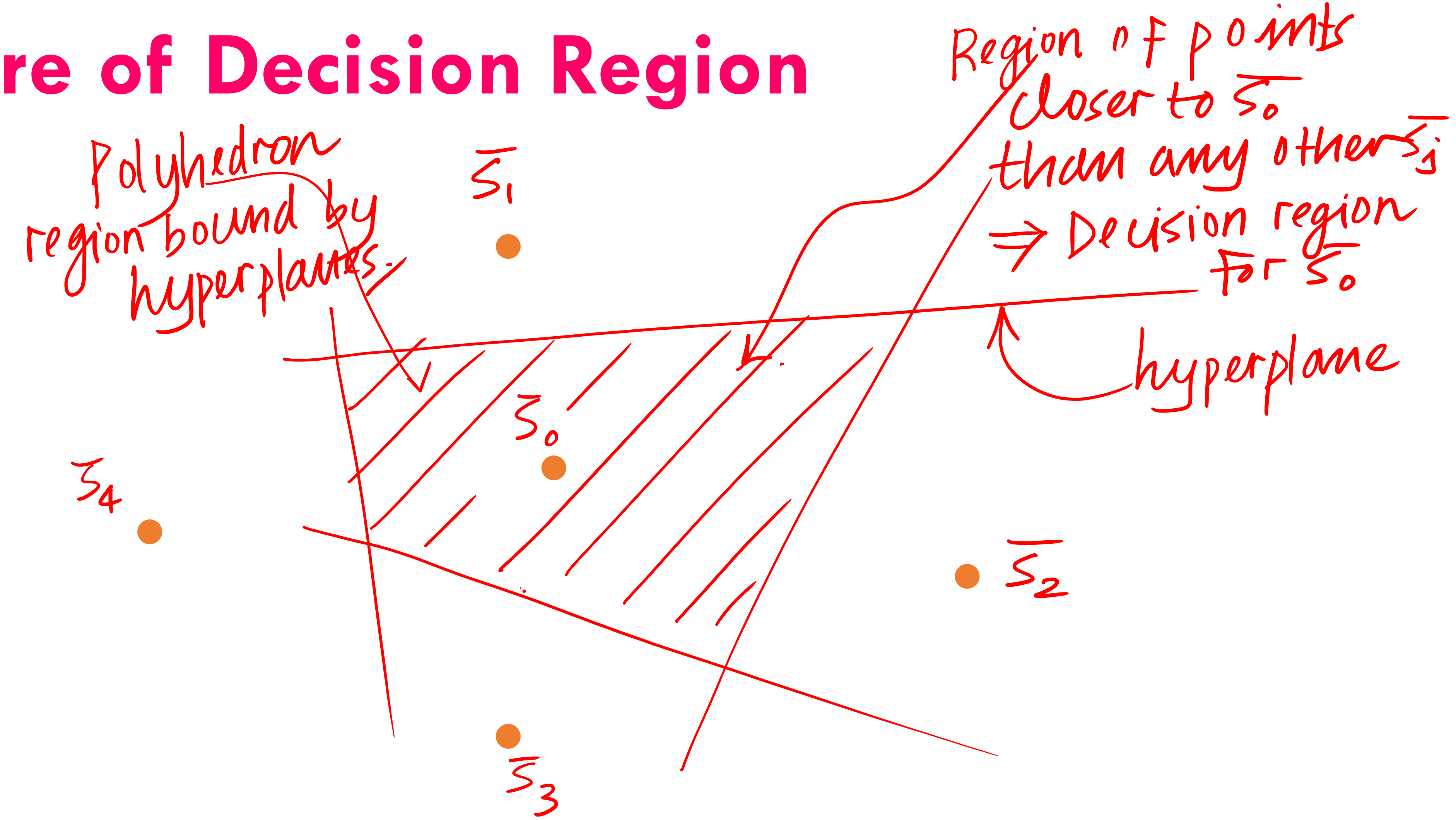
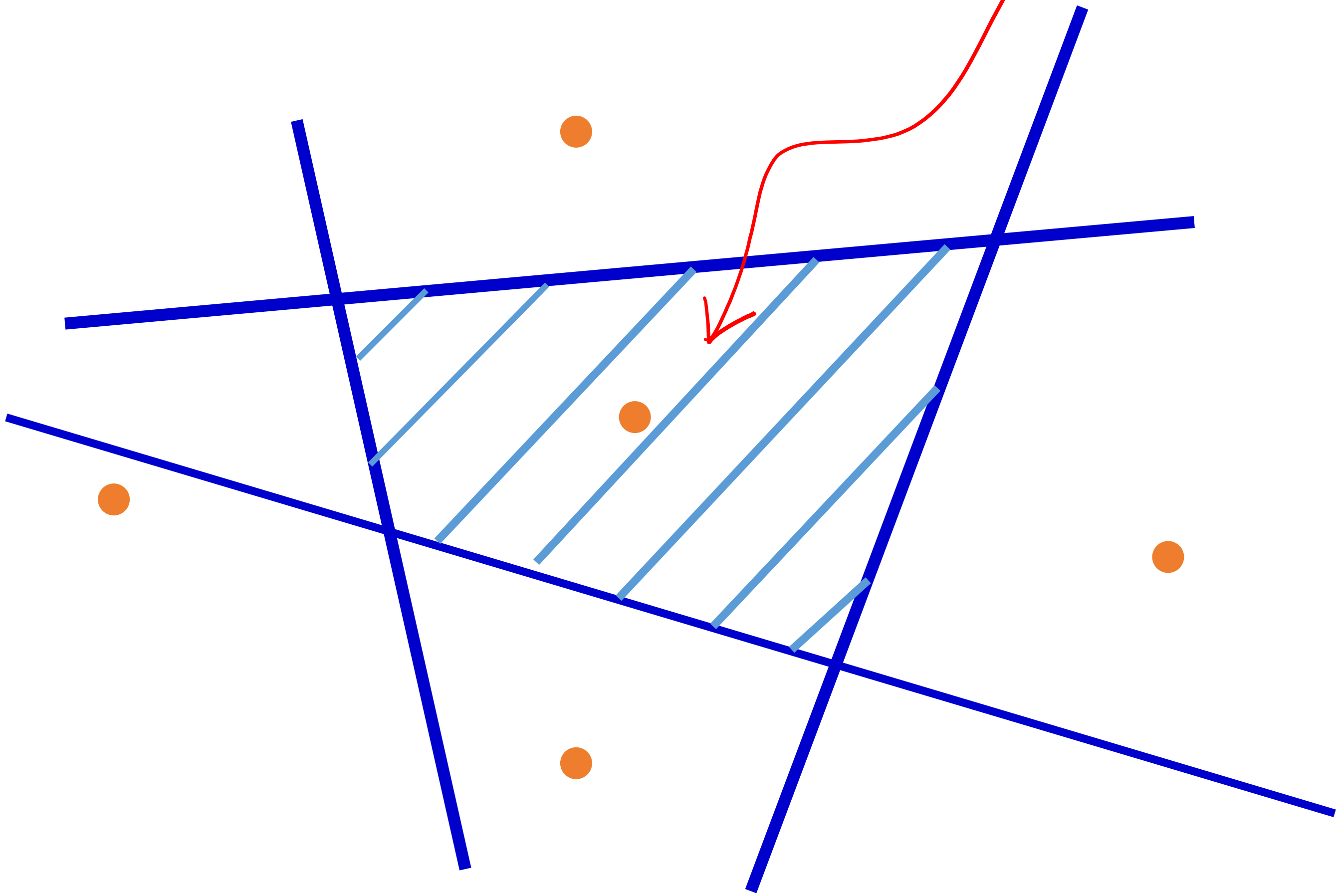


Figure of Decision Region

Decision region for $\bar{s}_0$



Multiple Symbol Constellation

- The decision region for each hypothesis is a ***polyhedron***.

Probability of Error

- What is the corresponding ***probability of error***?

P_e

Probability of Error of symbol decoding.

Probability of Error

distance between $\bar{s}_i, \bar{s}_j$

- **Confusion probability $i \to j$ is**

$$P_{\bar{s}_i - \bar{s}_j} = Q\left(\frac{\|\bar{s}_i - \bar{s}_j\|}{2\sigma}\right)$$

σ^2: Noise variance

Probability of Error

Confusion probability

- **Confusion probability** $i \to j$ is

$$P_{\bar{\mathbf{s}}_i \to \bar{\mathbf{s}}_j} = Q\left(\frac{\left\|\bar{\mathbf{s}}_i - \bar{\mathbf{s}}_j\right\|}{2\sigma}\right)$$

Probability of Error

$P(A \cup B \cup C \cdots) \leq P(A) + P(B) + P(C) + \cdots$

- **Probability of error for symbol i is**

union of confusion events.

$$P_{e,i} = P\left(\bigcup_{j \neq i} \bar{\mathbf{s}}_i \to \bar{\mathbf{s}}_j\right)$$

$$P_{e,i} \leq \sum_{j \neq i} P_{\bar{s}_i \to \bar{s}_j} = \sum_{j \neq i} Q\left(\frac{\|\bar{s}_i - \bar{s}_j\|}{2\sigma}\right)$$

Probability of Error

"Union bound"

- **Probability of error for symbol i is**

$$P_{e,i} = P\left(\cup_{j \neq i} \bar{\mathbf{s}}_i \to \bar{\mathbf{s}}_j\right)$$

Sum of confusion probabilities — Acts as upperbound for $P_{e,i}$

$$P_{e,i} \le \sum_{j \neq i} P(\bar{\mathbf{s}}_i \to \bar{\mathbf{s}}_j) = \sum_{j \neq i} Q\left(\frac{\|\bar{\mathbf{s}}_i - \bar{\mathbf{s}}_j\|}{2\sigma}\right)$$

Probability of Error

- Here we use the property

ignoring the probabilities of intersection of confusion events.

$$P(A \cup B) \leq P(A) + P(B)$$

Probability of Error

$P_i = \frac{1}{M}$.

- Considering equiprobable symbols, P_e is

$\sum_i P_i P_{e,i}$

$$P_e = \sum_i \frac{1}{M} P_{e,i} = \frac{1}{M} \sum_i P_{e,i} = \frac{1}{M} \sum_i \sum_{j \neq i} Q\left(\frac{\|\bar{s}_i - \bar{s}_j\|}{2\sigma}\right)$$

Probability of Error

These are termed: Nearest neighbors.

choose only $\bar{s}_j$ closest to $\bar{s}_i$

- Considering equiprobable symbols, P_e is

$$P_e = \sum_i \frac{1}{M} P_{e,i} = \frac{1}{M} \sum_i \sum_{j \neq i} Q\left(\frac{\|\bar{\mathbf{s}}_i - \bar{\mathbf{s}}_j\|}{2\sigma}\right)$$

$$= \frac{1}{M} \sum_i N_{min}^{i} \, Q\left(\frac{d_{min}^{i}}{2\sigma}\right)$$

Probability of Error

- This can be further simplified as

Number of nearest neighbors to s_i

distance of nearest neighbors to s_i

$$\underbrace{P_e = \frac{1}{M}\sum_i N^i_{min}\, Q\left(\frac{d^i_{min}}{2\sigma}\right)}_{\text{union bound for } P_e}$$

union bound for P_e.

Probability of Error

M dimensional constellation -

- This can be further simplified as

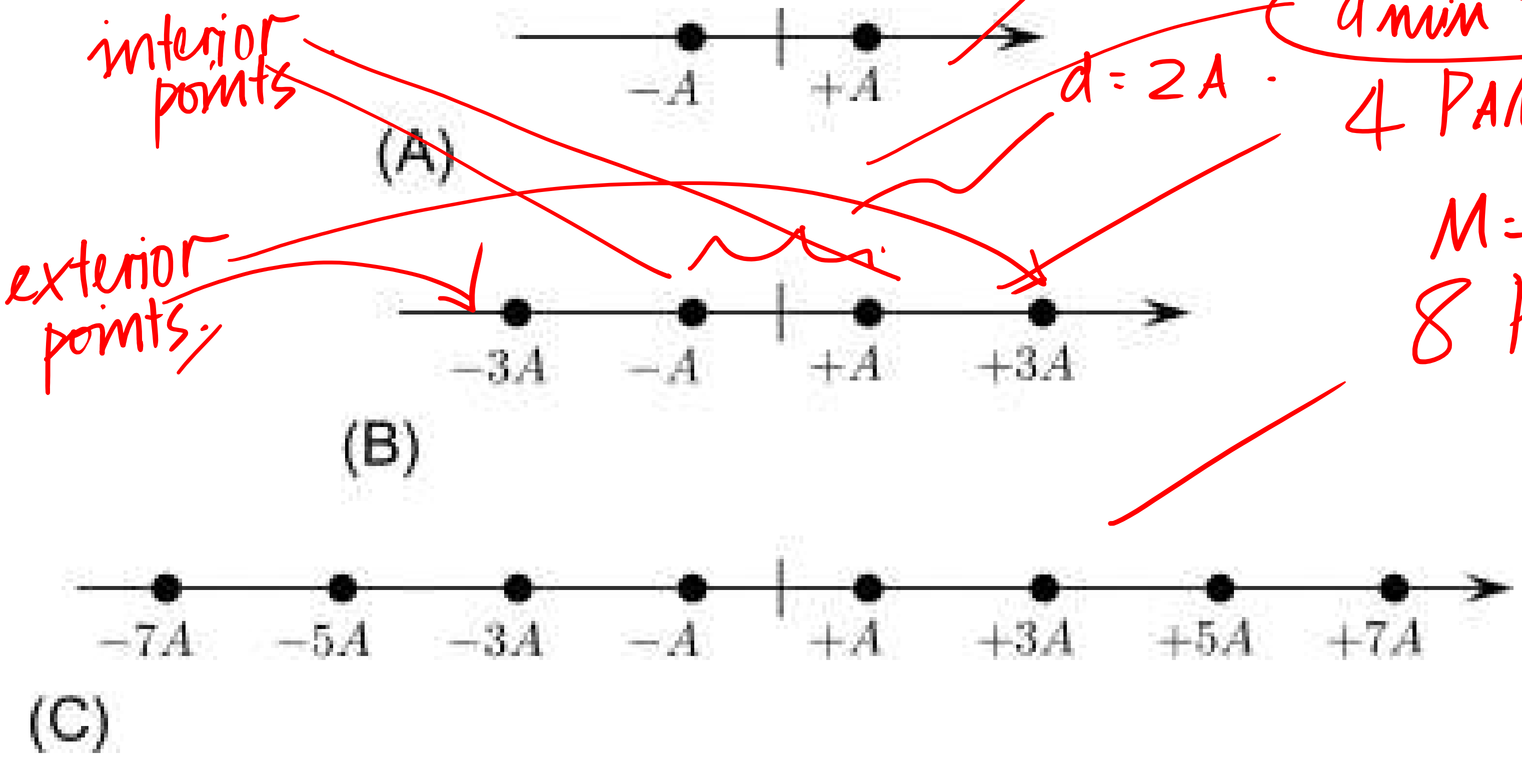
$$P_e = \frac{1}{M}\sum_{i}\sum_{j\neq i} Q\left(\frac{\|\bar{\mathbf{s}}_i - \bar{\mathbf{s}}_j\|}{2\sigma}\right)$$

$$= \frac{1}{M}\sum_{i} N_{min}^{i} Q\left(\frac{d_{min}^{i}}{2\sigma}\right)$$

M-ary PAM

- Consider now M –ary PAM

Binary Phase shift keying



interior points

exterior points.

$d_{min} = 2A$

$d = 2A$.

4 PAM

M = even & power of 2

integer bits/sym.

8 PAM.

M-ary PAM

- Ex 8 –ary PAM

$d_{min} = 2A$

i^{th} point of M-ary PAM

$-7A, -5A, -3A, -A, A, 3A, 5A, 7A$

$S_i = (2i - (M-1))A$

$i = 0, 1, \ldots, M-1$

M-ary PAM

- Ex $8-$ary PAM

$$\underbrace{-7A, -5A, -3A, -A, A, 3A, 5A, 7A}_{8-PAM}$$

M-ary PAM

M-ary hypothesis testing problem.

- This is a multiple hypothesis testing problem

$N = 1$

$$\mathcal{H}_0 : y = s_0 + v$$

$$\mathcal{H}_1 : y = s_1 + v$$

$$\vdots$$

$$\mathcal{H}_{M-1} : y = s_{M-1} + v$$

M-ary PAM

- This is a multiple hypothesis testing problem

8 ary PAM

$-7A, -5A, \ldots, 5A, 7A.$

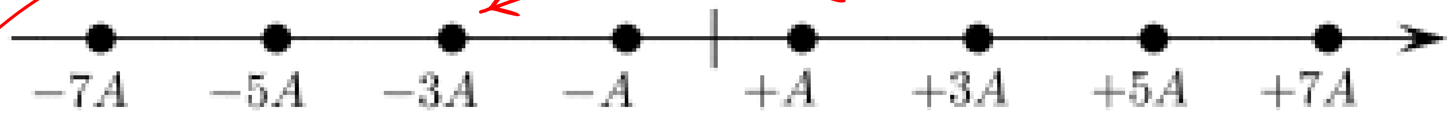
$$\mathcal{H}_0: y = s_0 + v$$
$$\mathcal{H}_1: y = s_1 + v$$
$$\vdots$$
$$\mathcal{H}_{M-1}: y = s_{M-1} + v$$
$$s_i = \big(2i - (M-1)\big)A$$

Symbol Error Rate

- Any interior point

$N_{min}^{i} = 2$

$d_{min}^{i} = 2A$



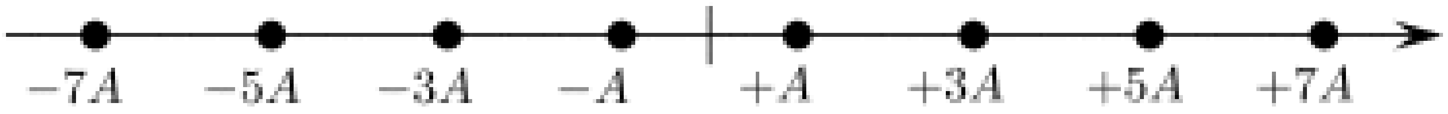
number of Nearest neighbors = 2

distance of Nearest neighbors = 2A

$$P_{e,i} = 2\,Q\left(\frac{2A}{2\sigma}\right) = 2\,Q\left(\frac{A}{\sigma}\right)$$

Symbol Error Rate

- Any interior point

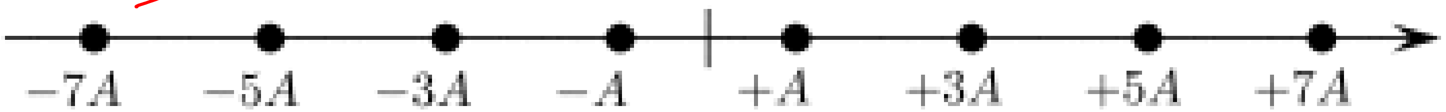


$$P_e \approx 2 \times Q\left(\frac{2A}{2\sigma}\right) = 2 \times Q\left(\frac{A}{\sigma}\right)$$

Symbol Error Rate

- Any boundary point

boundary points
Ex: $-7A, 7A$
Number of Nearest neighbors $= 1$
distance of nearest neighbors $= 2A$

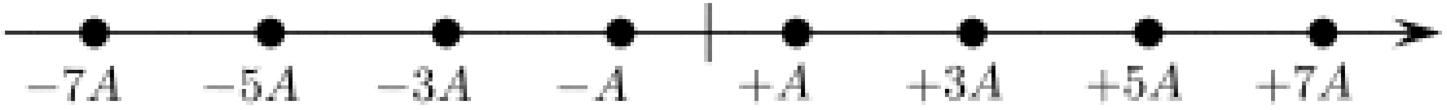


$$N_{min}^{i} = 1 \qquad d_{min}^{i} = 2A$$

$$P_{e,i} = Q\left(\frac{2A}{2\sigma}\right) = Q\left(\frac{A}{\sigma}\right)$$

Symbol Error Rate

- Any boundary point



$$P_e = Q\left(\frac{A}{\sigma}\right)$$

Symbol Error Rate

- Overall probability of error

$$P_e = \frac{1}{M} \sum_i N_{min}^i Q\left(\frac{d_{min}^i}{2\sigma}\right)$$

$$= \frac{1}{M}(M-2) \cdot 2\, Q\left(\frac{A}{\sigma}\right) + \frac{1}{M} \cdot 2\, Q\left(\frac{A}{\sigma}\right)$$

$$= \frac{1}{M}\left(2M-4+2\right) Q\left(\frac{A}{\sigma}\right) = 2\left(1-\frac{1}{M}\right) Q\left(\frac{A}{\sigma}\right)$$

Symbol Error Rate

$A = f(E_s)$.
$E_s = f^{-1}(A)$.

- Overall probability of error

$$\frac{(M-2)}{M} \times 2Q\left(\frac{A}{\sigma}\right) + \frac{2}{M} \times Q\left(\frac{A}{\sigma}\right)$$

$$P_e = 2\left(1 - \frac{1}{M}\right) Q\left(\frac{A}{\sigma}\right)$$

express A in terms of average symbol power E_s.

Symbol Error Rate

M-ary PAM

$S_i = (2i-(M-1))A$

- E_s Average symbol power

$E_s = \sum_i P(S_i)|S_i|^2 = \frac{1}{M}\sum_i |S_i|^2$

$$E_s = \frac{1}{M}\sum_{i=0}^{M-1}\left(2i-(M-1)\right)^2 A^2$$

$$= \frac{1}{M}\sum_{i=0}^{M-1}\left(4i^2+(M-1)^2-4i(M-1)\right)A^2$$

$$= \frac{A^2}{M}\cdot\left\{4\frac{(M-1)M(2M-1)}{6}+M(M-1)^2-4(M-1)\frac{(M-1)M}{2}\right\}$$

Symbol Error Rate

- E_s Average symbol power

$$E_s = \frac{A^2}{M}\left\{\frac{2}{3}(M-1)M(2M-1) - M(M-1)^2\right\}$$

$$= \frac{A^2}{\cancel{M}} \cdot \cancel{M}(M-1)\left\{\frac{2}{3}\cdot(2M-1) - (M-1)\right\}$$

$$= \frac{A^2(M-1)}{3}\left\{4M-2-3M+3\right\}$$

$$E_s = \frac{A^2}{3}(M^2-1).$$

Symbol Error Rate

- E_s Average symbol power

$$E_s = \frac{A^2}{3}\left(M^2 - 1\right).$$

$$A = \sqrt{\frac{3E_s.}{M^2-1}}$$

Symbol Error Rate

- E_s Average symbol power

$$E_s = \frac{1}{M}\sum_{i=0}^{M-1}\left(2i-(M-1)\right)^2 A^2$$

$$= \frac{A^2}{M}\sum_{i=0}^{M-1}\left(4i^2+(M-1)^2-4i(M-1)\right)$$

$$= \frac{A^2}{M}\left(\frac{4(M-1)M(2M-1)}{6}+M(M-1)^2 - \frac{4(M-1)(M-1)M}{2}\right)$$

Symbol Error Rate

- E_s Average symbol power

$$= \frac{A^2}{M}\left(\frac{2(M-1)M(2M-1)}{3} - M(M-1)^2\right)$$

$$= \frac{A^2}{3}\left(2(M-1)(2M-1) - 3(M-1)^2\right)$$

$$= \frac{A^2}{3}\left(4M^2 - 6M + 2 - 3M^2 - 3 + 6M\right)$$

$$E_s = \frac{A^2}{3}\left(M^2 - 1\right)$$

Symbol Error Rate

- Therefore we have

$$E_s = \frac{A^2}{3}(M^2-1)$$

$$A = \sqrt{\frac{3E_s}{M^2-1}}$$

Symbol Error Rate

- Therefore we have

$$E_s = \frac{A^2}{3}(M^2 - 1)$$

$$A = \sqrt{\frac{3E_s}{M^2 - 1}}$$

Symbol Error Rate (SER)

$\sigma^2 = \frac{N_0}{2}$.

- P_e reduces to

$$P_e = 2\left(1 - \frac{1}{M}\right) Q\left(\frac{A}{\sigma}\right) = 2\left(1 - \frac{1}{M}\right) Q\left(\sqrt{\frac{3E_s}{(M^2-1)\frac{N_0}{2}}}\right)$$

$$P_e = 2\left(1 - \frac{1}{M}\right) Q\left(\sqrt{\frac{6E_s}{(M^2-1)N_0}}\right)$$

Probability of symbol error for M-ary PAM.

Symbol Error Rate

- P_e reduces to

$$P_e = 2\left(1 - \frac{1}{M}\right) Q\left(\frac{A}{\sigma}\right) = 2\left(1 - \frac{1}{M}\right) Q\left(\frac{\sqrt{\frac{3E_s}{M^2 - 1}}}{\sigma}\right)$$

$$= 2\left(1 - \frac{1}{M}\right) Q\left(\sqrt{\frac{3E_s}{(M^2 - 1)\sigma^2}}\right)$$

$$= 2\left(1 - \frac{1}{M}\right) Q\left(\sqrt{\frac{6E_s}{(M^2 - 1)N_0}}\right)$$

QAM Constellation

Quadrature Amplitude Modulation.

M-ary QAM

- Consider now a QAM constellation

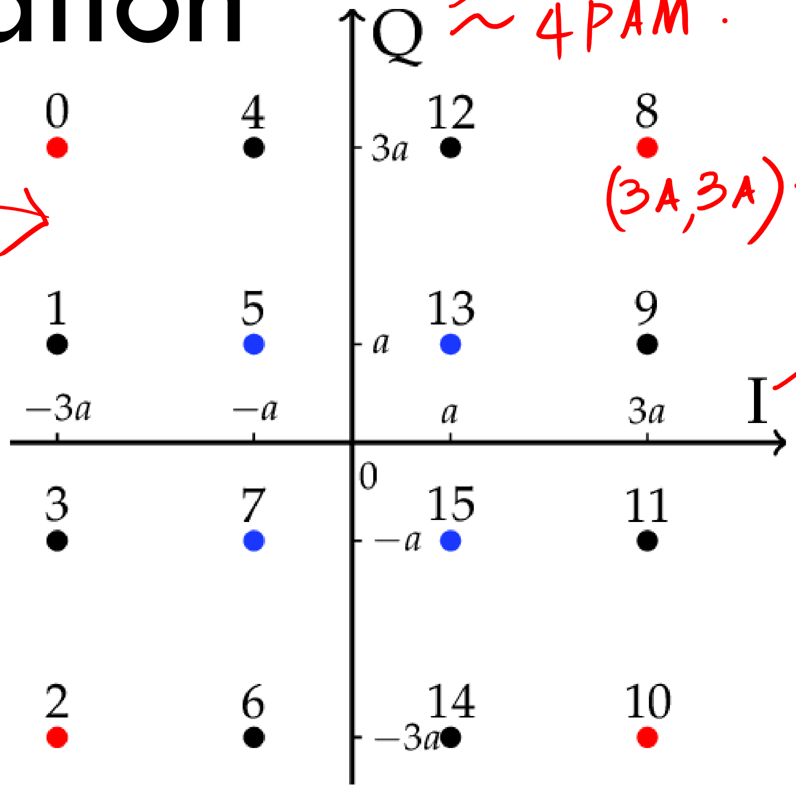
inphase + Quadrature

Quadrature.

~ 4PAM.

PAM inphase +j PAM Quadrature.

SQUARE constellation



(3A,3A).

inphase

4PAM.

$\Rightarrow 4 \times 4 = 16$ QAM.

QAM Constellation

$\sqrt{M} \times \sqrt{M}$ = Total M QAM symbols.

- M-ary QAM constellation can be modeled as

$$x_I + jx_Q$$

$\sqrt{M}$ PAM

Ex: $M = 16$.

$x_I \in$ 4 PAM $\{-3A, -A, A, 3A\}$

$x_Q \in$ 4 PAM $\{-3A, -A, A, 3A\}$.

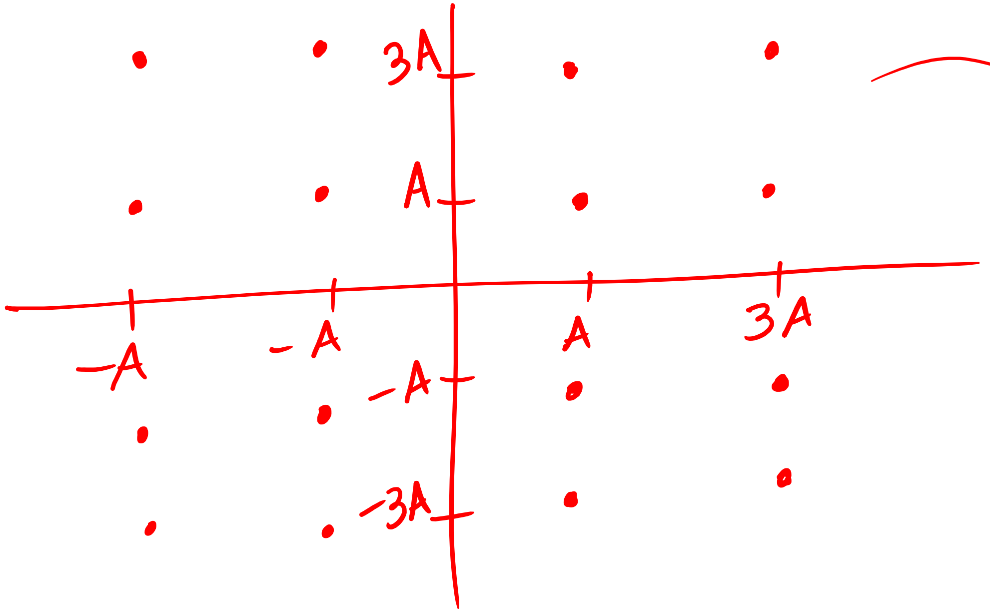
- $x_I, x_Q \in \sqrt{M}$-PAM

QAM Constellation

- Ex: 16 QAM

SQUARE constellation

$$x_I \in \{-3A, -A, A, 3A\}$$
$$x_Q \in \{-3A, -A, A, 3A\}$$



QAM Constellation

- Ex: 16 QAM

$$x_I, x_Q \in 4\ PAM$$
$$= \{-3A, -A, A, 3A\}$$

QAM Constellation

Total QAM Average symbol power E_s.

- E_s is the average symbol power

Each has half power!

- Power of x_I, x_Q: $\frac{E_s}{2}$

QAM Constellation

- SER of in-phase and Quadrature constellation

$\sqrt{M}$-PAM

$$P_{e,PAM} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{6\, E_s/2}{(M^2-1)N_0}}\right)$$

$$= 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3E_s}{(M^2-1)N_0}}\right)$$

QAM Constellation

- SER of in-phase and Quadrature constellation

$$P_{e,PAM} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3E_s}{(M-1)N_0}}\right)$$

QAM Constellation

- Probability of Symbol Error (SER) for the QAM?

Prob. x_I not in error $= 1 - P_{e,PAM}$
Prob x_Q not in error $= 1 - P_{e,PAM}$

Prob. both x_I, x_Q NOT in error
$= (1 - P_{e,PAM})^2$

$$P_{QAM} = 1 - \text{Prob. none of PAM in error}$$
$$= 1 - (1 - P_{e,PAM})^2$$

QAM Constellation

- Probability of Symbol Error (SER) for the QAM?

$$P_{e,QAM} = 1 - \left(1 - P_{e,PAM}\right)^2$$

$$= 1 - \left(1 + P_{e,PAM}^2 - 2P_{e,PAM}\right)$$

$$\approx 2P_{e,PAM} - P_{e,PAM}^2 \approx 2P_{e,PAM}.$$

QAM Constellation

- Probability of Symbol Error (SER) for the QAM?

$$P_{e,QAM} = 1 - \left(1 - P_{e,PAM}\right)^2$$
$$= 2P_{e,PAM} - P_{e,PAM}^2$$
$$\approx 2P_{e,PAM}$$

QAM Constellation

Symbol Error Rate OF M-ary QAM for given E_s.

- Probability of Symbol Error (SER) for the QAM is

$$P_{e,QAM} \approx 2P_{e,PAM}$$

$$= 2 \times 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3E_s}{(M^2-1)N_0}}\right)$$

$$= 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3E_s}{(M^2-1)N_0}}\right)$$

QAM Constellation

- Probability of Symbol Error (SER) for the QAM is

$$P_{e,QAM} \approx 2P_{e,PAM}$$

$$= 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3E_s}{(M-1)N_0}}\right)$$

Instructors may use this white area (14.5 cm / 25.4 cm) for the text.
Three options provided below for the font size.

Font: Avenir (Book), Size: 32, Colour: Dark Grey

Font: Avenir (Book), Size: 28, Colour: Dark Grey

Font: Avenir (Book), Size: 24, Colour: Dark Grey

Do not use the space below.