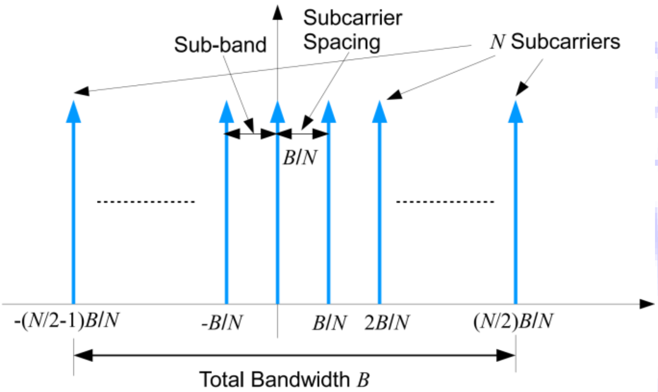
Live Interaction #5:

29th October 2023

E-masters Communication Systems

Estimation for Wireless

- **Orthogonal Frequency Division Multiplexing (OFDM).**



- OFDM Procedure

$$X_0, X_1, \ldots, X_{N-1}$$

$$\downarrow \textbf{IFFT}$$

$$x(0), x(1), \ldots, x(N-1)$$

$$\downarrow \textbf{CP addition}$$

$$\underbrace{x(N-L_{cp}), \ldots, x(N-2), x(N-1)}_{\text{Cyclic prefix}}, x(0), x(1), \ldots, x(N-1)$$

$$\downarrow \text{Transmit over channel}$$

$$\underbrace{\cancel{y(N-L_{cp}), \ldots, y(N-2), y(N-1)}}_{\text{Cyclic prefix}}, y(0), y(1), \ldots, y(N-1)$$

$$\downarrow \text{remove CP}$$

$$y(0), y(1), \ldots, y(N-1)$$

$$\downarrow \textbf{FFT}$$

$$Y(0), Y(1), \ldots, Y(N-1)$$

- OFDM Model:

$$y = h \circledast x + v$$
$$\downarrow FFT$$
$$Y_k = H_k \times X_k + V_k$$
$$k = 0,1,\ldots,N-1$$

- Number of subcarriers = N
- Some subcarriers are designated as **pilot sub-carriers**.

$$Y_k(1) = H_k X_k(1) + V_k(1)$$
$$Y_k(2) = H_k X_k(2) + V_k(2)$$
$$\vdots$$
$$Y_k(N_p) = H_k X_k(N_p) + V_k(N_p)$$

- How to estimate the channel?

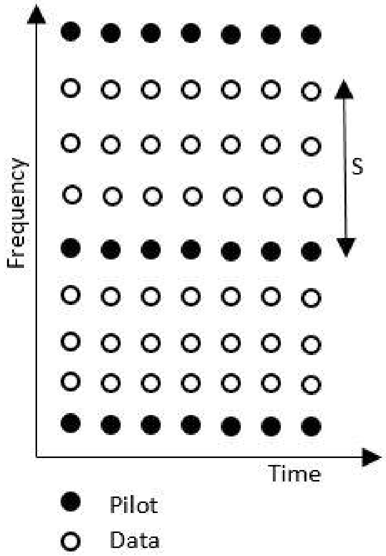
$$\underbrace{\begin{bmatrix} Y_k(1) \\ Y_k(2) \\ \vdots \\ Y_k(N_p) \end{bmatrix}}_{\bar{\bar{\mathbf{Y}}}_k} = \underbrace{\begin{bmatrix} X_k(1) \\ X_k(2) \\ \vdots \\ X_k(N_p) \end{bmatrix}}_{\bar{\bar{\mathbf{X}}}_k} H_k + \underbrace{\begin{bmatrix} V_k(1) \\ V_k(2) \\ \vdots \\ V_k(N_p) \end{bmatrix}}_{\bar{\bar{\mathbf{V}}}_k}$$

- What is the channel estimate $\widehat{H}_k$ for subcarrier k?

$$\widehat{H}_k = \frac{\sum_{i=1}^{N_p} X_k^*(i) Y_k(i)}{\sum_{i=1}^{N_p} X_k^*(i) X_k(i)} = \frac{\bar{\bar{\mathbf{X}}}_k^H \bar{\bar{\mathbf{Y}}}_k}{\bar{\bar{\mathbf{X}}}_k^H \bar{\bar{\mathbf{X}}}_k}$$

- What about the **non-pilot subcarriers**?
- We use **linear interpolation**.
- Consider subcarrier k between the pilot subcarriers k_1 and k_2. Given $\widehat{H}_{k_1}, \widehat{H}_{k_2}$.

$$\underbrace{\hat{H}_k = \hat{H}_{k_1} + \frac{(k - k_1)}{(k_2 - k_1)}\left(\hat{H}_{k_2} - \hat{H}_{k_1}\right)}_{\textbf{Linear Interpolation}}$$



Comb type channel estimation

- $\hat{H}_4 = -1 + 2j$
- $\hat{H}_8 = -4 - j$
- What is $\hat{H}_6 = ?$

$$\hat{H}_6 = \hat{H}_4 + \frac{1}{2}\left(\hat{H}_8 - \hat{H}_4\right)$$

$$= -1 + 2j + \frac{1}{2}(-3 - 3j)$$

$$= -\frac{5}{2} + \frac{1}{2}j$$

- **Equalization:**
- To suppress or eliminate the ISI.

$$y(k) = h(0)x(k) + h(1)x(k-1) + \cdots$$
$$+ h(L-1)x(k-L+1) + v(k)$$

- Simple system

$$y(k+1) = h(0)x(k+1) + h(1)x(k) + v(k+1)$$
$$y(k) = h(0)x(k) + h(1)x(k-1) + v(k)$$

- We can write the model

$$\underbrace{\begin{bmatrix} y(k+1) \\ y(k) \end{bmatrix}}_{\bar{\mathbf{y}}} = \underbrace{\begin{bmatrix} h(0) & h(1) & 0 \\ 0 & h(0) & h(1) \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x(k+1) \\ x(k) \\ x(k-1) \end{bmatrix}}_{\bar{\mathbf{x}}} + \underbrace{\begin{bmatrix} v(k+1) \\ v(k) \end{bmatrix}}_{\bar{\mathbf{v}}}$$

- Equalizer

$$c_0 y(k+1) + c_1 y(k)$$
$$= [c_0 \quad c_1] \begin{bmatrix} y(k+1) \\ y(k) \end{bmatrix} = \bar{\mathbf{c}}^T \bar{\mathbf{y}}$$
$$= \bar{\mathbf{c}}^T (\mathbf{H}\bar{\mathbf{x}} + \bar{\mathbf{v}})$$
$$= \bar{\mathbf{c}}^T \mathbf{H}\bar{\mathbf{x}} + \bar{\mathbf{c}}^T \bar{\mathbf{v}}$$
$$= \bar{\mathbf{c}}^T \mathbf{H} \begin{bmatrix} x(k+1) \\ x(k) \\ x(k-1) \end{bmatrix} + \bar{\mathbf{c}}^T \bar{\mathbf{v}}$$
$$= [0 \quad 1 \quad 0] \begin{bmatrix} x(k+1) \\ x(k) \\ x(k-1) \end{bmatrix} + \bar{\mathbf{c}}^T \bar{\mathbf{v}}$$

- Ideally,

$$\bar{\mathbf{c}}^T \mathbf{H} = [0 \quad 1 \quad 0]$$
$$\Rightarrow \mathbf{H}^T \bar{\mathbf{c}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

- Therefore, we write our least squares problem

$$\min \left\| \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \mathbf{H}^T \bar{\mathbf{c}} \right\|^2$$

$$\bar{\mathbf{c}} = \left(\mathbf{H}^{T^T} \mathbf{H}^T \right)^{-1} \mathbf{H}^{T^T} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= (\mathbf{H}\mathbf{H}^T)^{-1} \mathbf{H} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

- **Homework:**

$$y(k) = x(k) + \frac{1}{5} x(k-1) + v(k)$$

- Determine the **equalizer** $\bar{\mathbf{c}}$ for this.
- **Assignment #5 deadline: 4th November Saturday 11:59 PM**.
- **Live interaction 5th November 12:30 – 1:30 PM**.