Quiz-3 – 24th Feb 2023 – Multivariate Gaussian Covariance

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eMasters in Communication Systems, IITK
EE902: Advanced ML Techniques for Wireless Technology
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Question:
What is the mean and covariance matrix of multivariate Gaussian RV?
Solution:
Let's consider $\bar{x}$ is a multivariate Gaussian RV.
The mean/expected value of this GRV = $\boldsymbol{E\{\bar{x}\} = \bar{\mu}}$
Covariance of $\bar{x}$, $\boldsymbol{Cov(\bar{x}) = E\{(\bar{x} - \bar{\mu})(\bar{x} - \bar{\mu})^T\}}$
$\boldsymbol{Cov(\bar{x}) = E\{(\bar{x} - \bar{\mu})(\bar{x}^T - \bar{\mu}^T)\}}$
$\boldsymbol{=> E\{\bar{x}\bar{x}^T - \bar{\mu}\bar{x}^T - \bar{x}\bar{\mu}^T + \bar{\mu}\bar{\mu}^T\}}$
$\boldsymbol{=> E\{\bar{x}\bar{x}^T\} - E\{\bar{\mu}\bar{x}^T\} - E\{\bar{x}\bar{\mu}^T\} + E\{\bar{\mu}\bar{\mu}^T\}}$
$\boldsymbol{=> E\{\bar{x}\bar{x}^T\} - E\{\bar{\mu}\}E\{\bar{x}^T\} - E\{\bar{x}\}E\{\bar{\mu}^T\} + E\{\bar{\mu}\}.E\{\bar{\mu}^T\}}$

We know:

a) $E\{\bar{x}\} = \bar{\mu}$
b) $E\{\bar{x}^T\} = \bar{\mu}^T$
c) $E\{\bar{\mu}\} = \bar{\mu}$ - Mean is constant and hence expected value of the mean vector is the mean vector itself
d) $E\{\bar{\mu}^T\} = \bar{\mu}^T$ - Mean is constant hence expected value of the transpose of the mean vector is also the transpose of the mean vector

I recollect, now, that this was derived/addressed in one of prof.Aditya sir's classes last quarter.

$\therefore \boldsymbol{Cov(\bar{x}) = E\{\bar{x}\bar{x}^T\} - \bar{\mu}\bar{\mu}^T - \bar{\mu}\bar{\mu}^T + \bar{\mu}\bar{\mu}^T}$
$\boldsymbol{= E\{\bar{x}\bar{x}^T\} - \bar{\mu}\bar{\mu}^T}$

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