Assignment 4 Solution

Digital Communication System-I

May 27, 2023

1. (d) The CPM signal is a continuous signal as it has a continuous phase. Clearly the signal in option (d) is continuous at all values of t.

2. (c)

$$\begin{aligned} q(t) &= \int_0^t g(\tau)d\tau \\ q(3) &= \int_0^3 g(\tau)d\tau \\ &= \int_0^3 \frac{1}{9}d\tau \\ &= \frac{1}{9}\times 3 = \frac{1}{3} \end{aligned}$$

3. (a) A signal $s(t)$ with modulation index h can be written as

$$s(t) = \sqrt{\frac{2\mathcal{E}}{T}}\cos(2\pi f_c t + 2\pi h\sum_{k=-\infty}^{n} I_k q(t-kT))$$

   Comparing the above we get $h = \frac{1}{3}$.

4. (d)

$$S_t = \begin{cases} pM^{L-1} & \text{even } m \\ 2pM^{L-1} & \text{odd } m \end{cases}$$

$$L = 4, h = \frac{1}{3} = \frac{m}{p}$$

   M=2 for binary

   Therefore, total number of states, $S_t = 2pM^{L-1} = \left(2\times 3\times 2^3\right) = 48$.

5. (d) In this case, L = 2, h = $\frac{2}{3}$, $S_n = \left(\frac{\pi}{3}, +1\right)$, and $I_n = +1$. $\therefore \theta_n = \frac{\pi}{3}$ and $I_{n-1} = +1$. Thus we have $\theta_{n+1} = \theta_n + 2\pi h\left(I_{n-1}q(t-(n-1)T) + I_n q(t-nT)\right)$. The net change in the phase is given by $\frac{2\pi}{3}\left(I_{n-1}/2 + I_n\right)$. Thus if I_{n-1} and I_n have different signs, then the phase in next state is $= \frac{\pi}{3} + (1-\frac{1}{2})\frac{2\pi}{3} = 2\pi/3$. Therefore, the next state will be $S_{n+1} = (2\pi/3, -1)$.

6. (d)

$$\Theta_s = \left\{0, \frac{\pi m}{p}, \frac{2\pi m}{p}, \cdots, \frac{(p-1)\pi m}{p}\right\}$$

when m is even and

$$\Theta_s = \left\{0, \frac{\pi m}{p}, \frac{2\pi m}{p}, \cdots, \frac{(2p-1)\pi m}{p}\right\}$$

when m is odd. Therefore, the possible terminal phase states will be $\{0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}\}$

7. (d)

$S_t = 2p = 8$

8. (d)

9. (b) The correct option is

$\left\{\left(\frac{\pi}{4}, -1\right), \left(\frac{\pi}{4}, +1\right), \left(\frac{3\pi}{4}, -1\right), \left(\frac{3\pi}{4}, +1\right), \left(\frac{5\pi}{4}, -1\right), \left(\frac{5\pi}{4}, +1\right), \left(\frac{7\pi}{4}, -1\right), \left(\frac{7\pi}{4}, +1\right)\right\}$

Given, $L = 2, m = 1, p = 2$. The total number of states $= N_s = 2pM^{L-1}$ $= 2 \times 2 \times 2 = 8$.

We know $\theta_{n+1} = \theta_n + 2\pi h\left(I_{n-1}q(t-(n-1)T) + I_n q(t-nT)\right)$. The net change in the phase is given by

$$\frac{\pi}{2}hI_{n-1} + \frac{\pi}{2}hI_n$$

Thus if I_{n-1} and I_n have different signs, there is no net change in the phase. Otherwise, the net change in the phase is given by $\pi h\,\mathrm{sgn}(I_n)$

Initially, we assume that I_{n-1} is zero and we start from state 0, so we get to state $\frac{\pi}{4}$ or $\frac{7\pi}{4}$ (h is $\frac{1}{2}$ in this question) depending upon whether the input is $+1$ or -1. Subsequently, we will reach the following states $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ for both the inputs $+1$ and -1.

10. (b)