eMasters in Communication Systems

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Core Module:

Wireless Communication

Chapter 4

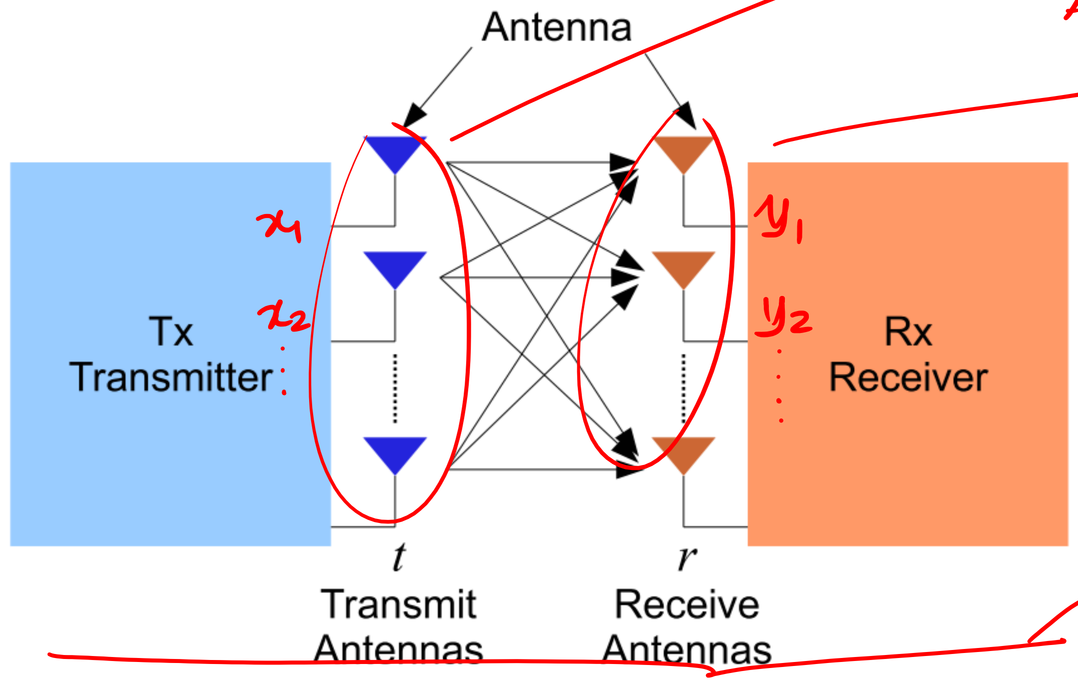
MIMO Technology

MULTIPLE INPUT
MULTIPLE OUTPUT
KEY WIRELESS TECHNOLOGY.

MIMO

- **MIMO stands for Multiple-Input Multiple-Output**
- Multiple Input ⇒ ***Multiple*** TRANSMIT ***antennas*** ⇒ Multiple input symbols. $x_1, x_2, \ldots$
- Multiple Output ⇒ ***Multiple*** RECEIVE ***antennas*** ⇒ Multiple output symbols. $y_1, y_2, \ldots$

Multiple-Input Multiple-Output



MULTIPLE TRANSMIT ANTENNAS.

MULTIPLE RECEIVE ANTENNAS.

MIMO WIRELESS SYSTEM.

MIMO

LONG TERM EVOLUTION

- **MIMO** is a key technology in **4G LTE** and **5G NR**

NEW RADIO.
MASSIVE MIMO
mmWAVE MIMO.

- It is also extensively used in **Wi-Fi**
  - 802.11n, 802.11 ac, 802.11 ax

WLAN. STANDARDS.

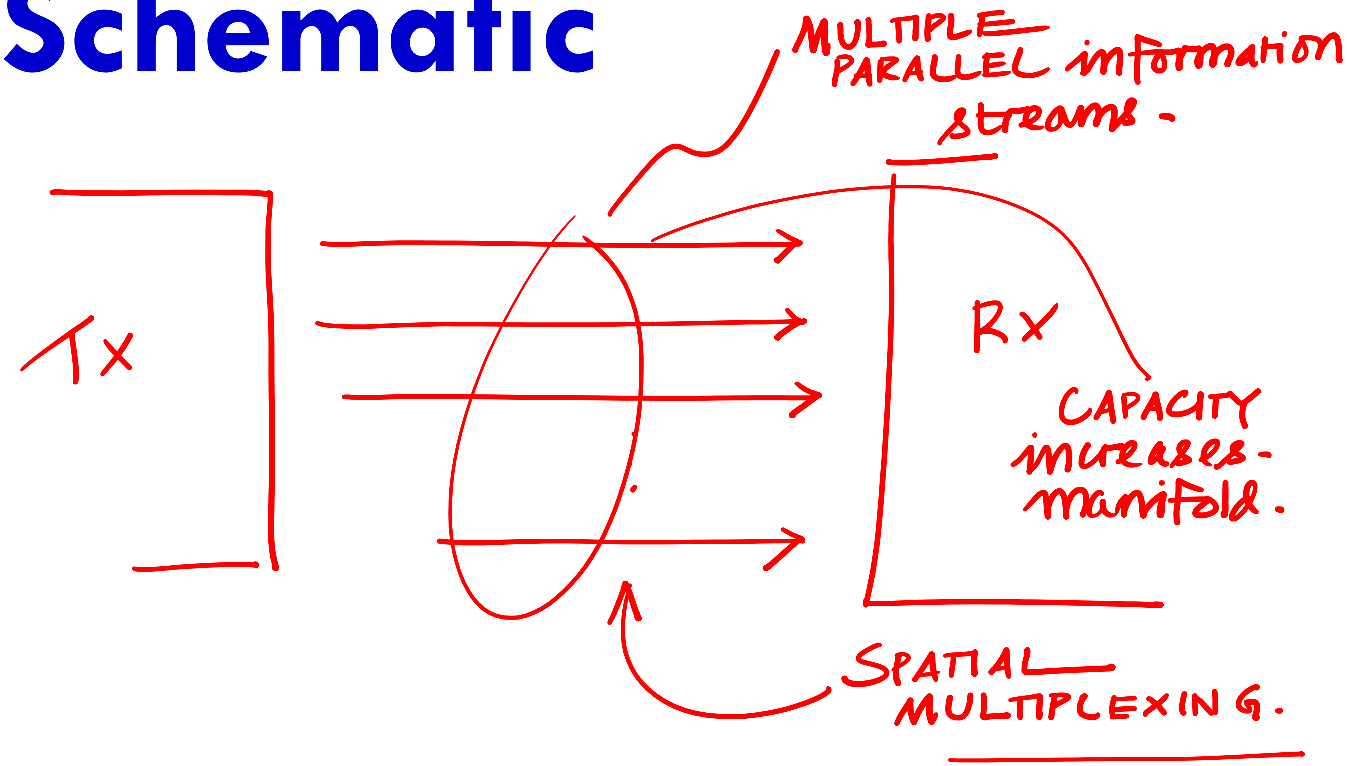
MIMO

MULTIPLEX SEVERAL INFORMATION STREAMS IN SPATIAL DOMAIN.

SAME TIME
SAME BW.

- MIMO can lead to **significant increase** in data rates
  - Via ***parallel transmission*** of multiple streams
- This is termed as SPATIAL MULTIPLEXING !

MIMO Schematic



MIMO System Model

- r is the number of RECEIVE antennas.
- t is the number of TRANSMIT antennas.

$r \times t$ MIMO system.

4×3 MIMO

$\Rightarrow$ # Rx antennas = 4

Tx antennas = 3.

MIMO System Model

- MIMO system model is **mathematically** described as follows

H MIMO CHANNEL MATRIX.

$$\overbrace{\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix}}_{\bar{y}}}^{r\times 1} = \overbrace{\underbrace{\begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1t} \\ h_{21} & h_{22} & \cdots & h_{2t} \\ \vdots & & & \\ h_{r1} & h_{r2} & \cdots & h_{rt} \end{bmatrix}}_{H}}^{r\times t} \overbrace{\underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix}}_{\bar{x}}}^{t\times 1} + \overbrace{\underbrace{\begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{bmatrix}}_{\bar{n}}}^{r\times 1}$$

$$\bar{y} = H\bar{x} + \bar{n}$$

MIMO System Model

- h_{ij} is the channel coefficient between ___ i^{th} ___ receive antenna and ___ j^{th} ___ transmit antenna

$i=3$ $j=2$ $\Rightarrow$ Rx antenna $=3$
Tx antenna $=2$

- Example: h_{32} is the channel coefficient between ___ 3^{rd} ___ receive antenna and ___ 2^{nd} ___ transmit antenna

MIMO System Model

r = # Receive antennas

t = # Transmit antennas.

- This is known as $r \times t$ MIMO system.
- Let us now do a **simple example** 3x2

MIMO Example

3 x 2

r = 3 Rx antennas.
t = 2 Tx antennas.

- Consider 3×2 MIMO system

outputs : y_1, y_2, y_3
inputs : x_1, x_2

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}}_{\bar{y}} = \underbrace{\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\bar{x}} + \underbrace{\begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}}_{\bar{n}}$$

3 x 2

h_{12} = coeff Between
Rx ant 1
Tx ant 2

MIMO Example

MIMO system of EQUATIONS.

$$
\left.
\begin{aligned}
y_1 &= h_{11} x_1 + h_{12} x_2 + n_1 \\
y_2 &= h_{21} x_1 + h_{22} x_2 + n_2 \\
y_3 &= h_{31} x_1 + h_{32} x_2 + n_3
\end{aligned}
\right\}
$$

3x2 MIMO system model.

MIMO System Model

- MIMO system model can be written compactly as

$$\underbrace{\bar{\mathbf{y}}}_{\substack{r \times 1 \\ \text{OUTPUT VECTOR}}} = \underbrace{\mathbf{H}}_{\substack{r \times t \\ \text{MIMO channel matrix}}} \underbrace{\bar{\mathbf{x}}}_{\substack{t \times 1 \\ \text{input vector}}} + \underbrace{\bar{\mathbf{n}}}_{\substack{r \times 1 \\ \text{NOISE VECTOR.}}}$$

MIMO Receiver

- Design the **MIMO Receiver** i.e., Given $\bar{\mathbf{y}}$ how to determine $\bar{\mathbf{x}}$?

GIVEN $\bar{y}$

HOW TO DETERMINE $\bar{x}$

$$\underbrace{\bar{\mathbf{y}}}_{r\times 1} = \mathbf{H} \underbrace{\bar{\mathbf{x}}}_{t\times 1}$$

r *Equations* $\qquad$ t *unknowns*

$t = \#$ unknowns.

$r = \#$ Equations.

MIMO Equations

SYSTEM OF LINEAR EQUATIONS.

- MIMO Equations can be written as

$t = \#$ unknowns.

$r = \#$ Equations.

$$
\left\{
\begin{aligned}
y_1 &= \overbrace{h_{11}x_1 + h_{12}x_2 + \cdots + h_{1t}x_t}^{t} \\
y_2 &= h_{21}x_1 + h_{22}x_2 + \cdots + h_{2t}x_t \\
&\vdots \\
y_r &= h_{r1}x_1 + h_{r2}x_2 + \cdots + h_{rt}x_t
\end{aligned}
\right.
$$

r outputs: $y_1, y_2, \ldots, y_r$

t inputs: $x_1, x_2, \ldots, x_t$

MIMO Receiver

#EQNS = #UNKNOWNS.

- r = Number of EQUATIONS
- t = Number of UNKNOWNS.
- Simple case: $r = t$. How to determine $\bar{\mathbf{x}}$ from $\bar{\mathbf{y}}$
  - In this case $\mathbf{H}$ is a SQUARE matrix

#rows = #columns.

MIMO Receiver

Det(H) ≠ 0

invertible.

- If $\mathbf{H}$ is **non-singular**, i.e. $\mathbf{H}^{-1}$ exists, $\bar{\mathbf{y}} = \mathbf{H}\bar{\mathbf{x}}$ has a ___UNIQUE___ ***solution.***
- The unique solution is given as

H^{-1} = inverse of H

$$H^{-1}H = I = HH^{-1}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2 x 2 identity matrix

ESTIMATED VECTOR.

$$\hat{\mathbf{x}} = \mathbf{H}^{-1}\bar{\mathbf{y}}$$

MIMO Inverse Example 2×2

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$\tilde{H} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$

$\det(\tilde{H}) = 2 - 2 = 0$

$\Rightarrow \tilde{H}$ NOT invertible!

$\det(H) = 1 \times 4 - 2 \times 3$

$= 4 - 6 = -2$

$\neq 0$

$\Rightarrow H$ is invertible

$$H^{-1} = \frac{1}{-2} \cdot \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$HH^{-1} = -\frac{1}{2} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = -\frac{1}{2} \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

MIMO Receiver

$\hat{x} = H^{-1}\bar{y}$

- What happens when $r > t$ i.e.

  $r \times t$
  3×2
  $r > t$

  # Equations > # Unknowns

  $H = [\quad]$ 3 ROWS 2 COLS. $\Rightarrow$ H is TALL

- **H** is **NOT invertible** $\Rightarrow$ HOW TO DETERMINE $\hat{x}$?
  - **H is a** ___TALL___ **matrix**

$3 \times 2 \Rightarrow$ NOT SQUARE

MIMO Receiver

$r > t$

#EQNS > #UNKNOWNS.

- Typically in such case **No solution**!!
- We try to find an **approximate solution**!

Find $\hat{x}$ such that error is minimum.

ERROR

minimize error!

$$\bar{\mathbf{y}} - \mathbf{H}\bar{\mathbf{x}} = \bar{\mathbf{e}}$$

LEAST SQUARES PROBLEM.

$$\min \|\bar{\mathbf{e}}\|^2 = \min . \|\bar{y} - H\bar{x}\|^2$$

LEAST SQUARE NORM ERROR.

MIMO Receiver

LS Problem

- This is termed the ***least-squares problem***.

MACHINE LEARNING.
LINEAR REGRESSION
DATA SCIENCE.

$$\min \|\bar{\mathbf{y}} - \mathbf{H}\bar{\mathbf{x}}\|^2$$

one of the most popular problems in signal. Processing.

MIMO Receiver

HERMITIAN FOR complex matrix

$$\hat{\mathbf{x}} = \left(H^H H\right)^{-1} H^H \bar{y}$$

ZF Receiver

- This is termed as the ZERO FORCING (ZF) Receiver

MIMO ZF Receiver

TALL MATRIX.
$\Rightarrow$ MORE ROWS THAN COLS.

- $(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$ is termed the **pseudo-inverse** of $\mathbf{H}$
- Why?

$$\underbrace{(H^H H)^{-1}}\cdot \underbrace{H^H \cdot \times H} = I$$

PSEUDO inverse

$$H^{+} = (H^H H)^{-1} H^H.$$

MIMO ZF Receiver Example

4 x 2
r=4 t=2
r>t

Example:

Consider

Output Vector

TALL matrix

$$\bar{\mathbf{y}} = \begin{bmatrix} -1 \\ 3 \\ 1 \\ -2 \end{bmatrix}, \mathbf{H} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}}_{r \times t}$$

4 x 2

What is $\hat{\mathbf{x}}$?

ESTIMATE OF INPUT VECTOR $\bar{x}$

MIMO ZF Receiver Example

FOR REAL matrix
$H^T = H^H$.

The **ZF estimate** can be calculated as follows

$$\mathbf{H}^T\mathbf{H} = \overbrace{\begin{bmatrix}1&1&1&1\\1&2&3&4\end{bmatrix}}^{H^H\ 2\times4} \overbrace{\begin{bmatrix}1&1\\1&2\\1&3\\1&4\end{bmatrix}}^{H \sim 4\times2} = H^H H.$$

$1\times1+2\times2+3\times3+4\times4$

$1\times1+1\times1+1\times1+1\times1$

$1\times1+1\times2+1\times3+1\times4$

1^{st} row $\times$ 2^{nd} col.

$$= \begin{bmatrix}4&10\\10&30\end{bmatrix}$$

MIMO ZF Receiver Example

$$\det(\mathbf{H}^T\mathbf{H}) = 4\times 30 - 10\times 10$$
$$= 120 - 100$$
$$= 20.$$

$$\mathbf{H}^T\mathbf{H} = \begin{bmatrix} 4 & 10 \\ 10 & 30 \end{bmatrix}$$

divide by determinant
interchange diagonal.
-ve of off diagonal.

$$(\mathbf{H}^T\mathbf{H})^{-1} = \underbrace{\frac{1}{20}\begin{bmatrix} 30 & -10 \\ -10 & 4 \end{bmatrix}}$$

inverse of H^TH.

MIMO ZF Receiver Example

$(H^H H)^{-1} H^H$

$$(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T = \frac{1}{20}\begin{bmatrix} 30 & -10 \\ -10 & 4 \end{bmatrix} \overbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}}^{H^T}$$

$$= \frac{1}{20}\begin{bmatrix} 20 & 10 & 0 & -10 \\ -6 & -2 & 2 & 6 \end{bmatrix}$$

$H^\dagger$ PSEUDO inverse of H.

$$= \underbrace{\begin{bmatrix} 1 & 1/2 & 0 & -1/2 \\ -3/10 & -1/10 & 1/10 & 3/10 \end{bmatrix}}_{\text{PSEUDO INVERSE}}$$

4 x 2

MIMO Pseudo Inverse

$$(H^TH)^{-1}H^T \cdot H$$

$$= \begin{bmatrix} 1 & \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{3}{10} & -\frac{1}{10} & \frac{1}{10} & \frac{3}{10} \end{bmatrix} \overbrace{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}}^{H} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$1+1+0-2=0$$

$$(H^TH)^{-1}H^TH = I$$

$$-\frac{3}{10}-\frac{1}{10}+\frac{1}{10}+\frac{3}{10} = 0$$

$$-\frac{3}{10}-\frac{2}{10}+\frac{3}{10}+\frac{12}{10} = \frac{10}{10} = 1$$

MIMO ZF Receiver Example

$$\hat{\mathbf{x}} = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\,\bar{\mathbf{y}}$$

$$= \begin{bmatrix} 1 & \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{3}{10} & -\frac{1}{10} & \frac{1}{10} & \frac{3}{10} \end{bmatrix}\begin{bmatrix} -1 \\ 3 \\ 1 \\ -2 \end{bmatrix}$$

ZF ESTIMATE

$$= \begin{bmatrix} -1 + \frac{3}{2} + 0 + 1 \\ \frac{3}{10} - \frac{3}{10} + \frac{1}{10} - \frac{6}{10} \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ -\frac{1}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \hat{x}$$

MIMO ZF Receiver Example

ZERO FORCING ESTIMATE

- Therefore, the **ZF estimate** is

$$\hat{\mathbf{x}} = \begin{bmatrix} \frac{3}{2} \\ -\frac{1}{2} \end{bmatrix}$$

LMMSE Receiver

- Another popular MIMO receiver is the **LMMSE Receiver**
- LMMSE:

LINEAR MINIMUM MEAN SQUARE ERROR.

LINEAR RECEIVER.

$$\hat{\mathbf{x}} = \mathbf{C}^H \bar{\mathbf{y}}\text{: Linear Estimate}$$

LINEAR TRANSFORMATION.

LMMSE Explained

LINEAR — ①

$$\min E\{\|\mathbf{C}^H\bar{\mathbf{y}} - \bar{\mathbf{x}}\|^2\}$$

SQUARE ERROR — ④

MEAN — ③

MINIMUM — ②

LINEAR MINIMUM MEAN SQUARE ERROR.

LMMSE Receiver

- Note the following quantities

$t \times t$

COVARIANCE MATRIX of $\bar{\mathbf{x}}$: $\mathbf{R}_{xx} = E\{\bar{x}\bar{x}^H\}$

$r \times r$

COVARIANCE MATRIX of $\bar{\mathbf{y}}$: $\mathbf{R}_{yy} = E\{\bar{y}\bar{y}^H\}$

LMMSE Receiver

t x r matrix

$\mathbf{R}_{xy} = E\{\bar{x}\bar{y}^H\}$

- This is termed as CROSS COVARIANCE MATRIX.

LMMSE Receiver

cross covariance matrix

- **LMMSE Receiver** is given as

$$\hat{\mathbf{x}} = \mathbf{R}_{xy}\mathbf{R}_{yy}^{-1}\bar{\mathbf{y}}$$

LMMSE Receiver

$E\{x_i x_j^*\} = 0$ if $i \neq j$

$E\{x_i x_j^*\} = E\{|x_i|^2\} = P$ if $i = j$

- Consider the transmit symbols to be i.i.d. mean 0, power P.

covariance matrix of $\bar{x}$

txt

$$\underbrace{E\{\bar{\mathbf{x}}\bar{\mathbf{x}}^H\}}_{\text{covariance matrix of } \bar{x}} = P\mathbf{I} = R_{xx}$$

LMMSE Receiver

COVARIANCE OF $\bar{y}$

- The covariance matrix of $\bar{\mathbf{y}}$ can be derived as

$$\mathbf{R}_{yy} = E\{\bar{\mathbf{y}}\bar{\mathbf{y}}^H\} = E\{(H\bar{x}+\bar{n})(H\bar{x}+\bar{n})^H\}$$

$$= E\{(H\bar{x}+\bar{n})(\bar{x}^H H^H+\bar{n}^H)\}$$

$$= E\{H\bar{x}\bar{x}^H H^H+\bar{n}\bar{x}^H H^H + H\bar{x}\bar{n}^H + \bar{n}\bar{n}^H\}$$

$$= H \underbrace{E\{\bar{x}\bar{x}^H\}}_{PI} H^H + \underbrace{E\{\bar{n}\bar{x}^H\}}_{0} H^H + H \underbrace{E\{\bar{x}\bar{n}^H\}}_{0} + \underbrace{E\{\bar{n}\bar{n}^H\}}_{N_0 I}$$

LMMSE Receiver

- The covariance matrix of $\bar{\mathbf{y}}$ can be simplified as

$$R_{yy} = E\{\bar{y}\bar{y}^H\} = H \cdot PI \cdot H^H + N_o I$$

$$= \underbrace{P \cdot HH^H + N_o I}_{R_{yy}}$$

LMMSE Receiver

- The **cross-covariance** matrix of $\bar{\mathbf{x}}, \bar{\mathbf{y}}$ can be derived as

$$\mathbf{R}_{xy} = E\{\bar{\mathbf{x}}\bar{\mathbf{y}}^H\} = E\{\bar{x}(H\bar{x}+\bar{n})^H\}$$

$$= E\{\bar{x}(\bar{x}^H H^H + \bar{n}^H)\}$$

$$= \underbrace{E\{\bar{x}\bar{x}^H\}}_{} . H^H + \cancel{E\{\bar{x}\bar{n}^H\}} = 0$$

$$= P . I . H^H$$

LMMSE Receiver

- The **cross-covariance** matrix can be simplified as

$$R_{xy} = E\{\bar{x}\bar{y}^H\} = P.\,I\,H^H$$

$$= P.\,H^H.$$

LMMSE Receiver

- Therefore, **LMMSE Receiver** is given as

$R_{xy} \cdot R_{yy}^{-1} \bar{y}$

$$\hat{\mathbf{x}} = \underbrace{P\mathbf{H}^H}_{R_{xy}} \underbrace{\overbrace{(P\mathbf{H}\mathbf{H}^H + N_0\mathbf{I})}^{r \times r \text{ matrix}}{}^{-1}}_{R_{yy}^{-1}} \bar{\mathbf{y}}$$

LMMSE Receiver

- Another expression for the LMMSE Receiver is

txt

$$\hat{\mathbf{x}} = P(P\mathbf{H}^H\mathbf{H} + N_0\mathbf{I})^{-1}\mathbf{H}^H\bar{\mathbf{y}}$$

$$= \left(H^H H + \underbrace{\frac{N_0}{P_u}}_{\frac{1}{SNR}} I\right)^{-1} H^H \bar{y} = \underbrace{\left(H^H H + \frac{1}{SNR}\right)^{-1} H^H \bar{y}}_{\text{LMMSE Receiver}}$$

LMMSE Receiver

I FORM: $P \cdot H^H \overbrace{\left(HH^H + N_0 I\right)^{-1}}^{r \times r} \bar{y}$

II FORM: $\underbrace{\left(H^H H + \frac{1}{SNR} I\right)^{-1}}_{t \times t} H^H \bar{y}$

MUCH LOWER COMPUTATIONAL COMPLEXITY

$r \geq t$

LMMSE Receiver

- As $SNR \to \infty \Rightarrow \frac{1}{SNR} \to 0$

high SNR

$$\hat{x} = \left(H^H H + \frac{1}{SNR}\right)^{-1} H^H \bar{y}$$

$$\left(H^H H + \underbrace{\frac{1}{SNR}}_{\approx 0} I\right)^{-1} H^H \bar{y} \to \underbrace{\left(H^H H\right)^{-1} H^H \bar{y}}_{\text{ZF Receiver}}$$

At very high SNR

LMMSE $\longrightarrow$ ZF

MIMO LMMSE Receiver Example

Example:

Consider

OUTPUT VECTOR

CHANNEL MATRIX

$$\bar{\mathbf{y}} = \begin{bmatrix} -1 \\ 3 \\ 1 \\ -2 \end{bmatrix}, \mathbf{H} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}}_{r \times t}$$

4×2

$\Rightarrow r = 4$
$t = 2$

LMMSE Estimate $\hat{x}$?

What is $\hat{\mathbf{x}}$ when $SNR = -3dB \approx \frac{1}{2}$

MIMO LMMSE Receiver Example

The **LMMSE estimate** can be calculated as follows

$$\mathbf{H}^T\mathbf{H} = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}}_{H^T} \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}}_{H}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = H^T H = 4I$$

$1+1+1+1$

$1-1+1-1$

$1-1+1-1$

$1+1+1+1$

MIMO LMMSE Receiver Example

- $SNR = -3\ dB \approx \frac{1}{2}$

2×2

$t \times t$

$$\mathbf{H}^T\mathbf{H} + \frac{1}{SNR}\mathbf{I} = \begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix} + \frac{1}{1/2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix} + 2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix} = 6I$$

MIMO LMMSE Receiver Example

$$\mathbf{H}^T\mathbf{H} + \frac{1}{SNR}\mathbf{I} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} = 6I$$

$$\left(\mathbf{H}^T\mathbf{H} + \frac{1}{SNR}\mathbf{I}\right)^{-1} = \frac{1}{6}I = \begin{bmatrix} 1/6 & 0 \\ 0 & 1/6 \end{bmatrix}$$

MIMO LMMSE Receiver Example

$$\left(\mathbf{H}^T\mathbf{H} + \frac{1}{\mathrm{SNR}}\mathbf{I}\right)^{-1}\mathbf{H}^T = \frac{\mathbf{1}}{\mathbf{6}}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

MIMO LMMSE Receiver Example

$$\hat{\mathbf{x}} = \left(\mathbf{H}^T\mathbf{H} + \frac{\mathbf{1}}{\mathrm{SNR}}\right)^{-1}\mathbf{H}^T\,\bar{\mathbf{y}}$$

$\bar{y}$

$$= \frac{1}{6}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \\ 1 \\ -2 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} -1+3+1-2=1 \\ -1-3+1+2=-1 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

MIMO LMMSE Receiver Example

- Therefore, the **LMMSE estimate** is

$$\hat{\mathbf{x}} = \frac{1}{6}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

LMMSE Estimate

OUTPUT OF MIMO LMMSE Receiver.

Instructors may use this white area (14.5 cm / 25.4 cm) for the text.
Three options provided below for the font size.

Font: Avenir (Book), Size: 32, Colour: Dark Grey

Font: Avenir (Book), Size: 28, Colour: Dark Grey

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