

## Example OFDM System

- OFDM system with BW: 10 MHz =  $B$ .
  - Number of subcarriers =  $1024 = N$
- $B = 10 \text{ MHz} = 10 \times 10^6 \text{ Hz}$
- $2^{10}$
- POWER OF 2
- $2^n$  256, 512, 1024, 2048, ...



## Example OFDM System

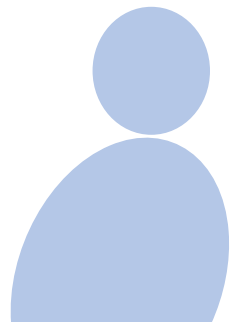
= # SUBCARRIERS.

- BW of each subcarrier

$$= \frac{B}{N} = \frac{10 \times 10^6}{1024} = 9.77 \text{ kHz} = \frac{B}{N}$$

- What is the size of IFFT at transmitter?

$$N = 1024$$



## Example OFDM System

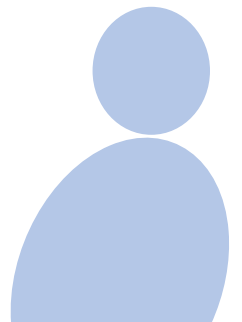
$$\underline{9.77 \times 10^3 \text{ Hz}}$$

- BW of each subcarrier

$$= 10 \times \frac{10^6}{1024} = 9.77 \text{ kHz} = \frac{B}{N}$$

- What is the size of *IFFT* at transmitter?

$$1024 = N.$$



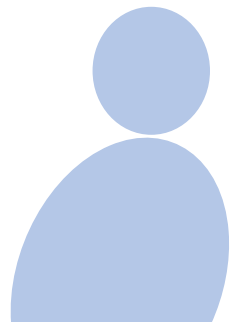
## Example OFDM System

WITHOUT CP -



- What is the **duration** of the OFDM symbol?

$$\frac{N}{B} = \frac{1024}{10 \times 10^6} = 102.4 \mu s$$
$$= 102.4 \times 10^{-6} s.$$



## Example OFDM System

$$\text{SYMBOL DURATION} = \frac{N}{B} = N \times \frac{1}{B}$$

#SAMPLES  
SAMPLE DURATION

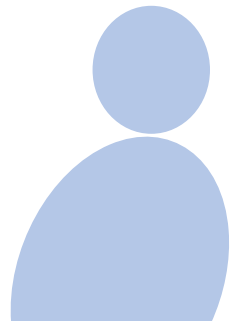
- What is the **duration** of the OFDM symbol?

$$\frac{1}{B} \times N = \frac{1024}{10 \times 10^6} = \underline{102.4 \mu s} = \frac{N}{B}$$

$$\text{SAMPLE DURATION} = \frac{1}{B}$$

$$x(0), x(1), \dots, x(N-1).$$

N samples.  
AFTER IFFT



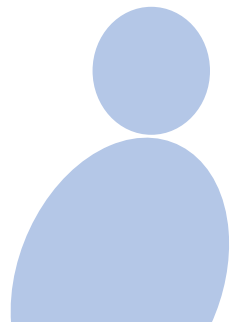
## Example OFDM System

NOW ADD CYCLIC PREFIX (CP)  $\nearrow N_{cp} \times \frac{1}{B} = \frac{N_{cp}}{B}$

- CP has  $N_{cp} = 80$  samples. What is duration of CP?

$$\frac{1}{B} \times N_{cp} = \frac{80}{10 \times 10^6} = 8 \mu s$$
$$= 8 \times 10^{-6}$$
$$= \text{CP DURATION.}$$

$$\frac{N_{cp}}{N} = \frac{80}{1024}$$



# Example OFDM System

$$\begin{array}{r} 1024 \\ + 80 \\ \hline 1104 \end{array}$$

- How many samples after addition of CP?

$$\underbrace{x(N - \tilde{L}), \dots, x(N - 2), x(N - 1)}_{\text{Cyclic Prefix} = 80}, \underbrace{x(0), x(1), x(2), \dots, x(N - 1)}_{\text{Original Samples} = 1024}$$

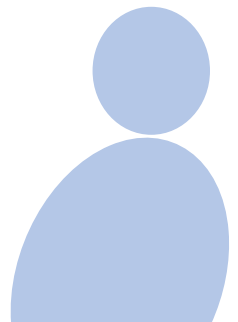
$$\text{Total} = \underline{1104 \text{ Samples}}$$

$$= N + N_{cp}$$

$$\begin{array}{r} 102.4 \\ 8.0 \\ \hline 110.4 \end{array} \mu s$$

= Total Duration  
with CP -

Total # Samples  
With CP -



# Example OFDM System

CYCLIC  
PREFIX .

80 samples .

$x(943), \dots, x(1022), x(1023), x(0), x(1), \dots, x(1023)$

- How many samples after addition of CP?

$x(N - \tilde{L}), \dots, x(N - 2), x(N - 1), x(0), x(1), x(2), \dots, x(N - 1)$

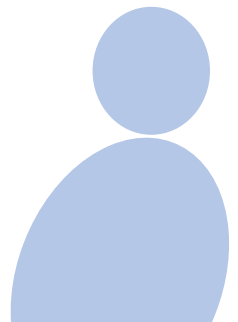
Cyclic Prefix=80 samples

Original Samples=1024

Total= 1104 samples

$$\begin{array}{r} 1023 \\ 80 \\ \hline 943 \end{array}$$

TIME DOMAIN.





## Example OFDM System

- Let us say each subcarrier is loaded with **QPSK symbols**. *2 Bits/symbol.*
- What is the effective bit-rate?

$$= \frac{2 \times N}{T + T_{cp}} = \frac{2 \times 1024 \text{ Bits}}{110.4 \times 10^{-6} \text{ s.}}$$

*$18.55 \times 10^6 \text{ Bits/s.}$*

$= 18.55 \text{ Mbps}$



## Example OFDM System

- Let us say each subcarrier is loaded with *QPSK symbols*.
- What is the effective bit-rate?

$$\frac{2 \frac{\text{bits}}{\text{subcarrier}} \times 1024 \text{ subcarriers}}{110.4 \mu s} = 18.55 \text{ Mbps}$$



# Example OFDM System

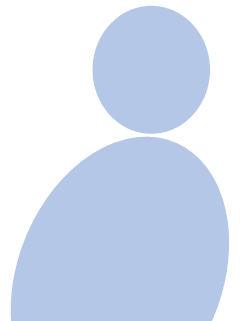
- **% Loss in spectral efficiency = ?**

$$\frac{N_{cp}}{N + N_{cp}} \times 100 = \frac{80}{80 + 1024} \times 100$$
$$= 7.24 \%$$

AS CP DURATION INCREASES  
LOSS IN SPECTRAL EFFICIENCY  
INCREASES!

LOSS IN SPECTRAL  
EFFICIENCY.

BECAUSE OF CYCLIC PREFIX



## Example OFDM System

- % **Loss in spectral efficiency** = ?

$$\frac{80}{1104} \times 100 = 7.24\%$$



# Example OFDM Transmission

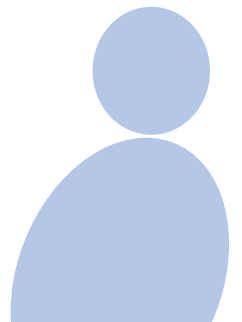
$N = 4$  subcarriers.

- Consider an  $N = 4$  subcarrier system

- Let

LOADED ON  
SUBCARRIERS.

$$\left\{ \begin{array}{l} X(0) = 1 + j \\ X(1) = 1 - j \\ X(2) = 1 + 2j \\ X(3) = 2 - j \end{array} \right.$$



# Example OFDM Transmission

- What are the time-domain samples?  $N=4$
- $x(0), x(1), x(2), x(3)$  are given by the IFFT

$$x(l) = \frac{1}{N} \sum_{k=0}^3 X_k \cdot e^{j2\pi \frac{kl}{N}}$$
$$= \frac{1}{4} \sum_{k=0}^3 X_k \cdot e^{j\pi \frac{1}{2} \cdot kl}$$

TIME DOMAIN SAMPLES.

IFFT  $N=4$

## Example OFDM Transmission

- What are the time-domain samples?
- $x(0), x(1), x(2), x(3)$  are given by the IFFT

$$x(l) = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{kl}{N}}$$

IFFT



## Example OFDM Transmission

$\ell = 0$

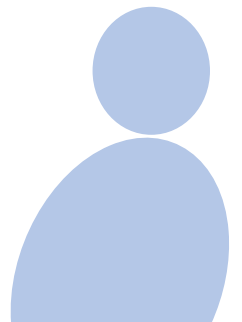
- What are the time-domain samples?
- $x(0), x(1), x(2), x(3)$  are given by the IFFT

$$x(0) = \frac{1}{4} (X(0) + X(1) + X(2) + X(3))$$

$$= \frac{1}{4} (1 + \cancel{j} + 1 - \cancel{j} + 1 + 2j + 2 - j)$$

$$= \frac{1}{4} (5 + j) = \frac{5}{4} + \frac{1}{4}j$$

$x(0)$





## Example OFDM Transmission

- What are the time-domain samples?
- $x(0), x(1), x(2), x(3)$  are given by the IFFT

$$\begin{aligned}x(0) &= \frac{1}{4} (X(0) + X(1) + X(2) + X(3)) \\&= \frac{1}{4} (1 + j + 1 - j + 1 + 2j + 2 - j) \\&= \frac{5}{4} + \frac{1}{4}j\end{aligned}$$



# Example OFDM Transmission

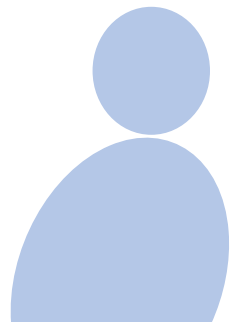
- What are the time-domain samples?
- $x(0), x(1), x(2), x(3)$  are given by the IFFT

$$x(1) = \frac{1}{4} \left( X(0) + jX(1) - X(2) - jX(3) \right)$$

$$= \frac{1}{4} \left( 1 + j + j(1-j) - (1+2j) - j(2-j) \right) \quad x(1)$$

$$= \frac{1}{4} \left( \cancel{1} + \cancel{j} + \cancel{j} + \cancel{1} - \cancel{1} - \cancel{2j} - \cancel{2j} - \cancel{1} \right)$$

$$= \frac{1}{4} (-2j) = -\frac{1}{2}j$$



## Example OFDM Transmission

- What are the time-domain samples?
- $x(0), x(1), x(2), x(3)$  are given by the IFFT

$$\begin{aligned}x(1) &= \frac{1}{4} (X(0) + jX(1) - X(2) - jX(3)) \\&= \frac{1}{4} (1 + j + (1 - j)j - (1 + 2j) - j(2 - j)) \\&= \frac{1}{4} (0 - 2j) = -\frac{1}{2}j\end{aligned}$$

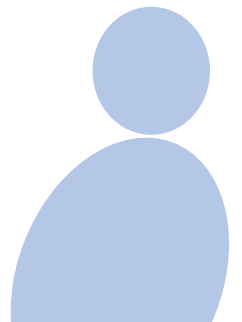


## Example OFDM Transmission

$$x(2) = -\frac{1}{4} + \frac{5}{4}j$$

Sample  $l=2$

$$x(3) = 0$$



# Example OFDM Transmission

- The transmit samples are

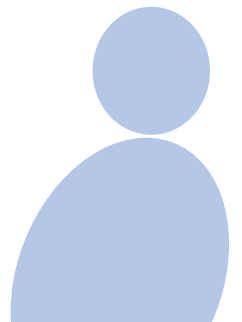
*Time Domain*

$$x(0), x(1), x(2), x(3)$$

Before Addition  
of CP -

$N = 4$   
Samples.

$$= \underline{\frac{5}{4} + \frac{1}{4}j, -\frac{1}{2}j, -\frac{1}{4} + \frac{5}{4}j, 0}$$



# Example OFDM Transmission

- Let **cyclic prefix** be of duration  $1$  sample  $\underline{N_{cp}}$ .
- The transmit frame is  $\rightarrow CP$ .

$x(3), x(0), x(1), x(2), x(3)$

$$N + N_{cp} = 4 + 1 = 5$$

$$0, \frac{5}{4} + \frac{1}{4}j, -\frac{1}{2}j, -\frac{1}{4} + \frac{5}{4}j, 0$$

TRANSMITTED OFDM BLOCK.  
WITH CP.

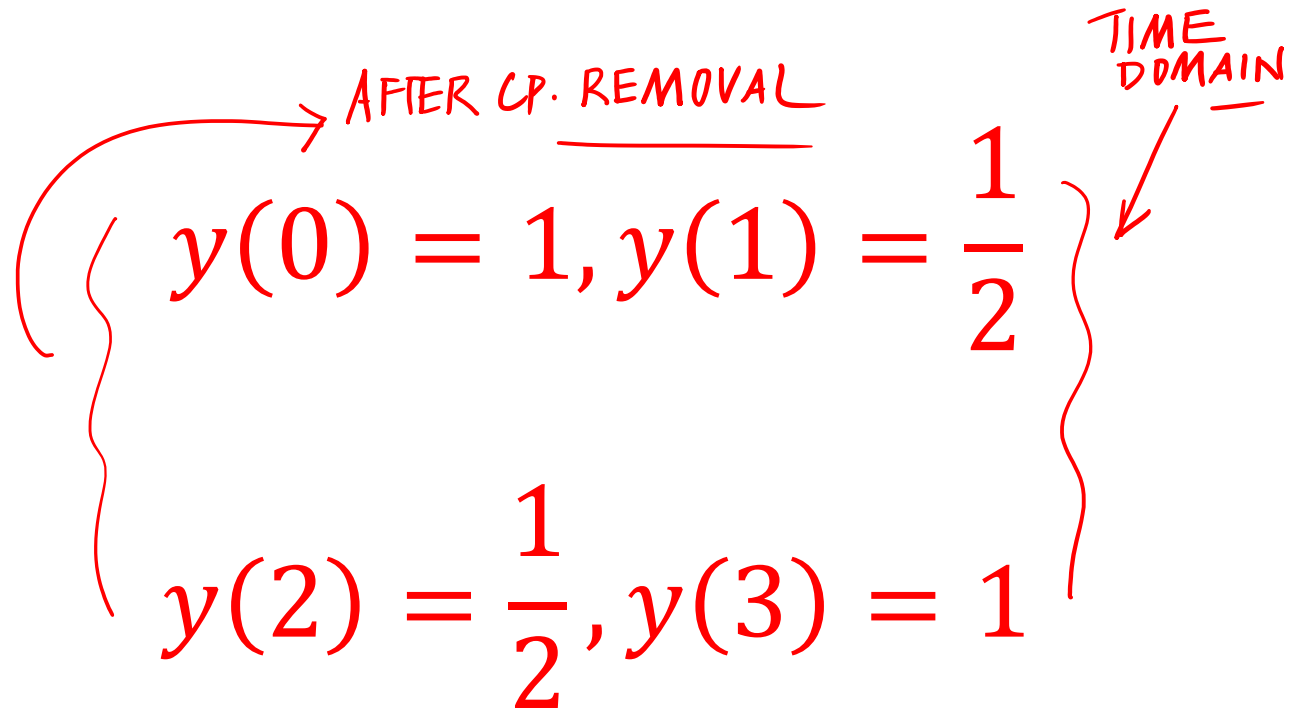
## Example OFDM Transmission Receiver?

- Consider the **output samples** given as

AFTER CP. REMOVAL

$$\left\{ \begin{array}{l} y(0) = 1, y(1) = \frac{1}{2} \\ y(2) = \frac{1}{2}, y(3) = 1 \end{array} \right\}$$

TIME DOMAIN

A red arrow points from the text 'AFTER CP. REMOVAL' to the first equation. A red bracket groups both equations. A red arrow points from the text 'TIME DOMAIN' to the bracketed equations.

# Example OFDM Transmission

- The subcarrier outputs are given by the FFT

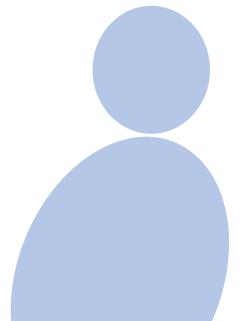
FAST FOURIER TRANSFORM

$N=4$

$$Y_k = \sum_{l=0}^{N-1} y(l) e^{-j 2\pi \frac{kl}{N}} = \sum_{l=0}^3 y(l) e^{-j \frac{\pi}{2} kl}$$

---

$N=4$ pt FFT  
AFTER CP REMOVAL





## Example OFDM Transmission

- The subcarrier outputs are given by the FFT

$$Y_k = \sum_{l=0}^{N-1} y(l) e^{-j2\pi \frac{kl}{N}}$$



# Example OFDM Transmission

$$Y_0 = y(0) + y(1) + y(2) + y(3)$$

→ OUTPUT ON SUBCARRIER. 0

$$= \left( 1 + \frac{1}{2} + \frac{1}{2} + 1 \right) = 3$$



## Example OFDM Transmission

$$Y_0 = y(0) + y(1) + y(2) + y(3)$$

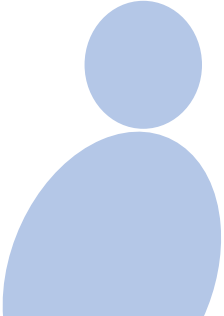
$$= 1 + \frac{1}{2} + \frac{1}{2} + 1 = 3$$



# Example OFDM Transmission


$$Y_1 = y(0) - jy(1) - y(2) + jy(3)$$
$$= 1 - j\frac{1}{2} - \frac{1}{2} + j1 = \frac{1}{2} + \frac{1}{2}j = Y_1$$

OUTPUT ON  
SUBCARRIER.



## Example OFDM Transmission

$$Y_1 = y(0) - jy(1) - y(2) + jy(3)$$

$$= 1 - j\frac{1}{2} - \frac{1}{2} + j1 = \frac{1}{2} + \frac{1}{2}j$$




## Example OFDM Transmission

- Similarly, one can determine

$$Y_2 = 0$$

$$Y_3 = \frac{1}{2} - \frac{1}{2}j$$



## Example OFDM Transmission

- Therefore, the subcarrier outputs are

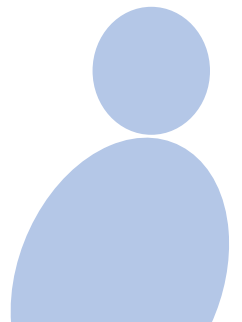
$$Y_0 = 3$$

$$Y_1 = \frac{1}{2} + \frac{1}{2}j$$

$$Y_2 = 0$$

$$Y_3 = \frac{1}{2} - \frac{1}{2}j$$

← OUTPUTS FOR  
N = 4 SUBCARRIERS -



## Example OFDM Transmission

- Consider the **channel taps** given as

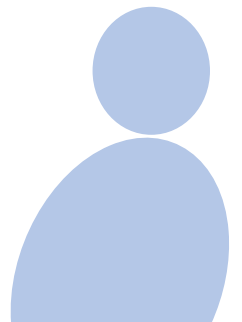
$$h(0) = 1, h(1) = \frac{1}{2}$$

*L = 2 channel taps.*

- $N = 4$  subcarriers

*H<sub>0</sub>, H<sub>1</sub>, H<sub>2</sub>, H<sub>3</sub>.*

- What are the **subcarrier channel coefficients**?





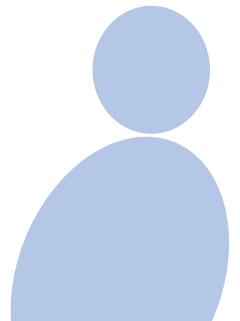
## Example OFDM Transmission

- The **subcarrier coefficients** are given by the zero padded FFT of channel taps

$$\begin{array}{c} \text{1} \\ \text{1, } \frac{\text{1}}{\text{2}}, \text{0, 0} \end{array} \quad \begin{array}{l} \text{N-L zeros -} \\ \text{N=4} \\ \text{L=2} \\ \text{4-2=2 zeros -} \end{array}$$

$\downarrow \text{FFT}$

$$H_0, H_1, H_2, H_3$$



## Example OFDM Transmission

- The **subcarrier channel coefficients** are

$$H_k = \sum_{l=0}^{L-1} h(l) e^{-j2\pi \frac{kl}{N}}$$

Zero PADDED FFT

$H_k$

$$= \sum_{l=0}^{L-1} h(l) e^{-j\frac{\pi}{2} kl} = h(0) + h(1) e^{-j\frac{\pi}{2} k}$$

## Example OFDM Transmission

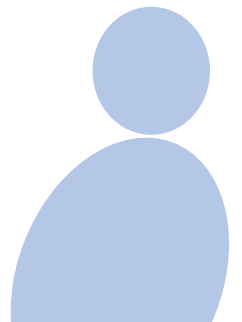
$$H_0 = h(0) + h(1)$$

$$= 1 + \frac{1}{2} = \frac{3}{2}$$



## Example OFDM Transmission

$$H_0 = h(0) + h(1) = 1 + \frac{1}{2} = \frac{3}{2}$$



## Example OFDM Transmission

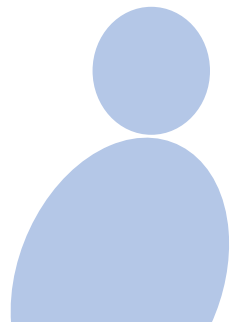
$$H_1 = h(0) - jh(1)$$

$$= 1 - \frac{1}{2}j$$



## Example OFDM Transmission

$$H_1 = h(0) - jh(1) = 1 - j\frac{1}{2}$$



# Example OFDM Transmission

- The **subcarrier channel coefficients** are

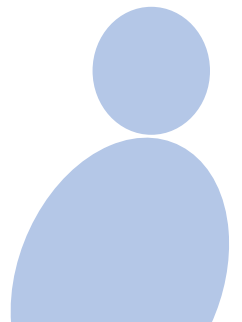
$$H_0 = \frac{3}{2}$$

$$H_1 = 1 - j\frac{1}{2}$$

$$H_2 = \frac{1}{2}$$

$$H_3 = 1 + j\frac{1}{2}$$

SUBCARRIER  
CHANNEL  
COEFFICIENTS.



## BER of OFDM

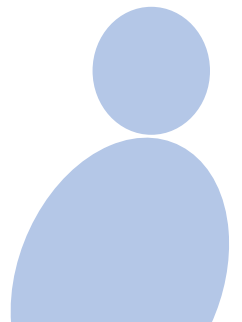
BIT ERROR RATE OF OFDM SYMBOL

- Consider the channel taps  $h(0), h(1), \dots, h(L-1)$
- Assume they are **Rayleigh fading** with unit power

$$E\{|h(l)|^2\} = 1$$

- Noise samples  $v(l)$  are i.i.d. with power  $N_0$

$$E\{|v(l)|^2\} = N_0.$$





# BER of OFDM

- Symbols loaded on subcarriers have power  $P$
- Define **effective SNR** for QPSK as

$$\rho_{eff} = \frac{L}{N} \times \frac{P}{N_0} = \frac{L}{N} \times SNR$$

NUMBER OF CHANNEL TAPS.

NUMBER OF SUBCARRIERS.

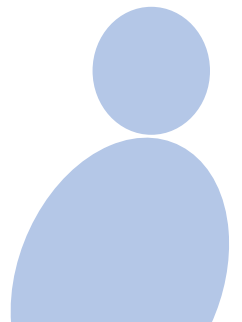
# BER of OFDM

- BER of OFDM for QPSK is

$$BER = \frac{1}{2} \left( 1 - \sqrt{\frac{P_{eff}}{2 + P_{eff}}} \right) \approx \frac{1}{2} \times \frac{1}{P_{eff}}$$

$$= \frac{1}{2} \cdot \frac{1}{\frac{L}{N} SNR}$$

$$BER = \frac{1}{2} \times \frac{N}{L \times SNR}$$

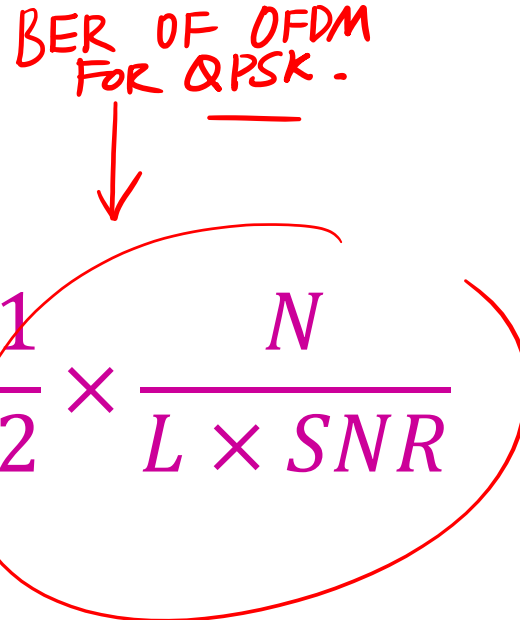


# BER of OFDM

- BER of OFDM for QPSK is

$$\frac{1}{2} \left( 1 - \sqrt{\frac{\rho_{eff}}{2 + \rho_{eff}}} \right) \approx \frac{1}{2} \times \frac{1}{\rho_{eff}} = \frac{1}{2} \times \frac{N}{L \times SNR}$$

BER OF OFDM  
FOR QPSK.





## BER of OFDM Example

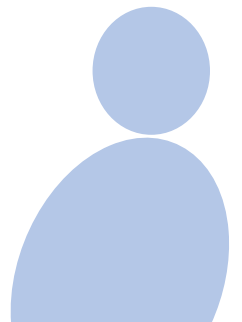
- What is the BER of OFDM for QPSK with SNR = 30 dB, L = 8 Channel taps and N = 64 subcarriers

$$SNR = 30 \text{ dB} = 10^3$$

$$L = 8$$
$$N = 64$$

$$\rho_{eff} = \frac{L}{N} \times \frac{P}{N_0} = \frac{L}{N} \times SNR$$

$$= \frac{8}{64} \times 10^3 = \frac{1}{8} \times 10^3 = \rho_{eff}$$



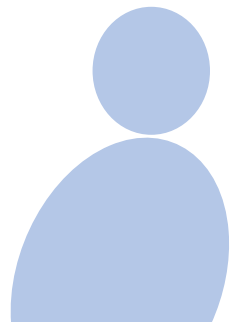
# BER of OFDM Example

- BER is

$$\frac{1}{2} \left( 1 - \sqrt{\frac{\rho_{eff}}{2 + \rho_{eff}}} \right) \approx \frac{1}{2} \times \frac{1}{\rho_{eff}}$$

$$= \frac{1}{2} \times \frac{1}{8 \times 10^3} = \frac{1}{2} \times \frac{8}{10^3}$$

$$= 4 \times 10^{-3} = \text{BER.}$$



# BER of OFDM Example

- **BER is**

$$\begin{aligned} \frac{1}{2} \left( 1 - \sqrt{\frac{\rho_{eff}}{2 + \rho_{eff}}} \right) &\approx \frac{1}{2} \times \frac{1}{\rho_{eff}} \\ &= \frac{1}{2} \times \frac{1}{\frac{10^3}{8}} = 4 \times 10^{-3} \end{aligned}$$



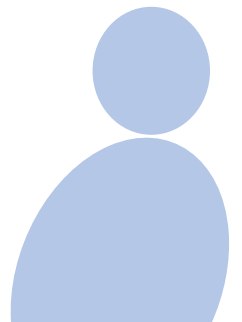
Instructors may use this white area (14.5 cm / 25.4 cm) for the text.  
Three options provided below for the font size.

Font: Avenir (Book), Size: 32, Colour: Dark Grey

Font: Avenir (Book), Size: 28, Colour: Dark Grey

Font: Avenir (Book), Size: 24, Colour: Dark Grey

Do not use the space below.



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