EMasters in Communication Systems. IITK
EE901: Probability and Random Processes
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Question set -1

Practice Questions
Don't submit the solutions for practice questions.

1. A biased dice is rolled. The probability that a number i appears is given as c/i. Compute the value of c.

   Let A denote the event that an even number occurs. Let B denote the event that a number less than 5 occurs.

   What is the probability of event A and $A \cap B$? What is probability of A^c?

2. A 6-faced dice is rolled. What is the smallest and biggest $\sigma-$ algebra possible?

3. If boys and and girls are born with same probability, what is the probability that in a family with three children, exactly one is a girl?

4. Let $\Omega = \{1, 2, 3\}$. If $P(\{1\}) = 1/3,\ P(\{2\}) = 1/2$ and $P(\{3\}) = \alpha$, find the value of α. Find $P(\{1, 2\}),\ P(\{1, 3\})$ and $P(\{2, 3\})$.

5. If $A \subset B$ and $P(A) \neq 0$. Then show that $P(B \mid A) = 1$.

6. A box contains three coins: two regular coins and one fake two-headed coin ($P(\{H\}) = 1$). You pick a coin at random and toss it. What is the probability that it lands heads up?

Assignment
Submit the solutions. All questions carry 4 marks.

1. A fair coin is tossed thrice. If we are interested in all outcomes. Find the sample space.

   If we are only interested in total number of heads, find the sample space.

2. Consider a dice roll experiment. Let A denote the event that an odd number occurs. Let B denote the event that an even number occurs. Let C denote the event that a number less than 3 occurs.

   Now compute $E_1 = A \cap C$ and $E_2 = B \cap C^c$.

   Compute $E_1 \cup E_2$.

3. Let $\Omega = (0, 1)$, which among the below are $\sigma-$ algebra?

$$\mathcal{F}_1 = \{\phi, \Omega, (0, 0.2), (0.2, 1)\}$$
$$\mathcal{F}_2 = \{\phi, \Omega, (0, 2/3), [2/3, 1), (0, 1/2), [1/2, 1]\}$$
$$\mathcal{F}_3 = \{\phi, \Omega, (0, 5/4), [5/4, 1)\}$$

4. Let $\Omega = \{A, B, C\}$. Obtain a $\sigma-$ algebra which contains at least $\{\{A\}, \{B\}\}$.

5. A computer picks a positive integer randomly. The probability that a number i appears is given as c/i^2. Compute the value of c.

   Let A denote the event that an even number occurs. Let B denote the event that a number less than 10 occurs.

   What is the probability of event A and $A \cap B$? What is probability of A^c?

6. A computer picks a real number between 0 and 1 randomly. The probability that a number in interal (a, b) appears is given as $c(b^2 - a^2)$. Compute the value of c.

   What is the probability of that a number less than 0.5 is picked?

7. You have two coins. We pick one coin, toss it and observe the outcome. The probability assignment is as follows

   The probability that fair coin is picked and head occurs = 1/4

   The probability that biased coin is picked and head occurs = 1/4

   The probability that fair coin is picked and tail occurs = 3/8

   The probability that biased coin is picked and tail occurs = 1/8

   Now compute the following:

   Given head appears, what is the probability that the coin is biased?

   Given biased coin is picked, what is the probability that head appears?

8. We pick a random number from 1 to 10, and call it N. Suppose that all outcomes are equally likely. Let A be the event that N is less than 7, and let B be the event that N is an even number. Are A and B independent?

9. We have three bags that each contain 100 marbles:

   Bag 1 has 75 red and 25 blue marbles

   Bag 2 has 60 red and 40 blue marbles;

   Bag 3 has 45 red and 55 blue marbles.

   We choose one of the bags at random and then pick a marble from the chosen bag, also at random. What is the probability that the chosen marble is red?

10. If $A \cap B = \phi$ and $P(B) \neq 0$. Then show that $P(A \mid B) = 0$.