Deep Q-Network

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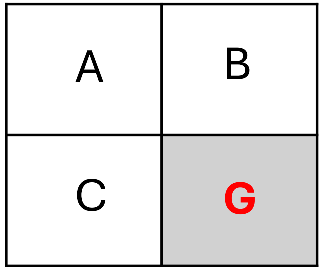
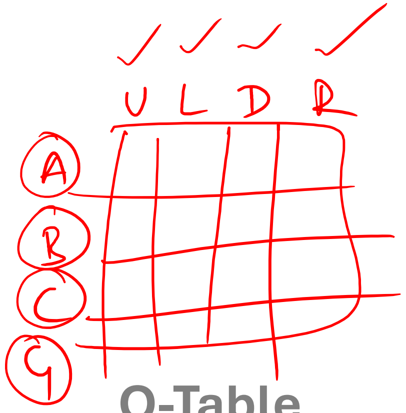
Infeasibility of Tabular Approaches

- Small state space: Q-Table Feasible



Grid Environment

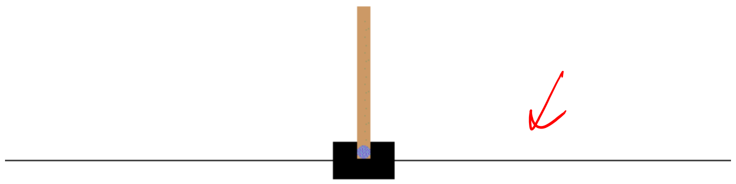
$\rightarrow Q(s, a)$



Q-Table

- Continuous or Large state space: Not feasible!

S



Cart-Pole

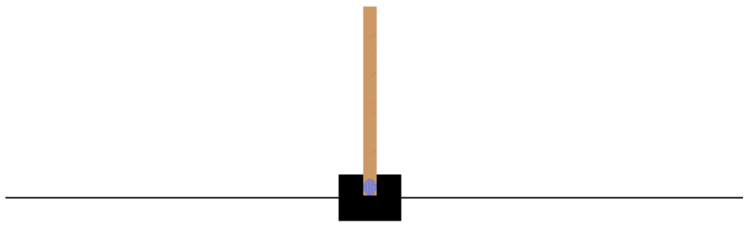
Go Game

Features: Cart-Pole Example

Cartpole: The goal is to balance the pole by applying forces in the left or right direction

State Features $s = (s_1, s_2, s_3, s_4)$ Q^* V

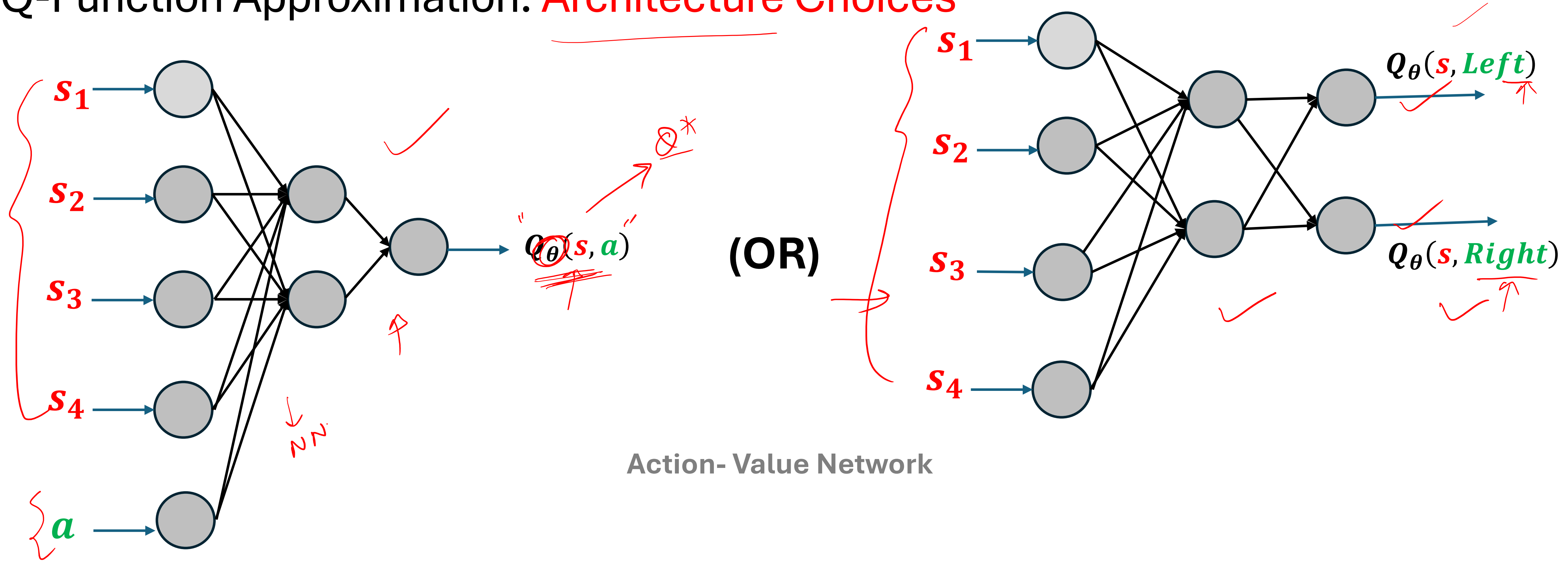
State $\bar{s}$	Min	Max
Cart Position s_1	-4.8	4.8
Cart Velocity s_2	-Inf	Inf
Pole Angle s_3	~ 24°	~ 24°
Pole Angular Velocity s_4	-Inf	Inf



a Action Features

0:	Push the cart to the LEFT
1:	Push the cart to the RIGHT

Q-Function Approximation: Architecture Choices



Neural Network-based Function Approximation

State Features $s = (s_1, s_2, s_3, s_4)$

Actions Features a

Neural Network Weights θ

Function approx. for Q^*

$$\boxed{\theta^* = \operatorname{argmin}_\theta \mathbb{E}\left[\left(Q^*(s,a) - Q_\theta(s,a)\right)^2\right]}$$

SGD: $\theta_{new} = \theta_{old} + 2\,\alpha \left(Q^*(s,a) - Q_\theta(s,a)\right) \nabla Q_\theta(s,a)$

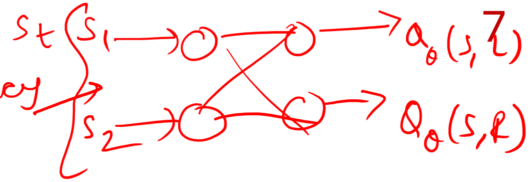
Challenge: Q^* unknown

$Q^*(s,a) = R_s^a + \sum P_{ss'}^a V^*(s')$

Bellman Equation: $Q^*(S_t, A_t) = \mathbb{E}\left[R_{t+1} + \gamma \max_{a'} Q^*(S_{t+1}, a')\right] \approx R_{t+1} + \gamma \max_{a'} Q_\theta(S_{t+1}, a')$

Solution: $\theta_{new} = \theta_{old} + 2\,\alpha \left(R_{t+1} + \gamma \max_{a'} Q_\theta(S_{t+1}, a') - Q_\theta(S_t, A_t)\right) \nabla Q_\theta(S_t, A_t)$

Q-Learning with Fn Approx: A Naïve Approach



- Initialize θ parameters randomly
- **Repeat** for each episode:
  - Initialize S_0 randomly
  - **Repeat** for each time-step t in the episode:
    - Obtain $Q_\theta(S_t, a)$ for all actions through a neural network forward pass
    - Sample action $A_t \sim \epsilon$-greedy w.r.t. $Q_\theta(S_t, a)$
    - Take action A_t and observe R_{t+1} and S_{t+1}
    - "$target = R_{t+1} + \gamma \max_{a'} Q_\theta(S_{t+1}, a')$"
    - Update $\theta = \theta + \alpha\left(target - Q_\theta(S_t, A_t)\right) \nabla Q_\theta(S_t, A_t)$ using backprop
- Output: $\pi^*(s) \approx greedy\,(Q_\theta(s, a))$

Q_θ

Q- $Q(s, a)$ → uniform random policy / ↳ ε-greedy Q

S_0, A_0

S_t

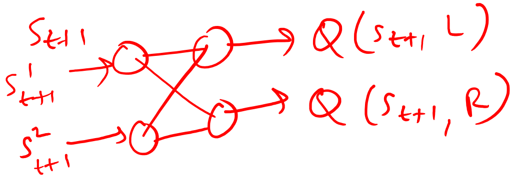
A_0

→ $Q^*(s_t, \text{left})$

→ $Q^*(s_t, \text{Right})$

A_t { w.p. $1-\varepsilon$ choose $\arg\max_a Q^*_\theta(s_t, a)$

ε → choose random action.



Issues with Naïve Q function approx.

$Q_\theta(s,a) = \theta_1 s + \theta_2 a$

1. "Non-Stationary Target"

- Minimize $\mathbb{E}\left[\left(Q^*(s,a) - Q_\theta(s,a)\right)^2\right]$
- Target $Q^*(s,a) \approx r + \gamma \max_{a'} Q_\theta(s',a')$
- Minimize $\mathbb{E}\left[\left(r + \gamma \max_{a'} Q_\theta(s',a') - Q_\theta(s,a)\right)^2\right]$

$Q_\theta(s_1,a_1) = 0$

$(-5-0)^2 = 25$

$(-5-(-3))^2 = 4$

$$N \begin{cases} (s_1, a_1) \to Q^*(s_1,a_1) = -5 \\ (s_2, a_2) \to \\ \vdots \\ (s_N, a_2) \to \end{cases}$$

$(-5-0)^2 = 25$

$(-10-(-3))^2 = 49$

Solution: Fixed-Target Q-Network

Issues with Naïve Q function approx.

1. Non-Stationary Target

- $target = R_{t+1} + \gamma \max_{a'} Q_\theta(S_{t+1}, a')$
- Update $\theta = \theta + \alpha \left(target - Q_\theta(S_t, A_t)\right) \nabla Q_\theta(S_t, A_t)$
- During the training process θ keeps changing
- The target depends on θ
- Since target keeps changing making it difficult to converge

Solution: Fixed-Target Q-Network

Fixed Target Q-Network

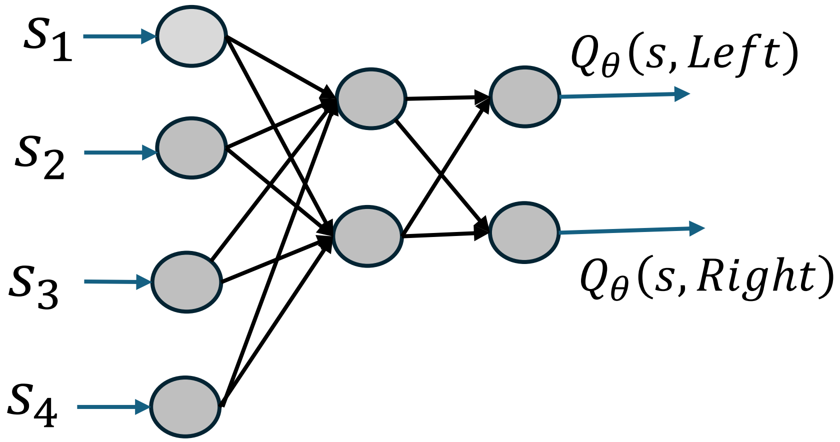
(s_1, a_1)

$(-5-0)^2$

$((-5)(-3))^2$

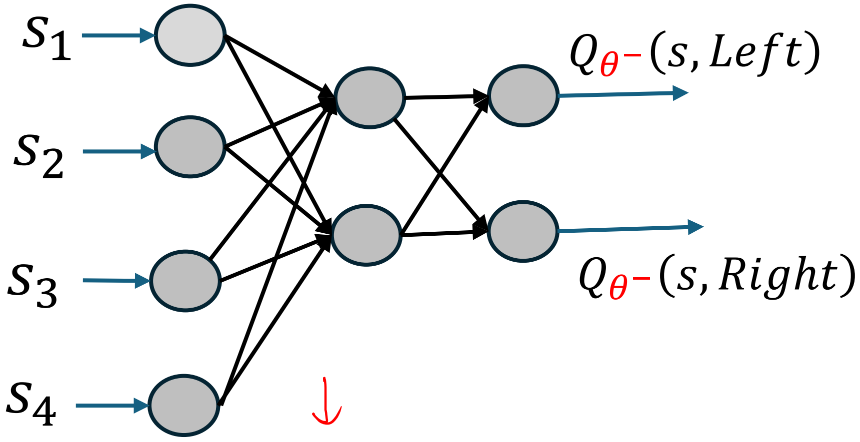
-10

Maintain an additional neural network for calculating target



Train Q-Network

Weights: θ



Target Q-Network

Weights: θ^-

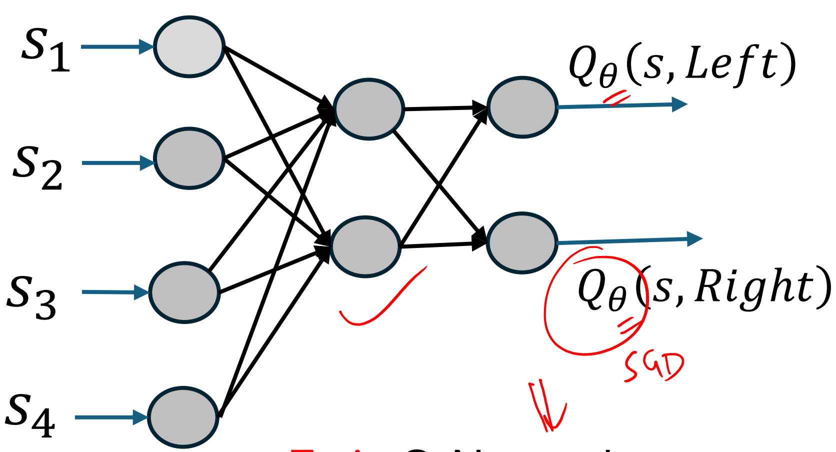
Calculate *target* $= R_{t+1} + \gamma \max_{a'} Q_{\theta^-}(S_{t+1}, a')$

Update train network $\theta = \theta + \alpha \left(\underbrace{target}_{Q^*(s,a)} - Q_\theta(S_t, A_t)\right) \nabla Q_\theta(S_t, A_t)$

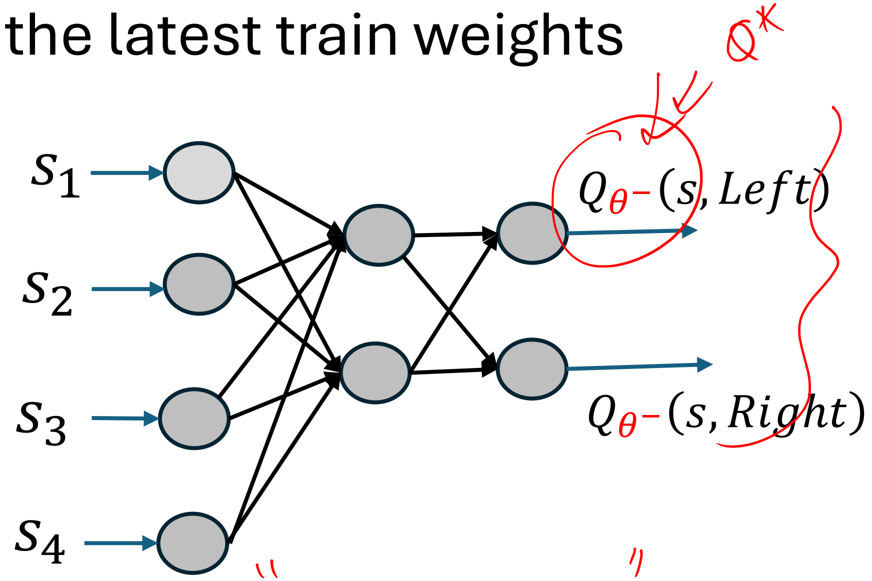
How to choose the target network weights $\boldsymbol{\theta^-}$?

How to choose target weights θ^-?

- Initialize $\theta^- = \theta$
- Repeat:
  - keep θ^- fixed for N time steps and update train weights θ
  - Update $\theta^- = \theta$ target weights to the latest train weights



Train Q-Network



"Target Q-Network"

2. Train for N time steps

$$target = R_{t+1} + \gamma \max_{a'} Q_{\theta^-}(S_{t+1}, a')$$

$$\theta = \theta + \alpha\left(target - Q_\theta(S_t, A_t)\right) \nabla Q_\theta(S_t, A_t)$$

1. Freeze

3. Update $\theta^- = \theta$

Issues with Naïve Q function approx.

SGD

(x_1, y_1)
(x_2, y_2)
$\vdots$
(x_n, y_n)
Training data.

θ_{new}

t $t+1$

2. Non i.i.d training samples: Leads to Instability

Sequence of samples during training in

- **Supervised learning:** i.i.d
- **Reinforcement learning:** Correlated

ε-greedy
(uniform rando

$Q(S_t, A_t), R_{t+1}, S_{t+1}, A_{t+1}, \ldots$

$S_t, A_t, R_{t+1}, S_{t+1}$

Solution:

- Store Last D time-steps data in a replay buffer
- Pick a random data sample from the replay buffer to train

Memory Replay Buffer

$(S_t, A_t, R_{t+1}, S_{t+1})$
$(S_{t-1}, A_{t-1}, R_t, S_t)$
$\vdots$
$\vdots$
$(S_{t-D-1}, A_{t-D-1}, R_{t-D}, S_{t-1})$

D

DQN Pseudo Code (with Target Network and Replay Buffer)

non-iid.

- Initialize train and target Q-network weights θ and θ^-
- **Repeat** for each episode:
  - Initialize S_0 randomly
  - **Repeat** for each time-step $\boldsymbol{t}$ in the episode:
    - Sample action $A_t \sim \epsilon$-greedy w.r.t. $Q_\theta(S_t, a)$
    - Take action A_t and observe R_{t+1} and S_{t+1}
    - Store the data $(S_t, A_t, R_{t+1}, S_{t+1})$ in the replay buffer
    - Select a random data sample (s, a, r, s') from the replay buffer
    - $target = r + \gamma \max_{a'} Q_{\theta^-}(s', a')$
    - Update train weights $\theta = \theta + \alpha\left(target - Q_\theta(s,a)\right) \nabla Q_\theta(s,a)$
    - If $t\ (mod)\ N == 0$:
      - Update target weights $\theta^- = \theta$
- Output: $\pi^*(s) \approx greedy\ (Q_\theta(s,a))$

Q

Enhancements to DQN

- Double DQN
  - DQN overestimates Q-values due to maximization bias

$$\boxed{\max_a Q(s,a)}$$

  - Uses two Q-networks to resolve bias

$$Q_1(s, \arg\max_a Q_2(s,a))$$

$Q_1 = Q_2 = Q$

- Duelling DQN
  - Splits Q-value into state-value $V(s)$ and advantage $A(s,a)$ functions:

$$Q(s,a) = V(s) + A(s,a)$$

  - This separation improves learning stability and efficiency