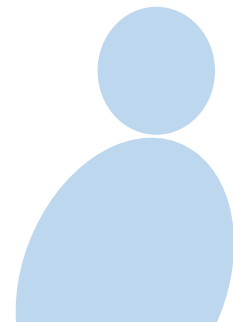


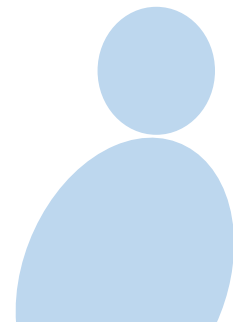
# **eMasters in Communication Systems**

**Prof. Aditya  
Jagannatham**



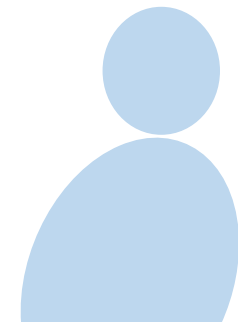
**Elective Module:**

**Advanced ML  
Techniques**



# Chapter 6

## Naiïve Bayes Examples

Two red curved lines, one above the other, spanning the width of the slide below the title.

# Naïve Bayes Example

Movies

- Consider a training set of **people's opinions**

- like** = 1 or **NOT** = 0 for the movies  $x_1$

NOT liking movie

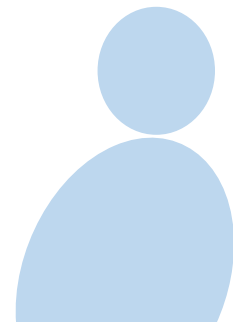
$x_2$  and  $y$

AVATAR

TG

Like movie

AV



# Naïve Bayes Example

- The estimates of the **prior probabilities** are as follows

	$x_1 = 0$	$x_1 = 1$	$p(x_1 = 0 y)$	$p(x_1 = 1 y)$
$y = 0$	3	10	$p(x_1 = 0 y = 0) = \frac{3}{13}$	$p(x_1 = 1 y = 0) = \frac{10}{13}$
$y = 1$	4	13	$p(x_1 = 0 y = 1) = \frac{4}{17}$	$p(x_1 = 1 y = 1) = \frac{13}{17}$

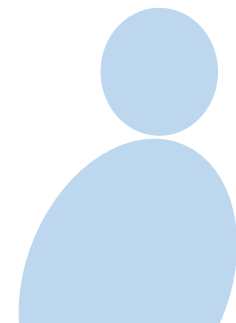
Number of people who  
Like  $x_1$  and dislike  $y$   
Total number who dislike  $y$

$x_1 = 0, y = 0 \Rightarrow$  DONOT Like  $x_1$  and  $y$

NOT Like  $x_1$   
Like  $y$

$x_1 = 1, y = 0$   
 $\Rightarrow$  Like  $x_1$ , but NOT  $y$

Like  $x_1$ , Like  $y$



# Naïve Bayes Example

- The estimates of the **prior probabilities** are as follows

	$x_1 = 0$	$x_1 = 1$	$p(x_1 = 0 y)$	$p(x_1 = 1 y)$
$y = 0$	3	10	$\frac{3}{13}$	$\frac{10}{13}$
$y = 1$	4	13	$\frac{4}{17}$	$\frac{13}{17}$

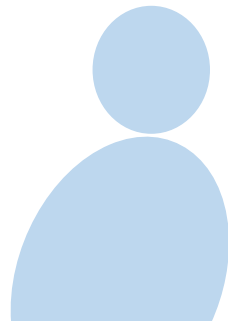
$$p(x_1=0|y=0)$$

$$p(x_1=1|y=0)$$

Prior probabilities

$$p(x_1=0|y=1)$$

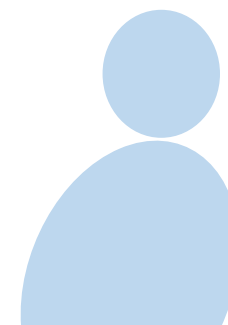
$$p(x_1=1|y=1)$$



# Naïve Bayes Example

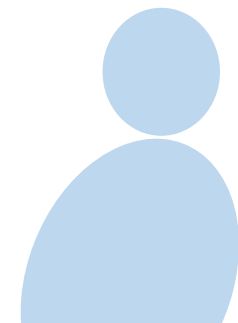
	$x_2 = 0$	$x_2 = 1$	<u><math>p(x_2 = 0 y)</math></u>	$p(x_2 = 1 y)$
$y = 0$	5	8	$p(x_2=0 y=0)$ $= \frac{5}{13}$	$p(x_2=1 y=0)$ $= \frac{8}{13}$
$y = 1$	7	10	$p(x_2=0 y=1)$ $= \frac{7}{17}$	$p(x_2=1 y=1)$ $= \frac{10}{17}$

Prior probabilities.



# Naïve Bayes Example

	$x_2 = 0$	$x_2 = 1$	$p(x_2 = 0 y)$	$p(x_2 = 1 y)$
$y = 0$	5	8	$\frac{5}{13}$	$\frac{8}{13}$
$y = 1$	7	10	$\frac{7}{17}$	$\frac{10}{17}$





# Naïve Bayes Example

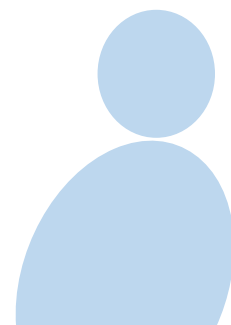
Number of people  
who dislike y.

	Count	$p(y)$
$y = 0$	13	$\frac{13}{30}$
$y = 1$	17	$\frac{17}{30}$

$$p(y=0) = \frac{\text{Number of people dislike } y}{\text{Total number of people.}}$$
$$= \frac{13}{30}$$

Number of  
people who like y

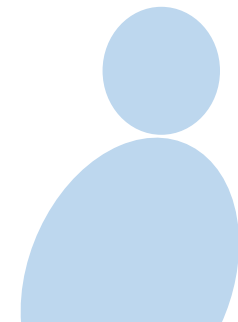
$$\frac{\text{Number like } y}{\text{Total Number of people.}}$$



# Naïve Bayes Example

	Count	$p(y)$
$y = 0$	13	$\frac{13}{30}$
$y = 1$	17	$\frac{17}{30}$

$p(y = v)$



# Naïve Bayes Example

person likes  $x_1$   
does NOT like  $x_2$

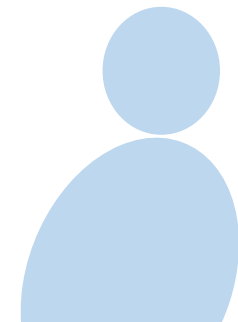
- What is the probability that a new person, who likes  $x_1 = 1$  but NOT  $x_2 = 0, \dots$

- will **like**  $y$ ?

A posteriori probability  
person likes  $y$   
given likes  $x_1$ , does NOT like  $x_2$

$$p(y=1 | x_1=1, x_2=0) = p(y=1 | \bar{x}=\bar{v})$$

$$\begin{aligned} &= \frac{p(x_1=1, x_2=0 | y=1) p(y=1)}{p(x_1=1, x_2=0)} \\ &= \frac{p(x_1=1 | y=1) \cdot p(x_2=0 | y=1) p(y=1)}{p(x_1=1, x_2=0)} \end{aligned}$$

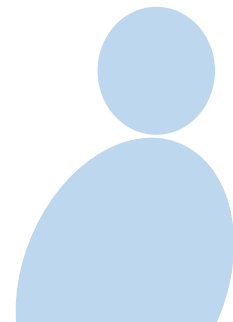


$$p(y=0|x_1=1, x_2=0)$$

A posteriori probability  
does NOT like  $y$   
given likes  $x_1$ , does NOT  
like  $x_2$

$$= \frac{p(x_1=1, x_2=0|y=0) p(y=0)}{p(x_1=1, x_2=0)}$$

$$= \frac{p(x_1=1|y=0) p(x_2=0|y=0) p(y=0)}{p(x_1=1, x_2=0)}$$



likes  $y$  if

$$p(y=1|x_1=1, x_2=0) > p(y=0|x_1=1, x_2=0)$$

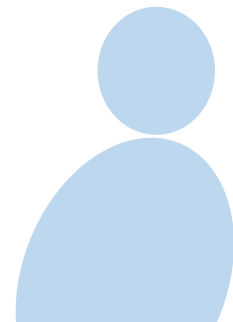
$$\Rightarrow \frac{p(x_1=1|y=1) p(x_2=0|y=1) p(y=1)}{p(x_1=1, x_2=0)} > \frac{p(x_1=1|y=0) p(x_2=0|y=0) p(y=0)}{p(x_1=1, x_2=0)}$$

$$\Rightarrow p(x_1=1|y=1) \cdot p(x_2=0|y=1) p(y=1) \xrightarrow{\text{LHS.}} > p(x_1=1|y=0) p(x_2=0|y=0) \xrightarrow{\text{RHS.}} \times p(y=0)$$

# Naïve Bayes Example

- We can readily compute this using the **Naïve Bayes formula!**

Aposteriori probabilities-

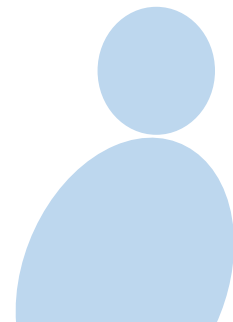


# Naïve Bayes Example

- For  $y = 1$ ,  $Q_1 =$

$$\underline{p(y = 1) \times p(x_1 = 1|y = 1) \times p(x_2 = 0|y = 1)}$$

$$= \frac{17}{30} \times \frac{13}{17} \times \frac{7}{17} = 0.178$$

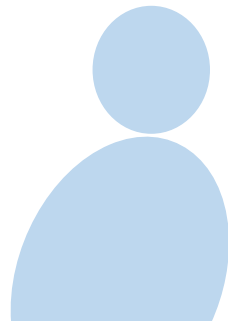


# Naïve Bayes Example

- For  $y = 1$ ,  $Q_1 =$

*Aposteriori  
for  $y = 1$ .*

$$\frac{p(y = 1) \times p(x_1 = 1|y = 1) \times p(x_2 = 0|y = 1)}{= \frac{17}{30} \times \frac{13}{17} \times \frac{7}{17} = 0.178}$$



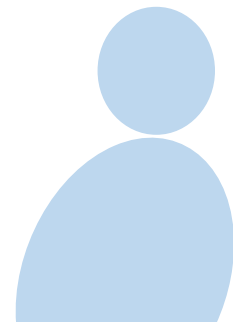


# Naïve Bayes Example

- For  $y = 0$ ,  $Q_0 =$

$$\underline{p(y = 0) \times p(x_1 = 1|y = 0) \times p(x_2 = 0|y = 0)}$$

$$= \frac{13}{30} \times \frac{10}{13} \times \frac{5}{13} = 0.128$$



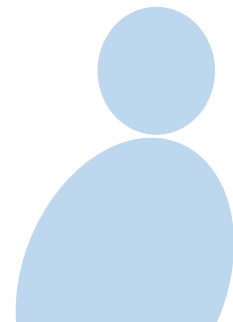
# Naïve Bayes Example

Aposteriori  
for  $y = 0$

- For  $y = 0$ ,  $Q_0 =$

$$p(y = 0) \times p(x_1 = 1|y = 0) \times p(x_2 = 0|y = 0)$$

$$= \frac{13}{30} \times \frac{10}{13} \times \frac{5}{13} = 0.128$$



# Naïve Bayes Example

- $Q_1$  is higher

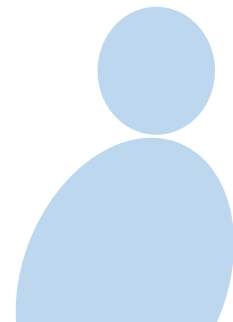
- $\Rightarrow$  new person is “**likely**” to like  $y$ !

$$\Rightarrow p(y=1 | x_1=1, x_2=0) > p(y=0 | x_1=1, x_2=0)$$

$\Rightarrow$  Person is likely to like  $y$ !

---

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# Laplace Smoothing Example

Avoid prior probability = 0

- The estimates of the **prior probabilities** are as follows

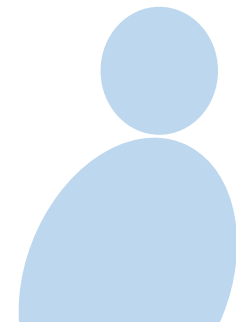
	$x_1 = 0$	$x_1 = 1$	$p(x_1 = 0 y)$	$p(x_1 = 1 y)$
$y = 0$	3	10	$p(x_1 = 0 y = 0)$ $= \frac{3+1}{13+2} = \frac{4}{15}$	$p(x_1 = 1 y = 0)$ $= \frac{10+1}{13+2} = \frac{11}{15}$
$y = 1$	4	13	$p(x_1 = 0 y = 1)$ $= \frac{4+1}{17+2} = \frac{5}{19}$	$p(x_1 = 1 y = 1)$ $= \frac{13+1}{17+2} = \frac{14}{19}$

Add 1 to numerator  
2 to denominator  
 $\frac{4}{15} + \frac{11}{15} = 1$

$\frac{5}{19} + \frac{14}{19} = 1$

Do NOT like  $x_1$  and  $y$   $1(x_1 = 0, y = 0)$

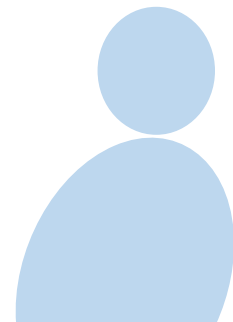
Movie recommendation



# Laplace Smoothing Example

- The estimates of the *prior probabilities* are as follows

	$x_1 = 0$	$x_1 = 1$	$p(x_1 = 0 y)$	$p(x_1 = 1 y)$
$y = 0$	3	10	$\frac{4}{15}$	$\frac{11}{15}$
$y = 1$	4	13	$\frac{5}{19}$	$\frac{14}{19}$



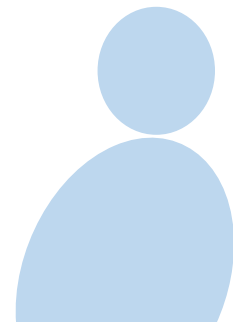
# Laplace Smoothing Example AVENGERS

	$x_2 = 0$	$x_2 = 1$	$p(x_2 = 0 y)$	$p(x_2 = 1 y)$
$y = 0$	5	8	$p(x_2=0 y=0) = \frac{5+1}{13+2} = \frac{6}{15}$	$p(x_2=1 y=0) = \frac{8+1}{13+2} = \frac{9}{15}$
$y = 1$	7	10	$p(x_2=0 y=1) = \frac{7+1}{17+2} = \frac{8}{19}$	$p(x_2=1 y=1) = \frac{10+1}{17+2} = \frac{11}{19}$

$$\frac{6}{15} + \frac{9}{15} = 1$$

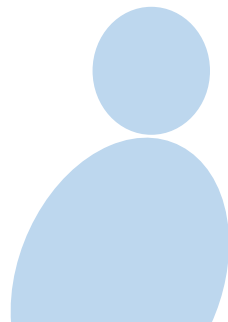
$$\frac{8}{19} + \frac{11}{19} = 1$$

do NOT like  $x_2$   
like  $y$



# Laplace Smoothing Example

	$x_2 = 0$	$x_2 = 1$	$p(x_2 = 0 y)$	$p(x_2 = 1 y)$
$y = 0$	5	8	$\frac{6}{15}$	$\frac{9}{15}$
$y = 1$	7	10	$\frac{8}{19}$	$\frac{11}{19}$



# Laplace Smoothing Example

TG. Avengers.

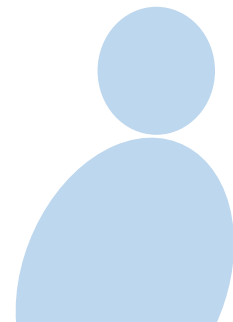
- What is the **probability** that a new person, who likes  $x_1 = 1$  but NOT  $x_2 = 0, \dots$

- will **like**  $y = 1$ ?

Avatar?

likes  $x_1 = TG$   
Does NOT like  $x_2$ : Avengers.

$P(\text{like } y: \text{Avatar})$ ?





# Laplace Smoothing Example

$$\frac{P(x_1=1|y=1) \times P(x_2=0|y=1)}{x P(y=1)}$$

$x_i$  are conditionally independent given  $y$ .

- We can readily compute this using the

Naïve Bayes formula!

Naïve Bayes

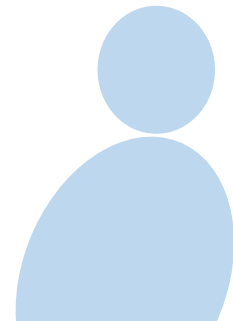
$$P(y=1|x_1=1, x_2=0) \propto P(x_1=1, x_2=0|y=1) P(y=1)$$

$$P(y=0|x_1=1, x_2=0) \propto P(x_1=1, x_2=0|y=0) P(y=0)$$

Naïve Bayes

$$P(x_1=1|y=0) P(x_2=0|y=0) P(y=0)$$

Aposteriori Probabilities.

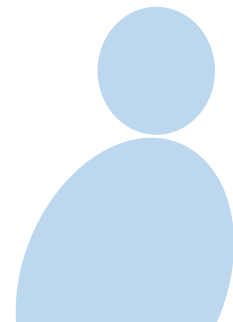


# Laplace Smoothing Example

- For  $y = 1$ ,  $Q_1 =$

$$p(y = 1) \times p(x_1 = 1|y = 1) \times p(x_2 = 0|y = 1)$$

$$= \frac{17}{30} \times \frac{14}{19} \times \frac{8}{19} = \underline{\underline{0.1758}}$$

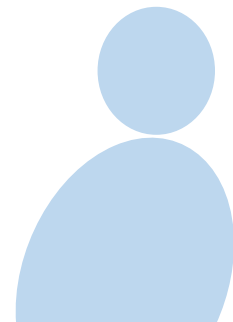


# Laplace Smoothing Example

- For  $y = 1$ ,  $Q_1 =$

$$\begin{aligned} & p(y = 1) \times p(x_1 = 1|y = 1) \times p(x_2 = 0|y = 1) \\ &= \frac{17}{30} \times \frac{14}{19} \times \frac{8}{19} = 0.1758 \end{aligned} \quad Q_1.$$

$Q_1 \propto$  Aposteriori prob  $y=1$   
given  $x_1=1, x_2=0$ .



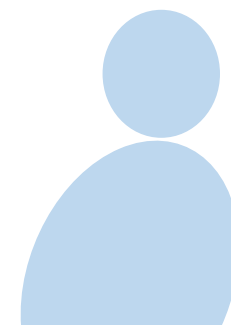
# Laplace Smoothing Example

- For  $y = 0$ ,  $Q_0 =$

$$p(y = 0) \times p(x_1 = 1|y = 0) \times p(x_2 = 0|y = 0)$$

$$= \frac{13}{30} \times \frac{11}{15} \times \frac{6}{15} = 0.1271 \quad \text{✓ } Q_0.$$

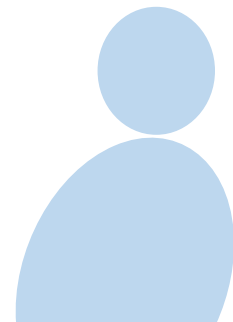
$Q_0 \propto$  Aposteriori probability  $y = 0$   
given  $x_1 = 1, x_2 = 0$ .



# Laplace Smoothing Example

- For  $y = 0$ ,  $Q_0 =$

$$\begin{aligned} & p(y = 0) \times p(x_1 = 1|y = 0) \times p(x_2 = 0|y = 0) \\ &= \frac{13}{30} \times \frac{11}{15} \times \frac{6}{15} = 0.1271 \end{aligned}$$



# Laplace Smoothing Example

- $Q_1$  is higher

Greater chance of  
liking avatar

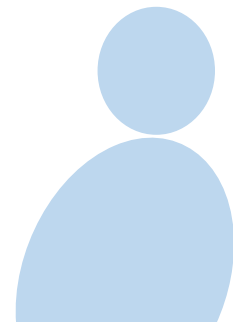
- $\Rightarrow$  new person is “likely” to like  $y$ !

$$Q_1 = 0.1758$$

$$Q_0 = 0.1271$$

$$Q_1 > Q_0 \Rightarrow$$

$$P(y=1 | x_1=1, x_2=0) > P(y=0 | x_1=1, x_2=0)$$



Instructors may use this white area (14.5 cm / 25.4 cm) for the text.  
Three options provided below for the font size.

Font: Avenir (Book), Size: 32, Colour: Dark Grey

Font: Avenir (Book), Size: 28, Colour: Dark Grey

Font: Avenir (Book), Size: 24, Colour: Dark Grey

Do not use the space below.

