EE910: Digital Communication Systems-I

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Lecture #3A: Digital Modulation: An Introduction

Introduction

- In a digital communication system, the source to be transmitted is discrete both in time and amplitude
- Digital information carrying signals must be first converted to an analog waveform prior to transmission
- At the receiving end, analog signals are converted back to a digital format before presentation to the end user
- The conversion process at the transmitting end is known as modulation
- The receiving end is known as demodulation or detection
- In digital wireless communication systems, the modulating signal may be represented as a time sequence of symbols or pulses, where each symbol has M finite states.
- Each symbol represents n bits of information where $n = \log_2 M$ bits/symbol.

Advantages of Digital Modulation over Analog

- Greater noise immunity (due to its finite process)
- Robustness to channel impairments
- Easier multiplexing of various forms of information like voice, data, video
- Security –by using coding techniques to avoid jamming
- Accommodation of digital error control codes which detect and/or correct transmission errors
- Equalization to improve the performance of over all communication link
- Supports complex signal conditioning and processing methods

Introduction

- Factors that influence digital modulation:
  - Low BER at low received signal-to-interference noise ratio (SINR)
  - Should performs well in multi-path and fading
  - Should have high spectral efficiency
  - Essentially, good BER performance at a low SINR under conditions of co-channel interference, and fading.
- The performance of a modulation scheme is often measured in terms of its power efficiency and bandwidth efficiency.
- The power efficiency is the ability of a modulation technique to preserve the fidelity (acceptable BER) of the digital message at low power levels
- Bandwidth efficiency is a measure of how many bits per symbol that we can reliably send over the communication system.

Introduction

- The source information is normally represented as a baseband (low-pass) signal
- Because of signal attenuation, it is necessary to move the baseband signal spectrum to reside at a much higher frequency band centered at f_c, called the carrier frequency, in the radio spectrum
- At the receiver end, the demodulation process removes the carrier frequency to recover the baseband information signal
- We choose different carrier frequencies for different signals
- Modulation/demodulation process facilitates channel assignment and reduces interference from other transmissions
- The modulation can be classified into two categories:
  - Linear modulation
  - Nonlinear modulation
- Also modulation can be memoryless or with memory.

Memoryless Modulation

- In a memoryless modulation scheme, the binary sequence is parsed into subsequences each of length k.
- Each sequence is mapped into one of the $s_m(t)$, $1 \leq m \leq 2^k$, signals regardless of the previously transmitted signals.
- This modulation scheme is equivalent to a mapping from $M = 2^k$ messages to M possible signals as shown in figure.

$$\overbrace{10\ldots1}^{k}\overbrace{00\ldots1}^{k}\overbrace{01\ldots0}^{k}\ldots \longrightarrow \boxed{\text{Modulator}} \longrightarrow s_m(t)$$

Modulation with memory

- In a modulation scheme with memory, the mapping is from the set of the current k bits and the past $(L-1)k$ bits to the set of possible $M = 2^k$ messages.
- In this case the transmitted signal depends on the current k bits as well as the most recent $L-1$ blocks of k bits. This defines a finite-state machine with $2^{(L-1)k}$ states.

Modulation with memory

- If at time instant $\ell - 1$ the modulator is in state $S_{\ell-1} \in \{1, 2, ..., 2^{(L-1)k}\}$ and the input sequence is $I_\ell \in \{1, 2, ..., 2^k\}$, then the modulator transmits the output $s_{m_\ell}(t)$ and moves to new state S_ℓ according to mappings

$$m_\ell = f_m(S_{\ell-1}, I_\ell) \tag{1}$$

$$S_\ell = f_s(S_{\ell-1}, I_\ell) \tag{2}$$

- Parameters k and L and functions $f_m(\cdot, \cdot)$ and $f_s(\cdot, \cdot)$ completely describe the modulation scheme with memory.
- Parameter L is called the constraint length of modulation.

Representation of digitally modulated signals

- The case of $L = 1$ corresponds to a memoryless modulation scheme.
- The modulator in a digital communication system maps a sequence of k binary symbols which in case of equiprobable symbols carries k bits of information into a set of corresponding signal waveforms $s_m(t)$, $1 \leq m \leq M$, where $M = 2^k$.

Representation of digitally modulated signals

- We assume that these signals are transmitted at every T_s seconds, where T_s is called the signaling interval.
- This means that in each second

$$R_s = \frac{1}{T_s} \tag{3}$$

symbols are transmitted.
- Parameter R_s is called the signaling rate or symbol rate.
- Since each signal carries k bits of information, the bit interval T_b, i.e., the interval in which 1 bit of information is transmitted, is given by

$$T_b = \frac{T_s}{k} = \frac{T}{\log_2 M} \tag{4}$$

Representation of digitally modulated signals

- The bit rate R is given by

$$R = kR_s = R_s \log_2 M \tag{5}$$

- If the energy content of $s_m(t)$ is denoted by $\mathcal{E}_m$, then the average signal energy is given by

$$\mathcal{E}_{avg} = \sum_{m=1}^{M} p_m \mathcal{E}_m \tag{6}$$

where p_m indicates the probability of the m-th signal (message probability).

Representation of digitally modulated signals

- In the case of equiprobable messages, $p_m = \frac{1}{M}$, and therefore,

$$\mathcal{E}_{avg} = \frac{1}{M}\sum_{m=1}^{M}\mathcal{E}_m \tag{7}$$

- If all signals have the same energy, then $\mathcal{E}_m = \mathcal{E}$ and $\mathcal{E}_{avg} = \mathcal{E}$.
- The average energy for transmission of 1 bit of information, or average energy per bit, when the signals are equiprobable is given by

$$\mathcal{E}_{bavg} = \frac{\mathcal{E}_{avg}}{k} = \frac{\mathcal{E}_{avg}}{\log_2 M} \tag{8}$$

Representation of digitally modulated signals

- If all signals have equal energy of $\mathcal{E}$, then

$$\mathcal{E}_b = \frac{\mathcal{E}}{k} = \frac{\mathcal{E}}{\log_2 M} \tag{9}$$

- If a communication system is transmitting an average energy of $\mathcal{E}_{bavg}$ per bit, and it takes T_b seconds to transmit this average energy, then the average power sent by the transmitter is

$$P_{avg} = \frac{\mathcal{E}_{bavg}}{T_b} = R\mathcal{E}_{bavg} \tag{10}$$

which for the case of equal energy signals becomes

$$P = R\mathcal{E}_b \tag{11}$$