1. The auto-correlation $R_{nn}(\tau)$ of white noise is

$$\frac{N_0}{2}\delta(\tau)$$

The PSD $S_{nn}(f)$ of white noise is the Fourier transform of the auto-correlation given as

$$\frac{N_0}{2}$$

Ans d

2. The symbol error rate (SER) for QPSK is approximately twice the BER. Hence, BER of QPSK is approximately half the SER
Ans c

3. The approximate probability of deep fade P_{DF} in the Rayleigh fading wireless channel is

$$\frac{1}{SNR}$$

Ans b

4. BER of **multiple antenna system** is given as

$$^{2L-1}C_{L-1} \times \frac{1}{2^L} \times \frac{1}{SNR^L}$$

Ans b

5. Inverse of a matrix exists for Only Non-singular square matrices
Ans c

6. The matrices $\mathbf{U}, \mathbf{V}$ in the SVD are Unitary
Ans c

7. As the bandwidth of the wireless channel increases, symbol duration decreases. This leads to Inter symbol interference
Ans a

8. In OFDM, IFFT and FFT are performed at the transmitter and receiver, respectively
Ans b

9. BER for BPSK modulation in the wireline system is given as $Q\left(\sqrt{SNR}\right)$. Therefore, SNR for a given BER is

$$BER = Q\left(\sqrt{SNR}\right) \Rightarrow SNR = \left(Q^{-1}(BER)\right)^2$$

Ans c

10. The Q −function is defined as $\int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$. Hence $Q(1)$ equals

$$\int_1^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

Ans a

11. The channel magnitude $a = |h|$ follows the PDF given as

$$2ae^{-a^2}, a \geq 0$$

Ans a

12. SER for $M = 256$ −QAM is

$$4\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3P}{N_0(M-1)}}\right)=4\left(1-\frac{1}{16}\right)Q\left(\sqrt{\frac{3P}{N_0\times 255}}\right)=\frac{15}{4}Q\left(\sqrt{\frac{P}{85N_0}}\right)$$

Ans d

13. Condition for deep fade in the wireless channel is

$$a^2<\frac{1}{SNR}\Rightarrow a<\frac{1}{\sqrt{SNR}}$$

Given $SNR=40\ dB=10^4$. Condition is

$$a<\frac{1}{\sqrt{10^4}}=\frac{1}{100}=0.01$$

Ans a

14. To prevent disruption in communication due to a single link in a deep fade one should implement

Ans d

15. The SNR at the output of the MRC beamformer is given as

$$\|\bar{\mathbf{h}}\|^2\frac{P}{N_0}$$

Ans b

16. The MRC beamformer is given as

$$\bar{\mathbf{w}}=\frac{\bar{\mathbf{h}}}{\|\bar{\mathbf{h}}\|}=\frac{1}{\sqrt{2}}\begin{bmatrix}-\sqrt{\frac{1}{2}}-\sqrt{\frac{1}{2}}j\\ -\sqrt{\frac{1}{2}}+\sqrt{\frac{1}{2}}j\end{bmatrix}=\begin{bmatrix}-\frac{1}{2}-\frac{1}{2}j\\ -\frac{1}{2}+\frac{1}{2}j\end{bmatrix}$$

Ans b

17. MIMO Technology is used in 4G LTE, 5G NR, 802.11 ax. Hence, answer is All of these

Ans a

18. Given output vector $\bar{\mathbf{y}}$ and MIMO channel $\mathbf{H}$

$$\bar{\mathbf{y}}=\begin{bmatrix}2\\-3\\-2\\1\end{bmatrix},\mathbf{H}=\begin{bmatrix}1&4\\1&3\\1&1\\1&2\end{bmatrix}$$

The ZF estimate can be evaluated as follows

$$\hat{\mathbf{x}}=(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\bar{\mathbf{y}}$$

$$\mathbf{H}^T\mathbf{H}=\begin{bmatrix}1&1&1&1\\4&3&1&2\end{bmatrix}\begin{bmatrix}1&4\\1&3\\1&1\\1&2\end{bmatrix}=\begin{bmatrix}4&10\\10&30\end{bmatrix}$$

$$(\mathbf{H}^T\mathbf{H})^{-\mathbf{1}}=\frac{1}{20}\begin{bmatrix}30&-10\\-10&4\end{bmatrix}$$

$$\hat{\mathbf{x}}=(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\bar{\mathbf{y}}$$

$$=\frac{1}{20}\begin{bmatrix}30 & -10\\ -10 & 4\end{bmatrix}\begin{bmatrix}1 & 1 & 1 & 1\\ 4 & 3 & 1 & 2\end{bmatrix}\begin{bmatrix}2\\ -3\\ -2\\ 1\end{bmatrix}$$

$$=\frac{1}{20}\begin{bmatrix}-10 & 0 & 20 & 10\\ 6 & 2 & -6 & -2\end{bmatrix}\begin{bmatrix}2\\ -3\\ -2\\ 1\end{bmatrix}=\frac{1}{20}\begin{bmatrix}-50\\ 16\end{bmatrix}=\frac{1}{10}\begin{bmatrix}-25\\ 8\end{bmatrix}$$

Ans a

19. The maximum rate of transmission for the ith MIMO mode is given as

$$\log_2\left(1+\sigma_i^2\times\frac{P_i}{N_0}\right)$$

Ans b

20. The matrix **U** contains **eigenvectors** of $\mathbf{HH}^H$

Ans c

21. The vector transmitted in the first time instant in the Alamouti code is given as

$$\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}1-2j\\ -3+j\end{bmatrix}$$

The vector transmitted in the second time instant is given as

$$\begin{bmatrix}-x_2^*\\ x_1^*\end{bmatrix}=\begin{bmatrix}3+j\\ 1+2j\end{bmatrix}$$

Ans c

22. The coefficient X_l can be extracted from the multi-carrier modulated signal as

$$f_0\int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} y(t)e^{-j2\pi l f_0 t}dt$$

Ans c

23. Given an OFDM system with number of subcarriers $N=512$ over a bandwidth 10 MHz with 25% CP. The duration of the OFDM symbol, after the addition of the CP, is

$$\frac{N}{B}+0.25\times\frac{N}{B}=1.25\times\frac{512}{10\times 10^6}=64\ \mu s$$

Ans d

24. Typical coherence bandwidth of the channel is approximately $200-300\ kHz$

Ans c

25. The Gaussian PDF with mean $\mu=2$ and variance $\sigma^2=2$ is

$$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(n-\mu)^2}{2\sigma^2}}=\frac{1}{\sqrt{4\pi}}e^{-\frac{(n-2)^2}{4}}$$

Ans a

26. Given $P=15\ dB$ and $\frac{N_0}{2}=6\ dB\Rightarrow N_0\approx 9\ dB$. SNR for BPSK modulation is approximately

$$\frac{P}{N_0/2}=15\ dB-6\ dB=9\ dB$$

Ans c

27. The SER of QPSK for $SNR=12\ dB=16$ can be evaluated as follows

$$SER=2\times Q\left(\sqrt{SNR}\right)=2Q(4)$$

Ans b

28. In 256 $-$QAM the number of bits per symbol is $\log_2 256=8$

Hence, number of bits per in-phase symbol is $\frac{8}{2} = 4$

Ans d

29. Overall SER of 64 −QAM in a Rayleigh fading wireless channel is

$$4\left(1 - \frac{1}{\sqrt{M}}\right) \times \frac{1}{2} \times \frac{(M-1)}{3 \times SNR} = 4\left(1 - \frac{1}{8}\right) \times \frac{1}{2} \times \frac{63}{3 \times SNR}$$
$$= \frac{7}{2} \times \frac{21}{2} \times \frac{1}{SNR} = \frac{147}{4 \times SNR}$$

Given $SNR = 147 \times 10^5$

$$SER = \frac{147}{4 \times 147 \times 10^5} = \frac{1}{4 \times 10^5} = 2.5 \times 10^{-6}$$

Ans d

30. Given $SNR = 40\ dB = 10^4$. The probability of deep fade P_{DF} in the Rayleigh fading wireless channel is

$$1 - e^{-\frac{1}{SNR}} = 1 - e^{-\frac{1}{10^4}} = 1 - e^{-0.0001}$$

Ans a

31. Given

$$\bar{\mathbf{h}} = \begin{bmatrix} -\sqrt{2} - \sqrt{2}j \\ -\sqrt{2} + \sqrt{2}j \end{bmatrix}$$

SNR = 9 dB = 8. Hence, output SNR is

$$SNR_o = 8 \times 8 = 18\ dB$$

Ans c

32. BER for a SIMO system with $L = 2$ antennas for $SNR = 27\ dB = 30 - 3\ dB = 5 \times 10^2$ is given as

$${}^3C_2 \times \frac{1}{2^2} \times \frac{1}{SNR^2} = \frac{3}{4} \times \frac{1}{SNR^2} = \frac{3}{4} \times \frac{1}{25 \times 10^4} = 3 \times 10^{-6}$$

Ans a

33. At $f_c = 6\ GHz$, the minimum antenna spacing is

$$\frac{\lambda}{2} = \frac{1}{2} \times \frac{3 \times 10^8}{6 \times 10^9} = 0.25 \times 10^{-1} m = 2.5\ cm$$

Ans b

34. Given the output vector $\bar{\mathbf{y}}$ and MIMO channel $\mathbf{H}$ and $SNR = -12\ dB = \frac{1}{16}$

$$\bar{\mathbf{y}} = \begin{bmatrix} 2 \\ -3 \\ -1 \\ 2 \end{bmatrix}, \mathbf{H} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \\ -1 & -1 \\ 1 & -1 \end{bmatrix}$$

At SNR The LMMSE estimate can be evaluated as shown below

$$\hat{\mathbf{x}} = \left(\mathbf{H}^T\mathbf{H} + \frac{1}{\text{SNR}}\mathbf{I}\right)^{-1} \mathbf{H}^T\bar{\mathbf{y}}$$

$$\mathbf{H}^T\mathbf{H} = \begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 1 \\ -1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\mathbf{H}^T\mathbf{H} + \frac{1}{\text{SNR}}\mathbf{I} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + 16\mathbf{I} = 20\mathbf{I}$$

$$\left(\mathbf{H}^T\mathbf{H}+\frac{1}{\text{SNR}}\mathbf{I}\right)^{-1}\mathbf{H}^T\bar{\mathbf{y}}=\frac{\mathbf{1}}{20}\mathbf{I}\times\begin{bmatrix}-1 & 1 & -1 & 1\\ 1 & 1 & -1 & -1\end{bmatrix}\begin{bmatrix}2\\-3\\-1\\2\end{bmatrix}$$

$$=\frac{1}{20}\begin{bmatrix}-2\\-2\end{bmatrix}=-\frac{1}{10}\begin{bmatrix}1\\1\end{bmatrix}$$

Ans b

35. Given SVD

$$\begin{bmatrix}\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2}\\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{bmatrix}\begin{bmatrix}-3 & 0\\ 0 & -6\\ 0 & 0\\ 0 & 0\end{bmatrix}\underbrace{\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}}_{\mathbf{V}^T}$$

At the receiver, we multiply the signal with **V**

$$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

Ans c

36. Given the MIMO channel with SVD below

$$\begin{bmatrix}\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2}\\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & \frac{1}{2\sqrt{2}} & 0\\ 0 & 0 & \frac{1}{4}\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{bmatrix}$$

The total power $P_T=12\,dB=16$ and noise power $N_0=0\,dB=1$. The optimal power values can be found as follows

$$P_1=\left(\frac{1}{\lambda}-\frac{N_0}{\sigma_j^2}\right)^+=\left(\frac{1}{\lambda}-\frac{1}{1}\right)^+=\left(\frac{1}{\lambda}-1\right)^+$$

$$P_2=\left(\frac{1}{\lambda}-\frac{1}{1/8}\right)^+=\left(\frac{1}{\lambda}-8\right)^+$$

$$P_3=\left(\frac{1}{\lambda}-\frac{1}{1/16}\right)^+=\left(\frac{1}{\lambda}-16\right)^+$$

Assume $\frac{1}{\lambda}\geq 16$

$$\frac{1}{\lambda}-1+\frac{1}{\lambda}-8+\frac{1}{\lambda}-16=16$$

$$\frac{1}{\lambda}=\frac{41}{3}\approx 14$$

$$14-16=-2\Rightarrow P_3=0$$

Assume $8\leq\frac{1}{\lambda}<16$

$$\frac{1}{\lambda}-1+\frac{1}{\lambda}-8=16$$

$$\frac{1}{\lambda} = \frac{25}{2}$$
$$P_2 = \frac{25}{2} - 8 = \frac{9}{2}$$
$$P_1 = \frac{1}{\lambda} - 1 = \frac{25}{2} - 1 = \frac{23}{2}$$

Ans a

37. Given the channel coefficients $h_1 = -1 - 2j,\ h_2 = 2 - j$. The Alamouti matrix is given as

$$\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} = \begin{bmatrix} -1 - 2j & 2 - j \\ 2 + j & 1 - 2j \end{bmatrix}$$

Ans c

38. Given bandwidth $B = 20\ MHz$ and number of subcarriers $N = 500$. The sampling rate of the OFDM system is $B = 20\ MHz$

Ans d

39. Given an $N = 4$ subcarrier OFDM system with symbols loaded on subcarriers given as

$$X_0 = -2, X_1 = 2j, X_2 = 2, X_3 = 2j$$

The time-domain sample $x(1)$ is given as

$$x(l) = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi\frac{kl}{N}}$$
$$x(2) = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi\frac{k3}{4}} = \frac{1}{4}\sum_{k=0}^{3} X_k e^{j\frac{3\pi}{2}k} = \frac{1}{4}(X_0 - jX_1 - X_2 + jX_3)$$
$$= \frac{1}{4}\big(-2 - j(2j) - 2 + j(2j)\big) = \frac{1}{4}(-2 + 2 - 2 - 2) = -1$$

Ans d

40. Given a vehicle moving at 36 km per hour at an angle of $\theta = 60^{\mathrm{o}}$. The carrier frequency is $f_c = 3.0\ GHz$. The Doppler shift of the signal is given as

$$f_D = \frac{v\cos\theta}{c} f_c$$
$$= 36 \times \frac{\frac{5}{18}}{3 \times 10^8} \times \frac{1}{2} \times 3 \times 10^9 = 50\ Hz$$
$$T_c = \frac{1}{4f_D} = \frac{1}{200} = 5\ ms$$

Ans b