EE900 - Quiz-1 Date: 23-7-202

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Solution (1)

~~Q1 solution~~ Q1 Solution at the end

~~(i) plane~~ ~~(ii) Hyperplane~~

Q2 solution

$$\|a_1\| = \sqrt{1+4+9} = \sqrt{14} = \sqrt{14}$$

$$\|a_2\| = \sqrt{1+0+25} = \sqrt{26}$$

$$\|a_3\| = \sqrt{0+4+16} = \sqrt{20} = 2\sqrt{5}$$

$$a_1 \cdot a_3 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix} = 0 - 4 + 12 = 8$$

$$a_2 \cdot (a_1 - a_3) = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} 1-0 \\ -2-2 \\ 3-4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -4 \\ -1 \end{bmatrix}$$

$$= 1 - 4 - 5 = -8$$

$$a_3 \cdot (2a_1 - 3a_2) = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 2-3 \\ -4-0 \\ 6-15 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -4 \\ -9 \end{bmatrix}$$

$$= 0 - 8 - 36 = -44$$

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Q3 Solution

Linear combination can be written as

$$x_1 a_1 + x_2 a_2 + x_3 a_3 = b \quad (1)$$

$\therefore$ ~~considering~~ $\cancel{X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}$

(1) can be written as

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 5 & -2 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 3 \end{bmatrix}$$

using elimination methodology (and back substitution)

$$\xrightarrow{r_3 - 2r_1} \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 5 & -2 & -7 \\ 0 & -2 & 1 & -1 \end{bmatrix}$$

$$\xrightarrow[r_3 + \frac{2}{5} r_2]{\cancel{r_3 + 2r_1}} \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 5 & -2 & -7 \\ 0 & 0 & -4/5 & -14/5 \end{bmatrix}$$

$$\therefore \quad -\frac{4}{5} x_3 = \frac{-14}{5}$$

$$\Rightarrow \boxed{x_3 = \frac{14}{4} = \frac{7}{2}}$$

$$5x_2 - 2x_3 = -7$$

$$5x_2 = -7 + 2x_3 = -7 + \cancel{2} \cdot \frac{7}{\cancel{2}} = 0$$

$$\boxed{x_2 = 0}$$

$x_1 + x_2 = 2$

$x_1 + 0 = 2$

$\therefore \boxed{x_1 = 2}$

$\therefore x_1 = 2, \quad x_2 = 0, \quad x_3 = 7/2$

The linear Combination is

$$\boxed{2a_1 + 0a_2 + \frac{7}{2}a_3 = b}$$

Q4 Solution

$Ax = 0$

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & -1 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

using elimination and back Sub stitution methodology

$$\begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -2 & 2 & 0 \end{bmatrix}$$

$$\xrightarrow{r_3 + 2r_2} \begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow \quad x_2 - x_3 = 0 \Rightarrow x_2 = x_3$

$x_1 - 2x_2 = 0 \Rightarrow x_1 = 2x_2$

1~~0~~ $\therefore$ The solution is:

$$x_1 = 2x_2$$

$$x_2 = x_3$$

$\boxed{\begin{array}{l}\text{This equation has infinite solutions} \\ \text{one solution is where } x_3 = 1 \\ \Rightarrow x_2 = 1,\ x_1 = 2 \\ \therefore (x_1, x_2, x_3) = (2, 1, 1)\end{array}}$

## Q5 Solution

(a) $XY = \begin{bmatrix} 1 & 5 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 3 \\ 5 & -10 \end{bmatrix}$$

$YX = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 5 & 0 \end{bmatrix}$

$$= \begin{bmatrix} \cancel{10}\,-9 & 5 \\ 5 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 3 \\ 5 & -10 \end{bmatrix} \neq \begin{bmatrix} -9 & 5 \\ 5 & 0 \end{bmatrix}$$

Hence $XY \neq YX$ is proved

Q5 Solution

(b)

$$(X+Y)^2 \neq (X^2+Y^2+2XY)$$

$$X+Y = \begin{bmatrix}1 & 5\\5 & 0\end{bmatrix} + \begin{bmatrix}1 & -2\\0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}2 & 3\\5 & 1\end{bmatrix}$$

$$(X+Y)^2 = \begin{bmatrix}2 & 3\\5 & 1\end{bmatrix}\begin{bmatrix}2 & 3\\5 & 1\end{bmatrix}$$

$$= \begin{bmatrix}4+15 & 6+3\\10+5 & 15+1\end{bmatrix} = \begin{bmatrix}19 & 9\\15 & 16\end{bmatrix}$$

$$X^2 = \begin{bmatrix}1 & 5\\5 & 0\end{bmatrix}\begin{bmatrix}1 & 5\\5 & 0\end{bmatrix} = \begin{bmatrix}1+25 & 5\\5 & 25\end{bmatrix} = \begin{bmatrix}26 & 5\\5 & 25\end{bmatrix}$$

$$Y^2 = \begin{bmatrix}1 & -2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & -2\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & -4\\0 & 1\end{bmatrix}$$

$$XY = \begin{bmatrix}1 & 5\\5 & 0\end{bmatrix}\begin{bmatrix}1 & -2\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 3\\5 & -10\end{bmatrix}$$

$$\Rightarrow (X^2+Y^2+2XY) = \begin{bmatrix}26 & 5\\5 & 25\end{bmatrix} + \begin{bmatrix}1 & -4\\0 & 1\end{bmatrix} + \begin{bmatrix}2 & 6\\10 & -20\end{bmatrix}$$

$$= \begin{bmatrix}26+1+2 & 5-4+6\\5+0+10 & 25+1-20\end{bmatrix} = \begin{bmatrix}29 & 7\\15 & 6\end{bmatrix}$$

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$$(X+Y)^2 = \begin{bmatrix} 19 & 9 \\ 15 & 16 \end{bmatrix}$$

$$(X^2+Y^2+2XY) = \begin{bmatrix} 29 & 7 \\ 15 & 6 \end{bmatrix}$$

$$\therefore (X+Y)^2 \neq (X^2+Y^2+2XY)$$

Hence proved

Q1 Solution

i) The $2^{nd}$ vector is a linear combination of first and third vector. Hence these vectors are linearly dependent and not independent and span $\mathbb{R}^2$

$$v_2 = -(v_1 + v_3)$$

ii) The linear combination span $\mathbb{R}^4$ and represent hyperplane