Live Interaction #2:

21st January 2024

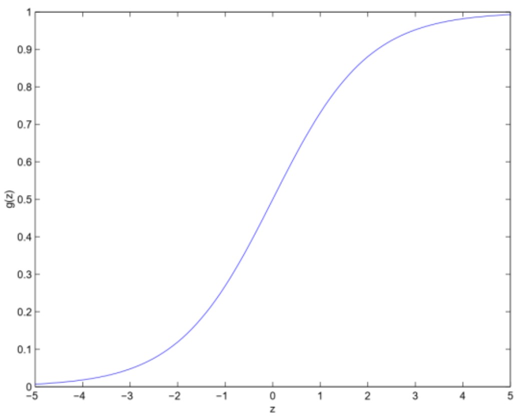
E-masters Next Generation Wireless Technologies

EE902 Advanced ML Techniques for Wireless Technology

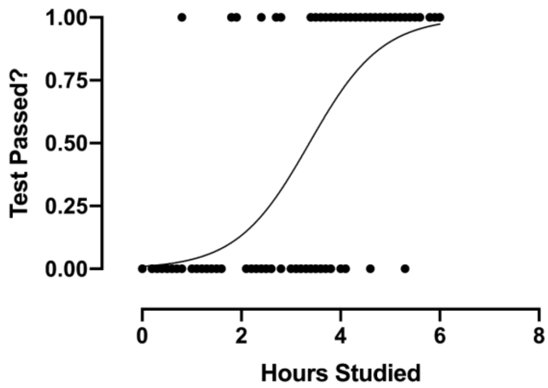
- **Logistic regression:**
- **Logistic function:**

$$f(z) = \frac{1}{1 + e^{-z}}$$

- $z \to -\infty, f(z) \to 0$
- $z \to \infty, f(z) \to 1$



- Linear regression: **continuous output**.
- Logistic regression: **Discrete output**.
- Application:



- Regressors: $x_1, x_2, \ldots, x_N$
- Regression coefficients: $h_0, h_1, h_2, \ldots, h_N$.

$$p(y = 1|\bar{\mathbf{x}}) = \frac{1}{1 + e^{-\bar{\mathbf{x}}^T \bar{\mathbf{h}}}}$$

$$p(y = 0|\bar{\mathbf{x}}) = 1 - \frac{1}{1 + e^{-\bar{\mathbf{x}}^T \bar{\mathbf{h}}}}$$

$$= \frac{e^{-\bar{\mathbf{x}}^T \bar{\mathbf{h}}}}{1 + e^{-\bar{\mathbf{x}}^T \bar{\mathbf{h}}}}$$

- What to learn?
- $h_0, h_1, \ldots, h_N$
- Start with:
- Training data

$$\begin{gathered} \bar{\mathbf{x}}(1), y(1) \\ \bar{\mathbf{x}}(2), y(2) \\ \vdots \\ \bar{\mathbf{x}}(k), y(k) \\ \vdots \\ \bar{\mathbf{x}}(M), y(M) \end{gathered}$$

- Likelihood:

$$p\big(y(k)|\bar{\mathbf{x}}(k)\big) = \left(\frac{1}{1+e^{-\bar{\mathbf{x}}^T\bar{\mathbf{h}}}}\right)^{y(k)} \left(\frac{e^{-\bar{\mathbf{x}}^T\bar{\mathbf{h}}}}{1+e^{-\bar{\mathbf{x}}^T\bar{\mathbf{h}}}}\right)^{1-y(k)}$$

- Joint PDF:

$$p(\bar{\mathbf{y}};\bar{\mathbf{h}}) = \prod_{k=1}^{M} \left(\frac{1}{1+e^{-\bar{\mathbf{x}}^T\bar{\mathbf{h}}}}\right)^{y(k)} \left(\frac{e^{-\bar{\mathbf{x}}^T\bar{\mathbf{h}}}}{1+e^{-\bar{\mathbf{x}}^T\bar{\mathbf{h}}}}\right)^{1-y(k)}$$

$$= \prod_{k=1}^{M} \big(g(\bar{\mathbf{x}})\big)^{y(k)} \big(1-g(\bar{\mathbf{x}})\big)^{1-y(k)}$$

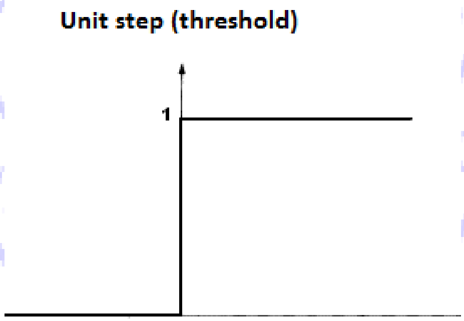
- Log-likelihood:

$$\ln p(\bar{\mathbf{y}};\bar{\mathbf{h}}) = \sum_{k=1}^{M} y(k)\ln\big(g(\bar{\mathbf{x}})\big) + \big(1-y(k)\big)\ln\big(1-g(\bar{\mathbf{x}})\big)$$

- Maximize the log-likelihood
- **Gradient ascent**: Update rule
  - $$\underbrace{\bar{\mathbf{h}}(k+1) = \bar{\mathbf{h}}(k) + \eta e(k+1)\bar{\mathbf{x}}(k+1)}_{\text{LMS update rule}}$$

$$e(k+1) = y(k+1) - g\big(\bar{\mathbf{x}}(k+1)\big)$$

- **Perceptron learning model**:



$$g(\bar{\mathbf{x}}) = \begin{cases} 1 & \bar{\mathbf{h}}^T\bar{\mathbf{x}} \geq 0 \\ 0 & \bar{\mathbf{h}}^T\bar{\mathbf{x}} < 0 \end{cases}$$

$$f(z) = \frac{1}{1+e^{-az}} \rightarrow \text{unit} - \text{step}$$

- Gradient ascent rule:

$$\bar{\mathbf{h}}(k+1) = \bar{\mathbf{h}}(k) + \eta e(k+1)\bar{\mathbf{x}}(k+1)$$

- **Assignment #2 deadline: 26th January 11:59 PM**.
- **Assignments 1, 2 discussion: 27th January 3-4 PM**.
- **Quiz #1: 27th January 4:30 – 5:30 PM**.