Live Interaction #3:

19th October 2023

E-masters Communication Systems

Detection for Wireless

- **Neyman-Pearson Criterion**:
- Optimal detector:
- Maximizes the P_D for a given P_{FA}.
- R_1 is the region corresponding to deciding H_1.

$$P_D = \int_{R_1} p(\bar{\mathbf{y}}; \mathcal{H}_1) d\bar{\mathbf{y}}$$

$$P_{MD} = \int_{R_0} p(\bar{\mathbf{y}}; \mathcal{H}_1) d\bar{\mathbf{y}}$$

$$P_{FA} = \int_{R_1} p(\bar{\mathbf{y}}; \mathcal{H}_0) d\bar{\mathbf{y}}$$

$$\max P_D$$

$$\text{s.t.} \; P_{FA} \leq \alpha$$

- **Lagrange multiplier**.

$$P_D + \lambda(\alpha - P_{FA})$$

- Optimal detector that maximizes P_D for a given P_{FA}

$$\Rightarrow \quad \underbrace{\frac{p(\bar{\mathbf{y}};\mathcal{H}_1)}{p(\bar{\mathbf{y}};\mathcal{H}_0)} \geq \alpha}_{\textbf{Likelihood Ratio Test (LRT)}}$$

- Signal detection:

$$P_{FA} = Q\left(\frac{\gamma}{\sigma\|\bar{\mathbf{s}}\|}\right) = \alpha$$

$$P_D = Q\left(\frac{\gamma - \|\bar{\mathbf{s}}\|^2}{\sigma\|\bar{\mathbf{s}}\|}\right)$$

$$\gamma = \sigma\|\bar{\mathbf{s}}\|Q^{-1}(\alpha)$$

$$P_D = Q\left(\frac{\sigma\|\bar{\mathbf{s}}\|Q^{-1}(\alpha) - \|\bar{\mathbf{s}}\|^2}{\sigma\|\bar{\mathbf{s}}\|}\right)$$

- **Generalized signal detection**:

$$\mathcal{H}_0: \bar{\mathbf{y}} = \bar{\mathbf{s}}_0 + \bar{\mathbf{v}}$$

$$\mathcal{H}_1: \bar{\mathbf{y}} = \bar{\mathbf{s}}_1 + \bar{\mathbf{v}}$$

- Optimal detector:
- Choose $\mathcal{H}_0$ if

$$\underbrace{(\bar{\mathbf{s}}_1 - \bar{\mathbf{s}}_0)^T\bar{\mathbf{y}} \leq \gamma}_{\text{Test statistic}}$$

$$P_{FA} = Q\left(\frac{\gamma}{\sigma\|\bar{\mathbf{s}}_1 - \bar{\mathbf{s}}_0\|}\right)$$

$$P_D = Q\left(\frac{\gamma - \|\bar{\mathbf{s}}_1 - \bar{\mathbf{s}}_0\|^2}{\sigma\|\bar{\mathbf{s}}_1 - \bar{\mathbf{s}}_0\|}\right)$$

$$P_e = Q\left(\frac{\|\bar{\mathbf{s}}_1 - \bar{\mathbf{s}}_0\|}{2\sigma}\right) = Q\left(\frac{d}{2\sigma}\right)$$

- **BPSK: Binary Phase Shift Keying**.

$$\mathcal{H}_0 : y = -A + v$$

$$\mathcal{H}_1 : y = A + v$$

- $E\{v^2\} = \sigma$

$$\underbrace{P_e = Q\left(\frac{A}{\sigma}\right) = Q\left(\frac{\sqrt{E_b}}{\sqrt{\frac{N_0}{2}}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}_{BPSK}$$

$$\underbrace{P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)}_{ASK}$$

- BPSK is **3 dB more efficient** than ASK.
- **Multiple Hypothesis Testing**.

$$\mathcal{H}_0 : \bar{\mathbf{y}} = \bar{\mathbf{s}}_0 + \bar{\mathbf{v}}$$

$$\mathcal{H}_1 : \bar{\mathbf{y}} = \bar{\mathbf{s}}_1 + \bar{\mathbf{v}}$$

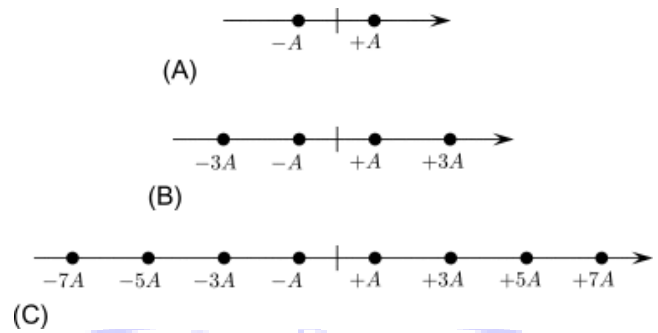
$$\vdots$$

$$\mathcal{H}_{L-1} : \bar{\mathbf{y}} = \bar{\mathbf{s}}_{L-1} + \bar{\mathbf{v}}$$

- How to decide? What is the ***detection rule***?

$$\text{choose } \mathcal{H}_j : j = \underset{i}{\text{argmin}} \|\bar{\mathbf{y}} - \bar{\mathbf{s}}_i\|$$

- **Minimum distance decoder**.
- $M-ary$ PAM



- Probability of error for interior point

$$2Q\left(\frac{A}{\sigma}\right)$$

- Probability of error for boundary point

$$Q\left(\frac{A}{\sigma}\right)$$

- **Probability of error**:

$$\underbrace{2\left(1-\frac{1}{M}\right)Q\left(\frac{A}{\sigma}\right)=2\left(1-\frac{1}{M}\right)Q\left(\frac{A}{\sqrt{N_0/2}}\right)}_{\text{Symbol Error Rate (SER)}}$$

- **Assignment 3 deadline 21st October 11:59 PM**.
- **Live interaction Friday 27th October 2023: 9-10 PM**.
- **Assignment 4 deadline 28th October 11:59 PM**
- **Assignment 3, 4 discussion 30th October 8:30 PM.**
- **Quiz #2 30th October Monday 9:00 PM.**