Introduce new variables

$$\min_{x} f_0(\underline{x}) \quad \Longleftrightarrow \quad \min_{x,t} t$$
$$f_i(x) \leq 0 \qquad\qquad f_0(x) \leq t$$

epigraph trick $\quad f_i(x) \leq 0$

Note when $f_0(x)$ convex then

$$\underbrace{f_0(x)}_{\text{convex}} - \underbrace{t}_{\text{affine}} \text{ also convex in } (\underline{x}, t)$$

Slack variables

$$a_i^T x - b_i \leq 0 \quad \Longleftrightarrow \quad a_i^T x - b_i + s_i = 0$$
$$s_i \geq 0$$

Equality constraints can be eliminated

$$\min_{x_1, x_2} f_1(x_1) + f_2(x_2) \quad \Longleftrightarrow \quad \min_{\theta} f_1(\theta) + f_2(1-\theta)$$
$$x_1 + x_2 = 1$$

$$x_1 = \theta$$
$$x_2 = 1 - \theta$$

General case

$$\min f(x)$$
$$Ax = b$$

$b \in \mathbb{R}^m \quad m < n$

$A \in \mathbb{R}^{m \times n}$

- suppose that $b \in R(A)$

(solution exists)

so $\exists\, x_0$ s.t. $Ax_0 = b$

feasible region : $\mathcal{X} = \{x \mid Ax = b\}$

$$x = x - x_0 + x_0$$
$$Ax = A(x - x_0) + Ax_0 = b$$
$$\Rightarrow \quad A(x - x_0) = 0$$
$$\Rightarrow \quad x - x_0 \in N(A)$$

$r = \text{rank}(A)$

suppose $N(A)$ has basis vectors $c_1, c_2 \dots c_{m-r}$

then $x - x_0 = Cu \quad u \in \mathbb{R}^{m-r}$

so

$$\min_{u} f(Cu + x_0)$$

Note : problem complexity may not reduce