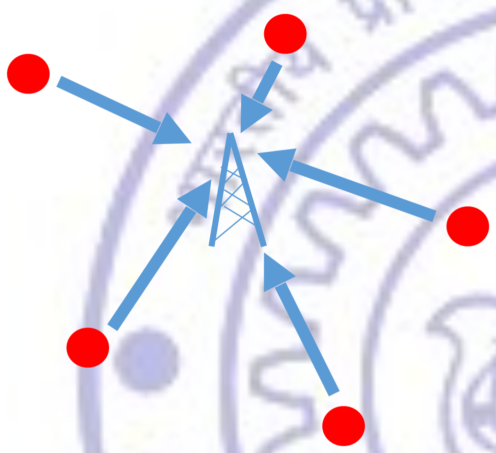
Live Interaction #1:

1st October 2023

E-masters Communication Systems

Estimation for Wireless

- Example



- Model for measurements:

$$y(1) = h + v(1)$$
$$y(2) = h + v(2)$$
$$\vdots$$
$$y(N) = h + v(N)$$

- h: unknown parameter
- $v(k)$ Gaussian noise, mean 0, variance σ^2
- What is the PDF of $y(k)$?

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(y(k)-h\right)^2}{2\sigma^2}}$$

- Joint PDF of $y(1), y(2), \ldots, y(N)$

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(y(1)-h\right)^2}{2\sigma^2}} \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(y(2)-h\right)^2}{2\sigma^2}} \times \ldots$$

$$\times \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(y(N)-h\right)^2}{2\sigma^2}}$$

$$\underbrace{p\left(\bar{\mathbf{y}}; h\right) = \left(\frac{1}{2\pi\sigma^2}\right)^{N/2} \times e^{-\frac{1}{2\sigma^2}\sum_{k=1}^{N}\left(y(k)-h\right)^2}}_{\textbf{Likelihood function}}$$

- How to determine the unknown parameter h?

$$\underbrace{\hat{h} = \operatorname{argmax} p\left(\bar{\mathbf{y}}; h\right)}_{\textbf{Maximim Likelihood (ML)}}$$

$$\min \sum_{k=1}^{N} \left(y(k) - h\right)^2$$

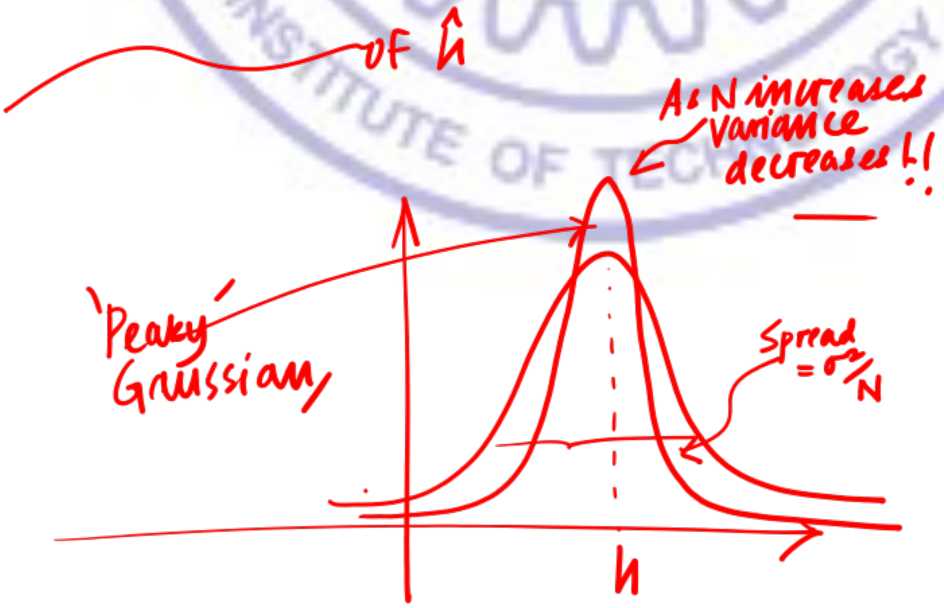
$$\underbrace{\hat{h} = \frac{1}{N}\sum_{k=1}^{N} y(k)}_{\text{ML Estimate}}$$

Sample mean

- Properties of ML Estimate:

$$\underbrace{E\left\{\hat{h}\right\} = h}_{\textbf{Unbiased estimator}}$$

$$E\left\{\left(\hat{h} - h\right)^2\right\} = \frac{\sigma^2}{N}$$



- $\hat{h}$ is Gaussian, with mean h and variance $\frac{\sigma^2}{N}$

Wireless communication:

- Channel estimation:
- For the purpose of channel estimation, we have to transmit **known symbols** – These are termed as **PILOT SYMBOLS**.

$$y(1) = hx(1) + v(1)$$
$$y(2) = hx(2) + v(2)$$
$$\vdots$$
$$y(N) = hx(N) + v(N)$$

- PDF

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(y(k) - hx(k)\right)^2}{2\sigma^2}}$$

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y(1) - hx(1))^2}{2\sigma^2}} \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y(2) - hx(2))^2}{2\sigma^2}} \times \ldots$$

$$\times \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y(N) - hx(N))^2}{2\sigma^2}}$$

$$\underbrace{p(\bar{\mathbf{y}}; h) = \left(\frac{1}{2\pi\sigma^2}\right)^{N/2} \times e^{-\frac{1}{2\sigma^2}\sum_{k=1}^{N}\left(y(k) - hx(k)\right)^2}}_{\textbf{Likelihood function}}$$

$$\min \sum_{k=1}^{N} \left(y(k) - hx(k)\right)^2$$

$$\hat{h} = \frac{\sum_{k=1}^{N} x(k)y(k)}{\sum_{k=1}^{N} x^2(k)} = \frac{\bar{\mathbf{x}}^T\bar{\mathbf{y}}}{\bar{\mathbf{x}}^T\bar{\mathbf{x}}}$$

$$E\left\{\hat{h}\right\} = h$$

$$E\left\{\left(\hat{h} - h\right)^2\right\} = \frac{\sigma^2}{\|\bar{\mathbf{x}}\|^2}$$

- Problem:

$$\bar{\mathbf{x}} = \begin{bmatrix} -2 \\ 3 \\ 1 \\ -2 \end{bmatrix}, \bar{\mathbf{y}} = \begin{bmatrix} -1 \\ 3 \\ -3 \\ 2 \end{bmatrix}, \sigma^2 = 2$$

- Calculate the estimate and MSE?

$$\text{Variance} = E\left\{\left(\hat{h} - E\left\{\hat{h}\right\}\right)^2\right\}$$

$$\text{MSE} = E\left\{\left(\hat{h} - h\right)^2\right\}$$

$$\hat{h} = \frac{\bar{\mathbf{x}}^T \bar{\mathbf{y}}}{\bar{\mathbf{x}}^T \bar{\mathbf{x}}} = \frac{4}{18} = \frac{2}{9}$$

$$\text{MSE} = \frac{2}{18} = \frac{1}{9}$$