1. The SVD of a matrix $\mathbf{H}$ is given as $\mathbf{U\Sigma V}^H$
   Ans a
2. The matrices $\mathbf{U}, \mathbf{V}$ in the SVD are Unitary
   Ans a
3. The SVD exists For any matrix
   Ans b
4. The matrix $\mathbf{U}$ contains **eigenvectors** of $\mathbf{HH}^H$
   Ans d
5. Given the decomposition below

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -3 & 0 \\ 0 & -6 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

   This is not a valid SVD since the singular values $-3, -6$ are negative
   Ans c
6. In SVD processing, at the transmitter, the symbol vector is processed a the receiver by multiplying with $\mathbf{U}^H$
   Ans c
7. The maximum rate of transmission for the ith MIMO mode is given as

$$\log_2\left(1 + \sigma_i^2 \times \frac{P_i}{N_0}\right)$$

   Ans b
8. The optimal power P_j to be allocated to the ith MIMO mode is given as

$$\left(\frac{1}{\lambda} - \frac{N_0}{\sigma_j^2}\right)^+$$

   Ans c
9. Given the MIMO channel with SVD below

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2\sqrt{2}} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

   The total power $P_T = 6\ dB\ =\ 4$ and noise power $N_0 = 6\ dB = 4$. The optimal power values can be found as follows

$$P_1 = \left(\frac{1}{\lambda} - \frac{N_0}{\sigma_j^2}\right)^+ = \left(\frac{1}{\lambda} - \frac{4}{1/2}\right)^+ = \left(\frac{1}{\lambda} - 8\right)^+$$

$$P_2 = \left(\frac{1}{\lambda} - \frac{4}{1/4}\right)^+ = \left(\frac{1}{\lambda} - 16\right)^+$$
$$P_3 = \left(\frac{1}{\lambda} - \frac{4}{1/8}\right)^+ = \left(\frac{1}{\lambda} - 32\right)^+$$

Assume $\frac{1}{\lambda} \geq 32$

$$\frac{1}{\lambda} - 8 + \frac{1}{\lambda} - 16 + \frac{1}{\lambda} - 32 = 16$$
$$\frac{1}{\lambda} = \frac{72}{3} = 24$$
$$24 - 32 = -8 \Rightarrow P_3 = 0$$

Assume $\frac{1}{\lambda} \geq 16$

$$\frac{1}{\lambda} - 8 + \frac{1}{\lambda} - 16 = 16$$
$$\frac{1}{\lambda} = \frac{40}{2} = 20$$
$$P_2 = \frac{1}{\lambda} - 16 = 20 - 16 = 4$$
$$P_1 = \frac{1}{\lambda} - 8 = 20 - 8 = 12$$

Ans b

10. Alamouti scheme is a Space-Time Block Code

Ans d