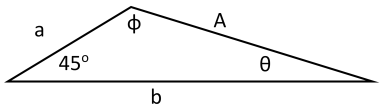
Assignment 3 Solution

Digital Communication System-II

May 27, 2023

1. (a) Consider the QAM constellation shown in the figure. Using the Pythagorean theorem we can find the radius of the inner circle as:

$$a^2 + a^2 = A^2 \Longrightarrow a = \frac{1}{\sqrt{2}}A$$



The radius of the outer circle can be found using the law of sines.

$$\begin{aligned}
\frac{A}{\sin 45^\circ} &= \frac{a}{\sin\theta} = \frac{b}{\sin\phi} \\
\frac{\sqrt{2}\alpha}{\left(\frac{1}{\sqrt{2}}\right)} &= \frac{a}{\sin\theta} \\
2 &= \frac{1}{\sin\theta} \\
\sin\theta &= \frac{1}{2} \\
\theta &= 30^\circ \\
\phi &= 180 - 45 - 30 \\
&= 105^\circ \\
b &= \left(\frac{A}{1/\sqrt{2}}\right)\sin 105^\circ \\
&= \sqrt{2}A\sin 105^\circ \\
&= \sqrt{2}A\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) \\
&= \frac{1+\sqrt{3}}{2}A
\end{aligned}$$

2. (b) Although it is possible to assign three bits to each point of the 8-PSK signal constellation so that adjacent points differ in only one bit, (e.g. going in a clockwise direction : 000, 001, 011, 010, 110, 111, 101, 100). this is not the case for the 8-QAM constellation. This is because there are fully connected graphs consisted of three points. To see this consider an equilateral triangle with vertices A, B and C. If, without loss of generality, we assign the all zero sequence $\{0, 0, \ldots, 0\}$ to point A, then point B and C should have the form

$$\mathrm{B} = \{0, \ldots, 0, 1, 0, \ldots, 0\} \quad \mathrm{C} = \{0, \ldots, 0, 1, 0, \ldots, 0\}$$

where the position of the 1 in the sequences is not the same, otherwise B = C. Thus, the sequences of B and C differ in two bits.

3. (b) Since each symbol conveys 3 bits of information, the resulted symbol rate is :

$$R_s = \frac{120 \times 10^6}{3} = 40 \times 10^6 \text{ symbols /sec}$$

4. (c) The minimum distance of the constellation for $M = 8$ is

$$d_{\min} = \sqrt{\frac{12}{M^2 - 1} E_{\text{avg}}} = \sqrt{\frac{12}{8^2 - 1} E_{\text{avg}}} = \sqrt{\frac{4}{21} E_{\text{avg}}}.$$

5. (b) Only the constellation diagrams for 4-ary QAM and QPSK are same.

6. (b)

$$f_2 - f_1 = \frac{k}{2T_b}$$
$$20\text{KHz} = \frac{k}{2T_b}$$
$$(40\text{KHz})\, T_b = k$$
$$k \text{ should be integer}$$

For option (b), k is integer.

7. (d) Please see the lecture slides.

8. (d) They are not orthogonal signals, e.g s_1 and s_3 are not orthogonal signals. So, they are clearly not bi-orthogonal signals which consists of orthogonal signals and their negatives. They are also not simplex signals which are obtained from orthogonal signals by subtracting the mean. Hence the correct option is (d).

9. (b) The cross-correlation of any pair of orthogonal signals in a simplex signalling with $M = 8$ is, $\rho = \frac{-1}{M-1}$. For $M = 8$, $\rho = \frac{-1}{7}$.

10. (b) The minimum distance for any M-ary PSK is $d_{min} = \sqrt{2\left(\sin^2 \frac{\pi}{M}\right) E_g}$.

For $M = 2$, $d_{min} = \sqrt{2E_g}$.

For $M = 4$, $d_{min} = \sqrt{E_g}$.

For $M = 8$, $d_{min} = 0.5412\sqrt{E_g}$.

For $M = 16$, $d_{min} = 0.2759\sqrt{E_g}$.