1. At $f_c = 10\ GHz$, the minimum antenna spacing is
$$\frac{\lambda}{2} = \frac{1}{2} \times \frac{3 \times 10^8}{10 \times 10^9} = 1.5\ cm$$
Ans d
2. MIMO Technology is used in 4G LTE, 5G NR, 802.11 ax. Hence, answer is All of the above
Ans d
3. MIMO can lead to a significant increase in the data rate of a system via Spatial Multiplexing
Ans c
4. In the MIMO channel matrix, h_{45} denotes the channel between 4th receive antenna and 5th transmit antenna
Ans d
5. Inverse of a matrix exists for Only Non-singular square matrices
Ans b
6. Given the channel
$$\mathbf{H} = \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$
Its inverse can be evaluated as follows
$$\mathbf{H}^{-1} = \frac{1}{2}\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$
Ans a
7. The solution to the least squares (LS) problem
$$\min \|\bar{\mathbf{y}} - \mathbf{H}\bar{\mathbf{x}}\|^2$$
for a complex matrix **H** is is given as
$$\hat{\mathbf{x}} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\bar{\mathbf{y}}$$
Ans d
8. Given output vector $\bar{\mathbf{y}}$ and MIMO channel **H**
$$\bar{\mathbf{y}} = \begin{bmatrix} 2 \\ -3 \\ -2 \\ 1 \end{bmatrix}, \mathbf{H} = \begin{bmatrix} 1 & 4 \\ 1 & 3 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$$
The ZF estimate can be evaluated as follows
$$\hat{\mathbf{x}} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\bar{\mathbf{y}}$$
$$\mathbf{H}^T\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 4 & 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 1 & 3 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ 10 & 30 \end{bmatrix}$$
$$(\mathbf{H}^T\mathbf{H})^{-1} = \frac{1}{20}\begin{bmatrix} 30 & -10 \\ -10 & 4 \end{bmatrix}$$
$$\hat{\mathbf{x}} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\bar{\mathbf{y}}$$
$$= \frac{1}{20}\begin{bmatrix} 30 & -10 \\ -10 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 4 & 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ -3 \\ -2 \\ 1 \end{bmatrix}$$

$$= \frac{1}{20}\begin{bmatrix} -10 & 0 & 20 & 10 \\ 6 & 2 & -6 & -2 \end{bmatrix}\begin{bmatrix} 2 \\ -3 \\ -2 \\ 1 \end{bmatrix} = \frac{1}{20}\begin{bmatrix} -50 \\ 16 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} -25 \\ 8 \end{bmatrix}$$

Ans c

9. For a complex MIMO channel matrix **H**, the LMMSE receiver is given as

$$\hat{\mathbf{x}} = \left(\mathbf{H}^H\mathbf{H} + \frac{1}{\mathrm{SNR}}\mathbf{I}\right)^{-1}\mathbf{H}^H\bar{\mathbf{y}}$$

Ans b

10. Given the output vector $\bar{\mathbf{y}}$ and MIMO channel **H** and $SNR = -6\ dB = \frac{1}{4}$

$$\bar{\mathbf{y}} = \begin{bmatrix} -3 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \mathbf{H} = \begin{bmatrix} -1 & 1 \\ -1 & 1 \\ -1 & -1 \\ -1 & -1 \end{bmatrix}$$

At SNR The LMMSE estimate can be evaluated as shown below

$$\hat{\mathbf{x}} = \left(\mathbf{H}^T\mathbf{H} + \frac{1}{\mathrm{SNR}}\mathbf{I}\right)^{-1}\mathbf{H}^T\bar{\mathbf{y}}$$

$$\mathbf{H}^T\mathbf{H} = \begin{bmatrix} -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -1 & 1 \\ -1 & -1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\mathbf{H}^T\mathbf{H} + \frac{1}{\mathrm{SNR}}\mathbf{I} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + 4\mathbf{I} = 8\mathbf{I}$$

$$\left(\mathbf{H}^T\mathbf{H} + \frac{1}{\mathrm{SNR}}\mathbf{I}\right)^{-1}\mathbf{H}^T\bar{\mathbf{y}} = \frac{\mathbf{1}}{8}\mathbf{I} \times \begin{bmatrix} -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} -3 \\ -1 \\ 2 \\ -1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 3 \\ -5 \end{bmatrix} = \begin{bmatrix} \frac{3}{8} \\ -\frac{5}{8} \end{bmatrix}$$

Ans a