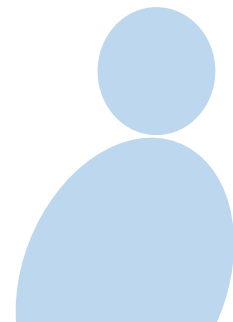


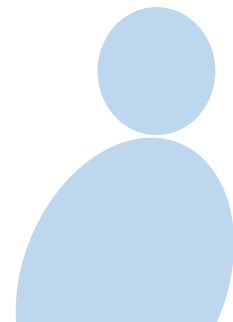
eMasters in Communication Systems

Prof. Aditya
Jagannatham



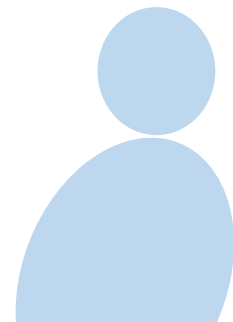
Elective Module:

**Detection for Wireless
Communication**



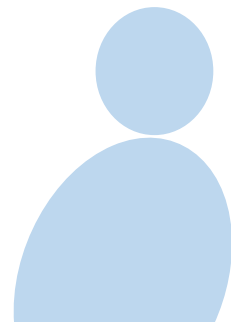
Chapter 2

Neyman Pearson (NP) Criterion



NP Criterion

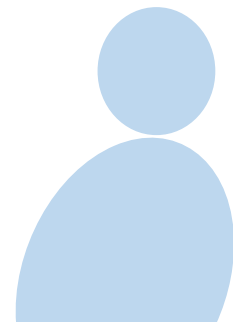
- The *optimal detector* maximizes the P_D for a given P_{FA}



NP Criterion

- The *optimal detector* maximizes the P_D for a given P_{FA}

$$\begin{aligned} & \max P_D \\ & \text{subject to: } P_{FA} = \alpha \end{aligned}$$

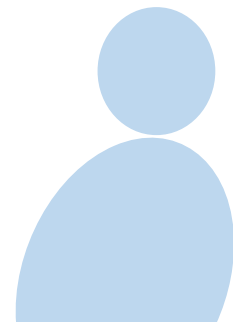


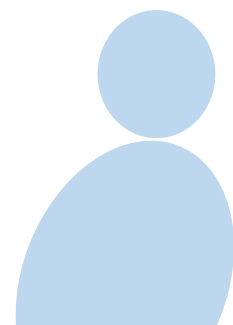
NP Criterion

- Let **optimal detector** choose

\mathcal{H}_1 :

\mathcal{H}_0 :



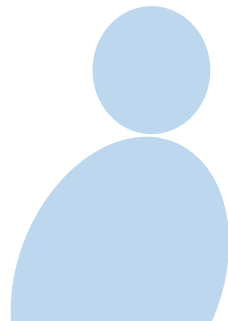


NP Criterion

- Let **optimal detector** choose

$$\mathcal{H}_1: \bar{\mathbf{y}} \in R_1 \subset R^{N \times 1}$$

$$\mathcal{H}_0: \bar{\mathbf{y}} \in R_0 = R^{N \times 1} - R_1$$

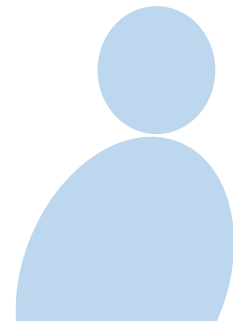


NP Criterion

- Therefore,

$$P_D = \Pr(\bar{\mathbf{y}} \in R_1; \mathcal{H}_1)$$

=

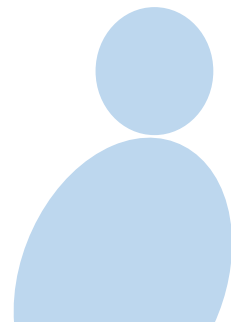


NP Criterion

- Therefore,

$$P_D = \Pr(\bar{\mathbf{y}} \in R_1; \mathcal{H}_1)$$

$$= \int_{R_1} p(\bar{\mathbf{y}}; \mathcal{H}_1) d\bar{\mathbf{y}}$$

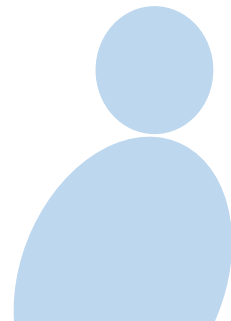


NP Criterion

- Therefore,

$$P_{FA} = \Pr(\bar{\mathbf{y}} \in R_1; \mathcal{H}_0)$$

=

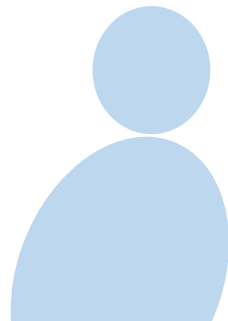


NP Criterion

- Therefore,

$$P_{FA} = \Pr(\bar{\mathbf{y}} \in R_1; \mathcal{H}_0)$$

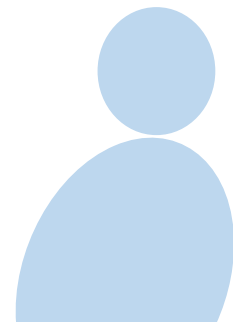
$$= \int_{R_1} p(\bar{\mathbf{y}}; \mathcal{H}_0) d\bar{\mathbf{y}}$$



NP Criterion

- How to solve the **constrained optimization** problem?

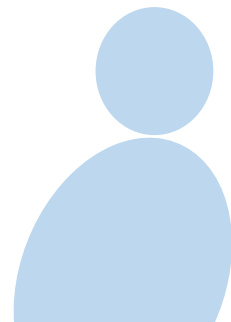
λ : Lagrange multiplier



NP Criterion

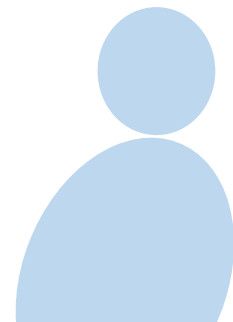
$$\begin{aligned} & \max P_D \\ & \text{subject to: } P_{FA} = \alpha \end{aligned}$$

Lagrangian is

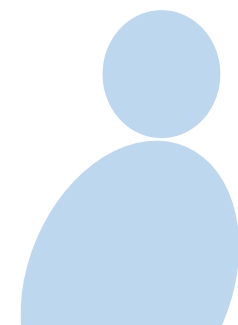


NP Criterion

$$\begin{aligned} & \max P_D \\ & \text{subject to: } P_{FA} = \alpha \\ & P_D + \lambda(\alpha - P_{FA}) \end{aligned}$$

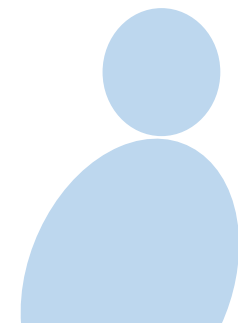


NP Criterion Lagrangian



NP Criterion Lagrangian

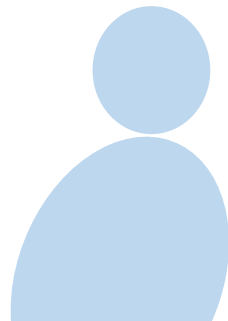
$$\begin{aligned} & \int_{R_1} p(\bar{\mathbf{y}}; \mathcal{H}_1) d\bar{\mathbf{y}} + \lambda \left(\alpha - \int_{R_1} p(\bar{\mathbf{y}}; \mathcal{H}_0) d\bar{\mathbf{y}} \right) \\ &= \int_{R_1} (p(\bar{\mathbf{y}}; \mathcal{H}_1) - \lambda p(\bar{\mathbf{y}}; \mathcal{H}_0)) d\bar{\mathbf{y}} + \lambda \alpha \end{aligned}$$



NP Criterion

$$\int_{R_1} (p(\bar{\mathbf{y}}; \mathcal{H}_1) - \lambda p(\bar{\mathbf{y}}; \mathcal{H}_0)) d\bar{\mathbf{y}} + \lambda \alpha$$

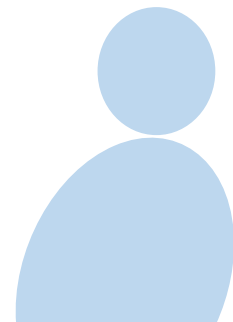
- How to maximize P_D ?



NP Criterion

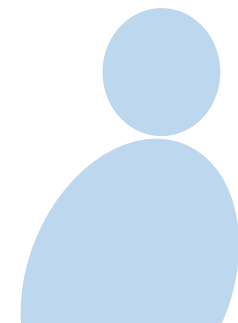
$$\int_{R_1} (p(\bar{\mathbf{y}}; \mathcal{H}_1) - \lambda p(\bar{\mathbf{y}}; \mathcal{H}_0)) d\bar{\mathbf{y}} + \lambda \alpha$$

- To maximize P_D include all points $\bar{\mathbf{y}}$ in R_1 such that
$$p(\bar{\mathbf{y}}; \mathcal{H}_1) > \lambda p(\bar{\mathbf{y}}; \mathcal{H}_0)$$



NP Criterion

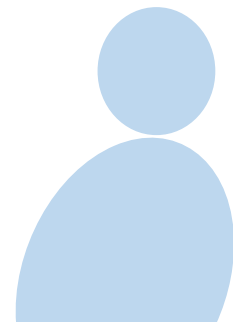
- Choose \mathcal{H}_1 if



NP Criterion

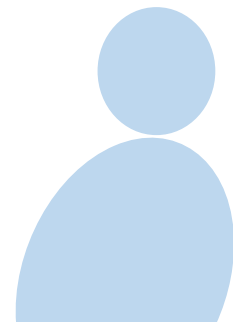
- Choose \mathcal{H}_1 if

$$p(\bar{\mathbf{y}}; \mathcal{H}_1) > \lambda p(\bar{\mathbf{y}}; \mathcal{H}_0)$$
$$\Rightarrow \frac{p(\bar{\mathbf{y}}; \mathcal{H}_0)}{p(\bar{\mathbf{y}}; \mathcal{H}_1)} < \frac{1}{\lambda}$$



NP Criterion

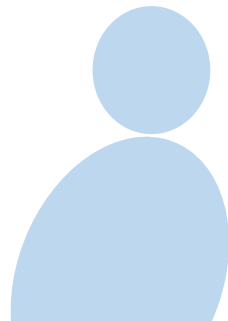
- Choose \mathcal{H}_0 if



NP Criterion

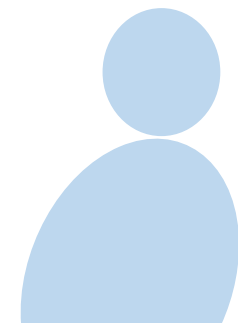
- Choose \mathcal{H}_0 if

$$p(\bar{\mathbf{y}}; \mathcal{H}_1) \leq \lambda p(\bar{\mathbf{y}}; \mathcal{H}_0)$$
$$\Rightarrow \frac{p(\bar{\mathbf{y}}; \mathcal{H}_0)}{p(\bar{\mathbf{y}}; \mathcal{H}_1)} \geq \frac{1}{\lambda} = \tilde{\gamma}$$



NP Criterion

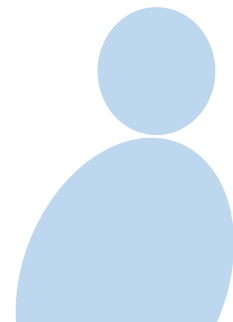
- How to choose λ ?



NP Criterion

- How to choose λ ?
- From the **constraint!!**

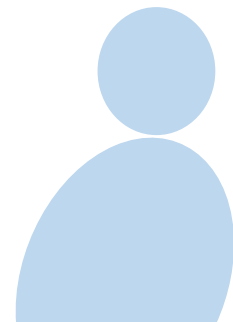
$$P_{FA} = \alpha$$



Constant Signal Detection

- Given $P_{FA} = \alpha$

$$P_{FA} = Q\left(\frac{\gamma}{\sigma \|\bar{\mathbf{s}}\|}\right) = \alpha$$

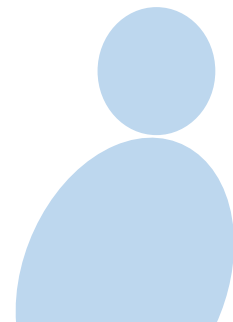


Constant Signal Detection

- Given $P_{FA} = \alpha$

$$P_{FA} = Q\left(\frac{\gamma}{\sigma \|\bar{\mathbf{s}}\|}\right) = \alpha$$

$$\Rightarrow \gamma = \sigma \|\bar{\mathbf{s}}\| Q^{-1}(\alpha)$$



Instructors may use this white area (14.5 cm / 25.4 cm) for the text.
Three options provided below for the font size.

Font: Avenir (Book), Size: 32, Colour: Dark Grey

Font: Avenir (Book), Size: 28, Colour: Dark Grey

Font: Avenir (Book), Size: 24, Colour: Dark Grey

Do not use the space below.

