Change of variables

$$\underline{z} = \phi(\underline{x}) \qquad \phi \text{ one-to-one}$$

unique z for each x

$$x = \phi^{-1}(z)$$

$\hat{f}_0(\underline{x})$

$$z^* = \arg\min_{z} f_0(\underline{z}) \quad \Leftrightarrow \quad x^* = \arg\min f_0(\phi(x)) \quad \Leftrightarrow \quad \boxed{x^* = \arg\min_x \hat{f}_0(x)}$$

$$f_i(\underline{z}) \leq 0 \qquad f_i(\phi(x)) \leq 0 \qquad \hat{f}_i(x) \leq 0$$

$$h_j(\underline{z}) = 0 \qquad h_j(\phi(x)) = 0 \qquad \hat{h}_j(x) = 0$$

$$z \in \mathcal{D} \qquad \phi(x) \in \mathcal{D} \qquad x \in \hat{\mathcal{D}}$$

could be convex

$\{x \mid \phi(x) \in \mathcal{D}\}$

Note: $x^* = \phi^{-1}(z^*)$

$z^* = \phi(x)$

Eg:

$$\min_{x \in \mathbb{R}^3} \; c_1 \log x_1 + c_2 \log x_2 + c_3 \log x_3$$

$$a_1 \log x_1 + a_2 \log x_2 + a_3 \log x_3 \leq b$$

$$x_i > 0 \qquad i = 1, 2, 3$$

concave

$c_i, a_i > 0$

– Not convex as objective & constraints are not convex

$$\phi(\cdot) = \log(\cdot) \qquad z_i = \log(x_i)$$

$$x_i = e^{z_i}$$

$$\left.\begin{aligned} z^* = \arg\min_z \; & c_1 z_1 + c_2 z_2 + c_3 z_3 \\ & a_1 z_1 + a_2 z_2 + a_3 z_3 \leq b \\ & z_i \geq 0 \end{aligned}\right\} \text{ linear program (LP)}$$

$$x_i^* = e^{z_i^*}$$