3. Examples

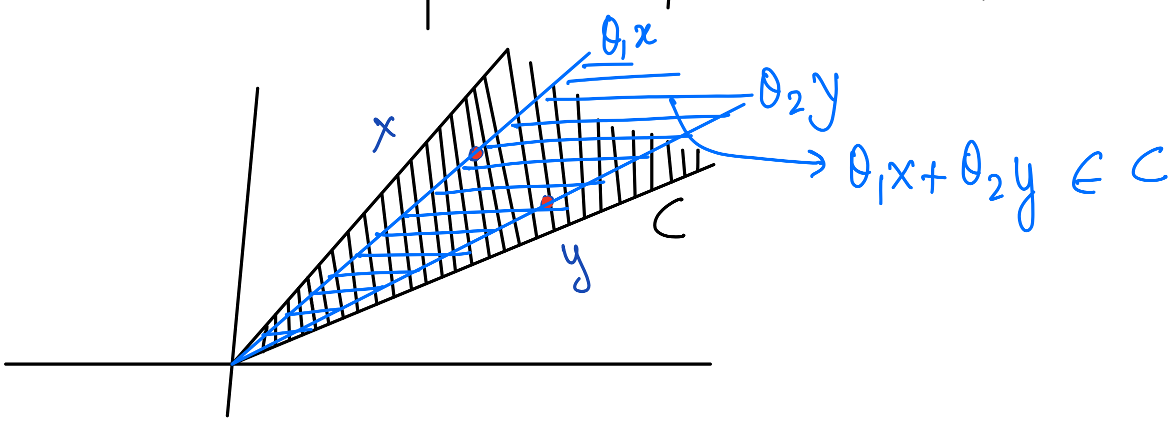
Convex cones : $x, y \in C$ then C convex cone

$\iff \theta_1 x + \theta_2 y \in C$

$\theta_1, \theta_2 \geq 0$

C is ... when $\theta_1 x + (1-\theta) y \in C$

	Affine	Convex	Convex Cone
θ_1	$\in \mathbb{R}$	$\in [0,1]$	≥ 0
θ_2	$= 1-\theta_1$	$= 1-\theta_1$	≥ 0



Q Are convex cones convex ? Yes

suppose that $x, y \in C$ (Convex cone)

then $\theta_1 x + \theta_2 y \in C \quad \forall \; \theta_1, \theta_2 \geq 0$

$\Rightarrow \theta_1 x + (1-\theta_1) y \in C$ use $\theta_2 = 1-\theta_1$ provided $\theta_1 \in [0,1]$

Recap
A is affine & convex
C is convex
CC is convex cone & convex , not affine

Hyperplane : $\{x \in \mathbb{R}^n \mid a^T x = b\}$ single restriction

$n=2$: $\{\underset{\sim}{x} \in \mathbb{R}^2 \mid x_1 = x_2\}$ line

$n=3$: $\{x \in \mathbb{R}^3 \mid x_3 = 0\}$ plane

$\vdots$

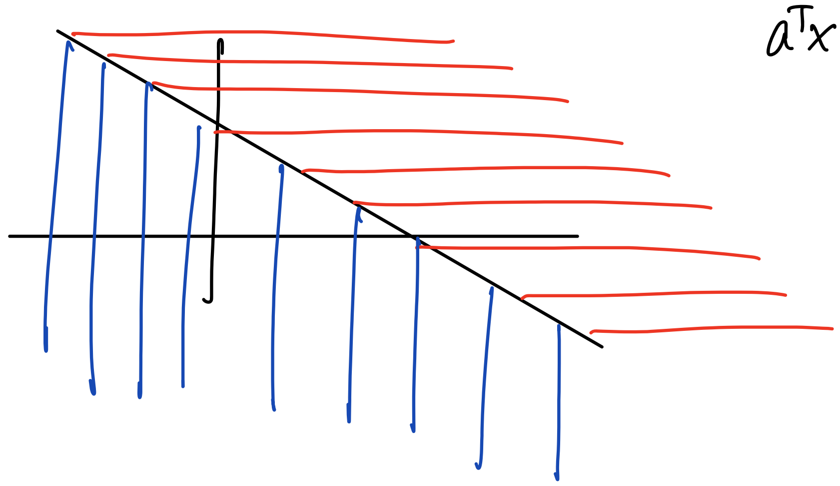
Half-space : $\{x \in \mathbb{R}^n \mid a^T x \le b\}$

Hyperplane divides space into 2 half spaces

$a^T x = b$

$a^T x \le b$

$a^T x \ge b$ or $-a^T x \le -b$



- not affine
- convex
- not convex cone unless $b = 0$ (prove)

Polyhedron : finite intersection of half-spaces & hyperplanes

$$\left\{x \in \mathbb{R}^n \;\middle|\; \begin{array}{l} a_1^T x = b_1,\ a_2^T x = b_2,\ \ldots\ a_m^T x \le b_m \\ c_1^T x \le d_1,\ c_2^T x \le d_2,\ \ldots,\ c_p^T x \le d_p \end{array}\right\}$$