eMasters in Communication Systems

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Elective Module:

Estimation for Wireless Communication

Chapter 4

Vector Parameter Estimation

Least Squares Solution

- The Least Squares solution can be **derived** as follows

Least Squares Solution

- The LS cost function is

Least Squares Solution

- The LS cost function is

$$\left\|\bar{\mathbf{y}} - \mathbf{X}\bar{\mathbf{h}}\right\|^2$$

Least Squares Solution

- The LS cost function can be simplified as

$$\left\|\bar{\mathbf{y}} - \mathbf{X}\bar{\mathbf{h}}\right\|^2$$

$$=$$

Least Squares Solution

Least Squares Solution

- The LS cost function can be simplified as

$$\left\|\bar{\mathbf{y}} - \mathbf{X}\bar{\mathbf{h}}\right\|^2$$
$$= \left(\bar{\mathbf{y}} - \mathbf{X}\bar{\mathbf{h}}\right)^T \left(\bar{\mathbf{y}} - \mathbf{X}\bar{\mathbf{h}}\right)$$
$$= \bar{\mathbf{y}}^T\bar{\mathbf{y}} - 2\bar{\mathbf{h}}^T\mathbf{X}^T\bar{\mathbf{y}} + \bar{\mathbf{h}}^T\mathbf{X}^T\mathbf{X}\bar{\mathbf{h}}$$

Least Squares Solution

- To minimize, we now calculate the **gradient** and set to zero

$$\nabla f\left(\bar{\mathbf{h}}\right) =$$

Least Squares Solution

- We now calculate the gradient

$$\nabla f(\bar{\mathbf{h}}) = \begin{bmatrix} \frac{\partial f(\bar{\mathbf{h}})}{\partial h_1} \\ \frac{\partial f(\bar{\mathbf{h}})}{\partial h_2} \\ \vdots \\ \frac{\partial f(\bar{\mathbf{h}})}{\partial h_M} \end{bmatrix}$$

Least Squares Solution

- We use the following principles

$$\bar{\mathbf{c}}^T\bar{\mathbf{h}} = c_1h_1 + \cdots + c_Mh_M = \bar{\mathbf{h}}^T\bar{\mathbf{c}}$$

$$\nabla\bar{\mathbf{c}}^T\bar{\mathbf{h}} = \nabla\bar{\mathbf{h}}^T\bar{\mathbf{c}} =$$

Least Squares Solution

- We use the following principles

$$\bar{\mathbf{c}}^T\bar{\mathbf{h}} = c_1 h_1 + \cdots + c_M h_M$$

$$\nabla \bar{\mathbf{c}}^T\bar{\mathbf{h}} = \nabla \bar{\mathbf{h}}^T\bar{\mathbf{c}} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_M \end{bmatrix}$$

Least Squares Solution

- For a symmetric matrix $\mathbf{P} = \mathbf{P}^T$

$$\nabla \bar{\mathbf{h}}^T \mathbf{P} \bar{\mathbf{h}} =$$

Least Squares Solution

- For a symmetric matrix $\mathbf{P} = \mathbf{P}^T$

$$\nabla \bar{\mathbf{h}}^T \mathbf{P} \bar{\mathbf{h}} = 2\mathbf{P}\bar{\mathbf{h}}$$

Least Squares Solution

- Therefore, it follows that

$$\nabla \left\| \bar{\mathbf{y}} - \mathbf{X}\bar{\mathbf{h}} \right\|^2$$

$$=$$

Least Squares Solution

- Therefore, it follows that

$$\nabla\left\|\bar{\mathbf{y}} - \mathbf{X}\bar{\mathbf{h}}\right\|^2 = \nabla\left(\bar{\mathbf{y}}^T\bar{\mathbf{y}} - 2\bar{\mathbf{h}}^T\mathbf{X}^T\bar{\mathbf{y}} + \bar{\mathbf{h}}^T\mathbf{X}^T\mathbf{X}\bar{\mathbf{h}}\right) = -2\mathbf{X}^T\bar{\mathbf{y}} + \mathbf{2X}^T\mathbf{X}\bar{\mathbf{h}}$$

Least Squares Solution

- Setting gradient equal to zero yields

Least Squares Solution

- Setting gradient equal to zero yields

$$-2\mathbf{X}^T\bar{\mathbf{y}} + \mathbf{2}\mathbf{X}^T\mathbf{X}\bar{\mathbf{h}} = \mathbf{0}$$

$$\Rightarrow \mathbf{X}^T\mathbf{X}\bar{\mathbf{h}} = \mathbf{X}^T\bar{\mathbf{y}}$$

$$\Rightarrow \hat{\mathbf{h}} = (\mathbf{X}^T\mathbf{X})^{-\mathbf{1}}\mathbf{X}^T\bar{\mathbf{y}}$$

Instructors may use this white area (14.5 cm / 25.4 cm) for the text. Three options provided below for the font size.

Font: Avenir (Book), Size: 32, Colour: Dark Grey

Font: Avenir (Book), Size: 28, Colour: Dark Grey

Font: Avenir (Book), Size: 24, Colour: Dark Grey

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