EE910: Digital Communication Systems-I

Adrish Banerjee

Department of Electrical Engineering
Indian Institute of Technology Kanpur
Kanpur, Uttar Pradesh
India

May 23, 2022

Lecture #7C: Differential Phase Shift Keying

Differential PSK

- In differentially encoded PSK, the information sequence determines the relative phase, or phase transition, between adjacent symbol intervals
- In M-ary DPSK, the carrier phase angle of the modulator for the nth symbol interval is given by

$$\theta_n = \left(\theta_{n-1} + x_n \frac{2\pi}{M} \right). \quad \text{modulo } 2\pi \tag{1}$$

where x_n is a modulator input symbol contained in $\{0, 1, \ldots, M-1\}$.

- The transmitted signal is the PSK waveform

$$s(t) = \left(\frac{2E_s}{T_s} \right)^{1/2} \cos\left(\omega_c t + \theta_n\right), \quad nT_s \le t \le (n+1)T_s. \tag{2}$$

- Thus, we implement M-ary PSK modulation, but with the phase differences $\delta_n = \theta_n - \theta_{n-1}$, modulo 2π, defined by the symbol sequence $\{x_n\}$.

Differential PSK

- At the receiver, due to an unknown (but assumed fixed) phase offset θ and the addition of white Gaussian noise, we observe

$$r(t) = \left(\frac{2E_s}{T_s} \right)^{1/2} \cos\left(\omega_c t + \theta_n + \theta\right) + n(t). \tag{3}$$

- For $M = 2$

Information Sequence		0	1	1	0	1
Carrier Phase at Modulator Output	0	0	π	0	0	π
Carrier Phase at Demodulator Output	θ	θ	$\pi+\theta$	θ	θ	$\pi+\theta$
Phase Difference		0	π	π	0	π
Output		0	1	1	0	1

Differential PSK

- Since in differential PSK the information is in the phase transitions and not in the absolute phase, the phase ambiguity from a PLL cancels between the two adjacent intervals and will have no effect on the performance of the system.
- The performance of the system is only slightly degraded due to the tendency of errors to occur in pairs, and the overall error probability is twice the error probability of a PSK system.

Differential PSK

- A differentially encoded phase-modulated signal also allows noncoherent detection.
- Since the information is in the phase transition, we have to do the detection over a period of two symbols.
- The vector representation of the lowpass equivalent of the mth signal over a period of two symbol intervals is given by

$$\mathbf{s}_{ml} = (\sqrt{2\mathcal{E}_s} \;\; \sqrt{2\mathcal{E}_s} e^{j\theta_m}), \qquad 1 \leq m \leq M \tag{4}$$

where $\theta_m = \frac{2\pi(m-1)}{M}$ is the phase transition corresponding to the mth message.

Differential PSK

- When $\mathbf{s}_{ml}$ is transmitted, the vector representation of the lowpass equivalent of the received signal on the corresponding two-symbol period is given by

$$\mathbf{r} = (r_1 \;\; r_2) = (\sqrt{2\mathcal{E}_s} \;\; \sqrt{2\mathcal{E}_s}e^{j\theta_m})e^{j\phi} + (n_{1l} \;\; n_{2l}), \quad 1 \leq m \leq M \quad (5)$$

  where n_{1l} and n_{2l} are two complex valued zero-mean circular Gaussian random variables each with variance $2N_0$ (variance N_0 for real and imaginary components) and ϕ is the random phase due to noncoherent detection.
- We assume that the phase offset ϕ remains the same over adjacent signaling periods.
- The optimal noncoherent receiver uses the following detection rule

$$\hat{m} = \arg\max_{1\leq m\leq M} I_0\left(\frac{|\mathbf{r}_l \cdot \mathbf{s}_{ml}|}{N_0}\right) \quad (6)$$

Differential PSK

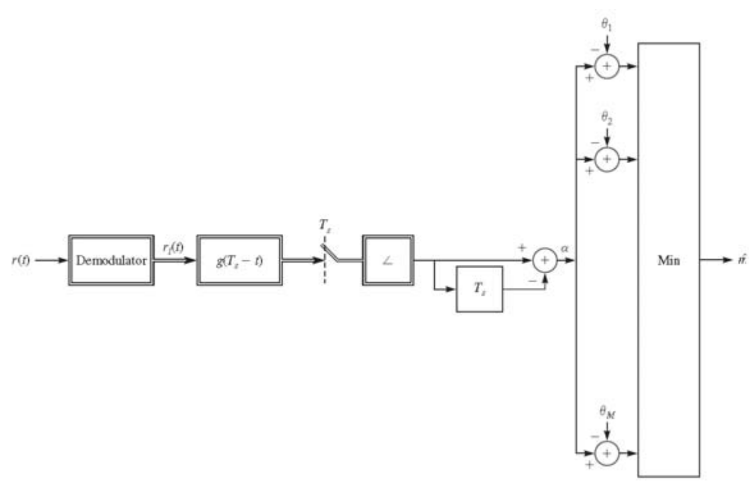
- Thus we have

$$\begin{aligned}
\hat{m} &= \arg\max_{1\leq m\leq M} |\mathbf{r}_l.\mathbf{s}_{ml}| \\
&= \arg\max_{1\leq m\leq M} \sqrt{2\mathcal{E}_s}|\mathbf{r}_1 + r_2 e^{-j\theta_m}| \\
&= \arg\max_{1\leq m\leq M} |r_1 + r_2 e^{-j\theta_m}|^2 \\
&= \arg\max_{1\leq m\leq M}(|r_1|^2 + |r_2|^2 + 2Re[r_1^* r_2 e^{-j\theta_m}]) \\
&= \arg\max_{1\leq m\leq M} Re[r_1^* r_2 e^{-j\theta_m}] \\
&= \arg\max_{1\leq m\leq M} |r_1 r_2| cos(\angle r_2 - \angle r_1 - \theta_m) \\
&= \arg\max_{1\leq m\leq M} cos(\angle r_2 - \angle r_1 - \theta_m) \\
&= \arg\min_{1\leq m\leq M} |\angle r_2 - \angle r_1 - \theta_m|
\end{aligned} \quad (7)$$

Differential PSK

- Note that $\alpha = \angle r_2 - \angle r_1$ is the phase difference of the received signal in two adjacent intervals
- The receiver computes this phase difference and compares it with $\theta_m = \frac{2\pi}{M}(m-1)$ for all $1 \leq m \leq M$ and selects the m for which θ_m is closest to α, thus maximizing $cos(\alpha - \theta_m)$.
- Differentially encoded PSK signal that uses this method for demodulation detection is called differential PSK (DPSK).
- This method of detection has lower complexity in comparison with coherent detection of PSK signals and can be used in situations where the assumption that ϕ remains constant over two-symbol intervals is valid.

Differential PSK



Differential PSK

- In binary DPSK the phase difference between adjacent symbols is either 0 or π, corresponding to a 0 or 1.
- The two lowpass equivalent signals are

$$\begin{aligned} \mathbf{s}_{1l} &= (\sqrt{2\mathcal{E}_s}\ \sqrt{2\mathcal{E}_s}) \\ \mathbf{s}_{2l} &= (\sqrt{2\mathcal{E}_s}\ -\sqrt{2\mathcal{E}_s}) \end{aligned} \tag{8}$$

- These two signals are noncoherently demodulated and detected using the general approach for optimal noncoherent detection.
- Note that the two signals are orthogonal on an interval of length $2T_s$.
- Therefore, the error probability can be obtained from the expression for the error probability of binary orthogonal signaling.
- The difference is that the energy in each of the signals $s_1(t)$ and $s_2(t)$ is $2\mathcal{E}_s$.

Differential PSK

- This is seen easily from Equation (8) which shows that the energy in lowpass equivalents is $4\mathcal{E}_s$.
- Therefore

$$\begin{aligned} P_b &= \frac{1}{2}e^{-\frac{2\mathcal{E}_s}{2N_0}} \\ &= \frac{1}{2}e^{-\frac{\mathcal{E}_b}{N_0}} \end{aligned} \tag{9}$$

- This is the bit error probability for binary DPSK.
- Comparing this result with coherent detection of BPSK where the error probability is given by

$$P_b = Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right) \tag{10}$$

we observe that by the inequality $Q(x) \leq \frac{1}{2}e^{-x^2/2}$,we have

$$P_b(\text{coherent}) \leq P_b(\text{noncoherent}) \tag{11}$$

Differential PSK