EE677: Applied Linear Algebra For Wireless Communications EE Dept. IITK

Quiz 1

Marks 20 *23 July 2023*

1. (4 marks) What does the linear combination of the following vectors represent in $\mathbf{R}^4$:

$$i) \begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 6 \\ -9 \\ 3 \end{bmatrix}, \text{ and } \begin{bmatrix} 2 \\ -4 \\ 6 \\ -2 \end{bmatrix} \qquad ii) \begin{bmatrix} 9 \\ 3 \\ 3 \\ 3 \end{bmatrix} \begin{bmatrix} -6 \\ 4 \\ -4 \\ 10 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 2 \\ -5 \end{bmatrix}, \text{ and } \begin{bmatrix} 3 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

2. For the given vectors:

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \ \mathbf{a}_2 = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}, \ \mathbf{a}_3 = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}.$$

   - (2 marks) Find the length of the vectors $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$.
   - (2 marks) Compute the following dot products $\mathbf{a}_1.\mathbf{a}_3$, $\mathbf{a}_2.(\mathbf{a}_1 - \mathbf{a}_3)$ and $\mathbf{a}_3.(2\mathbf{a}_1 - 3\mathbf{a}_2)$.

3. (3 marks) For the given vectors: $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix}$, $\mathbf{a}_3 = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$, find the linear combination of these vectors to give the vector $\mathbf{b} = \begin{bmatrix} 2 \\ -7 \\ 3 \end{bmatrix}$.

4. (4 marks) Find the solution for $\mathbf{Ax} = \mathbf{0}$, where $\mathbf{A} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & -1 \\ 0 & -2 & 2 \end{bmatrix}$, and $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

5. For the given matrices

$$\mathbf{X} = \begin{bmatrix} 1 & 5 \\ 5 & 0 \end{bmatrix} \text{ and } \mathbf{Y} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

   (a) (2.5 marks) Show that $\mathbf{XY} \neq \mathbf{YX}$.

   (b) (2.5 marks) Show that $(\mathbf{X} + \mathbf{Y})^2 \neq (\mathbf{X}^2 + \mathbf{Y}^2 + 2\mathbf{XY})$.