Live Interaction #6:

5th November 2023

E-masters Communication Systems

Detection for Wireless

- **Detection of Random Signals:**

$$\mathcal{H}_0 : \bar{\mathbf{y}} = \bar{\mathbf{v}}$$
$$\mathcal{H}_1 : \bar{\mathbf{y}} = \bar{\mathbf{s}} + \bar{\mathbf{v}}$$

- $\bar{\mathbf{s}}$ is a **random signal**.

$$s(1), s(2), \ldots \sim \mathcal{N}(0, \sigma_s^2)$$
$$v(i) \sim \mathcal{N}(0, \sigma^2)$$

- **Likelihoods:**

$$p(\bar{\mathbf{y}}; \mathcal{H}_0) = \left(\frac{1}{2\pi\sigma^2}\right)^{N/2} e^{-\frac{\|\bar{\mathbf{y}}\|^2}{2\sigma^2}}$$
$$p(\bar{\mathbf{y}}; \mathcal{H}_1) = \left(\frac{1}{2\pi(\sigma^2 + \sigma_s^2)}\right)^{N/2} e^{-\frac{\|\bar{\mathbf{y}}\|^2}{2(\sigma^2+\sigma_s^2)}}$$

- Choose $\mathcal{H}_1$:

$$p(\bar{\mathbf{y}}; \mathcal{H}_1) \geq p(\bar{\mathbf{y}}; \mathcal{H}_0)$$
$$\Rightarrow \|\bar{\mathbf{y}}\|^2 = \underbrace{|y(1)|^2 + \cdots + |y(N)|^2}_{\text{Energy detector}} > \gamma$$

- Optimal detector is given by the **Energy detector**.
- **Probability of False Alarm**:
- Under $\mathcal{H}_0$

$$\underbrace{\frac{|y(1)|^2}{\sigma^2} + \cdots + \frac{|y(N)|^2}{\sigma^2}}_{\chi_N^2} > \frac{\gamma}{\sigma^2}$$

$$P_{FA} = Q_{\chi_N^2}\left(\frac{\gamma}{\sigma^2}\right)$$

- χ_N^2: Chi-squared random variable with N degrees of freedom.

$$Q_{\chi_N^2}(x) = \frac{\Gamma\left(\frac{N}{2}, \frac{x}{2}\right)}{\Gamma\left(\frac{N}{2}\right)} = \frac{\int_{\frac{x}{2}}^{\infty} t^{\frac{N}{2}-1} e^{-t} dt}{\int_0^{\infty} t^{\frac{N}{2}-1} e^{-t} dt}$$

- Find P_{FA} for $N = 2$.

$$P_{FA} = Q_{\chi_N^2}\left(\frac{\gamma}{\sigma^2}\right)$$

$$Q_{\chi_2^2}(x) = \frac{\Gamma\left(1, \frac{x}{2}\right)}{\Gamma(1)} = \frac{\int_{\frac{x}{2}}^{\infty} e^{-t} dt}{\int_0^{\infty} e^{-t} dt} = \frac{e^{-\frac{x}{2}}}{1}$$

$$P_{FA} = Q_{\chi_2^2}\left(\frac{\gamma}{\sigma^2}\right) = e^{-\frac{\gamma}{2\sigma^2}}$$

- **Probability of detection:**

$$\underbrace{\underbrace{|y(1)|^2 + \cdots + |y(N)|^2}_{\text{Energy detector}} > \gamma}_{\mathcal{H}_1}$$

$$P_D = Q_{\chi_N^2}\left(\frac{\gamma}{\sigma^2 + \sigma_s^2}\right)$$

$$P_D = Q_{\chi_2^2}\left(\frac{\gamma}{\sigma^2 + \sigma_s^2}\right) = e^{-\frac{\gamma}{2(\sigma^2+\sigma_s^2)}}$$

- ROC?

$$P_{FA} = Q_{\chi_2^2}\left(\frac{\gamma}{\sigma^2}\right) = e^{-\frac{\gamma}{2\sigma^2}}$$

$$\Rightarrow \gamma = -2\sigma^2 \ln P_{FA}$$

$$P_D = e^{-\frac{\gamma}{2(\sigma^2+\sigma_s^2)}} = e^{\frac{2\sigma^2 \ln P_{FA}}{2(\sigma^2+\sigma_s^2)}}$$

$$= e^{\frac{\sigma^2}{(\sigma^2+\sigma_s^2)}\ln P_{FA}} = e^{\ln P_{FA}^{\frac{\sigma^2}{(\sigma^2+\sigma_s^2)}}}$$

$$P_D = P_{FA}^{\frac{\sigma^2}{(\sigma^2+\sigma_s^2)}}$$

- **Wireless channel:**
- **Average BER:**

$$\underbrace{y = hx + n}_{\text{Wireless Channel}}$$

- $|h| = a$: $f_A(a) = 2ae^{-a^2}, a \geq 0$

$$BER = E\left\{Q\left(\sqrt{a^2 \frac{2E_b}{N_0}}\right)\right\} = E\left\{Q\left(\sqrt{a^2\rho}\right)\right\}$$

$$\rho = \frac{2E_b}{N_0}$$

$$\int_0^\infty Q\left(\sqrt{a^2\rho}\right) 2ae^{-a^2} da = \frac{1}{2}\left(1 - \sqrt{\frac{\rho}{2+\rho}}\right) \approx \frac{1}{2\rho}$$

- BER of AWGN decreases **EXPONENTIALLY** $\sim e^{-\frac{1}{2}SNR}$.
- BER of Wireless channel decreases $\sim \frac{1}{SNR}$.

$$SNR = 20\ dB = 10^2 = 100$$

$$BER_{Wireless} \approx \frac{1}{2 \times SNR} = \frac{1}{200} = 5 \times 10^{-3}$$

$$BER_{Wireline} = Q\left(\sqrt{100}\right) = Q(10) = 7.6 \times 10^{-24}$$

- $BER_{Wireless}$ is **SIGNIFICANTLY HIGHER**!!
- This can be attributed Deep Fade phenomenon.
- How to overcome this?
- **DIVERSITY!!!!**
- **Assignment 6 deadline: Saturday 11th November 11:59 PM.**
- **Sunday 12th November: DIWALI HOLIDAY-No Live interaction.**
- **Saturday 18th November: Live interaction #7 – 2:30-3:30 PM**
- **Saturday 18th November: Assignment 5,6 Discussion – 3:30 – 4:00 PM**
- **Saturday 18th November: Quiz #3 – 4:30 – 5:15 PM**
- **Sunday 19th November: Live interaction #8 – 6:00-7:00 PM**