Quadratic Problems

(QP) $$\min_{x \in \mathbb{R}^n} \frac{1}{2}x^T P x + q^T x$$

convex $P \succeq 0$

$$\left.\begin{aligned} Gx &\le h \\ Ax &= b \end{aligned}\right\} \text{ linear}$$

Note : when no constraints

then

$$\min_x \frac{1}{2}x^T P x + q^T x$$

$$\Rightarrow \nabla\left(\frac{1}{2}x^T P x + q^T x\right) = 0$$

$$\text{or } Px + q = 0$$

QCQP :

quadratically constrained QP

$$\min \frac{1}{2}x^T P_0 x + q_0^T x$$

$$\frac{1}{2}x^T P_i x + q_i^T x + r_i \le 0$$

$$Ax = b$$

convex $P_0, P_i \succeq 0$

Eg: $w = \min_x x^T A x$ where $A \not\succeq 0$ not P.S.D. $A \in S^n$

$\Rightarrow \lambda_{\min}(A) < 0$ a negative eigenvalue

suppose $Au = \lambda_{\min}(A) u$

eigenvector

take $\underline{x} = \alpha \underline{u}$

$$\omega = \min x^T A x \quad \leq \quad \vartheta = \min_{\underline{x} = \alpha \underline{u}} x^T A x$$

→ additional constraint

$\omega \leq \vartheta$ more constraints may hurt

$$\vartheta = \min_{\alpha} \alpha^2 (u^T A u)$$

$$= \min_{\alpha} \underbrace{\alpha^2}_{\geq 0} \underbrace{\lambda_{min}(A)}_{<0} \underbrace{\|u\|^2}_{\geq 0} \rightarrow -\infty$$

as $\alpha \rightarrow \infty$

(unbounded below)

$$\omega \leq \vartheta \rightarrow -\infty$$

$$\Rightarrow \omega \rightarrow -\infty$$