Operations that preserve convexity

1. f_i convex $\forall\, i=1,2\ldots n \Rightarrow \sum w_i f_i(x)$ also convex for $w_i \geq 0$

- can verify using Hessian

2. f convex $\Rightarrow f(Ax+b)$ convex when $Ax+b \in \text{dom} f$

- can verify using zeroth/first order

3. Pointwise max. $\{f_i(\underline{x})\}_{i=1}^{n}$ convex

$$\Rightarrow g(\underline{x}) = \max_{1\leq i\leq n} \{f_i(\underline{x})\} \quad \text{convex}$$

Zeroth-order: $\underline{x}, y \in \text{dom}\, g = \bigcap_{i=1}^{n} \text{dom} f_i$

$$g(\theta\underline{x}+(1-\theta)y) = \max_i f_i(\theta x+(1-\theta)y)$$

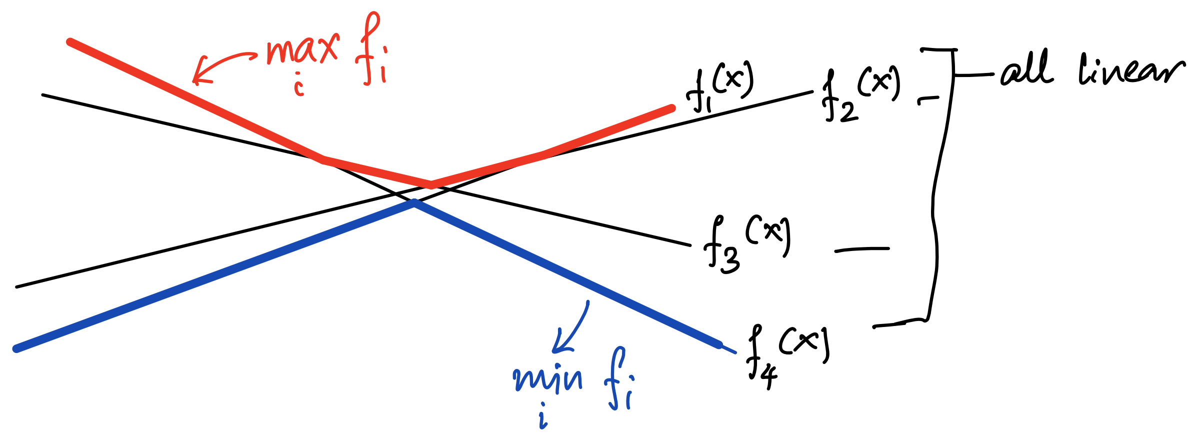
$$\leq \max_i \theta f_i(x) + (1-\theta) f_i(y)$$

$$\leq \theta \max_i f(x) + (1-\theta)\max_i f_i(y)$$

Recall: $\max_i \{a_i+b_i\} \le \max_i\{a_i\} + \max_i\{b_i\}$

$$= \theta g(x) + (1-\theta) g(y)$$

Eg



Since f_i are linear/affine $\quad f_i(x) = a_i^T x + b_i$

$\max_i \{a_i^T x + b_i\}$ convex

$\min_i \{a_i^T x + b_i\}$ concave

Support $\quad S_C(\underline{x}) = \max_{y \in C} \{x^T y\}$

max. of set C (can be arbitrary) need not be convex.

eg. $C = \{\underline{a} \mid \|a\| \le 1\}$

Eg. $\quad S_{B(0,1)}(\underline{x}) = \max_{\|y\| \le 1} x^T y \quad$ is convex