

Gaussian Discriminant Analysis (GDA)

Gaussian Discriminant Analysis (GDA) is a statistical classification algorithm that assumes the underlying probability distribution of the features is Gaussian (normal). It is a generative model, meaning it models the distribution of each class to make predictions. GDA has applications in pattern recognition, machine learning, and statistical classification.

Key Concepts:

1. Probabilistic Model:

GDA models the likelihood of observing features given a particular class using a Gaussian distribution.

For a single feature, the Gaussian probability density function is given by:

$$p(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where μ is the mean and σ^2 is the variance.

2. Assumption of Equal Covariance:

GDA assumes that all classes share the same covariance matrix.

This assumption simplifies the model and reduces the number of parameters to estimate.

3. Decision Rule - Maximum A Posteriori (MAP):

GDA uses the Maximum A Posteriori (MAP) decision rule to classify new instances.

Given a set of features x , the class y is predicted as:

$$\operatorname{argmax}_y P(Y = y|X = x)$$

The decision boundary is quadratic in nature due to the Gaussian distribution.

4. Parameter Estimation:

GDA involves estimating parameters such as the class priors, class means, and the shared covariance matrix from the training data.

The estimates are obtained by maximizing the likelihood function.

5. Training Procedure:

Given a dataset with features X and corresponding labels Y , GDA involves the following steps:

Estimate class priors: $P(Y = y)$.

Estimate class means: $\mu_y = \frac{1}{N_y} \sum_{i=1}^m 1\{y^{(i)} = y\}x^{(i)}$, where N_y is the number of instances in class y .

Estimate the shared covariance matrix: $\Sigma = \frac{1}{m} \sum_{i=1}^m (x^{(i)} - \mu_{y^{(i)}})(x^{(i)} - \mu_{y^{(i)}})^T$.

Advantages and Considerations:

1. Advantages:

GDA is relatively simple and often performs well, especially when the distribution of the data approximates a Gaussian form.

It is less prone to overfitting with a limited amount of training data compared to more complex models.

2. Considerations:

The assumption of equal covariance might not hold in some real-world scenarios, potentially limiting the model's accuracy.

GDA may not perform optimally in situations where the Gaussian assumption is violated.

Applications:

1. Face Recognition:

GDA has been used in face recognition systems, modeling the distribution of facial features for different individuals.

2. Medical Diagnosis:

In medical diagnosis, GDA can be applied to classify patients into different disease categories based on relevant features.

3. Speech Recognition:

GDA can be employed in speech recognition tasks, where the distribution of acoustic features is modeled for different spoken words.

Conclusion:

Gaussian Discriminant Analysis provides a probabilistic framework for classification based on the assumption of Gaussian distributions. While it makes simplifying assumptions, GDA is a powerful tool for various classification tasks, especially in scenarios where the Gaussian distribution is a reasonable approximation for the data. It strikes a balance between simplicity and performance, making it a valuable algorithm in the field of machine learning and statistical classification.