EE677: Applied Linear Algebra For Wireless Communications EE Dept. IITK

Mid Semester Exam

Marks 20 *13 August 2023*

1. (4 marks) Describe the smallest subspaces spanned by the following:

$$i) \begin{bmatrix}1\\0\\0\end{bmatrix} \text{ and } \begin{bmatrix}0\\-1\\0\end{bmatrix} \qquad ii) \begin{bmatrix}1 & 1\\0 & 0\end{bmatrix} \text{ and } \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}.$$

2. (6 marks) Reduce the following matrices in row-echelon form

$$\mathbf{A} = \begin{bmatrix}2 & 4 & 6\\-4 & -2 & -2\\3 & 6 & 12\end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix}4 & 12 & 16\\2 & -3 & 8\\4 & 12 & 5\end{bmatrix}.$$

3. (3 marks) For what condition on b_1, b_2, b_3 and b_4 the following system of equation is solvable:

$$\begin{aligned}x + y &= b_1\\2x + 3y &= b_2\\0.5x + y &= b_3\\5x + 10y &= b_4\end{aligned}$$

4. (3 marks) Project the vector $\mathbf{x} = \begin{bmatrix}1\\3\\2\end{bmatrix}$ on the line through $\mathbf{y} = \begin{bmatrix}1\\-1\\-1\end{bmatrix}$. Also verify that the projection error $\mathbf{e}$ is orthogonal to $\mathbf{y}$.

5. (4 marks) Find determinants for $\mathbf{A}$, $\mathbf{A}^{-1}$ and $\mathbf{A}^3$, where $\mathbf{A} = \begin{bmatrix}1 & 5\\4 & 0\end{bmatrix}$.