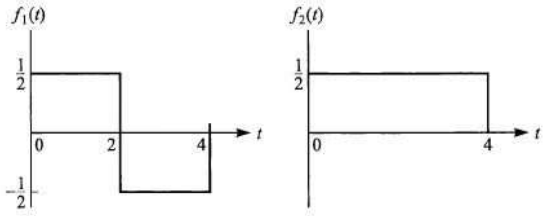
Assignment 2

Digital Communication System-I

April 2023

Please answer the following questions:

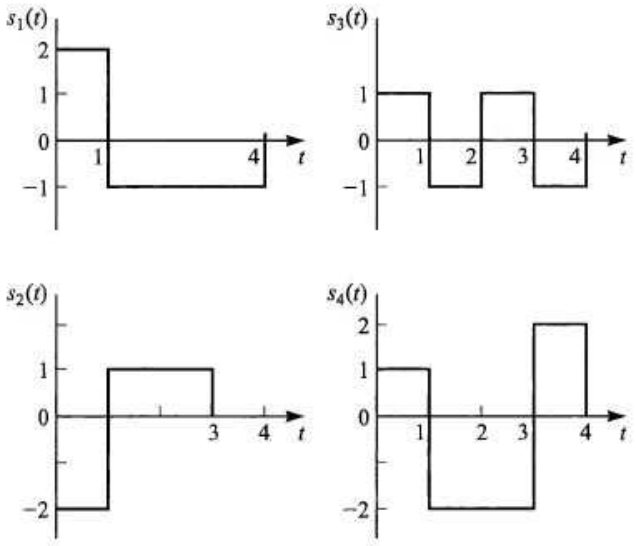
Q 1: Consider the following two waveforms $f_1(t)$ and $f_2(t)$. Are these waveforms orthonormal?



(a) True

(b) False

Q 2: Consider the four waveforms shown in the figure below. These are represented using four basis function as following vectors s_1, s_2, s_3, and s_4.



(a) $s_1 = [2\ -1\ -1\ -1]$, $s_2 = [-2\ 1\ 1\ 0]$, $s_3 = [1\ -1\ 1\ -1]$, $s_4 = [1\ -2\ -2\ 2]$.

(b) $s_1 = [2\ -1\ -1\ -1]$, $s_2 = [1\ -1\ 1\ -1]$, $s_3 = [-2\ 1\ 1\ 0]$, $s_4 = [1\ -2\ -2\ 2]$.

(c) $s_1 = [2\ -1\ -1\ -1]$, $s_2 = [1\ -1\ 1\ -1]$, $s_3 = [1\ -1\ 1\ 1]$, $s_4 = [1\ -2\ -2\ 2]$.

(d) None of the above.

Q 3: Suppose a DC signal x of amplitude A is transmitted over a additive white Gaussian noise (AWGN) channel where noise is zero mean Gaussian with variance σ^2. Then what will be the probability density function $f_Y(y)$ of the output y of the channel?

(a) $f_Y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(y-A)^2}{2\sigma^2}}$

(b) $f_Y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(y+A)^2}{2\sigma^2}}$

(c) $f_Y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-y^2}{2\sigma^2}}$

(d) None of the above

Consider the following statement for answering Q(4)-Q(5).

The joint probability density function (PDF) of two random variables (X and Y) are given by

$$f_{X,Y}(x,y) = \begin{cases} Ke^{-x-y}, & x \geq y \geq 0 \\ 0, & \text{otherwise} \end{cases}.$$

Q 4: Find the value of the constant K.

(a) 1

(b) 0.5

(c) $\frac{1}{e}$

(d) 2

Q 5: Find $E[X|Y = y]$.

(a) y

(b) $y + 1$

(c) $y - 1$

(d) $\exp(-y)$

Q 6: Let X be a Gaussian Random Variable with mean 1 and variance 9 i.e. $X \sim \mathcal{N}(1, 9)$. Calculate the value of $\mathbb{P}[X > 7]$

(a) $Q(1)$

(b) $Q(2)$

(c) $Q(0.5)$

(d) $Q(3)$

Q 7: Which of the following properties about complex Gaussian random variable $\mathbf{Z}$ is true?

(a) If $\mathbf{Z}$ is circularly symmetric, then it is zero-mean complex vector.

(b) If $\mathbf{Z}$ is circularly symmetric, then it is proper.

(c) If $\mathbf{Z}$ is zero mean proper complex vector, $\mathbf{Z}$ is circularly symmetric.

(d) All of these.

Q 8: Which of the following is an example of cyclostationary signal?

(a) $x(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT)$ where a_k is i.i.d. random variables.

(b) $x(t) = A\cos(2\pi ft + \theta)$ where $\theta \sim U(0, 2\pi)$

(c) $x(t) = a(t)\cos(2\pi ft + \theta)$ where $\theta \sim U(0, 2\pi)$, $a(t)$ is a periodic signal.

(d) All of these

Q 9: Let θ be a random variable uniformly distributed in $[0, 2\pi]$ and let $s(t) = A\sin(2\pi f_c t + \theta)$. Calculate the auto-correlation function of a random process $R_{ss}(t, t+\tau)$

(a) $\frac{A^2}{2}\sin(2\pi f_c \tau)$

(b) $\frac{A^2}{2}\cos(2\pi f_c \tau)$

(c) $A^2\cos(2\pi f_c \tau)$

(d) None of the above

Q 10: Let X, Y be independent standard normal random variables. Let

$$u(t) = X\cos(2\pi f_c t) + Y\sin(2\pi f_c t)$$

Then, which of the following are correct?

(a) $\mathbb{E}[u(t)] = 0$

(b) $R_{uu}(t, t+\tau) = \cos(2\pi f\tau)$

(c) $u(t)$ is a wide sense stationary Random process

(d) All of the above.