Example OFDM System

$B = 10\,MHz$
$= 10 \times 10^6\,Hz$

- **OFDM system with BW: 10 MHz** $= B$.
- Number of subcarriers $= 1024 = N$

2^{10}

POWER OF 2

2^n

256, 512, 1024, 2048....

Example OFDM System

= # SUBCARRIERS.

- **BW of each subcarrier**

$$= \underline{\frac{B}{N} = \frac{10\times10^{6}}{1024}} = 9.77\;kHz = \frac{B}{N}$$

- What is the size of $IFFT$ at transmitter?

$$\underline{N = 1024}$$

Example OFDM System

$9.77 \times 10^3\,Hz$

- BW of each subcarrier

$$= 10 \times \frac{10^6}{1024} = 9.77\,kHz = \frac{B}{N}$$

- What is the size of $IFFT$ at transmitter?

$1024 = N$.

Example OFDM System

WITHOUT CP.

- What is the **duration** of the OFDM symbol?

$$\frac{N}{B} = \frac{1024}{10 \times 10^{6}} = 102.4\ \mu s$$

$$= 102.4 \times 10^{-6}\ s.$$

Example OFDM System

SYMBOL DURATION $= \frac{N}{B} = N \times \frac{1}{B}$

#SAMPLES

SAMPLE DURATION

- What is the **duration** of the OFDM symbol?

$$\frac{1}{B} \times N = \frac{1024}{10 \times 10^6} = 102.4\ \mu s = \frac{N}{B}$$

SAMPLE DURATION $= \frac{1}{B}$

$$\underbrace{x(0),\ x(1),\ \ldots,\ x(N-1).}_{N \text{ samples.}}$$

AFTER IFFT

Example OFDM System

NOW ADD CYCLIC PREFIX (CP)

$N_{cp} \times \frac{1}{B}$

$= \frac{N_{cp}}{B}$.

- CP has N_{cp} =80 samples. What is duration of CP?

$$\frac{1}{B} \times N_{cp} = \frac{80}{10 \times 10^6} = 8\ \mu s$$

$= 8 \times 10^{-6}$.

$=$ CP DURATION.

$\frac{N_{cp}}{N} = \frac{80}{1024}$

Example OFDM System

$$\begin{array}{r} 1024 \\ +\ \ 80 \\ \hline 1104 \end{array}$$

- How many samples after addition of CP?

$$\underbrace{\underbrace{x(N-\tilde{L}),\ldots,x(N-2),x(N-1)}_{\text{Cyclic Prefix}=\underline{80}},\underbrace{x(0),x(1),x(2),\ldots,x(N-1)}_{\text{Original Samples}=\underline{1024}}}_{Total=\ \underline{1104\ \text{samples}}}$$

$= \underbrace{N + N_{cp}}_{\text{Total \# Samples with CP}}.$

$$\begin{array}{r} 102.4 \\ 8.0 \\ \hline 110.4\ \mu s \end{array}$$

= Total Duration with CP.

Example OFDM System

CYCLIC PREFIX.

80 samples.

$x(943), \ldots, x(1022), x(1023), x(0), x(1), \ldots, x(1023)$

- How many samples after addition of CP?

$$\underbrace{\underbrace{x(N-\tilde{L}), \ldots, x(N-2), x(N-1)}_{\textbf{Cyclic Prefix=80 samples}}, \underbrace{x(0), x(1), x(2), \ldots, x(N-1)}_{\textbf{Original Samples=1024}}}_{Total = 1104\ samples}$$

$$\begin{array}{r} 1023 \\ 80 \\ \hline 943 \\ \hline \end{array}$$

TIME DOMAIN.

Example OFDM System

- Let us say each subcarrier is loaded with **QPSK symbols.** 2 Bits/symbol.
- What is the **effective bit-rate**?

$$= \frac{2 \times N}{T + T_{cp}} = \frac{2 \times 1024 \text{ Bits}}{110.4 \times 10^{-6} \text{ s}}$$

$$18.55 \times 10^{6} \text{ Bits/s}$$

$$= 18.55\ Mbps$$

Example OFDM System

- Let us say each subcarrier is loaded with ***QPSK symbols.***
- What is the **effective bit-rate**?

$$\frac{2\,\dfrac{bits}{subcarrier} \times 1024\, subcarriers}{110.4\,\mu s}$$

$$= 18.55\, Mbps$$

Example OFDM System

- % **Loss in spectral efficiency** = ?

$$\frac{N_{cp}}{N + N_{cp}} \times 100 = \frac{80}{80+1024} \times 100$$

$$= 7.24\ \%$$

AS CP DURATION INCREASES LOSS IN SPECTRAL EFFICIENCY INCREASES!

LOSS IN SPECTRAL EFFICIENCY.

BECAUSE OF CYCLIC PREFIX

Example OFDM System

- % **Loss in spectral efficiency** = ?

$$\frac{80}{1104} \times 100 = 7.24\%$$

Example OFDM Transmission

N = 4 subcarriers.

- Consider an $N = 4$ subcarrier system
- Let

LOADED ON SUBCARRIERS.

$$X(0) = 1 + j$$
$$X(1) = 1 - j$$
$$X(2) = 1 + 2j$$
$$X(3) = 2 - j$$

Example OFDM Transmission

- **What are the time-domain samples?**
- $x(0), x(1), x(2), x(3)$ **are given by the IFFT**

$N=4$

$$x(l) = \frac{1}{N}\sum_{k=0}^{3} X_k \cdot e^{j2\pi\frac{kl}{N}}$$

$$= \underbrace{\frac{1}{4}\sum_{k=0}^{3} X_k \cdot e^{j\frac{\pi}{2}\cdot kl}}_{\text{IFFT } N=4}$$

TIME DOMAIN SAMPLES.

Example OFDM Transmission

- **What are the time-domain samples?**
- $x(0), x(1), x(2), x(3)$ **are given by the IFFT**

$$x(l) = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi\frac{kl}{N}}$$

IFFT

Example OFDM Transmission

$\ell = 0$

- **What are the time-domain samples?**
- $x(0), x(1), x(2), x(3)$ **are given by the IFFT**

$$x(0) = \frac{1}{4}\left(X(0) + X(1) + X(2) + X(3)\right)$$

$$= \frac{1}{4}\left(1 + \cancel{j} + 1 - \cancel{j} + 1 + 2j + 2 - j\right)$$

$x(0)$.

$$= \frac{1}{4}\left(5 + j\right) = \frac{5}{4} + \frac{1}{4}j$$

Example OFDM Transmission

- **What are the time-domain samples?**
- $x(0), x(1), x(2), x(3)$ **are given by the IFFT**

$$x(0) = \frac{1}{4}\big(X(0) + X(1) + X(2) + X(3)\big)$$

$$= \frac{1}{4}(1 + j + 1 - j + 1 + 2j + 2 - j)$$

$$= \frac{5}{4} + \frac{1}{4}j$$

Example OFDM Transmission

- **What are the time-domain samples?**
- $x(0), x(1), x(2), x(3)$ **are given by the IFFT**

$$x(1) = \frac{1}{4}\left(X(0) + jX(1) - X(2) - jX(3) \right)$$

$$= \frac{1}{4}\left(1 + j + j(1-j) - (1+2j) - j(2-j) \right)$$

$x(1)$

$$= \frac{1}{4}\left(\cancel{1} + \cancel{j} + \cancel{j} + \cancel{1} - \cancel{1} - \cancel{2j} - 2j - \cancel{1} \right)$$

$$= \frac{1}{4}(-2j) = -\frac{1}{2}j$$

Example OFDM Transmission

- **What are the time-domain samples?**
- $x(0), x(1), x(2), x(3)$ **are given by the IFFT**

$$x(1) = \frac{1}{4}\big(X(0) + jX(1) - X(2) - jX(3)\big)$$

$$= \frac{1}{4}\big(1 + j + (1 - j)j - (1 + 2j) - j(2 - j)\big)$$

$$= \frac{1}{4}(0 - 2j) = -\frac{1}{2}j$$

Example OFDM Transmission

Sample $l = 2$

$$x(2) = -\frac{1}{4} + \frac{5}{4}j$$

$$x(3) = 0$$

Example OFDM Transmission

- The transmit samples are

Before Addition of CP.

N = 4 Samples.

$$\overbrace{x(0), x(1), x(2), x(3)}^{\text{Time Domain}}$$

$$= \frac{5}{4} + \frac{1}{4}j, \ -\frac{1}{2}j, \ -\frac{1}{4} + \frac{5}{4}j, \ 0$$

Example OFDM Transmission

- Let **cyclic prefix** be of duration 1 sample

  N_{cp}.

- The transmit frame is

CP.

$$\underbrace{x(3),\ x(0),\ x(1),\ x(2),\ x(3)}_{N+N_{cp}=4+1=5}$$

$$\underbrace{0,\ \frac{5}{4}+\frac{1}{4}j,\ -\frac{1}{2}j,\ -\frac{1}{4}+\frac{5}{4}j,\ 0}_{\text{TRANSMITTED OFDM BLOCK WITH CP.}}$$

Example OFDM Transmission Receiver?

- Consider the **output samples** given as

AFTER CP. REMOVAL

TIME DOMAIN

$$y(0) = 1, y(1) = \frac{1}{2}$$

$$y(2) = \frac{1}{2}, y(3) = 1$$

Example OFDM Transmission

FAST FOURIER TRANSFORM

- The subcarrier outputs are given by the FFT

N=4

$$Y_k = \sum_{l=0}^{N-1} y(l) e^{-j2\pi \frac{kl}{N}} = \sum_{l=0}^{3} y(l) e^{-j\frac{\pi}{2}kl}.$$

N=4pt FFT

AFTER CP REMOVAL

Example OFDM Transmission

- The subcarrier outputs are given by the FFT

$$Y_k = \sum_{l=0}^{N-1} y(l) e^{-j2\pi \frac{kl}{N}}$$

Example OFDM Transmission

OUTPUT ON SUBCARRIER. 0

$$Y_0 = y(0) + y(1) + y(2) + y(3)$$

$$= \left(1 + \frac{1}{2} + \frac{1}{2} + 1\right) = 3$$

Example OFDM Transmission

$$Y_0 = y(0) + y(1) + y(2) + y(3)$$

$$= 1 + \frac{1}{2} + \frac{1}{2} + 1 = 3$$

Example OFDM Transmission

$Y_1 = \quad y(0) - jy(1) - y(2) + jy(3)$

OUTPUT ON SUBCARRIER.

$= \quad 1 - j\frac{1}{2} - \frac{1}{2} + j1 = \frac{1}{2} + \frac{1}{2}j = Y_1$

Example OFDM Transmission

$$Y_1 = y(0) - jy(1) - y(2) + jy(3)$$

$$= 1 - j\frac{1}{2} - \frac{1}{2} + j1 = \frac{1}{2} + \frac{1}{2}j$$

Example OFDM Transmission

- Similarly, one can determine

$$Y_2 = 0$$

$$Y_3 = \frac{1}{2} - \frac{1}{2}j$$

Example OFDM Transmission

- Therefore, the subcarrier outputs are

$$Y_0 = 3$$

$$Y_1 = \frac{1}{2} + \frac{1}{2} j$$

$$Y_2 = 0$$

$$Y_3 = \frac{1}{2} - \frac{1}{2} j$$

OUTPUTS FOR N=4 SUBCARRIERS.

Example OFDM Transmission

$L = 2$ channel taps.

- Consider the **channel taps** given as

$$h(0) = 1, h(1) = \frac{1}{2}$$

H_0, H_1, H_2, H_3.

- $N = 4$ subcarriers
- What are the **subcarrier channel coefficients**?

Example OFDM Transmission

- The **subcarrier coefficients** are given by the zero padded FFT of channel taps

$$1, \frac{1}{2}, \underbrace{0, 0}_{N-L \text{ zeros}}$$

N=4
L=2
4-2 = 2 zeros.

$$\downarrow FFT$$

$$H_0, H_1, H_2, H_3$$

Example OFDM Transmission

- The **subcarrier channel coefficients** are

ZERO PADDED FFT

$$H_k = \sum_{l=0}^{L-1} h(l) e^{-j2\pi\frac{kl}{N}}$$

H_k.

$$= \sum_{l=0}^{1} h(l) e^{-j\frac{\pi}{2}.kl.} = h(0) + h(1) e^{-j\frac{\pi}{2}k.}$$

Example OFDM Transmission

$$H_0 = h(0) + h(1)$$

$$= 1 + \frac{1}{2} = \frac{3}{2}$$

Example OFDM Transmission

$$H_0 = h(0) + h(1) = 1 + \frac{1}{2} = \frac{3}{2}$$

Example OFDM Transmission

$$H_1 = h(0) - j\,h(1)$$

$$= 1 - \frac{1}{2} j$$

Example OFDM Transmission

$$H_1 = h(0) - jh(1) = 1 - j\frac{1}{2}$$

Example OFDM Transmission

- The **subcarrier channel coefficients** are

SUBCARRIER
CHANNEL
COEFFICIENTS.

$$H_0 = \frac{3}{2}$$

$$H_1 = 1 - j\frac{1}{2}$$

$$H_2 = \frac{1}{2}$$

$$H_3 = 1 + j\frac{1}{2}$$

BER of OFDM

BIT ERROR RATE OF OFDM SYMBOL

- **Consider the channel taps** $h(0), h(1), \ldots, h(L-1)$
- Assume they are **Rayleigh fading** with unit power

$E\{|h(l)|^2\} = 1$

- Noise samples $v(l)$ are i.i.d. with power N_0

$E\{|v(l)|^2\} = N_0.$

BER of OFDM

- **Symbols loaded on subcarriers have power** P

- Define **effective SNR** for QPSK as

NUMBER OF CHANNEL TAPS.

$$\rho_{eff} = \frac{L}{N} \times \frac{P}{N_0} = \frac{L}{N} \times SNR$$

NUMBER OF SUBCARRIERS.

BER of OFDM

- **BER of OFDM for QPSK is**

$$BER = \frac{1}{2}\left(1 - \sqrt{\frac{\rho_{eff}}{2+\rho_{eff}}}\right) \approx \frac{1}{2} \times \frac{1}{\rho_{eff}}$$

$$= \frac{1}{2} \cdot \frac{1}{\frac{L}{N} SNR}$$

$$BER = \frac{1}{2} \times \frac{N}{L \times SNR}$$

BER of OFDM

- **BER of OFDM for QPSK is**

BER OF OFDM FOR QPSK.

$$\frac{1}{2}\left(1-\sqrt{\frac{\rho_{eff}}{2+\rho_{eff}}}\right) \approx \frac{1}{2} \times \frac{1}{\rho_{eff}} = \frac{1}{2} \times \frac{N}{L \times SNR}$$

BER of OFDM Example

- **What is the BER of OFDM for QPSK with** $SNR = 30\ dB, L = 8$ Channel taps and $N = 64$ subcariers

$SNR = 30\,dB = 10^3$

$L = 8$

$N = 64$

$$\rho_{eff} = \frac{L}{N} \times \frac{P}{N_0} = \frac{L}{N} \times SNR$$

$$= \frac{8}{64} \times 10^3 = \frac{1}{8} \times 10^3 = \rho_{eff}$$

BER of OFDM Example

- **BER is**

$$\frac{1}{2}\left(1-\sqrt{\frac{\rho_{eff}}{2+\rho_{eff}}}\right)\approx\frac{1}{2}\times\frac{1}{\rho_{eff}}$$

$$=\quad \frac{1}{2}\times\frac{1}{\frac{1}{8}\times 10^{3}}=\frac{1}{2}\times\frac{8}{10^{3}}$$

$$=\quad 4\times 10^{-3} = \text{BER.}$$

BER of OFDM Example

- **BER is**

$$\frac{1}{2}\left(1-\sqrt{\frac{\rho_{eff}}{2+\rho_{eff}}}\right)\approx\frac{1}{2}\times\frac{1}{\rho_{eff}}$$

$$=\frac{1}{2}\times\frac{1}{\frac{10^3}{8}}=4\times10^{-3}$$

Instructors may use this white area (14.5 cm / 25.4 cm) for the text.
Three options provided below for the font size.

Font: Avenir (Book), Size: 32, Colour: Dark Grey

Font: Avenir (Book), Size: 28, Colour: Dark Grey

Font: Avenir (Book), Size: 24, Colour: Dark Grey

Do not use the space below.

Instructors may use this white area (14.5 cm / 25.4 cm) for the text.
Three options provided below for the font size.

Font: Avenir (Book), Size: 32, Colour: Dark Grey

Font: Avenir (Book), Size: 28, Colour: Dark Grey

Font: Avenir (Book), Size: 24, Colour: Dark Grey

Do not use the space below.