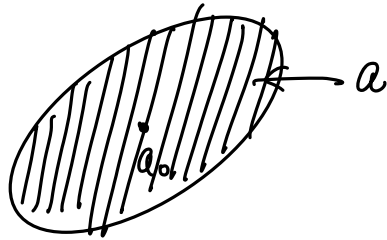


Ellipsoidal Uncertainty

$$(L.P) \quad \min c^T x \\ a^T x \leq b$$



$$a \in \mathcal{E}(a_0, P) \\ = \{ \underline{a} \in \mathbb{R}^n \mid (a - a_0)^T P^{-1} (a - a_0) \leq 1 \}$$

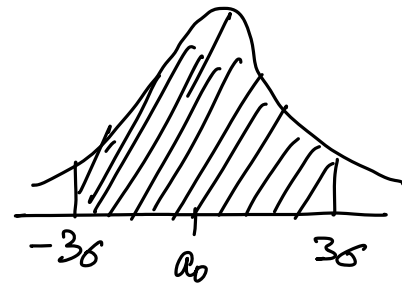
$$P \succ 0$$

$$\text{Eg: } n=1 \quad a \in [-1, 1]$$

interval

$$\text{Eg: } a \sim N(a_0, \sigma^2) \\ a_0 \in [a_0 - 3\sigma, a_0 + 3\sigma]$$

similarly for n



amounts to infinite # constraints

$$(n=1) \quad \min c x \\ a x \leq b \quad a \in [-1, 1]$$

$$\text{or} \quad \min c x \\ \left(\max_{a \in [-1, 1]} a x \right) \leq b$$

$$\text{Aside} \quad \max_{a \in [-1, 1]} a x \quad (\text{given } x) = |x|$$

$$a^* = \begin{cases} 1 & \text{when } x > 0 \\ -1 & \text{when } x < 0 \end{cases} \Rightarrow a^* x = |x|$$

$$\min_c cx$$

$$|x| \leq b$$

 \Leftrightarrow

$$\min_c cx$$

$$-b \leq x \leq b$$

$$\min_x c^T x$$

$$a^T x \leq b$$

 \Leftrightarrow

$$a \in \mathcal{E}(a_0, P)$$

$$\min_x c^T x$$

$$\left(\max_{a \in \mathcal{E}(a_0, P)} a^T x \right) \leq b$$

Aside

given x : $\max_a a^T x$

$$(a - a_0)^T P^{-1} (a - a_0) \leq 1$$

wrt a

$\rightsquigarrow g(x)$

$$g(x) \leq b$$

$$a \in \mathcal{E}(a_0, P) = \{ a_0 + \sqrt{P} u \mid \|u\|_2 \leq 1 \}$$

$$\max_{\substack{u \\ \|u\|_2 \leq 1}} (a_0 + \sqrt{P} u)^T x = a_0^T x + \max_{\substack{u \\ \|u\|_2 \leq 1}} u^T \sqrt{P} x$$

$$a_0^T x + \|\sqrt{P} x\|_2$$

$$\max_{\substack{u \\ \|u\|_2 \leq 1}} u^T w$$

$$w = \sqrt{P} x$$

$$u^* = \frac{w}{\|w\|}$$

$$u^{*T} w = \|w\|_2$$

$$\min_c c^T x$$

$$\|\sqrt{P} x\|_2 \leq b - a_0^T x \quad (\text{SOCP})$$