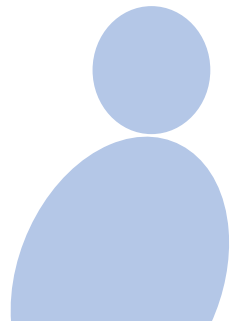


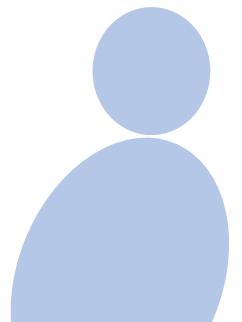
# eMasters in Communication Systems

Prof. Aditya  
Jagannatham



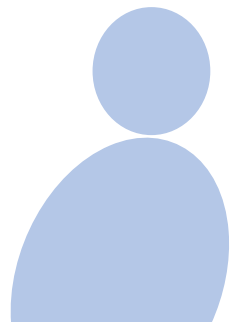
**Elective Module:**

**Estimation for Wireless  
Communication**



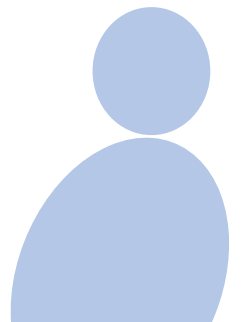
# Chapter 4

## Vector Parameter Estimation



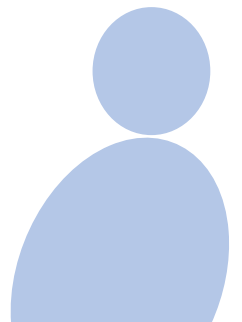
# Least Squares Solution

- The Least Squares solution can be **derived** as follows



# Least Squares Solution

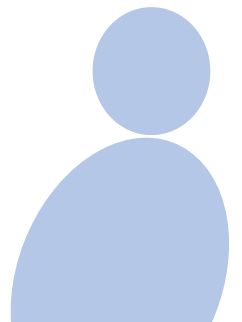
- The LS cost function is



# Least Squares Solution

- The LS cost function is

$$\|\bar{\mathbf{y}} - \mathbf{X}\bar{\mathbf{h}}\|^2$$

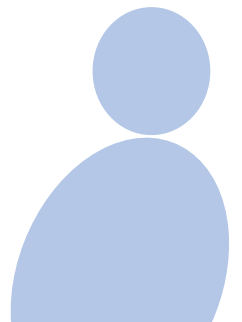


# Least Squares Solution

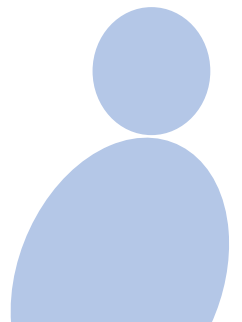
- The LS cost function can be simplified as

$$\|\bar{\mathbf{y}} - \mathbf{X}\bar{\mathbf{h}}\|^2$$

=



# Least Squares Solution





# Least Squares Solution

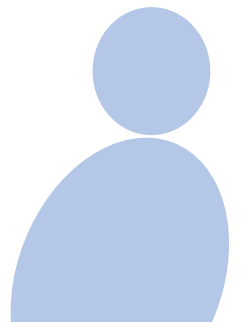
- The **LS cost function** can be simplified as

$$\begin{aligned} & \|\bar{\mathbf{y}} - \mathbf{X}\bar{\mathbf{h}}\|^2 \\ &= (\bar{\mathbf{y}} - \mathbf{X}\bar{\mathbf{h}})^T (\bar{\mathbf{y}} - \mathbf{X}\bar{\mathbf{h}}) \\ &= \bar{\mathbf{y}}^T \bar{\mathbf{y}} - 2\bar{\mathbf{h}}^T \mathbf{X}^T \bar{\mathbf{y}} + \bar{\mathbf{h}}^T \mathbf{X}^T \mathbf{X} \bar{\mathbf{h}} \end{aligned}$$

# Least Squares Solution

- To minimize, we now calculate the **gradient** and set to zero

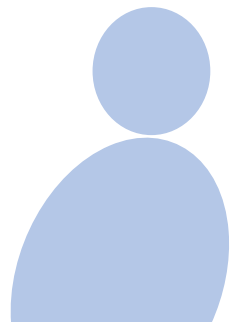
$$\nabla f(\bar{\mathbf{h}}) =$$



# Least Squares Solution

- We now calculate the gradient

$$\nabla f(\bar{\mathbf{h}}) = \begin{bmatrix} \frac{\partial f(\bar{\mathbf{h}})}{\partial h_1} \\ \frac{\partial f(\bar{\mathbf{h}})}{\partial h_2} \\ \vdots \\ \frac{\partial f(\bar{\mathbf{h}})}{\partial h_M} \end{bmatrix}$$

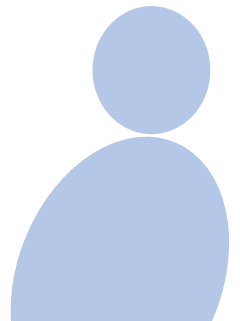


# Least Squares Solution

- We use the following principles

$$\bar{\mathbf{c}}^T \bar{\mathbf{h}} = c_1 h_1 + \cdots + c_M h_M = \bar{\mathbf{h}}^T \bar{\mathbf{c}}$$

$$\nabla \bar{\mathbf{c}}^T \bar{\mathbf{h}} = \nabla \bar{\mathbf{h}}^T \bar{\mathbf{c}} =$$

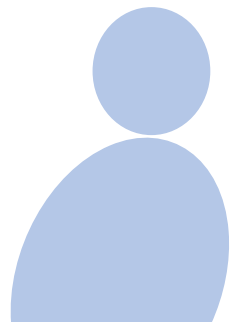


# Least Squares Solution

- We use the following principles

$$\bar{\mathbf{c}}^T \bar{\mathbf{h}} = c_1 h_1 + \cdots + c_M h_M$$

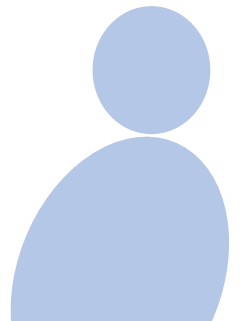
$$\nabla \bar{\mathbf{c}}^T \bar{\mathbf{h}} = \nabla \bar{\mathbf{h}}^T \bar{\mathbf{c}} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_M \end{bmatrix}$$



# Least Squares Solution

- For a symmetric matrix  $\mathbf{P} = \mathbf{P}^T$

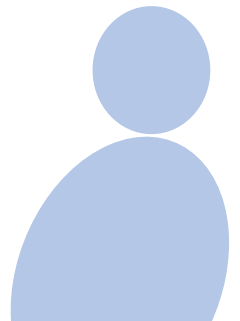
$$\nabla \bar{\mathbf{h}}^T \mathbf{P} \bar{\mathbf{h}} =$$



# Least Squares Solution

- For a symmetric matrix  $\mathbf{P} = \mathbf{P}^T$

$$\nabla \bar{\mathbf{h}}^T \mathbf{P} \bar{\mathbf{h}} = 2\mathbf{P} \bar{\mathbf{h}}$$

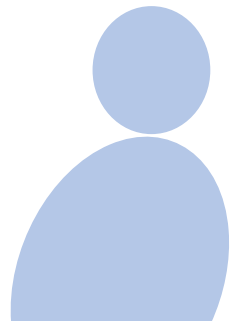


# Least Squares Solution

- Therefore, it follows that

$$\nabla \|\bar{\mathbf{y}} - \mathbf{X}\bar{\mathbf{h}}\|^2$$

=

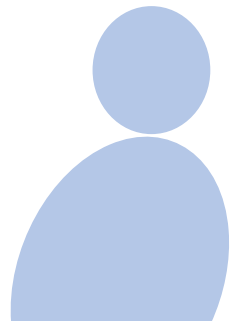




# Least Squares Solution

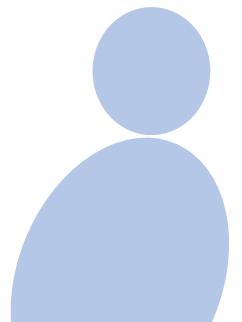
- Therefore, it follows that

$$\begin{aligned}\nabla \|\bar{\mathbf{y}} - \mathbf{X}\bar{\mathbf{h}}\|^2 &= \\ \nabla (\bar{\mathbf{y}}^T \bar{\mathbf{y}} - 2\bar{\mathbf{h}}^T \mathbf{X}^T \bar{\mathbf{y}} + \bar{\mathbf{h}}^T \mathbf{X}^T \mathbf{X} \bar{\mathbf{h}}) &= \\ = -2\mathbf{X}^T \bar{\mathbf{y}} + 2\mathbf{X}^T \mathbf{X} \bar{\mathbf{h}}\end{aligned}$$



# Least Squares Solution

- Setting gradient equal to zero yields



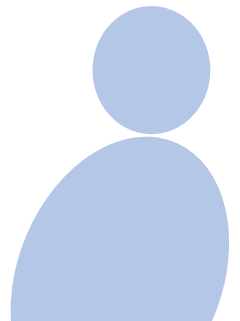
# Least Squares Solution

- Setting gradient equal to zero yields

$$-2\mathbf{X}^T \bar{\mathbf{y}} + 2\mathbf{X}^T \mathbf{X} \bar{\mathbf{h}} = \mathbf{0}$$

$$\Rightarrow \mathbf{X}^T \mathbf{X} \bar{\mathbf{h}} = \mathbf{X}^T \bar{\mathbf{y}}$$

$$\Rightarrow \hat{\mathbf{h}} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \bar{\mathbf{y}}$$



Instructors may use this white area (14.5 cm / 25.4 cm) for the text.  
Three options provided below for the font size.

Font: Avenir (Book), Size: 32, Colour: Dark Grey

Font: Avenir (Book), Size: 28, Colour: Dark Grey

Font: Avenir (Book), Size: 24, Colour: Dark Grey

Do not use the space below.

