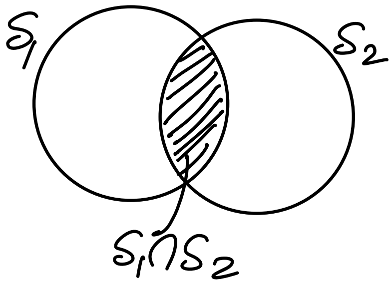
6. Operations:

Intersection : S_1, S_2 convex $\Rightarrow$ $S_1 \cap S_2$ convex



$X \in S_1 \cap S_2 \Rightarrow X \in S_1 \qquad X \in S_2$

$Y \in S_1 \cap S_2 \Rightarrow Y \in S_1 \qquad Y \in S_2$

$\theta X + (1-\theta) Y \in S_1 \qquad \theta X + (1-\theta) Y \in S_2$

$\theta X + (1-\theta) Y \in S_1 \cap S_2$

$\Rightarrow S_1 \cap S_2$ convex

also valid for infinite intersections

$$C(u) = \{ X \in S^n \mid u^T X u \geq 0 \}$$

set depends on u (given u)

polyhedron

Given u

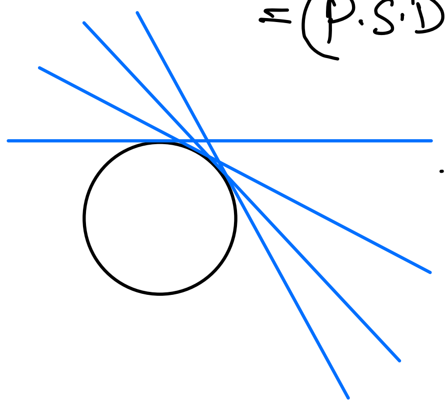
$u^T X u \geq 0$ half space

$\Rightarrow C(u)$ polyhedron

$$C = \bigcap_{u \in \mathbb{R}^n} C(u)$$

$$= \{ X \in \mathbb{R}^n \mid u^T X u \geq 0 \ \forall\, u \in \mathbb{R}^n \}$$

$$= (\text{P.S.D. Cone}) \quad S_+^n$$



Eg. norm ball = $\bigcap$ half spaces

Affine transformation $\quad a(x) = Ax+b \qquad b \in \mathbb{R}^m, x \in \mathbb{R}^n$

$$a : \mathbb{R}^n \longrightarrow \mathbb{R}^m$$

$$x \mapsto Ax+b$$

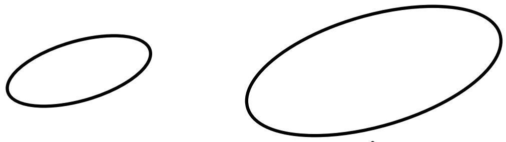
$C \subseteq \mathbb{R}^n \qquad A(C) = B = \{ a(\underline{x}) \mid \underline{x} \in C \}$

image of C under $Ax+b$

C convex $\Rightarrow$ $\overset{A(C)}{\text{image }(C)}$ convex

Eg: scaling $\quad A(C) = \{\alpha \underline{x} \mid \underline{x} \in C\}$

$$A = \alpha I, \quad b = 0$$



translation $\quad A(C) = \{x + x_0 / x \in C\}$

Projection $\quad A(C) = \{P\underline{x} \mid \underline{x} \in C\}$

$$x \in \mathbb{R}^n$$

$$P = \big[\underbrace{I_k}_{k \times k} \;\big|\; \underbrace{0}_{k \times (n-k)}\big]_{k \times n}$$

picks $x_1, x_2 \ldots x_k$ but drops $x_{k+1} \ldots x_n$

discard some components of x

Eg $\quad B(0,1) \longrightarrow \mathcal{E}(x_0, P)$

$$C = \{u \mid \|u\| \leq 1\} \quad = \quad B(0,1)$$

$$A(C) = \{\sqrt{P}u + x_0 \mid \|u\| \leq 1\} = \mathcal{E}(x_0, P)$$