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...dare to dream; care to win...

Class Assignment and Solution – From Live Interaction on 11th Feb 2024

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eMasters in Communication Systems, IITK
EE902: Advanced ML Techniques for Wireless Technology
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Question:
In GDA/LDA, a class is chosen that **maximizes the likelihood for a feature vector $\bar{x}$**. The classifier computes the probability of a data vector belonging to each class given its features, and then assigns the sample to the class with the highest probability. Assuming equable probabilities of the classes in a binary classification scenario ($\mathcal{C}_0, \mathcal{C}_1$), this can be mathematically represented as:

Choose $\boldsymbol{\mathcal{C}_0}$ if: $\boldsymbol{p(\bar{x};\mathcal{C}_0) \geq p(\bar{x};\mathcal{C}_1)}$

However, if the classes have prior probabilities:

$p(\mathcal{C}_0) = p_0$
$p(\mathcal{C}_1) = p_1$

Determine the linear classifier for the above case.

Solution:
The linear classifier needs to adjust the decision boundary to reflect the prior probabilities of the classes by incorporating these prior probabilities into the computation of class conditional probabilities to arrive at the ***posterior probabilities*** for the comparison and assigning the feature vector to the highest posterior probability class.

From the maximum likelihood rule with equi-prior probability classes:
Choose $\mathcal{C}_0$, if:

$$p(\bar{x};\mathcal{C}_0) \geq p(\bar{x};\mathcal{C}_1)$$
$$\Rightarrow \frac{1}{\sqrt{(2\pi)^n|R|}} e^{-\frac{1}{2}((\bar{x}-\bar{\mu}_0)^T R^{-1}(\bar{x}-\bar{\mu}_0)} \geq \frac{1}{\sqrt{(2\pi)^n|R|}} e^{-\frac{1}{2}((\bar{x}-\bar{\mu}_1)^T R^{-1}(\bar{x}-\bar{\mu}_1)}$$
$$\Rightarrow (\bar{x}-\bar{\mu}_0)^T R^{-1}(\bar{x}-\bar{\mu}_0) \leq (\bar{x}-\bar{\mu}_1)^T R^{-1}(\bar{x}-\bar{\mu}_1)$$

Simplified as:

Choose $\boldsymbol{\mathcal{C}_0}$: $\boldsymbol{\bar{h}^T(\bar{x}-\tilde{\mu}) \geq 0}$
Choose $\boldsymbol{\mathcal{C}_1}$: $\boldsymbol{\bar{h}^T(\bar{x}-\tilde{\mu}) < 0}$
Where:

$$\boldsymbol{\tilde{\mu} = \frac{1}{2}(\bar{\mu}_0 + \bar{\mu}_1)}$$
$$\boldsymbol{\bar{h} = R^{-1}(\bar{\mu}_0 - \bar{\mu}_1)}$$

Considering $p(\bar{x} \mid \mathcal{C}_i)$ as the PDF of feature vector $\bar{x}$ (Gaussian distribution), then the posterior probability of class $\mathcal{C}_i$ given feature set $\bar{x}$, as per Baye's theorem:

$$P(\mathcal{C}_i \mid \bar{x}) = \frac{p(\bar{x} \mid \mathcal{C}_i) \times P(\mathcal{C}_i)}{p(\bar{x})}$$

Where:

- $p(\bar{x} \mid \mathcal{C}_i)$ is the likelihood of observing the feature set $\bar{x}$ given $\mathcal{C}_i$ – modelled as Gaussian distribution
- $P(\mathcal{C}_i)$ is the prior probability of class $\mathcal{C}_i$
- $p(\bar{x})$ is the probability of observing feature set $\bar{x}$

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Then, the maximum likelihood rule with prior probability classes:
Choose $\mathcal{C}_0$, if:

$$p(\mathcal{C}_0 |\, \bar{x}) \geq p(\mathcal{C}_1 |\, \bar{x})$$

$$\Longrightarrow \frac{p(\bar{x} \mid \mathcal{C}_0) \times P(\mathcal{C}_0)}{p(\bar{x})} \geq \frac{p(\bar{x} \mid \mathcal{C}_1) \times P(\mathcal{C}_1)}{p(\bar{x})}$$

$$\Longrightarrow p(\bar{x} \mid \mathcal{C}_0) \times P(\mathcal{C}_0) \geq p(\bar{x} \mid \mathcal{C}_1) \times P(\mathcal{C}_1)$$

$$\Longrightarrow \frac{1}{\sqrt{(2\pi)^n |R|}} e^{-\frac{1}{2}((\bar{x}-\bar{\mu}_0)^T R^{-1} (\bar{x}-\bar{\mu}_0)} \times P(\mathcal{C}_0) \geq \frac{1}{\sqrt{(2\pi)^n |R|}} e^{-\frac{1}{2}((\bar{x}-\bar{\mu}_1)^T R^{-1} (\bar{x}-\bar{\mu}_1)} \times P(\mathcal{C}_1)$$

$$\Longrightarrow (\bar{x} - \bar{\mu}_0)^T R^{-1} (\bar{x} - \bar{\mu}_0) \times P(\mathcal{C}_0) \leq (\bar{x} - \bar{\mu}_1)^T R^{-1} (\bar{x} - \bar{\mu}_1) \times P(\mathcal{C}_1)$$

∴ The simplified linear classifier is:

Choose $\boldsymbol{\mathcal{C}_0}$: $\boldsymbol{\bar{h}^T (\bar{x} - \tilde{\mu}) \times P(\mathcal{C}_0) \geq 0}$

Choose $\boldsymbol{\mathcal{C}_1}$: $\boldsymbol{\bar{h}^T (\bar{x} - \tilde{\mu}) \times P(\mathcal{C}_1) < 0}$

Where:

$$\tilde{\mu} = \frac{1}{2}(\bar{\mu}_0 + \bar{\mu}_1)$$

$$\bar{h} = R^{-1}(\bar{\mu}_0 - \bar{\mu}_1)$$

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