eMasters in Communication Systems

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Elective Module:

Detection for Wireless Communication

Chapter 10

CR

5G/6G Technology

Cognitive Radio – Spectrum Sensing

Cognitive Radio

- **Traditional** spectrum allocation is STATIC and **inflexible**.
  - This leads to 'SPECTRAL HOLES'.

Cognitive Radio

- **Traditional** spectrum allocation is **static** and **inflexible**.
  - This leads to **spectral holes**.

Vacant spectrum

Cognitive Radio

- Radio spectrum allocated to licensed users/primary users.
  - cannot be utilized by unlicensed users secondary users.
  - even when it is **underutilized** or **vacant**!

Cognitive Radio

- Radio spectrum allocated to **licensed users**
  - cannot be utilized by **unlicensed users**
  - even when it is **underutilized** or **vacant**!

Cognitive Radio

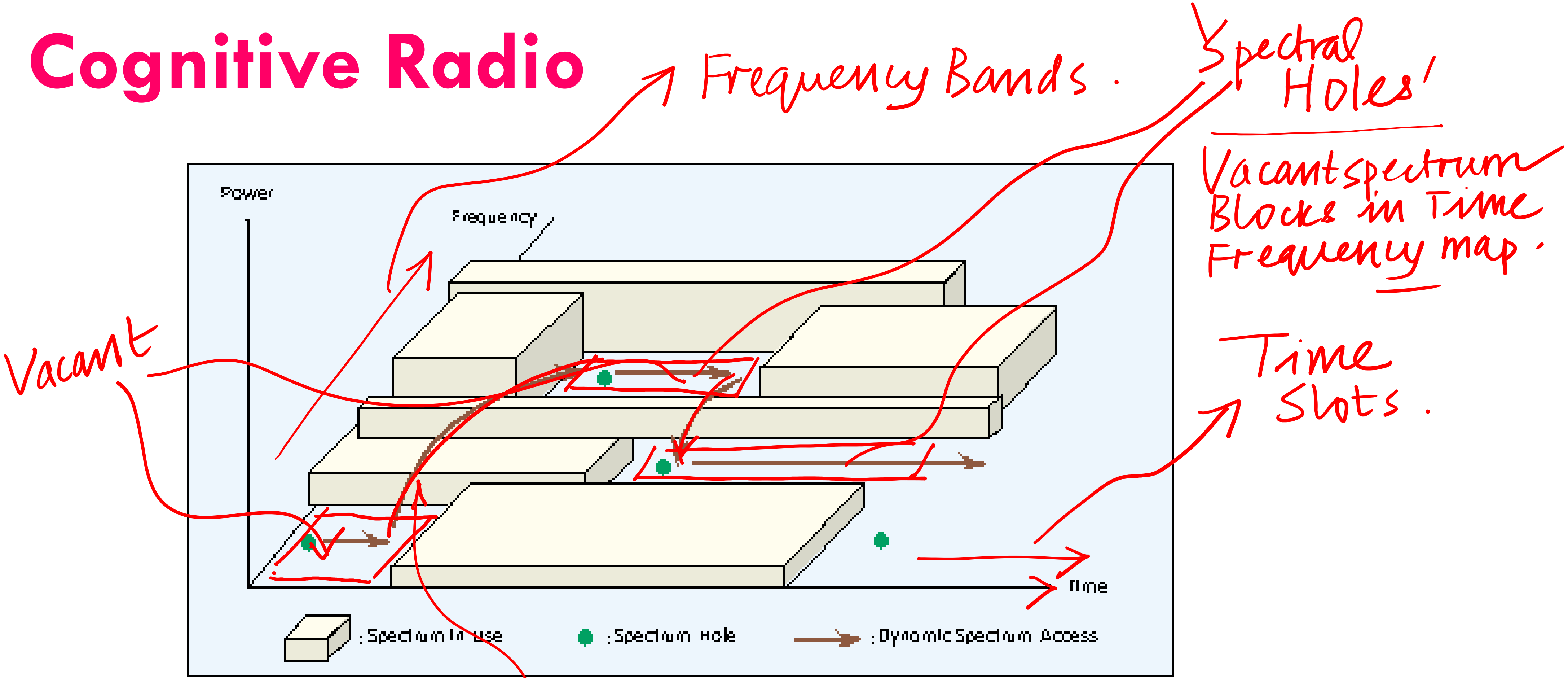
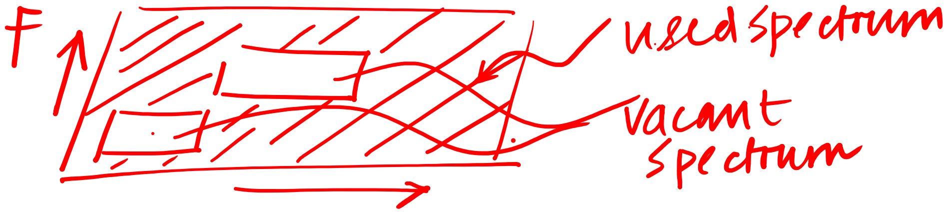


Figure 9. Spectrum hole.

Secondary users can dynamically access spectrum holes.



Cognitive Radio

- The main goal of cognitive radio is to enable Dynamic Spectrum Access.
  - by a few selected Secondary users.
- This **significantly improves** the efficiency of **spectrum utilization**

Efficiency of Spectrum Utilization η

$$\eta = \frac{\text{Shaded Area}}{\text{Total Area}}.$$

Cognitive Radio

unlicensed users.

- The main goal of cognitive radio is to enable **dynamic spectrum access**
  - by a few selected **secondary users**
- This **significantly improves** the efficiency of **spectrum utilization**

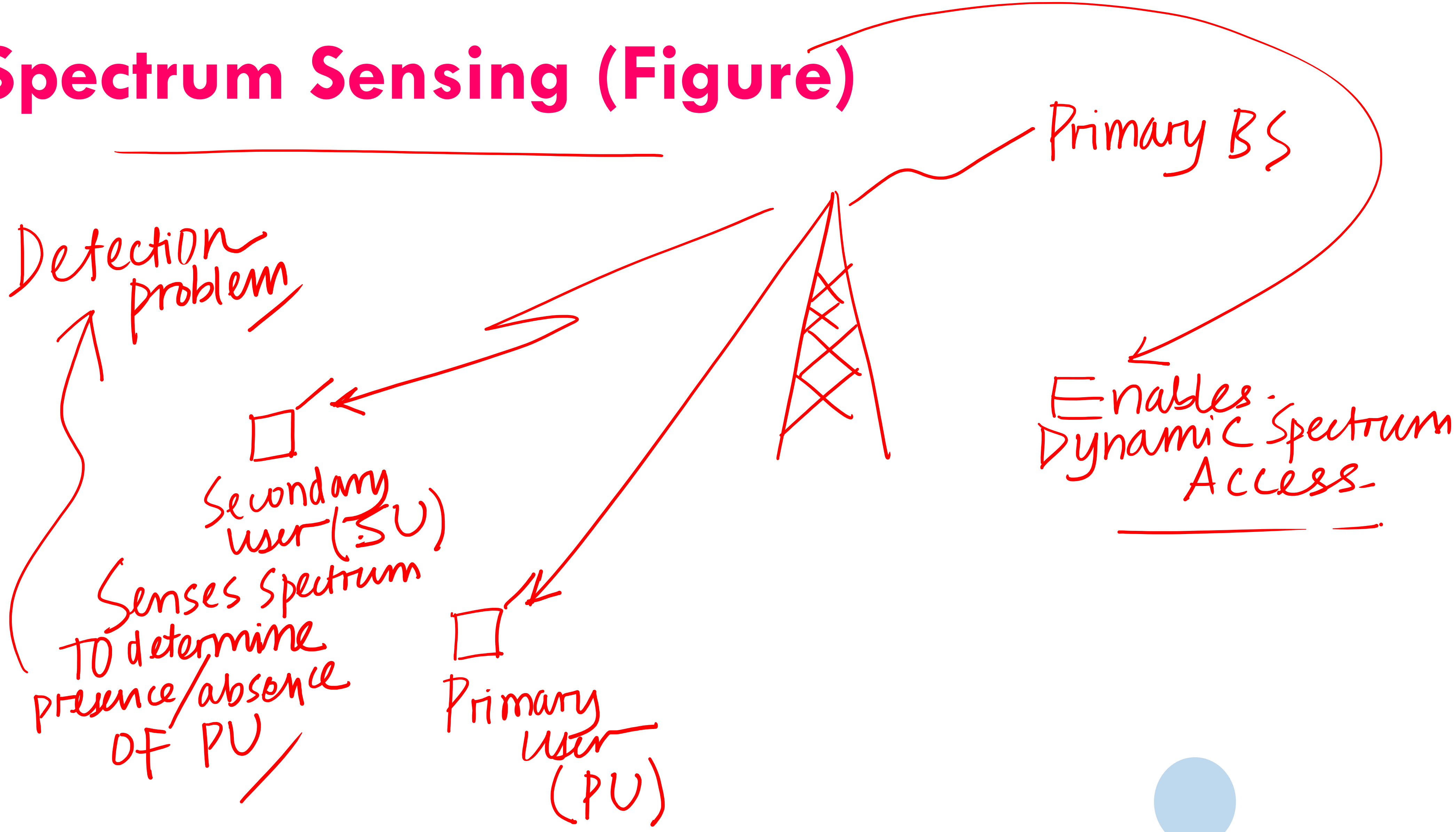
Cognitive Radio

- Therefore, SPECTRUM SENSING is a key aspect of CR…
  - To determine the **presence/ absence** of primary users

Cognitive Radio

- Therefore, **spectrum sensing** is a key aspect of CR…
  - To determine the **presence/ absence** of primary users

Spectrum Sensing (Figure)



Spectrum Sensing

- Let $y(1), y(2), \ldots, y(N)$ denote the **output symbols**.

Spectrum Sensing

- The corresponding **input-output model** is $\bar{y} = \bar{s} + \bar{v}$

Random Signal.

$$\begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(N) \end{bmatrix} = \begin{bmatrix} s(1) \\ s(2) \\ \vdots \\ s(N) \end{bmatrix} + \begin{bmatrix} v(1) \\ v(2) \\ \vdots \\ v(N) \end{bmatrix}$$

Spectrum Sensing

$$\underbrace{\begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(N) \end{bmatrix}}_{\bar{\mathbf{y}}} = \underbrace{\begin{bmatrix} s(1) \\ s(2) \\ \vdots \\ s(N) \end{bmatrix}}_{\bar{\mathbf{s}}} + \underbrace{\begin{bmatrix} v(1) \\ v(2) \\ \vdots \\ v(N) \end{bmatrix}}_{\bar{\mathbf{v}}}$$

Spectrum Sensing

- Let $s(i) = s_I(i) + js_Q(i)$

Complex Gaussian Signal. $\mathcal{CN}(0, \sigma_s^2)$

circularly symmetric complex Gaussian.

$s_I(i) \sim \mathcal{N}\left(0, \frac{\sigma_s^2}{2}\right)$

$s_Q(i) \sim \mathcal{N}\left(0, \frac{\sigma_s^2}{2}\right)$

independent identically distributed (iid)

Spectrum Sensing

complex: $E\{s(i)\} = 0$

$E\{|s(i)|^2\} = \sigma_s^2$

Signal Power

Complex Gaussian Signal samples.

- Let $s(i) = s_I(i) + j s_Q(i)$

inphase: $$s_I(i) \sim \mathcal{N}\left(0, \frac{\sigma_s^2}{2}\right)$$

Quadrature: $$s_Q(i) \sim \mathcal{N}\left(0, \frac{\sigma_s^2}{2}\right)$$

Spectrum Sensing

- Let $v(i) = v_I(i) + jv_Q(i)$

$\mathcal{CN}(0, \sigma^2)$

Zero mean circularly symmetric complex Gaussian

ZMCSCG.

$$v_I(i) \sim \mathcal{N}\left(0, \frac{\sigma^2}{2}\right)$$

$$v_Q(i) \sim \mathcal{N}\left(0, \frac{\sigma^2}{2}\right)$$

iid zero mean Gaussian.

Spectrum Sensing

inphase

Quadrature

- Let $v(i) = v_I(i) + jv_Q(i)$

$$v_I(i) \sim \mathcal{N}\left(0, \frac{\sigma^2}{2}\right)$$

$$v_Q(i) \sim \mathcal{N}\left(0, \frac{\sigma^2}{2}\right)$$

Spectrum Sensing

NULL Hypothesis.

- The **binary hypothesis testing** problem for **spectrum sensing** is

$$\mathcal{H}_0 : \bar{y} = \bar{v}$$

noise.

signal.

$$\mathcal{H}_1 : \bar{y} = \bar{s} + \bar{v}$$

$\bar{s}, \bar{v}$ are independent complex Gaussian.

Alternative hypothesis.

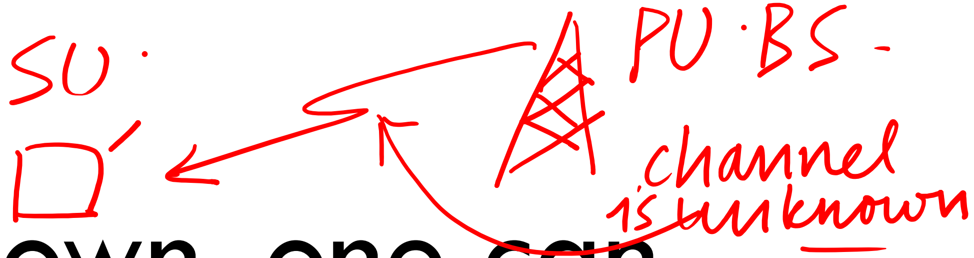
Spectrum Sensing

- The **binary hypothesis testing** problem for **spectrum sensing** is

$$\mathcal{H}_0 : \bar{\mathbf{y}} = \bar{\mathbf{v}}$$

$$\mathcal{H}_1 : \bar{\mathbf{y}} = \bar{\mathbf{s}} + \bar{\mathbf{v}}$$

Spectrum Sensing



- Since channel is unknown, one can use the **Energy Detector (ED)**
- **Test statistic is**

$$\|\bar{y}\|^2$$

$$= \underbrace{|y(1)|^2 + |y(2)|^2 + \cdots + |y(N)|^2}_{\text{Energy of Signal.}}$$

Spectrum Sensing

- Since channel is unknown, one can use the <u>**Energy Detector (ED)**</u>
- <u>**Test statistic is**</u>

$$\|\bar{\mathbf{y}}\|^2$$

Spectrum Sensing

Threshold.

- Compare this with a suitable threshold γ to get the **ED**

Choose $\mathcal{H}_1$: $\|\bar{y}\|^2 > \gamma$

Choose $\mathcal{H}_0$: $\|\bar{y}\|^2 \leq \gamma$

Spectrum Sensing

'Spectrum Sensing'

- Compare this with a suitable threshold γ to get the **ED**

$$\left.\begin{aligned} \|\bar{\mathbf{y}}\|^2 > \gamma &\Rightarrow \mathcal{H}_1 \\ \|\bar{\mathbf{y}}\|^2 \leq \gamma &\Rightarrow \mathcal{H}_0 \end{aligned}\right\}$$

Spectrum Sensing

- The **probability of false alarm** (P_{FA}) is

Signal is absent
But decision is signal present

$$P_{FA} = \Pr\left(\|\bar{y}\|^2 > \gamma ; \mathcal{H}_0\right)$$

Spectrum Sensing

- The **probability of false alarm** (P_{FA}) is

$$P_{FA} = \Pr(||\bar{\mathbf{y}}||^2 > \gamma|\mathcal{H}_0)$$

Spectrum Sensing

$$y(i) = y_I(i) + j y_Q(i)$$

$$= v_I(i) + j v_Q(i)$$

$$y_I(i), y_Q(i) \sim N\left(0, \frac{\sigma^2}{2}\right)$$

$$y_I(i) = v_I(i) \sim \mathcal{N}\left(0, \frac{\sigma^2}{2}\right)$$

$$y_Q(i) = v_Q(i) \sim \mathcal{N}\left(0, \frac{\sigma^2}{2}\right)$$

Spectrum Sensing

$$y(i) = y_I(i) + jy_Q(i)$$
$$= v_I(i) + jv_Q(i)$$

$$y_I(i), y_Q(i) \sim N\left(0, \frac{\sigma^2}{2}\right)$$

iid $\mathcal{N}\left(0, \frac{\sigma^2}{2}\right)$

Spectrum Sensing

$$\|\bar{\mathbf{y}}\|^2 > \gamma$$

$$= \sum_{i=1}^{N} |y_I(i)|^2 + |y_Q(i)|^2 > \gamma$$

$$= \sum_{i=1}^{N} \left|\frac{y_I(i)}{\sigma/\sqrt{2}}\right|^2 + \left|\frac{y_Q(i)}{\sigma/\sqrt{2}}\right|^2 > \frac{\gamma}{\sigma^2/2}$$

Spectrum Sensing

central chisquared RV
2N degrees of freedom.

$$\|\bar{\mathbf{y}}\|^2 > \gamma$$

iid Standard Gaussian RVs.

$$= \sum_{i=1}^{N} |y_I(i)|^2 + |y_Q(i)|^2 > \gamma$$

central.
$\Rightarrow \chi^2_{2N}$ RV

$$= \sum_{i=1}^{N} \left|\frac{y_I(i)}{\sigma/\sqrt{2}}\right|^2 + \left|\frac{y_Q(i)}{\sigma/\sqrt{2}}\right|^2 > \frac{\gamma}{\sigma^2/2}$$

Sum of squares of 2N
iid zero mean standard Gaussian
RVs.

Spectrum Sensing

- The **probability of false alarm** (P_{FA}) is

$$P_{FA} = \Pr(||\mathbf{y}||^2 > \gamma | \mathcal{H}_0)$$

$$= \Pr\left(\frac{||\mathbf{y}||^2}{\sigma^2/2} > \frac{\gamma}{\sigma^2/2} ; \mathcal{H}_0\right)$$

$$= Q_{\chi^2_{2N}}\left(\frac{\gamma}{\sigma^2/2}\right)$$

Complementary cumulative distribution Function (CCDF) of Chi squared RV 2N degrees of Freedom.

Spectrum Sensing

- The **probability of false alarm** (P_{FA}) is

$$P_{FA} = \Pr(||\mathbf{y}||^2 > \gamma | \mathcal{H}_0)$$

P_{FA} For Spectrum Sensing

$$= Q_{\chi^2_{2N}}\left(\frac{\gamma}{\sigma^2/2}\right)$$

Spectrum Sensing

- $Q_{\chi^2_{2N}}$ denotes the **complementary CDF** (CCDF) of a χ^2_{2N} random variable.

Spectrum Sensing

Signal is present
decision is $\mathcal{H}_1$.

- Consider next **hypothesis** $\mathcal{H}_1$.
- The <u>**probability of detection**</u> (P_D) is

$$P_D = \Pr\left(\|\bar{y}\|^2 > \gamma ; \mathcal{H}_1 \right)$$

Spectrum sensing

Spectrum Sensing

$$y(i) = y_I(i) + j y_Q(i)$$

$$= S_I(i) + j S_Q(i) + V_I(i) + j V_Q(i)$$

$$= \left(S_I(i) + V_I(i)\right) + j\left(S_Q(i) + V_Q(i)\right)$$

$$\mathcal{N}\left(0, \frac{\sigma_s^2 + \sigma^2}{2}\right)$$

$$y_I(i), y_Q(i) \sim$$

independent

$$S_I(i) \sim \mathcal{N}\left(0, \frac{\sigma_s^2}{2}\right)$$

$$V_I(i) \sim \mathcal{N}\left(0, \frac{\sigma^2}{2}\right)$$

$$S_I(i) + V_I(i) \sim \mathcal{N}\left(0, \frac{\sigma_s^2}{2} + \frac{\sigma^2}{2}\right)$$

Spectrum Sensing

$$y(i) = y_I(i) + j y_Q(i)$$
$$= s_I(i) + j s_Q(i) + v_I(i) + j v_Q(i)$$

$$y_I(i), y_Q(i) \sim N\left(0, \frac{\sigma_s^2 + \sigma^2}{2}\right)$$

inphase and quadrature components of signal.

Spectrum Sensing

$$\|\bar{\mathbf{y}}\|^2 > \gamma$$

$$= \sum_{i=1}^{N} |y_I(i)|^2 + |y_Q(i)|^2 > \gamma$$

$$= \underbrace{\sum_{i=1}^{N} \left| \frac{y_I(i)}{\sqrt{\frac{\sigma^2+\sigma_s^2}{2}}} \right|^2 + \left| \frac{y_Q(i)}{\sqrt{\frac{\sigma^2+\sigma_s^2}{2}}} \right|^2}_{} > \frac{\gamma}{(\sigma^2+\sigma_s^2)/2}$$

Sum of squares of 2N zero mean iid standard Gaussian RVs.

$\Rightarrow$ central χ^2_{2N} RV.

Spectrum Sensing

$$\|\bar{\mathbf{y}}\|^2 > \gamma$$

$$= \sum_{i=1}^{N} |y_I(i)|^2 + |y_Q(i)|^2 > \gamma$$

$$= \sum_{i=1}^{N} \left| \frac{y_I(i)}{\sqrt{\frac{\sigma_s^2 + \sigma^2}{2}}} \right|^2 + \left| \frac{y_Q(i)}{\sqrt{\frac{\sigma_s^2 + \sigma^2}{2}}} \right|^2 > \frac{\gamma}{\frac{\sigma_s^2 + \sigma^2}{2}}$$

Spectrum Sensing

- The **probability of detection** (P_{FA}) is

$$P_D = \Pr(||\mathbf{y}||^2 > \gamma|\mathcal{H}_1)$$

$$= \Pr\left(\frac{||\bar{y}||^2}{(\sigma_s^2+\sigma^2)/2} > \frac{\gamma}{(\sigma_s^2+\sigma^2)/2} ; \mathcal{H}_1\right)$$

$$= Q_{\chi^2_{2N}}\left(\frac{\gamma}{(\sigma_s^2+\sigma^2)/2}\right).$$

Spectrum Sensing

- The **probability of detection** (P_{FA}) is

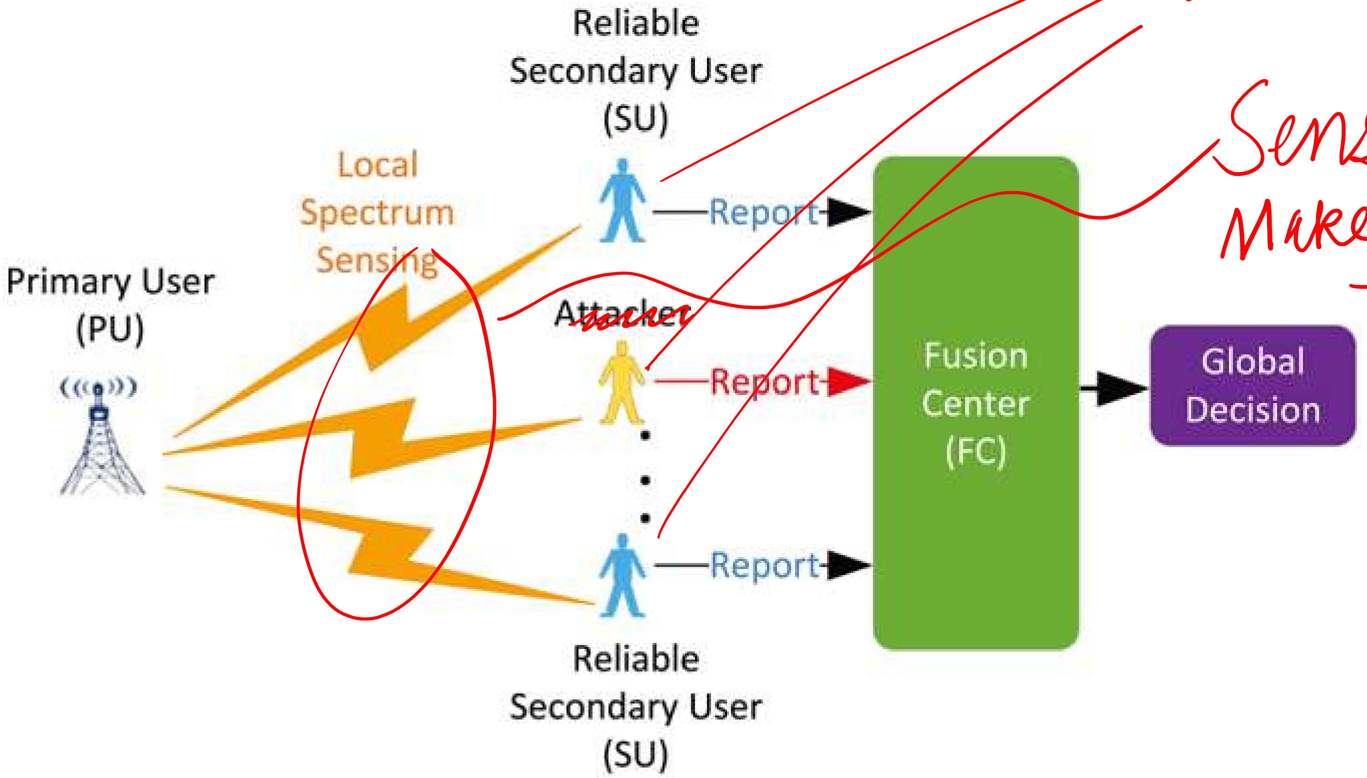
$$P_D = \Pr(||\mathbf{y}||^2 > \gamma|\mathcal{H}_1)$$

$$= Q_{\chi^2_{2N}}\left(\frac{\gamma}{(\sigma^2 + \sigma_s^2)/2}\right)$$

Cooperative Spectrum Sensing (Figure)

Fusion center (FC) makes a decision based on all decisions.

multiple secondary users



Sense spectrum
Make a decision
→ Transmit ~~Report~~
to Fusion center

Cooperative Spectrum Sensing

- In scenarios with **multiple SUs**,
  - each can communicate REPORT **sensing decision** to the FUSION CENTER.

Cooperative Spectrum Sensing

- In scenarios with **multiple SUs**,
  - each can communicate **sensing decision** to the **fusion center** (FC).

Cooperative Spectrum Sensing

- The FC can subsequently make a final decision
  - based on an appropriate FUSION RULE.

Cooperative Spectrum Sensing

- The FC can subsequently make a final decision
  - based on an appropriate **fusion rule**.

AND Rule

AND Fusion rule

- Fusion center **decides** H_1
  - only if **all sensors report** H_1 and H_0 otherwise.
  - i.e. FC performs **"AND"** of all decisions

Decision of FC = 1
Only if all sensors report 1
= 0 if one or more report 0
$\Rightarrow$ Optimistic rule.

AND Rule

Let number of sensors = K

- Let P_D, P_F denote the **probabilites of detection** and **false alarm** for **each sensor.**

AND Rule

- The **probability of false alarm** at the FC is

Only when all sensors Falsely report presence of PU. $\Rightarrow$ all sensors report 1 $\Rightarrow$ all sensors in FA!

independence of sensor decisions.

$$P_F^{FC} = \underbrace{P_{FA} \times P_{FA} \times \cdots \times P_{FA}}_{K \text{ Times}}$$

$$= P_{FA}^{K}.$$

AND Rule

Fusion center

K = # sensors.

- The **probability of false alarm** at the FC is

$$P_{FA}^{FC} = (P_{FA})^K = \left(Q\left(\frac{\gamma}{\chi^2_{2N}\sigma^2/2}\right)\right)^K.$$

For AND rule!

AND Rule

- The **overall** probability of detection for the FC is

Primary user present
each sensor reports presence of PU
$\Rightarrow$ Each sensor detects.

$$P_D^{FC} = \underbrace{P_D \times P_D \times \cdots \times P_D}_{K \text{ Times}}$$

$$= P_D^{K}$$

AND Rule

- The **overall** probability of detection for the FC is

Because each sensor has to detect presence of PU in and rule.

$$P_D^{FC} = (P_D)^K$$

Probability of detection using cooperative ~~sensing~~ AND rule & K sensors.

$$= Q\left(\frac{\gamma}{\chi^2_{2M}(\sigma_s^2+\sigma^2)/2}\right)^K$$

OR Rule

OR of all sensor decisions.

- FC decides H_1

⇒ FC decision = 1 if any sensor decision = 1
else if all sensors report 0 FC decision = 0.

- if at least **one of the sensors** reports H_1 and H_0 otherwise.

Conservative rule.

if any sensor Decision $D_i = 1$, FC decision $D_i = 1$
else if all $D_i = 0$, FC decision $D_i = 0$.

OR Rule

- The **probability of false alarm** is given as

False alarm at FC:
FC falsely detects if any sensor falsely detects!

Prob any sensor does NOT falsely detect $= 1 - P_{FA}$

$$P_F^{FC} =$$ Any sensor falsely detects

$= 1 -$ Prob. none falsely detects

$$= 1 - \underbrace{\left(1 - P_{FA}\right)^K}$$

Probability at least one sensor falsely detects.

$$= 1 - \left(1 - Q_{\chi^2_{2N}}\left(\frac{\gamma}{\sigma^2/2}\right)\right)^K.$$

OR Rule

- The **probability of false alarm** is given as

$$P_F^{FC} = 1 - (1 - P_F)^K$$

OR Rule

- Hence, **probability of detection** is given as

At least one sensor detects under $\mathcal{H}_1$.

Any sensor detects.

$$P_D^{FC} = \text{Probability at least one sensor detects.}$$

$$= 1 - \text{Probability none detects.}$$

$$= 1 - (1 - P_D)^K$$

$$= 1 - \left(1 - Q_{\chi^2_{2N}}\left(\frac{\gamma}{(\sigma_s^2+\sigma^2)/2}\right)\right)^K.$$

OR Rule

- Hence, **probability of detection** is given as

$$P_D^{FC} = 1 - (1 - P_D)^K$$

Generalized Fusion Rule

L sensors or more report H_1 decision $= H_1$
Else decision $= H_0$.

- FC decides H_1 only if **at least** L **sensors** report H_1
  - and H_0 otherwise.

Generalized Fusion Rule

- The **false alarm probability** is given as

$$P_F^{FC} = \sum_{k=L}^{K} \underbrace{{}^{K}C_k}_{\frac{K!}{k!(K-k)!}} \underbrace{(P_{FA})^k (1-P_{FA})^{K-k}}_{\text{Any set of k sensors report False alarm}}$$

k out of K sensors can be chosen as $^{K}C_k$.

Any set of k sensors report False alarm

Generalized Fusion Rule

- The **false alarm probability** is given as

$$P_F^{FC} = \sum_{k=L}^{K} {}^{K}C_k (P_F)^k \, (1 - \, P_F)^{K-k}$$

Generalized Fusion Rule

- The **probability of detection** is given as

$$P_D^{FC} = \sum_{k=L}^{K} {}^{K}C_k \,(P_D)^k (1-P_D)^{K-k}$$

k sensors correctly detect

K-k sensors fail to detect

Binomial Probability distribution.

Number of combinations of k sensors from total of K sensors.

Generalized Fusion Rule

- The **probability of detection** is given as

$$P_D = Q_{\chi^2_{2N}}\left(\frac{\gamma}{(\sigma_s^2+\delta^2)/2}\right).$$

$$P_D^{FC} = \sum_{k=L}^{K} {}^{K}C_k (P_D)^k (1 - P_D)^{K-k}$$

$Q_{\chi^2_{2N}}(x) =$ CCDF of chi squared RV with 2N degrees of freedom.

MIMO Spectrum Sensing

MIMO
Multiple input
Multiple output

- The model for **MIMO system** can be stated as follows

$r \times t$

r = # receive antennas
t = # Transmit antennas.

h_{ij} channel coeff between ith Rx antenna jth Tx antenna.

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix}}_{\bar{y}} = \underbrace{\begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1t} \\ h_{21} & & & \\ \vdots & & & \\ h_{r1} & \cdots & & h_{rt} \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix}}_{\bar{x}} + \underbrace{\begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{bmatrix}}_{\bar{n}}$$

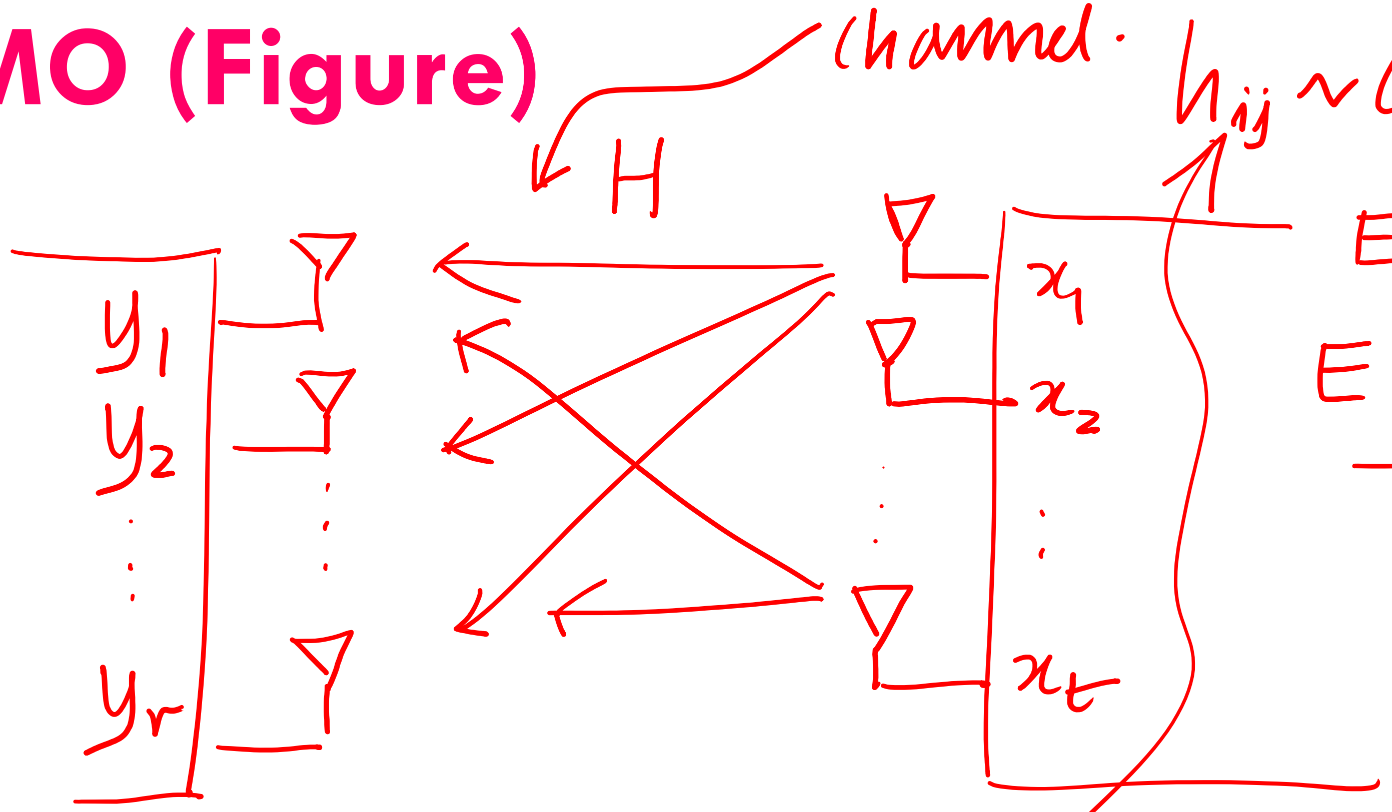
$$\bar{y} = H\bar{x} + \bar{n}$$

MIMO Spectrum Sensing

- The model for **MIMO system** can be stated as follows

$$\underbrace{\overset{\text{Receive vector}}{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix}}}_{\substack{\bar{\mathbf{y}} \\ r\times 1}} = \underbrace{\overbrace{\begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1t} \\ h_{21} & h_{22} & \cdots & h_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ h_{r1} & h_{r2} & \cdots & h_{rt} \end{bmatrix}}^{\text{channel matrix}}}_{\substack{\mathbf{H} \\ r\times t}} \underbrace{\overset{\text{Transmit vector}}{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix}}}_{\substack{\bar{\mathbf{x}} \\ t\times 1}} + \underbrace{\overset{\substack{\text{noise vector} \\ r\times 1}}{\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_r \end{bmatrix}}}_{\bar{\mathbf{v}}}$$

MIMO (Figure)



$$h_{ij} \sim \mathcal{CN}(0, 1)$$

$$E\{h_{ij}\} = 0$$

$$E\{|h_{ij}|^2\} = 1$$

Rayleigh Fading channel

$|h_{ij}| = a_{ij} \sim$ Rayleigh RV

MIMO Spectrum Sensing

- One can now use the statistic $\|\bar{\mathbf{y}}\|^2$
  - i.e. **energy** of the output

energy of output

$$\|\bar{y}\|^2 = |y_1|^2 + |y_2|^2 + \cdots + |y_r|^2$$

ENERGY.

MIMO Spectrum Sensing

- Compare this with a suitable threshold γ to get the **ED**

Threshold.

$$\underbrace{\begin{aligned} &\text{choose } \mathcal{H}_0 : \|\bar{y}\|^2 < \gamma \\ &\text{choose } \mathcal{H}_1 : \|\bar{y}\|^2 > \gamma \end{aligned}}_{\text{Energy detector}}$$

MIMO Spectrum Sensing

- Compare this with a suitable threshold γ to get the **ED**

$$\|\bar{\mathbf{y}}\|^2 > \gamma \Rightarrow \mathcal{H}_1$$

$$\|\bar{\mathbf{y}}\|^2 \leq \gamma \Rightarrow \mathcal{H}_0$$

Primary user present

Primary user absent

MIMO Spectrum Sensing

- Consider **hypothesis** $\mathcal{H}_0$
- The **probability of false alarm** (P_{FA}) is

Prob decision = $\mathcal{H}_1$
when PU is absent

$$P_{FA} = \Pr\left(\|\mathbf{y}\|^2 > \gamma ; \mathcal{H}_0\right)$$

MIMO Spectrum Sensing

- Consider **hypothesis** $\mathcal{H}_0$
- The **probability of false alarm** (P_{FA}) is

under $\mathcal{H}_0$.

$$P_{FA} = \Pr\left(\|\bar{\mathbf{y}}\|^2 > \gamma\right)$$

MIMO Spectrum Sensing

$$y_k = v_k = V_{k,I} + j V_{k,Q}$$

k^{th} receive antenna.

Signal is absent

PU is absent

Quadrature

inphase

$$v_{k,I} \sim \mathcal{N}\left(0, \frac{\sigma^2}{2}\right)$$

$$v_{k,Q} \sim \mathcal{N}\left(0, \frac{\sigma^2}{2}\right)$$

iid Gaussian

mean = 0

var = $\sigma^2/2$

MIMO Spectrum Sensing

inphase

$$y_k = v_k = v_{k,I} + j v_{k,Q}$$

Quadrature.

$$v_{k,I} \sim \mathcal{N}\left(0, \frac{\sigma^2}{2}\right)$$

$$v_{k,Q} \sim \mathcal{N}\left(0, \frac{\sigma^2}{2}\right)$$

MIMO Spectrum Sensing

$$\|\bar{\mathbf{v}}\|^2 > \gamma$$

r = # receive antennas.

$$\Rightarrow \sum_{k=1}^{r} |V_{k,I}|^2 + |V_{k,Q}|^2 > \gamma$$

$$\Rightarrow \underbrace{\sum_{k=1}^{r} \left|\frac{V_{k,I}}{\sigma/\sqrt{2}}\right|^2 + \left|\frac{V_{k,Q}}{\sigma/\sqrt{2}}\right|^2}_{\text{sum of squares of } 2r \text{ iid standard Normal RVs}} > \underbrace{\frac{\gamma}{\sigma^2/2}}_{\chi^2_{2r}}$$

sum of squares of $2r$ iid standard Normal RVs mean = 0, var = 1

Central. chi squared RV with $2r$ degrees of freedom!

MIMO Spectrum Sensing

$$\|\bar{\mathbf{v}}\|^2 > \gamma$$

$$\sum_{k=1}^{r} v_{k,I}^2 + v_{k,Q}^2 > \gamma$$

$$\sum_{k=1}^{r} \left|\frac{v_{k,I}}{\sigma/\sqrt{2}}\right|^2 + \left|\frac{v_{k,Q}}{\sigma/\sqrt{2}}\right|^2 > \frac{\gamma}{\sigma^2/2}$$

MIMO Spectrum Sensing

- The **probability of false alarm** (P_{FA}) is

$$P_{FA} = \Pr(\|\bar{\mathbf{y}}\|^2 > \gamma)$$

$$= \Pr(\|\bar{v}\|^2 > \gamma)$$

$$P_{FA} = \Pr\left(\frac{\|\bar{v}\|^2}{\sigma^2/2} > \frac{\gamma}{\sigma^2/2} \right)$$

$$= Q_{\chi^2_{2r}}\left(\frac{\gamma}{\sigma^2/2} \right)$$

CCDF of central chi squared RV with 2r degrees of freedom.

MIMO Spectrum Sensing

- The **probability of false alarm** (P_{FA}) is

$$P_{FA} = \Pr(\|\bar{\mathbf{y}}\|^2 > \gamma)$$
$$= \Pr(\|\bar{\mathbf{v}}\|^2 > \gamma)$$

2r degrees of Freedom

$$P_{FA} = \Pr\left(\frac{\|\bar{\mathbf{v}}\|^2}{\sigma^2/2} > \frac{\gamma}{\sigma^2/2}\right)$$

$$= Q_{\chi^2_{2r}}\left(\frac{\gamma}{\sigma^2/2}\right)$$

r = # Receive antennas in MIMO system.

MIMO Spectrum Sensing

decision = $\mathcal{H}_1$
under $\mathcal{H}_1$,
when PU is present

- Consider **hypothesis** $\mathcal{H}_1$.
- The **probability of detection** (P_D) is

$$P_D = \Pr\left(\|\bar{y}\|^2 > \gamma ; \mathcal{H}_1\right)$$

MIMO Spectrum Sensing

Under $\mathcal{H}_1$.

- Consider **hypothesis** $\mathcal{H}_1$.
- The **probability of detection** (P_D) is

$$P_D = \Pr(\|\bar{\mathbf{y}}\|^2 > \gamma)$$

MIMO Spectrum Sensing

power P $E\{x_i\}=0$.
$x_i \in \{\sqrt{P}, -\sqrt{P}\}$
$|x_i|^2 = P$.

- Consider symbols x_i to be BPSK $\pm\sqrt{P}$

$$E\{x_i\} = 0$$

$$|x_i|^2 = P.$$

MIMO Spectrum Sensing

Binary Phase Shift Keying

- Consider symbols x_i to be BPSK $\pm\sqrt{P}$

one bit/symbol.

$$E\{x_i\} = 0$$

$$|x_i|^2 = P$$

MIMO Spectrum Sensing

$$y_k = \sum_{l=1}^{t} h_{kl}\, x_l + v_k .$$

For each k.

symbol Transmitted on lth transmit antenna.

complex Gaussian

complex Gaussian.

- y_k is complex Gaussian $\mathcal{CN}$

MIMO Spectrum Sensing

linear combination of symmetric complex Gaussian RVs -

$$y_k = \sum_{l=1}^{t} h_{kl} x_l + v_k$$

zero mean circularly symmetric complex Gaussian.

- y_k is **complex Gaussian.**

MIMO Spectrum Sensing

$E\{v_k\} = 0.$

$E\{y_k\} = 0$

$E\{h_{kl}\} = 0$

$$y_k = \sum_{l=1}^{t} h_{kl} x_l + v_k$$

$$E\{|y_k|^2\} = \sum_{l=1}^{t} \underbrace{E\{|h_{kl}|^2\}}_{1} \underbrace{|x_l|^2}_{P} + \underbrace{E\{|v_k|^2\}}_{\sigma^2}$$

$$= tP + \sigma^2$$

MIMO Spectrum Sensing

$$y_k \sim \mathcal{CN}\left(0, \frac{tP+\sigma^2}{2}\right).$$

$$y_k = \sum_{l=1}^{t} h_{kl} x_l + v_k$$

$$E\{|y_k|^2\} = \sum_{l=1}^{t} E\{|h_{kl}|^2\}|x_l|^2 + E\{|v_k|^2\}$$

$$= tP + \sigma^2$$

MIMO Spectrum Sensing

crossterm.

$l \neq p$

iid channel coefficients.

$$E\{h_{kl} v_k^*\} = 0$$

$$E\{h_{kl} h_{kp}^*\} = 0$$

$h_{kl} \sim \mathcal{CN}(0,1)$

$$E\{|h_{kl}|^2\} = 1$$

$$E\{|v_k|^2\} = \sigma^2$$

MIMO Spectrum Sensing

$$E\{h_{kl}v_{kl}^*\} = 0$$

$$E\{h_{kl}h_{kp}^*\} = 0$$

$$E\{|h_{kl}|^2\} = 1$$

MIMO Spectrum Sensing

inphase

Quadrature

$$y_k = y_{kI} + j y_{k,Q}$$

$$y_{k,I} \sim \mathcal{N}\left(0, \frac{tP+\sigma^2}{2}\right)$$

$$y_{k,Q} \sim \mathcal{N}\left(0, \frac{tP+\sigma^2}{2}\right)$$

iid Gaussian

zero mean.

MIMO Spectrum Sensing

$$y_k = y_{k,I} + j y_{k,Q}$$

$$y_{k,I} \sim \mathcal{N}\left(0, \frac{tP + \sigma^2}{2}\right)$$

$$y_{k,Q} \sim \mathcal{N}\left(0, \frac{tP + \sigma^2}{2}\right)$$

MIMO Spectrum Sensing

$$\|\bar{\mathbf{y}}\|^2 > \gamma$$

$$\Rightarrow \sum_{k=1}^{r} |y_{k,I}|^2 + |y_{k,Q}|^2 > \gamma$$

$$\Rightarrow \underbrace{\sum_{k=1}^{r} \left|\frac{y_{k,I}}{\sqrt{\frac{tP+\sigma^2}{2}}}\right|^2 + \left|\frac{y_{k,Q}}{\sqrt{\frac{tP+\sigma^2}{2}}}\right|^2}_{\text{Sum of squares of } 2r \text{ iid zero mean standard Gaussian RVs.}} > \frac{\gamma}{\frac{tP+\sigma^2}{2}}$$

χ^2_{2r}

Central Chi Squared RV $2r$ degrees of freedom

MIMO Spectrum Sensing

$$\|\bar{\mathbf{y}}\|^2 > \gamma$$

$$\sum_{k=1}^{r} y_{k,I}^2 + y_{k,Q}^2 > \gamma$$

$$\sum_{k=1}^{r} \left|\frac{y_{k,I}}{\sqrt{tP+\sigma^2}/\sqrt{2}}\right|^2 + \left|\frac{y_{k,Q}}{\sqrt{tP+\sigma^2}/\sqrt{2}}\right|^2 > \frac{\gamma}{(tP+\sigma^2)/2}$$

χ^2_{2r}

MIMO Spectrum Sensing

- Consider **hypothesis** $\mathcal{H}_1$.
- The **probability of detection** (P_D) is

$$P_D = \Pr(||\bar{\mathbf{y}}||^2 > \gamma)$$

$$= \Pr\left(\frac{||\bar{y}||^2}{(tP+\sigma^2)/2} > \frac{\gamma}{(tP+\sigma^2)/2}\right)$$

$$P_D = Q_{\chi^2_{2r}}\left(\frac{\gamma}{(tP+\sigma^2)/2}\right)$$

CCDF of central chi-squared RV with 2r degrees of freedom.

Probability of Detection.

MIMO Spectrum Sensing

- Consider **hypothesis** $\mathcal{H}_1$.
- The **probability of detection** (P_D) is

$$P_D = \Pr(\|\bar{\mathbf{y}}\|^2 > \gamma)$$

$$= \Pr\left(\frac{\|\bar{\mathbf{y}}\|^2}{(tP + \sigma^2)/2} > \frac{\gamma}{(tP + \sigma^2)/2}\right)$$

$$P_D = Q_{\chi^2_{2r}}\left(\frac{\gamma}{(tP + \sigma^2)/2}\right)$$

MIMO Spectrum Sensing

- ROC can be found as follows
- The **probability of false alarm** (P_{FA}) is

$$P_{FA} = Q_{\chi^2_{2r}}\left(\frac{\gamma}{\sigma^2/2}\right)$$

$$\gamma = \frac{\sigma^2}{2} Q^{-1}_{\chi^2_{2r}}(P_{FA}).$$

$$P_D = Q_{\chi^2_{2r}}\left(\frac{\frac{\sigma^2}{\cancel{2}} Q^{-1}_{\chi^2_{2r}}(P_{FA})}{\frac{tP+\sigma^2}{\cancel{2}}} \right)$$

ROC

Receiver operating characteristic

$$P_D = Q_{\chi^2_{2r}}\left(\frac{Q^{-1}_{\chi^2_{2r}}(P_{FA})}{1 + t \times SNR} \right)$$

$$SNR = \frac{P}{\sigma^2}$$

MIMO Spectrum Sensing

- ROC can be found as follows
- The **probability of false alarm** (P_{FA}) is

$$P_{FA} = Q_{\chi^2_{2r}}\left(\frac{\gamma}{\sigma^2/2}\right)$$

$$\Rightarrow \gamma = \frac{\sigma^2}{2} Q^{-1}_{\chi^2_{2r}}(P_{FA})$$

$$P_D = Q_{\chi^2_{2r}}\left(\frac{\gamma}{(tP+\sigma^2)/2}\right) = Q_{\chi^2_{2r}}\left(\frac{\frac{\sigma^2}{2}Q^{-1}_{\chi^2_{2r}}(P_{FA})}{(tP+\sigma^2)/2}\right)$$

$$= Q_{\chi^2_{2r}}\left(\frac{\frac{\sigma^2}{2}Q^{-1}_{\chi^2_{2r}}(P_{FA})}{t\times SNR+1}\right)$$

$$SNR = \frac{P}{\sigma^2}$$

$E\{|V_k|^2\} = \sigma^2$

$|x_i|^2 = P$

Instructors may use this white area (14.5 cm / 25.4 cm) for the text.
Three options provided below for the font size.

Font: Avenir (Book), Size: 32, Colour: Dark Grey

Font: Avenir (Book), Size: 28, Colour: Dark Grey

Font: Avenir (Book), Size: 24, Colour: Dark Grey

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