Live Interaction #2:

8th October 2023

E-masters Communication Systems

Detection for Wireless

- **Binay hypothesis Testing**.

$$\mathcal{H}_0: \bar{\mathbf{y}} = \bar{\mathbf{v}}$$

$$\mathcal{H}_1: \bar{\mathbf{y}} = \bar{\mathbf{s}} + \bar{\mathbf{v}}$$

- **Probability of False Alarm**:

$$P_{FA} = Q\left(\frac{\gamma}{\sigma\|\bar{\mathbf{s}}\|}\right)$$

- **Probability of Detection**:

$$P_D = Q\left(\frac{\gamma - \|\bar{\mathbf{s}}\|^2}{\sigma\|\bar{\mathbf{s}}\|}\right)$$

- P_D versus P_{FA}: **Receiver Operating Characteristic**.

$$\gamma = \sigma\|\bar{\mathbf{s}}\|Q^{-1}(P_{FA})$$

$$P_D = Q\left(\frac{\sigma\|\bar{\mathbf{s}}\|Q^{-1}(P_{FA}) - \|\bar{\mathbf{s}}\|^2}{\sigma\|\bar{\mathbf{s}}\|}\right)$$

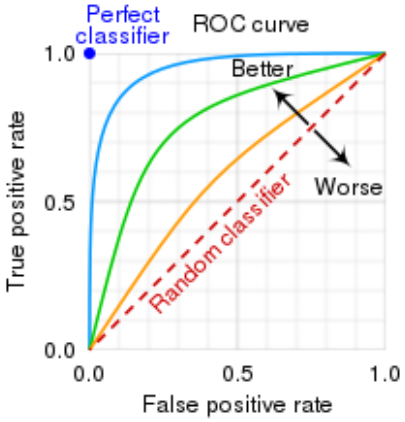
$$= Q\left(Q^{-1}(P_{FA}) - \frac{\|\bar{\mathbf{s}}\|}{\sigma}\right)$$

$$= Q\left(Q^{-1}(P_{FA}) - \sqrt{\frac{\|\bar{\mathbf{s}}\|^2}{\sigma^2}}\right)$$

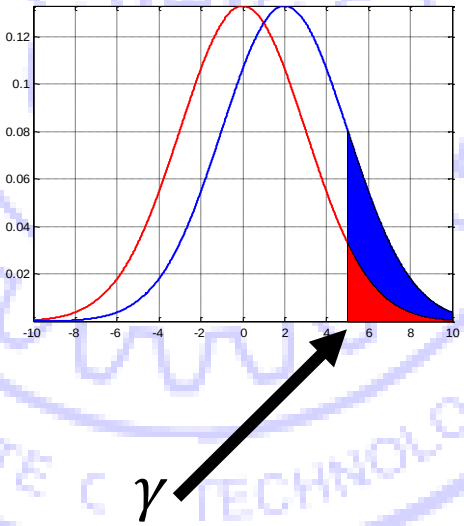
$$= Q\left(Q^{-1}(P_{FA}) - \sqrt{SNR}\right)$$

$$SNR = \frac{\|\bar{\mathbf{s}}\|^2}{\sigma^2}$$

- ROC



- $P_D = 0, P_{FA} = 0$ for $\gamma = \infty$
- $P_D = 1, P_{FA} = 1$ for $\gamma = -\infty$



- γ helps **tradeoff** P_D vs P_{FA}
- **Probability of error**:
- $\mathcal{H}_0$ there is False Alarm.
- $\mathcal{H}_1$ there is misdetection.

$$P_e = P(\mathcal{H}_0) \times P_{FA} + P(\mathcal{H}_1)(1 - P_D)$$

$$= P(\mathcal{H}_0) \times Q\left(\frac{\gamma}{\sigma\|\bar{\mathbf{s}}\|}\right) + \Pr(\mathcal{H}_1) \times \left(1 - Q\left(\frac{\gamma - \|\bar{\mathbf{s}}\|^2}{\sigma\|\bar{\mathbf{s}}\|}\right)\right)$$

- When

$$P(\mathcal{H}_0) = \Pr(\mathcal{H}_1) = \frac{1}{2}$$

- Optimal detector is ML detector.

$$\gamma = \frac{\|\bar{\mathbf{s}}\|^2}{2}$$

$$P_{FA} = Q\left(\frac{\frac{\|\bar{\mathbf{s}}\|^2}{2}}{\sigma\|\bar{\mathbf{s}}\|}\right) = Q\left(\frac{\|\bar{\mathbf{s}}\|}{2\sigma}\right) = P_{MD}$$

$$P_e = Q\left(\frac{\|\bar{\mathbf{s}}\|}{2\sigma}\right)$$

- Example:
- **ASK: Amplitude Shift Keying**.

$$\mathcal{H}_0 : y = v$$

$$\mathcal{H}_1 : y = A + v$$

$$P_e = Q\left(\frac{A}{2\sigma}\right) = Q\left(\frac{\sqrt{2E_b}}{2\sqrt{\frac{N_0}{2}}}\right)$$

$$= Q\left(\sqrt{\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{1}{2}SNR}\right)$$

- $\sigma^2 = \frac{N_0}{2} \Rightarrow \sigma = \sqrt{\frac{N_0}{2}}$

$$SNR = \frac{E_b}{\frac{N_0}{2}} = \frac{2E_b}{N_0}$$

$$\Rightarrow \frac{E_b}{N_0} = \frac{1}{2} SNR$$

- E_b: **Average energy per bit**.

$$E_b = \frac{A^2}{2} \Rightarrow A = \sqrt{2E_b}$$

- $SNR = 14\ dB$. What is the probability of error?

$$SNR = 14\ dB$$

$$10 \log_{10} SNR = 14 \Rightarrow SNR = 10^{1.4} = 25.118$$

$$P_e = Q\left(\sqrt{\frac{1}{2} \times 25.118}\right) = Q(3.54)$$

$$= 2 \times 10^{-4}$$

- **Assignment 1 deadline: 14th October Saturday 11:59 PM**.
- **Assignment 2 deadline: 14th October Saturday 11:59 PM**.
- **Assignment discussion: 15th October Sunday 4:30-5:00 PM**.
- **Quiz 1: 15th October Sunday 5:00-5:45 PM**.
- **Live interaction 19th Thursday 9:00 PM.**