Norm is any function that satisfies: $\|\cdot\|$

[without any domain restriction : defined $\forall x$ }
e.g. $\underline{x} \in \mathbb{R}^n$ then $\|x\|$ defined $\forall x \in \mathbb{R}^n$

(1) Non-negative $\|x\| \geq 0 \;\; \forall \underline{x}$

(2) Definite $\|x\| = 0$ only if $x = 0$

(3) Homogeneity $\|t\underline{x}\| = |t| \, \|x\|$ $t \in \mathbb{R}$

absolute value

(4) Triangle inequality $\|\underline{x}+y\| \leq \|\underline{x}\| + \|y\|$

norm ball : $B = \{ x \mid \|x\| \leq 1 \}$

Eg: vector norm $\|x\|_2$ ℓ_2-norm $\sqrt{\langle x, x \rangle}$

ℓ_1-norm : $\|x\|_1 = \sum_{i=1}^{n} |x_i|$

$\|x\|_\infty = \max_{1 \leq i \leq n} |x_i|$

(no inner product) Chebyshev Norm

ℓ_p-norm : $\|x\|_p = \sqrt[p]{\sum_{i=1}^{n} |x_i|^p}$

Matrix norms $\|X\|_F$ Frobenius

Sum- Absolute Value (SAV) $\|X\|_{sav} = \sum_{i=1}^{n} |x_{ij}|$

Equivalence of norms $\forall a, b \geq 1$

$$\alpha \|x\|_a \leq \|x\|_b \leq \beta \|x\|_a$$

do not depend on x
but depend on n or a, b

$\|x\|_b$ small $\Rightarrow$ $\|x\|_a$ small
(large) (large)