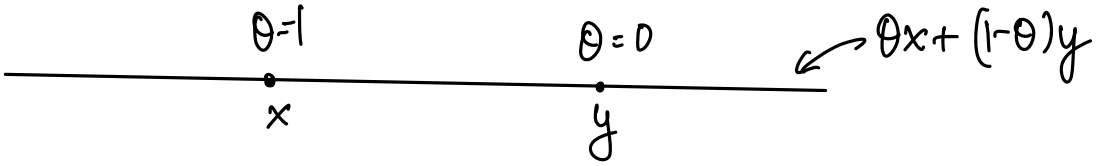
# 1. Affine Sets

$C$ affine $\equiv$ line through $x, y \in C$ also lies in $C$

$$\theta x + (1-\theta) y \in C \quad \forall \ \theta \in \mathbb{R}$$

↓

line through $x, y$




Eg: $C = \{ x \in \mathbb{R}^n \mid Ax = b \}$ solution set of a system of linear eqns.

$A \in \mathbb{R}^{m \times n}$ $b \in \mathbb{R}^m$

suppose $x, y \in C \Rightarrow Ax = b$, $Ay = b$

suppose $z = \theta x + (1-\theta) y$

↑ parameter

Does $z \in C$ ? or $Az = b$

or $A(\theta x + (1-\theta)y) = b$

or $\theta(Ax) + (1-\theta)(Ay)$

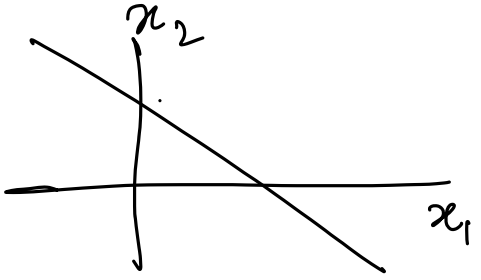
$= \theta b + (1-\theta)b = b$ ✓

$\Rightarrow z \in C$ $\quad \boxed{\forall\, \theta \in \mathbb{R}}$

for any $\theta \in \mathbb{R}$

(no restriction on $\theta$)

Eg: $n = 2$ $\quad \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2 \;\middle|\; x_1 + x_2 = 1 \right\}$




line is affine

$n = 3$ $\quad \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; x_3 = 0 \right\}$

plane in 3D

Eg: $D = \{ x \in \mathbb{R}^n \mid Ax \leq b \}$

solution set of inequalities $\quad b \in \mathbb{R}^m$, $A \in \mathbb{R}^{m \times n}$

$x, y \in D$

$z = \theta x + (1-\theta)y \in D$ ?

$x \in D \Rightarrow Ax \leq b$

$y \in D \Rightarrow Ay \leq b$

$$Az = A(\theta x + (1-\theta)y) = \theta(Ax) + (1-\theta)(Ay)$$

In general : $Ax \leq b \not\Rightarrow \theta(Ax) \leq \theta b$

not true for $\theta < 0$

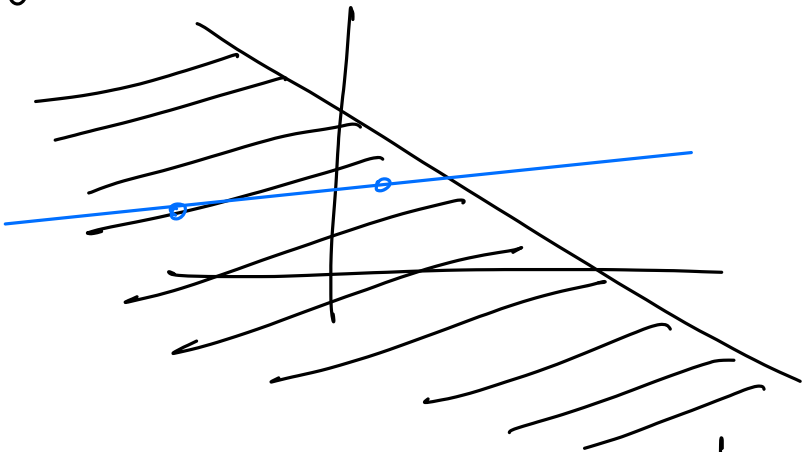
$\Rightarrow Az \leq b$ only when $\theta \geq 0$ & $1-\theta \geq 0$

$\Rightarrow z \notin D$ when $\theta < 0$ or $\theta > 1$

not true for arbitrary $\theta$

$\Rightarrow$ D not affine

Eg $n=2, m=1$ $\quad \{x \in \mathbb{R}^2 \mid x_1 + x_2 \leq 1\}$



half-space

not affine

$n=1, m=1$ $\quad \{x \in \mathbb{R} \mid x \leq 1\}$

half line not affine