eMasters in Communication Systems

Prof. Aditya Jagannatham

Elective Module:

Estimation for Wireless Communication

Chapter 7

OFDM

OFDM

- OFDM stands for Orthogonal Frequency Division Multiplexing.

Technology
Multicarrier modulation Technique.

OFDM

- OFDM stands for **Orthogonal Frequency Division Multiplexing**

OFDM

- Orthogonal Frequency Division Multiplexing (OFDM) is one of the most **extensively used** wireless technologies

- OFDM is used in 4G LTE (Long Term Evolution), 5G NR (New Radio.)
- Wi-Fi – **802.11n, 802.11 ac, 802.11ax…**

OFDM

4G ~ 150-200 Mbps.
5G ~ 1 Gbps.

- OFDM is widely employed in most of the ***modern cellular*** and ***Wi-Fi systems.***
- OFDM enables **ultrahigh** Data Rates. ________________.

conventional/orthodox/Traditional.

Single Carrier Modulation

- Consider bandwidth $\frac{B}{2}$ and a *single* carrier.

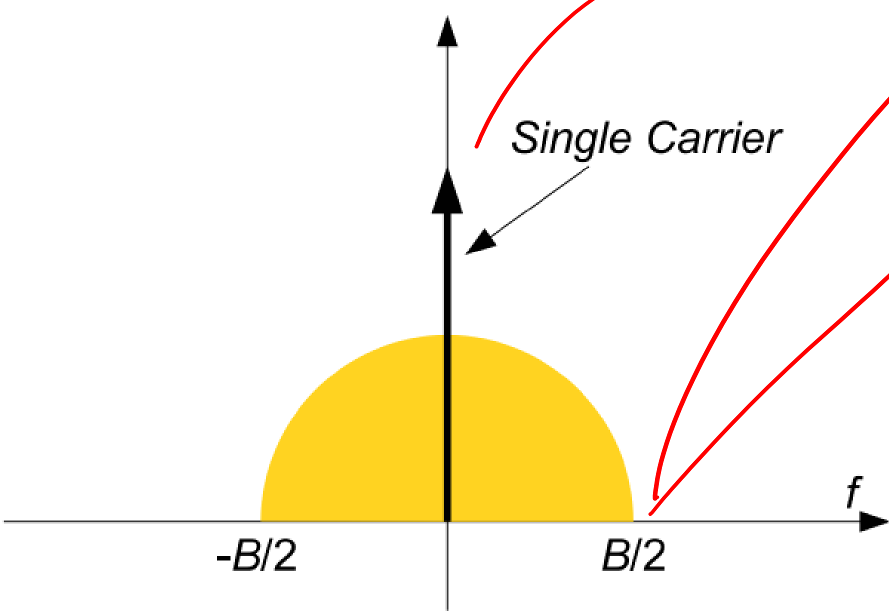
$\propto \frac{1}{B}$

- Symbol duration = $\frac{1}{B}$ $\Rightarrow$ as Bandwidth increases. symbol duration decreases.

Single carrier.

One sided. Bandwidth.

$\frac{B}{2}$ = maximum. Frequency



Single Carrier Modulation

$B = 10\,MHz$

$T_{sym} = \frac{1}{B} = \frac{1}{10\,MHz} = 0.1\,\mu s.$

- The symbol duration above is ***extremely small***. !!
- What happens when the **symbol duration** is extremely small?

$\Rightarrow$ ISI.

intersymbol interference

Delay spread ~ 2-3 μs = T_d.

Symbol Duration = $0.1\,\mu s \ll T_d$.

$\Rightarrow$ ISI.

Signicant ISI!

Multicarrier Modulation

avoid ISI?

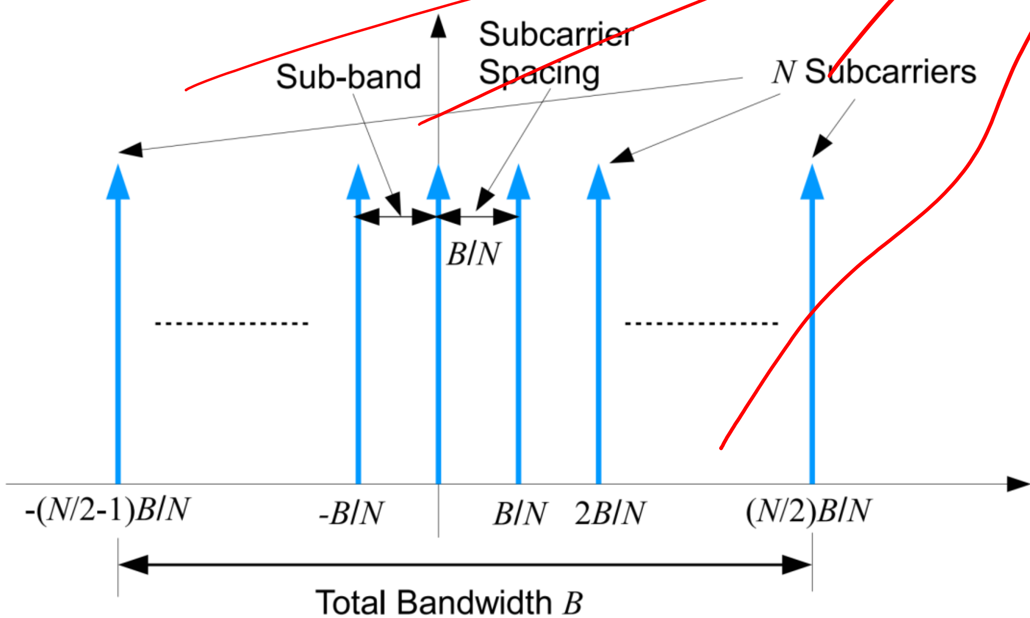
- How to ***avoid ISI***?

Multicarrier system.

- Instead of using one carrier, use N SUBCARRIERS.

N subcarriers.

N subbands.

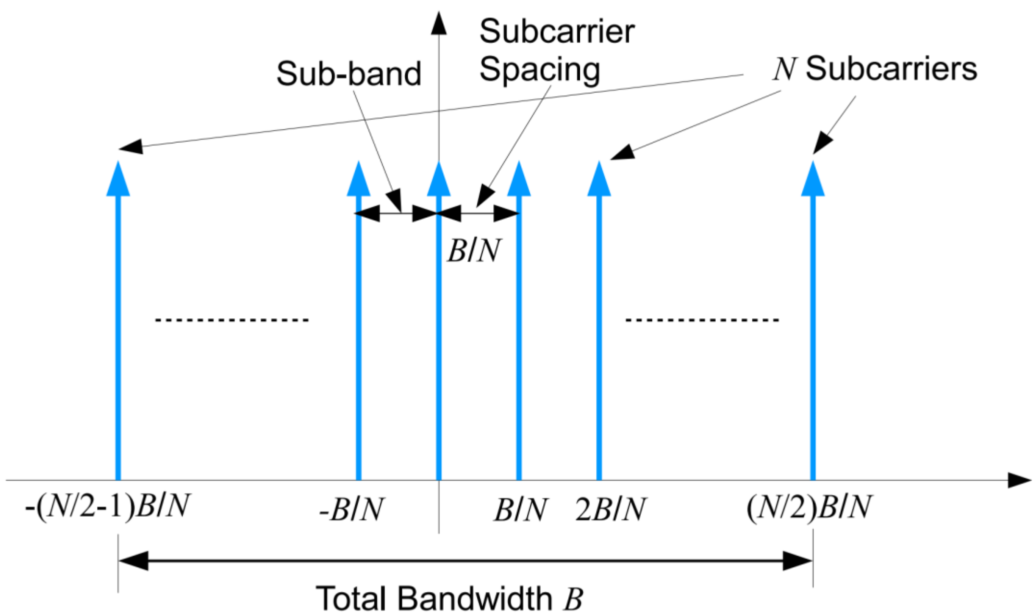


Multicarrier Modulation

Multiple subcarriers.

- This is also known as ***multicarrier modulation***.
- Divide the bandwidth into N subbands.

  B/N ______________________. Bandwidth of each subband.
- Width of each subband is $\frac{B}{N}$



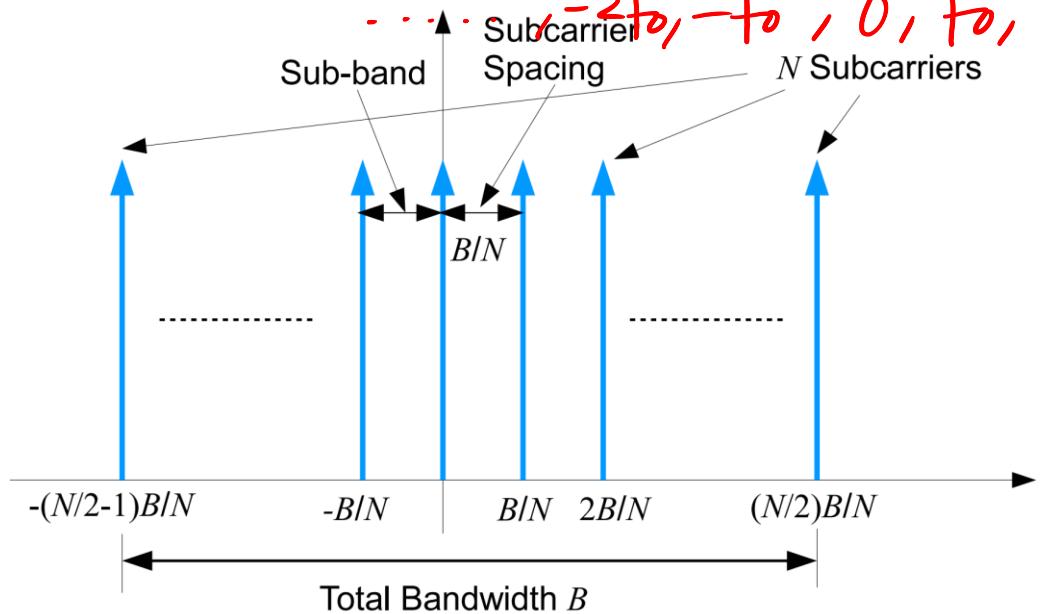
Multicarrier Modulation - Example

- What is the ***subcarrier spacing***?

$\frac{B}{N} = F_0$

$\ldots, -\frac{2B}{N}, -\frac{B}{N}, 0, \frac{B}{N}, \frac{2B}{N}, \frac{3B}{N}, \ldots$

$\ldots, -2f_0, -f_0, 0, f_0, 2f_0, \ldots$

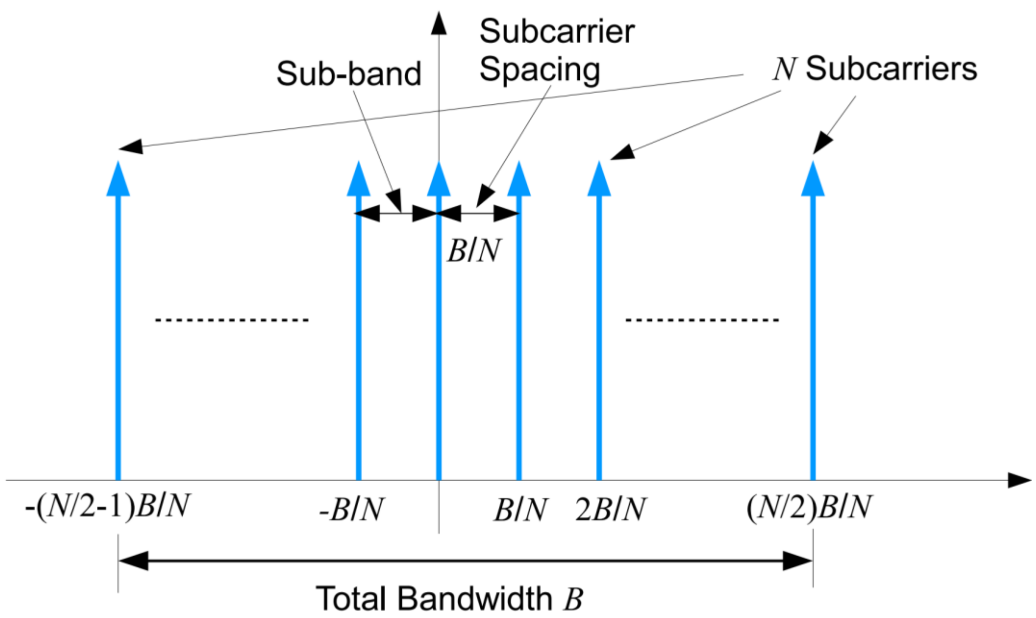


Multicarrier Modulation

- $\frac{B}{N} = f_0$
- Subcarriers are placed at

f_0 and integer multiples of f_0.

$\ldots, -2f_0, -f_0, 0, f_0, 2f_0, \ldots$



Multicarrier Modulation

- $\frac{B}{N} = f_0$
- Subcarriers are placed at

$$\dots, -2f_0, -f_0, 0, f_0, 2f_0, \dots$$

Multicarrier Modulation

- **Modulate** the symbol X_k on the kth subcarrier

kth subcarrier

$$X_k \cdot e^{j2\pi k f_0 t}$$

Multicarrier Modulation

- Take the ***sum of the signals*** across all the subcarriers.
- The **transmit signal** is given as

Transmit signal.

Normalization factor.

Sum across all subcarriers.

$$x(t) = \frac{1}{N}\sum_{k} X_k e^{j2\pi k f_0 t}$$

$$x(t) = \frac{1}{N} \sum_{k} X_k e^{j2\pi k f_0 t}$$

Multicarrier Demodulation

- $e^{j2\pi k f_0 t}$, $e^{j2\pi l f_0 t}$ are ORTHOGONAL

Multicarrier Demodulation

Subcarriers are Orthogonal!

- **Orthogonality** can be seen as follows

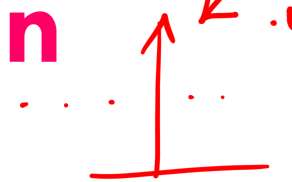
$$F_0 \int_0^{\frac{1}{F_0}} e^{j2\pi k F_0 t} \cdot \left(e^{j2\pi l F_0 t}\right)^* dt$$

$$= F_0 \int_0^{\frac{1}{F_0}} e^{j2\pi(k-l)F_0 t} dt = \begin{cases} 1 & \text{if } k = l \\ 0 & \text{if } k \neq l \end{cases}$$

Multicarrier Demodulation

- This explains the name **OFDM**
- **Orthogonal** $\Rightarrow$ Subcarriers Orthogonal.
- **Frequency Division** $\Rightarrow$ Dividing Frequency into N subbands.
- **Multiplexing** $\Rightarrow$ Simultaneous transmission over N subcarriers.

Multicarrier Demodulation

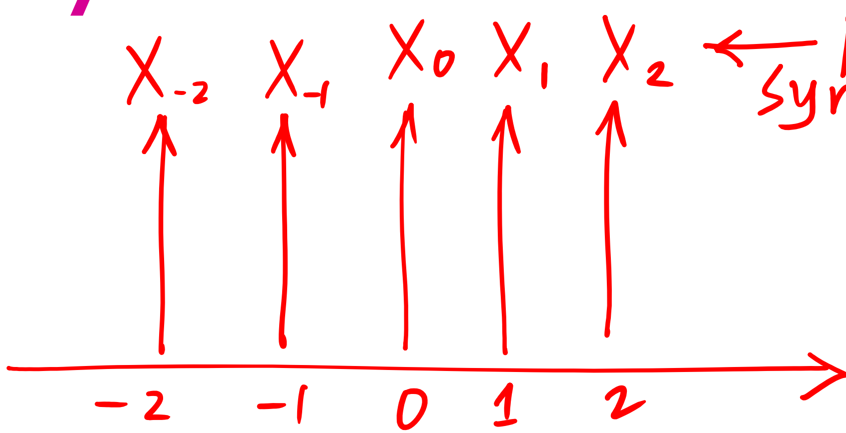


loding X_k over kth subcarrier.

- **Orthogonal**: Subcarriers are **ORTHOGONAL**.
- **Frequency Division**: Dividing the band into ***multiple subbands***.

N parallel subcarriers.

- **Multiplexing**: Simultaneous transmission of **multiple symbols** over same channel.



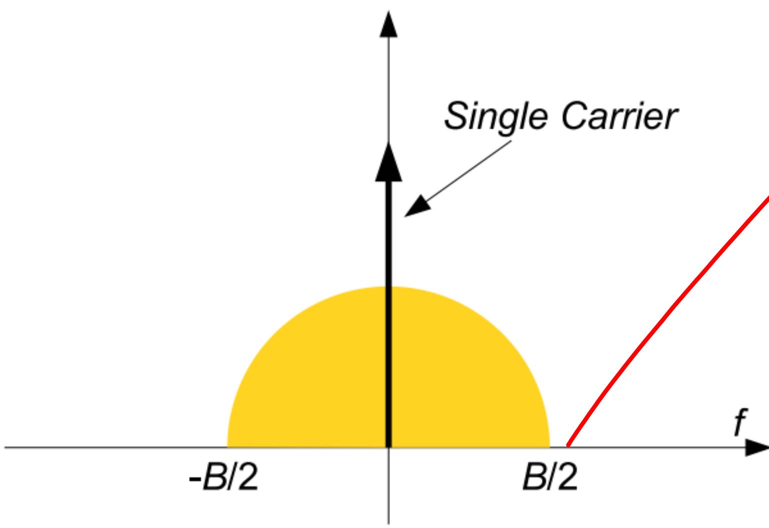
Loading symbols over subcarriers.

OFDM Generation

- Signal **bandlimited** to $\frac{B}{2}$.

$2 \times$ maximum Freq $= 2 \times \frac{B}{2} = B.$

- What is the minimum **sampling frequency**?



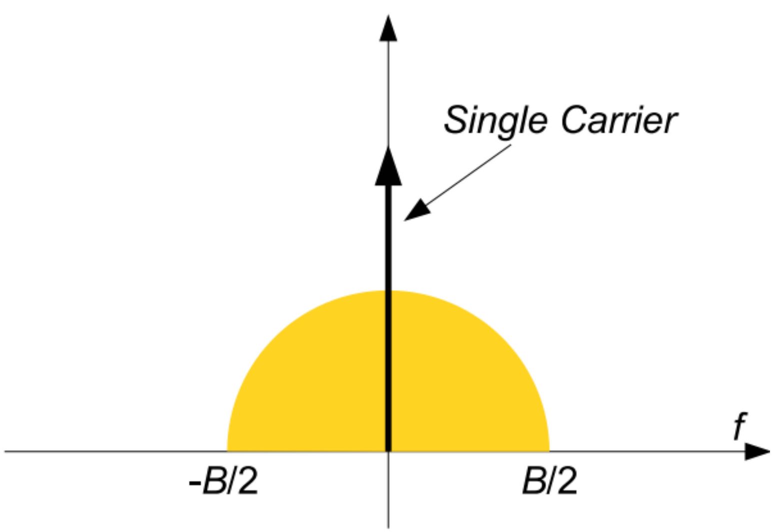
$f_m =$ maximum Freq $= \frac{B}{2}.$

OFDM Generation

NYQUIST Sampling Theory

- $f_s = 2 \times \frac{B}{2} =$ B : This is termed NYQUIST criterion!

- **Sampling duration** $T_s =$ $\frac{1}{f_s} = \frac{1}{B}$.



OFDM Generation

$= l \cdot T_s = l \times \frac{1}{B}$

- lth sample will be at $t = \frac{l}{B}$

Sampled OFDM Signal can be generated via IFFT

$$x(l) = \frac{1}{N} \sum_k X_k \cdot e^{j2\pi k \frac{\cancel{B}}{N} \frac{l}{\cancel{B}}}$$

$$= \frac{1}{N} \sum_k X_k e^{j2\pi kl/N}$$

IFFT operation!

OFDM Generation

OFDM samples -

- lth sample will be at $t = \frac{l}{B}$

IDFT

IFFT

$$x(l) = \frac{1}{N}\sum_{k} X(k)e^{j2\pi k\frac{\cancel{B}}{N}\frac{l}{\cancel{B}}}$$

$$\underbrace{x(l) = \frac{1}{N}\sum_{k} X(k)e^{j2\pi\frac{kl}{N}}}_{IDFT}$$

OFDM Generation

inverse Fast Fourier Transform

- What is the advantage of IDFT?
  - We can implement is very efficiently using **IFFT**.

Fast Fourier Transform = inverse IFFT !!

- How to recover symbols at receiver?
  - Use **FFT!!!**

OFDM Generation

- **Samples** of the OFDM signal can be generated very efficiently using the IFFT algorithm!!

$$\underbrace{X_0, X_1, \ldots, X_{N-1}}_{\text{Freq Domain}}$$

Freq Domain Loaded on subcarriers !

$$\downarrow IFFT$$

$x(0)\ x(1)\ \ldots\ldots\ x(N-1)$

Time Domain transmitted over channel.

Time Domain Samples

ISI Channel Model

intersymbol
interference channel.

The **ISI channel model** is given as

channel-
taps. channel. Filter

$$y(k) = h(0)x(k) + h(1)x(k-1) + \cdots + h(L-1)x(k-L+1) + v(k)$$

Linear convolution.

$$= \sum_{l=0}^{L-1} h(l)x(k-l) + v(k).$$

ISI Channel Model

Linear convolution

$$y(k) = h * x + v(k)$$

Frequency Selective. ISI channel.

Thus the channel performs

LINEAR. convolution

Cyclic Prefix

introduces circular symmetry!

Prior to transmission we add the cyclic Prefix to an OFDM block

cyclic Prefix:
Cycle and Prefix at Head.

Few Samples from tail.

$$\underbrace{x\left(N-\tilde{L}\right),\dots,x(N-2),x(N-1)}_{\textbf{Cyclic Prefix}},\underbrace{x(0),x(1),x(2),\dots,x(N-1)}_{\textbf{Original Samples}}$$

N pt IFFT.

Cyclic Prefix

Linear convolution Becomes circular convolution!

- This converts

circular convolution

$$y(l) = h \circledast x + v(l)$$

after removal of CP at receiver

Cyclic Prefix

FFT at Receiver

- What happens when we take the **FFT**?

$$y(l) = h \circledast x + v(l)$$

$$\downarrow FFT$$ @ RX . channel.

$$Y_k = H_k \times X_k + V_k$$

symbol on subcarrier k.

circular convolution in Time.

⟹ Product in Freq Domain.

OFDM Model

$Y_k = H_k X_k + V_k.$

- Therefore we have?

$$
\begin{aligned}
Y_0 &= H_0 \times X_0 + V_0 \\
Y_1 &= H_1 \times X_1 + V_1 \\
&\vdots \\
Y_{N-1} &= H_{N-1} \times X_{N-1} + V_{N-1}
\end{aligned}
$$

symbol on subcarrier k.

output on subcarrier k.

channel on subcarrier k.

N outputs.

N channel coefficients.

N symbols loaded on subcarriers.

N noise samples.

OFDM Model

in OFDM

- How to perform channel estimation?
  - Transmit **pilots** on each subcarrier

Pilot subcarriers.

OFDM Model

k^{th} subcarrier

- Therefore we have

$$Y_k(1) = H_k X_k(1) + V_k(1)$$

$$Y_k(2) = H_k X_k(2) + V_k(2)$$

$$\vdots$$

$$Y_k(N_p) = H_k X_k(N_p) + V_k(N_p).$$

N_p = # pilot symbols.

OFDM Model

Pilot outputs - Pilots -

- Therefore we have

$$Y_k(1) = H_k X_k(1) + V_k(1)$$
$$Y_k(2) = H_k X_k(2) + V_k(2)$$
$$\vdots$$
$$Y_k(N_p) = H_k X_k(N_p) + V_k(N_p)$$

channel coefficient -

OFDM Model

- This can be written in vector form as

$$\underbrace{\begin{bmatrix} Y_k(1) \\ Y_k(2) \\ \vdots \\ Y_k(N_p) \end{bmatrix}}_{\substack{\bar{Y}_k \\ \text{Pilot output} \\ \text{Vector}}} = H_k \underbrace{\begin{bmatrix} X_k(1) \\ X_k(2) \\ \vdots \\ X_k(N_p) \end{bmatrix}}_{\substack{\bar{X}_k \\ \text{Pilot} \\ \text{Vector}}} + \underbrace{\begin{bmatrix} V_k(1) \\ V_k(2) \\ \vdots \\ V_k(N_p) \end{bmatrix}}_{\substack{\text{Noise} \\ \text{vector}}}$$

OFDM Model

- The channel estimate for the kth subcarrier is

$$\hat{H}_k = \frac{\sum_{i=1}^{N_p} X_k^*(i) Y_k(i)}{\sum_{i=1}^{N_p} X_k^*(i) X_k(i)}$$

$$= \frac{\sum_{i=1}^{N_p} X_k^*(i) Y_k(i)}{\sum_{i=1}^{N_p} |X_k(i)|^2}.$$

OFDM Model

Estimate for subcarrier k.

- The channel estimate for the kth subcarrier is

$$\widehat{H}_k = \frac{\sum_{i=1}^{N_p} X_k^*(i) Y_k(i)}{\sum_{i=1}^{N_p} |X_k(i)|^2}$$

OFDM Channel Estimation

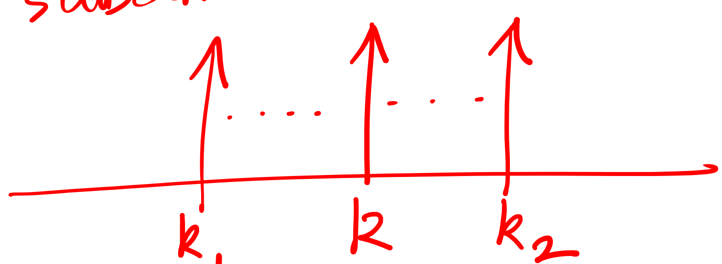
- Coefficients on rest of the subcarriers can be estimated via **linear interpolation**

Because subcarriers are closely spaced.

$$\hat{H}_k = \hat{H}_{k_1} + \frac{(k - k_1)}{(k_2 - k_1)} \left(\hat{H}_{k_2} - \hat{H}_{k_1} \right)$$

k_1, k_2 = Pilot subcarriers

k ~ intermediate subcarrier.



OFDM Channel Estimation

- Coefficients on rest of the subcarriers can be estimated via linear interpolation

$$\widehat{H}_k = \widehat{H}_{k_1} + \frac{(k - k_1)}{k_2 - k_1}\left(\widehat{H}_{k_2} - \widehat{H}_{k_1}\right)$$

OFDM Channel Estimation Example

- Consider

$$\bar{X}_k = \begin{bmatrix} 1-j \\ 1+j \\ 1-j \\ 1+j \end{bmatrix}$$

$N_p = 4$
Pilot
symbols.

OFDM Channel Estimation Example

- Consider

Pilot output vector.

$$\bar{Y}_k = \begin{bmatrix} -1 \\ -j \\ j \\ -1 \end{bmatrix}$$

OFDM Channel Estimation Example

- The channel estimate is given as

$$\hat{H}_k = \frac{\sum_{i=1}^{N_p} X_k^*(i) Y_k(i)}{\sum_{i=1}^{N_p} |X_k(i)|^2}$$

$$= \frac{(1+j)(-1) + (1-j)(-j) + (1+j)(j) + (1-j)(-1)}{2+2+2+2}$$

$$= \frac{-1 \cancel{-j} \cancel{-j} -1 \cancel{+j} -1 -1 \cancel{+j}}{8}$$

$$= \frac{-4}{8} = -\frac{1}{2}$$

channel Estimate subcarrier k.

OFDM Channel Estimation Example

- The channel estimate is given as

$$\hat{H}_k = \frac{\sum_{i=1}^{N_p} X_k^*(i) Y_k(i)}{\sum_{i=1}^{N_p} |X_k(i)|^2}$$

$$= \frac{(1+j)(-1) + (1-j)(-j) + (1+j)j + (1-j)(-1)}{2+2+2+2} = \frac{-4}{8} = -\frac{1}{2}$$

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