Alamouti Code

MULTIPLE ANTENNAS.

- It is an **Orthogonal Space Time Block Code**

$r = 1$ 1×2 SYSTEM.

$t = 2$

- For a system with 2 transmit antennas and 1 receive antenna

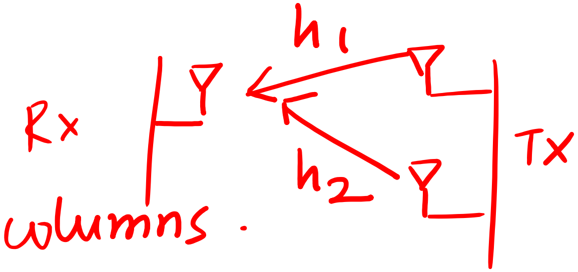
Alamouti Code

- Consider now a $r = 1, t = 2$ system
- This is a ____1x2____ system

MULTIPLE INPUT SINGLE OUTPUT.

- Also known as a ____MISO____ system

Alamouti Code



1 row

2 columns.

- The MISO channel is given as

2 channel coeffs.

$$\overbrace{[h_1 \quad h_1]}^{1\times 2} = \bar{\mathbf{h}}^T$$

Alamouti Code

- h_1 is the **channel coefficient** between __RX ANTENNA__ and __TX ANTENNA 1__

- h_2 is the **channel coefficient** between __RX ANTENNA__ and __TX ANTENNA 2__

Alamouti Code

First Transmit instant

- In the **first time instant** consider the transmit vector

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

TRANSMIT ANTENNA 1

TRANSMIT ANTENNA 2

Alamouti Code

- x_1 is transmitted from Transmit antenna 1

- x_2 is transmitted from Transmit antenna 2

Alamouti Code

- Total power is split equally between x_1 and x_2

SPLIT EQUALLY.

$$\overbrace{E\{|x_1^2|\} = E\{|x_2^2|\}}^{\text{POWER} = P} = \frac{P}{2}$$

Each symbol has half Transmit power

Alamouti Code

- Therefore, output in **time instant** 1 is given as

$$y_1 = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$

OUTPUT IN FIRST TIME INSTANT

$$= h_1 x_1 + h_2 x_2 + n_1 = y_1$$

Alamouti Code

Time instant 2

- In the **second time instant** consider the transmit vector

Transmit antenna 1

$$\begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix}$$

Transmit antenna 2

Alamouti Code

- $-x_2^*$ is transmitted from ~~Transmit antenna 1~~ Transmit antenna 1

- x_1^* is transmitted from Transmit antenna 2

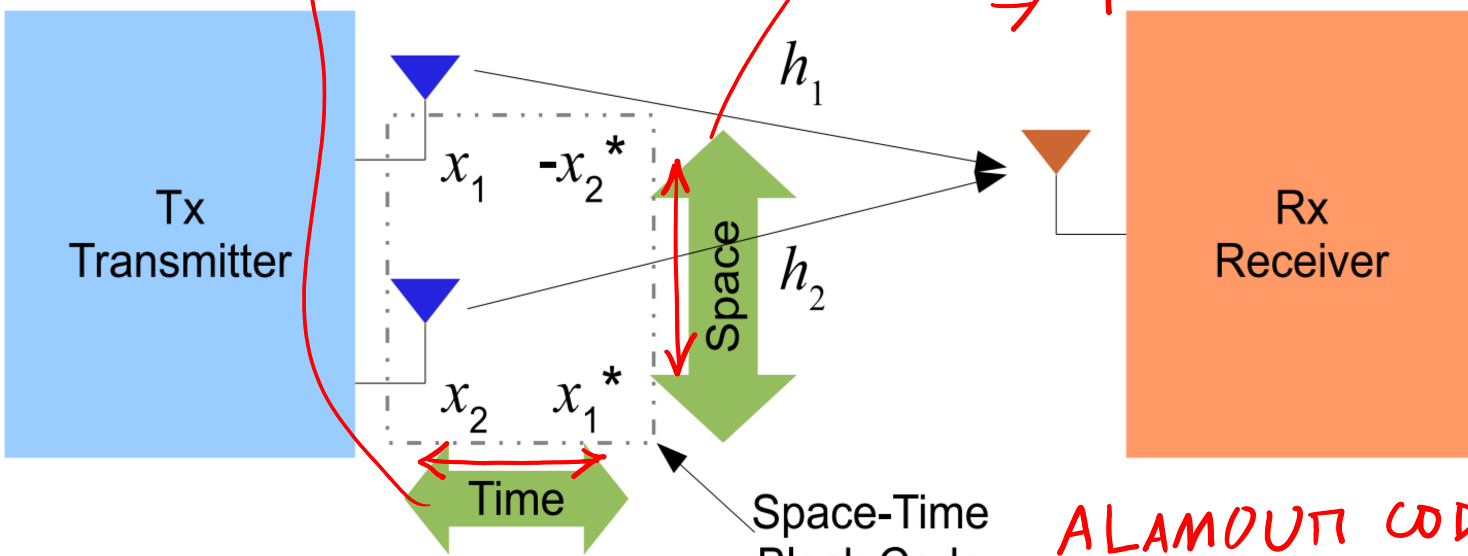
STBC

2 Transmit Time instants.

Space ⇒ 2 Antennas.

⇒ Space + Time coding

⇒ Space Time Block Code (STBC)



ALAMOUTI CODE is an STBC.

STBC

- Coding across SPACE and TIME

- Hence termed SPACE TIME BLOCK CODE (STBC)

Alamouti Code

- Therefore, output in **time instant** 2 is given as

NOISE AT TIME = 2

$$y_2 = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} + n_2$$

OUTPUT AT TRANSMIT INSTANT 2

$$= -h_1 x_2^* + h_2 x_1^* + n_2 = y_2$$

Alamouti Code

$y_2 = -h_1 x_2^* + h_2 x_1^* + n_2$

- Consider now y_2^*

complex conjugate of output at time = 2

y_2^*

$$y_2^* = -h_1^* x_2 + h_2^* x_1 + n_2$$

$$= \begin{bmatrix} h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2^*$$

$$= h_2^* x_1 + (-h_1^*) x_2 + n_2^*$$

Alamouti Code

- The net system model is given as

Very interesting matrix !!!

$$\underbrace{\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix}}_{\text{output Vector}} = \underbrace{\begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix}}_{2\times 2 \text{ Alamouti matrix}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\text{Symbol Vector}} + \underbrace{\begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}}_{\text{Noise Vector}}$$

Alamouti Code

- What is the interesting aspect of the matrix below?

$$\overbrace{\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}}^{\text{2x2 ALAMOUTI MATRIX}}$$

INTERESTING!

Alamouti Code

$\bar{u}, \bar{v}$

$\bar{u}^H \bar{v} = $ inner product

$\bar{u}, \bar{v}$ orthogonal

$\Rightarrow$ inner product $= 0$.

$\bar{u}^H \bar{v} = 0$

- The columns of the matrix below are ORTHOGONAL

$\Rightarrow$ inner product of column vectors $= 0$

$$\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$$

Alamouti Code

- This can be verified as follows

$$\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$$

$$\bar{c}_1 = \begin{bmatrix} h_1 \\ h_2^* \end{bmatrix} \qquad \bar{c}_2 = \begin{bmatrix} h_2 \\ -h_1^* \end{bmatrix}$$

Alamouti Code

$$\bar{c}_1^H \bar{c}_2 = \begin{bmatrix} h_1^* & h_2 \end{bmatrix} \begin{bmatrix} h_2 \\ -h_1^* \end{bmatrix}$$

$$= h_1^* h_2 - h_2 h_1^* = 0$$

$\Rightarrow$ $\bar{c}_1, \bar{c}_2$ ARE orthogonal!!

MAKES DECODER VERY EASY!

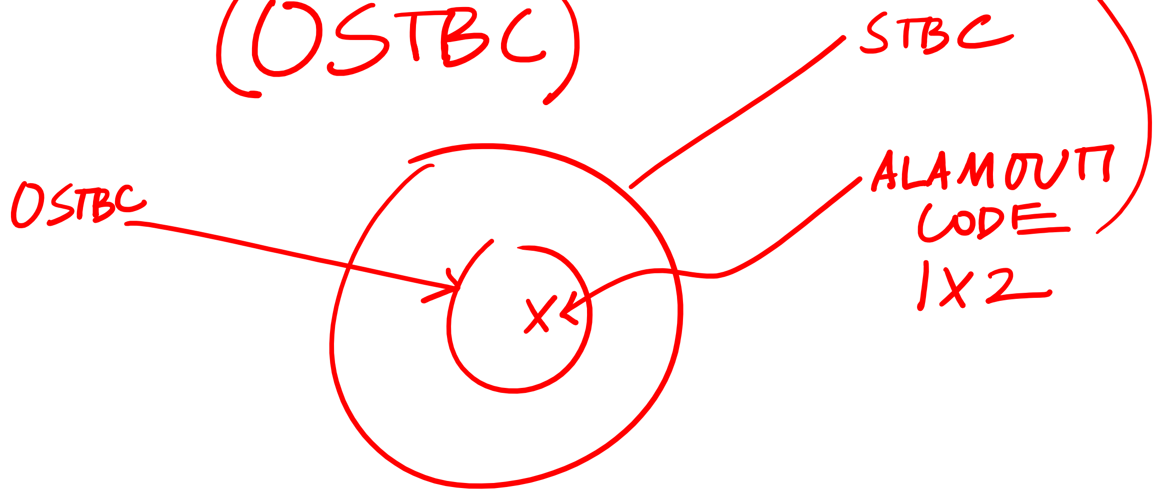
Alamouti Code

Can be extended TO arbitrary # Rx antennas.

- Hence Alamouti code is termed as

ORTHOGONAL SPACE TIME BLOCK CODE

(OSTBC)



Alamouti Code

$H = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$ $H^T = \begin{bmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{bmatrix}$

- Since matrix is **orthogonal**, decoding can be simply performed by **multiplying by inverse** given as

$$\frac{1}{\|\bar{\mathbf{h}}\|^2}\mathbf{H}^H = \overbrace{\frac{1}{\|\bar{h}\|^2} \underbrace{\begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix}}_{H^H}} = \frac{H^H}{\|\bar{h}\|^2}$$

$H^H = (H^T)^*$

$\|\bar{h}\|^2 = |h_1|^2 + |h_2|^2$

inverse of Alamouti matrix

Alamouti Code

$H = 2 \times 2$ ALAMOUTI Matrix

- Since matrix is **orthogonal**, decoding can be simply performed by **multiplying by inverse** given as

$$H^{-1} = \frac{1}{\|\bar{\mathbf{h}}\|^2}\mathbf{H}^H = \frac{1}{\|\bar{\mathbf{h}}\|^2}\begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix}$$

$|h_1|^2 + |h_2|^2$

$$\bar{y} = \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix}$$

$$H^{-1}\bar{y} = \frac{1}{\|\bar{h}\|^2} \cdot \underbrace{H^H \bar{y}}$$

Decoder Operation

Alamouti Code Example

Example for Alamouti code.

multiple input single output.

1x2 MISO channel.

- Consider the MISO channel

$$\left[\underbrace{\overbrace{1-2j}^{h_1} \quad \overbrace{-2+j}^{h_2}}_{1\times 2}\right] \quad \bar{h}^T$$

$\|\bar{h}\|^2 = |h_1|^2 + |h_2|^2$

$= 1+4+4+1 = 10.$

Alamouti Code

- $h_1 = \underline{\ 1-2j\ }$ is the **channel coefficient** between Rx antenna, Tx antenna 1

- $h_2 = \underline{\ -2+j\ }$ is the **channel coefficient** between Rx antenna, Tx antenna 2

Alamouti Code

$x_1 = -1 - j$
$x_2 = 1 + j$

- In the **first time instant** consider the transmit vector

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 - j \\ 1 + j \end{bmatrix}$$

x_1

Transmit instant $= 1$

x_2

Alamouti Code

- $x_1 = \underline{\ -1-j\ \ \ \ \ \ \ \ \ \ \ \ }$ is transmitted from Transmit antenna 1
- $x_2 = \underline{\ \ \ \ \ 1+j\ \ \ \ \ \ \ \ \ \ \ \ }$ is transmitted from Transmit antenna 2

Alamouti Code

$x_2 = 1+j \quad x_2^* = 1-j$

$-x_2^* = -1+j$

- In the **second time instant** consider the transmit vector

$x_1 = -1-j \Rightarrow x_1^* = -1+j$

$$\begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} = \begin{bmatrix} -1+j \\ -1+j \end{bmatrix} \begin{array}{l} -x_2^* \\ x_1^* \end{array}$$

Transmit instant 2

Alamouti Code

Transmit antenna 1

- In the **second time instant** consider the transmit vector

$-x_2^*$

$$\begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} = \begin{bmatrix} -1+j \\ -1+j \end{bmatrix}$$

x_1^*

Transmit antenna 2

Alamouti Code

- $-x_2^* =$ _____$-1+j$_____ is transmitted from Tx Antenna 1

- $x_1^* =$ _____$-1+j$_____ is transmitted from Tx antenna 2

Alamouti Code

- Alamouti matrix is given as

$$\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} = \underbrace{\begin{bmatrix} 1-2j & -2+j \\ -2-j & -1-2j \end{bmatrix}}_{\text{2X2 ALAMOUTI MATRIX.}}$$

2x2 Alamouti matrix

$h_2^* = -2-j$

$h_1 = 1-2j \quad h_2 = -2+j$

$h_1^* = 1+2j \quad -h_1^* = -1-2j$

$\bar{c}_1, \bar{c}_2$ orthogonal.
Because inner product $= 0$.

$$\bar{c}_1 = \begin{bmatrix} 1-2j \\ -2-j \end{bmatrix} \quad \bar{c}_2 = \begin{bmatrix} -2+j \\ -1-2j \end{bmatrix}$$

$$\begin{aligned} \bar{c}_1^H \bar{c}_2 &= (1+2j)(-2+j) + (-2+j)(-1-2j) \\ &= (1+2j)(-2+j) - (-2+j)(1+2j) \\ &= 0 \end{aligned}$$

Alamouti Code

- The columns can be seen to be orthogonal as follows

$$\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$$

$$\bar{c}_1 \qquad \bar{c}_2$$

$\bar{c}_1^H \bar{c}_2 = 0$

$\bar{c}_1, \bar{c}_2$ are Orthogonal.

$$\bar{c}_1^H \bar{c}_2 = h_1^* h_2 + h_2(-h_1^*) = h_1^* h_2 - h_2 h_1^*$$
$$= 0$$

Alamouti Code

$$\overbrace{\begin{bmatrix} 1-2j & -2+j \\ -2-j & -1-2j \end{bmatrix}}^{2\times 2 \text{ Alamouti matrix}}$$

h_1 $\quad$ h_2

h_2^* $\quad$ $-h_1^*$

$$(1+2j)(-2+j)+(-2+j)(-1-2j)$$

$$=(1+2j)(-2+j)$$

$$\underbrace{-(-2+j)(1+2j)=0}_{\bar{c}_1^H \bar{c}_2 = 0 \;\Rightarrow\; \text{columns are Orthogonal!!!}}$$

OSTBC

Alamouti Code

$H = \begin{bmatrix} 1-2j & -2+j \\ -2-j & -1-2j \end{bmatrix}$

- Decoding can be simply performed by **multiplying by inverse** given as

Decoding matrix

$$\frac{1}{\|\bar{\mathbf{h}}\|^2}\mathbf{H}^H = \frac{1}{10}\underbrace{\begin{bmatrix} 1+2j & -2+j \\ -2-j & -1+2j \end{bmatrix}}_{H^H} = H^{-1}$$

$\|\bar{h}\|^2 = |h_1|^2 + |h_2|^2$

$= |1-2j|^2 + |-2+j|^2$

$= 1 + 4 + 4 + 1 = 10$

Alamouti Code

- Decoding can be simply performed by **multiplying by inverse** given as

$$\frac{1}{\|\bar{\mathbf{h}}\|^2}\mathbf{H}^H = \frac{1}{10}\begin{bmatrix} 1+2j & -2+j \\ -2-j & -1+2j \end{bmatrix}$$

Alamouti BER

- The output SNR for each stream is given as

Splitting power
Each stream has
$\frac{1}{2}$ power

$E\{|x_1|^2\} = E\{|x_2|^2\}$
$= \frac{P}{2}$

$$SNR_o = \|\bar{\mathbf{h}}\|^2 \frac{\frac{P}{2}}{N_0} = \frac{1}{2} \times \|\bar{\mathbf{h}}\|^2 \times SNR$$

$$= \frac{1}{2} \cdot \left(|h_1|^2 + |h_2|^2\right) \times SNR.$$

Alamouti BER

QPSK. $SNR = \frac{P}{N_0}$.

BPSK. $SNR = \frac{2P}{N_0}$.

- The output BER for BPSK is given as

ADVANTAGE:

DOES NOT NEED CHANNEL STATE INFORMATION @ TRANSMITTER.

$$BER = \frac{3}{SNR^2}$$

BER is decreasing as $\frac{1}{SNR^2}$

$\Rightarrow$ Diversity order = 2

$= \frac{3}{SNR^2}$

Alamouti Code Does NOT need knowledge of channel. @ Transmitter for Beamforming.

Alamouti BER

$10 \log_{10} SNR = 20 \Rightarrow \log_{10} SNR = 2 \Rightarrow SNR = 10^2$

- Example: Evalate BER for SNR=20 dB= 10^2

BPSK. – Binary Phase Shift Keying.

$$BER = \frac{3}{SNR^2} = \frac{3}{(10^2)^2}$$

$= 3 \times 10^{-4} = BER$

Alamouti BER

- BER for SNR=20 dB equals

$$3 \times 10^{-4}$$

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