eMasters in Communication Systems

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Core Module:

Wireless Communication

Chapter 6

Wireless Channel Characterization

Wireless Channel Model

- Wireless ***channel model***

L = NUMBER OF MULTIPATH COMPONENTS.

channel.

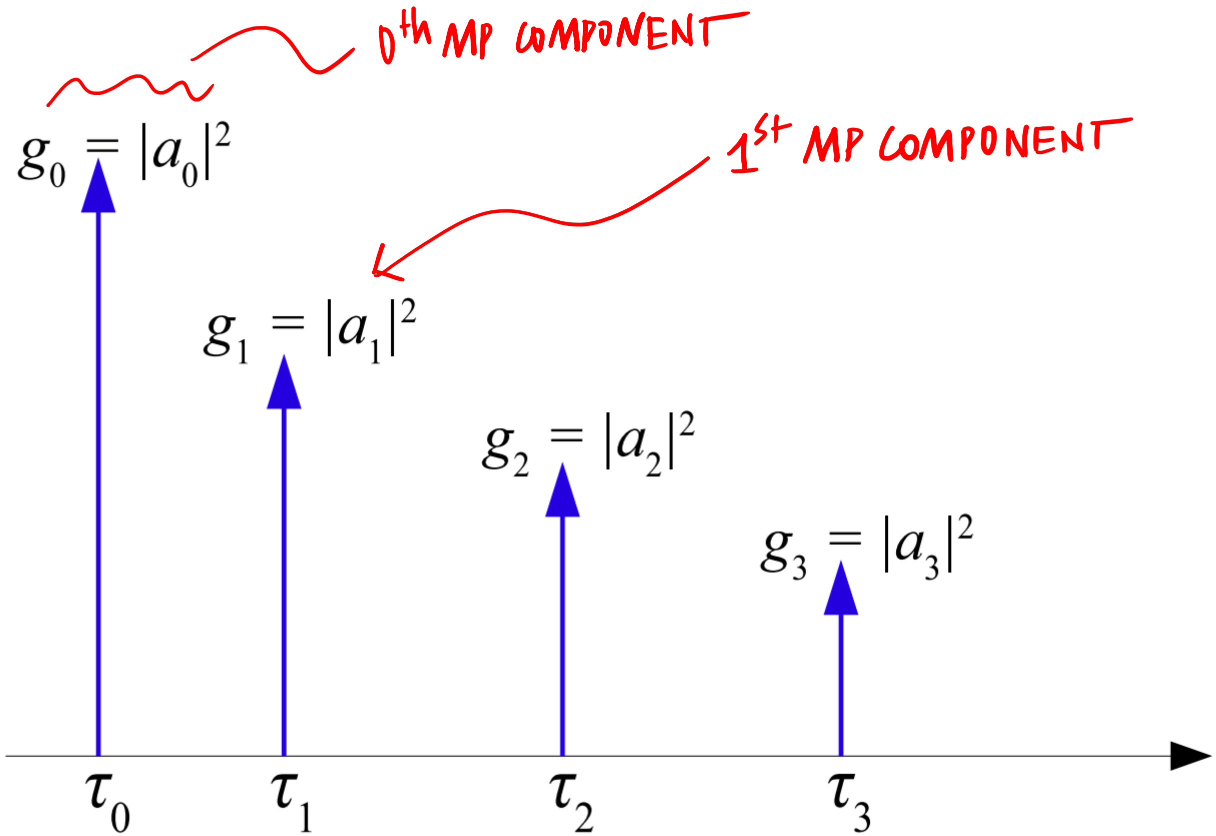
$$h = \sum_{i=0}^{L-1} a_i \delta(t - \tau_i)$$

DELAY.

ATTENUATION

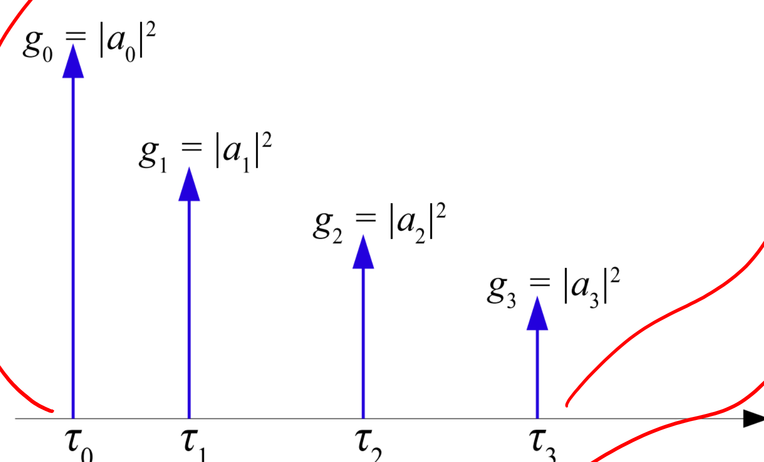
- τ_i: **Delays** of the multipath components

Wireless Channel Model



Wireless Channel Model

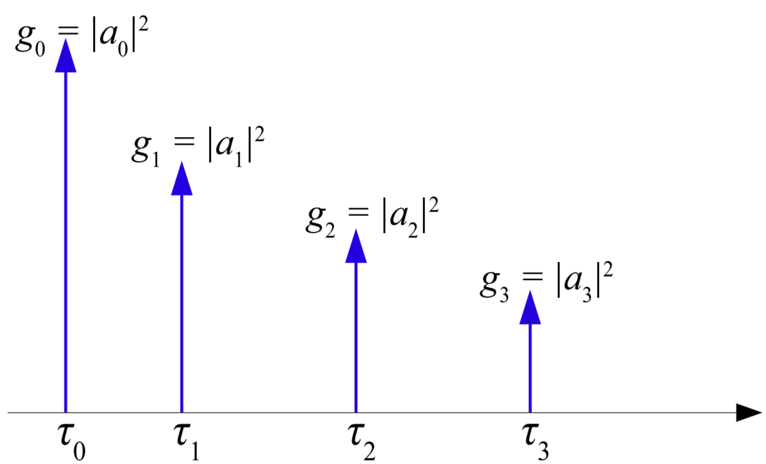
- How many **multipath components**? 4
- What is the **least delay**? τ_0 EARLIEST
- What is the **maximum delay**? τ_3 LAST



SPREAD OF TIME

Wireless Channel Model

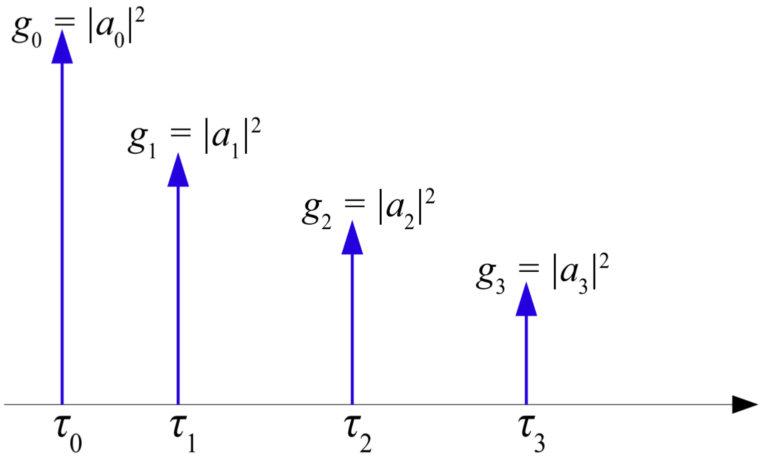
- Multipath components are arriving over a SPREAD OF TIME.
- Or in other words, the delays are **spread over time**.
  - This is termed the DELAY SPREAD.



Delay Spread

DELAY. SPREAD

- How to characterize this?



Delay Spread

- **Max delay spread** = Maximum Delay Spread.

$$T_d = \tau_{L-1} - \tau_0 = T_{d,max}.$$

Delay of Last MP component

Delay of First MP component

Delay Spread

Spread of Time

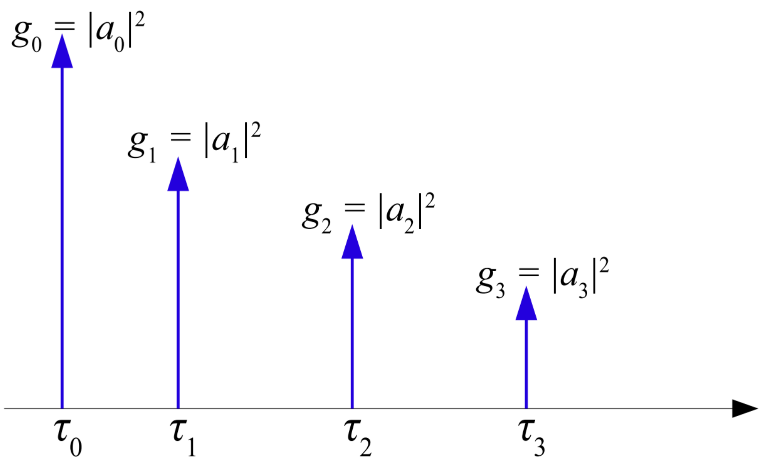
- Max delay spread

$$T_d =$$

Delay Spread

- **Max delay spread**

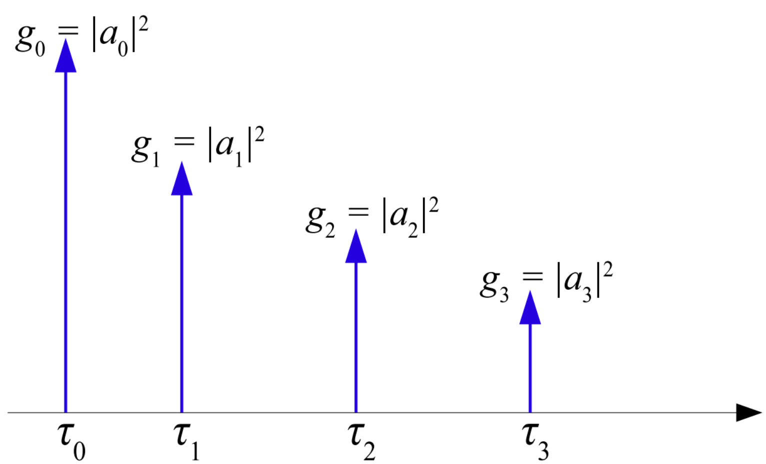
$$T_d = \tau_{L-1} - \tau_0$$



RMS Delay Spread

Root Mean Square.

- Another metric for the delay spread is the **RMS delay spread.**



RMS Delay Spread

- This is defined as follows
- Let $g = |a_i|^2$

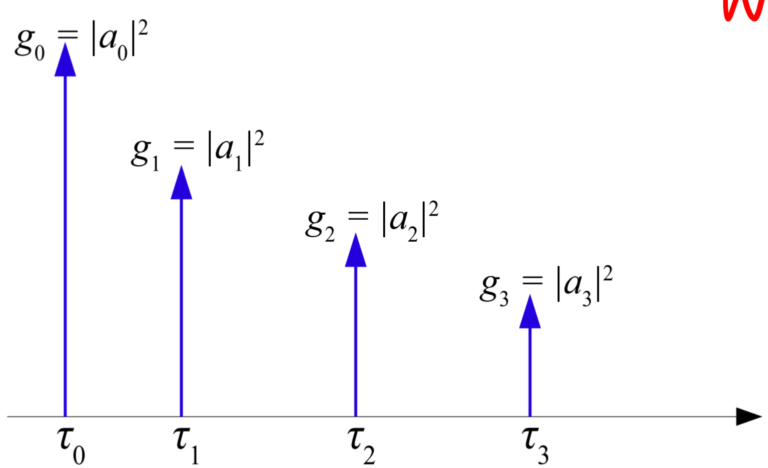
gain of ith multipath component

$g_i = |a_i|^2$

$\bar{\tau} =$

$$\bar{\tau} = \frac{\sum_i g_i \tau_i}{\sum_i g_i}$$

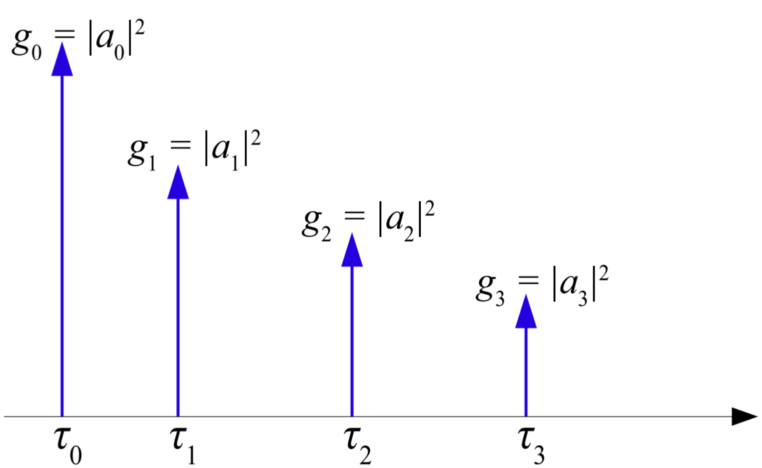
Mean delays
Weighted Average of Delay.



RMS Delay Spread

- This is defined as follows
- Let $g = |a_i|^2$

$$\bar{\tau} = \frac{\sum_i g_i \tau_i}{\sum_i g_i}$$



RMS Delay Spread

$$\underbrace{T_{d,rms} = \sqrt{\frac{\sum_i g_i(\tau_i - \bar{\tau})^2}{\sum_i g_i}}}_{\text{RMS Delay Spread.}}$$

RMS Delay Spread

$$T_{d,rms} = \sqrt{\frac{\sum_i g_i(\tau_i - \bar{\tau})^2}{\sum_i g_i}}$$

Delay Spread

- What is typical **delay/ delay spread** in wireless channel?

Order of distances in the wireless channel.

$\mathcal{O}(\mu s)$

$$\approx \frac{1\,\text{km}}{3 \times 10^{8}\,\text{m/s}} \approx 3.3\,\mu s$$

$\approx 2 - 3\,\mu s.$

Velocity of EM wave.

Delay Spread

- What is typical **delay/ delay spread** in wireless channel?

$$\frac{1km}{3 \times 10^8 m/s} = \underline{\underline{3.3\ \mu s}}$$

Delay Spread

- Typical **delay spread**

OUTDOOR CHANNEL

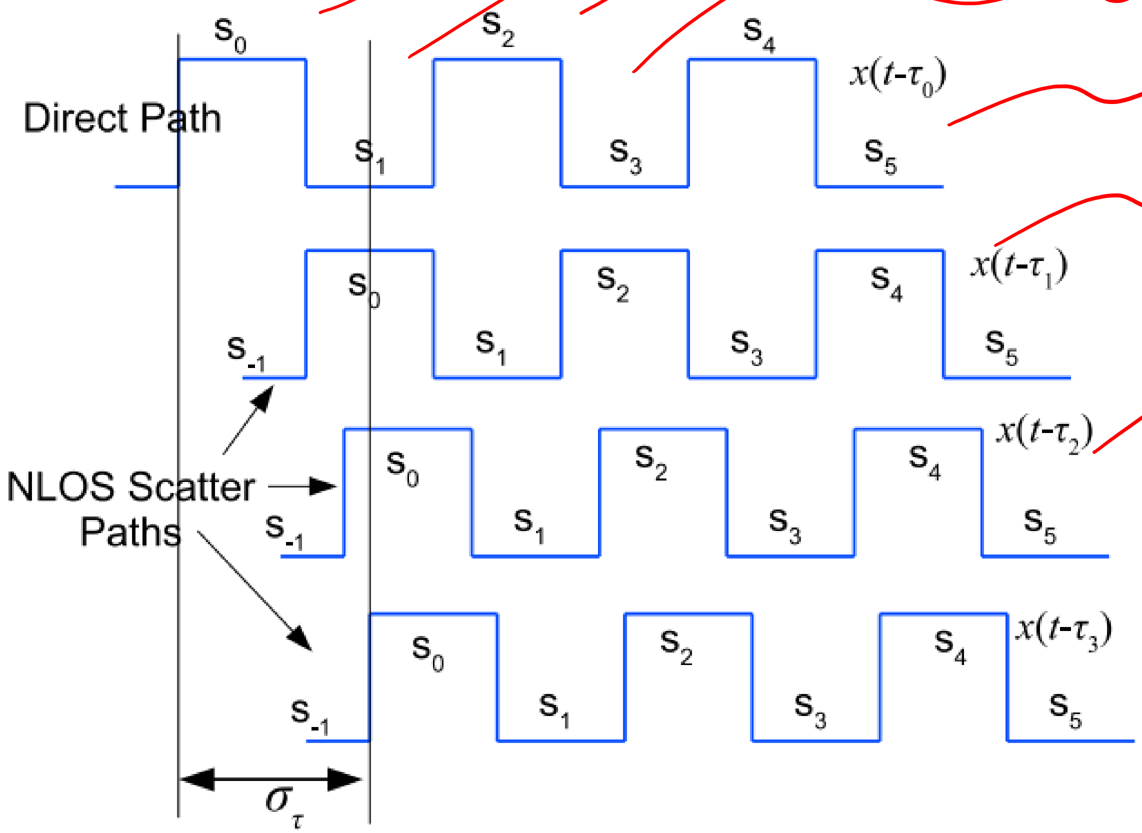
$$\approx 2 - 3\ \mu s$$

Delay Spread

SYMBOLS .

- What is the **impact** of delay spread?

inter symol interference (ISI)



τ_0 .

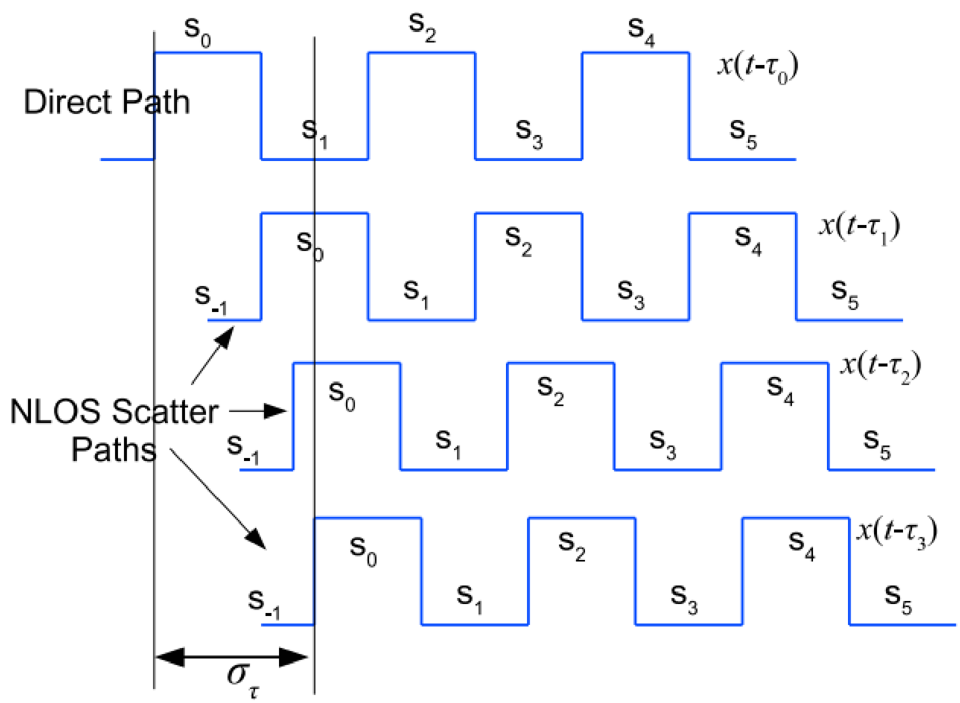
τ_1

τ_2

SUPERPOSE AT RECEIVER

Delay Spread

- Large delay spread leads to **Inter-symbol interference** (**ISI**) ISI



Delay Spread

- When does ISI occur?

T_d = Delay Spread.
T_s = Symbol Time

$$T_d \geq \frac{1}{2} T_s \Rightarrow \text{ISI}$$

$$T_d < \frac{1}{2} T_s \Rightarrow \text{No ISI.}$$

Delay Spread

- When does ISI occur?

DELAY SPREAD

SYMBOL TIME

$$T_d \geq \frac{1}{2} T_s$$

Delay Spread

- When does ISI occur?
- Set $T_d = 2\mu s$

$$\overbrace{T_d \geq \frac{1}{2} T_s}^{\text{ISI Occurs.}}$$

ISI occurs when $\underline{T_s \leq 4\mu s}$.

$$T_d = 2\mu s \geq \frac{1}{2} T_s$$

$$\Rightarrow T_s \leq 2T_d = 4\mu s.$$

Delay Spread

- When does ISI occur?
- Set $T_d = 2\mu s$

$$T_d \geq \frac{1}{2} T_s$$

ISI occurs.

$$T_d = 2\mu s \geq \frac{1}{2} T_s$$

$$\Rightarrow T_s \leq 4\mu s = 2T_d$$

No-ISI Channel Model

- No ISI channel model is

Channel coefficient

current symbol.

$$\underbrace{y(k) = h\,x(k) + n(k)}_{\text{No ISI}}$$

- $y(k)$ depends only on $x(k)$

No-ISI Channel Model

- No ISI channel model is

$$y(k) = hx(k) + n(k)$$

ISI Channel Model

current symbol

Previous symbols.

- **ISI channel model** is

$$y(k) = h(0)x(k) + h(1)x(k-1) + h(2)x(k-2) + \cdots + h(L-1)x(k-L+1) + n(k)$$

$$= h * x + n$$

- $y(k)$ depends on $x(k), x(k-1), \ldots$

$\underbrace{h(0), h(1), \ldots, h(L-1)}_{\text{Channel Taps.}}$: ISI channel.

ISI Channel Model

- ISI channel model is

channel.
Taps

Taps of
channel. Filter
FIR Filter

$$y(k) = h(0)x(k) + h(1)x(k-1) + \cdots + n(k)$$

Delay Spread

$B = 2$ sided BW

$T_d = 2\mu s = 2 \times \frac{B}{2}$

- What is the bandwidth?

Symbol Time

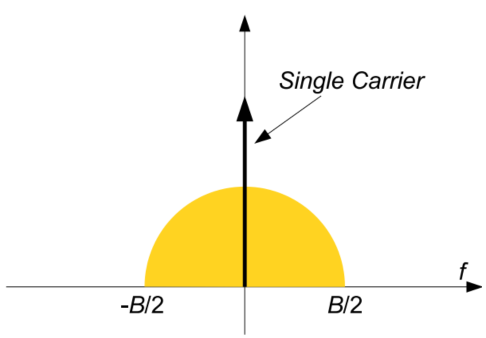
$= \frac{1}{B} = \frac{1}{2 \times B/2}$

$$T_s \le 4\mu s = 2T_d$$

$$\Rightarrow \frac{1}{B} \le 4\mu s$$

B_c of channel
= coherence BW of channel.

$$\Rightarrow B \ge \frac{1}{2T_d} = \frac{1}{4\mu s} = \underline{250\,kHz}.$$



Delay Spread

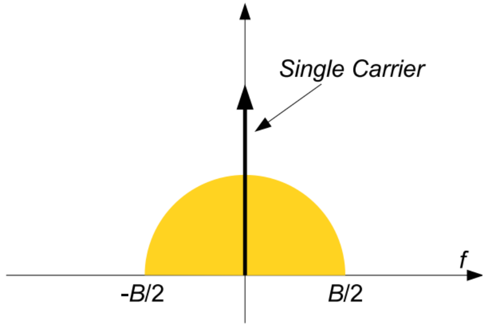
- What is the bandwidth?

ISI occurs if
$B \geq 250 kHz$

$$T_s \leq 4\mu s$$

$$\Rightarrow \frac{1}{B} \leq 4\mu s$$

$$\Rightarrow B \geq \frac{1}{2T_d} = \frac{1}{4\mu s} = 250\,kHz$$



Delay Spread

coherence BW

$$\boxed{B_c = \frac{1}{2T_d}}$$

T_d = channel delay spread.

- ISI occurs if

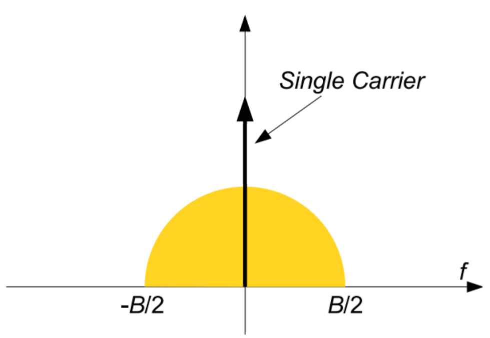
$$\Rightarrow B \geq \frac{1}{4\mu s} = \frac{1}{2T_d} = 250\ kHz = B_c$$

- This is termed the **Coherence Bandwidth** of channel.

ISI occurs if

$$\left.\begin{array}{l} T_d \geq \frac{1}{2} T_s \\ B \geq B_c = \frac{1}{2T_d} \end{array}\right\}$$

EQUIVALENT CONDITIONS.



Delay Spread

- One can define

$$B_c \approx \frac{1}{2T_d}$$

coherence BW
~ 250kHz.

Delay
Spread.
2~3μs.

Delay Spread

- Therefore,

$$B_c \propto \frac{1}{T_d}.$$

Coherence BW is inversely prop to Delay spread.

Delay Spread

- As delay spread increases…
  - Coherence BW DECREASES.

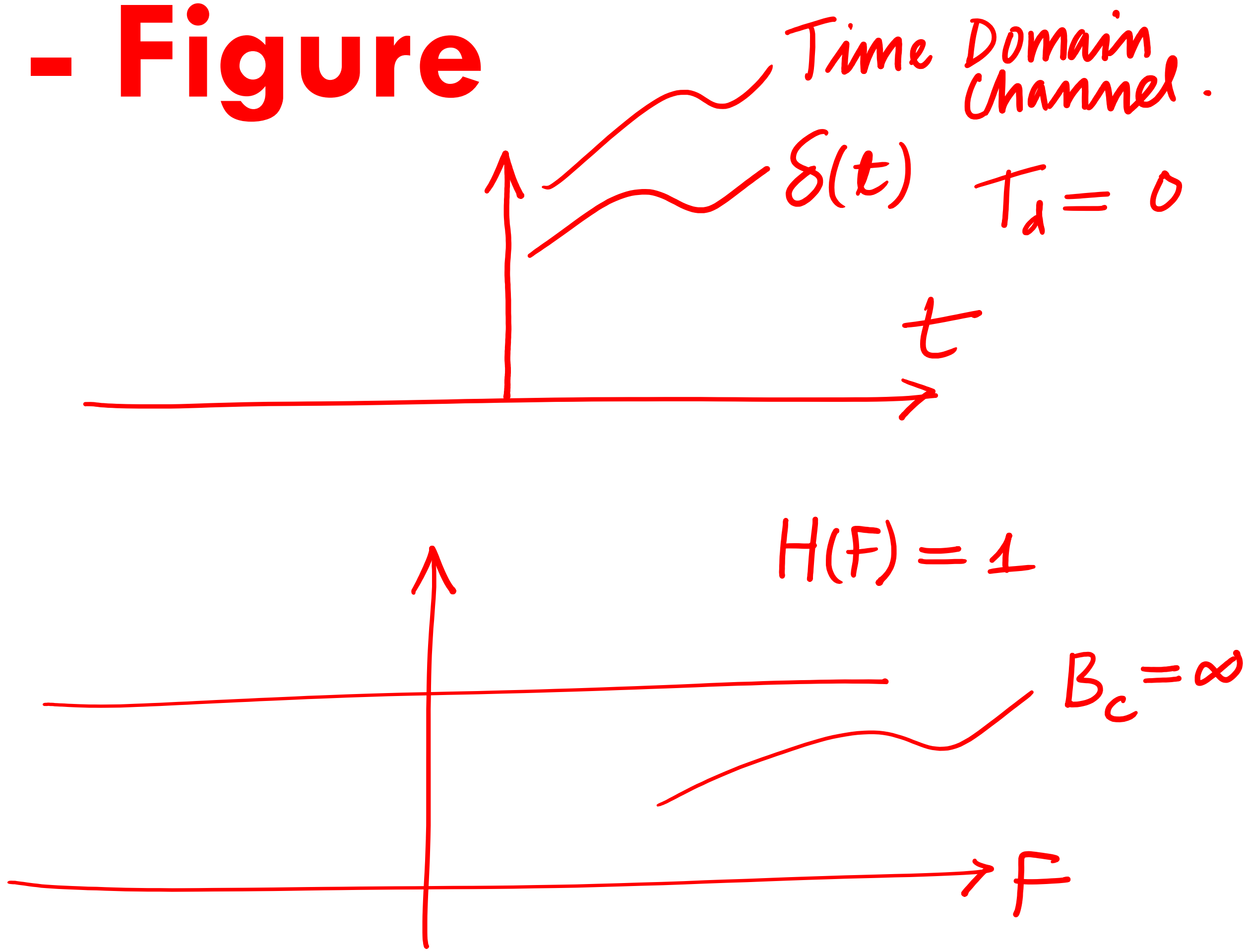
$$B_c \approx \frac{1}{2T_d}.$$

Delay Spread

$h(t) = \delta(t)$

- Delay spread $T_d = 0$.
- Fourier transform $|H(f)| = 1$
  - BW $= B_C = \infty$

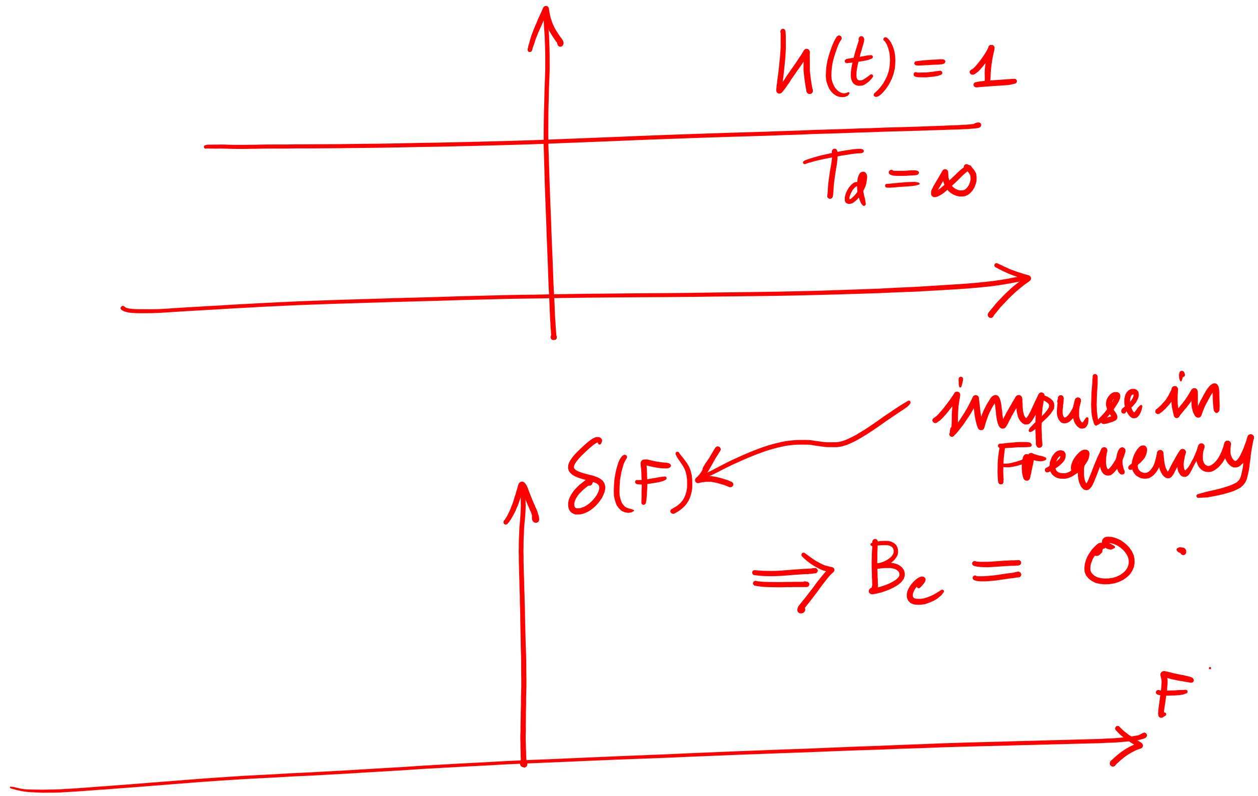
$T_d = 0$ - Figure



Delay Spread

- Delay spread $T_d = \infty$. $h(t) = 1$ impulse ~~in~~ frequency
- Fourier transform $|H(f)| =$ $\delta(F)$
  - BW $= B_c =$ 0

$T_d = \infty$ - Figure



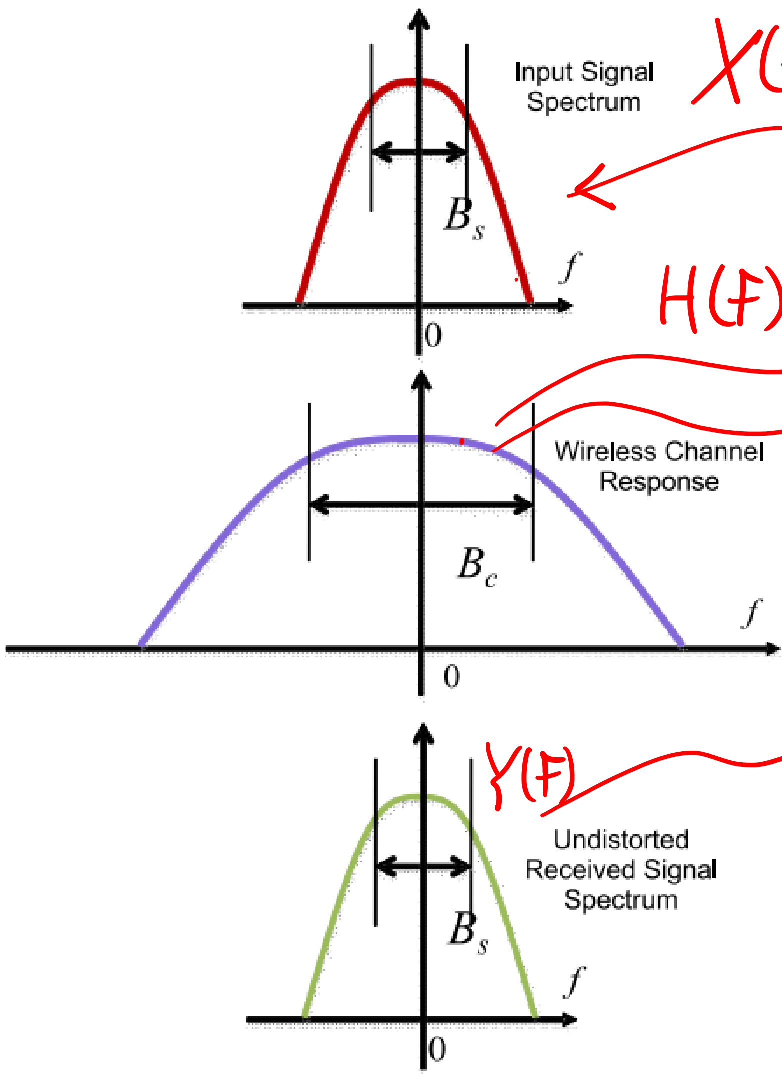
Coherence Bandwidth

- What is the frequency domain interpretation? of B_c.

Coherence Bandwidth

Signal Spectrum Lies in Flat Region of channel Response

B.



X(F)

input Spectrum

H(F)

BW = Bc

BW OF channel.

Response OF channel is Flat.

Y(F)

undistorted!

$B_s << B_c$

Coherence Bandwidth

- When $B_s < B_c$, Output spectrum is **undistorted**.

  Signal BW < coh BW.

  - Channel response is FLAT over signal bandwidth.
  - Such a channel is termed FLAT FADING..

    $B < B_c$.

  - $\Rightarrow$ No ISI $\Rightarrow$ NO DISTORTION.

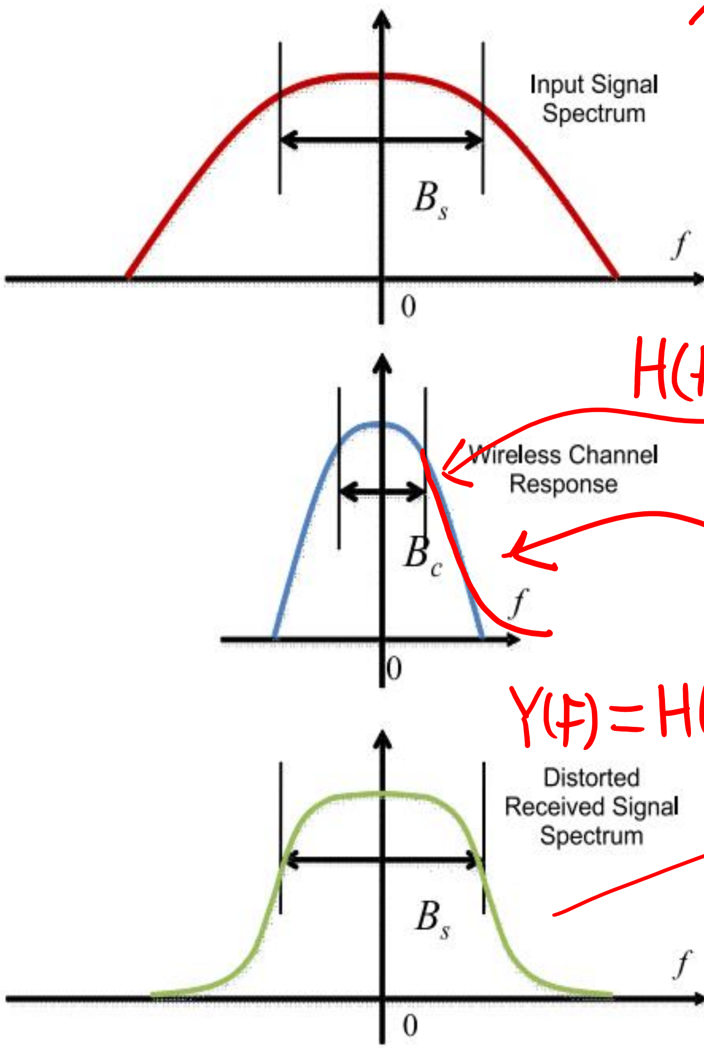
Coherence Bandwidth

$X(F)$

$BW = B$.

$\Rightarrow$ ISI in Time Domain

$B \geq B_c$



$H(F)$

Wireless channel.

$BW = B_c$

Response of channel.

NOT Flat.

Distorted version OF input spectrum

$Y(F) = H(F)X(F)$

Coherence Bandwidth

$B \geq B_c$

signal BW

coherence BW

- When $B_s \geq B_c$, Output spectrum is **Distorted**.
  - Fading is FREQUENCY SELECTIVE.
  - Channel response is VARYING. over signal bandwidth.
- $\Rightarrow$ ISI

inter symbol interference.

Coherence Bandwidth

- When $B_s \geq B_c$, Output spectrum is **Distorted.**

  Output spectrum is distorted version of input spectrum.

  - Fading is___ **frequency selective fading**.
  - Channel response is **varying** over signal bandwidth.
- $\Rightarrow$ ISI

  inter symbol interference.

Summary…

Flat Fading	Frequency Selective
$T_d < \frac{1}{2} T_s$	$T_d \geqslant \frac{1}{2} T_s$
No ISI	ISI Occurs
$B_s < B_c$	$B_s = B \geq B_c$
Output spectrum UNDISTORTED.	Output spectrum distorted

Summary…

Output depends only on current input symbol.

Flat Fading	Frequency Selective
Flat−fading channel	Frequency−selectiv channel
$y(k) = hx(k) + n(k)$	$y(k) = h(0)x(k) + h(1)x(k-1) + h(2)x(k-2) + \cdots + h(L-1)x(k-L+1) + n(k)$.

Output depends also on previous symbols.

Summary…

Flat Fading	Frequency Selective
Flat−fading channel	Frequency−selectiv channel
$y(k) = hx(k) + n(k)$	$y(k) = h(0)x(k) + h(1)x(k-1) + \cdots + h(L-1)x(k-L+1) + n(k)$

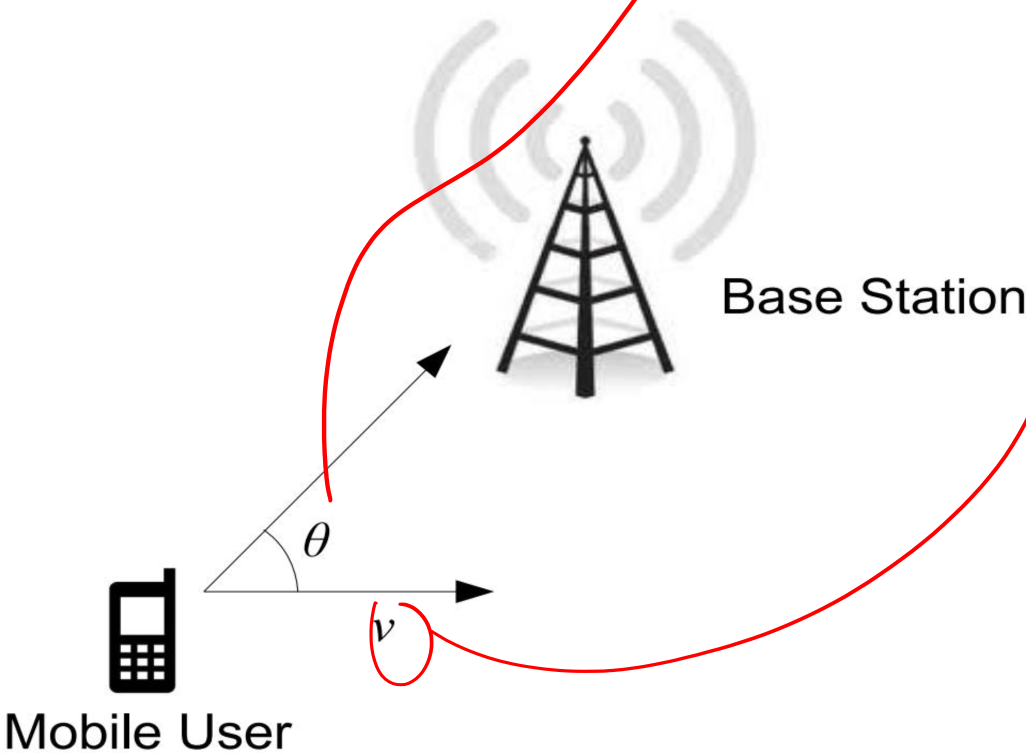
Doppler Shift

- Mobile or source is **moving** towards or away from the receiver

BS

angle = $\underline{\theta}$.

Velocity = $\underline{V}$



Doppler Shift

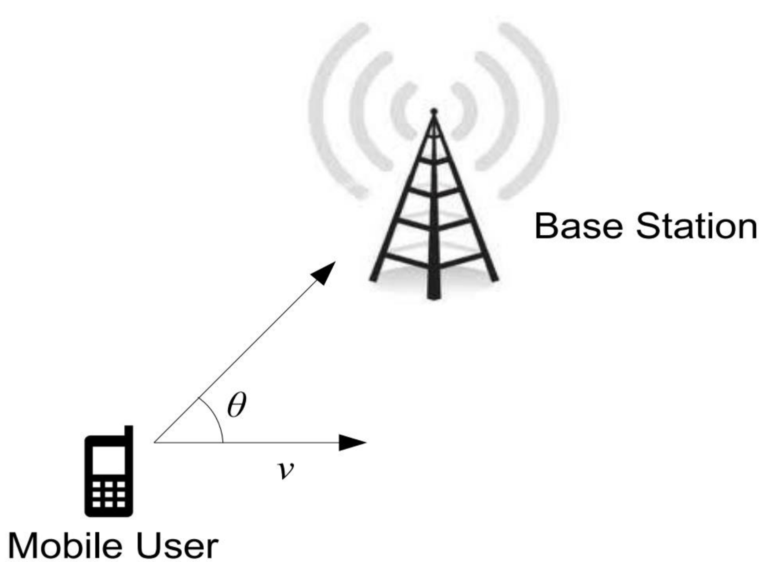
angle between velocity & Line joining Mobile to BS.

- There is a change in the frequency which is termed the DOPPLER SHIFT.

carrier frequency

$$f_D = \frac{V \cos\theta}{C} \times F_c$$

= Velocity of Light

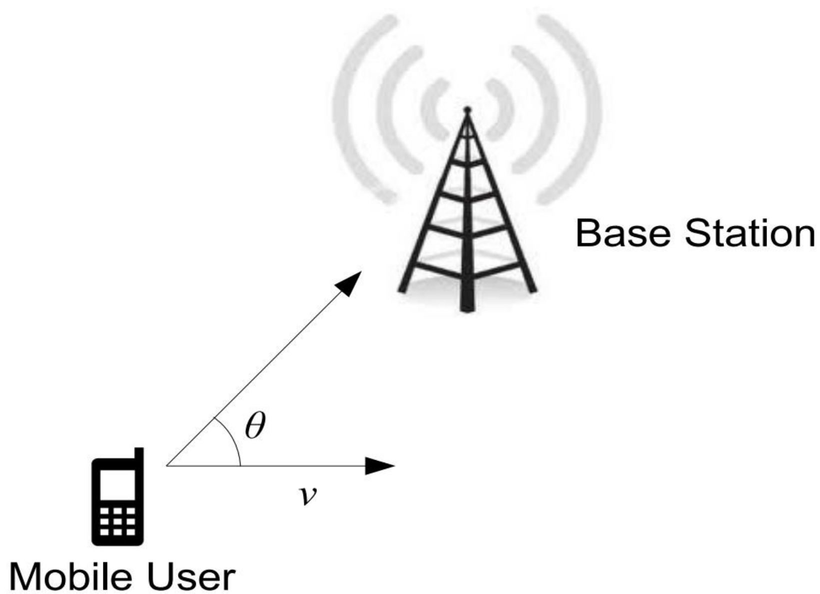


Doppler Shift

Formula for Doppler shift

- There is a change in the frequency which is termed the **Doppler shift**.

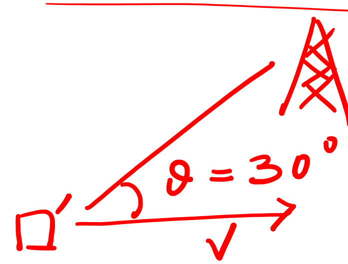
$$f_D = \frac{v \cos\theta}{c} f_c$$



Doppler Shift-Example

$60 \times \frac{5}{18}$ m/s.

- Consider a vehicle moving a 60 km per hour
- at an angle of $\theta = 30^\circ$



- **Carrier frequency** of fc = 2 GHz

$= 2 \times 10^{9}$ Hz

- Compute the **Doppler shift** of the received signal at a.

Doppler Shift-Example

$$f_D = \frac{v\cos\theta}{c} f_c$$

$$= \frac{60 \times \frac{5}{8} \times \frac{\sqrt{3}}{2}}{3 \times 10^8} \times 2 \times 10^9$$

$$= 96.22\ Hz \quad = F_D.$$

Doppler Shift-Example

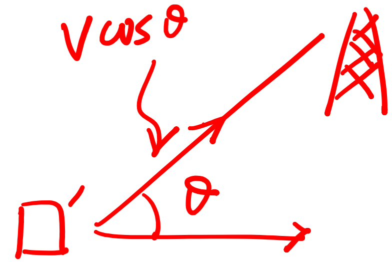
$$f_D = \frac{v\cos\theta}{c} f_c$$

V in m/s.   cos 30°.   Fc.

$$= \underbrace{60 \times \frac{5}{18}}_{V \text{ in m/s}} \times \frac{\underbrace{\frac{\sqrt{3}}{2}}_{\cos 30^\circ}}{\underbrace{3 \times 10^8}_{=c}} \times \underbrace{2 \times 10^9}_{F_c}$$

$$= 96.22\ Hz$$

Impact on Channel



- Consider path i

original distance

$$\tau_i(t) = \frac{d_i - (v\cos\theta)t}{c}$$

New Delay.

Delay depends on Time

Time varying delay

$$h(t) = \sum_{i=0}^{L-1} a_i \delta(\tau - \tau_i(t)) = \sum_{i=0}^{L-1} a_i \delta\underbrace{\left(\tau - \frac{d_i - v\cos\theta\, t}{c}\right)}_{\tau_i(t)}$$

channel is a Function of Time!!

Impact on Channel

- Because of Doppler, what is **happening to channel**?
  - Channel is **time-varying!!!**.
  - This is termed as TIME SELECTIVE CHANNEL.

- **Doppler** → TIME SELECTIVE CHANNEL

Impact on Channel

- Because of Doppler, what is **happening to channel**?
  - Channel is **time-varying!!!**.
  - This is termed as ***time-selective channel***.

- **Doppler** → **Time Selective Channel**

Impact on Channel

- Let T_c denote the time over which channel is constant.
  - This is termed as the COHERENCE TIME.

How FAST is channel varying?

Impact on Channel

coherence Time is inversely proportional to Doppler

$$T_c = \frac{1}{4f_D} \propto \frac{1}{f_D}$$

- In previous example

approx constant for 2.6 ms.

$$T_c = \frac{1}{4f_D} = \frac{1}{4 \times 96.22} = 2.6\,ms$$

Impact on Channel

$$T_c = \frac{1}{4f_D} \propto \frac{1}{f_D}$$

- In previous example

$T_c \approx \underline{O(ms)}$.

$$T_c = \frac{1}{4f_D} = \frac{1}{4 \times 96.22\ Hz} = 2.6\ ms$$

Impact on Channel

$F_D \propto v$

- Higher **velocity** $\Rightarrow$ Doppler shift higher
- $\Rightarrow$ channel changing **faster**
- $\Rightarrow$ Channel constant over a very small period of time
- $\Rightarrow$ COHERENCE TIME. is small !

$$\Rightarrow T_C \propto \frac{1}{f_D}$$

Impact on Channel

- Higher **velocity** $\Rightarrow$ Higher **Doppler shift** f_D
- $\Rightarrow$ channel changing **faster**
- $\Rightarrow$ Channel constant over a very small period of time
- $\Rightarrow$ **Coherence time** is small

$$\Rightarrow T_c \propto \frac{1}{f_D}$$

Impact on Channel

- $2f_D$: **Doppler bandwidth**. B_D
- For previous example

Doppler Bandwidth of channel.

$$B_D = \underline{2 \times 96.22 \text{ Hz}} = 192.44\ Hz$$

Impact on Channel

- $2f_D$: **Doppler bandwidth**. B_D
- For previous example

T_c is inversely proportional to Doppler BW.

$$B_D = 2 \times 96.22 = 192.44\ Hz$$

$$T_c = \frac{1}{4f_D} = \frac{1}{2B_D}.$$

$$T_c \propto \frac{1}{B_D}.$$

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