# eMasters in Communication Systems

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# Elective Module:

# Advanced ML Techniques

# Chapter 6

# Naïve Bayes Examples

# Naïve Bayes Example

Movies

- Consider a training set of ***people's opinions***

NOT liking movie

  - **like** $= 1$ or **NOT** $= 0$ for the movies $x_1$, $x_2$ and $y$

AVATAR

TG

AV

Like movie

# Naïve Bayes Example

$$\frac{\text{Number of people who like } x_1 \text{ and dislike } y}{\text{Total number who dislike } y}$$

- The estimates of the ***prior probabilities*** are as follows

$x_1=0, y=0 \Rightarrow$ DO NOT Like $x_1$ and $y$

| | $x_1 = 0$ | $x_1 = 1$ | $p(x_1 = 0 \mid y)$ | $p(x_1 = 1 \mid y)$ |
|---|---|---|---|---|
| $y = 0$ | 3 | 10 | $p(x_1=0 \mid y=0) = \frac{3}{13}$ | $p(x_1=1 \mid y=0) = \frac{10}{13}$ |
| $y = 1$ | 4 | 13 | $p(x_1=0 \mid y=1) = \frac{4}{17}$ | $p(x_1=1 \mid y=1) = \frac{13}{17}$ |

NOT Like $x_1$
Like y

$x_1=1, y=0$
$\Rightarrow$ Like $x_1$ but NOT y

Like $x_1$, Like y

# Naïve Bayes Example

$p(x_1=0|y=0)$

$p(x_1=1|y=0)$

- The estimates of the ***prior probabilities*** are as follows

Prior Probabilities

| | $x_1 = 0$ | $x_1 = 1$ | $p(x_1 = 0\|y)$ | $p(x_1 = 1\|y)$ |
|---|---|---|---|---|
| $y = 0$ | 3 | 10 | $\frac{3}{13}$ | $\frac{10}{13}$ |
| $y = 1$ | 4 | 13 | $\frac{4}{17}$ | $\frac{13}{17}$ |

$p(x_1=0|y=1)$ $p(x_1=1|y=1)$

# Naïve Bayes Example

| | $x_2 = 0$ | $x_2 = 1$ | $p(x_2 = 0\|y)$ | $p(x_2 = 1\|y)$ |
|---|---|---|---|---|
| $y = 0$ | 5 | 8 | $p(x_2=0\|y=0) = \frac{5}{13}$ | $p(x_2=1\|y=0) = \frac{8}{13}$ |
| $y = 1$ | 7 | 10 | $p(x_2=0\|y=1) = \frac{7}{17}$ | $p(x_2=1\|y=1) = \frac{10}{17}$ |

Prior probabilities.

# Naïve Bayes Example

| | $x_2 = 0$ | $x_2 = 1$ | $p(x_2 = 0 \mid y)$ | $p(x_2 = 1 \mid y)$ |
|---|---|---|---|---|
| $y = 0$ | 5 | 8 | $\frac{5}{13}$ | $\frac{8}{13}$ |
| $y = 1$ | 7 | 10 | $\frac{7}{17}$ | $\frac{10}{17}$ |

# Naïve Bayes Example

Number of people who dislike y.

$p(y=0)$

$=$ Number of people dislike y / Total number of people.

$= \frac{13}{30}$

| | Count | $p(y)$ |
|---|---|---|
| $y = 0$ | 13 | 13/30 |
| $y = 1$ | 17 | 17/30 |

Number of people who like y

Number like y / Total Number of people.

# Naïve Bayes Example

| | Count | $p(y)$ |
|---|---|---|
| $y = 0$ | 13 | $\frac{13}{30}$ |
| $y = 1$ | 17 | $\frac{17}{30}$ |

$p(y=0)$

# Naïve Bayes Example

person likes $x_1$
does NOT like $x_2$

- What is the ***probability*** that a new person, who likes $x_1 = 1$ but NOT $x_2 = 0$,…
  - will **like** $y$?

Aposteriori probability
person likes $y$
given likes $x_1$, does NOT like $x_2$

$$p(y=1 \mid x_1=1, x_2=0) = p(y=u \mid \bar{x}=\bar{v})$$

$$= \frac{p(x_1=1, x_2=0 \mid y=1)\, p(y=1)}{p(x_1=1, x_2=0)}$$

$$= \frac{p(x_1=1 \mid y=1) \cdot p(x_2=0 \mid y=1)\, p(y=1)}{p(x_1=1, x_2=0)}$$

$$p(y=0 \mid x_1=1, x_2=0)$$

Aposteriori probability
does NOT like y
given likes $x_1$, does NOT like $x_2$

$$= \frac{p(x_1=1, x_2=0 \mid y=0)\, p(y=0)}{p(x_1=1, x_2=0)}$$

$$= \frac{p(x_1=1 \mid y=0)\, p(x_2=0 \mid y=0)\, p(y=0)}{p(x_1=1, x_2=0)}$$

likes $y$ if

$$p(y=1 \mid x_1=1, x_2=0) > p(y=0 \mid x_1=1, x_2=0)$$

$$\Rightarrow \frac{p(x_1=1 \mid y=1)\, p(x_2=0 \mid y=1)\, p(y=1)}{\cancel{p(x_1=1, x_2=0)}} > \frac{p(x_1=1 \mid y=0)\, p(x_2=0 \mid y=0)\, p(y=0)}{\cancel{p(x_1=1, x_2=0)}}$$

LHS. RHS.

$$\Rightarrow p(x_1=1 \mid y=1) \cdot p(x_2=0 \mid y=1)\, p(y=1)$$

$$> p(x_1=1 \mid y=0)\, p(x_2=0 \mid y=0) \times p(y=0)$$

# Naïve Bayes Example

- We can readily compute this using the **Naïve Bayes formula**!

Aposteriori probabilities

# Naïve Bayes Example

- For $y = 1$, $Q_1 =$

$$p(y = 1) \times p(x_1 = 1|y = 1) \times p(x_2 = 0|y = 1)$$

$$= \frac{17}{30} \times \frac{13}{17} \times \frac{7}{17} = 0.178$$

# Naïve Bayes Example

Aposteriori for $y = 1$.

- For $y = 1$, $Q_1 =$

$$p(y=1) \times p(x_1 = 1|y=1) \times p(x_2 = 0|y=1)$$

$$= \frac{17}{30} \times \frac{13}{17} \times \frac{7}{17} = 0.178$$

# Naïve Bayes Example

- For $y = 0$, $Q_0 =$

$$p(y = 0) \times p(x_1 = 1|y = 0) \times p(x_2 = 0|y = 0)$$

$$= \frac{13}{30} \times \frac{10}{13} \times \frac{5}{13} = 0.128$$

# Naïve Bayes Example

Aposteriori for $y = 0$

- For $y = 0$, $Q_0 =$

$$p(y = 0) \times p(x_1 = 1|y = 0) \times p(x_2 = 0|y = 0)$$

$$= \frac{13}{30} \times \frac{10}{13} \times \frac{5}{13} = 0.128$$

# Naïve Bayes Example

- $Q_1$ is **higher**
  - $\Rightarrow$ new person is **"likely"** to like $y$!

$\Rightarrow p(y=1 \mid x_1=1, x_2=0) > p(y=0 \mid x_1=1, x_2=0)$

$\Rightarrow$ Person is likely to like y !

# Laplace Smoothing Example

Avoid prior probability = 0.

- The estimates of the ***prior probabilities*** are as follows

Add 1 to numerator
2 to denominator

$\frac{4}{15}+\frac{11}{15}=1$

| | $x_1=0$ | $x_1=1$ | $p(x_1=0\|y)$ | $p(x_1=1\|y)$ |
|---|---|---|---|---|
| $y=0$ | 3 | 10 | $p(x_1=0\|y=0) = \frac{3+1}{13+2} = \frac{4}{15}$ | $p(x_1=1\|y=0) = \frac{10+1}{13+2} = \frac{11}{15}$ |
| $y=1$ | 4 | 13 | $p(x_1=0\|y=1) = \frac{4+1}{17+2} = \frac{5}{19}$ | $p(x_1=1\|y=1) = \frac{13+1}{17+2} = \frac{14}{19}$ |

$\frac{5}{19}+\frac{14}{19} = 1$

$1(x_1=0, y=0)$

DO NOT like $x_1$ and $y$

Movie recommendation

# Laplace Smoothing Example

- The estimates of the ***prior probabilities*** are as follows

| | $x_1 = 0$ | $x_1 = 1$ | $p(x_1 = 0 \mid y)$ | $p(x_1 = 1 \mid y)$ |
|---|---|---|---|---|
| $y = 0$ | 3 | 10 | $\frac{4}{15}$ | $\frac{11}{15}$ |
| $y = 1$ | 4 | 13 | $\frac{5}{19}$ | $\frac{14}{19}$ |

# Laplace Smoothing Example

AVENGERS.

$\frac{6}{15}+\frac{9}{15}=1$

| | $x_2 = 0$ | $x_2 = 1$ | $p(x_2 = 0 \mid y)$ | $p(x_2 = 1 \mid y)$ |
|---|---|---|---|---|
| $y = 0$ | 5 | 8 | $p(x_2=0 \mid y=0) = \frac{5+1}{13+2} = \frac{6}{15}$ | $p(x_2=1 \mid y=0) = \frac{8+1}{13+2} = \frac{9}{15}$ |
| $y = 1$ | 7 | 10 | $p(x_2=0 \mid y=1) = \frac{7+1}{17+2} = \frac{8}{19}$ | $p(x_2=1 \mid y=1) = \frac{10+1}{17+2} = \frac{11}{19}$ |

$\frac{8}{19}+\frac{11}{19}=1$

DO NOT like $x_2$
like y

# Laplace Smoothing Example

| | $x_2 = 0$ | $x_2 = 1$ | $p(x_2 = 0 \vert y)$ | $p(x_2 = 1 \vert y)$ |
|---|---|---|---|---|
| $y = 0$ | 5 | 8 | $\frac{6}{15}$ | $\frac{9}{15}$ |
| $y = 1$ | 7 | 10 | $\frac{8}{19}$ | $\frac{11}{19}$ |

# Laplace Smoothing Example

TG.

Avengers.

- What is the **probability** that a new person, who likes $x_1 = 1$ but NOT $x_2 = 0$,…
  - will **like** $y = 1$?

Avatar?

likes $x_1$ = TG

Does NOT like $x_2$: Avengers.

P(like y: Avatar) ?

# Laplace Smoothing Example

$x_i$ are conditionally independent given $y$.

- We can readily compute this using the **Naïve Bayes formula**!

$$P(x_1=1|y=1) \times P(x_2=0|y=1) \times P(y=1).$$

Naive Bayes

$$P(y=1|x_1=1, x_2=0) \propto P(x_1=1, x_2=0|y=1)\,P(y=1)$$

$$P(y=0|x_1=1, x_2=0) \propto P(x_1=1, x_2=0|y=0)\,P(y=0)$$

Naive Bayes

$$P(x_1=1|y=0)\,P(x_2=0|y=0)\,P(y=0)$$

Aposteriori Probabilities.

# Laplace Smoothing Example

- For $y = 1$, $Q_1 =$

$p(y = 1) \times p(x_1 = 1|y = 1) \times p(x_2 = 0|y = 1)$

$= \frac{17}{30} \times \frac{14}{19} \times \frac{8}{19} = 0.1758$

# Laplace Smoothing Example

- For $y = 1$, $Q_1 =$

$$p(y=1) \times p(x_1 = 1|y=1) \times p(x_2 = 0|y=1)$$

$$= \frac{17}{30} \times \frac{14}{19} \times \frac{8}{19} = 0.1758$$

$Q_1$.

$Q_1 \propto$ Aposteriori prob $y=1$ given $x_1 = 1$, $x_2 = 0$.

# Laplace Smoothing Example

- For $y = 0$, $Q_0 =$

$$p(y = 0) \times p(x_1 = 1|y = 0) \times p(x_2 = 0|y = 0)$$

$$= \frac{13}{30} \times \frac{11}{15} \times \frac{6}{15} = 0.1271 \; : Q_0.$$

$Q_0 \propto$ Aposteriori probability $y = 0$ given $x_1 = 1$, $x_2 = 0$.

## Laplace Smoothing Example

- For $y = 0$, $Q_0 =$

$$p(y=0) \times p(x_1 = 1|y=0) \times p(x_2 = 0|y=0)$$

$$= \frac{13}{30} \times \frac{11}{15} \times \frac{6}{15} = 0.1271$$

# Laplace Smoothing Example

- $Q_1$ is **higher**
  - $\Rightarrow$ new person is **"likely"** to like $y$!

Greater chance of liking avatar

$Q_1 = 0.1758$
$Q_0 = 0.1271$

$Q_1 > Q_0 \Rightarrow$

$P(y=1 \mid x_1=1, x_2=0)$
$> P(y=0 \mid x_1=1, x_2=0).$

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Font: Avenir (Book), Size: 28, Colour: Dark Grey

Font: Avenir (Book), Size: 24, Colour: Dark Grey

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