1. PDF of a Gaussian RV is

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

   PDF of a Gaussian RV with mean $\mu = 2$ and variance $\sigma^2 = 2$ is

$$f_X(x) = \frac{1}{\sqrt{4\pi}} e^{-\frac{(x-2)^2}{4}}$$

   Ans b

2. The **mean** and **covariance matrix** of the multivariate Guassian are defined as

$$E\{\bar{\mathbf{x}}\} = \bar{\boldsymbol{\mu}}, E\{(\bar{\mathbf{x}} - \bar{\boldsymbol{\mu}})(\bar{\mathbf{x}} - \bar{\boldsymbol{\mu}})^T\} = \mathbf{R}$$

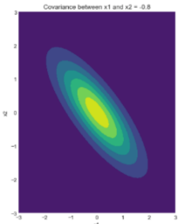
   Ans a

3. PDF of a ***Gaussian random vector*** is

$$\frac{1}{\sqrt{(2\pi)^n |\mathbf{R}|}} e^{-\frac{1}{2}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}})^T \mathbf{R}^{-1} (\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}})}$$

   Ans c

4. The contours of a 2D Gaussian with unequal variances of different components are given as



   The PDF of a multivariate Gaussian with unequal variances of components is

$$\frac{1}{\sqrt{(2\pi)^2 \sigma_1^2 \sigma_2^2}} e^{-\frac{1}{2}\left(\frac{(x_1-\mu_1)^2}{\sigma_1^2} + \frac{(x_2-\mu_2)^2}{\sigma_2^2}\right)}$$

   Hence contours of equal PDF satisfy

$$\frac{(x_1 - \mu_1)^2}{\sigma_1^2} + \frac{(x_2 - \mu_2)^2}{\sigma_2^2} = C$$

   which are basically ellipses

   Ans a

5. Given

$$\bar{\boldsymbol{\mu}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{R} = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$$

$$|\mathbf{R}| = \left| \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \right| = 9 - 1 = 8$$

$$\mathbf{R}^{-1} = \frac{1}{8} \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

$$(\bar{\mathbf{x}} - \bar{\boldsymbol{\mu}})^T \mathbf{R}^{-1} (\bar{\mathbf{x}} - \bar{\boldsymbol{\mu}})$$

$$= [x_1 - 1 \quad x_2 - 1] \frac{1}{8} \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 - 1 \\ x_2 - 1 \end{bmatrix}$$

$$= \frac{1}{8}\left(3(x_1 - 1)^2 + 3(x_2 - 1)^2 + 2(x_1 - 1)(x_2 - 1)\right)$$

$$= \frac{1}{8}(3x_1^2 + 3x_2^2 - 8x_1 - 8x_2 + 2x_1x_2 + 8)$$

$$\frac{1}{\sqrt{(2\pi)^n |\mathbf{R}|}} e^{-\frac{1}{2}(\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}})^T \mathbf{R}^{-1} (\bar{\mathbf{x}}-\bar{\boldsymbol{\mu}})} = \frac{1}{\sqrt{(2\pi)^2 \times 8}} e^{-\frac{1}{2} \times \frac{1}{8}(3x_1^2 + 3x_2^2 - 8x_1 - 8x_2 + 2x_1x_2 + 8)}$$

$$= \frac{1}{\sqrt{32\pi^2}} e^{-\frac{1}{16}(3x_1^2+3x_2^2-8x_1-8x_2+2x_1x_2+8)}$$

Ans c

6. In LDA , we **choose** $\mathcal{C}_0$ if

$$p_0 \times p(\bar{\mathbf{x}}; \mathcal{C}_0) > p_1 \times p(\bar{\mathbf{x}}; \mathcal{C}_1)$$

Ans a

7. The Gaussian discriminant classifier can be simplified as Choose $\mathcal{C}_0$ if

$$\bar{\mathbf{h}}^T(\bar{\mathbf{x}} - \widetilde{\boldsymbol{\mu}}) \geq 0, \widetilde{\boldsymbol{\mu}} = \frac{1}{2}(\bar{\boldsymbol{\mu}}_0 + \bar{\boldsymbol{\mu}}_1), \bar{\mathbf{h}} = \mathbf{R}^{-1}(\bar{\boldsymbol{\mu}}_0 - \bar{\boldsymbol{\mu}}_1)$$

Ans b

8. Given two classes $\mathcal{C}_0, \mathcal{C}_1$ distributed as

$$\mathcal{C}_0 \sim N\left(\begin{bmatrix} -8 \\ -6 \end{bmatrix}, \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}\right), \mathcal{C}_1 \sim N\left(\begin{bmatrix} 8 \\ 6 \end{bmatrix}, \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}\right)$$

The Gaussian classifier is given as follows: Choose $\mathcal{H}_0$ if

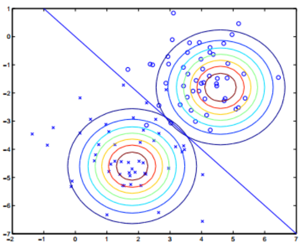
$$(\bar{\boldsymbol{\mu}}_0 - \bar{\boldsymbol{\mu}}_1)^T \mathbf{R}^{-1}\left(\bar{\mathbf{x}} - \frac{1}{2}(\bar{\boldsymbol{\mu}}_0 + \bar{\boldsymbol{\mu}}_1)\right) \geq \ln\frac{p_1}{p_0} = \ln\left(\frac{\frac{1}{2}}{\frac{1}{2}}\right) = \ln 1 = 0$$

$$\Rightarrow [-16 \quad -12]\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}\left(\bar{\mathbf{x}} - \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) \geq 0$$

$$\Rightarrow -32x_1 - 48x_2 \geq 0 \Rightarrow 2x_1 + 3x_2 \leq 0$$

Ans a

9. Gaussian discriminant classifier is shown by the picture



Ans b

10. LDA can be imported in PYTHON as
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
Ans c