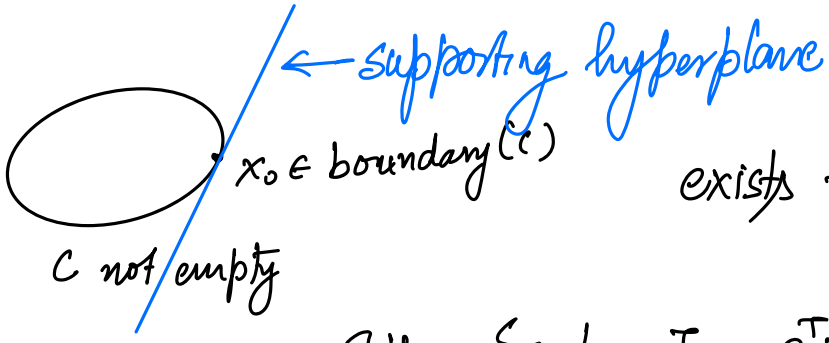
7. Supporting hyperplane



exists $\forall\ x_0 \in bd\ C$

$$S.H = \{y \mid a^T y = a^T x_0\}$$

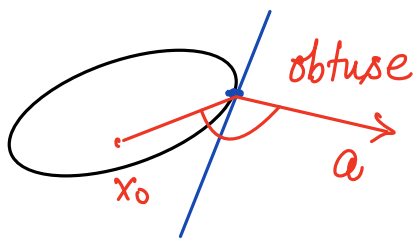
when $a \neq 0$ &

$$a^T x \leq a^T x_0 \quad \forall\ x \in C$$

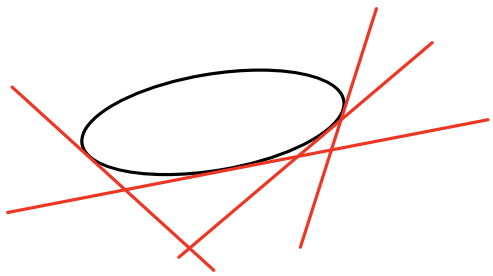
C lies on one side of S.H.

$$\Rightarrow \quad a^T(x - x_0) \leq 0$$

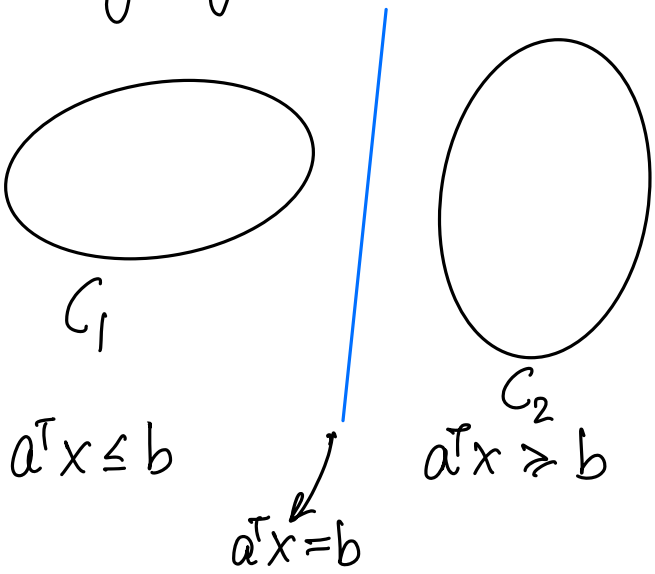
$\angle a, x - x_0$ obtuse



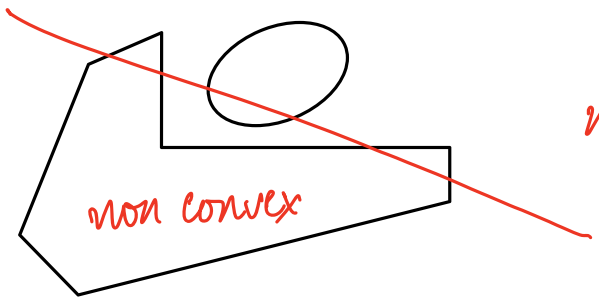
any convex set = $\bigcap$ half spaces



Separating hyperplane



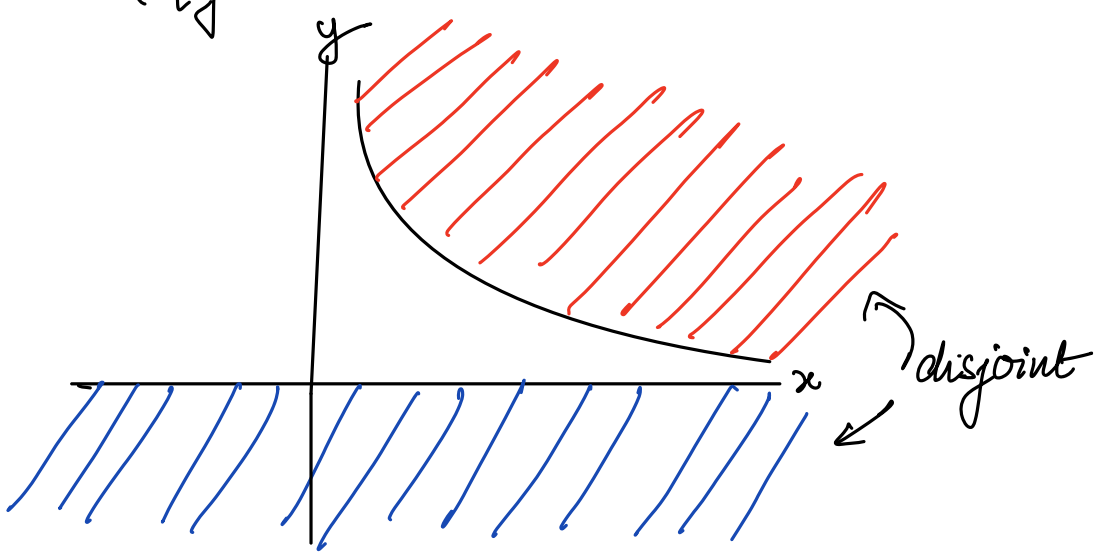
s.t. $C_1 \cap C_2 = \emptyset$



no separating hyperplane

$$C = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2_+ \mid xy \ge 1 \right\}$$

$$D = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \mid y \le 0 \right\}$$



Separating hyperplane : $\{ y = 0 \}$