SVD

SINGULAR VALUE DECOMPOSITION

- SVD stands for **Singular Value Decomposition**
- One of the most important techniques for **MIMO processing**

WARNING!!!
PLEASE DO NOT
CONFUSE WITH
EIGENVALUE DECOMPOSITION

SVD

r = # Receive antennas.
t = # Transmit antennas.

- Given matrix $\mathbf{H}$ with $r \geq t$ SVD is defined as

$$\mathbf{H} = \mathbf{U}_{r\times r}\boldsymbol{\Sigma}_{r\times t}\mathbf{V}^{H}_{t\times t}$$

r×t

t×t SQUARE

r×r SQUARE

- SVD **always exists**!!!

SVD Properties

$H = U \Sigma V^H$

$r \times r$

$t \times t$

- The matrices $\mathbf{U}, \mathbf{V}$ satisfy the property

$$U^H U = U U^H = I$$
$$V^H V = V V^H = I$$

unitary matrix

$AA^H = A^H A = I$

- These are ___UNITARY.___ matrices

SVD Properties

- The matrix Σ of size $r \times t$ has the structure

$r \times t$ rectangular matrix

$r \geq t$

Tall matrix $r \geq t$

t singular values.

$$\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \sim 4 \times 2 \qquad r = 4,\ t = 2$$

$$\Sigma = \underbrace{\left[\begin{array}{ccc} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & \\ 0 & & \ddots \\ \vdots & & \sigma_t \\ \hline 0 & & 0 \\ \vdots & & \vdots \\ 0 & & 0 \end{array}\right]}_{r \times t}$$

$t \times t$ DIAGONAL matrix.

$(r-t) \times t$

SVD Properties

SINGULAR VALUES.

- The diagonal values σ_i are $\underline{\sigma_i \geq 0}$

σ_i = REAL & NON-NEGATIVE

- These are arranged in ~~DECREASING ORDER~~ order

$$\underbrace{\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_t \geq 0}_{\text{Decreasing order.}}$$

SVD Properties

- Number of **non-zero singular values** σ_i equals the rank of H

rank(H) = # non-zero singular values.

$\sigma_1, \sigma_2, \sigma_3, \sigma_4$ rank = 3.

$3.8 \geq 1.6 \geq 0.35 \geq 0$

SVD Properties

$r \times r$ UNITARY MATRIX.

- What is the property of **columns** of **U**?

ORTHONORMAL

$\bar{u}_i^H \bar{u}_j = 0$ if $i \neq j$

$\|\bar{u}_i\|^2 = 1 \Rightarrow \|\bar{u}_i\| = 1$

- These are ORTHONORMAL.

$$\mathbf{U} = [\bar{\mathbf{u}}_1 \quad \bar{\mathbf{u}}_2 \quad \dots \quad \bar{\mathbf{u}}_r]$$

$U^H U = U U^H = I$

SVD Properties

$\bar{u}_1$ DOMINANT Left Singular Vector

- $\mathbf{U}$ is known as the LEFT SINGULAR **matrix**.
- $\mathbf{u}_i$ are known as LEFT SINGULAR **vectors**.

Left Singular Vectors -

$$\mathbf{U} = [\bar{\mathbf{u}}_1 \quad \bar{\mathbf{u}}_2 \quad \ldots \quad \bar{\mathbf{u}}_r]$$

r x 1.

SVD Properties

$\bar{v}_i^H \bar{v}_j = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$

- What is the property of **columns** of $\mathbf{V}$?

$\bar{v}_i^H \bar{v}_j = 0$ if $i=j$

$\|\bar{v}_i\|^2 = 1$

- These are ORTHONORMAL.

$t \times 1$

$$\mathbf{V} = [\bar{\mathbf{v}}_1 \quad \bar{\mathbf{v}}_2 \quad \ldots \quad \bar{\mathbf{v}}_r]$$

Dominant Right singular vector.

SVD Properties

- $\mathbf{V}$ is known as the Right Singular **matrix**.
- $\bar{\mathbf{v}}_i$ are known as Right Singular **vectors**.

Dominant Right Singular vector

$$\mathbf{V} = [\bar{\mathbf{v}}_1 \quad \bar{\mathbf{v}}_2 \quad \cdots \quad \bar{\mathbf{v}}_r]$$

SVD Properties

Relation to Eigenvalue Decomposition

- Note that

$t \times t$

$$\mathbf{H}^H\mathbf{H} = \mathbf{V}\begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_t^2 \end{bmatrix}\mathbf{V}^H$$

$\sigma_1^2, \sigma_2^2, \ldots, \sigma_t^2$ are eigenvalues of $H^H H$

$\bar{v}_1, \bar{v}_2, \ldots \bar{v}_t$ are Eigenvectors.

SVD Properties

t eigenvalues.

- $\sigma_1^2, \sigma_2^2, \ldots, \sigma_t^2$ are **eigenvalues** of $\mathbf{H}^H\mathbf{H}$

$\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_t$

- $\mathbf{V}$ contains **eigenvectors** of $\mathbf{H}^H\mathbf{H}$

RIGHT SINGULAR VECTORS.

SVD Properties

HH^H

$r \times r$

t Diagonal values.

(r-t) diagonal values = 0

$$\mathbf{U}\begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_t^2 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{r\times r}\mathbf{U}^H$$

SVD Properties

Rest $(r-t)$ eigenvalues of HH^H are zero!

- $\sigma_1^2, \sigma_2^2, \ldots, \sigma_t^2$ are non-zero **eigenvalues** of $\mathbf{HH}^H$

$r \times r$

- $\mathbf{U}$ contains **eigenvectors** of $\mathbf{HH}^H$

$\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_r$ LEFT SINGULAR VECTORS OF H.

SVD Example

SINGULAR VALUE DECOMPOSITION.

$H = U \Sigma V^H$

Unitary

$UU^H = U^H U = I$

V is unitary

$VV^H = V^H V = I$

$\Sigma \sim r \times t$

Diagonal matrix

$\sigma_i \geq 0$

σ_i in DECREASING ORDER.

- Consider the matrix below

$$\mathbf{H} = \underbrace{\begin{bmatrix} 1 & 2 \\ 1 & -2 \\ 1 & -2 \\ 1 & 2 \end{bmatrix}}_{4 \times 2}$$

SVD Example

- The SVD is given as

$$\overbrace{\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}}^{U \sim 4 \times 4} \overbrace{\begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}^{\Sigma \sim 4 \times 2} \overbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}^{V \sim 2 \times 2}$$

SVD Example

- Let us explore this..

$$\Sigma = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$4^2 = 16,\ 2^2 = 4$

Eigenvalues of $H^H H, HH^H$

$4 \times 2 = r \times t$

$\sigma_1 = 4 \geq 0$

$\sigma_2 = 2 \geq 0$

$\sigma_1 \geq \sigma_2 \geq 0$

σ_i ARE NON NEGATIVE

DECREASING order

2 nonzero singular values

$\Rightarrow \text{rank}(H) = 2$

SVD Example

- Let us explore this..

$$\mathbf{V} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$\bar{v}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ~ Dominant Right Singular vector

V in unitary

$VV^H = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$V^H V = I_{2\times 2}$

columns are ORTHONORMAL

$\|\bar{v}_1\|^2 = 0 + 1 = 1$

$= \|\bar{v}_2\|^2$

$\bar{v}_1^H \bar{v}_2 = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$

SVD Example

- Let us explore this..

Dominant Left Singular vector

$$\mathbf{U} = \begin{array}{c} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \\ \begin{array}{cccc} \bar{u}_1 & \bar{u}_2 & \bar{u}_3 & \bar{u}_4 \end{array} \end{array}$$

$r = 4$

$r \times r$

4×4

matrix

$\bar{u}_1^H \cdot \bar{u}_2$

$= \frac{1}{4} - \frac{1}{4} - \frac{1}{4} + \frac{1}{4}$

$= 0$

Easy to verify

$U^H U = U U^H = I$

$\|\bar{u}_1\|^2 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$

$= \|\bar{u}_2\|^2 = \|\bar{u}_3\|^2 = \|\bar{u}_4\|^2$

columns of U are ORTHONORMAL

SVD MIMO Processing

- SVD can be used for MIMO processing as follows

channel matrix

$$H = U \Sigma V^H$$

SVD MIMO Processing

- Consider the model

Output vector

Transmit vector

Noise vector

$$\underbrace{\bar{\mathbf{y}}_{r\times 1} = \mathbf{H}_{r\times t}\bar{\mathbf{x}}_{t\times 1} + \bar{\mathbf{n}}_{r\times 1}.}_{\text{MIMO channel model.}}$$

$H = U \Sigma V^H.$

SVD MIMO Processing

- Substitute the SVD of $\mathbf{H} = U \Sigma V^H$

$$\bar{\mathbf{y}} = \underline{\; U \Sigma V^H \;} \, \bar{\mathbf{x}} + \bar{\mathbf{n}}$$

SVD MIMO Processing

- At the receiver, process using $\mathbf{U}^H$

$\tilde{n} = U^H \bar{n}$

- This gives

Note: $U^H U = I$

$$\tilde{\mathbf{y}} = \mathbf{U}^H \bar{\mathbf{y}}$$

$$= U^H(H\bar{x} + \bar{n}) = U^H(U\Sigma V^H \bar{x} + \bar{n})$$

$$= \Sigma V^H \bar{x} + \tilde{n} = \tilde{y}$$

SVD MIMO Processing

- At the receiver, we multiply by $\mathbf{U}^H$.

$U^H = \begin{cases} \text{COMBINER} \\ \text{RECEIVE BEAMFORMER.} \end{cases}$

- This is also called a **combiner** or RECEIVE **beamformer**.

SVD MIMO Processing

Processing Prior To Transmission

- At the transmitter, pre-process using $\mathbf{V}$

Precoding.

$$\bar{x} = V\tilde{x}$$

Transmit vector

- This gives

$$\bar{\mathbf{x}} = \mathbf{V}\tilde{\mathbf{x}}$$

since $V^H V = I$

$$\tilde{\mathbf{y}} = \Sigma V^H \bar{x} + \tilde{n} = \Sigma V^H V \tilde{x} + \tilde{n} = \Sigma \tilde{x} + \tilde{n}$$

$$\tilde{y} = \Sigma \tilde{x} + \tilde{n}$$

SVD MIMO Processing

- At the transmitter, we ***premultiply*** or ***preprocess*** using the matrix $\mathbf{V}$.
- This is termed PRECODER.
- Also the TRANSMIT beamforming matrix

SVD MIMO Processing

Very very interesting model !!!

- As a result we now have the model

$$\tilde{\mathbf{y}} = \mathbf{\Sigma}\tilde{\mathbf{x}} + \tilde{\mathbf{n}}$$

SVD MIMO Processing

- This can be explicitly written as

$$\tilde{\mathbf{y}} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_r \end{bmatrix} = \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & & \\ \vdots & & \ddots & \\ & & & \sigma_t \\ \hline 0 & 0 & \cdots & 0 \\ 0 & & & \vdots \\ \vdots & & & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \vdots \\ \tilde{n}_r \end{bmatrix}$$

$\tilde{y}_{t+1}, \tilde{y}_{t+2}, \ldots$ IGNORED

ONLY Noise

SVD MIMO Processing

ONLY NOISE

- Note that $\tilde{y}_{t+1}, \tilde{y}_{t+2}, \dots, \tilde{y}_r$ are **only noise**

$(r-t) \rightarrow$ IGNORED.

$$\tilde{y}_{t+1} = \tilde{n}_{t+1}$$
$$\tilde{y}_{t+2} = \tilde{n}_{t+2}$$
$$\vdots$$
$$\tilde{y}_r = \tilde{n}_r$$

- These can be ***ignored***

SVD MIMO Processing

- The rest of the outputs can be explicitly written as

$$\tilde{y}_1 = \sigma_1 \tilde{x}_1 + \tilde{n}_1$$
$$\tilde{y}_2 = \sigma_2 \tilde{x}_2 + \tilde{n}_2$$
$$\vdots$$
$$\tilde{y}_t = \sigma_t \tilde{x}_t + \tilde{n}_t$$

SPATIAL MULTIPLEXING.

t Parellel information streams.

t DECOUPLED CHANNELS.

VERY HIGH DATA RATE

SVD MIMO Processing

- Consider the ith channel

i^{th} mode

i^{th} stream

$$\tilde{\mathbf{y}}_i = \sigma_i \tilde{\mathbf{x}}_i + \tilde{\mathbf{n}}_i$$

N_0.

$\sigma_i^2 =$ Gain for i^{th} mode.

$E\{|\tilde{x}_i|^2\} = P_i$

- The SNR is given as

$$SNR_0^i = \sigma_i^2 \cdot \frac{P_i}{N_0} = SNR_0^i$$

output SNR of i^{th} stream.

MIMO Capacity

Maximum rate for Error free Transmission

- The Shannon capacity is given as

$$\log_2\left(1 + \sigma_i^2 \times \frac{P_i}{N_0}\right)$$

SNR_o^i

$$= \log_2\left(1 + \mathrm{SNR}_o^i\right)$$

$\log_2(1 + \mathrm{SNR})$

MIMO Capacity

- The sum capacity of the MIMO channel is

$$\sum_{i=1}^{t} R_i = \underbrace{\sum_{i=1}^{t} \log_2\left(1+SNR_o^i\right)}_{\text{Sum of the maximum rates of all } i \text{ channels.}}$$

$$= \sum_{i=1}^{t} \log_2\left(1+\sigma_i^2 \frac{P_i}{N_o}\right)$$

MIMO Capacity

- There is a ***maximum transmit power*** for every transmitter. Let us call this as P_0.

maximum Transmit Power

- This is the ***total permissible*** transmit power.

MIMO Capacity

Sum of all powers of MIMO modes.

Maximum Transmit Power

- Therefore, it follows that

$$P_1 + P_2 + \cdots + P_t = \underline{\quad \sum_{i=1}^{t} P_i \quad} = \underline{\quad P_0 \quad}$$

MIMO Capacity

What is the Maximum Transmission rate?

- **Maximum** possible transmission rate of the MIMO channel?

MIMO Capacity

- The **optimization problem** is given as

Constrained Optimization Problem,

$$\begin{cases} \text{Max.} & \displaystyle\sum_{i=1}^{t} \log_2\left(1+\sigma_i^2 \frac{P_i}{N_0}\right) \\ \text{Subject to} & \displaystyle\sum_{i=1}^{t} P_i = P_0 \end{cases}$$

P_0 = maximum Transmit power

MIMO Capacity

Lagrange multiplier

- This is the constrained optimization problem for ***maximum sum-rate of the MIMO system***.
  - **Capacity** of the MIMO system.

objective

Lagrange multiplier

constraint

$F(x) + \lambda g(x)$

MIMO Capacity

- Solving this optimization problem gives the following expression for the **optimal power**.

$$P_j = \left(\frac{1}{\lambda} - \frac{N_0}{\sigma_j^2}\right)^+$$

$$x^+ = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

MIMO Capacity

$\left(\frac{1}{\lambda} - \frac{N_0}{\sigma_j^2}\right)^+ = P_j$

- This quantity $P_j = \left(\frac{1}{\lambda} - \frac{N_0}{\sigma_j^2}\right)^+$ can be explicitly written as

$$P_j = \begin{cases} \frac{1}{\lambda} - \frac{N_0}{\sigma_j^2} & \text{if } \frac{1}{\lambda} \geq \frac{N_0}{\sigma_j^2} \Rightarrow \frac{1}{\lambda} - \frac{N_0}{\sigma_j^2} \geq 0 \\ 0 & \text{if } \frac{1}{\lambda} < \frac{N_0}{\sigma_j^2} \Rightarrow \frac{1}{\lambda} - \frac{N_0}{\sigma_j^2} < 0 \end{cases}$$

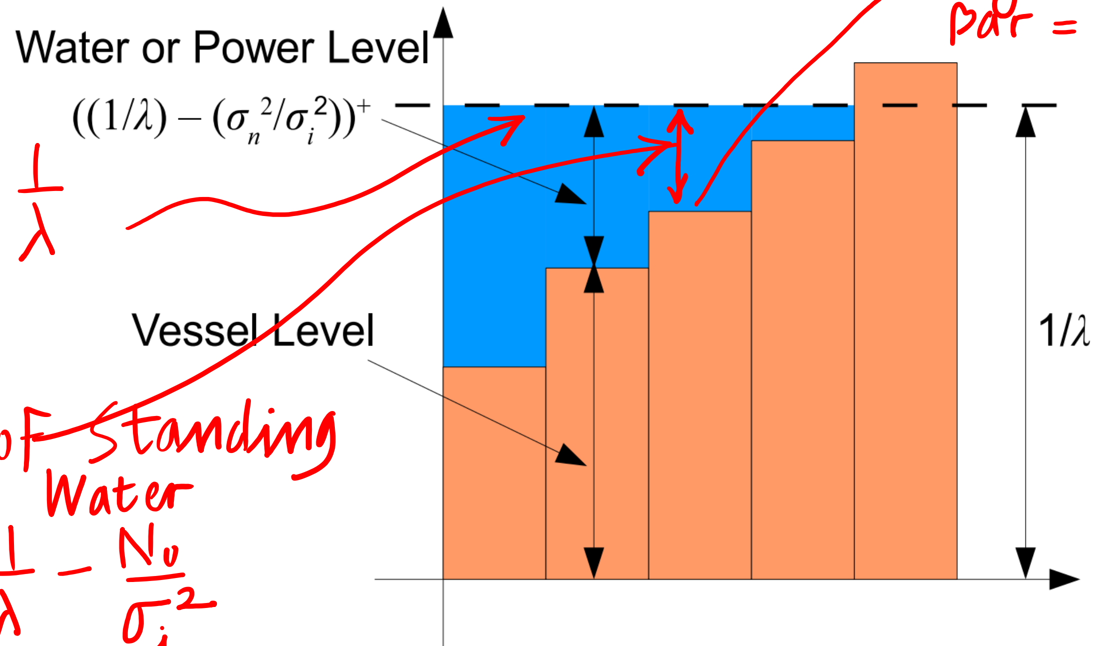
MIMO Capacity — information Theory

Water Pouring

- This is also termed **water-filling power allocation**

Height of pdr $= \frac{N_0}{\sigma_j^2}$

Water Level $= \frac{1}{\lambda}$

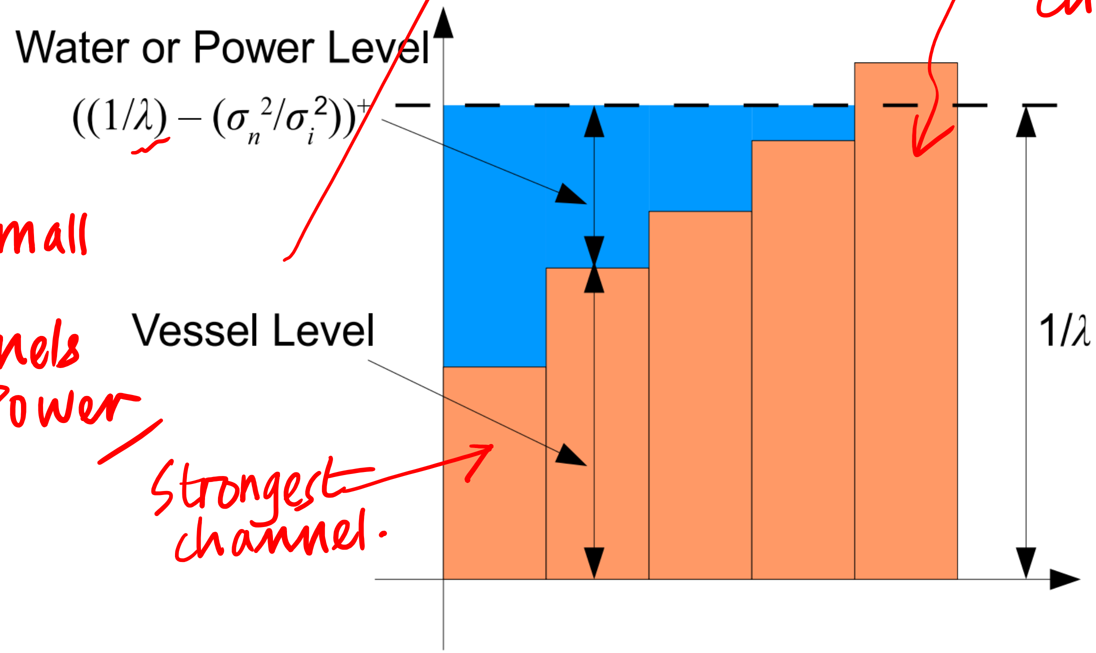


Level of Standing Water $= \frac{1}{\lambda} - \frac{N_0}{\sigma_j^2}$

MIMO Capacity

- This is also termed **water-filling power allocation**

Stronger channels allocated more power



σ_j = Small

$\Rightarrow \frac{1}{\lambda} - \frac{N_0}{\sigma_j^2}$ = small

$\Rightarrow$ Weaker channels allocated Lower Power

MIMO Capacity

- $\frac{1}{\lambda}$: termed as the **water-level**.

- Power is non-zero only if $\frac{N_0}{\sigma_j^2}$ is **less than the water-level** $\frac{1}{\lambda}$ i.e. $\frac{1}{\lambda} \geq \frac{N_0}{\sigma_j^2}$

- Otherwise power is 0, i.e. $\frac{1}{\lambda} \leq \frac{N_0}{\sigma_j^2}$.

MIMO Capacity Example

- Consider the channel matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 2 \\ 1 & -2 \\ 1 & -2 \\ 1 & 2 \end{bmatrix}$$

4×2

$r = 4$

$t = 2$

MIMO Capacity Example

- $N_0 = 12\ dB =$ $\underline{10^{1.2} = (10^{0.3})^4 \approx 2^4 = 16}$
- **Total Transmit power** $P_0 = 3\ dB =$ $\underline{2\qquad}$

$10 \log_{10} N_0 = 12$

$\Rightarrow N_0 = 10^{1.2} \approx 16$

MIMO Capacity Example

- Recall SVD is given as

SVD OF $H = \begin{bmatrix} 1 & 2 \\ 1 & -2 \\ 1 & -2 \\ 1 & 2 \end{bmatrix}$

$U \sim 4 \times 4$

$\Sigma \sim 4 \times 2$

$\sigma_1 = 4$

$\sigma_2 = 2$

$V \sim 2 \times 2$

$$\overbrace{\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}}^{U} \underbrace{\overbrace{\begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}^{\Sigma}}_{\Sigma} \overbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}^{V}$$

MIMO Capacity Example

- The singular values are

$$\sigma_1 = \underline{\quad 4 \quad}$$

$$\sigma_2 = \underline{\quad 2 \quad}$$

MIMO Capacity Example

- The powers are given as

$$P_1 = \left(\frac{1}{\lambda} - \frac{N_0}{\sigma_1^2}\right)^+ = \left(\frac{1}{\lambda} - \frac{16}{4^2}\right)^+ = \left(\frac{1}{\lambda} - 1\right)^+$$

$$P_2 = \left(\frac{1}{\lambda} - \frac{N_0}{\sigma_2^2}\right)^+ = \left(\frac{1}{\lambda} - \frac{16}{2^2}\right) = \left(\frac{1}{\lambda} - 4\right)^+$$

MIMO Capacity Example

- Use now the **power constraint**

$$P_1 + P_2 = P_0 = 2$$

$$\Rightarrow \underbrace{\left(\frac{1}{\lambda} - 1\right)^+}_{P_1} + \underbrace{\left(\frac{1}{\lambda} - 4\right)^+}_{P_2} = 2$$

MIMO Capacity Example

- Case 1: Consider $\frac{1}{\lambda} \geq 4$

ASSUME $\frac{1}{\lambda} \geq 4$

$\Rightarrow \frac{1}{\lambda} - 4 \geq 0$

$\Rightarrow \frac{1}{\lambda} - 1 \geq 0$

- **This yields** $\left(\frac{1}{\lambda} - 1\right)^+ + \left(\frac{1}{\lambda} - 4\right)^+ = 2$

$\Rightarrow \frac{1}{\lambda} - 1 + \frac{1}{\lambda} - 4 = 2$

$\Rightarrow \frac{2}{\lambda} = 7 \Rightarrow \frac{1}{\lambda} = 3.5$

MIMO Capacity Example

- However, we now see that $P_2 < 0$

$$P_2 = \frac{1}{\lambda} - 4 = 3.5 - 4 = -0.5 < 0$$

THIS IS NOT ACCEPTABLE!!!
THIS IS NOT VALID.

MIMO Capacity Example

- This is invalid!
- Thus we have to set

$$P_2 = 0$$

MIMO Capacity Example

Now use same equation. $P_1 + P_2 = P_0$ ^{0}

- This implies that

$$P_1 + P_2 = \overbrace{\frac{1}{\lambda} - 1}^{P_1} + \overbrace{0}^{P_2} = 2$$

$$\frac{1}{\lambda} = \underline{3} \Rightarrow P_1 = \frac{1}{\lambda} - 1 \underline{\underline{= 2}}$$

MIMO Capacity Example

- What are the **optimal power values**?

$$P_1 = \underline{\quad 2 \quad} = \underline{\quad 3 \quad}\, dB, P_2 = \underline{\quad 0 \quad}$$

OPTIMAL <u>POWER ALLOCATION</u>.

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Font: Avenir (Book), Size: 28, Colour: Dark Grey

Font: Avenir (Book), Size: 24, Colour: Dark Grey

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