Assignment - 4

Que 1:- Given, min $c^T x$ st $Ax \leq b$

A is square & full rank

(a) When is the problem infeasible?

Soln:- Infeasible if no x will satisfy constraint

& for $x \to \infty$

if $c_i > 0$ & assuming $x_i > 0$

It will become infeasible as for $x \to \infty$

(b) When is problem unbounded below?

Soln:- Keep on minimizing the fun^k upto $-\infty$

minimize for $x_i = 0$ 1, 2, 3, 4, 1000

$c_i < 0$ $x_i \cancel{>} 0$

As x_i increases the fun^k will keep on minimizing.

(c) finite soln?

minimized $= 0$ if $x \geq 0$

$c_i \geq 0 \ \forall i$

taking $x_i = 0$

minimum value of object $= 0$ when $x_i = 0$

Que 2:- Show that any linear prog.

$$\min c^T x$$
st $A x = b$
$x_i \geq 0 \quad i = 1, \ldots, n$

lets take some problems which has constraints like
$$A x \leq b$$
using slack variable, we can rewrite this as
$$A x + s = b, \quad s \geq 0$$

we can have constraints like:
$$A x \geq b$$
using slack variable, we can rewrite this as
$$A x - s = b, \quad s \geq 0$$

change objective
to minimize, replace $+c$ to $-c$.

we can eliminate free variables
if x_i unconstrained, replacing it with $x_i^+ - x_i^-$
with $x_i^+ \geq 0$
$x_i^- \geq 0$

Transformation to standard form example

minimize $2x_1 + 4x_2$.
st
$x_1 + x_2 \geq 3$
$3x_1 + 2x_2 = 14$
$x_1 \geq 0$

No constraint on x_2
x_2 can be eliminated

Minimize, $2x_1 + 4x_2^+ - 4x_2^-$

$x_1 + x_2^+ - x_2^- = 3$

~~$3x_1 + 2x_2^+ - 2x_2^-$~~

$3x_1 + 2x_2^+ - 2x_2^- = 14$

$x_1 \geq 0$

$x_2^+ \geq 0$ $\qquad x_3 \geq 0$

$x_3 \geq 0$

$x_2^- \geq 0$

Que 3:- Consider program

$$\min c^T x$$
$$\text{st } Ax = b$$

(a) when is the program infeasible?

Soln: If $c_i > 0$ & assuming $x_i > 0$
It will become infeasible as $f_0(x) \to \infty$

(b) when is the program unbounded below?
Soln: keep on minimizing the fun all the way to $-\infty$.

Minimize for $x_i = 0$ $\qquad i = 1, 2, 3, \ldots$
$c_i < 0$, $x_i > 0$

As x_i increases & reaches to ∞, the fun will keep on minimizing.

(i) When will the solution be finite & what?

Soln:- finite Minimum soln is 0

taking $x_i = 0$ ~~$c_i \forall$~~ $\forall\ c_i \geq 0\ \forall_i$

Minimum value of objective is 0