Assignment 1 Solution

Digital Communication System-I

April 2023

1. (b) Statement 1 is false, and can be verified from the Figure 1. While,

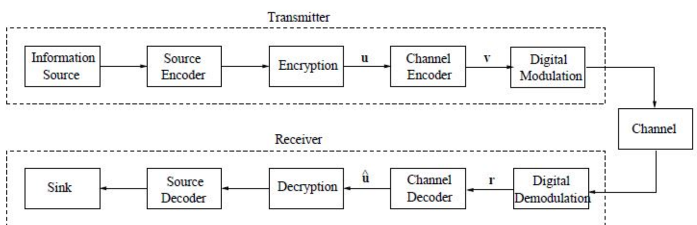

Figure 1: Communication block diagram.

   the function of source encoder is to minimize the number of bits per unit time required to represent the source output, and this process is known as source coding or data compression. Hence, statement 2 is correct.

2. (a) Code-rate = length of information bits/length of coded bits = 4/7 = 0.5714.

3. (d) The output signal is represented by $g(t) = 0.3 \times f(t) + n$. Hence, the maximum amplitude of the output signal waveform is $g(t)_{\max} = 0.3 \times 1 + 0.2 = 0.5$.

4. (a) The energy of the signal $x(t) = \sin(t)$ is

$$E = \int_{-\infty}^{\infty} |X(f)|^2 df = \int_{-1}^{1} df = 2.$$

5. (b) Noise is typically assumed to be Additive White Gaussian Noise.

6. (c) The output waveform after passing through the "Hilbert transform" block is $y(t) = f(t) * h(t)$ with $h(t)$ being the Hilbert transformer. As the convolution in time domain is the multiplication in frequency domain,

therefore the output waveform becomes $Y(\omega) = F(\omega)H(\omega)$, where $H(\omega)$ is defined as

$$
\begin{aligned}
H(\omega) &= \begin{cases} -j, & \omega > 0 \\ 0, & \omega = 0 \\ j, & w < 0 \end{cases} \\
&= \exp\left(-j\frac{\pi}{2} sgn(\omega)\right).
\end{aligned}
$$

And, the fourier transform (FT) of the input signal f(t) can be given by

$$
F(\omega) = \frac{1}{2}\left[\delta(\omega - 2) + \delta(\omega + 2)\right].
$$

Hence, the output signal can be given by

$$
Y(\omega) = \frac{-j}{2}\left[\delta(\omega - 2) - \delta(\omega + 2)\right] \xrightarrow{IFT} \sin(2t). \tag{1}
$$

7. (c)

$$
\begin{aligned}
E &= \lim_{T\to\infty} \int_0^T 4/\sqrt{t}dt \\
&= \lim_{T\to\infty} \left[8\sqrt{t}\right]_0^T \\
&= \lim_{T\to\infty} 8\sqrt{T} \\
&= \infty
\end{aligned} \tag{2}
$$

And, to find the power

$$
\begin{aligned}
P &= \lim_{T\to\infty} \frac{1}{T} \int_{-T}^T 2/\sqrt{t}dt \\
&= \lim_{T\to\infty} \frac{4}{T}\sqrt{T} \\
&= 0
\end{aligned} \tag{3}
$$

Hence, it's neither power nor energy signal.

8. (b) $x(t) = x^*(t)$, where $x^*(t)$ is the conjugate of signal $x(t)$. And, $x(t + T_0) \neq x(t)$, for $-\infty < t < \infty$ and $T_0 \neq 0$. Hence, this is a real-aperiodic signal.

   The signal can be represented by a mathematical function and re- mains constant for same value of t every time. Hence, the signal is deterministic. Also, $x(t)$ is defined for all t contained in some interval on the real line. Hence, its is a continuous signal.

9. (d) The energy of the signal is

$$E = \int_{-\infty}^{\infty} |X(f)|^2 df = \int_{-1}^{1} |1 - f|^2 df = 2/3.$$

Let us assume that Bp is the frequency, where 99% of the energy is preserved. Hence, we can compute the fraction of energy by

$$\frac{\int_{-B_p}^{B_p} (|1 - f|)^2 df}{2/3} = 0.99$$
$$\implies 3 \int_{0}^{B_p} (1 - f)^2 df = 0.99$$
$$\therefore B_p = 0.7846.$$

10. (b) Please see the lecture slides.