Linear Programs

History: aircraft scheduling (WW2)

Applications: expenditure planning

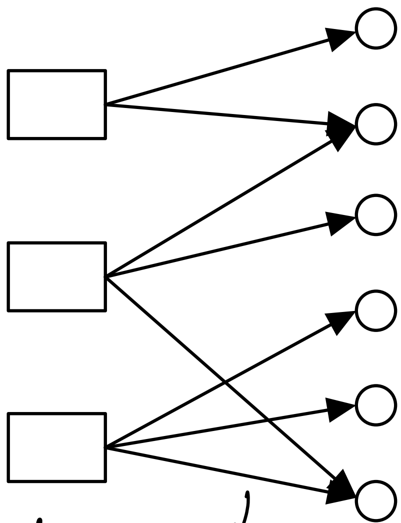
logistics supply chain

e-commerce

financial planning

portfolio optimization

Eg



warehouse $i = 1, 2 \ldots W$

flow of goods

outlet $j = 1, 2 \ldots O$

c_{ij} = cost of transport (per unit) from warehouse i to outlet j

x_{ij} = qty. from $i \to j$

s_i = supply/availability at warehouse i

d_j = demand at outlet j

Define N_i = neighbors of i (outlets connected to i)

N_j = warehouses connected to outlet j

(LP)

$$\min_{\{x_{ij}\}} \sum_{i=1}^{W} \sum_{j \in N_i} c_{ij} x_{ij}$$

$$x_{ij} \geq 0$$

total qty shipped to outlet j $\qquad \sum_{k \in N_j} x_{kj} \geq d_j \qquad$ demand at outlet j

total qty. shipped by i $\qquad \sum_{j \in N_i} x_{ij} \leq s_i \qquad$ supply at i

oldest algo : Simplex

$$\begin{array}{c} \min c^T x \\ Gx \leq h \\ Ax = b \end{array} \quad \xrightarrow{\ ?\ } \quad \boxed{\begin{array}{c} \min c^T y \\ Hy = d \\ y \geq 0 \end{array}}$$

(a) $\quad Gx + s = h \; , \; s \geq 0$

x could be -ve

(b) $\quad x = u - v \quad$ where $\quad u \geq 0 , v \geq 0$