

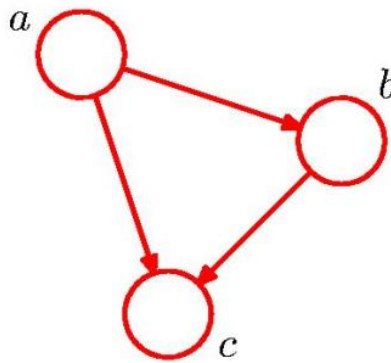
## Live Interaction #8:

3<sup>rd</sup> March 2024

### E-masters Next Generation Wireless Technologies

## EE902 Advanced ML Techniques for Wireless Technology

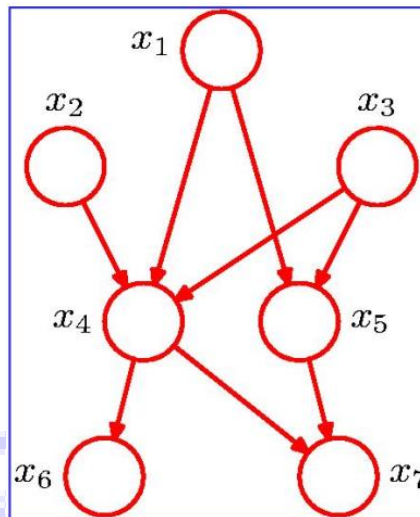
### ► Probabilistic Graphical Models:



- Nodes: **Random variables**.
- Edges: Probabilistic relationships.
- Joint PDF

$$p(x_1, x_2, \dots, x_K) = p(x_1) \times p(x_2|x_1) \times p(x_3|x_1, x_2) \\ \times p(x_4|x_1, x_2, x_3) \times \dots \\ \times p(x_K|x_1, x_2, \dots, x_{K-1})$$

- **Bayesian Network:**



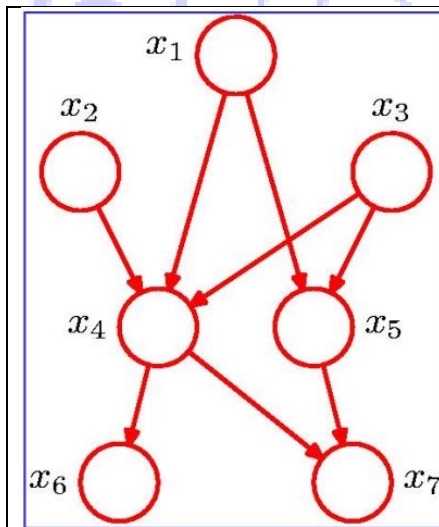
► **Parent nodes:**

$$x_5$$

$$\mathcal{P}_5 = \{x_1, x_3\}$$

$$p(x_k | x_1, x_2, \dots, x_{k-1}) = p(x_k | \mathcal{P}_k)$$

► Given parents,  $x_k$  is **conditionally independent** of others!



$$p(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$$

$$= p(x_1) \times p(x_2 | x_1)$$

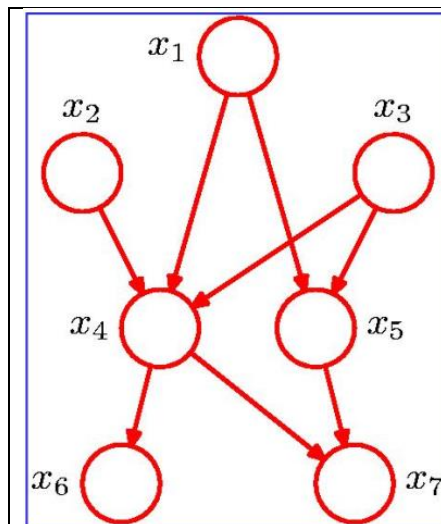
$$\times p(x_3 | x_1, x_2)$$

$$\times p(x_4 | x_1, x_2, x_3)$$

$$\times p(x_5 | x_1, x_2, x_3, x_4)$$

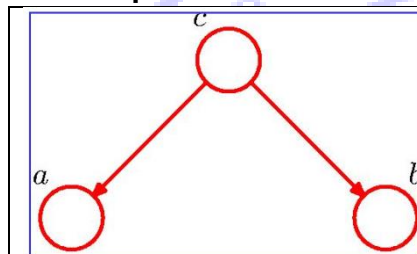
$$\times p(x_6 | x_1, x_2, x_3, x_4, x_5)$$

$$\times p(x_7 | x_1, x_2, x_3, x_4, x_5, x_6)$$

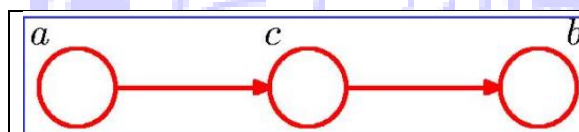


$$\begin{aligned}
 p(x_1, x_2, x_3, x_4, x_5, x_6, x_7) \\
 &= p(x_1) \times p(x_2) \times p(x_3) \\
 &\quad \times p(x_4|x_1, x_2, x_3) \\
 &\quad \times p(x_5|x_1, x_3) \\
 &\quad \times p(x_6|x_4) \times p(x_7|x_4, x_5)
 \end{aligned}$$

► Example



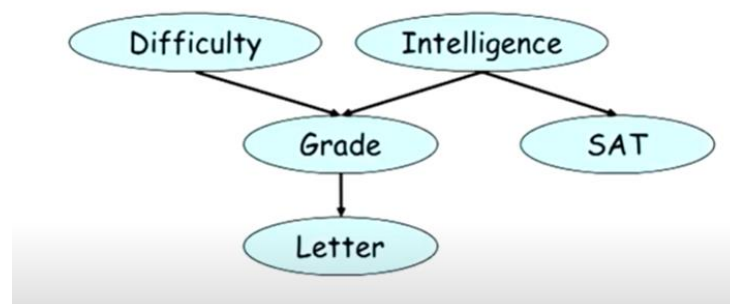
$$p(c) \times p(a|c) \times p(b|c)$$



Markov chain

$$\begin{aligned}
 p(a, b, c, d) \\
 &= p(a) \times p(c|a) \times p(b|c) \\
 &\quad \times p(d|b)
 \end{aligned}$$

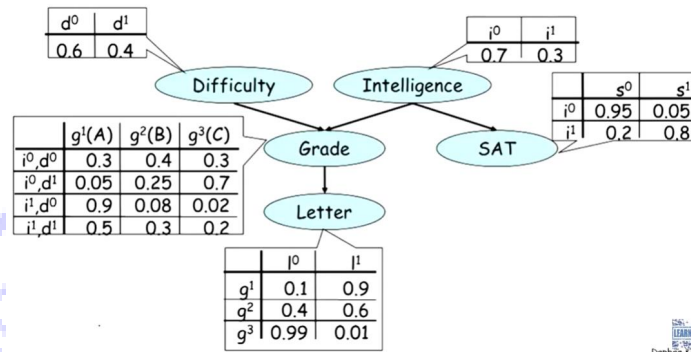
► Student example:



$$p(D, I, G, L, S) = p(D) \times p(I) \times p(G|D, I), p(L|G)$$

$$\begin{aligned}
 & \times p(S|I) \\
 p(D, I, G, L, S) &= p(D) \times p(I|D) \times p(G|D, I) \times p(L|G, D, I) \\
 & \times p(S|D, I, G, L)
 \end{aligned}$$

► Probabilities



- BN aids in compact representation!
- Computing any **joint probability**

$$\begin{aligned}
 & p(d^1, i^0, g^3, s^1, l^1) \\
 &= 0.4 \times 0.7 \times 0.7 \times 0.05 \times 0.01 \\
 &= 0.000098
 \end{aligned}$$

- Computing the conditional probability:

$$p(i^1 | s^1, l^0) = \frac{p(i^1, s^1, l^0)}{p(s^1, l^0)}$$

$$= \frac{\sum_{D, G} p(i^1, s^1, l^0, D, G)}{\sum_{I, D, G} p(s^1, l^0, I, D, G)}$$

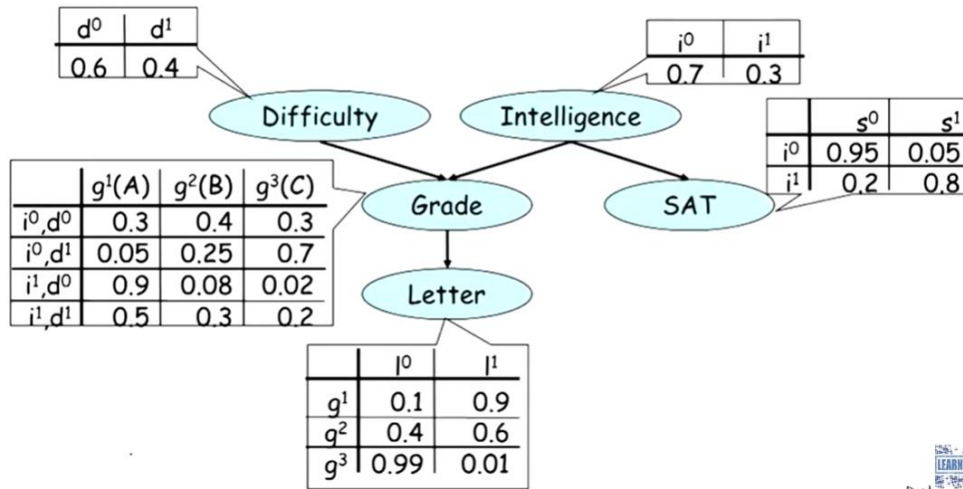
$$\sum_{D, G} p(i^1, s^1, l^0, D, G)$$

$$\begin{aligned}
 &= p(i^1, s^1, l^0, d^0, g^1) + p(i^1, s^1, l^0, d^0, g^2) + p(i^1, s^1, l^0, d^0, g^3) \\
 &+ p(i^1, s^1, l^0, d^1, g^1) + p(i^1, s^1, l^0, d^1, g^2) + p(i^1, s^1, l^0, d^1, g^3)
 \end{aligned}$$

- **Causal reasoning:**

$$p(G|D), p(L|I)$$

## ► Evidential reasoning:



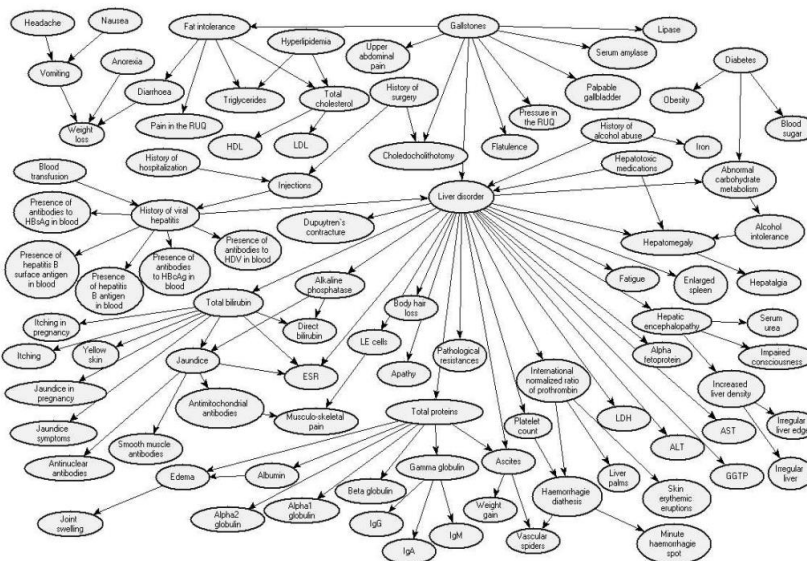
LEARNING  
Daphne Koller

$$p(I|L), p(D|L)$$

## ► Intercausal reasoning:

$$p(I|D, L)$$

## ► Medical diagnosis:



- **Assignment #8 Deadline: 7<sup>th</sup> March Thursday 11:59 PM.**
- **Assignment #7, 8 Discussion: 8<sup>th</sup> March Friday 8:00 PM – 8:30 PM.**
- **Quiz #4: 8<sup>th</sup> March Friday 9:00 - 9:45 PM.**
- **Final Exam: 10<sup>th</sup> March Sunday 9:00 AM – 12:00 PM. (Please check!!)**