Robustness to parameter variations

Eg(1) logistics : how much to ship

but allow minor variations in traffic / demand etc.

(2) communications : variations in channel gains

how to allow for a margin of error ?

$$\min \; c^T x$$
$$Ax \leq b \qquad\qquad A \in \mathcal{A} = \{A_1, A_2 \ldots A_k\}$$
$$a_i^T x \leq b_i \quad i = 1 \ldots m$$

not too large

Formulation : constraint should be satisfied always

$$\boxed{\begin{array}{l} \min \; c^T x \\ A_i x \leq b \qquad i = 1, \ldots k \end{array}}$$

constraint satisfied
in the worst case

Note : more constraints $\Rightarrow$ min value $\uparrow$

very robust $\Rightarrow$ higher cost

L.S. Example:

$$\min \|Ax - b\|_2$$
$$A \in \mathcal{A} = \{A_1, \ldots, A_k\}$$

worst case error:

$$e_{wc}(x) = \max_i \|A_i x - b\|_2$$

$$\min_x e_{wc}(x) = \min_{x,t} t$$
$$\max_i \|A_i x - b\|_2 \le t$$

$$\Updownarrow$$

(SOCP)

$$\min_{(x,t)} t$$
$$\|A_i x - b\|_2 \le t \qquad i = 1 \ldots k$$