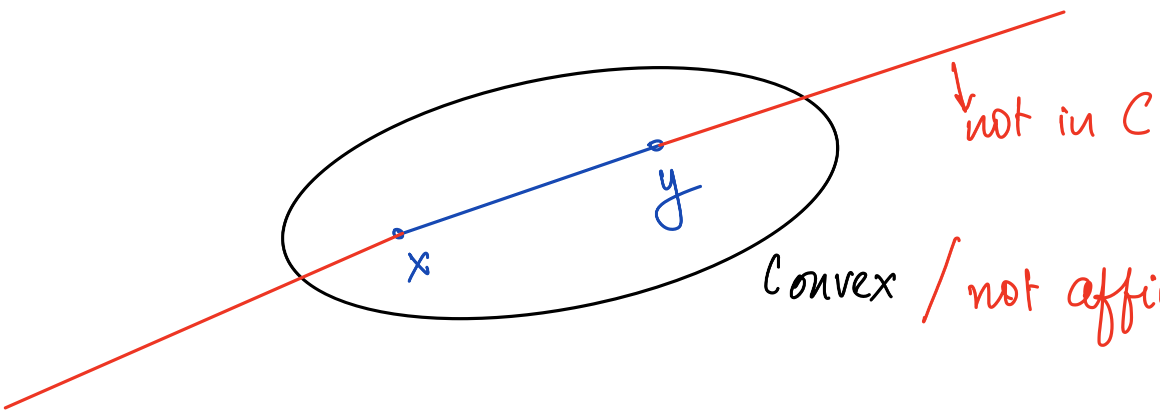
2. Convex Sets

C convex $\equiv$ line between $x, y \in C$ lies in C

$x, y \in C$ then

C convex $\Leftrightarrow$ $\underbrace{\theta x + (1-\theta) y \in C \quad \theta \in [0,1]}$

line between x, y
(restriction on θ)



Convex / not affine

a convex set may not be affine

Q: Are affine sets always convex?

A affine $\Leftrightarrow$ $x, y \in A$ then

$$\theta x + (1-\theta) y \in A \quad \forall\ \theta \in \mathbb{R}$$

$$\Rightarrow \theta x + (1-\theta) y \in A \quad \forall\ \theta \in [0,1]$$

$$\Rightarrow A \text{ convex}$$

$\Rightarrow$ All affine sets are also convex

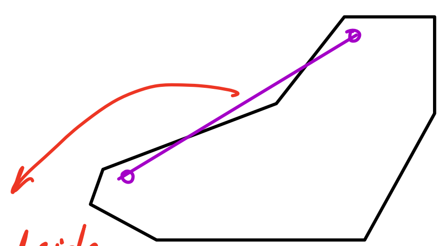
Eg $\{x \mid Ax = b\}$ affine $\Rightarrow$ convex

$C = \{x \mid Ax \leq b\}$ $x, y \in C$

$$A(\theta x + (1-\theta)y) = \theta(Ax) + (1-\theta)(Ay)$$
$$\leq \theta b + (1-\theta)b \quad \text{since} \quad \begin{matrix}\theta > 0 \\ 1-\theta > 0\end{matrix}$$
$$= b$$

$\Rightarrow$ C convex (but not affine)

Not convex



line segment outside

$\{x \in \mathbb{R}_{++} \mid \log(x) \leq 2\}$ $\Leftrightarrow$ $\{0 < x \leq e^2\}$

$x > 0$

interval

convex