eMasters in Communication Systems

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Elective Module:

Detection for Wireless Communication

Chapter 2

Neyman Pearson (NP) Criterion

NP Criteron

- The ***optimal detector*** maximizes the P_D for a given P_{FA}

NP Criteron

- The ***optimal detector*** maximizes the P_D for a given P_{FA}

$$\max P_D$$
$$\text{subject to: } P_{FA} = \alpha$$

NP Criteron

- Let **optimal detector** choose

$\mathcal{H}_1$:

$\mathcal{H}_0$:

NP Criteron

- Let **optimal detector** choose

$$\mathcal{H}_1 : \bar{\mathbf{y}} \in R_1 \subset R^{N\times 1}$$

$$\mathcal{H}_0 : \bar{\mathbf{y}} \in R_0 = R^{N\times 1} - R_1$$

NP Criteron

- Therefore,

$$P_D = \Pr(\bar{\mathbf{y}} \in R_1; \mathcal{H}_1)$$

$$=$$

NP Criteron

- Therefore,

$$P_D = \Pr(\bar{\mathbf{y}} \in R_1; \mathcal{H}_1)$$

$$= \int_{R_1} p(\bar{\mathbf{y}}; \mathcal{H}_1)\, d\bar{\mathbf{y}}$$

NP Criteron

- Therefore,

$$P_{FA} = \Pr(\bar{\mathbf{y}} \in R_1; \mathcal{H}_0)$$

$$=$$

NP Criteron

- Therefore,

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$$= \int_{R_1} p(\bar{\mathbf{y}}; \mathcal{H}_0)\, d\bar{\mathbf{y}}$$

NP Criteron

- How to solve the **constrained optimization** problem?

λ: Lagrange multiplier

NP Criteron

$$\max P_D$$
$$\text{subject to: } P_{FA} = \alpha$$

Lagrangian is

NP Criteron

$$\max P_D$$

$$\text{subject to: } P_{FA} = \alpha$$

$$P_D + \lambda(\alpha - P_{FA})$$

NP Criteron Lagrangian

NP Criteron Lagrangian

$$\int_{R_1} p(\bar{\mathbf{y}};\mathcal{H}_1)\,d\bar{\mathbf{y}} + \lambda\left(\alpha - \int_{R_1} p(\bar{\mathbf{y}};\mathcal{H}_0)\,d\bar{\mathbf{y}}\right)$$

$$= \int_{R_1} \big(p(\bar{\mathbf{y}};\mathcal{H}_1) - \lambda p(\bar{\mathbf{y}};\mathcal{H}_0)\big)d\bar{\mathbf{y}} + \lambda\alpha$$

NP Criteron

$$\int_{R_1} \left(p(\bar{\mathbf{y}};\mathcal{H}_1) - \lambda p(\bar{\mathbf{y}};\mathcal{H}_0)\right) d\bar{\mathbf{y}} + \lambda\alpha$$

- How to maximize P_D ?

NP Criteron

$$\int_{R_1} \left(p(\bar{\mathbf{y}};\mathcal{H}_1) - \lambda p(\bar{\mathbf{y}};\mathcal{H}_0)\right) d\bar{\mathbf{y}} + \lambda\alpha$$

- To maximize P_D include all points $\bar{\mathbf{y}}$ in R_1 such that

$$p(\bar{\mathbf{y}};\mathcal{H}_1) > \lambda p(\bar{\mathbf{y}};\mathcal{H}_0)$$

NP Criteron

- Choose $\mathcal{H}_1$ if

NP Criteron

- Choose $\mathcal{H}_1$ if

$$p(\bar{\mathbf{y}};\mathcal{H}_1) > \lambda p(\bar{\mathbf{y}};\mathcal{H}_0)$$

$$\Rightarrow \frac{p(\bar{\mathbf{y}};\mathcal{H}_0)}{p(\bar{\mathbf{y}};\mathcal{H}_1)} < \frac{1}{\lambda}$$

NP Criteron

- Choose $\mathcal{H}_0$ if

NP Criteron

- Choose $\mathcal{H}_0$ if

$$p(\bar{\mathbf{y}};\mathcal{H}_1) \le \lambda p(\bar{\mathbf{y}};\mathcal{H}_0)$$

$$\Rightarrow \frac{p(\bar{\mathbf{y}};\mathcal{H}_0)}{p(\bar{\mathbf{y}};\mathcal{H}_1)} \ge \frac{1}{\lambda} = \tilde{\gamma}$$

NP Criteron

- How to choose λ?

NP Criteron

- How to choose λ?
- From the **constraint**!!

$$P_{FA} = \alpha$$

Constant Signal Detection

- Given $P_{FA} = \alpha$

$$P_{FA} = Q\left(\frac{\gamma}{\sigma\|\bar{\mathbf{s}}\|}\right) = \alpha$$

Constant Signal Detection

- Given $P_{FA} = \alpha$

$$P_{FA} = Q\left(\frac{\gamma}{\sigma\|\bar{\mathbf{s}}\|}\right) = \alpha$$

$$\Rightarrow \gamma = \sigma\|\bar{\mathbf{s}}\|Q^{-1}(\alpha)$$

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Three options provided below for the font size.

Font: Avenir (Book), Size: 32, Colour: Dark Grey

Font: Avenir (Book), Size: 28, Colour: Dark Grey

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