eMasters in Communication Systems

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Core Module:

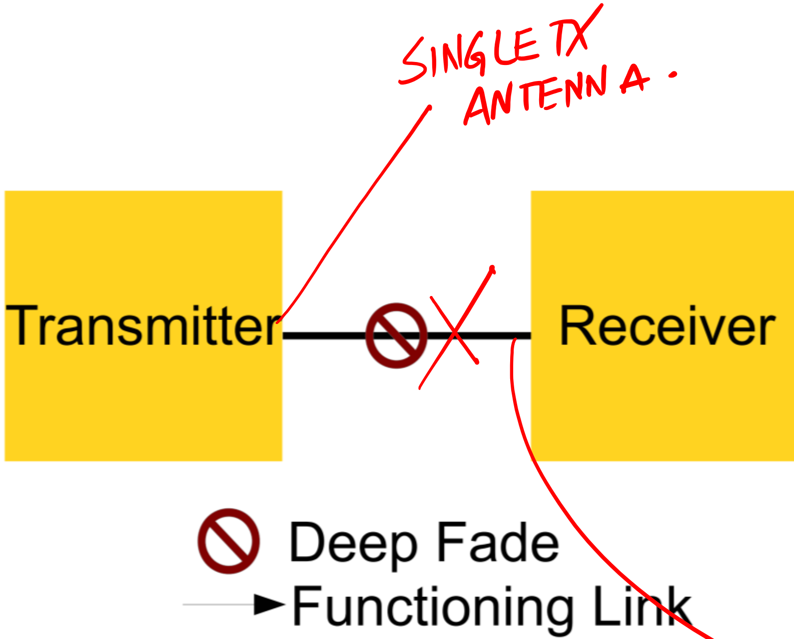
Wireless Communication

Chapter 3

Multiple Antennas and Diversity

Wireless Challenge

- Problem with wireless system: **BER is very high**!!!
- This arises because of ***DF*** = DEEP FADE.
- In a deep fade, there is a ***sharp drop*** in the OUTPUT signal power,…
  - because of which the ***reliability of communication is affected.***



Single link

- In typical wireless systems, there is a ***single link***.
- When this link is in a **deep fade**, communication is ***disrupted***.

SINGLE RX ANTENNA

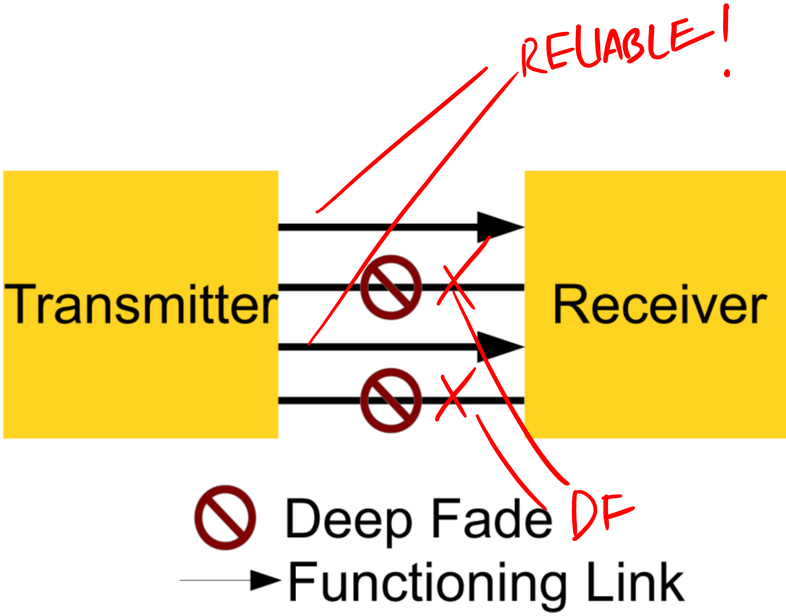
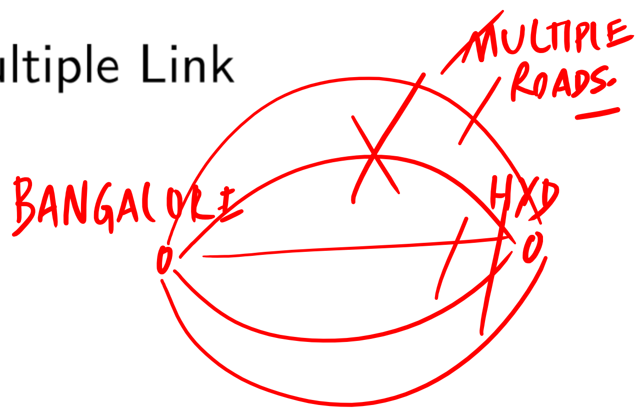


Figure : Multiple Link System



Multiple Links

DF

- In order to prevent this, one should have ***multiple functioning links***!!!

LINKS -

- Even if **one or few are in a deep fade**, the rest can be used for signal communication.

Diversity

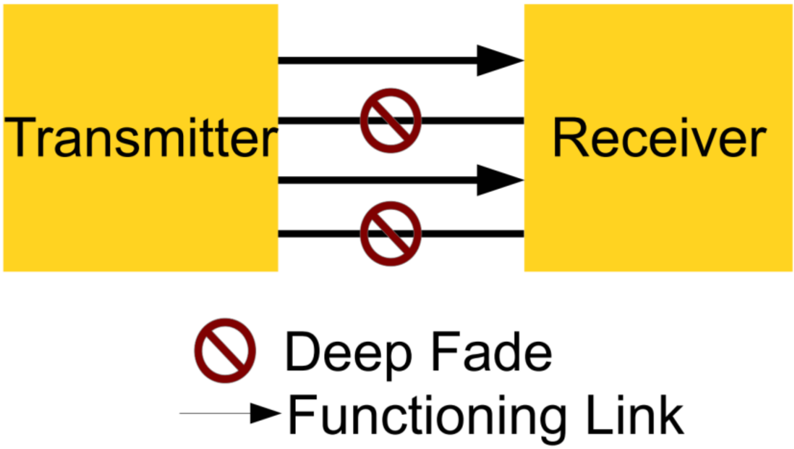


Figure : Multiple Link System

- Essentially we need ***diverse links*** in a system.
- This principle is termed as __DIVERSITY__.

Multiple Antennas

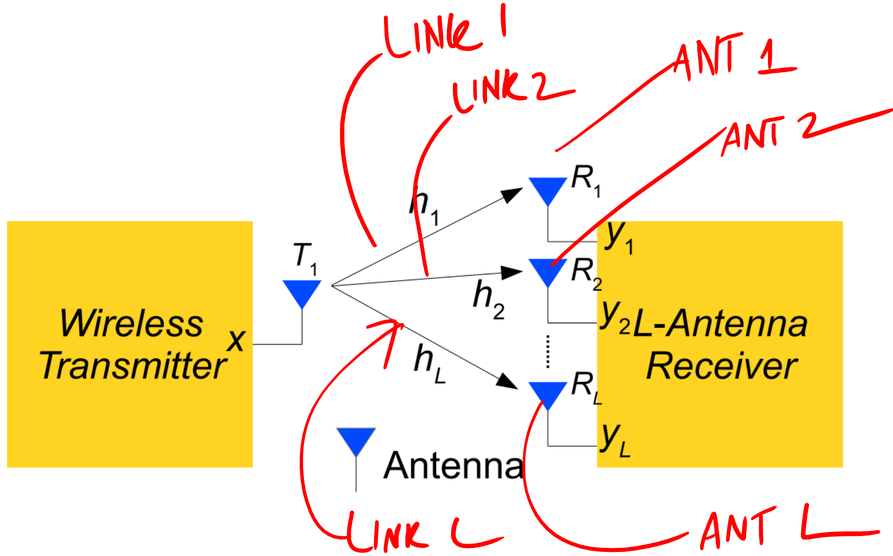


Figure : L Antenna Receiver

INCREASING # RX ANTENNAS TO . L

- One simple technique for diversity is to use ***multiple antennas***.

  L = # RECEIVE ANTENNAS.

- Single transmit antenna, but ***multiple receive antennas***.
- So there is one link between the transmit antenna and ***each receive antenna.***

  TOTAL # LINKS = L

SIMO System

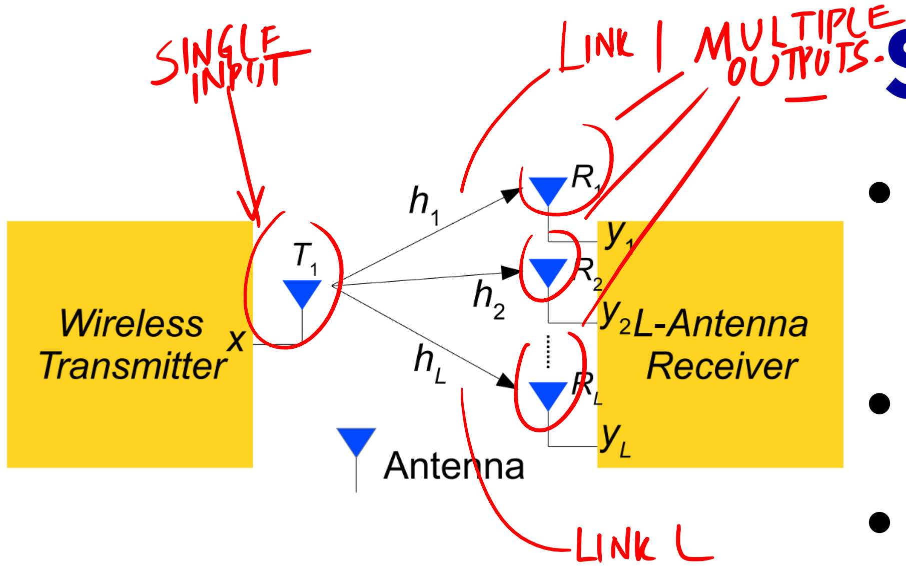


Figure : L Antenna Receiver

- For L **receive antennas**, how many links are there?
- There are L links.
- This is also known as a...

SINGLE INPUT MULTIPLE OUTPUT (SIMO) SYSTEM.

Multiple Antenna System Model

- Let the **transmitted symbol** be x INPUT SYMBOL
- Let the received symbol at **receive antenna** i be denoted by y_i OUTPUT SYMBOL ANTENNA i

$$y_i = h_i \times x + n_i$$

NOISE AT ANTENNA i

CHANNEL COEFFICIENT BETWEEN THE TX ANTENNA, RX ANTENNA i

Multiple Antenna System Model

- For L **antennas**, the model is

$$
\begin{aligned}
y_1 &= h_1 \times x + n_1 \\
y_2 &= h_2 \times x + n_2 \\
&\vdots \\
y_L &= h_L \times x + n_L
\end{aligned}
$$

OUTPUT ON ANTENNA L

Multiple Antenna System Model

SIMO SYSTEM.

- This can be written in **vector form** as

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix}}_{\bar{y}} = \underbrace{\begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix}}_{\bar{h}} x + \underbrace{\begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_L \end{bmatrix}}_{\bar{n}}$$

$\bar{y}$: $L \times 1$ OUTPUT VECTOR

$\bar{h}$: $L \times 1$ CHANNEL VECTOR

$\bar{n}$: $L \times 1$ NOISE VECTOR

Multiple Antenna System Model

- This can be written in **vector form** as

$$\bar{y} = \bar{h}x + \bar{n}$$

OUTPUT VECTOR

CHANNEL VECTOR

NOISE VECTOR

SIMO SYSTEM.

Multiple Antenna System Model

- Thus it can be written in the ***compact form***

OUTPUT VECTOR

INPUT SYMBOL

$$\bar{\mathbf{y}} = \bar{\mathbf{h}}\,x + \bar{\mathbf{n}}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} \quad \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix} \quad \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_L \end{bmatrix}$$

Multiple Antenna Processing

- Now, one has to ***process these samples***

$$y_1, y_2, \cdots, y_L$$

OUTPUT SYMBOLS

- How to ***process these samples***?

Multiple Antenna Processing

- The most ***popular and efficient*** technique to process the samples is via **linear combining**.

$$\underbrace{w_1^* y_1 + w_2^* y_2 + \cdots + w_L^* y_L}_{\text{Weighted Linear Combination}}$$

FILTERING OPERATION.

Multiple Antenna Processing

- This can represented in a ***compact fashion*** in ***vector form*** as

$$w_1^* y_1 + w_2^* y_2 + \cdots + w_L^* y_L$$

$$= \underbrace{\begin{bmatrix} w_1^* & w_2^* & \cdots & w_L^* \end{bmatrix}}_{\bar{w}^H} \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix}}_{\bar{y}} = \bar{w}^H \bar{y}$$

$\bar{w}^H$ HERMITIAN OPERATOR

TAKE TRANSPOSE + COMPLEX CONJUGATE.

Multiple Antenna Processing

$$\bar{\mathbf{w}} = \underbrace{\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix}}_{L \times 1}$$

BEAMFORMER

$$\bar{w}^H = \overbrace{[w_1^* \; w_2^* \cdots w_L^*]}^{1 \times L}$$

HOW TO CHOOSE $\bar{w}$?

- $\bar{\mathbf{w}}$: is also known as the ***beamforming vector*** or ***beamformer***.

Multiple Antenna Processing

- An **efficient** way to choose the beamformer is to …
  - maximize the ***signal to noise power ratio (SNR)*** of the output.

CHOOSE BEAMFORMER TO MAXIMIZE SNR.

Multiple Antenna Processing

- Let us examine the ***output of the beamformer***.

$\bar{y} = \bar{h} x + \bar{n}$

$$\bar{\mathbf{w}}^H \bar{\mathbf{y}} = \bar{\mathbf{w}}^H \left(\bar{\mathbf{h}}\, x + \bar{\mathbf{n}} \right)$$
$$= \underbrace{\bar{\mathbf{w}}^H \bar{\mathbf{h}}\, x}_{\text{signal}} + \underbrace{\bar{\mathbf{w}}^H \bar{\mathbf{n}}}_{\text{noise}}$$

Multiple Antenna Processing

$$\bar{w}^H \bar{y} = \underbrace{\bar{w}^H \bar{h} x}_{\text{SIGNAL PART}} + \underbrace{\bar{w}^H \bar{n}}_{\text{NOISE PART}}$$

OUTPUT OF BEAMFORMER

$$\text{SIGNAL POWER} = |\bar{w}^H \bar{h}|^2 \cdot P$$

$$\text{NOISE POWER} = E\{|\bar{w}^H \bar{n}|^2\}.$$

$$= ?$$

Multiple Antenna Processing

- ***Output Noise power*** $E\{|\bar{\mathbf{w}}^H\bar{\mathbf{n}}|^2\}$
- We now evaluate this

$$E\{|\bar{w}^H\bar{n}|^2\}$$

$$= E\left\{\left|\begin{bmatrix} w_1^* & \cdots & w_L^* \end{bmatrix}\begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_L \end{bmatrix}\right|^2\right\}$$

$$= E\{|w_1^* n_1 + \cdots + w_L^* n_L|^2\}$$

Multiple Antenna Processing

INDEPENDENT IDENTICALLY DISTRIBUTED.

$n_1, n_2, \ldots, n_L$

- let the noise samples be **i.i.d. across the different antennas**.

$$E\{n_i n_j^*\} = \begin{cases} 0 & \text{if } i \neq j \\ N_0 & \text{if } i = j \end{cases}$$

Multiple Antenna Processing

$$E\{n_i n_j^*\} = \underbrace{E\{n_i\} E\{n_j^*\}}_{\text{SINCE THEY ARE INDEPENDENT.}} \quad \text{if } i \neq j$$

$$\underbrace{E\{n_i\} = E\{n_j^*\} = 0}_{\text{ZERO MEAN. NOISE}}$$

$$= 0 \times 0 = 0$$

$$E\{n_i n_j^*\} = E\{|n_i|^2\} \quad \text{if } i = j$$

$$= N_0.$$

Multiple Antenna Processing

$$E\{n_i n_j^*\} = \begin{cases} 0 & \text{if } i \neq j \\ N_0 & \text{if } i = j \end{cases}$$

i.i.d. NOISE SAMPLES ACROSS ANTENNAS.

Multiple Antenna Processing

- Using this property, **output noise power** can be derived as.

$$E\{|\bar{\mathbf{w}}^H\bar{\mathbf{n}}|^2\} = N_0\|\bar{\mathbf{w}}\|^2$$

$$\|\bar{\mathbf{w}}\|^2 = |w_1|^2 + |w_2|^2 + \cdots + |w_L|^2$$

$$\|\bar{w}\| = \sqrt{|w_1|^2 + |w_2|^2 + \cdots + |w_L|^2}$$

Multiple Antenna Processing

- Consider the output AFTER BEAMFORMING

$$\underbrace{\bar{\mathbf{w}}^H \bar{\mathbf{h}}\, x}_{\text{signal}} + \underbrace{\bar{\mathbf{w}}^H \bar{\mathbf{n}}}_{\text{noise}}$$

Multiple Antenna Processing

- Output SNR is given as

$$SNR_o = \frac{\left|\bar{\mathbf{w}}^H \bar{\mathbf{h}}\right|^2 \overbrace{E\{|x|^2\}}^{P}}{\underbrace{N_0 \|\bar{\mathbf{w}}\|^2}_{\text{OUTPUT NOISE POWER}}}$$

Multiple Antenna Processing

SYMBOL POWER

- Output SNR is given as

$$SNR_o = \frac{\left|\bar{\mathbf{w}}^H \bar{\mathbf{h}}\right|^2 E\{|x|^2\}}{N_0 \|\bar{\mathbf{w}}\|^2} = \frac{\left|\bar{\mathbf{w}}^H \bar{\mathbf{h}}\right|^2 P}{N_0 \|\bar{\mathbf{w}}\|^2}$$

Multiple Antenna Processing

INNER PRODUCT

- We now use the Cauchy Schwarz inequality

$$\left|\bar{\mathbf{w}}^H \bar{\mathbf{h}}\right|^2 \le \|\bar{w}\|^2 \|\bar{h}\|^2$$

$$(\bar{w} \cdot \bar{h})^2 \leqslant \|\bar{w}\|^2 \|\bar{h}\|^2$$

Multiple Antenna Processing

- Output SNR can be simplified as

$$\frac{|\bar{\mathbf{w}}^H\bar{\mathbf{h}}|^2 P}{N_0\|\bar{\mathbf{w}}\|^2} \le \frac{\cancel{\|\bar{w}\|^2}\|\bar{h}\|^2 P}{N_0\cancel{\|\bar{w}\|^2}} = \overbrace{\frac{\|\bar{h}\|^2 \cdot P}{N_0}}^{\text{maximum output SNR}}$$

$\bar{w} \propto \bar{h}$ $\quad$ $\bar{w} = k\bar{h}$

Maximum output SNR occurs when $\bar{w} \propto \bar{h}$

Multiple Antenna Processing

BPSK. $SNR_o = \|\bar{h}\|^2 \cdot \frac{2P}{N_o}$.

- **Maximum output SNR** is

$$SNR_o = \|\bar{\mathbf{h}}\|^2 \frac{P}{N_0}$$ — QPSK.

OCCURS FOR $\bar{w} \propto \bar{h}$

Multiple Antenna Processing

- When is the **maximum achieved**?

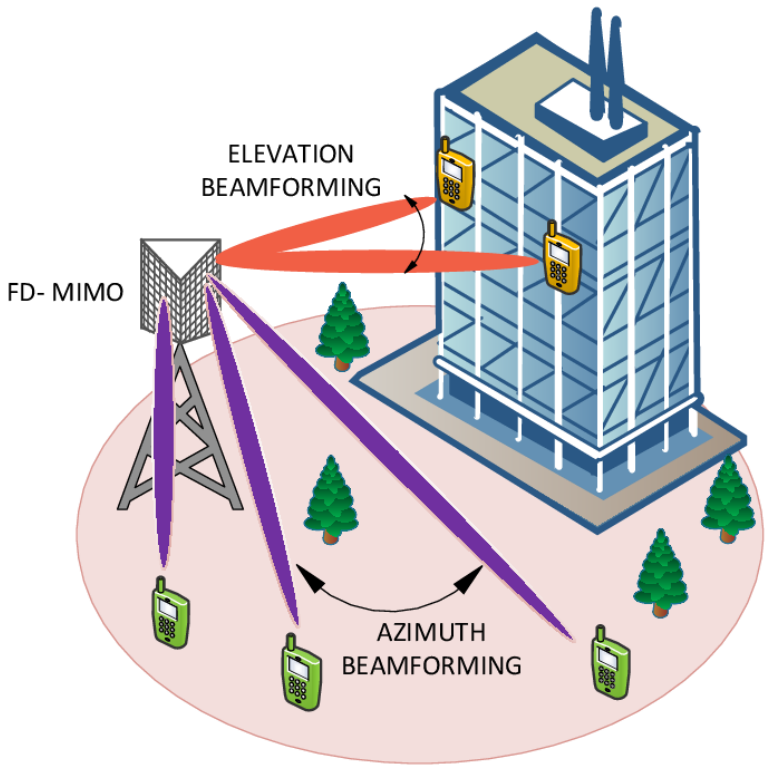
$$\bar{\mathbf{w}} \propto \bar{\mathbf{h}}$$

$$\bar{\mathbf{w}} = \frac{\bar{h}}{\|\bar{h}\|} \quad : \underbrace{\mathbf{unit-norm}\ \text{beamformer}}_{\|\bar{w}\| = 1}$$

Multiple Antenna Processing

MRC maximizes output SNR

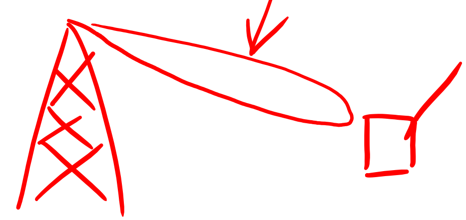
- Since $\frac{\bar{\mathbf{h}}}{\|\bar{\mathbf{h}}\|}$ is the ***combiner that*** ***maximizes the signal to noise power ratio***,
- this is known as the MAXIMAL RATIO COMBINER (MRC) **(MRC)**



Beamformer

- Beamformer forms a ***narrow beam*** to a particular user
- Therefore, it **maximizes the SNR**

NARROW BEAM.



Output SNR

$E\{|x|^2\}$.

- Output SNR of ***maximal ratio combiner*** is

$$SNR_{\text{o}} = \frac{P}{N_0}\left\|\bar{\mathbf{h}}\right\|^2$$

MRC Output SNR Example

$|h_1|^2 = 2+2 \quad |h_2|^2 = 2+2$

Consider the **channel vector**

$$\bar{\mathbf{h}} = \begin{bmatrix} \sqrt{2}-\sqrt{2}j \\ \sqrt{2}+\sqrt{2}j \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

$h_1 = \sqrt{2}-\sqrt{2}j \quad h_2 = \sqrt{2}+\sqrt{2}j$

$\|\bar{h}\| = \sqrt{|h_1|^2+|h_2|^2} = \sqrt{2+2+2+2}$

$= \sqrt{8} = 2\sqrt{2}$

MRC Output SNR Example

Maximal Ratio Combiner

What is the MRC beamformer

$$\bar{\mathbf{w}} = \frac{\bar{\mathbf{h}}}{\|\bar{\mathbf{h}}\|} = \frac{1}{2\sqrt{2}} \cdot \bar{h}$$

$$\|\bar{\mathbf{h}}\| = \sqrt{2+2+2+2}$$

$$= \sqrt{8} = 2\sqrt{2} = \|\bar{h}\|$$

MRC Output SNR Example

MRC beamformer is

Unit Norm Beamformer

$$\bar{\mathbf{w}} = \frac{\bar{\mathbf{h}}}{\|\bar{\mathbf{h}}\|} = \frac{1}{2\sqrt{2}}\begin{bmatrix}\sqrt{2}-\sqrt{2}j \\ \sqrt{2}+\sqrt{2}j\end{bmatrix}$$

$$= \begin{bmatrix}\frac{1}{2}-\frac{1}{2}j \\ \frac{1}{2}+\frac{1}{2}j\end{bmatrix}$$

$$\|\bar{w}\|^2 = \frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4} = 1 \Rightarrow \|\bar{w}\| = 1$$

MRC Output SNR Example

$\bar{w}^H \bar{y} = w_1^* y_1 + w_2^* y_2$

What is the processing at the receiver

$$\bar{\mathbf{w}}^H\,\bar{\mathbf{y}} = \begin{bmatrix} \frac{1}{2}+\frac{1}{2}j & \frac{1}{2}-\frac{1}{2}j \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= \left(\frac{1}{2}+\frac{1}{2}j\right) y_1 + \left(\frac{1}{2}-\frac{1}{2}j\right) y_2$$

MRC Output SNR Example

$\frac{P}{N_0} = 3\,dB \approx 2$

If SNR = 3 dB, what is output SNR?

$$SNR = 3\,dB \approx 2$$

$\|\bar{h}\|^2 \cdot \frac{P}{N_0}$

$$SNR_0 = \|\bar{\mathbf{h}}\|^2 SNR$$

$$= \underline{\quad 8 \times 2 = 16 \quad}$$

$$= 16 \approx \underline{2^4 \approx 12dB}(dB)\_\_\_$$

BER of Multiple Antenna System

BPSK : Binary Phase Shift Keying

BER of **multiple antenna system** is given as

$$ {}^{2L-1}C_{L-1} \times \frac{1}{2^L} \times \frac{1}{SNR^L} $$

$$ {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!} $$

BER of Multiple Antenna System

$$ {}^{2L-1}C_{L-1} = \frac{(2L-1)!}{(L-1)! \times L!} $$

Example: $L = 3$

$$ {}^{2L-1}C_{L-1} = {}^{5}C_{2} = \frac{5!}{2!\,3!} = \frac{5 \times \cancel{4}^{2} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{2} \times 1 \times \cancel{3} \times \cancel{2} \times \cancel{1}} $$

$$ = \underline{10} $$

BER of Multiple Antenna System $\underline{L=2}$

BER for $L = 2$ antennas is $\frac{3}{4} \cdot \frac{1}{SNR^2}$

$$ {}^{2L-1}C_{L-1} \times \frac{1}{2^L} \times \frac{1}{SNR^L} $$

$$ = {}^{3}C_{1} \times \frac{1}{2^2} \times \frac{1}{SNR^2} = \frac{3}{4} \cdot \frac{1}{SNR^2} $$

BER of Multiple Antenna System $L = 2$

BPSK ·

BER for $L = 2$ antennas is $\underline{SNR = 20\ dB}$

$$SNR = 10^2 = 100$$

$$BER = \frac{3}{4}\cdot\frac{1}{(10^2)^2} = \frac{3}{4}\times\frac{1}{10^4}$$

$$= 7.5 \times 10^{-5}$$

HW. Problem: What is BER in $L = 2$ system for $SNR = 15\ dB$?

BER of Multiple Antenna System $L = 3$

BER for $L = 3$ antennas is $\frac{5}{4} \cdot \frac{1}{SNR^3}$

$$ {}^{2L-1}C_{L-1} \times \frac{1}{2^L} \times \frac{1}{SNR^L} = BER $$

$$ = {}^{5}C_{2} \times \frac{1}{2^3} \times \frac{1}{SNR^3} = \frac{10}{8} \cdot \frac{1}{SNR^3} $$

$$ = \frac{5}{4} \cdot \frac{1}{SNR^3} $$

BER of Multiple Antenna System $L = 3$

Example:

BER for $L = 3$ antennas is $SNR = 20\ dB$

$$SNR = 10^2 = 100$$

$$BER = \frac{5}{4} \cdot \frac{1}{SNR^3} = \frac{5}{4} \cdot \frac{1}{100^3}$$

$$= 1.25 \times 10^{-6}$$

BER of Single Antenna System

$L = 1$
$2L - 1 = 1$
$L - 1 = 0$

${}^{2L-1}C_{L-1}$
$= {}^{1}C_0 = 1$

Recall BER for $L = 1$ antennas is

$$ {}^{2L-1}C_{L-1} \times \frac{1}{2^L} \times \frac{1}{SNR^L} $$

$$ = \ {}^{1}C_0 \times \frac{1}{2^1} \times \frac{1}{SNR^1} = \frac{1}{2} \times \frac{1}{SNR} $$

BER of Multiple Antenna System $L = 1$

BER for $L = 1$ antennas is $SNR = 20\ dB$

$$SNR = 10^2 = 100$$

$$BER = \frac{1}{2} \times \frac{1}{SNR} = \frac{1}{2} \times \frac{1}{100}$$

$$= 5 \times 10^{-3}$$

BER of Comparison

AS NUMBER OF ANTENNAS INCREASES, BER DECREASES.

BER TABLE

# ANTENNAS	BER FORMULA	BER @ 20 dB
$L = 1$	$\frac{1}{2} \times \frac{1}{SNR}$	5×10^{-3}
$L = 2$	$\frac{3}{4} \times \frac{1}{SNR^2}$	7.5×10^{-5}
$L = 3$	$\frac{5}{4} \times \frac{1}{SNR^3}$	1.25×10^{-6}

INCREASING ANTENNAS

BER DECREASES

BER of Comparison

What is happening as the ***number of antennas is increasing***? **BER decreases**!!!

BER of Comparison

- How is the BER decreasing with the **number of antennas**?

$$L = 1: BER \propto \frac{1}{2} \times \frac{1}{SNR} \propto \frac{1}{SNR}$$

$$L = 2: BER \propto \frac{3}{4} \times \frac{1}{SNR^2} \propto \frac{1}{SNR^2}$$

$$L = 3: BER \propto \frac{5}{4} \cdot \frac{1}{SNR^3} \propto \frac{1}{SNR^3}$$

$$\vdots$$

$$L: BER \propto {}^{2L-1}C_{L-1} \cdot \frac{1}{2^L} \cdot \frac{1}{SNR^L} \propto \frac{1}{SNR^L}$$

BER of Comparison

- As the number of antennas is increasing, the **BER is decreasing at a faster rate**.
  - L is termed the **diversity order**.

FOR. L ANTENNAS, BER $\propto \frac{1}{SNR^L}$

Diversity Order

- If the BER decreases as $\frac{1}{SNR^d}$, then ***diversity order*** is d

EXAMPLE: $L = 2$ ANTENNAS

$BER \propto \frac{1}{SNR^2} \Rightarrow$ DIVERSITY ORDER = 2

Deep Fade Analysis

- Deep fade occurs when **signal is buried in noise**.

$$\text{i.e. } SNR_o \leq 1 \Rightarrow \text{OUTPUT SIGNAL POWER} \leq \text{OUTPUT NOISE POWER}$$

$$\frac{P}{N_0}\|\bar{\mathbf{h}}\|^2 \leq 1$$

$$|h_1|^2 + |h_2|^2 + \cdots + |h_L|^2$$

Deep Fade Analysis

$$g = \|\bar{\mathbf{h}}\|^2 \leq \frac{1}{SNR}$$

CHI-SQUARED.
RANDOM VARIABLE.

The PDF of g is given as

$$f_G(g) = \frac{g^{L-1}e^{-g}}{(L-1)!}$$

$$P_r\left(g \leq \frac{1}{SNR}\right)$$

$$= \int_0^{\frac{1}{SNR}} f_G(g)\,dg$$

Deep Fade Analysis

Probability of deep fade is given as

$$P_{DF} = \Pr\left(g \leq \frac{1}{SNR}\right)$$

$$= \int_0^{\frac{1}{SNR}} f_G(g)\,dg$$

$$= \int_0^{1/SNR} \frac{g^{L-1} e^{-g}}{(L-1)!}\,dg$$

At high SNR

$\frac{1}{SNR} \rightarrow 0$

$\Rightarrow e^{-g} \approx 1$

Deep Fade Analysis

$$\approx \int_0^{\frac{1}{SNR}} \frac{g^{L-1}}{(L-1)!}\, dg$$

$P_{DF} \propto \frac{1}{SNR^L}$

$$P_{DF} = \left.\frac{g^L}{L!}\right|_0^{\frac{1}{SNR}} = \frac{1}{L!} \times \frac{1}{SNR^L} \propto \frac{1}{SNR^L}$$

Deep Fade Analysis

- Therefore, once again, ***not surprisingly***, we observe

$$BER \propto P_{DF} \propto \frac{1}{SNR^L}$$

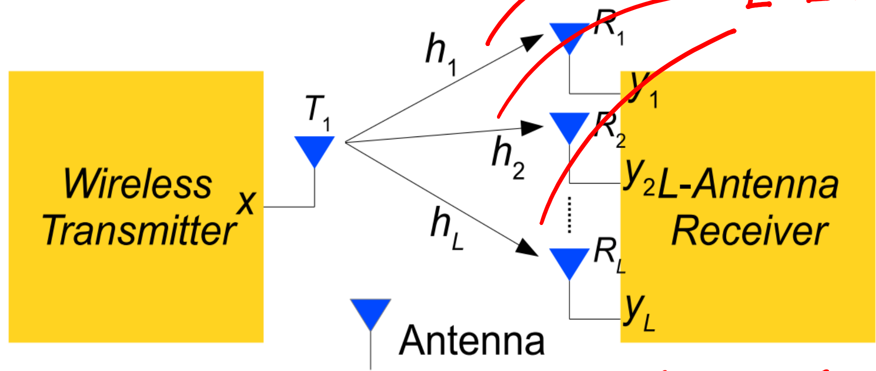
$BER = \frac{1}{2} \cdot \frac{1}{SNR}$

$\propto P_{DF} = \frac{1}{SNR}.$

Deep Fade Justification

- Consider the multiple antenna system

L LINKS



SIMO SYSTEM
SINGLE INPUT MULTIPLE OUTPUT SYSTEM.

Figure : L Antenna Receiver

Deep Fade Justification

- Let the event that link i is in deep fade be E_i

PROBABILITY THAT A SINGLE LINK IS IN DEEP FADE

$$\Pr(E_i) \propto \frac{1}{SNR}$$

Deep Fade Justification

$E_1 \cap E_2 \cap E_3 \cdots \cap E_L$: ALL THE LINKS ARE IN DEEP FADE.

- If the system has to be in **deep fade**, then

$$P_{DF} = \Pr(E_1 \cap E_2 \cap \cdots \cap E_L)$$

$$= \Pr(E_1) \times \Pr(E_2) \times \cdots \times \Pr(E_L)$$

ASSUMING LINKS ARE INDEPENDENTLY FADING.

$$P_{DF} \propto \frac{1}{SNR} \times \frac{1}{SNR} \times \cdots \times \frac{1}{SNR} = \frac{1}{SNR^L}$$

$$\Rightarrow P_{DF} \propto \frac{1}{SNR^L}$$

MULTIPLE ANTENNA SYSTEM.

INDEPENDENT EVENTS:

IF A, B ARE INDEPENDENT EVENTS.

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B).$$

ASSUMPTION: LINKS ARE INDEPENDENTLY FADING:

Independent Fading

- If $E_1, E_2, \dots, E_L$ are **independent**!!!
- The different links are ***independently fading***…

  λ = WAVELENGTH OF SIGNAL
  $\lambda = c/f_c$.

  - For this, ***antenna spacing has to be large***…
- Good **rule of thumb** is $> \frac{\lambda}{2}$.

ANTENNA SPACING $\geq \frac{\lambda}{2}$

λ = WAVELENGTH OF SIGNAL

FREE SPACE

$$\lambda = \frac{c}{F_c} = 3 \times 10^8 \, m/s$$

c = VELOCITY OF EM WAVE
VELOCITY OF LIGHT

F_c = CARRIER FREQUENCY.

Antenna Spacing

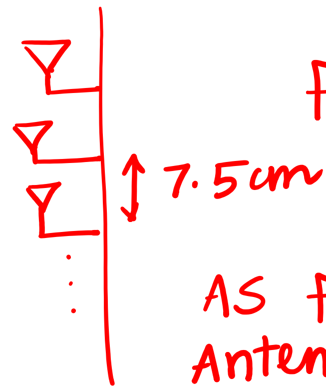
$2 \times 10^9 \, Hz$ ANTENNA

- At $f_c = 2\,GHz$ calculate the **minimum spacing**

$\lambda = c/f_c = \frac{3 \times 10^8}{2 \times 10^9}$

$$\frac{\lambda}{2} = \frac{1}{2} \cdot \frac{3 \times 10^8}{2 \times 10^9} = \underline{7.5\,cm}$$

Antenna Spacing



$f_c = 2 \times 10^9$ Hz

AS f_c increases.
Antenna spacing decreases.

Antenna Spacing

mmWave system

$25\times10^{9}\ Hz$

- At $f_c = 25\ GHz$ calculate the **minimum spacing**

$\lambda = \frac{c}{f_c} = \frac{3\times10^{8}}{25\times10^{9}}$

$$\frac{\lambda}{2} = \frac{1}{2}\times\frac{3\times10^{8}}{25\times10^{9}} = 0.6\ cm = 6\ mm$$

AS f_c INCREASES,

ANTENNA SPACING DECREASES!

$\Rightarrow$ MORE ANTENNAS CAN BE EMBEDDED ON DEVICE OF SAME SIZE

Instructors may use this white area (14.5 cm / 25.4 cm) for the text.
Three options provided below for the font size.

Font: Avenir (Book), Size: 32, Colour: Dark Grey

Font: Avenir (Book), Size: 28, Colour: Dark Grey

Font: Avenir (Book), Size: 24, Colour: Dark Grey

Do not use the space below.

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