Linear Algebra Application

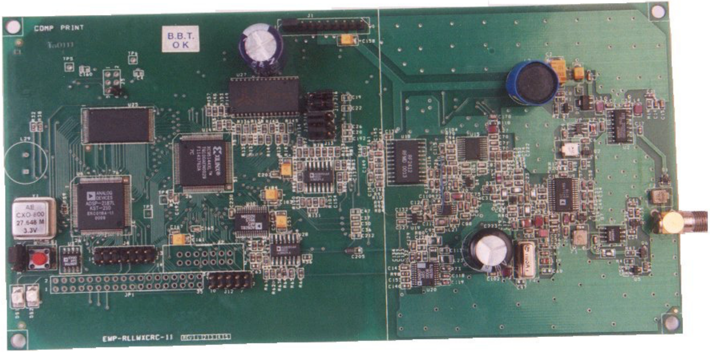
Rohit Budhiraja, IITK

Applied Linear Algebra for Wireless Communications

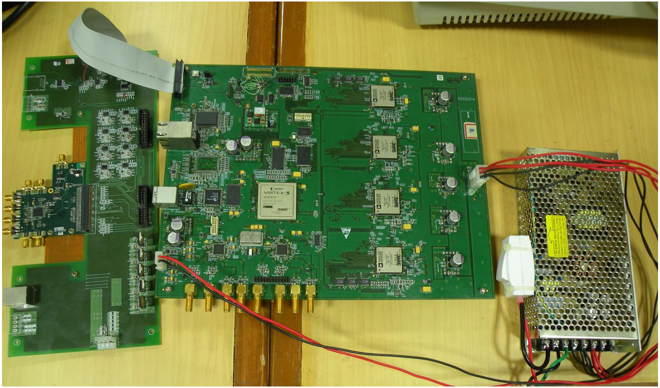
Agenda for today's class

- Apply linear algebra to one application in 4G/5G wireless systems

3G wireless systems – single antenna

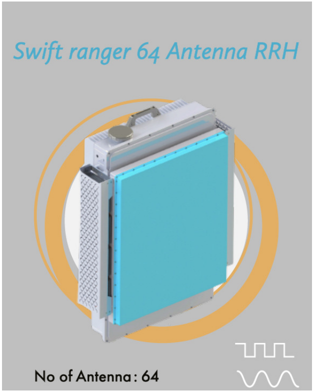


4G wireless systems – two/four antennas



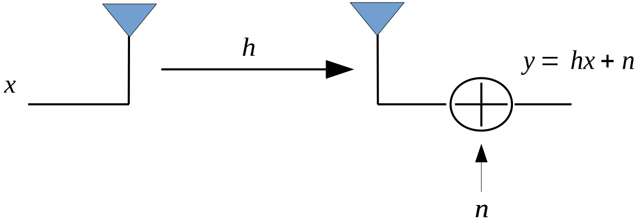
- Designed this system in 2010

5G base station at IIT Kanpur – 64 antennas



- Designed this system in 2020

Single antenna wireless system

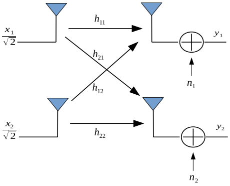


- Consider 1 transmit antenna and 1 receive antennas
- System is mathematically represented as

$$y = hx + n$$

- x is transmit symbol, n is receiver thermal noise
- h is wireless channel between the transmitter and receiver

Multiple transmit and receive antenna system (1)



- Transmit $\frac{x_1}{\sqrt{2}}$ from first antenna and $\frac{x_2}{\sqrt{2}}$ from second antenna
- Received signal is

$$
\begin{aligned}
y_1 &= \frac{h_{11}x_1}{\sqrt{2}} + \frac{h_{12}x_2}{\sqrt{2}} + n_1 \\
y_2 &= \frac{h_{21}x_1}{\sqrt{2}} + \frac{h_{22}x_2}{\sqrt{2}} + n_2 \\
\mathbf{y} &= \mathbf{Hx} + \mathbf{n}
\end{aligned}
$$

Multiple transmit and receive antennas (3)

- $\mathbf{x} = [x_1/\sqrt{2}\ x_2/\sqrt{2}]^T$ is transmit vector
- $\mathbf{y} = [y_1\ y_2]^T$ is receiver vector
- $\mathbf{n} = [n_1\ n_2]^T$ is the receiver noise vector

  $\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$ is channel matrix
- Two symbols in transmit vector $\mathbf{x}$ interfere with each other at the receiver
  - Need to design a receiver to recover $\mathbf{x}$ from $\mathbf{y}$
- Most common receiver is zero forcing $\mathbf{W} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T$

$$\begin{aligned} \mathbf{W}\mathbf{y} = \tilde{\mathbf{y}} &= \mathbf{W}\mathbf{H}\mathbf{x} + \mathbf{W}\mathbf{n} \\ &= (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{H}\mathbf{x} + \underbrace{\mathbf{W}\mathbf{n}}_{\tilde{\mathbf{n}}} \\ &= \mathbf{x} + \tilde{\mathbf{n}} \end{aligned}$$

- We have learn this operation – left-inverse of a matrix

Other applications

- Linear algebra is used in the subjects which you will learn in emasters
- Wireless communications
- Machine learning for wireless
- Convex optimization for signal processing
- Coding theory