## **EE932 Assignment-2 Solution**

## eMasters in Communication Systems, IITK **EE932:** Introduction to Reinforcement Learning Instructor: Prof. Subrahmanya Swamy Peruru Student Name: Venkateswar Reddy Melachervu

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## **Question 9:**

In the Bellman expectation equation, we related  $V^{\pi}$  in terms of  $V^{\pi}$ . Write down an equation that expresses  $Q^{\pi}$  in terms of  $Q^{\pi}$ . Hint: Refer to the Week 3 (Part 2) slide with the title "Relating  $Q^{\pi}$  and  $V^{\pi}$ 

## Solution:

$$\begin{aligned} Q_{\pi}(s, a) &= \mathbb{E}_{\pi} \{ G_{t} \mid S_{t} = s, A_{t} = a \} \\ &= \mathbb{E}_{\pi} \{ R_{t+1} + \gamma G_{t+1} \mid S_{t} = s, A_{t} = a \} \\ &= \mathbb{E} \{ R_{t+1} \mid S_{t} = s, A_{t} = a \} + \gamma \mathbb{E}_{\pi} \{ G_{t+1} \mid S_{t} = s, A_{t} = a \} \\ &= R_{s}^{a} + \gamma \sum_{s'} P_{ss'}^{a} \mathbb{E}_{\pi} \{ G_{t+1} \mid S_{t+1} = s', A_{t+1:\infty} \sim \pi \} \\ &= R_{s}^{a} + \gamma \sum_{s'} P_{ss'}^{a} V_{\pi}(s') \end{aligned}$$

Substituting  $V_{\pi}(s) = \sum_{a} \pi(a \mid s) Q_{\pi}(s, a)$  into above  $Q_{\pi}$  equation,

$$\begin{split} Q_{\pi}(s,a) &= R_s^a + \gamma \sum_{s'} P_{ss'}^a \sum_{a'} \pi(a' \mid s') Q_{\pi}(s',a') \\ \text{Substituting} \sum_{a'} P_{ss'}^a \pi(a' \mid s') &= P_{ss'}^\pi \end{split}$$

$$Q_{\pi}(s,a) = R_s^a + \gamma \sum_{s',a'} P_{ss'}^{\pi} Q_{\pi}(s',a')$$

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