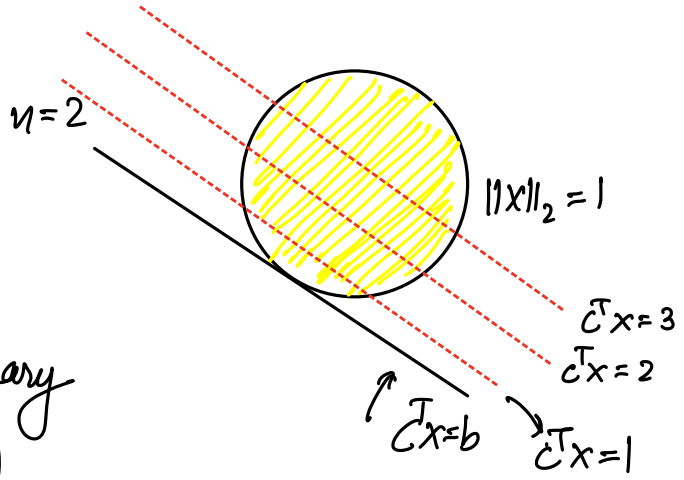
Minimising linear objective over a norm ball

$$\min\ c^T x \quad \|x\|_2 \le 1$$



Solution on the boundary satisfies $\|x^*\|_2 = 1$

C.S. $\quad c^T x^* \ge -\|c\|_2 \|x^*\|_2 = -\|c\|_2$

$\downarrow$

equality when $x^* = -\alpha c \quad \Rightarrow \quad -c^T(\alpha c) = -\|c\|_2$

$$\Rightarrow \alpha = \frac{1}{\|c\|_2}$$

$$\Rightarrow x^* = -\frac{c}{\|c\|_2}$$

closed-form solution

Key steps : establish a bound

show that it is achieved