Live Interaction #7:

18th November 2023

E-masters Communication Systems

Detection for Wireless

- **GLRT -Generalized Likelihood Ratio Test:**

$$\mathcal{H}_0: \bar{\mathbf{y}} = \bar{\mathbf{v}}$$

$$\mathcal{H}_1: \bar{\mathbf{y}} = A\bar{\mathbf{s}} + \bar{\mathbf{v}}$$

- Parameter A is unknown.
- $p(\bar{\mathbf{y}}; A, \mathcal{H}_1)$

$$p(\bar{\mathbf{y}}; \mathcal{H}_1) = \max_A p(\bar{\mathbf{y}}; A, \mathcal{H}_1)$$

- We employ the ML estimate of A corresponding to the $\mathcal{H}_1$.

$$\hat{A} = \frac{\bar{\mathbf{s}}^T \bar{\mathbf{y}}}{\|\bar{\mathbf{s}}\|^2}$$

- **Likelihood Ratio Test (LRT)**: Choose $\mathcal{H}_0$

$$\underbrace{p(\bar{\mathbf{y}}; \mathcal{H}_0) > p\left(\bar{\mathbf{y}}; \hat{A} = \frac{\bar{\mathbf{s}}^T \bar{\mathbf{y}}}{\|\bar{\mathbf{s}}\|^2}, \mathcal{H}_1\right)}_{\textbf{Generalized Likelihood Ratio Test}}$$

- Choose $\mathcal{H}_1$ if

$$|\bar{\mathbf{s}}^T \bar{\mathbf{y}}| > \gamma$$

$$\Rightarrow \bar{\mathbf{s}}^T \bar{\mathbf{y}} > \gamma \text{ or } \bar{\mathbf{s}}^T \bar{\mathbf{y}} < -\gamma$$

- What is the P_{FA}?

$$P_{FA} = 2Q\left(\frac{\gamma}{\sigma\|\bar{\mathbf{s}}\|}\right)$$

- What is the P_D?

$$P_D = Q\left(\frac{\gamma - A\|\bar{\mathbf{s}}\|^2}{\sigma\|\bar{\mathbf{s}}\|}\right) + Q\left(\frac{\gamma + A\|\bar{\mathbf{s}}\|^2}{\sigma\|\bar{\mathbf{s}}\|}\right)$$

- **ROC – Receiver Operating Characteristic**:

$$P_{FA} = 2Q\left(\frac{\gamma}{\sigma\|\bar{\mathbf{s}}\|}\right)$$

$$\Rightarrow \gamma = \sigma\|\bar{\mathbf{s}}\|Q^{-1}\left(\frac{P_{FA}}{2}\right)$$

$$P_D = Q\left(\frac{\sigma\|\bar{\mathbf{s}}\|Q^{-1}\left(\frac{P_{FA}}{2}\right) - A\|\bar{\mathbf{s}}\|^2}{\sigma\|\bar{\mathbf{s}}\|}\right) + Q\left(\frac{\sigma\|\bar{\mathbf{s}}\|Q^{-1}\left(\frac{P_{FA}}{2}\right) + A\|\bar{\mathbf{s}}\|^2}{\sigma\|\bar{\mathbf{s}}\|}\right)$$

$$P_D = Q\left(Q^{-1}\left(\frac{P_{FA}}{2}\right) - \frac{A\|\bar{\mathbf{s}}\|}{\sigma}\right) + Q\left(Q^{-1}\left(\frac{P_{FA}}{2}\right) + \frac{A\|\bar{\mathbf{s}}\|}{\sigma}\right)$$

- **Chi-square approximation**:

$$\chi_N^2 = \underbrace{x_1^2 + x_2^2 + \cdots + x_N^2}_{\text{Sum of squares of } N \text{ zero-mean unit-variance Gaussians}} \rightarrow \mathcal{N}(N, 2N)$$

- P_D, P_{FA}

$$P_D = Q_{\chi_N^2}\left(\frac{\tilde{\gamma}}{\sigma^2 + \sigma_s^2}\right) = Q\left(\frac{\frac{\tilde{\gamma}}{\sigma^2 + \sigma_s^2} - N}{\sqrt{2N}}\right)$$

$$P_{FA} = Q_{\chi_N^2}\left(\frac{\tilde{\gamma}}{\sigma^2}\right) = Q\left(\frac{\frac{\tilde{\gamma}}{\sigma^2} - N}{\sqrt{2N}}\right)$$

- $Q(\cdot)$ is the CCDF of the standard Gaussian.

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt$$

- Derivation:
- Mean

$$E\{x_1^2 + x_2^2 + \cdots + x_N^2\}$$
$$= E\{x_1^2\} + E\{x_2^2\} + \cdots + E\{x_N^2\} = N$$

- Variance

$$E\{(x_1^2 + x_2^2 + \cdots + x_N^2)^2\}$$
$$= \sum_i E\{x_i^4\} + 2 \sum_i \sum_{j<i} E\{x_i^2 x_j^2\}$$
$$= \sum_i E\{x_i^4\} + 2 \sum_i \sum_{j<i} E\{x_i^2\} E\{x_j^2\}$$
$$= \sum_i E\{x_i^4\} + N(N-1)$$
$$= 3N + N(N-1) = N^2 + 2N$$
$$\text{Variance} = N^2 + 2N - N^2 = 2N$$

- **Assignment #7 deadline: 20th November 11:59 PM.**
- **Live interaction #8: 25th November 7:30-8:30 PM.**
- **Assignment #8 deadline: 25th November 11:59 PM**
- **Assignment #7, #8 Discussion: 26th November 4:30-5:00 PM.**
- **Quiz #4: 26th November 5:30-6:30 PM.**

Final Exam:

- **Total questions in Final: 40**

- **Multiple choice questions with four given options and only one correct option.**
- **NO negative marking**
- **Closed-book exam**
- **Duration is 3 hrs**
- **One mark per question**
- **Question Paper PATTERN**
- **8 questions: Recall type (Purely formula), one from each week**
- **16 questions: Seen, Directly from assignments, two from each week**
- **16 questions: Unseen, Roughly 2 from every week based on assignment questions**
- Weightage:

	Proposed Weightage
Assignments (Theory)	20%
Quizzes	30%
End-Sem	40%
Attendance Minimum 80% attendance	10%

- **Best 3 out of four quizzes**
- **Best 6 out of 8 assignments**