EE901
Probability and Random Processes

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1

What is Probability?

- Events occurring in our surroundings are uncertain.
- Common analysis method for uncertain situations:

?	Will our team score a goal in today's match?

  - Use "long-term averages", i.e., probabilities.

2

What is Probability?

- Common approach for making a decision under uncertainty
  - Optimizing according to the "average value" of the result of the decision.
  - perhaps subject to some risk constraints.
- Probability theory deals with the phenomena
  - where outcomes are not fully predictable,
  - but exhibit some regularity when observed many times.

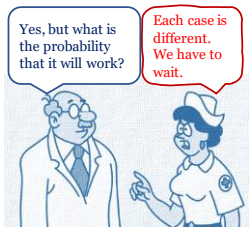
3

A Dialogue for Uncertainty and Probability

A patient is administered (potentially) life-saving drug.

4

A Dialogue for Uncertainty and Probability



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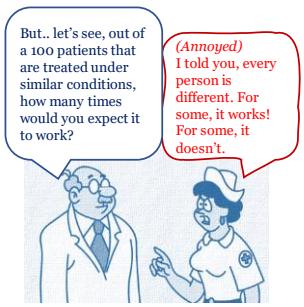
A Dialogue for Uncertainty and Probability



6

A Dialogue for Uncertainty and Probability

7

A Dialogue for Uncertainty and Probability

8

Probability Interpretations

Classical Interpretation

- For any random experiment, find the collections of atomic and equi-probable outcomes.
- Probability of any event is equal to the ratio of the number of outcomes in favor of this and the number of total outcomes.

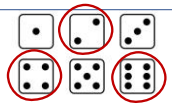
9

Probability Interpretations

Classical Interpretation

Roll a dice, what is the probability to get an even number?

- 6 Equally likely outcomes
- 3 outcomes are favorable
- Probability is 3/6



Limitations!

10

Probability Interpretations

Frequentism Interpretation

- If the experiment is repeated many times, how many times the favorable event occurs?

Toss a coin many times. Count the number of heads.

- Reality may or may not involve repetition!

11

Probability Interpretations

Subjective Interpretation

- An individual's opinion or belief

Who will win today?

What is the probability that the sun will rise tomorrow?

- How much you can bet!

12

History of Probability Theory

Games of chances

Cardano (16th century):
- First systematic treatment for his gambling interests

Pascal and Fermat
(Question of division of stakes)


13

History of Probability Theory

Christiaan Huygens (first book on probability)

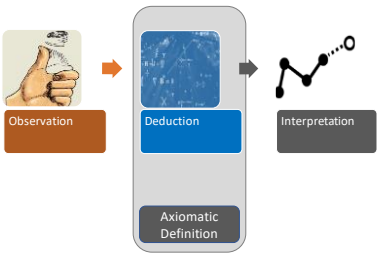
(18-19 Century)
- Jacob Bernoulli
- Laplace: Applied to many practical problems, classic interpretation
- Poisson, Gauss (18-l9th Century): Mathematical organization

Kolmogorov (20th Century): Axiomatization
Definition of probability that is precise enough for use in mathematics, yet comprehensive enough to be applicable to a wide range of phenomena

14

Application of Probability Theory

In a game, two players A and B toss a coin one by one. The player getting the head first, wins. What is the chance A wins?

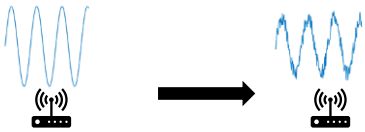


15

Applications in Engineering

- Outputs of real-world systems are generally random.

Example: Transmission of a message.



What is the probability that signal is distorted beyond recovery?

16

Applications

Engineering

- Communications, information theory
- Signal processing and control
- Queuing theory and modeling computer systems
- Decision/resource allocation under uncertainty
- Reliability

Statistics

- Collection and organization of data so that useful inferences can be drawn from them
- Games prediction

Physics

- Statistical mechanics
- Thermodynamics
- Quantum mechanics

Computer Science and ML

- Randomized algorithms, random search
- Bioinformatics
- Machine Learning
- Optimization under noise
- Prediction

Economics and Finance

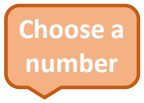
- Investment/insurance risk assessment
- Gambling

17

Models

18

Random Experiments



Finite number of possibilities	Countable number of possibilities	Uncountable number of possibilities

19

Sample Space

- Set of all possible outcomes

$$\Omega = \{H, T\}$$

20

Random Variables

- Assign a number to each outcome
- Why?
  - For tractability and mathematical notation.
  - Order to outcomes.
  - Quantify distributions.
- Can be a physical interpretation, intuitive or any arbitrary assignment.

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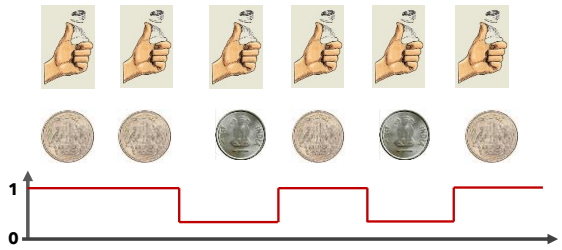
21

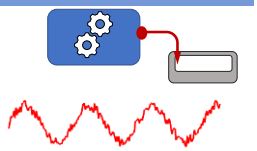
Repeated Trials



22

Random Processes

- Outcomes in form of waveforms/signals



- Continuous process
- Can be used to model noise, random behavior of systems, packet arrival process, unknown parameters process, perturbed process, time series models, spatial process (to model BS locations), and speech signal.

23

Course Content

Module Number	Name
1	Introduction to Probability and Probability Space,
2	Random Variables, Continuous and Discrete Random Variables,
3	CDF and PDF/PMF,
4	Expectation, Variance, MGF
5	Functions of Random Variables and Multiple RVs
6	Random Variable Transformation
7	Sampling of random variable and empirical statistics using computer simulations
8	Conditional Expectation Distribution
9	Law of Large Number, Central Limit Theorem
10	Introduction to Random Processes and Examples
11	Distribution of Random Processes
12	Random Processes via Linear Systems

24

References

Class notes

Books

- A. Papoulis and S. Pillai, ``Probability, Random Variables, and Stochastic Processes," McGraw-Hill, 4th Edition.
- Bruce Hajek,``An Exploration of Random Processes for Engineers." Available Online at http://www.ifp.illinois.edu/~hajek/Papers/randomprocJuly14.pdf

25