eMasters in Communication Systems, IITK
EE901: Probability and Random Processes

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Assignment – 1 - Question Set -1 - Solution

Q1. A fair coin is tossed thrice. If we are interested in all outcomes. Find the sample space. If we are only interested in total number of heads, find the sample space.
Solution:
Let's say H – Heads, T – Tails
Since each coin toss has two possible outcomes (H or)
➔ All outcomes for 3 tosses =$2^3 = 8$

All outcomes sample space:
All heads – HHH
Two heads and one tail – HHT, HTH, THH
One head and two tails – HTT, THT, TTH
All tails - TTT
∴ **All outcomes sample space** $\boldsymbol{\Omega} = \{\boldsymbol{HHH}, \boldsymbol{HHT}, \boldsymbol{HTH}, \boldsymbol{THH}, \boldsymbol{HTT}, \boldsymbol{THT}, \boldsymbol{TTH}, \boldsymbol{TTT}\ \}$
Total number of heads sample space
From the total sample space, we can see that there can be :
$\Omega = \{0, 1, 2, 3\}$ where the number represents the total number of heads

Q2: Consider a dice roll experiment. Let A denote the event that an odd number occurs. Let B denote the event that an even number occurs. Let C denote the event that a number less than 3 occurs. Now compute $\boldsymbol{E_1 = A \cap C}$ and $\boldsymbol{E_2 = B \cap C^c}$. Compute $\boldsymbol{E_1 \cup E_2}$.
Solution:
A die roll can result in one of the possible outcome numbers = $\Omega = \{1,2,3,4,5,6\}$
Odd number set = $A = S_{Odd} = \{1,3,5\}$
Even number set = B = $S_{Even} = \{2,4,6\}$
Less than 3 set = C = $S_{LT3} = \{1,2\}$
$\boldsymbol{E_1 = A \cap C = \{1\}}$

$C^c = \Omega - C = \{3,4,5,6\}$
$\boldsymbol{E_2 = B \cap C^c = \{4, 6\}}$
∴ $\boldsymbol{E_1 \cup E_2 = \{1, 4, 6\}}$

Q3: Let = (0; 1), which among the below are σ −algebra?
$\mathcal{F}_1 = \{\phi, \Omega, (0,0.2), (0.2,1)\}$
$\mathcal{F}_2 = \{\phi, \Omega, \left(0, \frac{2}{3}\right), \left[\frac{2}{3}, 1\right), \left(0, \frac{1}{2}\right), [\frac{1}{2}, 1)\}$
$\mathcal{F}_3 = \{\phi, \Omega, (0, 5/4\ ), \left[\frac{5}{4}, 1\right)\}$
Solution:
$\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3$

Q4: Let $\boldsymbol{\Omega} = \{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}\}$. Obtain a $\boldsymbol{\sigma}$-algebra which contains at least $\{\{\boldsymbol{A}\}, \{\boldsymbol{B}\}\}$

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Solution:
Let's construct the sigma algebra containing at least set A and set B

- Empty and entire set Ω - $\{\phi, \Omega\}$
- The asked sets themselves - $\{\phi, \Omega, \{A\}, \{B\}\}$
- The complements of sets A and B - $\{\phi, \Omega, \{A\}, \{B\}, A^c, B^c\}$
- The union of sets of A and B - $\{\phi, \Omega, \{A\}, \{B\}, \{C\}, \{A, B\}\}$

Q5: A computer picks a positive integer randomly. The probability that a number i appears is given as c/i^2. Compute the value of c. Let A denote the event that an even number occurs. Let B denote the event that a number less than 10 occurs.

What is the probability of event A and $A \cup B$? What is probability of A^c?
Solution:
Sum of all probabilities is 1

$$\therefore\ c * \sum \frac{1}{i^2} = 1$$

The series of sum of reciprocal square integers is basel problem which is $\pi^2/6$,

$$\therefore c * \frac{\pi^2}{6} = 1 => c = \frac{6}{\pi^2}$$

$$P(A) = c * \left[\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \frac{1}{8^2} + \frac{1}{10^2} + \frac{1}{12^2} + \cdots\right]$$

- ⇨ $P(A) = c * \frac{1}{4} * [\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\ ]$
- ⇨ $P(A) = c * \frac{1}{2} * \frac{\pi^2}{6}$

Similarly,

$$P(B) = c * [\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{9^2}]$$

As per Riemann zeta function evaluated at 2,

$$P(B) = c * \frac{\pi^2}{6}$$

By probability axioms, $P(A) + P(B) = 1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A \cap B)$ is the probability the number is even and less than 10 is same as $P(A)$
$P(A\ \cap B) = P(A)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A) = P(B) = c * \frac{\pi^2}{6} = 1$$

$$P(A^c) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$

Q6: A computer picks a real number between 0 and 1 randomly. The probability that a number in interval (a, b) appears is given as $c(b^2 - a^2)$. Compute the value of c.
Solution:
The sum of probabilities between 0 and 1 is 1

$$\therefore \int_0^1 c(b^2 - a^2)dbda = c[\int_0^1 b^2db - \int_0^1 a^2da] = c[2b - 2a] = 1\ \ => c = \frac{1}{2(b-a)}$$

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Q7 Solution

Say f is fair coin, b is biased coin

$P_f(H) = \frac{1}{4}$

$P_b(H) = \frac{1}{4}$

$P_f(T) = \frac{3}{8}$

$P_b(T) = \frac{1}{8}$

Given head appears, what is the probability that the coin is biased=> $\frac{1}{2}$

Given biased coin is picked, what is the probability that head appears=> $\frac{2}{3}$

Q8 Solution

Are A and B independent? No. A influences B so when A occurs and it is an even number, B is automatically occurred

Q9 Solution

Probability of red marble

$$\frac{1}{3} * \left(\frac{75}{100} + \frac{60}{100} + \frac{45}{100}\right) = \frac{1}{3}\left(\frac{3}{4} + \frac{3}{5} + \frac{9}{20}\right) = \frac{3}{5}$$

Q10 Solution

If $A \cap B = \phi$ and $P(B) \neq 0$, then show that P(A|B) = 0

Given that A ∩ B = ∅ (the intersection of events A and B is an empty set) and P(B) ≠ 0 (the probability of event B occurring is not equal to 0), we want to show that P(A | B) = 0.

The conditional probability P(A | B) is defined as:

P(A | B) = P(A ∩ B) / P(B)

Since A ∩ B = ∅ (empty set), its probability is 0:

P(A ∩ B) = 0

Now, since P(B) ≠ 0, we can divide by P(B):

P(A | B) = 0 / P(B)

Since any number divided by a non-zero number is 0:

P(A | B) = 0

Therefore, we have shown that P(A | B) = 0, as required.

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