1. Solve these problems and submit by 5th May (Saturday) before the discussion session.
2. There is no penalty for submitting incorrect attempts
3. However, plagiarism will result in serious penalties, such as an F grade.

2 1. Consider the following linear program

$$\begin{aligned} &\min\ \mathbf{c}^T\mathbf{x} \\ &\text{s. t. } \mathbf{A}\mathbf{x} \leq \mathbf{b} \end{aligned}$$

where $\mathbf{A}$ is square and full rank.

(a) When is the problem infeasible?

(b) When is the problem unbounded below?

(c) When does the problem have a finite solution, and what is it?

2 2. Show that any linear programming problem can be expressed as

$$\begin{aligned} &\min\ \mathbf{c}^T\mathbf{x} \\ &\text{s. t. } \mathbf{A}\mathbf{x} = \mathbf{b} \\ &\quad x_i \geq 0 \qquad i = 1, \ldots, n \end{aligned}$$

2 3. Consider the following linear program

$$\begin{aligned} &\min\ \mathbf{c}^T\mathbf{x} \\ &\text{s. t. } \mathbf{A}\mathbf{x} = \mathbf{b} \end{aligned}$$

(a) When is the problem infeasible?

(b) When is the problem unbounded below?

(c) When does the problem have a finite solution, and what is it?

2 4. Consider the following linear program

$$\begin{aligned} &\min\ \mathbf{c}^T\mathbf{x} \\ &\text{s. t. } \mathbf{a}^T\mathbf{x} \leq b \end{aligned}$$

where $\mathbf{a} \neq 0$.

(a) When is the problem infeasible?

(b) When is the problem unbounded below?

(c) When does the problem have a finite solution, and what is it?

2 5. Solve the following optimization problem for $\mathbf{A} \succ 0$,

$$\begin{aligned} &\min\ \mathbf{x}^T\mathbf{A}\mathbf{x} \\ &\text{s. t. } \|\mathbf{x}\|_2^2 = 1 \end{aligned}$$

Hint: Given that the eigenvalue decomposition $\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^T$, use the change of variable $\mathbf{y} = \mathbf{U}^T\mathbf{x}$.