A Hybrid Optimization Algorithm for Solving Constrained Engineering Design Problems

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***Abstract*—This paper introduces a new hybrid optimization procedure named GA-PSO-SQP for resolution constrained engineering design problems. Certifiable issues in the engineering field are typically expansive scale or nonlinear or constrained optimization problems, very well-designed numerical procedure is this way needed. This coupling is based on the genetic algorithm, the sequential quadratic programming and on the particle swarm optimization united with a projected gradient algorithm to deal constrained optimization problems. Numerical results based on well-known held back engineering design problems are reported and compared. The solutions acquired by the proposed technique are generally improved than those results given by other known methodologies in the open literature.**

***Index Terms*—Hybrid optimization, Border Correction Procedure, Genetic Algorithm, Sequential Quadratic Programming, Particle Swarm Optimization, Constraint Optimization.**

I. Introduction

Constrained optimization constitutes a major part of many problems in engineering and industry. Most of real world design optimization problems can be defined as the process of finding the optimal parameters, which provide the maximum or minimum value of an objective function, according to a set of specific requirements called constraints. Such an optimization problem is well-known as constrained optimization problems or non-linear programming problemsIn late decades a large number of meta heuristic algorithms been created and applied in order to solve constrained optimization problems. Mostly of these algorithms are inspired form natural phenomena and based on a combination of various rules and randomness. The following algorithms are the mostly used ones: Genetic Algorithm (GA) [1], Particle Swarm Optimization [2], differential evolution with dynamic stochastic selection (DEDS) [3], modified differential evolution (MDE) [4], mine blast algorithm (MBA) [5], league championship algorithm (LCA) [6] etc.

As an alternative to the conventional mathematical approaches, heuristic techniques are very reasonable and amazing for getting the solution of optimization issues. In spite of the fact that, they are approximate methods they do not have much mathematical requirements about the optimization problem. Also, these strategies reduce the need for persistent capacity in terms of costs and factors used for mathematical optimization methods. As exhibited in the writing optimization techniques referenced above have been effectively connected to constrained optimization problems. However, performance while achieving an optimal or near optimal solution shows a significant difference. Therefore, previously developed algorithms, such as heuristic algorithms have weaknesses that promote uncertainty, premature convergence of the solution, do not use prior knowledge, do not exploit local search information, difficulty managing large scale optimization issues, violate constraints and many given solutions remote from the domain of constraints.

To overcome these difficulties, a new optimization methodology is introduced herein based on the coupling of various optimization algorithms, Namely, the GA, the sequential quadratic programming (SQP) [7] and PSO joined with a projected gradient technique [8] so as to address the arrangements out of the domain and send them to the search space's border. As of late, PSO has developed as a promising algorithm in taking care of different optimization issues in a wide assortment of uses. On the other hand, the SQP strategy that is an iterative technique for nonlinear optimization, has ended up being considerably more proficient than those dependent on the gradient.

Based on the advantages of these algorithms, this work aims to develop an efficient hybrid technique named GA-PSO-SQP combined with a projected gradient algorithm to locate the ideal solution of the constraint nonlinear optimization problems. This paper is sorted out as pursue: Section II introduce the general formulation of the optimization problem and correction procedures. In section III, the hybrid methodology is described. The structural design problem is discussed and compared with existing methods in Section IV. The main conclusions are given in section V.

II. Non-linear constrained optimization problem

Generally, an engineering design optimization issue can be planned as a nonlinear programming (NLP) problem. Contrary to generic NLP problems that only include continuous or

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integer variables, engineering design optimizations usually involve continuous, binary, discrete and integer variables. In formulating the engineering design problem to choose elective alternatives the twofold factors are normally included. The discrete variables are used to represent standardization constraints such as the diameters of standard sized bolts. Integer variables habitually come when the numbers of objects are design variables, such as the number of gear teeth. In view of on blended factors an engineering optimization problem is defined as

$$\begin{array}{ll} \min\limits_{x} & F(x) \\ \text{subject to} & h_k(x) = 0 \ ; \quad k = 1, 2, ..., p \\ & g_j(x) \le 0 \ ; \quad j = 1, 2, ..., p \\ & l_i \le x_i \le u_i \ ; \quad i = 1, 2, ..., n \end{array} \tag{1}$$

where $x = [x_1, x_2, ..., x_n]^T$ denotes the decision allowable values for variable solution vector; F is the objective function; l_i and u_i are the minimum and maximum allowable values for ith variable respectively; p is the number of equality constraints and q is the number of inequality constraints. Let $S = \{x \in \mathbb{R} / g_i(x) \le 0; j = 1, 2, , ..., p + q (= M)\}$ be the set of the feasible solutions, so problem 1 can be reformulated as

$$\begin{array}{ll} \min\limits_{x} & F(x) \\ \text{subject to} & g_j(x) \le 0 \ ; \quad j = 1, 2, ..., M \\ & l_i \le x_i \le u_i \ ; \quad i = 1, 2, ..., n \end{array}$$

The main task While paying attention to solving the constraint optimization problem is to deal with the constraint. In the constrained optimization problem, it is difficult to locate the attainable solution of the issue. To deal with these constraints, many methodological approach has been proposed. On the other hand, these proposed procedures have some shortcoming, for example, the lower strength premature convergence of the solution, not utilizing an earlier knowledge, not exploiting local search information that may not necessarily be the expected optimum and many given solutions remote from the domain of constraints. To overcome this limitation, we elaborate a novel hybrid algorithm for solving the constrained optimization problems.

III. Hyrbid GA-PSO-SQP approach

Most optimization methods are local methods which can not converge towards a global optimum in the case where the objective function is non-convex. So, they have the disadvantage of buckling, and for constantly returning to the same point. These algorithms (from the gradient family or Newton-Raphson) are based on the gradient evaluation to determine the direction of search for the optimum. To stay away from these burdens a few authors have considered utilizing stochastic methods such as GA and PSO that don't require any smooth theory of the objective function or on constraints.

The GA and PSO demonstrate their heartiness particularly, when the objective function isn't differentiable and when the search intervals are excessively small. But, these methods have some disadvantages if the search interval is large, it may not converge and give solutions remote from the overall optimum. Also, these methods have additionally the disservices of premature convergence, which amounts to a nearby hunt to a local search around a minimum that is not necessarily the expected optimum. To avoid this drawback, we elaborated herein a coupling methodology based on GA, the PSO and on the SQP.

The projection subroutine of the projected gradient algorithm that comprises projecting an infeasible point on the limit of the constraints [9], [10] will be used. It ought to be noticed that the main advantage of the presented projection method is to correct the solutions out of search domain and send them to the search space border. This correction effect will be mainly used in the following coupling procedure.

A. *Correction to the border*

Once new point x_{t_k} is determined, we will see, if $x_{t_k}$is an infeasible point, the correction is performed by using an iterative method: We generate a sequence of points $\{x^{(j)}\}_{j \ge 0}$, such that $x^{(0)} = x_{t_k}$ and

$$x^{(j+1)} = x^{(j)} - \varphi_k(\varphi), \quad \varphi_k(\varphi) = A_k^T V_k g_k(x^{(j)}) \tag{2}$$

where

$$A_k = A_a, \ V_k = V_a, \ g_k(x^{(j)}) = g_a(x^{(j)}), \quad if \quad \|P_k d\| \ge \beta_k;$$

$$A_k = A_{a-1}, \ V_k = V_{a-1}, \ g_k(x^{(j)}) = g_{a-1}(x^{(j)}), \ \ if \ \ \|P_k d\| < \beta_k;$$

where

$$\frac{V_a A_a d}{2\sqrt{V_{aii}(x)}} = \beta_k$$

where A_a, d, g_a the Jacobian matrix, the gradient vector of the objective function and constraints active respectively, and $V_a = (A_a A_a^T)^{-1}$ and V_{aii} are the elements of the diagonal of V_a. The projection matrix $P_a(x)$ defined by:

$$P_a(x) = Id - A_a(x)^T V_a(x) A_a(x) \tag{3}$$

Let $x_k \in S$ and $\delta > 0$ verify

$$\|g_a(k_k)\| \le \delta \le \frac{b_k^2}{48\lambda}; \tag{4}$$

where b_k is defined by

$$b_k = \frac{1}{2} \max\{\|P_a d\|, \beta_k\}$$

then [39]

$$b_k \le 1, \quad \forall x_k \in \partial S$$

and j satisfies

$$j \ge 1.443 log(24\lambda\delta)^{-1} \tag{5}$$

The convergence to a limit point is ensured by the accompanying theorems see [11].

Theorem 2.1.1 Let Equations 4 and 5 be satisfied. Assume that $2b_k = \|P_k d\| \ge \beta_k$.

then

$$\|g_a(x^{(j)})\| \le \delta. \tag{6}$$

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Moreover,

$$F(x_k) - F(x^{(j)}) \geq \frac{3}{2}\eta_k b_k \geq 12\alpha\delta. \quad (7)$$

Theorem 2.1.2 Let Equations 4 and 5 be satisfied. Assume that $2b_k = \beta_k \geq \|P_k d\|$.

then

$$\|g_{a-1}(x^{(j)})\| \leq \delta. \quad (8)$$

and

$$F(x_k) - F(x^{(j)}) \geq \frac{3}{2}\eta_k b_k \geq 12\alpha\delta. \quad (9)$$

If, $\beta_k \leq (4\alpha)^{-1}$, then

$$g_l(x^{(j)}) \leq -\frac{\eta_k b_k}{2} \leq -4\alpha\delta. \quad (10)$$

This correction effect will be mainly used in the following coupling procedure based on some well known optimization methods.

B. Coupling Algorithm

In the accompanying, we consider a coupling between the PSO, SQP and the GA method. It is notable that the main step of the GA method is crossover mutation and selection. Classically the Crossover is using arithmetic crossover, the two parents creates randomly give two children, Mutation With a mutation probability, some genes of the children are changed randomly. The idea of this coupling procedure is to apply the PSO method in the step crossing to an exactness without moving toward the optimum then utilize the SOP in the step. The benefit of our coupling is to manufacture an effective algorithm to avoid local solutions and to find the global solutions. Our coupling is adaptive for any optimization problem. It gives more accurate results satisfying the constraints, using the correction method applied whenever the solutions are out of the domain and to bring them to the domain's border.

Initially, we generate a population $S_0 = \{x_1^0, ..., x_N^0\}$ of points achievable as follows:

- Generating a random population of points $\{x_1^0, ..., x_N^0\}$ in the search interval.
- Each point is brought back to the border, to be a feasible point, using the correction method

The iterations generate S_1, S_2, $...S_N$ as follows: at the iteration k, $S_k = \{x_1^k, ..., x_N^k\}$ is given Generate $S_{k+1} = \{x_1^{k+1}, ..., x_N^{k+1}\}$ using three operators : **R** (Crossover), **M** (mutation), **S**(selection).

The application of these operators is made as follows:

- Apply the crossover operator to the population S_{k+1} and generate r elements which are the results R_k of the method PSO. The correction method is applied to each point of our results R_k if that are out of the domain and bring them to the domain's border.
- Apply the mutation operator to the population R_k obtained in the crossover with PSO method : Generate t elements using the method SQP applied to all the elements of R_k. Then these elements are corrected using the iterative method, to have a feasible set M_k.
- Apply the selection operator to the population: we choose the best N elements of the set $X_k = S_k \cup R_k \cup M_k$, which will give a new realizable set.
$$S_{k+1} = \{x_1^{k+1}, ..., x_N^{k+1}\}$$

The iterations are stopped if

$$\| S_{k+1} - S_k \| \leq \epsilon \quad or \quad \epsilon \ll 1$$

IV. Numerical Examples

In order to validate and to demonstrate the performance of the proposed methodological approach many examples have been elaborated from the optimization problem given in the literature. The presented algorithm is executed in MATLAB and the program has been kept running a PC, 2GHz Intel Core (TM) 3 Duo processor with 4GB of Random Access Memory (RAM). In each example, 30 independent runs are made which involve 30 different initial trial solutions with a randomly generated population of size $50 \times D$ for the optimization, where D is the dimension of the problem. In the PSO settings the cognitive and social components c_1 and c_2 are constants that can be utilized to change the weighting among personal and population experience, respectively. In our numerical results, cognitive and the social components are both taken to 1.4962. Inertia weight (w) is represented by the expression $w = 2(\varphi - 2 + \sqrt{\varphi^2 - 4\varphi})$. Here, φ is set to 4.1.

A. Constrained optimization problem

1) Himmelblau's non-linear optimization problem: Before solving the structural engineering problems, the presented coupling algorithm was benchmarked using a Himmelblau's problem. This issue has initially been proposed by himmelblau [12] and it has been broadly utilized as a benchmark nonlinear constrained optimization problem. In this problem there are five positive design factors $X = [x1, x2, x3, x4, x5]$, six nonlinear imbalance constraints and ten boundary conditions. The issue can be mathematically defined as:

$$\begin{aligned}
\min_x F(X) \quad &= 5.3578547x_3^2 + 0.8356891x_1x_5 + 37.293239x_1 - 40792.141 \\
\text{s.t.} \quad & 0 \leq g_1(X) \leq 92 \\
& 90 \leq g_2(X) \leq 110 \\
& 20 \leq g_3(X) \leq 25
\end{aligned}$$

where

$$g_1(X) = 85.334407 + 0.0056858x_2x_5 + \mathbf{0.0006262}x_1x_4 - 0.0022053x_3x_5$$
$$g_2(X) = 80.51249 + 0.0071317x_2x_5 + 0.0029955x_1x_2 - 0.0021813x_3^2$$
$$g_3(X) = 9.300961 + 0.0047026x_3x_5 + 0.0012547x_1x_3 + 0.0019085x_3x_4$$
$$78 \leq x_1 \leq 102; \quad 33 \leq x_2 \leq 45; \quad 27 \leq x_3, x_4, x_5 \leq 45$$

This problem has been used before as a benchmark by practicing many other methods such as GA [13], [14]. harmony search algorithm [15], PSO-GA [16] and PSO [17], Cuckoo search [18], simplex search [19] etc. Also, a portion of the researchers [20]–[23] have tried their algorithm in another variety of this issue (named as version II), where a parameter 0.0006262 (type-set bold in the constraint g_1) has been taken as 0.00026.

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The proposed coupling methodological approach has been tried on both versions and compare the obtained best solution of the problem with previous best solution in literature (see Table I)

In light of the above simulation results and comparisons it very well may be accomplished that our proposed algorithm has a high performance and efficiency in nature and has better searching quality.

B. Structural optimization problems

In this subsection two surely constrained engineering design problems have been exhibited to demonstrate the proficiency and robustness of the proposed algorithm.

1) Design of pressure vessel: Let us consider a compressed air storage tank with working pressure of 2000 psi and maximum volume of 750 ft^3 [24], as shown in Figure 1. The pressure vessel issue is to minimize the all cost of material, forming and welding of a cylindrical vessel. There are four design factors related with it to be specific the thickness of the pressure vessel $T_s = x_1$, thickness of the head $T_h = x_2$, inner radius of the vessel $R = x_3$, and length of the vessel without heads $L = x_4$. The issue can be mathematically defined as:

$$\begin{aligned}
\min_X \quad & F(X) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3 \\
\text{s.t.} \quad & g_1(X) = -x_1 + 0.0193x_3 \leq 0 \\
& g_2(X) = -x_2 + 0.0095x_3 \leq 0 \\
& g_3(X) = -\pi x_3^2x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \leq 0 \\
& g_4(X) = x_4 - 240 \leq 0 \\
& 1 \times 0.0625 \leq x_1, x_2 \leq 99 \times 0.0625, \quad 10 \leq x_3, x_4 \leq 200
\end{aligned}$$

For example view authors have solved this problem utilizing diverse algorithms such as augmented lagrangian multiplier approach [24], hybrid PSO with a feasibility based guideline for constrained optimization [25], co-evolutionary PSO [26] and hybrid PSO-GA [16]. The proposed methodological approach has been applied to this optimization problem the got outcomes are

$$X = (0.880944, 0.433702, 45.634248, 137.249985)$$

with corresponding function value equals to

$$F(X) = 5798.79899568$$

and constraints

$$[g_1, g_2, g_3, g_4] = [-0.0002, -0.00017, -4.787, -102.750]$$

Comparison of our outcomes to those acquired by various authors are given in Table II. It is plainly expressed that the proposed algorithm leads to better outcomes.

2) Speed reducer design optimization problem: The speed reducer optimization problem shown in Figure 2 is considered [34]. The aim is to minimize the weights of the speed reducer subject to constraints on bending stress of the gear teeth, surface stress, transverse deflections and stresses in the shafts. Structure parameters of the speed reducer issue, the face width (b), module of teeth (m), number of teeth in the pinion (z), length of the first shaft between bearings (l_1), length of the second shaft (l_2) and the diameter of the first shaft (d_1) and second shaft (d_2) correspond to $x_1, x_2, ..., x_7$, respectively. The corresponding optimization problem can be formulated as [34]:

$$\begin{aligned}
\min_X \quad & F(X) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) \\
& \quad - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2) \\
\text{s.t.} \quad & g_1(X) = \frac{27}{x_1x_2^2x_3} - 1 \leq 0 \\
& g_2(X) = \frac{397.5}{x_1x_2^2x_3^2} - 1 \leq 0 \\
& g_3(X) = \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \leq 0 \\
& g_4(X) = \frac{1.93x_5^3}{x_2x_3x_7^4} - 1 \leq 0 \\
& g_5(X) = \frac{\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 16.9 \times 10^6}}{110.0x_6^3} - 1 \leq 0 \\
& g_6(X) = \frac{\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 157.5 \times 10^6}}{85.0x_7^3} - 1 \leq 0 \\
& g_7(X) = \frac{x_2x_3}{40} - 1 \leq 0 \\
& g_8(X) = \frac{5x_2}{x_1} - 1 \leq 0 \\
& g_9(X) = \frac{x_1}{12x_2} - 1 \leq 0 \\
& g_{10}(X) = \frac{1.5x_6 + 1.9}{x_4} - 1 \leq 0 \\
& g_{11}(X) = \frac{1.1x_7 + 1.9}{x_5} - 1 \leq 0 \\
& 2.6 \leq x_1 \leq 3.6, \quad 0.7 \leq x_2 \leq 0.8, \quad 17 \leq x_3 \leq 28, \quad 7.3 \leq x_4 \leq 8.3 \\
& 7.8 \leq x_5 \leq 8.3, \quad 2.9 \leq x_6 \leq 3.9, \quad 5 \leq x_3 \leq 5.5
\end{aligned}$$

The speed reducer issue in this situation was optimized

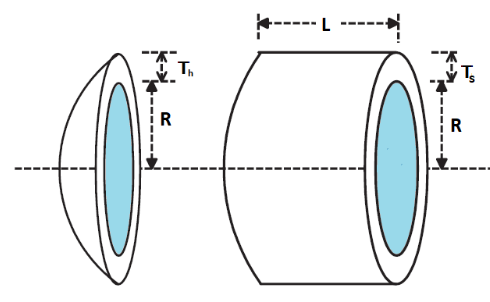

Fig. 1. Pressure vessel design problem

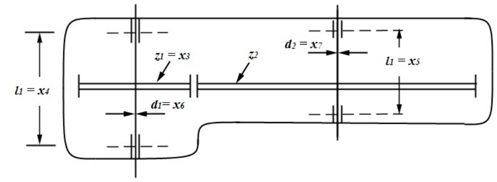

Fig. 2. Speed reducer design problem

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TABLE I

OPTIMAL RESULTS FOR HIMMELBLAU'S NONLINEAR OPTIMIZATION PROBLEM (NA MEANS NOT AVAILABLE)

Version	Method	Design variables					Cost	Constraints		
		x_1	x_2	x_3	x_4	x_5	$F(X)$	$0 \leq g_1(X) \leq 92$	$90 \leq g_2(X) \leq 110$	$20 \leq g_3(X) \leq 25$
I	Himmelblau [12]	NA	NA	NA	NA	NA	-30373.9490	NA	NA	NA
	Homaifar et al. [14]	80.39	35.07	32.05	40.33	33.34	-30005.700	91.65619	99.53690	20.02553
	Deb [13]	NA	NA	NA	NA	NA	-30665.539	NA	NA	NA
	Lee and Geem [15]	78.00	33.00	29.995	45.00	36.776	-30665.500	NA	98.84051	NA
	He et al. [17]	78.00	33.00	29.995256	45.00	36.7758129	-30665.539	NA	100.40478	20.00000
	Gandomi et al. [18]	78.00	33.00	29.99616	45.00	36.77605	-30665.233	91.99996	98.84067	20.0003
	Mehta and Dasgupta [19]	78.00	33.00	29.995256	45.00	36.775813	-30665.538741	NA	NA	NA
	Garg [16]	78.00	33.00	29.995174	45.00	36.7757340	-30665.56614	91.99999	94.915394	NA
	Present study	78	33	29.993037	45	36.774332	-30665.767138	92	94.9153437	20
II	Shi and Eberhart [23]	78.00	33.00	27.07099	45.00	44.969	-31025.561	NA	100.40473	NA
	Coello [20]	78.0495	33.007	27.081	45.00	44.94	-31020.859	NA	100.40786	20.00191
	Hu et al. [22]	78.00	33.00	27.070997	45.00	44.9692425	-31025.5614	92	100.404784	20
	Fesanghary et al. [21]	78.00	33.00	27.085149	45.00	44.925329	-31024.3166	NA	100.39612	20.00000
	Garg [16]	78.00	33.00	27.07095	45.00	44.9691668	-31025.57471	91.999994	97.207689	NA
	Present study	78	33	27.068972	45	44.967673	-31025.668919	92	97.207571	20.000000

TABLE II

COMPARISON OF THE BEST SOLUTION FOR PRESSURE VESSEL DESIGN PROBLEM FOUND BY DIFFERENT METHODS (NA MEANS NOT AVAILABLE)

Method	Design variables				Cost	Constraints			
	x_1	x_2	x_3	x_4	$F(X)$	g_1	g_2	g_3	g_4
Sandgren [27]	1.125000	0.625000	47.700000	117.701000	8129.1036	-0.2043900	-0.1718500	NA	-122.2990000
Deb and Gene [28]	1.125000	0.500000	48.329000	112.679000	6410.3811	-0.1922503	-0.0408745	-3652.87832	-127.3210000
Coello [20]	0.812500	0.437500	40.323900	200.000000	6288.7445	-0.0342487	-0.0544229	-304.4020515	-40.0000000
Coello and Montes [29]	0.812500	0.437500	42.097398	176.654050	6059.9460	-0.0000202	-0.03757471	-24.8997585	-63.3459499
Montes and Coello [30]	0.812500	0.437500	42.098087	176.640518	6059.7456	-0.0000069	-0.0375681	NA	-63.35948200
Kaveh and Talatahari [25]	0.812500	0.437500	42.103566	176.573220	6059.0925	NA	-0.0375161	-0.3553866	-63.4267800
Kaveh and Talatahari [31]	0.812500	0.437500	42.098353	176.637751	6059.7258	-0.0000017	-0.037565	-0.0431147	-63.3622489
Cagnina et al. [32]	0.812500	0.437500	42.098445	176.636595	6059.7143	-0.000000011	-0.0375647	NA	-63.3634050
Coello [26]	0.812500	0.437500	42.098400	176.637200	6059.7208	-0.0000008	-0.037565	-0.2179852	-63.3627999
Garg [33]	0.778197	0.384665	40.321054	199.980236	5885.4032	-0.0000013	-0.0016155	-1.1378705	-40.0197632
Garg [16]	0.778168	0.384649	40.319618	200.000000	5885.3327	0	0	-4.656×10^{-10}	-40.0000000
Present study	0.880944	0.433702	45.634248	137.249985	5798.7989	-0.000203	-0.000176	-4.787782	-102.750015

utilizing differential evolution with dynamic stochastic selection (DEDS) [3], modified differential evolution (MDE) [4], PSO with differential evolution (PSO-DE) [35], water cycle algorithm (WCA) [36], mine blast algorithm (MBA) [5], league championship algorithm (LCA) [6], accelerated particle swarm optimization algorithm (APSO) [37] and improved accelerated particle swarm optimization algorithm (IAPSO) [38]. The acquired outcomes, based on the previous algorithms, are reported in Table III and compared with results obtained by the proposed mythological approach. Our proposed algorithm converges again to a new solution, better than all known solutions so far in literature. In light of the above reproduction results and examinations, it tends to be reasoned that the proposed algorithm is of superior searching quality and robustness and strength and can be utilized for constrained engineering design issues.

V. CONCLUSION

In this paper introduces, an effective optimization approach is developed based on a coupling procedure of GA, SQP and PSO joined with projected gradient algorithm. This methodological approach comprises projecting an infeasible point to the search space boundary of the constraints to solve the constrained optimization problems. To evaluate the performance and robustness of the proposed algorithm we compared it to other optimization methods. Engineering problems, which include pressure vessel design and speed reducer design are elaborated. In these optimization problems, the objective is to minimize the cost of the design subservient to many nonlinear constraints. The comparison results with others heuristic methods demonstrate that the proposed GA-PSO-SQP algorithm demonstrates to be extremely effective and robust for finding the global solution or finds a near-global solution in each problem.

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REFERENCES

[1] Z. Michalewicz, D. Dasgupta, R. LeRiche and M. Schoenauer, "Evolutionary algorithms for constrained engineering problems," Computers Engineering, vol. 3, pp. 851–870, 1996.

[2] J. Kennedy and R. Eberhart, " Particle Swarm Optimization," Proceedings of IEEE International Conference on Neural Networks, pp. 1942–1948, 1995.

[3] M. Zhang, W. Luo and X. Wang, "Differential evolution with dynamic stochastic selection for constrained optimization,"Information Sciences, vol. 178, pp. 3043–3074, 2008.

[4] E. Mezura Montes and J. Velazquez Reyes and C. A. Coello Coello, "Modified Differential Evolution for Constrained Optimization,"J. Ind .Manag. Optim, pp. 25–32, 2006.

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TABLE III

COMPARISON OF THE BEST SOLUTION OBTAINED FROM VARIOUS STUDIES FOR THE SPEED REDUCER DESIGN OPTIMIZATION PROBLEM

Variables	DEDS [3]	MDE [4]	PSO-DE [35]	WCA [36]	MBA [5]	LCA [6]	APSO [37]	IAPSO [38]	Present study
x_1	3.5	3.5	3.5	3.5	3.5	3.5	3.50	3.5	3.5
x_2	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
x_3	17	17	17	17	17	17	18	17	17
x_4	7.3	7.3	7.3	7.3	7.3	7.3	8.1	7.3	7.3
x_5	7.7153190	7.8000270	7.8000000	7.7153190	7.7157720	7.8	8.0421210	7.71531990	7.71531991
x_6	3.350214	3.350221	3.350214	3.350214	3.350218	3.350214666096	3.352446	3.3502146660960	3.350214660964
x_7	5.286654	5.286685	5.2866832	5.286654	5.286654	5.28668322975	5.287076	5.286654464970	5.28665446498
g_1	-0.0739	-0.0739	-0.9907	-0.739	-0.0739	-0.0739	-0.1256	-0.9893	-0.0739
g_2	-0.1979	-0.1980	-0.9919	-0.1979	-0.1979	-0.1979	-0.2849	-0.9907	-0.1979
g_3	-0.4991	-0.4991	-0.99499	-0.4991	-0.4991	-0.5805	-0.3488	-0.4991	-0.4991
g_4	-0.9045	-0.9014	-0.9901	-0.9046	-0.9046	-0.9014	-0.8980	-0.9046	-0.9046
g_5	NA	-0.5×10^{-6}	-0.0060	NA	-0.2×10^{-6}	-0.1245	-0.0013	NA	-0.1×10^{-14}
g_6	NA	-0.9×10^{-7}	-0.0007	NA	NA	-0.4×10^{-14}	-0.0002	NA	0
g_7	-0.7025	-07025	AN	-0.7025	-0.7025	-0.7025	-0.6850	-0.7025	-0.7025
g_8	0	-0.2×10^{-6}	NA	0	0	0	-0.0003	-0.9885	0
g_9	-0.5833	-5833	-0.9583	-0.583	-0.583	-0.5833	-0.5831	NA	-0.5833
g_{10}	-0.0513	-0.0513	-0.0513	-0.0513	-0.0513	-0.0201	-0.1475	-0.0513	-0.0513
g_{11}	NA	-0.108	-0.0108	NA	-0.5×10^{-5}	-0.0108	-0.0405	NA	0
$F(X)$	2994.471066	2996.356689	2996.348167	2994.471066	2994.482453	2994.471066	3187.630486	2994.471066	2994.443555

[5] "A. Sadollah and A. Bahreininejad and H. Eskandar and M. Hamdi, “"Mine blast algorithm: A new population based algorithm for solving constrained engineering optimization problems,"Applied Soft Computing, vol. 13, pp. 2592–2612, 2013.

[6] A. Husseinzadeh Kashan, “An efficient algorithm for constrained global optimization and application to mechanical engineering design: League championship algorithm (LCA),"Computer-Aided Design, vol. 43, pp. 1769–1792, 2011.

[7] K. Schittkowski, “ NLPQL: A FORTRAN-Subroutine Solving Constrained Nonlinear Programming Problems, Annals of Operations Research," Annals of Operations Research, vol. 5 pp. 485–500, 1985.

[8] M. Z. Es-sadek, “ "Contribution a l'optimisation globale. Approche déterministe et stochastique et application," Université Mohammed V Agdal, Rabat, Maroc, 2009.

[9] Y. Belkourchia, L. Azrar, and E. M. Zeriab, “Hybrid optimization procedure applied to optimal location finding for piezoelectric actuators and sensors for active vibration control," Applied Mathematical Modelling, Vol. 262, pp. 701–716,2018.

[10] Y. Belkourchia, L. Azrar and E. M. Zeriab, “Hybrid optimization procedure and active vibration control of beams with piezoelectric patches," Optimization and Applications (ICOA), 2018 4th International Conference on. IEEE, pp. 1–7,2018.

[11] M. Bouhadi, “Contribution to Global Optimization with Constraints. Stochastic approach," PhD thesis, Mohammed V University, Faculty of Sciences, Rabat, Morocco, 1997.

[12] D. M. Himmelblau, “ Applied Nonlinear Programming," McGraw-Hill, New York, vol. 4, pp. 166–124, 1972.

[13] K. Deb, “An efficient constraint handling method for genetic algorithms," Computer Methods in Applied Mechanics and Engineering, vol. 186, pp. 311–338, 2000.

[14] A. Homaifar, S. H. Y. Lai and X. Qi,, “Constrained optimization via genetic algorithms," Simulation, vol. 62, pp. 242–254, 1994.

[15] K. S. Lee and Z. W. Geem, “ new meta-heuristic algorithm for continuous engineering optimization: harmony search theory and practice," A, Computer Methods in Applied Mechanics and Engineering, vol. 194, pp. 3902–3933, 2005.

[16] H. Garg, “A hybrid PSO-GA algorithm for constrained optimization problems," Applied Mathematics and Computation, vol. 274, pp. 292–305, 2016.

[17] S. He, E. Prempain and Q. H. Wu, “An improved particle swarm optimizer for mechanical design optimization problems," Engineering Optimization, vol. 36, pp. 585–605, 2004.

[18] A. H. Gandomi, X. S. Yang and A. Alavi, “ A Cuckoo search algorithm: a metaheuristic approach to solve structural optimization problems," Engineering with Computers, vol. 29, pp. 17–35, 2003.

[19] V. K. Mehta and B. Dasgupta, “ A constrained optimization algorithm based on the simplex search method," Engineering Optimization, vol. 44, pp. 537–550, 2012.

[20] C. A. C. Coello,“ Use of a self-adaptive penalty approach for engineering optimization problems," Computers in Industry, vol. 41, pp. 113–127, 2000.

[21] M. Fesanghary, M. Mahdavi, M. Minary-Jolandan and Y. Alizadeh, “ Hybridizing harmony search algorithm with sequential quadratic programming for engineering optimization problems," Computer Methods in Applied Mechanics and Engineering, vol. 197, pp. 3080–3091, 2008.

[22] X. H. Hu, R. C. Eberhart and Y. H. Shi, “ Engineering optimization with particle swarm," Proceedings of the 2003 IEEE Swarm Intelligence Symposium, pp. 53–57, 2003.

[23] Y. Shi and R. C. Eberhart, “A modified particle swarm optimizer," IEEE International Conference on Evolutionary Computation, Engineering Optimization, Piscataway, NJ: IEEE Press, pp. 69–73, 1998.

[24] B.K. Kannan and S.N. Kramer, “An augmented lagrange multiplier based method for mixed integer discrete continuous optimization and its applications to mechanical design," Trans. ASME, J. Mech. Des., Vol. 116, pp. 318–320, 1994.

[25] A. Kaveh and S. Talatahari, “Engineering optimization with hybrid particle swarm and ant colony optimization," Asian J. Civil Eng., vol. 10, pp. 611–628, 2009.

[26] L. S. Coelho, “Gaussian quantum-behaved particle swarm optimization approaches for constrained engineering design problems," Expert Syst. Appl., vol. 2, pp. 1676–1683, 2010.

[27] E. Sandgren, ‘Nonlinear integer and discrete programming in mechanical design," Journal of Mechanical Design, vol. 112, pp. 223–229, 1990.

[28] K. Deb and A.S. Gene, “A robust optimal design technique for mechanical component design," Evolutionary Algorithms in Engineering Applications, Springer, Berlin, vol. 21, pp. 497–514, 1997.

[29] Carlos A. Coello Coello and Efren Mezura Montes, “Constraint handling in genetic algorithms through the use of dominance-based tournament selection,"Adv. Eng. Inf., vol. 16, pp. 193–203, 2002.

[30] Carlos A. Coello Coello and Efren Mezura Montes, “An empirical study about the usefulness of evolution strategies to solve constrained optimization problems,"Int. J. General Syst., vol. 4, pp. 443–473, 2008.

[31] A. Kaveh and S. Talatahari, “An improved ant colony optimization for constrained engineering design problems,"Eng. Comput., vol. 1, pp. 155–182, 2010.

[32] Leticia C. Cagnina and Susana C. Esquivel and Carlos A. Coello Coello, “Solving engineering optimization problems with the simple constrained particle swarm optimizer,"Informatica, vol. 32, pp. 319–326, 2008.

[33] H. Garg, “Solvings tructural engineering design optimization problems using an artificial bee colony algorithm,"J. Ind .Manag. Optim, vol. 3, pp. 777–794, 2014.

[34] J. Golinski, “An adaptive optimization system applied to machine synthesis,"Mech. Mach. Theory, vol. 8, pp. 419–436, 1973.

[35] H. Liu and Z. Cai and Y. Wang, “Hybridizing particle swarm optimization with differential evolution for constrained numerical and engineering optimization,"Applied Soft Computing, vol. 10, pp. 629–640, 2010.

[36] "H. Eskandar and A. Sadollah and A. Bahreininejad and M. Hamdi, “Water cycle algorithm - A novel metaheuristic optimization method

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for solving constrained engineering optimization problems,”Computers and Structures, vol. 110-111, pp. 151–166, 2012.
[37] X. S. Yang, “Engineering Optimization: an Introduction with Metaheuristic Applications,” 2010.
[38] N. B. Guedri, “Improved accelerated PSO algorithm for mechanical engineering optimization problems,”Applied Soft Computing, vol. 40, pp. 455–467, 2016.
[39] J.E. Souza De Cursi, R. Ellaia and M. Bouhadi, “Global Optimization under nonlinear restrictions by using stochastic perturbations of the projected gradient,” Frontiers in Global Optimization, vol. 1, pp. 541–561, 2003.

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