EE910: Digital Communication Systems-I

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Lecture #4B: Continuous-Phase Modulation (CPM)

Continuous-Phase Modulation (CPM)

- CPFSK becomes a special case of a general class of continuous-phase modulated (CPM) signals in which the carrier phase is

$$\phi(t; I) = 2\pi \sum_{k=-\infty}^{n} I_k h_k q(t - kT), \qquad nT \le t \le (n+1)T \tag{1}$$

where

  - $\{I_k\}$ is the sequence of M-ary information symbols selected from the alphabet $\pm 1, \pm 3, \cdots, \pm(M-1)$,
  - $\{h_k\}$ is a sequence of modulation indices, and
  - $q(t)$ is some normalized waveform shape.

Continuous-Phase Modulation (CPM)

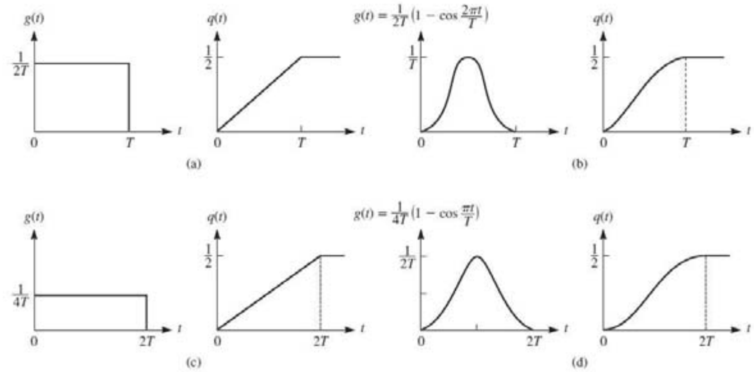
- When $h_k = h$ for all k, the modulation index is fixed for all symbols.
- When the modulation index varies from one symbol to another, the signal is called multi-h CPM.
- The waveform $q(t)$ may be represented in general as the integral of some pulse $g(t)$, i.e.

$$q(t) = \int_0^t g(\tau) d\tau \tag{2}$$

- If $g(t) = 0$ for $t > T$, the signal is called full-response CPM.
- If $g(t) \neq 0$ for $t > T$, the modulated signal is called partial-response CPM.

Continuous-Phase Modulation (CPM)

- Figure illustrates several pulse shapes for $g(t)$ and the corresponding $q(t)$.



Continuous-Phase Modulation (CPM)

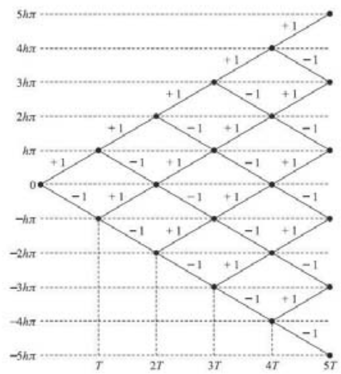
- Three popular pulse shapes are given in Table

LREC	$g(t) = \begin{cases} \frac{1}{2LT} & 0 \le t \le LT \\ 0 & \text{otherwise} \end{cases}$
LRC	$g(t) = \begin{cases} \frac{1}{2LT}(1 - cos\frac{2\pi t}{LT}) & 0 \le t \le LT \\ 0 & \text{otherwise} \end{cases}$
GMSK	$g(t) = \dfrac{Q(2\pi B(t - \frac{T}{2})) - Q(2\pi B(t + \frac{T}{2}))}{\sqrt{ln 2}}$

(3)

Continuous-Phase Modulation (CPM)

- One can sketch the set of phase trajectories $\phi(t; I)$ generated by all possible values of the information sequence $\{I_n\}$.
- For example, in the case of CPFSK with binary symbols $I_n = \pm 1$, the set of phase trajectories beginning at time $t = 0$ is shown in Figure.



Continuous-Phase Modulation (CPM)

- Simpler representations for the phase trajectories can be obtained by displaying only the terminal values of the signal phase at the time instants $t = nT$.
- In this case, we restrict the modulation index of the CPM signal to be rational.
- In particular, let us assume that $h = m/p$, where m and p are relatively prime integers, then a full-response CPM signal at the time instants $t = nT$ will have the terminal phase states

$$\Theta_s = \left\{0, \frac{\pi m}{p}, \frac{2\pi m}{p}, \cdots, \frac{(p-1)\pi m}{p}\right\} \tag{4}$$

when m is even and

$$\Theta_s = \left\{0, \frac{\pi m}{p}, \frac{2\pi m}{p}, \cdots, \frac{(2p-1)\pi m}{p}\right\} \tag{5}$$

when m is odd.

Continuous-Phase Modulation (CPM)

- On the other hand, when the pulse shape extends over L symbol intervals (partial-response CPM), the number of phase states may increase up to a maximum of S_t , where

$$S_t = \begin{cases} pM^{L-1} & \text{even } m \\ 2pM^{L-1} & \text{odd } m \end{cases} \tag{6}$$

where M is the alphabet size.

Continuous-Phase Modulation (CPM)

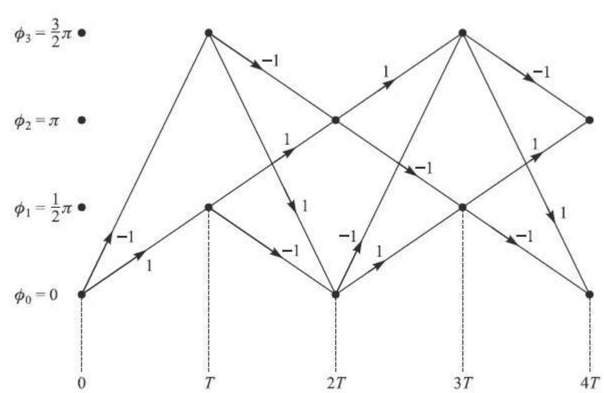
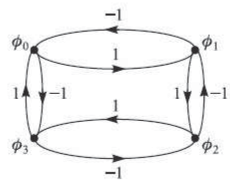


Figure: The state trellis for the binary CPFSK signal (full-response, rectangular pulse) with $h = \frac{1}{2}$

Continuous-Phase Modulation (CPM)

- An alternative representation to the state trellis is the state diagram, which also illustrates the state transitions at the time instants $t = nT$.
- In this representation, only the possible (terminal) phase states and their transitions are displayed.
- For example, the state diagram for the CPFSK signal with $h = \frac{1}{2}$ is shown in Figure .



Continuous-Phase Modulation (CPM)

- Determine the number of states in the state trellis diagram for a full-response binary CPFSK with $h = \frac{2}{3}$ or $\frac{3}{4}$.
- $h = \frac{2}{3}$**:** There are no correlative states in this system, since it is a full response CPM.
- Recall, a full-response CPM signal at the time instants $t = nT$ will have the terminal phase states

$$\Theta_s = \left\{0, \frac{\pi m}{p}, \frac{2\pi m}{p}, \cdots, \frac{(p-1)\pi m}{p}\right\}$$

when m is even

- Thus, we obtain the phase states as:

$$\Theta_s = \left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$$

Continuous-Phase Modulation (CPM)

- $h = \frac{3}{4}$: Recall, a full-response CPM signal at the time instants $t = nT$ will have the terminal phase states $\Theta_s = \left\{0, \frac{\pi m}{p}, \frac{2\pi m}{p}, \cdots, \frac{(2p-1)\pi m}{p}\right\}$ when m is odd.
- In this case, we obtain the phase states :

$$\Theta_s = \left\{0, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{9\pi}{4} \equiv \frac{\pi}{4}, \pi, \frac{15\pi}{4} \equiv \frac{7\pi}{4}, \frac{18\pi}{4} \equiv \frac{\pi}{2}, \frac{21\pi}{4} \equiv \frac{5\pi}{4}\right\}$$

Continuous-Phase Modulation (CPM)

- Determine the number of states in the state trellis diagram for a partial-response $L = 3$ binary CPFSK with $h = \frac{2}{3}$ or $\frac{3}{4}$.
  1. The combined states are $S_n = (\theta_n, I_{n-1}, I_{n-2})$, where $\{I_{n-1}, I_{n-2}\}$ take the values ± 1. Hence there are $3 \times 2 \times 2 = 12$ combined states in all.
  2. The combined states are $S_n = (\theta_n, I_{n-1}, I_{n-2})$, where $\{I_{n-1}, I_{n-2}\}$ take the values ± 1. Hence there are $8 \times 2 \times 2 = 32$ combined states in all.

Continuous-Phase Modulation (CPM)

- Determine the number of terminal phase states in the state trellis diagram for a full-response binary CPFSK with $h = \frac{2}{3}$ or $\frac{3}{4}$.
- We know that

$$\phi(t; \mathbf{I}) = 2\pi h \sum_{k=-\infty}^{n} I_k q(t - kT)$$

- Full response binary CPFSK ($q(T) = 1/2$) :
  1. $h = 2/3$. At the end of each bit interval the phase is : $2\pi \frac{2}{3} \frac{1}{2} \sum_{k=-\infty}^{n} I_k = \frac{2\pi}{3} \sum_{k=-\infty}^{n} I_k$. Hence the possible terminal phase states are $\{0, 2\pi/3, 4\pi/3\}$.
  2. $h = 3/4$. At the end of each bit interval the phase is : $2\pi \frac{3}{4} \frac{1}{2} \sum_{k=-\infty}^{n} I_k = \frac{3\pi}{4} \sum_{k=-\infty}^{n} I_k$. Hence the possible terminal phase states are $\{0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4\}$