Live Interaction #4:

4th February 2024

E-masters Next Generation Wireless Technologies

EE902 Advanced ML Techniques for Wireless Technology

- **Naïve Bayes:**
- **Outcome is discrete.**
- **Input vectors also discrete.**
- Example: Spam or genuine e-mail.

$$y = \begin{cases} 1 & \text{Spam} \\ 0 & \text{Genuine} \end{cases}$$

- Feature vector:

$$\begin{array}{c} \begin{bmatrix} \vdots \\ \vdots \\ \vdots \\ 0 \\ 1 \\ 1 \\ 0 \\ \vdots \end{bmatrix} \end{array} \begin{array}{c} \vdots \\ \vdots \\ \vdots \\ \text{able} \\ \text{above} \\ \text{abroad} \\ \text{access} \\ \vdots \end{array}$$

- **What is the size of the feature vector**?
- We have the compute the **prior probabilities**.

- $p(x_j = 1|y = 1)$: j[th] word occurs in a **spam email**.

$$p(x_j = 1|y = 1) = \frac{\text{Number of spam e} - \text{mails containing jth word}}{\text{Total number of spam e} - \text{mails}}$$
$$= \frac{\sum_{i=1}^{M} 1(x_j(i) = 1, y(i) = 1)}{\sum_{i=1}^{M} 1(y(i) = 1)}$$

- $p(x_j = 1|y = 0)$ = Probability that j[th] word occurs in a **genuine email**.

$$p(x_j = 1|y = 0) = \frac{\text{Number of genuine e} - \text{mails containing jth word}}{\text{Total number of genuine e} - \text{mails}}$$
$$= \frac{\sum_{i=1}^{M} 1(x_j(i) = 1, y(i) = 0)}{\sum_{i=1}^{M} 1(y(i) = 0)}$$

- $p(y = 1)$: Probability that e-mail is a spam e-mail.

$$p(y = 1) = \frac{\text{Number of spam e} - \text{mails}}{\text{Total number of e} - \text{mails}}$$
$$p(x_j = 0|y = 1) = 1 - p(x_j = 1|y = 1)$$
$$p(x_j = 0|y = 0) = 1 - p(x_j = 1|y = 0)$$
$$p(y = 0) = 1 - p(y = 1)$$

- **Naïve Bayes Assumption**:
- The words are **conditionally independent** given the label.

$$p(\bar{\mathbf{x}} = \bar{\mathbf{v}}|y = 1)$$
$$= p(x_1 = v_1, x_2 = v_2, \ldots, x_N = v_N|y = 1)$$

$$= p(x_1 = v_1|y=1) \times p(x_2 = v_2|y=1) \times \ldots \times p(x_N = v_N|y=1)$$

- What do we want to determine?
- Given a feature vector $\bar{\mathbf{x}} = \bar{\mathbf{v}}$ to determine **probabilities** of it being a spam e-mail or genuine e-mail.
  - These are termed as the **posterior probabilities**.

$$p(y = 1|\bar{\mathbf{x}} = \bar{\mathbf{v}})$$

$$p(y = 0|\bar{\mathbf{x}} = \bar{\mathbf{v}})$$

$$P(B|A) = \frac{P(A|B) \times P(B)}{P(A)}$$

$$p(y = 1|\bar{\mathbf{x}} = \bar{\mathbf{v}}) = \frac{p(\bar{\mathbf{x}} = \bar{\mathbf{v}}|y=1)p(y=1)}{p(\bar{\mathbf{x}} = \bar{\mathbf{v}})}$$

$$p(y = 0|\bar{\mathbf{x}} = \bar{\mathbf{v}}) = \frac{p(\bar{\mathbf{x}} = \bar{\mathbf{v}}|y=0)p(y=0)}{p(\bar{\mathbf{x}} = \bar{\mathbf{v}})}$$

- After computing posteriors

$$\underbrace{p(y = 1|\bar{\mathbf{x}} = \bar{\mathbf{v}}) > p(y = 0|\bar{\mathbf{x}} = \bar{\mathbf{v}})}_{\text{Classified as spam}}$$

$$\underbrace{p(y = 1|\bar{\mathbf{x}} = \bar{\mathbf{v}}) \leq p(y = 0|\bar{\mathbf{x}} = \bar{\mathbf{v}})}_{\text{Classified as genuine}}$$

$$\frac{p(\bar{\mathbf{x}} = \bar{\mathbf{v}}|y=1)p(y=1)}{\cancel{p(\bar{\mathbf{x}} = \bar{\mathbf{v}})}} > \frac{p(\bar{\mathbf{x}} = \bar{\mathbf{v}}|y=0)p(y=0)}{\cancel{p(\bar{\mathbf{x}} = \bar{\mathbf{v}})}}$$

- LHS

$$p(\bar{\mathbf{x}} = \bar{\mathbf{v}}|y=1)p(y=1)$$

$$\underbrace{= p(x_1 = v_1|y=1) \times p(x_2 = v_2|y=1) \times \ldots \times p(x_N = v_N|y=1) \times p(y=1)}_{Q_1}$$

- RHS

$$\underbrace{\begin{gathered} p(\bar{\mathbf{x}}=\bar{\mathbf{v}}|y=0)p(y=0) \\ = p(x_1=v_1|y=0)\times p(x_2=v_2|y=0)\times \dots \\ \times p(x_N=v_N|y=0)\times p(y=0) \end{gathered}}_{Q_0}$$

- If $Q_1 \geq Q_0$ then **spam e-mail**.
- If $Q_1 < Q_0$ then **genuine e-mail**.
- Naïve Bayes principle: Features are conditionally independent given label value.

$$\underbrace{\cancel{p(\bar{\mathbf{x}}=\bar{\mathbf{v}}) = p(x_1=v_1)\times p(x_2=v_2)\times \dots \times p(x_N=v_N)}}_{\cancel{\text{independence}}}$$

$$p(\bar{\mathbf{x}}=\bar{\mathbf{v}}|y=1)$$

$$\underbrace{= p(x_1=v_1|y=1)\times p(x_2=v_2|y=1)\times \dots \times p(x_N=v_N|y=1)}_{\text{Conditional independence}}$$

- **Laplacian smoothing: To avoid zero prior probabilities** for features.

$$p(x_j=1|y=1) = \frac{\begin{matrix}\text{Number of spam e} - \text{mails} \\ \text{containing jth word}\end{matrix} + 1}{\text{Total number of spam e} - \text{mails} + 2}$$

$$= \frac{\sum_{i=1}^{M} 1\left(x_j(i)=1, y(i)=1\right)+1}{\sum_{i=1}^{M} 1(y(i)=1)+2}$$

$$p(x_j=1|y=0) = \frac{\begin{matrix}\text{Number of genuine e} - \text{mails} \\ \text{containing jth word}\end{matrix} + 1}{\text{Total number of genuine e} - \text{mails} + 2}$$

$$= \frac{\sum_{i=1}^{M} 1\left(x_j(i)=1, y(i)=0\right)+1}{\sum_{i=1}^{M} 1(y(i)=0)+2}$$

$$p(y=1) = \frac{\text{Number of spam e} - \text{mails} + 1}{\text{Total number of e} - \text{mails} + 2}$$

- **Assignment #4 Deadline: 9th Feb Friday 11:59 PM.**
- **Assignment #3, 4 Discussion: 10th Feb Saturday 2 PM - 3.00 PM.**
- **Quiz #2: 10th February 3:30 - 4:30 PM.**
- **Live interaction #5: 11 February Sunday 2:00 – 3:00 PM.**