Online Example

Online Estimation

- Consider the problem

Pilot matrix

$$\bar{\mathbf{y}} = \begin{bmatrix} -2 \\ -3 \\ 1 \\ -2 \end{bmatrix}, \mathbf{X} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -1 & -1 \\ -1 & 1 \end{bmatrix}$$

Online Example

Error covariance at $N=4$.

- Let $\sigma^2 = 4$. Error covariance is

$$\mathbf{P}(N) \overset{N=4}{=} \sigma^2 (\mathbf{X}^T\mathbf{X})^{-1} = \mathbf{I}$$

$$= 4\,(4I)^{-1} = 4 \times \frac{1}{4} I$$

$$= I = P(4).$$

Online Example

- Consider now a new input-output

output. input

$$y(5) = -2, \bar{\mathbf{x}}(5) = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

Online Example

Gain N+1=5

- The estimate $\hat{\mathbf{h}}(5)$ can be evaluated as follows

$$\bar{\mathbf{k}}(5) = \frac{\mathbf{P}(N)\bar{\mathbf{x}}(N+1)}{\sigma^2 + \bar{\mathbf{x}}^T(N+1)\mathbf{P}(N)\bar{\mathbf{x}}(N+1)}$$

$$= \frac{\mathbf{I} \times \begin{bmatrix} -2 \\ 2 \end{bmatrix}}{4 + \begin{bmatrix} -2 & 2 \end{bmatrix} \mathbf{I} \times \begin{bmatrix} -2 \\ 2 \end{bmatrix}} = \frac{1}{12}\begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

Error Covariance Update

- The error covariance update for time $N+1$ is

$$P(N+1) = \left(I - \bar{k}(N+1)\bar{x}^T(N+1)\right)P(N)$$

$$P(5) = \left(I - \bar{k}(5)\bar{x}^T(5)\right)P(4).$$

Error Covariance Update

- The error covariance update for time $N+1$ is

$$\mathbf{P}(N+1) = \left(\mathbf{I} - \bar{\mathbf{k}}(N+1)\bar{\mathbf{x}}^T(N+1)\right)\mathbf{P}(N)$$

Error Covariance Update

- The error covariance for time $N+1$ can be evaluated as

$$\mathbf{P}(N+1)=\left(\overbrace{\mathbf{I}-\bar{\mathbf{k}}(N+1)\bar{\mathbf{x}}^T(N+1)}^{2\mathrm{X}2}\right)\mathbf{P}(N)$$

$$=\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\frac{1}{12}\begin{bmatrix}-2\\ 2\end{bmatrix}\begin{bmatrix}-2 & 2\end{bmatrix}\right)\cdot\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}\frac{1}{3} & -\frac{1}{3}\\ -\frac{1}{3} & \frac{1}{3}\end{bmatrix}\right)=\begin{bmatrix}\frac{2}{3} & \frac{1}{3}\\ \frac{1}{3} & \frac{2}{3}\end{bmatrix}=P(5).$$

Error covariance $N+1=5$

Error Covariance Update

- The error covariance for time $N+1$ can be evaluated as

h(5) = Estimate at N+1=5
P(5) = Error cov at N+1=5

$$\mathbf{P}(N+1) = \left(\mathbf{I} - \bar{\mathbf{k}}(N+1)\bar{\mathbf{x}}^T(N+1)\right)\mathbf{P}(N)$$

$$= \left(\mathbf{I} - \frac{1}{12}\begin{bmatrix}-2 \\ 2\end{bmatrix}\begin{bmatrix}-2 & 2\end{bmatrix}\right)\mathbf{I} = \begin{bmatrix}\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3}\end{bmatrix}$$