MIMO-OFDM

SPATIAL MUX

- Combines the benefits of **MIMO + OFDM**

FREQ. MUX

- Exploits **Space + Frequency** division Multiplexing

4G LTE ~ 100 Mbps
5G NR ~ 1 Gbps.

  - This leads to **ultra high** data rates!!!

MIMO-OFDM Channel Model

- **Channel taps** between Receive antenna i and transmit antenna j

MIMO – r = # Receive Antennas.
t = # Transmit Antennas.

$$\overbrace{h_{ij}(0), h_{ij}(1), \dots, h_{ij}(L-1)}^{L \text{ taps}}$$

Total #taps = rtL

$h_{ij}(l)$ = l^{th} channel tap
Between i^{th} Rx Antenna
j^{th} Tx Antenna,

MIMO-OFDM

in MIMO OFDM.

- Transmission can be done as follows

MIMO-OFDM Transmission

Perform IFFT

- On each transmit antenna j **load the subcarriers** as shown below

$$\overbrace{X_j(0), X_j(1), \ldots, X_j(N-1)}^{\text{SYMBOLS LOADED ON SUBCARRIERS FOR TRANSMIT ANTENNA } j}$$

$X_j(k)$ = Symbol Loaded on Subcarrier k @ Transmit antenna j

Total # symbols $= Nt$

MIMO-OFDM Transmisson

ON EACH TRANSMIT ANTENNA.

- **IFFT** can be performed as shown below

FREQ DOMAIN SAMPLES.

$$X_j(0), X_j(1), \ldots, X_j(N-1)$$

$$\downarrow IFFT$$

TIME DOMAIN SAMPLES ON TRANSMIT ANTENNA j

$$\underbrace{x_j(0), x_j(1), \ldots, x_j(N-1)}_{\text{N SAMPLES.}}$$

MIMO-OFDM Transmisson

- How many IFFTs?

ONE for each Transmit Antenna

$\Rightarrow t$ IFFTs.

MIMO-OFDM Transmission

ON EACH TX ANTENNA

- Add the **cyclic prefix** as shown below

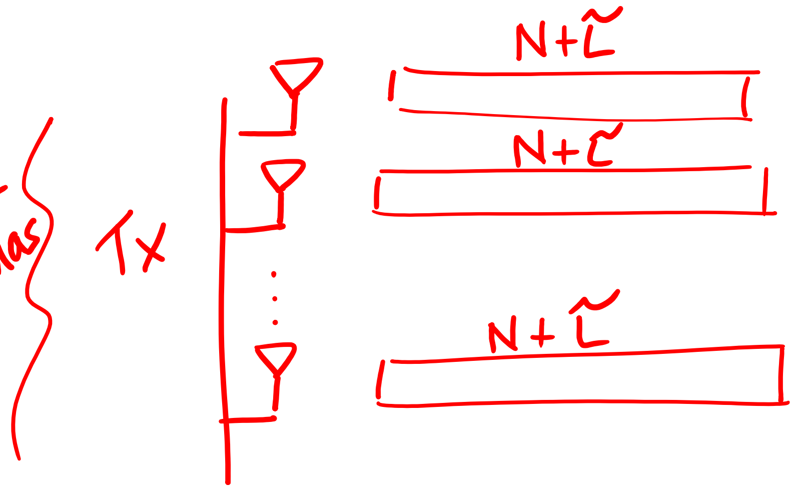
$$\underbrace{\underbrace{\underbrace{x_j(N-\tilde{L}),\ldots,x_j(N-2),x_j(N-1)}_{\text{CP}}}_{\tilde{L}},\underbrace{\underbrace{x_j(0),x_j(1),\ldots,x_j(N-1)}_{\text{Original samples}}}_{N}}_{N+\tilde{L}\ \text{samples on each Tx antenna.}}$$

MIMO-OFDM Transmission

- What is the size of the total **transmission block**?

in one OFDM BLOCK.

t = # Transmit Antennas



$\Rightarrow$ Total # Samples

$$= (N+\tilde{L})t$$

$$= (N+N_{cp})t$$

MIMO-OFDM Channel

- After removal of CP, **Linear Convolution** becomes **Circular Convolution**- Why?

Channel between RX antenna i TX antenna j

Samples on TX antenna j

$$y_i(k) = \sum_{j=1}^{t} h_{ij} \circledast x_j + w_i(k)$$

Circular convolution

MIMO-OFDM Receiver

Fast Fourier Transform

- Performing **FFT** at each receive antenna yields

SAMPLES ON RX antenna i after CP removal.

$$y_i(0), y_i(1), \ldots, y_i(N-1)$$

$$\downarrow FFT$$

$$Y_i(0), Y_i(1), \ldots, Y_i(N-1)$$

$Y_i(k)$ = OUTPUT ON SUBCARRIER k.

MIMO-OFDM Receiver

- How many FFTs?

r = # Receive antennas.

One FFT @ each Rx antenna

$\Rightarrow$ r FFTs.

MIMO-OFDM

For subcarrier k.

- The net MIMO-OFDM system model is

$$\begin{bmatrix} Y_1(k) \\ Y_2(k) \\ \vdots \\ Y_r(k) \end{bmatrix} = \underbrace{\begin{bmatrix} H_{11}(k) & H_{12}(k) & \cdots \\ H_{21}(k) & \ddots & \\ \vdots & & \ddots \end{bmatrix}}_{\substack{r \times t \text{ matrix on} \\ \text{SUBCARRIER } k.}} \begin{bmatrix} X_1(k) \\ X_2(k) \\ \vdots \\ X_t(k) \end{bmatrix} + \begin{bmatrix} W_1(k) \\ W_2(k) \\ \vdots \\ W_r(k) \end{bmatrix}$$

MIMO-OFDM

r x1 vector of outputs for all Rx antennas on subcarrier k.

r x t MIMO channel for subcarrier k.

- The net MIMO-OFDM system model is

$$\underbrace{\begin{bmatrix} Y_1(k) \\ Y_2(k) \\ \vdots \\ Y_r(k) \end{bmatrix}}_{\mathbf{Y}(k)} = \underbrace{\begin{bmatrix} H_{11}(k) & H_{12}(k) & \dots & H_{1t}(k) \\ H_{21}(k) & H_{22}(k) & \dots & H_{2t}(k) \\ \vdots & \vdots & \ddots & \vdots \\ H_{r1}(k) & H_{r2}(k) & \dots & H_{rt}(k) \end{bmatrix}}_{\mathbf{H}(k)} \underbrace{\begin{bmatrix} X_1(k) \\ X_2(k) \\ \vdots \\ X_t(k) \end{bmatrix}}_{\mathbf{X}(k)} + \underbrace{\begin{bmatrix} W_1(k) \\ W_2(k) \\ \vdots \\ W_r(k) \end{bmatrix}}_{\mathbf{W}(k)}$$

r x1 Noise vector for subcarrier k.

t x1 vector of symbols loaded on subcarrier k for all Transmit antennas.

MIMO-OFDM

- $\mathbf{Y}(k)$: $r \times 1$ OUTPUT VECTOR SUBCARRIER k
- $\mathbf{H}(k)$: $r \times t$ CHANNEL MATRIX SUBCARRIER k
- $\mathbf{X}(k)$: $t \times 1$ Transmit vector subcarrier k.
- $\mathbf{W}(k)$: $r \times 1$ Noise vector subcarrier k.

MIMO-OFDM Channel Coefficients

$H_{ij}(k)$

Frequency Domain Subcarrier k.

ith RX antenna

jth TX antenna.

$$h_{ij}(0), h_{ij}(1), \ldots, h_{ij}(L-1), \overbrace{0, 0, \ldots, 0}^{N-L \text{ zeros.}}$$

$$\Big\downarrow \text{FFT} \leftarrow \text{N-pt FFT}$$

FREQ DOMAIN COEFFICIENTS.

$$H_{ij}(0), H_{ij}(1), \ldots \quad H_{ij}(N-1)$$

MIMO-OFDM

$r \times 1$

- $\mathbf{Y}(k)$: Output symbol vector for subcarrier k

$r \times t$

- $\mathbf{H}(k)$: Channel matrix for subcarrier k

$t \times 1$

- $\mathbf{X}(k)$: Symbol vector for subcarrier k

$r \times 1$

- $\mathbf{W}(k)$: Noise vector for subcarrier k

FLAT MIMO CHANNEL

$$Y(k) = H(k)X(k) + W(k)$$

$$k = 0, 1, \ldots, N-1$$

N PARALLEL MIMO CHANNELS.

MIMO-OFDM

- **MIMO-OFDM** Model for Subcarrier k is

$$\underbrace{Y(k) = H(k)X(k) + W(k).}_{\text{FREQUENCY DOMAIN MODEL}}$$

MIMO-OFDM

- **MIMO-OFDM** Model for Subcarrier k is

$$\mathbf{Y}(k) = \mathbf{H}(k)\mathbf{X}(k) + \mathbf{W}(k)$$

MIMO-OFDM

- How many such **parallel MIMO systems** are there?

ONE FOR EACH SUBCARRIER

$\Rightarrow$ N. PARALLEL MIMO CHANNELS.

MIMO-OFDM Parallel Channels

$$\left.\begin{aligned}
\mathbf{Y}(0) &= H(0)X(0)+W(0)\\
\mathbf{Y}(1) &= H(1)X(1)+W(1)\\
&\vdots\\
\mathbf{Y}(N-1) &= H(N-1)X(N-1)+W(N-1)
\end{aligned}\right\}\ \underline{\text{N parallel MIMO channels.}}$$

MIMO-OFDM Parallel Channels

$$\mathbf{Y}(0) = \mathbf{H}(0)\mathbf{X}(0) + \mathbf{W}(0)$$
$$\mathbf{Y}(1) = \mathbf{H}(1)\mathbf{X}(1) + \mathbf{W}(1)$$
$$\vdots$$
$$\mathbf{Y}(N-1) = \mathbf{H}(N-1)\mathbf{X}(N-1) + \mathbf{W}(N-1)$$

MIMO-OFDM

$\underline{Y(k) = H(k) X(k) + W(k)}.$

- How to recover $\mathbf{X}(k)$? ZF Receiver!

$$\widehat{\mathbf{X}}(k) = \mathbf{H}^{\dagger}(k)\mathbf{Y}(k)$$

$$= \underbrace{\hat{X}(k) = \underbrace{\left(H^{H}(k) H(k)\right)^{-1} H^{H}(k)}_{\text{Pseudo inverse}} Y(k).}_{\text{ZF Receiver}}$$

MIMO-OFDM

Linear Minimum Mean Square Error

- One can also use the **LMMSE Receiver**

$$\hat{\mathbf{X}}(k) = \underbrace{\left(\mathbf{H}^H(k)\,\mathbf{H}(k) + \frac{1}{SNR}\mathbf{I}\right)^{-1}\mathbf{H}^H(k)}_{\text{LMMSE Receiver}}\,\mathbf{Y}(k).$$

Instructors may use this white area (14.5 cm / 25.4 cm) for the text.
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