Please submitted by Saturday, 5 Aug. 2023, 11 am, right before the discussion hour.

1. Attempt all 5 problems. There is no penalty for submitting incorrect attempts
2. However, plagiarism will result in serious penalties, such as an F grade.

1. Consider the following linear program

$$\min \ \mathbf{c}^T\mathbf{x} \tag{1}$$
$$\text{s. t. } \mathbf{A}\mathbf{x} \leq \mathbf{b} \tag{2}$$

where $\mathbf{A}$ is square and full rank.

(a) When is the problem infeasible?

(b) When is the problem unbounded below?

(c) When does the problem have a finite solution, and what is it?

2. Show that any linear programming problem can be expressed as

$$\min \ \mathbf{c}^T\mathbf{x} \tag{3}$$
$$\text{s. t. } \mathbf{A}\mathbf{x} = \mathbf{b} \tag{4}$$
$$x_i \geq 0 \qquad i = 1, \ldots, n \tag{5}$$

3. Consider the following linear program

$$\min \ \mathbf{c}^T\mathbf{x} \tag{6}$$
$$\text{s. t. } \mathbf{A}\mathbf{x} = \mathbf{b} \tag{7}$$

(a) When is the problem infeasible?

(b) When is the problem unbounded below?

(c) When does the problem have a finite solution, and what is it?

4. Consider the following linear program

$$\min \ \mathbf{c}^T\mathbf{x} \tag{8}$$
$$\text{s. t. } \mathbf{a}^T\mathbf{x} \leq b \tag{9}$$

where $\mathbf{a} \neq 0$.

(a) When is the problem infeasible?

(b) When is the problem unbounded below?

(c) When does the problem have a finite solution, and what is it?

5. Solve the following optimization problem for $\mathbf{A} \succ 0$,

$$\min \ \mathbf{x}^T\mathbf{A}\mathbf{x} \tag{10}$$
$$\text{s. t. } \|\mathbf{x}\|_2^2 = 1 \tag{11}$$

Hint: Given that the eigenvalue decomposition $\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^T$, use the change of variable $\mathbf{y} = \mathbf{U}^T\mathbf{x}$.