eMasters in Communication Systems

Core Module:

Wireless Communication

- What is this **module** about?
- Introduce you to cutting-edge wireless technologies
  - **Multiple-antenna systems**
  - **MIMO (Multiple Input Multiple Output) Technology**
  - **OFDM (Orthogonal Frequency Division Multiplexing) System**

- These form the basis of modern wireless **cellular** and **Wi-Fi** systems
  - **LTE**
  - **5G-NR**
  - **802.11 ac, 802.11 ax**

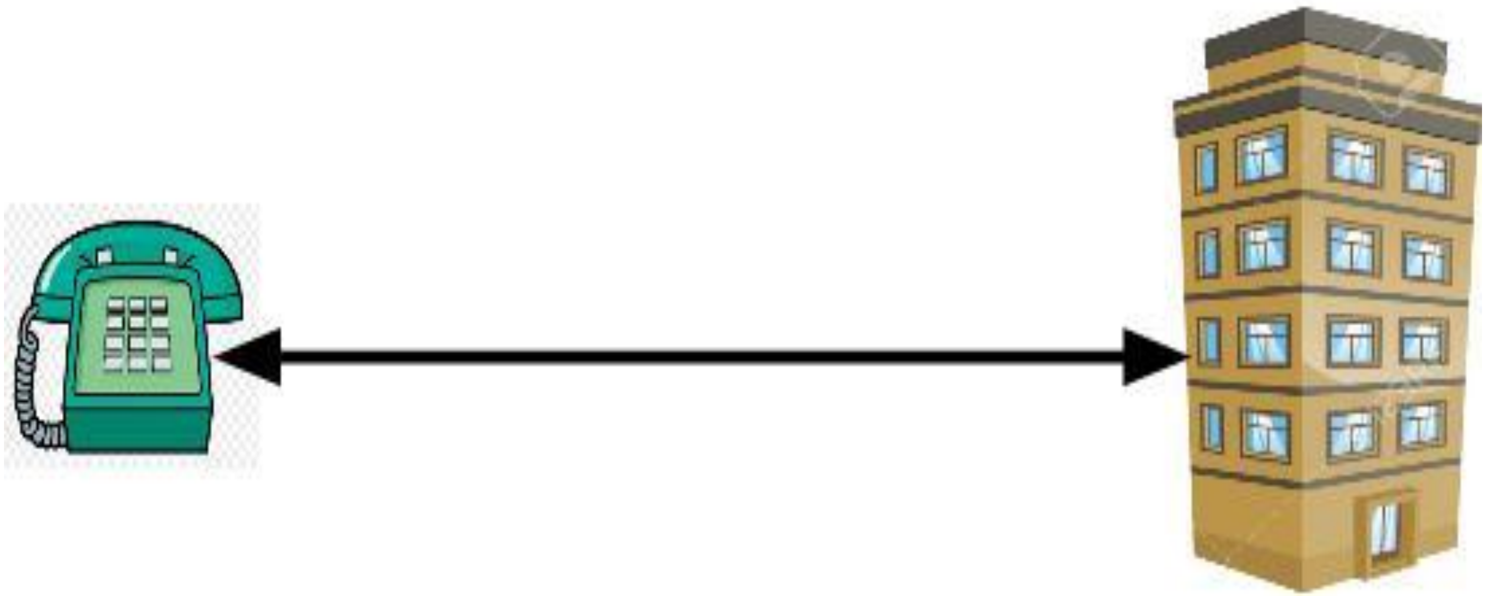
- However, in order to achieve these goals…
  - We have to first understand the basic **principles** and **models** of wireless systems

- Also, since this is a formal course…
  - We will also use a lot of **mathematical tools and analysis**

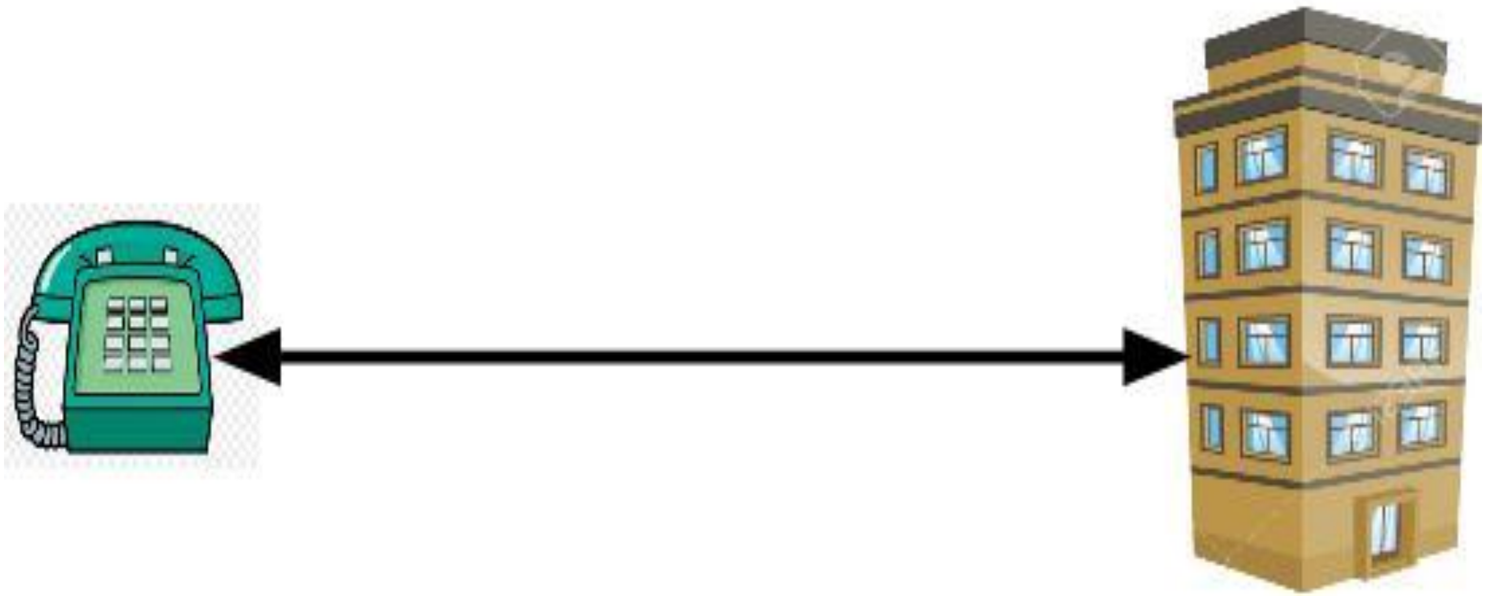
Chapter 1:

Basic Digital Communication

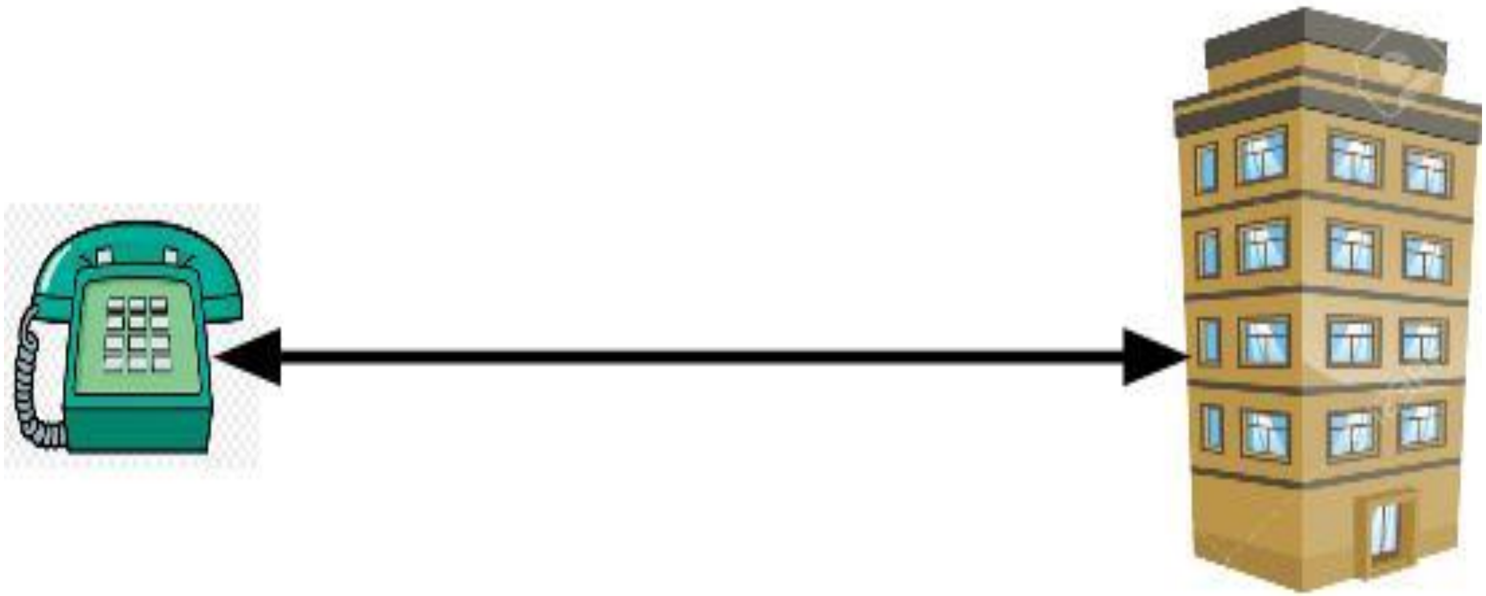
Conventional Comm



- Why is **conventional wireline communication** different from **wireless**?



Reason: **Channel is FIXED**!!



What are the **implications** of this?

To understand this, we have to develop a model for the **wireline digital system**.

Model for Digital Wireline System

Comm system has 4 components

1. Received signal y
2. Transmit signal x
3. Noise n

SNR – **Signal to Noise Power Ratio**.

- Very very **IMPORTANT** quantity in communication
- Which we will refer to frequently…

A Simple Model...

$$y = x + n$$

Signal

- **Signal power** is defined as
$$E\{|x|^2\} = P$$
- $E\{\cdot\}$ denotes the **expected value** or **average**

Signal

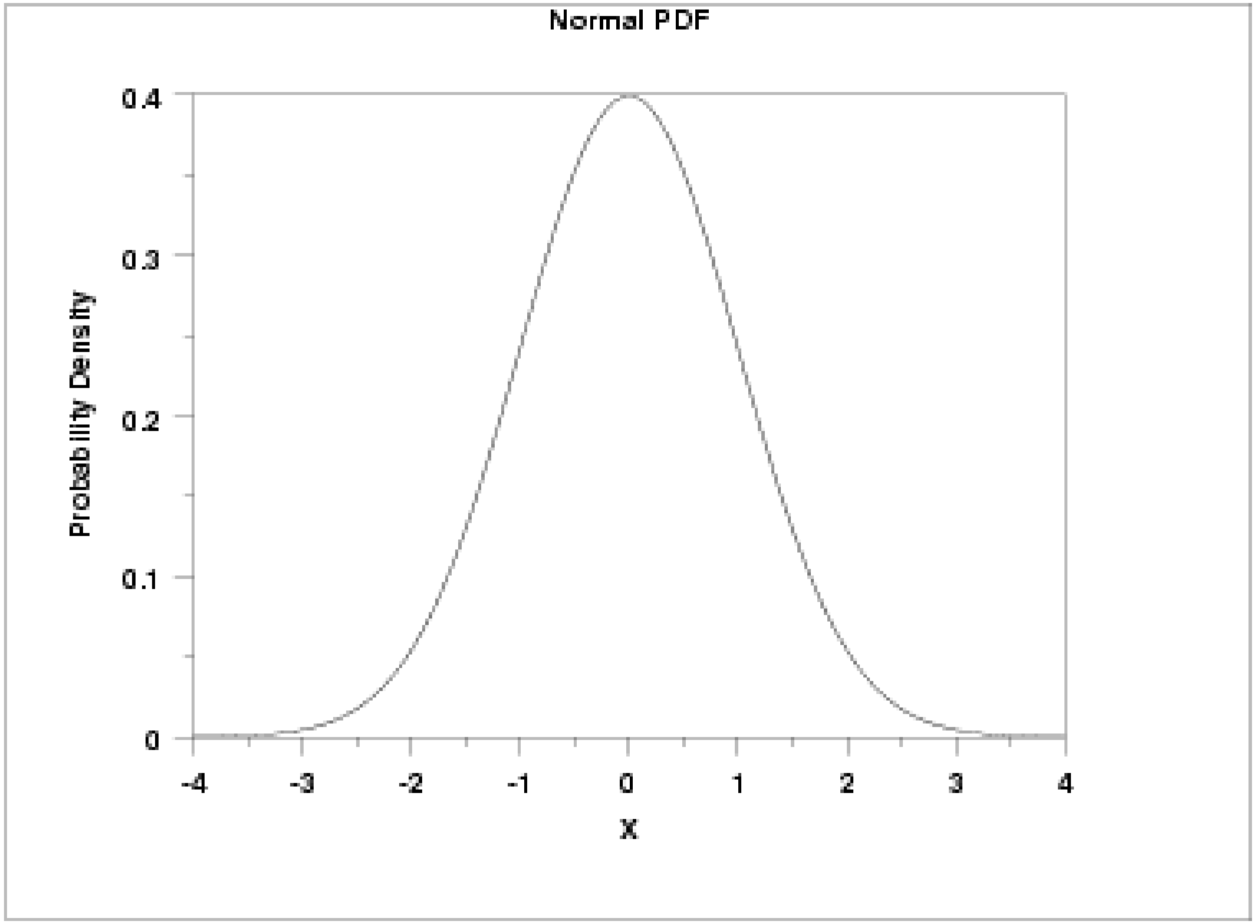
- But **NOT any signal** is permitted
- Communication signals have to have a **specific structure** to convey maximum **information(bits)**…
- This is termed **MODULATION**… which we will come to later

Noise

- The noise n is termed AWGN – **Additive White Gaussian Noise**.
- Noise is **Additive**.

Some Mathematical Analysis...

- **Gaussian noise** $\Rightarrow$
- Noise PDF (**Probability Density Function**) is **Gaussian**


Normal PDF
Probability Density
0.4
0.3
0.2
0.1
0
-4
-3
-2
-1
0
1
2
3
4
X

Gaussian PDF: Shape ?

$$f_N(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n-\mu)^2}{2\sigma^2}}$$

- $\mu =$ **Mean** $= E\{N\}$
- $\sigma^2 =$ **Variance** $= E\{(N-\mu)^2\}$

Gaussian PDF: Shape ?

$$f_N(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{n^2}{2\sigma^2}}$$

- **Typically** $\mu = 0$ **for noise**
- $\sigma^2 = \frac{N_0}{2}$ **Variance** $= E\{|N|^2\}$

Noise

- **White Noise** $\Rightarrow$
- Noise **PSD**(**Power Spectral Density**) is flat or **constant** across the frequency spectrum

White Noise

$$S_{nn}(\Omega) = \frac{N_0}{2} = \text{constant}$$

- Similar to **white light**!
- Contains all **frequencies**

PSD

- How is PSD defined?
- Fourier transform of **auto-correlation**

$$\mathrm{R}_{nn}(l) = E\{n(k)n(k+l)\} = \frac{N_0}{2}\delta(l)$$

SNR – Signal to Noise Power Ratio

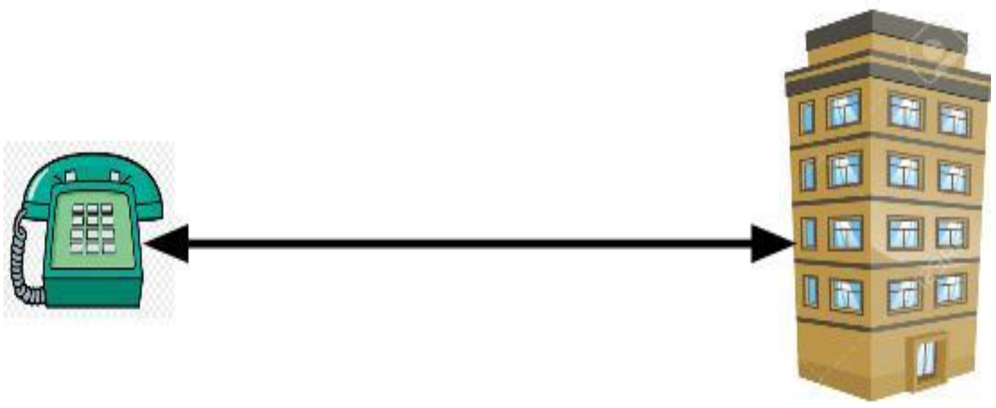
- SNR is the ratio of **signal** to **noise power**

$$y = x + n$$

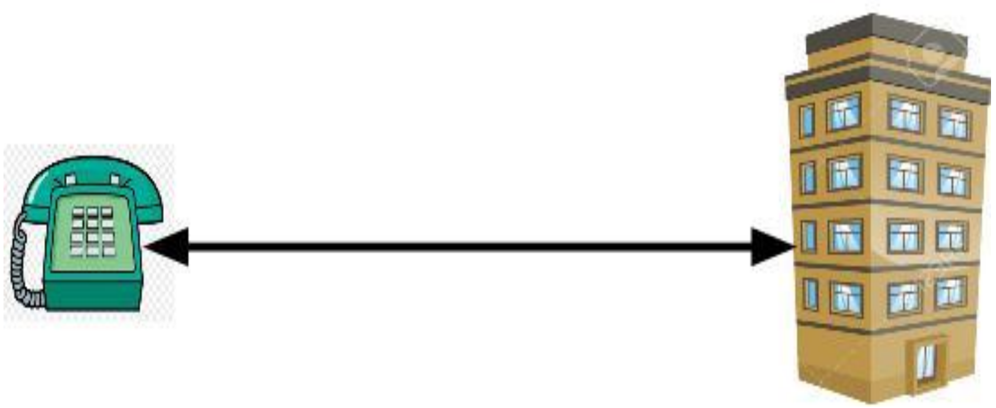
$$\mathrm{SNR} = \frac{E\{|x|^2\}}{E\{|n|^2\}} = \frac{P}{N_0/2} = \frac{2P}{N_0}$$

SNR Property of Wireline Channel

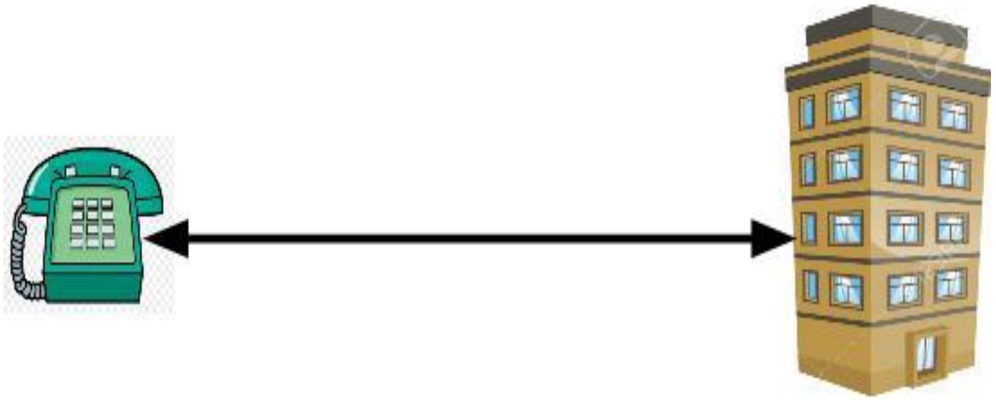
- SNR of the wireline channel is **approximately constant**!!
- Why?



- Because the channel is **FIXED**.
- Therefore, no **variations** or **fluctuations** in SNR



- This is a very **important property** of a **wireline comm** system



Performance of Communication Systems

- How to characterize the **performance** of a comm system?
- **BER (Bit-Error Rate)** is an **important metric** for any comm system

- What is the BER?
- **Probability** that a single bit is in error
- Example: $\mathrm{BER} = 10^{-2} \Rightarrow$
  - **Approx 1%** bits are in error
  - **10 in every 1000 bits** on an average are in error

Digital Modulation

- Mapping **information bits** to signals that can be **transmitted** over the channel
- There are various formats for **Digital Modulation**
  - **Examples: BPSK, QPSK, QAM**

BPSK

- **Binary Phase Shift Keying**
- $x \in \{+A, -A\}$: **Two phases** - 0^{o} and 180^{o}
- $\{+A, -A\}$: **Signal constellation**

Bit Mapping

- **Information bits** can be mapped as follows

$$0 \rightarrow +A$$
$$1 \rightarrow -A$$

- Consider now **signal power** P
- It follows that $A = \sqrt{P}$
- $x \in \{A, -A\} = \{\sqrt{P}, -\sqrt{P}\}$

$$E\{|x|^2\} = P$$

Performance of BPSK for Wireline

- **BER for BPSK** over wireline channel is given as

$$BER = Q\left(\sqrt{\frac{2P}{N_0}}\right) = Q\left(\sqrt{\mathrm{SNR}}\right)$$

Gaussian Q-function

- What is $Q(\cdot)$?
- This is the **Gaussian Q function**

- $Q(\cdot)$ is the **Complementary Cumulative Distribution Function** (CCDF) of the standard Gaussian RV

$\text{CDF} = \Pr(X \leq x) = F_X(x)$

$\text{CCDF} = \Pr(X > x) = \bar{F}_X(x)$

$= 1 - \text{CDF}$

- **Standard Gaussian RV**: Mean $\mu = 0$, Variance $\sigma^2 = 1$

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

PDF

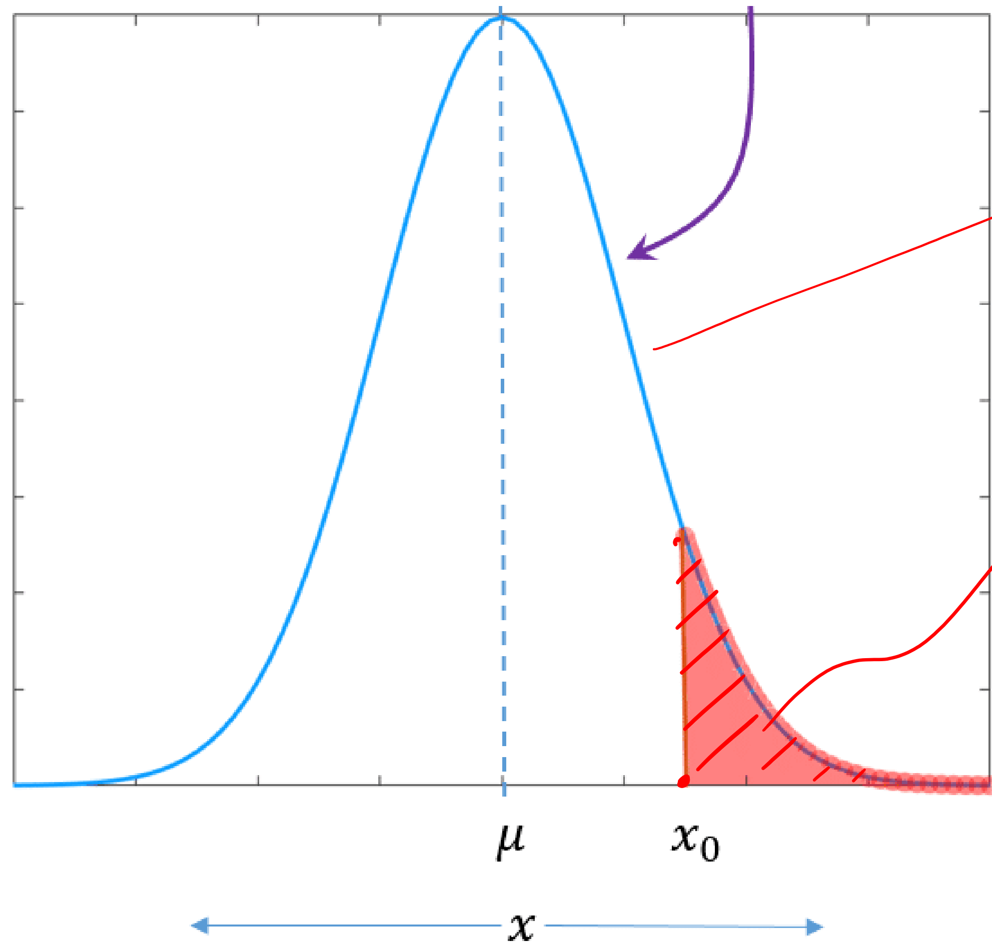
CDF

$f_X(x) = \frac{dF_X(x)}{dx}$

- $Q(x)$ is defined as

$$\overset{Q(x)}{P(X \geq x)} = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

$Q(x)$
DECREASING FUNCTION.



Standard Gaussian PDF

$Q(-\infty) = ?$
$Q(0) = ?$

$$P(X \geq x) = \int_x^\infty f_X(x)\,dx.$$

BER Example

$Q(\sqrt{SNR})$.

- Evaluate the **BER of a wireline channel** with **BPSK transmission** $\text{SNR} = 12\ dB$

$10\log_{10} SNR = SNR \text{ in } dB$.

$SNR = 10^{SNR_{dB}/10}$.

- This can be evaluated as follows

$$10\log_{10} SNR = 12dB$$
$$\Rightarrow \log_{10}(SNR) = 1.2$$
$$\Rightarrow SNR = 10^{1.2} = 15.85$$

- Hence, the **BER** is given as

HOW TO CALCULATE?

$$\begin{aligned}\mathrm{BER} &= Q\left(\sqrt{SNR}\right)\\ &= Q\left(\sqrt{15.85}\right)\\ &\approx 3.44\times 10^{-5}\end{aligned}$$

NO CLOSED FORM EXPRESSION!

TABLES OR ONLINE CALCULATOR

BER Assignment

- Evaluate the **BER of a wireline channel** with **BPSK transmission** $\mathrm{SNR} = 15\ dB$

- Please do this as homework

90°

QPSK

- **Quadrature Phase Shift Keying**
- **QPSK constellation** is given as

$x_I(t) + j x_Q(t)$

$$x_I + j x_Q$$

- x_I: **In phase** component $(\cos(2\pi f_c t))$
- x_Q: **Quadrature** component $(\sin(2\pi f_c t))$

ORTHOGONAL CARRIERS.

QCM

QUADRATURE CARRIER MULTIPLEXING.

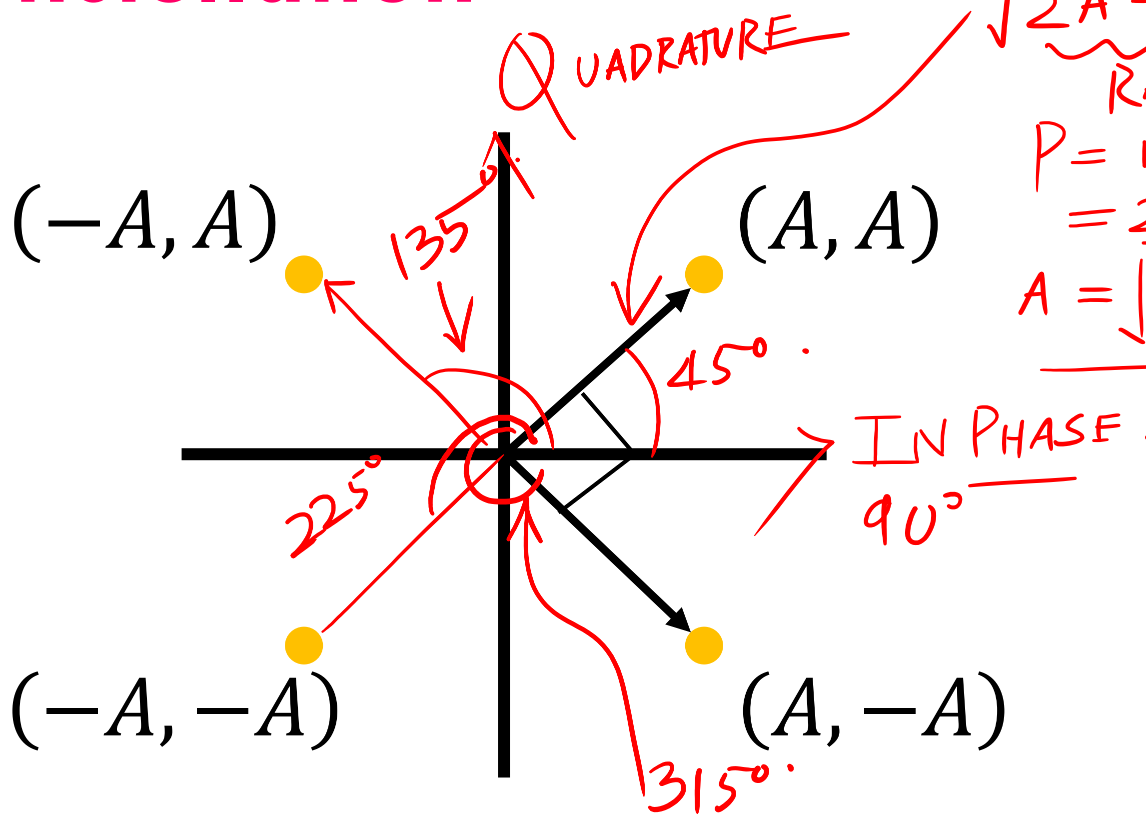
- $x_I \in \{+A, -A\}$
- $x_Q \in \{+A, -A\}$
- Therefore, **QPSK constellation** is $x_I + jx_Q$

$$\underbrace{\{A + jA, A - jA, -A + jA, -A - jA\}}_{M=4}$$

M = 4 SYMBOLS IN QPSK.

- QPSK has **$\log_2 M = \log_2 4 =$ 2 bits** per symbol

QPSK constellation



$$\underbrace{\sqrt{2}A = r}_{\text{RADIUS}}$$

$$P = r^2$$
$$= 2A^2$$

$$A = \sqrt{\frac{P}{2}}$$

- Note that the **phases** of the symbols are $45^o, 135^o, 225^o, 315^o$

$135^\circ - 45^\circ = 90^\circ$

- i.e. **Phase differences** are 90^o
- Hence termed **Quadrature Phase Shift Keying**

$M = 4$

$\log_2 M = \log_2 4 = 2$

- QPSK has **2 bits** per symbol
- The **mapping** can be done as

$A \rightarrow 0$

$-A \rightarrow 1$

$$(A, A) \rightarrow 00$$
$$(A, -A) \rightarrow 01$$
$$(-A, A) \rightarrow 10$$
$$(-A, -A) \rightarrow 11$$

- Model for this **communication system** is given as

$y = x + n.$

$$\underbrace{(y_I + j y_Q)}_{y} = \underbrace{(x_I + j x_Q)}_{x} + \underbrace{(n_I + j n_Q)}_{n}$$

$\frac{N_0}{2}$ $\frac{N_0}{2}$

COMPLEX BB. NOISE!

TOTAL POWER $= N_0.$

- Observe now that y, x, n are **complex**!!!

- For power P

$$A = \sqrt{\frac{P}{2}}$$

- In phase and Quadrature components each have **half the power.** $= \frac{P}{2}$

- Noise n_I, n_Q are **Gaussian** with power $\frac{N_0}{2}$
  - Total **noise power** $= N_0$ $= \frac{N_0}{2} + \frac{N_0}{2}$
- SNR for this system is given as

$$SNR = \frac{P}{N_0} = \frac{P}{N_0} = \frac{\text{SIGNAL POWER}}{\text{NOISE POWER}}$$

- Observe that QPSK comprises of **2 parallel BPSK streams**

$$\left.\begin{aligned} y_I &= x_I + n_I \\ y_Q &= x_Q + n_Q \end{aligned}\right\} \text{2 PARALLEL STREAMS.}$$

MULTIPLEXING.

- BER of each **BPSK stream** is

$$Q\left(\sqrt{SNR}\right) = Q\left(\sqrt{\frac{P}{N_0}}\right)$$

BER FOR INPHASE OR QUADRATURE

- QPSK symbol is in error when **either of the bits** is in error
- **Symbol Error Rate (SER)** of QPSK

$x_I + j x_Q$

$1-\left(1-Q\left(\sqrt{SNR}\right)\right)^2$

EXACT SER EXPRESSION

$\approx 2Q\left(\sqrt{SNR}\right)$

$$\mathrm{SER} \approx 2 \times \mathrm{BER}$$

$$= \underbrace{2Q\left(\sqrt{\mathrm{SNR}}\right)}_{\text{SER FOR QPSK}} = 2Q\left(\sqrt{\frac{P}{N_0}}\right)$$

- **QPSK Example**
- Given $\mathrm{SNR} = 15\, dB$, what is the BER and SER for **QPSK transmission** over an AWGN channel?
- This can be calculated as follows

$$10\log_{10}\mathrm{SNR} = 15dB$$
$$\Rightarrow \log_{10}(\mathrm{SNR}) = 1.5$$
$$\Rightarrow \mathrm{SNR} = 10^{1.5} = 31.62$$

SNR
$10\sqrt{10}$.

- Therefore, the **SER** and **BER** are given as

$$\mathrm{BER} = Q\left(\sqrt{31.62}\right) = \overbrace{9.37 \times 10^{-9}}^{\text{BER}}$$

$$\mathrm{SER} = 2 \times \mathrm{BER} = \underbrace{1.87 \times 10^{-8}}_{\text{SER}}$$

QAM

2^{2n}. $n=1$ $=4$

GENERAL

- **Quadrature Amplitude Modulation**
- QAM is one of the **most important constellations**
- Used in 4G, 5G etc
- Examples: **16-QAM**, **64-QAM**

LTE = LONG TERM EVOLUTION

NEW RADIO = NR

2^{2n} QAM

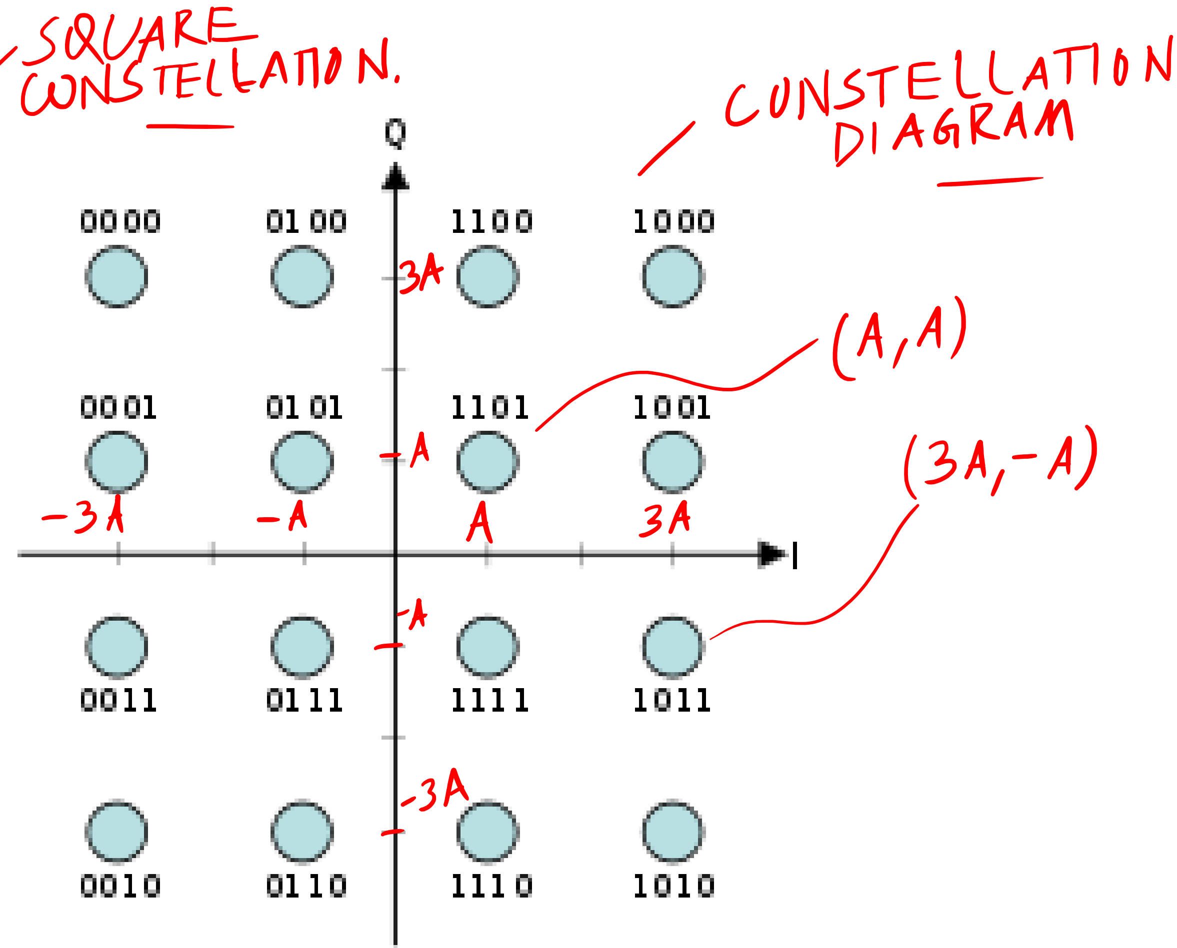
4 QAM

QPSK.

- QAM is also known as M **−QAM**
- M is the **number of symbols**
- Number of bits per symbol is $\log_2 M$
- **Example**: $M = 16 \Rightarrow \log_2 M = 4$

BITS/SYM.

16 QAM



SQUARE CONSTELLATION.
CONSTELLATION DIAGRAM
Q
I
0000
0100
1100
1000
0001
0101
1101
1001
0011
0111
1111
1011
0010
0110
1110
1010
3A
A
-A
-3A
-3A
-A
A
3A
(A, A)
(3A, -A)

16 QAM Example

DEPENDS ON POWER!

$$x_I \in \{-3A, -A, A, 3A\}$$

$$x_Q \in \{-3A, -A, A, 3A\}$$

$$x_I + jx_Q = \underbrace{-3A - j3A, -3A - jA, \ldots}_{\text{16 POSSIBLE COMBINATIONS.}}$$

INPHASE.

QUADRATURE

16 QAM.

- QAM allows to transmit **very high bit-rates**

HOM = HIGHER ORDER MODULATION.

- Example: **1024 QAM** has **$\log_2 1024$ = 10** bits per symbol!

AMC = ADAPTIVE MODULATION & CODING.

Symbol Error Rate (SER)

SYMBOL POWER

- SER for M −QAM is given as

$P = E\{|x|^2\} = E_S$

$N_0 = E\{|n|^2\}$ = NOISE POWER.

MODULATION ORDER.

$$\text{SER} \approx 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3P}{N_0(M-1)}}\right)$$

Thank You!