Inner Product : $\langle \underline{x}, y \rangle = \underline{x}^T y = \sum_{i=1}^{n} x_i y_i$

$x, y \in \mathbb{R}^n$

Euclidean norm $\|x\|^2 = \langle \underline{x}, \underline{x} \rangle = \sum x_i^2 = x^T x$

Cauchy-Schwarz inequality

$$\boxed{-\|x\| \|y\| \leq \langle x, y \rangle \leq \|x\| \|y\|}$$

scalar

$=$ if & only if $\underline{x} = \alpha y$

$$\alpha \langle x, x \rangle = \alpha \|x\| \|x\|$$

<u>Angle</u>

$\underline{x}, y \neq 0$ then $\theta = \cos^{-1}\left(\frac{\langle x, y \rangle}{\|\underline{x}\| \|y\|}\right)$

orthogonal if $\langle x, y \rangle = 0$ $\quad x \perp y$

Eg: $e_i = \begin{bmatrix} 0 \\ 0 \\ \\ 1 \\ 0 \\ \vdots \end{bmatrix}$ ← i-th location

$$\langle e_i, e_j \rangle = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

$e_1, e_2 \ldots e_n$ orthogonal (basis)

Matrix $\langle X, Y \rangle = tr(X^T Y) = \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} Y_{ij}$

$X, Y \in \mathbb{R}^{m \times n}$

Norm $\|X\|_F^2 = tr(X^T X) = \langle X, X \rangle = \sum_{i,j} X_{ij}^2$

Frobenius

Symmetric matrices: $S^n = \{ X \in \mathbb{R}^{n \times n} \mid X = X^T \}$