eMasters in Communication Systems

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Core Module:

Wireless Communication

Chapter 2

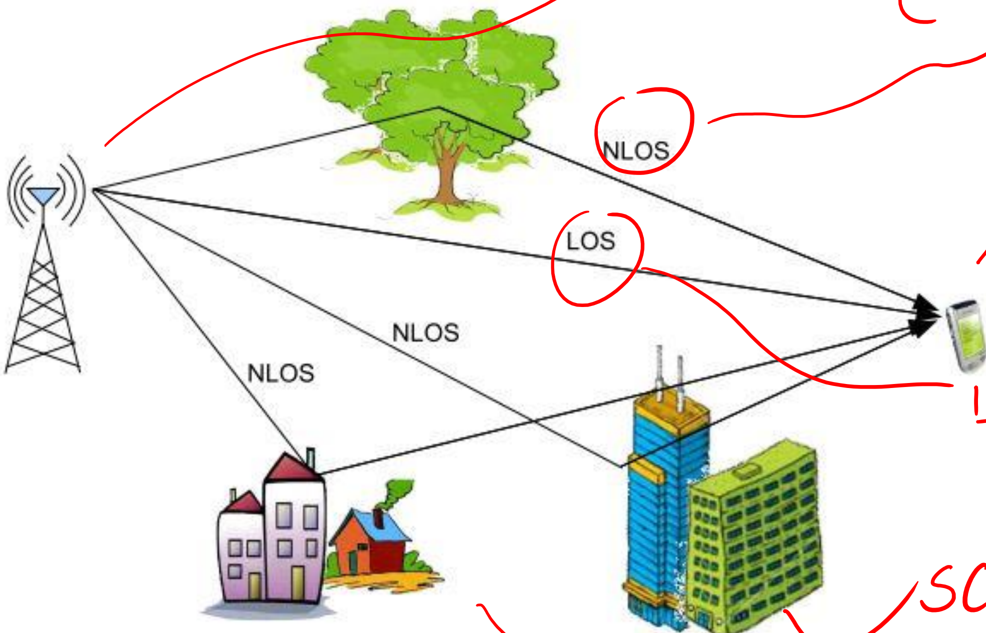
Wireless Channel and Performance

Wireless Channel

BASE STATION (BS)

NON-LINE OF SIGHT

MOBILE STATION (MS)



LOS = LINE OF SIGHT

SCATTERERS.

Figure: Multipath Propagation

Wireless channel

LOS
+SEVERAL NLOS.

- Fundamental Difference – There are **multiple propagation paths**
  - This is termed **multipath propagation**

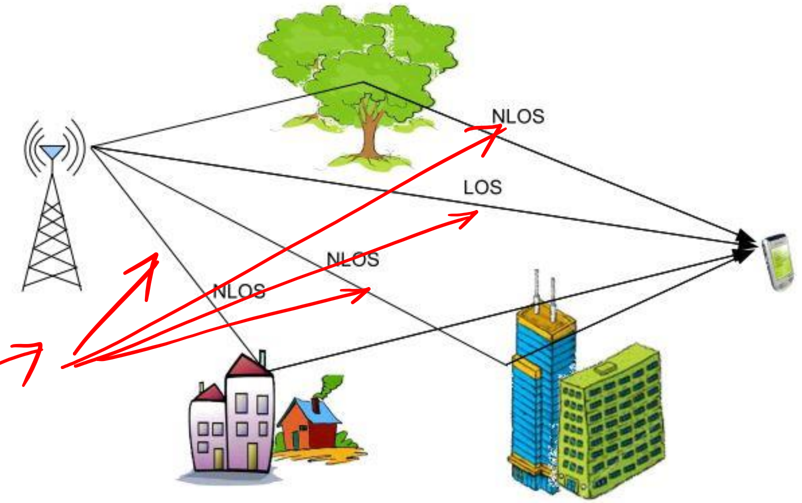


Figure: Multipath Propagation

Wireless channel

- Multipath arises due to **large objects**
  - Example: Trees, Buildings etc
  - These are termed **scatterers**

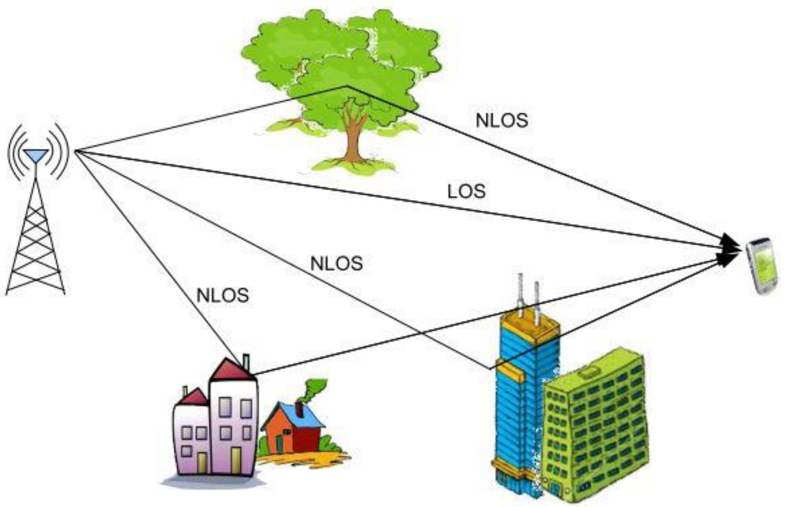


Figure: Multipath Propagation

Multipath Propagation

- This leads to **multiple copies** of the signal at the receiver
- This causes **superposition** of the signals…
  - Resulting in **interference**

MULTIPLE SIGNAL COPIES

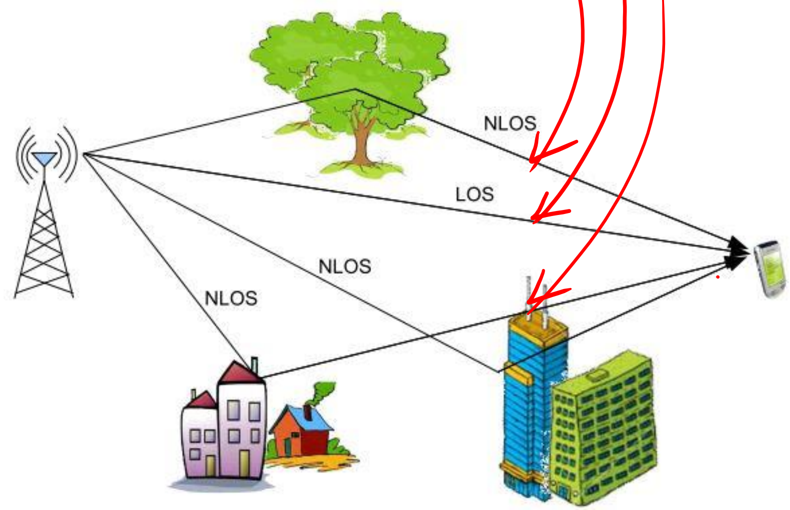


Figure: Multipath Propagation

Multipath Interference

- The interference can be constructive or destructive
- Because of this, the SNR varies or fluctuates.

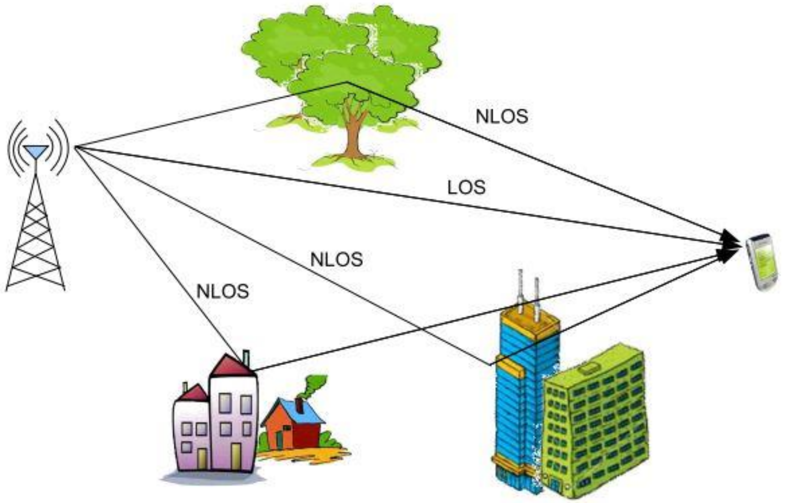


Figure: Multipath Propagation

Channel Fluctuation

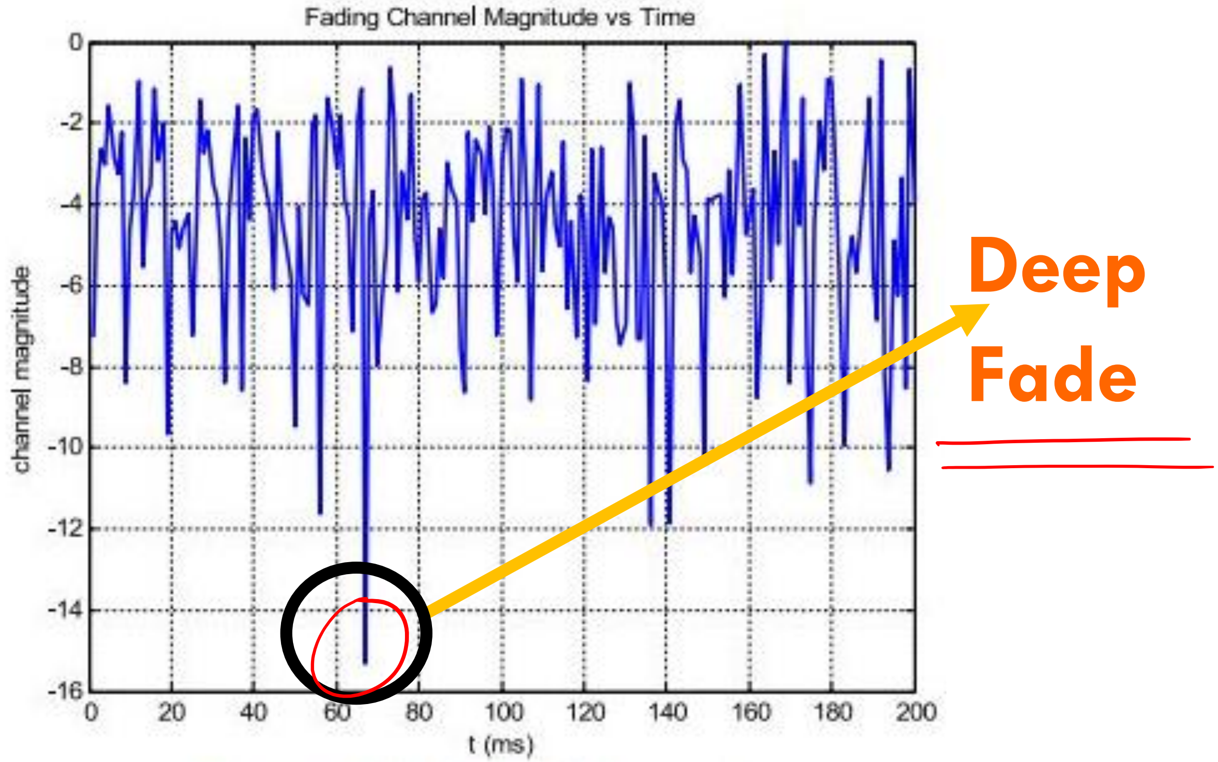


Figure: Fading Channel

Fading

FLUCTUATES.

- The received signal power varies or **fades**...
  - Therefore, the wireless channel is also termed as a **fading channel**
- Where the received power dips significantly is termed as a **DEEP FADE**

Wireless Channel Model

- The **wireless channel model** is given as

$$y = hx + n$$

COMPLEX

$y = x + n$

WIRELINE

- h is the **fading channel coefficient**

$u + jv$

Wireless Channel Model

$$y = hx + n$$

OUTPUT POWER

SIGNAL OUTPUT $= hx$

$P_o = |h|^2 \cdot P$

- Note that h determines the **output power**
  - Power is large $|h|$ is **large**
  - Else small if $|h|$ is **small**

Fading Channel Coefficient

- The fading **channel coefficient is random in nature**
- It is modeled as

RANDOM.

h_I

$$h = u + jv$$

h_Q

$E\{u\} = E\{v\} = 0$

REAL PART

IMAGINARY PART

- u, v are **independent Gaussian RVs**.
- Their mean $= 0$ and variance $= \frac{1}{2}$

$E\{u^2\} = E\{v^2\} = \frac{1}{2}$

Fading Coefficient – Amplitude and Phase

- Thus, h is a **complex Gaussian random variable** (RV)

$E\{h\} = E\{u\} + jE\{v\} = 0$.
$E\{|h|^2\} = E\{u^2 + v^2\}$
$= E\{u^2\} + E\{v^2\}$
$= \frac{1}{2} + \frac{1}{2} = 1$

- Another representation of h is

$$h = ae^{j\phi}$$

MAGNITUDE.

- where a is the **amplitude** and ϕ is the **phase**

$$a = |h|$$
$$\phi = \measuredangle h$$

$a = \sqrt{u^2 + v^2}$

Fading Channel Coefficient – Amplitude

RAYLEIGH FADING CHANNEL

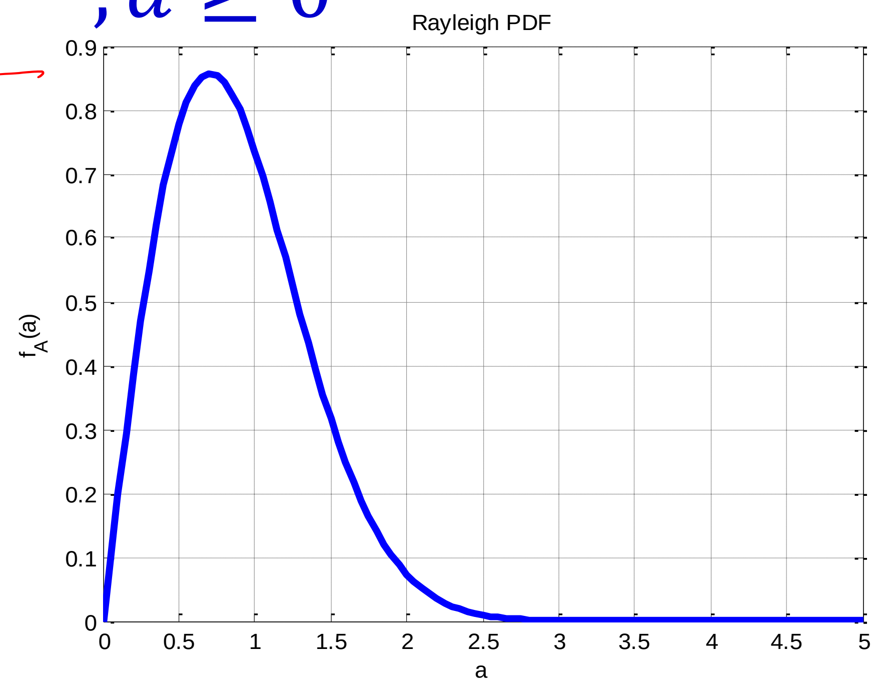
- a follows the **Rayleigh PDF**

$$f_A(a) = 2ae^{-a^2}, a \geq 0$$

$a = |h|.$

$f_A(a) = 0$

for $a < 0.$



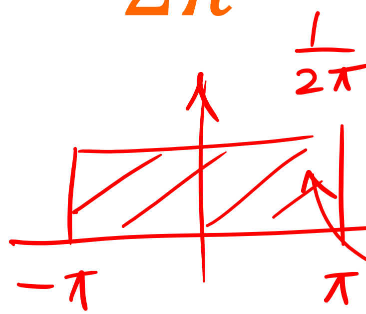
Fading Channel Coefficient – Phase

$[-\pi, \pi]$

- Phase ϕ is **uniformy distributed** in $[\pi, \pi]$

$$f_\Phi(\phi) = \frac{1}{2\pi}, -\pi < \phi \leq \pi$$

$\phi = \angle h.$

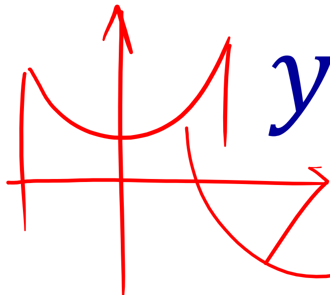


UNIFORM PDF

AREA = 1

Wireless Channel: Symbol Detection

- Consider the **BPSK constellation** $x \in \{+A, -A\}$
- **Symbol detection** can be carried as follows



$$y = hx + n$$

ESTIMATE h
CHANNEL ESTIMATION

$\frac{1}{|H(f)|}$

INVERT EFFECT OF CHANNEL h.

- At the receiver, first carry out **equalization**

$$z = \frac{1}{h} \times y = \frac{1}{h}(hx + n) = x + \frac{n}{h}$$

UNDISTORTED SIGNAL

Wireless Channel: Symbol Detection

- Recall $x \in \{+A, -A\}$
- Therefore, **detection** can be carried out as follows

DETECTED SYMBOL

$$z \geq 0 \Rightarrow \hat{x} = +A \longrightarrow 0/1$$

THRESHOLD = 0

$$z < 0 \Rightarrow \hat{x} = -A \longrightarrow 1/0$$

ESTIMATE OF x

- This is termed as a **Threshold Detector**
- It is also the **Maximum Likelihood (ML) Detector**

Wireless Channel: Output SNR

- Consider the wireless system

$$y = hx + n$$

$P_o = |h|^2 \cdot P$.

$a = |h|$.

- Recall, the **output power** is

$$|h|^2 \times E\{|x|^2\} = |h|^2 \times P = a^2 P.$$

$\cancel{h^2 \cdot P}$

Wireless Channel: Output SNR

OUTPUT POWER / NOISE POWER.

- The **output SNR** is defined as

$$SNR_o = |h|^2 \times \frac{P}{N_0/2} = |h|^2 \times SNR$$

$\frac{P}{N_0/2}$

$= a^2 SNR.$

$y = x + n$

COEFF $= 1$.

Wireless Channel Performance

$Q(\sqrt{SNR_o})$

- **BER** for BPSK is given as

$$Q\left(\sqrt{SNR_o}\right) = Q\left(\sqrt{|a|^2 \times SNR}\right)$$

$= Q(\sqrt{a^2 \times SNR})$

- However, this depends on a, which is a **random quantity**

MAGNITUDE
$a = |h|$.

- Hence, to calculate the actual BER, one has to **average** with respect to PDF of a

$f_A(a) = 2a e^{-a^2}$

Wireless BER

$\text{AVERAGE} = \int_0^\infty g(a) f_A(a)\,da.$

- Therefore, **BER for BPSK** modulation for the wireless channel is given as

FUNCTION OF a.

PDF OF a.

$$\int_{-\infty}^{\infty} Q\left(\sqrt{a^2 \times SNR}\right) f_A(a)\,da$$

PDF

$SNR = \frac{2P}{N_0}$. BPSK.

$$= \int_{-\infty}^{\infty} Q\left(\sqrt{a^2 \times SNR}\right) 2ae^{-a^2}\,da$$

$a = |h|$

$$= \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right) = \frac{1}{2}\left(1 - \sqrt{\frac{2P/N_0}{2+2P/N_0}}\right)$$

WIRELINE $\sim Q(\sqrt{SNR})$

Wireless BER Example

$10\log_{10} SNR = 12$
$\Rightarrow \log_{10} SNR = 1.2$
$SNR = 10^{1.2}$

- Example: Evaluate the **BER of a wireless channel** with **BPSK transmission** $\mathrm{SNR} = 12\ dB$

$$\mathrm{SNR} = 10^{1.2} = 15.85$$

$$\frac{1}{2}\left(1-\sqrt{\frac{15.85}{2+15.85}}\right)$$

$$\mathrm{BER} = \frac{1}{2}\left(1-\sqrt{\frac{SNR}{2+SNR}}\right) = 0.0288$$

$= 2.88 \times 10^{-2}$

BER

$$= 2.88 \times 10^{-2}$$

Wireless BER Example

12 dB

- Recall BER of wireline channel for same SNR is 3.44×10^{-5}

BER OF WIRELESS SIGNIFICANTLY HIGHER THAN BER OF WIRELINE !!

- BER of wireless channel is **very high!!**

$$Q\left(\sqrt{15.85}\right) = 3.44 \times 10^{-5} = BER_{WIRELINE}$$

$$BER_{WIRELESS} = 2.88 \times 10^{-2}$$

$$BER_{WIRELESS} \gg BER_{WIRELINE} \quad !!$$

Another comparison...

$10 \log_{10} SNR = 20.$

- Consider **SNR = 20 dB = 10^2 = 100**

$$\mathrm{BER}_{\mathrm{wireline}} = Q\left(\sqrt{100}\right) = 7.62 \times 10^{-24} \approx 10^{-24}$$

$$\mathrm{BER}_{\mathrm{wireless}} = \frac{1}{2}\left(1 - \sqrt{\frac{100}{102}}\right) \approx 5 \times 10^{-3} \approx 10^{-3}$$

$\mathrm{BER}_{\mathrm{WIRELESS}} \approx 10^{21} \times \mathrm{BER}_{\mathrm{WIRELINE}}$

- **Why is the BER of wireless so high?**

$\frac{1}{2}\left(1 - \sqrt{SNR/(2+SNR)}\right)$

Wireline BER Simplified

$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$

SIMPLE.
BOUND FOR
Q-FUNCTION.

- Note that

$$Q(x) \leq \frac{1}{2} e^{-\frac{1}{2}x^2}$$

- The **wireline BER** can be simplified as follows

$$Q\left(\sqrt{SNR}\right) \leq \frac{1}{2} e^{-\frac{1}{2}SNR}$$

$\Rightarrow$ DECREASES EXPONENTIALLY!

$BER_{WIRELINE}$.

$\approx e^{-\frac{1}{2}SNR}$

e^{-kx} $k > 0$.

- Therefore, wireline BER **decreases exponentially!!!**

Wireless BER Simplified

- The expression for the **wireless BER** can be simplified as follows

AT HIGH SNR. $\frac{2}{SNR} \to 0$ VERY SMALL!

$$\frac{1}{2}\left(1-\sqrt{\frac{SNR}{2+SNR}}\right)=\frac{1}{2}\left(1-\sqrt{\frac{1}{\frac{2}{SNR}+1}}\right)$$

$(1+x)^{-\frac{1}{2}} \approx 1-\frac{1}{2}x$

FIRST ORDER TAYLOR SERIES APPROXIMATION

$$=\frac{1}{2}\left(1-\left(\frac{2}{SNR}+1\right)^{-\frac{1}{2}}\right)\approx\frac{1}{2}\left(\cancel{1}-\left(\cancel{1}-\frac{1}{\cancel{2}}\times\frac{\cancel{2}}{SNR}\right)\right)$$

$$BER_{WIRELESS} \approx =\frac{1}{2}\times\frac{1}{SNR} \approx \frac{1}{SNR}$$

$\left(1+\frac{2}{SNR}\right)^{-1/2} \approx 1-\frac{1}{2}\cdot\frac{2}{SNR} = 1-\frac{1}{SNR}$

- **Wireless BER** only decreases as $\frac{1}{SNR}$

VERY SLOW!

Another view…

- Another way to look at this is as follows
- Consider BER = 10^{-6}. — BER $= 10^{-6}$.
- Let us calculate **SNR required** to achieve this in both wired and wireless channels

WIRELINE

Wired Channel

BER $= 10^{-6}$.

- The SNR for **wired channel** can be calculated as follows

inverse Q function
- TABLES.
- MATLAB.
- PYTHON

$$Q\left(\sqrt{SNR}\right) = 10^{-6}$$

$$\Rightarrow \sqrt{SNR} = Q^{-1}\left(10^{-6}\right) \approx 4.75$$

$$\Rightarrow SNR = 4.75^2 = 22.56$$

$$= 10\log_{10} 22.56\,dB = 13.53\,dB$$

SNR in dB REQD. TO ACHIEVE BER $= 10^{-6}$ IN WIRED.

SNR WIRELINE

Wireless Channel

- The SNR for **wireless channel** can be calculated as follows

$BER_{WIRELESS}$.

$$\frac{1}{2}\left(1-\sqrt{\frac{SNR}{2+SNR}}\right)=10^{-6}$$

$$1-\sqrt{\frac{SNR}{2+SNR}}=2\times 10^{-6}$$

$$\sqrt{\frac{SNR}{2+SNR}}=1-2\times 10^{-6}.$$

$$\Rightarrow \sqrt{\frac{SNR}{2+SNR}}=1-2\times 10^{-6}$$

Wireless Channel

SNR in dB REQD TO ACHIEVE BER $=10^{-6}$ IN WIRELESS.

$$\frac{SNR}{2+SNR} = \left(1-2\times10^{-6}\right)^2$$

$$\Rightarrow SNR = 2\times\frac{\left(1-2\times10^{-6}\right)^2}{1-\left(1-2\times10^{-6}\right)^2}$$

$$= 5\times10^5$$

$$\Rightarrow SNR = 10\log_{10}5\times10^5 \approx 57\,dB$$

SNR WIRELESS.

Wireless Channel

- We could have also done this calculation in **a much simpler fashion**
- Recall, **BER approximation** for wireless channel is

$$BER_{WIRELESS} \approx \frac{1}{2} \times \frac{1}{SNR} = 10^{-6} = \text{TARGET BER}$$

$$\Rightarrow SNR = \frac{1}{2 \times 10^{-6}}.$$

Wireless Channel

- Therefore, we have

$$\frac{1}{2} \times \frac{1}{SNR} = 10^{-6}$$

$$\Rightarrow SNR = \frac{1}{2 \times 10^{-6}} = 5 \times 10^5 = 57\ dB$$

$10 \log_{10} 5 \times 10^5$
$= 57 dB.$

$$10\log_{10}5\times10^5 = 10\log_{10}\tfrac{1}{2}\times\overline{10\times10^5}$$

$$10^6.$$

$$= 10\log_{10}\tfrac{1}{2}\times10^6$$

$$= \underbrace{10\log_{10}\tfrac{1}{2}} + \underbrace{10\log_{10}10^6}_{=10\times6=60.}$$

$$= -\underbrace{10\log_{10}2} + 60$$

$$= -3+60 = \underline{\underline{57\,dB}}$$

$$10\log_{10}2 \approx 3\,dB$$

Wireless versus Wired Comparison

TARGET BER = 10^{-6}

- Let us now compare **wired versus wireless**

BER = 10^{-6}.

2.256×10^1.

$$SNR_{wired} = 13.53\ dB = 22.56 \approx 10^1$$

$$SNR_{wireless} = 57\ dB = 5 \times 10^5 \approx 10^5$$

$SNR = \dfrac{\text{SIGNAL POWER}}{\text{NOISE POWER}}$.

Wireless versus Wired Comparison

RATIO OF SNRS.

$$\frac{P_{WIRELESS}}{P_{WIRED}}$$

$$\frac{SNR_{wireless}}{SNR_{wired}} = \frac{5 \times 10^5}{22.56} \approx 22{,}000$$

$$= 2.2 \times 10^4$$

$$\sim 10^4$$

$$SNR = \frac{P}{N_0/2}$$

SIGNIFICANTLY HIGH POWER!

- This implies wireless system requires **22,000 times** more transmit power!

$$\frac{P_{WIRELESS}}{P_{WIRED}} \sim 10^4$$

What is the Reason?

- This is a consequence of the DEEP FADE (DF) phenomenon

$P_{WIRELESS} \gg P_{WIRELINE}$?

Deep Fade

- Recall, in a wireless system, the SNR **fluctuates**.

FADING.

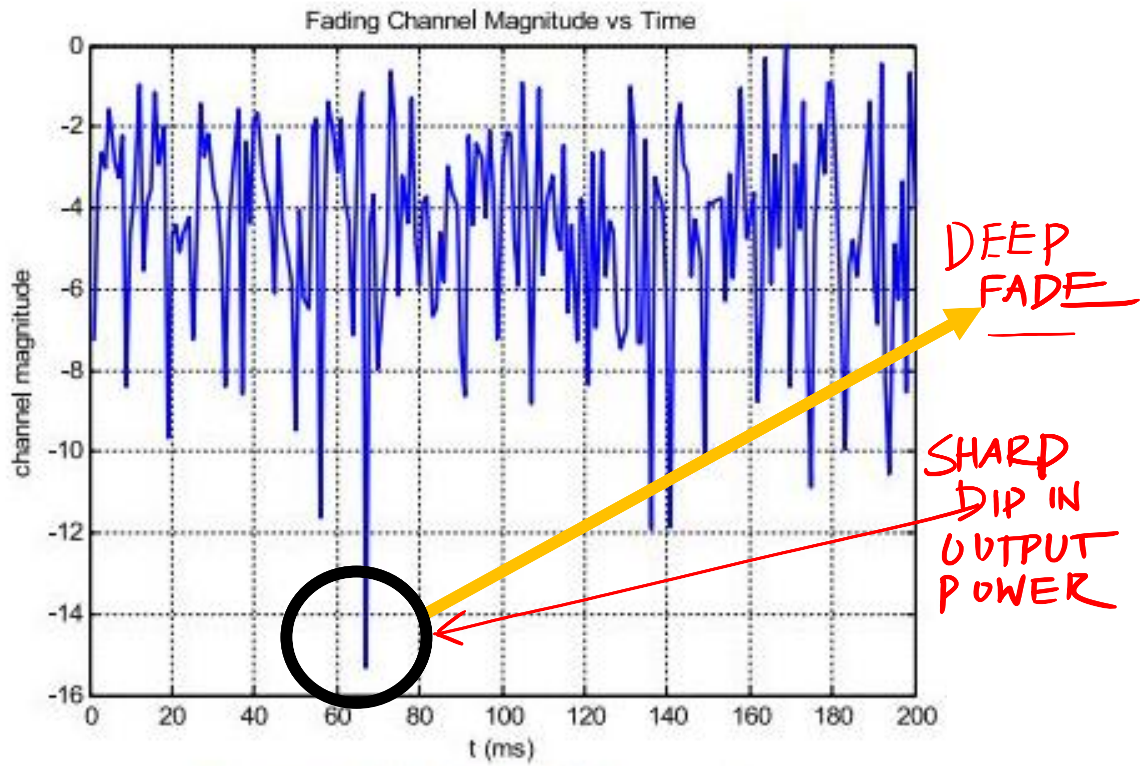


Figure: Fading Channel

Deep Fade

- **Deep fade** is a **sharp drop** in the SNR

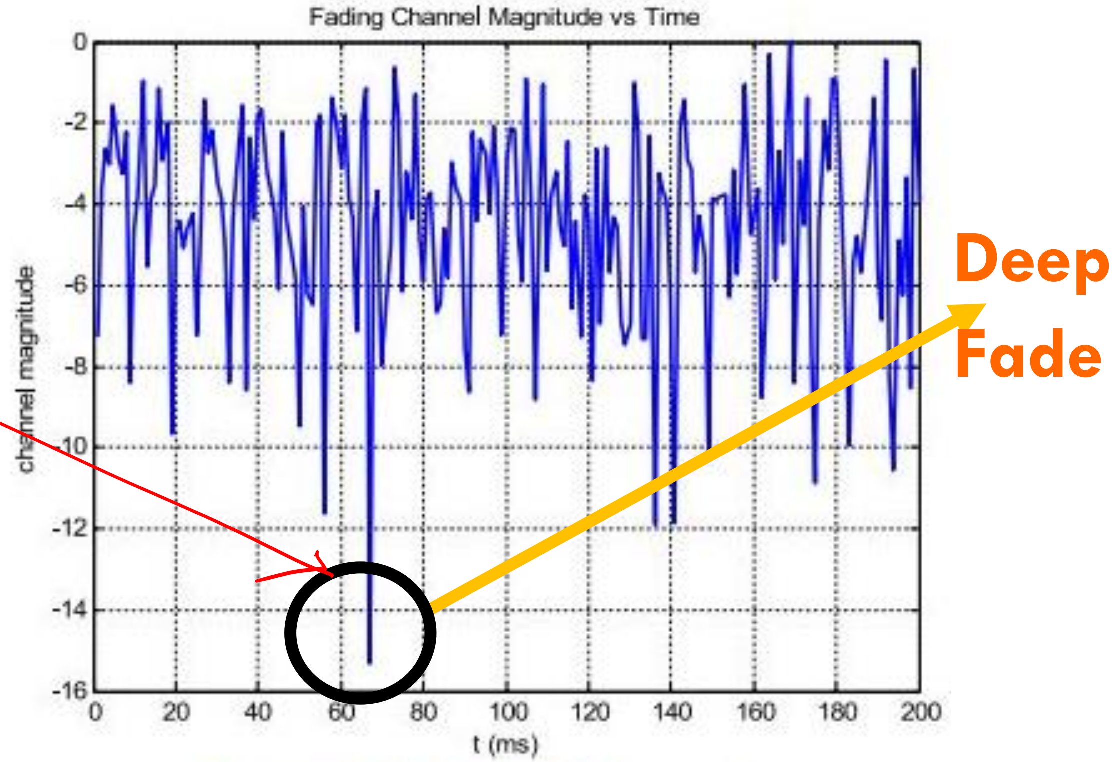


Figure: Fading Channel

Deep Fade

- This implies the **signal is buried in noise**
- i.e., **Output signal power < Noise Power**

$\Rightarrow |h|^2 P < N_0$

NOISE POWER

OUTPUT SIGNAL POWER

NOISE LEVEL

SIGNAL LEVEL

Deep Fade

$a^2 < \frac{1}{SNR}$

CONDITION FOR DEEP FADE

- The condition for this is

$$|h|^2 P < N_0$$

$$\Rightarrow |h|^2 = a^2 < \frac{N_0}{P} = \frac{1}{SNR}$$

Probability of Deep Fade

$P_{DF} = ?$

- One can now evaluate the **probability of deep fade** P_{DF} as follows

$$P_{DF} = \Pr\left(a^2 < \frac{1}{SNR}\right)$$

$$= \Pr\left(a < \frac{1}{\sqrt{SNR}}\right)$$

$a = |h|$.

a = RAYLEIGH RV

Probability of Deep Fade

$f_A(a)$ = PDF of a.

$$P_{DF} = \Pr\left(a < \frac{1}{\sqrt{SNR}}\right)$$

$$= \int_0^{\frac{1}{\sqrt{SNR}}} f_A(a)\,da = \int_0^{\frac{1}{\sqrt{SNR}}} 2a e^{-a^2}\,da$$

$$= -e^{-a^2}\Big|_0^{\frac{1}{\sqrt{SNR}}} = 1 - e^{-\frac{1}{SNR}} = P_{DF}$$

Probability of Deep Fade

$e^{-x} \approx 1 - x$

$x \approx 0$

$$P_{DF} = 1 - e^{-\frac{1}{SNR}}$$

$x = \frac{1}{SNR}$

$SNR \to \infty$

$\frac{1}{SNR} \to 0.$

$$\approx \cancel{1} - \left(\cancel{1} - \frac{1}{SNR}\right) \cong \frac{1}{SNR} = P_{DF}$$

- This result is very interesting, because…

DEEP FADE.

$$BER = \frac{1}{2\,SNR} = \frac{1}{2} P_{DF}$$

$BER = \frac{1}{2} \cdot \frac{1}{SNR} = \frac{1}{2} \times P_{DF}.$

Probability of Deep Fade

- This implies that

$$BER \propto \frac{1}{2} P_{DF}$$

$BER \propto P_{DF}$

Bit Error Rate $\propto$ PROB. OF DEEP. FADE $\propto \frac{1}{SNR.}$

- Thus, the **high BER** of the wireless channel is a direct consequence of **deep fade**!

Deep Fade

- **Deep fade** causes a **huge spike** in the BER
- This **dominates** the BER

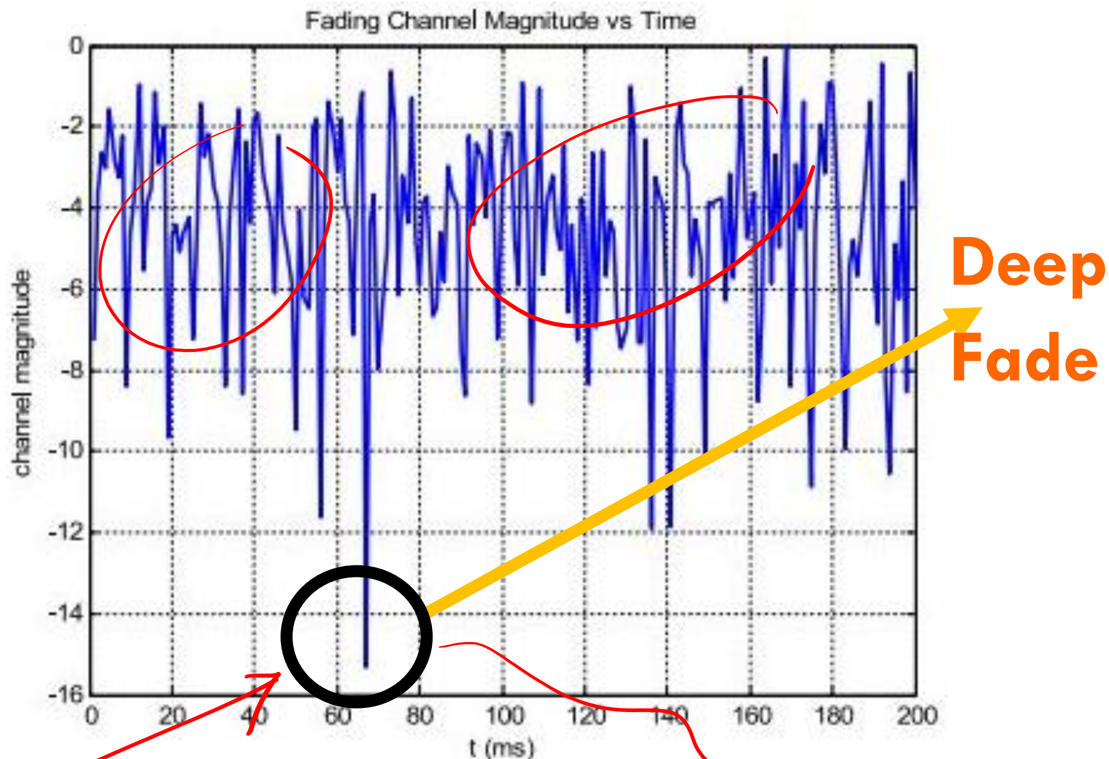


Figure: Fading Channel

DOMINATES BER.

EXTREMELY HIGH BER

Deep Fade

- How to **improve the BER** of wireless?
- One has to **overcome** the problem of deep fade!

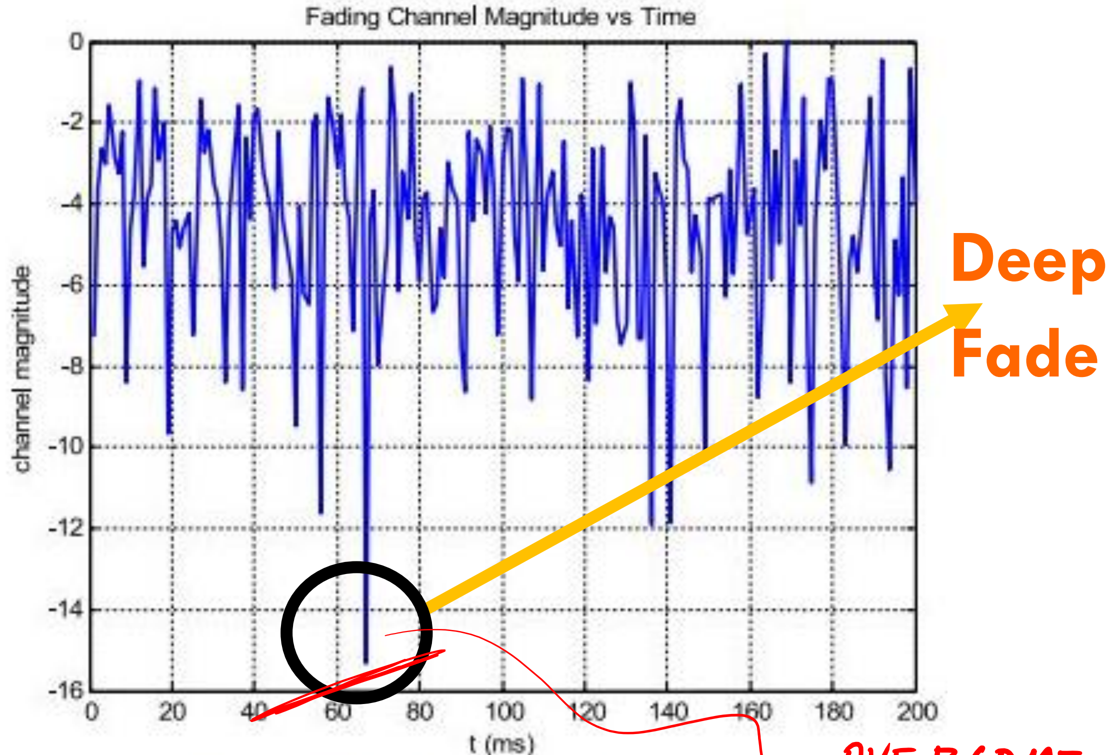


Figure: Fading Channel

OVERCOME DEEP-FADE.

BER and SER for QPSK and QAM

BIT ERROR RATE

SYMBOL ERROR RATE

- Before we conclude,…
- the BER and SER expressions for QPSK and QAM in a wireless system are as follows

QPSK = QUADRATURE PSK.

QAM = QUADRATURE AMPLITUDE MODULATION

BER and SER of QPSK

$x_I + j x_Q$

BPSK.

BPSK.

$SNR = \frac{P}{N_0}$.

- BER of each **BPSK stream** is

$$= \frac{1}{2} \times \frac{1}{SNR} = \frac{1}{2} \times \frac{N_0}{P} = \frac{1}{2} \times \frac{1}{SNR}$$

- Overall SER of the QAM is

$$SER \approx 2 \times BER = \frac{N_0}{P} = \frac{1}{SNR}$$

ONLY DECREASES AS $\frac{1}{SNR}$.

SER of QAM

Mary − QAM

#bits/sym = $\log_2 M$.

$\frac{P}{N_0} = SNR$.

- SER of M −QAM is

$$4\left(1-\frac{1}{\sqrt{M}}\right)\times\frac{1}{2}\times\frac{1}{\frac{3P}{N_0(M-1)}}$$

BPSK
QPSK
QAM
$\propto \frac{1}{SNR}$.

$$=4\left(1-\frac{1}{\sqrt{M}}\right)\times\frac{1}{2}\times\frac{(M-1)}{3\times SNR} \propto \frac{1}{SNR}$$

BER and SER of QPSK/QAM

- Thus, one can see that BER and SER are **once again proportional to** $\frac{1}{SNR}$.

- Thus, one cannot improve the performance by simply **changing the modulation**.

How to Improve Wireless Performance?

- What then is the **solution** to improving wireless performance?
- This can be achieved via DIVERSITY!…
  - which is our next focus.

DIVERSITY! FUNDAMENTAL TO WIRELESS SYSTEM.

Thank You!

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