STAT 305 2024S2 **Instructor: Vivian Meng**
Lecture 6.2 **Date: July 16, 2024**

Learning Goals

G1. Bayesian prior to posterior update

General formula for Bayesian prior to posterior update

Theorem 6.2 of course notes:
Let observable Y be a RV with pmf/pdf $f_{Y|\Theta}(y|\theta)$, and Θ a RV representing our uncertainty about the model parameter value θ. Given prior distribution $p_\Theta(\theta)$ for Θ, the conditional/posterior distribution of θ given Y takes value y is

$$p_{\Theta|Y}(\theta|y) = \frac{f_{Y|\Theta}(y|\theta)p_\Theta(\theta)}{f_Y(y)}$$

where

$$f_Y(y) = \begin{cases} \sum_\theta f_{Y|\Theta}(y|\theta)p_\Theta(\theta) & \text{(if } \Theta \text{ is discrete)} \\ \int f_{Y|\Theta}(y|\theta)p_\Theta(\theta)\mathrm{d}\theta & \text{(if } \Theta \text{ is continuous),} \end{cases}$$

and the summation or integration is over all values θ with $p_\Theta(\theta) > 0$.

Let's examine a couple of things:

- $p_\Theta(\theta)$ is a

- $f_{Y|\Theta}(y|\theta)$ is a

- $f_Y(y)$ is

- Posterior distribution of Θ given $Y = y$ is

What is the form for the posterior distribution for observing $Y_1 = y_1, \ldots, Y_n = y_n$ if the joint likelihood depends on a parameter θ?

Example: small sample Bayesian inference

Let X_i represent the lifespan (measured in years) of the i-th sampled specimen of a bird of the species *Turdus philomelos* (Song Thrush). Goal is to infer average lifespan.

- We assume X_i, $i = 1, \ldots, 3$ to be i.i.d. samples from $\mathcal{E}(\lambda)$ distribution. Let $\theta = \dfrac{1}{\lambda}$. We will use the likelihood function written in terms of θ parameterization.
- We model uncertainty with regards to θ via RV Θ with prior distribution

$$p_\Theta(\theta) = \frac{\exp\left(-\frac{1}{2(1.5)^2}(\theta-3)^2\right)\mathbb{1}_{(0,\infty)}(\theta)}{\int_0^\infty \exp\left(-\frac{1}{2(1.5)^2}(\theta-3)^2\right)\mathrm{d}\theta},$$

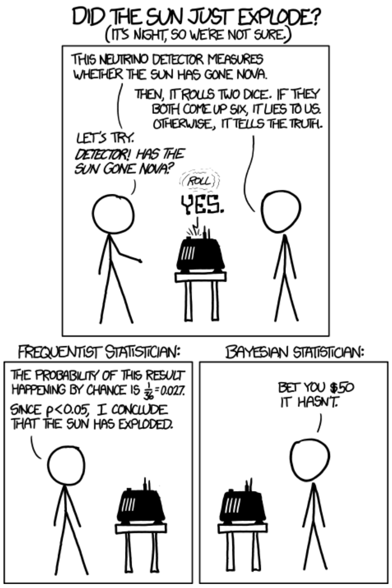
  noting

$$\mathbb{1}_{(0,\infty)}(\theta) = \begin{cases} 1 & \text{if } \theta \in (0,\infty) \\ 0 & \text{otherwise.} \end{cases}$$

- Visualize the prior $p_\Theta(\theta)$
- Write down the posterior distribution of Θ given $X_1 = 3, X_2 = 7, X_3 = 3.2$, then visualize this posterior distribution.

Lecture 6.2 Student Activities

1. Explain the following XKCD comic strip in terms of Bayesian prior to posterior update. (Source: https://xkcd.com/1132/)



2. A typical Bayesian application question. We will use Bayesian inference/learning to update our view about the proportion of UBC students who use their u-pass more than 3 days a week during the Winter terms in light of survey data. The survey consists of n randomly sampled UBC students, and let

$$X_i = \begin{cases} 1 & \text{if student uses uPass on more than 3 days a week on average} \\ 0 & \text{otherwise,} \end{cases}$$

and are assumed to be i.i.d. for all $i = 1, \dots, n$.

   (a) State your data model and reasoning.

   (b) What are you trying to learn in terms of the data-model parameter?

   (c) Write down the (joint)-likelihood for the data given parameter.

   (d) Write down your prior distribution for the model parameter and justify your choice, including hyper-parameters. You should choose one of the distributions in Table 1.4.

   (e) Write down the posterior distribution of the model parameter given data? (Simplify it as much as possible... but we will learn more about techniques in Lecture 6.3.)