Modeling Philippine inflation using Fourier transformation: A new approach

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1 Introduction

Inflation is an economic phenomenon in the form of price increase (Mankiw, 2018). Most economic institutions in the Philippines adopted inflation targeting as framework for monetary policymaking. Since then, inflation forecasting models have been an integral component of the policymaking process. These institutions employ a suite of models to nowcast and forecast inflation and other key macroeconomic variables such as output and money supply. These models range from simple autoregressive integrated moving average (ARIMA) models to a more complex macroeconomic model with an endogenous monetary policy rule.

The current models used to analyze inflation is in the form of time series. Time series is the collection of observation in the time domain (Hamilton, 2020). It may contain trend, seasonality, cycle, and stochastic component. Thus, a large body of empirical literature is devoted to modeling inflation. With regards to time series data, the simplest and perhaps most commonly used statistical tools to forecast inflation are the auto regressive (AR) and ARIMA models (Moser, Rumler and Scharler, 2007), (Baciu, 2015). More recently there have been attempts to look inflation in a different domain, the frequency domain (Ftiti, Essaadi et al., 2008), (Iacobucci, 2005). In this study, we try to use frequency domain to model the Philippine aggregate inflation series.

When we look at time sereies in the frequency domain, a stationary series has a wave-like pattern as it has constant mean and constant variance. Waves have a period which is the distance between peaks or time between two peaks of a wave. Frequency is the closely related property of the wave period and is just the proportion of cycle that occurs during one observation. Frequency domain is used to analyze the data with respect to frequency.

The most common way to transform time domain function into a frequency domain function is through Fourier transformation developed by Joseph Fourier in 1822 (Culling, 1960). This frequency domain analysis is also known as spectral analysis or Fourier analysis. In this paper, spectral analysis approach with periodogram analysis is used to model the monthly inflation rate of the Philippines. Further, the resulting model will be compared with autoregressive integrated moving average (ARIMA) which is the commonly used tool by Economic institutions to model inflation.

This paper develops a model based on Fourier analysis to model the Philippine inflation and compares it with the commonly used statistical model for forecasting time series in econometrics - ARIMA, developed by Box and Jenkins (Box and Jenkins, 1976). Their performance will be assessed using performance metrics. The remainder of the paper is as follows: Section 2 describes briefly discuss the mathematical theory of Fourier analysis, Section 3 gives a brief literature review.

2 Mathematical theory on Fourier analysis development

This section briefly provides mathematical considerations on the Fourier development.

Let a function $f : R \to R$, with f and f' piecewise continous on R and periodic with period T, therefore $f(x+T) = f(x) \forall x \in R$ Considering Fourier series associated with the function $f : F(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty}(a_k \cos \frac{2k\pi x}{T} + b_k \sin \frac{2k\pi x}{T})$ we then have the following,

$$\int_{\frac{-T}{2}}^{\frac{T}{2}} f(x) \cos \frac{2n\pi x}{T} dx = \frac{a_n T}{2}, n \geq 0, \int_{\frac{-T}{2}}^{\frac{T}{2}} f(x) \sin \frac{2n\pi x}{T} dx = \frac{b_n T}{2}, n \geq 1 \tag{1}$$

From Fourier series expression, it is observed that $F(x+T) = F(x) \forall x \in R$ so its sum is also a periodic function of period T

The Dirichlet's Theorem (Weisstein, 2004) states that in the conditions above, the Fourier series should converge punctually to f in the points of continuity and to $\frac{f(x+0)+f(x-0)}{2}$ in the discontinuity points.

Considering the partial sum of order n corresponding to the series of function F, the n-th Fourier polynomials is then given by

$$F_n(x) = \frac{a_0}{2} + \sum_{k=1}^{n}(a_k \cos \frac{2k\pi x}{T} + b_k \sin \frac{2k\pi x}{T}) \tag{2}$$

The Fourier polynomials have the property of approximating the function through one periodical with the observation that the absolute error tends to fall (due to the convergence points) with the rise of n.

Due to the existence of an important number of cyclical phenomena in many scientific fields, we intend, below, to approximate their development by means of Fourier polynomials of degree conveniently chosen.

In the case of the discretized phenomenons, we put the problem in the generation of functions that will pass through a series of data points. A very useful tool is the Lagrange interpolation polynomial. Therefore, considering a set of data $(x_i, y_i), i = 1, k+1$, the Lagrange interpolation polynomial takes the form given in **Equation 3**. Further, **Equation 3** is the polynomial of minimum degree k passing through the data points.

$$L + n(x) = \sum_{i=1}^{k+1} \frac{(x - x_1)...(x - x_{i-1})(x - x_{i+1})...(x - x_n)}{(x_i - x_1)...(x_i - x_{i-1})(x_i - x_{i+1})...(x_i - x_n)} y_i \tag{3}$$

3 Literature review

There is a growing body of literature on the empirical application of frequency domain analysis for modeling in the fields of economics and finance. Results thus far have generally been favorable attesting to the usefulness of said techniques for these fields.

One study (Konarasinghe, Abeynayake and Gunaratne, 2016) was conducted to develop a model to forecast Returns of Sri Lankan share market using Fourier transform. The model was named as “Cirucular model (CM)" which forecasts individual company returns. The study revealed that the CM was successful in forecasting monthly returns of 80% of the companies. They have concluded that CM is a suitable forecasting technique for Sri Lankan share market. However, the authors of the paper have noted that CM is only appropriate if the data shows no trend (detrended).

Another study was conducted by Thomson and Vuuren in 2016 (Thomson and Van Vuuren, 2016) to propose a spectral analysis to determine the duration of South African business cycle which is measured by using log changes in nominal gross domestic product (GDP). Three dominant cycles are used to forecast log monthly nominal GDP and forecasts are compared with historical data. The authors found that spectral analysis is more effective in estimating the business cycle length as well as in determining the position of the economy in the business cycle.

A study by Sella et al. in 2016 (Sella et al., 2010) which applies several advanced spectral analysis methods to analyse GDP fluctuations in Italy, Netherlands, and the United Kingdom. They have noted that these analysis tools allowed them to spectrally decompose, as well as reconstruct GDP time series from the data. They further noted that the models are well adapted to the analysis of short and noisy data like the GDP time series in their particular study.

To go further, a study (de Melo et al., 2015) was also conducted to use wavelet transform which is an extensiion of frequency domain models in economics and finanace. This study considered a generalized wavelet that in a particular case of unit amplitudes reduces to the Morlet wavelet. The introduced wavelet appears to be handy for the wavelet processing as it enables even very small details. The study shows some remarkable properties of frequency domain models which make them powerful tools to be used in economics. Thus, it can be seen that there have been attempts to integrate frequency domain analysis in the field of economics.

4 Research Methodology

The main objective of this study is to develop a significant model to forecast inflation in the Philippines. For this purpose, monthly consumer price index (CPI) data is gathered from the Philippine Statistics Authority (PSA) and covers the period from January 1994 to December 2020.

First, data preparation is used to transform the CPI data to inflation rates. Then, periodogram analysis is used to find inflation cycle as inflation is affected by many factors which may cause seasonality and periodicity in the series. After which, the period or frequency of sereis, significance of selected period is tested by a test developed by Fisher in 1929 (Fisher, 1929). Fourier analysis is then used to model inflation by utilizing selected frequency as Fourier frequency. On the other hand, ARIMA methodology will also be used to model the inflation data. However, in this study, performing ARIMA is not extensively discussed as this methodology has been well elaborated in several economic literatures. To assess the performance of the two models, this paper will utilize root mean square error (RMSE) and Mean Absolute Error (MAE) as the performance metrics (see **Equations 4** & **5**). By using these accuracy measures, we can conclude which model is superior based on smaller values of RMSE and MAE.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_i^2} \tag{4}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n} |e_i| \tag{5}$$

4.1 Data preparation

Inflation is defined as the percent difference between the current CPI and the CPI of the previous year as seen in **Equation 6** below:

$$\pi_{year-on-year} = \frac{CPI_t - CPI_{t-1}}{CPI_{t-1}} * 100 \tag{6}$$

where CPI_t is the CPI of the current month while CPI_{t-1} is the same month's CPI from the previous year.

4.2 Periodogram analysis

Periodogram is a tool that partitions the total variance of a time series into component variances. This method is similar to ANOVA. The longer cycle shares large variance in the series. In general practice, when periods of cycles are not known then Periodogram is utilized to identify dominant cyclic behavior in the series. In periodogram, time series can be viewed as

$$Y_t = T_t + \sum_{i=1}^{N}(a_i \cos w_i t + b_i \sin w_i t) + \epsilon_t \tag{7}$$

where T_t is trend, N is total number of observations, a_i and b_i are coefficients, w_i is angular frequency in radians and ϵ_t is the error term. The coefficients are calculated which is presented in **Equations 8** & **9** below.

$$a_i = \frac{2}{N}\sum_{t=1}^{N}(Y_t - \hat{T}_t)\cos w_i t \tag{8}$$

$$b_i = \frac{2}{N}\sum_{t=1}^{N}(Y_t - \hat{T}_t)\sin w_i t \tag{9}$$

The calculated coefficients are then used to obtain Intensity of Periodogram coordinate at frequency f_i as

$$I(f_i) = \frac{2}{N}(a_i^2 + b_i^2) \tag{10}$$

where $i = 1, 2, 3, .q$. In case of even number of observations $N = 2q, q = n/2$ and for odd number of observations $N = 2q + 1, q = (N - 1)/2$. The period against the largest Intensity that is actually the largest sum of squares is selected as cycle period of series.

4.3 Significance test

The variability in the sizes of sum of squares may be due to just sampling error. The largest ordinate must indicate strong periodicity even for white noise series. Therefore it is necessary to test the significance of largest periodic component in white noise. A test developed by Fisher in 1929 (Fisher, 1929) provides a reasonable method for testing significance of such periodic components. To perform Fisher test, g statistic is computed which is the ratio of largest sum of squares (or intensity ordinate) to the total sum of squares. A table of reference is given by Russel (Russel, 1985) which is the critical values for the test statistic. The null hypothesis of white noise series is rejected, if the value of g statistic is greater than critical value.

4.4 Spectral analysis

The basic idea of spectral analysis is to transform the time domain series into frequency domain series, which determines the importance of each frequency in the original series. This target is achieved by using Fourier transformation.

The general Fourier series model that contain components of time series is given by

$$Y_t = T_t + \sum_{i=1}^{k} (\alpha_i \cos iwt + \beta_i \sin iwt) + \epsilon_t \tag{11}$$

where $w = 2\pi f$, $k = n/2$ in case of even number, $k = (n-1)/2$ in case of odd number, n is the number of observations per season or cycle length, f is the Fourier frequency of number of peaks in series, k is the harmoninic of w - a more detailed explanation for this is given in the study by Delurgio (Armstrong, 2001), t is the time index, α_i and β_i are amplitudes which are estimated through multiple linear regression analysis.

4.5 Development of control model

The control model for this study is ARIMA presented by Box & Jenkins (Box and Jenkins, 1976). The model requires the time series to be stationarity. Afterwards parameters p, d, and q, the autoregressive terms, times differenced, and moving average terms respectively. This model is governed by a linear equation in which predictors are lags of dependent variable and lags of error terms, presented in **Equation 12** below.

$$\phi_p(B)\Delta^d Y = \theta_q(B)\epsilon_t \tag{12}$$

where Y_t is the present value, d is the difference, B is the backshift operator, and ϵ_t is the error term.

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