MT18242_A4_VM

November 8, 2019

1 DSc Assignment 4

1.1 Survival Analysis

[2]:
```
# Data taken from http://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_Survival/
↪BS704_Survival_print.html
```

[3]:
```
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt
from random import sample
```

[4]:
```
time = [0, 1, 2, 3, 5, 6, 9, 10, 11, 12, 13, 14, 17, 18, 19, 21, 23, 24]
nt =  [20, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 7, 6, 5, 4, 3]
dt = [0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0]

df = pd.DataFrame({'Time':time,
                   'Number at Risk':nt,
                   'Number of Deaths':dt})
```

[5]:
```
df
```

[5]:

	Time	Number at Risk	Number of Deaths
0	0	20	0
1	1	20	1
2	2	19	0
3	3	18	1
4	5	17	1
5	6	16	0
6	9	15	0
7	10	14	0
8	11	13	0
9	12	12	0
10	13	11	0
11	14	10	1
12	17	9	1
13	18	7	0
14	19	6	0

15	21	5	0
16	23	4	1
17	24	3	0

$S_{t-1} = S_t * ((N_{t+1} - D_{t+1})/N_{t+1})$

[6]:
```
def survival(df):
    # see formula above
    surv = [1]
    for i in range(1,df.shape[0]):
        temp = surv[i-1] * ((df.iloc[i,1] - df.iloc[i,2])/ df.iloc[i,1])
        surv.append(temp)
    return surv
```

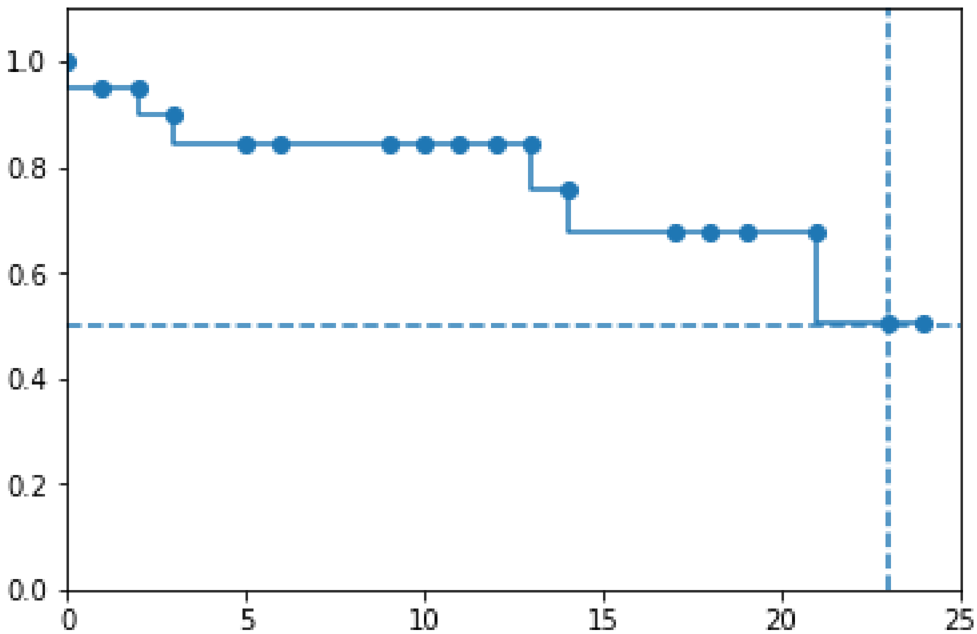
[7]:
```
surv = survival(df)
```

[8]:
```
surv
```

[8]:
```
[1,
 0.95,
 0.95,
 0.8972222222222221,
 0.8444444444444443,
 0.8444444444444443,
 0.8444444444444443,
 0.8444444444444443,
 0.8444444444444443,
 0.8444444444444443,
 0.8444444444444443,
 0.7599999999999999,
 0.6755555555555555,
 0.6755555555555555,
 0.6755555555555555,
 0.6755555555555555,
 0.5066666666666666,
 0.5066666666666666]
```

[9]:
```
plt.step(df[['Time']], surv, linestyle = '-', marker = 'o')
plt.ylim([0,1.1])
plt.xlim([0,25])
plt.axhline(0.5,linestyle = '--')
plt.axvline(23, linestyle = '--')
```

[9]:
```
<matplotlib.lines.Line2D at 0x1f6ba6a9cc0>
```



[10]:
```
# The median survival time is 23 years
```

2 Logrank Test

[11]:
```
# For the logrank test, we take two groups and compare their survival
# data is again taken from the above mentioned link
```

[12]:
```
time = [8,12,14,21,26,27,28,33,41]
n1t = [10,8,7,5,4,3,2,1,0]
n2t = [10,10,10,10,8,8,8,7,5]
nt = [20,18,17,15,12,11,10,8,5]
o1t = [1,1,1,1,1,1,0,0,0]
o2t = [0,0,0,0,0,0,1,1,1]
ot = [1,1,1,1,1,1,1,1,1]

lr_df = pd.DataFrame({'Time':time,
                     'Number_atRisk_Grp1':n1t,
                     'Number_atRisk_Grp2':n2t,
                     'Total_Number_atRisk':nt,
                     'Number_ofEvents_Grp1':o1t,
                     'Number_ofEvents_Grp2':o2t,
                     'Total_Number_ofEvents':ot})
```

[13]:
```
lr_df
```

[13]:

	Time	Number_atRisk_Grp1	Number_atRisk_Grp2	Total_Number_atRisk
0	8	10	10	20
1	12	8	10	18
2	14	7	10	17
3	21	5	10	15
4	26	4	8	12
5	27	3	8	11
6	28	2	8	10
7	33	1	7	8
8	41	0	5	5

	Number_ofEvents_Grp1	Number_ofEvents_Grp2	Total_Number_ofEvents
0	1	0	1
1	1	0	1
2	1	0	1
3	1	0	1
4	1	0	1
5	1	0	1
6	0	1	1
7	0	1	1
8	0	1	1

$E_{1t} = N_{1t} * (O_t/N_t)$

[14]:
```
e1t = []
e2t = []
for i in range(lr_df.shape[0]):
    e1t.append(n1t[i]*(ot[i]/nt[i]))
    e2t.append(n2t[i]*(ot[i]/nt[i]))
```

$\chi^2 = \sum \frac{(\sum O_{jt} - \sum E_{jt})^2}{\sum E_{jt}}$

[15]:
```
def logrank(df, exp1, exp2):
    ojt = sum(df['Number_ofEvents_Grp1'].tolist())
    ojt2 = sum(df['Number_ofEvents_Grp2'].tolist())
    e1t = sum(exp1)
    e2t = sum(exp2)
    temp = [((ojt - e1t)**2)/(e1t),((ojt2 - e2t)**2)/(e2t)]
    return sum(temp)
```

[16]:
```
logrank(lr_df, e1t, e2t)
```

[16]: 6.148087536256202

We now need the critical value of the χ^2 distribution at $\alpha = 0.05$. Tabular $\chi^2_{0.05} = 3.841$ Since the calculated value is higher than the tabular value, we reject the null hypothesis that there is no difference in the two groups.

[17]:
```
new_df = pd.DataFrame({'Time':lr_df.iloc[:,0],
                       'At_risk':lr_df.iloc[:,2],
                       'Death':lr_df.iloc[:,5]})
```

[18]:
```
survival(new_df)
```

[18]:
```
[1, 1.0, 1.0, 1.0, 1.0, 1.0, 0.875, 0.75, 0.6000000000000001]
```

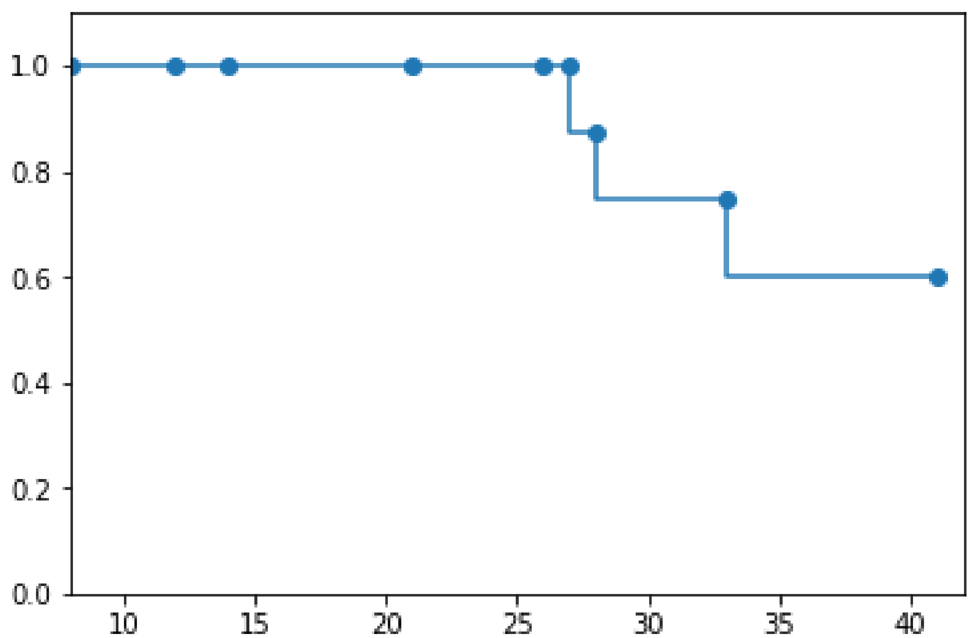
[19]:
```
new_df
```

[19]:

	Time	At_risk	Death
0	8	10	0
1	12	10	0
2	14	10	0
3	21	10	0
4	26	8	0
5	27	8	0
6	28	8	1
7	33	7	1
8	41	5	1

[21]:
```
plt.step(new_df.iloc[:,0], survival(new_df), linestyle = '-', marker = 'o')
plt.xlim([8,42])
plt.ylim([0,1.1])
```

[21]:
```
(0, 1.1)
```



[22]:
```
# We never reach the median value for this dataset, we may assume it as
# the value at 0.6, that is 41 years.
```

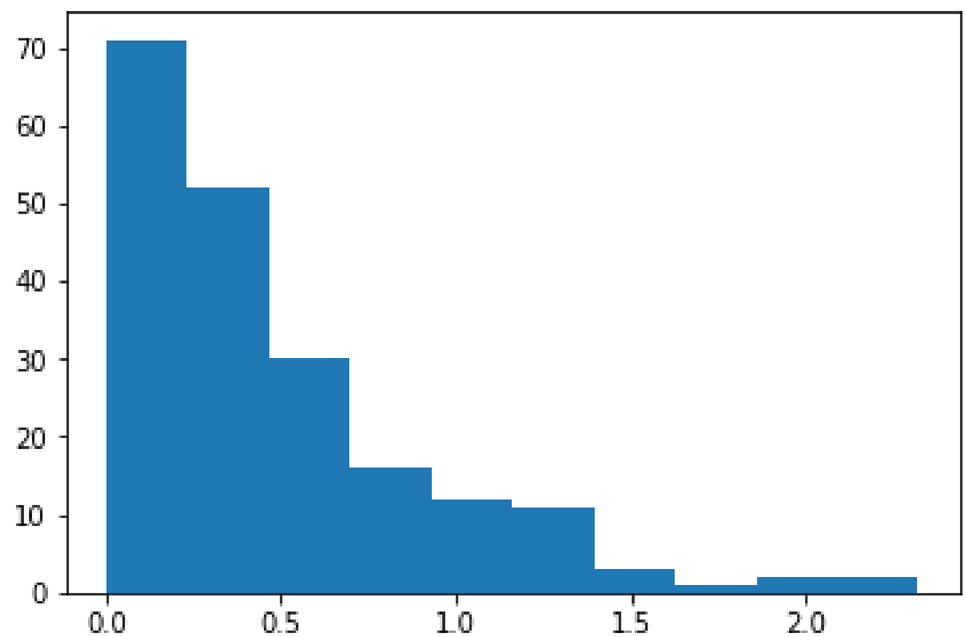
3 Ques 2

Simulating data from an exponential distribution $f(x; \frac{1}{\beta}) = \frac{1}{\beta}\exp(-\frac{x}{\beta})$ Taken from numpy's page

[56]:
```
# lambda = 2
data1 = np.random.exponential(scale = 0.5, size = 200).tolist()
# lambda = 4
data2 = np.random.exponential(scale = 0.25, size = 200).tolist()
```

[57]:
```
plt.hist(data1)
```

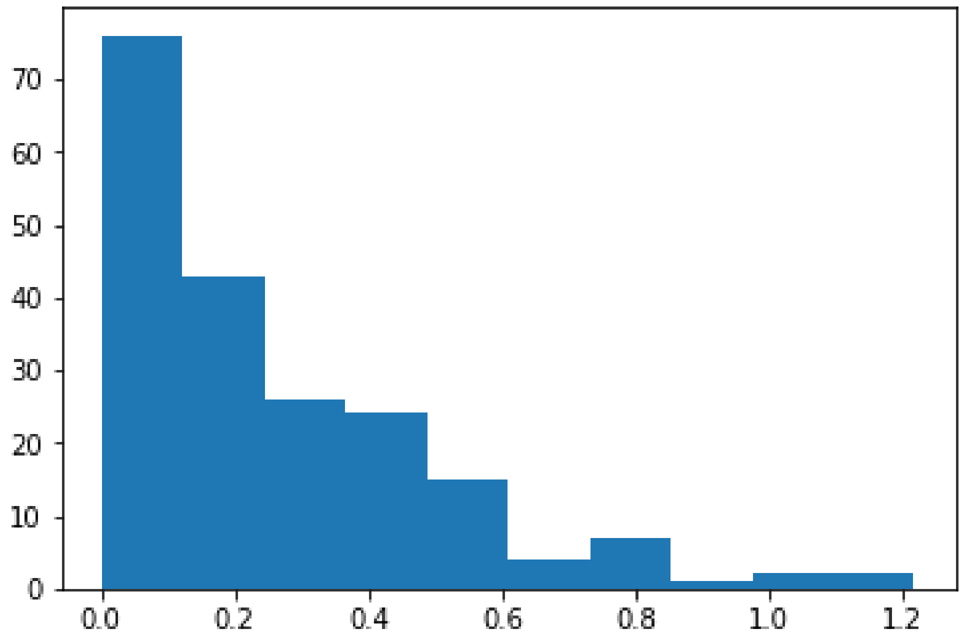
[57]:
```
(array([71., 52., 30., 16., 12., 11.,  3.,  1.,  2.,  2.]),
 array([1.86068052e-03, 2.33623903e-01, 4.65387125e-01, 6.97150347e-01,
        9.28913569e-01, 1.16067679e+00, 1.39244001e+00, 1.62420324e+00,
        1.85596646e+00, 2.08772968e+00, 2.31949290e+00]),
 <a list of 10 Patch objects>)
```



[58]:
```
plt.hist(data2)
```

[58]:

```
(array([76., 43., 26., 24., 15.,  4.,  7.,  1.,  2.,  2.]),
 array([0.00177864, 0.12328252, 0.24478639, 0.36629026, 0.48779413,
        0.609298  , 0.73080187, 0.85230574, 0.97380961, 1.09531348,
        1.21681735]),
 <a list of 10 Patch objects>)
```



[59]:

```
# censoring with a probability of 10%
censor1 = np.random.choice(2, size = 100, p = [0.9,0.1]).tolist()
censor2 = np.random.choice(2, size = 100, p = [0.9, 0.1]).tolist()
```

[60]:

```
int_data1 = [int(x*1000) for x in data1]
int_data2 = [int(x*1000) for x in data2]
```

[61]:

```
set1 = set(sorted(int_data1))
set2 = set(sorted(int_data2))
```

[62]:

```
sample1 = sorted(sample(list(set1), 100))
sample2 = sorted(sample(list(set2), 100))
```

[63]:

```
# deaths
death1 = np.random.choice(2, size = 100, p = [0.3, 0.7]).tolist()
death2 = np.random.choice(2, size = 100, p = [0.3, 0.7]).tolist()
```

[64]:

```
mera_df = pd.DataFrame({'Time':sample1,
                        'Risk1':[100 for _ in range(100)],
```

```
                          'Death1':death1,
                          'Censor1':censor1})
mera_df2 = pd.DataFrame({'Time':sample2,
                          'Risk2':[100 for _ in range(100)],
                          'Death2':death2,
                          'Censor2':censor2})
```

$N_{t+1} = N_t - D_t - C_t$

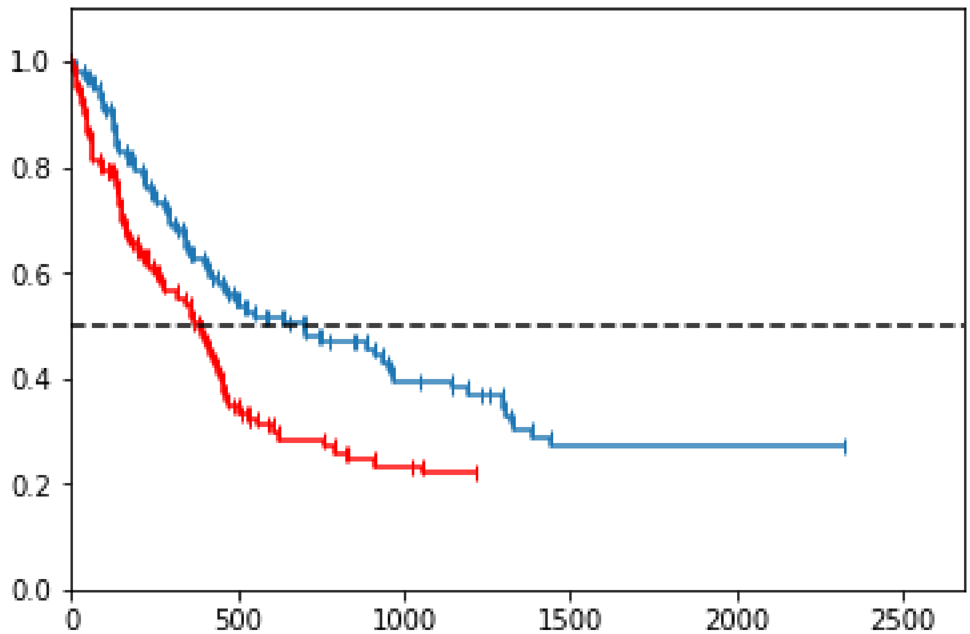
[65]:
```
for i in range(1,mera_df.shape[0]):
    mera_df.iloc[i,1] = mera_df.iloc[i-1,1] - mera_df.iloc[i-1,2] - mera_df.
 ↪iloc[i-1,3]
for i in range(1,mera_df2.shape[0]):
    mera_df2.iloc[i,1] = mera_df2.iloc[i-1,1] - mera_df2.iloc[i-1,2] - mera_df2.
 ↪iloc[i-1,3]
```

[66]:
```
mera_df.iloc[99,3] = mera_df.iloc[99,1]
mera_df2.iloc[99,3] = mera_df2.iloc[99,1]
```

[70]:
```
surv1 = survival(mera_df)
surv2 = survival(mera_df2)
```

[71]:
```
plt.step(mera_df.iloc[:,0], surv1, linestyle = '-', marker = '|')
plt.step(mera_df2.iloc[:,0], surv2, linestyle = '-', marker = '|', color =␣
 ↪'red')
plt.axhline(0.5, linestyle = '--', color = 'black')
plt.ylim([0,1.1])
plt.xlim([0, 2681])
```

[71]: (0, 2681)



[39]:
```
#surv1.index( 0.505895437823829)
```

[40]:
```
#surv2.index( 0.5098612291287132)
```

[41]:
```
mera_df.iloc[92,:]
```

[41]:
```
Time        1259
Risk1         28
Death1         1
Censor1        0
Name: 92, dtype: int64
```

[42]:
```
mera_df2.iloc[70,:]
```

[42]:
```
Time        355
Risk2        44
Death2        0
Censor2       0
Name: 70, dtype: int64
```

[43]:
```
# For choice of stochastic parameters -
# The median survival for group 1 is 1287 units
# The median survival for group 2 is 286 units.
```

[44]:
```
# Logrank test
```

[45]:
```
mera_df.iloc[1:5,:]
```

[45]:

	Time	Risk1	Death1	Censor1
1	12	99	0	0
2	18	99	1	0
3	31	98	1	0
4	32	97	1	0

[46]:
```
mera_df2.iloc[1:5,:]
```

[46]:

	Time	Risk2	Death2	Censor2
1	3	99	0	0
2	4	99	1	1
3	9	97	1	0
4	11	96	1	0

[47]:
```
e1t = []
e2t = []
for i in range(lr_df.shape[0]):
    e1t.append(n1t[i]*(ot[i]/nt[i]))
    e2t.append(n2t[i]*(ot[i]/nt[i]))

def logrank(df, exp1, exp2):
    ojt = sum(df['Number_ofEvents_Grp1'].tolist())
    ojt2 = sum(df['Number_ofEvents_Grp2'].tolist())
    e1t = sum(exp1)
    e2t = sum(exp2)
    temp = [((ojt - e1t)**2)/(e1t),((ojt2 - e2t)**2)/(e2t)]
    return sum(temp)

logrank(lr_df, e1t, e2t)
```

[47]: 6.148087536256202

[51]:
```
e1t = []
e2t = []
for i in range(mera_df.shape[0]):
    e1t.append(mera_df.iloc[i,1] * ((mera_df.iloc[i,2]+mera_df2.iloc[i,2])/
 ↪(mera_df.iloc[i,1]+mera_df2.iloc[i,1])))
    e2t.append(mera_df2.iloc[i,1] * ((mera_df.iloc[i,2]+mera_df2.iloc[i,2])/
 ↪(mera_df.iloc[i,1]+mera_df2.iloc[i,1])))
```

[52]:
```
def logrank2(mera_df, mera_df2, exp1, exp2):
    ojt = sum(mera_df['Death1'].tolist() + mera_df['Censor1'].tolist())
    ojt2 = sum(mera_df2['Death2'].tolist() + mera_df2['Censor2'].tolist())
    e1t = sum(exp1)
    e2t = sum(exp2)
```

```python
    temp = [((ojt - e1t)**2)/(e1t),((ojt2 - e2t)**2)/(e2t)]
    return sum(temp)
```

[53]:
```python
logrank2(mera_df, mera_df2, e1t, e2t)
```

[53]: 24.375952598006688

We now need the critical value of the χ^2 distribution at $\alpha = 0.05$. Tabular $\chi^2_{0.05} = 3.841$ Since the calculated value is higher than the tabular value, We reject the null hypothesis that there is no difference in the two groups.