

# Prethermalization by Random Multipolar Driving on a 78-Qubit Superconducting Processor

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Time-dependent drives hold the promise of realizing non-equilibrium many-body phenomena that are absent in undriven systems. Yet, drive-induced heating normally destabilizes the systems, which can be parametrically suppressed in the high-frequency regime by using periodic (Floquet) drives. It remains largely unknown to what extent highly controllable quantum simulators can suppress heating in non-periodically driven systems. Using the 78-qubit superconducting quantum processor, *Chuang-tzu 2.0*, we report the first experimental observation of the long-lived prethermal phases with tunable heating rates in many-body systems, driven by structured random protocols, characterized by the  $n$ -multipolar temporal correlations. By measuring both the particle imbalance and the subsystem entanglement entropy, we trace out the entire heating process over 1,000 driving cycles and confirm the existence of the prethermal plateau. Its lifetime grows algebraically with the driving frequency and the scaling exponent is approximately  $2n+1$ , tunable by the multipolar order. Using quantum state tomography on different subsystems, we demonstrate a non-uniform spatial entanglement distribution and observe a crossover from the area-law to the volume-law entanglement scaling. We utilize all 78 qubits and 137 couplers in a 2D configuration, and their entire far-from-equilibrium heating dynamics are beyond the reach of simulation using tensor-network numerical techniques. Our work underscores superconducting quantum processors as a powerful platform for exploring non-equilibrium phases of matter in driven systems.

Periodically driven (Floquet) systems can host novel far-from-equilibrium phenomena that are absent in thermal equilibrium [1–3]. Prominent examples include the discrete-time crystals (DTC) [4–10], Floquet topological matter [11–14] and dynamical phase transitions [15–17]. Periodic drives have also been widely employed for Floquet engineering of many-body interactions [18–20] and mitigating environment-induced decoherence [21, 22], serving as a robust and versatile approach to stabilize and control modern quantum simulators [23–31]. Explorations of non-periodic driving have surged in recent years, resulting in the rich discovery of non-equilibrium phenomena beyond the Floquet lore [32–47]. For instance, quasi-periodic and structured random drives can lead to the appearance of discrete-time quasi-crystals [33, 48–50] and time rondeau crystals [41, 51], notably enriching the possible forms of temporal order in non-equilibrium settings.

Due to the lack of energy conservation, generic time-dependent many-body systems are inherently susceptible to heating, eventually ending in the featureless infinite-temperature state [52, 53], where the subsystem entanglement entropy also reaches its maximum – the Page value [54]. This heating effect thus poses a fundamental challenge for utilizing and stabilizing large-scale quantum simulators, especially over long timescales. In Floquet systems, heating can be efficiently suppressed by many-body localization (MBL) induced via strong spatial disorder [55–57]. In clean systems, on the other hand, heating can also be exponentially suppressed by using high-frequency drives [58, 59], leading to the transient but long-lived prethermal regime before the eventual heat death [60–68]. In contrast, stabilizing non-periodically driven systems is a notoriously difficult task, especially when the driving protocol involves temporal randomness. It typically opens deleterious energy absorption channels, which even MBL cannot prevent, and thus heating occurs swiftly.

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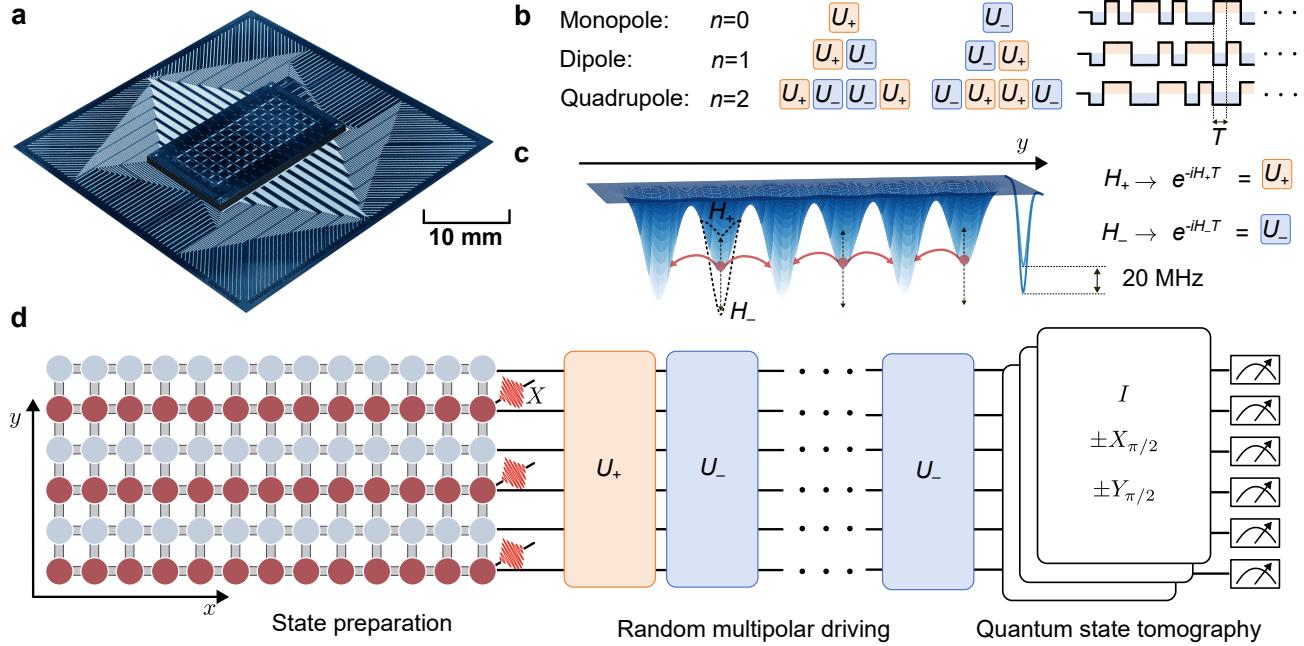
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In this work, we experimentally demonstrate the existence of the long-lived prethermal phenomenon in systems driven by random but structured protocols, with a high degree of tunability in the heating rate. We use a superconducting quantum



**FIG. 1. Quantum processor and experimental scheme.** **a**, Optical micrograph of the 78-qubit superconducting processor, *Chuang-tzu* 2.0. The processor is designed as a 6-by-13 square lattice, comprising 78 qubits interconnected by 137 couplers that link all adjacent qubits. **b**, Schematic diagram of the random multipolar driving (RMD) protocol, characterized by the temporal multipolar order  $n$ . The 0-RMD is constructed by randomly selecting elements from the two elementary operators  $\{\hat{U}_+, \hat{U}_-\}$ , and  $n$ -RMD sequence consists of a random selection of two  $n$ -multipoles, which are obtained by anti-aligning two  $(n-1)$ th order operators. **c**, The elementary operators are generated as  $\hat{U}_\pm = \exp\{-i\hat{H}_\pm T\}$  with the driving period  $T$ , where  $H_\pm$  differs in the staggered potential in  $y$ -direction. **d**, Experimental procedure. First, we initialize the system in a Néel-type state, where even sites along the  $y$ -axis are occupied. Next, we implement the RMD protocol that destabilizes the Néel order and the system heats up. Finally, we use multiquantum state tomography to determine the entanglement entropy and perform particle number measurement to characterize the non-equilibrium time evolution.

processor, *Chuang-tzu* 2.0 as shown in Fig. 1, that involves 78 qubits and 137 couplers. Leveraging the precise control and flexibility of this device, we accurately implement stable long-term drives and perform large-scale quantum simulations of the two-dimensional hard-core Bose-Hubbard system.

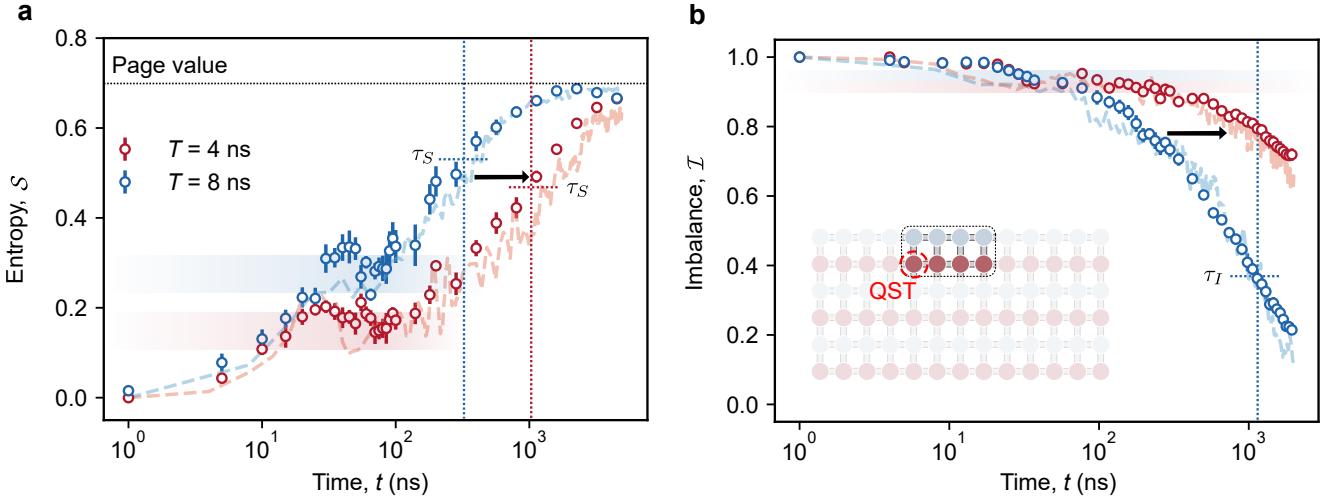
Specifically, we implement a family of structured random protocols, known as random multipolar driving (RMD) [36, 69]. As illustrated in Fig. 1b, the protocol involves two elementary evolution operators  $\hat{U}_+$  and  $\hat{U}_-$ , generated by two Hamiltonians  $\hat{H}_\pm$  that differ in the chemical potential along the  $y$ -axis, see Fig. 1b and c. Starting from an initial density-wave state, the random modulation of  $\hat{U}_\pm$  destabilizes the system and hence induces heating. The heating rate can be significantly suppressed by imposing a dipolar structure into the random driving sequence, such that the elementary building blocks now read  $\hat{U}_+\hat{U}_-$  and  $\hat{U}_-\hat{U}_+$ . Similarly, the  $n$ -th multipole can be recursively constructed by anti-aligning two  $(n-1)$ -th order operators, and in the  $n \rightarrow \infty$  limit it converges to the self-similar Thue-Morse driving [32, 46, 69].

We first benchmark our experiments on 8 qubits, then we gradually enlarge the system size up to 78 qubits. To quantify the heating process, we experimentally monitor the decay of the particle imbalance. Crucially, we also measure the growth of the entanglement entropy, thus going beyond established experimental results on driven systems where typically only

the evolution of local observables is accessible [63–66]. We perform more than 1,000 driving cycles and such a long time scale allows us to capture the long-lived prethermal plateau in the high-frequency regime. More importantly, we confirm the crucial role of the temporal multipolar correlation in stabilizing the system: The heating rates follow a power-law dependence on the driving frequency, with a universal scaling exponent of approximately  $2n+1$ , in accordance with the Fermi's golden rule (FGR) [36] and a generalized Floquet perturbation theory prediction [69].

Furthermore, by selecting different subsystem configurations, we demonstrate a non-uniform spatial entanglement distribution and observe the crossover from the area-law to the volume-law entanglement scaling within the prethermal regime. The onset of heating further accelerates entanglement growth, posing a significant challenge in numerical simulations within a reasonable computational time [70, 71]. Advanced tensor network numerical techniques, such as grouped matrix product states (GMPS) and projected entangled pair states (PEPS), struggle to keep pace with the rapid entanglement growth. Therefore, our experiment potentially demonstrates the quantum advantage in emulating the entire heating dynamics towards the maximally entangled infinite-temperature state.

## Experimental setup



**FIG. 2. Prethermalization by random multipolar driving (RMD) in an 8-qubit system.** **a** (**b**), Experimental data showing the dynamics of the subsystem entanglement entropy (imbalance), for driving periods  $T = 4$  ns and  $8$  ns, where the prethermal plateau is clearly visible. Both the lifetime of the entanglement plateau,  $\tau_S$ , and the imbalance decay time scale,  $\tau_I$ , can be prolonged by increasing the driving frequency. We implement the 1-RMD protocol and the system comprises 8 qubits  $\{Q_{x,y}\}$  (the inset in **b**), indexed by  $x = \{5, 6, 7, 8\}$  and  $y = \{1, 2\}$ . We use quantum state tomography (QST) to determine the entanglement entropy of the qubit  $\{Q_{5,2}\}$ . Error bars indicate one standard error of experimental results averaged over 10 independent RMD sequences. The light dashed curves depict numerical results for benchmarking.

Our experiments are performed on a flip-chip superconducting processor arranged in a 6-by-13 square lattice array (Fig. 1a), with 78 qubits and 137 couplers between all neighbor qubits. The qubits are labeled as  $Q_{x,y}$  with  $x$  varied from 1 to 13 and  $y$  from 1 to 6, see Fig. 1d. Since the anharmonicity is designed much larger than the hopping strength, with an average value of  $-200$  MHz, the system can be described as the non-integrable 2D hard-core Bose-Hubbard model [72, 73]. The effective Hamiltonian reads ( $\hbar = 1$ ):

$$\hat{H} = \sum_j \omega_j \hat{n}_j + \sum_{\langle i,j \rangle} g_{i,j} (\hat{a}_i^\dagger \hat{a}_j + \text{H.c.}), \quad (1)$$

where  $i, j$  labels the qubit in the group  $\{Q_{x,y}\}$ ,  $\hat{n}_j$  denotes particle number operator at the  $j$ -th site,  $\hat{a}_j^\dagger(\hat{a}_j)$  is the creation (annihilation) operator,  $\omega_j$  is the on-site potential, and  $g_{i,j}$  is the hopping strength between two nearest neighbor qubits. Device information can be found in [74]. Owing to significant progress in coherence time, scalability, and controllability of superconducting quantum circuits [75–77], this platform has emerged as a powerful system for exploring complex quantum phenomena that demand precise manipulation.

Our target elementary operators in the  $n$ -RMD protocol are generated as  $\hat{U}_\pm = \exp(-i\hat{H}_\pm T)$ , where  $T$  denotes the characteristic timescale (noted as the driving period below). In our experiments,

$$\hat{H}_\pm = \hat{H}_\kappa + (1 \pm \delta h) \hat{H}_p, \quad (2)$$

with a dimensionless scaling factor  $\delta h$  characterizing the driving amplitude, the on-site term exhibiting a uniform potential along the  $x$ -axis and a staggered arrangement along the  $y$ -axis

$$\hat{H}_p = h_0 \sum_{x,y} [1 - (-1)^y] \hat{a}_{x,y}^\dagger \hat{a}_{x,y}, \quad (3)$$

and the hopping term isotropic in both spatial directions

$$\hat{H}_\kappa = J \sum_{x,y} (\hat{a}_{x,y}^\dagger \hat{a}_{x+1,y} + \hat{a}_{x,y}^\dagger \hat{a}_{x,y+1} + \text{H.c.}). \quad (4)$$

The parameters are chosen as  $J/2\pi = 2$  MHz,  $\delta h = 1.2$ ,  $h_0/2\pi = 10$  MHz, and  $T$  ranges from 3 ns to 8 ns. The average decoherence time ( $T_1$ ) of our device is  $26.4 \mu\text{s}$  [74], allowing us to experimentally implement more than 1000 driving cycles before any notable decoherence occurs.

For 1-RMD, the protocol involves a random sequence of two dipolar operators  $\hat{U}_+ \hat{U}_-$  and  $\hat{U}_- \hat{U}_+$ ; and for 2-RMD, the elementary building blocks become  $\hat{U}_+ \hat{U}_- \hat{U}_- \hat{U}_+$  and  $\hat{U}_- \hat{U}_+ \hat{U}_+ \hat{U}_-$ . Experimentally, implementing the driving protocol requires precise temporal modulation of the pulse signal, especially in the high-frequency regime ( $T \approx 3$  ns): Qubits with odd  $y$ -index are biased to the working point, while the pulse signal following an RMD sequence is implemented on qubits with even  $y$ -index; Through careful calibration of Z pulse distortion and crosstalk, combined with precise timing alignment via Floquet engineering, we achieve highly accurate RMD control, see details in the Supplementary Material (SM) [74].

As shown in Fig. 1d, we initialize the system as a Néel-type ordered product state  $|\psi_0\rangle$ , where all lattice sites with even  $y$ -index are occupied, resulting in the maximal Hilbert space dimension. In the high-frequency limit,  $T \rightarrow 0$ , the early time evolution of the system can be simply described by an effective Hamiltonian,  $\hat{H}_{\text{eff}} = (\hat{H}_+ + \hat{H}_-)/2$ , where the staggered potential along the  $y$ -axis, Supplementary Eq. 3, stabilizes the initial Néel order. However, for any finite  $T$ , the random modulation in the lattice potential destabilizes it and eventually, the system heats up to the infinite-temperature state where the Néel order no longer survives. Therefore, the particle number

imbalance serves as a good diagnostics of this heating process

$$\mathcal{I} = \frac{1}{N_0} \sum_{j \in \{x,y\}} \langle \psi_0 | Z_j(t) Z_j(0) | \psi_0 \rangle, \quad (5)$$

with  $Z_j = 2\hat{n}_j - 1$ , and the initial particle number  $N_0$ . In addition to these local observables, we also study the time evolution of the entanglement entropy,  $S = -\text{Tr}[\rho \ln \rho]$  where  $\rho$  denotes the reduced density matrix of a subsystem, to capture the growth of non-local correlations between a subsystem and its complement [56]. In the experiment, we perform quantum state tomography (QST) [78] on a subsystem to reconstruct  $\rho$  at different stages of the heating process.

### Characteristics of prethermalization by RMD

We first benchmark our experiments on 8 qubits, which can be efficiently simulated numerically. Then, we gradually increase the system size, as shown in Fig. 3a, and use all 78 qubits, allowing for a detailed analysis of heating dynamics and the entanglement entropy growth.

Fig. 2a displays the measured entanglement entropy dynamics of a single qubit under 1-RMD, in a small-scale system of 8 qubits and 10 couplers, for  $T=4$  ns and  $T=8$  ns respectively. The entropy evolution exhibits several distinct features: (1) Starting from  $S=0$ , as expected for a product initial state, the entanglement entropy rapidly increases within a short time; (2) A distinct prethermal plateau appears within the time interval  $20 \lesssim t \lesssim 100$  ns, hallmarking the notable suppression of energy absorption. This plateau can be described by a quasi-conserved effective Hamiltonian which can be perturbatively constructed using a generalized Floquet-Magnus expansion [69, 74]; (3) At later times,  $S$  deviates from the plateau and continues increasing. Around  $t \gtrsim 2,000$  ns, it saturates at  $S_M \approx 0.7$ , in accordance with the Page value,  $S_P \approx L \ln 2$  [54] where  $L$  denotes the subsystem size, indicating that the system completely thermalizes. We also perform exact numerical simulations (light dashed lines in Fig. 2a) which match well with our experimental observations. The lifetime of the prethermal plateau,  $\tau_S$ , can be quantitatively determined by numerically fitting the entanglement entropy growth to the ansatz  $S \sim S_M(1 - e^{-t/\tau_S})$  [79] in the intermediate region  $100 \lesssim t \lesssim 2,000$  ns. By reducing the driving period  $T=8$  ns to  $T=4$  ns,  $\tau_S$  significantly increases from  $0.25 \mu\text{s}$  to  $1.03 \mu\text{s}$ , indicating a strong suppression of the heating rate.

A similar feature also appears in the decay of the imbalance. As shown in Fig. 2b, we observe that, starting from the initial value  $\mathcal{I}=1$ , the imbalance decays towards 0 at later times, and a high-frequency drive significantly slows down this decay. The characteristic decay time scale  $\tau_I$  can be extracted by fitting the imbalance evolution to an exponentially decaying function  $\mathcal{I} \sim e^{-t/\tau_I}$ . For  $T = 8$  ns and  $T = 4$  ns, the time scales are  $1.2 \mu\text{s}$  and  $7.4 \mu\text{s}$ , respectively.

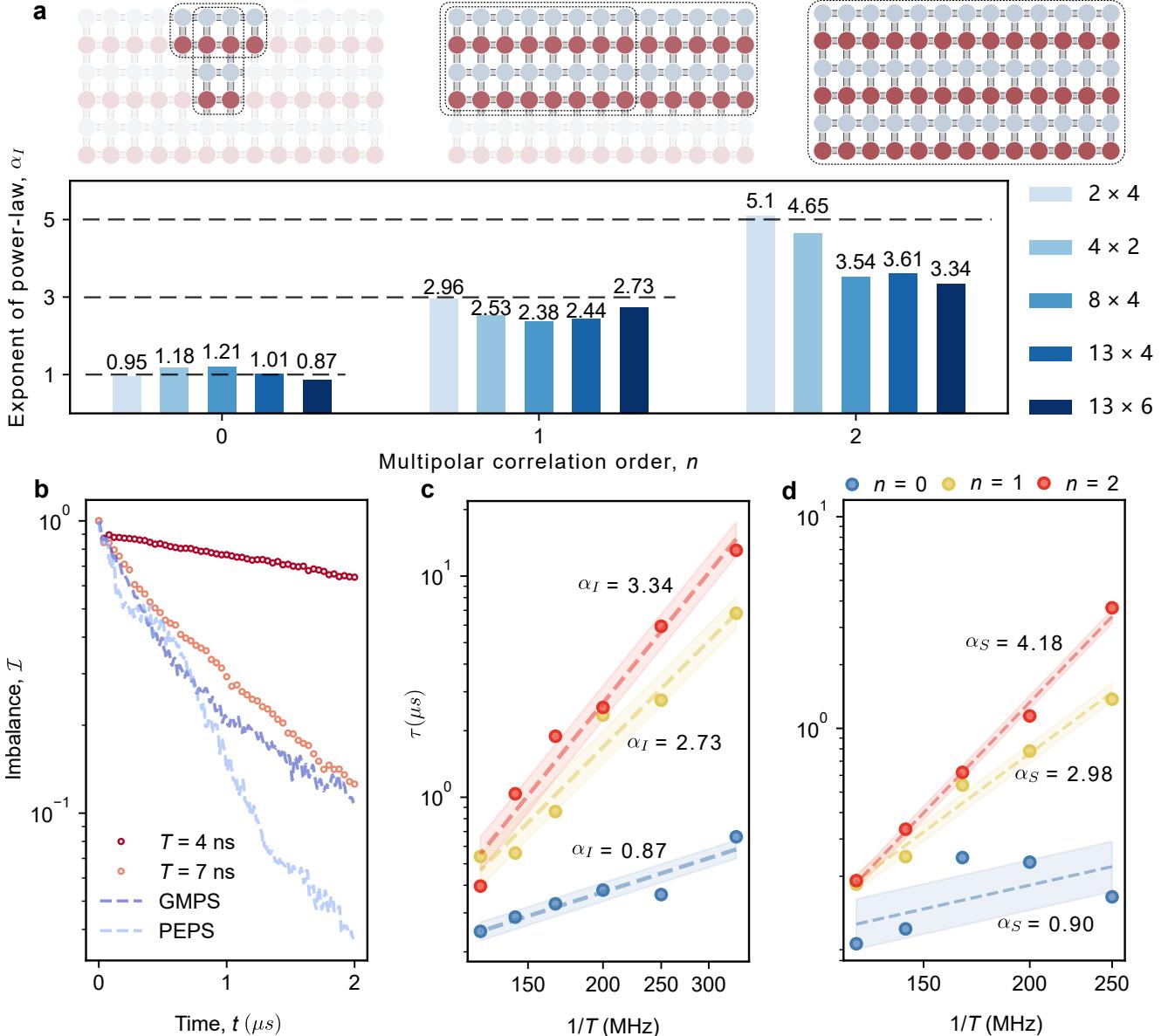
### Tunable heating rate by RMD

To further study the suppression of the heating rate, we perform a similar RMD protocol and examine the dependence of the prethermal lifetime on the driving frequency for different multipolar order  $n$ . Experimentally, we implement the

driving protocol on larger systems of varying geometries, as shown in Fig. 3a. In particular, in Fig. 3b, we present the time evolution of the imbalance for the 78-qubit system driven by the 2-RMD protocol using two different values of  $T$  (orange and red dots). Both decay exponentially, in agreement with both numerical [79] and experimental observations during the heating process [63]. In Fig. 3c and 3d, we use a log-log scale and show the prethermal lifetimes  $\tau_I$  and  $\tau_S$  for different  $1/T$ . Data points approximately follow a straight line, suggesting a power-law dependence,  $\tau \sim (1/T)^\alpha$ . This power-law behavior is in sharp contrast to conventional Floquet systems with local interactions, where the prethermal lifetime scales exponentially with the driving frequency  $\tau \sim e^{1/T}$  [64, 66]. Crucially, the scaling exponent  $\alpha$  is now tunable and exhibits a strong dependence on the multipolar order in the high-frequency regime. This is a special feature of RMD systems, which cannot be achieved by either Floquet or other quasi-periodic drives. The reason is that the multipolar structure suppresses the low-frequency components of the drive, thereby constraining heating. A generalized Floquet-Magnus theory and an FGR argument predicts the relation  $\alpha=2n+1$  [36, 69].

Experimentally, for the 78-qubit system and  $n=\{0, 1, 2\}$ , we numerically fit the exponent based on the prethermal lifetime of the entanglement entropy plateau and obtain  $\alpha_S = \{0.904, 2.976, 4.182\}$ . Based on the imbalance decay, we obtain the exponents  $\alpha_I = \{0.871, 2.727, 3.344\}$ . Also, in Fig. 3a, we summarize the scaling exponents  $\alpha_I$ , and our results reveal a positive correlation between  $\alpha_I$  and  $n$  across all investigated system sizes and configurations. Experimental data on the corresponding time evolution can be found in [74]. In particular, the scaling exponents for  $n = 0$  and 1 closely follow the theoretical prediction (black dashed lines). For  $n = 2$ , it is particularly challenging to obtain a converging scaling exponent, which requires an extremely fast drive and long simulation time, see detailed discussions in SM [74]. Here, we implement approximately 1,000 driving cycles to clearly distinguish the scaling behaviors for  $n = 1$  and 2. This also highlights the exceptional long-time stability and controllability of *Chuang-tzu 2.0*, which allows us to precisely engineer the temporal correlations and manipulate the quantum thermalization process.

To validate our experimental observations, we employ advanced numerical techniques based on tensor network representations [80] to approximately simulate the many-body dynamics. We initially developed the projected entangled pair states (PEPS) with a single update for simulations. PEPS is a two-dimensional tensor network and is feasible for our circuit system. However, due to the lack of canonical form, PEPS is not accurate in long-time simulations. The simulation results are shown in Fig. 3b, where we can see that shortly after a few driving cycles ( $t \approx 0.1 \mu\text{s}$ ), the PEPS simulations exhibit significant deviations from the experimental data. To improve the numerical accuracy, we further developed the grouped MPS (GMPS) method. Compared with the conventional matrix product states (MPS) method, the GMPS method merges certain tensors in the conventional MPS into a giant tensor, thereby avoiding truncations inside the giant tensor and the swapping operations in MPS. This strategy is



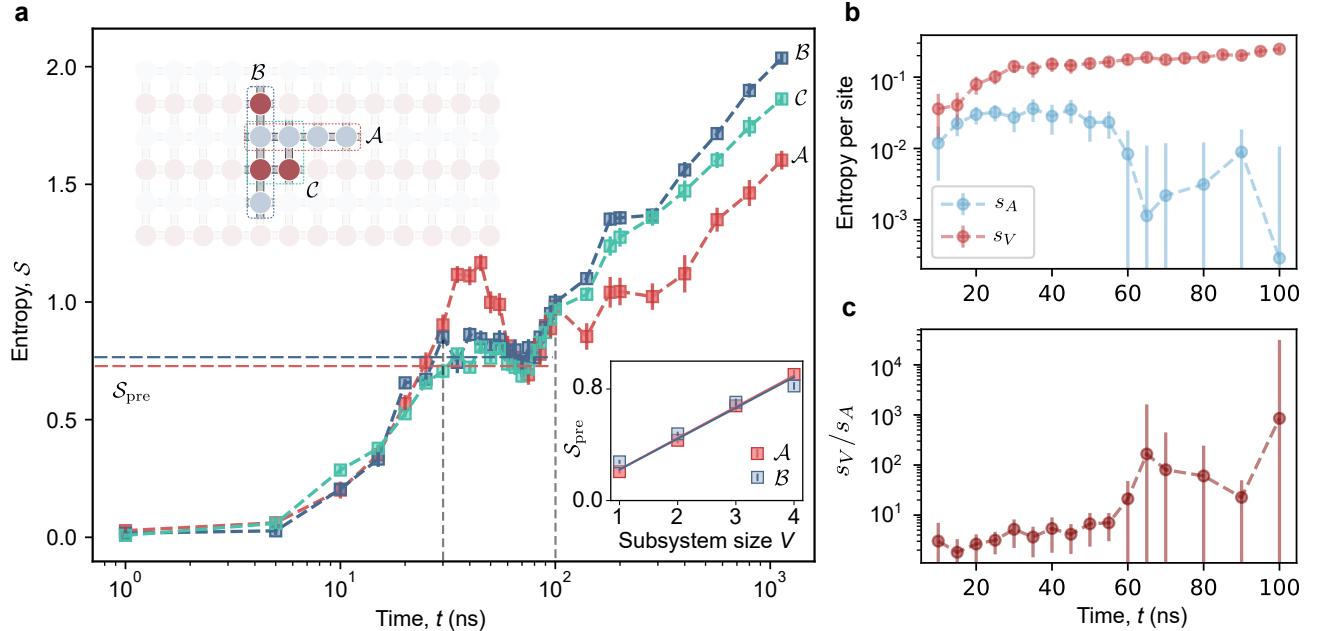
**FIG. 3. Scaling behavior of the prethermal lifetime with driving frequency.** **a**, Various system configurations and their corresponding power-law scaling exponents. These scaling exponents closely follow the theoretically predicted value of  $\alpha(n)=2n+1$  (the dashed lines). **b**, Decay of the imbalance shown in a log scale. The markers represent the experimental data, while the dashed curves are obtained numerically using the grouped matrix product states (GMPS) with bond dimension  $\chi=96$  and projected entangled pair states (PEPS), for  $T=7 \text{ ns}$  and  $4 \text{ ns}$ , respectively, with 2-RMD drives. Numerical methods can only capture the early time evolution. **c** (**d**), The power-law scaling of the prethermal lifetime obtained from the dynamics of the imbalance  $\tau_I$  (the subsystem entanglement entropy  $\tau_S$ ) in the 78-qubit system. A larger multipolar order  $n$  significantly suppresses the heating rates. The subsystem for calculating the entanglement entropy comprises qubits  $\{Q_{5,3}, Q_{6,3}\}$ .

particularly efficient in simulating quasi-1D systems such as our system. As shown in Fig. 3b, GMPS correctly reproduces the experimental data at early times,  $t \lesssim 0.3 \mu\text{s}$ , and later deviations from the experimental data also become visible. In the SM, we systematically test the performance of GMPS and observe that its estimated fidelity significantly decays as heating occurs (fidelity drops below 0.05 when  $t \approx 1.5 \mu\text{s}$ ). The rapid growth of entanglement during this process limits both its accuracy and efficiency. Therefore, our quantum processor potentially demonstrates a quantum advantage in simulating

the entire heating dynamics towards the highly-entangled infinite temperature states [81–86].

### Entanglement dynamics and scaling

We further investigate the subsystem size dependence of the entanglement entropy, which provides valuable information for estimating the numerical complexity in simulating the many-body dynamics [87–90]. Specifically, we perform QST of various subsystems, each comprising up to 4 sites within the 78-qubit system. As shown in Fig. 4a, these configura-



**FIG. 4. Entanglement dynamics and volume law scaling.** **a**, Dynamics of the entanglement entropy for different subsystem configurations involving 4 qubits in the 78-qubit system. All subsystems enter a long-lived prethermal regime, where  $\mathcal{A}$  exhibits pronounced oscillatory dynamics. The inset depicts the entanglement entropy averaged over the prethermal regime ( $30 \text{ ns} \leq t \leq 100 \text{ ns}$ , two black dashed lines in panel **a**), as a function of volume  $V$  for subsystems  $\mathcal{A}$  and  $\mathcal{B}$ . **b**, The volume ( $s_V$ ) and the area entanglement entropy ( $s_A$ ) per site, numerically fitted by analyzing subsystems of varying volumes and areas at different times. The error bars correspond to one standard error of the fit parameter. **c**, The ratio  $s_V/s_A$  at different times, serving as a quantitative measure to distinguish between the area-law and the volume-law scaling. As time evolves, a clear crossover from the area-law to volume-law scaling is observed. We use 1-RMD protocol and  $T=4 \text{ ns}$  to perform the experiment.

tions exhibit distinct spatial arrangements: the subsystem  $\mathcal{A}$  is aligned with the  $x$ -axis,  $\mathcal{B}$  is oriented along the  $y$ -axis, and  $\mathcal{C}$  forms a  $2 \times 2$  square lattice. Fig. 4a depicts the entanglement entropy dynamics for different subsystems, and all of them exhibit the long-lived prethermal regime. Interestingly, the entanglement dynamics strongly depend on the specific choice of subsystems:  $\mathcal{A}$  exhibits pronounced oscillatory dynamics during the prethermal regime, while  $\mathcal{B}$  and  $\mathcal{C}$  quickly saturates at a plateau, suggesting a non-uniform entanglement distribution in 2D. Such oscillations should originate from the coherent particle exchange between qubits with even and odd  $y$ -index, stabilized by the staggered potential in the effective Hamiltonian  $\hat{H}_{\text{eff}}$ , and can also be validated by GMPS simulations, see [74].

To quantify the entanglement entropy scaling, we calculate the averaged entropy,  $S_{\text{pre}}$ , during the prethermal regime to reduce the temporal fluctuations. In the inset of Fig. 4a, we show  $S_{\text{pre}}$  as a function of the linear subsystem size  $V$  for  $\mathcal{A}$  (red) and  $\mathcal{B}$  (blue), and linear dependence can be observed. For other possible subsystem configurations, in practice, one can approximately distinguish different contributions using the ansatz [90]

$$S_{\text{pre}}(\rho_X) = s_A A_X + s_V V_X, \quad (6)$$

where  $\rho_X$  denotes the reduced density matrix of a subsystem  $X$ ,  $A_X$  and  $V_X$  correspond to the subsystem's area and volume, respectively. The ratio  $s_V/s_A$  quantifies the degree to

which the state adheres to area-law or volume-law entanglement scaling. By analyzing 12 non-repetitive subsystems with varying volumes and areas, we numerically fit  $s_V$  and  $s_A$  and show their time evolution in Fig. 4b, and the ratio  $s_V/s_A$  in Fig. 4c. At early times ( $t \leq 30 \text{ ns}$ ), the system stays close to the Néel ordered product state and  $s_V/s_A$  remains small. A notable increase occurs around  $t \approx 50 \text{ ns}$  and this ratio becomes substantially larger than 1, suggesting a crossover from the area-law to volume-law scaling. At later times, entanglement increases and still obeys the volume-law scaling, and therefore, the system's dynamics become further computationally intractable.

## Conclusion and outlook

We present a systematic experimental study on the non-equilibrium dynamics of 2D interacting systems driven by the  $n$ -RMD protocol on a 78-qubit superconducting processor. Its precise and stable pulse sequence allows for the demonstration of the long-lived prethermal plateau and suppressed heating rates, which exhibit the characteristic algebraic scaling  $T^{2n+1}$  in the high-frequency regime. Although our experiment is performed on a superconducting processor, the underlying heating control mechanism is readily applicable to different quantum simulator platforms. The possibility of driving closed quantum systems with temporal randomness while avoiding heating paves the way for engineering novel non-equilibrium phases of matter beyond the conventional Floquet

paradigm.

By performing QST, we trace out the growth of the subsystem entanglement entropy during the entire heating process. Although the system eventually evolves towards the featureless infinite-temperature state, during the prethermal regime we observe a non-uniform spatial entanglement distribution. In particular, subsystem  $\mathcal{A}$  exhibits coherent oscillatory entanglement dynamics. These observations shed light on the understanding of the microscopic generation of entanglement, especially in higher dimensions where the subsystem configurations can exhibit a rich geometric structure.

Furthermore, by analyzing the entanglement entropy for various subsystems, we demonstrate the crossover from area-

law to volume-law entanglement scaling as time evolves. Such rapid entanglement growth poses significant challenges in simulating our experiments using classical algorithms. Therefore, our work demonstrates a potential quantum advantage in emulating the entire non-equilibrium heating dynamics in driven systems.

Looking ahead, it will be interesting to further explore the initial state dependence and the spatial non-homogeneity in the heating process, as well as the stability of MBL in randomly driven systems. Also, identifying different prethermalization mechanisms for much broader classes of non-periodic drives remains an open and interesting direction.

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### Competing interests

The authors declare no competing interests.

*Supplementary Materials for:*  
**Prethermalization by Random Multipolar Driving on a 78-Qubit Superconducting Processor**

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**S1. PRETHERMALIZATION AND NUMERICAL SIMULATION**

**A. Prethermalization in Floquet systems**

Periodically driven (Floquet) quantum systems can engineer novel far-from-equilibrium phases of matter absent in static systems [2]. These system can be analysed within the framework of Floquet theory. Consider the Hamiltonian  $\hat{H}$  with temporal periodicity,  $\hat{H}(t) = \hat{H}(t + T)$ , the Floquet theorem states that the time evolution operator can be written as

$$\hat{U}(t) = \hat{P}(t)e^{-i\hat{H}_F t}, \quad (1)$$

where  $\hat{H}_F$  is the time-independent Floquet Hamiltonian, and  $\hat{P}(t)$  is the stroboscopic operator with period  $T$ . In the high frequency regime, the dynamics can be approximated by the Hamiltonian  $\hat{H}_F$ , known as the effective Hamiltonian. In general, an exact closed-form expression for  $\hat{H}_F$  is difficult to obtain, and a common scheme is to use the Floquet-Magnus expansion (FME), which is a perturbative expansion in the driving frequency

$$\hat{H}_F = \sum_{n=0}^{\infty} \frac{1}{\Omega^n} \hat{H}_F^{(n)}, \quad (2)$$

with the driving frequency  $\Omega = 2\pi/T$ . The lowest-order effective Hamiltonian is the time average of system Hamiltonian  $\hat{H}$  over a period  $T$

$$\hat{H}_F^{(0)} = \frac{1}{T} \int_0^T dt \hat{H}(t). \quad (3)$$

Unlike static systems that conserve energy, Floquet systems typically absorb energy from the drive [3, 53]. The nature of equilibrium states under such heating becomes clearer through the analysis of FME convergence. When the FME converges, the effective Hamiltonian  $\hat{H}_F$  can be effectively represented by a summation of local operators. In this case, the system is expected to thermalize to a steady state that, at the local level, becomes effectively indistinguishable from the microcanonical ensemble of a static system described by  $\hat{H}_F$ . However, it is widely believed that the FME diverges in generic many-body interacting systems, indicating the absence of a quais-local  $\hat{H}_F$ . For a non-integrable Floquet system, heating leads to the eventual thermalization where the system reaches the featureless infinite-temperature state.

Theoretical studies on the long-time behaviors of Floquet systems reveal that, the FME may converge to the time-averaged Hamiltonian when driving periods are sufficiently short, such that the driving frequency  $\Omega$  is much larger than the local energy scale of many-body system. In this regime, Floquet prethermalization appears before the onset of noticeable heating, giving rise to an exponentially long-lived prethermal regime governed by the prethermal Hamiltonian

$$\hat{H}_{\text{pre}} = \sum_{n=0}^N \frac{1}{\Omega^n} \hat{H}_F^{(n)}, \quad (4)$$

obtained by truncating FME at the order  $N$ . In contrast to using many-body localization (MBL) to suppress heating [56], Floquet prethermalization is generally applicable to clean systems without the need of strong spatial disorder.

The exponential suppression of the heating rate can be justified by noticing the residual part of the Floquet Hamiltonian satisfying  $\|\hat{H}_F - \hat{H}_{\text{pre}}\| < e^{-O(1/\Omega)}$ , with  $\|\cdot\|$  denoting the norm of an operator. Higher-order processes lead to exponentially small corrections that become noticeable after an exponentially long time scales [59]. Alternatively, one can also use a simple linear response theory—the Fermi’s golden rule (FGR) [36, 58]—to understand this exponential suppression. We start with a generic Floquet system

$$\hat{H} = \hat{H}_0 + g(t)\hat{K}, \quad (5)$$

where  $g(t)$  is taken the Fourier series form  $g(t) = \sum_{m>0} g_m \sin(m\Omega t)$ ,  $\hat{H}_0$  and  $\hat{K}$  are time-independent operators. For Fourier mode  $m$ , the heating rate is given by

$$\Gamma_m(t) = \frac{\dot{E}_m(t)}{E_\infty - E(t)}, \quad (6)$$

where  $E_\infty$  denotes the energy at infinite temperature with  $E_\infty = 0$  for our model. The transition rate of the mode  $\dot{E}_m(t)$  can be obtained from the FGR

$$\dot{E}_m(t) = 2\pi g_m^2 \sum_{i,f} |\langle E_f^0 | \hat{K} | E_i^0 \rangle|^2 (E_f^0 - E_i^0) P_i^0(t) \cdot \delta(E_f^0 - E_i^0 \pm m\Omega), \quad (7)$$

with  $E_i^0, E_f^0$  being eigenstates of  $\hat{H}_0$ , and  $P_i^0(t) = \langle E_i^0 | \rho(t) | E_i^0 \rangle$ . Combining all Fourier modes, the heating rate of the system is

$$\Gamma(t) = \sum_{m>0} \Gamma_m(t), \quad (8)$$

with the  $m$ -mode rate

$$\Gamma_m = g_m^2 A e^{-m\Omega/\epsilon}, \quad (9)$$

where  $A$  and  $\epsilon$  are  $\Omega$ -independent constants.

## B. Prethermalization by random multipolar driving

Non-periodically driven systems have attracted notable research interest during recent years, which go beyond the constraint imposed by the strict temporal periodicity. We focus on a family of random but structured driving protocol, random multipolar driving (RMD), characterized by the multipolar correlation  $n$ . This protocol involves two elementary evolution operators  $\hat{U}_+$  and  $\hat{U}_-$ , generated by two Hamiltonians  $\hat{H}_\pm$ . A random selection of  $U_\pm$  leads to 0-RMD sequence and normally this quickly destabilizes the system and induces heating. The heating rate can be significantly suppressed by imposing a dipolar structure into the random driving sequence, such that the elementary building blocks now read  $\hat{U}_+ \hat{U}_-$  and  $\hat{U}_- \hat{U}_+$ . 1-RMD protocol

then contains a random sequence of these two building blocks. Similarly, the  $n$ -th multipole can be recursively constructed by anti-aligning two  $(n - 1)$ -th order operators, and in the  $n \rightarrow \infty$  limit it converges to the self-similar Thue-Morse driving [69]. A detailed discussion on the properties of RMD sequences is presented in [36] and here we elaborate on how to estimate the heating rate.

A purely random sequence exhibits a flat and continuous distribution of different frequencies in the Fourier space. Similarly,  $n$ -RMD sequence also exhibits a continuous spectrum. However, the overall envelope of the distribution is modified to a power law  $x^n$  [36], due to the presence of the multipolar correlation in time. Therefore, to estimate the heating rate one can first approximate the time-dependent RMD signal as

$$g(\tau)\hat{K} = \int dx g_x \sin(x\Omega\tau)\hat{K}, \quad (10)$$

where  $g_x = x^n$  captures the suppressed spectrum distribution. According to the linear response theory, each frequency mode  $x$  contributes to the heating rate separately, following Supplementary Eq. 9 as in Floquet systems. Therefore, to obtain the entire heating rates from all frequencies, one can simply perform a integral

$$\Gamma = \int_0^\infty dx g_x^2 A e^{-x\Omega/\epsilon} \propto \Omega^{-2n-1} (2n)!, \quad (11)$$

where we use the Gamma function,  $F(n) = \int_0^\infty x^{n-1} e^{-x} dx = (n-1)!$ . Its inverse defines the heating time scale,  $1/\Gamma \propto \Omega^{2n+1}$ , which has been observed in our experiments. In fact, a generalized Floquet-Magnus expansion can be employed to investigate higher order processes. It goes beyond this simple linear response theory and still leads to the same prediction on the heating rates, see details in [69].

### C. Numerical results

We adopt two advanced tensor network algorithms, Projected Entangled Pair States (PEPS) and grouped Matrix Product States (GMPS), to simulate the dynamics generated by RMD protocols. Tensor networks are powerful methods to simulate quantum many-body physics and quantum circuits, providing highly efficient classical computational frameworks. Notably, they have demonstrated the capacity to surpass Google's quantum supremacy claims [91, 92]. Although tensor network algorithms are limited to simulating dynamics within constrained Hilbert spaces, they allow us to extract critical features from the numerical results. In this section, we initially present the Trotter-Suzuki decomposition, an effective method for approximately factoring time-evolution operators into products of local terms, thus making them suitable for tensor network simulations. Subsequently, we introduce these two methods, respectively, and compare their performance with our experimental observations. These numerical simulations will support our experimental observation, including  $T^{2n+1}$  scaling in heating rates, oscillatory dynamics in entanglement entropy during the prethermal regime, as well as the rapid entanglement growth induced by heating.

#### 1. Trotter-Suzuki Decomposition

The time-dependent evolution operators are supported over the entire Hilbert space, applying directly to a quantum state is nearly infeasible. To address this challenge, we introduce the second-order Trotter-Suzuki decomposition [93]. This method approximates the short-time evolution operator  $e^{-i\hat{H}\delta t}$  as a product of operators with finite support, thereby facilitating practical computation and implementation. Specifically, we split the Hamiltonian as

$$\hat{H} = \hat{H}^1 + \hat{H}^2 + \cdots + \hat{H}^m, \quad (12)$$

where  $\hat{H}^i$  is a sum of local operators  $\hat{H}^i = \sum_{x,y} \hat{h}_{x,y}^i$ . The second-order Trotter-Suzuki decomposition approximates the time-evolution operators as

$$e^{-i\hat{H}\delta t} = \prod_{k=m}^1 e^{-i\hat{H}^k \delta t/2} \prod_{k=1}^m e^{-i\hat{H}^k \delta t/2} + \mathcal{O}(\delta t^3). \quad (13)$$

The selection of  $\hat{H}^k$  guarantees that each term  $\hat{h}_{x,y}^k$  commutes pairwise. This commutativity implies that each operator on the right-hand side of Supplementary Eq. 13 can be further decomposed into a product of local operators.

$$e^{-i\sum_{x,y} \hat{h}_{x,y}^k \delta t/2} = \prod_{x,y} e^{-i\hat{h}_{x,y}^k \delta t/2}. \quad (14)$$

We express our two-dimensional Hamiltonian as a sum of five distinct components.

$$\hat{H}_\pm^1 = \sum_{x,y} (1 \pm \delta h) h_0 [1 - (-1)^y] a_{x,y}^\dagger a_{x,y}, \quad (15)$$

$$\hat{H}^2 = \sum_{x \text{ is odd}} \sum_y J a_{x+1,y}^\dagger a_{x,y} + \text{H.c.}, \quad (16)$$

$$\hat{H}^3 = \sum_{x \text{ is even}} \sum_y J a_{x+1,y}^\dagger a_{x,y} + \text{H.c.}, \quad (17)$$

$$\hat{H}^4 = \sum_x \sum_{y \text{ is odd}} J a_{x,y+1}^\dagger a_{x,y} + \text{H.c.}, \quad (18)$$

$$\hat{H}^5 = \sum_x \sum_{y \text{ is even}} J a_{x,y+1}^\dagger a_{x,y} + \text{H.c..} \quad (19)$$

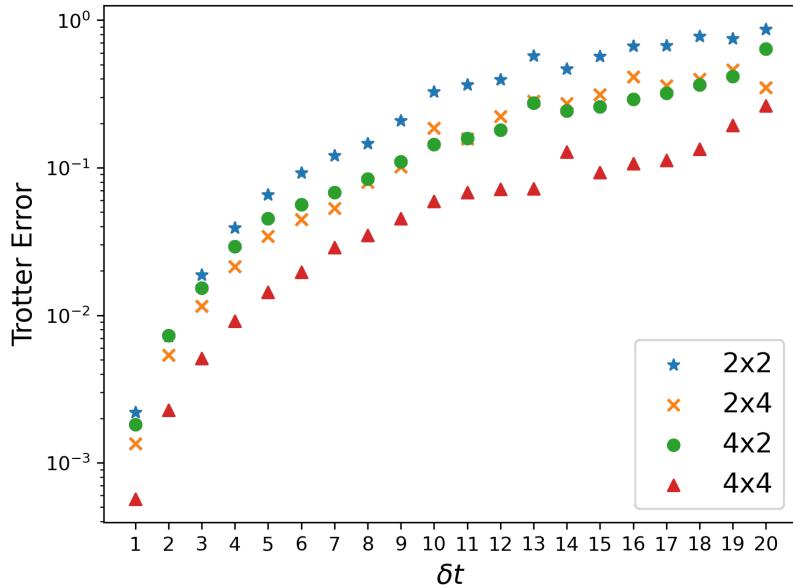
Here,  $\hat{H}^1$  is the sum of single-site operators while  $\hat{H}^2$  to  $\hat{H}^5$  are the sum of two-site operators.

For extended time intervals  $t$ , we introduce timestamps  $\delta t$ ,  $2\delta t$ ,  $3\delta t \dots$  and  $T$ ,  $2T$ ,  $3T \dots$ , and label them sequentially as  $t_1, t_2, \dots$  in a chronological order, with  $t_0 = 0$  and  $t_N = t$ . This construction guarantees that each time interval  $t_{i+1} - t_i$  is less than  $\delta t$  and ensures that the Hamiltonian remains time-independent.

We use a raw tensor to estimate the Trotter errors, defined as the maximum discrepancy between the imbalance computed using the exact Hamiltonian and that obtained from an approximate Hamiltonian,

$$\text{Trotter Error} = \|\mathcal{I}_{\text{raw}} - \mathcal{I}_{\text{exa}}\|_\infty. \quad (20)$$

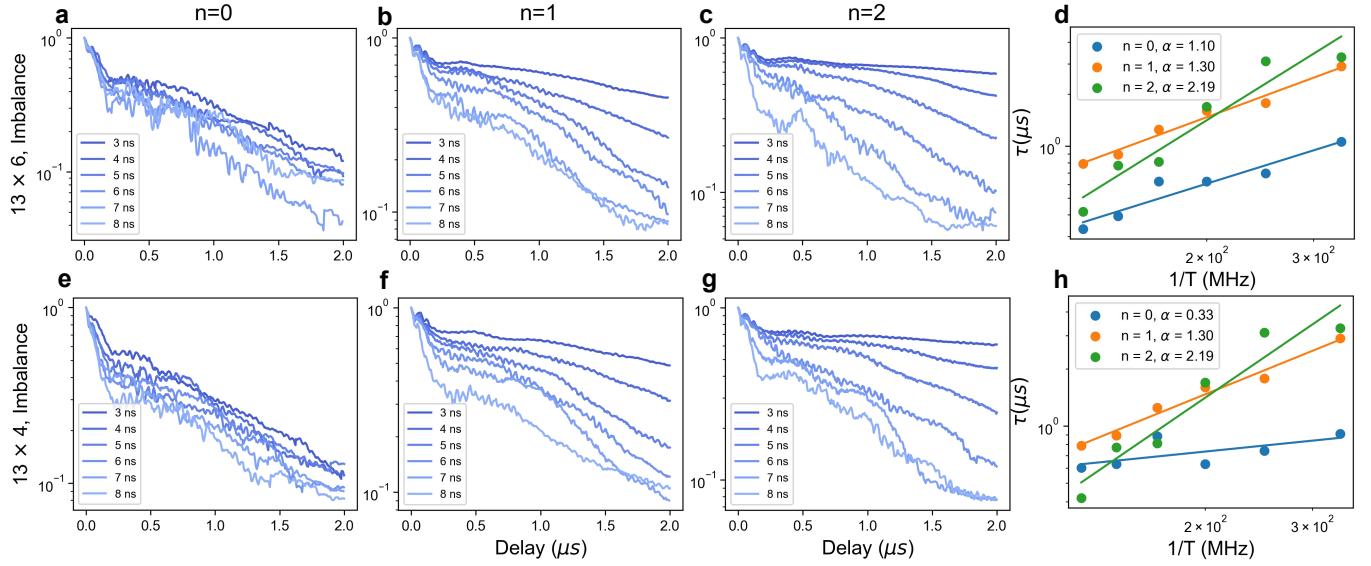
Here  $\mathcal{I}_{\text{raw}}$  is calculated using raw tensors in conjunction with the Trotter-Suzuki decomposition. We compute the Trotter errors exactly for the parameters  $T = 20$  ns and  $n = 2$  on  $2 \times 2$ ,  $2 \times 4$ ,  $4 \times 2$  and  $4 \times 4$  lattices. The results are depicted in Supplementary Fig. 1. In our subsequent simulations, we select  $\delta t = 3$  ns, such that the Trotter errors remain negligible.



Supplementary Fig. 1. **Estimating Trotter errors (Eq. 20).** Evaluate the Trotter errors with different time steps on  $2 \times 2$ ,  $2 \times 4$ ,  $4 \times 2$  and  $4 \times 4$  lattices. The driving period is set to  $T = 20$  ns and  $n = 2$ .

## 2. Project Entangled Pair States

PEPS presents a way to parameterize many-body wave functions in a tensor network whose structure matches the two-dimensional underlying geometry of size  $L_x \times L_y$ . This approach is scalable in 2D and expressive, making it a potent tool for investigating the physics of strongly correlated systems [94, 95] and for simulating quantum circuits [96].



**Supplementary Fig. 2. The dynamics of imbalance in the system of size \$13 \times 6\$ and \$13 \times 4\$ obtained by PEPS.** The maximum bond dimensions are \$\chi = 4\$. **a-c**, Size \$13 \times 6\$, \$n = 0, 1, 2\$ respectively, imbalance dynamics for \$T\$ ranging from 3 ns to 8 ns. **d**, We use a linear fitting to determine the power-law scaling, \$\tau \propto T^{-\alpha}\$, in the log-log scale. **e-h**, Size \$13 \times 4\$, same as **a-d**. All results are computed on Intel Xeon Platinum 8268 CPUs.

We adopt the same decomposition strategy as Supplementary Eqs. 15-19. Choose the basis \$|0\rangle \leftrightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}\$ for the unoccupied state and \$|1\rangle \leftrightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}\$ for the occupied one. The factorized time-evolution operators in this basis are represented as

$$e^{-i\delta t h a_i^\dagger a_i} = \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\delta t h} \end{bmatrix}. \quad (21)$$

$$e^{-i\delta t J(a_i^\dagger a_j + a_j^\dagger a_i)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\delta t J) & -i \sin(\delta t J) & 0 \\ 0 & -i \sin(\delta t J) & \cos(\delta t J) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (22)$$

Single-site operators are \$2 \times 2\$ matrices and thus can be directly applied to the single physical bonds. The two-site operators are \$4 \times 4\$ full-rank matrices, which will increase the bond dimensions. We apply the two-site time evolution operators using simple update methods [97, 98], simply applying the operators on local tensors without accounting for extra environments, and then truncate to retain the largest bond dimension \$\chi\$. This approach ensures that the computational cost is managed while maintaining the essential features of the system's states.

For generic PEPS, computing the expectation value of a single-site observable is computationally expensive due to the fact that fully contracting a PEPS is a \$\#P\$-hard problem. To address this, we employ the standard approximate method known as Time-Evolving Block Decimation (TEBD), which was originally designed to simulate the time evolution of matrix product states [99, 100], and can also be utilized to contract two-dimensional PEPS. The computational complexity of this procedure scales as \$\mathcal{O}(\chi^{12})\$, making it the most resource-intensive step. The dynamics of imbalance from PEPS of \$\chi = 4\$ are shown in Supplementary Fig. 2. Each curve is obtained by averaging over ten different random realizations. Although PEPS captures the signal of prethermalization over a very short time scale, the systems experience accelerated thermalization during extended dynamic processes, manifesting as rapid decays of the imbalance. Furthermore, the observed scaling law of the thermalization lifetime is significantly shorter than theoretical predictions. Accurately modeling longer dynamics would incur a substantially higher computational cost compared to experiments.

### 3. Grouped Matrix Product States

Given the inherent limitations of PEPS methods, particularly the lack of a canonical form, we employ MPS. MPS are renowned for their efficiency and high accuracy in representing one-dimensional gapped quantum phases [99]. With the availability of a

canonical form, the expectation values of local operators can be computed by contracting only the local tensors, rather than the entire tensor networks.

There are two possible MPS representations for a two-dimensional lattice. The first approach involves winding the MPS across the two-dimensional lattice, effectively forcing the system into a one-dimensional configuration. This transformation makes connections in the 2D lattice non-local, depending on the chosen 1D ordering. To apply non-local hopping operators in this configuration, additional SWAP gates are required to bring two sites into adjacency. We use the second approach, known as the grouped MPS method. In this approach, we group  $L_y$  lattice sites in a single column, treating them as a single site with a physical dimension of  $2^{L_y}$ . This allows us to represent the system as a MPS of length  $L_x$ . The advantage of this method is that it preserves locality. However, the trade-off is that it is not scalable in the  $y$ -direction.

We use the Trotter-Suzuki decomposition as follows

$$\hat{H}_\pm^1 = \sum_{x=1}^{L_x} \sum_{y=1}^{L_y} h_\pm a_{x,y}^\dagger a_{x,y} + J \left( a_{x,y+1}^\dagger a_{x,y} + \text{H.c.} \right), \quad (23)$$

$$\hat{H}^2 = \sum_{x=1}^{L_y} \sum_{y=1}^{L_y} J \left( a_{x+1,y}^\dagger a_{x,y} + \text{H.c.} \right). \quad (24)$$

The single-site operators are  $2^{L_y} \times 2^{L_y}$  matrices and can be directly applied to each tensor without affecting the canonical position. For two-site hopping terms, we take  $L_y = 4$  and  $\hat{U} = \exp(-iJ\delta t(a_{j=i+1,4}^\dagger a_{i,4} + \text{H.c.}))$  as an example and summarize update rules below. We also illustrate them in Supplementary Fig.3.

1. Perform QR decompositions on each tensor to isolate the active physical bonds ( $i_4$  and  $j_4$  here) into  $R$  tensors. This step is essential for ensuring that the calculations are in the most computationally efficient manner.
2. Apply the two-site time evolution operator and contract into a single tensor.
3. Execute a singular value decomposition (SVD) and eliminate the singular values that are smaller than a specified threshold. Subsequently, truncate the results by retaining the largest  $\chi$  singular values.
4. Absorb the singular values and vectors into the updated tensors.

Owing to the canonical form in MPS, it is possible to monitor fidelity throughout the update process. During each truncation step, the singular values  $s_1 \geq s_2 \geq \dots \geq s_D$  are arranged in descending order, and only the top  $\chi (< D)$  singular values are kept. Consequently, the fidelity is given by the ratio:

$$\frac{\sum_{k=1}^{\chi} s_k^2}{\sum_{k=1}^D s_k^2}, \quad (25)$$

This expression provides a measure of how much of the original state's information is preserved after truncation.

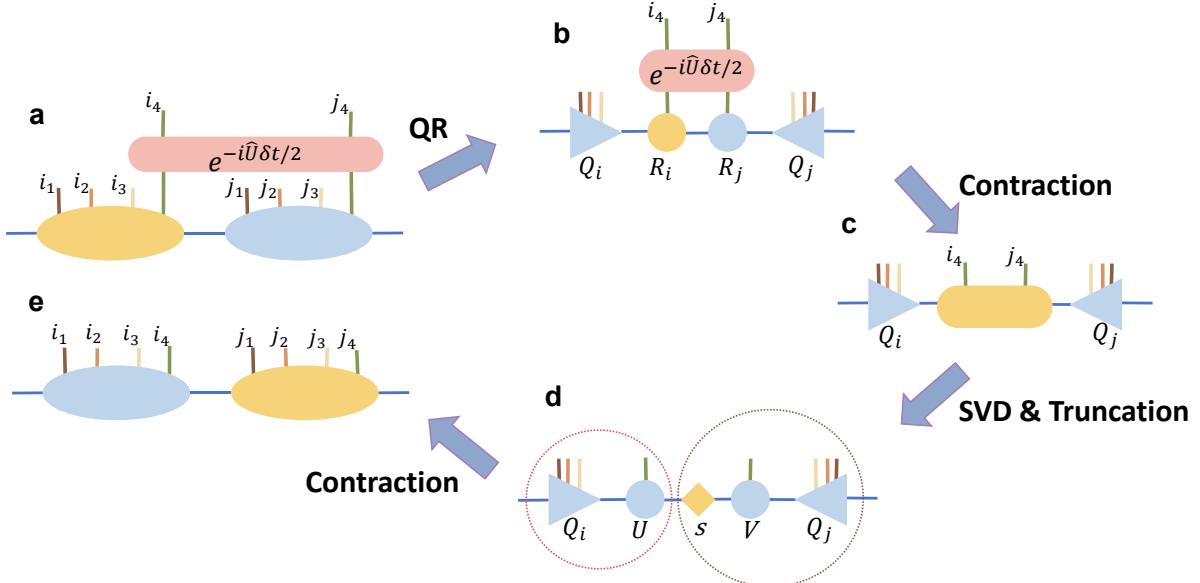
The numerical results of the dynamics of the  $13 \times 6$  system are shown in Supplementary Fig. 4. All computations are performed on Intel Xeon Phi 7285 CPUs and it takes about ten hours to complete one trajectory simulation. We compute the imbalance (Supplementary Fig. 4a-c), the entropy of the upper leftmost site (Supplementary Fig. 4d-f), and the fidelity (Supplementary Fig. 4g-i). Each curve is obtained by averaging over ten trajectories. The results show that the imbalance exhibits exponential decay at short times ( $t \lesssim 0.5$  ns, shown by the red dashed lines) and subsequently slows down. This time scale is also consistent with the deceleration as observed in the entropy growth. The short-time imbalance data are used to estimate the decay rate, as shown in Supplementary Fig. 4j, which exhibits the expected algebraic scaling  $T^{2n+1}$  approximately.

The decay of the estimated fidelity is also consistent with the heating process. During the prethermal regime, the system remains close to the Néel ordered state and the growth of the entanglement is slow. Therefore, MPS exhibits a high accuracy in numerical simulations. However, heating destabilizes the Néel order and at the same time, fidelity quickly drops. In particular, for  $T = 7$  ns and  $n = 2$  (corresponding to the orange experimental data in Fig.3), as shown in Supplementary Fig. 4i, a notable decay of the estimated fidelity is visible already at early times and it drops below 0.05 when  $t \approx 1.5$   $\mu$ s. This highlights that the non-equilibrium dynamics of a driven quantum system quickly become numerically intractable, especially for 2D systems like ours.

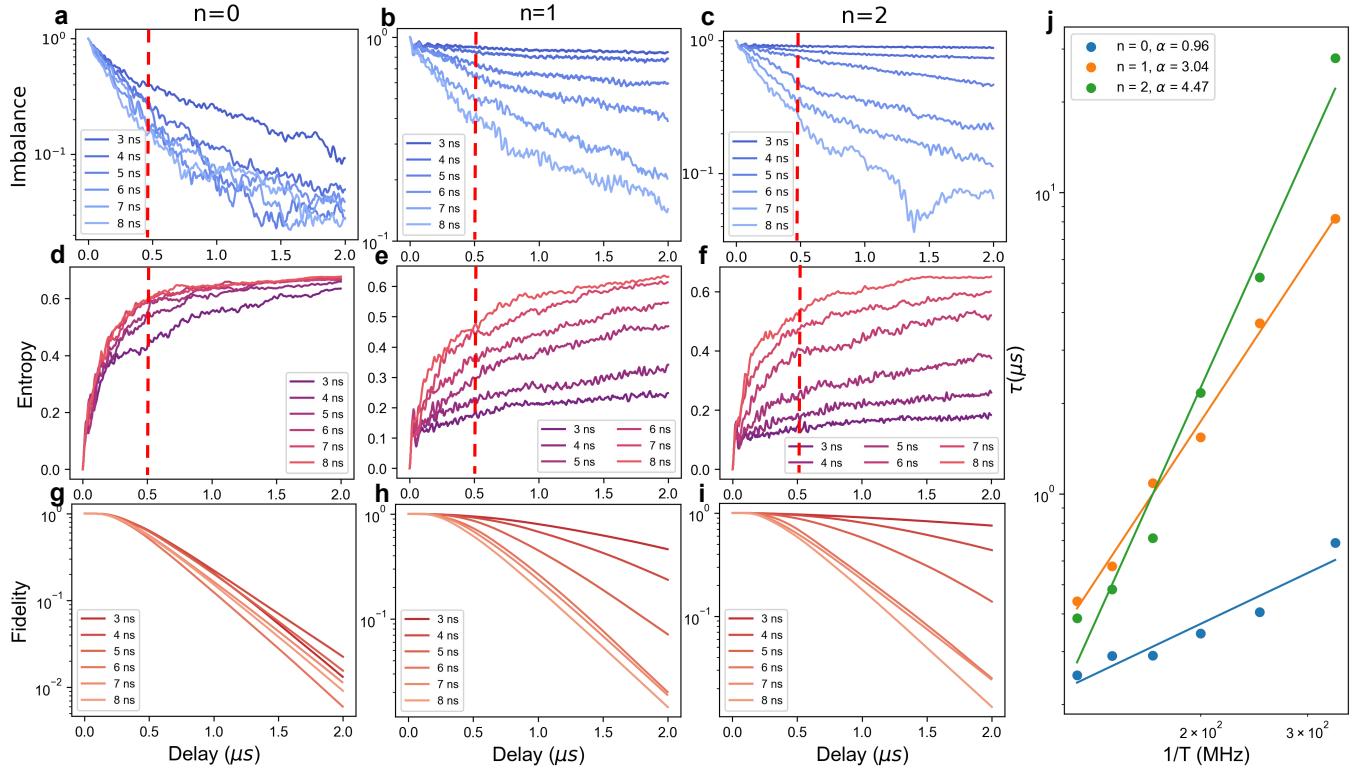
Utilizing the canonical form we can conveniently calculate the von Neumann entropy of a subsystem

$$S = -\text{Tr}(\rho \log \rho), \quad (26)$$

with the reduced density matrix  $\rho$ . For instance, when calculating the von Neumann entropy of the subsystem  $\mathcal{B}$  as depicted in Fig. 4a in the main text, all of the four qubits in  $\mathcal{B}$  are in one column and grouped into the fifth MPS tensor from left to



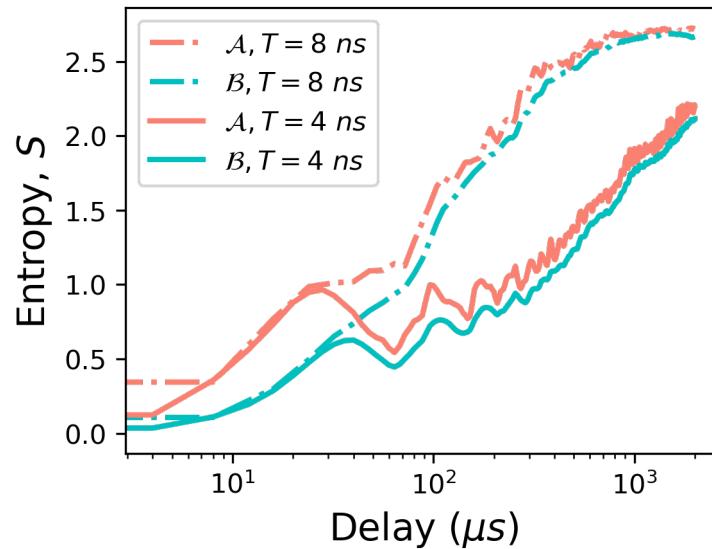
Supplementary Fig. 3. **A single step of time evolution under two-site operators.**  $i_1 \sim i_4$  and  $j_1 \sim j_4$  are labels for the local physical bonds along the  $y$ -direction. The canonical tensors are labeled in yellow. **a-b** QR decomposition on each tensor to extract the physical bonds  $i_4$  and  $j_4$  into  $R_{i(j)}$  tensors. **b-c** Contract  $R_{i(j)}$  tensors together with the time-evolution operator. **c-d** Apply singular value decomposition and keep the largest  $\chi$  singular values. **d-e** Absorb singular values and vectors into  $Q_{i(j)}$  to obtain the updated tensors.



Supplementary Fig. 4. **Grouped MPS results for a  $13 \times 6$  system.** The maximum bond dimension is set to  $\chi = 96$ , and we average ten random trajectories for each parameter being studied. **a-c**, Imbalance dynamics for various  $T$ , ranging from 3 ns to 8 ns, and  $n = 0, 1, 2$  respectively. **d-f**, Single-site entropy for various  $T$  ranging from 3 ns to 8 ns, and  $n = 0, 1, 2$  respectively. The red dashed line approximately marks the time where noticeable errors appear. **g-i**, The corresponding fidelity of the MPS simulation. **j**, We employ the short-time imbalance dynamics ( $t < 0.5$  ns) to determine the lifetime  $\tau$ . It exhibits a power-law dependence  $\tau \propto (1/T)^\alpha$ . The slopes are  $\alpha_{n=0} = 0.96$ ,  $\alpha_{n=1} = 3.04$ ,  $\alpha_{n=2} = 4.47$ , close to the theoretical prediction,  $2n+1$ .

right. We begin by moving the canonical site to the fifth tensor. This arrangement ensures that when tracing out the tensors 1~4 and 6~13 tensors we obtain two identity matrices separately. Then we reshape the fifth tensor to a tensor of dimensions  $(\chi, 2, \mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{2}, 2, \chi)$  and modify it to a  $16 \times 4\chi^2$  matrix  $C$ . Here, the bold numbers signify the physical dimensions of the subsystem  $\mathcal{B}$ . The reduced density matrix is  $\rho_{\mathcal{B}} = CC^\dagger$  and von Neumann entropy can be calculated straightforwardly. For the subsystem  $\mathcal{A}$ , computation of the reduced density matrix is more complicated. We still benefit from the canonical form by tracing out the 1~4 and 9~13 tensors, but need to contract the 5~8 tensors explicitly to get  $\rho_{\mathcal{A}}$ . For  $\chi = 96$ , the evaluation of  $S_{\mathcal{A}}$  takes approximately 6.9 seconds, whereas the evaluation of  $S_{\mathcal{B}}$  only takes about 0.54 seconds on a personal computer.

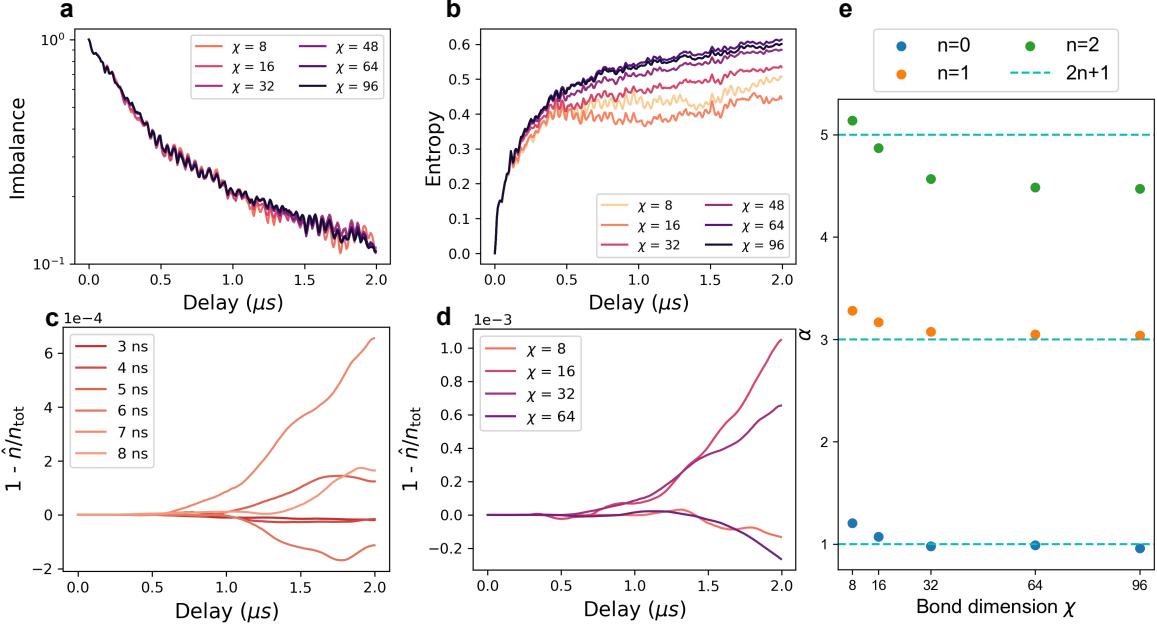
For simplicity, we first analyse the dynamics of the entanglement entropy of the top leftmost single site, as shown in Supplementary Fig. 4d-f. Entanglement entropy grows at different rates for different multipolar order and the driving period,  $T$ . Crucially, for multipolar order  $n \geq 1$ , the increase of entanglement entropy can be substantially suppressed by using a fast drive, highlighting the importance of the temporal correlation in stabilizing the system. However, heating is still inevitable and leads to notable entanglement generation at later times, significantly increasing the computational resources in simulating the time evolution.



Supplementary Fig. 5. **Entanglement dynamics.** The dynamics of entanglement entropy for different subsystems  $\mathcal{A}$  and  $\mathcal{B}$ .  $\mathcal{A}$  and  $\mathcal{B}$  both exhibits oscillatory behaviors when  $T = 4$  ns. In the experiment, however, we only observe the pronounced oscillatory dynamics in subsystem  $\mathcal{A}$ . For subsystem  $\mathcal{B}$ , due to the presence of noise and decoherence, as shown in Fig.4a, its entanglement entropy quickly saturates at a prethermal plateau in real experiments. We use 1-RMD drives.

We further analyze the entanglement dynamics of subsystems  $\mathcal{A}$  and  $\mathcal{B}$ , as illustrated in Supplementary Fig. 5. For a rapid drive with  $T = 4$  ns, our numerical simulation successfully reproduces the oscillation behavior of entanglement within the prethermal regime. It is worth noting that, the subsystem  $\mathcal{A}$  exhibits a more pronounced oscillation amplitude than the subsystem  $\mathcal{B}$ , indicating a non-uniform entanglement distribution in this prethermal plateau. This behavior has also been experimentally detected, as shown in Fig.4a in the main text. However, due to the presence of noise and decoherence, the oscillatory dynamics in subsystem  $\mathcal{B}$  becomes invisible experimentally. Also, for a slower drive (dashed dotted line in Supplementary Fig. 5), the system heats up swiftly without exhibiting the prethermal plateau.

Moreover, we investigate the performance of grouped MPS with different maximum bond dimensions. In Supplementary Fig. 6a-b, the congruence of these imbalance and entropy is sustained only for a short time, beyond which discrepancies appear at later times. Varying values of  $\chi$  also affects the largest reachable entanglement entropy. Even with  $\chi = 96$  the entropy is still far from the Page value at long times, suggesting the computational challenge in simulating the heating process. We calculate the scaling exponents by extracting the prethermal lifetime for different  $T$  based on the imbalance decay and present the results in Supplementary Fig. 6e. For  $n = 0$  and 1 the exponents converge to the theoretically prediction  $\alpha = 2n + 1$  (dashed lines) when  $\chi \geq 32$ . However, for  $n = 2$ , the exponent converges to around 4.5 which is still smaller than 5. It happens because for large  $n$ , typically one needs an extremely long time scale to extract the scaling exponent, as shown in [36], which is far beyond the reach of MPS simulations. In fact, similar behavior also appears in our experiments. As shown in Fig.3 a, due to the presence of noise and decoherence, for large system sizes the scaling exponent for  $n = 2$  is around 3.6.



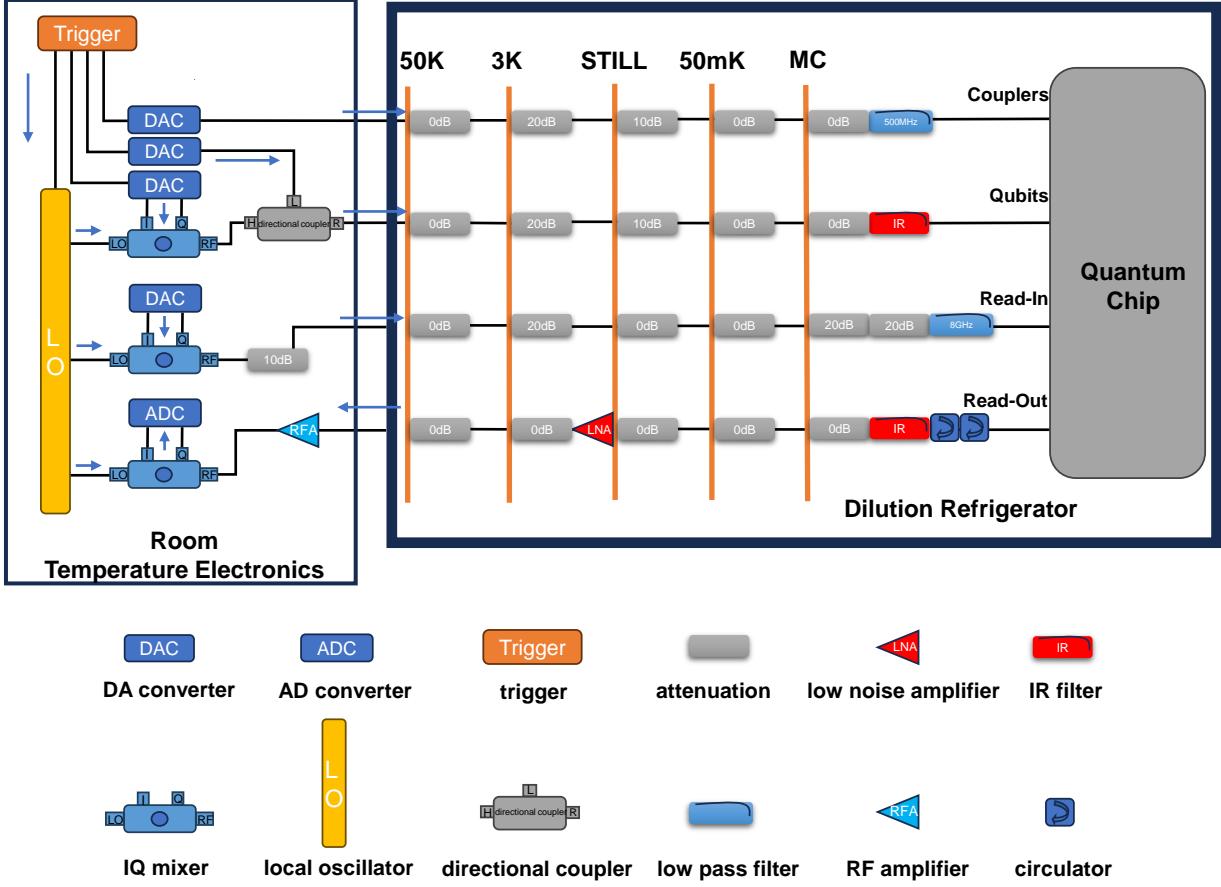
**Supplementary Fig. 6. Grouped MPS with varying maximum bond dimensions.** **a-b,** Depicts the imbalance and single-site entropy for a  $13 \times 6$  lattice, the time period is  $T = 7$  ns and with 2-RMD drives. **c,** Particle numbers with 2-RMD drives and different periods. The relative errors are at scale  $10^{-4}$  and the particle numbers are well preserved. **d,** Particle numbers at different maximum bond dimension, the time period is  $T = 7$  ns and with 2-RMD drives. **e,** Displays the exponents  $\alpha$  obtained through fitting with imbalance across various bond dimensions  $\chi$ ; the dots represent the fitted values, while the dashed lines correspond to  $2n + 1$ .

We also check the conservation of total particle number

$$\hat{n} = \sum_{x,y} a_{x,y}^\dagger a_{x,y}, \quad (27)$$

during the MPS simulation. It remains almost conserved at the value  $n_{\text{tot}} = L_x L_y / 2$  throughout the driven dynamics. This quantity serves as a critical criterion for the performance of our algorithms. Our analysis reveals that the relative errors in particle numbers are around  $10^{-4}$  (Supplementary Fig. 6c), indicating that convergence is robustly achieved. Furthermore, this conservation is consistently maintained across different values of  $\chi$  (Supplementary Fig. 6d).

In summary, we perform extensive numerical simulations via tensor network techniques. GMPS exhibits a better performance than PEPS for our systems. GMPS is capable of capturing the early time dynamics, based on these data we confirm the  $T^{2n+1}$  in the suppressed heating rates. However, the appearance of heating accelerates the growth of the entanglement entropy and GMPS method requires significant amount of computational resources in emulating our 78-qubit experiments over 1,000 driving cycles. Therefore, our experiment potentially demonstrates a quantum advantage in studying the non-equilibrium many-body dynamics in driven systems.

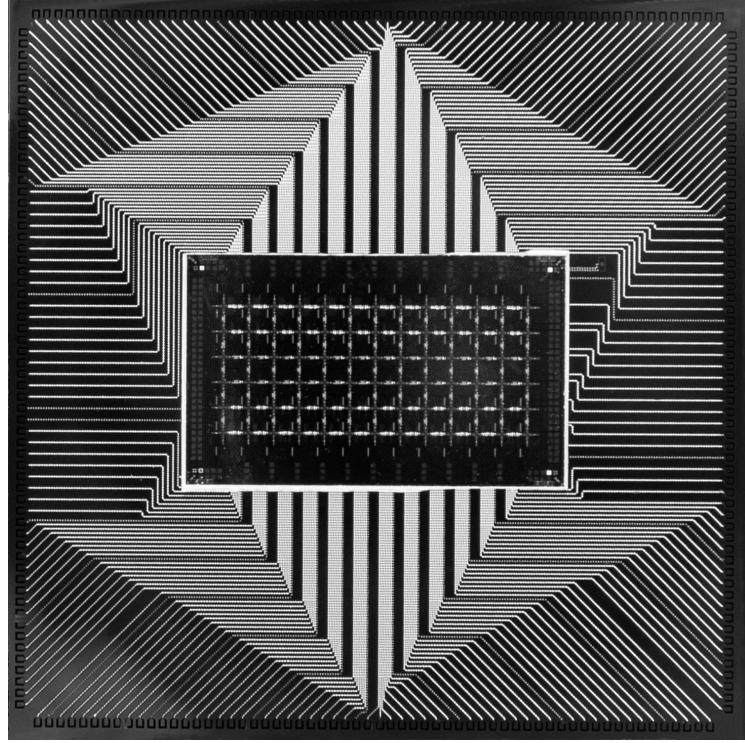


Supplementary Fig. 7. Room temperature and cryogenic wiring.

## S2. EXPERIMENTAL SETUP

Our experiments are performed on a superconducting quantum processor, named *Chuang-tzu 2.0*, which comprises 78 qubits arranged in a square lattice configuration with 6 rows and 13 columns. Each pair of nearest-neighbor qubits is interconnected by an adjustable coupler, resulting in a total of 137 couplers within the processor. In Supplementary Fig. 7, we show the room temperature and cryogenic wiring layout. The cryogenic environment, maintained at approximately 12 mK, is achieved using a BlueFors dilution refrigerator (model XLD1000). A carefully designed arrangement of attenuators and filters has been implemented to effectively suppress thermal noise, ensuring precise control of qubits and couplers.

To meet the high-frequency requirements of RMD, we utilize a dual-channel digital-to-analog converter (DAC) operating at 2 Gs/s with 16-bit vertical resolution and dual-channel analog-to-digital converter (ADC) at 1 Gs/s with 14-bit vertical resolution, controlled by a field-programmable gate array (FPGA) chip. This system, functioning as the Z control for both qubits and couplers, is capable of generating nanosecond-scale pulses, ensuring the precision and speed necessary for advanced quantum operations. Moreover, when combined with a microwave source acting as the local oscillator (LO), the two DAC channels can modulate the in-phase and quadrature components of an IQ mixer, enabling frequency up-conversion to generate signals in the GHz range. These signals are critical for qubit XY control and dispersive readout operations. We also perform zero calibration of each IQ mixer to reduce intrinsic and mirror leakages. The custom DAC/ADC devices are synchronized using a trigger signal and a 250 MHz reference clock. Communication between the host computer and the devices is achieved through a high-speed local area Ethernet network, leveraging fiber-optic connections with a bandwidth of 10,000 Mbps.



Supplementary Fig. 8. Optical micrograph of the processor *Chuang-tzu 2.0*.

### S3. DEVICE FABRICATION AND PERFORMANCE

#### A. Device fabrication

The optical micrograph of the processor is displayed in Supplementary Fig. 8. It is noteworthy that the qubit connectivity in our processor, while maintaining a square lattice structure, differs slightly from processors such as Sycamore [81], which feature a square lattice with zig-zag edges. We employ a tunable coupler with a capacitively connecting pad (TCCP) architecture [77], which facilitates a  $1200\text{-}\mu\text{m}$  spacing between adjacent qubits, thereby ensuring sufficient wiring space. The qubits and couplers are fabricated on the qubit layer chip, whereas the control lines, readout lines, and readout cavities are integrated into the wiring layer chip. These two chips are interconnected using flip-chip bonding technology. The readout cavities for the 6 qubits in each column are multiplexed onto a single readout line and are capacitively coupled to the qubits through interfacial capacitance. The control lines are similarly coupled to the qubits and couplers, enabling excitation and biasing via interfacial capacitance and mutual inductance.

The superconducting qubit processor utilized in the experiment was fabricated using an aluminum film deposited on a sapphire substrate, combined with a flip-chip bonding process. The detailed fabrication procedure is outlined as follows:

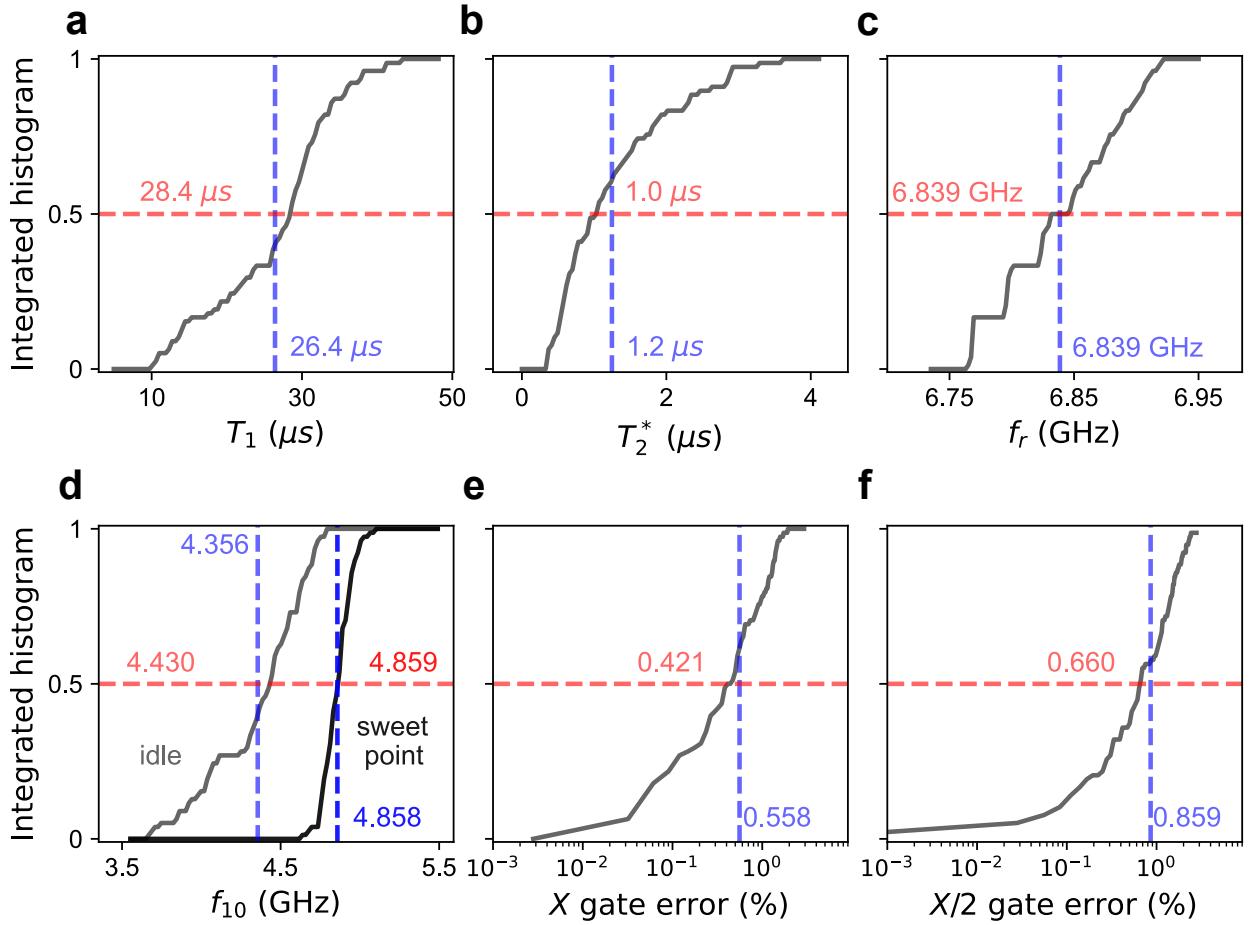
1. Two two-side polished sapphire wafers are annealed by electron beam evaporation equipment. Then 100-nm aluminum is evaporated, followed by in-situ oxidation to generate a dense aluminum oxide. This dense aluminum oxide can both protect the aluminum film and make subsequent etching processes more uniform.
2. Apply SPR-955 photoresist to the wafer surface. Use laser direct writing to expose patterns of the qubit layer and wiring layer on two wafers respectively. Develop with Tetramethylammonium hydroxide solution and etch with heated one. Finally, use the N-Methylpyrrolidone to remove the photoresist to obtain two the qubit wafer and wiring wafer.
3. We need to prepare niobium on the wafers as a transition layer between aluminum and indium, as well as for the alignment mark of the electron beam exposure. Spin-coat LOR-5A and SPR-955 photoresists sequentially, then expose and develop to obtain patterns with undercut structures. Use magnetron sputtering to in-situ argon ion cleaning to remove the aluminum surface aluminum oxide layer and then deposit 100-nm niobium. Finally, remove the photoresist and lift-off the excess niobium film from the surface of the wafers.
4. Prepare Josephson junctions on the qubit wafers. Spin-coat MAA-EL9 and PMMA-A5 bilayer resist sequentially and then deposit 10-nm aluminum for conductivity. Perform multiple exposures with different doses using electron beam

lithography, and then remove the surface 10-nm aluminum, and develop using MIBK solution diluted with isopropanol. Employ in-situ argon ion cleaning with electron beam evaporation to remove the surface aluminum oxide layer, followed by double-angle evaporation with 65-nm and 100 nm aluminum respectively. Introduce in-situ oxidation between the two evaporation steps to form the insulating region in the Josephson junction's two-layer superconductors. By controlling the oxidation time, the Josephson junction's resistance and Josephson energy can be managed. Finally, remove the photoresist and lift-off the excess aluminum film, and its resistance is tested using a probe station to ensure it meets the design specifications.

5. Both the qubit wafer and wiring layer wafer need to prepare air-bridges to achieve better ground sharing and reduce various parasitic modes. Start by spin-coating SPR-220 photoresist, exposing and developing the pillar patterns for the air-bridges. Then perform reflow to create arched air-bridges surfaces. Use electron beam evaporation to deposit 500-nm aluminum after in-situ argon ion cleaning, followed by in-situ oxidation. Spin-coat S-1813 photoresist, expose and develop the areas outside the air-bridges, and then etch using heated aluminum etchant type-D. Clean with pure oxygen using reactive ion etching and then remove the resist to obtain the air-bridges.
6. Prepare indium bumps for mechanically and superconductively connecting the two chips. Spin-coat AZ-4620 photoresist for the first exposure to create the undercut region. Wait for a period to allow the photoresist to rehydrate and degas, then spin-coat S-1813 photoresist for the second exposure in the indium bump area. After development, a structure with a deep undercut can be obtained. Use a indium thermal evaporation equipment to deposit 7- $\mu\text{m}$  of indium after in-situ argon ion cleaning. Clean with a reactive ion etching using a mix of argon and oxygen, and remove the photoresist and lift-off the excess indium film to obtain the indium bumps.
7. Finally, we will dice the wafers into chips and proceed with the packaging. Spin-coat AZ-4620 to protect the wafer, then use a dicing saw to cut the wafer into chips for the qubit layer (11mm  $\times$  18mm) and wiring layer (35mm  $\times$  35mm). After removing the photoresist, the two chips are bonded using flip-chip bonding equipment. Lastly, the chips are packaged into sample boxes using an ultrasonic wire bonder. After verifying that all connections from the sample box to the chip are functioning correctly using a multimeter, the chip can undergo low-temperature testing.

## B. Device performance

In Supplementary Fig. 9, we display the basic performance of qubits, including mean energy relaxation time  $\bar{T}_1$ , Ramsey dephasing time at idle frequency  $T_2^*$ , qubit readout frequency  $f_r$ , idle frequency  $f_{10}$ , and single-qubit gate error. The flux crosstalk between neighbouring qubits is below  $10^{-3}$ . The hopping strength between neighboring qubits can be precisely adjusted using couplers, with a tunable range spanning from -10 MHz to 3 MHz. This flexibility allows for straightforward achievement of the target value,  $J/2\pi = 2$  MHz. Notably, the average anharmonicity is  $U/2\pi = -200$  MHz, with a ratio  $|U/J| \approx 100 \gg 1$ , our processor can be regarded as a hard-core bosonic system [72]. All measured qubit probabilities are corrected to mitigate the readout errors.



Supplementary Fig. 9. **Device performance of the processor *Chuang-tzu 2.0*.** **a**, Distribution of the mean energy relaxation time  $\bar{T}_1$ . **b**, Distribution of the dephasing time  $\bar{T}_2^*$  measured by Ramsey experiments at idle points. **c**, Distribution of the readout frequency  $f_r$ . **d**, Distribution of qubit frequency of sweet points (the darker curve) and idle points (the lighter curve). **e**, Distribution of the X gate error. **f**, Distribution of the X/2 gate error. The single-qubit gates are characterized by quantum process tomography (QPT). In these panels, the blue vertical dashed lines indicate the mean values, while the red horizontal dashed lines represent the median values.

## S4. DEVICE CONTROL AND CALIBRATION

### A. Pulse calibration

When signals propagate from the DAC device to the qubits and couplers, the pulse shape undergoes distortion arising from the finite bandwidth limitations of various electrical components. The distortion of the Z pulses can induce an unintended frequency drift prior to the readout process. In our experiments, we implement precise correction protocol to effectively compensate for the distortion of Z pulses.

Typically, the measurement circuit can be modeled as a linear time-invariant (LTI) system, where the output is fully determined by its impulse response  $h(t)$ :

$$y(t) = S[x(t)] = \int_{-\infty}^{+\infty} d\tau x(\tau)h(t - \tau) = x(t) * h(t), \quad (28)$$

where  $f * g$  denotes the convolution between  $f$  and  $g$ ,  $h(t)$  represents the system's output when subjected to a Dirac delta function  $\delta(t)$  as the input

$$h(t) = S[\delta(t)]. \quad (29)$$

Alternatively, the unit step response of the Z pulse during propagation can be measured, i.e.,  $s(t) = S[\theta(t)]$ , with the unit step function  $\theta(t)$ , offering a more practical and convenient implementation, and  $h(t)$  is given by

$$h(t) = \dot{s}(t). \quad (30)$$

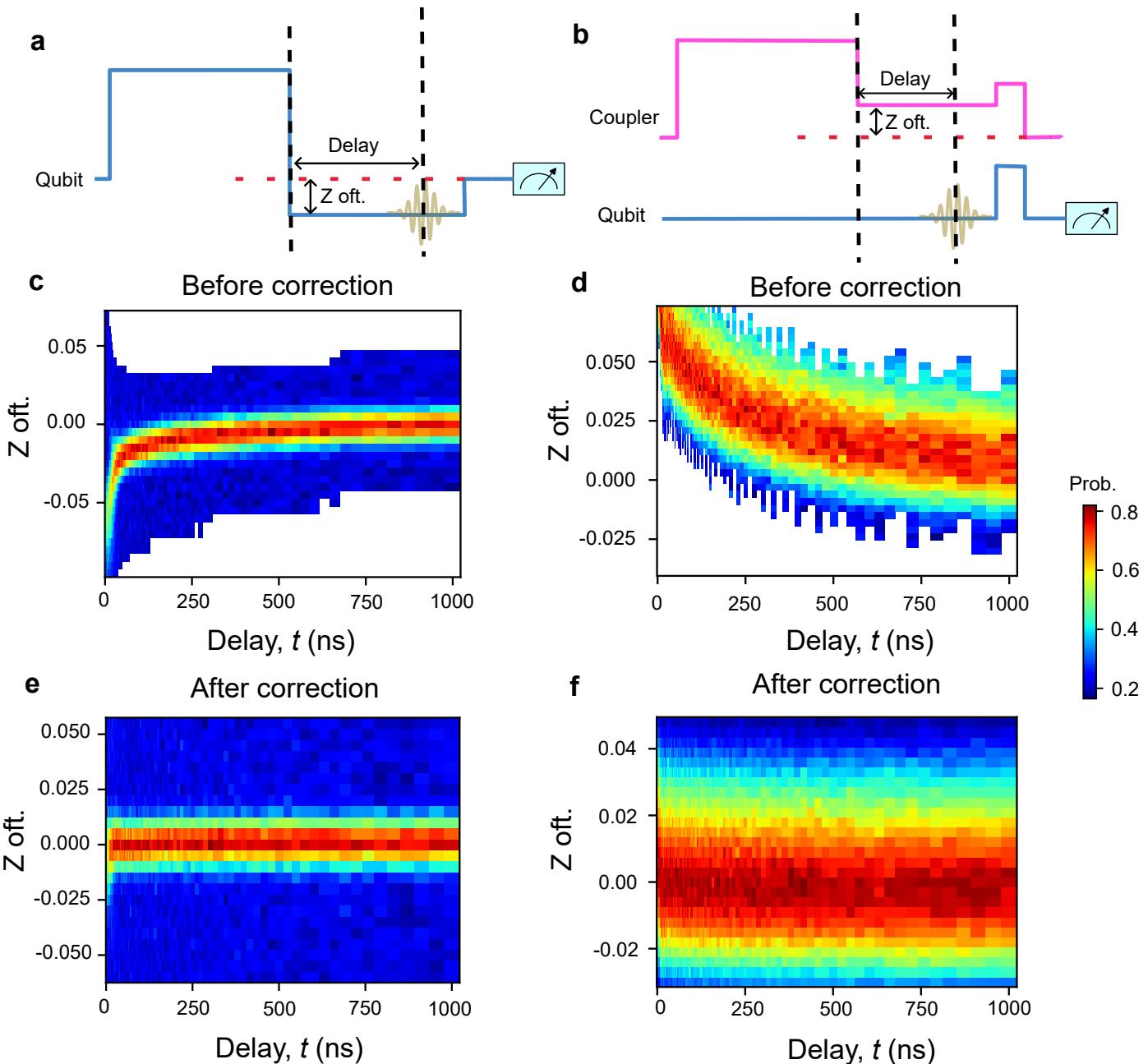
In frequency domain, we have Fourier transformations  $Y(j\omega) = \mathcal{F}[y(t)]$ ,  $H(j\omega) = \mathcal{F}[h(t)]$  and  $X(j\omega) = \mathcal{F}[x(t)]$ , such that Supplementary Eq. (28) is transformed as

$$Y(j\omega) = H(j\omega)X(j\omega), \quad (31)$$

with  $H(j\omega) \approx 1$ . Thus, the target signals can be effectively reconstructed through deconvolution with the inverse of the frequency response. According to Supplementary Eq. (30), the key procedure for distortion correction is to obtain the unit step response  $s(t)$ . The pulse sequences for correcting the Z pulse distortion in qubits and couplers [101] is shown in Supplementary Fig. 10a and b, respectively.

To correct the distortion in a qubit, a long pulse with duration about  $3 \mu s$  and a fixed amplitude is applied to the Z channel, generating a step signal. After a specific delay, a  $\pi$ -pulse is applied to the qubit, which can only excite the qubit to the  $|1\rangle$  state when its frequency matches that of the  $\pi$ -pulse. The probability of the qubit being in the  $|1\rangle$  state is then measured, as illustrated in Supplementary Fig. 10c. By determining the peak population of the  $|1\rangle$ -state for each delay, the Z offset data can be fitted using an exponential decay function. By employing the deconvolution method, we generate a corrected waveform that is subsequently transmitted to the arbitrary waveform generator (AWG). As a result, the qubit experiences a significantly flattened Z pulse, as demonstrated in Supplementary Fig. 10e.

In addition, the Z distortion in the coupler can be effectively corrected by exploiting its interaction with the neighboring qubit. This is achieved by first applying a step signal to the coupler, followed by a  $\pi$ -pulse to the neighboring qubit after a specific delay. The qubit and coupler are then biased into resonance to enable efficient energy exchange by a fine-tuned swapping duration and amplitude. Following this swapping process, the probability of the  $|1\rangle$  state is measured. Similarly, the population of the  $|1\rangle$  state is maximized only when the coupler frequency precisely matches the qubit frequency. As demonstrated in Supplementary Fig. 10d and f, the peak probability of the  $|1\rangle$  state can be fitted to accurately characterize and correct the Z distortion in the couplers.



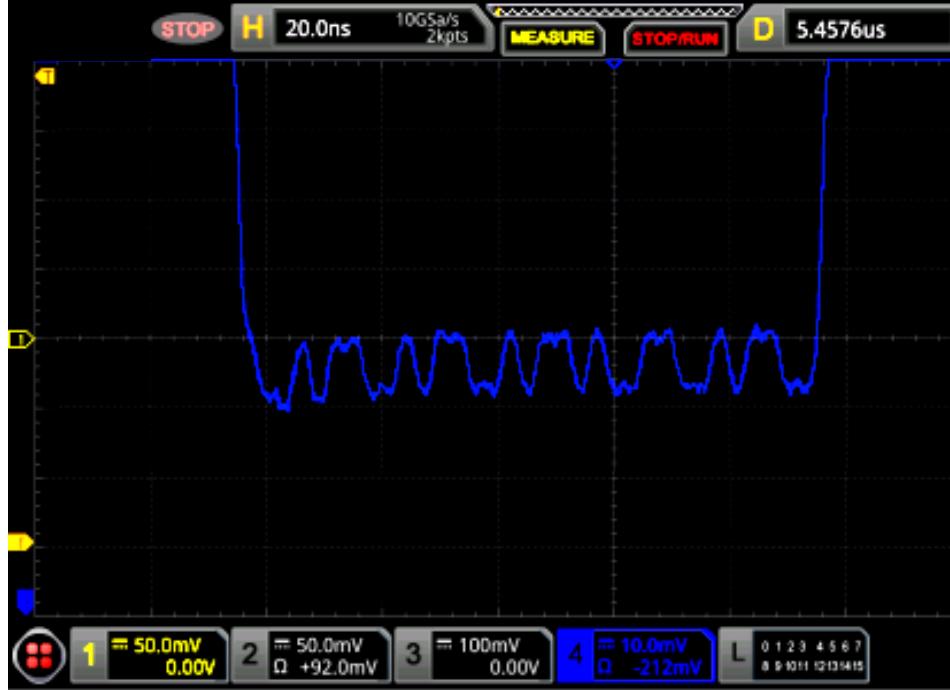
Supplementary Fig. 10. **Calibration of the Z pulse distortion for qubits and couplers.** **a**, Schematic of the experimental pulse sequence for correcting Z distortion of qubits. **b**, Schematic of the experimental pulse sequence for correcting Z distortion of couplers. **c** and **d**, The  $|1\rangle$  state probability before correction for qubits and couplers, respectively. **e** and **f**, The  $|1\rangle$  state probability after correction for qubits and couplers, respectively.

## B. The RMD pulse

To implement the Hamiltonian described in the main text, the qubit frequency  $\omega_q$  needs to be rapidly modulated between two values  $\omega_q/2\pi = \omega_c + \delta h \cdot h_0$  for  $U_+$ , while  $\omega_q/2\pi = \omega_c - \delta h \cdot h_0$  for  $U_-$ , where  $\omega_c$  is the common qubit frequency. For the frequency-tunable transmon qubit, the relationship between  $\omega_q$  and the amplitude of Z pulse  $z$  is

$$\omega_q = \sqrt{8E_{JJ}E_C|\cos(kz+b)|} - E_C, \quad (32)$$

where  $E_{JJ}$  denotes the Josephson energy,  $E_C$  is the charging energy, and  $kz+b = \Phi_{\text{ext}}/\Phi_0$  with the weak external flux  $\Phi_{\text{ext}}$ . The RMD pulse consists of a series of square Z waves with duration of  $T$ . Owing to the constrained sampling rate of the DAC, both the falling and rising edge durations are inherently limited to a minimum of about 0.5 ns. In our experiments, these square waves are substituted to trapezoidal waves with edges of 0.5 ns. Additionally, we present the waveform captured by the oscilloscope in Supplementary Fig. 11.



Supplementary Fig. 11. Waveforms obtained from oscilloscope measurements.

To further characterize the RMD pulse, we measure the dynamical phase induced during its operation. Initially, the qubit is prepared at its idle point and excited using an  $X/2$  gate. Then, the qubit is biased to the working point using the RMD pulse and the flat pulse following a delay, respectively. After turning off all Z pulses to tune the qubit back to its idle point, we apply another rotation  $R_\phi(\frac{\pi}{2})$ , with  $\phi$  ranging from 0 to  $2\pi$ . The population of  $|1\rangle$  state reaches its maximum only when  $\phi$  compensates for the accumulated dynamical phase. In Supplementary Figs. 12a and b, we show the accumulated phase in the RMD case

$$\varphi_r(t) = \int_0^t dt (\omega_r(t) - \omega_{\text{idle}}), \quad (33)$$

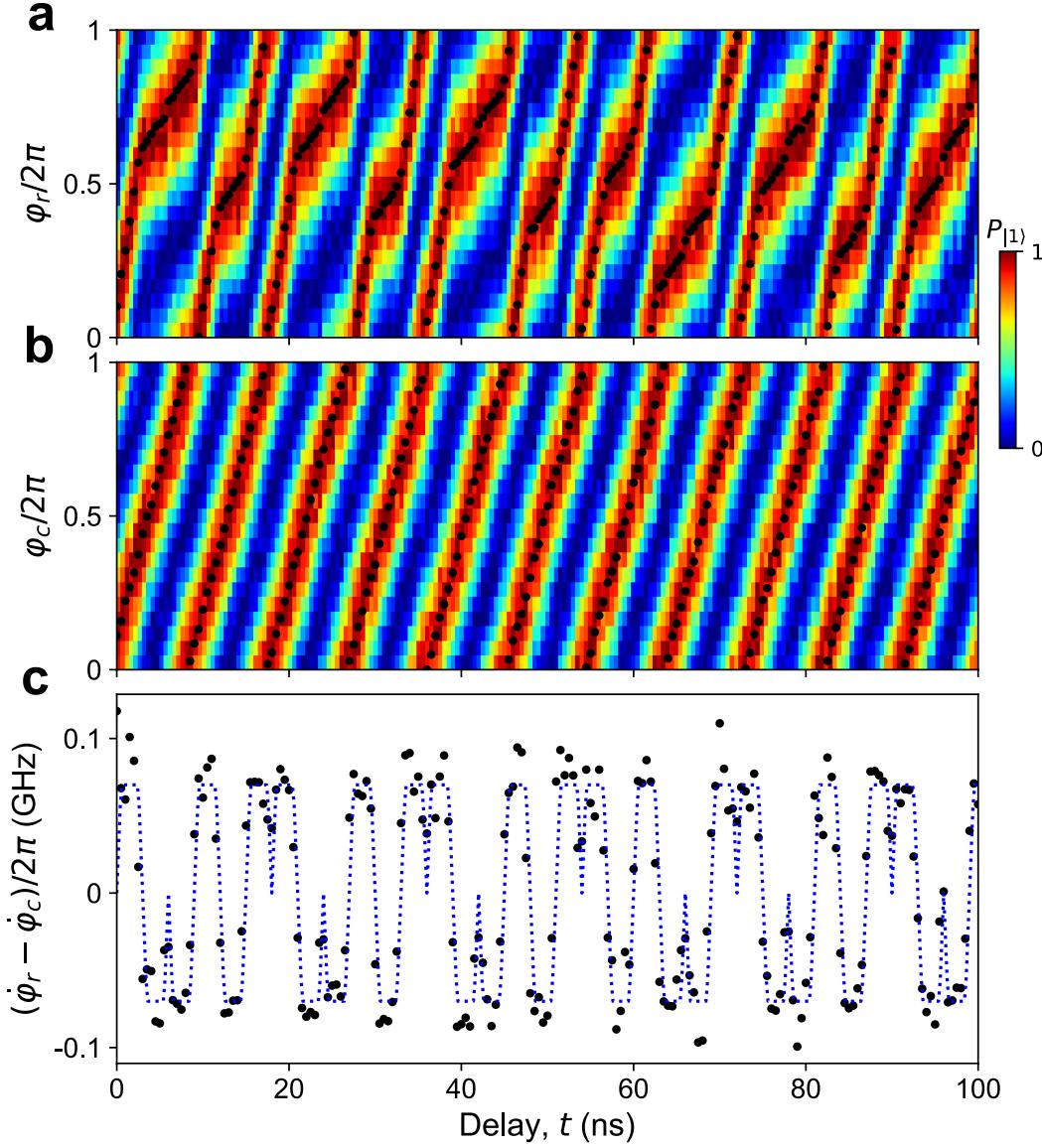
where  $\omega_r(t)$  is the qubit frequency under RMD, and the phase in the flat case

$$\varphi_c(t) = \int_0^t dt (\omega_c - \omega_{\text{idle}}). \quad (34)$$

The RMD sequence is precisely recovered by differentiating  $\varphi_r - \varphi_c$  with respect to time

$$\omega_r(t) - \omega_c = \frac{d}{dt}(\varphi_r(t) - \varphi_c(t)). \quad (35)$$

The result, as depicted in Supplementary Fig. 12c, is well in accordance with the engineered sequence.



Supplementary Fig. 12. **Characterization of the random multipolar driving (RMD) pulse.** **a**, The measured phase  $\varphi_r$ , when employing RMD pulse, as a function of RMD pulse duration. **b**, The measured phase,  $\varphi_c$ , obtained by applying rectangle Z pulse, versus the pulse duration. **c**, The qubit frequency, derived by differentiating  $\varphi_r - \varphi_c$  with respect to time, as a function of the delay. The dashed curve represents the engineered RMD sequence, with the characteristic timescale  $T = 3$  ns,  $n = 1$  and the amplitude of  $\delta h \cdot h/2\pi = 70$  MHz. Specially, the RMD sequence reads  $\{U_+U_-U_-U_+U_-U_+\dots\}$ .

### C. Timing by Floquet engineering

Another challenge in achieving precise RMD control lies in the timing misalignment between adjacent qubits, when applying rapid pulse with the short period less than 5 ns.

Here, we employ Floquet engineering [26] to synchronize the timing sequence of the flux bias pulses between qubit-qubit pairs. As shown in Supplementary Fig. 13a, the Floquet sequence  $\{U_-U_+U_-U_+U_-U_+-\dots\}$ , with the same characteristic time  $T$ , is simultaneously applied to two adjacent qubits. The coupling strength is dynamically modulated according to the degree of timing misalignment between the qubits. To mimic the the modulation dynamics, we use a sinusoidal waveform

$(j = 1, 2)$

$$\omega_j(t) = \omega_c + A \sin(\mu t + \varphi_j), \quad (36)$$

where  $A = \delta h \cdot h$  is the driving amplitude,  $\mu = \pi/T$  is the driving frequency, and  $\varphi_j$  characterizes the timing offset. The Hamiltonian reads

$$\hat{H} = -\frac{1}{2}\omega_1(t)\sigma_1^z - \frac{1}{2}\omega_2(t)\sigma_2^z + g(\sigma_1^+\sigma_2^- + \text{H.c.}), \quad (37)$$

with Pauli matrices  $\sigma_{x,y,z}$ ,  $\sigma_{\pm} = (\sigma_x \pm i\sigma_y)/2$ , and the direct coupling strength  $g$ . By applying the unitary transformation

$$\hat{U} = \exp \left\{ i \int_0^t dt [\omega_1(t) + \omega_2(t)] \right\} = \exp \left\{ -\frac{1}{2}i \sum_j \left[ \omega_c t + \frac{A}{\mu} \cos(\mu t + \varphi_j) \right] \sigma_j^z \right\}, \quad (38)$$

the effective Hamiltonian can be calculated as

$$\hat{H}' = \hat{U} \hat{H} \hat{U}^\dagger + i\partial_t \hat{U} \hat{U}^\dagger \quad (39a)$$

$$= g \exp \left\{ i \frac{A}{\mu} [\cos(\mu t + \varphi_1) - \cos(\mu t + \varphi_2)] \right\} \sigma_1^+ \sigma_2^- + \text{H.c.} \quad (39b)$$

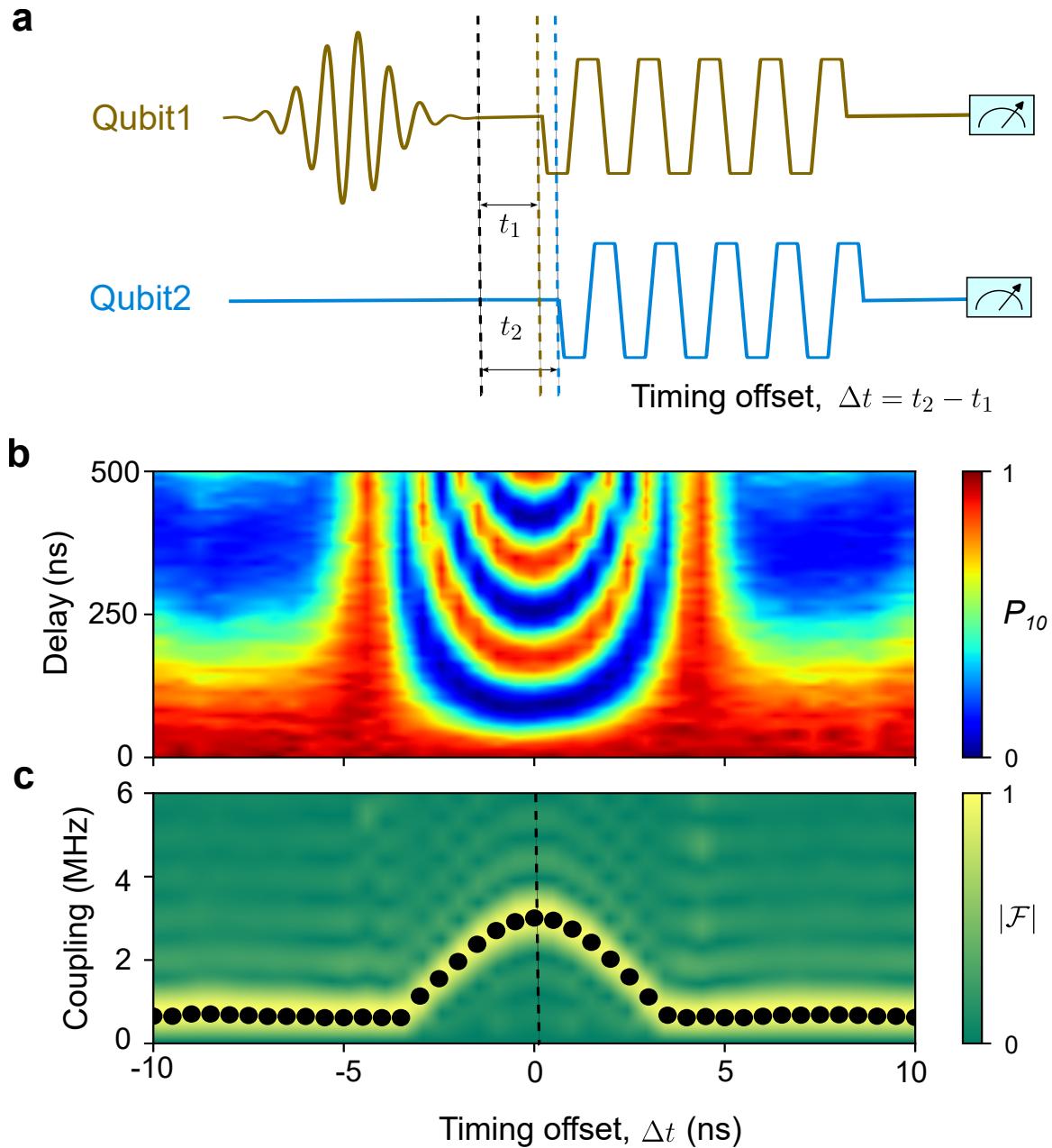
$$= g \exp \left\{ 2i \frac{A}{\mu} \sin\left(\frac{\Delta\varphi}{2}\right) \sin\left(\mu t + \frac{\varphi_1 + \varphi_2}{2}\right) \right\} \sigma_1^+ \sigma_2^- + \text{H.c.} \quad (39c)$$

$$= g \sum_{n=-\infty}^{\infty} J_n \left( 2 \frac{A}{\mu} \sin\left(\frac{\Delta\varphi}{2}\right) \right) e^{in\theta} \sigma_1^+ \sigma_2^- + \text{H.c.}, \quad (39d)$$

where  $\Delta\varphi = \varphi_2 - \varphi_1$  is the phase difference,  $\theta = \mu t + (\varphi_1 + \varphi_2)/2$ ,  $J_n(\cdot)$  is the  $n$ -order Bessel function, and the last line is from Jacobi-Anger series  $e^{ix \sin \theta} = \sum_n J_n(x) e^{in\theta}$ . Under rotating-wave approximation (RWA), when the condition  $\mu \gg g$  is satisfied, only the lowest frequency term  $n = 0$  remains, leading to the effective coupling strength

$$g^{\text{eff}} = g J_0 \left( 2 \frac{A}{\mu} \sin\left(\frac{\Delta\varphi}{2}\right) \right). \quad (40)$$

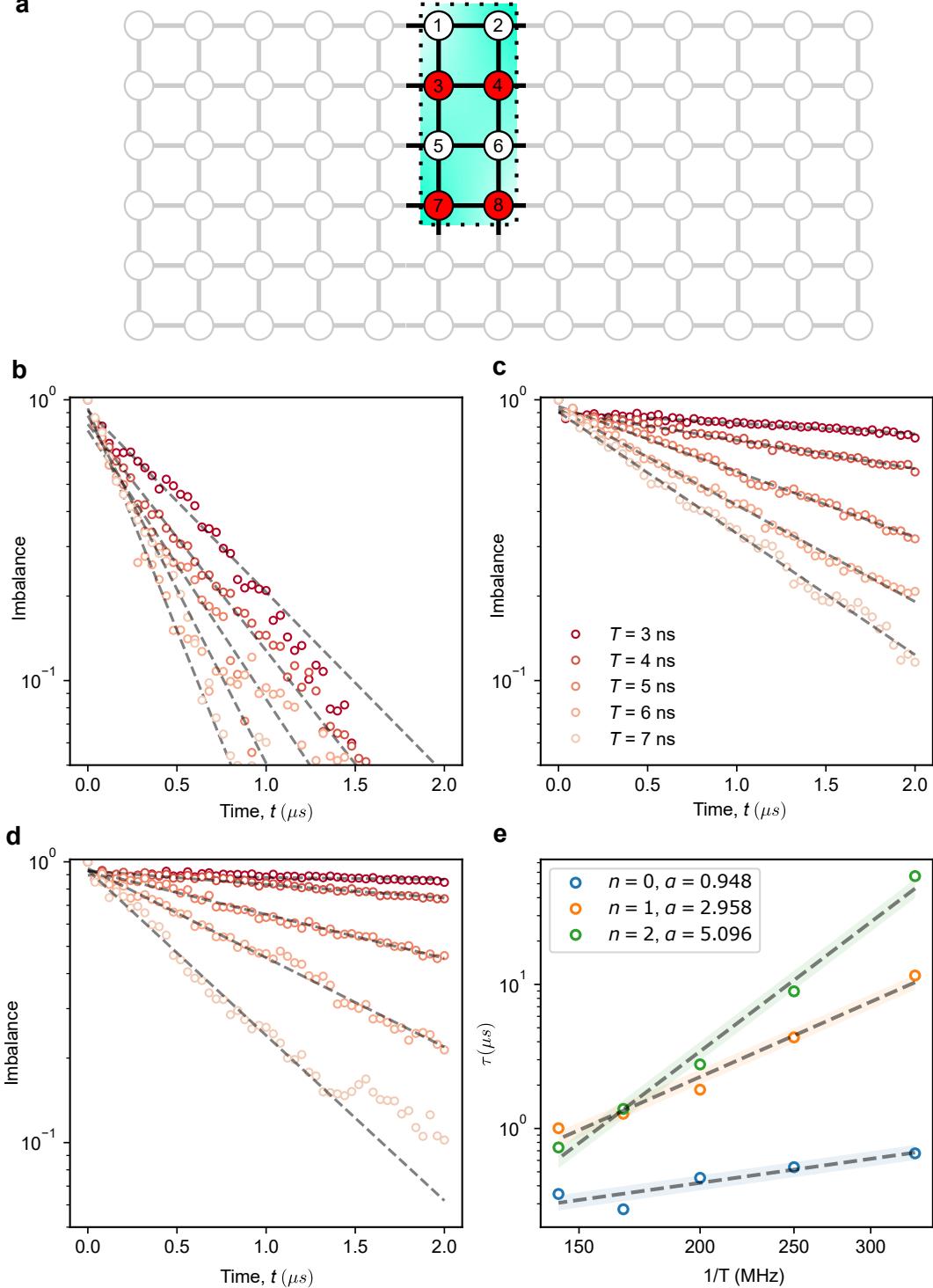
The effective coupling strength reaches its maximum value  $g$ , if and only if the phase difference satisfies  $\Delta\varphi = 2m\pi$  with an arbitrary integer  $m$ . This relationship enables us to achieve precise timing alignment by systematically characterizing the coupling strength as a function of the phase difference  $\Delta\varphi$  or, equivalently, the timing offset  $\Delta t$  in the Floquet sequence implementation. A typical result is shown in Supplementary Fig. 13**b** and **c**.



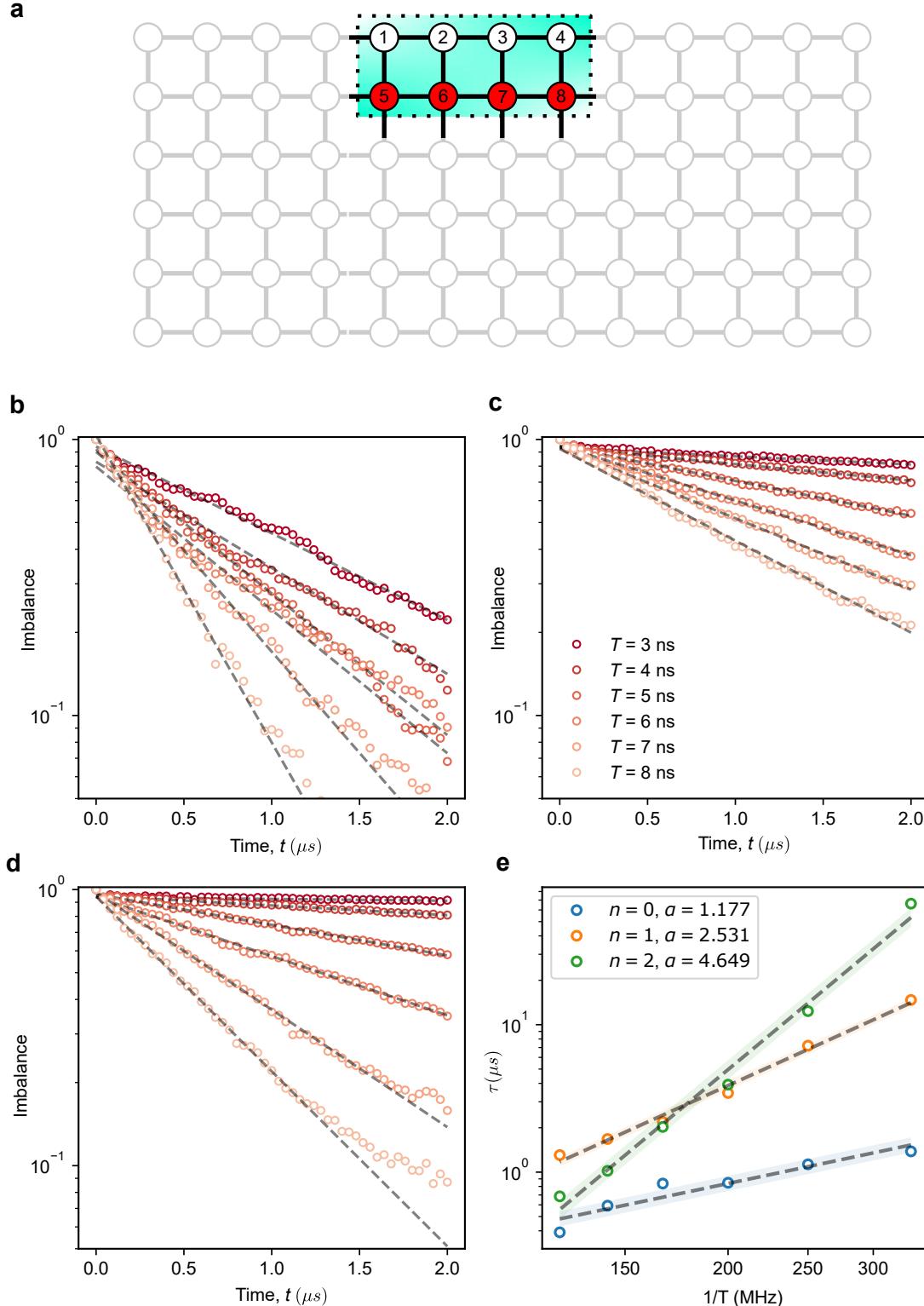
Supplementary Fig. 13. **Synchronization of the flux bias pulses between qubit-qubit pairs by Floquet engineering.** **a**, Experimental pulse sequences for calibrating the timing between the flux bias pulses of a qubit-qubit pair. The qubit is excited to the state  $|1\rangle$  using a  $\pi$  pulse, followed by the measurement of the coupling strength as a function of the timing offset. **b**, The experimentally measured  $|10\rangle$  probability, of the qubit versus the timing offset  $\Delta t$ . **c**, Fourier spectrum of the data presented in **b**. The characteristic time  $T$  is chosen as 5 ns.

## S5. SUPPLEMENTARY DATA

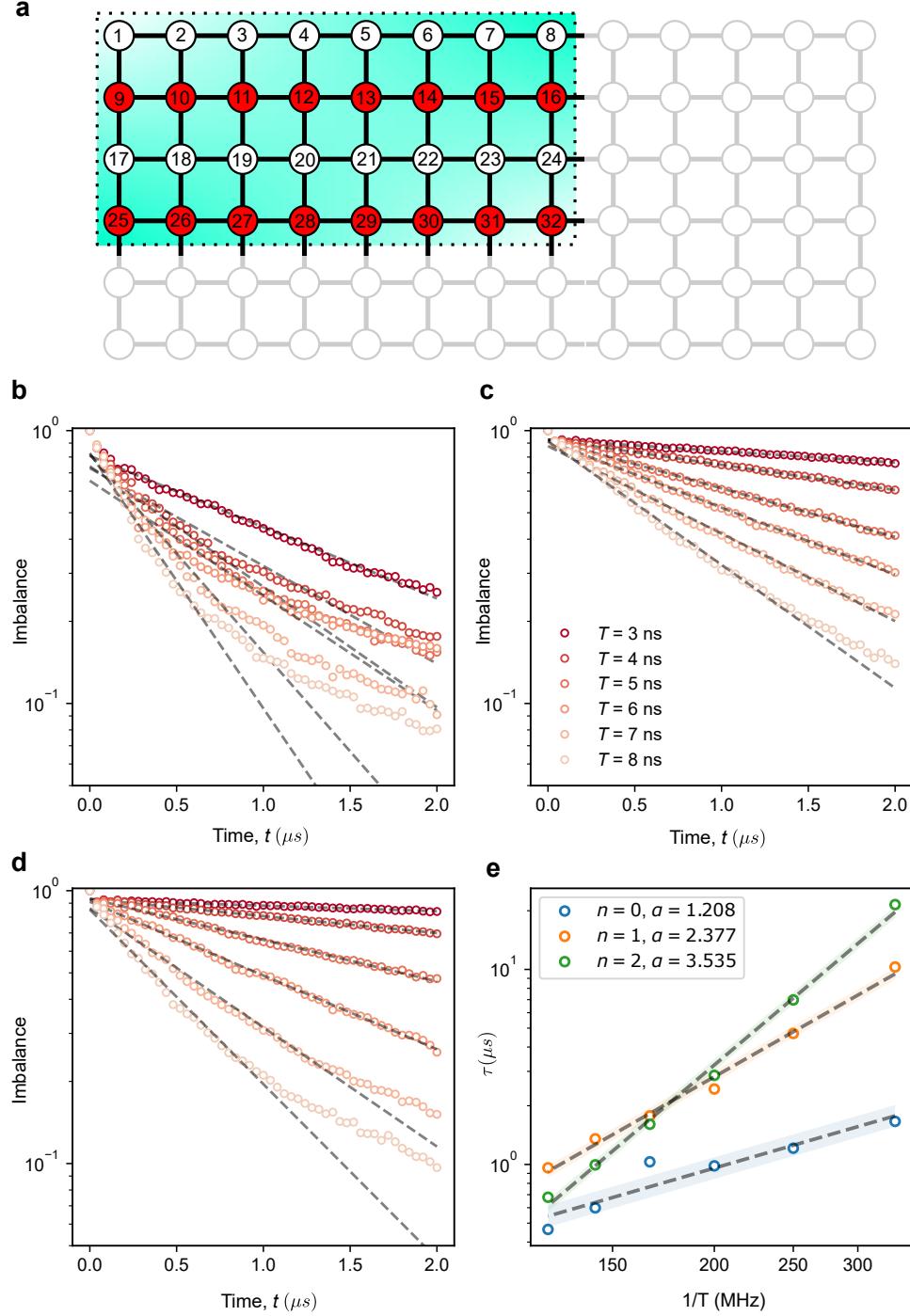
## A. Dynamics of imbalance in systems with different sizes



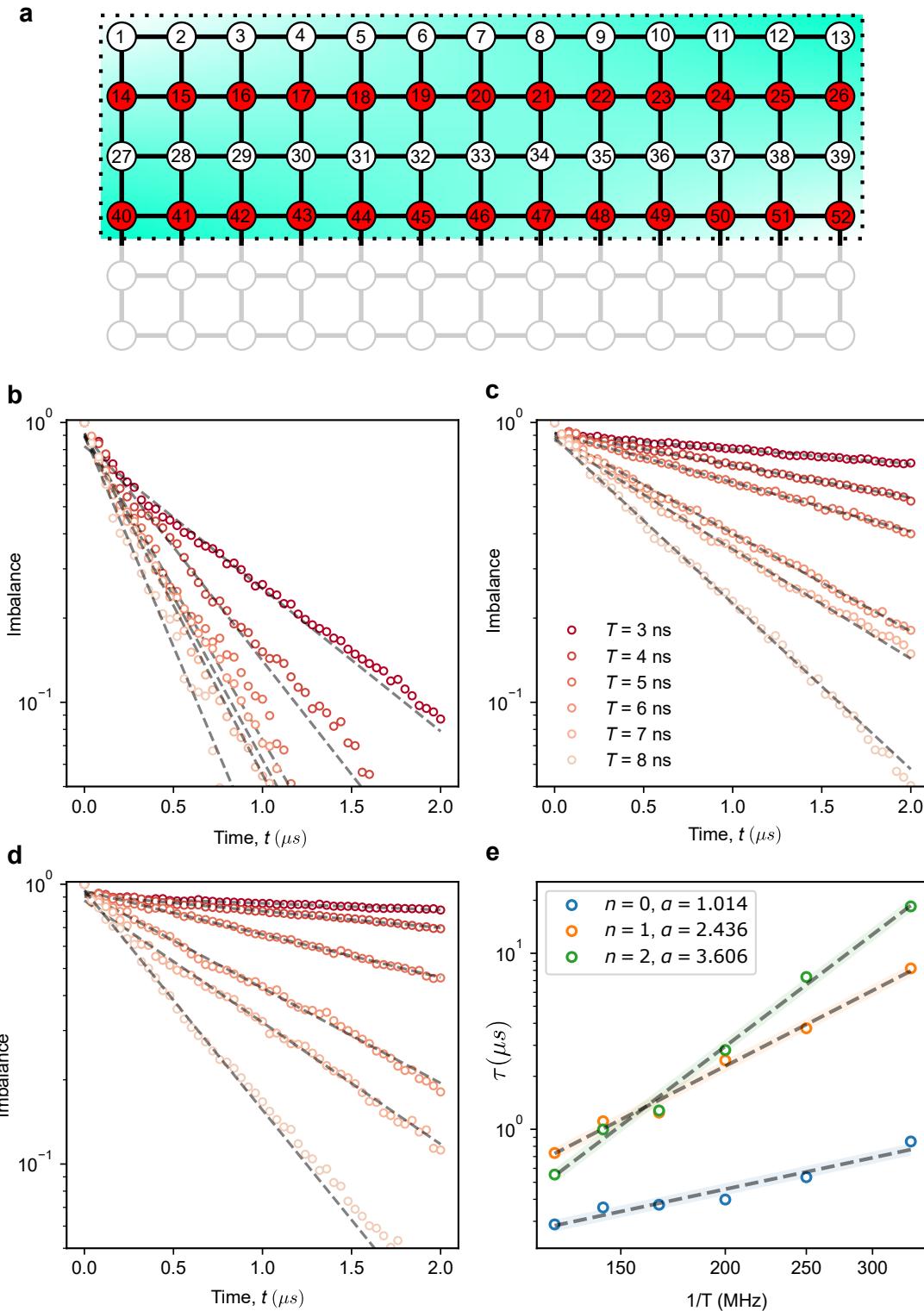
Supplementary Fig. 14. **The dynamics of imbalance in the system of size  $2 \times 4$ .** **a**, Spatial configuration of the  $2 \times 4$  system within the processor. **b-d**, Time evolution of the imbalance for driving periods  $T$  ranging from 3 ns to 7 ns, corresponding to  $n = 0$ ,  $n = 1$ ,  $n = 2$ , individually. Markers represent experimental data, while dashed lines denote fitted curves used to extract the decay rate  $-1/\tau_I$ , where  $\tau_I$  is the characteristic decay time. **e**, Power-law scaling of  $\tau_I$ , presented on a log-log scale. The scaling exponents,  $\alpha(n = 0) = 0.948$ ,  $\alpha(n = 1) = 2.958$ ,  $\alpha(n = 2) = 5.096$ , follow the theoretical prediction,  $\alpha = 2n + 1$ .



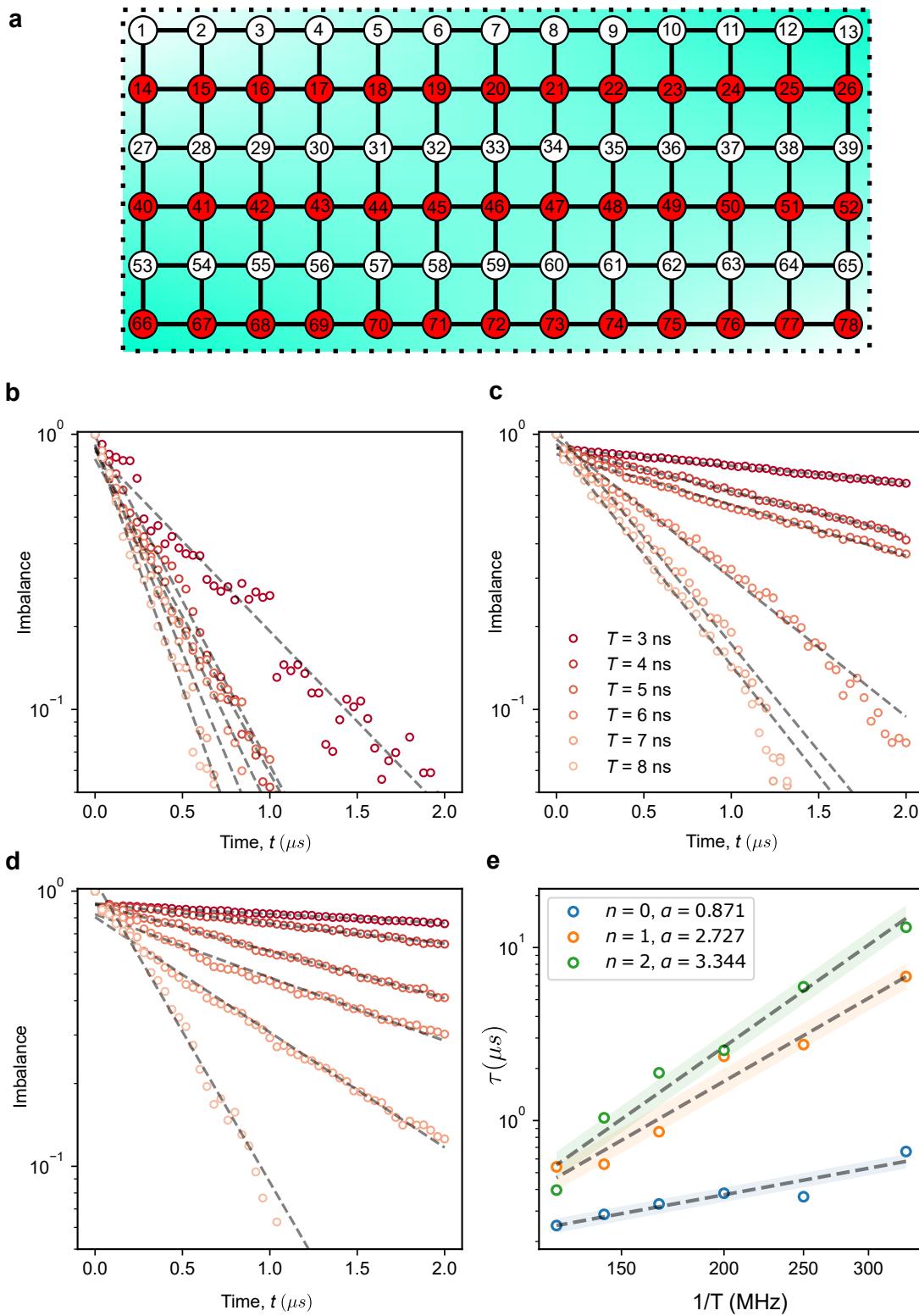
Supplementary Fig. 15. **The dynamics of imbalance in the system of size  $4 \times 2$ .** **a**, Spatial configuration of the  $4 \times 2$  system within the processor. **b-d**, Time evolution of the imbalance for driving periods  $T$  ranging from 3 ns to 8 ns, corresponding to  $n = 0$ ,  $n = 1$ ,  $n = 2$ , individually. Markers represent experimental data, while dashed lines denote fitted curves used to extract the decay rate  $-1/\tau_I$ , where  $\tau_I$  is the characteristic decay time. **e**, Power-law scaling of  $\tau_I$  in a log-log scale, with extracted exponents  $\alpha(n = 0) = 1.177$ ,  $\alpha(n = 1) = 2.531$ ,  $\alpha(n = 2) = 4.649$ .



Supplementary Fig. 16. **The dynamics of imbalance in the system of size  $8 \times 4$ .** **a**, Spatial configuration of the  $8 \times 4$  system within the processor. **b-d**, Time evolution of the imbalance for driving periods  $T$  ranging from 3 ns to 8 ns, corresponding to  $n = 0$ ,  $n = 1$ ,  $n = 2$ , individually. **e**, Power-law scaling of the imbalance decay time, presented in a log-log scale, with extracted exponents  $\alpha(n = 0) = 1.208$ ,  $\alpha(n = 1) = 2.377$ ,  $\alpha(n = 2) = 3.535$ .

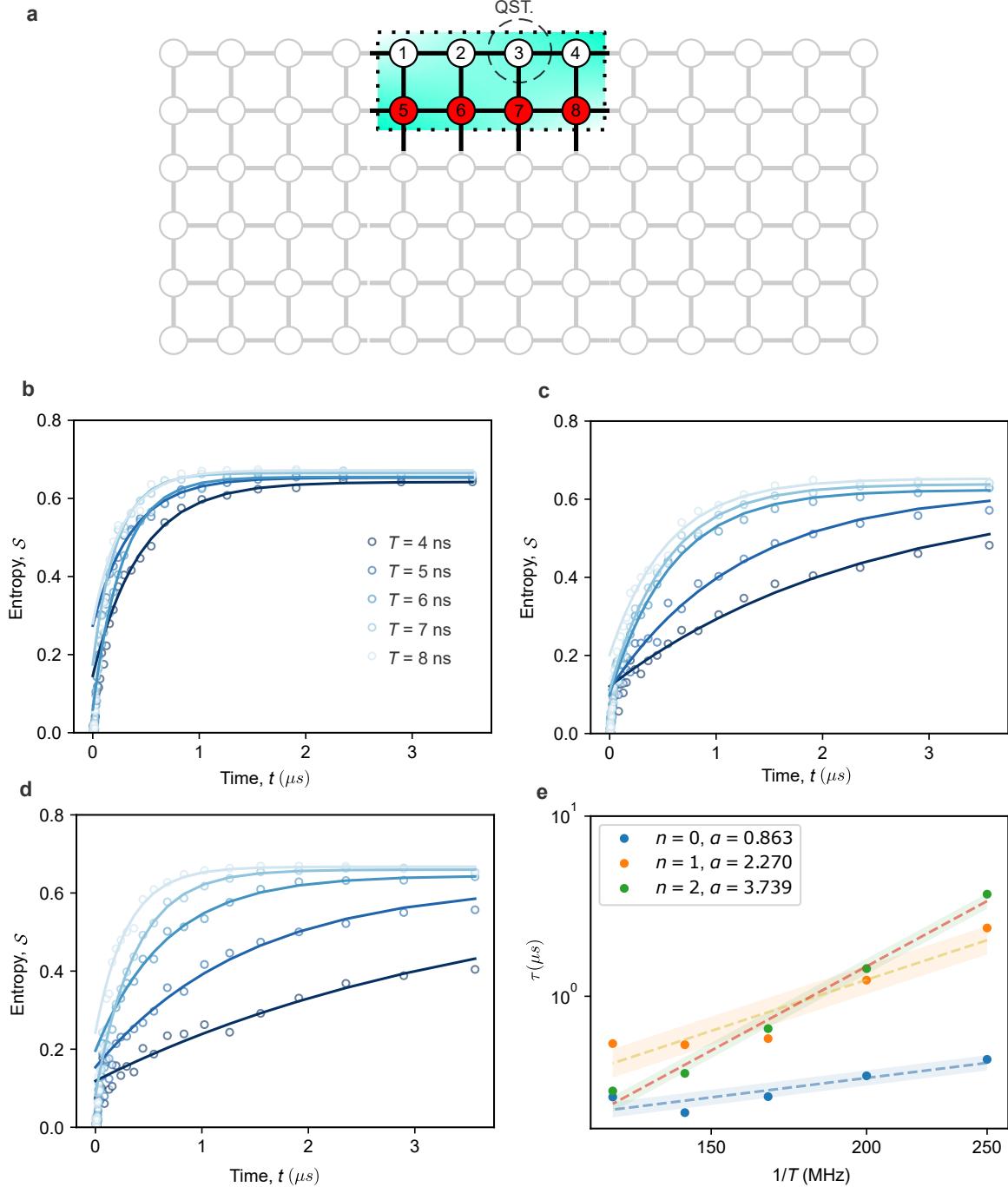


Supplementary Fig. 17. **The dynamics of imbalance in the system of size  $13 \times 4$ .** **a**, Spatial configuration of the  $13 \times 4$  system within the processor. **b-d**, Time evolution of the imbalance for driving periods  $T$  ranging from 3 ns to 8 ns, corresponding to  $n = 0$ ,  $n = 1$ ,  $n = 2$ , individually. Markers represent experimental data, while dashed lines denote fitted curves used to extract the decay rate  $-1/\tau_I$ , where  $\tau_I$  is the characteristic decay time. **e**, Power-law scaling of  $\tau_I$ , presented on a log-log scale, with extracted exponents  $\alpha(n = 0) = 1.014$ ,  $\alpha(n = 1) = 2.436$ ,  $\alpha(n = 2) = 3.606$ .

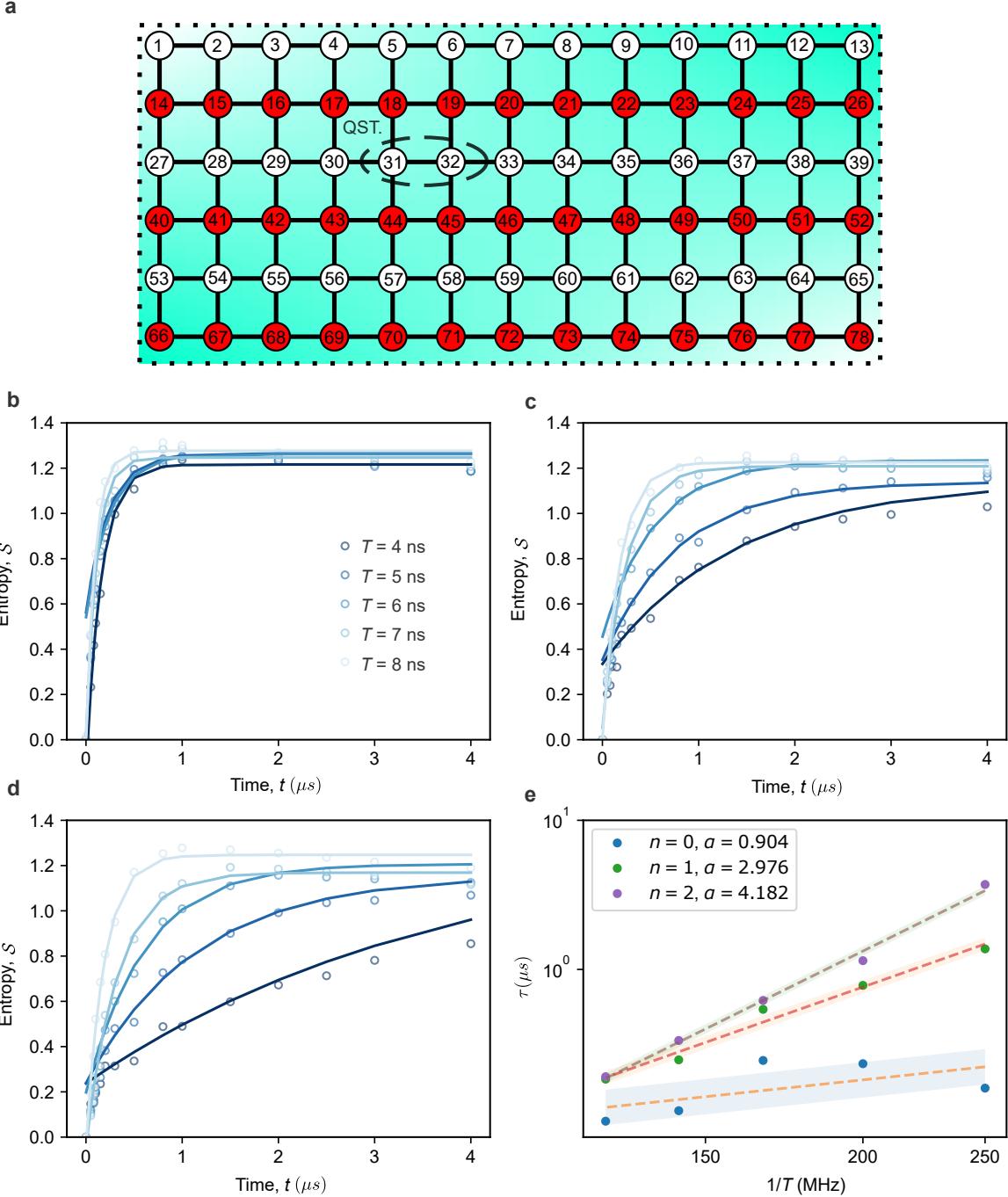


Supplementary Fig. 18. **The dynamics of imbalance in the system of size  $13 \times 6$ .** **a**, Spatial configuration of the  $13 \times 6$  system within the processor. **b-d**, Time evolution of the imbalance for driving periods  $T$  ranging from 3 ns to 8 ns, corresponding to  $n = 0$ ,  $n = 1$ ,  $n = 2$ , individually. Markers represent experimental data, while dashed lines denote fitted curves used to extract the decay rate  $-1/\tau_I$ , where  $\tau_I$  is the characteristic decay time. **e**, Power-law scaling of  $\tau_I$ , presented on a log-log scale, with extracted exponents  $\alpha(n = 0) = 0.871$ ,  $\alpha(n = 1) = 2.727$ ,  $\alpha(n = 2) = 3.344$ .

### B. Dynamics of entropy in systems with different sizes



Supplementary Fig. 19. **The dynamics of entropy in the system of size  $4 \times 2$ .** **a**, Spatial configuration of the  $4 \times 2$  system and the subsystem for performing quantum state tomography within the processor. **b-d**, Time evolution of the subsystem entanglement entropy for driving periods  $T$  ranging from 4 ns to 8 ns, corresponding to  $n = 0, n = 1, n = 2$ , individually. **e**, Power-law scaling of the prethermal lifetime presented on a log-log scale, with extracted exponents  $\alpha(n = 0) = 0.863, \alpha(n = 1) = 2.270, \alpha(n = 2) = 3.739$ . The curves in **b-d** are the fitted results using the form  $S \sim S_M(1 - e^{-t/\tau_S})$ .



Supplementary Fig. 20. **The dynamics of entropy in the system of size  $13 \times 6$ .** **a**, Spatial configuration of the  $13 \times 6$  system and the subsystem for performing quantum state tomography (QST) within the processor. **b-d**, Time evolution of the subsystem entanglement entropy for driving periods  $T$  ranging from 4 ns to 8 ns, corresponding to  $n = 0, n = 1, n = 2$ , individually. **e**, Power-law scaling of the prethermal lifetime in a log-log scale, with extracted exponents  $\alpha(n = 0) = 0.904, \alpha(n = 1) = 2.976, \alpha(n = 2) = 4.182$ . The curves in **b-d** are the fitted results using the form  $S \sim S_M(1 - e^{-t/\tau_S})$ .