

Effective action for ϕ^4 -Yukawa theory via 2PI formalism in the inflationary de Sitter spacetime

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March 28, 2025

Abstract

We consider a scalar field theory with quartic self interaction, coupled to fermions via the Yukawa interaction in the inflationary de Sitter spacetime background. The scalar has a classical background plus quantum fluctuations, whereas the fermions are taken to be quantum. We derive for this system the effective action and the effective potential via the two particle irreducible (2PI) formalism. This formalism provides an opportunity to find out resummed, non-perturbative results. We have considered the two loop vacuum graphs and have computed the local part of the effective action. The various resummed counterterms corresponding to self energies, vertex functions and the tadpole have been explicitly found. The variation of the renormalised effective potential for massless fields has been investigated numerically. We show that for the potential to be bounded from below, we must have $\lambda \gtrsim 16g^2$, where λ and g are respectively the quartic and Yukawa couplings. We emphasise the qualitative differences of this non-perturbative calculation with that of the standard perturbative ones in de Sitter. The qualitative differences of our result with that of the flat spacetime has also been pointed out.

Keywords : 2PI formalism, ϕ^4 -Yukawa interaction, effective action, secular logarithm

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1 Introduction

The standard hot big bang model of cosmology has been greatly successful in explaining the redshift of light coming from a galaxy, the origin of the cosmic microwave background radiation, abundance of light elements, the large scale cosmic structure formation etc.. However, this model cannot explain a few puzzling features of our universe, such as its spatial flatness, the horizon problem associated with the spatial isotropy at very large scales and the hitherto unobserved relics such as the magnetic monopoles. The epoch of primordial cosmic inflation is a proposed phase of a very rapid, near exponential accelerated expansion of our very early universe. Such inflationary phase not only gives answers to these three problems, but also provides a suitable framework to generate primordial quantum density perturbations and correlation functions, as seeds to the large scale cosmic structures we observe today in the sky, see [1] and references therein for various theoretical and observational aspects of cosmic inflation.

The inflation requires an exotic matter field with negative isotropic pressure. Traditionally, this is explained by a scalar field called inflaton, slowly moving down a potential. Also, after sufficient number of e-foldings, the universe must gracefully exit the inflationary period. This graceful exit also specifies the current observed value of the cosmological constant. It turns out that only 10% change to this value would modify the evolution of our universe greatly, known as the cosmic coincidence problem. We refer our reader to e.g. [2, 3, 4, 5, 6, 7, 8] and references therein for various attempts to address this issue.

The de Sitter (Eq. (1)) or quasi de Sitter spacetime is believed to be the metric appropriate for the inflationary phase. For such time dependent background, understanding quantum fluctuations is an important task. We refer our reader to [9, 10, 11, 12, 13, 14] for discussion on field quantisation in de Sitter background. The dynamics of a light scalar field can be very non-trivial in the inflationary background. In particular, a massless but non-conformally invariant (such as a massless minimally coupled scalar, gravitons) field cannot have a de Sitter invariant Wightman function [13, 14]. This leads to the appearance of logarithm of the scale factor in quantum amplitudes and thereby breaking down the perturbative expansion at late times, known as the secular effect [15]. Such large logarithms are chiefly related to the super-Hubble, deep infrared fluctuations at late times. They can lead to dynamical generation of the field mass at late times. For various aspects of this non-perturbative effect including mass generation, cosmic decoherence and entanglement, we refer our reader to e.g. [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37] and references therein. Resummation of these secular logarithms has been attempted in various instances via numerous approaches chiefly inspired from renormalisation group, see e.g. [38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53] and references therein. Even though there has been a considerable progress to understand the resummation in the context of an interacting scalar field theory, the same issue for field theories with derivative interactions such as gravity, remains largely as an open question. Also, for scalar field

theories with non-derivative interaction, the late time stochastic formalism is an excellent way to do resummation [54] (see also e.g. [55, 56, 57, 58, 59] and references therein).

The effective action technique can be a very strong tool to address the early inflationary quantum fluctuations. In this work we wish to derive the effective action for a ϕ^4 -Yukawa theory in the de Sitter background by using the non-perturbative, 2 particle irreducible (2PI) (see e.g. [60] for a vast review) formalism. Earlier works on renormalisation and resummation, self energy computation, fermion mass generation and decoherence for the Yukawa theory in de Sitter can be seen in [61, 62, 63, 64, 65]. See [66] for one loop effective action of Yukawa theory in de Sitter and also [67] for inclusion of gauge coupling. Effective action and correlation function for the Yukawa theory using the influence functional technique can be seen in [68]. For a renormalisation group improvement of the ultraviolet (UV) limit of the one loop effective action for the ϕ^4 -Yukawa theory in de Sitter can be seen in [69]. We also refer our reader to [70] for a study on the effect of Yukawa coupling on cosmological correlation functions. We further refer our reader to [71, 72, 73, 74, 75], for discussion on effective action with Yukawa and four fermion interactions in general curved spacetimes using the Schwinger-DeWitt local expansion technique. This technique is UV effective, and hence cannot probe the large scale deep infrared physics.

Earlier, the 2PI technique was used in de Sitter to compute the non-perturbative effective action for ϕ^4 or $O(N)$ scalar field theory at two loop Hartree approximation in [76, 77, 78]. We will focus only on the local part of the effective action in this work. In particular, we will see below the renormalisation in the ϕ^4 -Yukawa theory has non-trivial features, which is qualitatively very different from that of the standard perturbative approach. We will also emphasise in the due course, the non-trivial results such non-perturbative technique may bring in, in the de Sitter background. We will also emphasise the qualitative differences of our result with that of the flat spacetime.

The rest of the paper is organised as follows. In the next section we sketch the basic technical framework we will be working in. In Section 3.1, following [76, 77, 78] we briefly discuss the derivation of the two loop 2PI effective action for ϕ^4 theory in the Hartree approximation. Here we also derive the two and three loop vacuum graphs (both local and non-local parts) for a massless and minimal scalar without any background field, in the leading power of the secular logarithm, using the Schwinger-Keldysh formalism (e.g. [79, 80], and references therein). Next in Section 3.2, we compute the local part of the 2PI effective action for the ϕ^4 -Yukawa theory at two loop. We focus on computing the local part of the effective action. Finally we conclude in Section 4. The four appendices contain some technical detail of the calculations. We will work with the mostly positive signature of the metric in $d = 4 - \epsilon$ ($\epsilon = 0^+$) dimensions and will set $c = 1 = \hbar$ throughout. Also for the sake of brevity and to save space, we will denote for powers of propagators and logarithms respectively as, $(i\Delta)^n \equiv i\Delta^n$ and also $(\ln a)^n \equiv \ln^n a$.

2 The basic set up

We wish to briefly review below the basic framework we will be working in, following chiefly [17, 18, 66]. The metric for the inflationary de Sitter spacetime reads respectively in the cosmological and conformal temporal coordinates

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2) \quad (1)$$

where $a(t) = e^{Ht}$ or $a(\eta) = -1/H\eta$ is the de Sitter scale factor and $H = \sqrt{\Lambda/3}$ is the de Sitter Hubble rate, with Λ being the cosmological constant. We have the range $0 \leq t < \infty$, so that $-H^{-1} \leq \eta < 0^-$. Note that any temporal level of the initial hypersurface can be achieved as we wish, by exploiting the time translation symmetry (along with a scaling of the spatial coordinates) of de Sitter.

The bare action corresponding to the matter sector reads,

$$S = \int a^d d^d x \left[-\frac{1}{2} g^{\mu\nu} (\nabla_\mu \Phi') (\nabla_\nu \Phi') - \frac{1}{2} m_0^2 \Phi'^2 - \frac{\lambda_0}{4!} \Phi'^4 - \frac{\beta_0}{3!} \Phi'^3 - \tau_0 \Phi' + i \bar{\Psi} \not{\nabla} \Psi - M_0 \bar{\Psi} \Psi - g_0 \bar{\Psi} \Psi \Phi' \right] \quad (2)$$

where $\tilde{\nabla} = \tilde{\gamma}^\mu \nabla_\mu$, and $\tilde{\gamma}^\mu$ and ∇_μ are respectively the curved space gamma matrices and the spin covariant derivative. Also, since we are working with the mostly positive signature of the metric, we will take the anti-commutation relation for the γ -matrices as

$$[\tilde{\gamma}_\mu, \tilde{\gamma}_\nu]_+ = -2g_{\mu\nu} \mathbf{I}_{d \times d} = -2a^2 \eta_{\mu\nu} \mathbf{I}_{d \times d} \quad (3)$$

Thus we may choose, $\tilde{\gamma}_\mu = a(\eta)\gamma_\mu$, where γ_μ 's are the flat space gamma matrices. Defining the field strength renormalization, $\varphi = \Phi'/\sqrt{Z}$ and $\psi = \Psi/\sqrt{Z_f}$, we have

$$S = \int a^d d^d x \left[-\frac{Z}{2} \eta^{\mu\nu} a^{-2} (\partial_\mu \varphi)(\partial_\nu \varphi) - \frac{1}{2} Z m^2 \varphi^2 - \frac{Z^2 \lambda_0}{4!} \varphi^4 - \frac{\beta_0 Z^{3/2}}{3!} \varphi^3 - \tau \sqrt{Z} \varphi \right. \\ \left. - i Z_f \bar{\psi} \tilde{\nabla} \psi - M Z_f \bar{\psi} \psi - g_0 Z_f \sqrt{Z} \bar{\psi} \psi \varphi \right] \quad (4)$$

We next write

$$\begin{aligned} Z &= 1 + \delta Z & Z m^2 &= m_0^2 + \delta m^2 & Z^2 \lambda_0 &= \lambda + \delta \lambda & \beta_0 Z^{3/2} &= \delta \beta \\ \tau \sqrt{Z} &= \delta \tau & Z_f &= 1 + \delta Z_f & M Z_f &= M_0 + \delta M & g_0 Z_f \sqrt{Z} &= g + \delta g \end{aligned} \quad (5)$$

The above decomposition splits Eq. (4) as

$$S = \int a^d d^d x \left[-\frac{1}{2} \eta^{\mu\nu} a^{-2} (\partial_\mu \varphi)(\partial_\nu \varphi) - \frac{1}{2} m_0^2 \varphi^2 - \frac{\lambda}{4!} \varphi^4 - \delta \tau \varphi + i \bar{\psi} \tilde{\nabla} \psi - M_0 \bar{\psi} \psi - g \bar{\psi} \psi \varphi \right. \\ \left. - \frac{\delta Z}{2} \eta^{\mu\nu} a^{-2} (\partial_\mu \varphi)(\partial_\nu \varphi) - \frac{1}{2} \delta m^2 \varphi^2 - \frac{\delta \lambda}{4!} \varphi^4 - \frac{\delta \beta}{3!} \varphi^3 + i \delta Z_f \bar{\psi} \tilde{\nabla} \psi - \delta M \bar{\psi} \psi - \delta g \bar{\psi} \psi \varphi \right] \quad (6)$$

Let us now come to the issue of the scalar field. The exact propagator for a massive scalar can be seen in Eq. (98) of Appendix B. Although we will chiefly work with a massive scalar in this paper, we also wish to additionally compute some vacuum graphs for a massless and minimally coupled scalar field (Appendix A), which is of special interest in de Sitter. The relevant propagator such a scalar reads [17],

$$i\Delta(x, x') = A(x, x') + B(x, x') + C(x, x') \quad (7)$$

where

$$\begin{aligned} A(x, x') &= \frac{H^{2-\epsilon} \Gamma(1-\epsilon/2)}{4\pi^{2-\epsilon/2}} \frac{1}{y^{1-\epsilon/2}} = \frac{\Gamma(1-\epsilon/2)}{2^2 \pi^{2-\epsilon/2}} \frac{(aa')^{-1+\epsilon/2}}{(\Delta x)^{2-\epsilon}} \\ B(x, x') &= \frac{H^{2-\epsilon} \Gamma(3-\epsilon)}{2^{4-\epsilon} \pi^{2-\epsilon/2} \Gamma(2-\epsilon/2)} \left[-\frac{2\Gamma(3-\epsilon/2)\Gamma(2-\epsilon/2)(aa'H^2/4)^{\epsilon/2}}{\epsilon \Gamma(3-\epsilon)} (\Delta x^2)^{\epsilon/2} + \left(\ln(aa') + \frac{2}{\epsilon} \right) \right] \\ C(x, x') &= \frac{H^{2-\epsilon}}{(4\pi)^{2-\epsilon/2}} \sum_{n=1}^{\infty} \left[\frac{\Gamma(3-\epsilon+n)}{n \Gamma(2-\epsilon/2+n)} \left(\frac{y}{4} \right)^n - \frac{\Gamma(3-\epsilon/2+n)}{(n+\epsilon/2)\Gamma(2+n)} \left(\frac{y}{4} \right)^{n+\epsilon/2} \right] \end{aligned} \quad (8)$$

The de Sitter invariant biscalar interval reads

$$y(x, x') = aa' H^2 \Delta x^2 = a(\eta) a(\eta') H^2 [|\vec{x} - \vec{x}'|^2 - (\eta - \eta')^2] \quad (9)$$

The $\ln(aa')$ term appearing in the $B(x, x')$ term appearing in Eq. (8) makes the propagator non-invariant under de Sitter symmetry transformations. This is a unique feature of an exactly massless and minimal scalar field in de Sitter. In other words, a smooth $m^2 \rightarrow 0$ limit for a scalar in de Sitter may not even exist.

There are four propagators pertaining to the in-in or the Schwinger-Keldysh formalism one needs to use in a cosmological framework e.g. [79] (Appendix A), characterised by suitable four complex distance functions, Δx^2 ,

$$\begin{aligned} \Delta x_{++}^2 &= [|\vec{x} - \vec{x}'|^2 - (|\eta - \eta'| - i\epsilon)^2] = (\Delta x_{--}^2)^* \\ \Delta x_{+-}^2 &= [|\vec{x} - \vec{x}'|^2 - ((\eta - \eta') + i\epsilon)^2] = (\Delta x_{-+}^2)^* \quad (\epsilon = 0^+) \end{aligned} \quad (10)$$

The first two correspond respectively to the Feynman and anti-Feynman propagators, whereas the last two correspond to the two Wightman functions. From Eq. (8), we have in the coincidence limit for all the four propagators

$$i\Delta(x, x) = \frac{H^{2-\epsilon}\Gamma(2-\epsilon)}{2^{2-\epsilon}\pi^{2-\epsilon/2}\Gamma(1-\epsilon/2)} \left(\frac{1}{\epsilon} + \ln a \right) \quad (11)$$

Using the above expression, we may compute, for example, the one loop self energy bubble corresponding to the quartic self interaction diagrams. The corresponding one loop mass renormalisation counterterms read [18]

$$\delta m_\lambda^2 = -\frac{\lambda H^{2-\epsilon}\Gamma(2-\epsilon)}{2^{3-\epsilon}\pi^{2-\epsilon/2}\Gamma(1-\epsilon/2)\epsilon} \quad (12)$$

3 Computation of the local effective action

3.1 Warm up – a scalar with ϕ^4 self-interaction

The computation of the local, non-perturbative effective action in Hartree approximation in de Sitter was done earlier in [76, 77, 78]. For our future purpose, we wish to briefly sketch below the outline of that computation in our own notation. We begin with the action relevant for our purpose

$$S[\varphi] = - \int a^d d^d x \left[\frac{1}{2} (\nabla_\mu \varphi)(\nabla^\mu \varphi) + \frac{1}{2} (m_0^2 + \delta m^2) \varphi^2 + \frac{(\lambda + \delta \lambda) \varphi^4}{4!} + \delta \xi R \varphi^2 \right] \quad (13)$$

We decompose the field as

$$\varphi = v + \phi$$

where v is the background field and ϕ is the quantum fluctuation. The corresponding 2PI effective action reads [60]

$$\Gamma_{2\text{PI}}[v, iG] = S[v] - \frac{i}{2} \ln \det iG(x, x') + \frac{1}{2} \text{Tr} \int (aa')^d d^d x d^d x' i\Delta^{-1}(x, x') iG(x', x) + i\Gamma_2[v, iG] \quad (14)$$

where $i\Delta(x, x')$ is the free whereas $iG(x, x')$ is the exact propagator for the scalar. The above expression explicitly reads for our theory

$$\begin{aligned} \Gamma_{2\text{PI}}[v, iG] = & - \int a^d d^d x \left[\frac{1}{2} (\nabla_\mu v)(\nabla^\mu v) + \frac{1}{2} (m_0^2 + \delta m_1^2) v^2 + \frac{\lambda_1 v^4}{4!} + \delta \xi_1 R v^2 \right] \\ & - \frac{1}{2} \ln \det iG(x, x) + \frac{1}{2} \int a^d d^d x \left[\square - (m_0^2 + \delta m_2^2) - 2\delta \xi_2 R - \frac{\lambda_2 v^2}{2} \right] iG(x, x) + i\Gamma_2[v, iG] \end{aligned} \quad (15)$$

$i\Gamma_2[v, iG]$ generates the 2PI vacuum graphs, and the vertex counterterms are contained within λ_1, λ_2 . At the leading order and at two loop, we have

$$i\Gamma_2[v, iG] = -\frac{\lambda_3}{2^3} \int a^d d^d x iG^2(x, x) \quad (16)$$

Note that the above double bubble vacuum graph is purely local (the first of Fig. 1). This is called the Hartree approximation. We also have

$$\lambda_i = \lambda + \delta \lambda_i \quad i = 1, 2, 3$$

Let us consider the equation of motion satisfied the Green function found from Eq. (15),

$$\left[\square - (m_0^2 + \delta m_2^2) - 2\delta \xi_2 R - \frac{\lambda_2 v^2}{2} \right] iG(x, x') = \frac{i\delta^d(x - x')}{a^d} - 2 \int a''^d d^d x'' \frac{\delta i\Gamma_2}{\delta iG(x, x'')} iG(x'', x') \quad (17)$$

where we have used

$$\int d^d x'' a''^d iG(x, x'') iG^{-1}(x'', x') = \frac{\delta^d(x - x')}{a^d}$$

The term appearing in the last term on the right hand side of Eq. (17), $2\delta i\Gamma_2/\delta iG(x, x')$ is basically $-i$ times the $\mathcal{O}(\lambda)$ 1PI self energy. This whole last term explicitly reads

$$\frac{\lambda_3}{2} iG(x, x) iG(x, x') \quad (18)$$

Since $iG(x, x)$ is purely local, Eq. (17) gives us the opportunity to resum it. The resummed local self energy dynamically generates a rest mass at late times, say m_{dyn}^2 . Accordingly, we assume that Eq. (17) at late times takes the form

$$\left[\square - \frac{\lambda v^2}{2} - m_{\text{dyn}}^2 \right] iG(x, x') = \frac{i\delta^d(x - x')}{a^d} \quad (19)$$

Let us now come to the non-perturbative renormalisation scheme. From Eq. (104) (Appendix B) valid for an arbitrary massive scalar, we write for the sake of brevity

$$iG(x, x) = m_{\text{dyn,eff}}^2 f_d + H^2 f'_d + f_{\text{fin}} \quad (20)$$

where

$$\begin{aligned} f_d &= -\frac{H^{-\epsilon}}{2^{3-\epsilon}\pi^{2-\epsilon/2}\epsilon} & f'_d &= -\frac{H^{-\epsilon}}{2^{2-\epsilon}\pi^{2-\epsilon/2}\epsilon} \left(1 - \frac{\gamma\epsilon}{2}\right) \\ f_{\text{fin}} &= \frac{H^2}{2^3\pi^2} \left(1 - \frac{1}{2} \frac{m_{\text{dyn,eff}}^2}{H^2}\right) \left[\frac{1}{s} - \frac{2}{1-s} - (\psi(1+s) + \psi(1-s)) \right], \end{aligned} \quad (21)$$

where ψ is the digamma function, and we have abbreviated,

$$s = \frac{3}{2} - \left(\frac{9}{4} - \frac{m_{\text{dyn,eff}}^2}{H^2} \right)^{1/2} \quad (22)$$

From the field equation for the propagator, Eq. (17), Eq. (19), we write

$$m_{\text{dyn,eff}}^2 = m_0^2 + \delta m_2^2 + \frac{(\lambda + \delta\lambda_2)v^2}{2} + \frac{\lambda + \delta\lambda_3}{2} iG(x, x) + 2\delta\xi_2 R \quad (23)$$

where we have defined

$$m_{\text{dyn,eff}}^2 = m_0^2 + \frac{\lambda f_{\text{fin}}}{2} + \frac{\lambda v^2}{2} \quad (24)$$

We next plug in Eq. (20) into Eq. (23). We first infer that

$$\delta\lambda_2 = \delta\lambda_3, \quad \delta\xi_2 = 0 \quad (25)$$

We also have the self consistency condition

$$\delta m_2^2 + \frac{1}{2}(\lambda + \delta\lambda_2)(f_d m_0^2 + H^2 f'_d) + \frac{v^2}{2} \left(\delta\lambda_2 + \frac{\lambda}{2} (\lambda + \delta\lambda_2) f_d \right) + \frac{1}{2} \left(\delta\lambda_2 + \frac{\lambda}{2} (\lambda + \delta\lambda_2) f_d \right) f_{\text{fin}} = 0 \quad (26)$$

By individually setting the coefficients of v^2 and f_{fin} to zero, we obtain the non-perturbative renormalisation counterterms

$$\delta\lambda_2 = -\frac{\lambda^2 f_d}{2 \left(1 + \frac{\lambda f_d}{2}\right)}, \quad \delta m_2^2 = -\frac{\lambda(f_d m_0^2 + H^2 f'_d)}{2 \left(1 + \frac{\lambda f_d}{2}\right)} \quad (27)$$

We now write down the effective action as

$$\begin{aligned}\Gamma_{2\text{PI}}[v, iG] = & - \int a^d d^d x \left[\frac{1}{2} (\nabla_\mu v) (\nabla^\mu v) + \frac{1}{2} (m_0^2 + \delta m_1^2) v^2 + \frac{(\lambda + \delta \lambda_1) v^4}{4!} \right] - \frac{1}{2} \int a^d d^d x \int dm_{\text{dyn,eff}}^2 (m_{\text{dyn,eff}}^2 f_d \\ & + H^2 f'_d + f_{\text{fin}}) + \frac{\lambda + \delta \lambda_2}{2^3} \int a^d d^d x (m_{\text{dyn,eff}}^2 f_d + H^2 f'_d + f_{\text{fin}})^2\end{aligned}\quad (28)$$

For the last two terms in the effective action, we respectively have

$$\begin{aligned}\int dm_{\text{dyn,eff}}^2 iG(x, x) &= \frac{1}{2} f_d m_{\text{dyn,eff}}^4 + H^2 m_{\text{dyn,eff}}^2 f'_d + \int dm_{\text{dyn,eff}}^2 f_{\text{fin}} \\ &= \frac{1}{2} m_0^4 f_d + v^2 \cdot \frac{1}{2} m_0^2 \lambda f_d + v^4 \cdot \frac{1}{8} \lambda^2 f_d + v^2 f_{\text{fin}} \cdot \frac{1}{4} \lambda^2 f_d + f_{\text{fin}} \cdot \frac{1}{2} m_0^2 \lambda f_d \\ &+ f_{\text{fin}}^2 \cdot \frac{1}{8} \lambda^2 f_d + H^2 \left(m_0^2 + \frac{1}{2} \lambda v^2 + \frac{1}{2} \lambda f_{\text{fin}} \right) f'_d + \int dm_{\text{dyn,eff}}^2 f_{\text{fin}}\end{aligned}\quad (29)$$

and

$$\begin{aligned}iG^2(x, x) = & m_0^4 f_d^2 + v^2 \cdot m_0^2 \lambda f_d^2 + v^4 \cdot \frac{1}{4} \lambda^2 f_d^2 + v^2 f_{\text{fin}} \cdot \lambda f_d \left(1 + \frac{1}{2} \lambda f_d \right) + f_{\text{fin}} \cdot 2 m_0^2 f_d \left(1 + \frac{1}{2} \lambda f_d \right) \\ & + f_{\text{fin}}^2 \cdot \left(1 + \frac{1}{2} \lambda f_d \right)^2 + H^4 f_d'^2 + 2 H^2 f'_d \left[\left(m_0^2 + \frac{1}{2} \lambda v^2 + \frac{1}{2} \lambda f_{\text{fin}} \right) f_d + f_{\text{fin}} \right]\end{aligned}\quad (30)$$

Plugging the two above expressions into Eq. (28), we first conclude that

$$\delta \lambda_1 = 3 \delta \lambda_2, \quad \delta m_2^2 = \delta m_1^2, \quad \delta \xi_1^2 = 0 \quad (31)$$

We now collect and group terms proportional to v^2 , v^4 , $v^2 f_{\text{fin}}$, f_{fin} and f_{fin}^2 in Eq. (28). Using Eq. (24), Eq. (26), Eq. (27) and Eq. (31), we obtain the following respective coefficients after a little algebra

$$\begin{aligned}v^2 : & \frac{1}{8} (\lambda + \delta \lambda_2) m_0^2 \lambda f_d^2 - \frac{1}{4} m_0^2 \lambda f_d - \frac{1}{2} \delta m_2^2 + \frac{1}{4} (\lambda + \delta \lambda_2) H^2 f'_d \left(1 + \frac{1}{2} \lambda f_d \right) - \frac{1}{4} H^2 \lambda f'_d = 0 \\ v^4 : & -\frac{\delta \lambda_2}{8} - \frac{1}{16} \lambda^2 f_d + \frac{1}{32} (\lambda + \delta \lambda_2) \lambda^2 f_d^2 = 0 \\ f_{\text{fin}} : & \frac{1}{8} (\lambda + \delta \lambda_2) 2 m_0^2 f_d \left(1 + \frac{1}{2} \lambda f_d \right) - \frac{1}{4} m_0^2 \lambda f_d + \frac{1}{4} H^2 f'_d \left(1 + \frac{1}{2} \lambda f_d \right) (\lambda + \delta \lambda_2) - \frac{1}{4} H^2 \lambda f'_d = 0 \\ f_{\text{fin}}^2 : & \frac{1}{8} (\lambda + \delta \lambda_2) \left(1 + \frac{1}{2} \lambda f_d^2 \right)^2 - \frac{1}{16} \lambda^2 f_d = \frac{\lambda}{8} \\ v^2 f_{\text{fin}} : & \frac{1}{8} (\lambda + \delta \lambda_2) \lambda f_d \left(1 + \frac{1}{2} \lambda f_d \right) - \frac{1}{8} \lambda^2 f_d = 0\end{aligned}\quad (32)$$

Putting everything together, the 2PI effective action now reads,

$$\begin{aligned}\Gamma_{2\text{PI}}[v, iG] = & - \int a^d d^d x \left[\frac{1}{2} (\nabla_\mu v) (\nabla^\mu v) + \frac{1}{2} m_0^2 v^2 + \frac{\lambda}{4!} v^4 - \frac{\lambda}{8} f_{\text{fin}}^2 + \frac{1}{2} \int dm_{\text{dyn,eff}}^2 f_{\text{fin}} \right. \\ & \left. + \left(\frac{1}{8} (\lambda + \delta \lambda_2) m_0^4 f_d^2 - \frac{1}{4} m_0^4 f_d + \frac{1}{8} (\lambda + \delta \lambda_2) H^4 f_d'^2 + \frac{1}{4} (\lambda + \delta \lambda_2) H^2 m_0^2 f_d f'_d - \frac{1}{2} H^2 m_0^2 f_d' \right) \right]\end{aligned}\quad (33)$$

The v -independent divergences appearing in the second line above can be absorbed in a cosmological constant counterterm in the gravitational action. The renormalised effective action, now regarded as the non-perturbative 1PI effective action, as it is no longer a function of the Green function, reads under the local or Hartree approximation

$$\Gamma_{1\text{PI}}[v]_{\text{Ren.}} = - \int a^d d^d x \left[\frac{1}{2} (\nabla_\mu v) (\nabla^\mu v) + \frac{1}{2} m_0^2 v^2 + \frac{\lambda}{4!} v^4 - \frac{\lambda}{8} f_{\text{fin}}^2 + \frac{1}{2} \int dm_{\text{dyn,eff}}^2 f_{\text{fin}} \right] \quad (34)$$

where f_{fin} is given by Eq. (21), Eq. (22) and $m_{\text{dyn,eff}}^2$ is given by Eq. (24). The 1PI effective potential corresponding to Eq. (34) reads

$$\bar{V}_{\text{eff}}(\bar{v}) = \frac{V_{\text{eff}}(v)}{H^4} = \left(\frac{1}{2} \bar{m}_0^2 \bar{v}^2 + \frac{\lambda}{4!} \bar{v}^4 - \frac{\lambda}{8} \bar{f}_{\text{fin}}^2 - \frac{1}{2} \int d\bar{m}_{\text{dyn,eff}}^2 \bar{f}_{\text{fin}} \right), \quad (35)$$

where the bar over quantities denotes scaling with respect to appropriate power of H^2 .

The explicit form of $\bar{V}_{\text{eff}}(\bar{v})$ has to be found by numerical analyses. Note from Eq. (24) that the dynamically generated mass also needs to be evaluated numerically in general. However, when $m_{\text{dyn,eff}}^2$ is small compared to the Hubble rate, $m_{\text{dyn,eff}}^2/H^2 \ll 1$, we may find an analytic expression for it in the leading approximation,

$$\bar{m}_{\text{dyn,eff}}^2 \simeq \frac{2\pi(\bar{m}_0^2 + \lambda \bar{v}^2/2) + \sqrt{3\lambda + 4\pi^2(\bar{m}_0^2 + \lambda \bar{v}^2/2)}}{4\pi} \quad (36)$$

Note that for $\lambda = 0$, $m_{\text{dyn,eff}}^2 = m_0^2$, i.e. there is no dynamically generated mass. This corresponds trivially to the fact that in the absence of interaction, there is no self energy. Next, for $m_0^2 = 0 = \bar{v}$, we have $m_{\text{dyn,eff}}^2 = \sqrt{3\lambda}/4\pi$. This reproduces the result of [54] and later confirmed by many others using different methods, e.g. [51] and references therein.

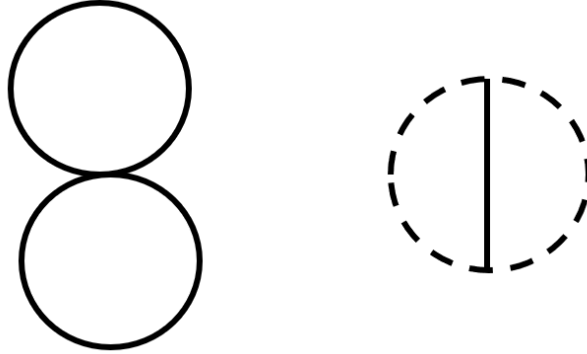


Figure 1: (Left) The two loop (lowest order) 2PI vacuum diagram for the quartic self interaction. (Right) The two loop (lowest order) 2PI diagram for the Yukawa interaction. Solid line stands for scalar, whereas a dashed line stands for fermion. The propagators are exact here.

The variation of the effective potential, Eq. (35), with respect to the background field v has been depicted in Fig. 2, Fig. 3. In the first, we see a non-trivial behaviour for a negative rest mass squared, first reported in [76, 77, 78].

Before we end this section, we note from our preceding discussion that in the presence of a background field, due to the $\lambda v^2/2$ term, the scalar cannot be treated as effectively massless, even if its rest mass is vanishing. However, given its special status in de Sitter pertaining to the isometry breaking and the non-existence of any smooth $m^2 \rightarrow 0$ limit [18] (cf., the discussion in Section 2), we wish to briefly make some comments about the vacuum graphs of a massless and minimal scalar field in this background. This corresponds to setting $v = 0 = m_0^2$. We wish to compute the 2PI vacuum graphs using the *tree level* or free propagator, Eq. (8).

For the double bubble given by the first of Fig. 1, we have using Eq. (11)

$$i\Gamma_2|_{2\text{-loop}} = -\frac{\lambda}{2^3} \int a^d d^d x i\Delta_{++}^2(x, x) = -\frac{H^{4-2\epsilon}\Gamma^2(2-\epsilon)}{2^{7-2\epsilon}\pi^{4-\epsilon}\Gamma^2(1-\epsilon/2)} \left(\frac{1}{\epsilon^2} + \frac{2\ln a}{\epsilon} + \ln^2 a \right) \quad (37)$$

The first divergence appearing above can be absorbed in a cosmological constant counterterm, whereas the second can be cancelled by the vacuum graph contribution generated by the one loop mass renormalisation counterterm, Eq. (12).

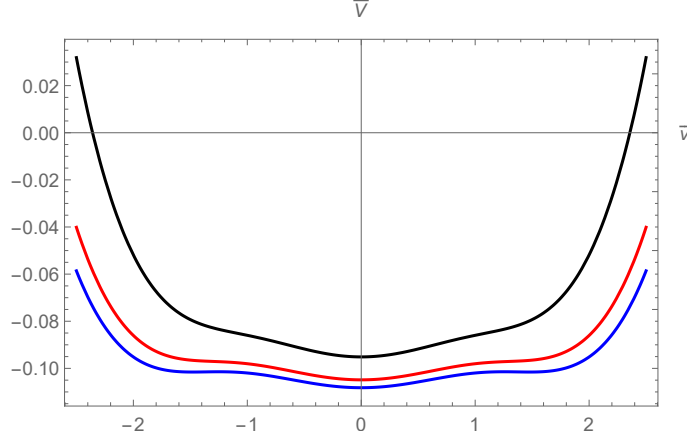


Figure 2: Variation of the effective potential Eq. (35), with respect to the background field v , for negative rest mass squared $m_0^2 \sim -0.063H^2$. The bar over the quantities denote scaling with respect to appropriate power of the de Sitter Hubble rate, H . The blue, red and black curves respectively correspond to λ values 0.10, 0.11 and 0.15 respectively. This non-trivial feature was reported first in [76, 77, 78].

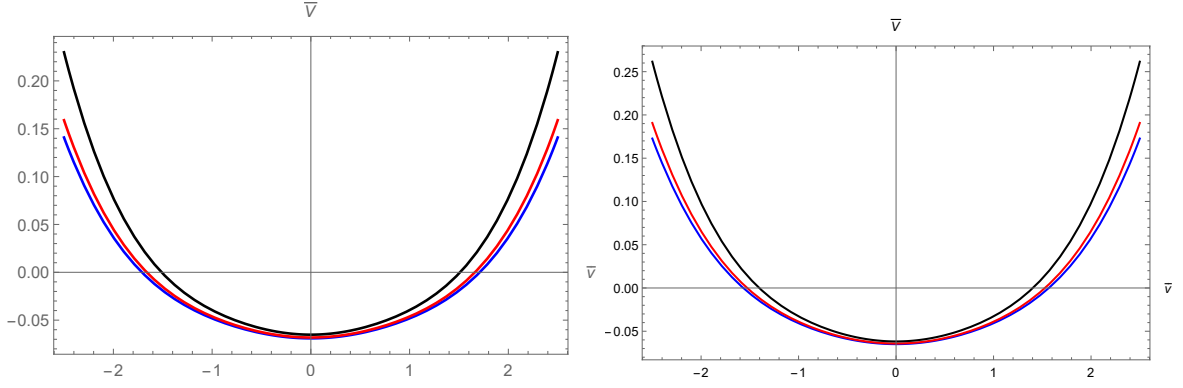


Figure 3: Variation of the effective potential Eq. (35), with respect to the background field v . The left and right set of curves correspond to $m_0^2 = 0$ and $m_0^2 \sim 0.01H^2$ respectively. The blue, red and black curves respectively correspond to λ values 0.10, 0.11 and 0.15.

The three loop vacuum graph is given by Fig. 4. Unlike the two loop case, it contains both local and non-local contributions. The former gives divergences as well as subleading secular logarithms, whereas the latter yields the leading, deep infrared secular contribution. Even though we are chiefly interested in the local parts of the vacuum graphs in this paper, we will compute the non-local part of Fig. 4 as well. The corresponding renormalised contribution, computed in Appendix A using the Schwinger-Keldysh or in-in formalism, equals

$$i\Gamma_{2|3\text{-loop, Ren.}} = \frac{\lambda^2}{2^{10} \times 3\pi^6} \int a^4 d^4x \left[\frac{1}{6} \ln^4 a + \ln^3 a + \mathcal{O}(\ln^2 a) \right] \quad (38)$$

To the best of our knowledge, the diagram topology like Fig. 4 has not been computed earlier in de Sitter.

The issue of the secular logarithms does not bother us in Eq. (34), simply because we confined ourselves to two loop Hartree approximation there and second, the scalar field was not effectively massless there due to the background field, no matter

whether its rest mass is zero or not. We wish to show below that this will not be the case if we include the Yukawa interaction. Even though we shall focus on computing only the local part of $i\Gamma_2$, it will contain secular logarithm originating from the ultraviolet terms. Thus in order to reach any meaningful conclusion about the effective potential in this case, we must associate some finite value to such large logarithm.

3.2 Inclusion of the Yukawa interaction

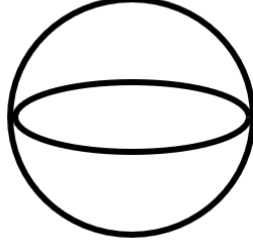


Figure 4: Three loop (next to the leading order) 2PI vacuum diagram for ϕ^4 self interaction. The other three loop diagram is a connected three-bubble, which is not 2PI. Although we have not considered this graph for non-perturbative computations in this paper, we have computed its renormalised expression for a massless minimal scalar in [Appendix A](#), with the tree level propagator, [Eq. \(8\)](#). See main text for discussion.

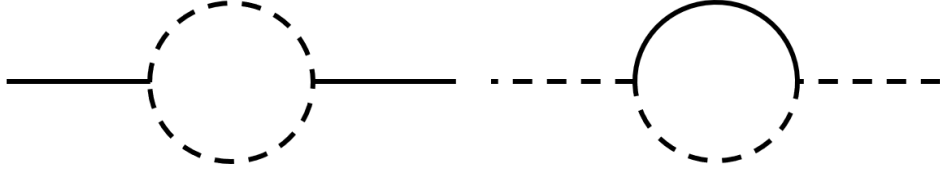


Figure 5: One loop self energy diagrams for the Yukawa interaction. Solid and dashed lines respectively correspond to scalar and fermion propagators. The propagators are exact here.

We now wish to add the effect of fermions to the effective potential of [Eq. \(35\)](#). The bare action is given by [Eq. \(6\)](#). We assume that there is no background fermion field, i.e., the fermion is purely quantum. Then the general 2PI effective action is given by [\[60\]](#),

$$\begin{aligned} \Gamma_{2\text{PI}}[v, iG] = & S[v] - \frac{i}{2} \ln \det iG(x, x) + i \ln \det iS(x, x) + \frac{1}{2} \text{Tr} \int (aa')^d d^d x d^d x' i\Delta^{-1}(x, x') iG(x', x) \\ & - \text{Tr} \int (aa')^d d^d x d^d x' iS_0^{-1}(x, x') iS(x, x') + i\Gamma_2[v, iG] \end{aligned} \quad (39)$$

where $i\Delta(x, x')$ and $iS_0(x, x')$ are respectively the tree level scalar and fermion propagators, whereas $iG(x, x')$ and $iS(x, x')$

are their exact forms. The above expression explicitly reads for our theory

$$\begin{aligned} \Gamma_{2\text{PI}}[v, iG] = & - \int a^d d^d x \left[(1 + \delta Z_1) \frac{1}{2} (\nabla_\mu v) (\nabla^\mu v) + \frac{1}{2} (m_0^2 + \delta m_1^2) v^2 + \frac{\delta \beta_1 v^3}{3!} + \frac{\lambda_1 v^4}{4!} + \delta \xi_1 R v^2 \right] \\ & - \frac{i}{2} \ln \det iG(x, x) + i \ln \det iS(x, x) + \frac{1}{2} \int a^d d^d x \left[(1 + \delta Z_2) \square - (m_0^2 + \delta m_2^2) - 2\delta \xi_2 R - \frac{\lambda_2 v^2}{2} - \delta \beta_2 v \right] iG(x, x) \\ & - \int a^d d^d x \left[(1 + \delta Z_f) i\nabla - M_0 - \delta M - g_2 v \right] iS(x, x) + i\Gamma_2[v, iG], \end{aligned} \quad (40)$$

where the 2PI vacuum contribution, Fig. 1, in this case reads at two loop

$$i\Gamma_2[v, iG] = -\frac{\lambda_3}{2^3} \int a^d d^d x iG^2(x, x) - \frac{ig_3^2}{2} \text{Tr} \int (aa')^d d^d x d^d x' iS(x, x') iS(x', x) iG(x, x'), \quad (41)$$

and $g_2 = g + \delta g_2$ and $g_3 = g + \delta g_3$. Fig. 5 shows the one loop scalar and fermion self energies corresponding to the two loop Yukawa vacuum graph.

We begin with the equation of motion for the fermion propagator,

$$\left[(1 + \delta Z_f) i\nabla - M_0 - \delta M - (g + \delta g_2) v \right] iS(x, x') = \frac{i\delta^d(x - x')}{a^d} - i(g + \delta g_3)^2 \int a''^d d^d x'' (iS(x, x'') iG(x, x'')) iS(x'', x') \quad (42)$$

As of the scalar field theory, we assume that the above equation is reduced to

$$\left[i\nabla - M_{\text{dyn,eff}} \right] iS(x, x') = \frac{i\delta^d(x - x')}{a^d} + \text{non-local terms} \quad (43)$$

where

$$M_{\text{dyn,eff}} = M_0 + gv + M_{\text{dyn}}$$

The dynamical mass in Eq. (43) originates from the integral of Eq. (42) containing the fermion self energy. The non-local term in Eq. (43), originates from the non-local part of the self energy, unlike the previously discussed case of scalar field theory in the Hartree approximation. However, this non-local contribution will not explicitly concern us in this work. Accordingly, we break the fermion propagator into two parts, and let $iS_l(x, x')$ be the part that satisfies the differential equation

$$\left[i\nabla - M_{\text{dyn,eff}} \right] iS_l(x, x') = \frac{i\delta^d(x - x')}{a^d}$$

Thus $iS_l(x, x')$ is simply the propagator for a fermion with mass $M_{\text{dyn,eff}}$. Now, since we are interested only in the local part of the self energy, we will only need the fermion propagator at small scales, so that we obtain a δ -function in the integrand in the self energy integral of Eq. (42), giving rise to the dynamically generated fermion mass. We take the fermion to be light. Then the part of the propagator *relevant* for our present purpose can be red off from Eq. (108) of Appendix B ([66]),

$$iS_l(x, x') = -\frac{i\Gamma(2 - \epsilon/2)}{2\pi^{2-\epsilon/2}(aa')^{3/2-\epsilon/2}} \frac{\Delta x}{(\Delta x^2)^{2-\epsilon/2}} + \frac{\Gamma(1 - \epsilon/2)}{2^2\pi^{2-\epsilon/2}(aa')^{3/2-\epsilon/2}} \frac{aM_{\text{dyn,eff}}}{(\Delta x^2)^{1-\epsilon/2}} \mathbf{I}_{d \times d}, \quad (44)$$

where Δx contains contraction with respect to the flat space gamma matrices. Note also that since we are working in local approximation, the propagators appearing above are all Feynman propagators, and we do not need to consider the Wightman functions necessary for the in-in formalism described in Appendix A, just like the ϕ^4 -theory in the two loop Hartree approximation discussed earlier.

Using Eq. (44), let us now compute the self energy term appearing in Eq. (42) under our local approximation. Using the expression Eq. (101) for the scalar propagator and Eq. (83), we have

$$\begin{aligned} & \int a''^d d^d x'' (iS(x, x'') iG(x, x'')) iS(x'', x') \\ &= \int a^{-d} (aa'')^{3/2} d^d x'' \left[\frac{\mu^{-\epsilon} \Gamma(1 - \epsilon/2) \not{\partial} \delta^d(x - x'')}{2^4 \pi^{2-\epsilon/2} (1 - \epsilon) \epsilon} + \frac{i \mu^{-\epsilon} \Gamma(1 - \epsilon/2) a M_{\text{dyn,eff}} \delta^d(x - x'')}{2^3 \pi^{2-\epsilon/2} (1 - \epsilon) \epsilon} \right] iS(x'', x') + \text{non-local terms} \end{aligned} \quad (45)$$

Note that $iS_l(x, x')$, Eq. (44), also yields non-local terms via the logarithms of Eq. (83), which we are not considering here. We now plug the above expression into Eq. (42). The divergence associated with the first term on the right hand side of Eq. (45) can be tackled by a fermion field strength renormalisation counterterm, δZ_f . Following [61], we write

$$i\delta Z_f \not{\partial} iS(x, x') = i\delta Z_f \int d^d x'' a^{-d} (aa'')^{(d-1)/2} \not{\partial} \delta^d(x - x'') iS(x'', x') \quad (46)$$

where in $\not{\partial}$, contraction with respect to the flat space gamma matrix has been used. This immediately yields

$$\delta Z_f = -\frac{\mu^{-\epsilon} (g + \delta g_3)^2 \Gamma(1 - \epsilon/2)}{2^4 \pi^{2-\epsilon/2} (1 - \epsilon) \epsilon} \quad (47)$$

Eq. (42) can now be rewritten after a little algebra as (suppressing the non-local terms for the sake of brevity)

$$\left[i\not{\nabla} - M_0 - \delta M - (g + \delta g_2)v - \frac{\mu^{-\epsilon} (g + \delta g_3)^2 \Gamma(1 - \epsilon/2) M_{\text{dyn,eff}}}{2^3 \pi^{2-\epsilon/2} (1 - \epsilon)} \left(\frac{1}{\epsilon} + \ln a \right) \right] iS_l(x, x') = \frac{i\delta^d(x - x')}{a^d} \quad (48)$$

We are yet to explicitly determine δg_3 and $M_{\text{dyn,eff}}$ appearing in Eq. (47), Eq. (48). In order to do this, we group the non-derivative terms appearing on the left hand side of the above equation as

$$\delta M + \left(M_0 + gv + \frac{g^2 M_{\text{dyn,eff}}}{2^3 \pi^2} \ln a \right) + \delta g_2 v + \frac{\mu^{-\epsilon} \Gamma(1 - \epsilon/2) M_{\text{dyn,eff}}}{2^2 \pi^{2-\epsilon/2} (1 - \epsilon)} \left(g\delta g_3 \ln a + \frac{\delta g_3^2 \ln a}{2} + \frac{(g + \delta g_3)^2}{2\epsilon} \right) \quad (49)$$

Thus we identify

$$\begin{aligned} M_{\text{dyn,eff}} &= M_0 + gv + \frac{g^2 M_{\text{dyn,eff}}}{2^3 \pi^2} \ln a \\ \Rightarrow M_{\text{dyn,eff}} &= \frac{M_0 + gv}{1 - \frac{g^2 \ln a}{2^3 \pi^2}} \end{aligned} \quad (50)$$

Writing now $M_{\text{dyn,eff}} = M_0 + gv + M_{\text{dyn}}$, we have the non-perturbative expression

$$M_{\text{dyn}} = \frac{g^2 (M_0 + gv) \ln a}{2^3 \pi^2 \left(1 - \frac{g^2 \ln a}{2^3 \pi^2} \right)} \quad (51)$$

Thus for a massless fermion, there can be no dynamical generation of mass in de Sitter space if $v = 0$, at least at two loop. This is in contrast to a massless minimally coupled scalar and should be attributed to the conformal invariance of a massless fermion. The above expression also shows that in flat spacetime ($a = 1$) there can be no dynamical mass generation.

We next substitute the expression for $M_{\text{dyn,eff}}$ from Eq. (50) into the last term of Eq. (49), to regroup them all as

$$\begin{aligned} & M_{\text{dyn,eff}} + \left(\delta M + \frac{\mu^{-\epsilon} M_0 (g + \delta g_3)^2 \Gamma(1 - \epsilon/2)}{2^3 \pi^{2-\epsilon/2} (1 - \epsilon) \epsilon} \right) + \left(\delta g_2 + \frac{\mu^{-\epsilon} g (g + \delta g_3)^2 \Gamma(1 - \epsilon/2)}{2^3 \pi^{2-\epsilon/2} (1 - \epsilon) \epsilon} \right) v \\ &+ \frac{\mu^{-\epsilon} \Gamma(1 - \epsilon/2)}{2^3 \pi^{2-\epsilon/2} (1 - \epsilon)} \left((g + \delta g_3)^2 - g^2 + \frac{g^2 (g + \delta g_3)^2}{2^3 \pi^2 \epsilon} \right) M_{\text{dyn,eff}} \ln a \end{aligned} \quad (52)$$

Since the above expression must equal $M_{\text{dyn,eff}}$, the terms within each bracket must be vanishing. This yields the counterterms

$$\begin{aligned}\delta g_3 &= -g + \frac{g}{\left(1 + \frac{g^2}{2^3 \pi^2 \epsilon}\right)^{1/2}} \\ \delta g_2 &= -\frac{\mu^{-\epsilon} g(g + \delta g_3)^2 \Gamma(1 - \epsilon/2)}{2^3 \pi^{2-\epsilon/2} (1 - \epsilon) \epsilon} = -\frac{\mu^{-\epsilon} g^3 \Gamma(1 - \epsilon/2)}{2^3 \pi^{2-\epsilon/2} (1 - \epsilon) \epsilon \left(1 + \frac{g^2}{2^3 \pi^2 \epsilon}\right)} \\ \delta M &= -\frac{\mu^{-\epsilon} M_0 (g + \delta g_3)^2 \Gamma(1 - \epsilon/2)}{2^3 \pi^{2-\epsilon/2} (1 - \epsilon) \epsilon} = -\frac{\mu^{-\epsilon} M_0 g^2 \Gamma(1 - \epsilon/2)}{2^3 \pi^{2-\epsilon/2} (1 - \epsilon) \epsilon \left(1 + \frac{g^2}{2^3 \pi^2 \epsilon}\right)}\end{aligned}\quad (53)$$

It is clear that the above counterterms are non-perturbative. Using the first of the above expression, [Eq. \(47\)](#) becomes

$$\delta Z_f = -\frac{\mu^{-\epsilon} g^2 \Gamma(1 - \epsilon/2)}{2^4 \pi^{2-\epsilon/2} \left(1 + \frac{g^2}{2^3 \pi^2 \epsilon}\right) (1 - \epsilon) \epsilon} \quad (54)$$

Before we proceed, let us estimate the dynamically generated effective mass of the fermion appearing in [Eq. \(50\)](#). In an eternal de Sitter spacetime, clearly $M_{\text{dyn,eff}}$ has many unsettling features, owing to the appearance of the secular logarithm. For example, it diverges when the denominator of [Eq. \(50\)](#) is vanishing, becomes negative afterward and then becomes asymptotically vanishing. However, inflation cannot last forever, and we must put a cut off to the time scale of it and estimate the value of the logarithm at that time scale. For example, we can take it to be just the standard number of e -foldings. Alternatively, and perhaps albeit naively, we may attempt to associate the $\ln a$ term with some ratio of the momentum/wavelength scale as follows. Since we are dealing with the secular logarithm associated with the ultraviolet or local terms, let us begin with a momentum k at $a = 1$. The proper momentum is k/a . The ratio of the initial and late time value of this proper momentum would be $k/(k/a) = (ka)/k = a$. Inspired from this we write

$$\ln a \sim \ln \frac{k_{\text{end}}}{k_{\text{pivot}}}$$

where $k_{\text{end}} \sim 10^{23} \text{Mpc}^{-1}$ is the scale of the horizon exit associated with the CMB, and k_{pivot} is some pivot scale which is taken as $\sim 0.05 \text{Mpc}^{-1}$, estimated from the reheating temperature, e.g. [\[58\]](#) and references therein. We thus have $\ln a \sim 55.262$ at the end of the inflation. Note that this is also approximately the standard number of e -foldings. We do *not* claim this estimation to be accurate. In particular as of now, we do not see in this framework any formal way to do resummation of this logarithm as opposed to e.g. [\[47, 51\]](#), where autonomous differential equations to sum perturbation series was constructed. On the other hand, the present formalism involves non-perturbative propagators only. Possibly one then needs to think about additional equations to achieve any such formal resummation, if any. This remains as a possible caveat to our analysis. We also refer our reader to e.g. [\[50\]](#) and references therein for discussion on the relationship of the deep infrared secular logarithms and ratio of different momentum scale.

Putting things together now, we have at the end of inflation,

$$M_{\text{dyn,eff}} \simeq \frac{M_0 + gv}{1 - 0.7g^2} \quad (55)$$

Thus for example for the range $g \sim 0.1 - 0.5$, we see that the logarithm term contributes at most $0.7 - 17\%$ to $M_0 + gv$. Due this reason we shall take $M_{\text{dyn,eff}} \simeq M_0 + gv$ in the following, and will ignore the dynamically generated fermion mass.

Let us now consider the equation of motion for the Green function of the scalar field

$$\begin{aligned}
& \left[\square - (m_0^2 + \delta m_2^2) - 2\delta\xi_2 R - \frac{(\lambda + \delta\lambda_2)v^2}{2} - \delta\beta_2 v \right] iG(x, x') = \frac{i\delta^d(x - x')}{a^d} + \frac{(\lambda + \delta\lambda_3)}{2} iG(x, x) iG(x, x') \\
& + i(g + \delta g_3)^2 \int d^d x'' a''^d \text{Tr}(iS(x, x'') iS(x'', x)) iG(x'', x') \\
& = \frac{i\delta^d(x - x')}{a^d} + \frac{(\lambda + \delta\lambda_3)}{2} [m_{\text{dyn,eff}}^2 f_d + H^2 f'_d + f_{\text{fin}}] iG(x, x') \\
& + \frac{(g + \delta g_3)^2}{2\pi^2} \left[\frac{\mu^{-\epsilon}(M_{\text{dyn,eff}}^2 + H^2)\Gamma(1 - \epsilon/2)(1 - \epsilon/4)}{\pi^{-\epsilon/2}(1 - \epsilon)\epsilon} + (H^2 + M_{\text{dyn,eff}}^2) \ln a \right] iG(x, x') + \text{non-local terms} \quad (56)
\end{aligned}$$

where we have used [Eq. \(20\)](#), [Eq. \(21\)](#), [Eq. \(111\)](#). We will also take $M_{\text{dyn,eff}} \simeq M_0 + gv$, as argued above. We also have suppressed the scalar field strength renormalisation counterterm as it will not be necessary for our present purpose.

Before we proceed, first we note from [Eq. \(53\)](#) that the quantity $(g + \delta g_3)^2$ is not only finite, but also vanishingly small as $\mathcal{O}(\epsilon)$. Thus the term $(g + \delta g_3)^2/\epsilon$ is a non-vanishing and *finite* quantity as $\epsilon \rightarrow 0$. This suggests that the finite quantities appearing within the square bracket on the last line of [Eq. \(56\)](#) can safely be ignored, and the rest of the terms, containing $(g + \delta g_3)^2/\epsilon$, are finite as $\epsilon \rightarrow 0$. Note that analogous thing also happens for the pure quartic self interaction. For example, $\delta\lambda_2$ is finite and $(\lambda + \delta\lambda_2) \rightarrow 0$ as $\epsilon \rightarrow 0$, [Eq. \(27\)](#). It can then be seen from [Eq. \(32\)](#) that the product of this term and the divergent $(1 + \lambda f_d/2)$ eventually yields ultraviolet finite non-vanishing terms.

It seems *a priori* that we have two most natural ways to handle the issue of renormalisation of [Eq. \(56\)](#). First, we renormalise all the terms containing $(g + \delta g_3)^2/\epsilon$ via mass, cubic and quartic coupling renormalisations (respectively, δm_2^2 , $\delta\beta_2$, $\delta\lambda_2$ in [Eq. \(56\)](#)) through some finite counterterms. Alternatively, we keep them as it is in order to include them in the effective dynamical mass squared of the scalar field.

However, following either of these paths leads to divergent term in the effective action which cannot be renormalised away. This will be clear from our analysis below. In order to tackle this issue, we break the Yukawa coupling term in [Eq. \(56\)](#) as

$$(g + \delta g_3)^2 = (k + (1 - k))(g + \delta g_3)^2 \quad (57)$$

where k is a number to be determined. We renormalise away all the terms containing $(1 - k)(g + \delta g_3)^2$. Accordingly, the rest containing $k(g + \delta g_3)^2$ contributes to the scalar field's effective dynamical mass squared. With this, we note from [Eq. \(56\)](#) the renormalisation conditions

$$\begin{aligned}
& \delta m_2^2 + \frac{(\lambda + \delta\lambda_3)}{2} \left[\left(m_0^2 + \frac{k\mu^{-\epsilon}(M_0^2 + H^2)(g + \delta g_3)^2\Gamma(1 - \epsilon/2)(1 - \epsilon/4)}{2\pi^{2-\epsilon/2}(1 - \epsilon)\epsilon} \right) f_d + H^2 f'_d \right] \\
& + \frac{\mu^{-\epsilon}(1 - k)(g + \delta g_3)^2\Gamma(1 - \epsilon/2)(1 - \epsilon/4)(M_0^2 + H^2)}{2\pi^{2-\epsilon/2}\epsilon(1 - \epsilon)} = 0 \\
& \delta\lambda_2 + (\lambda + \delta\lambda_3) \left(\frac{\lambda}{2} + \frac{k\mu^{-\epsilon}g^2(g + \delta g_3)^2\Gamma(1 - \epsilon/2)(1 - \epsilon/4)}{2\pi^{2-\epsilon/2}\epsilon(1 - \epsilon)} \right) f_d + \frac{\mu^{-\epsilon}(1 - k)g^2(g + \delta g_3)^2\Gamma(1 - \epsilon/2)(1 - \epsilon/4)}{\pi^{2-\epsilon/2}\epsilon(1 - \epsilon)} = 0 \\
& \delta\beta_2 + \frac{\mu^{-\epsilon}(1 - k)g(g + \delta g_3)^2\Gamma(1 - \epsilon/2)(1 - \epsilon/4)M_0}{\pi^{2-\epsilon/2}\epsilon(1 - \epsilon)} + \frac{k\mu^{-\epsilon}gM_0(\lambda + \delta\lambda_3)(g + \delta g_3)^2 f_d \Gamma(1 - \epsilon/2)(1 - \epsilon/4)}{2\pi^{2-\epsilon/2}(1 - \epsilon)\epsilon} = 0, \\
& \delta\lambda_3 = -\frac{\lambda^2 f_d}{2 \left(1 + \frac{\lambda f_d}{2} \right)}, \quad \delta\xi_2 = 0. \quad (58)
\end{aligned}$$

It is clear that renormalising the Yukawa contribution entirely or keeping it entirely corresponds respectively to $k = 1$ or $k = 0$ in the above equation. The above decomposition corresponds to the dynamical effective scalar mass squared

$$m_{\text{dyn,eff}}^2 = m_0^2 + \frac{\lambda v^2}{2} + \frac{\lambda f_{\text{fin}}}{2} + \frac{\mu^{-\epsilon}k(g + \delta g_3)^2((M_0 + gv)^2 + H^2)\Gamma(1 - \epsilon/2)(1 - \epsilon/4)}{2\pi^{2-\epsilon/2}(1 - \epsilon)\epsilon} \quad (59)$$

Using Eq. (21), Eq. (53), the counterterms appearing in Eq. (58) can explicitly be written as

$$\begin{aligned}
\delta m_2^2 &= \frac{\lambda H^{-\epsilon}}{2^4 \pi^{2-\epsilon/2} \epsilon \left(1 - \frac{\lambda H^{-\epsilon}}{2^4 \pi^{2-\epsilon/2} \epsilon}\right)} \left(m_0^2 + H^2(2 - \gamma\epsilon) + \frac{k \mu^{-\epsilon} (M_0^2 + H^2) g^2 \Gamma(1 - \epsilon/2)(1 - \epsilon/4)}{2 \pi^{2-\epsilon/2} (1 - \epsilon) \epsilon \left(1 + \frac{g^2}{2^3 \pi^2 \epsilon}\right)} \right) \\
&\quad - \frac{\mu^{-\epsilon} (1 - k) g^2 (M_0^2 + H^2) \Gamma(1 - \epsilon/2)(1 - \epsilon/4)}{2 \pi^{2-\epsilon/2} \epsilon (1 - \epsilon) \left(1 + \frac{g^2}{2^3 \pi^2 \epsilon}\right)}, \\
\delta \beta_2 &= - \frac{\mu^{-\epsilon} (1 - k) g^3 M_0 \Gamma(1 - \epsilon/2)(1 - \epsilon/4)}{\pi^{2-\epsilon/2} \epsilon (1 - \epsilon) \left(1 + \frac{g^2}{2^3 \pi^2 \epsilon}\right)} + \frac{k (\mu H)^{-\epsilon} M_0 \lambda g^3 \Gamma(1 - \epsilon/2)(1 - \epsilon/4)}{2^{4-\epsilon} \pi^{4-\epsilon} (1 - \epsilon) \epsilon^2 \left(1 + \frac{g^2}{2^3 \pi^2 \epsilon}\right) \left(1 - \frac{\lambda H^{-\epsilon}}{2^{4-\epsilon} \pi^{2-\epsilon/2} \epsilon}\right)}, \\
\delta \lambda_2 &= \frac{\lambda H^{-\epsilon}}{2^{4-\epsilon} \pi^{2-\epsilon/2} \epsilon \left(1 - \frac{\lambda H^{-\epsilon}}{2^{4-\epsilon} \pi^{2-\epsilon/2} \epsilon}\right)} \left(\lambda + \frac{k \mu^{-\epsilon} g^4 \Gamma(1 - \epsilon/2)(1 - \epsilon/4)}{\pi^{2-\epsilon/2} \epsilon (1 - \epsilon) \left(1 + \frac{g^2}{2^3 \pi^2 \epsilon}\right)} \right) - \frac{\mu^{-\epsilon} (1 - k) g^4 \Gamma(1 - \epsilon/2)(1 - \epsilon/4)}{\pi^{2-\epsilon/2} \epsilon (1 - \epsilon) \left(1 + \frac{g^2}{2^3 \pi^2 \epsilon}\right)}, \\
\delta \lambda_3 &= \frac{\lambda^2 H^{-\epsilon}}{2^{4-\epsilon} \pi^{2-\epsilon/2} \epsilon \left(1 - \frac{\lambda H^{-\epsilon}}{2^{4-\epsilon} \pi^{2-\epsilon/2} \epsilon}\right)}, \quad \delta \xi_2 = 0. \tag{60}
\end{aligned}$$

Note that $\delta \lambda_2 \neq \delta \lambda_3$ here, unlike the earlier case of pure scalar field theory. The structure of the counterterms appearing above suggest various non-trivial self energy and vertex corrections and their resummation. For example, the last term on the right hand side in the expression for $\delta \lambda_2$ corresponds to the renormalisation of the box diagram corresponding to the scalar four point function of the Yukawa theory. On the other hand, the second term on the right hand side of the same corresponds to the resummed renormalisation of the quartic scalar vertex correction due to the insertion of the Yukawa vertices, Fig. 6.

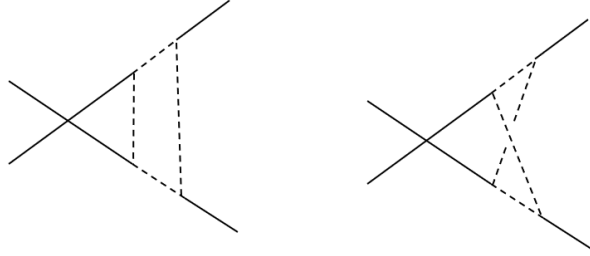


Figure 6: A non-trivial scalar quartic vertex function correction and its corresponding resummed counterterm generated by the 2PI formalism, Eq. (60). Solid and dashed lines represent respectively, scalar and fermions.

With this, Eq. (56) can be rewritten as (suppressing the non-local terms)

$$(\square - m_{\text{dyn,eff}}^2) iG_{\text{loc}}(x, x') = \frac{i \delta^d(x - x')}{a^d}, \tag{61}$$

with $m_{\text{dyn,eff}}^2$ being given by Eq. (59).

We are yet to clarify the necessity of introducing the constant k and its value appearing in Eq. (58), Eq. (60), Eq. (59). In order to see this, let us now consider the effective action Eq. (39), which, after using the equations of motion for the propagators,

Eq. (42), Eq. (56), becomes

$$\begin{aligned} \Gamma_{2\text{PI}}[v, iG] = & - \int a^d d^d x \left[\frac{1}{2} (\nabla_\mu v) (\nabla^\mu v) + \frac{1}{2} (m_0 + \delta m_1^2) v^2 + \frac{(\lambda + \delta \lambda_1) v^4}{4!} + \frac{\delta \beta_1 v^3}{3!} + \delta \xi_1 R v^2 + \gamma_{\text{tad}} v \right] \\ & - \frac{1}{2} \int d^d x a^d \int d m_{\text{dyn,eff}}^2 iG(x, x) + \int d^d x a^d \int d M_{\text{dyn,eff}} iS(x, x) \\ & + \frac{\lambda + \delta \lambda_3}{2^3} \int a^d d^d x iG^2(x, x) + i(g + \delta g_3)^2 \text{Tr} \int (aa')^d d^d x d^d x' iS(x, x') iS(x', x) iG(x, x') \end{aligned} \quad (62)$$

We now wish to compute the vacuum loop integrals appearing in the last line of Eq. (62). Let us for the sake of brevity, abbreviate in Eq. (59),

$$\frac{\mu^{-\epsilon} (g + \delta g_3)^2 \Gamma(1 - \epsilon/2) (1 - \epsilon/4)}{2\pi^{2-\epsilon/2} (1 - \epsilon) \epsilon} = C \text{ (say)} \quad (63)$$

Using next Eq. (20), Eq. (59) and Eq. (111) of Appendix C, we obtain for Eq. (62) after a little algebra

$$\begin{aligned} & \frac{\lambda + \delta \lambda_3}{2^3} \int a^d d^d x iG^2(x, x) + i(g + \delta g_3)^2 \text{Tr} \int (aa')^d d^d x d^d x' iS(x, x') iS(x', x) iG(x, x') \\ = & \frac{\lambda}{2^3 (1 + \lambda f_d/2)} \int a^d d^d x \left[(m_0^2 f_d + H^2 f_d')^2 + \frac{\lambda^2 v^4}{4} f_d^2 + k^2 C^2 ((M_0 + gv)^2 + H^2)^2 f_d^2 + \left(1 + \frac{\lambda f_d}{2}\right)^2 f_{\text{fin}}^2 \right. \\ & + \lambda f_d (m_0^2 f_d + H^2 f_d') v^2 + 2kC f_d (m_0^2 f_d + H^2 f_d') ((M_0 + gv)^2 + H^2) + 2(m_0^2 f_d + H^2 f_d') \left(1 + \frac{\lambda f_d}{2}\right) f_{\text{fin}} \\ & + \lambda f_d \left(1 + \frac{\lambda f_d}{2}\right) v^2 f_{\text{fin}} + \lambda v^2 kC f_d^2 ((M_0 + gv)^2 + H^2) + 2kC ((M_0 + gv)^2 + H^2) \left(1 + \frac{\lambda f_d}{2}\right) f_{\text{fin}} f_d \Big] \\ & + \frac{C}{2} \int a^d d^d x (H^2 + (M_0 + gv)^2) \left(1 + \epsilon \ln a + \frac{1}{2} \epsilon^2 \ln^2 a + \mathcal{O}(\epsilon^3)\right) \left[\left(m_0^2 + \frac{\lambda v^2}{2} + \frac{\lambda f_{\text{fin}}}{2} + Ck (H^2 + (M_0 + gv)^2)\right) f_d \right. \\ & \left. + H^2 f_d' + f_{\text{fin}} \right] \end{aligned} \quad (64)$$

Now we note in the above expression that the most problematic divergence appears in the penultimate line, given by

$$\frac{C\lambda}{4} (H^2 + (M_0 + gv)^2) f_{\text{fin}} f_d,$$

Given the structure of f_{fin} , Eq. (20), the above divergence cannot be absorbed by any counterterm we know of. This is an overlapping divergence and is absent in the traditional computations. Recall also that we do not have the freedom here to add any further graph in the effective action to attempt a cancellation. Putting things together, we make the choice $k = -1$ in Eq. (58), Eq. (60) and Eq. (64), so that the above overlapping divergence gets cancelled by the last term of the fourth line of Eq. (64). It is the necessity of cancellation of this divergence that led us to introduce the constant k . Any other choice than $k = -1$ (like $k = 1$ or $k = 0$) would not have served our purpose. Note also that this problem would not have arisen if the sign in front of the last term on the right hand side of the effective action Eq. (62) was opposite. However, we have made it sure that there is no error in any sign.

Putting things together, we now rewrite [Eq. \(64\)](#) as

$$\begin{aligned}
& \frac{\lambda + \delta\lambda_3}{2^3} \int a^d d^d x \, iG^2(x, x) + i(g + \delta g_3)^2 \text{Tr} \int (aa')^d d^d x d^d x' iS(x, x') iS(x', x) iG(x, x') \\
&= \frac{\lambda}{2^3(1 + \lambda f_d/2)} \int a^d d^d x \left[(m_0^2 f_d + H^2 f'_d)^2 + \frac{\lambda^2 v^4}{4} f_d^2 + C^2 ((M_0 + gv)^2 + H^2)^2 f_d^2 + \left(1 + \frac{\lambda f_d}{2}\right)^2 f_{\text{fin}}^2 \right. \\
&\quad + \lambda f_d (m_0^2 f_d + H^2 f'_d) v^2 - 2C f_d (m_0^2 f_d + H^2 f'_d) ((M_0 + gv)^2 + H^2) + 2(m_0^2 f_d + H^2 f'_d) \left(1 + \frac{\lambda f_d}{2}\right) f_{\text{fin}} \\
&\quad \left. + \lambda f_d \left(1 + \frac{\lambda f_d}{2}\right) v^2 f_{\text{fin}} - \lambda v^2 C f_d^2 ((M_0 + gv)^2 + H^2) \right] \\
&\quad + \frac{C}{2} \int a^d d^d x [H^2 + (M_0 + gv)^2] \left[\left(m_0^2 + \frac{\lambda v^2}{2} - C(H^2 + (M_0 + gv)^2)\right) f_d + H^2 f'_d + f_{\text{fin}} \right. \\
&\quad \left. - \left(m_0^2 + \frac{\lambda v^2}{2} + \frac{\lambda f_{\text{fin}}}{2} - C(H^2 + (M_0 + gv)^2) + 2H^2\right) \frac{\ln a}{2^3 \pi^2} \right] \tag{65}
\end{aligned}$$

We next compute for [Eq. \(62\)](#) using the unrenormalised coincidence limit expressions [Eq. \(20\)](#) and [Eq. \(108\)](#),

$$\begin{aligned}
& \frac{1}{2} \int d^d x a^d \int dm_{\text{dyn,eff}}^2 iG(x, x) - \int d^d x a^d \int dM_{\text{dyn,eff}} iS(x, x) \\
&= \int d^d x a^d \left[\frac{m_{\text{dyn,eff}}^4}{4} f_d + \frac{H^2 m_{\text{dyn,eff}}^2}{2} f'_d + \frac{1}{2} \int dm_{\text{dyn,eff}}^2 f_{\text{fin}} \right] \\
&\quad - \int d^d x a^d \left[\left(-\frac{2}{\epsilon} + \frac{3}{2} + \gamma\right) \frac{M_{\text{dyn,eff}}^2 H^{2-\epsilon}}{2^{1-\epsilon/2} \pi^{2-\epsilon/2}} + \left(-\frac{2}{\epsilon} - \frac{3}{2} + \gamma\right) \frac{M_{\text{dyn,eff}}^4 H^{-\epsilon}}{2^{2-\epsilon/2} \pi^{2-\epsilon/2}} + \int dM_{\text{dyn,eff}} F_{\text{fin}} \right], \tag{66}
\end{aligned}$$

where we have abbreviated from [Eq. \(108\)](#),

$$F_{\text{fin}} = \frac{M_{\text{dyn,eff}} H^2}{\pi^2} \left(1 + \frac{M_{\text{dyn,eff}}^2}{H^2}\right) \left[\psi\left(1 + \frac{iM_{\text{dyn,eff}}}{H}\right) + \psi\left(1 - \frac{iM_{\text{dyn,eff}}}{H}\right) \right], \tag{67}$$

where ψ is the digamma function. Also recall that we have taken $M_{\text{dyn,eff}} \simeq M_0 + gv$.

In order to find out the effective action Eq. (62), we next subtract Eq. (66) from Eq. (65), and obtain after a little algebra

$$\begin{aligned}
& \int a^4 d^4x \left[\int dM_{\text{dyn,eff}} F_{\text{fin}} - \frac{1}{2} \int dm_{\text{dyn,eff}}^2 f_{\text{fin}} + \frac{\lambda f_{\text{fin}}^2}{8} \right. \\
& + \frac{C}{2} [H^2 + (M_0 + gv)^2] \left\{ f_{\text{fin}} - \left(m_0^2 + \frac{\lambda v^2}{2} + \frac{\lambda f_{\text{fin}}}{2} - C(H^2 + (M_0 + gv)^2) + 2H^2 \right) \frac{\ln a}{2^3 \pi^2} \right\} \\
& + \int a^d d^d x \left[\left\{ \frac{\lambda((m_0^2 f_d + H^2 f'_d) - C(M_0^2 + H^2) f_d)^2}{2^3(1 + \lambda f_d/2)} - \frac{m_0^2}{2} (m_0^2 f_d + 2H^2 f'_d) + \frac{M_0^2 H^{-\epsilon}}{2^{1-\epsilon/2} \pi^{2-\epsilon/2}} \left(\frac{M_0^2}{2} \left(-\frac{2}{\epsilon} - \frac{3}{2} + \gamma \right) \right. \right. \right. \\
& + H^2 \left(-\frac{2}{\epsilon} + \frac{3}{2} + \gamma \right) \left. \right\} + \frac{C}{2} (H^2 + M_0^2) ((m_0^2 - C(H^2 + M_0^2)) f_d + H^2 f'_d) \left. \right\} \\
& - \frac{\lambda(m_0^2 f_d + H^2 f'_d) v^2}{4(1 + \lambda f_d/2)} - \frac{\lambda^2 f_d v^4}{16(1 + \lambda f_d/2)} \left. \right] \\
& + \int a^d d^d x M_0 g \left[\frac{H^{-\epsilon}}{2^{-\epsilon/2} \pi^{2-\epsilon/2}} \left\{ H^2 \left(-\frac{2}{\epsilon} + \frac{3}{2} + \gamma \right) + M_0^2 \left(-\frac{2}{\epsilon} - \frac{3}{2} + \gamma \right) \right\} \right. \\
& + \frac{\lambda C}{2(1 + \lambda f_d/2)} (C(H^2 + M_0^2) - (m_0^2 f_d + H^2 f'_d) f_d) + C((m_0^2 f_d + H^2 f'_d) - 2C(H^2 + M_0^2) f_d) \left. \right] v \\
& + \int d^d x a^d \left[\frac{g^2 H^{-\epsilon}}{2^{1-\epsilon/2} \pi^{2-\epsilon/2}} \left\{ H^2 \left(-\frac{2}{\epsilon} + \frac{3}{2} + \gamma \right) + 3M_0^2 \left(-\frac{2}{\epsilon} - \frac{3}{2} + \gamma \right) \right\} \right. \\
& + \frac{\lambda}{4(1 + \lambda f_d/2)} \left((3M_0^2 + H^2) C^2 g^2 f_d^2 - C f_d g^2 (m_0^2 f_d + H^2 f'_d) - \frac{\lambda C f_d^2}{2} (M^2 + H_0^2) \right) \\
& + \frac{C}{2} \left(g^2 ((m_0^2 - C(H^2 + M_0^2)) f_d + H^2 f'_d) + \frac{\lambda}{2} (H^2 + M_0^2) f_d - 4g^2 C M_0^2 f_d \right) \left. \right] v^2 \\
& + \int a^d d^d x M_0 g \left[C f_d \left\{ -\frac{\lambda^2 f_d}{4(1 + \lambda f_d/2)} + \left(\frac{\lambda}{2} - 2Cg^2 \right) \right\} + \left(-\frac{2}{\epsilon} - \frac{3}{2} + \gamma \right) \frac{g^2 H^{-\epsilon}}{2^{-\epsilon/2} \pi^{2-\epsilon/2}} \right] v^3 \\
& + \int a^d d^d x g^2 \left[\frac{\lambda f_d^2 C}{1 + \lambda f_d/2} (Cg^2 - \lambda) + \frac{C}{2} \left(\frac{\lambda}{2} - Cg^2 \right) f_d + \left(-\frac{2}{\epsilon} - \frac{3}{2} + \gamma \right) \frac{g^2 H^{-\epsilon}}{2^{2-\epsilon/2} \pi^{2-\epsilon/2}} \right] v^4, \tag{68}
\end{aligned}$$

where $M_{\text{dyn,eff}}$ and F_{fin} is given by Eq. (67), $m_{\text{dyn,eff}}^2$ by Eq. (59) (with $k = -1$), f_{fin} , f_d and f'_d by Eq. (21) and Eq. (22) and C is defined in Eq. (63). The first two lines of the above equation are finite. The fifth line gives the divergences corresponding to the quartic self interaction for mass and coupling constant renormalisation. The remaining lines give the divergences corresponding to the Yukawa or overlapping Yukawa- ϕ^4 interactions, and they all vanish in the absence of the Yukawa coupling. When Eq. (68) is plugged into Eq. (62), the terms proportional to v , v^2 , v^3 and v^4 respectively give the tadpole (γ_{tad}), the mass renormalisation (δm_1^2), the cubic coupling ($\delta \beta_1$) and the quartic coupling ($\delta \lambda_1$) counterterms. We also set $\delta \xi_1 = 0$. Note also that for vanishing fermion rest mass, we have $\delta \beta_1 = 0 = \gamma_{\text{tad}}$. Also, the third and fourth lines are field independent and hence must be absorbed in a cosmological constant counterterm in the gravitational action.

Putting things together, we now have the renormalised 1PI effective action, only as a function of the background scalar field,

$$\begin{aligned}
\Gamma_{\text{1PI}}[v]_{\text{Ren}} = & - \int a^d d^d x \left[\frac{1}{2} (\nabla_\mu v) (\nabla^\mu v) + \frac{1}{2} m_0 v^2 + \frac{\lambda v^4}{4!} - \int dM_{\text{dyn,eff}} F_{\text{fin}} + \frac{1}{2} \int dm_{\text{dyn,eff}}^2 f_{\text{fin}} \right. \\
& - \frac{\lambda f_{\text{fin}}^2}{8} - \frac{C}{2} [H^2 + (M_0 + gv)^2] \left\{ f_{\text{fin}} - \left(m_0^2 + \frac{\lambda v^2}{2} + \frac{\lambda f_{\text{fin}}}{2} - C(H^2 + (M_0 + gv)^2) + 2H^2 \right) \frac{\ln a}{2^3 \pi^2} \right\} \left. \right] \\
& + \text{non-local contributions.} \tag{69}
\end{aligned}$$

The first line of the above equation corresponds to the standard tree plus one loop effective action, whereas the second line gives the contribution due to integrating out the 2PI vacuum graphs, Fig. 1. In the absence of the fermion, the above reduces

to the scalar field theory result, [Eq. \(34\)](#), whereas setting $a = 1$ yields the result of the flat spacetime.

What will be the differences of the renormalisation procedure and the final result if we instead compute the two loop effective action for the ϕ^4 -Yukawa theory via standard 1PI perturbation theory? This issue has been outlined in [Appendix D](#). Chiefly, owing to the fact that one uses the tree level propagators in such computations, we show that the renormalisation procedure is qualitatively very different in the standard 1PI technique. In particular, we show that there is no necessity of the constant k , as of [Eq. \(58\)](#). Second, we show that the finite part of the two loop effective action is also different, and in fact the secular logarithm is quadratic there opposed to [Eq. \(69\)](#).

Now, even though renormalised, [Eq. \(69\)](#) has the obvious problematic feature associated with the secular logarithm, which grows monotonically, and eventually becomes large at late times. As earlier, we will assign with it the value we estimated earlier, $\ln a \sim 55.262$ (cf, the discussion below [Eq. \(54\)](#)). In particular unlike what we did for fermion's effective dynamical mass, since this logarithm is not multiplied with any g^2 , we cannot ignore it compared to the f_{fin} term appearing in the curly bracket of [Eq. \(69\)](#). Substituting this estimated value of $\ln a$ into [Eq. \(69\)](#), we have

$$\begin{aligned} \Gamma_{\text{1PI}}[v]_{\text{Ren}} = & - \int d^d x \left[\frac{1}{2} (\nabla_\mu v)(\nabla^\mu v) + \frac{1}{2} m_0 v^2 + \frac{\lambda v^4}{4!} - \int dM_{\text{dyn,eff}} F_{\text{fin}} + \frac{1}{2} \int dm_{\text{dyn,eff}}^2 f_{\text{fin}} \right. \\ & \left. - \frac{\lambda f_{\text{fin}}^2}{8} - \frac{C}{2} [H^2 + (M_0 + gv)^2] \left\{ f_{\text{fin}} - 0.7 \left(\frac{\lambda v^2}{2} + \frac{\lambda f_{\text{fin}}}{2} - C (2M_0 gv + g^2 v^2) \right) \right\} \right] \\ & + \text{non-local contributions}, \end{aligned} \quad (70)$$

where we have absorbed further some field independent constant terms into redefinition of the cosmological constant in the gravitational action. [Eq. \(70\)](#) is the final result of this paper. Note that owing to our estimation of the secular logarithm, the above effective action is valid only towards the end of the inflation. At some earlier stage, one may just ignore the $\ln a$ term.

Let us now specialise to the massless cases, $M_0 = 0 = m_0$, so that the local effective potential corresponding to [Eq. \(70\)](#) reads

$$\begin{aligned} V_{\text{eff,loc}} = & \frac{\lambda v^4}{4!} - \int dM_{\text{dyn,eff}} F_{\text{fin}} + \frac{1}{2} \int dm_{\text{dyn,eff}}^2 f_{\text{fin}} - \frac{\lambda f_{\text{fin}}^2}{8} + \frac{0.7C}{4} (\lambda - 2Cg^2) v^2 (H^2 + (M_0 + gv)^2) \\ & - \frac{C}{2} f_{\text{fin}} \left(1 - \frac{0.7\lambda}{2} \right) (H^2 + (M_0 + gv)^2), \end{aligned} \quad (71)$$

where C is given by [Eq. \(63\)](#), and we recall once again that by the virtue of the first of [Eq. \(53\)](#), it is absolutely finite as $\epsilon \rightarrow 0$. By setting $g = C = F_{\text{fin}} = 0$, we reproduce the result for the pure scalar field theory, [Eq. \(35\)](#).

Before we investigate the behaviour of [Eq. \(71\)](#), let us first take the high field limit of [Eq. \(71\)](#), $|v|/H \gg 1$. Using the asymptotic expansion for the digamma function,

$$\psi(1+x)|_{|x| \gg 1} \simeq \ln x + \mathcal{O}\left(\frac{1}{x}\right),$$

we have from [Eq. \(67\)](#), [Eq. \(59\)](#) (with $k = -1$) the leading expression

$$V_{\text{eff,loc}}|_{|v|/H \gg 1} \simeq \left[\frac{\lambda v^4}{4!} + \frac{1}{2\pi^2} \left(\frac{(\lambda - 8g^2)^2}{64} - g^4 \right) v^4 \ln \frac{|v|}{H} \right] \quad (72)$$

Thus if we want our potential to be bounded from below in order to avoid any runaway disaster for the system, we must have,

$$\lambda \gtrsim 16g^2 \quad (73)$$

Although this high field limit is similar to that of the standard one loop effective action, the coefficients of the $v^4 \ln v$ term contains contribution beyond one loop. Note from [Eq. \(73\)](#) that if we want λ to be less than unity, we must have $g \lesssim 0.3$.

This is consistent with our argument $M_{\text{dyn,eff}} \simeq M_0 + gv$, we made earlier for the fermion (cf, the discussion below Eq. (54)). Finally, we also note that $|v|/H \gg 1$ can also be interpreted as the flat spacetime ($H \rightarrow 0$) limit of Eq. (71). The constraint Eq. (73) ensures that Eq. (72) has the standard Coleman-Weinberg spontaneous symmetry breaking feature, as depicted in Fig. 7.

We have depicted the behaviour of the non-perturbative effective potential for massless fields, Eq. (71), in Fig. 8, with $\lambda > 16g^2$, Eq. (73). Let us compare them with the pure scalar field theory results, Fig. 2, Fig. 3. For a scalar with zero or positive rest mass squared, Fig. 3 are rather similar to the tree level potential. Fig. 8 on the other hand, shows behaviour qualitatively similar to that of Fig. 2, which corresponds to negative rest mass square for the scalar field in a pure ϕ^4 -theory. Fig. 9 shows the feature of the non-perturbative effective potential with $\lambda = 16g^2$. Note that for this equality, the $H \rightarrow 0$ limit of the effective potential is just the tree level quartic potential, which is very different from that of Fig. 9. The above discussion shows that the non-perturbative quantum effects can bring in novel features in spacetimes like the de Sitter. We also recall that for very high energy phenomenon occurring at very small length scales one usually ignores gravity and spacetime curvature. Then the comparison of our non-perturbative results with that of the $H \rightarrow 0$ limit also gives us one example of how quantum physics can be very different at short and large scales.

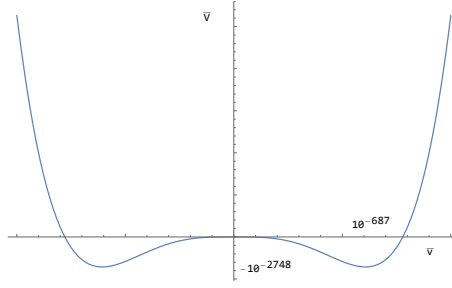


Figure 7: Spontaneous symmetry breaking feature of the flat spacetime limit ($H \rightarrow 0$) of the effective potential, Eq. (72), with respect to the background scalar field v . Bar over the quantities denotes that they are dimensionless. See main text for discussion. We have taken the coupling values $g = 0.06$ and $\lambda = 0.1$.

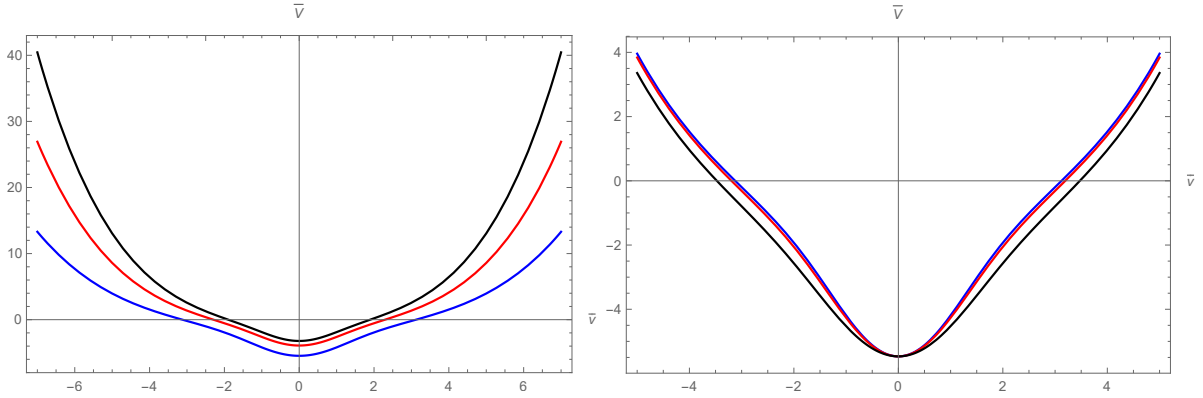


Figure 8: Variation of the effective potential Eq. (71), with respect to the background field \bar{v} , for $\lambda > 16g^2$, Eq. (73). Both left and right set of curves, correspond to massless cases. For the left figure, the blue, red and black curves respectively correspond to λ values 0.1, 0.2 and 0.3, with $g = 0.01$. For the right figure, the blue, red and black curves respectively correspond to g values 0.01, 0.03 and 0.06, with $\lambda = 0.1$.

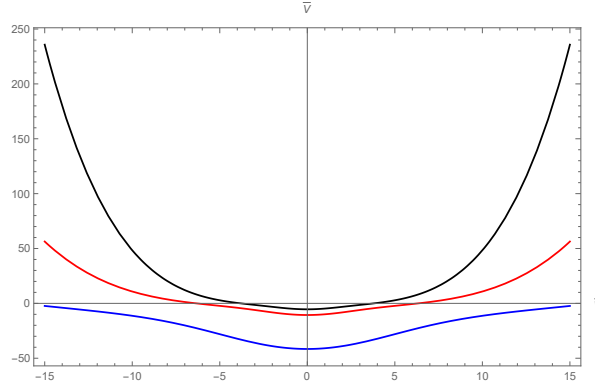


Figure 9: Variation of the effective potential Eq. (71), with respect to the background field \bar{v} , for $\lambda = 16g^2$, Eq. (73). The blue, red and black curves respectively correspond to g values 0.01, 0.04 and 0.08. Note the qualitative difference with that of the $\lambda = 16g^2$ limit of the flat spacetime limit of the effective potential, Eq. (72).

4 Conclusions

In this paper we have obtained the local part of the two loop effective action for the ϕ^4 -Yukawa theory via non-perturbative 2PI effective action formalism. In Section 3.1, we briefly sketch the derivation for the pure ϕ^4 theory. In Section 3.2, we discuss the inclusion of the Yukawa interaction. This paper is an extension of the previous works on the scalar field theory in the two loop Hartree approximation [76, 77, 78]. The Schwinger-Dyson equations satisfied by the scalar and fermion propagators have been constructed and various non-trivial non-perturbative counterterms has been found out explicitly, e.g. Eq. (53), Eq. (58). The dynamically generated masses of the scalar and the fermions have also been computed. We have pointed out non-trivial feature associated with cancellation of an overlapping divergence in the effective action (cf., discussion below Eq. (56), Eq. (64)), absent in the standard 1PI perturbative formalism. Eq. (69) is the renormalised result thus found. This contains a secular logarithm, $\ln a$, which is absent in the scalar sector. We also have shown briefly that in the perturbative technique, the finite part actually contains a quadratic power of $\ln a$, Appendix D, owing to the different renormalisation procedure. We next explicitly investigate the behaviour of the effective potential for zero rest mass cases. For the effective potential to be bounded from below, we find the constraint, $\lambda \gtrsim 16g^2$, Eq. (73). Fig. 8 and Fig. 9 shows the variation of the effective potential with respect to the background field. We note in particular the qualitative similarity of these plots with that of Fig. 2, which corresponds to *negative* rest mass squared of the scalar field. Putting things together, the present work perhaps shows the non-triviality of the non-perturbative effects in a spacetime like de Sitter. The discussion towards the end of the preceding section also shows how the results of the curved and flat spacetimes can be qualitatively very different.

We have also computed the 2PI two and three loop vacuum graphs for ϕ^4 theory towards the end of Section 3.1 and Appendix A, for a massless and minimally coupled quantum scalar with *vanishing* background field. The background field term ($\lambda v^2/2$) acts like an effective mass term in the propagator. In the absence of it the ‘purely massless’ scalar’s propagator behaves in a qualitatively different manner in de Sitter from that of a massive scalar, no matter how tiny its mass is [18]. We have computed in this particular case both the local and leading non-local parts of the vacuum loops using the Schwinger-Keldysh formalism. To the best of our knowledge, diagram topology like Fig. 4 where product of four propagators are present, has not been attempted before in de Sitter.

We believe the above results are interesting in their own right.

These results can be applied or extended in different directions. For example, what should be the scalar correlation functions with the effective potential of Eq. (71)? How do they differ quantitatively and qualitatively from that of the simple $\lambda\phi^4$ potential? Inclusion of gauge fields would also be interesting. Finally and most importantly, we have focussed only on the

local part of the effective potential or part of the propagator relevant for that. How do we compute the non-local parts? This involves solving the Schwinger-Dyson equations with non-local self energy contributions, and we need to use the Schwinger-Keldysh formalism. For scalar field theory some asymptotic analysis can be seen in [43, 51], for the self energy. All these seem to be challenging as well as important tasks and we hope to come back to at least some of them in the near future.

Acknowledgements

KR's research is supported by the fellowship from Council of Scientific and Industrial Research, Government of India (File No. 09/096(0987)/2019-EMR-I).

A Three loop 2PI vacuum diagram at $\mathcal{O}(\lambda^2)$

In this appendix we wish to compute the three loop 2PI vacuum diagram (Fig. 4). We shall ignore any rest mass or background field term, and hence treat the scalar to be *purely* massless and minimally coupled. We wish to compute both local and non-local contributions from Fig. 4, and hence we need to use the Schwinger-Keldysh, or in-in formalism. Let us first very briefly review this formalism, referring our reader to [17, 79, 80] for detail.

We recall that the standard time ordered functional integral representation of the standard in-out matrix elements for an observable of the field $A[\phi]$ read

$$\langle \phi | T A[\phi] | \psi \rangle = \int \mathcal{D}\phi \, e^{i \int_{t_i}^{t_f} \sqrt{-g} d^d x \mathcal{L}[\phi]} \Phi^*[\phi(t_f)] A[\phi] \Psi[\phi(t_i)] \quad (74)$$

where $|\phi\rangle$, $|\psi\rangle$ are field base-kets, $\Phi[\phi]$, $\Psi[\phi]$ are wave functionals. The above matrix elements are well defined only if the asymptotic states are stable. However in a dynamical background such as the de Sitter, the initial vacuum state is not stable owing to the particle pair creation issues. Also, in such backgrounds, the interaction cannot be turned on and off and it should be omnipresent. In such non-equilibrium or dynamical scenario, one instead needs to resort to the Schwinger-Keldysh formalism in order to compute any expectation value meaningfully.

In order to introduce the in-in formalism, we write from Eq. (74) for the anti-time ordering

$$\langle \psi | \bar{T} A[\phi] | \phi \rangle = (\langle \phi | \bar{T} A[\phi] | \psi \rangle)^* = \int \mathcal{D}\phi \, e^{-i \int_{t_i}^{t_f} \sqrt{-g} d^d x \mathcal{L}[\phi]} \Phi[\phi(t_i)] A[\phi] \Psi^*[\phi(t_f)] \quad (75)$$

We have the completeness relationship on the final hypersurface at $t = t_f$,

$$\int \mathcal{D}\phi \, \Phi[\phi_-(t_f)] \Phi[\phi_+(t_f)] = \delta(\phi_+(t_f) - \phi_-(t_f)) \quad (76)$$

From Eq. (74), Eq. (75) and using Eq. (76), we have the in-in matrix elements

$$\langle \psi | \bar{T}(A) T(A) | \psi \rangle = \int \mathcal{D}\phi_+ \mathcal{D}\phi_- \delta(\phi_+(t_f) - \phi_-(t_f)) e^{i \int_{t_i}^{t_f} \sqrt{-g} d^d x (\mathcal{L}[\phi_+] - \mathcal{L}[\phi_-])} \Psi^*[\phi_-(t_i)] A[\phi_-] A[\phi_+] \Psi[\phi_+(t_i)] \quad (77)$$

Note that the field ϕ_+ makes the forward time evolution, whereas ϕ_- makes backward time evolution. We will take $|\psi\rangle$ to be the initial Bunch-Davies vacuum for a massless and minimal scalar field in de Sitter, $\nabla^2 \phi = 0$, corresponding to the mode function

$$u_k(\eta) = \frac{H\eta}{\sqrt{2k}} \left(1 - \frac{i}{k\eta} \right) e^{-ik\eta} \quad (78)$$

where k is the modulus of the spatial momentum, \vec{k} .

The three loop contribution to the 2PI effective action then reads (the other three loop contribution is a connected three bubble, which is not 2PI),

$$i\Gamma_2^{3\text{-loop}}(\lambda^2) = \frac{i\lambda^2}{48} \int (aa')^d d^d x d^d x' [i\Delta_{++}^4(x, x') - i\Delta_{+-}^4(x, x')] \quad (79)$$

where $i\Delta_{++}(x, x')$ and $i\Delta_{+-}(x, x') = \langle \phi_+(x) \phi_-(x') \rangle$ are respectively the free Feynman propagator and the negative frequency Wightman function, [Section 2](#). Recall also that in our notation, $i\Delta^n \equiv (i\Delta)^n$.

Generically, from [Eq. \(7\)](#), [Eq. \(8\)](#), we have the different segments for the fourth power of the propagators,

$$i\Delta^4(x, x') = A^4 + 4A^3B + 6A^2B^2 + 4AB^3 + B^4 + 4A^3C \quad (80)$$

The rest of the terms containing $C(x, x')$ are finite and drop out as $\epsilon \rightarrow 0$. We have

$$\begin{aligned} A^4 &= \frac{\Gamma^4(1-\epsilon/2)}{2^8 \pi^{8-2\epsilon}} \frac{(aa')^{-4+2\epsilon}}{\Delta x^{8-4\epsilon}} \\ A^3B &= \frac{H^{2-\epsilon} \Gamma^2(1-\epsilon/2) \Gamma(2-\epsilon) (aa')^{-3+3\epsilon/2}}{2^{9-\epsilon} \pi^{8-2\epsilon}} \left[-\frac{2\Gamma(3-\epsilon/2) \Gamma(2-\epsilon/2) (aa' H^2/4)^{\epsilon/2}}{\epsilon \Gamma(3-\epsilon)} \frac{1}{\Delta x^{6-4\epsilon}} + \left(\frac{2}{\epsilon} + \ln(aa') \right) \frac{1}{\Delta x^{6-3\epsilon}} \right] \\ A^2B^2 &= \frac{H^{4-2\epsilon} \Gamma^2(3-\epsilon) (aa')^{-2+\epsilon}}{2^{12-2\epsilon} \pi^{8-2\epsilon} (1-\epsilon)} \left[\frac{2^{2-2\epsilon} H^{2\epsilon} \Gamma^2(3-\epsilon/2) \Gamma^2(2-\epsilon/2)}{\epsilon^2 \Gamma^2(3-\epsilon)} \frac{(aa')^\epsilon}{\Delta x^{4-4\epsilon}} + \frac{(\ln(aa') + 2/\epsilon)^2}{\Delta x^{4-2\epsilon}} \right. \\ &\quad \left. - \frac{2^{2-\epsilon} H^\epsilon \Gamma(3-\epsilon/2) \Gamma(2-\epsilon/2)}{\epsilon \Gamma(3-\epsilon)} \frac{(aa')^{\epsilon/2} (\ln(aa') + 2/\epsilon)}{\Delta x^{4-3\epsilon}} \right] \\ AB^3 &= -\frac{H^6}{2^{11} \pi^8 aa'} \frac{\ln^3 \sqrt{e} H^2 \Delta x^2/4}{\Delta x^2} \\ B^4 &= \frac{H^8}{2^{12} \pi^8} \ln^4 \frac{\sqrt{e} H^2 \Delta x^2}{4} \\ A^3C &= \frac{\Gamma^3(1-\epsilon/2)}{2^6 \pi^{6-3\epsilon/2}} \frac{(aa')^{-3+3\epsilon/2} C(x, x')}{\Delta x^{6-3\epsilon}} \end{aligned} \quad (81)$$

Following [\[18\]](#), we next note for $d = 4 - \epsilon$,

$$\begin{aligned} \frac{1}{\Delta x^{8-4\epsilon}} &= \frac{1}{2(3-2\epsilon)(4-3\epsilon)} \partial^2 \frac{1}{\Delta x^{6-4\epsilon}} = \frac{1}{2^2(3-2\epsilon)(4-3\epsilon)(2-2\epsilon)(2-3\epsilon)} \partial^4 \frac{1}{\Delta x^{4-4\epsilon}} \\ &= -\frac{1}{2^3 3(3-2\epsilon)(4-3\epsilon)(2-2\epsilon)(2-3\epsilon)(1-2\epsilon)\epsilon} \partial^6 \frac{1}{\Delta x^{2-4\epsilon}} \\ \frac{1}{\Delta x^{6-4\epsilon}} &= -\frac{1}{2^2 \cdot 3(2-2\epsilon)(2-3\epsilon)(1-2\epsilon)\epsilon} \partial^4 \frac{1}{\Delta x^{2-4\epsilon}} \\ \frac{1}{\Delta x^{6-3\epsilon}} &= -\frac{1}{2^3(2-2\epsilon)(2-3\epsilon/2)(1-3\epsilon/2)\epsilon} \partial^4 \frac{1}{\Delta x^{2-3\epsilon}} \\ \frac{1}{\Delta x^{4-4\epsilon}} &= -\frac{1}{2 \cdot 3(1-2\epsilon)\epsilon} \partial^2 \frac{1}{\Delta x^{2-4\epsilon}} \\ \frac{1}{\Delta x^{4-3\epsilon}} &= -\frac{1}{2^2(1-3\epsilon/2)\epsilon} \partial^2 \frac{1}{\Delta x^{2-3\epsilon}} \\ \frac{1}{\Delta x^{4-2\epsilon}} &= -\frac{1}{2(1-\epsilon)\epsilon} \partial^2 \frac{1}{\Delta x^{2-2\epsilon}} \end{aligned} \quad (82)$$

We also note from [Eq. \(10\)](#) that

$$\partial^2 \frac{1}{\Delta x_{++}^{2-\epsilon}} = \frac{4i\pi^{2-\epsilon/2}}{\Gamma(1-\epsilon/2)} \delta^d(x-x')$$

Thus from Eq. (82), we have

$$\begin{aligned}
\frac{1}{\Delta x_{++}^{8-4\epsilon}} &= -\frac{i\mu^{-3\epsilon}\pi^{2-\epsilon/2}}{6(3-2\epsilon)(4-3\epsilon)(2-2\epsilon)(2-3\epsilon)(1-2\epsilon)\epsilon\Gamma(1-\epsilon/2)}\partial^4\delta^d(x-x') - \frac{1}{3\times 2^8}\partial^6\frac{\ln\mu^2\Delta x_{++}^2}{\Delta x_{++}^2} \\
\frac{1}{\Delta x_{++}^{6-4\epsilon}} &= -\frac{i\mu^{-3\epsilon}\pi^{2-\epsilon/2}}{3(2-2\epsilon)(2-3\epsilon)(1-2\epsilon)\epsilon\Gamma(1-\epsilon/2)}\partial^2\delta^d(x-x') - \frac{1}{2^5}\partial^4\frac{\ln\mu^2\Delta x_{++}^2}{\Delta x_{++}^2} \\
\frac{1}{\Delta x_{++}^{6-3\epsilon}} &= -\frac{i\mu^{-2\epsilon}\pi^{2-\epsilon/2}}{2(2-2\epsilon)(2-3\epsilon/2)(1-3\epsilon/2)\epsilon\Gamma(1-\epsilon/2)}\partial^2\delta^d(x-x') - \frac{1}{2^5}\partial^4\frac{\ln\mu^2\Delta x_{++}^2}{\Delta x_{++}^2} \\
\frac{1}{\Delta x_{++}^{4-4\epsilon}} &= -\frac{2i\mu^{-3\epsilon}\pi^{2-\epsilon/2}}{3(1-2\epsilon)\epsilon\Gamma(1-\epsilon/2)}\delta^d(x-x') - \frac{1}{2^2}\partial^2\frac{\ln\mu^2\Delta x_{++}^2}{\Delta x_{++}^2} \\
\frac{1}{\Delta x_{++}^{4-3\epsilon}} &= -\frac{i\mu^{-2\epsilon}\pi^{2-\epsilon/2}}{(1-3\epsilon/2)\epsilon\Gamma(1-\epsilon/2)}\delta^d(x-x') - \frac{1}{2^2}\partial^2\frac{\ln\mu^2\Delta x_{++}^2}{\Delta x_{++}^2} \\
\frac{1}{\Delta x_{++}^{4-2\epsilon}} &= -\frac{2i\mu^{-\epsilon}\pi^{2-\epsilon/2}}{(1-\epsilon)\epsilon\Gamma(1-\epsilon/2)}\delta^d(x-x') - \frac{1}{2^2}\partial^2\frac{\ln\mu^2\Delta x_{++}^2}{\Delta x_{++}^2} \\
\frac{1}{\Delta x_{++}^{6-2\epsilon}} &= -\frac{i\mu^{-\epsilon}\pi^{2-\epsilon/2}}{(1-\epsilon)(2-\epsilon)^2\Gamma(1-\epsilon/2)\epsilon}\partial^2\delta^d(x-x') - \frac{1}{2^5}\partial^4\frac{\ln\mu^2\Delta x_{++}^2}{\Delta x_{++}^2}
\end{aligned} \tag{83}$$

where μ is an arbitrary scale having the dimension of mass. Note in the above expressions, that the terms associated with the δ -functions are local and carry divergences, whereas the logarithms are non-local. The δ -functions arise from the $(|\eta-\eta'|\mp i\epsilon)$ terms appearing in $\Delta_{++}^2(x, x')$ or $\Delta_{--}^2(x, x')$, Eq. (10). Thus terms containing $\Delta x_{\pm\mp}^2$ cannot yield any local contribution or divergences, but they contribute to the deep infrared.

We wish to first compute below the local contribution in Eq. (79), arising from the Feynman propagator $(++)$. This will require renormalisation, following which we will obtain finite local contribution. After this, we shall compute the aforementioned deep infrared, non-local contribution separately. Thus for $i\Delta_{++}^4(x, x')$ in Eq. (79), the contribution from the A_{++}^4 term, Eq. (81), reads after using Eq. (83),

$$\begin{aligned}
&\frac{i\lambda^2\Gamma^4(1-\epsilon/2)}{2^{12}\times 3\pi^{8-2\epsilon}}\int d^d x d^d x' (aa')^d \frac{(aa')^{-4+2\epsilon}}{\Delta x_{++}^{8-4\epsilon}} \\
&= \frac{\mu^{-3\epsilon}\lambda^2\Gamma^3(1-\epsilon/2)}{2^{13}\times 9\pi^{6-3\epsilon/2}(3-2\epsilon)(4-3\epsilon)(2-2\epsilon)(2-3\epsilon)(1-2\epsilon)\epsilon}\int d^d x d^d x' (aa')^\epsilon \partial^4\delta^d(x-x') = \frac{\lambda^2 H^4}{2^{15}\times 9\pi^6}\int d^4 x a^4
\end{aligned} \tag{84}$$

Note that the above term is free of any ultraviolet divergence.

We next consider the $4A_{++}^3 B_{++}$ term in Eq. (79), Eq. (81). Using the second and the third of Eq. (83), we have for the local

contribution

$$\begin{aligned}
& \frac{i\lambda^2 H^{2-\epsilon} \Gamma^2(1-\epsilon/2) \Gamma(2-\epsilon)}{2^{11-\epsilon} \cdot 3\pi^{8-2\epsilon}} \int d^d x d^d x' (aa')^{d-3+3\epsilon/2} \left[-\frac{2\Gamma(3-\epsilon/2) \Gamma(2-\epsilon/2) (aa' H^2/4)^{\epsilon/2}}{\epsilon \Gamma(3-\epsilon)} \frac{1}{\Delta x_{++}^{6-4\epsilon}} \right. \\
& \left. + \left(\frac{2}{\epsilon} + \ln(aa') \right) \frac{1}{\Delta x_{++}^{6-3\epsilon}} \right] \\
& = \frac{i\lambda^2 H^{2-\epsilon} \Gamma^2(1-\epsilon/2) \Gamma(2-\epsilon)}{2^{11-\epsilon} \cdot 3\pi^{8-2\epsilon}} \int d^d x d^d x' (aa')^{d-3+3\epsilon/2} \left[\frac{i\mu^{-3\epsilon} H^\epsilon 2^{-\epsilon} \pi^{2-\epsilon/2} \Gamma(3-\epsilon/2) (aa')^{\epsilon/2}}{6\Gamma(2-\epsilon)(1-\epsilon)(2-3\epsilon)(1-2\epsilon)\epsilon^2} \partial^2 \delta^d(x-x') \right. \\
& \left. - \left(\frac{2}{\epsilon} + \ln(aa') \right) \frac{i\mu^{-2\epsilon} \pi^{2-\epsilon/2}}{2(2-2\epsilon)(2-3\epsilon/2)(1-3\epsilon/2)\epsilon \Gamma(1-\epsilon/2)} \partial^2 \delta^d(x-x') \right] \\
& = -\frac{\lambda^2 \mu^{-3\epsilon} H^2 \Gamma^2(1-\epsilon/2) \Gamma(3-\epsilon/2)}{2^{12} \times 9\pi^{6-3\epsilon/2} (1-\epsilon)(2-3\epsilon)(1-2\epsilon)\epsilon^2} \int d^d x d^d x' (aa')^{1+\epsilon} \partial^2 \delta^d(x-x') \\
& + \frac{\lambda^2 \mu^{-2\epsilon} H^{2-\epsilon} \Gamma(1-\epsilon/2) \Gamma(1-\epsilon)}{2^{13-\epsilon} \cdot 3\pi^{6-3\epsilon/2} (2-3\epsilon/2)(1-3\epsilon/2)\epsilon} \int d^d x d^d x' (aa')^{1+\epsilon/2} \left(\frac{2}{\epsilon} + \ln(aa') \right) \partial^2 \delta^d(x-x') \\
& = \frac{\lambda^2 \mu^{-2\epsilon} H^{4-\epsilon} (1+\epsilon)(3+2\epsilon) \Gamma^2(1-\epsilon/2) \Gamma(3-\epsilon/2)}{2^{11} \cdot 9\pi^{6-3\epsilon/2} (1-\epsilon)(2-3\epsilon)(1-2\epsilon)\epsilon^2} \left(\frac{\mu}{H} \right)^{-\epsilon} \int d^d x a^d \left(1 + 3\epsilon \ln a + \frac{9\epsilon^2}{2} \ln^2 a + \mathcal{O}(\epsilon^3) \right) \\
& - \frac{\lambda^2 \mu^{-2\epsilon} H^{4-\epsilon} (2+\epsilon)(3+\epsilon) \Gamma(1-\epsilon/2) \Gamma(1-\epsilon)}{2^{12-\epsilon} \cdot 3\pi^{6-3\epsilon/2} (2-3\epsilon/2)(1-3\epsilon/2)\epsilon^2} \int d^d x a^d (1 + 2\epsilon \ln a + 2\epsilon^2 \ln^2 a + \mathcal{O}(\epsilon^3)) \\
& - \frac{\lambda^2 \mu^{-2\epsilon} H^{4-\epsilon} \Gamma(1-\epsilon/2) \Gamma(1-\epsilon)}{2^{12-\epsilon} \cdot 3\pi^{6-3\epsilon/2} (2-3\epsilon/2)(1-3\epsilon/2)\epsilon} \int d^d x a^d [(2+\epsilon)(3+\epsilon) \ln a (1 + 2\epsilon \ln a) + (5+2\epsilon)(1 + 2\epsilon \ln a) + \mathcal{O}(\epsilon^2)]
\end{aligned} \tag{85}$$

Let us now consider the $6A^2B^2$ term in Eq. (81). Using the last three equations of Eq. (83), we have for the local contribution

$$\begin{aligned}
& \frac{i\lambda^2 H^{4-2\epsilon} \Gamma^2(3-\epsilon)}{2^{15-2\epsilon} \pi^{8-2\epsilon} (1-\epsilon)} \int d^d x d^d x' (aa')^{d-2+\epsilon} \left[\frac{2^{2-2\epsilon} H^{2\epsilon} \Gamma^2(3-\epsilon/2) \Gamma^2(2-\epsilon/2)}{\epsilon^2 \Gamma^2(3-\epsilon)} \frac{(aa')^\epsilon}{\Delta x_{++}^{4-4\epsilon}} + \frac{(\ln(aa') + 2/\epsilon)^2}{\Delta x_{++}^{4-2\epsilon}} \right. \\
& \left. - \frac{2^{2-\epsilon} H^\epsilon \Gamma(3-\epsilon/2) \Gamma(2-\epsilon/2)}{\epsilon \Gamma(3-\epsilon)} \frac{(aa')^{\epsilon/2} (\ln(aa') + 2/\epsilon)}{\Delta x_{++}^{4-3\epsilon}} \right] \\
& = \frac{\lambda^2 \mu^{-2\epsilon} H^{4-\epsilon} \Gamma^2(3-\epsilon/2) \Gamma^2(2-\epsilon/2)}{2^{12} \cdot 3\pi^{6-3\epsilon/2} (1-\epsilon)(1-2\epsilon) \Gamma(1-\epsilon/2) \epsilon^3} \int d^d x a^d \left(1 + 3\epsilon \ln a + \frac{9\epsilon^2}{2} \ln^2 a + \frac{9\epsilon^3}{2} \ln^3 a + \mathcal{O}(\epsilon^4) \right) \\
& + \frac{\lambda^2 \mu^{-2\epsilon} H^{4-2\epsilon} \Gamma^2(3-\epsilon)}{2^{12-2\epsilon} \pi^{6-3\epsilon/2} (1-\epsilon)^2 \Gamma(1-\epsilon/2) \epsilon^3} \int d^d x a^d \left(1 + 3\epsilon \ln a + \frac{7\epsilon^2}{2} \ln^2 a + \frac{13\epsilon^3}{6} \ln^3 a + \mathcal{O}(\epsilon^4) \right) \\
& - \frac{\lambda^2 \mu^{-2\epsilon} H^{4-\epsilon} \Gamma(3-\epsilon) \Gamma(3-\epsilon/2) \Gamma(2-\epsilon/2)}{2^{12-\epsilon} \pi^{6-3\epsilon/2} (1-\epsilon)(1-3\epsilon/2) \Gamma(1-\epsilon/2) \epsilon^3} \int d^d x a^d \left(1 + 3\epsilon \ln a + 4\epsilon^2 \ln^2 a + \frac{10\epsilon^3}{3} \ln^3 a + \mathcal{O}(\epsilon^4) \right)
\end{aligned} \tag{86}$$

It can be shown that the AB^3 and B^4 terms do not contribute to any local terms in the effective action. The only remaining term relevant for this purpose will be the $4A^3C$ term in Eq. (81),

$$\begin{aligned}
& \frac{i\lambda^2 \Gamma^3(1-\epsilon/2)}{2^8 \cdot 3\pi^{6-3\epsilon/2}} \int d^d x d^d x' (aa')^{d-3+3\epsilon/2} \frac{C_{++}(x, x')}{\Delta x_{++}^{6-3\epsilon}} \\
& = \frac{\lambda^2 \mu^{-2\epsilon} \Gamma^3(1-\epsilon/2)}{2^{10} \cdot 3\pi^{4-\epsilon} (1-\epsilon)(2-3\epsilon/2)(1-3\epsilon/2) \Gamma(3-\epsilon/2) \epsilon} \int d^d x d^d x' (aa')^{d-3+3\epsilon/2} C_{++}(x, x') \partial^2 \delta^d(x-x')
\end{aligned} \tag{87}$$

We integrate the above equation by parts. From Eq. (8) we note that in $3-\epsilon$ spatial dimensions,

$$\partial^2 \Delta x_{++}^2 = \vec{\partial}^2 |\vec{x} - \vec{x}'|^2 + \partial_0^2 |\eta - \eta'|^2 = 2(3-\epsilon) + 4(\eta - \eta') \delta(\eta - \eta') + 2(\text{sgn}(\eta - \eta'))^2 \tag{88}$$

where the sgn stands for the sign of $(\eta - \eta')$ and it is zero for $\eta = \eta'$. Putting things together now, [Eq. \(87\)](#) becomes

$$\frac{\lambda^2 \mu^{-2\epsilon} H^{4-\epsilon} \Gamma^2(1-\epsilon/2)(1-\epsilon/3)}{2^{15-\epsilon} \pi^{6-3\epsilon/2} (1-\epsilon)(2-3\epsilon/2)(1-3\epsilon/2)\Gamma(3-\epsilon/2)\epsilon} \left(\frac{\Gamma(4-\epsilon)}{\Gamma(3-\epsilon/2)} - \frac{\Gamma(4-\epsilon/2)}{2^\epsilon(1+\epsilon/2)\Gamma(3)} \right) \int d^d x a^d a^{2\epsilon} \quad (89)$$

It is easy to check by expanding the Gamma functions within parenthesis that the above term is not divergent. Thus we may set $d = 4$, $a^{2\epsilon} = 1$ above and hence the whole term can be absorbed in a finite cosmological constant counterterm, $\delta\Lambda/8\pi G$. Combining now [Eq. \(84\)](#), [Eq. \(85\)](#), [Eq. \(86\)](#) and [Eq. \(89\)](#), we have the three loop, local contribution

$$\begin{aligned} \Gamma_{2, \text{loc}}^{3-\text{loop}}(\lambda^2) &= \frac{\lambda^2 H^4}{2^{15} \pi^6} \left[\frac{1}{9} + \frac{1}{4\epsilon} \left(\frac{\Gamma(4-\epsilon)}{\Gamma(3-\epsilon/2)} - \frac{\Gamma(4-\epsilon/2)}{2^\epsilon(1+\epsilon/2)\Gamma(3)} \right) \right] \int d^4 x a^4 \\ &+ \frac{\lambda^2 \mu^{-2\epsilon} H^{4-\epsilon}}{2^{11} \pi^{6-3\epsilon/2}} \left[\frac{(1+\epsilon)(3+2\epsilon)\Gamma^2(1-\epsilon/2)\Gamma(3-\epsilon/2)}{9(1-\epsilon)(2-3\epsilon)(1-2\epsilon)\epsilon^2} \left(\frac{\mu}{H} \right)^{-\epsilon} - \frac{(2+\epsilon)(3+\epsilon)\Gamma(1-\epsilon/2)\Gamma(1-\epsilon)}{2^{1-\epsilon} 3(2-3\epsilon/2)(1-3\epsilon/2)\epsilon^2} \right. \\ &- \frac{\Gamma(1-\epsilon/2)\Gamma(1-\epsilon)}{2^{1-\epsilon} 3(2-3\epsilon/2)(1-3\epsilon/2)\epsilon} + \frac{\Gamma^2(3-\epsilon/2)\Gamma^2(2-\epsilon/2)}{6(1-\epsilon)(1-2\epsilon)\Gamma(1-\epsilon/2)\epsilon^3} + \frac{\Gamma^2(3-\epsilon)}{2^{1-2\epsilon}(1-\epsilon)^2\Gamma(1-\epsilon/2)\epsilon^3} \\ &- \left. \frac{\Gamma(3-\epsilon)\Gamma(3-\epsilon/2)\Gamma(2-\epsilon/2)}{2^{1-\epsilon}(1-\epsilon)(1-3\epsilon/2)\Gamma(1-\epsilon/2)\epsilon^3} \right] \int d^d x a^d \\ &+ \frac{\lambda^2 \mu^{-2\epsilon} H^{4-\epsilon}}{2^{11} \pi^{6-3\epsilon/2}} \left[\frac{(1+\epsilon)(3+2\epsilon)\Gamma^2(1-\epsilon/2)\Gamma(3-\epsilon/2)}{3(1-\epsilon)(2-3\epsilon)(1-2\epsilon)} \left(\frac{\mu}{H} \right)^{-\epsilon} - \frac{(2+\epsilon)(3+\epsilon)\Gamma(1-\epsilon/2)\Gamma(1-\epsilon)}{2^{-\epsilon} 3(2-3\epsilon/2)(1-3\epsilon/2)} \right. \\ &- \frac{\Gamma(1-\epsilon/2)\Gamma(1-\epsilon)}{2^{-\epsilon}(2-3\epsilon/2)(1-3\epsilon/2)} + \frac{\Gamma^2(3-\epsilon/2)\Gamma^2(2-\epsilon/2)}{2(1-\epsilon)(1-2\epsilon)\Gamma(1-\epsilon/2)\epsilon} + \frac{3\Gamma^2(3-\epsilon)}{2^{1-2\epsilon}(1-\epsilon)^2\Gamma(1-\epsilon/2)\epsilon} \\ &- \left. \frac{3\Gamma(3-\epsilon)\Gamma(3-\epsilon/2)\Gamma(2-\epsilon/2)}{2^{1-\epsilon}(1-\epsilon)(1-3\epsilon/2)\Gamma(1-\epsilon/2)\epsilon} \right] \int d^d x a^d \ln a + \frac{\lambda^2 \mu^{-2\epsilon} H^{4-\epsilon}}{2^{12} \pi^{6-3\epsilon/2}} \left[\frac{3\Gamma^2(3-\epsilon/2)\Gamma^2(2-\epsilon/2)}{2(1-\epsilon)(1-2\epsilon)\Gamma(1-\epsilon/2)} \right. \\ &+ \left. \frac{7\Gamma^2(3-\epsilon)}{2^{1-2\epsilon}(1-\epsilon)^2\Gamma(1-\epsilon/2)} - \frac{4\Gamma(3-\epsilon)\Gamma(3-\epsilon/2)\Gamma(2-\epsilon/2)}{2^{-\epsilon}(1-\epsilon)(1-3\epsilon/2)\Gamma(1-\epsilon/2)} \right] \int d^d x a^d \ln^2 a + \frac{\lambda^2 H^4}{2^{10} \cdot 3\pi^6} \int d^4 x a^4 (\ln^3 a + \mathcal{O}(\ln^2 a)) \end{aligned} \quad (90)$$

The divergence associated with the $\ln^2 a$ term can be absorbed by adding with the above the contribution from a quartic vertex counterterm

$$-\frac{\delta\lambda}{4!} \int a^d d^d x \langle \phi^4 \rangle = -\frac{\delta\lambda}{2^3} \int a^d d^d x i\Delta^2(x, x) = -\frac{\delta\lambda H^{4-2\epsilon} \Gamma^2(2-\epsilon)}{2^{7-2\epsilon} \pi^{4-\epsilon} \Gamma^2(1-\epsilon/2)} \int a^d d^d x \left(\frac{1}{\epsilon^2} + \frac{2\ln a}{\epsilon} + \ln^2 a \right)$$

where we have used [Eq. \(11\)](#). This leads to the choice,

$$\delta\lambda = \frac{\mu^{-2\epsilon} \lambda^2 H^\epsilon \Gamma^2(1-\epsilon/2)}{2^{5+2\epsilon} \pi^{2-\epsilon/2} \Gamma^2(2-\epsilon)\epsilon} \left[\frac{3\Gamma^2(3-\epsilon/2)\Gamma^2(2-\epsilon/2)}{2(1-\epsilon)(1-2\epsilon)\Gamma(1-\epsilon/2)} + \frac{7\Gamma^2(3-\epsilon)}{2^{1-2\epsilon}(1-\epsilon)^2\Gamma(1-\epsilon/2)} - \frac{4\Gamma(3-\epsilon)\Gamma(3-\epsilon/2)\Gamma(2-\epsilon/2)}{2^{-\epsilon}(1-\epsilon)(1-3\epsilon/2)\Gamma(1-\epsilon/2)} \right] \quad (91)$$

The divergence with the $\ln a$ term can be cancelled via a mass renormalisation counterterm,

$$-\frac{\delta m^2}{2} \int a^d d^d x \langle \phi^2 \rangle = -\frac{\delta m^2 H^{2-\epsilon} \Gamma(2-\epsilon)}{2^{3-\epsilon} \pi^{2-\epsilon/2} \Gamma(1-\epsilon/2)} \int a^d d^d x \left(\ln a + \frac{1}{\epsilon} \right) \quad (92)$$

with the choice

$$\begin{aligned} \delta m^2 &= \frac{\delta\lambda H^{2-\epsilon} \Gamma(2-\epsilon)}{2^{3-\epsilon} \pi^{2-\epsilon/2} \Gamma(1-\epsilon/2)\epsilon} - \frac{\mu^{-2\epsilon} \lambda^2 H^2 \Gamma(1-\epsilon/2)}{2^{8+\epsilon} \pi^{4-\epsilon} \Gamma(2-\epsilon)\epsilon} \left[\frac{(1+\epsilon)(3+2\epsilon)\Gamma^2(1-\epsilon/2)\Gamma(3-\epsilon/2)}{3(1-\epsilon)(2-3\epsilon)(1-2\epsilon)} \left(\frac{\mu}{H} \right)^{-\epsilon} \right. \\ &- \frac{(2+\epsilon)(3+\epsilon)\Gamma(1-\epsilon/2)\Gamma(1-\epsilon)}{2^{-\epsilon} \cdot 3(2-3\epsilon/2)(1-3\epsilon/2)} - \frac{\Gamma(1-\epsilon/2)\Gamma(1-\epsilon)}{2^{-\epsilon}(2-3\epsilon/2)(1-3\epsilon/2)} + \frac{\Gamma^2(3-\epsilon/2)\Gamma^2(2-\epsilon/2)}{2(1-\epsilon)(1-2\epsilon)\Gamma(1-\epsilon/2)\epsilon} \\ &+ \left. \frac{3\Gamma^2(3-\epsilon)}{2^{1-2\epsilon}(1-\epsilon)^2\Gamma(1-\epsilon/2)\epsilon} - \frac{3\Gamma(3-\epsilon)\Gamma(3-\epsilon/2)\Gamma(2-\epsilon/2)}{2^{1-\epsilon}(1-\epsilon)(1-3\epsilon/2)\Gamma(1-\epsilon/2)\epsilon} \right] \end{aligned} \quad (93)$$

The remaining constant terms can be absorbed in the renormalisation of the cosmological constant. Thus at three loop we have the renormalised local expression for the vacuum graph, Fig. 4

$$i\Gamma_2^{3\text{-loop}}|_{\text{loc.,ren.}} = \frac{\lambda^2 H^4}{2^{10} \times 3\pi^6} \int a^4 d^4 x [\ln^3 a + \mathcal{O}(\ln^2 a)] \quad (94)$$

Let us now come to the non-local part of $\Gamma_2^{3\text{-loop}}$. This will yield the leading deep infrared contribution of $\mathcal{O}(\ln^4 a)$. We shall use the in-in formalism to do this. Now, instead of using the exact expressions of Eq. (83), we wish to use the IR effective formalism described in e.g. [50]. We have from Eq. (79) appropriate for the in-in and non-local contribution

$$i\Gamma_2^{3\text{-loop}}(\lambda^2)_{\text{non-loc}} = \frac{i\lambda^2}{48} \int (aa')^d d^d x d^d x' [i\Delta_{++}^4(x, x') - i\Delta_{+-}^4(x, x')] = \frac{i\lambda^2}{48} \int (aa')^4 d^4 x d^4 x' [i\Delta_{-+}^4(x, x') - i\Delta_{+-}^4(x, x')] \quad (95)$$

where we have used for the Feynman propagator

$$i\Delta_{++}(x, x') = \theta(\eta - \eta') i\Delta_{-+}(x, x') + \theta(\eta' - \eta) i\Delta_{+-}(x, x')$$

We now rewrite Eq. (95) in the spatial momentum space. In the deep IR, super-Hubble limit, we have $H \lesssim k \lesssim Ha$, where $k = |\vec{k}|$.

$$\begin{aligned} i\Gamma_2^{3\text{-loop}}|_{\text{leading,IR}} &= \frac{i\lambda^2}{48} \int a^4 d^4 x \int a'^4 d\eta' \frac{d^3 \vec{k}_1 d^3 \vec{k}_2 d^3 \vec{k}_3}{(2\pi)^9} (i\Delta_{-+}(k_1, \eta, \eta') - \text{c.c.}) (i\Delta_{-+}(k_2, \eta, \eta') + \text{c.c.}) \\ &\times \left(i\Delta_{-+}(k_3, \eta, \eta') i\Delta_{-+}(|\vec{k}_1 + \vec{k}_2 + \vec{k}_3|, \eta, \eta') + \text{c.c.} \right) \Theta(Ha' - k_1) \Theta(Ha' - k_2) \Theta(Ha' - k_3) \Theta(Ha' - |\vec{k}_1 + \vec{k}_2 + \vec{k}_3|) \\ &\simeq \frac{\lambda^2 H^4}{2^{11} \times 9\pi^6} \int a^4 d^4 x \ln^4 a \end{aligned} \quad (96)$$

where we have used $\eta \gtrsim \eta'$, or η as the final time, and for the Wightman function,

$$i\Delta_{+-}(k, \eta, \eta') = \frac{H^2 \Theta(Ha' - k) \Theta(Ha - k)}{2k^3} \left(1 - \frac{ik^3}{3H^3 a'^3} \right) = (i\Delta_{-+}(k, \eta, \eta'))^* \quad (97)$$

Eq. (96), combined with Eq. (94) gives the renormalised, leading late time expression for $i\Gamma_2^{3\text{-loop}}$ at $\mathcal{O}(\lambda^2)$, yielding Eq. (38).

B Propagators and their coincidence limits

The Green function for a scalar field is given by [10]

$$iG(x, x') = \frac{H^{2-\epsilon}}{2^{4-\epsilon} \pi^{2-\epsilon/2}} \frac{\Gamma(\frac{3}{2} + \nu - \frac{\epsilon}{2}) \Gamma(\frac{3}{2} - \nu - \frac{\epsilon}{2})}{\Gamma(2 - \frac{\epsilon}{2})} {}_2F_1\left(\frac{3}{2} + \nu - \frac{\epsilon}{2}, \frac{3}{2} - \nu - \frac{\epsilon}{2}, 2 - \frac{\epsilon}{2}, 1 - \frac{y}{4}\right) \quad (98)$$

where $\nu = ((d-1)^2/4 - m^2/H^2)^{1/2}$ and the parameter $\epsilon = 0^+$ has been kept for regularisation purpose. Using the transformation formula for the hypergeometric function [81],

$${}_2F_1(\alpha, \beta, \gamma, z) = \frac{\Gamma(\gamma)\Gamma(\gamma - \alpha - \beta)}{\Gamma(\gamma - \alpha)\Gamma(\gamma - \beta)} {}_2F_1(\alpha, \beta, \alpha + \beta - \gamma + 1, 1 - z) + (1 - z)^{\gamma - \alpha - \beta} \frac{\Gamma(\gamma)\Gamma(\alpha + \beta - \gamma)}{\Gamma(\alpha)\Gamma(\beta)} {}_2F_1(\gamma - \alpha, \gamma - \beta, \gamma - \alpha - \beta + 1, 1 - z), \quad (99)$$

we have

$$\begin{aligned} {}_2F_1\left(\frac{3}{2} + \nu - \frac{\epsilon}{2}, \frac{3}{2} - \nu - \frac{\epsilon}{2}, 2 - \frac{\epsilon}{2}, 1 - \frac{y}{4}\right) &= \frac{\Gamma(2 - \epsilon/2)\Gamma(-1 + \epsilon/2)}{\Gamma(1/2 - \nu)\Gamma(1/2 + \nu)} {}_2F_1\left(\frac{3}{2} + \nu - \frac{\epsilon}{2}, \frac{3}{2} - \nu - \frac{\epsilon}{2}, 2 - \frac{\epsilon}{2}, \frac{y}{4}\right) \\ &+ \left(\frac{y}{4}\right)^{-1+\epsilon/2} \frac{\Gamma(1 - \epsilon/2)\Gamma(2 - \epsilon/2)}{\Gamma(3/2 + \nu - \epsilon/2)\Gamma(3/2 - \nu - \epsilon/2)} {}_2F_1\left(\frac{1}{2} - \nu, \frac{1}{2} + \nu, \frac{\epsilon}{2}, \frac{y}{4}\right) \end{aligned} \quad (100)$$

For our purpose of computing local parts of the vacuum diagrams, we wish to make an expansion of the Green function for small separation, y . We have

$$\begin{aligned} iG(y \rightarrow 0) &= \frac{\Gamma(1 - \epsilon/2)}{2^2 \pi^{2-\epsilon/2}} \frac{(aa')^{-1+\epsilon/2}}{\Delta x^{2-\epsilon}} + \frac{H^{2-\epsilon}}{2^{4-\epsilon} \pi^{2-\epsilon/2}} \frac{\Gamma(3/2 + \nu - \epsilon/2) \Gamma(3/2 - \nu - \epsilon/2)}{\Gamma(1/2 - \nu) \Gamma(1/2 + \nu)} \Gamma(-1 + \epsilon/2) + \mathcal{O}(y) \\ &= \frac{\Gamma(1 - \epsilon/2)}{2^2 \pi^{2-\epsilon/2}} \frac{(aa')^{-1+\epsilon/2}}{\Delta x^{2-\epsilon}} + \frac{H^{2-\epsilon}}{2^{3-\epsilon} \pi^{2-\epsilon/2}} \left[\frac{2}{\epsilon} - \gamma - \frac{\bar{m}^2}{\epsilon} + \left(\frac{1}{2} \bar{m}^2 - 1 \right) \left[\psi \left(\frac{1}{2} + \nu \right) + \psi \left(\frac{1}{2} - \nu \right) \right] \right] + \mathcal{O}(\epsilon) \end{aligned} \quad (101)$$

where ψ stands for the digamma function, and the bar denotes scaling with respect to H^2 . Thus in the coincidence limit, we have (under the dimensional regularisation scheme),

$$iG(x, x) = \frac{H^{2-\epsilon}}{2^{3-\epsilon} \pi^{2-\epsilon/2}} \left[\frac{2}{\epsilon} - \gamma - \frac{\bar{m}^2}{\epsilon} + \left(\frac{1}{2} \bar{m}^2 - 1 \right) \left[\psi \left(\nu + \frac{1}{2} \right) + \psi \left(\frac{1}{2} - \nu \right) \right] \right] + \mathcal{O}(\epsilon) \quad (102)$$

We now define a parameter

$$s = \frac{3}{2} - \nu = \frac{3}{2} - \left(\frac{9}{4} - \bar{m}^2 \right)^{1/2} \quad (103)$$

Thus we have

$$\begin{aligned} iG(x, x) &= \frac{H^{2-\epsilon}}{2^{3-\epsilon} \pi^{2-\epsilon/2}} \left[\frac{2}{\epsilon} - \gamma - \frac{\bar{m}^2}{\epsilon} + \left(\frac{1}{2} \bar{m}^2 - 1 \right) [\psi(2+s) + \psi(-1+s)] \right] + \mathcal{O}(\epsilon) \\ &= \frac{H^{2-\epsilon}}{2^{3-\epsilon} \pi^{2-\epsilon/2}} \left[\frac{2}{\epsilon} - \gamma - \frac{\bar{m}^2}{\epsilon} + \left(\frac{1}{2} \bar{m}^2 - 1 \right) \left(\psi(1+s) + \psi(1-s) + \frac{2}{1-s} - \frac{1}{s} \right) \right] + \mathcal{O}(\epsilon), \end{aligned} \quad (104)$$

where we have used

$$\psi(1+x) = \psi(x) + \frac{1}{x}$$

Eq. (104) can be renormalised by multiplying it with $m^2/2$ and then using the cosmological constant counterterm, giving

$$iG(x, x)_{\text{Ren.}} = \langle \phi^2 \rangle_{\text{Ren.}} = \frac{H^2}{2^3 \pi^2} \left(1 - \frac{1}{2} \bar{m}^2 \right) \left[\frac{1}{s} - \frac{2}{1-s} - (\psi(1+s) + \psi(1-s)) \right] \quad (105)$$

In particular, when the field is light i.e. \bar{m} is small, we may expand the digamma function as

$$\psi(1+s) = -\gamma + \sum_{n=2}^{\infty} (-1)^n \zeta(n) s^{n-1}$$

to have

$$iG(x, x) = \langle \phi^2 \rangle = \frac{H^{2-\epsilon}}{2^{3-\epsilon} \pi^{2-\epsilon/2}} \left[\frac{2}{\epsilon} - \gamma - \frac{\bar{m}^2}{\epsilon} + \left(\frac{1}{2} \bar{m}^2 - 1 \right) \left(-2\gamma - 2 \sum_{n=1}^{\infty} \zeta(2n+1) s^{2n} + \frac{2}{1-s} - \frac{1}{s} \right) \right] + \mathcal{O}(\epsilon) \quad (106)$$

Likewise, the propagator for a fermion field with mass M for small separation reads, e.g. [21],

$$iS(x, x') = \frac{\Gamma(1 - \epsilon/2)}{2^2 \pi^{2-\epsilon/2} (aa')^{3/2-\epsilon/2}} [i\cancel{\partial} + aM] \frac{1}{\Delta x^{2-\epsilon}} + \frac{MH^{2-\epsilon}}{2^{2-\epsilon/2} \pi^{2-\epsilon/2}} \frac{\Gamma(\frac{d}{2} + i\bar{M}) \Gamma(\frac{d}{2} - i\bar{M})}{\Gamma(1 + i\bar{M}) \Gamma(1 - i\bar{M})} \left(-\frac{2}{\epsilon} + \gamma - 1 \right) \times \mathbf{I}_{d \times d} + \mathcal{O}(y) \quad (107)$$

Hence we have in the dimensional regularisation scheme

$$\begin{aligned}
\langle \bar{\psi}\psi \rangle &= -\text{Tr}iS(x, x) = -\frac{MH^{2-\epsilon}}{2^{-\epsilon/2}\pi^{2-\epsilon/2}} \frac{\Gamma(\frac{d}{2} + i\bar{M}) \Gamma(\frac{d}{2} - i\bar{M})}{\Gamma(1 + i\bar{M})\Gamma(1 - i\bar{M})} \left(-\frac{2}{\epsilon} + \gamma - \frac{1}{2} \right) \\
&= -\frac{\bar{M}H^{3-\epsilon}}{2^{-\epsilon/2}\pi^{2-\epsilon/2}} \left(-\frac{2}{\epsilon} + \gamma - \frac{1}{2} \right) \left[1 - \epsilon + \bar{M}^2 - (1 + \bar{M}^2) (\psi(1 + i\bar{M}) + \psi(1 - i\bar{M})) \frac{\epsilon}{2} \right] \\
&= -\frac{\bar{M}H^{3-\epsilon}}{2^{-\epsilon/2}\pi^{2-\epsilon/2}} \left[\left(-\frac{2}{\epsilon} + \frac{3}{2} + \gamma \right) + \left(-\frac{2}{\epsilon} - \frac{1}{2} + \gamma \right) \bar{M}^2 + (1 + \bar{M}^2) (\psi(1 + i\bar{M}) + \psi(1 - i\bar{M})) \right] \quad (108)
\end{aligned}$$

In particular, when \bar{M} is small, we have

$$\langle \bar{\psi}\psi \rangle = -\frac{\bar{M}H^{3-\epsilon}}{2^{-\epsilon/2}\pi^{2-\epsilon/2}} \left[\left(-\frac{2}{\epsilon} + \frac{3}{2} - \gamma \right) + \left(-\frac{2}{\epsilon} - \frac{1}{2} - \gamma \right) \bar{M}^2 - 2(1 + \bar{M}^2) \sum_{n=1}^{\infty} \zeta(2n+1)(i\bar{M})^{2n} \right] \quad (109)$$

If we multiply [Eq. \(108\)](#) by the fermion mass M , the corresponding quantity represents the trace of the free fermion's energy-momentum tensor. Accordingly, it can be renormalised via a cosmological constant counterterm, giving

$$\langle \bar{\psi}\psi \rangle_{\text{Ren.}} = -\frac{\bar{M}H^3}{\pi^2} \left[\left(\frac{3}{2} + \gamma \right) - \left(\frac{1}{2} - \gamma \right) \bar{M}^2 + (1 + \bar{M}^2) (\psi(1 + i\bar{M}) + \psi(1 - i\bar{M})) \right] \quad (110)$$

For non-perturbative computations in the main text, we need to replace the mass terms appearing above by the respective effective dynamical masses.

C Local self energy integral with fermion propagators for [Eq. \(56\)](#)

We consider the self energy integral with two fermion propagators appearing in [Eq. \(56\)](#), and compute *only* its local part for our present purpose,

$$\begin{aligned}
&i \int d^d x'' a''^d \text{Tr}(iS(x, x'') iS(x'', x)) iG(x'', x') \\
&= \frac{i}{\pi^{4-\epsilon}} \left\{ \text{Tr} \int d^d x'' \frac{a''^d}{(aa'')^{3-\epsilon}} \left[\frac{\Gamma^2(2-\epsilon/2)}{2^2} \frac{\gamma_\mu \gamma_\nu \Delta x^\mu \Delta x^\nu}{\Delta x^{8-2\epsilon}} + \frac{\Gamma^2(1-\epsilon/2) a^2 M^2}{2^4 \Delta x^{4-2\epsilon}} \right] \right\} iG(x'', x') \\
&= \frac{\mu^{-\epsilon} \Gamma(1-\epsilon/2)(1-\epsilon/4)}{2^2 \pi^{2-\epsilon/2} (1-\epsilon)\epsilon} \int d^d x'' \frac{a''^d}{(aa'')^{3-\epsilon}} [-\partial^2 \delta^d(x-x'') + 2a^2 M^2 \delta^d(x-x'')] iG(x'', x') + \text{non-local terms} \\
&= \frac{\mu^{-\epsilon} (M^2 + H^2) \Gamma(1-\epsilon/2)(1-\epsilon/4)}{2\pi^{2-\epsilon/2} (1-\epsilon)} \left(\frac{1}{\epsilon} + \ln a + \frac{\epsilon \ln^2 a}{2} + \mathcal{O}(\epsilon^2) \right) iG(x, x') + \text{non-local terms} \quad (111)
\end{aligned}$$

where the fermion propagator is given by [Eq. \(108\)](#). The gamma matrices appearing above are flat space, satisfying $[\gamma^\mu, \gamma^\nu]_+ = -2\eta^{\mu\nu} \mathbf{I}_{d \times d}$, appropriate for our mostly positive metric signature. We have also used [Eq. \(83\)](#) in the second equality.

For the two loop Yukawa vacuum graph, we need to put $x = x'$ in [Eq. \(111\)](#), where the coincidence scalar propagator $iG(x, x)$ is given by [Eq. \(104\)](#). Also, while using in the main text, the mass terms appearing above needs to be replaced by the respective effective dynamical masses.

D On the difference of results between 2PI renormalisation and the standard 1PI perturbative method

In this appendix we wish to sketch very briefly about what happens if we instead attempt to compute the standard perturbative 1PI effective action for the ϕ^4 -Yukawa theory at two loop. First, all the propagators involved here will be tree level, $i\Delta(x, x')$,

and there is no Schwinger-Dyson equations containing self energies (Eq. (42), Eq. (56)). This means, most importantly, there is *no* finite loop contribution like $\lambda f_{\text{fin}}/2$ in the mass term of the scalar propagator, and hence we simply take, $m_{\text{dyn,eff}}^2 = m_0^2 + \lambda v^2/2$ in $iG(x, x)$, Eq. (20), Eq. (21) and Eq. (22). (The fermion effective mass has already been taken to be, $M_0 + gv$, i.e., the usual one for perturbative computations, cf., the discussion below Eq. (55)). Second, in Eq. (62), we must have $\delta\lambda_3 = 0 = \delta g_3$. Accordingly, the constant C is entirely divergent now, Eq. (63). We must also set $k = 0$ in Eq. (64) for this perturbative computations, as the scalar mass is now the tree level. Putting everything together, the Yukawa term in Eq. (64) in this case reads

$$\frac{\mu^{-\epsilon} g^2 \Gamma(1 - \epsilon/2)(1 - \epsilon/4)}{2^2 \pi^{2-\epsilon/2} (1 - \epsilon) \epsilon} \int a^d d^d x (H^2 + (M_0 + gv)^2) \left(1 + \epsilon \ln a + \frac{1}{2} \epsilon^2 \ln^2 a + \mathcal{O}(\epsilon^3) \right) \left[\left(m_0^2 + \frac{\lambda v^2}{2} \right) f_d + H^2 f'_d + f_{\text{fin}} \right]$$

In this perturbative procedure one then adds to the effective action the one loop counterterm contributions, generated by the divergence of the second line of Eq. (62). After that one sets the two loop mass and vertex counterterms. However, even after doing that, there remains a non-trivial divergent term in the above expression, explicitly reading,

$$\frac{\mu^{-\epsilon} g^2 \Gamma(1 - \epsilon/2)(1 - \epsilon/4)}{2^2 \pi^{2-\epsilon/2} (1 - \epsilon)} \int a^d d^d x (H^2 + (M_0 + gv)^2) \ln a \left[\left(m_0^2 + \frac{\lambda v^2}{2} \right) f_d + H^2 f'_d \right]$$

Note that there is no such divergence in flat spacetime ($a = 1$). In order to tackle this, we add with the above the two loop vacuum graph generated by a *finite* quartic vertex counterterm *and* that of the quadratic term containing $(H^2 + (M_0 + gv)^2)$, looking explicitly like,

$$\frac{\delta\lambda_{\text{fin}}}{2} \int (aa')^d d^d x d^d x' (H^2 + (M_0 + gv)^2) i\Delta^2(x, x') i\Delta(x', x')$$

Using now Eq. (8) and Eq. (83), one can see that the square of the Feynman propagator generates a secular logarithm, and by choosing then $\delta\lambda_{\text{fin}}$ appropriately, we can remove the aforementioned divergence. We refer our reader to [65] and references therein for the standard two loop renormalisation of the ϕ^4 -Yukawa theory in de Sitter. We note that the finite secular logarithm term generated by the Yukawa vacuum loop is quadratic, opposed to the non-perturbative result of Eq. (69).

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