Low-cost nonlinear optics experiment for undergraduate instructional laboratory and lecture demonstration: A second experiment

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Low-cost nonlinear optics experiment for undergraduate instructional laboratory and lecture demonstration: A second experiment

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This communication presents a second simple and affordable experiment on nonlinear optics which is suitable for an undergraduate educational laboratory and is useful as a tabletop demonstration in a lecture on nonlinear optics. The experiment is based on the spatial self-phase modulation of a laser beam that generates in the far-field a typical pattern of interference rings. Such a nonlinear optical effect is due to thermally induced changes on the phase of a Gaussian laser beam that impinges on an optically absorbing medium. As the absorber, we propose to use a commercial soy sauce or grape juice, both of which are sold in supermarkets and exhibit strong optical absorption. The activities proposed for the experiment allow us to obtain the nonlinear refractive index of the sample.
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I. INTRODUCTION

Typically, linear optics experiments are the entrance door for first contact with the physics laboratory of a high school and in the first years of coursework at the undergraduate level. Linear optics experiments are usually restricted to demonstrating phenomena of geometrical optics like the reflection and the refraction of light and the properties of lenses. Other experiments of physical optics include creating diffraction by a slit and a circular hole.[1] In fact, teaching about nonlinear optics (the field of optics that studies the modifications of the optical properties of a medium by light impinging on it and the consequences on the light itself and its propagation through the medium) is frequently disregarded in spite of its importance in our day-to-day technology.[2] Examples of nonlinear optical effects are harmonic generation, Kerr effect, frequency mixing, and nonlinear absorption.[3] Although nonlinear optical experiments were proposed for introducing the nonlinear optics field in undergraduate physics laboratories, these experiments typically require sophisticated equipment and expensive and/or hazardous materials.[4–7]

In a previous work, we proposed an introductory experiment in nonlinear optics, which is found in the observation of the thermal lens effect, observable when a beam with a Gaussian spatial intensity profile (Gaussian beam) impinges on an absorbing medium.[8] The nonradiative relaxation of the medium leads to an increase in its temperature and a change in the refractive index. Such a change makes the medium behave like a lens. In the current work, we propose a second introductory experiment in nonlinear optics, one that is based on the spatial self-phase modulation (SSPM) of an incident Gaussian beam on an absorbing medium.[9] The SSPM gives rise to a set of interference cones that produce an amazing pattern of rings when encountering a screen. Such a ring pattern does not bear similarities to the diffraction pattern of a circular aperture.

It is worth pointing out that for an introductory laboratory, a simple and feasible experimental setup in the spirit of a plug-and-play experiment is recommended. In this communication, we present a simple and inexpensive experimental setup for observing the diffraction pattern due to the spatial self-phase modulation of a laser beam. The proposed nonlinear medium is commercial soy sauce (as proposed in the previous work), which is a highly light-absorbing fluid, although other media can also be used such as whole grape juice. First, we provide an introduction to the Gaussian beam and the spatial self-phase modulation effect. Next, we give a step-by-step procedure for observing the spatial self-phase modulation and provide instructions on how to get some parameters of the medium from the experiment.

II. THEORETICAL BACKGROUND

A. Gaussian beam

Lasers used in research laboratories typically have a beam, called a Gaussian beam, with a Gaussian spatial intensity profile, which corresponds to the fundamental mode of an optical oscillator (TEM_{00} mode). The general mathematical expression of the intensity of a Gaussian beam is given by

$$I(r) = I_o\, e^{-r^2/w^2}, \tag{1}$$

where I_o is the intensity on the propagation axis $(r = 0)$ and w, called the Gaussian beam radius, is the radial distance at which the intensity has decreased to $1/e$ of its axial or peak value.[10] Figure 1 shows the 3D spatial distribution of a Gaussian beam.

For a focused Gaussian beam propagating along the z-axis and at a distance z from the focus $(z = 0)$, the value of the beam radius w is given by

$$[w(z)]^2 = w_o^2\left[1 + \left(\frac{\lambda z}{\pi w_o^2}\right)^2\right] \tag{2}$$

and the radius of curvature R of the wave front, in the paraxial approximation, is given by

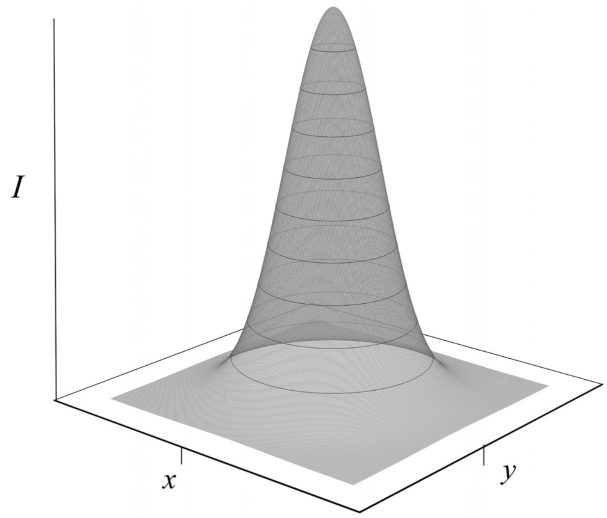

Fig. 1. 3D graph of the spatial intensity profile of a Gaussian beam. The radial distance r is given by $r^2 = x^2 + y^2$.

$$R(z) = z\left[1 + \left(\frac{\pi w_o^2}{\lambda z}\right)^2\right], \tag{3}$$

where w_o is the beam waist, the minimum radius of the Gaussian beam, which is located at the focus, and λ is the wavelength of the laser beam.[11] Equation (3) shows that at the focus, $R \rightarrow \infty$ (i.e., the wave front is planar at the focus of the converging lens). For a Gaussian beam, it can also be shown that the power P of the laser beam is related to the on-axis intensity by

$$P = I_o \pi w_o^2 / 2. \tag{4}$$

B. Spatial self-phase modulation

The spatial self-phase modulation effect is a phenomenon easily observable using a laser beam focused on a light absorbing sample. Qualitatively, the effect can be explained as follows. The refractive index n of a sample depends on, among other factors, temperature T. Thus, a change δT in the temperature of the sample will alter its refractive index by $\delta n = (dn/dT)\delta T$, meaning an increment or a decrement of its value at initial temperature (room temperature) according to whether the sign of dn/dT, called the thermo-optic coefficient, is positive or negative.[12] On the other hand, when an absorbing medium is illuminated by light of wavelength λ, the local temperature will rise according to the optical absorption coefficient $\alpha(\lambda)$ and the light beam intensity. Thus, an absorbing medium illuminated by a Gaussian beam will suffer a modification of its refractive index that will follow approximately the spatial profile of the laser beam intensity (a precise determination of the spatial profile of δT should take into account the thermal diffusion properties of the medium and the boundary conditions). The thermally induced change in the refractive index leads to an additional phase shift $\psi(r)$ to the phase Ψ_o of the incident plane wave that, at the exit plane of an isotropic thin sample of thickness L, is given by (Fig. 2)

$$\psi(r) \simeq \frac{2\pi}{\lambda}\delta n(r)\, L \propto \alpha I(r). \tag{5}$$

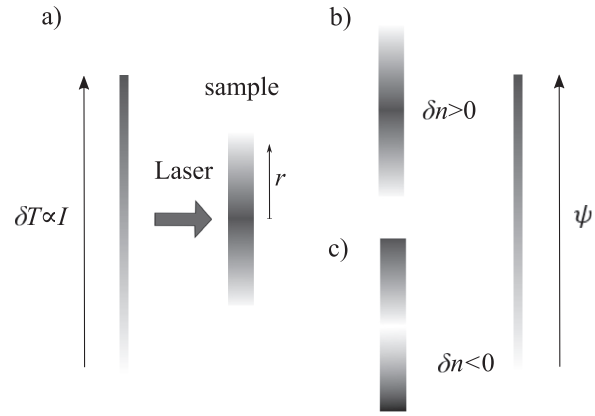

Fig. 2. (a) Spatial profile of the change in temperature δT of an optically absorbing sample illuminated by a Gaussian laser beam of intensity I and the corresponding phase change ψ of the same beam at the exit plane of the sample for (b) $\delta n > 0$ and (c) $\delta n < 0$.

Hence, it is possible to write

$$\psi(r) = \psi_o\, e^{-r^2/\omega^2}, \tag{6}$$

where ψ_o is the maximum change that occurs along the propagation axis. As seen above, at the focal point of the converging lens, the undistorted wavefront of the laser beam fulfills the condition of a plane wave (i.e., the wave vector $\vec{k}$ is parallel to the propagation direction). (To better understand the following, it could be useful to review the concept of the eikonal function.) The time-independent solution of the wave equation is given by $\vec{E} = \vec{E}_o\, e^{-i\Psi(\vec{r})} = \vec{E}_o\, e^{-ik_o S(\vec{r})}$ for the electric field (similar expressions can be found for the magnetic field $\vec{H}$), where $\vec{E}_o$ is the amplitude of the electric field, $\Psi(\vec{r})$ is a phase, k_o is the free-space wave vector, and $S(\vec{r})$ is the optical path length and is often called the eikonal function. The locus of constant phase $(\Psi(\vec{r}) = \text{cte})$, and hence constant optical path $(S(\vec{r}) = \text{cte})$, is the geometrical wavefront.[13] Substituting the solutions of the fields in Maxwell's equations and taking the limit $\lambda_o \rightarrow 0$ (geometrical optics), the eikonal function satisfies the equation $|\nabla S(\vec{r})|^2 = n^2(\vec{r})$, called the eikonal equation. The unit vector $\hat{s}(\vec{r})$ defined by $\hat{s}(\vec{r}) = (\nabla S(\vec{r})/n) = (\nabla\Psi(\vec{r})/k_o n)$ is normal to the wavefront (see Fig. 3) and gives the propagation direction of the beam (the ray direction) at point $\vec{r}$. Hence, it is possible to write $\hat{s} = \vec{k}/k$, where $\vec{k}$ is the wave vector.[14]

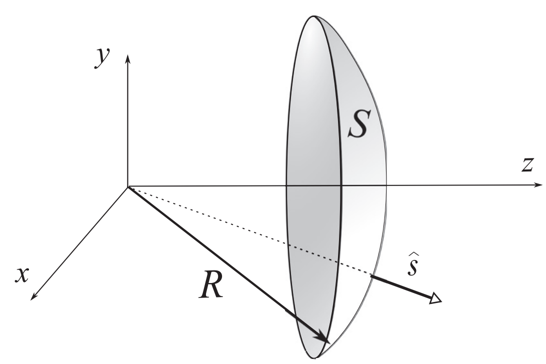

Fig. 3. Section of a spherical wavefront, $S(r) = cte$, of radius R. The unit vector $\hat{s}$ normal to the wavefront at a given point gives the ray direction at that point.

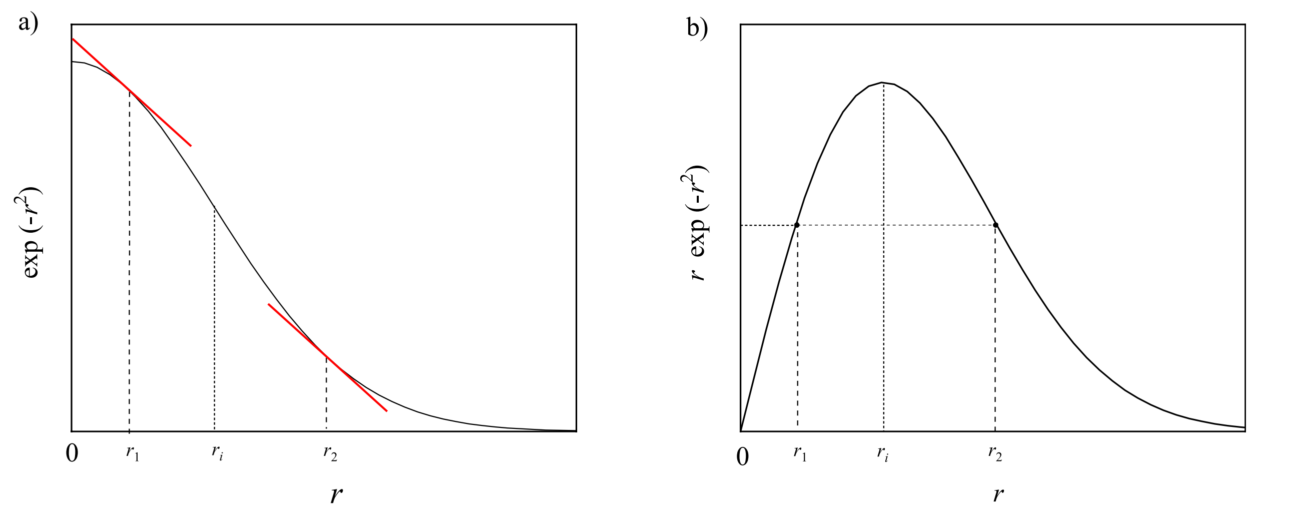

Fig. 4. (a) Graph of function $f(r) = e^{-r^2}$. Red lines shows the tangents with the same slope; (b) graph of function $r\,e^{-r^2}$, proportional to df/dr. Note that, generally, df/dr will assume the same value for two different values of r. The radii r_1 and r_2 are two arbitrary values of r for which df/dr assumes the same value and r_i is the inflection point of $f(r)$.

So, at a distance r from the center of the beam, the change $\delta n(r)$ in the refractive index leads to a transversal component of the wave vector, $k_\perp$, besides the k_z component. For the radial component, and assuming a cylindrical symmetry, we can write (see the Appendix)

$$k_\perp \propto \nabla_r \psi(r) = -\frac{2\psi_o}{w^2}\, r\, e^{-\left(r^2/\omega^2\right)}, \tag{7}$$

where ∇_r stands for $\partial/\partial r$. For understanding the implication of Eq. (7), let us consider the Gaussian function e^{-r^2} and the function proportional to its derivative, $r\,e^{-r^2}$, shown in Figs. 4(a) and 4(b), respectively. Figure 4(b) shows that the function $r\,e^{-r^2}$ displays the same value for pairs of values of r, with r_1 and r_2 being two of them. Therefore, from Eq. (7), it is possible to show that there will be two different rays emerging at distances r_1 and r_2 from the propagation axis along the same radial direction, which propagate in the direction given by the wave vector $\vec{k} = \vec{k}_z + \vec{k}_\perp$ (see Fig. 4).

The interference between these two rays will be totally constructive or destructive if $\psi(r_2) - \psi(r_1) = m\pi$, with m being an even or odd integer, respectively. Following this, ψ_o, the maximum value of the phase difference (see Eq. (6)), is given by $\psi_o = \psi(0) - \psi(\infty)$. We note that $\psi(\infty) = 0$, $\psi_o = \psi(0)$. If $\psi_o > 2\pi$, the number N of bright rings can be estimated by $N \simeq \psi_o/2\pi$. The diverged beams will be distributed into a series of coaxial diffraction cones of half-angle θ, which will produce concentric bright and dark rings when projected onto a plane far from the lens.

Under an intense light beam, the refractive index of a medium is given by $n = n_o + n_2 I$, where n_o is the linear refractive index and n_2 is the nonlinear refractive index, which can be positive or negative. The signal of n_2 is related to the formation of a converging ($n_2 > 0$) or diverging ($n_2 < 0$) thermal lens in the sample, respectively. Thus, it is possible to write $\delta n(r) = n_2 I(r)$ and $\psi_o = (2\pi/\lambda) n_2 I_o L$. Hence, we can write

$$N = \frac{n_2 2L}{\lambda \pi w_o^2} P, \tag{8}$$

where Eq. (4) was taken into account. It is important to notice that Eq. (8) allows the determination of the magnitude of n_2 but not its sign because a phase difference of $\pm m 2\pi$ would lead to the same number of bright rings. To determine the sign of n_2, the procedure indicated in Ref. 8 can be followed to determine which type of lens, converging or diverging, is formed under the incidence of the laser beam.

The deflection angle θ of the beam (see Fig. 5), for $\theta < 1$, can be written as

$$\theta \cong \left|\frac{k_\perp}{k_z}\right|, \tag{9}$$

and considering just the radial dependence of $\nabla_r \psi$ from Eq. (7),

$$\theta \propto r\, e^{-\left(r^2/w^2\right)}. \tag{10}$$

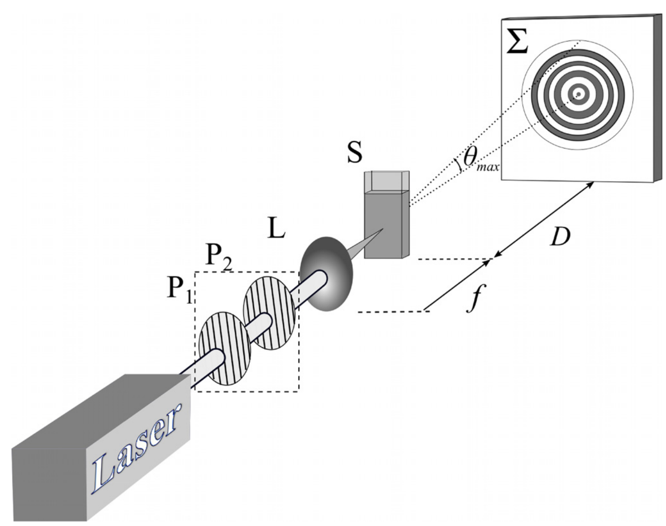

Fig. 5. Scheme of the basic experimental setup for observing SSPM: lens (L) of focal length f, with sample (S) and screen (Σ) positioned at a distance D of approximately 2 m from the sample. Polarizers P_1 and P_2, shown inside the dotted square, are optional and can be used to control the intensity of the laser on the sample when the laser has a constant power: just P_1 for a linearly polarized laser and the set of P_1 and P_2 for a nonpolarized laser. θ is the deflection of rays that originate due to SSPM ($\theta = 0$ corresponds to the initial propagation direction, and θ_{max} is its maximum value).

From Fig. 4, it is possible to notice that the maximum deflection angle (θ_{max}) occurs for rays that emerge at the radial distance corresponding to approximately the inflection point r_i. Because θ does not change significantly around θ_{max} and the interfering rays that emerge with those angles have roughly the same intensity, the outermost ring is the broadest and brightest ring in the pattern.

III. EXPERIMENTAL DETAILS

The basic experimental setup includes a laser with a Gaussian beam, a converging lens, and a screen. Figure 5 shows a scheme of the experimental setup, which can be placed on a board. In our experimental assembly, we used a continuous wave laser ($\lambda = 532$ nm, Ventus, Laser Quantum), a converging lens of focal length $f = 10$ cm, producing a beam waist w_o, waist at focus, of about 33 μm, and have placed the observation screen at a distance D from the sample about 2 m.

A concomitant effect to the SSPM is the thermal lens effect; hence, for observing just the SSPM effect, the cell containing the sample must be positioned at the focus of the converging lens. A ray through the center of a thin circular lens will be undeflected; thus, even if the laser beam induces a thermal lens in the medium, the beam will not suffer an additional perturbation due to the thermal lens effect.

For observing the spatial self-phase modulation, we propose to use commercial soy sauce (shoyu sauce) or whole grape juice, fluids that have a strong optical absorption in the visible range of the electromagnetic spectrum. Tap water can be used as a control. The sample can also be diluted with tap water. In our experiment, we have prepared three samples, 100% (undiluted), 80%, and 60%. The absorbing and the control media were encapsulated into an $\sim$200 μm-thick cell. For a medium of rather low optical absorption, it is possible to use both a higher power laser and a thicker cell, such as a spectrophotometer cuvette with a path length of 0.5–1 cm. Details regarding the construction of the 200 μm-thick cell can be found elsewhere.[8] For controlling the incident power of the laser beam on the sample if it does not have its own power control, it is possible to use just one linear polarizer for a linearly polarized laser beam or—for a randomly polarized (unpolarized) laser—a set of two linear polarizers, positioned between the laser and the lens, by taking advantage of Malus's law (see Fig. 5).[1] Maintaining one of the polarizers at a fixed orientation and rotating the second one, it is possible to control the incident power of the laser on the sample.

IV. RESULTS AND DISCUSSION

With the laser focused on the absorbing sample and at a low power, the typical spot of the laser can be observed. By increasing the power, the concentric rings of the far-field diffraction pattern by a circular aperture (Airy disc) could also be observed. Such a pattern corresponds to the diffraction of a plane wave passing through a circular aperture with roughly the size of the beam waist. However, this pattern must not be confused with the pattern due to self-phase modulation.

Figure 6 shows the ring patterns by spatial self-phase modulation of the undiluted sample of shoyu sauce at different powers of the incident laser. All the pictures were taken from the same distance to the screen. By increasing the power of the laser from the lowest value, the bright circle of the laser beam spot that is seen initially is replaced by a luminous circle with a central dark spot, thus forming the first bright ring. Further increasing the beam power increases both the number and the diameter of the rings. It is worth noticing that by increasing the power, the circularity of the rings in the pattern can be gradually lost, acquiring a D-shape. This effect is observed at the higher powers in Fig. 6, where the upper part of the ring pattern is gradually flattened. The origin of the effect is the emergence of free convection inside the sample due to a density gradient.[15] To avoid such an effect, it is possible to modify the experimental setup by using a laser beam propagating vertically, for example, upward, making it to fall upon the sample from below and observing the ring pattern on a screen positioned above the sample.[16]

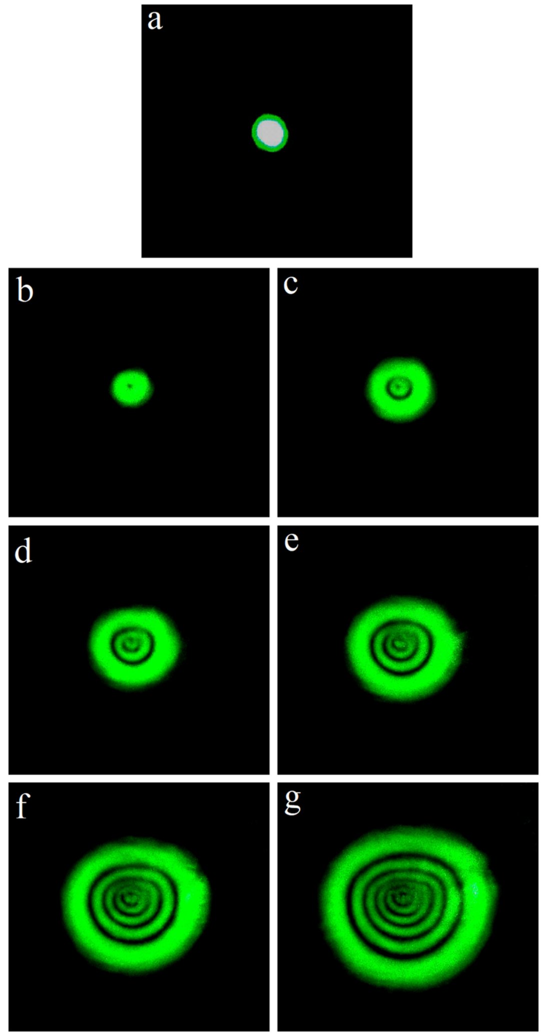

Fig. 6. (a) Spot of the laser beam on the screen obtained with pure water at 50 mW where no ring pattern is observed. A similar image can be seen with the undiluted shoyu sauce at powers lower than 20 mW. (b)–(g) pattern of rings by SSPM obtained at increasing values of the beam power with the undiluted sample: (b) 21 mW, (c) 40 mW, (d) 50 mW, (e) 66 mW, (f) 80 mW, and (g) 95 mW.

Figure 7 illustrates the number of rings as a function of the incident power for different sample concentrations. In the same figure, linear fittings of data are also shown. For each sample, it is possible to observe the linear trend of the number of rings as a function of the power of the laser. The linear coefficients obtained from the fittings allowed us to obtain the nonlinear refractive index n_2 of the samples (see Eq. (8)), the values of which are shown in Table I. The

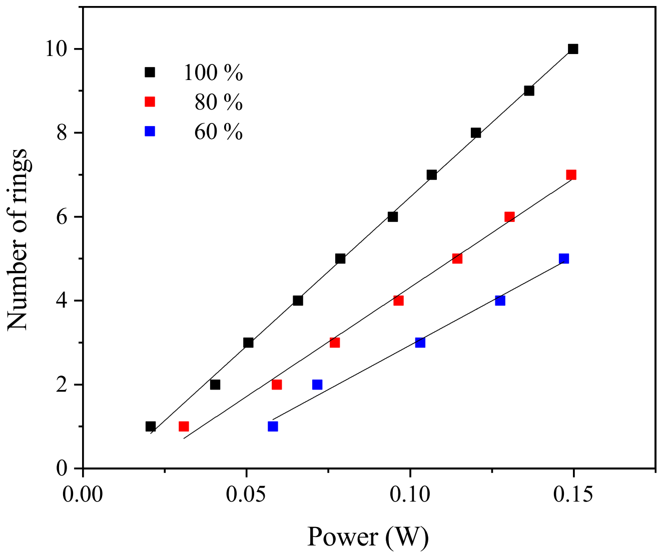

Fig. 7. Graph of the number of rings in the SSPM pattern as a function of beam power for different concentrations of shoyu sauce. Solid lines show linear fitting to data.

negative signs attributed to the values of n_2 were derived from the experiments depicted in Ref. 8.

Figure 7 also illustrates that there is a minimum power of the incident laser beam for the appearance of the first ring. Such power, indicated as P_{th}, depends on the sample concentration, increasing its value as this concentration decreases. Such a finding has been used as one of the criteria for determining the possible adulteration of a commercial fluid.[17] The values of P_{th} for the samples employed in this communication are shown in Fig. 8.

The activities proposed in this paper can be employed to study a wide range of materials with the only condition being the ability to absorb the laser's light. For observing the spatial self-phase modulation in low absorbers, it is possible to use a more powerful laser or sample holders of longer path lengths such as 1 cm-thick spectroscopy cuvettes if a stronger laser is not available. However, in such conditions, the supposition of thin samples is not valid, and any quantitative analysis should be based on a more rigorous theory.[18]

Finally, some of the numerous technological applications of self-phase modulation (SPM) are worth mentioning. Besides the thermal origin of the SPM observed in the experiment proposed in this paper, SPM of electronic origin can also been observed with short pulses of light in a medium, known as a Kerr medium, which exhibits the optical Kerr effect ($n(I) = n_o + n_2 I$). For a short pulse of light, the intensity will be a function of time $I(t)$, and accordingly, the phase will also be time-dependent (see Eq. (5)). Thus, there will be a change in the instantaneous frequency of the optical beam $\Delta\omega(t) \propto dI/dt$. The extra frequencies lead to a broader spectrum of the pulse of light, and in a dispersive medium,

Table I. Value of the nonlinear refractive index of shoyu sauce for different concentrations.

Concentration (%)	Linear coefficient (W^{-1})	$n_2(\times 10^{-10} m^2 W^{-1})$
60	71.03	–2.4
80	52.02	–1.7
100	42.30	–1.4

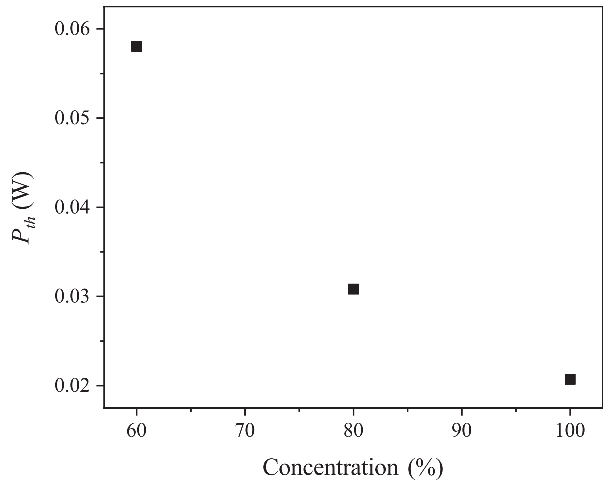

Fig. 8. Values of P_{th} for different concentrations of shoyu sauce.

this effect produces a reshaping of the pulse (a stretching or a compression). This effect has been used to produce, for example, ultrashort pulses of light at a high repetition rate, optical solitons, and white-light continuum generation.[19,20]

V. CONCLUSIONS

In this communication, we proposed a second experiment for illustrating a nonlinear optical effect that is possible to implement with a low-cost and nontoxic material. Such an experiment is based on the observation at the far field of the typical pattern of rings of the self-phase modulation of a Gaussian laser beam that propagates through a sample of commercial soy sauce. The simplicity of the experimental setup will provide both undergraduate and graduate students with not only a first contact with the field of nonlinear optics but also the possibility of initiating serious and fascinating research on the nonlinear optical properties of materials.

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APPENDIX: DERIVATION OF THE WAVEVECTOR FROM THE EIKONAL

The direction of the ray is that of the gradient of the geometric wavefronts, which are given by $S(\vec{r}) = cte$, and can be expressed from the eikonal equation as

$$\begin{aligned}\nabla S &= n\,\hat{s}\\ &= n\frac{\vec{k}}{k}\\ &= n\frac{\vec{k}_\perp + \vec{k}_z}{k}\\ &= \frac{\vec{k}_\perp + \vec{k}_z}{k_o},\end{aligned}$$

where $\vec{k}_\perp$ is the radial component of $\vec{k}$, and it was assumed to be a medium with cylindrical symmetry (i.e., there is not an angular dependence). For such a medium,

$$\nabla S = \nabla_r S + \nabla_z S = \frac{\partial S}{\partial r}\hat{r} + \frac{\partial S}{\partial z}\hat{z},$$

and hence we can write

$$\frac{k_\perp}{k_o} = \frac{\partial S}{\partial r} = \frac{1}{nk_o}\frac{\partial(\Psi_o + \psi)}{\partial r} = \frac{1}{nk_o}\frac{\partial(\psi)}{\partial r} = \frac{1}{nk_o}\nabla_r \psi.$$

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