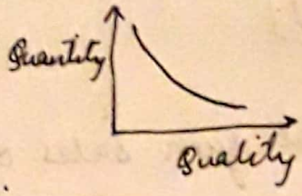
Requirements:

① safety

safe operation of a chemical plant is primary requirement for well-being of the people in the plant & for continued contribution to the economy.

~~Requirements~~

② Production specification

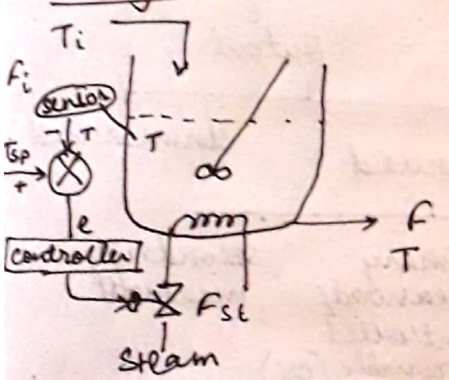


③ Environmental regulations:

④ operational constraint

⑤ optimum operating condition

Heating Tank



Control objective: $T = T_{sp}$ (set point)

start up ⟶ SS ⟶ switch "on" controller

$e = T_{sp} - T$ ← comparator

$e > 0$ ⇒ more steam

$e < 0$ ⇒ less steam.

External disturbances cause change in steady stat.
Hence, controller is required.

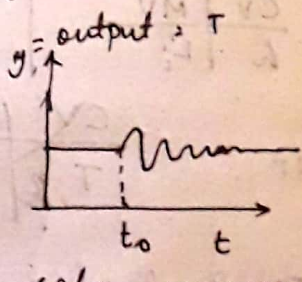
$F_{st} = K_c e$.

→ effect of external disturbance

$T_i \downarrow$ $T \downarrow$

$F_i \uparrow$ $T \downarrow$

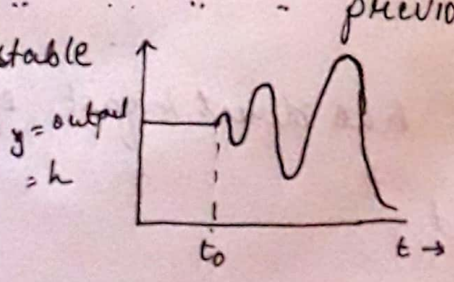
→ stability :



stable / self regulating

If stability is the sole concern, controller is not required.
If we wish to reach stability earlier, ... required.
.. .. previous steady state, ...

– unstable

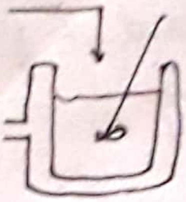


CSTR (continuous stirrer tank reactor)

$A \rightarrow B \rightarrow C$ [Exothermic]

Reactant product waste.

coolant



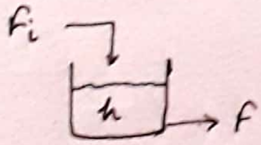
Profit (ϕ) $= \int_0^{t_R} [(\text{revenue from sales of product}) - (\text{raw material cost}) - (\text{energy cost}) - \text{other cost}]\, dt$

Constraint: g raw material utilization

~~Variable~~:

- Variable
  - Input
    - Manipulated (MV) / control ~~(CV)~~
    - Load variable (LV) / Disturbance.
      - Measured LV
      - Unmeasured LV
  - Output.
    - Measured
      - Primary measured / Controlled variable (CV)
      - Secondary measured
    - Unmeasured

<u>Example-1</u>



Control objective: $h = h_{sp}$.

I/p = F_i

O/p = h, F.

CV	MV
h	F_i

CV	MV	LV
h	F	F_i

<u>Ex-2</u> → Heating Tank

I/p → F_i, T_i, F_{ST}.

O/p → F, T, h

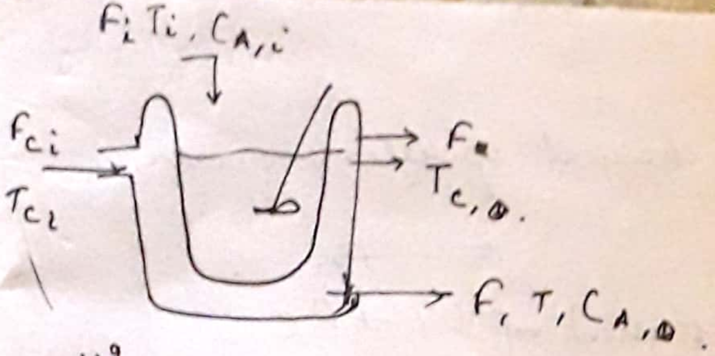
CV	MV	LV
T, h	~~F~~ F_{st} ~~F_i~~	T_i

Control objective: $T = T_{sp}$ & $h = h_{sp}$.

CV	MV	LV
T	F_{ST}, ~~F_i~~	~~F_i~~ F_i
h.	F_{st}	T_i

→ MV is selected such that it has direct & fast effect on CV.

superheated or saturated



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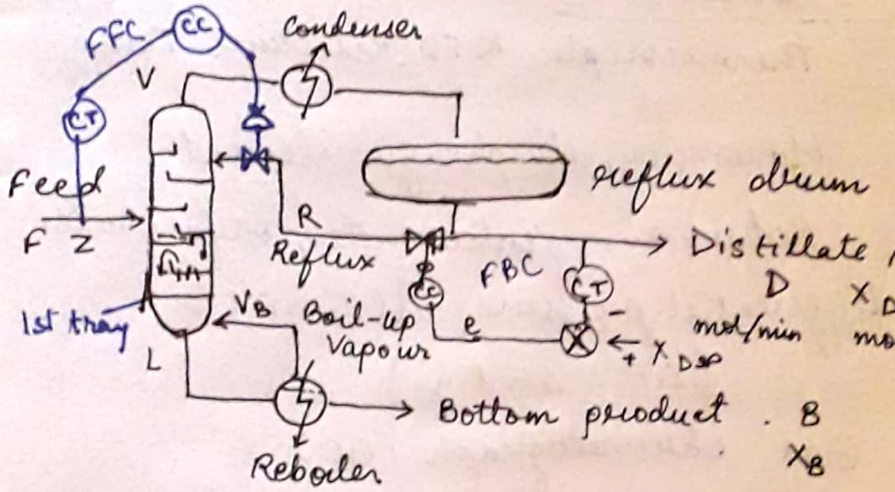
Control Systems

Feedback control

CT → Composition transmitter



There are pores in the tray. Because vapour is having higher pressure than the liquid, it is not allowing liquid to pass through the pores.

Control objective: $X_D = X_{DSP}$

$X_B = X_{BSP}$.

FBC → Feedback control

FFC → Feed follower control

CV	MV
X_D	R
X_B	V_B

FBC & FFC are not employed simultaneously.

In FBC, output variable is measured (controlled variable)

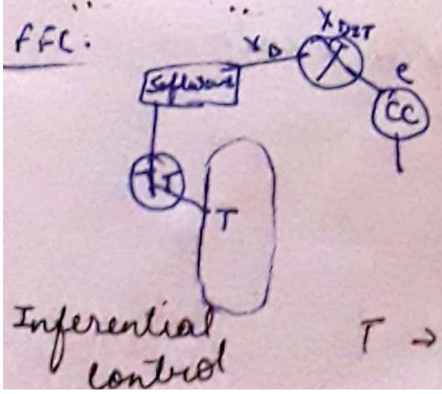
In FFC, load variable is measured.

Drawback:

Gas chromatograph involves delayed response.

" " " huge investment & maintenance cost

FFC:



Inferential control

In this case, software is used to convert T to corresponding X_D.

A corelation is fed into it

Software : $X_D = f(T)$

T → secondary measured variable

Hardware elements

In process, physical & chemical operations occur.

① The process
② Measuring device (sensor) – CV - FBC; LV - FFC; Secondary measured – inferential control (IC)

Variable	sensor
T	Thermocouple, RTD (Resistance Temp.
P	Manometer, diaphragm element
F	Rotameter, venturimeter, orificemeter
h	Differential pressure cell (DPC) ($\Delta P = \Delta h \rho g$)
composition	Gas chromatograph (GC)

③ Transducer

Measurement Signal $\rightarrow$ $\boxed{\text{Trans.}}$ $\rightarrow$ physical quantity (V, I, pneumatic signal)

④ Transmission line
Sometimes the measurement signal is weak (in mV), due to which they can't be transmitted over large distances. Hence, ~~used~~ amplifier is used.

⑤ Controller
- Receives measurement signal
- Calculates control action

⑥ FCE (Final control element)
eg - control valve, variable spin pump - variable spin compressor.

⑦ Recording device
- Display unit

Modelling

$$\text{Input} \rightarrow \boxed{\text{set up}} \rightarrow \text{Output}$$
$$\quad \hookrightarrow \boxed{\text{Math}} \rightarrow \text{Output}$$

Rigorous : A model is rigourous if both the setup output & mathematical output are ~~pr~~ precisely same.

Model is a mathematical representation of the process intended to promote understanding of the real system.

Types of model :

① Empirical model → It is based on input-output data sets. Easy to develop.

② Theoretical model → Based on conservation of mass, energy & momentum.

Adv :

→ Can be engaged to a number of conditions
→ physical insight into the process.

Disadvantage :

① Time consuming. ② Complex

③ Hybrid model / mixed model

Accumulation = Input − Output + generation
(based on energy conservation)

$\Delta H = \int C_p \, dT$; $C_p = a + bT + CT^2$ [Empirical model]

$\cancel{\dfrac{d(mH)}{dt}}$ $\dfrac{dC_A}{dt} = (-r_A) + \text{Input} + O/P$ [Theoretical]

$-r_A = k C_A^n$

$k = k_0 \exp\left(-\dfrac{E}{RT}\right) \leftarrow$ Empirical model.

Model

Algebraic eq^n — Differential eq^n

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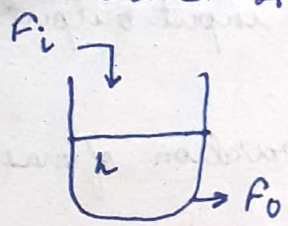
State variables & equations

- natural state
- mass, energy, momentum
- V, T, composition, ρ @ state variables

The equations which are derived by the application of conservation principle on fundamental properties to relate the state variables with other state variables are called ~~other~~ state ~~variables~~ equations.



$\cancel{\rho A h = (F_i - F_0) t}$.

$$\frac{d}{dt}(A h \rho) = (F_i - F_0)\rho$$ [F → volume flow rate]

$$\boxed{A\frac{dh}{dt} = F_i - F_0}$$ state eq^n.

Degrees of freedom

$F = V - E$

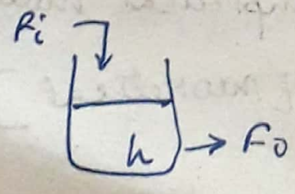
V → no. of variables independent to each other.

E → no. of equations

(i) $F = 0$, $V = E$ specified ← Ideal

(ii) $F > 0$, $V > E$ underspecified ← common

(iii) $F < 0$, $V < E$ overspecified ← Exceptional



$V = (F_i, F_0, h) = 3$.

$E = 1$.

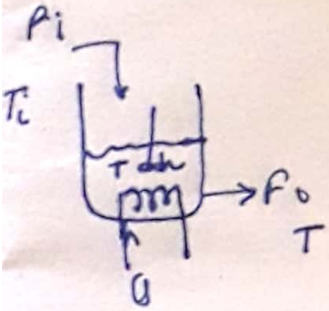
$F = V - E = 2$.

CV	MV	LV
h	F_0	F_i →Measure

$F_0 = F_{0s} + K_c(h_{sp} - h)$.

$F = 1 - 1 = 0$.

Model development



Assumption:

$mc_p\, dT.$

1. Perfect mixing
2. No Heat loss
3. ρ, C_p are constant
4. T_i, T are constant

Mass balance: $A \frac{dh}{dt} = F_i - F_o$. — (1)

Energy balance: $\cancel{F_i \rho C_p}$

$$hA\rho C_p \frac{d(T-T_{ref})}{dt} = F_i \rho C_p (T_i - T_{ref}) - F_o \rho C_p (T - T_{ref}) + Q .$$

For $T_{ref} = 0$.

$$hA\rho C_p \frac{dT}{dt} = F_i \rho C_p T_i - F_o \rho C_p T + Q .$$

$$A \frac{dT}{dt} = \cancel{(F_i - F_o)} \frac{F_i T_i}{h} - \frac{F_o T}{h} + \frac{Q}{\rho C_p h} . \quad — (2)$$

$E = 2$.

$V = F_i, F_o, T, h, Q$

$$A\rho C_p \left[h\frac{dT}{dt} + T\frac{dh}{dt} \right] = F_i \rho C_p T_i - F_o \rho C_p T_o + Q$$

$$\Rightarrow Ah\frac{dT}{dt} + AT\frac{dh}{dt} = F_i T_i - F_o T_o + \frac{Q}{\rho C_p}$$

$$\Rightarrow Ah\frac{dT}{dt} + T(F_i - F_o) = F_i T_i - F_o T_o + \frac{Q}{\rho C_p} .$$

CV	MV	LV
h	F	F_i
T	Q	T_i

2 control eq^n

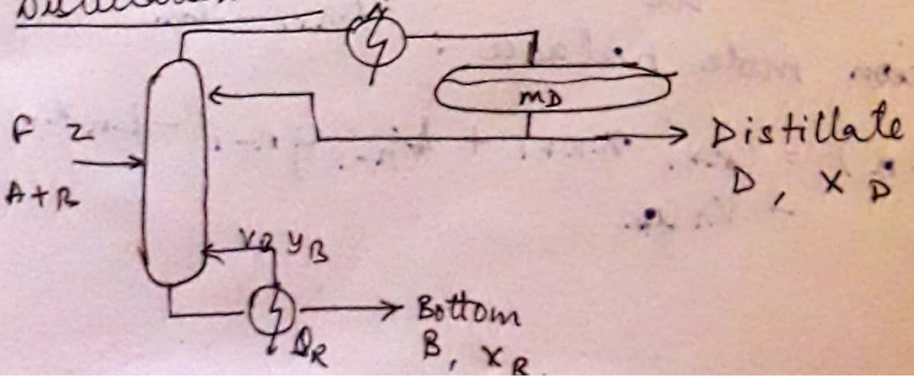
$V = 6 \rightarrow F_i, T_i, F, T, h, Q$.

$E = 2$

$f = 4$

T_i → 2 specified variable

Distillation



D → Dis. flow rate, mol/min
F → Feed .. ..
B → Bottoms .. ..
L → liquid .. ..
V → vapour .. ..

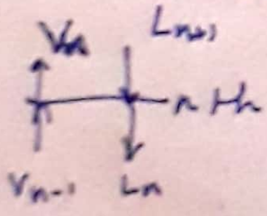
x : mole fraction of liquid
y : .. .. vapour
z : .. .. feed
Q_R → heat input to the reboiler
m → liq. hold up, mole.

<u>Assumptions</u> :

① No heat loss
② No vap. holdup on the tray
③ variable liq. holdup.
④ Molar heats of vaporization of both A & B are approximately equal. 1 mole of condensing vapour releases enough heat to evaporate enough heat to evaporate 1 mole of liquid.

$V_1 = V_2 = \cdots = V_{20} = V_B$

⑤ All trays are equally ~~eff~~ & 100 % efficient.
⑥ Relative volatility (α) constant.
⑦ L + V → Ideal phase. ⑧ Feed = saturated liquid



Mole balance :

$$\frac{dm_n}{dt} = L_{n+1} + V_{n-1} - L_n - V_n$$
$$= L_{n+1} - L_n$$

Composition mole balance :

$$\frac{d}{dt}(m_n x_n) = L_{n+1} x_{n+1} + V_{n-1} y_{n-1} - L_n x_n - V_n y_n$$

Mass balance in reflux drum.

$\cancel{V_{20}}$

$$\frac{dm_D}{dt} = V_{20} - D - R$$

Component balance

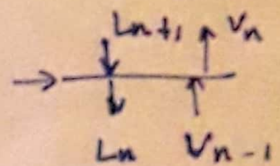
$$\frac{d}{dt}(m_D x_D) = V_{20}\, y_{20} - R x_D - D x_D .$$

<u>Equations for Reboiler:</u>

$$\frac{dm_B}{dt} = L_1 - B - V_B$$

$$\frac{d}{dt}(m_B x_B) = L_1 x_1 - B x_B - V_B y_B$$

<u>Eq^n for tray having feed stream</u>



$$\frac{dm_n}{dt} = L_{n+1} + V_{n-1} + F - V_n - \cancel{V_n}\, L_n$$

$$\frac{d(m_n x_n)}{dt} = L_{n+1} x_{n+1} + V_{n-1} y_{n-1} + F x - V_n y_n - L_n x_n$$

Simulation

$m \rightarrow$ Total mole

$x \rightarrow$ component ..

$y \rightarrow$

$L \rightarrow$

$V \rightarrow$

Const. relative volatility

$$\alpha_{ij} = \frac{K_i}{K_j} = \frac{y_i/x_i}{y_j/x_j} = \frac{y_i/x_i}{(1-y_i)/(1-x_i)}$$

$k \rightarrow$ vapour - liquid eq^n constant $= \frac{y}{x}$

$$\alpha_{ij} = \frac{y_i}{1-y_i} \cdot \frac{1-x_i}{x_i}$$

$$\Rightarrow \frac{1-y_i}{y_i} = \frac{1-x_i}{x_i \alpha_{ij}}$$

$$\Rightarrow \frac{1}{y_i} = 1 + \frac{1-x_i}{x_i \alpha_{ij}}$$

$$\Rightarrow y_i = \frac{x_i \alpha_{ij}}{x_i \alpha_{ij} + 1 - x_i} = \frac{x_i \alpha_{ij}}{1 + (\alpha_{ij} - 1) x_i} \quad \text{.. VLE}$$

In this eq^n, α_{ij} is known, x_i can be calculated.

vapour : $P_i = y_i P_t$

Liquid : $P_i = x_i P_i^{\circ}$

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$K_i = \frac{y_i}{x_i} = \frac{P_i^0}{P_t} = \frac{\quad}{P_t}$

$= f(T, P)$

<u>Francis Weir eq^n</u>

$L_n = \underbrace{L_{no} + \frac{m_n - m_{no}}{\beta}}_{SS \text{ values.}}$

β : hydraulic time constant
~ 3-5 sec.

In β time period, $m_n - m_{no}$ flow through $L_n - L_{no}$.

$V_1 = V_2 = \ldots = V_{20} = V_B$.

Reboiler: $V_B = \frac{Q_R}{\lambda_B}$

No

$\lambda = \Sigma y_i \lambda_i$

$\lambda_i = RT^2 \left[\frac{B_i}{(T + C_i)^2} \right]$ [using clausius clapeyron eq^n]

Degrees of freedom

No. of eq^n	origin
N+1	$y = \frac{\alpha x}{1 + (\alpha - 1) x}$
N (D & B are calculated separately)	Francis - weir (L)
N+2	Total mole (m)
N+2	comp mol. (x)
E = 4N + 5	

$x, y, L, M, \cancel{t}$ → 22, 21, 20, 22

$y \rightarrow N+1$

$L \rightarrow N$

$m \rightarrow N+2$

$x \rightarrow N+2$

F, z, D, R, B, VB → 6

$V = 4N + 11$

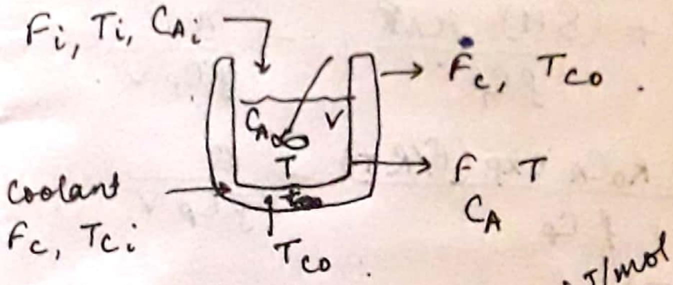
LV = F, z @ measured

CV	MV
x_D	R
x_B	V_B
m_D	D
m_B	B.

$f = 0.$

Process modelling

~~lig. l~~ Continuous Stirred tank reactor.



$A \rightarrow B$ [exothermic]
[1st order]

$\delta H \rightarrow J/mol$

$R_A \rightarrow$ Rate of appearance of A. $\left(\frac{mole}{m^3}\right)$ $\frac{m^3}{s} \cdot \frac{mol}{m^3}$

Assumption:

- ~~Ex~~ No heat loss.
- Perfect mixing.
- ρ, C_p, (δH) are constant 3

$R_A = K C_A$.

$K = K_0 \exp\left(-\frac{E_a}{RT}\right)$

$$\frac{dV}{dt} = F_i - F \quad -① \text{ [Total mole balance.]}$$

$$\frac{d(C_A \cdot V)}{dt} = F_i \cdot C_{Ai} - F \cdot C_A$$

$$\Rightarrow V\frac{dC_A}{dt} + C_A\frac{dV}{dt} = F_i \cdot C_{Ai} - F C_A + R_A \cdot V.$$

$$\Rightarrow V\frac{dC_A}{dt} + C_A(F_i - F) = F_i \cdot C_{Ai} - F C_A + V K_0 C_A \exp\left(\frac{-E_a}{RT}\right)$$

$$\Rightarrow V\frac{dC_A}{dt} + C_A F_i - \cancel{C_A F} = C_{Ai} F_i - \cancel{C_A F} + K_0 V \exp\left(-\frac{E_a}{RT}\right) C_A$$

$$\Rightarrow V\frac{dC_A}{dt} + F_i(C_A - C_{Ai}) = K_0 V C_A \exp\left(\frac{-E_a}{RT}\right)$$

$$\Rightarrow \frac{dC_A}{dt} = \frac{F_i(C_{Ai} - C_A)}{V} + K_0 C_A \exp\left(\frac{-E}{RT}\right) \quad -②.$$

Energy balance:

$$\cancel{\rho C_p \frac{d(T - T_{ref})}{dt} = \rho F_i C_p (T_i - T_{ref}) - \rho F C_p (T - T_{ref}) + Q + H.}$$

~~$Q \rightarrow$ Heat generated (J), $H \rightarrow$ heat transferred (J).~~

$$\rho C_p F_i T \rightarrow \frac{mole}{m^3} \cdot \frac{J}{mol \cdot K} \cdot \frac{m^3}{s} \cdot K \rightarrow \frac{J}{s}.$$

$$\frac{d}{dt}(\rho V C_p T) = F_i \rho C_p T_i - F \rho C_p T + (-\delta H)(-r_A V) - Q.$$

$$\Rightarrow \frac{d}{dt}(VT) = F_i T_i - F T + (\delta H)\left(\frac{r_A V}{\rho C_p}\right) - \frac{Q}{\rho C_p}.$$

$$\Rightarrow T\frac{dV}{dt} + V\frac{dT}{dt} = F_i T_i - F T + (\delta H)\left(\frac{r_A V}{C_p \rho}\right) - Q.$$

$(F_i - F)T + V\frac{dT}{dt} = F_i T_i - FT + (\delta H)\left(\frac{r_A}{C_p \rho}\right) - \frac{q}{\rho C_p}$

$\Rightarrow V\frac{dT}{dt} = \frac{F_i}{V}(T_i - T) + \frac{(\delta H)\, r_A V}{\rho C_p} - \frac{q}{\rho C_p V}$

$$\frac{dT}{dt} = \frac{F_i}{V}(T_i - T) + \frac{(-\Delta H)\, k_0 C_A \exp(E/RT)}{\rho C_p} - \frac{q}{\rho C_p V}$$

$Q = F_c\, Cp_c \rho_c (T_{co} - T)$

$Q = UA(T - T_{co})$

<u>Laplace Transform</u>

$L[f(t)] = \bar{f}(s) = \int_0^\infty f(t)\, e^{-st}\, dt.$

$f(t) = 1 \qquad \frac{1}{s}$

$= t^n e^{-at}. \qquad \frac{\lfloor n}{(s+a)^{n+1}}$

$e^{-at} \sin \omega t \qquad \frac{\omega}{(s+a)^2 + \omega^2}$

$\cancel{\frac{\lfloor a}{\omega^2}}$

$e^{-at} \cosh \omega t \qquad \frac{(s+a)}{(s+a)^2 - \omega^2}.$

$$L\left[\frac{d^n f}{dt^n}\right] = s^n \bar{f}(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - f^{n-1}(0)$$

<u>Deviation / Perturbation variable</u> : $f = f_{msr} - f_{ss}$

$f(0) = 0$

$\Rightarrow L\left[\frac{d^n f}{dt^n}\right] = s^n \bar{f}(s)$, for process at steady state at $t = 0$

$L\left[\int_0^t f(t)\, dt\right] = \frac{1}{s} \bar{f}(s)$

Initial value theorem: ~~f(t)~~: $\lim_{t \to 0} f(t) = \lim_{s \to \infty} s \bar{f}(s)$

Final " " : $\lim_{t \to \infty} f(t) = \lim_{s \to 0} s \bar{f}(s)$

$f(t) \longrightarrow$ [Process] $\longrightarrow y(t)$

Forcing function $\rightarrow$ Input variable

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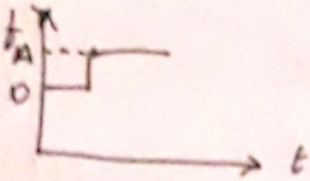
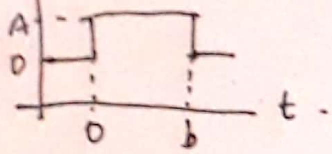
① Ideal step input



$$f(t) = \begin{cases} 0 & t < 0 \\ A & t > 0 \end{cases}$$

$F(s) = \frac{A}{s}$ For $A = 1$, $F(s) = \frac{1}{s}$ [unit step function]

② Ideal pulse / rectangular input



$$f(t) = \begin{cases} 0, & t<0 \\ A, & 0<t<b \\ 0, & t>b \end{cases}$$

$f_1 = \begin{cases} 0, & t < 0 \\ A, & t > 0 \end{cases}$; $f_2 = \begin{cases} 0, & t<0 \\ A, & t > b \end{cases}$

$= -f_1(t-b)$.

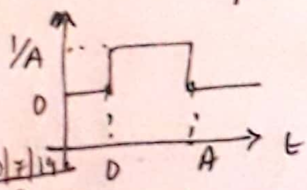
$f(t) = f_1(t) + f_2(t)$.

$= f_1(t) - f_1(t-b)$

$F(s) = F_1(s) - F_1(s) \cdot e^{-bs}$

$= \frac{A}{s} - \frac{A}{s} \cdot e^{-bs} = \frac{A}{s}(1 - e^{-bs})$.

For unit pulse function, the area must be 1.



$\therefore F(s) = \frac{1 - e^{-As}}{As} = \delta_A$

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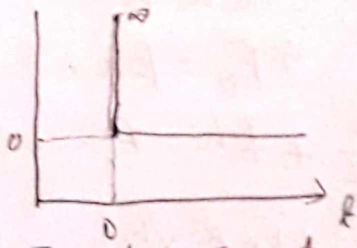
③ Impulse input

④ Ramp input.

~~Unit pulse input~~ :



Impulse Input.

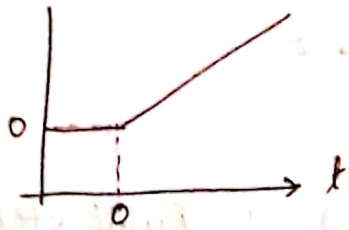
Impulse : $\begin{cases} \infty, & t = 0 \\ 0, & \text{otherwise} \end{cases}$

$\lim_{A \to 0} \delta_A(t)$.

$L[\delta(t)] = L\left[\lim_{A\to 0} \delta_A\right] = \lim_{A \to 0} L[\delta_A]$.

$= \lim_{A\to 0} L\left[\frac{1 - e^{-As}}{As}\right]$:

Ramp function:



$f(t) = At$

$L[f(t)] = \frac{A}{s^2}$

eg:

Linearization:

$\frac{dx}{dt} = f(x)$ non linear model (single variable system)

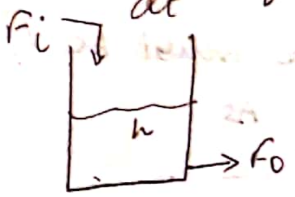
Taylor series:

$f(x) = f(x_0) + \frac{df}{dx}\Big|_{x_0} \frac{(x-x_s)}{1!} + \frac{d^2 f}{dx^2}\Big|_{x_0} \frac{(x-x_s)^2}{2!} + \dots$

$\longrightarrow$ Neglecting

$f(x) = \frac{dx}{dt} = f(x_s) + \frac{df}{dx}\Big|_{x_s} (x - x_s) \quad \dots (1)$

Ex:



Model:

$A\frac{dh}{dt} = F_i - F_0$

Case 1: $F_0 \propto h$.

$\Rightarrow F_0 = \beta h$.

$A\frac{dh}{dt} + \beta h = F_i$ ... linear

Case 2: $F_0 \propto \sqrt{h}$.

$F_0 = \cancel{\beta h}\ \alpha\sqrt{h}$.

$A\frac{dh}{dt} + \alpha\sqrt{h} = F_i$ .... non-linear.

$\cancel{\sqrt{h}}$ $f(h) = \sqrt{h}$. $\frac{1}{2\sqrt{h}}$

$f(x) = f(x_s) + \left(\frac{df}{dx}\right)_{x=x_s} (x - x_s)$

$\sqrt{h} = \sqrt{h_0} + \frac{1}{2\sqrt{h_0}}(h - h_0)$.

$\Rightarrow \sqrt{h} - \sqrt{h_0} = \frac{1}{2\sqrt{h_0}}(h-h_0) \Rightarrow \cancel{2\sqrt{h h_0}} - 2h_0 = h - h_0$.

$\Rightarrow h - 2\sqrt{h h_0} + h_0 = 0$

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$$A\frac{dh}{dt} + \alpha\sqrt{h_0} + \frac{\alpha}{2\sqrt{h_0}}(h - h_0) = F_i$$

$$\Rightarrow A\frac{dh}{dt} + \frac{\alpha}{2\sqrt{h_0}}h = F_i + \frac{\alpha\sqrt{h_0}}{2} - \alpha\sqrt{h_0}$$

$$= F_i - \frac{\alpha\sqrt{h_0}}{2}.$$

Dynamic model :

$$f(x) : \frac{dx}{dt} = f(x_s) + \left(\frac{df}{dx}\right)_{x=x_s}(x - x_s) \ldots ①.$$

Steady state model :

$$\frac{dx_s}{dt} = f(x_s). \ldots ②$$

① − ②

$$\frac{dx'}{dt} = \left(\frac{df}{dx}\right)_{x_s} x' \quad \ldots ③$$ [Linearized model for deviation variable]

$$A\frac{dh}{dt} = F_i - F_o.$$

$$A\frac{dh}{dt} + \frac{\alpha}{2\sqrt{h_0}}h = F_i - \frac{\alpha\sqrt{h_0}}{2}.$$

Steady state,

$$A\frac{dh_s}{dt} + \frac{\alpha}{2}\frac{h_s}{\sqrt{h_s}} = F_{i,s} - \frac{\alpha\sqrt{h_s}}{2}.$$

$$A\frac{dh'}{dt} + \frac{\alpha}{2\sqrt{h_s}}(h - h_s) = 0 ..$$

$$\Rightarrow A\frac{dh'}{dt} = -\frac{\alpha}{2\sqrt{h_s}}h' + F_i'$$

<u>Multivariable system</u> :

$$\frac{dx_1}{dt} = f_1(x_1, x_2)$$

$$\frac{dx_2}{dt} = f_2(x_1, x_2)$$

$$f_1(x_1, x_2) = f_1(x_{1s}, x_{2s}) + \left(\frac{\partial f_1}{\partial x_1}\right)_{x_{1s}, x_{2s}}\frac{(x_1 - x_{1s})}{1!} + \left(\frac{\partial f_1}{\partial x_2}\right)\frac{(x_2 - x_{2s})}{1!}$$

$$+ \left(\frac{\partial^2 f_1}{\partial x_1^2}\right)_{x_s}\frac{(x_1 - x_{1s})^2}{2!} + \left(\frac{\partial^2 f_1}{\partial x_2^2}\right)_{x_s}\frac{(x_2 - x_{2s})^2}{2!} + \frac{\partial^2 f_1}{\partial 2}$$

Neglecting

$$\left(\frac{\partial^2 f_1}{\partial x_1 \partial x_2}\right)_{x_s}(x_1 - x_{1s})(x_2 - x_{2s})$$

$$\frac{dx_1}{dt} = f_1(x_{1s}, x_{2s}) + \left(\frac{\partial f_1}{\partial x_1}\right)_{x_s}(x_1 - x_{1s}) + \left(\frac{\partial f_1}{\partial x_2}\right)_{x_s}(x_2 - x_{2s})$$

$$\frac{dx_2}{dt} = f_2(x_{1s}, x_{2s}) + \left(\frac{\partial f_2}{\partial x_1}\right)_{x_s}(x_1 - x_{1s}) + \left(\frac{\partial f_2}{\partial x_2}\right)_{x_s}(x_2 - x_{2s})$$

Dynamic

$$\frac{dx_{1s}}{dt} = f_1(x_{1s}, x_{2s}) + \cancel{\partial f_1}\ 0.$$

$$\therefore \frac{dx_1'}{dt} = \left(\frac{\partial f_1}{\partial x_1}\right)_{x_s} x_1' + \left(\frac{\partial f_1}{\partial x_2}\right)_{x_s} x_2'.$$

$$\frac{dx_2'}{dt} = \left(\frac{\partial f_2}{\partial x_1}\right)_{x_s} x_1' + \left(\frac{\partial f_1}{\partial x_2}\right)_{x_s} x_2'$$

$$\frac{dx_1'}{dt} = a_{11} x_1' + a_{12} x_2'$$

$$\frac{dx_2'}{dt} = a_{21} x_1' + a_{22} x_2'.$$

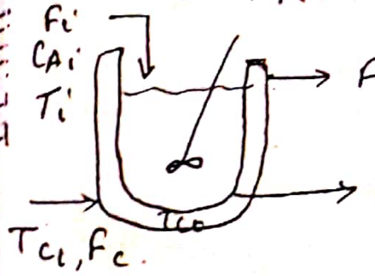
$$a_{ij} = \left(\frac{\partial f_i}{\partial x_j}\right)_{x_s}.$$

$$\begin{bmatrix} \dot{x}_1' \\ \dot{x}_2' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \end{bmatrix}$$

a_{ij}: row, column

Ex: CSTR.

Model:



Assuming constant volume,

$$\frac{dC_A}{dt} = \frac{F_i}{V}(C_{Ai} - C_A) - K_0 C_A e^{-E/RT}$$

$$\frac{dT}{dt} = \frac{F_i}{V}(T_i - T) - \frac{Q}{\rho C_p V} + \frac{(-\Delta H) K_0 C_A e^{-E/RT}}{\rho C_p}$$

Using Taylor series.

$$\cancel{f(C_A, T) = C_A e^{-E/RT}}.$$

$$\frac{dC_A}{dt} = \cancel{\left(C_A e^{-E/RT}\right)_{T_s, C_{As}} +}$$

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$$\frac{dC_A}{dt} = f_1(C_A, T)$$

$$\frac{dT}{dt} = f_2(C_A, T)$$

$$\cancel{f_1(C_A, T) =}$$

$$f(C_A, T) = C_A e^{-E/RT}$$

$$C_A e^{\frac{-E}{RT}} = \left(C_A e^{-E/RT}\right)_{T_s, C_{As}} + \frac{\partial (C_A e^{-E/RT})}{\partial C_A}(C_A - C_{As})$$

$$+ \frac{\partial (C_A e^{-E/RT}}{\partial T}\Bigg|_{T_s, C_{As}} (T - T_s)$$

$$\Rightarrow C_A e^{-E/RT} = C_{As} e^{-E/RT_s} + e^{-E/RT_s}(C_A - C_{As})$$

$$\cancel{+ C_A\left(\frac{-E}{RT_s}\right)}$$

$$+ C_{As} e^{-E/RT_s}\left(\frac{E}{RT_s^2}\right)(T - T_s)$$

$$\cancel{\frac{dC_A}{dt} = \frac{F_i}{V}(C_{Ai} - C_A) - k_0}$$

$$\frac{dC_A}{dt} = \frac{F_i}{V}(C_{Ai} - C_A) - k_0 C_{As} e^{-E/RT_s} - k_0 e^{-E/RT_s}(C_A')$$

$$- k_0 C_{As} e^{-E/RT_s}\left(\frac{E}{RT_s^2}\right) T'$$

$$\frac{dC_A'}{dt} = -\frac{F_i}{V} C_A' - k_0 e^{-E/RT_s} C_A' - \cancel{k_0 C_{As} e^{-E/RT_s}}$$

$$- k_0 C_{As} e^{-E/RT_s}\left(\frac{E}{RT_s^2}\right) T' + \frac{F_i}{V}(C_{Ai}').$$

$$\frac{dT}{dt} = \frac{F_i}{V}(T_i - T) - \frac{Q}{\rho C_p V} + \frac{(-\Delta H) k_0}{\rho C_p}\Big[C_{As} e^{-E/RT_s}$$

$$+ e^{-E/RT_s}(C_A - C_{As}) + C_{As} e^{-E/RT_s}\left(\frac{E}{RT_s^2}\right)(T - T_s)\Big]$$

$$\cancel{\frac{dT'}{dt} = \frac{F_i'}{V} T' - \frac{Q'}{\rho C_p}}$$

$$\frac{dT'}{dt} = \frac{F_i}{V}(T_i' - T') - \frac{Q'}{\rho C_p V} + \frac{(-\Delta H) k_0}{\rho C_p}\Big[e^{-\frac{E}{RT_s}} C_A'$$

$$+ C_{As} e^{-E/RT_s}\left(\frac{E}{RT_s^2}\right)(T')\Big]$$

$$\tau = \frac{V}{F_i}$$

$$\frac{dC_A'}{dt} = \frac{1}{\tau}(C_{Ai}' - C_A') - K_0 e^{-E/RT_s} C_A' - \frac{K_0 C_{As} E}{RT_s^2}$$

$$\frac{dT'}{dt} = \frac{1}{\tau}(T_i' - T') - \frac{Q'}{\rho C_p V} - \frac{K \Delta H}{\rho C_p} e^{-E/RT_s}\left(C_A' + \frac{C_{As} E}{RT_s^2}\right)$$

Transfer function:

$$f(t) \longrightarrow \boxed{\text{Process}} \longrightarrow y(t)$$

SISO system
(single input single output)

$$\bar{f}(s) \longrightarrow \boxed{G(s)} \longrightarrow \bar{y}(s) \qquad \text{Block giagram}$$

$$G(s) = \frac{\bar{y}(s)}{\bar{f}(s)} \equiv TF$$

SISO (linearized)

$s^{n-1}\bar{y}(s)$ $\quad s\bar{y}(s)$ $\quad a_0\bar{y}(s)$

$$a_n \frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \dots + a_1 \frac{dy}{dt} + a_0 y = b f(t)$$

$s^n \bar{y}(s)$

$y, f \Rightarrow$ deviation variables.

$$\frac{\bar{y}(s)}{\bar{f}(s)} = \frac{b}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0} = G(s) = TF.$$

MISO system

$$\begin{matrix} f_1(t) \\ f_2(t) \end{matrix} \rightrightarrows \boxed{\text{Process}} \longrightarrow y(t)$$

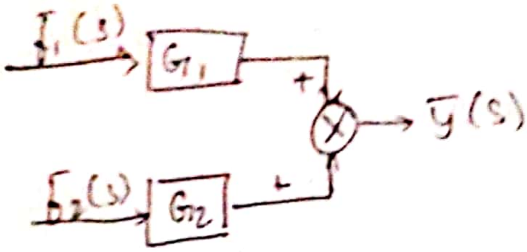
$$a_n \frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \dots + a_1 \frac{dy}{dt} + a_0 y = b_1 f_1(t) + b_2 f_2(t)$$

$$\cancel{\frac{\bar{y}(s)}{\bar{f}_1(s)}} \quad \bar{y}(s) = \frac{b_1 \bar{f}_1(s) + b_2 \bar{f}_2(s)}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}$$

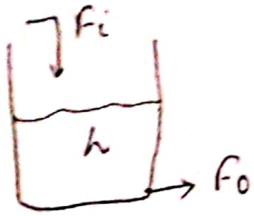
$$\bar{y}(s) = G_1(s)\,\bar{f}_1(s) + G_2(s)\,\bar{f}_2(s).$$

$$G_1(s) = \frac{b_1}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}$$

$$G_2(s) = \frac{b_2}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}$$



Ex :



Model : $A\frac{dh}{dt} = F_i - F_o$.

Develop
(a) Transfer function
(b) Block diagram .

CV	MV
h	F_o

$f(t) = F_o$; $y(t) = h$.

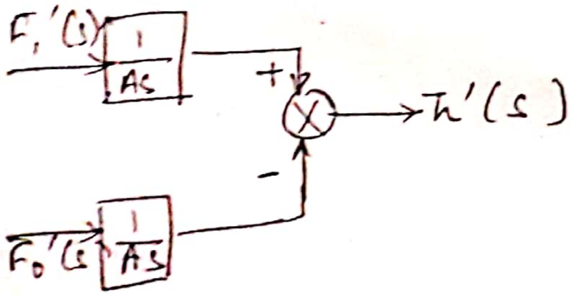
$A\frac{dy}{dt} = F_i - f(t)$.

$A s\,\bar{y}(s) = \frac{F_i}{s} - \bar{f}(s)$.

$A\frac{dh'}{dt} = F_i' - F_o'$

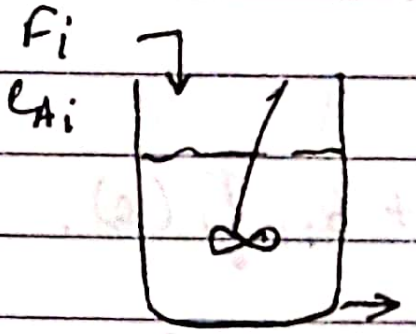
$h'(s) = \frac{1}{As}\bar{F}_i'(s) - \frac{1}{As}\bar{F}_o'(s)$.

$G_1 = \frac{1}{As}$; $G_2 = \frac{1}{As}$



Transfer function:

Ex 2: Isothermal reactor.



$A \rightarrow B$

1st order.

For $F_i = F =$ const.

$$A\rho \frac{dh}{dt} = F_i - F \qquad \therefore A\rho\frac{dh}{dt} = 0.$$

$$\cancel{C_A A\rho \frac{dh}{dt} = F_i C_{Ai} - F C_A}$$

$$\cancel{F_i C_{Ai} - F C_A = C_A V \frac{dV}{dt}}$$

$$\frac{d(VC_A)}{dt} = F_i C_{Ai} - F C_A + R_A V$$

$$= F_i C_{Ai} - F C_A - K C_A V$$

$$\frac{dC_A}{dt} = \frac{F_i C_{Ai} - F C_A}{V} + K C_A.$$

$$G(s) = \frac{\frac{1}{\tau}}{s + \frac{1}{\tau} + K} = \frac{C_A'(s)}{C_{Ai}'(s)}$$

MIMO system (2×2)

$$\begin{bmatrix} \frac{dy_1}{dt} \\ \frac{dy_2}{dt} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

No

$$\frac{dy_1}{dt} = a_{11}y_1 + a_{12}y_2 + b_{11}f_1 + b_{12}f_2 .$$

$$\frac{dy_2}{dt} = a_{21}y_1 + a_{22}y_2 + b_{21}f_1 + b_{22}f_2 .$$

$$y_1(0) = y_2(0) = 0 .$$

$$\bar{y}_1(s) = G_{11}\bar{f}_1(s) + G_{12}\bar{f}_2(s) .$$
$$\bar{y}_2(s) = G_{21}\bar{f}_1(s) + G_{22}\bar{f}_2(s)$$

$$s\bar{y}_1(s) = a_{11}\bar{y}_1(s) + a_{12}\bar{y}_2(s) + b_{11}\bar{f}_1(s) + b_{12}\bar{f}_2(s) .$$
$$s\bar{y}_2(s) = a_{21}\bar{y}_1(s) + a_{22}\bar{y}_2(s) + b_{21}\bar{f}_1(s) + b_{22}\bar{f}_2(s)$$

$\cancel{s\bar{y}_2(s) = a_{12}y}$

$\cancel{s^2 y_2}$

$$s\bar{y}_2(s) = \frac{a_{12}\bar{y}_2(s) + b_{11}\bar{f}_1(s) + b_{12}\bar{f}_2(s)}{s - a_{11}} \cdot a_{21} + a_{22}\bar{y}_2(s) + b_{21}\bar{f}_1(s) + b_{22}\bar{f}_2(s) ,$$

$$\Rightarrow (s - a_{22} - a_{21}\cdot a_{12})\bar{y}_2(s) = \cancel{b_{11}+}$$
$$= \cancel{b_{11} - b_{2}+}$$
$$\left(\frac{b_{11}a_{21}}{s - a_{11}} + b_{21}\right)\bar{f}_1(s) + \left(\frac{b_{12}a_{21}}{s - a_{11}} + b_{22}\right)\bar{f}_2(s)$$

$$G_{21} = \frac{(s - a_{11})b_{21} + a_{21}b_{11}}{P(s)}$$

$$G_{22} = \frac{b_{22}(s - a_{11}) + b_{12}a_{21}}{P(s)}$$

$$P(s) = s^2 - (a_{11} + a_{22})s - (a_{12}a_{21} - a_{11}a_{22})$$

No

$$s\bar{y}_1(s) - a_{11}\bar{y}_1(s) = \frac{a_{12}}{s-a_{22}-a_{21}.a_{12}}\left(\frac{b_{11}a_{21}}{s-a_{11}}+b_{21}\right)f_1(s)$$

$$+\frac{b_{12}}{s-a_{22}-a_{12}.a_{21}}\left(\frac{b_{12}a_{21}}{s-a_{11}}+b_{22}\right)f_2(s)$$

$$+b_{11}f_1(s)+b_{12}f_2(s).$$

$$G_{11}(s) = \frac{(s-a_{22})b_{11}+a_{12}b_{21}}{P(s)}$$

$$G_2(s) = \frac{(s-a_{22})b_{12}+a_{12}b_{22}}{P(s)}$$

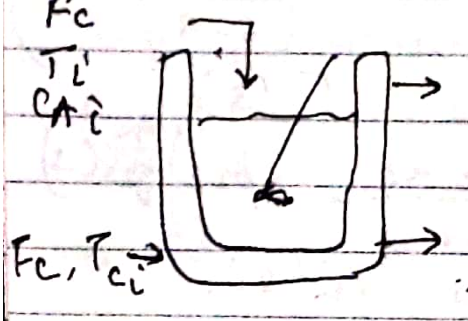
$$\frac{a_{12}b_{11}a_{21}+b_{21}a_{12}(s-a_{11})}{P(s)}+b_{11}$$

$$\frac{a_{12}b_{11}a_{21}+b_{22}a_{12}(s-a_{11})+b_{11}(s-a_{11})(s-a_{22}-a_{21}a_{12})}{P(s)}$$

$$= a_{12}b_{11}a_{21}+b_{21}a_{12}(s-a_{11})+b_{11}(s-a_{22})(s-a_{11})$$

No

Ex - 3 - CSTR



$q = UA(T - T_c)$

$S = \dfrac{-\Delta H}{\rho C_p}$

$\tau = \dfrac{V}{F_i}$

$$\frac{dC_A'}{dt} + \left[\frac{1}{\tau} + k_0 e^{-E/RT_0}\right] C_A' + \left[\frac{k_0 E}{RT_0^2} e^{-E/RT_0} C_{A0}\right] T' = \frac{1}{\tau} C_{Ai}'$$

$$\frac{dT'}{dt} + \left[\frac{1}{\tau} - \frac{S k_0 E}{RT_0^2} e^{-E/RT_0} C_{A0} + \frac{UA}{V\rho C_p}\right] T' - \left[S k_0 e^{-E/RT_0}\right] C_A' = \frac{1}{\tau} T_i' + \frac{UA}{V\rho C_p} T_c'$$

$$\frac{dC_A'}{dt} + a_{11} C_A' + a_{12} T' = b_1 C_{Ai}'$$

$$\frac{dT'}{dt} + a_{21} C_A' + a_{22} T' = b_1 T_i' + b_2 T_c'$$

$$\bar{C}_A'(s) = G_{11} \bar{C}_{Ai}'(s) + G_{12}(s) \bar{T}_i'(s) + G_{13} \bar{T}_c'$$

$$\bar{T}'(s) = G_{21} \bar{C}_{Ai}'(s) + G_{22} \bar{T}_i'(s) + G_{23} \bar{T}_c'(s)$$

$$s\bar{C}_A'(s) + a_{11} \bar{C}_A'(s) + a_{12} \bar{T}'(s) = b_1 \bar{C}_{Ai}'(s) \;\&$$

$$s\bar{T}'(s) + a_{21} \bar{C}_A'(s) + a_{22} \bar{T}'(s) = b_1 \bar{T}_i'(s) + b_2 \bar{T}_c'(s)$$

Zeroes & Poles

5/2/19

$$\xrightarrow{f(s)} \boxed{G(s)} \rightarrow y(s)$$

$\frac{\bar{y}(s)}{\bar{f}(s)} = G(s) = \frac{Q(s)}{P(s)}$ = ratio of two polynomials

root of $Q(s) \equiv$ "zeroes" $\quad Q(s) = 0$

$s = ???$

root of $P(s) \equiv$ "poles" $\quad P(s) = 0$

$s = ??$

$G(s) = \frac{s-1}{(s-2)(s-3)}$ $\quad$ zeroes = 1 @ $G(s) = 0$

poles = 2, 3 @ $G(s) \to \infty$

General

$$a_n \frac{d^n y}{dy^n} + a_{n-1} \frac{d^{n-1} y}{dy^n} + \dots + a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m f}{dt^m} + b_{m-1} \frac{d^{m-1} f}{df^m} + \dots + b_1 \frac{df}{dt} + b_0 f$$

$y(0) = f(0) = 0$

$$\frac{\bar{y}(s)}{\bar{f}(s)} = \frac{b_m(s-z_1)(s-z_2)\dots(s-z_m)}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}$$

$$= \frac{b_m (s-z_1)(s-z_2)\dots(s-z_m)}{a_n (s-p_1)(s-p_2)\dots(s-p_n)} = \frac{Q(s)}{P(s)}$$

Order of $Q(s) = m$.

" " $P(s) = n$.

$n < m \qquad n = 0\ ;\ m = 1$.

From eq^n (1), $\quad a_0 y = b_1 \frac{df}{dt} + b_0 f$.

$$\bar{y}(s) = \frac{b_1}{a_0} s \bar{f}(s) + \frac{b_0}{a_0} \bar{f}(s)$$

$\bar{y}(s) = \frac{b_1}{a_0} + \frac{b_0}{a_0} \frac{1}{s}$ [Putting $\bar{f}(s) = \frac{1}{s}$].

$$y(t) = \frac{b_1}{a_0} \delta(t) + \frac{b_0}{a_0}$$

This is physically not possible.

The step change is 1 & change in $y(t)$ is infinite.

This is called improper system.

For $n > m$, $\Rightarrow$ Proper system. ~~$Z_{p_1}s$ bc~~

$$\frac{\bar{y}(s)}{\bar{f}(s)} : \frac{b_1}{a_0}s + \frac{b_0}{a_0} = \frac{Z_{p_1}s+1}{Z_{p_2}s+1}$$

For $n = m$ $\Rightarrow$ semiproper.

Cumulative Analysis : Response

$$\bar{f}(s) \rightarrow \boxed{G(s)} \rightarrow \bar{y}(s) \quad ; \quad \bar{y}(s) = G(s).\bar{f}(s)$$

$\bar{f}(s)$ is known as the change introduced is known to us.

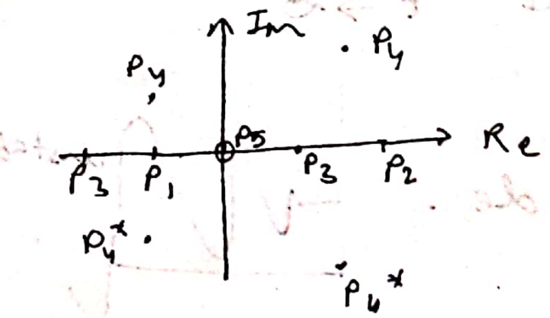
$G(s)$ is known for the system.

$$\bar{y}(s) = \frac{Q(s)}{P(s)} \cdot \frac{R(s)}{q(s)}$$

$y(t) = ??$

By ~~k~~ knowing the poles, we can know if the system is stable or not & determine qualitative characteristics of process response against a particular change of input without additional computation

$$G(s) = \frac{C_1}{s-P_1} + \frac{C_2}{s-P_2} + \left\{\frac{G_{31}}{s-P_3} + \frac{G_{32}}{(s-P_3)^2} + \dots + \frac{G_{3m}}{(s-P_3)^m}\right\}$$
$$+ \frac{C_4}{s-P_4} + \frac{C_4^*}{s-P_4^*} + \frac{C_5}{s-P_5}$$



Case 1: Two distinct real poles

$$G(s) = \frac{C_1}{s-P_1} + \frac{C_2}{s-P_2}$$

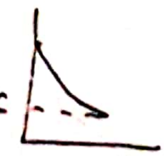
$$V = L^{-1}[G(s)] = C_1 e^{P_1 t} + C_2 e^{P_2 t}$$

i) $P_1 < 0$ $\quad t \rightarrow \infty \quad C_1 e^{P_1 t} \rightarrow 0$.

ii) $P_2 > 0$ $\quad t \rightarrow \infty \quad C_2 e^{P_2 t} \rightarrow \infty$

two poles are $-ve$ @ $V = 0$ ss

$\Rightarrow$ stable



If both poles are +ve @ $v \to \infty$ unstable.

one +ve, one −ve @ $v \to \infty$ "

Case 2: Multiple real poles.

$$G(s) = \frac{G_{31}}{s-P_3} + \frac{G_{32}}{(s-P_3)^2} + \dots + \frac{G_{3m}}{(s-P_3)^m}$$

$$\cancel{L^{-1}[G(s)] = G_{31}\, e^{P_3 t} + \frac{G_{32}\, t\, e}{\lfloor 1}}$$

$$L^{-1}[G(s)] = \left[G_{31} + \frac{G_{32}\, t}{\lfloor 1} + \frac{G_{33}\, t^2}{\lfloor 2} + \dots + \frac{G_{3m}\, t^{m-1}}{\lfloor m-1}\right] e^{P_3 t}$$

$t \to \infty$ $\qquad \downarrow \to \infty$

(i) $P_3 > 0$ $\quad t \to \infty\ e^{P_3 t} \to \infty$ $\quad v \to \infty$ unstable.

(ii) $P_3 < 0$ $\quad t \to \infty$ $\quad e^{P_3 t} \to 0$ $\quad v \to 0$. stable.

Case 3: ~~Two complex~~ conjugate pole

$$G(s) = \frac{C_4}{s - P_4} + \frac{C_4^*}{s - P_4^*}$$

$P_4 = \alpha + j\beta$; $P_4^* = \alpha - j\beta$

$$L^{-1}[G(s)] = (\quad) e^{\alpha t} \sin(\beta t + \phi)$$

① $\alpha > 0$ Right half plane.

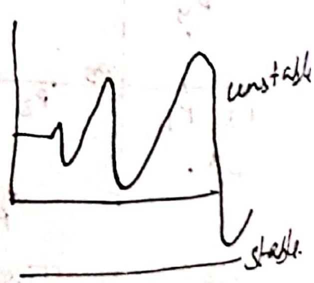
$t \to \infty$ RHP ~~$t \to \infty$~~

② $\alpha < 0$ Left half plane $t \to 0$.

③ $\alpha = 0$ $t \to \infty$

oscillate with constant amplitude.

(marginally stable)



④ pole is at origin

$$G(s) = \frac{C_5}{s - P_5} = \frac{C_5}{s}$$

$$L^{-1}[G(s)] = C_5$$

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6/8/19 First order system

$f \rightarrow \boxed{\text{System}} \rightarrow y$

$a_1 \frac{dy}{dt} + a_0 y = b f(t)$ $a, b \rightarrow$ constant
$y, f \rightarrow$ deviation variable

Case 1: $a_0 \neq 0$

$\frac{a_1}{a_0}\frac{dy}{dt} + y = \frac{b}{a_0} f(t)$ $\frac{a_1}{a_0} = \tau_p =$ time constant

$\tau_p \frac{dy}{dt} + y = K_p f(t)$. $\frac{b}{a_0} = K_p =$ steady state gain or static gain

$\tau_p s \bar{y}(s) + \bar{y}(s) = K_p \bar{f}(s)$.

$\Rightarrow \boxed{\frac{\bar{y}(s)}{\bar{f}(s)} = \frac{K_p}{s\tau_p + 1} = G(s)} \equiv$ T.F. of 1st order lag system

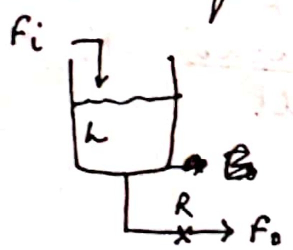
$\left(\frac{\Delta \text{output}}{\Delta \text{input}}\right)_{ss} = \frac{b}{a_0} = \frac{y}{f(t)}$ [as $\frac{dy}{dt} = 0$ at ss]

Case 2: $a_0 = 0$ [Purely integrator / purely capacitive]

$a_1 s \bar{y}(s) = b \bar{f}(s)$

$\frac{\bar{y}(s)}{\bar{f}(s)} = \frac{b}{a_1 s} = \frac{K_p'}{s}$; $K_p' = \frac{b}{a_1}$

Ex 1: Liquid tank system.



$A\frac{dh}{dt} = F_i - F_0$; $F_0 = \frac{h}{R}$

$\Rightarrow A\frac{dh}{dt} = F_i - \frac{h}{R}$

$\Rightarrow AR\frac{dh'}{dt} + h' = F_i' R$

$\tau_p = AR$

$\Rightarrow \frac{dh'}{dt} + \frac{h'}{AR} =$

$K_p = R$

$R \rightarrow \frac{s}{m^2}$

$AR = s \Rightarrow m^2 R = s \Rightarrow R = \frac{s}{m^2}$

$\frac{\bar{h}'(s)}{\bar{F}_i'(s)} = \frac{K_p}{\tau_p s + 1}$

$\tau_p = AR =$ (storage capacitance) X (Resistance).

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Ex 2 :

V = constant
∄ no phase change
No heat loss.

$$Q = \rho V \cancel{dt} C_p \frac{dT}{dt}$$

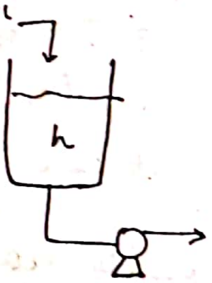
$$\Rightarrow \rho V C_p \frac{dT}{dt} = UA_t (T_{st} - T)$$

$$\Rightarrow \rho V C_p s \bar{T}(s) = UA_t [\bar{T}_{st}(s) - \bar{T}(s)]$$

$$\Rightarrow \frac{\bar{T}(s)}{\bar{T}_{st}(s)} = \frac{UA_t}{\rho V C_p s + UA_t} = \frac{1}{\frac{\rho V C_p s}{UA_t} + 1} = \frac{K_p}{\tau_p s + 1}$$

$K_p = 1$; $\tau_p = \frac{(\rho V C_p)}{UA_t}$ = storage capacitance × Resistance

Ex-3 : Pure integrator



constant displacement pump (RPM is fixed)

$$A \frac{dh}{dt} = F_i - F_0$$

$$A \frac{dh_0}{dt} = F_{i_0} - F_0$$

$$A \frac{d(h - h_0)}{dt} = F_i - F_{i_0}$$

Model : $A \frac{dh'}{dt} = F_i' \cancel{F_0}$

$$\frac{\bar{h}'(s)}{\bar{F}_i'(s)} = \frac{1/A}{s} = \frac{K_p'}{s}$$

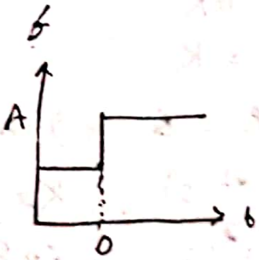
Dynamic response

$$\frac{\bar{y}(s)}{\bar{f}(s)} = \frac{K_p'}{s} \quad \ldots \text{Pure Integrator}$$

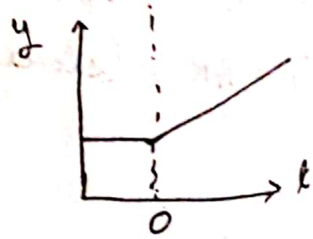
$$\bar{y}(s) = \frac{K_p'}{s} \bar{f}(s) \quad ; \quad \bar{f}(s) = \frac{A}{s}$$

$$= \frac{K_p'}{s} \cdot \frac{A}{s} = \frac{K_p' A}{s^2}$$

$$y(t) = t K_p' A$$



Non-self regulating system



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$$\frac{\bar{y}(s)}{\bar{f}(s)} = \frac{K_p}{\tau_p s + 1}$$

$$\bar{y}(s) = \frac{K_p}{\tau_p s + 1}\,\bar{f}(s) \quad ; \quad \bar{f}(s) = \frac{A}{s}$$

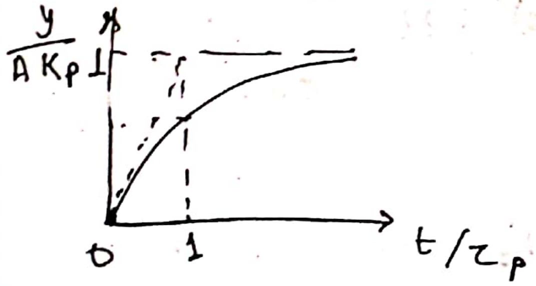
$$\cancel{\bar{y}(s)} \quad \cancel{y(t)}\ \bar{y}(s) = \frac{K_p}{\tau_p s + 1} \cdot \frac{A}{s}.$$

$$= \cancel{\frac{\tau_p}{\tau_p s + 1} - \frac{K_p A}{s}}$$

$$= -K_p A \left[\frac{\tau_p}{\tau_p s + 1} - \frac{1}{s} \right]$$

$$= K_p A \left[\frac{1}{s} - \frac{\tau_p}{\tau_p s + 1} \right]$$

$$y(t) = K_p A \left[1 - e^{-\frac{t}{\tau_p}} \right]$$



$\cancel{K_p} =$

$$\text{Slope} = \left[\frac{d(y/AK_p)}{d(t/\tau_p)} \right]_{t=0}$$

$$= 1 = \tan\theta.$$

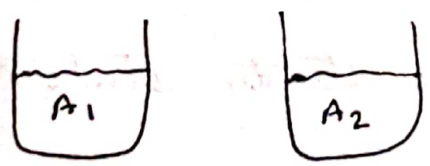
$$\Rightarrow \frac{t}{\tau_p} = 1 \Rightarrow t = \tau_p.$$

If initial rate of change of y were to be maintained, the response would reach its final value in ~~at~~ one time constant.

$$y(\tau_p) = K_p A \left[1 - \frac{1}{e} \right] = 0.632\, K_p A$$

t/τ_p	1	2	3	4	5
y/AK_p	63.2	86.5	95	98	99

Problem 1 :

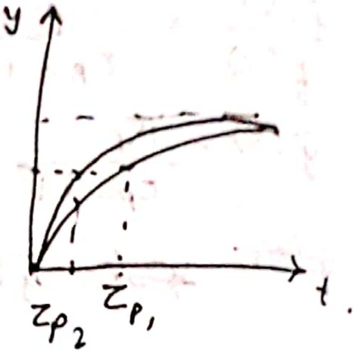


$A_1 > A_2$
$R_1 = R_2$.

K_p
τ_p.

$$A_1 R_1 \frac{dh'}{dt} + h' = F_i' R$$

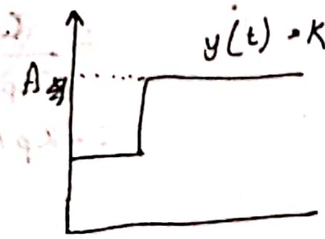
$$A_2 R_2 \frac{dh'}{dt} + h' = F_i' R$$



$$y_1(t) = K_p A [1 - e^{-t/A_1R_1}]$$

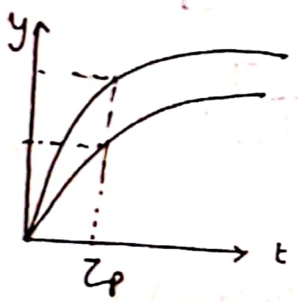
$$y_2(t) = K_p A [1 - e^{-t/A_2R_2}]$$

$y(t) = K_pA(1-e^{-t/\tau_p})$



$A_1 > A_2 \quad R_1 \neq R_2$

$\tau_{p_1} = \tau_{p_2} \rightarrow R_1 < R_2$.



Model : $A\frac{dh}{dt} = F_i - F_o$; $F_o \propto \sqrt{h}$.

Find $G(s) = ??$

K_p : ; τ_p :

$$\cancel{A\frac{dh'}{dt} = F_i'}$$

$$A\frac{dh}{dt} + \frac{\alpha}{2\sqrt{h_o}} h = F_i - \frac{\alpha\sqrt{h_o}}{2}$$

$$A\frac{dh'}{dt} + \frac{\alpha}{2\sqrt{h_o}} h' = F_i'$$

$$A s \bar{h}'(s) + \frac{\alpha}{2\sqrt{h_o}} \bar{h}'(s) = F_i'(s)$$

$$\Rightarrow \frac{\bar{h}'(s)}{\bar{F}_i'(s)} = \frac{1}{As + \frac{\alpha}{2\sqrt{h_o}}} = \frac{2\sqrt{h_o}/\alpha}{\frac{2\sqrt{h_o}A}{\alpha}s + 1}$$

$$\tau_p = \frac{2\sqrt{h_o}A}{\alpha}$$

$$K_p = \frac{2\sqrt{h_o}}{\alpha}$$

τ_p & K_p vary with the steady state values.

8/8/19 2nd order system : whose output model can be represented by a 2nd order D.E.

$$a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = bf(t).$$

$a_0 \neq 0$; $\frac{a_2}{a_0} \frac{d^2y}{dt^2} + \frac{a_1}{a_0} \frac{dy}{dt} + y = bf(t)$

$\frac{a_2}{a_0} = \tau^2$; τ = natural period of oscillation

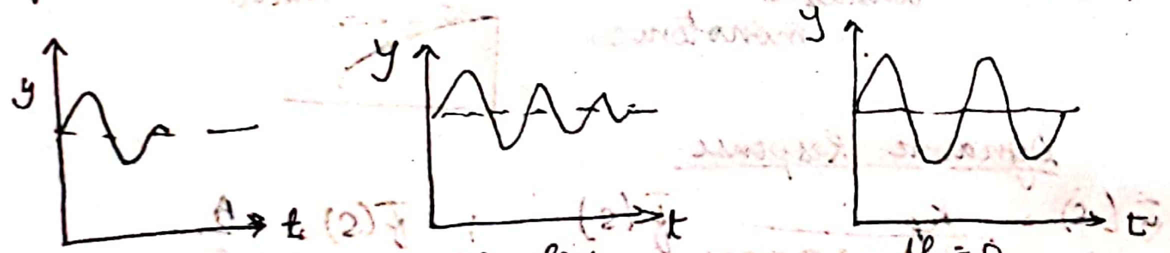
$\frac{a_1}{a_0} = 2\psi\tau$; ψ = damping factor.

$\frac{b}{a_0} = K_p$; K_p = static gain .

$$\frac{\bar{y}(s)}{\bar{f}(s)} = \frac{b}{a_2 s^2 + a_1 s + a_0} = \frac{\overbrace{b/a_0}^{K_p}}{\underbrace{\frac{a_2}{a_0}}_{\tau^2} s^2 + 2\psi\tau s + 1}$$

$$\boxed{\frac{\bar{y}(s)}{\bar{f}(s)} = \frac{K_p}{\tau^2 s^2 + 2\psi\tau s + 1}}$$ ← T.F. for 2nd order system

ψ provides measure for the amount of damping in the process response. This is the degree of oscillation ~~of~~ in the process response against a ~~pertur~~ perturbation.



$\psi \to$ more $\qquad \psi$ = less $\qquad \psi = 0 \Rightarrow$ undamped.

$\psi > 0 \Rightarrow$ oscillation with decreasing amplitude

$\psi < 0 \Rightarrow$ " " increasing "

Ex of 2nd order :

① 1st order + 1st - order.

② Inherently ⇒ eqn having accn .

③ 1st order process + controller

$$\frac{\bar{y}(s)}{\bar{f}(s)} = \frac{K_p}{\tau^2 s^2 + 2\psi\tau s + 1} = \frac{Q(s)}{P(s)} \quad ; \quad \tau^2 s^2 + 2\psi\tau s + 1 = 0$$

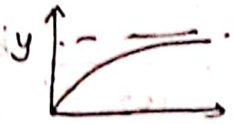
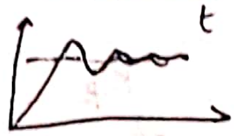
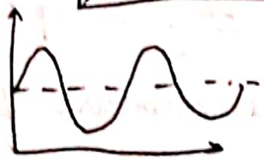
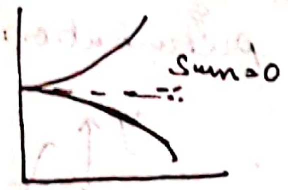
$$s = \frac{-2\psi\tau \pm \sqrt{(2\psi\tau)^2 - 4\tau^2}}{2\tau^2}$$

$$s = -\frac{\varphi}{\tau} \pm \frac{\sqrt{\varphi^2 - 1}}{\tau}$$

$$\frac{\bar{y}(s)}{\bar{f}(s)} = \frac{K_p/\tau^2}{(s-p_1)(s-p_2)}$$

1) $\varphi > 1$ — two distinct poles — Overdamped
2) $\varphi = 1$ — equal poles $\left(-\frac{1}{\tau}\right)$ — Critically damped
3) $\varphi < 1$ — conjugate poles — Underdamped

$$p = -\frac{\varphi}{\tau} \pm i\frac{\sqrt{1-\varphi^2}}{\tau}$$

φ	system	nature
$\varphi \geq 1$	stable & monotonic	
$0 < \varphi < 1$	stable & oscillating	
$\varphi = 0$	Undamped (marginally stable)	
$-1 < \varphi < 0$	system unstable & oscillatory.	
$\varphi < 1$	unstable & monotonic	

Dynamic Response

$$\bar{y}(s) = \frac{K_p}{\tau^2 s^2 + 2\varphi\tau s + 1}\bar{f}(s) \quad ; \quad \bar{f}(s) = \frac{A}{s}$$

$$= \frac{K_p}{\tau^2 s^2 + 2\varphi\tau s + 1} \cdot \frac{A}{s}$$

$\varphi > 1$

$$y(t) = K_p A\left[1 - e^{-\varphi t/\tau}\left\{\cosh\left(\frac{\sqrt{\varphi^2-1}}{\tau}t\right) + \frac{\varphi}{\sqrt{\varphi^2-1}}\sinh\frac{\sqrt{\varphi^2-1}}{\tau}t\right\}\right]$$

(Overdamped) 1st + 1st = 2nd

$0 < \varphi < 1$

$$y(t) = K_p A\left[1 - e^{-\varphi t/\tau}\left\{\cos\frac{\sqrt{1-\varphi^2}}{\tau}t + \frac{\varphi}{\sqrt{1-\varphi^2}}\sin\frac{\sqrt{1-\varphi^2}}{\tau}t\right\}\right]$$

$$= K_p A\left[1 - \frac{1}{\sqrt{1-\varphi^2}}e^{-\varphi t/\tau}\sin(\omega t + \phi)\right]$$

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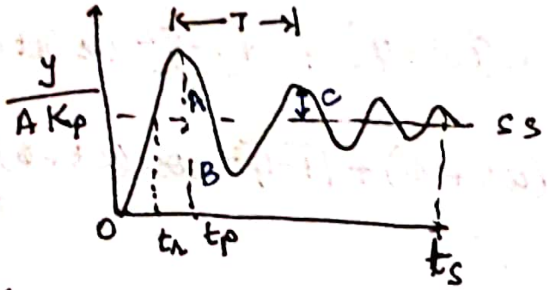
ω = radian frequency (rad / time) $= \frac{\sqrt{1-\varphi^2}}{\tau}$

ϕ = phase angle $= \tan^{-1}\left(\frac{\sqrt{1-\varphi^2}}{\varphi}\right)$



t_r : rise time $= T/4$

$= \frac{\pi\tau}{2\sqrt{1-\varphi^2}}$

= time required to reach the final value 1st time.

t_p = peak time $\simeq T/2$ (approx.) [For sinusoidal wave]

= time required by the process output to reach its maximum value

$\omega = \frac{\sqrt{1-\varphi^2}}{\tau}$; $\omega T = 2\pi$; $T = \frac{2\pi\tau}{\sqrt{1-\varphi^2}}$.

$t_p = \frac{\pi\tau}{\sqrt{1-\varphi^2}}$.

t_s = settling time

13/8/19

2nd order system (underdamped)

$$y(t) = A \cdot K_p \left[1 - e^{-\varphi t/\tau}\left\{\cos \omega t + \frac{\varphi}{\sqrt{1-\varphi^2}} \sin \omega t\right\}\right]$$

$$= AK_p\left[1 - \frac{1}{\sqrt{1-\varphi^2}} e^{-\varphi t/\tau} \sin(\omega t + \phi)\right]$$

$\omega = \frac{\sqrt{1-\varphi^2}}{\tau}$; $\phi = \tan^{-1}\left(\frac{\sqrt{1-\varphi^2}}{\varphi}\right)$

$\frac{dy(t)}{dt} = 0$.

$$\Rightarrow e^{-\varphi t/\tau}\left[-\frac{\varphi}{\tau}\left\{\cancel{\cos \frac{\sqrt{1-\varphi^2}}{\tau} t} + \frac{\varphi}{\sqrt{1-\varphi^2}} \sin \frac{\sqrt{1-\varphi^2}}{\tau} t\right\} - \sin \frac{\sqrt{1-\varphi^2}}{\tau} t \cdot \frac{\sqrt{1-\varphi^2}}{\tau} + \cancel{\frac{\varphi}{\tau} \cos \frac{\sqrt{1-\varphi^2}}{\tau} t}\right] = 0$$

$$\Rightarrow \cancel{-\frac{\varphi}{\tau} \cos \frac{\sqrt{1-\varphi^2}}{\tau} t} + \frac{-\varphi^2}{\tau\sqrt{1-\varphi^2}} \sin \frac{\sqrt{1-\varphi^2}}{\tau} \cdot t - \frac{\sqrt{1-\varphi^2}}{\tau} \sin \frac{\sqrt{1-\varphi^2}}{\tau} \cdot t = 0$$

$$\Rightarrow \frac{dy(t)}{dt} = \frac{K_p A e^{-\varphi t/\tau}}{\tau\sqrt{1-\varphi^2}} \sin \frac{\sqrt{1-\varphi^2}}{\tau} t = 0.$$

$$\frac{-AK_p}{\sqrt{1-\varphi^2}} \cdot e^{-\varphi t/\tau}\left[-\frac{\varphi}{\tau}\sin(\omega t+\phi) + \cancel{\tau}\cos(\omega t+\phi)\right]$$

$$= \frac{-AK_p}{\sqrt{1-\varphi^2}} e^{-\varphi t/\tau}\left[\frac{-\varphi}{\tau}\sin(\omega t+\phi) + \frac{\sqrt{1-\varphi^2}}{\tau}\cos(\omega t+\phi)\right]$$

$$= \frac{AK_p}{\tau\sqrt{1-\varphi^2}} e^{-\varphi t/\tau}\left[\varphi \sin(\omega t+\phi) + \sqrt{1-\varphi^2}\cos(\omega t+\phi)\right]$$

Let $\varphi = \sin x$

$\cos x = \sqrt{1-\varphi^2}$

$\sin \omega t = \sin \pi$

$\Rightarrow t = t_p = \frac{\pi}{\omega} = \frac{\pi\tau}{\sqrt{1-\varphi^2}}$

$\frac{A}{B}$ = Overshoot.

$A + B = \left[\frac{y}{AK_p}\right]_{t=t_p}$, $\quad \frac{t_p\varphi}{\tau} = \frac{\pi\varphi}{\sqrt{1-\varphi^2}}$

$\frac{\sqrt{1-\varphi^2}}{\tau} t_p = \pi$

$= \left[\cancel{1 - e^{-t\varphi/\tau}}\right.$

$$= \left[1 - e^{\frac{-\pi\varphi}{\sqrt{1-\varphi^2}}}\left\{\cos\pi + \frac{\varphi}{\sqrt{1-\varphi^2}} \cancel{\tan} \sin\pi\right\}\right]$$

$$= 1 + e^{\frac{-\pi\varphi}{\sqrt{1-\varphi^2}}}$$

$y(t_\infty) = AK_p$

$\therefore B = 1$

$\therefore \boxed{\frac{A}{B} = e^{\frac{-\pi\varphi}{\sqrt{1-\varphi^2}}}}$. max^m deviation w.r.t. steady state value.

For rise time, $\frac{y(t)}{AK_p} = 1$.

$$1 - e^{-\varphi t/\tau}\left\{\cos\omega t + \frac{\varphi}{\sqrt{1-\varphi^2}}\sin\omega t\right\} = 1.$$

$$\Rightarrow -e^{-\varphi t/\tau}\left\{\cos\omega t + \frac{\varphi}{\sqrt{1-\varphi^2}}\sin\omega t\right\} = 0.$$

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$$\Rightarrow \tan\omega t + \frac{\sqrt{1-\psi^2}}{\psi} = 0.$$

$$\Rightarrow \omega t = \tan^{-1}\left(-\frac{\sqrt{1-\psi^2}}{\psi}\right)$$

$$\Rightarrow t_r = \frac{1}{\omega}\tan^{-1}\left(-\frac{\sqrt{1-\psi^2}}{\psi}\right)$$

or $\sin(\omega t+\phi) = \sin\pi$.

$\tan\phi = \frac{\sqrt{1-\psi^2}}{\psi}$

$$\Rightarrow \omega t + \phi = \pi.$$

$\sqrt{1-\psi^2+\psi^2} = 1$

$\sin\phi = \sqrt{1-\psi^2}$

$$\Rightarrow t_r = \frac{1}{\omega}(\pi-\phi)$$

$$= \frac{\tau}{\sqrt{1-\psi^2}}\left[\pi - \tan^{-1}\frac{\sqrt{1-\psi^2}}{\psi}\right]$$

4) Decay ratio

$$\frac{C}{A}$$

$$\sin\omega t' = \sin 3\pi.$$

$$\Rightarrow \omega t' = 3\pi \quad \Rightarrow (t') = \frac{3\pi}{\omega}.$$

$$\frac{y(t')}{AK_p} = 1 - \frac{1}{\sqrt{1-\psi^2}} e^{-\frac{\psi}{\tau}\cdot\frac{3\pi}{\omega}} \sin(3\pi+\phi)$$

$$= 1 - \frac{1}{\sqrt{1-\psi^2}} e^{-\frac{3\pi\psi}{\sqrt{1-\tau^2}}} \sin(\pi+\phi).$$

$$= 1 + \frac{1}{\sqrt{1-\psi^2}} e^{-\frac{3\pi\psi}{\sqrt{1-\tau^2}}} \sin\phi$$

$$= 1 + e^{-\frac{3\pi\psi}{\sqrt{1-\tau^2}}}.$$

$$\therefore C = e^{-\frac{3\pi\psi}{\sqrt{1-\tau^2}}}$$

$$\frac{C}{A} = \frac{e^{-\frac{3\pi\psi}{\sqrt{1-\tau^2}}}}{e^{-\frac{\pi\psi}{\sqrt{1-\psi^2}}}} = e^{-\frac{2\pi\psi}{\sqrt{1-\tau^2}}}$$

5) $\omega = 2\pi f = \frac{2\pi}{T}$; f = cyclical frequency.

$$f = \frac{\omega}{2\pi} = \frac{\sqrt{1-\psi^2}}{2\pi\tau} \quad ; \quad T = \frac{2\pi}{\omega} = \frac{2\pi\tau}{\sqrt{1-\psi^2}}.$$

6) Natural frequency.

$$s^2 + \frac{1}{\tau^2}.$$

$$\left(s+\frac{i}{\tau}\right)\left(s-\frac{i}{\tau}\right)$$

$$G(s) = \frac{K_p}{\tau^2 s^2 + 2\varphi\tau s + 1}$$

$$G(s) = \frac{K_p/\tau^2}{\left(s-\frac{i}{\tau}\right)\left(s+\frac{i}{\tau}\right)}$$

$$\omega_n = \frac{\sqrt{1-\varphi^2}}{\tau} = \frac{1}{\tau} \quad ; \quad f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi\tau}.$$

$$T_n = 2\pi\tau.$$

$$G(s) = \frac{\bar{y}(s)}{\bar{f}(s)} = \frac{4}{s^2+1.6s+4} = \frac{1}{\frac{s^2}{4}+\frac{1.6}{4}s+1}$$

$$\tau^2 = \frac{1}{4} \Rightarrow \tau = \frac{1}{2} \qquad 2\varphi\tau = \frac{1.6}{4}.$$

$$\Rightarrow \varphi = \frac{1.6}{4} = 0.4$$

$$\therefore s = -\frac{4}{4} \pm \frac{2\sqrt{21}}{5}i \quad \Rightarrow \text{underdamped.}$$

$$\text{overshoot} = e^{\frac{-\pi\varphi}{\sqrt{1-\varphi^2}}} = e^{\frac{-\pi\,0.4}{\sqrt{1-0.16}}} = \cancel{0.163} \; 0.2538$$

Max value of $y(t)$, given $f(t)=10$.

B $\quad A=10$

$$y(t) = AK_p\left[1 + e^{\frac{-\pi\varphi}{\sqrt{1-\varphi^2}}}\right]$$

$$= 10\,[1+0.2538] = 12.538.$$

~~Rise time~~ $= \cancel{\frac{\tau\theta}{\sqrt{1-\varphi^2}}}$

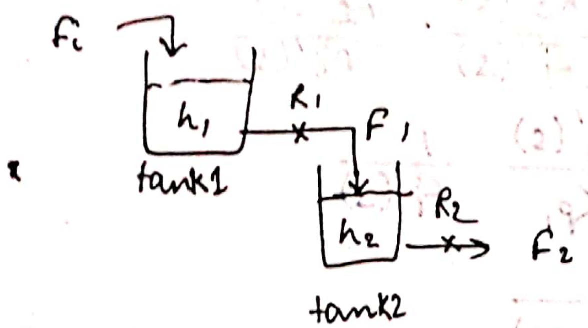
$$B = \lim_{s\to 0} s\,\bar{y}(s) = \lim_{t\to\infty} y(t) = 10$$

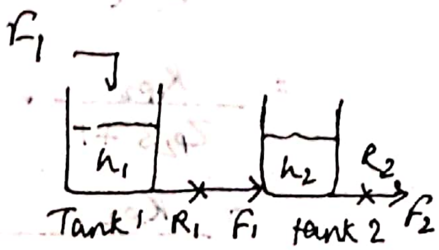
$$\text{Rise time} = \frac{\tau}{\sqrt{1-\varphi^2}}\left[\pi - \tan^{-1}\frac{\sqrt{1-\varphi^2}}{\varphi}\right]$$

$$= \frac{0.5}{\sqrt{1-0.16}}\left[\pi - \tan^{-1}\frac{\sqrt{1-0.16}}{0.4}\right]$$

ANSHIKA SHRISTI

1st order + 1st order = 2nd order.





One way interrupting system: Interacting
or non ~~interrupting~~ interacting system

$$\bar{f}_1 \longrightarrow \boxed{G_1} \xrightarrow{\bar{y}_1} \boxed{G_2} \longrightarrow \bar{y}_2$$

1st 1st

General Case : $G_1 = \dfrac{\bar{y}_1}{\bar{f}_1} = \dfrac{K_{p_1}}{\tau_{p_1} s+1}$

$$G_2 = \frac{\bar{y}_2}{\bar{y}_1} = \frac{K_{p_2}}{\tau_{p_2} s+1}$$

overall T.F. $= G_0(s) = \dfrac{\bar{y}_2}{\bar{f}_1} = G_1 \cdot G_2 = \dfrac{K_{p_1} \cdot K_{p_2}}{(\tau_{p_1} s+1)(\tau_{p_2} s+1)}$

Remarks:

① G_0 = 2nd order

② $s = -\dfrac{1}{\tau_{p_1}}, \dfrac{-1}{\tau_{p_2}}$ ⇒ overdamped.

③ $G_0 = G_1 G_2 G_3 \ldots G_n = \dfrac{K_{p_1} K_{p_2} \ldots K_{p_n}}{(\tau_{p_1} s+1) \ldots (\tau_{p_n} s+1)}$

Non interacting :

tank 1 : $G_1 = ??$ $\quad A_1 \dfrac{dh_1}{dt} = F_i - F_1 = F_i - \dfrac{h_1}{R_1}$ $\quad \Big| \quad \tau_{p_1} = A_1 R_1$

$A_1 R_1 \dfrac{dh_1'}{dt} + h_1' = F_i R_1'$ $\quad K_p = R_1$

~~tan 2 : G_2 = ??~~ $\quad \tau_{p_1} \dfrac{dh_1}{dt} + h_1' = K_{p_1} F_i'$

$$G_1(s) = \frac{\bar{h}_1'(s)}{\bar{F}_i'(s)} = \frac{K_{p_1}}{s\tau_{p_1}+1}$$

~~overall~~

tank 2 : $A_2 \dfrac{dh_2}{dt} = F_1 - F_2 = F_1 - \dfrac{h_2}{R_2}$.

⇒ $A_2 R_2 \dfrac{dh_2'}{dt} + h_2' = F_1' R_2$

⇒ $G_2(s) = \dfrac{\bar{h}_2'(s)}{\bar{F}_i(s)} = \dfrac{R_2 K_{p_2}}{\tau_{p_1} s+1}$

Scanned by CamScanner

$$G_0(s) = \frac{R_2 h_2'(s)}{F_i'(s)} = \frac{\bar{h}_2'(s)}{\bar{F}_1'(s)} \cdot \frac{\bar{F}_1'(s)}{\bar{F}_i'(s)}$$

$$= \frac{K_{p2}}{\tau_{p2}s+1} \cdot \frac{\bar{h}_1'(s)}{K_{p1}} \cdot \frac{1}{\bar{F}_i'(s)}$$

$$= \frac{K_{p2}}{(\tau_{p2}s+1)(\tau_{p1}s+1)}$$

$F_1 = \frac{h_1 - h_2}{R_1}$ $\Rightarrow$ 2nd order ② overdamped

② Interacting:

$$F_1 = \frac{h_1 - h_2}{R_1}$$

$$G_1(s) = \cancel{\frac{F_1(s)}{F_i}} \frac{\bar{h}_1'(s)}{\bar{F}_i'(s)}$$

$$A\frac{dh_1}{dt} = F_i - F_1 = F_i - \frac{h_1 - h_2}{R_1}$$

$$\Rightarrow A\frac{dh_1}{dt} + \frac{h_1}{R_1} = k F_i + \frac{h_2}{R_1}$$

$$\Rightarrow A_1 R_1 \frac{dh_1'}{dt} + h_1' = F_i' R_1 + h_2'$$

Tank 2: $A_2 \frac{dh_2}{dt} = \frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2}$

Interacting: $\frac{\bar{h}_2'(s)}{\bar{F}_i'(s)} = \frac{K_{p1}}{\tau_{p1}\tau_{p2}s^2 + (\tau_{p1}+\tau_{p2}-A_1R_2)s + 1}$

Remarks: ① 2nd order

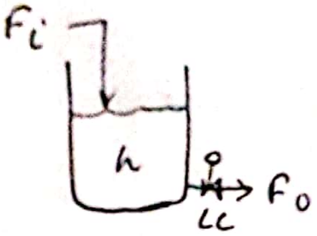
②

$$S_2 = \frac{-(\tau_{p1}+\tau_{p2}-A_1R_2) \pm \sqrt{(\tau_{p1}+\tau_{p2}+A_1R_2)^2 - 4\tau_{p1}\tau_{p2}}}{2\tau_{p1}\tau_{p2}}$$

ANSHIKA SHRISTI

19/8/19 Multicapacity process

1st order + controller = 2nd order



$$A\frac{dh'}{dt} = F_i' - F_o'$$

CV	MV
h	Fo

$F_o' = F_o - F_{os}$

LC → Level controller

Controller : $F_o = F_{os} + K_c h' + \frac{K_c}{\tau_i}\int h'\,dt$.

(i) $h' = 0 \Rightarrow F_o = F_{os}$
$\Rightarrow h = h_s$.

(ii) $h' < 0 \Rightarrow$ ~~$F_o < F_{os}$~~ $F_o < F_{os}$

(iii) $h' > 0$ $F_o > F_{os}$

φ

$$A\frac{dh'}{dt} + K_c h' + \frac{K_c}{\tau_i}\int h'\,dt = F_i'$$

$$\frac{\bar{h}'(s)}{\bar{F}_i'(s)} = \frac{1}{As + K_c + \frac{K_c}{s\tau_i}\cdot\frac{1}{s}}.$$

$$= \frac{\tau_i s}{\frac{A\tau_i}{K_c}s^2 + \tau_i s + 1} = \frac{K_p}{\tau^2 s^2 + 2\varphi\tau s + 1}$$

$$\tau = \sqrt{\frac{A\tau_i}{K_c}}$$

$2\varphi\tau = \tau_i$

$\Rightarrow \varphi = \frac{\tau_i}{2}\sqrt{\frac{K_c}{A\tau_i}}$

$= \frac{1}{2}\sqrt{\frac{K_c\tau_i}{A}}$

$$\varphi = \frac{1}{2}\sqrt{\frac{K_c\tau_i}{A}}.$$

Remarks.

① 2nd-order process

② $\sqrt{\frac{K_c\tau_i}{A}}$ = 2 critically
> 2 overdamped
< 2 underdamped.

Higher order :

1. 1st + 1st + 1st = 3rd.

   1st + 2nd = 3rd
2. Process + dead time = higher order.
3. Process + inverse response :

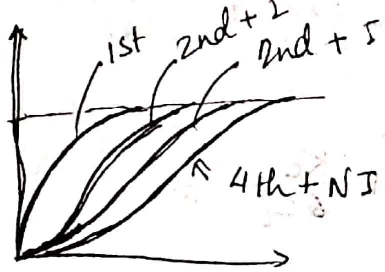
$$f_1 \to \boxed{G_1} \xrightarrow{y_1} \boxed{G_2} \xrightarrow{y_2} \cdots \xrightarrow{y_{n-1}} \boxed{G_N} \to y_N$$

Non-interacting

1. $G_0(s) = G_1 \cdot G_2, \cdots G_N(s)$

$$= \frac{K_{p_1} \cdot K_{p_2} \cdots K_{pN}}{(\tau_{p_1} s+1)(\tau_{p_2}+1)\cdots(\tau_{pN} s+1)}$$

2. Response = Response of overdamped system

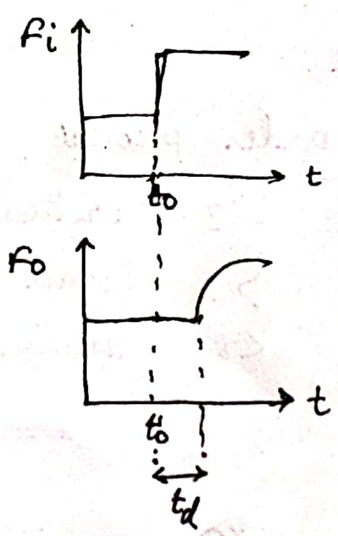


3. If order increases, sluggishness increases.

Interacting :

1. $G_0(s) \equiv$ complex [compared to non-interacting]
2. Interaction ~~reduces~~ increases sluggishness.

Dead time



$$f \to \boxed{\text{Process}} \xrightarrow{y} \boxed{\text{Dead time}} \to y(t-t_d)$$

$$L[f(t)] \rightarrow \boxed{\text{Process}} \xrightarrow{L[y(t)]} \underset{e^{-t_d s}}{\boxed{\text{Dead time}}} \rightarrow L[y(t-t_d)].$$

$$G_p : \frac{L[y(t)]}{L[f(t)]} = \frac{K_p}{\tau_p s+1} \leftarrow \text{1st order}$$

$$G_{dead} = \frac{L[y(t-t_d)]}{L[y(t)]} = e^{-t_d s}$$

$$G_o(s) : \frac{L[y(t-t_d)]}{L[f(t)]} = \frac{K_p e^{-t_d s}}{\tau_p s+1} \leftarrow \text{1st order + dead time (FOPDT) system.}$$

$$G_o(s) = \frac{K_p e^{-t_d s}}{\tau^2 s^2+2\varphi\tau s+1} \leftarrow \text{2nd order + dead time system (SOPDT)}$$

$$\equiv \frac{Q(s)}{P(s)}$$

Approximation of $e^{-t_d s}$

Pade approximation : $e^{-t_d s} = \dfrac{1-\frac{t_d}{2}s}{1+\frac{t_d}{2}s}$ ---. 1st order

$$= \frac{1-\frac{t_d}{2}s+\frac{t_d^2 s^2}{12}}{1+\frac{t_d}{2}s+\frac{t_d^2 s^2}{12}} \text{ ... 2nd order}$$

Taylor series : $e^{-t_d s} \approx \cancel{1+} 1-t_d s \approx \dfrac{1}{1+t_d s}$

3) $G(s) = \dfrac{K_p(-0.2s+1)}{(6s+1)(3s+1)(s+1)}$

FOPDT = ??

$$= \frac{K_p}{(6s+1)} \cdot \frac{(-0.2s+1)}{(3s+1)(s+1)}$$

$$e^{-0.2s} \cdot e^{-3s} \cdot e^{-s} = e^{-4.2s}$$

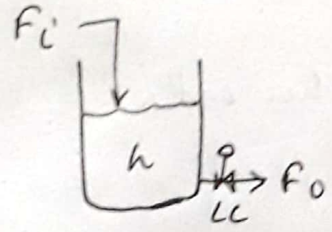
$$= \frac{K_p}{6s+1} \cdot e^{-4.2s}.$$

Skogestad half-rule : $3 \rightarrow \frac{3}{2}+\frac{3}{2}$

$$G(s) = \cancel{\frac{K_p\, e^{-2.7s}}{(6s+1)\left(\frac{3s}{2}+1\right)}} \quad \frac{K_p e^{-2.7s}}{(7.5s+1)}$$

19/8/19 Multicapacity process

1st order + controller = 2nd order



$$A\frac{dh'}{dt} = F_i' - F_0'$$

CV	MV
h	F_0

$F_0' = F_0 - F_{0s}$.

LC → Level controller

Controller : $F_0 = F_{0s} + K_c h' + \frac{K_c}{\tau_i}\int h' dt$.

(i) $h' = 0$ $\Rightarrow F_0 = F_{0s}$
$\Rightarrow h = h_s$.

(ii) $h' < 0$ $\Rightarrow$ ~~$F_0 < F_{0s}$~~ $F_0 < F_{0s}$

(iii) $h' > 0$ $F_0 > F_{0s}$

φ

$$A\frac{dh'}{dt} + K_c h' + \frac{K_c}{\tau_i}\int h' dt = F_i'$$

$$\frac{\bar{h}'(s)}{\bar{F}_i'(s)} = \frac{1}{As + K_c \cdot \cancel{\frac{1}{s}} + \frac{K_c}{\cancel{s}\tau_i}\cdot\frac{1}{s}}.$$

$$= \frac{\tau_i s}{\frac{A\tau_i}{K_c}s^2 + \cancel{K_c}\,\tau_i s + 1} = \frac{\cancel{\ }\,K_p}{\tau^2 s^2 + 2\varphi\tau s + 1}$$

$\tau = \sqrt{\frac{A\tau_i}{K_c}}$

$2\varphi\tau = \tau_i$

$\Rightarrow \varphi = \frac{\tau_i}{2}\sqrt{\frac{K_c}{A\tau_i}}$
$= \frac{1}{2}\sqrt{\frac{K_c\tau_i}{A}}$

$\varphi = \frac{1}{2}\sqrt{\frac{K_c\tau_i}{A}}$.

Remarks :

① 2nd-order process

② $\sqrt{\frac{K_c\tau_i}{A}}$ = 2 critically
> 2 overdamped
< 2 underdamped.

Higher order:

1. 1st + 1st + 1st = 3rd

   1st + 2nd = 3rd
2. Process + dead time = higher order.
3. Process + inverse response.

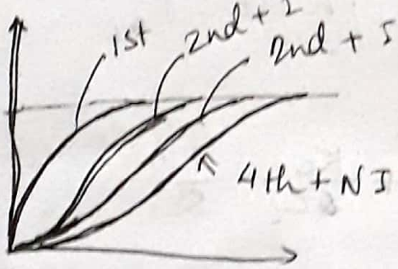
$$\xrightarrow{\delta_1} \boxed{G_1} \xrightarrow{y_1} \boxed{G_2} \xrightarrow{y_2} \cdots \xrightarrow{y_{N-1}} \boxed{G_N} \rightarrow y_N$$

I. Non-interacting

1. $G_0(s) = G_1 \cdot G_2 , \ldots G_N(s)$

$$= \frac{K_{P_1} \cdot K_{P_2} \cdot \ldots \cdot K_{PN}}{(\tau_{P_1} s+1)(\tau_{P_2}+1) \ldots (\tau_{PN} s+1)}$$

2. Response = Response of overdamped system

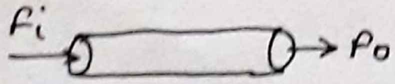


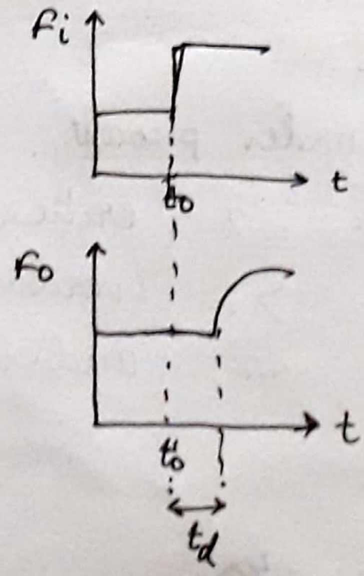
3. If order increases, sluggishness increases.

Interacting:

1. $G_0(s)$ = complex [compared to non-interacting]
2. Interaction ~~reduces~~ increases sluggishness.

Dead time





$$\xrightarrow{\delta} \boxed{\text{Process}} \xrightarrow{y} \boxed{\text{Dead time}} \rightarrow y(t-t_d)$$

$L[f(t)] \rightarrow \boxed{\text{Process}} \xrightarrow{L[y(t)]} \boxed{\begin{matrix}\text{Dead}\\ \text{time}\end{matrix}} \rightarrow L[y(t-t_d)]$

$e^{-t_d s}$

$$G_p = \frac{L[y(t)]}{L[f(t)]} = \frac{K_p}{\tau_p s+1} \quad \leftarrow \text{1st order}$$

$$G_{dead} = \frac{L[y(t-t_d)]}{L[y(t)]} = e^{-t_d s}$$

$$G_o(s) = \frac{L[y(t-t_d)]}{L[f(t)]} = \frac{K_p e^{-t_d s}}{\tau_p s+1} \quad \leftarrow \text{1st order + dead time system (FOPDT)}$$

$$G_o(s) = \frac{K_p e^{-t_d s}}{\tau^2 s^2 + 2\zeta\tau s+1} \quad \leftarrow \text{2nd order + dead time system (SOPDT)}$$

$$= \frac{Q(s)}{P(s)}$$

Approximation of $e^{-t_d s}$

Pade approximation: $e^{-t_d s} = \dfrac{1-\frac{t_d}{2}s}{1+\frac{t_d}{2}s}$ ... 1st order

$$= \frac{1-\frac{t_d}{2}s+\frac{t_d^2 s^2}{12}}{1+\frac{t_d}{2}s+\frac{t_d^2 s^2}{12}} \quad \text{... 2nd order}$$

Taylor series: $e^{-t_d s} \approx 1-t_d s \approx \dfrac{1}{1+t_d s}$

Q) $G(s) = \dfrac{K_p(-0.2s+1)}{(6s+1)(3s+1)(s+1)}$

FOPDT = ??

$$= \frac{K_p}{(6s+1)} \cdot \frac{(-0.2s+1)}{(3s+1)(s+1)}$$

$$e^{-0.2s} \cdot e^{-3s} \cdot e^{-s} = e^{-4.2s}$$

$$= \frac{K_p}{6s+1} e^{-4.2s}.$$

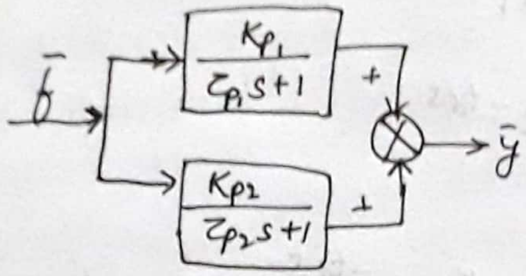
Skogestad half-rule: $3 \rightarrow \frac{3}{2}+\frac{3}{2}$

$$G(s) = \cancel{\frac{K_p e^{-2.7s}}{(6s+1)(\frac{3s}{2}+1)}} \quad \frac{K_p e^{-2.7s}}{(7.5s+1)}$$

20/8/19

Inverse Response:

- ~~systems connected in series~~
- process



$$\frac{\bar{y}}{\bar{f}}(s) = \frac{K_{p_1}}{\tau_{p_1}s+1} + \frac{K_{p_2}}{\tau_{p_2}s+1}$$

$$= \frac{(K_{p_1}\tau_{p_2} + K_{p_2}\tau_{p_1})s + K_{p_1} + K_{p_2}}{(\tau_{p_1}s+1)(\tau_{p_2}s+1)}$$

$$= \frac{[K_{p_1}+K_{p_2}]\left[\frac{(K_{p_1}\tau_{p_2} + K_{p_2}\tau_{p_1})s}{K_{p_1}+K_{p_2}} + 1\right]}{(\tau_{p_1}s+1)(\tau_{p_2}s+1)}$$

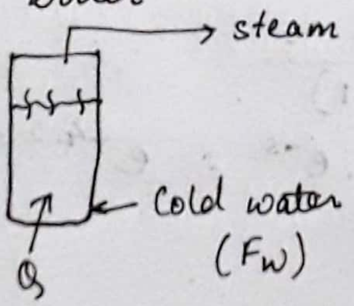
$$= \frac{K_p(\tau s+1)}{(\tau_{p_1}s+1)(\tau_{p_2}s+1)}$$

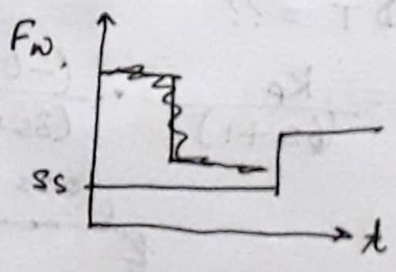
For inverse response, ~~zs~~ zeroes $\to$ RHP $\Rightarrow \tau < 0$

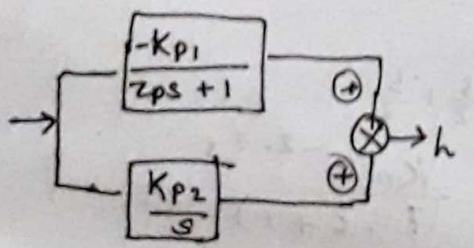
$$K_{p_1}\tau_{p_2} + K_{p_2}\tau_{p_1} < 0$$

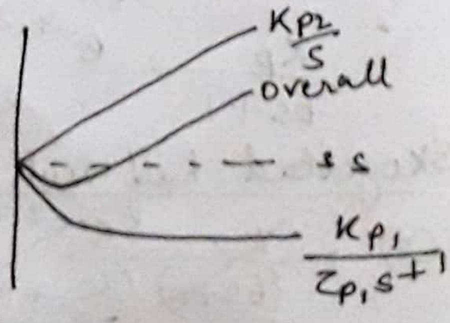
$$\Rightarrow -\frac{K_{p_2}}{K_{p_1}} > \underbrace{\frac{\tau_{p_2}}{\tau_{p_1}}}_{\text{positive}} \Rightarrow K_{p_1} \text{ \& } K_{p_2} \text{ have opposite signs.}$$

Ex: Boiler





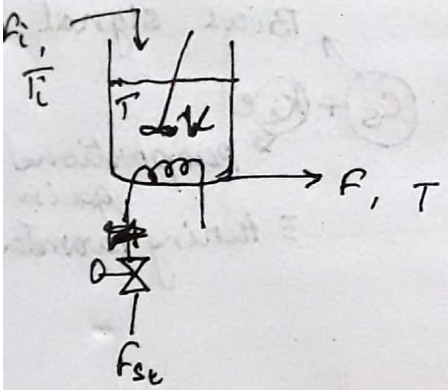




$$G(s) = \frac{K_{p2}}{s} - \frac{K_{p1}}{\tau_{p1}s+1} = \frac{(K_{p2}\tau_{p1} - K_{p1})s + K_{p2}}{s(\tau_{p1}s+1)}$$

* If poles are present in RHP, ~~poles~~ system is unstable, but if zeroes are present in RHP, process shows inverse response (the system may or may not be stable).

Feedback control:



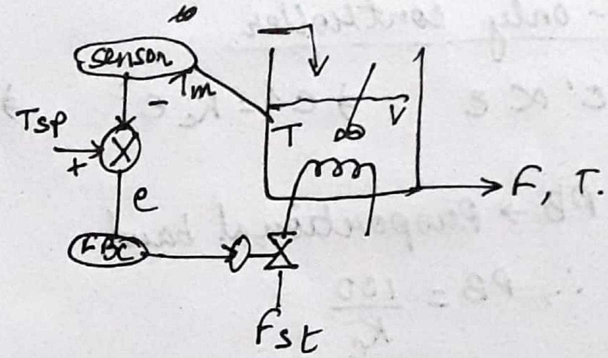


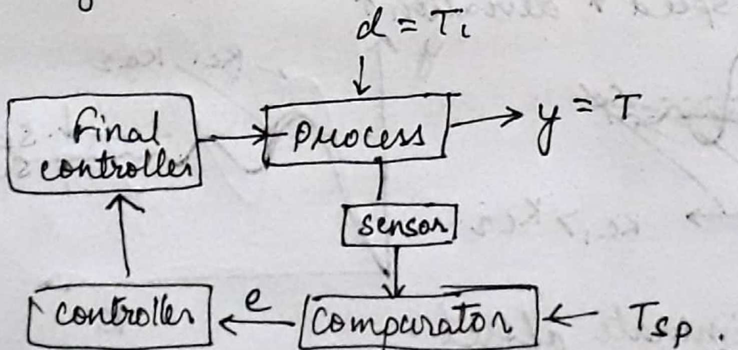
Fig: open loop process. (process without controller)

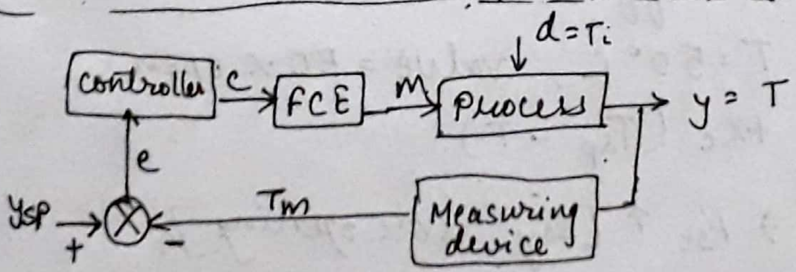
closed loop process. (process with controller)

$F_i = F = \text{const}$ @ $V = \text{const}$.

CV	MV	LV
T	F_{st}	T_i

Block diagram:





$$e \xrightarrow{} \boxed{\text{Controller}} \rightarrow c'$$

$$e = y_{sp} - y$$

P = Proportional action
I = Integral "
D = Derivative "

P- only controller

$c' \propto e \Rightarrow c' = K_c e. \Rightarrow c = C_s + K_c e$

C_s → Bias signal

K_c → Proportional gain ≡ tuning parameter

PB → Proportional band

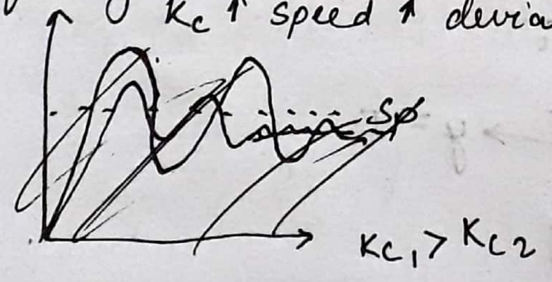
$\therefore PB = \dfrac{100}{K_c}$

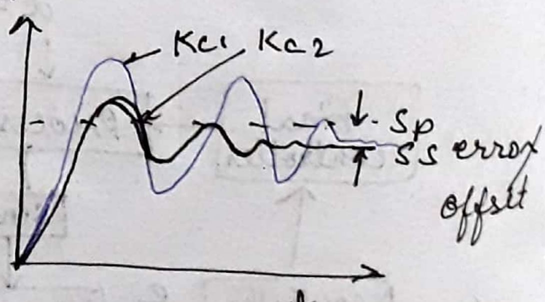
① Transfer function $G_c = \dfrac{\bar{c}'(s)}{\bar{e}(s)} = K_c$.

② With the increase in value of K_c, P- only controller becomes more sensitive to error signal.

③ Speed of response gets improved at the expense of higher deviation & longer oscillation.

$K_c \uparrow$ speed $\uparrow$ deviation $\uparrow$





④ cannot eliminate offset.

Ex : $T_i = 25°C$, $T = 50°C$, valve = 50% open.

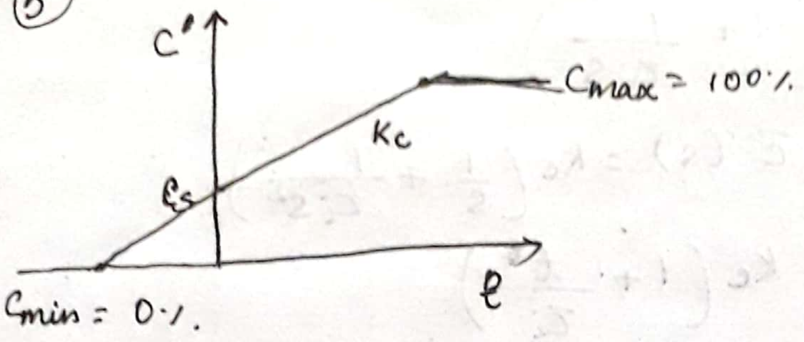
P-only : $F_{st} = F_{st_s} + K_c (T_{sp} - T)$

If $T_i \downarrow$ $T \downarrow$ $\Rightarrow F_{st} \uparrow$ [say, 60% opening].

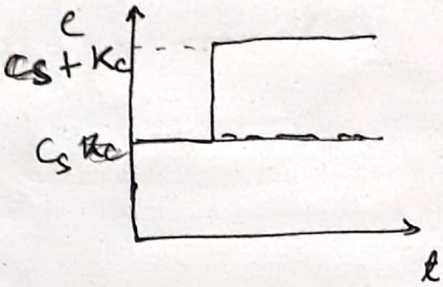
$60 = 50 + 10$ $\Rightarrow K_c e = 10$

$\Rightarrow e \neq 0$.

⑤



⑥ $e(t) = 1$



$$\frac{\bar{c}'(s)}{1/s} = K_c \Rightarrow \bar{c}'(s) = \frac{K_c}{s} \Rightarrow c'(t) = K_c$$
$$\Rightarrow c(t) = c_s + K_c$$

⑦ If K_c is very very large, controller behaves as a switch.

PI controller

$$c'(t) = K_c\, e(t) + \frac{K_c}{\tau_I}\int e\, dt.$$

τ_i = Integral time constant

$\frac{1}{\tau_i}$ = reset rate ≡ PB

$$TF = G_c(s) = \frac{\bar{c}'(s)}{\bar{e}(s)} = K_c + \frac{K_c}{\tau_i}\frac{1}{s} = K_c\left(1 + \frac{1}{\tau_i s}\right)$$

Remarks:

① P ≡ speeds up.

② offset @ eliminate [effect of I].

③ 1st ~~process~~ open loop process + PI ≡ 2nd closed loop process.

⇒ system response becomes sluggish.

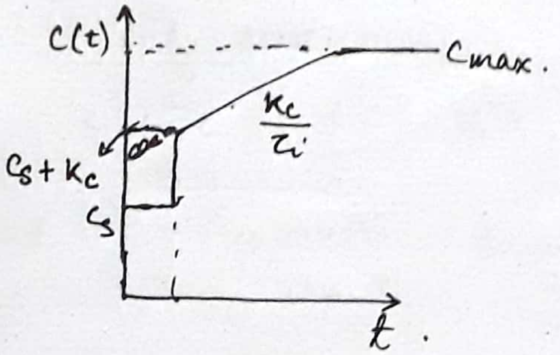
④ $\tau_i \downarrow$ speed ↑ deviation ↑.

(5) $\dfrac{\bar{c}'(s)}{\bar{e}(s)} = K_c\left(1 + \dfrac{1}{\tau_i s}\right)$

At $\bar{e}(s) = \dfrac{1}{s}$, $\bar{c}'(s) = K_c\left(\dfrac{1}{s} + \dfrac{1}{\tau_i s^2}\right)$

$\Rightarrow c'(t) = K_c\left(1 + \dfrac{t}{\tau_i}\right)$

$\Rightarrow c(t) = c_s + K_c\left(1 + \dfrac{t}{\tau_i}\right) = K_c + c_s + \dfrac{K_c}{\tau_i} t$.



Reset time is the time needed by the controller to repeat its initial proportional action change in its output.

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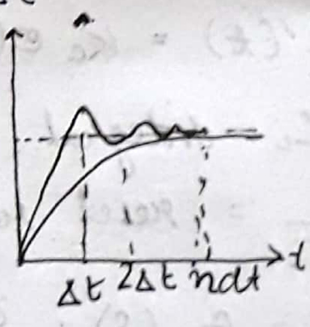
$c(t) = c_s + K_c e + \dfrac{K_c}{\tau_i}\int e\,dt$.

$\dfrac{K_c}{\tau_i}\int e\,dt = \dfrac{K_c}{\tau_i}\sum_{i=1}^{n} e(i\Delta t)\,\Delta t$

$$\left\{ c(t) = c_s + \frac{K_c}{\tau_i}\Delta t\left[e(\Delta t) + e(2\Delta t) + \dots + e(n\Delta t)\right] \right\} \text{PI}$$



P : $c(t) = c_s + \dfrac{K_c}{\tau_i} e(t)$

$\Rightarrow c'(t) = \dfrac{K_c}{\tau_i} \times 10^{-5}$

PI:

$c'(t) = \dfrac{K_c}{\tau_i}\Delta t\left[e(\Delta t) + e(2\Delta t) + \dots + e(n\Delta t)\right]$.

$= \dfrac{K_c}{\tau_i}\Delta t\left[1 + 0.8 + \dots + 10^{-5}\right] \approx 10 + 10^{-5}$

Although error is small, the integral action keeps on adding and the action is large. thus, it eliminates the offset in case of PI controller.

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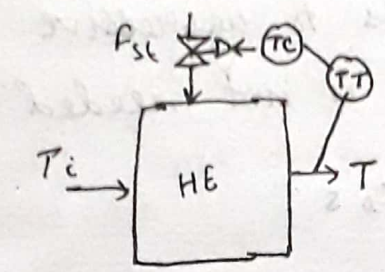
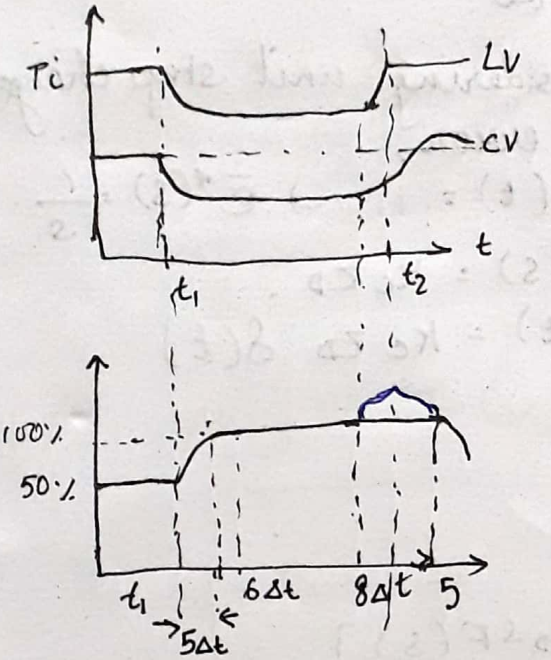
Reset windup [drawback of PI controller]

CV	MV
T	F_{st}





Control eq^n	valve opening
5	100%
5.8	100%
6.5	100%
⋮	
8	100%
7.5	100%
⋮	⋮
5	100%
4.5	95%

When mathematical expression reaches 5, valve opening becomes 100%. (assumed).

$$\left.\begin{array}{ll} \text{if } c > c_{max} & c = c_{max} \\ \text{else if } c < c_{min} & c = c_{min} \end{array}\right\} \text{Anti reset windup.}$$

PID controller

$$c(t) = \underbrace{c_s + K_c e}_{P} + \underbrace{\frac{K_c}{\tau_i}\int e\,dt}_{I} + \underbrace{K_c \tau_D \frac{de}{dt}}_{D}$$

$$G(s) = \frac{\bar{c}'(s)}{\bar{e}(s)} = K_c + \frac{K_c}{\tau_i}\cdot\frac{1}{s} + \tau_D K_c \cdot s$$

$$= K_c\left[1 + \frac{1}{\tau_i s} + \tau_D s\right]$$

$\tau_D \rightarrow$ Derivative time

= tuning parameter

Remarks:

① If e = constant, D. term does not exist.
⇒ cannot eliminate offset.

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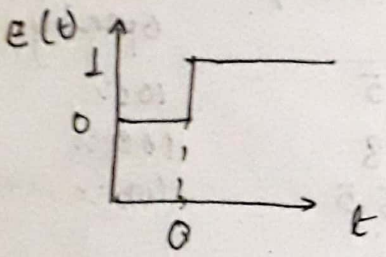
② for a noisy response, with almost zero error, derivative term leads to aggressive control action, although it is not needed at all.

D - term : $\frac{\bar{c}'(s)}{\bar{\epsilon}(s)} = k_c \tau_D s$.

$\Rightarrow \bar{c}'(s) = k_c \tau_D s \, \bar{\epsilon}(s)$.

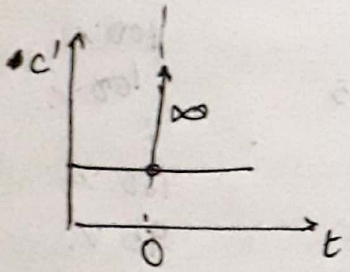


considering unit step change in error,

$e(t) = 1 \Rightarrow \bar{\epsilon}(s) = \frac{1}{s}$.

$\bar{c}'(s) = k_c \tau_D$.

$\Rightarrow c'(t) = k_c \tau_D \, \delta(t)$



$G_c(s) = K_c\left[1 + \frac{1}{\tau_i s} + \tau_D s F(s)\right]$

$F(s)$ = T. F. of the filter.

$= \frac{1}{(\tau_p s+1)^n}$; n = order of the filter

To make it semiproper, n = 1.

$G_c(s) = k_c\left[1 + \frac{1}{\tau_i s} + \frac{\tau_D s}{\tau_p s+1}\right]$

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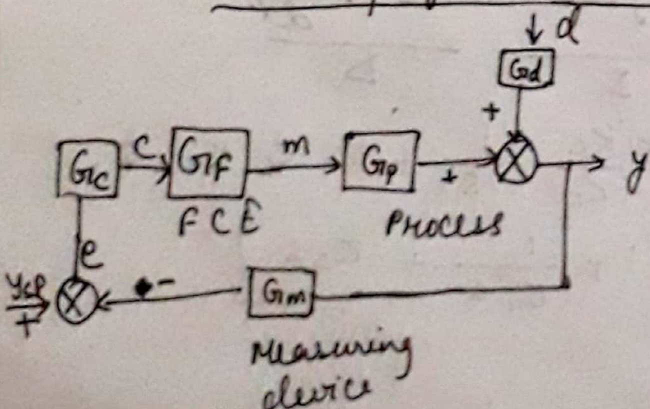
<u>closed-loop process - Dynamic response</u>



Process : $\bar{y} = \bar{m} G_p + \bar{d} G_d$ — ①

Sensor : $\bar{y}_m = G_m \bar{y}$; Controller : $\bar{e} = \bar{y}_{sp} - \bar{y}_m$

$\bar{c} = G_c \bar{\epsilon}$

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FCE : $\bar{m} = G_f \bar{c} = G_f G_c \bar{e}$

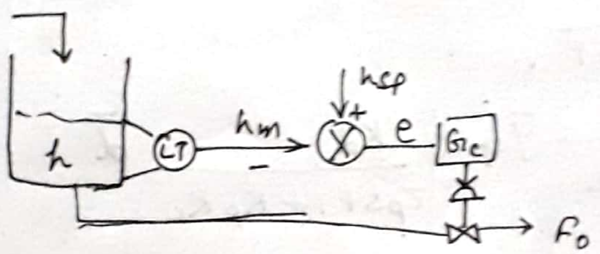
$= G_f G_c (\bar{y}_{sp} - \bar{y} G_m)$

$$\bar{y} = \frac{G_p G_f G_c \bar{y}_{sp} + \bar{d} G_d}{G_p G_f G_c G_m + 1}$$

$$= \cancel{\frac{G_p}{G_m} \bar{y}_{sp} + \frac{G_d}{G_f G_c G_m}} \bar{d}$$

✓ $$\bar{y} = \underbrace{\frac{G_p G_f G_c}{1 + G_m G_f G_c G_p}}_{(servo)} \bar{y}_{sp} + \underbrace{\frac{G_d}{1 + G_m G_f G_c G_p}}_{(Regulating)} \bar{d}$$ --- CLTF.

Ex :



CV	MV	LV
h	Fo	Fi

$\cancel{G_p}$ $F_i - F_o = A \frac{dh}{dt} \Rightarrow \bar{F}_i' - \bar{F}_o = A s \bar{h}$

$G_p = \cancel{\frac{\bar{h}}{\bar{F}_o'}} \Rightarrow \bar{h} = \frac{\bar{F}_i}{As} - \frac{\bar{F}_o}{As}$

$G_p = \frac{-1}{As}$; $G_d = \frac{1}{As}$

<u>Measuring device</u> ! (Differential Pressure cell) (DP cell)

$$\tau^2 \frac{d^2 h_m}{dt^2} + 2\psi\tau \frac{dh_m}{dt} + h_m = K_p \Delta P = K_p \alpha h.$$

$\Delta P = h\rho g \propto h = \alpha h.$

$$\frac{\bar{h}_m}{\bar{h}} = \frac{K_p \alpha}{\tau^2 s^2 + 2\psi\tau s + 1}$$

<u>Effect of P action</u> : $\rightarrow G_p = \frac{K_p}{\tau_p s + 1}$

Process — 1st order / 2nd order

controller : $G_c = K_c$

FCE, sensor : $G_f = G_m = 1$

$$\bar{y} = \frac{\frac{K_p}{\tau_p s + 1} \times 1 \times K_c}{1 + 1 \times 1 \times K_c \times \frac{K_p}{\tau_p s + 1}} \bar{y}_{sp} + \frac{G_d}{1 + 1 \times 1 \times K_c \times \frac{K_p}{\tau_p s + 1}} \bar{d}$$

$$\bar{y} = \frac{K_p K_c}{\tau_p s + 1 + K_p K_c}\,\bar{y}_{sp} + \frac{G_d(\tau_p s+1)}{\tau_p s + 1 + K_p K_c}\,\bar{d}$$

$$\tau_p \frac{dy}{dt} + y = K_p \underset{MV}{m} + K_d \underset{LV}{d}$$

K_p: process gain w.r.t. MV; K_d: process gain w.r.t. LV

$$\bar{y}(s) = \frac{K_p}{\tau_p s+1}\,\bar{m} + \frac{K_d}{\tau_p s+1}\,\bar{d}$$

$$\bar{y} = G_p \bar{m} + G_d \bar{d}$$

$$\therefore G_d = \frac{K_d}{\tau_p s + 1}$$

$$\bar{y} = \frac{K_p K_c}{\tau_p s + 1 + K_p K_c}\,\bar{y}_{sp} + \frac{K_d}{\tau_p s + 1 + K_p K_c}\,\bar{d}$$

$$\frac{\frac{K_p K_c}{\tau_p s+1}}{\frac{K_p K_c}{\tau_p s+1} + 1}\,\bar{y}_{sp} + \frac{K_d}{}$$

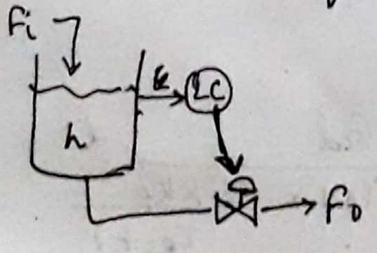
$$\bar{y} = \frac{\frac{K_p K_c}{1+K_p K_c}}{\frac{\tau_p s}{1+K_p K_c} + 1}\,\bar{y}_{sp} + \frac{\frac{K_d}{1+K_p K_c}}{\frac{\tau_p s}{1+K_p K_c} + 1}\,\bar{d}$$

$$= \left(\frac{K_p'}{\tau_p' s + 1}\right)\bar{y}_{sp} + \left(\frac{K_d'}{\tau_p' s+1}\right)\bar{d}$$

$$K_p' = \frac{K_p K_c}{1+K_p K_c}\ ; \quad \tau_p' = \frac{\tau_p}{1+K_p K_c}\ ; \quad K_d' = \frac{K_d}{1+K_p K_c}$$

Remarks:

1. No change in order. Hence no sluggishness.
2. $K_p > K_p'$ if $K_p K_c$ is positive



$h\uparrow \quad F_o \uparrow \quad \exists$ Direct actions

1) As the input signal to the controller increases, the output signal ~~function~~ from the controller must increase (for direct action).

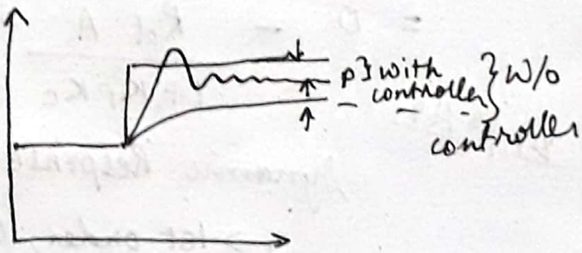
P-only : $f_o = f_{os} + K_c \underbrace{(h_{sp} - h)}_{e}$

② $-ve$ K_c

Reverse action : $h \uparrow$ $f_i \downarrow$, $+ve$ K_c.

CV	MV	K_c	K_p $\left(\frac{\Delta h}{\Delta f_o}\right)$	$K_c K_p$
h	f_o	$-ve$	$-ve$	$+ve$
h	f_i	$+ve$	$+ve$ $\left(\frac{\Delta h}{\Delta f_i}\right)$	$-ve$

$$K_p' = \frac{K_p}{K_p + \frac{1}{K_c}}$$



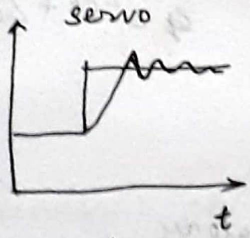
③ $K_d > K_d'$

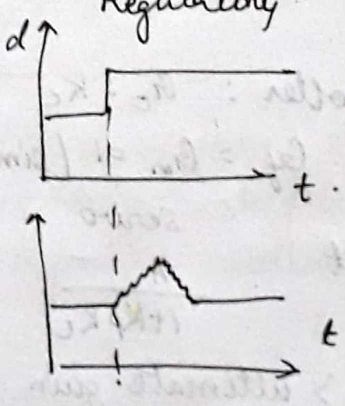
④ $\tau_p > \tau_p'$

less time constant ⇒ higher speed

⇒ Response is fast.

③ Offset :





CLTF : $\bar{y} = \left(\frac{K_p'}{\tau_p' s + 1}\right) \frac{A}{s}$: servo.

$y_{sp} = \frac{A}{s}$.

Offset = New set point − ultimate value of response

$= A - \lim_{s \to 0} s\bar{y}(s)$.

$= A - \lim_{s \to 0} \frac{K_p' A}{\tau_p' s + 1}$

$= A(1 - K_p') = A\left(1 - \frac{K_p}{K_p + \frac{1}{K_c}}\right) = \frac{A\left(\frac{1}{K_c}\right)}{K_p + \frac{1}{K_c}}$

$= \frac{A}{K_p K_c + 1} \neq 0$

For regulatory case :

$$\bar{y} = \frac{K_d'}{\tau_p' s + 1} \bar{d}$$

If $\bar{d} = \frac{A}{s}$.

~~offset = A - lim s→0~~

offset = New sp - ultimate value of response

$$= \cancel{A - \lim_{s \to 0} s\bar{y}(s)}$$

$$= A - \lim_{s \to 0} \frac{K_d' s}{\tau_p' s + 1}$$

$$\cancel{= A - \lim} \quad \cancel{= A}$$

$$= 0 - \frac{K_d A}{1 + K_p K_c} \neq 0$$

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Dynamic Response

Process — 1st order, $G_p = \frac{K_p}{\tau_p s + 1}$

Process — 2nd order, $G_p = \frac{K_p}{\tau^2 s^2 + 2\xi \tau s + 1}$

Controller : $G_c = K_c$

$G_f = G_m = 1$ (simplicity)

	Servo	regulatory
offset	$\frac{A}{1+K_p K_c}$	$\frac{-K_d A}{1+K_p K_c}$

If $K_c \to \infty$, offset $\to 0$

$K_c >$ ultimate gain (K_u) @ unstable

$$\bar{y} = \frac{G_p G_c G_f}{1 + G_p G_c G_f G_m} \bar{y}_{sp}$$

$$= \frac{\frac{K_p}{\tau^2 s^2 + 2\xi \tau s + 1} \times K_c}{1 + \frac{K_p K_c}{\tau^2 s^2 + 2\xi \tau s + 1} \cancel{\times K_c}} \bar{y}_{sp}$$

$$= \frac{K_p K_c}{\tau^2 s^2 + 2\varphi\tau s + 1 + K_p K_c} \bar{y}_{sp}$$

$$= \frac{\frac{K_p K_c}{1+K_p K_c}}{\frac{\tau^2 s^2}{1+K_pK_c} + \frac{2\varphi\tau}{1+K_pK_c} s + 1} \bar{y}_{sp} = \frac{K_p'}{(\tau_p')^2 s^2 + 2\varphi'\tau' s + 1} \bar{y}_{sp}$$

$$K_p' = \frac{K_p K_c}{1+K_p K_c} \; ; \; \tau_p' = \frac{\tau}{\sqrt{1+K_pK_c}}$$

$$= \cancel{\frac{K_p}{\tau_p'^2 s^2 -}}$$

$$\varphi' = \frac{\varphi}{\sqrt{1+K_pK_c}}$$

$$\bar{y}_{sp} = \frac{A}{s}$$

offset = New set point − ultimate value of response

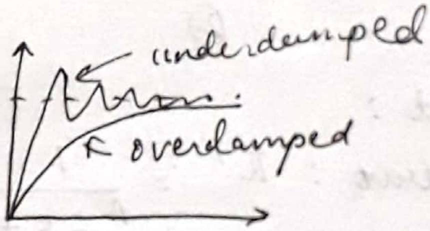
$$= A - \lim_{s\to 0} s\,\bar{y}(s)$$

$$= A - \cancel{K_p}\,\cancel{\lim_{s\to}} \frac{K_pK_c}{1+K_pK_c} A$$

$$= A\left[1 - \frac{K_pK_c}{1+K_pK_c}\right] = \frac{A}{1+K_pK_c}$$

Remarks :

① No change in order.



② $K_p' < K_p$

$\tau_p' < \tau_p$

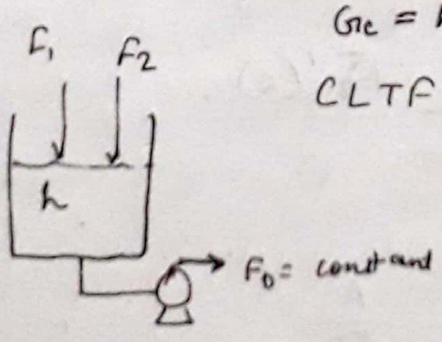
$\varphi' < \varphi$

overdamped → underdamped
$\varphi > 1$ $0 < \varphi < 1$

[As φ is decreasing, the process may turn from overdamped to underdamped]

③ offset $= \dfrac{A}{1+K_pK_c} \neq 0$

④)



$G_c = K_c$; $G_f = G_m = 1$

CLTF : $\bar{y} = \dfrac{G_p G_c G_f}{1+G_p G_c G_f G_m} \bar{y}_{sp} + \dfrac{G_d}{1+G_p G_f G_c G_m} \bar{d}$

CV	MV	LV
h	F_1	F_2

$$A\frac{dh}{dt} = F_1 + F_2 - F_0 .$$

$$A\frac{dh'}{dt} = F_1' + F_2' \Rightarrow A s \bar{h}'(s) = \bar{F}_1'(s) + \bar{F}_2'(s)$$

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$G_p = \cancel{\frac{\bar{F}_1'(s)}{y}} \quad \frac{\bar{h}'(s)}{\bar{F}_1'(s)}$:

$G_p = G_d = \frac{1}{As}$

CLTF, $\bar{y} = \frac{K_c \cdot \frac{1}{As} \cdot 1}{1 + K_c \frac{1}{As} \cdot 1 \cdot 1} \bar{y}_{sp} + \frac{\frac{1}{As}}{1 + K_c \cdot \frac{1}{As} \cdot 1 \cdot 1} \bar{d}$

$\Rightarrow \bar{y} = \frac{K_c}{\cancel{1 + \frac{K_c}{As}}\ As + K_c} \bar{y}_{sp} + \frac{1}{As + K_c} \bar{d}$

$\Rightarrow \bar{h}' = \frac{1}{\frac{A}{K_c} s + 1} \bar{h}_{sp} + \frac{1/K_c}{\frac{A}{K_c} s + 1} \bar{F}_2'(s)$

$\tau_p = \frac{A}{K_c}$; $K_p = 1$

Remarks : ① No change in order.

②

offset :

Servo : $\bar{h}' = \frac{1}{\frac{A}{K_c} s + 1} \bar{h}_{sp}$.

$\bar{h}_{sp} = \frac{A}{s}$

offset : $A - \lim_{s \to 0} s \bar{h}'(s)$.

$= A - \lim_{s \to 0} \frac{A}{\frac{A}{K_c} s + 1} = 0\%$

Regulatory : $\bar{h}' = \frac{1/K_c}{\frac{A}{K_c} s + 1} \bar{F}_2'(s)$.

$F_2'(s) = \frac{A}{s}$.

offset = ~~A - lim~~ $0 - \lim_{s \to 0} \frac{\frac{A}{K_c}}{\frac{A}{K_c} s + 1}$

$= -\frac{A}{K_c}$ //

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This process itself has integrating action. Hence P-only controller can be used.

Effect of I-action:

Process : $G_p = \frac{K_p}{\tau_p s+1}$; $G_c = \frac{K_c}{\tau_i s}$; $G_f = G_m = 1$

CLTF : $\bar{y} = \frac{G_p G_c G_f}{1+G_p G_c G_f G_m} \bar{y}_{sp} + \frac{G_d}{1+G_p G_c G_f G_m} d$

Servo : $\bar{y} = \frac{\frac{K_p}{\tau_p s+1} \cdot \frac{K_c}{\tau_i s}}{1+\frac{K_p}{\tau_p s+1} \cdot \frac{K_c}{\tau_i s}} \bar{y}_{sp}$

$$= \frac{K_p K_c}{(\tau_p s+1)\tau_i s + K_p K_c} \bar{y}_{sp} = \frac{K_p K_c \, \bar{y}_{sp}}{\tau_p \tau_i s^2 + \tau_i s + K_p K_c}$$

$$= \frac{1}{\frac{\tau_p \tau_i}{K_p K_c} s^2 + \frac{(\tau_i)}{K_p K_c} s + 1} \bar{y}_{sp}$$

$\tau = \sqrt{\frac{\tau_p \tau_i}{K_p K_c}}$; $2\psi\tau = \frac{\tau_i}{K_p K_c}$ $\Rightarrow 2\psi \sqrt{\frac{\tau_p \tau_i}{K_p K_c}} = \frac{\tau_i}{K_p K_c}$

$\Rightarrow \psi = \frac{1}{2}\sqrt{\frac{\tau_i}{\tau_p K_p K_c}}$

Remarks: ① Order increases by 1.
② Speed of response gets decreased
To increase speed, increase K_c or decrease τ_i

③ offset :
servo : $\bar{y} = A - \frac{A}{1} = 0$

Effect of D-action:

Process : $G_p = \frac{K_p}{\tau_p s+1}$; $G_c = K_c \tau_d s$.

CLTF (servo) : $\bar{y} = \frac{\frac{K_p}{\tau_p s+1} \cdot K_c \tau_d s}{1+\frac{K_p}{\tau_p s+1} \cdot K_c \tau_d s} \bar{y}_{sp}$

$$= \frac{K_p K_c \tau_d s}{\underbrace{(\tau_p + K_p K_c \tau_d)}_{\tau_p'} s + 1} \bar{y}_{sp}$$

Remarks :

~~① Order : $\tau_p' > \tau_p$~~

① Order remains same

$\tau_p' > \tau_p$; stability ; Robustness

Consolidated effect -

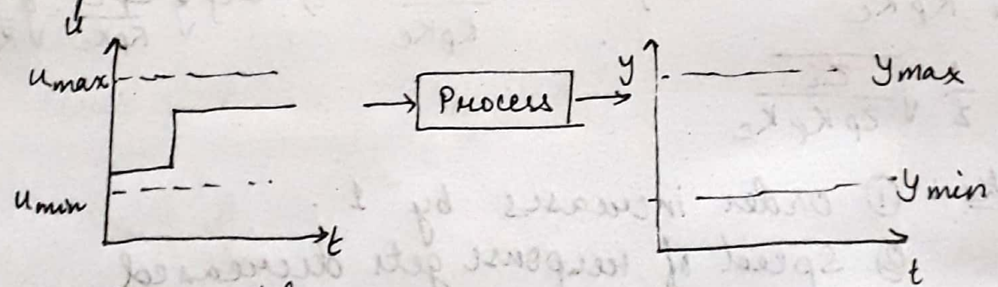
① ~~Prop~~ P - action : speeds up the response ; $K_c \uparrow \Rightarrow$ factor
offset can't be eliminated

② P + I : offset can be eliminated ;
sluggish [$K_c \uparrow$ or/and $\tau_i \downarrow$ to speed up]
High $K_c \Rightarrow$ unstable .

③ P + I + D : speeds up :
offset eliminated
stable .

2/9/19

A dynamic system is said to be stable when for any bounded input, it provided bounded output.



LHP @ stable

RHP @ unstable

On imaginary axis @ merginary stable .

Closed loop stability :

servo : $\frac{\bar{y}}{\bar{y}_{sp}} = \frac{G_c G_p G_f}{1 + G_c G_p G_f G_m} = \frac{q(s)}{P(s)}$

$\underbrace{1 + G_c G_p G_f G_m}_{G_{OL}} = 0$ ---- characteristic eq^n

$\Rightarrow 1 + G_{OL} = 0$

$\bar{y} = \frac{5}{s-2}\bar{m} + \frac{10}{s-2}\bar{d}$; $G_c = K_c$

$G_f = G_m = 2$

① open loop process → unstable.

② Condition @ closed loop process is stable.

$1 + G_c G_p G_f G_m = 0$

$\Rightarrow 1 + K_c \frac{K_p}{\tau_p s + 1} \cdot 2 \times 2 = 0$

$\Rightarrow 1 + K_c \frac{5}{s-2} \times 4 = 0$

Pole < 0.

$\Rightarrow \frac{s - 2 + 20 K_c}{s-2} = 0$

$\cancel{s-2}$ $s < 0$.

$s - 2 + 20 K_c = 0 \Rightarrow s = -20 K_c + 2$.

$\Rightarrow s < 0 \quad \Rightarrow -20 K_c + 2 < 0 \quad \Rightarrow K_c > \frac{1}{10}$

Routh ~~Hurwitz~~ Hurwitz Test

① Algebraic test

② No. of poles present in RHP.

③ Applicable to polynomial form of characteristic eqn.

Steps:

① Develop the CLTF

② CE : $1 + G_c G_p G_m G_f = 0 \Rightarrow a_0 s^n + a_1 s^{n-1} + \dots + a_n = 0$

- a_0 is +ve.
- Inspect all the coefficients.

If one coefficient is −ve, at least one pole is present in the RHP ⇒ unstable.

If all are +ve @ further analysis.

③ Routh array

	①	②	③	
Row 1	a_0	a_2	a_4	--- a_{n-1}
2	a_1	a_3	a_5	--- a_n
3	A_1	A_2	A_3	---
4	B_1	B_2	B_3	...
⋮				
$n+1$	Z_1	Z_2	Z_3	

$A_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$; $A_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$

④ For stable system, all values in the 1st column must be positive

⑤ No. of poles present in the RHP equal to no. of sign changes involved in 1st column

Q) C.E. : $s^3+2s^2+(2+K_c)s+\frac{K_c}{\tau_i}=0$; $\tau_i = 0.1$

$a_0 = 1$; $a_1 = 2$; $a_2 = 2+K_c$; $a_3 = \frac{K_c}{\tau_i}$.

$2+K_c > 0$ $\quad \frac{K_c}{\tau_i} > 0 \Rightarrow K_c > 0$

$$A_1 = \frac{2(2+K_c) - \left(\frac{K_c}{\tau_i}\right)}{2} > 0.$$

$\Rightarrow 4 + 2K_c - \frac{K_c}{\tau_i} > 0$

$\Rightarrow K_c\left(2 - \frac{1}{\tau_i}\right) > -4 \Rightarrow K_c(-8) > -4$

$\Rightarrow K_c < 0.5$

Row			
1	1	$2+K_c$	0
2	2	K_c/τ_i	0
3	$\frac{2(2+K_c)-\frac{K_c}{\tau}}{2}$	0	
4	K_c/τ_i	0	

$\therefore$, $0 < K_c < 0.5 \rightarrow$ stable

$> 0.5 \rightarrow$ unstable

$= 0.5 \rightarrow$ critical stability

$s^3 + 2s^2 + 2.5s + 5 = 0.$

$\Rightarrow s^2(s+2) + 2.5(s+2) = 0.$

$\Rightarrow (s^2+2.5)(s+2) = 0 \Rightarrow s^2+2.5 = 0$ or $s = -2$

$s = \pm i\sqrt{2.5}$ or -2.

n-th row has 1st element as 0,

$2s^2 + 5 = 0.$

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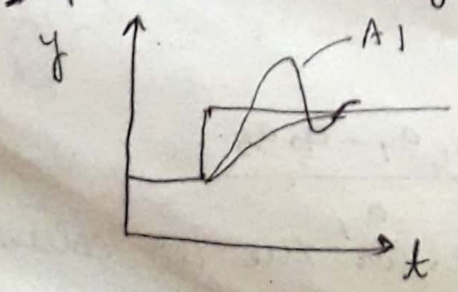
Tuning Method:



$A_1 \Rightarrow K_{c1}$ $\tau_{i,1}$

$A_2 \Rightarrow K_{c2}$ τ_{i2}

Criteria 1 : Minimum rise time ⇒ K_{c_1}, τ_{i_1}

2 : Minimum overshoot ⇒ K_{c_2}, τ_{i_2}

Time integral performance :

① Integral of square error, $ISE = \int_0^t e^2\,dt$

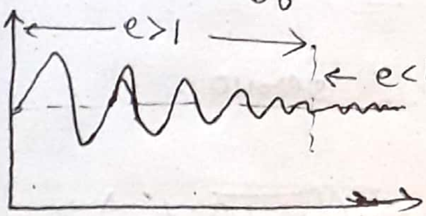
② Integral of the absolute value of the error

$$IAE = \int_0^t |\epsilon(t)|\,dt$$

error	e^2	Criterion
large	larger	ISE
small	smaller	IAE

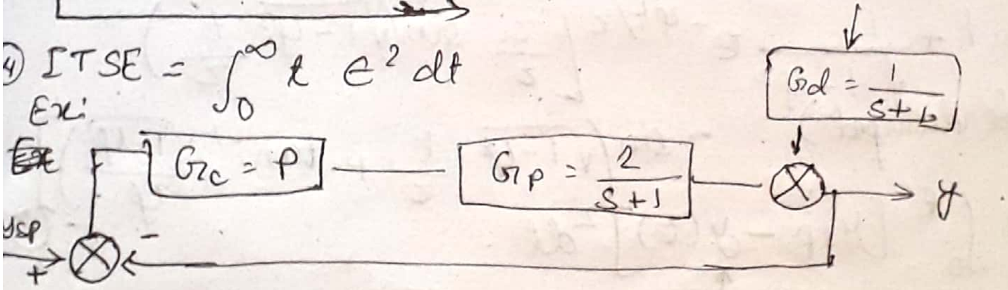
③ Integral of the time weighted absolute error :

$$ITAE = \int_0^\infty t|e|\,dt$$



④ $ITSE = \int_0^\infty t\, e^2\,dt$

Ex:



$$\bar{y} = \frac{G_c G_f G_p}{1+G_{OL}} \cdot \bar{y}_{sp} + \frac{G_d}{1+G_{OL}} \cdot \bar{d}$$

$$\bar{y} = \frac{K_c \cdot 1 \cdot \frac{2}{s+1}}{1+K_c \cdot 1 \cdot \frac{2}{s+1} \cdot 1}\,\bar{y}_{sp} + \frac{\frac{1}{s+1}}{1+K_c \cdot 1 \cdot \frac{2}{s+1} \cdot 1}\,\bar{d}$$

$$= \frac{2K_c}{2K_c + s+1}\,\bar{y}_{sp} + \frac{1}{2K_c+s+1}\,\bar{d}$$

$$= \frac{\frac{2K_c}{2K_c+1}}{\frac{s}{2K_c+1}+1}\,\bar{y}_{sp} + \frac{\frac{1}{2K_c+1}}{\frac{s}{2K_c+1}+1}\,\bar{d}$$

For PI : $G_c = \cancel{\frac{K_c}{\tau_i s}}\ K_c\left(1+\frac{1}{\tau_i s}\right)$

$$\bar{y} = \frac{\frac{K_c}{\tau_i s}\cdot 1 \times \frac{2}{s+1}}{1+\frac{K_c}{\tau_i s}\cdot 1\cdot\frac{2}{s+1}}\,\bar{y}_{sp} + \frac{\frac{1}{s+1}\cdot\cancel{\frac{K_d}{\tau_d s}}}{1+\frac{K_c}{\tau_i s}\cdot\frac{1}{s+1}}\,\bar{d}$$

$$= \frac{2K_c}{2K_c+\tau_i s(s+1)}\,\bar{y}_{sp} + \frac{K_c\,\tau_i s}{K_c+\tau_i s(s+1)}\cdot\bar{d}$$

$$= \frac{2K_c}{\tau_i s^2+\tau_i s+2K_c}\,\bar{y}_{sp}$$

$$\bar{y} = \frac{\tau_i s+1}{\tau^2 s^2+2\varphi\tau s+1}\,\bar{y}_{sp} + \frac{(\tau_i/2K_c)s}{\tau^2 s^2+2\varphi\tau s+1}\,\bar{d} \quad \text{..(1)}$$

$$\tau = \sqrt{\frac{\tau_i}{2K_c}}\ ; \quad \varphi = \frac{1}{2}\sqrt{\frac{\tau_i}{2K_c}}\,(1+2K_c)$$

(3) $$\bar{y} = \frac{\tau_i s+1}{\tau^2 s^2+2\varphi\tau s+1}\cdot\frac{1}{s} \quad \text{--- servo.}$$

(4) $$y(t) = 1+\frac{1}{\sqrt{1-\varphi^2}}\,e^{-\varphi t/\tau}\left[\frac{\tau_i}{\tau}\sin\left(\sqrt{1-\varphi^2}\,\frac{t}{\tau}\right) - \sin\left(\sqrt{1-\varphi^2}\,\frac{t}{\tau}+\tan^{-1}\frac{\sqrt{1-\varphi^2}}{\varphi}\right)\right] \quad \text{--(2)}$$

(underdamped)

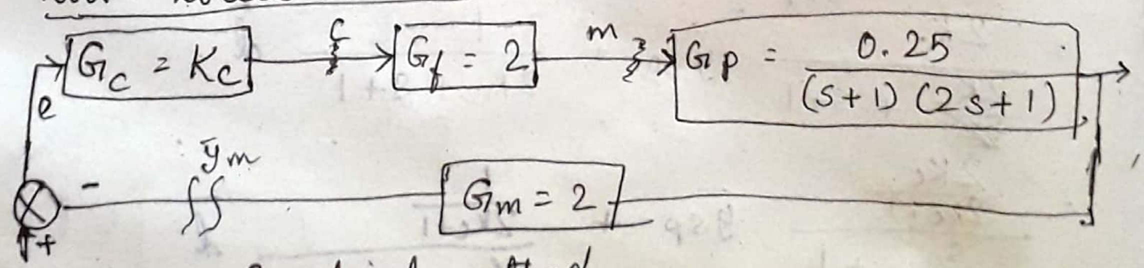
(5) $$ISE = \int_0^t \left[y_{sp} - y(t)\right]^2 dt .$$

↑ eqⁿ (1)

(6) $$\frac{d(ISE)}{dK_c} = \frac{d(ISE)}{d\tau_i} = 0$$

(7) $$\tau^* = \sqrt{\frac{\cancel{K_c}\tau_i}{2\cancel{K_c}}}\ ; \quad \varphi^* = \frac{1}{2}\sqrt{\frac{\cancel{K_c}\tau_i}{\cancel{2K_i}\ 2K_c}}\,(1+2K_c)$$

Root Locus Method



— Graphical method

$$CE : 1+G_{OL} = 0 \Rightarrow 1+\frac{\cancel{0.5}\ 5\,K_c}{(2s+1)(s+1)} = 0$$

$$G_{OL} = \frac{K_c}{(s+1)(2s+1)}$$

$$G_{OL} = \frac{\bar{y}_m}{\bar{y}_{sp}} = G_c\, G_p\, G_f\, G_m .$$

$(\text{poles})_{OL} = -1, -\frac{1}{2}$; $(\text{poles})_{CL} = -1$

$$\bar{y} = \frac{\frac{0.25}{(s+1)(2s+1)} \cdot 2 \cdot 2 \cdot K_c}{1 + \frac{K_c}{(s+1)(2s+1)}} \cdot \bar{y}_{sp}$$

$$= \frac{K_c}{(s+1)(2s+1) + K_c}\, \bar{y}_{sp} = \frac{K_c / K_c + 1}{\frac{2s^2}{K_c+1} + \frac{3s+1}{K_c+1}}$$

$2s^2 + 3s + K_c + 1 = 0$.

$$\Rightarrow s = \frac{-3 \pm \sqrt{9 - 4\times 2(K_c+1)}}{2\times 2} = \underbrace{\frac{-3 \pm \sqrt{1 - 8K_c}}{4}}_{\text{poles}}$$

K_c	P_1	P_2
0	$-\frac{1}{2}$	-1
$\frac{1}{16}$	$\frac{-3+\sqrt{\frac{1}{2}}}{4}$	$\frac{-3-\sqrt{\frac{1}{2}}}{4}$
$\frac{1}{8}$	$-\frac{3}{4}$	$-\frac{3}{4}$
$\frac{1}{4}$	$\frac{-3+i}{4}$	$\frac{-3-i}{4}$
$\frac{1}{2}$	$\frac{-3+i\sqrt{3}}{4}$	$\frac{-3-i\sqrt{3}}{4}$
1	$\frac{-3+i\sqrt{7}}{4}$	$\frac{-3-i\sqrt{7}}{4}$

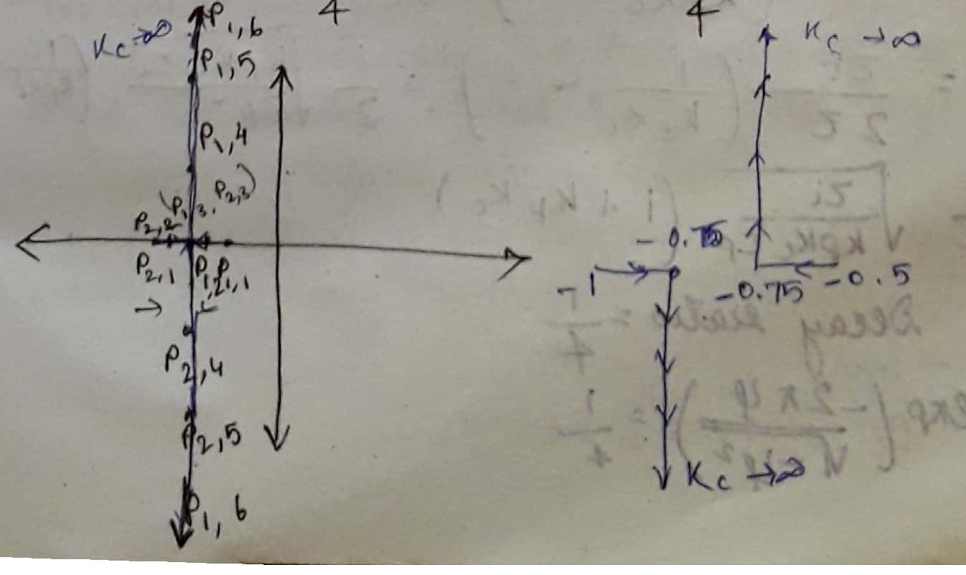
$\frac{-3\pm 1}{4}$

$= \frac{-3 \pm \sqrt{1-\frac{1}{2}}}{4}$

$= -3 \pm$

$\frac{1}{\sqrt{2}}$

1.4



Remarks:

① No. of root loci = No. of poles of G_{OL}

② Originates : poles of G_{OL}

③ Stable

④ PI: fix one & vary another from $0 \to \infty$

One quarter decay ratio method

$G_c \equiv PI \quad ; \quad G_p = \dfrac{K_p}{\tau_p s + 1}$

$G_f = G_m = 1$

$$CLTF\,(servo) = \frac{K_c\left(1+\frac{1}{\tau_i s}\right)\cdot\frac{K_p}{\tau_p s+1}}{1+K_c\left(1+\frac{1}{\tau_i s}\right)\left(\frac{K_p}{\tau_p s+1}\right)}$$

$$= \frac{K_p K_c(\tau_i s+1)}{(\tau_p s+1)\tau_i s + K_c(\tau_i s+1)(K_p)}$$

$$= \frac{K_p K_c(\tau_i s+1)}{\tau_p \tau_i s^2 + (\tau_i + K_p K_c \tau_i)s + K_p K_c}$$

$$= \frac{\cancel{K_p}\quad \tau_i s+1}{\frac{\tau_p \tau_i}{K_p K_c}s^2 + \tau_i\left(\frac{1}{K_p K_c}+1\right)s+1}$$

$$\tau = \sqrt{\frac{\tau_p \tau_i}{K_p K_c}} \; ;$$

$$2\psi\tau = \tau_i\left(\frac{1}{K_p K_c}+1\right)$$

$$\Rightarrow \psi = \frac{\tau_i}{2\tau}\left(\frac{1}{K_p K_c}+1\right) = \frac{1}{2}\sqrt{\frac{K_p K_c \tau_i}{\tau_p}}\left(\frac{1}{K_p K_c}+1\right)$$

$$= \frac{1}{2}\sqrt{\frac{\tau_i}{K_p K_c \tau_p}}\left(1+K_p K_c\right)$$

Decay ratio $= \dfrac{1}{4}$

$$\exp\left(\frac{-2\pi\psi}{\sqrt{1-\psi^2}}\right) = \frac{1}{4}$$

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$$\Rightarrow \frac{-\cancel{2}\pi \frac{1}{\cancel{2}} \sqrt{\frac{\tau_i}{K_pK_c\tau_p}}(1+K_pK_c)}{\sqrt{1-\frac{1}{4}\frac{\tau_i}{K_pK_c\tau_p}(1+K_pK_c)^2}} = \ln\left(\frac{1}{4}\right)$$

$$\Rightarrow \frac{\pi \sqrt{\frac{\tau_i}{K_pK_c\tau_p}}(1+K_pK_c)}{\sqrt{\frac{4K_pK_c\tau_p - \tau_i(1+K_pK_c)^2}{K_pK_c\tau_p}}} = \ln 4$$

$$\Rightarrow \frac{\pi\sqrt{\tau_i}(1+K_pK_c)}{\sqrt{4K_pK_c\tau_p - \tau_i(1+K_pK_c)^2}} = \ln 4.$$

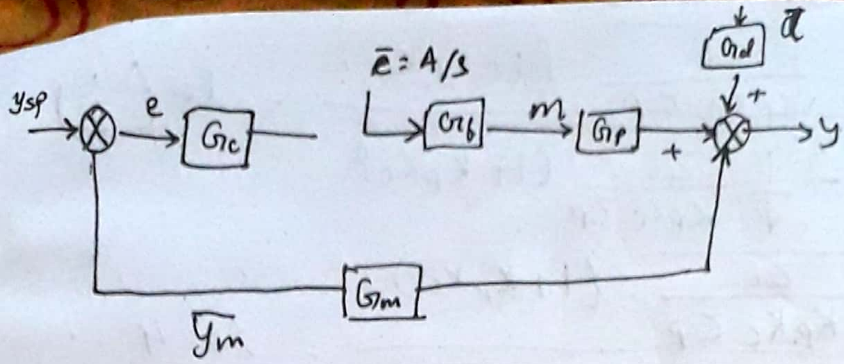
Unknown = K_c, τ_i → 2
Equations → 1 ⇒ underdetermined.
[$K_p = 0.1$, $\tau_p = 10$]

For $K_c = 1$, $\tau_i = 0.153$ $K_c = 10$, $\tau_i = 0.348$ $K_c = 50$, $\tau_i = 0.258$.

Remarks:
1. No clear cut guideline for selecting K_c.
2. PID : fixed : 2 parameters.
3. 1st order + P-only → Not applicable. i.e., valid only for 2nd order.
4. Time integral methods are better

5/9/19 **Cohen - Coon method**

1. Routh & Root locus method - these can find range of K_c only ; these methods can be used only for stability test.
2. One - quarter decay ratio method
   - No clear cut guideline for K_c
   - Considers response for two time instants, due to which we cannot expect very good performance
3. Time integral performance criteria
   - Perfect modelling is not possible. Hence, there is an error associated with it.



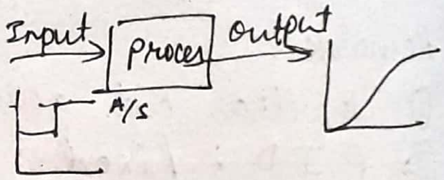
$G_{PRC} = \dfrac{\bar{y}_m}{\bar{c}} = G_m G_p G_f$ for $d = 0$

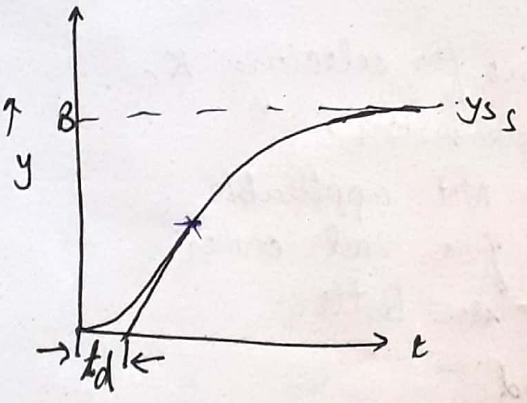
PRC → Process reaction curve

Q) This method is called PRC method, why?

→ For most of the chemical process, it is observed that the PRC response is same as that of 1st order + dead time.

$$G_{PRC} = \frac{\bar{y}_m}{\bar{c}} = G_p G_f G_m = \frac{k e^{-t_d s}}{\tau s + 1}$$





$k = \dfrac{B}{A}$

$\tau = \dfrac{B}{s}$

s = slope at $t = t_d$

$\therefore \tau = \dfrac{B}{s} = \dfrac{B}{\left.\frac{dy}{dt}\right|_{t = t_d}}$

	K_c	τ_i	τ_D
P	$\frac{1}{k}\frac{\tau}{t_d}\left(1 + \frac{t_d}{3\tau}\right)$	–	–
PI	$\frac{1}{k}\frac{\tau}{t_d}\left(0.9 + \frac{t_d}{12\tau}\right)$	$\dfrac{30 + 3\frac{t_d}{\tau}}{9 + 20\frac{t_d}{\tau}} t_d$	–
PID	$\frac{1}{k}\frac{\tau}{t_d}\left(\frac{4}{3} + \frac{t_d}{4\tau}\right)$	$t_d\left(\dfrac{32 + 6\frac{t_d}{\tau}}{13 + 8 t_d/\tau}\right)$	$t_d \dfrac{4}{11 + 2 t_d/\tau}$

Ex: $G_p = \dfrac{1}{(s+3)^2}$; G_c = PID ; $G_f = G_m = 1$

Tune the PI controller.

$$G_{PRC} = G_p G_f G_m = \frac{1}{(s+3)^2} = \frac{1}{s^2+6s+9}$$

$$\cancel{= \frac{1}{s+3} e^{-3s} = \frac{\frac{1}{3}}{\frac{s}{3}+1} e^{-3s}}.$$

$$\cancel{\tau = \frac{1}{3} \; ; \; K = \frac{1}{3} \; ; \; t_d = 3}$$

$$\frac{\bar{y}_m}{\bar{c}} = \frac{1}{(s+3)^2} \Rightarrow \bar{y}_m = \frac{1}{s(s+3)^2}$$

$$\bar{y}_m = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{(s+3)^2} = \frac{1}{9s} + \frac{1}{9(s+3)}$$

$$= \frac{A(s+3)^2 + Bs(s+3) + Cs}{s(s+3)^2} = \frac{1}{s(s+3)^2}.$$

$$= A(s^2+6s+9) + B(s^2+3s) + Cs.$$

$A + B = 0$

$A = \frac{1}{9}$

$6A + 3B + C = 0$

$\Rightarrow 3A + C = 0$

$\Rightarrow C = -3A = \frac{-1}{3}$

$$y(t) = \frac{1}{9} - \frac{1}{9} e^{-3t} - \frac{1}{3} t e^{-3t}.$$

Inflection point : $\ddot{y}_m = 0 . \Rightarrow t = 1/3$

slope $= \cancel{\frac{1}{3} e^{-3t}} \cancel{- \frac{1}{3} e^{-3t}} + t e^{-3/t}.$

$$= \frac{e^{-1}}{3} = \frac{1}{3e} = 0.123$$

$$[y_m]_{t=\frac{1}{3}} = \cancel{te} \; \frac{1}{9} - \frac{1}{9} e^{-1} - \frac{1}{9} e^{-1}$$

$$= \frac{1}{9}\left(1 - \frac{2}{e}\right) = 0.029$$

$$\text{slope} = \frac{0.0294 - 0}{\frac{1}{3} - t_d} = 0.123.$$

$$\Rightarrow \frac{1}{3} - t_d = \frac{0.0294}{0.123} \; \cancel{0.123}$$

$$\Rightarrow t_d = \cancel{0.427}, \; 0.0943.$$

$$K = \frac{B}{A} = \lim_{t \to \infty} \frac{y(t)}{1} = \frac{1}{9}.$$

$$\tau = \frac{B}{S} = \frac{\frac{1}{9}}{t e^{-3/t}} \quad \frac{\frac{1}{9}}{0.123}$$

$t_d = \cancel{0.0943}$

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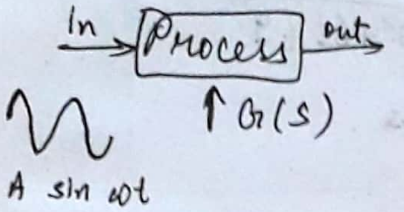
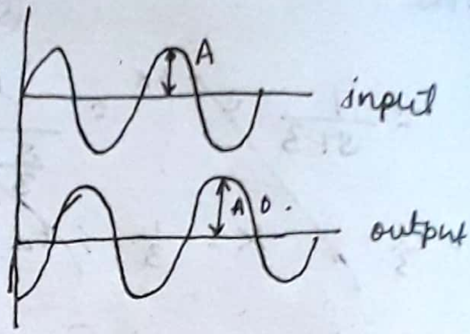
Frequency Response

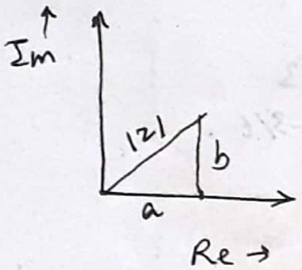




O/p : $A_0 \sin(\omega t + \phi)$

① If i/p is sinusoidal, o/p is sinusoidal as well.
② Output is out of phase by ϕ.
③ $\frac{A_0}{A}$ = Amplitude Ratio (AR).
④ ~~AR & ϕ @ ω.~~
AR & ϕ vary with ω.

Eg :- $z = a + jb$.



Magnitude of $z = |z| = \sqrt{a^2+b^2}$
Arg. of z : $\arg(z) = \angle z = \tan^{-1}\frac{b}{a}$.
$AR = |G(j\omega)|$
Why is jω taken
$\phi = \angle z = \underline{\angle G(j\omega)}$

Ex : 1st order system.

$G(s) = \frac{K}{\tau s+1}$; $G(j\omega) = \frac{K}{\tau j\omega + 1}$ = Frequency response transfer func^n

Laplace domain T.F.

$G(j\omega)$ — Cartesian : $G(j\omega) = Re(\omega) + j\, Im(\omega)$
$AR = |G(j\omega)| = \sqrt{Re^2 + Im^2}$; $\phi = \tan^{-1}\left(\frac{Im}{Re}\right)$
— Polar : $G(j\omega) = AR e^{j\phi}$

$$G(j\omega) = \frac{K}{\tau j\omega+1} = \frac{Kj}{\cancel{\tau}j - \tau\omega} = \frac{Kj\;(-j - \tau\omega)}{1+\tau^2\omega^2}$$

$$= \frac{K(1-j\tau\omega)}{1+\tau^2\omega^2} = \frac{K}{1+\tau^2\omega^2} - j\frac{K\tau\omega}{1+\tau^2\omega^2}.$$

$\therefore$ $Re(\omega) = \frac{K}{1+\tau^2\omega^2}$; $Im(\omega) = -\frac{K\tau\omega}{1+\tau^2\omega^2}$.

$$A.R. = \sqrt{\left(\frac{K}{1+\tau^2\omega^2}\right)^2 + \left(\frac{K\tau\omega}{1+\tau^2\omega^2}\right)^2} = \frac{K\sqrt{1+\tau^2\omega^2}}{1+\tau^2\omega^2}$$

$$= \frac{K}{\sqrt{1+\tau^2\omega^2}}$$

$\cancel{\phi = \tan^{-1}(-\tau\omega)}$ $\quad \phi = \tan^{-1}(-\tau\omega) = -\tan^{-1}(\tau\omega)$

$\therefore$, In polar form, $G(j\omega) = \dfrac{k}{\sqrt{1+\tau^2\omega^2}}\, e^{-j\tan^{-1}(\tau\omega)}$

$$G(j\omega) = \frac{N(j\omega)}{D(j\omega)} = \left|\frac{N(j\omega)}{D(j\omega)}\right| \frac{e^{j\angle N(j\omega)}}{e^{j(\angle D(j\omega))}}$$

$$= \underbrace{\left|\frac{N(j\omega)}{D(j\omega)}\right|}_{AR} e^{j\underbrace{(\angle N(j\omega) - \angle D(j\omega))}_{\phi}}$$

$$AR = \frac{k}{\sqrt{1+\tau^2\omega^2}} \quad \left\{ \left|\frac{k}{\tau j\omega+1}\right| \right.$$

$\therefore$, $N(j\omega) = k$
$D(j\omega) = \dfrac{K}{\cancel{\tau j\omega}\ 1+j\tau\omega}$.

$$\phi = \angle N(j\omega) - \angle D(j\omega) = -\tan^{-1}(\tau\omega)$$

2nd order : $G(s) = \dfrac{K_p}{\tau^2 s^2 + 2\varphi\tau s + 1}$

$$G(j\omega) = \frac{K_p}{-\tau^2\omega^2 + 2\varphi\tau j\omega + 1} = \frac{K_p}{1-\tau^2\omega^2 + j(2\varphi\tau\omega)}.$$

$N(j\omega) = K_p$; $D(j\omega) = 1-\tau^2\omega^2 + j(2\varphi\tau\omega)$.

$$\left|\frac{N(j\omega)}{D(j\omega)}\right| = \frac{K_p}{\sqrt{(1-\tau^2\omega^2)^2 + (2\varphi\tau\omega)^2}}$$

$$\phi = \angle N(j\omega) - \angle D(j\omega) = 0 - \tan^{-1}\left(\frac{2\varphi\tau\omega}{1-\tau^2\omega^2}\right) = -\tan^{-1}\left(\frac{2\varphi\tau\omega}{1-\tau^2\omega^2}\right)$$

$\therefore$, $\cancel{G(j}$ $AR = \dfrac{K_p}{\sqrt{(1-\tau^2\omega^2)^2 + (2\varphi\tau\omega)^2}}$

$$\phi = -\tan^{-1}\left(\frac{2\varphi\tau\omega}{1-\tau^2\omega^2}\right).$$

<u>N systems in series</u> :

$$\bar{u} \longrightarrow \boxed{G_1} \longrightarrow \boxed{G_2} \longrightarrow \cdots \longrightarrow \boxed{G_N} \longrightarrow \bar{y}_N$$

$$\frac{\bar{y}_N(s)}{\bar{u}(s)} = G(s) = G_1 G_2 \ldots G_N.$$

$$AR\, e^{j\phi} = (AR_1 e^{j\phi_1})(AR_2 e^{j\phi_2}) \ldots (AR_N e^{j\phi_N})$$

$$AR = AR_1 \cdot AR_2 \ldots AR_N$$

① $\log AR = \log AR_1 + \log AR_2 + \ldots + \log AR_N$

② $\phi = \phi_1 + \phi_2 + \ldots + \phi_N$.

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Bode plot

① AR vs ω (log - log)
② ϕ vs ω (semi - log)

Ex : Pure gain system

$G(s) = k$

$G(j\omega) = k + 0.(j\omega)$

$AR = k \Rightarrow \frac{AR}{k} = 1 =$ magnitude ratio

$\phi = 0 \qquad \Rightarrow \log(MR) = \log 1 = 0.$

Ex : 1st order system

$G(s) = \frac{k}{\tau s + 1}$

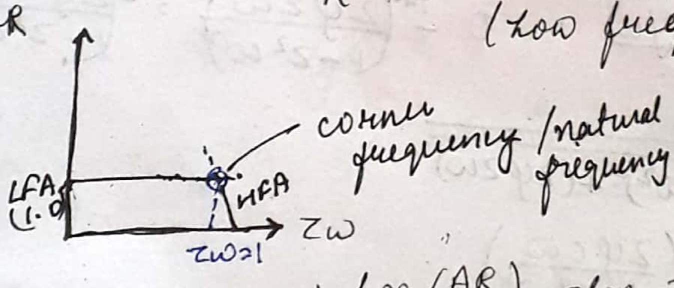
$AR = \frac{k}{\sqrt{1+\tau^2\omega^2}} \qquad \phi = -\tan^{-1}(\tau\omega).$

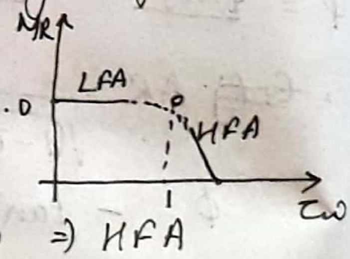
~~$\log(AR) = \log k + \frac{1}{2}\log$~~

$\log\left(\frac{AR}{k}\right) = \frac{1}{2}\log(1+\tau^2\omega^2)$

<u>Asymtotic considerations</u> :

(i) $\tau\omega \to 0$; $\frac{AR}{k} \to 1$. $\log MR = \log 1$.
(Low frequency asymtote)(LFA



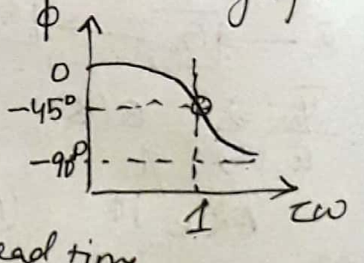


② $\tau\omega \to \infty \qquad \Rightarrow \log\left(\frac{AR}{k}\right) = -\log \tau\omega \Rightarrow HFA$

$\Rightarrow \phi = -90°$ High frequency asymptote

$\omega = \frac{1}{\tau}$

For $\tau\omega = 1 \Rightarrow \psi = 0 \Rightarrow$ No damping



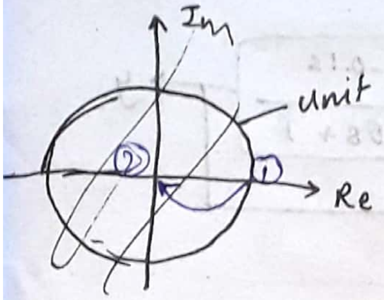
Repeat this for PI, 1st order + dead time, 2nd order

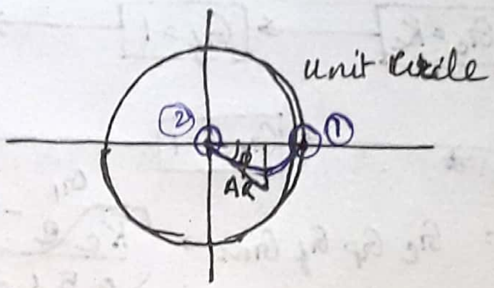
Nyquist plot

- Alternative of Bode
- Frequency response characteristics at dynamic state

$G(s) \longrightarrow G(j\omega) = Re + j\,Im$

$AR = \sqrt{Re^2 + Im^2}$; $\phi = \tan^{-1}\left(\frac{Im}{Re}\right)$





Ex 1 : 1st order system

$G(s) = \frac{K}{\tau s + 1}$

$AR = \frac{k}{\sqrt{1+\tau^2\omega^2}}$; $\phi = -\tan^{-1}(\tau\omega)$.

Let $k = \tau = 1$.

① starts $\omega \to 0$.

$AR = \frac{1}{\sqrt{1+\omega^2}}$ as $\omega \to 0$, $AR = 1$; $\phi = 0$.

② terminates $\omega \to \infty$.

$AR = 0$; $\phi = -\frac{\pi}{2}$

For 2nd order system.

$G(s) = \frac{K}{\tau^2 s^2 + 2\varphi\tau s + 1}$; $k = \tau = 1$!

$\therefore,\ G(s) = \frac{1}{s^2 + 2\varphi s + 1}$

$AR = \frac{1}{\sqrt{(1-\omega^2)^2 + (2\varphi\omega)^2}} = \cancel{\sqrt{1+\omega^4 - 2}}$

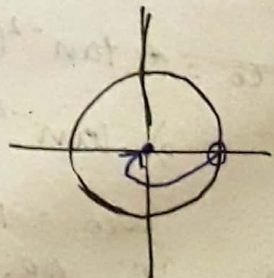
$\frac{2\varphi}{\cancel{2}-2\omega} = \frac{\varphi}{\omega}$

$\phi = -\tan^{-1}\left(\frac{2\varphi\omega}{1-\omega^2}\right)$

① At $\omega \to 0$, $AR = 1$; $\phi = 0^\circ$

② At $\omega \to \infty$, $AR = 0$; $\phi = -180^\circ$

At $\omega = 1$, $AR = \frac{1}{2\varphi}$; $\phi = -\frac{\pi}{2}$.

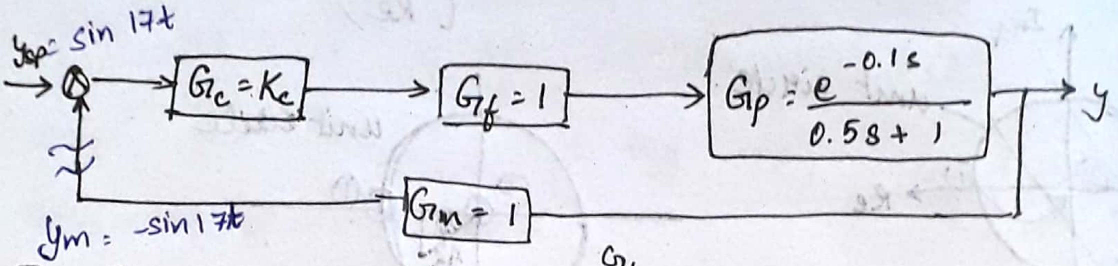


Ex 3: $G(s) = \dfrac{1}{(\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)}$

$\omega \to 0$; $AR = 1$; $\phi = 0°$

$\omega \to \infty$; $AR = 0$; $\phi = -270°$

<u>Bode stability criteria</u> :



$\dfrac{y_m}{y_{sp}} = G_{OL} = G_c G_p G_f G_m = \underbrace{\dfrac{K_c}{0.5s+1}}_{G_1} e^{-0.1s} = G_1 \cdot G_2$

$G_{OL}(j\omega) = K_c \dfrac{e^{-0.1j\omega}}{0.5j\omega+1}$

$\phi = \phi_1 + \phi_2$

$\dfrac{K_c}{0.5j\omega+1} = \dfrac{K_c(1-0.5j\omega)}{1+(0.5\omega)^2}$

$\phi_1 = \tan^{-1}(-0.5\omega)$

$e^{-0.1j\omega}$ $\therefore \phi_2 = -0.1\omega$

$\therefore \phi = \tan^{-1}(-0.5\omega) - 0.1\omega \times \dfrac{180°}{\pi}$

~~For $\phi = 180°$,~~

The frequency at $\phi = -180°$ is called cross over frequency (ω_{co}) $\phi < 180° \to$ unstable

$\phi > 180° \to$ stable

$\tan^{-1}(0.5\omega) + \dfrac{0.1 \times 180°}{\pi}\omega = 180$.

$\Rightarrow \tan^{-1}\left(\dfrac{\omega_{co}}{2}\right) + \dfrac{18}{\pi}\omega_{co} = 180$

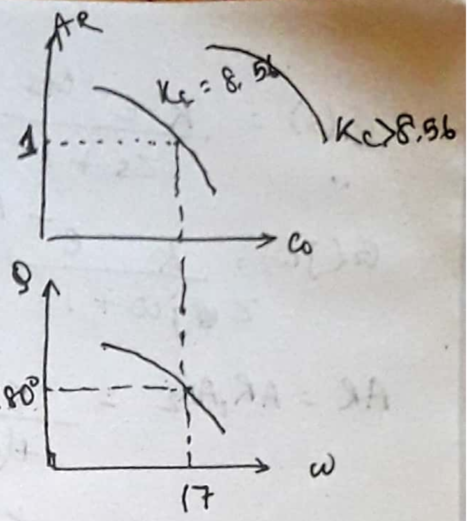
$\omega_{co} = 17$ rad/min

$AR = AR_1 \cdot AR_2 = \left(\dfrac{K_c}{1+(0.5\omega)^2}\right)\sqrt{1+(0.5\omega)^2} \cdot 1.$

$= \dfrac{K_c}{\sqrt{1+(0.5\omega)^2}} = \dfrac{1}{\cancel{\sqrt{1+(0.5\times17)^2}}}$

$\phi = -180^\circ$; AR = 1.

$\Rightarrow K_c = 8.56$.



Open loop

① $\phi = -180^\circ$, AR = 1

② $y_{sp} = A \sin \omega t$
$= \sin 17t \quad (A = 1)$.

Output = ~~y~~ ~~y~~ $A_0 \sin(\omega t + \phi)$

~~$\bar{y}_m = G_{OL} \bar{y}_{sp}$.~~

~~$= \frac{K_c \cdot e^{-0.1s}}{0.5s+1}$~~ .

$AR = \frac{A_0}{A} = 1 \Rightarrow A_0 = A = 1$.

$\phi = -180^\circ$

$\therefore$ o/p = $\sin(\omega t - 180^\circ)$ $= \sin(\omega t + 180^\circ)$

$= -\cancel{\cos}\ \sin\ \omega t$. $= -\sin 17t$

① close the loop

② $y_{sp} = 0$

AR = 1.2

Input	output
$\sin 17t$	$-1.2 \sin 17t$.
$1.2 \sin 17t$	$-1.44 \sin 17t$.
$1.44 \sin 17t$	$-(1.2)^3 \sin 17t$

$\phi = -180^\circ$ —
- AR > 0 → unstable
- AR = 1 → marginally stable
- AR < 1 → stable.

A feedback system is said to be unstable if the AR of the corresponding open loop transfer function is greater than one at cross over frequency.

Ex 1: 1st order system

$G(s) = \frac{K}{\tau s + 1}$; $0 < \omega < \infty$

$1 > AR > 0$

$AR = \frac{K_c}{\sqrt{1 + \tau^2 \omega^2}}$ $\quad 0^\circ > \phi > -90^\circ$.

Ex 2: 2nd order system.

$G(s) = \frac{K}{\tau^2 \omega^2 + 2\zeta \tau \omega + 1}$; $0 < \omega < \infty$; $0^\circ > \phi > -180^\circ$

$1 > AR > 0$

For 3rd order system ; $0 < \omega < \infty$

$1 > AR > 0$

$0^\circ > \phi > -270^\circ$

$$G(s) = \frac{K e^{-t_d s}}{\tau s + 1}$$

$$G(j\omega) = \frac{K}{\tau j\omega + 1} e^{-t_d j\omega} = \frac{K(1 - \tau j\omega)}{1 + (\tau\omega)^2} e^{-t_d j\omega}.$$

$$AR = AR_1 AR_2 = \frac{K}{1+(\tau\omega)^2}\sqrt{1+(\tau\omega)^2} \quad ; \quad 1.$$

$$= \frac{K}{\sqrt{1+\tau^2\omega^2}}$$

$$\phi = \phi_1 + \phi_2 = \tan^{-1}(-\tau\omega) - \omega t_d \cdot \frac{180}{\pi}$$

$0 < \omega < \infty$

$1 > AR > 0$

$0 > \phi > -90° - \infty$

$0 > \phi > -\infty$

R_i θ h_2

$\theta = 270° - \alpha_T$.

$\cos\theta = \cos(270° - \alpha_T) \approx -\sin\alpha_T$

$H = h_1 + h_2$

$h_1 = R_1 \sin\alpha_S$; $h_2 = R_i \cos\theta$

$= R_i \cos(270 - \alpha_T)$

$\approx R_i \sin\alpha_T$

Q7) $D = 2000$ mm

$d = 100$ mm

$n : 15$ rpm

$\frac{mv^2}{R} = mg\cos\theta + N$

$m\omega^2 r = mg\cos\theta + N$.

$\omega^2 r = K$.

$$\cos\theta = \frac{[2\pi(R-r)\omega]^2}{9.81(R-r)60^2} ; \quad V_c = \frac{42.3}{\sqrt{D-d}}$$

$$V_c = \frac{42.3}{\sqrt{2000-100}} = 0.97$$

$$\frac{V_c}{rpm_1} = \frac{V_c'}{rpm_2}. \qquad rpm_2 = \frac{V_c'}{V_c} \cdot rpm_1$$

$$= \left(\frac{\sqrt{2000-50}}{\sqrt{2000-100}}\right) \times 15$$

~~15.196~~. 14.79 rpm

MK 26/9/19

$F_{drag, unit\ movement} = \left(f\left(\mu_i, \dot{m}_{belt}, \dot{m}_{powder}, \dot{m}_{other\ component}\right)\right)g$ $\rightarrow \tan\phi_i$

$\Rightarrow$ Power $= F_{drag} \cdot v$

Q, $q_{powder} = \frac{(\dot{m}_{powder})\,g}{v}$

$$q_{us,i} = \frac{m_{qs,i}\, g}{l_{us,i}}$$

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