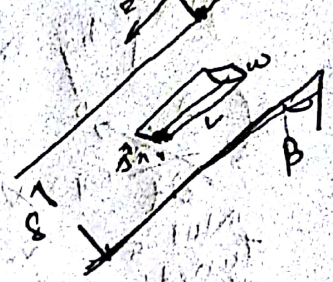
# FLUID MECHANICS




1.

Momentum balance

$M^2_{in} - M^2_{out} + \Sigma F = 0$

$\Rightarrow$ $L \times W \times \tau_{x+\Delta x} - L W \tau_x + g \times \rho \Delta x L W \cos\beta = 0$

$\Rightarrow$ $(\tau_{x+\Delta x} - \tau_x) = -\rho \Delta x g \cos\beta$

$\Rightarrow$ $\frac{\partial \tau}{\partial x} = -\rho g \cos\beta$

$\Rightarrow$ $\tau = -\rho g x \cos\beta + C$

$\tau = -\mu\left(\frac{\partial v}{\partial x}\right)$ $\Rightarrow$ $-\mu\left(\frac{\partial v}{\partial x}\right) = -\rho g x \cos\beta + C$

$\Rightarrow$ $V = \left(-\frac{\rho g x^2}{2} \cos\beta + C x + d\right)\frac{-1}{\mu}$

Boundary equations

At $x = 0$, $\tau = 0$

$\Rightarrow$ $C = 0$

At $x = \delta$, $V = 0$

$\Rightarrow$ $d = -\frac{\rho g \delta^2}{2} \cos\beta$

$\Rightarrow$ $V = -\frac{\rho g (x^2 - \delta^2)}{\mu \; 2} \cos\beta$

$= \frac{\rho g (\delta^2 - x^2)}{2\mu} \cos\beta = \frac{\rho g \delta^2 \left(1 - \left(\frac{x}{\delta}\right)^2\right)\cos\beta}{2\mu}$

Scanned by CamScanner

ii) Solving by Navier Stokes equation.

$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}}^{0} + \cancel{v_x}^{0}\frac{\partial v_z}{\partial x} + v_y\cancel{\frac{\partial v_z}{\partial y}}^{0} + v_z\cancel{\frac{\partial v_z}{\partial z}}^{0}\right)$$

Not a function of time

$v_x = 0$

Not a function of z

$$= -\cancel{\frac{dP}{\partial z}}^{0} + \rho g_z + \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \cancel{\frac{\partial^2 v_z}{\partial y^2}}^{0} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}^{0}\right)$$

No imposed pressure grad

$$\Rightarrow -\mu\frac{\partial^2 v_z}{\partial x^2} = \rho g \cos\beta$$

$$\Rightarrow \underbrace{\mu\frac{\partial v_z}{\partial x}}_{\tau} = \rho g x \cos\beta$$

↓ (same equations obtained hence verified.

$Q$ = flow rate = $\langle v_z\rangle \times$ flow area

$$\langle v_z\rangle = \frac{\int_0^w\int_0^\delta v_z\,dx\,dy}{\int_0^w\int_0^\delta dx\,dy} = \frac{\rho g\delta^2}{2\mu}\cos\beta \frac{x\,w\left(1-\left(\frac{x}{\delta}\right)^2\right)\Big|_0^\delta}{(w\times\delta)}$$

$$= \frac{\rho g\delta^2}{2\mu}\cos\beta\times\underbrace{\left(x - \frac{x^3}{3\delta^2}\right)}_{\frac{2\delta^2}{3}}$$

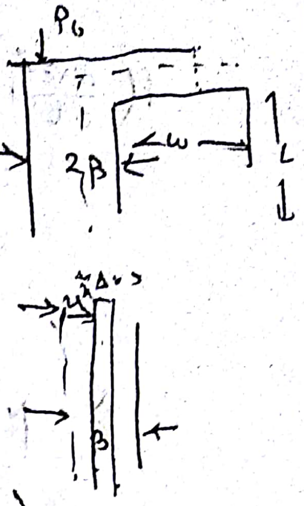
$$= \frac{\rho g\delta^2\cos\beta}{3\mu}$$

$$Q = \frac{\rho g\delta^2\cos\beta}{3\mu}\times\delta\times w = \frac{\rho g\delta^3\times w\cos\beta}{3\mu}$$

Scanned by CamScanner

② Laminar flow in narrow slit

① Momentum balance



$$\tau_x \times (L W)\Big|_{x=x} - L W \times \tau_{x+\Delta x}\Big|_{x=x+\Delta x}$$

$$+ \rho g \times (\Delta x) \times L \times W$$

$$+ (P_0 - P_L) W \Delta x$$

$$\Rightarrow \frac{\tau_{x+\Delta x} - \tau_x}{\Delta x} = \left(\rho g + \frac{P_0 - P_L}{L}\right)$$

$$\Rightarrow \frac{\partial \tau}{\partial x} = \left[\rho g + \left(\frac{P_0 - P_L}{L}\right)\right]$$

$$\Rightarrow \tau = \left[\rho g + \left(\frac{P_0 - P_L}{L}\right)\right] x + C$$

$$\Rightarrow -\mu \frac{\partial v_z}{\partial x} = \left[\rho g + \left(\frac{P_0 - P_L}{L}\right)\right] x + C$$

$$\Rightarrow v_z = \frac{-1}{\mu}\left(\left[\rho g + \frac{(P_0 - P_L)}{L}\right]\frac{x^2}{2} + C x + d\right)$$

Boundary conditions

at $x = \beta$, $v = 0$

$x = 0$, $\tau = 0$

$$\Rightarrow C = 0$$

$$d = \frac{1}{\mu}\left[\rho g + \frac{(P_0 - P_L)}{L}\right]\frac{\beta^2}{2}$$

$$v_z = \frac{1}{\mu}\left[\rho g + \frac{P_0 - P_L}{L}\right] \times \frac{\beta^2}{2} \times \left(1 - \frac{x^2}{\beta^2}\right)$$

② Using Navier Stoke's equation.

$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}} + \cancel{v_x\frac{\partial v_z}{\partial x}} + \cancel{v_y\frac{\partial v_z}{\partial y}} + \cancel{v_z\frac{\partial v_z}{\partial z}}\right)$$

$$= -\frac{dP}{\partial z} + \rho g + \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2}^{0} + \frac{\partial^2 v_z}{\partial z^2}^{0}\right)$$

$$\Rightarrow -\mu\left(\frac{\partial^2 v_z}{\partial x^2}\right) = -\frac{(P_L - P_0)}{L} + \rho g$$

$$\Rightarrow -\frac{\partial \tau}{\partial x} = \rho g + \left(\frac{P_0 - P_L}{L}\right)$$

Average

$$\langle v_z \rangle = \frac{\int_0^{W}\int_0^{\beta} v_z\, \partial x\, dy}{\int_0^{W}\int_0^{\beta} \partial x\, dy}$$

$$= \frac{\frac{\beta^2}{2\mu}\left[\rho g + \frac{P_0 - P_L}{L}\right]\left(1 - \frac{x^2}{\beta^2}\right)\partial x \times W}{\beta \times W}$$

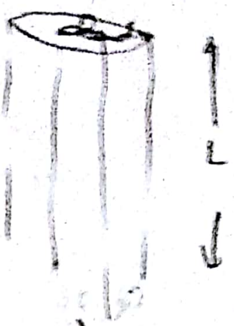
$$= \frac{\beta^2}{3\mu} \times \left(\rho g + \frac{P_0 - P_L}{L}\right)$$

$$Q = \frac{\beta^2}{3\mu} \times \left(\rho g + \frac{P_0 - P_L}{L}\right) \times (2\beta) \times W$$

$$= \frac{2\beta^3 W}{3\mu}\left(\rho g + \frac{P_0 - P_L}{L}\right)$$

③ Liquid flowing through a pipe

Using Navier-Stokes equation




$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}}^{\,0} + v_r \cancel{\frac{\partial v_z}{\partial r}}^{\,0} + \frac{v_\theta}{r}\cancel{\frac{\partial v_z}{\partial \theta}}^{\,0} + v_z \cancel{\frac{\partial v_z}{\partial z}}^{\,0}\right)$$

$$= \rho g - \frac{\partial P}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\cancel{\frac{\partial^2 v_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right)$$

$$\Rightarrow -\mu\left(\frac{1}{r}\frac{\partial}{\partial r} r\left(\frac{\partial v_z}{\partial r}\right)\right) = \rho g - \left(\frac{P_L - P_0}{L}\right)$$

$$= \rho g + \left(\frac{P_0 - P_L}{L}\right)$$

$$\Rightarrow -\mu\frac{\partial}{\partial r}\left(r\left(\frac{\partial v_z}{\partial r}\right)\right) = \left(\rho g + \frac{P_0 - P_L}{L}\right) r$$

$$\Rightarrow -\mu\left(r\frac{\partial v_z}{\partial r}\right) = \left(\frac{P_0 - P_L}{L}\right)\frac{r^2}{2} + C$$

$$\Rightarrow \frac{\partial v_z}{\partial r} = \frac{-1}{\mu}\left[\left(\frac{P_0 - P_L}{L}\right)\frac{r}{2} + \frac{C}{r}\right]$$

$$\Rightarrow \tau = \left(\frac{P_0 - P_L}{L}\right)\frac{r}{2} + C/r$$

$$v_z = \frac{-1}{\mu}\left[\left(\frac{P_0 - P_L}{L}\right)\frac{r^2}{4} + C\ln r + d\right]$$

At $r = 0$, $\tau = 0$ $\Rightarrow C = 0$

At $r = R$, $v = 0$

$$-\left(\frac{P_0 - P_L}{L}\right)\frac{R^2}{4} = d$$

$$v_z = \frac{1}{\mu}\left[\left(\frac{P_0 - P_L}{L}\right)\frac{R^2 - r^2}{4}\right]$$

$$= \frac{(P_0 - P_L)}{4\mu L} \times R^2 \left(1 - r^2/R^2\right)$$

$$Q = \langle v_z \rangle \times \pi R^2$$

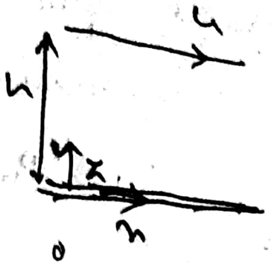
$$\frac{\int_0^{2\pi}\int_0^{R} v_z \, r \, dr \, d\theta}{\pi R^2} = \frac{(P_0 - P_L)}{4\mu L} \times R^2 \times \frac{2\pi R^2}{2}\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$= \frac{P_0 - P_L}{4\mu L} \times R^2 \times \frac{R^2}{2^2} \times \frac{2\pi}{\pi R^2}$$

$$\langle v_z \rangle = \frac{(P_0 - P_L) R^2}{8\mu L}$$

$$\boxed{Q = \frac{(P_0 - P_L)\pi R^4}{8\mu L}}$$

(4). Couette flow




$$\frac{\partial \rho}{\partial t} + \rho \Big( \partial$$

$$\rho\left(\cancel{\frac{\partial v_x}{\partial t}} + v_z \cancel{\frac{\partial v_x}{\partial z}} + v_y \cancel{\frac{\partial v_x}{\partial y}} + v_x \cancel{\frac{\partial v_x}{\partial x}}\right)$$

$$= \frac{-dP}{dx} + \mu\left[\cancel{\frac{\partial^2 v_x}{\partial x^2}} + \cancel{\frac{\partial^2 v_x}{\partial y^2}} + \frac{\partial^2 v_x}{\partial z^2}\right] + \cancel{\rho g_x}$$

If there is no imposed pressure gradient,

$$\mu \frac{\partial^2 v_x}{\partial z^2} = 0$$

$$\Rightarrow \mu \frac{\partial v_x}{\partial z} = c$$

$$\Rightarrow v_x = (cz + d)$$

Boundary conditions

At $z = 0$, $v_x = 0$ ; At $z = h$, $v_x = u$

$\Rightarrow V_x = cz$

At $z = h$, $\tau = 0$

$\Rightarrow u = ch$

$\Rightarrow \left(c = \frac{u}{h}\right)$

$$\Rightarrow \left(v_x = \frac{u}{h} z\right)$$

$$\left(\tau = \frac{-u\mu}{h}\right) \quad \left(-\mu \frac{\partial v_x}{\partial z}\right)$$

Pressure gradient $= \frac{P_L - P_0}{L}$

Favourable when

$P_0 > P_L$

Let in couette flow, there exists a pressure gradient

$\Rightarrow \frac{dP}{\partial x} = \mu \frac{\partial^2 v_x}{\partial z^2}$ Let $\frac{dP}{\partial x} = c$

$\Rightarrow c = \mu\left(\frac{\partial^2 v_x}{\partial z^2}\right)$

$\Rightarrow (cz + d) = \mu \frac{\partial v_x}{\partial z}$

$\Rightarrow \left(\frac{cz^2}{2} + dz + e\right) = \mu(v_x)$

∴ Same boundary conditions

At, $z = 0$, $v = 0$

$\Rightarrow e = 0$

$z = h$, $v = u$

$\left(\frac{ch^2}{2} + dh\right) = \mu u$

~~$z=0$, at $z=h$~~

~~$-(ch+d) = 0$~~

$\Rightarrow (d = -ch)$

$\Rightarrow d = \frac{1}{h}\left(\mu u - \frac{ch^2}{2}\right)$

$\Rightarrow v_x = \frac{1}{\mu}\left[\frac{cz^2}{2} + \frac{1}{h}\left(\mu u - \frac{ch^2}{2}\right)\right]$

Now, let an unfavourable pressure gradient is applied so that net flow rate = 0

$\Rightarrow$

$$\langle v_x \rangle = 0$$

$$\Rightarrow \frac{\int_0^h \frac{1}{\mu}\left[\frac{cz^2}{2} + \frac{1}{h}\left(\mu u - \frac{ch^2}{2}\right)z\right]dz}{h} = 0$$

$$\Rightarrow \frac{\left(\frac{cz^3}{6} + \frac{\mu z^2 u}{2h} - \frac{ch z^2}{4}\right)}{h}$$

$$\Rightarrow \frac{1}{h}\left(\frac{ch^3}{6} + \frac{\mu h^2 u}{2} - \frac{ch^3}{4}\right) = 0$$

$$\Rightarrow \frac{\mu u}{2} = ch^2\left(\frac{1}{4} - \frac{1}{6}\right)$$

$$= \frac{ch^2}{12}$$

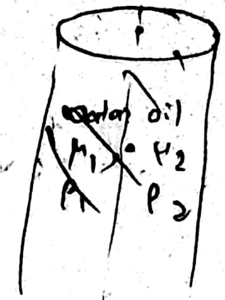
$$\Rightarrow \boxed{c = \frac{6\mu u}{h^2}}$$

we can find fav. pressure gradient for v to be max.

similarly,

equating $\frac{\partial v_x}{\partial z} = 0$

# Remember




If there is an interface of two liquids flowing, then at that section, the velocities of both the fluids are the same.

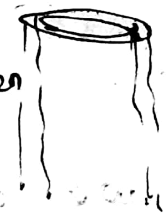
→ Also, the stress at interface are same.

6. Magnet wire being coated with varnish in a tube wire assembly. Find the force required to pull.

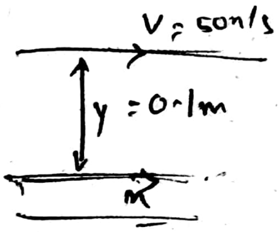
$d = 0.8\,mm$ $\quad L = 20\,mm$

$D = 0.9\,mm$ $\quad \mu = 20\,cP$

$v = 50\,m/s$



→ "When the separation between two layers is very small, we can convert the cylindrical co-ordinate system to cartesian co-ordinate system.




$$\rho\left(\cancel{\frac{\partial v_x}{\partial t}} + \cancel{v_x\frac{\partial v_x}{\partial x}} + \cancel{v_y\frac{\partial v_x}{\partial y}} + \cancel{v_z\frac{\partial v_x}{\partial z}}\right)$$

$$= \frac{-dP}{\partial x} + \mu\left[\cancel{\frac{\partial^2 v_x}{\partial x^2}} + \frac{\partial^2 v_x}{\partial y^2} + \cancel{\frac{\partial^2 v_x}{\partial z^2}}\right] + \cancel{\rho g}$$

$$\Rightarrow \frac{\partial p}{\partial x} = \mu \frac{\partial^2 v_x}{\partial y^2}$$

It is similar to a couette flow.

$$\Rightarrow \mu \frac{\partial^2 v_x}{\partial y^2} = 0$$

$$\Rightarrow \frac{\partial v_x}{\partial y} = c$$

$$\Rightarrow v_x = cy + d$$

At $y = 0$, $v_x = 0$

$\Rightarrow$ At $y =$ ~~0.05~~ $0.05\,mm$

$v_x = 50\,m/s$

$$\Rightarrow c = \frac{50}{0.05 \times 10^{-3}} = (\cancel{1 \times 10^5}) = 1 \times 10^6$$

$$v_x = cy$$

$$-\mu \frac{\partial v_x}{\partial y} = -\mu c = \frac{\cancel{20}}{\cancel{100}} \times 1 \times 10^6 = (\cancel{10^5}) = 2 \times 10^5$$

$\cancel{(F \times 2\pi DL)} = \tau$

$$\Rightarrow \cancel{F = \frac{10^5}{2\pi \times (0.8 \times 10^{-3}) \times 20 \times 10^{-3}}}$$

$(\tau \times \pi DL) = F$

$$\Rightarrow F = 2 \times 10^5 \times \pi \times d \times L$$

$$= 2 \times 10^5 \times \pi \times 0.8 \times 10^{-3} \times 20 \times 10^{-3}$$

$$= 10.05\,N$$

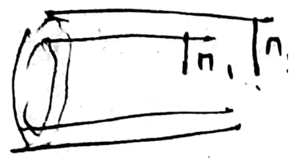
TRYING TO SOLVE IN CYLINDRICAL SYSTEM OF CO-ORDINATE

$v_z$ is the direction of flow here.

$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}}^{0} + v_r\frac{\partial v_z}{\partial r}^{0} + v_z\cancel{\frac{\partial v_z}{\partial z}}^{0} + \frac{v_\theta}{r}\cancel{\frac{\partial v_z}{\partial \theta}}^{0}\right)$$

$$= \cancel{\rho g_z}^{0} - \cancel{\frac{\partial p}{\partial z}}^{0} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\cancel{\frac{\partial^2 u_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 u_z}{\partial z^2}}^{0}\right]$$

$$\Rightarrow \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\left(\frac{\partial v_z}{\partial r}\right)\right) = 0$$




$$\Rightarrow r\left(\frac{\partial v_z}{\partial r}\right) = c$$

$$\Rightarrow -\mu\frac{\partial v_z}{\partial r} = -\mu\frac{c}{r}$$

$$\Rightarrow v_z = (c\ln r) + d$$

At $r = r_2$, $V_z = u$

$r = r_1$, $v_z = 0$

$$\Rightarrow c\ln r_2 + d = u$$

$$c\ln r_1 + d = 0$$

$$\Rightarrow c\ln\left(r_2/r_1\right) = u$$

$$\Rightarrow c = \frac{u}{\ln\left(r_2/r_1\right)}$$

Scanned by CamScanner

$$\Rightarrow v_2 = \frac{u(\ln r)}{\ln(r_2/r_1)} + d$$

$$d = -c \ln r_1$$

$$\Rightarrow v_2 = \frac{u}{\ln(r_2/r_1)}\left[\ln r/r_1\right]$$

$$= \frac{u(\ln r - \ln r_1)}{(\ln r_2 - \ln r_1)}$$

$$\boxed{\tau = \mu\left[\frac{1}{r}\,\partial(r v_z)\right]}$$

$$= \frac{-\mu}{(\ln r_2 - \ln r_1)} \times u \frac{(\ln r + 1 - \ln r_1)}{r}$$

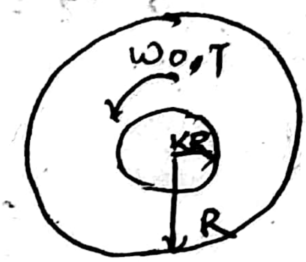
At $r = r_1$, $\tau =$

$$= \frac{-\mu \times \cancel{u}}{r_1(\ln r_2/r_1)}$$

$$F = \frac{\mu}{\cancel{r_1} \times \ln(r_2/r_1)} \times 2\pi \cancel{r_1} \times L \times u$$

$$= \frac{2\pi \times \mu u \times L}{\ln(r_2/r_1)} = 10.66\ N$$

7. Two cylinders with inner one rotating by application of a known torque T. Find expression of velocity in terms of T.




Here, $v_\theta$ is to be considered as velocity change occurs along $\theta$.

$$\rho\left(\cancel{\frac{\partial v_\theta}{\partial t}}^{0} + \cancel{v_r}^{0}\cancel{\frac{\partial v_\theta}{\partial r}}^{0} + \frac{v_\theta}{r}\cancel{\frac{\partial v_\theta}{\partial \theta}} - \cancel{\frac{v_\theta v_r}{r}} + v_z\cancel{\frac{\partial v_\theta}{\partial z}}\right)$$

$\frac{\partial v_\theta}{\partial \theta}$ → $V_\theta$ changes with $\theta$

$$= \cancel{\rho g_\theta}^{0} - \frac{1}{r}\cancel{\frac{\partial p}{\partial \theta}}^{0} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right)\right.$$

Doubt

$$\left(-\cancel{\frac{v_\theta}{r^2}}^{0}\right) + \frac{1}{r^2}\cancel{\frac{\partial^2 v_\theta}{\partial \theta^2}}^{0} + \cancel{\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}}^{0}$$

$$\left. + \cancel{\frac{\partial^2 v_\theta}{\partial z^2}}\right]$$

(Diff. eqn)

$\Rightarrow \mu\left[\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\ldots}\right.\right.\right.$

$$\mu \frac{\partial}{\partial r}\left(\frac{1}{r}\left(\frac{\partial}{\partial r} r V_\theta\right)\right) = 0$$

$$\Rightarrow \frac{\partial}{\partial r}(r V_\theta) = C r$$

$$\Rightarrow r V_\theta = \frac{C r^2}{2} + d$$

$$\Rightarrow V_\theta = \frac{C r}{2} + \frac{d}{r}$$

B.C

At $r = KR$, $V_\theta = (\omega_0 \times KR)$

$$\Rightarrow \left(\omega_0 KR = \frac{CKR}{2} + \frac{d}{KR}\right)$$

$$\Rightarrow \omega_0 KR = \left(\frac{CKR}{2} - \frac{CR}{2K}\right)$$

$r = R$, $V_\theta = 0$

$$\Rightarrow C = \frac{\omega_0 KR}{\left(\frac{K}{2} - \frac{1}{2K}\right)}$$

$$= \frac{2\omega_0 R}{\left(1 - 1/K^2\right)}$$

$$\Rightarrow \frac{CR}{2} + \frac{d}{R} = 0$$

$$\Rightarrow d = -\frac{CR^2}{2}$$

$$V_\theta = \frac{\omega_0 R}{\left(1 - 1/K^2\right)} \cdot r - \frac{\omega_0 R^3}{\left(1 - 1/K^2\right) \times r}$$

$$= \frac{\omega_0 R^2}{\left(1 - 1/K^2\right)}\left[\frac{r}{R} - \frac{R}{r}\right]$$

$$\tau_{r\theta} = -\mu\left[r \frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right)\right]$$

$$= \left(\frac{\omega_0 R}{1 - 1/K^2}\right) \cdot -\mu\left[r \times \frac{\partial}{\partial r}\left(\frac{1}{R} - \frac{R}{r^2}\right)\right]$$

$$= \frac{-\mu r \times \omega_0 R}{1 - 1/K^2} \times \frac{-2R}{r^3} = \frac{2\mu\omega_0 R^2}{\left(1 - 1/K^2\right) r^2}$$

$$= \frac{\mu \omega_0 2R^2}{\left(1-\frac{1}{K^2}\right) r^2}$$

In fig.,

$$T = \text{torque} = F \times d$$

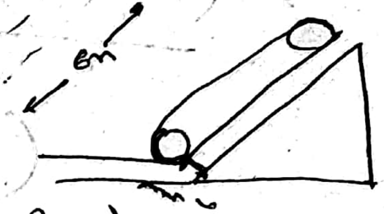
$$= \left[\tau_{r\theta} \times (2\pi K R) \times L\right] \times KR$$

$$= \frac{\mu \omega_0}{\left(1-\frac{1}{K^2}\right)} \times \frac{4\pi K^2 R^4 L}{r^2}$$

$$= \frac{\mu \omega_0}{\left(1-\frac{1}{K^2}\right)} \times \frac{4\pi K^2 R^4 L}{(KR)^2}$$

$$= \frac{\mu \omega_0 \times 4\pi R^2 L}{\left(1-\frac{1}{K^2}\right)}$$

6.




$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial t} + v_y\frac{\partial v_z}{\partial t} + v_z\frac{\partial v_z}{\partial t}\right)$$

$$= -\frac{\partial P}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 v_z}{\partial x^2}\right)$$

$$\Rightarrow \mu\left(\frac{\partial^2 v_z}{\partial x^2}\right) = \left(\frac{\partial P}{\partial z} - \rho g_z\right)$$

$$\Rightarrow \mu\frac{\partial^2 v_z}{\partial x^2} = (C - \rho g_z)$$

$$\Rightarrow \mu\left(\frac{\partial v_z}{\partial x}\right) = \left(C - \frac{\rho g}{2}\right)x + d$$

$$\Rightarrow \mu v_z = \left(C - \frac{\rho g}{2}\right)\frac{x^2}{2} + dx + C$$

At $x = 0$,

$v = 0$

$$\Rightarrow v_z = \frac{1}{\mu}\left[\left(\cancel{C}^{0} - \frac{\rho g}{2}\right)\frac{x^2}{2} + dx\right]$$

At $v = u$, $x = 2\,mm$

$$\Rightarrow v_z = -\frac{\rho g}{4\mu}x^2 + Cx$$

At $x = 2\,mm$, $v_z = 3\,m/s$

Scanned by CamScanner

$$\Rightarrow \frac{3 + \frac{\rho g}{4\mu} x^2}{x} = c_1 x$$

$$\Rightarrow c_1 = \cancel{(436\,40)}$$

$$\cancel{v} = \cancel{\left(\frac{\rho g x^2}{4\mu} - 436\,40 x\right)} -$$

$$\frac{\rho g}{4\mu} = 2.1 \times 10^{7}$$

$$v_z = \frac{\rho g \cos\beta}{2\mu} + C x$$

10 poise = 1 N

1 po ($10^{-1}$)

$\mu = 10^{-2}$

($10^{-1}$)

($2.1 \times 10^{5}$)

$$\Rightarrow C = \frac{\left(v_z - \frac{\rho g \cos\beta}{2\mu}\right)}{x}$$

$$= \cancel{(\ldots 21 \ldots)} \,/\, 1080$$

$$\frac{\left(\frac{2.1 \times 10^{5} x^3}{3} + \frac{C x^2}{2}\right)}{2 \times 10^{-3}} = \frac{\left(\frac{2.1 \times 10^{3}}{3} x + \frac{C}{2}\right) x}{140}$$

$$v_z = \frac{\rho g \cos\beta}{2\mu} + C x$$

$$\Rightarrow C = \left(v_z - \frac{\rho g \cos\beta}{2\mu}\right) \frac{1}{x}$$

$$\Rightarrow C = 1080$$

$$\langle v_z \rangle = \left(\frac{2.1 \times 10^{3}}{3} x + \frac{C}{2}\right) x$$

$$= 1.36$$

$$Q = 1.36 \times 10 \times 10^{-3}$$

$$= 0.0136$$