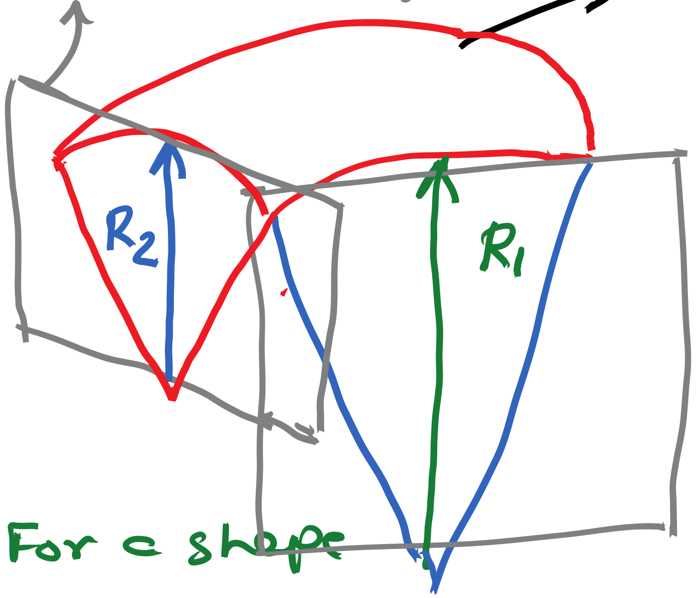
* If the Curvature at two different planes orthogonal to each other is known at any point, then the Local Curvature at that point is known.

16/03/2022

Second Plane (Radius R_2)

Arbitrarily Curved Surface



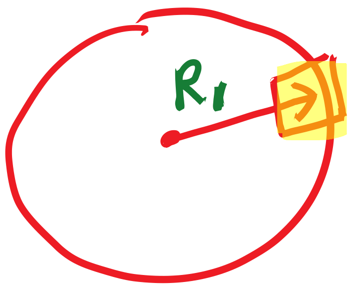
1st Plane

$\overline{R_1}$

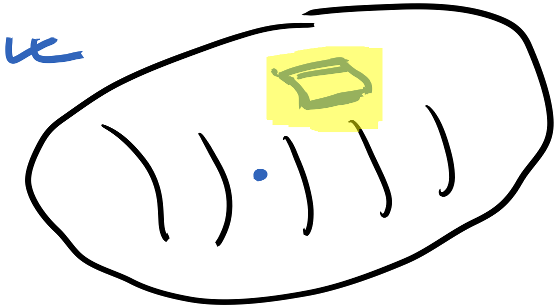
For a shape

At all points radius is R_1, $\rightarrow$ Then it is a sphere

$$\frac{1}{R_1} + \frac{1}{R_2}$$



ie

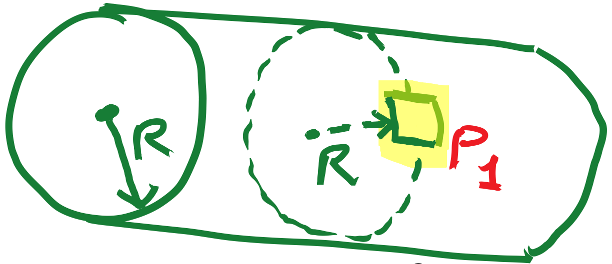


Local Curvature changes from point to point.

Cylinder

Point P_1 $\rightarrow$

Curvature at the Radius at the Point



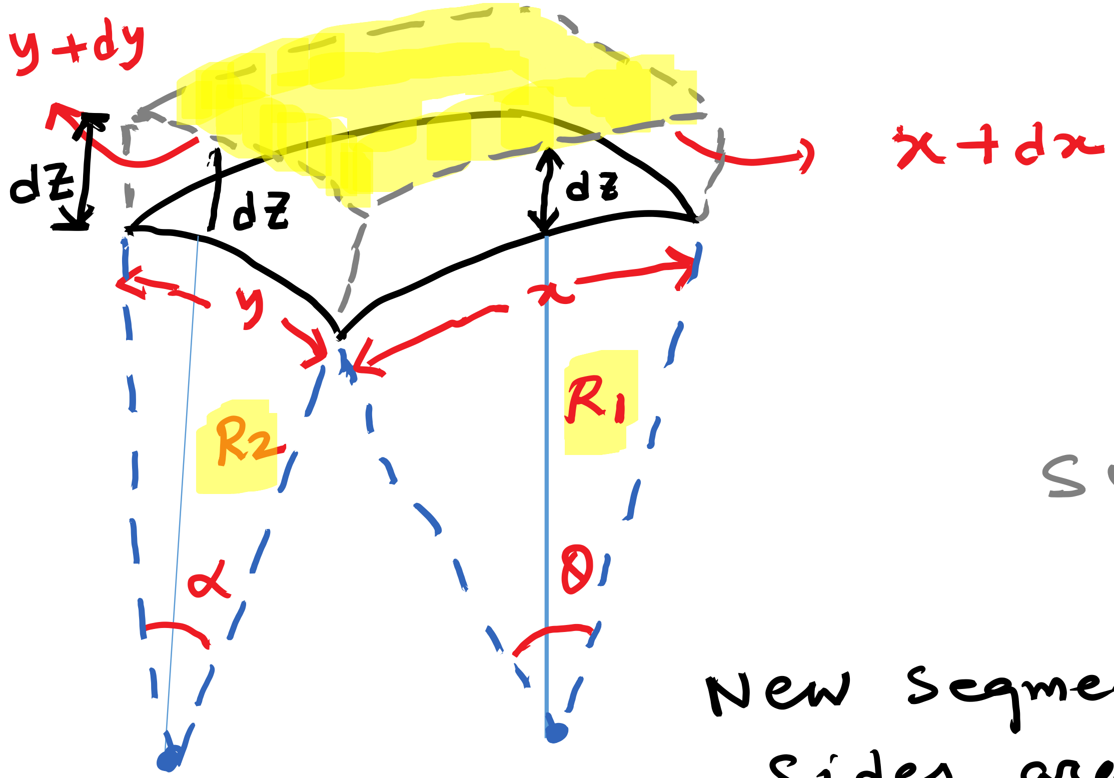
Radius is R in θ direction

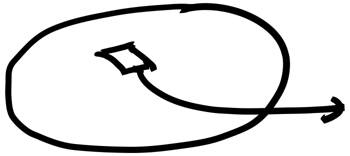
Radius is ∞ in Z direction

Date: 16.03.2022

Lecture - 26

Let us consider an An area of an arbitrarily Curved Surface.





→ Local Curvature is given as $\frac{1}{R_1} + \frac{1}{R_2}$

ΔP → Across the interface.

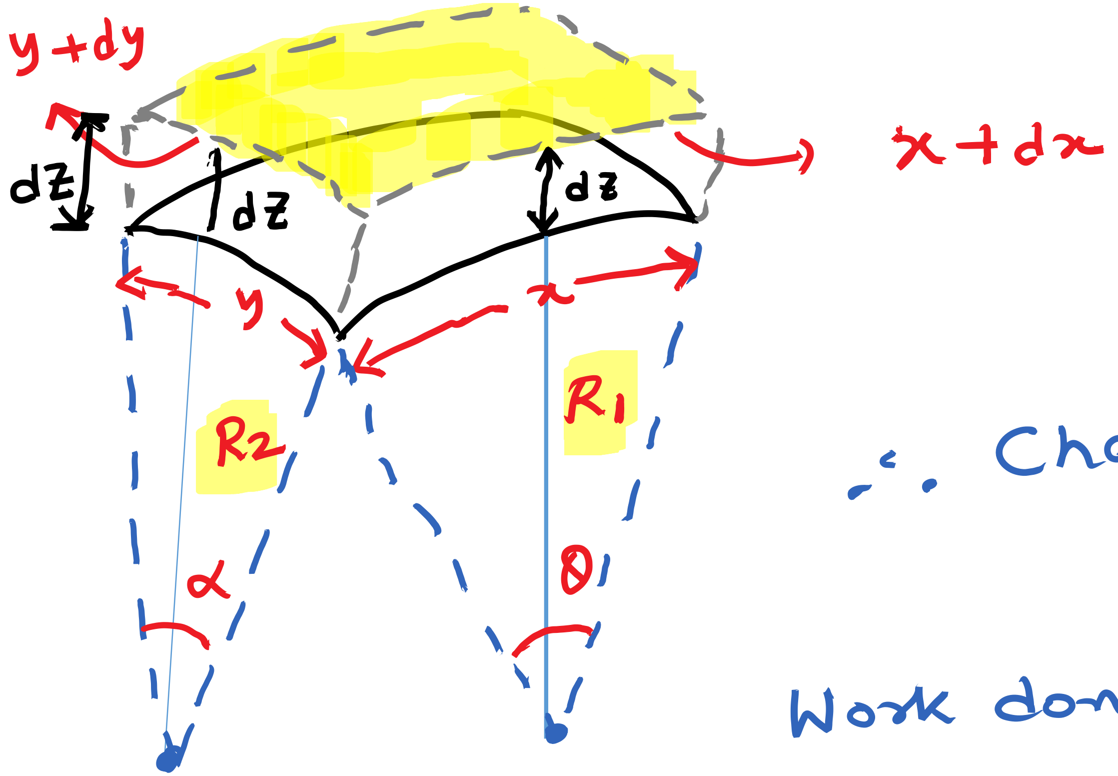
Supply some energy → Expansion takes place

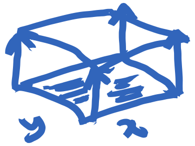
New segment of the surface (Yellow highlight)

Sides are $x + dx$, $y + dy$

Radius are $\underline{R_1 + dZ}$ and $\underline{R_2 + dZ}$

Let us consider an An area of an arbitrarily curved surface.





$\rightarrow xy.dZ$

Change in the Surface Area

$$dA = (x+dx)(y+dy) - xy$$

$$= x\,dy + y\,dx \qquad \cancel{dx\,dy}^{\,0}$$

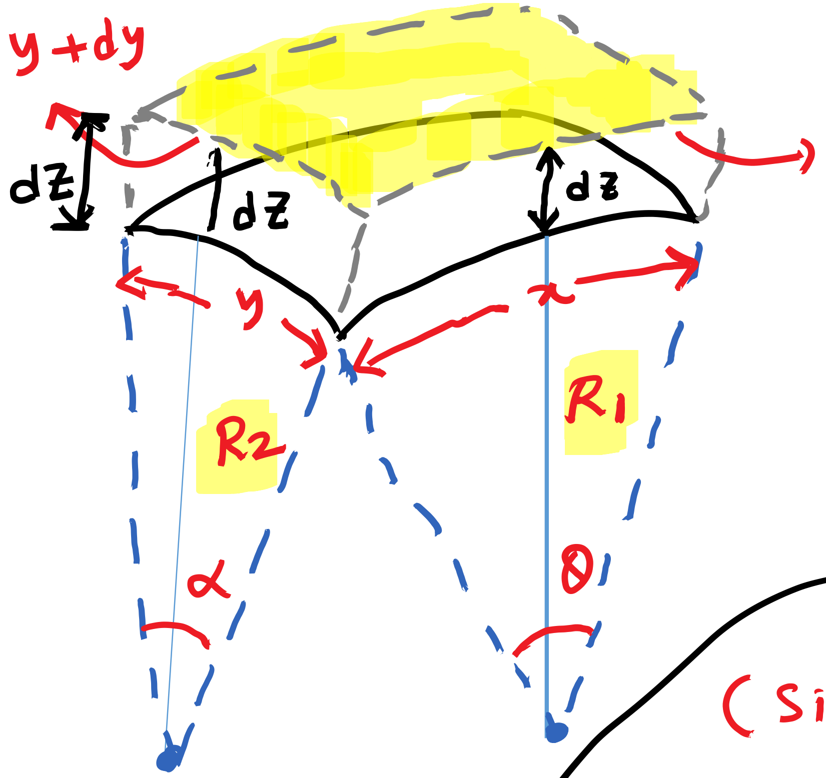
∴ Change in Surface Energy

$$dE_s = \gamma.(x\,dy + y\,dx) \quad \text{———} \quad (1)$$

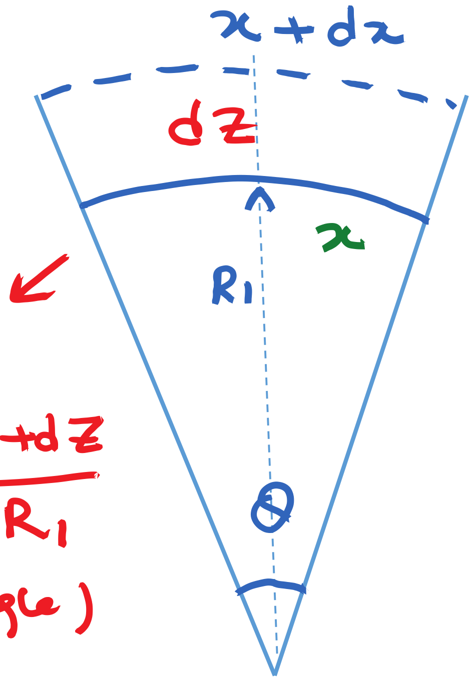
Work done for this Expansion

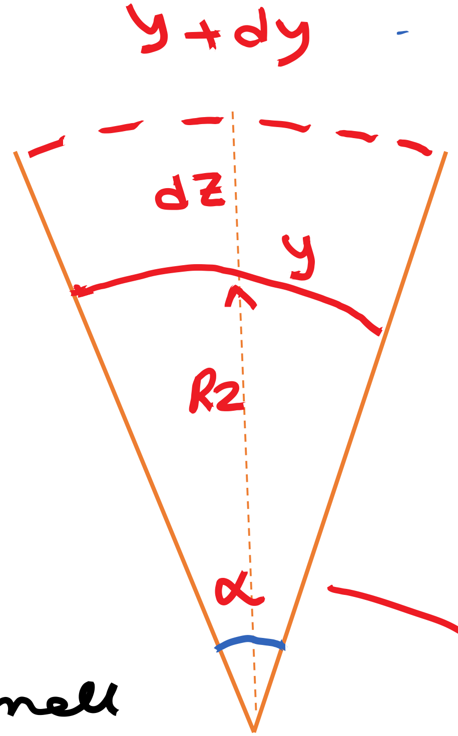
$$\delta W = \Delta P.dV = \Delta P(xy\,dZ) \quad \text{———} \quad (2)$$

Let us consider an An area of an arbitrarily Curved Surface.



Two Sections





$$\frac{x+dx}{x} = \frac{R_1+dZ}{R_1}$$

(Similar Triangle)

Since the area (x and y) are very small so we neglect the curvature effect & use property of similar triangle

$$dx = \frac{x\,dZ}{R_1} \quad \text{----}(3)$$

$$\frac{y+dy}{y} = \frac{R_2+dZ}{R_2}$$

$$dy = \frac{y\,dZ}{R_2} \quad (4)$$

$$dE_s = \gamma\,(x\,dy + y\,dx)$$

$$dx = \frac{x\,dZ}{R_1}, \quad dy = \frac{y\,dZ}{R_2}$$

$$= \gamma\left(\frac{xy\,dZ}{R_2} + \frac{xy\,dZ}{R_1}\right)$$

$$\delta W = (\Delta P)\cdot xy\,dZ$$

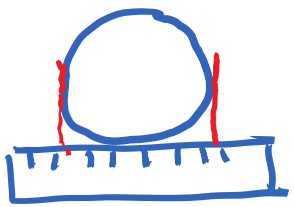
$$\delta W = dE_s$$

$$(\Delta P)\cdot xy\,dZ = \gamma\left(\frac{xy\,dZ}{R_2} + \frac{xy\,dZ}{R_1}\right)$$

$$\Rightarrow \boxed{\Delta P = \gamma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

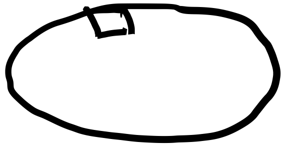
Young Laplace Eqn.

If at each point R_1 and R_2 is known, then we can calculate ΔP.



For the Spherical drop $\boxed{\Delta P = \frac{2\gamma}{R}}$

For a Sphere $R_1 = R_2$



American Football

Point by point R_1 and R_2 cannot be determined.

Axi-Symmetric Surfaces:-

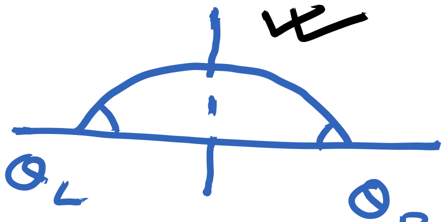
Surfaces that have an axis of symmetry.

Center Line for a Cylinder is the axis (of Rotation) of a Line

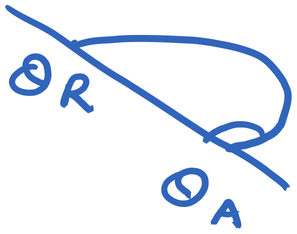
Rotate by 360° → Cylinder
↓
the Line

Center Line

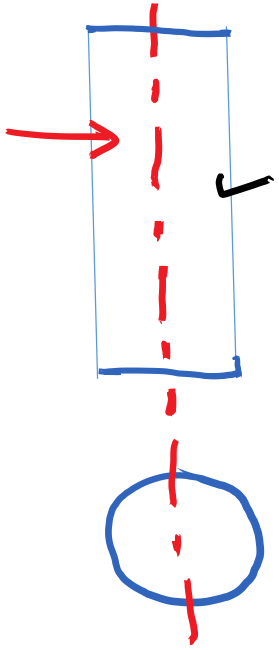
Line ∥ CL



✓ Center Line

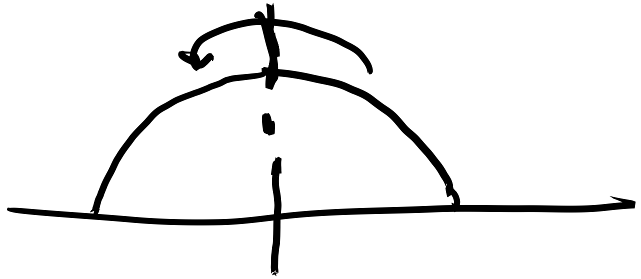


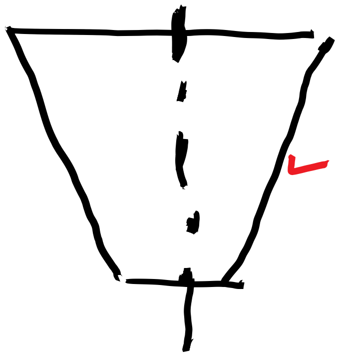
× CL (No symmetry)

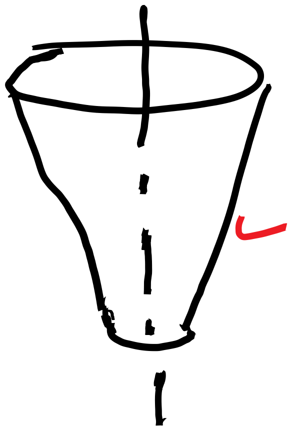


Cylinder

If the Line is NOT Parallel to the axis of symmetry and you rotate →



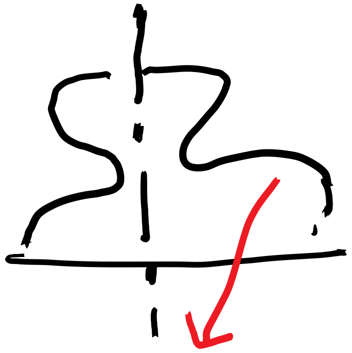


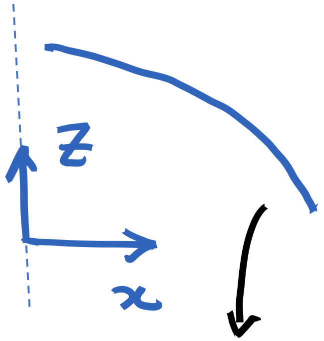


Axi - Symmetric Surface → We have a line or a Curve.

Rotate by 360° → We get the desired surface

(Surface) - 3D → it is related to something that Can be represented on a 2D Plane





Eqn. of this Line is Known

$$Z = f(x)$$

For the boundary $Z = f(x)$ is Known

x | Z

Numerically generate the functionality.

If we now have an axi-symmetric (Liquid) Surface

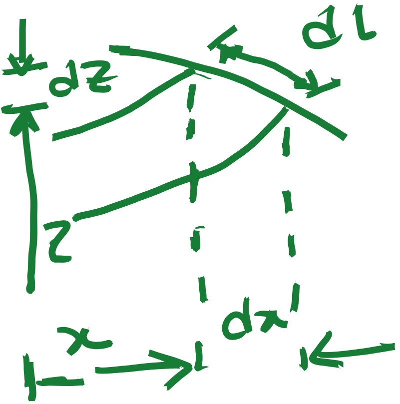
↳ We get a functional form of the Y-L Eqn.

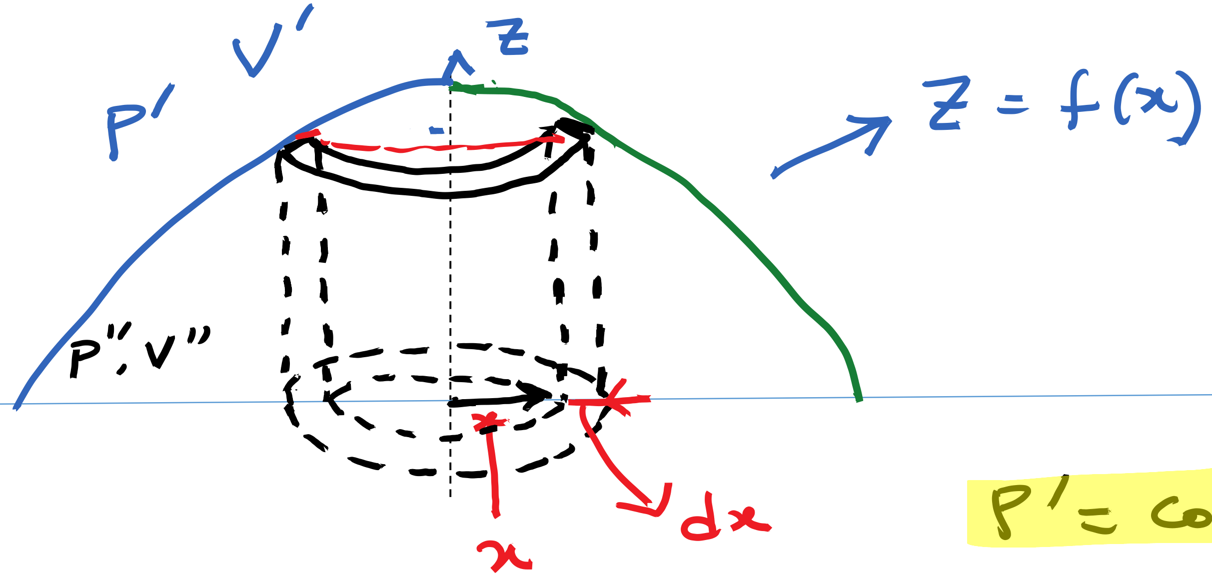
$$\Delta P = \gamma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\left[Z, \text{ or } \frac{dZ}{dx} \text{ or } \frac{d^2Z}{dx^2} \right]$$

Generalized Expression of Young Laplace Eqn for an axi-Symmetric Surface

Atm $P' = \text{const}$





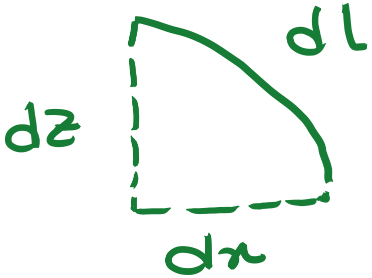
$P'' \rightarrow Pr$] Inside
$V'' \rightarrow Vol$

P'] → out Side
V'

$V = V' + V''$

$P' = \text{const.}$

$V = \text{Const}$
$T = \text{Const}$
$V'' = \text{const}$

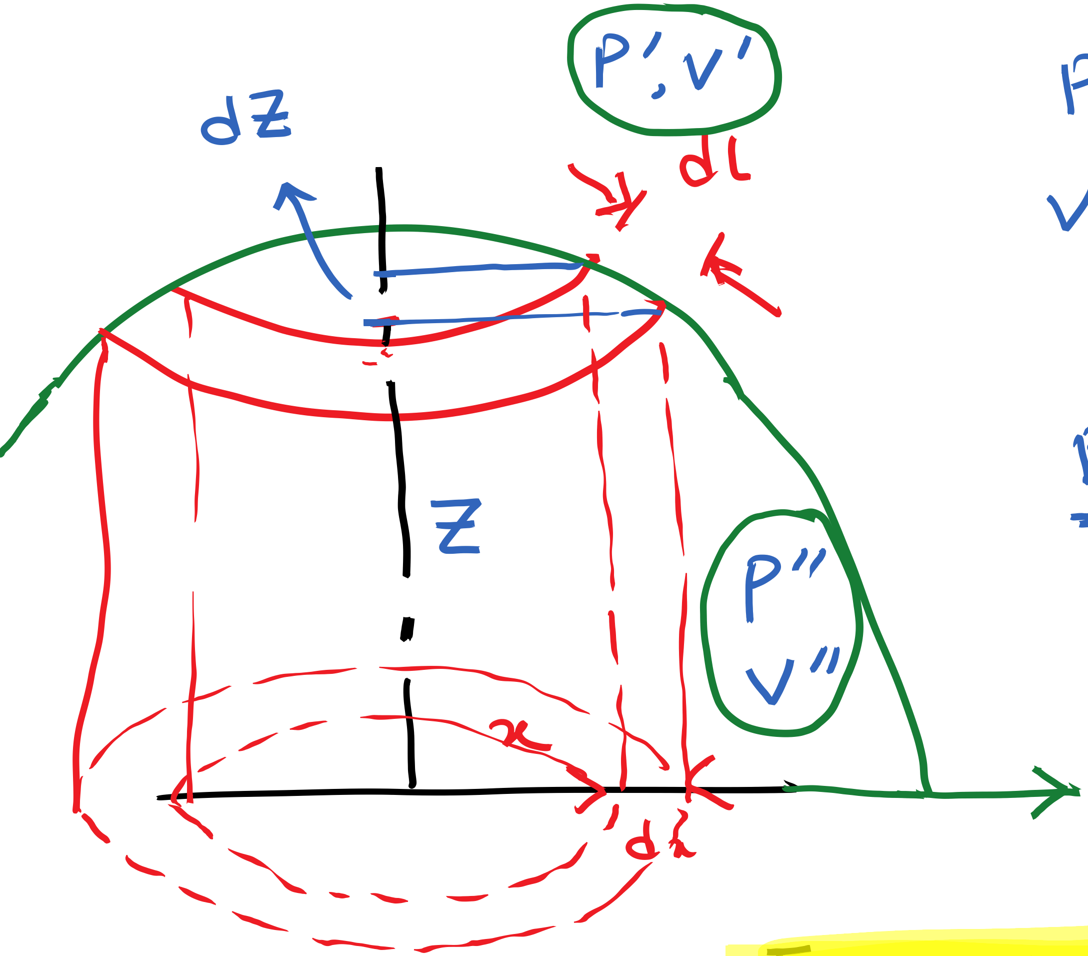


$V'' = \text{constant}$

↳ Incompressible, Non Evaporating

Laplace Pressure :→ Pressure diff across a curved Surface (Liquid) at Equilibrium. → We are going to obtain the shape of the Interface at Eqlbm

Cond for Eqlbm:→
Minimization of Free Energy



$P' = $ Const

$V = V' + V'' = $ const , $V'' = $ const

$\therefore V' = $ const.

P'' ?? Is it Constant?

↳ P'' → Not Constant → Now we know about Laplace Pr.

Free Energy of the System
(Helmholtz Free Energy)
Symbol : F

$F = U - TS$

Modified Helmholtz Free Energy for a Colloidal System

$$F = U - TS + \sum_j \gamma_j A_j$$

j = No of Interfaces.

dx, dZ, x, Z, dl → we will use all of this for Calculating A & V'' Later.

Helmholtz Free Energy:

$$F = U - TS + \sum_j \mu_j A_j = U - TS + \mu A$$

μ = Here Surface Tension

μ = Surface Tension

$$H = U + PV$$

$$G = H - TS$$

$$\Rightarrow \quad H = G + TS$$

$$U + PV = G + TS$$

$$\Rightarrow \quad U = G + TS - PV$$

$$\Rightarrow \quad F = G + \cancel{TS} - PV - \cancel{TS} + \mu A$$

$$= G - PV + \mu A$$

$$G = \sum \mu_i x_i = \text{Const}$$

μ_i: Chemical Potential

Since for the System Composition does not change, So $G = \text{Const}$

$$F = G - pV + \gamma A \qquad V = V' + V''$$

$$= G - p'V' - p''V'' + \gamma A.$$

Since $p' = \text{const}$, $V = \text{const}$

$$F = G - p''V'' - p'(V - V'') + \gamma A.$$

$$= \gamma A - p''V'' + p'V'' - \underbrace{p'V}_{C} + \underbrace{G}_{C}$$

$$F = \gamma \underline{\underline{A}} + \underline{\underline{V''}} \underbrace{(p' - p'')}_{\Delta P \to \text{Across the Interface}} + C \longrightarrow$$

Minimization of this F will give the Shape at Eqlbm.

Free Energy for an Axi Symmetric Surface as a function of Surface Area and Laplace Pressure.

$\rightarrow$ Surface Area changes, $\Delta P \rightarrow$ also changes & F changes and for the combination of A and ΔP for which F is going to be minimum $\rightarrow$ (Equilibrium) [for a specific Shape.]