$\cancel{\theta_a(0) = \int_0^{x} \exp}$

$\theta_a(0) \sim \sqrt{Pr}$

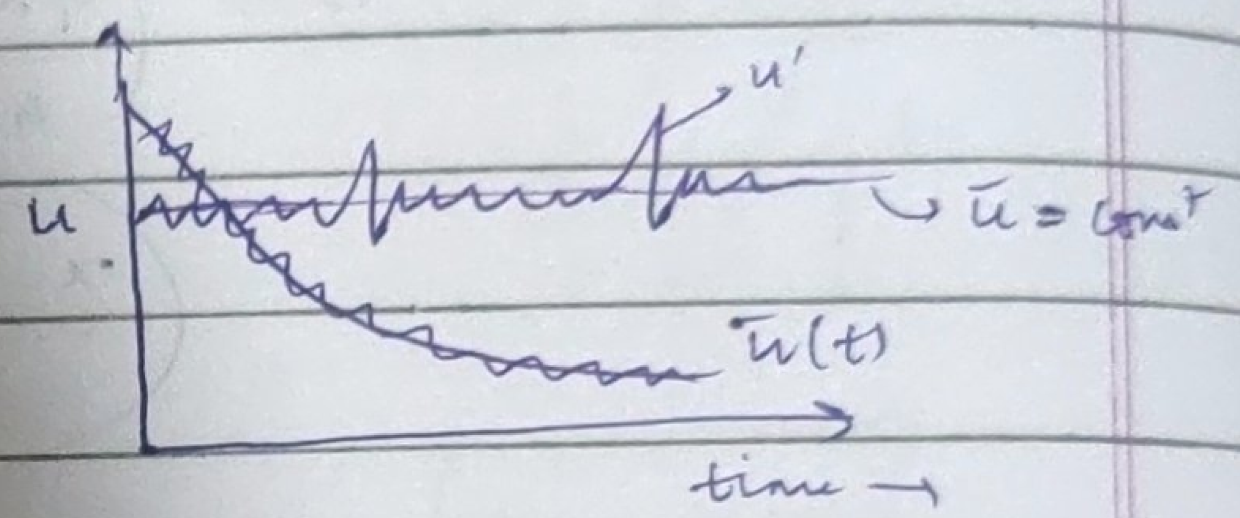
Sound velocity $\rightarrow$ $a = \sqrt{\gamma R T}$

$$T_w = T_1\left[1 + 0.167\, Ma^2\right] \underset{Ma = 0.8}{\Rightarrow} 302.178$$

TURBULENT BOUNDARY LAYER (chap 6)

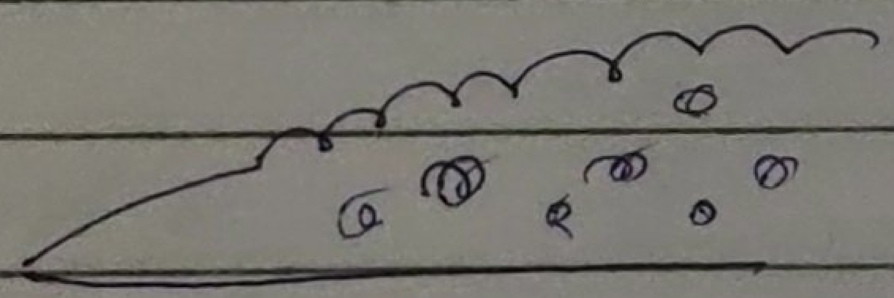
$$u = \bar{u} + u'$$

$$\bar{u} = \frac{1}{t_p}\int_0^{t_p} u \, dt_p$$



take time frame such that avg of fluctuating component $= 0$

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0$$



$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \cancel{\bar{w}\frac{\partial \bar{u}}{\partial z}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + \nu \nabla^2 \bar{u}_i - \cancel{\frac{\partial}{\partial x}\overline{u'^2}} - \cancel{\frac{\partial}{\partial z}\left(\overline{u' w'}\right)}$$

0 z symmetry $\qquad \sim 0 \qquad$ 0 z - symmetry

$$o(u') \sim o(v')$$

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{d\bar{p}}{dx} + \nu\frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\partial}{\partial y}\left(\overline{u' v'}\right)$$

$u' \geq 0 \,,\; v' < 0$

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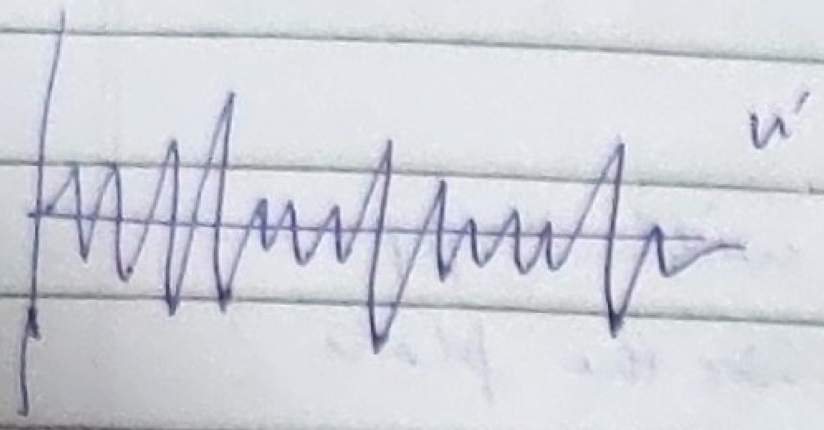
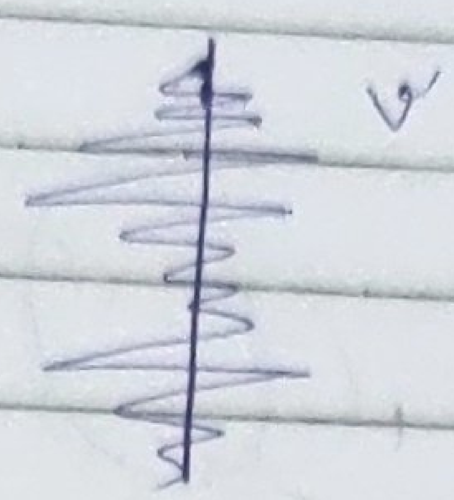




$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0 .$$

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{d\bar{P}}{dx} + \nu\frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\partial}{\partial y}\left(\overline{u'v'}\right)$$

$$\bar{u}\frac{\partial \bar{T}}{\partial x} + \bar{v}\frac{\partial \bar{T}}{\partial y} = \alpha\frac{\partial^2 \bar{T}}{\partial y^2} - \frac{\partial}{\partial y}\underbrace{\left(\overline{v'T'}\right)}_{=(-ve)}$$

$=(-ve)$. Why?

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0$$

$$\frac{\partial u'}{\partial x} = -\frac{\partial v'}{\partial y}$$

(if one is +ve, other is −ve)

Reynold's stress.

$$-\overline{u'v'} = \epsilon_M \frac{\partial \bar{u}}{\partial y}$$

due to eddies

eddy momentum diffusivity

analogous to ν

$$-\overline{v'T'} = \epsilon_H \frac{\partial \bar{T}}{\partial y}$$

analogous to α.

eddy thermal diffusivity

$\mu \to$ system property

$\epsilon_M \to$ not system property

depends on the eddies

Momentum eqn $\cdots\cdots = -\frac{1}{\rho}\frac{d\bar{P}}{dx} + \nu\frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial}{\partial y}\left(\epsilon\frac{\partial \bar{u}}{\partial y}\right)$

$$\cdots + \frac{\partial}{\partial y}\left(\underbrace{(\nu + \epsilon_M)\frac{\partial \bar{u}}{\partial y}}_{\tau_{app}}\right)$$

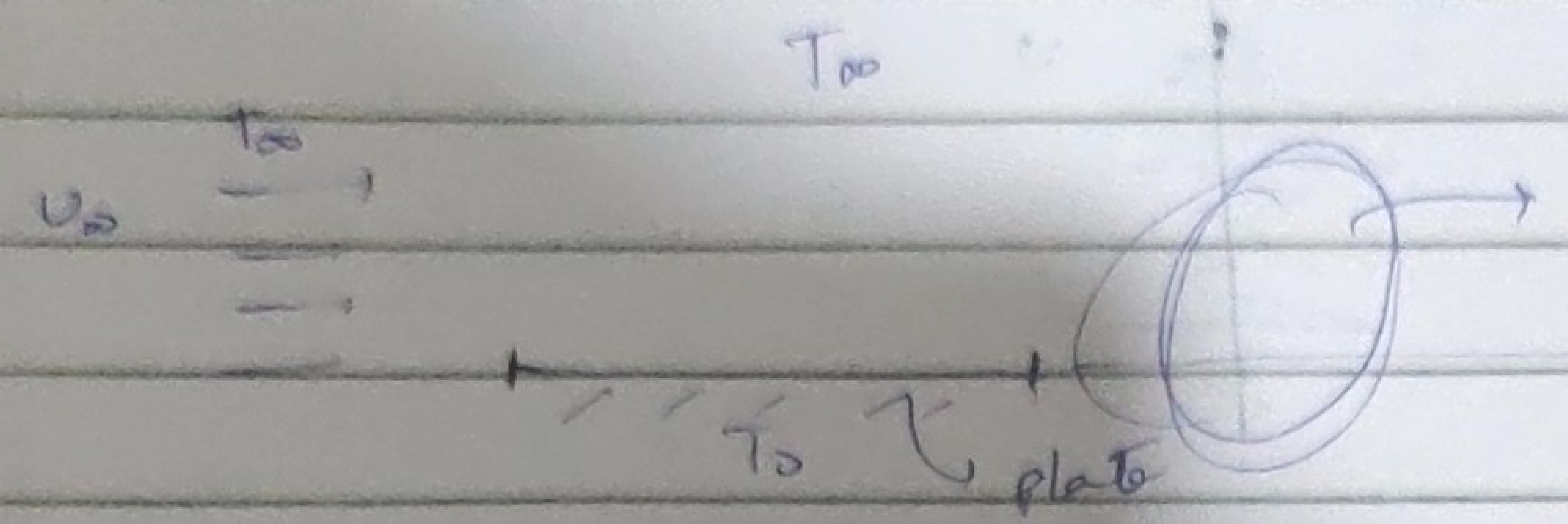
τ_{app} — apparent

Energy eqn :— $\cdots\cdots \frac{\partial}{\partial y}\left((\alpha + \epsilon_H)\frac{\partial \bar{T}}{\partial y}\right)$

$\epsilon_m, \epsilon_H, \bar{u}, \bar{v}, \bar{P}, \bar{T} \to$ unknown.

take $\bar{P}$ = const

P_∞ ✓.



what will happen away from the plate?

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{P}_\infty}{\partial x} + \left\{\nu\frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\partial(\overline{u'v'})}{\partial y}\right\} \quad \frac{1}{\rho}\left[\frac{\partial}{\partial y}\left\{\mu\frac{\partial \bar{u}}{\partial x} - \underbrace{(\rho\overline{u'v'})}_{\text{reynolds stress}}\right\}\right]$$

τ_{app}

$$= \rho\,\epsilon_M\frac{\partial \bar{u}}{\partial y}$$

$$\ldots + \left[\frac{\partial}{\partial y}\left\{(\nu + \epsilon_M)\frac{\partial \bar{u}}{\partial y}\right\}\right]$$

$$\Rightarrow \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = \cancel{-\frac{1}{\rho}\frac{d\bar{P}_\infty}{dx}} + \left[\frac{\partial}{\partial y}\left\{(\mu + \epsilon_M)\frac{\partial \bar{u}}{\partial y}\right\}\right]$$

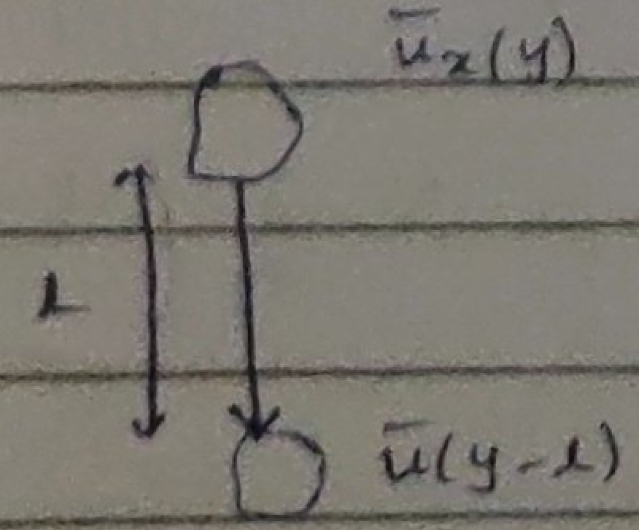
≈ 0

$$\Rightarrow \bar{u}\frac{\partial \bar{T}}{\partial x} + \bar{v}\frac{\partial \bar{T}}{\partial y} = \left[\frac{\partial}{\partial y}\left\{(\alpha + \epsilon_H)\frac{\partial \bar{T}}{\partial y}\right\}\right]$$

$$\Rightarrow \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0$$

Unknowns: $\bar{u}$, $\bar{v}$, $\bar{T}$, ϵ_M, ϵ_H 5 3 equations

Mixing length model



$$\sim(\bar{u}_x(y) - \bar{u}_x(y-l)) \sim O'(u') \sim l\frac{\partial \bar{u}}{\partial y}$$

$$O(u') \sim O(v')$$

$$-\overline{u'v'} \sim l^2\left(\frac{\partial \bar{u}}{\partial y}\right)^2$$

$$-\rho\overline{u'v'} = \rho\, \epsilon_M \frac{\partial \bar{u}}{\partial y}$$

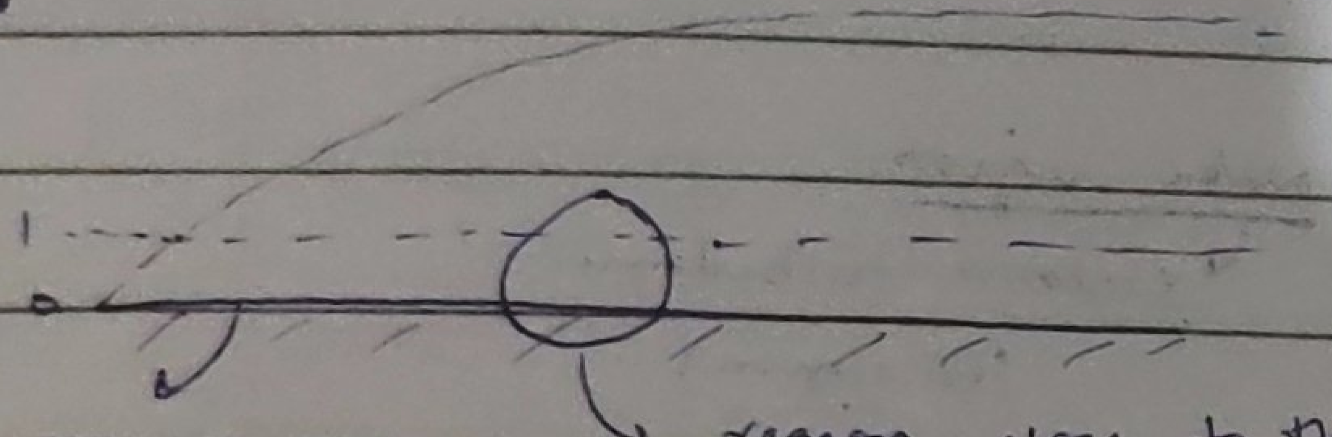
$$\epsilon_M = l^2 \left|\frac{\partial \bar{u}}{\partial y}\right|$$

→ $l \propto y \sim \kappa y$

κ → kappa. const

→ We need the velocity profile to find the wall shear stress.

⇒



τ_0 wall shear stress

region close to the wall

(We are assuming that the apparent shear stress is const)

$\tau_{app} \approx$ const

we will define wall coordinate system.

$$\cancel{\tau_{app} = \nu \frac{\partial \bar{u}}{\partial y} - \overline{u'v'} = \nu \frac{\partial \bar{u}}{\partial y}} + \epsilon_M \frac{\partial \bar{u}}{\partial y}$$

$$\boxed{\tau_{app} = \rho(\nu + \epsilon_M)\frac{\partial \bar{u}}{\partial y}} \qquad \epsilon_M = l^2\left|\frac{\partial \bar{u}}{\partial y}\right|$$

$$\tau_{app} = \rho(\nu + \epsilon_M)\frac{\partial \bar{u}}{\partial y} = \tau_0$$

⇒ Why do we use wall coordinate system?

$\cancel{u^* = \bar{u}/u^*}$

$$u^+ = \frac{\bar{u}}{u^*}, \qquad v^+ = \frac{\bar{v}}{u^*}$$

$$y^+ = \frac{y u^*}{\nu} \qquad x^+ = \frac{x u^*}{\nu}$$

$$u^* = \left(\frac{\tau_0}{\rho}\right)^{1/2}$$

$$(\nu + \epsilon_M)\frac{\partial \bar{u}}{\partial y} = \frac{\tau_0}{\rho}$$

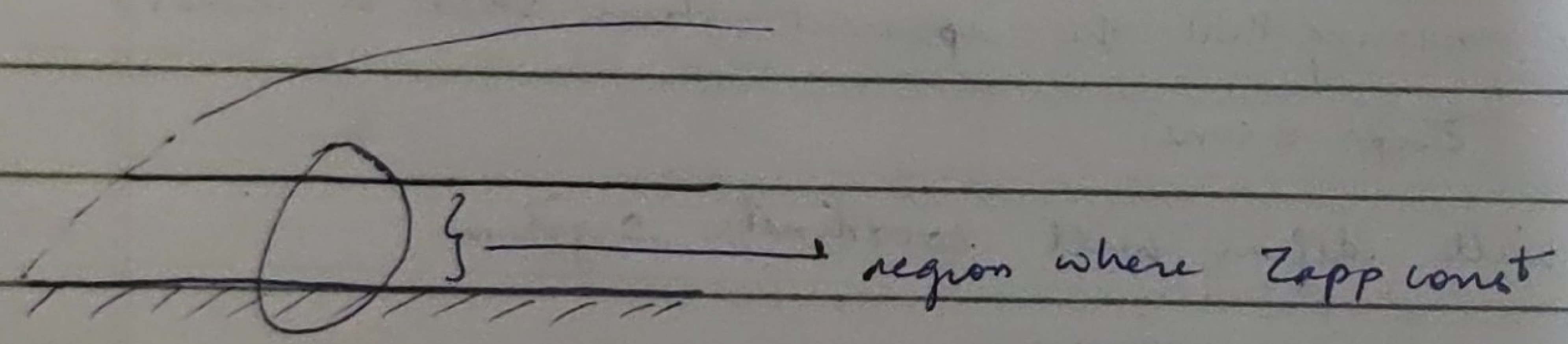
$$(\nu + \epsilon_M)\frac{(u^*)^2}{\nu}\frac{\partial u^+}{\partial y^+} = (u^*)^2$$

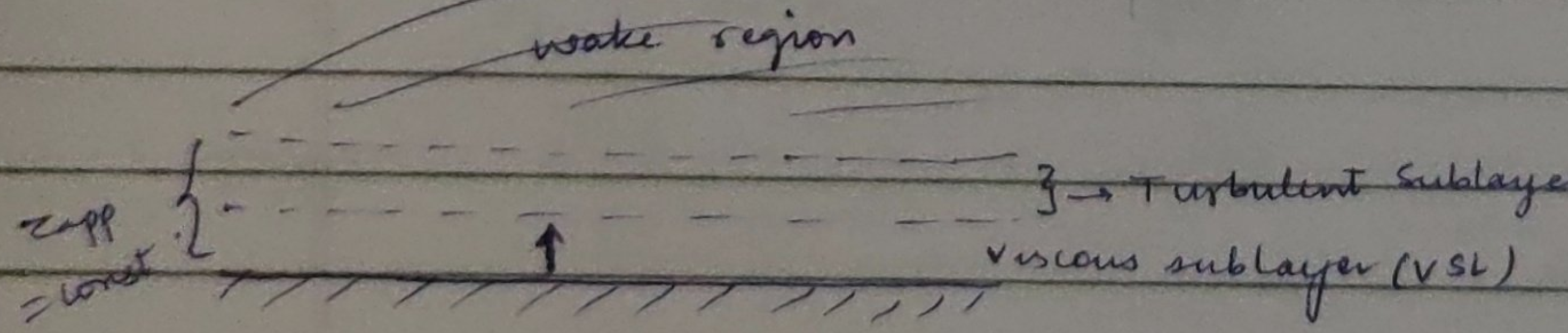
$$\frac{\partial u^+}{\partial y^+} = \frac{1}{\left(1 + \frac{\epsilon_M}{\nu}\right)}$$

$$\left(1 + \frac{\epsilon_M}{\nu}\right)\frac{\partial u^+}{\partial y^+} = 1$$

valid only where τ_{app} is const

– region outside wall coordinate system is wake region → excessive turbulence (BT region)





$\frac{\epsilon_M}{\nu}$ → will determine which layer will dominate (TSL or VSL)

$\nu = \frac{\mu}{\rho}$ → molecular diffusivity

$\gamma >> \epsilon_M \rightarrow VSL$

$$\frac{\partial u^+}{\partial y^+} = 1$$

$$\int_0^{u^+} \partial u^+ = \int_0^{y^+} \partial y^+$$

$$u^+ = y^+ \rightarrow \underline{0 < y^+ < y^+_{VSL}}$$

$\Rightarrow \epsilon_M >> \gamma$

$$\frac{\epsilon_M}{\gamma} \frac{\partial u^+}{\partial y^+} = 1$$

$$\epsilon_M = l^2 \frac{\partial \bar{u}}{\partial y}$$

$l \sim k.y$

$$\epsilon_M = k^2 y^2 \frac{\partial \bar{u}}{\partial y}$$

$$k^2 (y^+)^2 \frac{\partial u^+}{\partial y^+} \cdot \frac{\partial u^+}{\partial y^+} = 1$$

$$\epsilon_M = k^2 \left(\frac{y^+ \gamma}{u^*}\right)^2 \frac{\partial u^+ \times u^*}{\partial y^+ \frac{\gamma}{u^*}}$$

$$\epsilon_M = k^2 \frac{\gamma u^*}{u^*} (y^+)^2 \frac{\partial u^+}{\partial y^+}$$

$$\frac{\epsilon_M}{\gamma} = k^2 \frac{u^*}{u^*} (y^+)^2 \frac{\partial u^+}{\partial y^+}$$

$$\frac{\partial u^+}{\partial y^+} = \frac{1}{k y^+}$$

$$\int_{u^+_{VSL}}^{u^+} du^+ = \int_{y^+_{VSL}}^{y^+} \frac{dy^+}{k y^+}$$

$$\boxed{\underbrace{u^+ - u^+_{VSL}}_{y^+_{VSL}} = \frac{1}{k} \ln\left(\frac{y^+}{y^+_{VSL}}\right)}$$

$$u^+ = y^+_{VSL} + \frac{1}{k} \ln\left(\frac{y^+}{y^+_{VSL}}\right)$$

$y^+_{VSL} = 11.6$

$k = 0.4$

$$u^+ = \underbrace{y^+_{VSL} - \frac{1}{k} \ln(y^+_{VSL})}_{const} + \underbrace{\frac{1}{k}}_{const} \ln y^+$$

$$u^+ = \underbrace{A}_{\sim 5.5} + \underbrace{B}_{\sim 2.5} \ln y^+$$

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$u^+ = \frac{\ }{u^+}$

Turbulent BL

friction velocity (τ_0 is unknown)

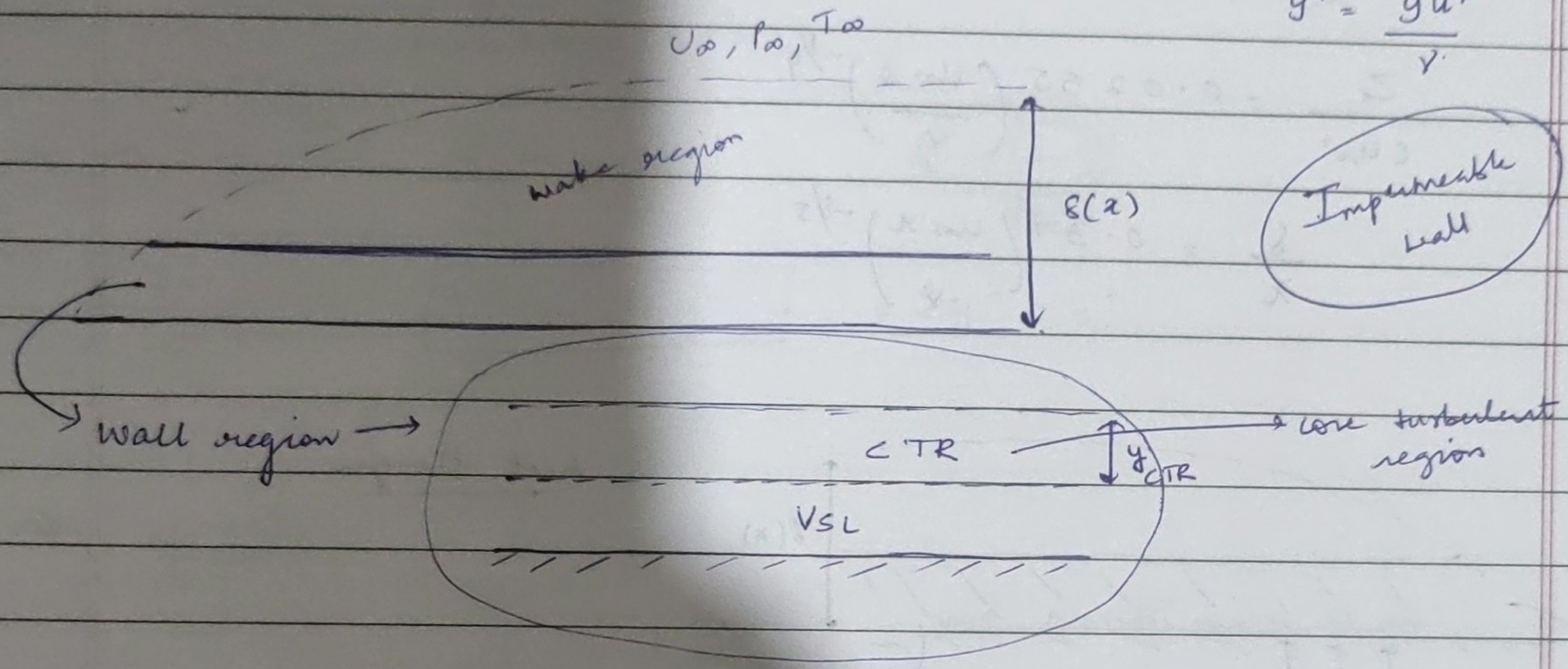
$$u^* = \left(\frac{\tau_0}{\rho}\right)^{1/2}$$

$$u^+ = y^+ \qquad 0 < y^+ \le y^+_{VSL}$$

$$u^+ = A + B\ln y^+ \qquad y^+_{VSL} \le y^+ \le y^+_{CTR}$$

$$u^+ = \frac{\bar{u}}{u^*}$$

$$y^+ = \frac{y u^*}{\nu}$$



τ_{app} = constant

How to check whether the function is correct within the domain?

① experimental values ② check the BC.

→ assume $u^+ = f_u(y^+)$, $\frac{\bar{u}}{u^*} = f_u\left(\frac{y u^*}{\nu}\right)$ → inside the Boundary layer

at the edge of BL → $\frac{u_\infty}{(\tau_0/\rho)^{1/2}} = f_u\left(\frac{\delta(\tau_0/\rho)^{1/2}}{\nu}\right)$ → ①

Prandtl's 1/7 th law → $u^+ = 8.7\,(y^+)^{1/7}$ $\quad$ τ_0, δ are unknowns.

Use integral approach → $\bar{u}\frac{\partial \bar{u}}{\partial y} + \bar{v}\frac{\partial \bar{u}}{\partial x} = -\frac{1}{\rho}\frac{dp^\infty}{dx} + \nu\frac{\partial^2 \bar{u}}{\partial y^2}$ ($\frac{dp^\infty}{dx} \to 0$)

$Pr \sim 1$ → for Turbulent BL → assumption

$$\int_0^\delta \bar{u}\frac{\partial \bar{u}}{\partial y}dy + \int_0^\delta \bar{v}\frac{\partial \bar{u}}{\partial y}dy = \int_0^\delta \nu\frac{\partial^2 \bar{u}}{\partial y^2}dy$$

$$\frac{d}{dx}\int_0^\delta \bar{u}^2 dy + \left[\bar{v}\bar{u}\right]_0^\delta = \left[\nu\frac{\partial \bar{u}}{\partial y}\right]_0^\delta$$

$$\frac{d}{dx}\int_0^\delta \bar{u}^2 dy + U_\infty \bar{v}\big|_\delta = -\frac{1}{\rho}(\tau_0)$$

$$\frac{d}{dx}\int_0^\delta \bar{u}^2 dy + U_\infty\left[-\frac{d}{dx}\int_0^\delta \bar{u}\,dy\right] = -\frac{\tau_0}{\rho}$$

$$\frac{d}{dx}\int_0^{\delta} \bar{u}(u_\infty - \bar{u})\, dy = \tau_0/\rho \longrightarrow (2) \qquad \text{use } u^+ = 8.7(y^+)^{1/7}$$

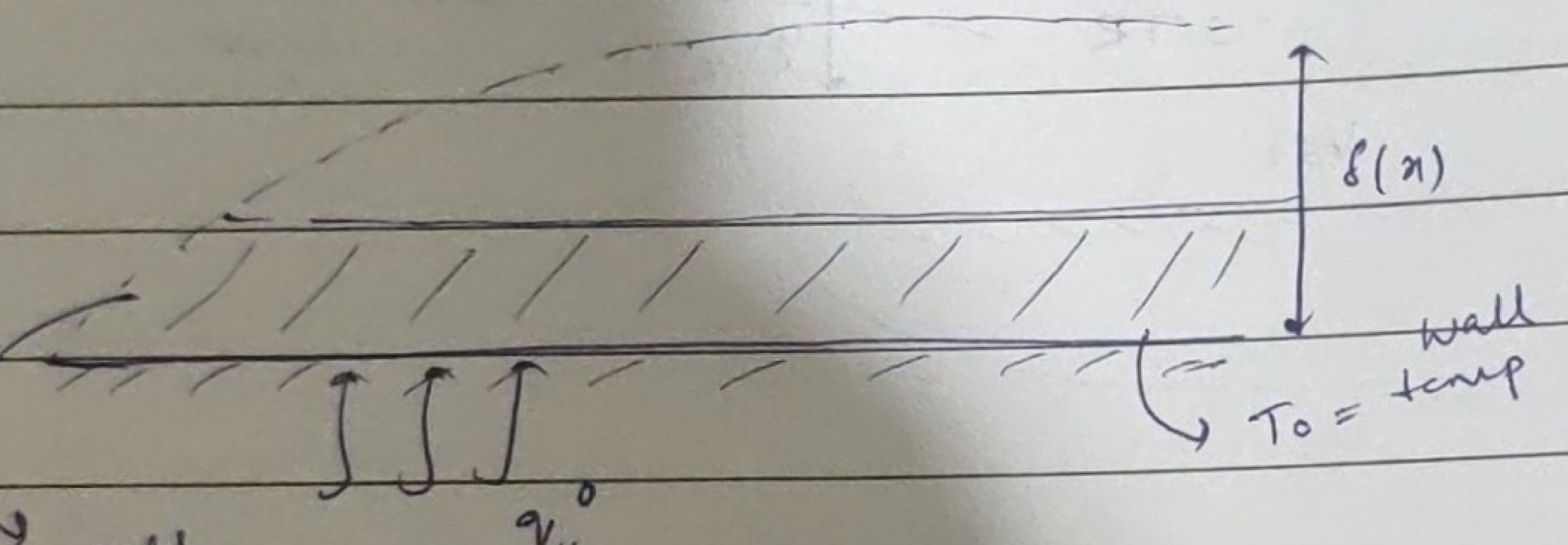
After integrating $\quad v_\infty = f(\delta, \tau_0) \longrightarrow (3)$

Compare eqn (3) & (1)

$$\frac{\tau_0}{\rho u_\infty^2} = 0.0255\left(\frac{u_\infty \delta}{\nu}\right)^{-1/4}$$

$$\frac{\delta}{x} = 0.37\left(\frac{u_\infty x}{\nu}\right)^{-1/5}$$

heat transfer



wall region (apparent heat flux is const)

$$(\alpha + \epsilon_H)\frac{\partial \bar{T}}{\partial y} = \frac{-q''_{0,app}}{\rho c_p}$$

$q_{app} = \text{const} \rightarrow \neq f(y)$

$$q''_{0,app} = q''_0 = -\rho c_p(\alpha + \epsilon_H)\frac{\partial \bar{T}}{\partial y}$$

$$y^+ = \frac{y u^*}{\nu} \qquad \frac{\nu}{u^*}\partial y^+ = \partial y$$

$$\frac{\partial \bar{T}}{\partial y^+}\left(\frac{\rho c_p u^*}{-q''_0}\right) = \frac{1}{\left(\frac{\alpha}{\nu} + \frac{\epsilon_H}{\nu}\right)}$$

$$\frac{\partial(T_0 - \bar{T})}{\partial y^+} \times \frac{\rho c_p u^+}{q''_0} =$$

$\hookrightarrow T^+(y^+, x^+)$

$$\frac{\partial T^+}{\partial y^+} = \frac{1}{\left(\frac{1}{Pr} + \frac{\epsilon_M}{\nu}\times\frac{\epsilon_H}{\epsilon_M}\right)}$$

eddies are transferring the energy & momentum

In the absence of eddies → α, ν → resemblance (ϵ_H, ϵ_M)

$$\frac{\epsilon_H}{\epsilon_M} = \frac{1}{Pr_t}$$

ϵ_H → eddy thermal diffusivity

ϵ_M → eddy momentum diffusivity

Pr_t → not a const (depends on ...) → turbulent Pr no.

Pr_t → due to eddies

Pr → due to molecular

$$\frac{\partial T^+}{\partial y^+} = \left(\frac{1}{\frac{1}{Pr} + \frac{\epsilon_M}{\nu}\frac{1}{Pr_t}} \right)$$

close to the wall → VSL → due to molecular movement we have momentum transfer.

At conduction sub layer → $\frac{\epsilon_M}{\nu}\frac{1}{Pr_t} << \frac{1}{Pr}$

↳ similar to viscous sub layer

$\frac{\epsilon_M}{\nu}$ ↳ ratio is very small (?) (Some assumption?) (check)

At CSL, $\frac{\partial T^+}{\partial y^+} = Pr$

$$\boxed{T^+ = Pr\, y^+} \quad 0 < y^+ < y^+_{CSL}$$

→ at CTR

$$\frac{\partial T^+}{\partial y^+} = \frac{1}{\frac{\epsilon_M}{\nu} \times \frac{1}{Pr_t}}$$

use prandtl mixing length. $l = Ky$

$$\frac{\epsilon_M}{\nu} = K^2 y_+^2 \frac{du^+}{dy^+} = K y_+$$

$$u^+ = A + B \ln y^+$$

$$\frac{du^+}{dy^+} = \frac{B}{y^+} \qquad B = \frac{1}{K}$$

(at a particular y^+ → Pr_t is taken to be const)

$$\frac{\partial T^+}{\partial y^+} = \frac{Pr_t}{K y^+}$$

Amt of energy & momentum transferred by eddies is same. ↳ wrt y

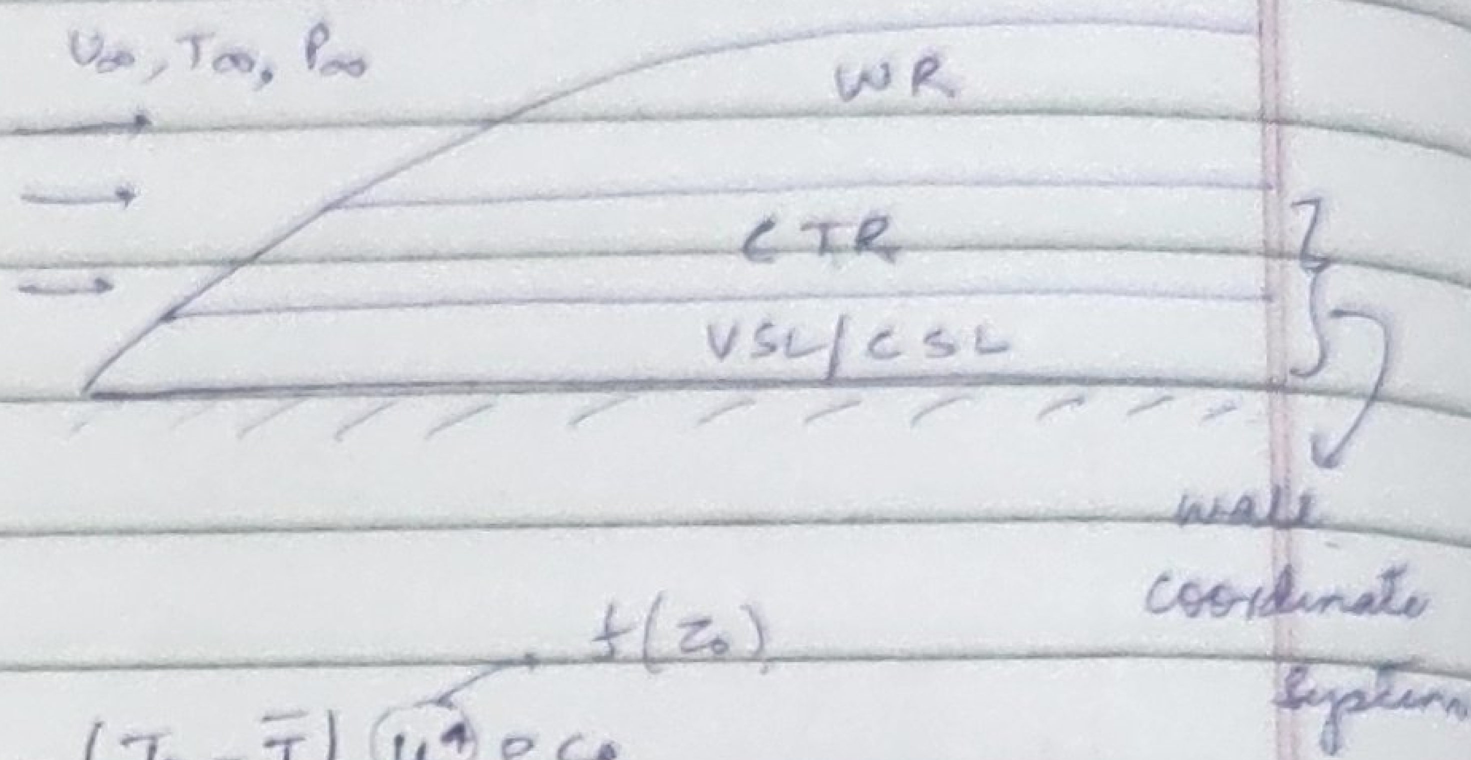
$$\int_{T_{CSL}}^{T^+} dT^+ = \frac{Pr_t}{K}\int_{y^+_{CSL}}^{y^+} \frac{dy^+}{y^+}$$

$$T^+ = Pr\, y^+ + \frac{Pr_t}{K} \ln\left(\frac{y^+}{y^+_{CSL}}\right)$$

TURBULENT BL

$T^+ = Pr\, y^+ \qquad 0 < y^+ < y^+_{CSL}$

$T^+ = Pr\, y^+_{CSL} + \frac{Pr_t}{k} \ln\left(\frac{y^+}{y^+_{CSL}}\right) \qquad y^+_{CSL} < y^+ < y^+$



$U^+ = y^+ \qquad 0 < y^+ < y^+_{VSL}$

$U^+ = A + B \ln y^+ \qquad y^+_{VSL} < y^+$

$u^* = (\tau_0/\rho)^{1/2}$ friction velocity

velocity based on the wall shear stress

$T^+ = \frac{(T_0 - \bar{T})\, \overbrace{u^*}^{f(\tau_0)}\, \rho c_p}{q''}$

→ apply this at the edge of BL becoz we know the boundary conditions at the edge of the BL (CTR)

$$T^+ = Pr\, y^+_{CSL} + \frac{Pr_t}{k}\left[\ln y^+ - \ln y^+_{CSL}\right]$$

$$\frac{(T_0 - \bar{T})\, u^* \rho c_p}{q''} = Pr\, y^+_{CSL} + \frac{Pr_t}{k}\left[\ln \frac{y\, u^*}{\nu} - \ln y^+_{CSL}\right]$$

at BL $\bar{T} = T_\infty$

extend this at the edge of BL

$$\frac{(T_0 - T_\infty)\, u^* \rho c_p}{q''} = Pr\, y^+_{CSL} + \frac{Pr_t}{k}\left[\ln \frac{\delta\, u^*}{\nu} - \ln y^+_{CSL}\right] \longrightarrow (1)$$

$\frac{1}{h}$ ~

$\frac{\delta}{x} = 0.37\left(\frac{U_\infty x}{\nu}\right)^{-1/5}$ → Cannot use this ?? — why?

→ we have assumed a velocity & got this δ (now we won't assume any velocity profile)

$u^+ = A + B \ln y^+$

$u^+ = y_{vsl}^+ + \frac{1}{k} \ln\left(\frac{y^+}{y^+_{vsl}}\right)$

$u^+ = y_{vsl}^+ - \frac{1}{k} \ln y_{vsl}^+ + \frac{1}{k} \ln y^+$

$$u^+ = B + \frac{1}{k} \ln y^+$$

$$\frac{\bar{u}}{u^*} = \frac{1}{k} \ln \frac{y u^*}{\nu} + B$$

extend this at the edge of BL.

$$\boxed{\frac{u_\infty}{u^*} = \frac{1}{k} \ln\left(\frac{\delta u^*}{\nu}\right) + B} \longrightarrow (2)$$

$$\left(\frac{u_\infty}{u^*}\right)^2 = \frac{u_\infty^2}{\tau_0/\rho} = \frac{2}{\left(\tau_0/\frac{1}{2}\rho u_\infty^2\right)} = \frac{2}{C_{f,x}}$$

$$\boxed{\left(\frac{u_\infty}{u^*}\right) = \left(\frac{2}{C_{f,x}}\right)^{1/2}}$$

$$\ln\left(\frac{\delta u^*}{\nu}\right) = k\left[\frac{u_\infty}{u^*} - B\right]$$

$$\frac{h}{\rho C_p U_\infty} = St_x = \frac{Nu}{Pe} = \frac{Nu}{Re_x Pr} \rightarrow \checkmark$$

$Pr \sim 1$

Pr should be close to 1

$$St_x \propto \frac{1}{2} C_{f,x}$$
$$= f(Pr)\, C_{f,x}$$

$$\frac{C_{f,x}}{2} = St_x (Pr)^{2/3}$$

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Von karman assumed 3 layers → Buffer layer

→ $Nu_x = \frac{1}{2} C_f Pr^{1/3} \quad 0 < Pr < 0.5$

$\simeq 0.0296 Re_x^{4/5} Pr^{1/3}$.

$\overline{Nu} = 0.037 Re_L^{4/5} Pr^{1/3} \quad Pr \gtrsim 0.5$

$\frac{d\bar{u}}{dy}(\nu + \epsilon_M) = \tau_0/\rho$

$\frac{d\bar{T}}{dy}(\alpha + \epsilon_H) = \frac{-q_0''}{\rho C_p}$.

assume :- $Pr_t = 1 \Rightarrow \epsilon_H = \epsilon_M \quad \} Pr \approx 1 \quad \alpha = \nu$.

$\frac{d\bar{T}}{d\bar{u}} = \frac{-q_0''}{\tau_0 C_p}$

$$\int_{T_0}^{T_\infty} d\bar{T} = -\frac{1}{C_p}\frac{q_0''}{\tau_0}\int_0^{U_\infty} d\bar{u}$$

$$\frac{T_\infty - T_0}{-q_0''} = \frac{+1}{C_p \tau_0} U_\infty$$

$$C_p\left(\frac{T_0 - T_\infty}{q_0''}\right) = \frac{U_\infty}{\tau_0}$$

$$\frac{\rho U_\infty C_p (T_0 - T_\infty)}{q_0''} = \frac{\rho U_\infty^2}{\tau_0}$$

$$\frac{\rho U_\infty C_p}{h} = \frac{\frac{1}{2}\rho U_\infty^2}{\frac{1}{2}\tau_0}$$

$$\frac{h}{\rho U_\infty C_p} = \frac{C_{f,x}}{2}$$

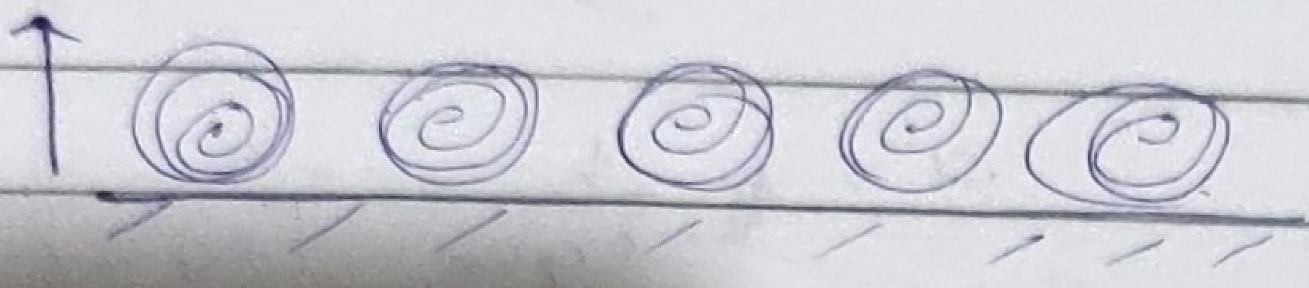
$$St_x = \frac{1}{2} C_{f,x}$$

reynolds analogy assumption ($Pr_t = 1$)

$Pr \approx 1$

assume this for many equation

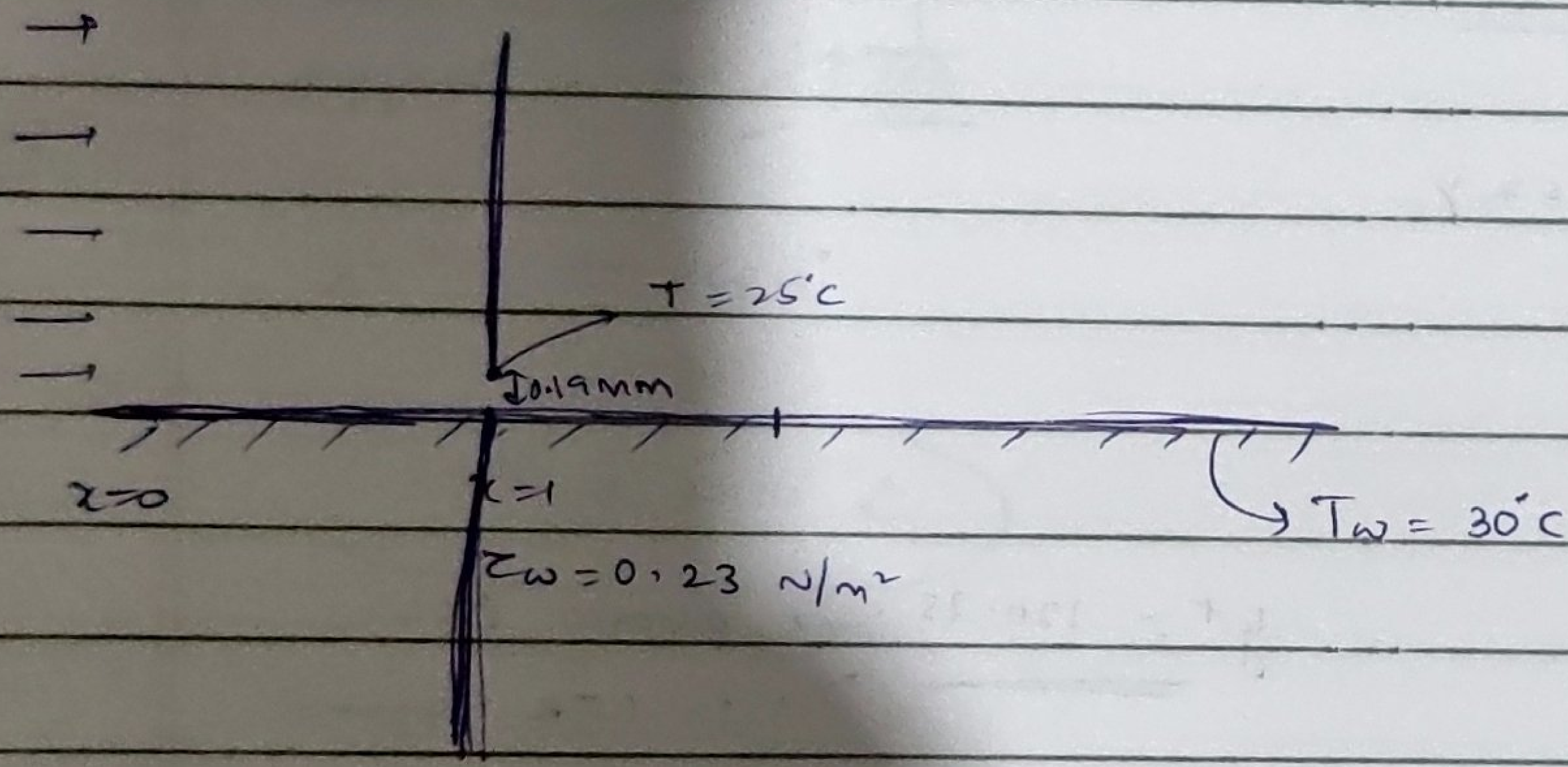
$Pr_t = 1$



amount of momentum transfered by eddies = amt of energy transfered by eddies

$\epsilon_M \approx \epsilon_H$

Q) $U_\infty = 10\,m/s$, $T_\infty = 10^\circ C$



$y^+_{CSL} = 13.2$

~~14mm → CTR region~~

$q_0'' = h\,\Delta T$

h

$y^+ = y\frac{u^*}{\nu}$

$u^* = \left(\frac{\tau_0}{\rho}\right)^{1/2}$

$y^+ = 5.1$

within CSL

$u^* = \left(\frac{0.23}{1.13}\right)^{1/2}$

$\nu = 16.7\times10^{4}$

$q_0'' = -k\left(\frac{d\bar{T}}{dy}\right)_{y=0 = y=0.19mm}$

$$\boxed{q_0'' = -k\left[\frac{5}{0.19\times10^{-3}}\right]} = 15.67\%$$

(b) $St = Pr^{-2/3} \frac{C_{fx}}{2}$

$St = 2.53 \times 10^{-3}$

$C_{f,x} = \frac{\tau_0}{\frac{1}{2}\rho u_\infty^2} = \frac{0.23}{\frac{1}{2} \times 1.13 \times 16^2}$

$St = \frac{h}{\rho C_p U_\infty} = \frac{q_w''}{\rho C_p U_\infty (T_w - T_\infty)}$

find h

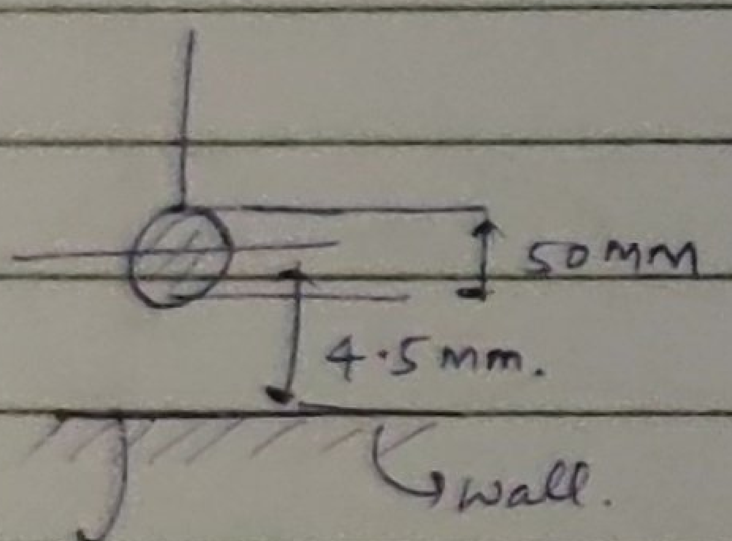
$h(T_w - T_\infty) = q_w''$

$q_w'' = 3 \times 10^{-3}$

$\text{error} = \frac{3 - 2.53}{3} \times 100$

$= 15.67\%$

→



$T_w = 30°C$

$y^+ = 120.78$ → above CSL
in CTR regime

$T^+ = Pr\, y^+_{CSL} + \frac{Pr_t}{K} \ln\left(\frac{y^+}{y^+_{CSL}}\right)$

$Pr_t = 0.9$

$K = 0.41$

$y^+_{CSL} = 13.2$

$T^+ = 2.195 y^+ + 358$

$T^+ = 14.10$

$T^+ = \frac{(T_w - \bar{T})\rho C_p u^*}{q''} = 14.10$

$\bar{T} = 11°C$

$$\frac{dT^+}{dy^+} = 2.195\frac{1}{y^+}$$

$$\left.\frac{dT^+}{dy^+}\right|_{y^+=121} = \frac{2.195}{121} = 0.018$$

distance is small → assume linear variation

$$\frac{\Delta T^+}{d^+} = 0.018 \qquad d^+ = \frac{d \times u^*}{\nu} = \underline{1.35}$$

$$\underline{\Delta T^+ = 0.024}$$

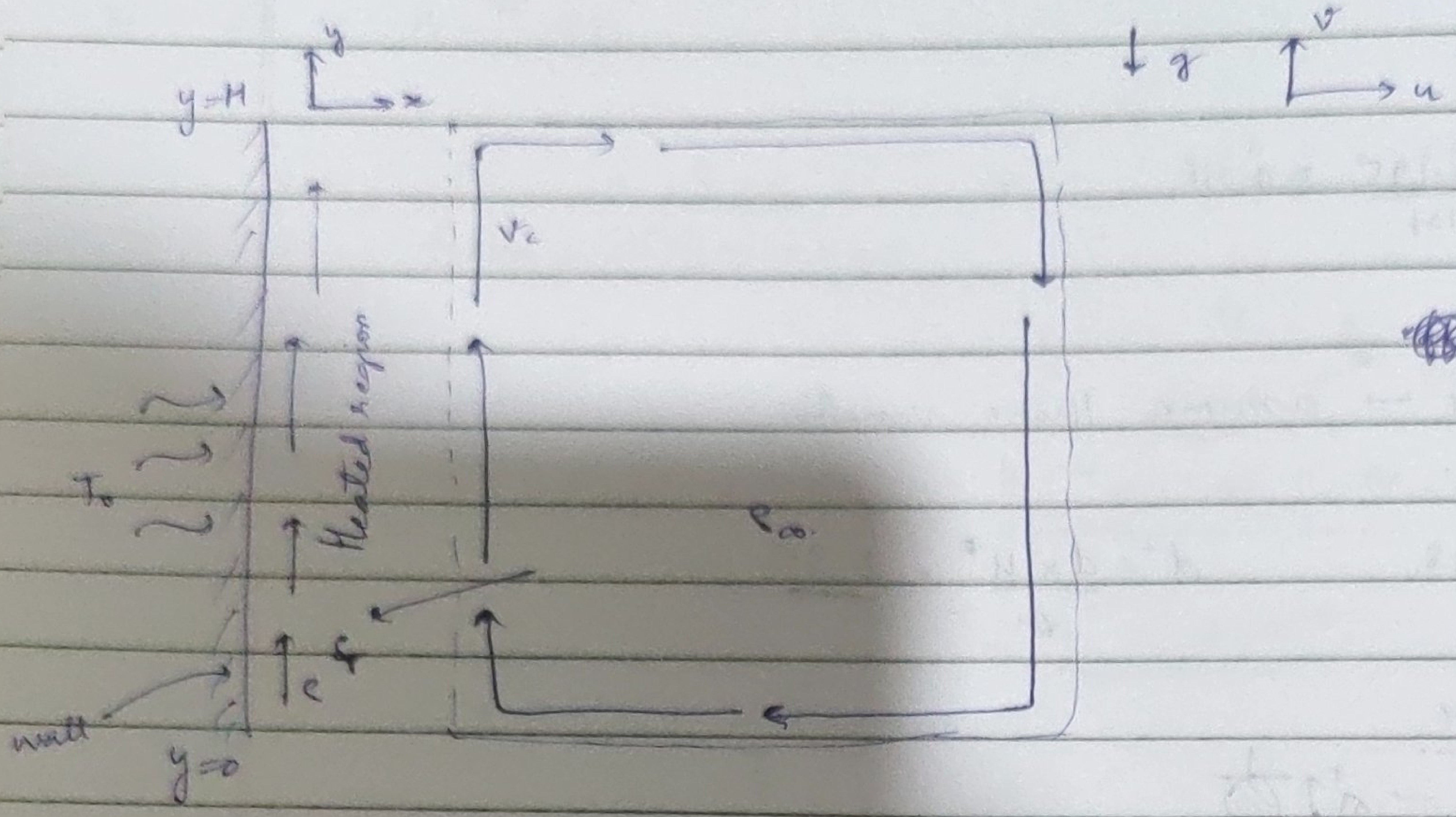
→ difference → ΔT^+

$$\Delta T^+ = \frac{\Delta T\, \rho c_p u^*}{q''}$$

$$\Delta T = 3.2 \times 10^{-2}\ ^\circ C$$

(e) Yes or No both
(Pr_t → will change)

Natural Convection



temperature difference is the main cause of velocity

$\delta_T << H$

$$Q = h(T_0 - T_\infty)$$

from solid to liq

$h \to h_{0-H}$ average heat transfer coeff.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \implies \frac{\partial p}{\partial y} = \frac{dp_\infty}{dy}$$

not required

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \text{Additional term}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \cancel{\frac{\partial^2 v}{\partial y^2}}_{\,0}\right) - \rho g$$

$= \rho_\infty$ being Boussinesq approx

$$\frac{dp_\infty}{dy} = -\rho_\infty g$$

$$\Rightarrow \rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \mu\frac{\partial^2 v}{\partial x^2} + \underbrace{g(\rho_\infty - \rho)}_{\text{bouyancy}}$$

Assumption: Effect of temp on density in bouyancy force >> effect of temp on any other physical property (ρ, α, μ)

Boussinesq Approx

$$\frac{1}{\vartheta}\frac{D\vartheta}{Dt} \to 0 \longrightarrow \text{incompressible fluid} \Longrightarrow \frac{D\rho}{Dt} = 0.$$

rate of change of volume of packet of fluid ~~unit~~ with respect to single ~~packet~~ packet

$$\beta_\infty = \frac{1}{\vartheta}\frac{\partial \vartheta}{\partial T} = \frac{-1}{\rho_\infty}\left.\frac{d\rho_\infty}{dT}\right|_\infty$$

$$\rho = \rho_\infty + \left.\frac{\partial \rho}{\partial T}\right|_\infty (T - T_\infty) + \text{higher order term}$$

$$\rho = \rho_\infty - \rho_\infty \beta_\infty (T - T_\infty)$$

$$\underline{(\rho_\infty - \rho) = \rho_\infty \beta_\infty (T - T_\infty)}$$

$$\rightarrow \rho_\infty\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \mu\frac{\partial^2 v}{\partial x^2} + \rho_\infty\beta_\infty(T - T_\infty) g.$$

$$\underbrace{u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}}_{\text{Inertia}} = \underbrace{\nu}_{\frac{\mu_\infty}{\rho_\infty}}\frac{\partial^2 v}{\partial x^2} + \underbrace{\beta_\infty (T - T_\infty) g}_{\text{bouyancy.}}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \underbrace{\alpha}_{\text{const}}\frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

$$u^* = \frac{u}{v_c} \qquad v^* = \frac{v}{v_c} \qquad x^* = \frac{x}{H} \qquad y^* = \frac{y}{H}$$

$$\theta = \frac{T - T_\infty}{T_0 - T_\infty}$$

$$\left[\frac{v_c^2 u^*}{H}\frac{\partial u^*}{\partial x^*} + \frac{v_c^2 v^*}{H}\frac{\partial v^*}{\partial y^*}\right] = \left[\frac{v_c \nu}{H^2}\frac{\partial^2 v^*}{\partial x^{*2}} + \beta_\infty \theta g (T_0 - T_\infty)\right]\frac{H}{v_c^2}$$

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial^2 v^*}{\partial y^*} = \frac{H}{v_c^2} \times \frac{v_c}{H^2}\nu\frac{\partial^2 v^*}{\partial x^{*2}} + \underbrace{\left[\frac{\beta_\infty g (T_0 - T_\infty) H}{v_c^2}\right]}_{\text{will tell you whether natural convection will dominate or not}}\theta$$

natural v/s forced

$$\frac{\beta_\infty g \Delta T H}{\nu^2} \times \frac{\nu^2}{v_c^2 H^2}$$

$\frac{\beta_\infty g \Delta T H}{\nu^2} \rightarrow Gr_H$

$\frac{\nu^2}{v_c^2 H^2} \rightarrow \frac{1}{Re^2}$

$\frac{Gr_H}{Re^2} = Ri$ (richardson's number)

Ri will tell you whether natural or forced convection dominates.

Scaling analysis

$x \sim \delta_T$

$y \sim H$

$v \sim v_c$ (characteristic velocity)

$u \sim u$

$\Delta T \sim T_0 - T_\infty$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\sim \frac{u}{\delta_T} \sim \frac{v_c}{H} \qquad u \sim \frac{v_c \delta_T}{H}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} \right)$$

$$\sim u \frac{\Delta T}{\delta_T} \sim v_c \frac{\Delta T}{H} \sim \alpha \frac{\Delta T}{\delta_T^2}$$

$u \rightarrow \frac{v_c \delta_T}{H}$

$u \frac{\Delta T}{\delta_T} \sim \frac{v_c \Delta T}{H}$

$$\frac{v_c \Delta T}{H} \sim \alpha \frac{\Delta T}{\delta_T^2}$$

$$\delta_T \sim \sqrt{\frac{\alpha H}{v_c}}, \qquad v_c \sim \frac{\alpha H}{\delta_T^2}$$

δ_T, v_c are unknown

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$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 v}{\partial x^2} + g\beta(T - T_\infty)$$

$$\sim \frac{u\, v_c}{\delta_T} \quad \sim \frac{v_c v_c}{H} \qquad \sim \nu\frac{v_c}{\delta_T^2} \qquad \sim g\beta\Delta T$$

$$\underbrace{\sim \frac{v_c^2}{H} \qquad \sim \frac{v_c^2}{H}}_{\text{Inertia}} \qquad \underbrace{\sim \nu\frac{v_c}{\delta_T^2}}_{\text{Viscous/friction}} \qquad \underbrace{\sim g\beta\Delta T}_{\text{Bouyancy}}$$

Bouyancy → omnipresent

$$\frac{\text{Inertia}}{\text{Bouyancy}} = \frac{\frac{v_c^2}{H}}{g\beta\Delta T} = \frac{\frac{\alpha^2 H^2}{\delta_T^4}\times\frac{1}{H}}{g\beta\Delta T} = \frac{\alpha^2 H}{\delta_T^4 (g\beta\Delta T)} \times \frac{H^3\nu}{H^3\nu}$$

$$= \frac{\overset{\frac{1}{Pr}}{\alpha/\nu}}{\underbrace{\frac{g\beta\Delta T\, H^3}{\alpha\nu}}_{\frac{1}{Ra_H}}}\left(\frac{H}{\delta_T}\right)^4$$

$$= Ra_H^{-1}\, Pr^{-1}\left(\frac{H}{\delta_T}\right)^4$$

$$\frac{\text{Friction}}{\text{Bouyancy}} = \frac{\nu\frac{v_c}{\delta_T^2}}{g\beta\Delta T}$$

$$= \frac{\nu\frac{\alpha H}{\delta_T^4}}{g\beta\Delta T} \times \frac{H^3}{H^3}$$

$$= \frac{1}{\frac{g\beta\Delta T\, H^3}{\nu\alpha}} \times \left(\frac{H}{\delta_T}\right)^4 = Ra_H^{-1}\left(\frac{H}{\delta_T}\right)^4$$

$$\Rightarrow \sim Ra_H^{-1} Pr^{-1}\left(\frac{H}{\delta_T}\right)^4 \sim Ra_H^{-1}\left(\frac{H}{\delta_T}\right)^4 \sim 1$$

high $Pr \rightarrow Pr >> 1$ — Inertia has very low magnitude.

$$Ra_H^{-1}\left(\frac{H}{\delta_T}\right)^4 \sim 1 \qquad \boxed{\delta_T \sim H \times Ra_H^{-1/4}} \qquad h \sim \frac{k}{\delta_T}$$

$$Nu \sim \frac{hH}{k} \sim Ra_H^{1/4}$$

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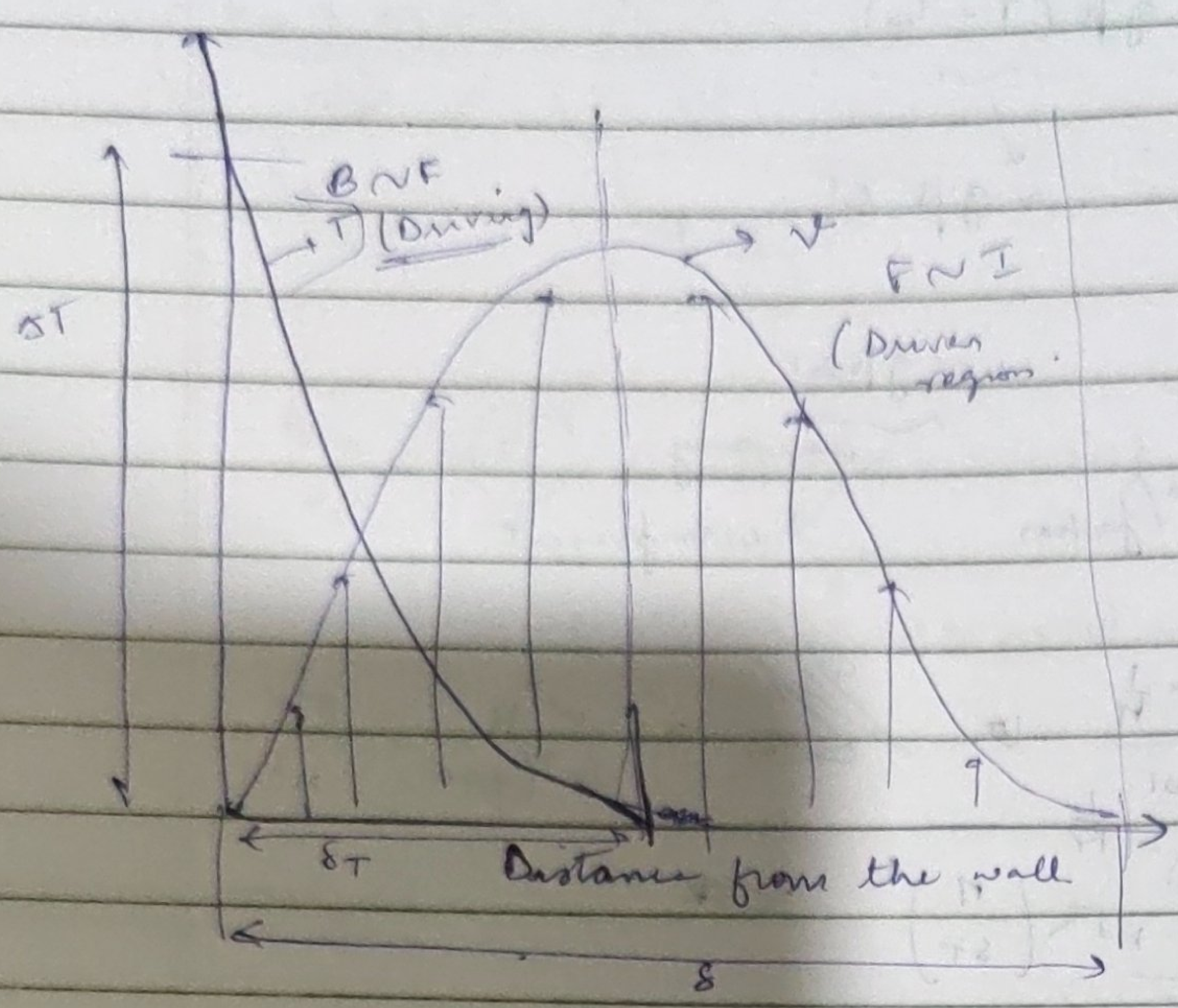

Buoyancy → B
Friction → F
Inertia → I

Friction ∼ Inertia

$$\frac{\nu v_c}{\delta^2} \sim \frac{v_c^2}{H}$$

$$\delta^2 \sim \frac{\nu H}{\frac{\alpha H}{\delta_T^2}}$$

$$\delta^2 \sim \left(\frac{\nu}{\alpha}\right) \times \delta_T^2$$

$$\delta^2 \sim Pr\, Ra_H^{-1/2} H^2.$$

$$\delta \sim Pr^{1/2} Ra_H^{-1/4} H$$

$$\frac{\delta}{\delta_T} \sim \frac{Pr^{1/2} H Ra_H^{-1/4}}{H Ra_H^{-1/4}} \sim Pr^{1/2}$$

low Pr $Pr <<< 1$

$$Ra_H^{-1} Pr^{-1} \left(\frac{H}{\delta_T}\right)^4 \sim 1$$

$\delta_T \sim H (Bo)^{-1/4}$ Bo → Boussinesq number.

$$Nu \sim Bo^{1/4}$$

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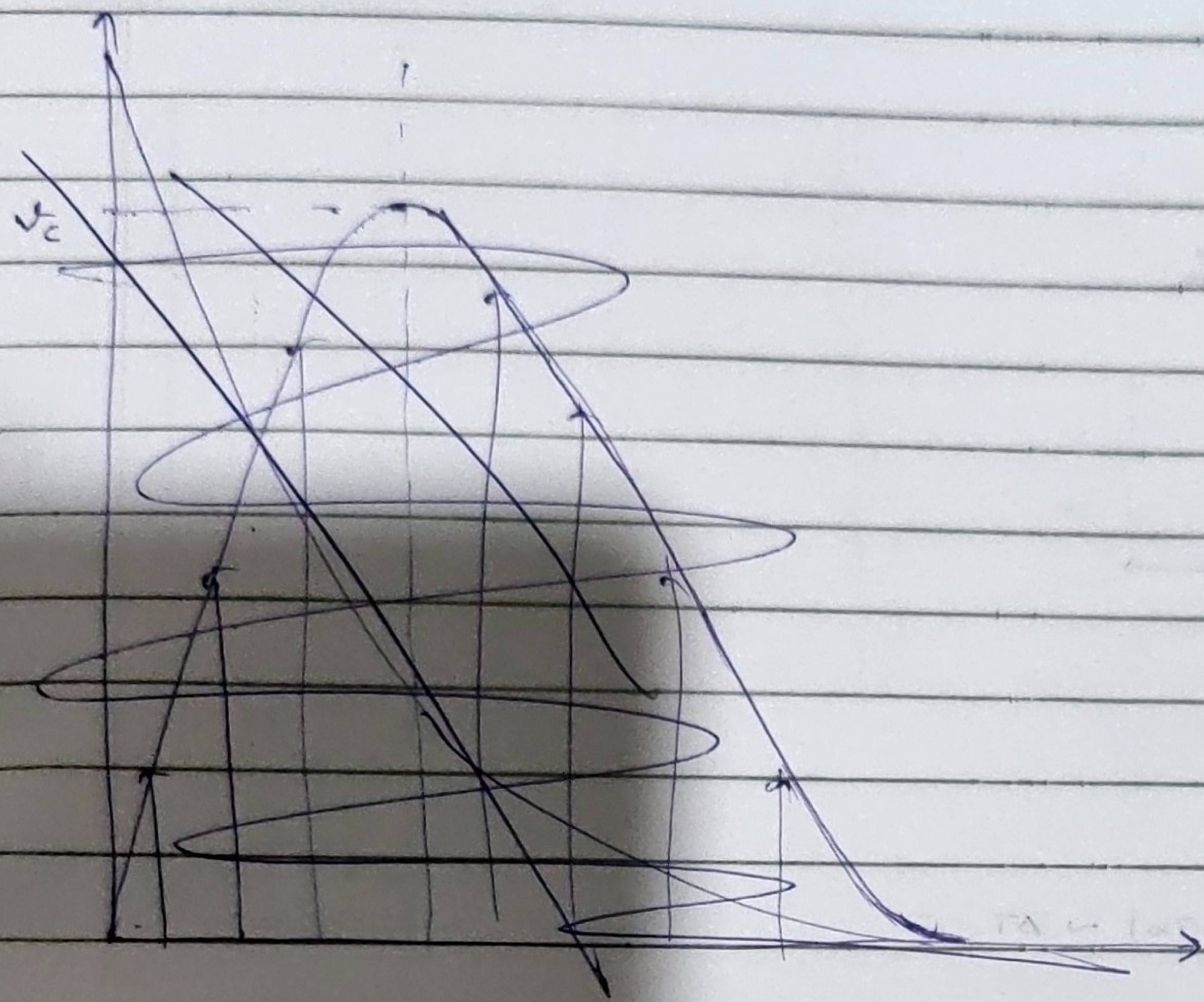
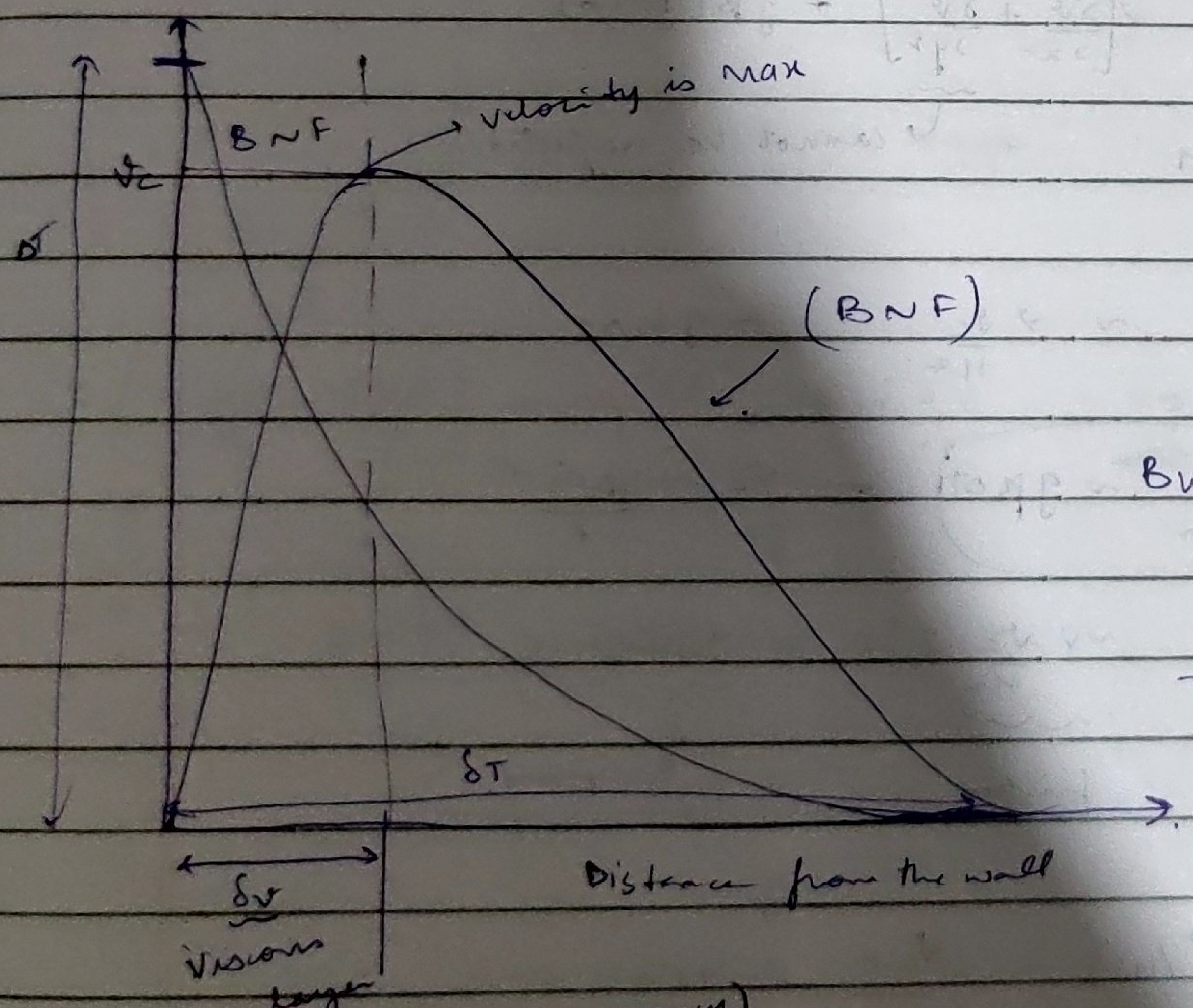




(viscous force v/s Buoyancy)

$v_c \sim \frac{\alpha H}{\delta_T^2}$

Gr → idea about scale of viscous sub layer.

~~Ran~~

$Gr \approx \frac{B}{F} \sim 10^8 - 10^{10}$

$Gr^{1/4} = \frac{H}{\delta_v}$

$Ra_H^{1/4} = \frac{H}{\delta_T}$ $Pr > 1$

$Bo^{1/4} = \frac{H}{\delta_T}$ $Pr < 1$

Buoyancy ~ Friction

$g\beta\Delta T \sim \nu \frac{v_c}{\delta_v^2}$

$\delta_v^2 \sim \frac{\nu \frac{\alpha H}{\delta_T^2}}{g\beta\Delta T}$

$\delta_v^2 \sim \frac{H^4}{\frac{g\beta\Delta T H^3}{\nu\alpha} \times \delta_T^2}$

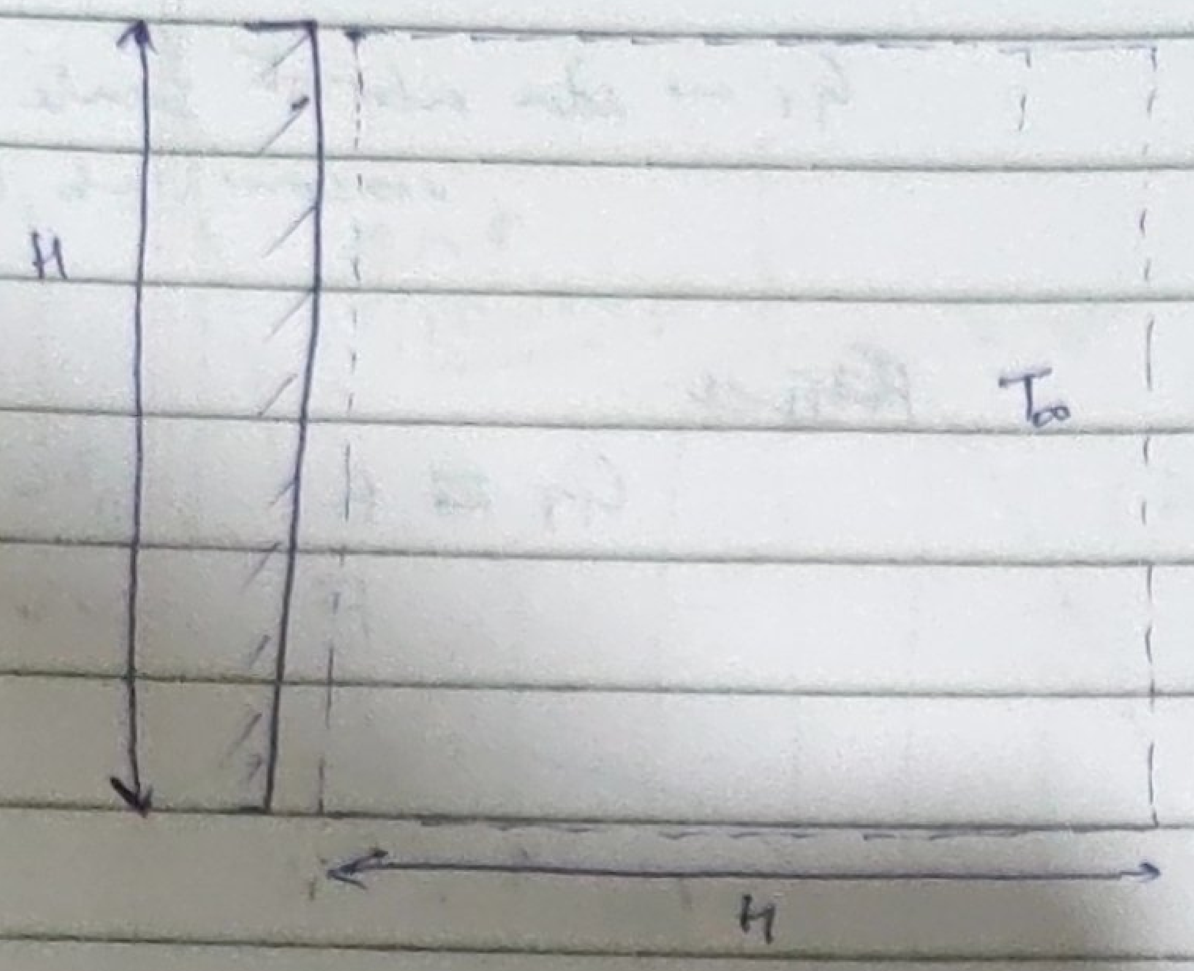
$\delta_v^2 \sim H^2 Pr^{1/2} Ra_H^{-1/2}$

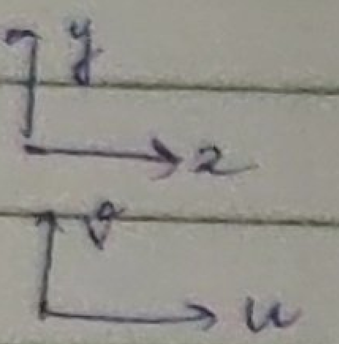
$\delta_v^2 \sim H^2 \left(\frac{Pr}{Ra_H}\right)^{1/2}$

$\delta_v^2 \sim H^2 (Gr)^{-1/2}$

$\delta_v \sim H (Gr)^{-1/4}$

8)





$u \sim u$

$v \sim v_c$

$(T - T_\infty) \sim (T_0 - T_\infty) \sim \Delta T$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \left[\frac{\partial^2 v}{\partial x^2} + \underbrace{\frac{\partial^2 v}{\partial y^2}}_{\text{cannot be neglected}} \right] + g\beta (T - T_\infty)$$

$x \sim H, \quad y \sim H$

$$\sim u \frac{v_c}{H} \qquad \sim \frac{v_c^2}{H} \qquad \sim \nu \frac{v_c}{H^2} \qquad \sim g\beta\Delta T$$

$$\nu \frac{v_c}{H^2} \sim g\beta\Delta T \quad \rightarrow \quad v_c \sim \frac{g\beta\Delta T H^2}{\nu}$$

$$\underbrace{\sim \frac{v_c^2}{H}}_{\text{inertia}} \qquad \underbrace{\sim \nu \frac{v_c}{H^2}}_{\text{friction}} \qquad \underbrace{\sim g\beta\Delta T}_{\text{Buoyancy}}$$

$$\frac{v_c^2/H}{\nu \frac{v_c}{H^2}} = \frac{v_c H}{\nu} = Gr.$$

$$\Rightarrow \quad u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial x^2}$$

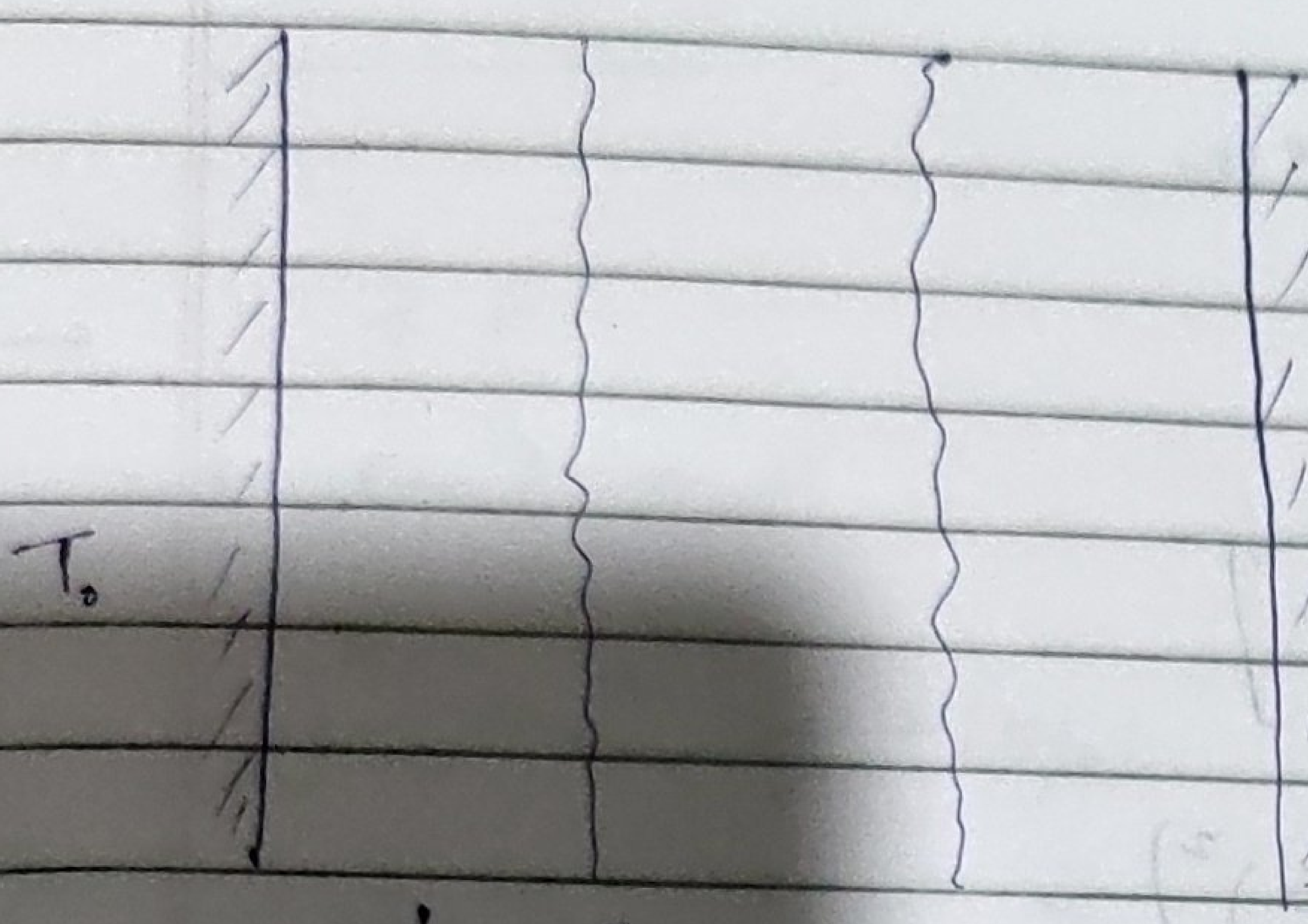
$$\sim u \frac{\Delta T}{H} \qquad \sim v_c \frac{\Delta T}{H} \qquad \sim \alpha \frac{\Delta T}{H^2}$$

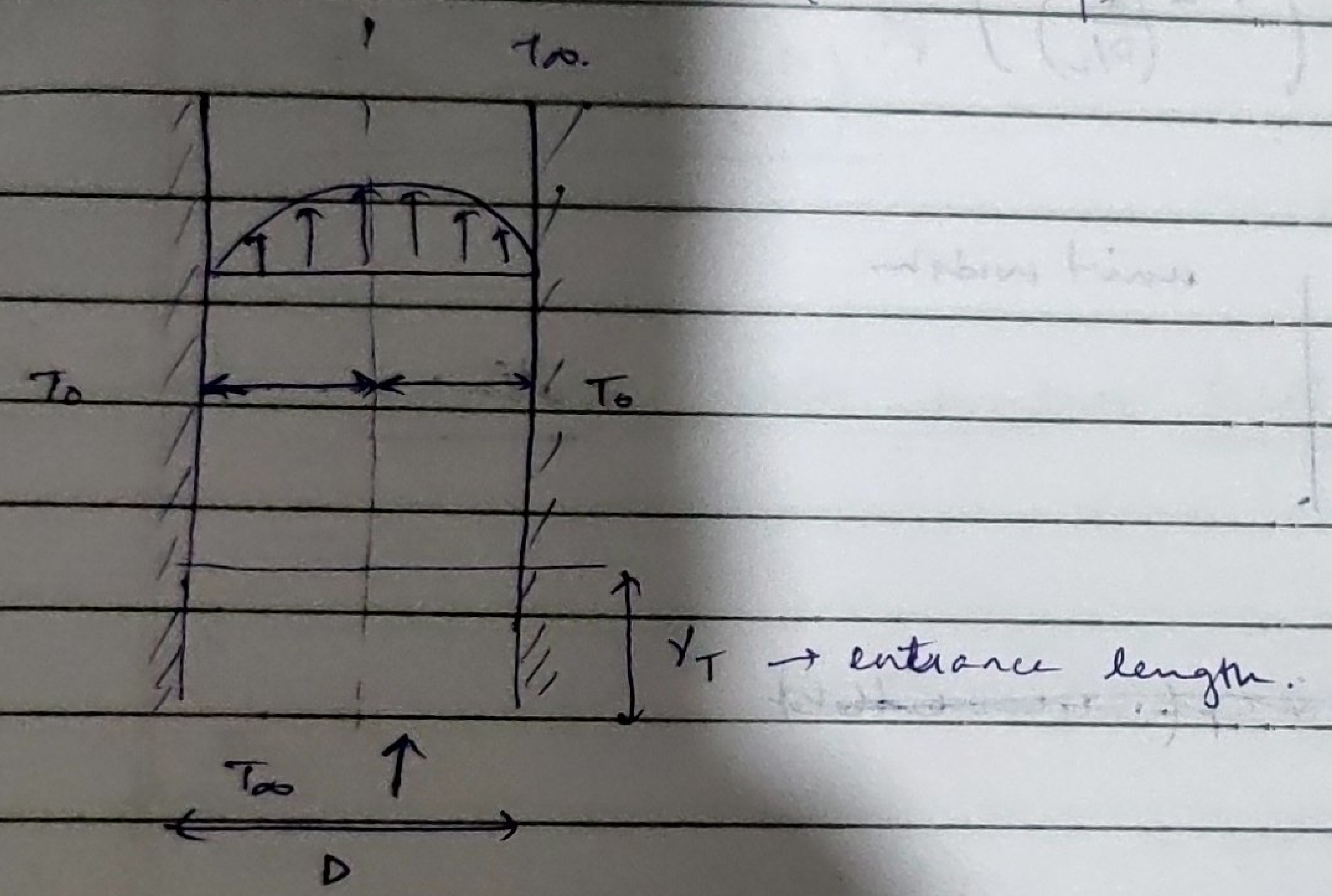
$$\sim v_c \frac{\Delta T}{H} \qquad \sim \alpha \frac{\Delta T}{H^2}$$

$$v_c \sim \frac{\alpha}{H}$$

18th April end sem (24)

Q) Solar chimney





$$\rho\left(\cancel{u\frac{\partial v}{\partial x}} + \cancel{v\frac{\partial v}{\partial y}}\right) = -\frac{dP_\infty}{dy} + \mu\frac{\partial^2 v}{\partial x^2} - \rho g$$

$u\frac{\partial v}{\partial x} \approx 0$, $u << v$; $v\frac{\partial v}{\partial y} = 0$ fully developed. ; $-\frac{dP_\infty}{dy} = -\rho_\infty g$

$$= \nu\frac{\partial^2 v}{\partial x^2} + g(\rho - \rho_\infty)$$

$$= \nu\frac{\partial^2 v}{\partial x^2} + g\beta_\infty(T - T_\infty)$$

$$0 = \nu\frac{d^2 v}{dx^2} + g\beta_\infty(T - T_\infty)$$

$$\nu\frac{d^2 v}{dx^2} = -g\beta_\infty(T - T_\infty)$$ → we need energy eqn to solve this eqn.

BC → $x = \frac{D}{2}$, $v = 0$

$x = 0$, $\frac{\partial v}{\partial x} = 0$

$(T - T_\infty) << (T_0 - T_\infty)$ → assumption

$$\nu \frac{\partial^2 v}{\partial x^2} = -g\beta_\infty(T_0 - T_\infty)$$

$$v(x) = -\frac{g\beta_\infty(T_0 - T_\infty)}{2\nu}x^2 + C_1x + C_2$$

$(C_1 = 0)$

$$v(x) = +\frac{g\beta_\infty(T_0 - T_\infty)}{\nu}\left[\frac{D^2}{8} - \frac{x^2}{2}\right]$$

$$v(x) = \frac{g\beta_\infty(T_0 - T_\infty)D^2}{8\nu}\left(1 - \left(\frac{x}{D/2}\right)^2\right)$$

mass flowrate $= \rho V_{avg} A$

$$\dot{m} = 2\rho \int_0^{D/2} v(x)\,dx.$$

unit width.

width = w.

$$\dot{q} = \dot{m}\,c_p\,\underbrace{\Delta T}_{(out-in.)}$$

~~$\dot{q} = \dot{m} c_p$ (unit width)~~

average heat flux $\rightarrow$ $q''_{0-H} = \frac{\dot{q}}{2\times(w\times H)}$

$w = 1.$

$$Nu_{0-H} = \frac{h_{0-H}\times H}{k} = \frac{\rho_\infty g\beta_\infty(\Delta T)^2 D^3 c_p}{24\times H\times k\,\Delta T} = \frac{Ra_D}{24.}$$

$(y_T < H)$.

$\Rightarrow$ $Pr > 1$, $\delta_T \sim H\,Ra_H^{-1/4}$

$$\frac{D}{2} \sim y_T\,Ra_H^{-1/4}. \qquad \left(y_T \sim \frac{D}{2}Ra_H^{1/4}\right)$$

$\Rightarrow$ $y_T < H \longrightarrow \frac{D}{2}Ra_H^{1/4} < H.$

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$$\frac{Ra_H}{Ra_D} = \left(\frac{H}{D}\right)^3.$$

Problem 4.13

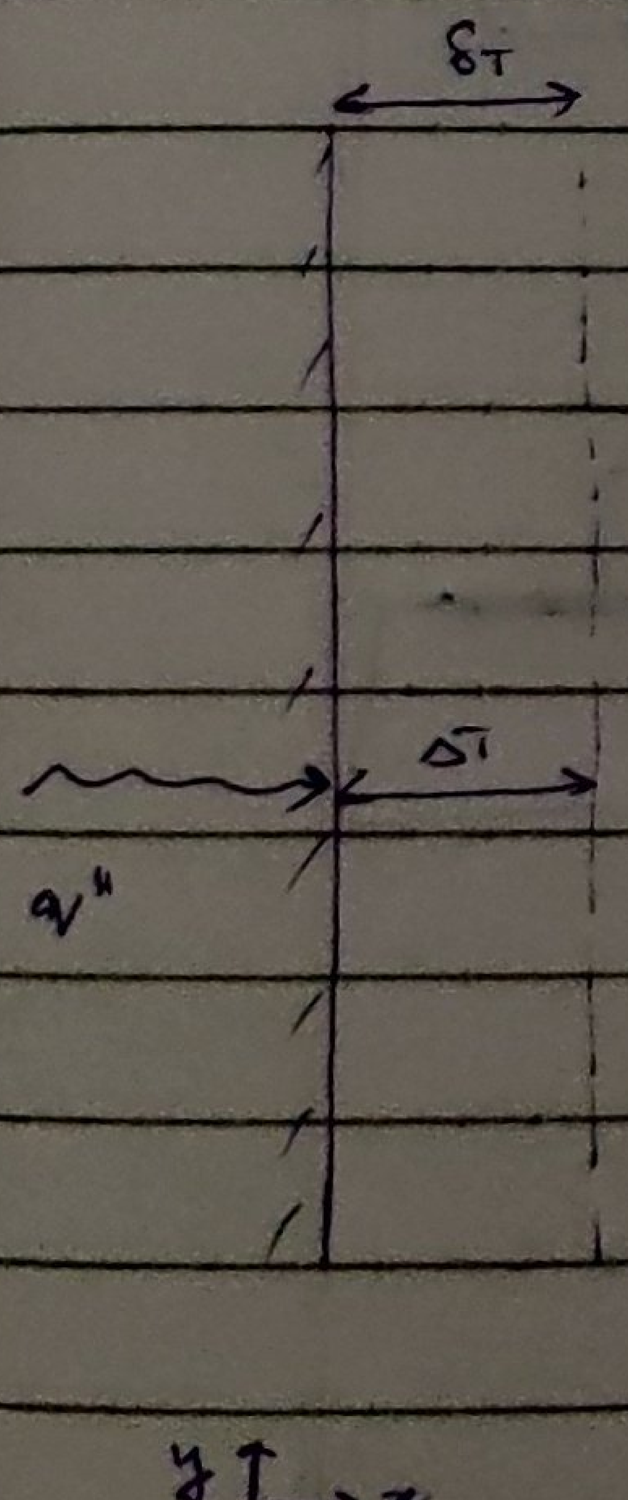
$\Rightarrow \quad y_T < H$

$$\frac{D}{2} Ra_H^{1/4} < H$$

$$\left(\frac{H}{D}\right)^{3/4} Ra_D^{1/4} < 2(H/D)$$

$$Ra_D^{1/4} < 2\left(\frac{H}{D}\right)^{1/4}.$$

Natural convection with uniform heat flux



$q'' \sim k \dfrac{\Delta T}{\delta_T} \rightarrow$ from scaling

$\Delta T \sim \dfrac{q'' \delta_T}{k}$

$(\Delta T \sim f(y) \rightarrow$ at the wall$)$

$Pr > 1$

$$\delta_T \sim H (Ra_H)^{-1/4}$$

$$\sim H \left(\frac{g\beta \Delta T H^3}{\nu\alpha}\right)^{-1/4}$$

$$\delta_T^4 \sim H^4 \left[\frac{g\beta q'' \delta_T H^3}{k}\right]^{-1}$$

$$\delta_T^5 \sim H^5 \left[\frac{\nu\alpha k}{\underbrace{g\beta \Delta T q'' H^4}_{Ra_H^*}}\right]$$

$Ra_H^* \rightarrow$ Rayleigh number based on heat flux

$$\boxed{\delta_T \sim H (Ra_H^*)^{-1/5}}$$

$Pr < 1$

$$\delta_T \sim H(Pr\, Ra_H^*)^{-1/5}$$

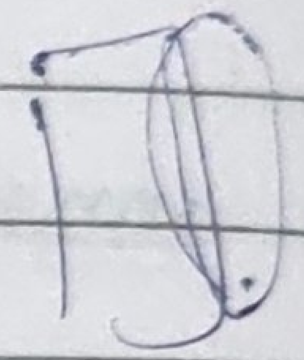
Problem 3 — Problem 4.17

Assumption

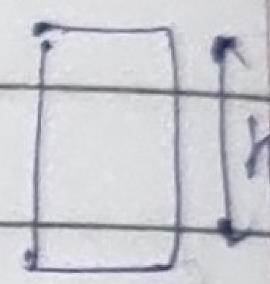
1. resistance of bottle is negligible
2. Natural convection only
3. lump capacitance model ($T = f(t) \rightarrow$ only)

$$\dot{Q} = m_{cd} c_{p_{cd}} \left.\frac{dT}{dt}\right|_{cd}$$ (cd → cold drink)

rate at which cold drink is losing heat



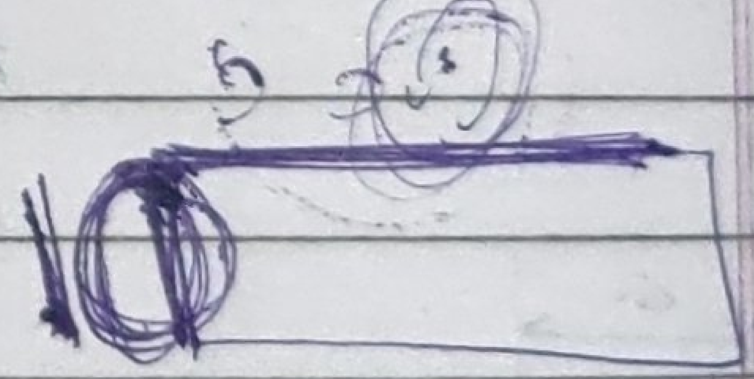
$$\dot{Q} = h \underbrace{A}_{\text{area of bottle}} (T_{cd} - T_{air})$$



$$m_{cd} c_{p_{cd}} \left.\frac{dT}{dt}\right|_{cd} = h A (T_{cd} - T_{air}) \rightarrow \text{at SS}$$

h → will change if the orientation of bottle changes.

$$\frac{1}{h} = \frac{1}{h_{air}} + \frac{1}{h_{cd}}$$



$$\frac{mc_p \Delta T}{t} \sim h A \Delta T$$ → scaling

$$t \sim \frac{mc_p}{hA}$$

t_1 → time taken to cool → vertical

t_2 → horizontal

$$\frac{t_1}{t_2} = \frac{h_2}{h_1}$$

$Nu|_{air}$

$Nu|_{cd}$

$$\frac{h_{cd}}{h_{air}} = \frac{k(Ra_H)^{1/4}}{k(Ra_H)^{1/4}} \sim 58.1$$

$$\frac{1}{h} = \frac{1}{h_{air}} + \cancel{\frac{1}{h_{cd}}}^{\approx 0}$$

$$h \approx h_{air}$$

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