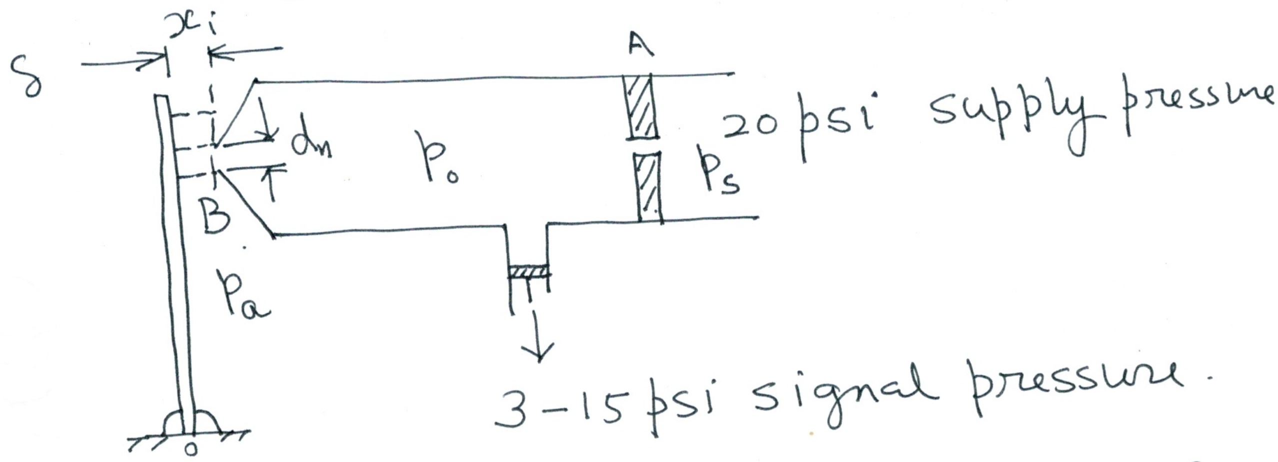
Complete Hardware of a Pneumatic Control system

1. Flapper nozzle system: Translates the displacement produced by a sensor (or any other device) into a pneumatic signal.



The signal pressure depends on the gap δ.
More gap, less pressure.

Steady behavior

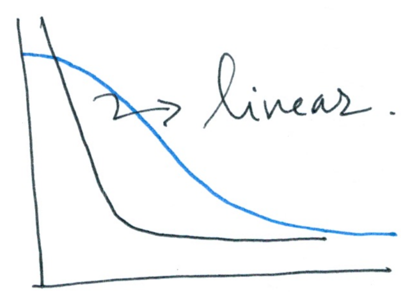
Velocity through orifice: $u_0 = \dfrac{C_0}{\sqrt{1-\beta^4}} \sqrt{\dfrac{2(P_a - P_b)}{\rho}}$.

$\therefore$ mass flow through the orifice at A:

$$\dot{m}_A = \frac{\pi d_o^2}{4} \sqrt{\rho} \cdot C_d \cdot \sqrt{p_s - p_o}$$

$$\dot{m}_B = \pi d_n x_i \sqrt{\rho}\, C_n \sqrt{p_o}$$

$C_n \approx C_d \approx 1$.



Equating

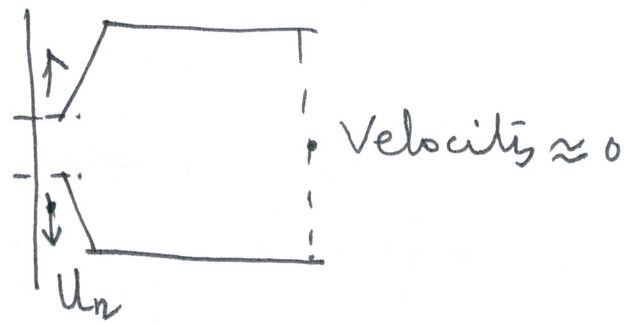
$$d_o^2/4 \cdot \sqrt{p_s - p_o} = d_n x_i \sqrt{p_o}$$

$$d_o^4/16 \cdot (p_s - p_o) = d_n^2 x_i^2 p_o$$

$$\left(\frac{p_s}{p_o} - 1\right) = \frac{16 d_n^2 x_i^2}{d_o^4} \quad \therefore \frac{p_o}{p_s} = \frac{1}{1 + \frac{16 d_n^2}{d_o^4} \cdot x_i^2}$$

$d_n \approx d_o \approx 1 \times 10^{-3}\,m$.

$$\frac{p_o}{p_s} = \frac{1}{1 + 16 \times 10^6 \cdot x_i^2}$$



Bernoulli's eqn

$$\frac{p_0}{\rho} + 0 = \frac{0}{\rho} + \frac{U_n^2}{2}$$

$$\therefore U_n = \sqrt{\frac{2p_0}{\rho}}$$

$$\therefore \dot{m}_B = \pi d_n \cdot x_i \cdot \sqrt{\frac{2p_0}{\rho}} \cdot \rho \approx \pi x_i d_n \sqrt{\rho}\sqrt{2p_0}$$

$$\dot{m}_A = \frac{\pi d_0^2}{4} \cdot \sqrt{\rho} \cdot \sqrt{2(p_s - p_0)}$$

Equating:

$$\cancel{\pi} x_i d_n \cancel{\sqrt{\rho}} \cdot \sqrt{\cancel{2} p_0} = \frac{\cancel{\pi} d_0^2}{4} \cdot \cancel{\sqrt{\rho}} \cdot \sqrt{\cancel{2}(p_s - p_0)}$$

$$\sqrt{\frac{p_s - p_0}{p_0}} = \frac{4 x_i d_n}{d_0^2}$$

$$\therefore \left(\frac{p_s}{p_0} - 1\right) = \frac{16 x_i^2 d_n^2}{d_0^4}$$

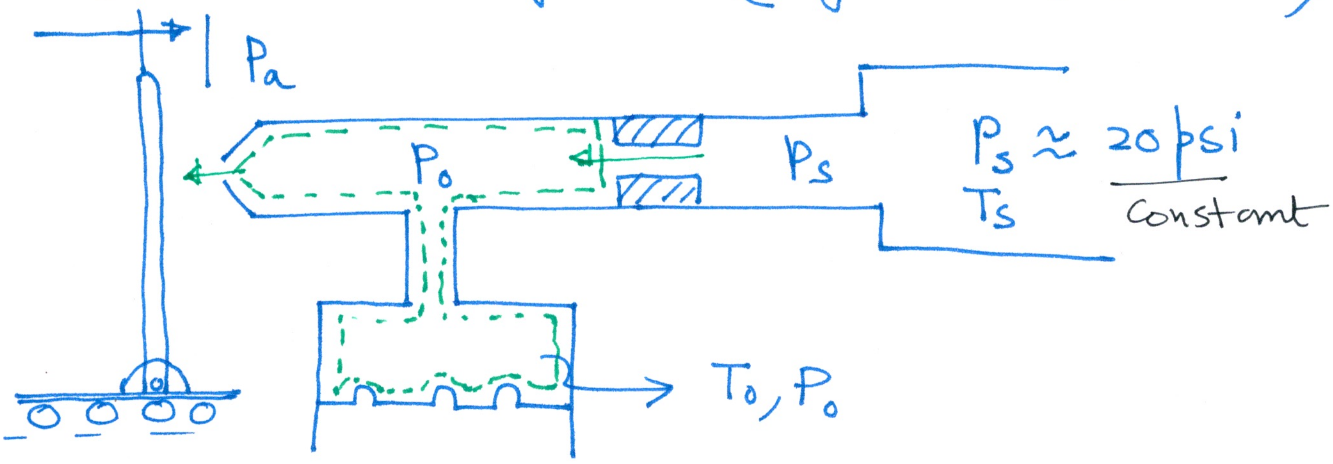
$$\frac{p_s}{p_0} = \frac{16 x_i^2 d_n^2 + d_0^4}{d_0^4}$$

$$\frac{p_0}{p_s} = \frac{d_0^4}{16 x_i^2 d_n^2 + d_0^4}$$

$$\therefore \boxed{p_0 = p_s \cdot \frac{d_0^4}{16 x_i^2 d_n^2 + d_0^4}}$$

plot the steady state in - out relation for typical values of d_0 & d_n.

Quantitative analysis of Flapper nozzle system (Pg-322 of Doebelin) ①



1) $G_s \equiv$ Mass flow rate (supply side orifice)

$$G_s(P_o) = G_{s,o} + \left.\frac{dG_s(P_o)}{d(P_o)}\right|_{P_{o,o}} (P_o - P_{o,o})$$

2) Nozzle side mass flow rate:

$$G_n(P_o, x_i) = G_{n,o} + \left.\frac{\partial G_n}{\partial P_o}\right|_{P_{o,o}, x_{i,o}} (P_o - P_{o,o}) + \frac{\partial G_n}{\partial x_i} (x_i - x_{i,o})$$

3) Rate of change of mass in the control volume:

$$P_o V = \frac{M}{M_w} RT$$

$$\frac{dM}{dt} = \frac{V M_w}{RT} \frac{dP_o}{dt}$$

Rate of change of mass = Rate of mass in − Rate of mass out

(Perturbation analysis)

$$\therefore \frac{VM_w}{RT_0}\frac{dp_0}{dt} = \cancel{G_{s,0}} + \left.\frac{dG_s}{dp_0}\right|_{p_{0,0}}(p_0 - p_{0,0}) - \cancel{G_{n,0}} \pm \left.\frac{\partial G_n}{\partial p_0}\right|_{p_{0,0}}(p_0 - p_{0,0}) - \left.\frac{\partial G_n}{\partial x_i}\right|_{\substack{x_{i,0}\\ p_{0,0}}}(x_i - \cancel{x_{i0}})$$

Set to be zero.

at s.s.
$G_{s,0} = G_{n,0}$

Writing in terms of deviation variables:

$$\frac{VM_w}{RT_0}\cdot\frac{dp_0}{dt} = \Bigg\{\underbrace{\left.\frac{dG_s}{dp_0}\right|_{p_{0,0}}}_{K_s} - \underbrace{\left.\frac{\partial G_n}{\partial p_0}\right|_{p_{0,0}}}_{K_n}\Bigg\}(p_0 - p_{0,0}) - \left.\frac{\partial G_n}{\partial x_i}\right|_{\substack{x_{i,0}=0\\ p_{0,0}}}\cdot x_i$$

$$\frac{V.M_w}{RT_0}\cdot\frac{d\bar{p}_0}{dt} + (K_n - K_s)\bar{p}_0 = \left[-K_{nx}\right]x_{i,p}.$$

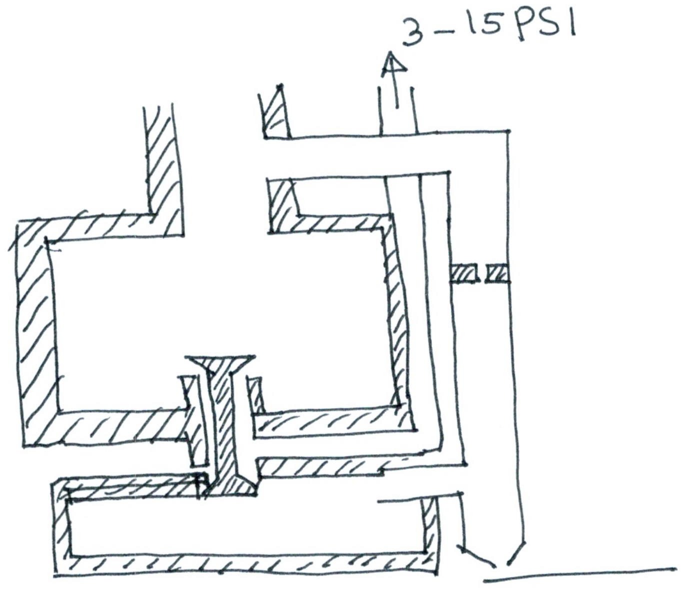
First order system.

$$\underbrace{\frac{VM_w}{RT_0(K_n - K_s)}}_{\tau_p}\frac{d\bar{p}_0}{dt} + \bar{p}_0 = \underbrace{\frac{-K_{nx}}{K_n - K_s}}_{K_p}x_{i,p}.$$

$$\boxed{\tau_p\frac{d\bar{p}_0}{dt} + \bar{p}_0 = K_p\, x_{ip}}$$

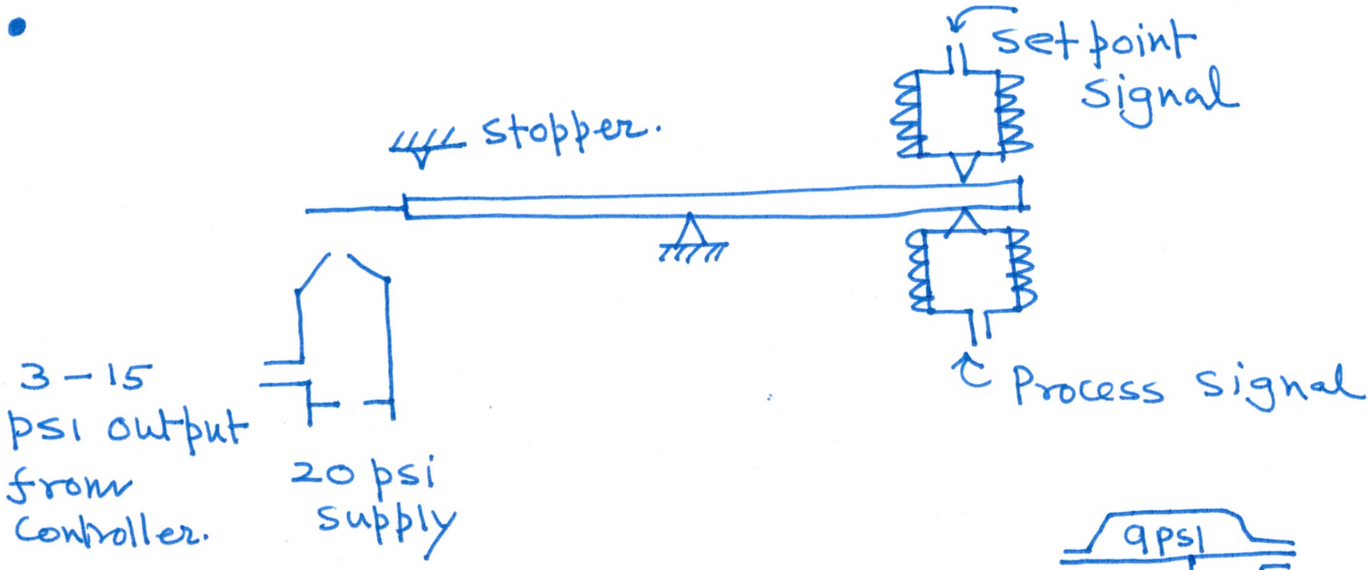
Hence the volume should be very small for quick response of the system. Usually, the volume is kept very small and the response time of the instrument is much smaller than the dynamics. Hence, it is reasonable to assume quasi steady behavior.

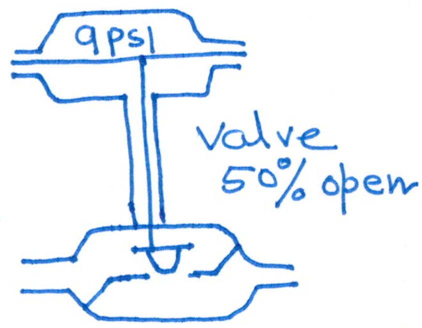
Because the volume of this air is very small, the flapper nozzle cannot produce much air volume for controller/control valve. Hence, an amplifier is needed.



Air relay.

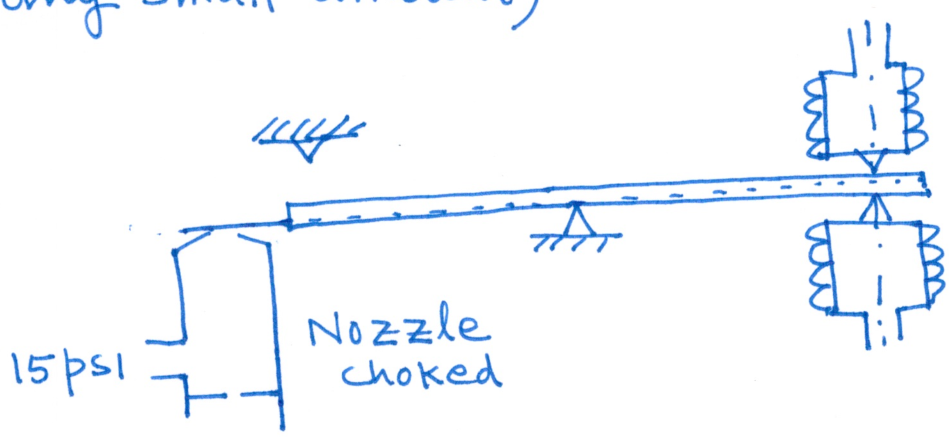
Pneumatic controller: on-off control:

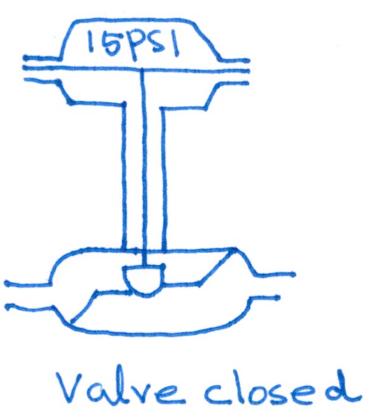
- 



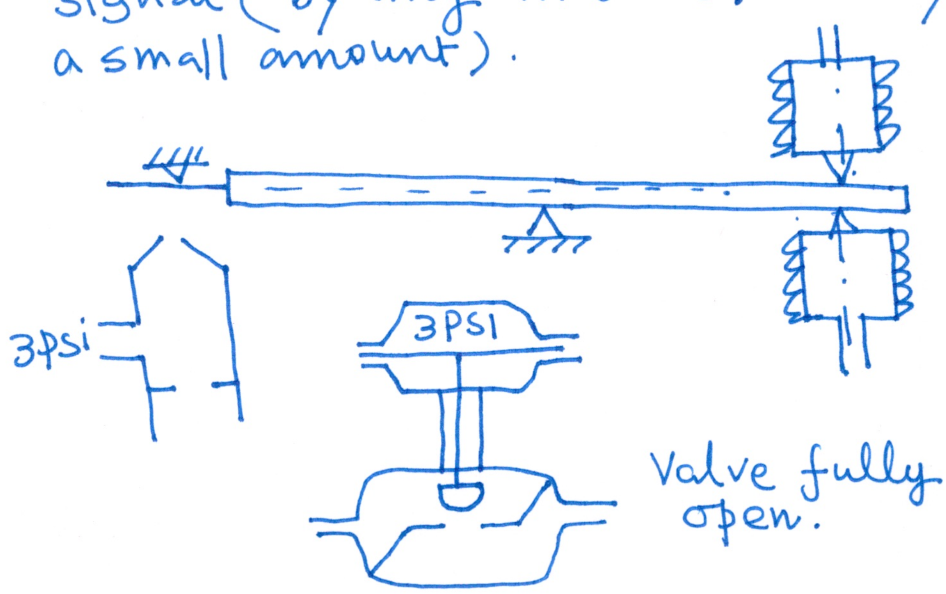
Position when error = 0
output of controller = 9 psi

- When process signal > set point signal. (even by any small amount)



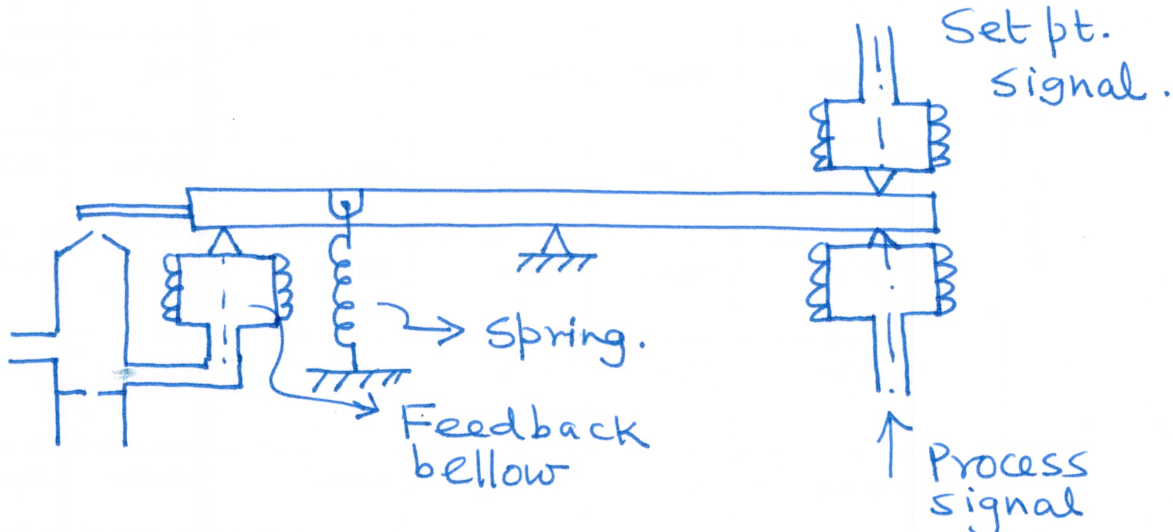


- When process signal < Set point signal (by any amount: even by a small amount).



Pneumatic Control : proportional controller

(The movement of flapper has to be proportional to the error signal. We already know that the output signal from the flapper nozzle system is proportional (approximately) to the separation distance x_i. Hence, if the movement of the flapper is proportional to the error signal, we have achieved proportional control.

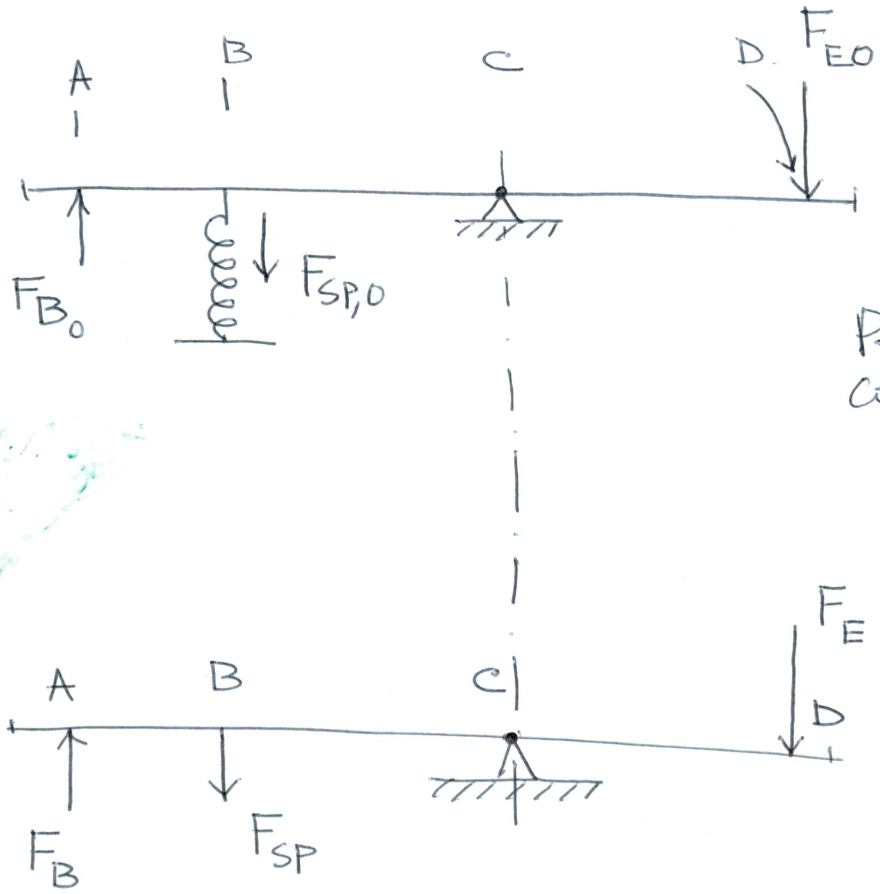


When flapper tries to move towards the nozzle, the movement is opposed by both feedback bellow & spring. When it tries to move away from the nozzle, the movement is opposed by the spring. This arrangement provides proportional action.

When error is zero, the torque created by the spring & feedback bellow balances each other.

Lever Law (when Error signal = 0)

$$-\overline{CD}\times F_{E0} + F_{SP,0}\,\overline{BC} - F_{B0}\,\overline{AC} = 0$$



Proportional controller.

Lever law applied (when error signal ≠ 0)

$$-\overline{AC}\cdot F_B - \overline{CD}\,F_E + F_{SP}\cdot\overline{BC} = 0$$

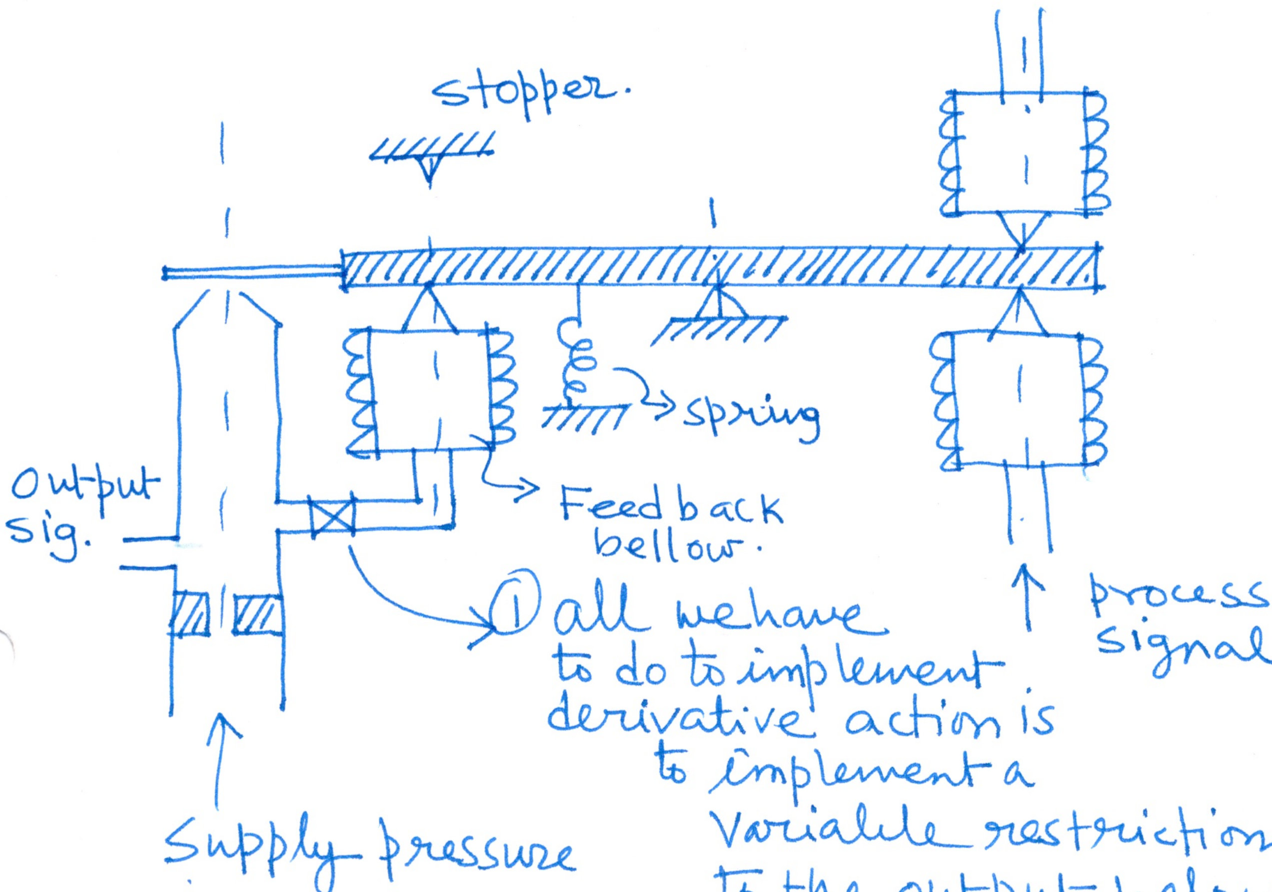
Subtract:

$$-\overline{AC}\left(F_B - F_{B0}\right) - \overline{CD}\left(F_E - F_{E0}\right) + \overline{BC}\left(F_{SP} - F_{SP0}\right) = 0$$

$$\begin{aligned}
\overline{CD}\,\varepsilon &= \overline{BC}\left(F_{SP} - F_{SP0}\right) - \overline{AC}\left(F_B - F_{B0}\right)\\
&= \overline{BC}\left(k_{SP}\bar{x}\right) - \overline{AC}\cdot A_{Blow}\left(P_0 - P_{0,0}\right)\\
&= \overline{BC}\,k_{SP}\,\bar{x} - \overline{AC}\,A_{Bellow}\left(-k_{FN}\right)\bar{x}\cdot k_{geo}\\
&= \left(k_{SP}\overline{BC} + \overline{AC}\cdot A_{Bellow}K_{FN}K_{geo}\right)\bar{x}
\end{aligned}$$

Pneumatic Controller:

Proportional & Derivative action.

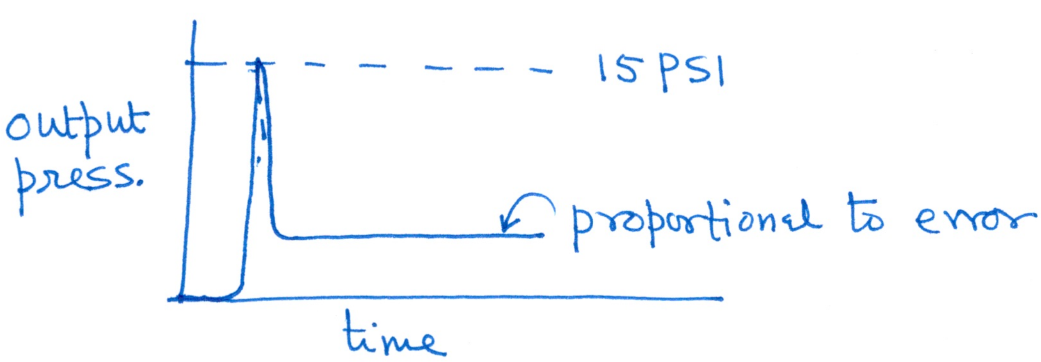


1. all we have to do to implement derivative action is to implement a variable restriction to the output below

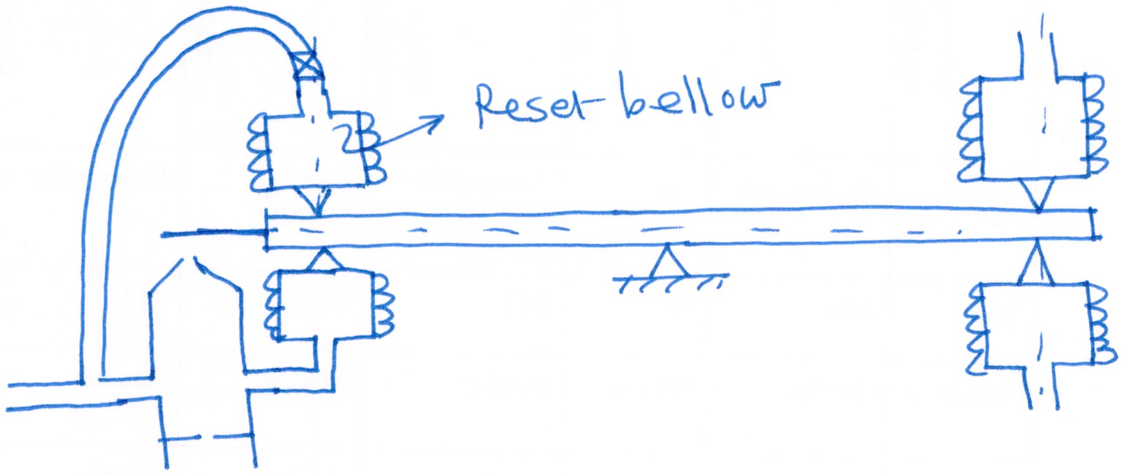
2. Because of the restriction, there will be a delay in the action of the output bellow.

3. Hence, as soon as the step change occurs, it will choke the nozzle, shooting up the output pressure to the maximum possible.

4. Soon the output bellow will apply restoring force and only the proportional action will remain.



Proportional Integral action



When step change is given in the error signal, the reset bellow cannot kick in immediately because of the restriction present. Hence, initially only proportional action is present.

But the increased signal output eventually reaches the reset bellow and keep pushing down the flapper. More it pushes the flapper, the pressure increases and it can be shown that the output will look like: