Advanced heat transfer

Thermodynamics will not tell about mode of heat transfer and rate

→ **Heat conduction**

free electrons > lattice vibration → contribution

opaque solid → heat transfer is due to conduction

CONTINUUM CONCEPT

thermophysical properties ← Heat conduction problem can be solved based on molecular structure

→ consider matter as a continuous medium instead of discrete particles

→ phenomenological approach → we will follow

→ continuum concept works if (mean free path of the molecules) < (other dimension of the body)

→ Temperature field → $T(\underline{r}, t)$

$$\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

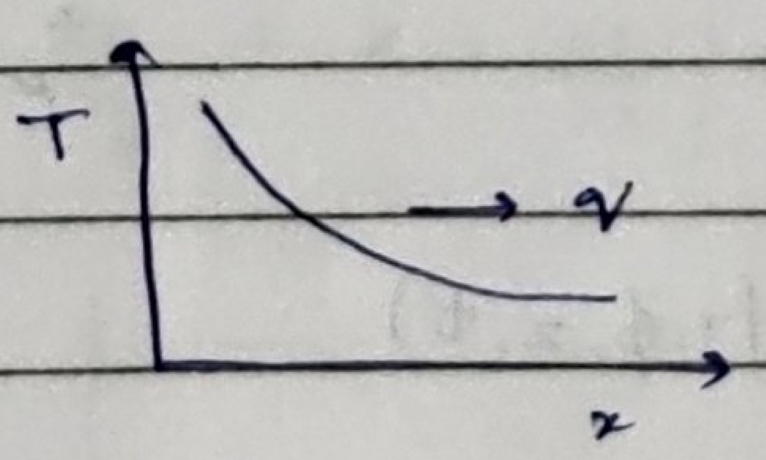
→ Isothermal surface → same temperature points

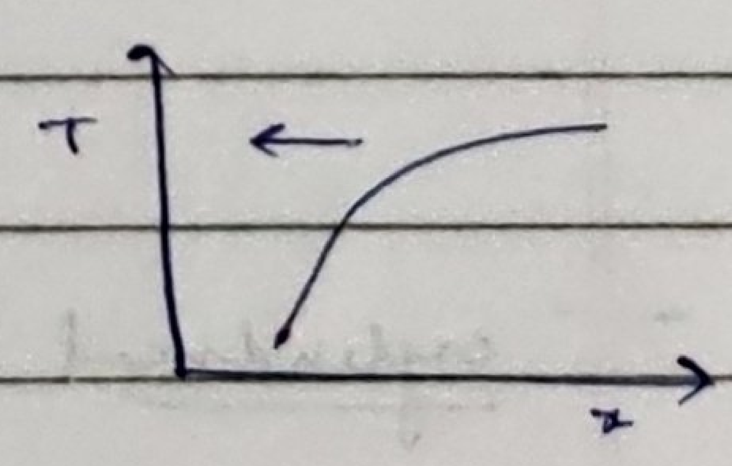
No 2 isotherms will intersect

family of isothermal curves.

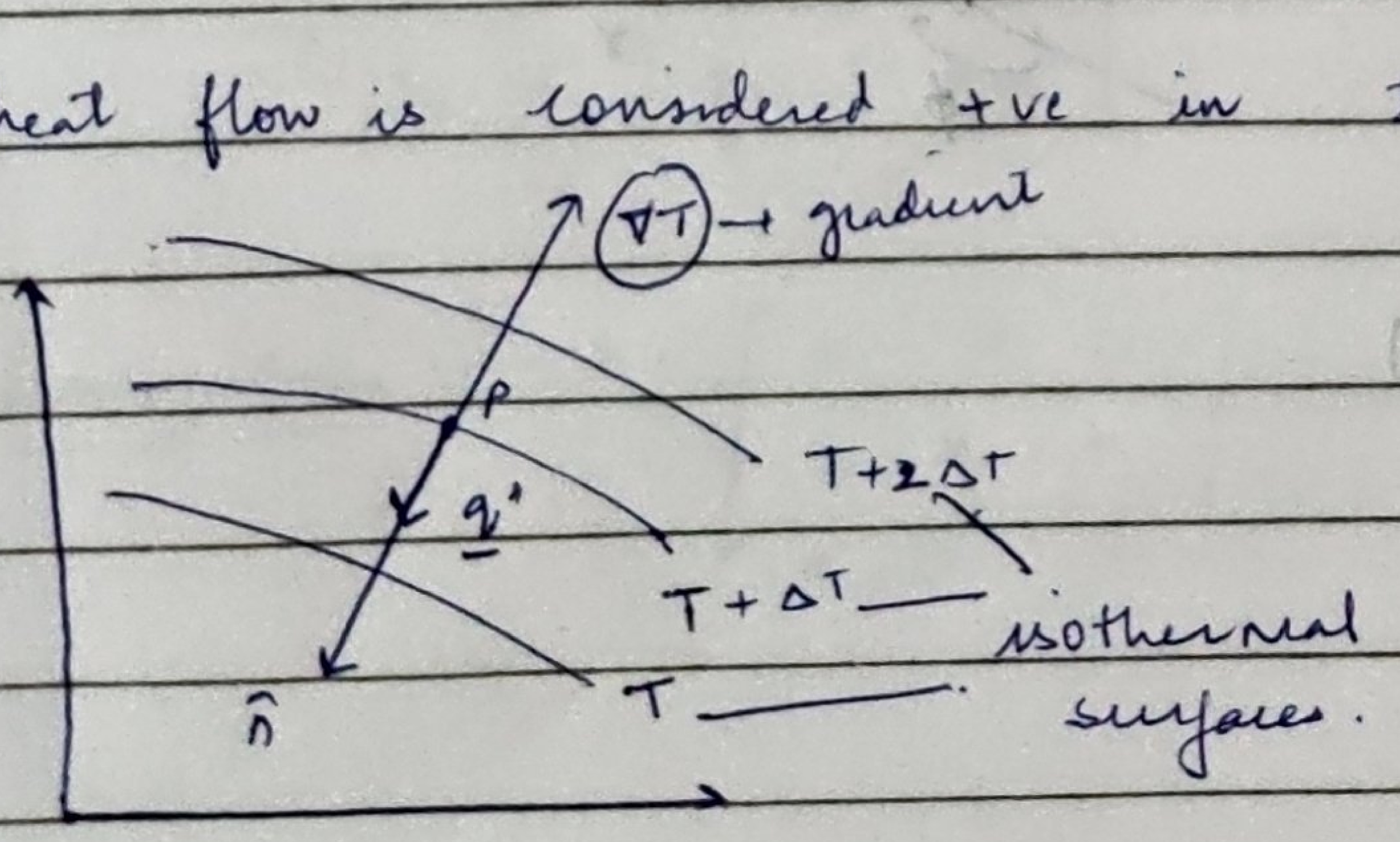
→ **Fourier's law** :- Vector form

$$q'' = -k\frac{dT}{dx}$$





heat flow is considered +ve in the +ve direction of x.



q'' → vector

$\hat{n}$ = normal vector

→ gradient points to the direction of increasing function value.

→ $q_n'' = -k \frac{\partial T}{\partial n}$ — fourier's law

→ For rectangular coordinate

$$q_n'' = -k\left[\frac{\partial T}{\partial x}\frac{dx}{dn} + \frac{\partial T}{\partial y}\frac{dy}{dn} + \frac{\partial T}{\partial z}\frac{dz}{dn}\right]$$

$$= -k\left[\frac{\partial T}{\partial x}\cos\alpha + \frac{\partial T}{\partial y}\cos\beta + \frac{\partial T}{\partial z}\cos\gamma\right]$$

$$\hat{n} = \hat{i}\cos\alpha + \hat{j}\cos\beta + \hat{k}\cos\gamma$$

→ using vector calculus

$$q_n'' = -k\left[\hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}\right]\cdot\hat{n}$$

$$\boxed{q_n'' = -k\,\nabla T\cdot\hat{n}}$$

↳ heat flux magnitude.

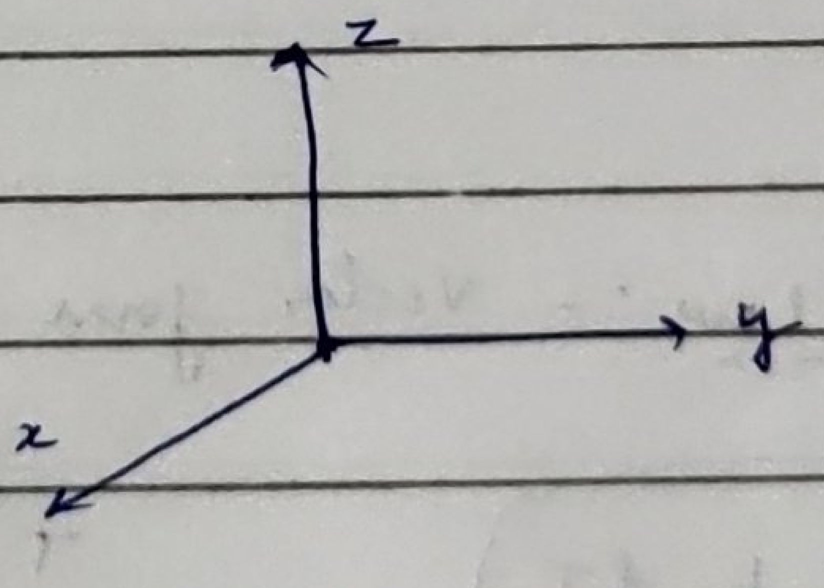
Heat flux vector → $\vec{q}'' = \hat{n}\, q_n''$

$$\boxed{\vec{q}'' = -k\,\nabla T}$$

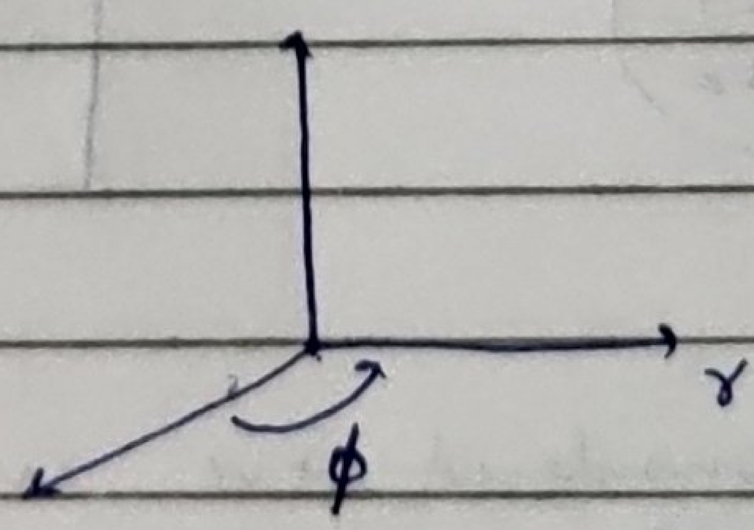
vector

$\vec{q}''$

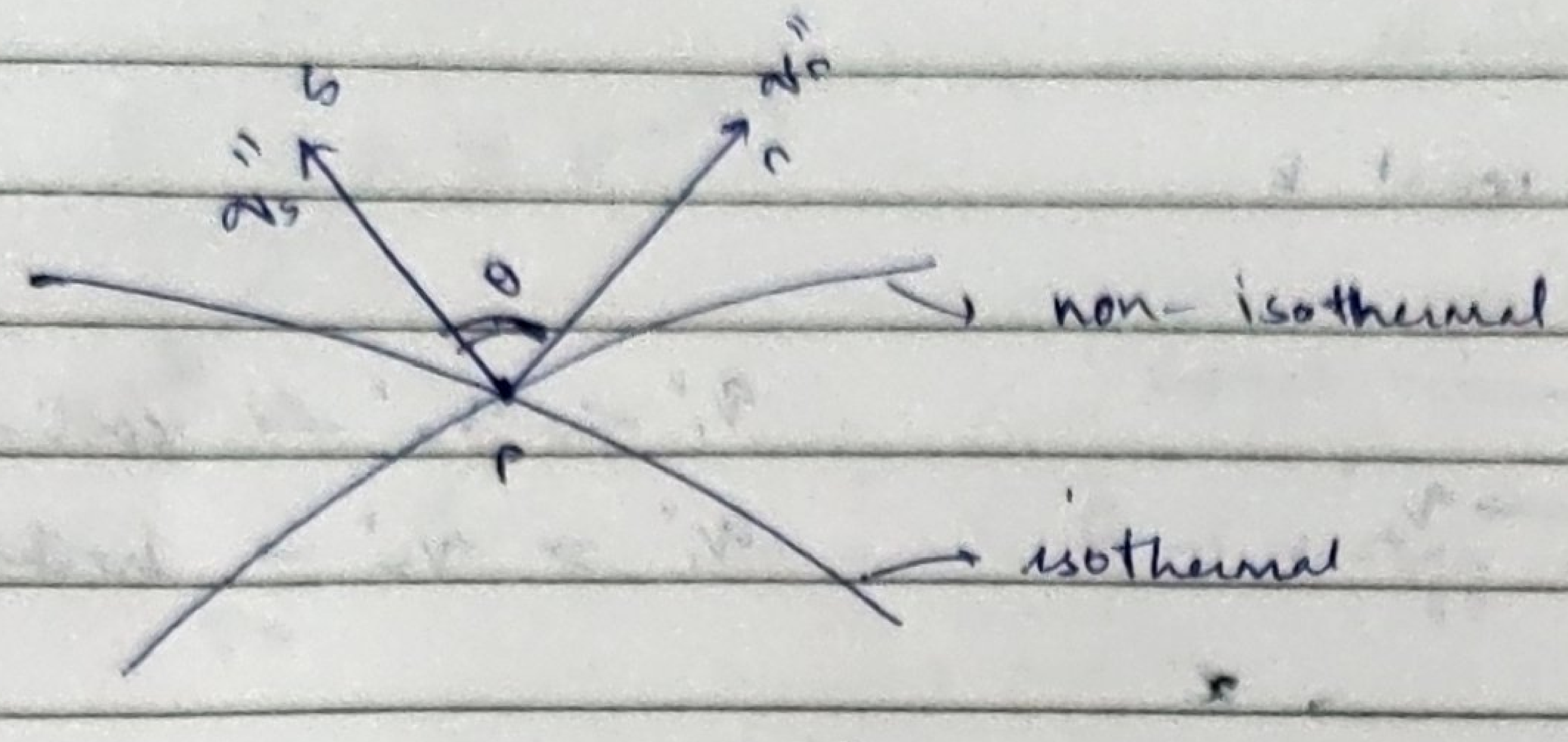
→ Rectangular :- $T(x, y, z, t)$



→ cylindrical :- $T(r, \phi, z, t)$



→ spherical : $T(r, \theta, \phi)$



$$q_s'' = q'' \cdot \hat{s} = q_n'' \, \hat{n} \cdot \hat{s} = q_n'' \cos\theta = -k \frac{\partial T}{\partial n} \cos\theta$$

$$q_s'' = \underbrace{(-k \nabla T) \cdot \hat{s}}_{\frac{\partial T}{\partial s}} = = -k \frac{\partial T}{\partial s}$$

→ Homogeneous solid → k is same everywhere

Heterogeneous solid → k is not same everywhere

isotropic → k is same in all directions

anisotropic → k is not same in all directions → solid with fibrous structure

→ Anisotropic solid

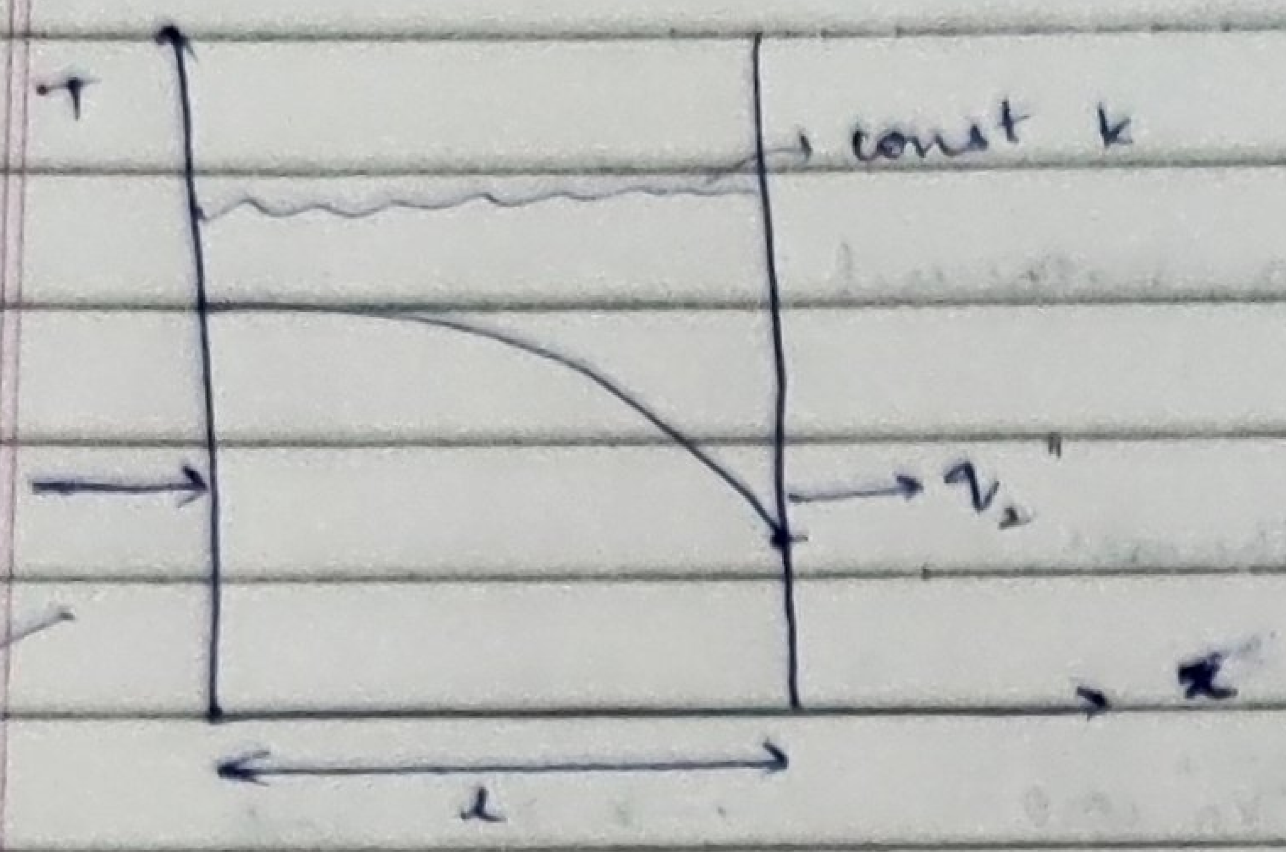
$$q_x'' = -\left[k_{11} \frac{\partial T}{\partial x} + k_{12} \frac{\partial T}{\partial y} + k_{13} \frac{\partial T}{\partial z} \right]$$

$$q_y'' = -\left[k_{21} \frac{\partial T}{\partial x} + k_{22} \frac{\partial T}{\partial y} + k_{23} \frac{\partial T}{\partial z} \right]$$

$$q_z'' = -\left[k_{31} \frac{\partial T}{\partial x} + k_{32} \frac{\partial T}{\partial y} + k_{33} \frac{\partial T}{\partial z} \right]$$

$$k = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$$

→ Homogeneous :- $k = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$



$q_1'' < q_2'' \rightarrow$ cooling

$q_1'' > q_2'' \rightarrow$ heating

$q_1'' < q_2''$

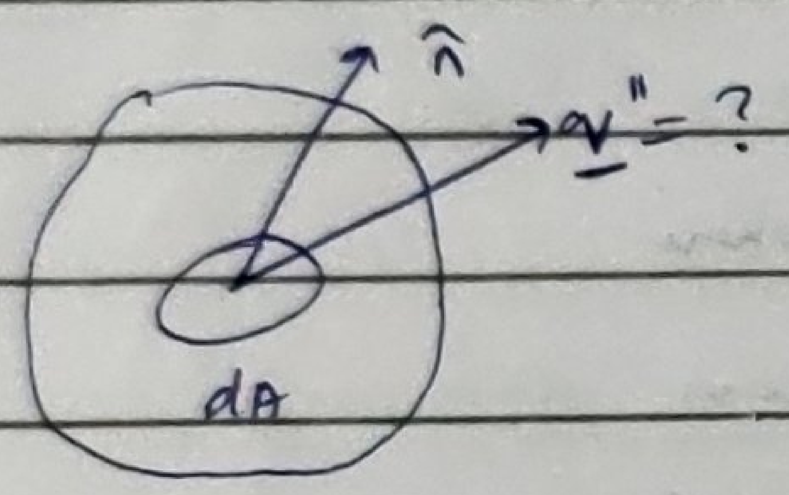
$\frac{\partial T}{\partial x} \neq 0$

$\therefore$ slab is cooling

$$q_1'' = -k \frac{\partial T}{\partial x}\Big|_{x=0}$$

$$q_2'' = -k \frac{\partial T}{\partial x}\Big|_{x=l}$$

$\Rightarrow$



find total heat ~~flux through~~ the bounding surface?

Total amount of heat transfer in time t_1 to t_2 $= \int_{t_1}^{t_2} Q\,dt = \int_{t_1}^{t_2}\int_A \underline{q}''\cdot\hat{n}\,dA\,dt$

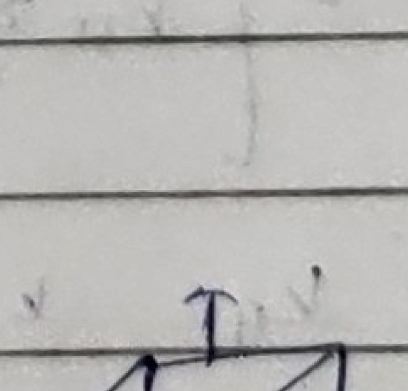
Total heat outflow through dA per unit time $= \underline{q}''\cdot\hat{n}\,dA$

$\rightarrow$ rate of heat flowing through the entire bounding surface $= \int_A \underline{q}''\cdot\hat{n}\,dA$

$$Q = \int_A \underline{q}''\cdot\hat{n}\,dA = \int_A q_n''\,dA$$

$$\rightarrow \int_{t_1}^{t_2} Q\,dt = -\int_{t_1}^{t_2}\left[\int_A \left(k\frac{\partial T}{\partial n_i}\,dA\right)\right]dt$$

$\hookrightarrow$ Consider a cube



$$\sum_{i=1}^{6} k\frac{\partial T}{\partial n_i}$$

$$\Rightarrow Q = -k\int_{t_1}^{t_2}\left(\sum_{i=1}^{6}\int_A\left(k\frac{\partial T}{\partial n_i}\,dA\right)\right)dt$$

for a parallel piped with 6 surfaces

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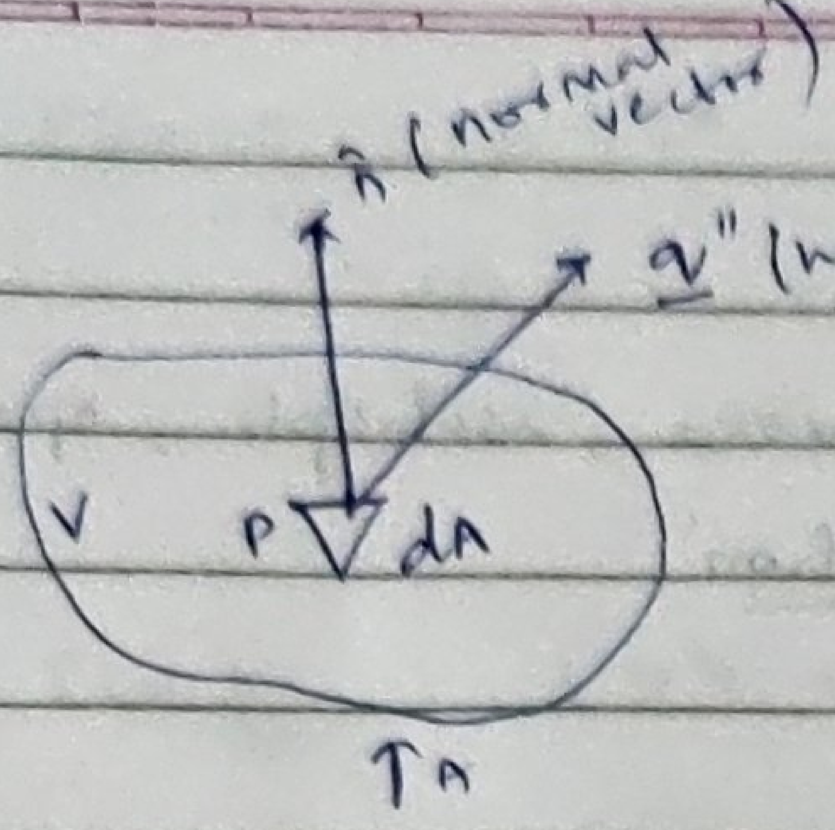
→



stationary, homogeneous, isotropic, opaque solid

constant thermophysical properties

const volume, mass.

V = volume of the system = fixed

$\dot{E} \rightarrow$ energy of the system

$$\frac{dE}{dt} = \underbrace{\dot{Q}}_{\text{net rate of heat transfer.}}$$

uniform volumetric heat generation $= \dot{q}$ (W/m^3)

$e \rightarrow$ specific energy.

$c_p \rightarrow$ heat capacity.

$$\frac{dE}{dt} = \int_V \rho \frac{de}{dt} dv$$

$$\frac{dE}{dt} = \int_V \rho c_p \frac{\partial T}{\partial t} dv$$

$\dot{Q}$ = net heat flow into the solid by conduction through bounding surface A

$$\dot{Q} = -\int_A \underline{q}'' \cdot \hat{n}\, dA + \int_V \dot{q}\, dv$$

(opp to normal vector.)

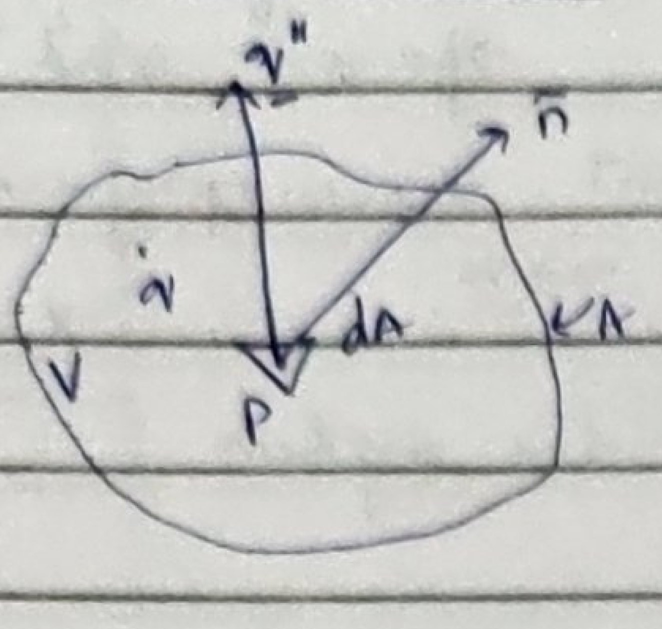
$$\boxed{\int_V \rho c_p \frac{\partial T}{\partial t} dv = -\int_A \underline{q}'' \cdot \hat{n}\, dA + \int_V \dot{q}\, dv}$$

convert Area integral to vol integral using divergence.

$$\int_A f \cdot \hat{n}\, dA = \int_V \nabla \cdot f\, dv$$

$$\int_V \rho c_p \frac{\partial T}{\partial T} dv = -\int_V \nabla \cdot \underline{q}''\, dv + \int_V \dot{q}\, dv.$$

General Heat Conduction equation



→ stationary, homogeneous, isotropic, opaque solid with heat generation
- → chemical
- → Nuclear
- → electrical
- → infrared

Vol of system fixed
No work transfer
Mass const

$\dot{q}$ = uniform volumetric heat generation.
↳ $\dot{q}(\underline{r}, t)$

$$\underbrace{\frac{dE}{dt}}_{\int_V \rho \frac{\partial e}{\partial t} dv} = \dot{Q}$$

$e \rightarrow$ specific energy
$\dot{Q}$ = Net rate of heat input through the surface A + $\dot{q}$

$$= \int_V \rho c_p \frac{\partial T}{\partial t} dv$$

$$\dot{Q} = -\int_A \underline{q}'' \cdot \hat{n}\, dA + \int \dot{q}(\underline{r}, t)\, dv$$

$$\Rightarrow \int_V \rho c_p \frac{\partial T}{\partial t} dv = -\int_A \underline{q}'' \cdot \hat{n}\, dA + \int_V \dot{q}\, dv$$

$$\int_A \underline{F} \cdot \hat{n}\, dA = \int_V \nabla \cdot \underline{F}\, dv$$
↳ vector field

$$\rightarrow \int_V \rho c_p \frac{\partial T}{\partial t} dv = -\int_V \nabla \cdot \underline{q}''\, dv + \int_V \dot{q}\, dv$$

$$\int_V \left(\rho c_p \frac{\partial T}{\partial t} + \nabla \cdot \underline{q}'' - \dot{q} \right) dv = 0$$

$$\Rightarrow \rho c_p \frac{\partial T}{\partial t} + \nabla \cdot \underline{q}'' - \dot{q} = 0$$

use :- $\underline{q}'' = -k \nabla T$

$$\boxed{\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \dot{q}}$$

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$\Rightarrow \rho C_p \frac{\partial T}{\partial t} = k \nabla^2 T + \nabla k \cdot \nabla T + \dot{q}$

if k = const $\rightarrow$ $\nabla k \cdot \nabla T = 0$

$$\rho C_p \frac{\partial T}{\partial t} = k \nabla^2 T + \dot{q}$$

$\rightarrow \rho C_p \frac{\partial T}{\partial t} = k \nabla^2 T + \dot{q}$

$\rightarrow$ if $k = f(T)$ the PDE will be non-linear

$$\boxed{\frac{1}{\alpha}\frac{\partial T}{\partial t} = \nabla^2 T + \frac{\dot{q}}{k}} \rightarrow \text{Fourier Biot equation}$$

α = thermal diffusivity $\frac{k}{\rho C_p}$

↳ High α means high k. → the material has less capacity to store energy (dissipate heat very quickly)

↳ conduct heat easily.

$\Rightarrow$ No heat source :- $\frac{1}{\alpha}\frac{\partial T}{\partial t} = \nabla^2 T$ → heat diffusion equation

↳ homogeneous.

$\Rightarrow$ steady state, heat source : $\nabla^2 T + \frac{\dot{q}}{k} = 0$ → Poisson equation

$\Rightarrow$ steady state, no heat sorce :- $\nabla^2 T = 0$ → laplace equation

↳ (No thermophysical property) (k)

steady state heat conduction is independent of thermophysical property; Depends only on geometry

→ Rectangular :- $T = T(x, y, z, t)$ $\quad \nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$

→ cylindrical :- $T = T(r, \theta, z, t)$ $\quad \nabla^2 T = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$

→ spherical :- $T = T(r, \theta, \phi, t)$ $\quad \nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}$

k

isotropic solid → scalar

Anisotropic solid → 2nd order tensor.

$$\underline{k} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix}$$

$$\underline{\underline{k}} = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}$$

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→ Let us now consider a solid is moving with a velocity $\underline{u}$

$\underline{u} = (u_x, u_y, u_z)$ ↳ Both conduction & convection

$$q_x'' = -k\frac{\partial T}{\partial x} + \rho c_p T u_x$$

$$q_y'' = -k\frac{\partial T}{\partial y} + \rho c_p T u_y$$

$$q_z'' = \underbrace{-k\frac{\partial T}{\partial z}}_{\text{conduction}} + \underbrace{\rho c_p T u_z}_{\text{convection}}$$

$$\Rightarrow \rho c_p \frac{\partial T}{\partial t} + \nabla \cdot \underline{q}'' - \dot{q} = 0$$

$$\boxed{\underline{q}'' = -k\nabla T + \rho c_p T \underline{u}}$$

$$-\nabla \cdot q'' = -\nabla \cdot \left(-k\frac{\partial T}{\partial x} + \rho c_p T u_x\right) \quad \text{for 1D}$$

$$= k\frac{\partial^2 T}{\partial x^2} - \rho c_p u_x \frac{\partial T}{\partial x}.$$

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p u_x \frac{\partial T}{\partial x} + \rho c_p u_y \frac{\partial T}{\partial y} + \rho c_p u_z \frac{\partial T}{\partial z} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \dot{q}$$

$$\rho c_p \left(\frac{DT}{Dt}\right) = k\left(\nabla^2 T\right) + \dot{q}$$

$$\frac{1}{\alpha}\frac{DT}{Dt} = \nabla^2 T + \dot{q}$$

↳ Substantial derivation

Total derivative

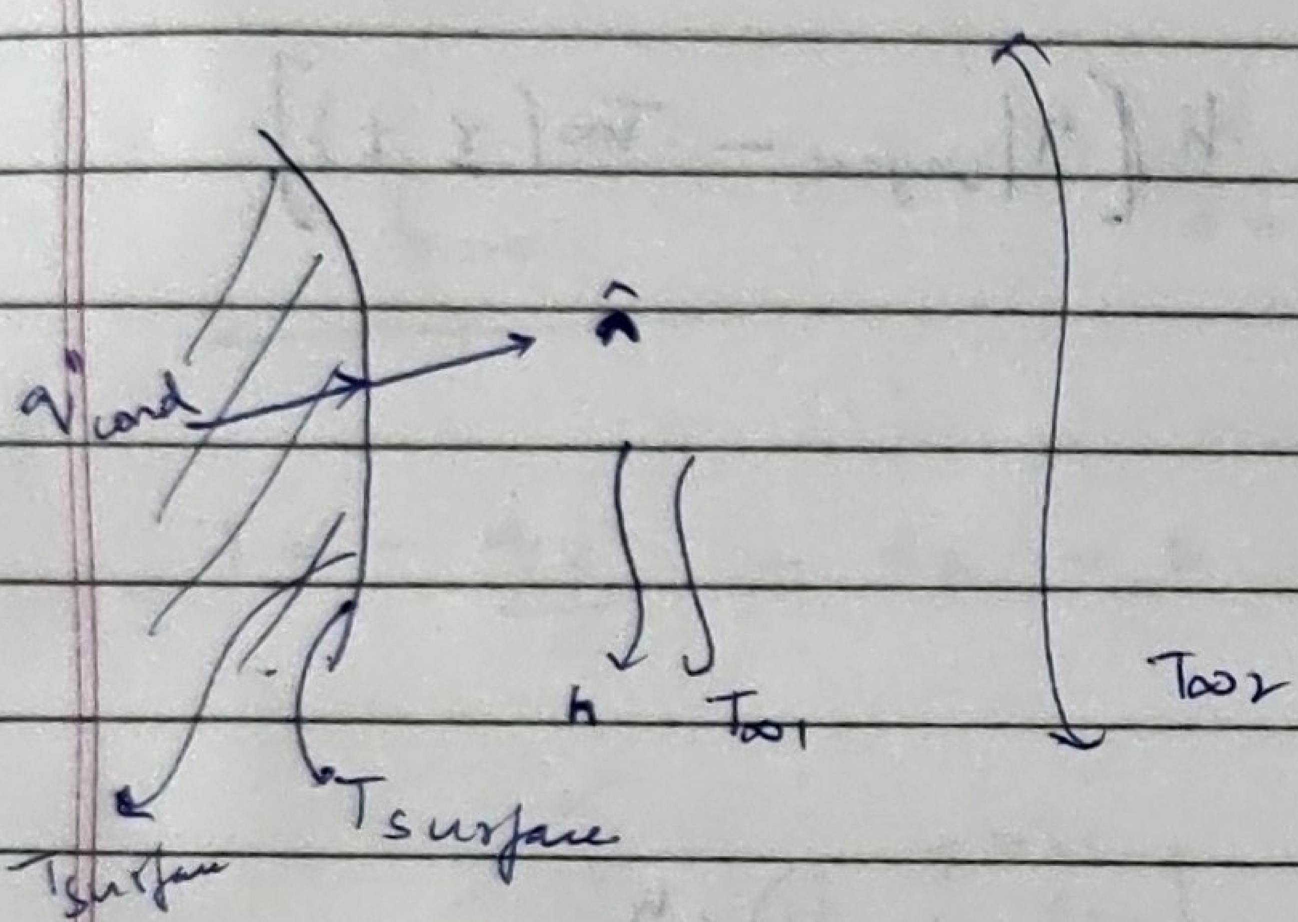
(derivative along the direction of motion)

INITIAL AND BOUNDARY CONDITION

Initial condition :- $T(\underline{r}, t)\Big|_{t\to 0} = T_0(\underline{r})$ $\underline{r}$ → position vector

→ Boundary condition : $-k\dfrac{\partial T}{\partial n}\Big|_{surface} = h\left(T\big|_{surface} - T_{\infty 1}\right) + \underbrace{\varepsilon\sigma\left(T^4\big|_{surface} - T^4_{\infty 2}\right)}_{radiation}$

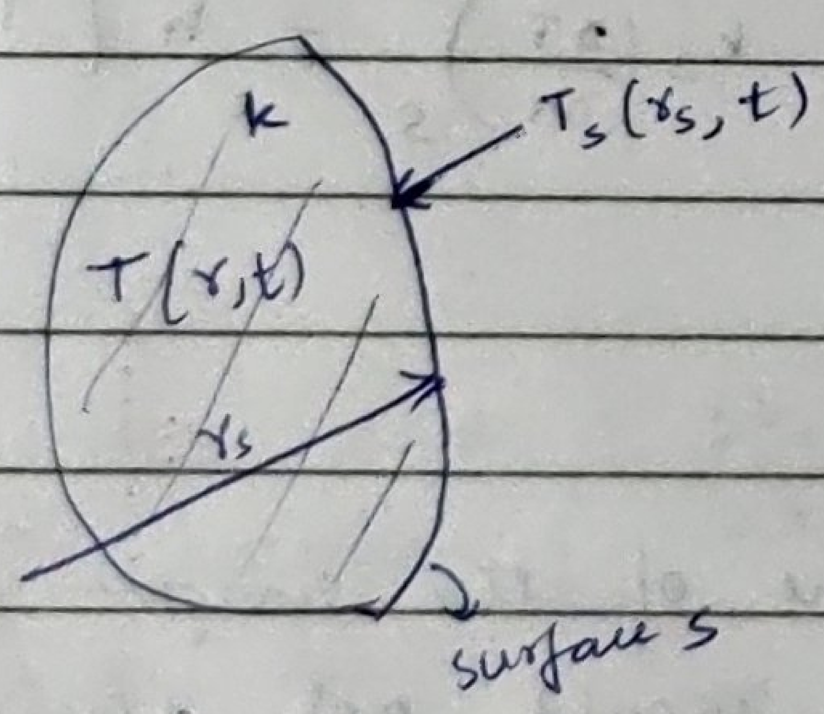
→ Non linear BC.



$$q''_{in} = q''_{out}$$

→ Dirichlet BC (BC of 1st kind)

Prescribed temp.

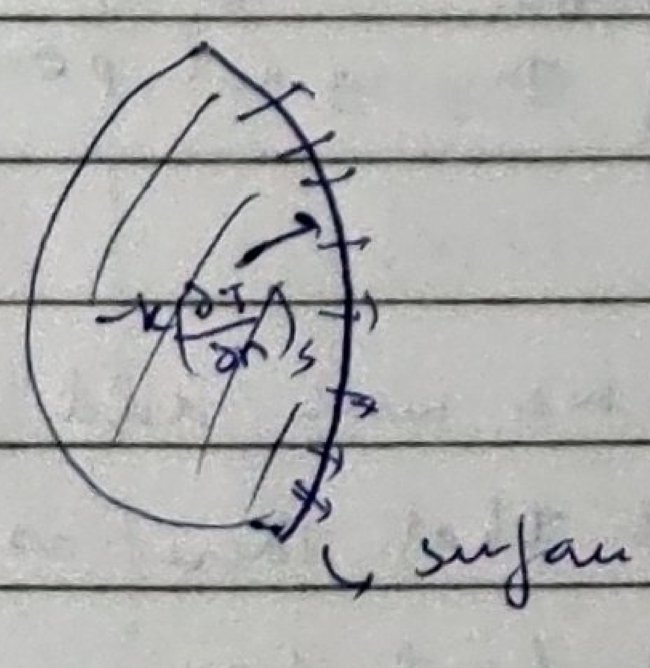


$$T\big|_{surface} = T_0$$

$$T\big|_{surface} = f(\underline{r}, t)$$

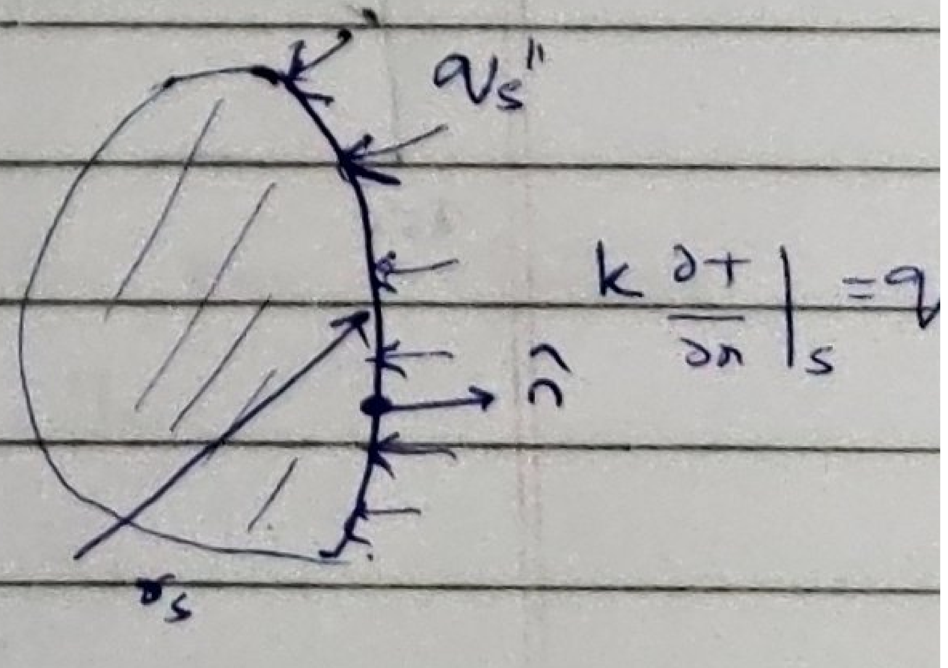
$$T\big|_{surface} = 0 \rightarrow \text{homogeneous}$$

→ Neumann BC (BC of 2nd kind)



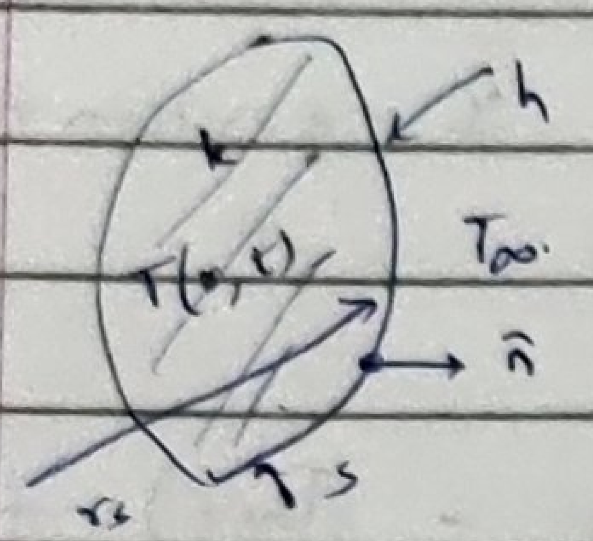
$$-k\frac{\partial T}{\partial n}\Big|_{surface} = f(r_s, t)$$

$$-k\frac{\partial T}{\partial n}\Big|_{surface} = q''_0$$



$\dfrac{\partial T}{\partial n}\Big|_{surface} = 0$ → homogeneous BC → (perfectly insulated or adiabatic)

Robin mixed BC (BC of third kind)



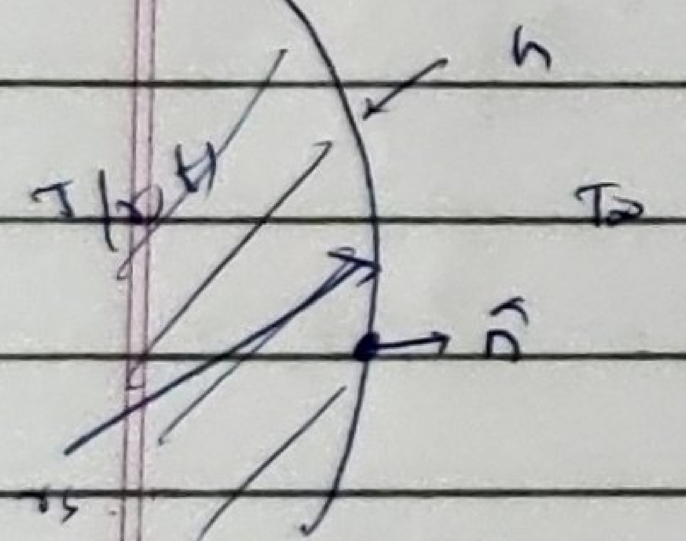
$$-k\frac{\partial T}{\partial n}\Big|_s = h\left(T|_s - T_\infty\right)$$

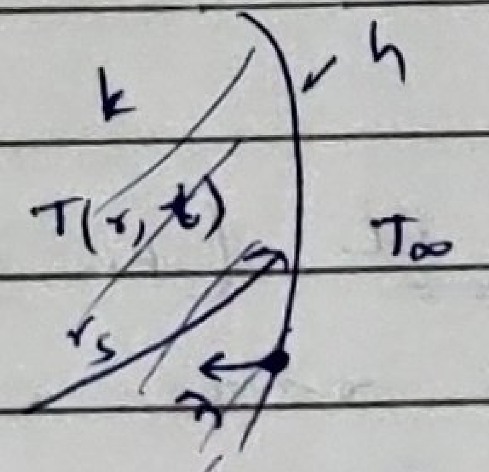
$T_\infty \to$ may be $f(\underline{r}, t)$

$$-k\frac{\partial T}{\partial n}\Big|_s = h\left[T|_{surface} - T_\infty(\underline{r}, t)\right]$$

homogeneous BC of 3rd kind

$$-k\frac{\partial T}{\partial n}\Big|_s = hT|_s.$$

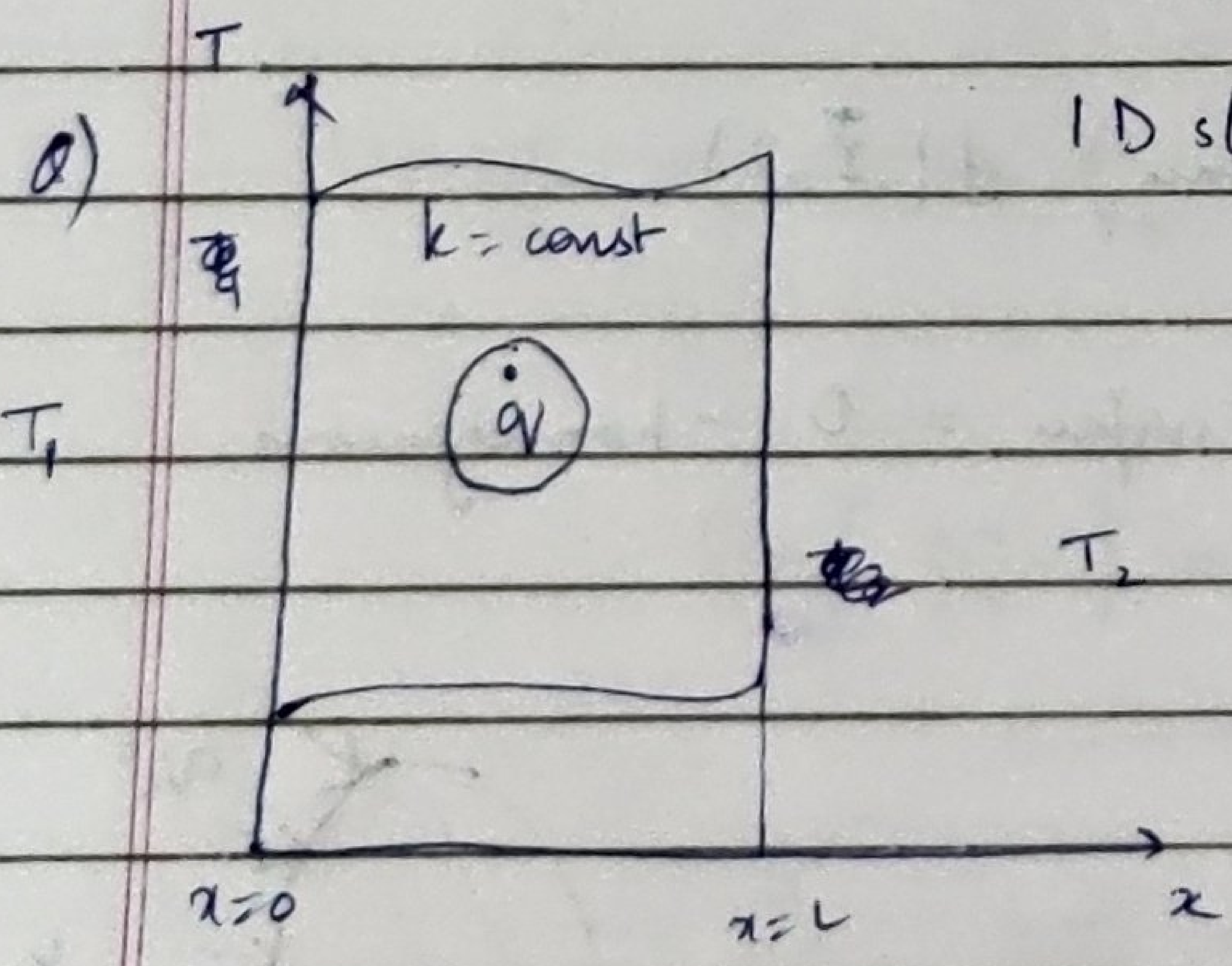




$$-k_s\frac{\partial T}{\partial n}\Big|_s = h\left(T(r_s,t) - T_\infty\right)$$

$$k_s\left(\frac{\partial T}{\partial n}\right)_s = h\left(T(r_s,t) - T_\infty\right)$$

Q) 1D slab.



one of the surface should not dissipate heat

$T_1 > T_2$

at $x=0 \to$ No dissipation

[insulated]

assume

this is not possible

$\Rightarrow \therefore$ we will add heat ($\dot{q}$) uniform

such that there is no elimination of heat loss.

→ $\frac{1}{\alpha}\cancel{\frac{\partial T}{\partial t}}_{0} = \nabla^2 T + \frac{\dot q}{k}$

steady state

<u>1D</u>

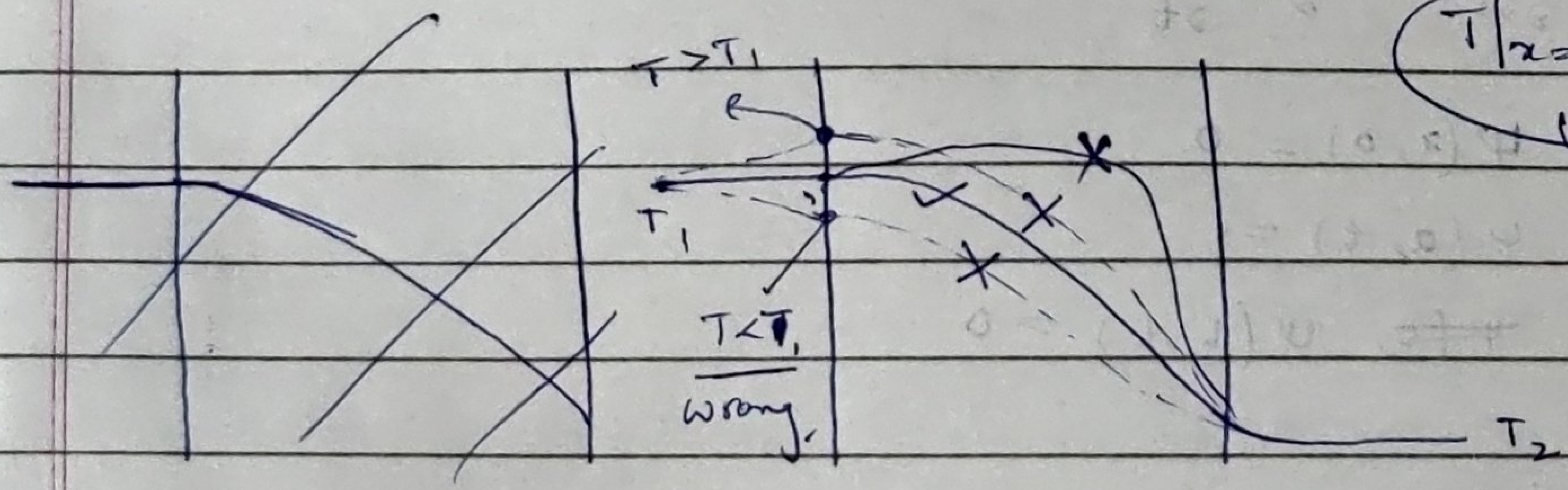
$$\frac{d^2T}{dx^2} + \frac{\dot q}{k} = 0.$$

$$\left.\frac{dT}{dx}\right|_{x=0} = 0, \quad -k\left.\frac{dT}{dx}\right|_{x=L} = h\left[T|_{x=L} - T_2\right]$$

$$T = -\frac{\dot q x^2}{2} + Ax + B \quad \rightarrow T(x) ✓$$

temp at $x=0$ = $T = T_1$ ✓.

$T|_{x=0} = T_1$ ✓ → to get correct soln.



($\dot q$ is also unknown)

Q) show that the solution to the diff equation is unique

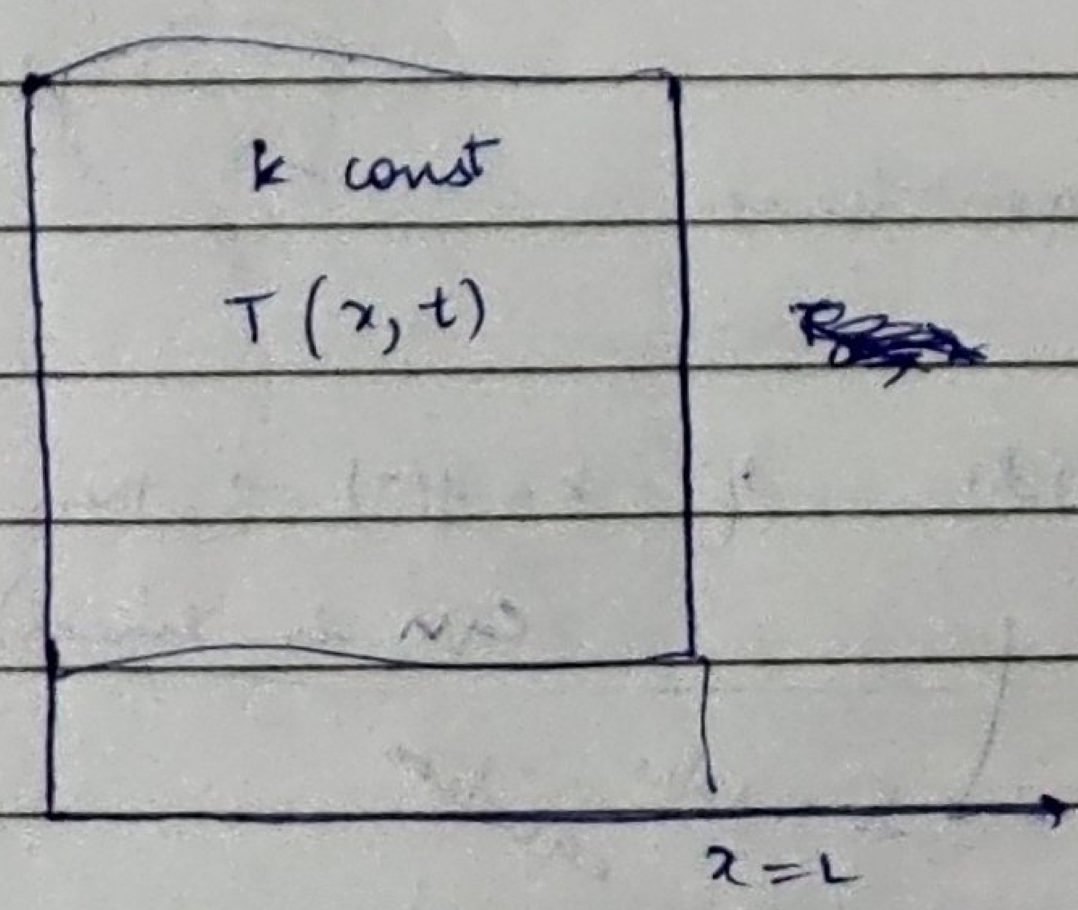
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

A

$T(x,0) = T_i(x)$

$T(0,t) = \phi_1$

$T(L,t) = \phi_2$



Let there be two solutions T_1, T_2

$$\frac{\partial^2 T_1}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T_1}{\partial t}$$

$T_1(x,0) = T_i(x)$

$T_1(0,t) = \phi_1$

$T_1(L,t) = \phi_2$

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Q) $\cancel{\frac{\partial^2 T}{\partial x^2} = 1}$

$$\frac{\partial^2 T_2}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T_2}{\partial t}$$

$T_2(x,0) = T_1(x)$

$T_2(0,t) = \phi_1$

$T_2(L,t) = \phi_2$

linear problem,

$\psi = (T_1 - T_2)$ should also satisfy (A).

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{\alpha}\frac{\partial \psi}{\partial t}$$

$\psi(x,0) = 0$

$\psi(0,t) = 0$

~~$\psi(L$~~, $\psi(L,t) = 0$

$\therefore\ \psi = 0$

$\therefore\ T_1 = T_2$

Unique soln

KIRCHHOFF TRANSFORMATION

$$\rho C_p \frac{\partial T}{\partial t} = \nabla \cdot k \nabla T + \dot{q}$$

If k is $k(T)$ is non-linear

$\Rightarrow$ Mean temperature, $k_m = \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} k(T)\,dT$ If $k = f(T)$ → then k = const (eqn is linear)

to make the equation linear

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Define $\theta(\underline{r}, t) = \frac{1}{k_R} \int_{T_R}^{T(\underline{r},t)} k(T')\,dT'$

T_R = reference temperature

$\underline{k_R = k(T_R)}$

$T(\underline{r}, t) \rightarrow \theta(\underline{r}, t)$

$$\frac{\partial \theta(\underline{r}, t)}{\partial t} = \frac{\partial}{\partial t} \cdot \frac{1}{k_R} \int_{T_R}^{T} k(T')\,dT$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{k_R} \, k(T) \frac{\partial T}{\partial t}$$

$$\nabla \theta = \frac{k(T)}{k_R} \nabla T$$

$$\rho c_p \frac{k_R}{k(T)} \frac{\partial \theta}{\partial t} = \nabla \cdot (k_R \nabla \theta) + \dot{q}$$

$$\frac{k_R \rho c_p}{k(T)} \frac{\partial \theta}{\partial t} = k_R \nabla^2 \theta + \dot{q}$$

1st, 2nd kind BC → will not change after transformation

3rd kind → there is a problem

Convert BC also in terms of $\underline{\theta}$

$$\boxed{\frac{1}{\alpha} \frac{\partial \theta}{\partial t} = \nabla^2 \theta + \frac{\dot{q}}{k_R}}$$

→ Kirchoff transformed equation

→ Non-linear

① $\alpha = \alpha(T)$ → The equation is still non-linear, better form.

② $\alpha = \frac{k(T)}{\rho(T) c_p(T)}$, α might be weak function of temperature ⇒ the equation is approximately linear.

$\uparrow \quad \uparrow$

③ steady state equation is always linear

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→ Cole - Hopf transformation

→ BC of 1st kind

$$T = f_i(\underline{r}, t)$$

$$k = k_o(1+\beta T)$$

$$\theta = \frac{1}{k_R}\int_{T_R}^{T} k(T')\,dT'$$

T_o = reference temp

$$\theta = \frac{1}{k_o}\int_{T_o}^{T} k_o(1+\beta T')\,dT'$$

$$\theta = \left[T' + \frac{\beta T'^2}{2}\right]_{T_o}^{T}$$

Let $T_o = 0$

$$\boxed{\theta = T + \frac{\beta T^2}{2}}$$

$$\boxed{\theta = f_i(\underline{r}, t) + \frac{\beta}{2} f_i^2(\underline{r}, t)}$$ → BC of 1st kind after transformation

→ BC of 2nd kind

$$k(T)\frac{\partial T}{\partial x_i} = \psi_i(\underline{r}, t)$$

$$\theta = \frac{1}{k_o}\int_{0}^{T} k(T)\,dT$$

$$\frac{\partial \theta}{\partial x_i} = \frac{k(T)}{k_o}\frac{\partial T}{\partial x_i}$$

$$\frac{\partial \theta}{\partial x_i} = \frac{1}{k_o}\psi_i(\underline{r}, t)$$ → BC of 2nd kind

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→ 3rd kind → not easy (may not be possible.

Q) $\dot{q} = ?$

$r_0 = 1$ cm → radius

$T_W = 350°C.$

$k = \frac{3167}{T+273}$

~~Assume SS~~

~~$0 = \nabla \cdot k \nabla T + \dot{q}$~~

~~use kirchoff transformation~~

~~$\nabla^2 \theta + \frac{\dot{q}}{k_R} = 0$~~

~~$\theta = \frac{1}{k_R}$~~

$\frac{1}{r}\frac{d}{dr}\left\{ r\, k(T) \frac{dT}{dr} \right\} + \dot{q} = 0$ → (1D)

$\theta = \frac{1}{k_W}\int_{T_W}^{T} k(T')\,dT'$

$\left(\frac{dT}{dr}\right)_{r=0} = 0$

$T(r_0) = T_W$

→ BC.

T_W → reference. $k_W = k(T_W)$

$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right) + \frac{\dot{q}}{k_W} = 0$

$\left.\frac{d\theta}{dr}\right|_{r=0} = 0 \qquad \theta(r_0) = 0$

$\theta = \frac{\dot{q}\, r_0^2}{4 k_W}\left[1 - \left(\frac{r}{r_0}\right)^2\right] = \frac{1}{k_W}\int_{T_W}^{T} k(T')\,dT'$

→ substitute $r = 0$.

$\int_{T_W}^{T_c} k(T')\,dT' = \frac{\dot{q}\, r_0^2}{4}.$ (T_c ← Centerline)

$\int_{T_W}^{T_c} \frac{3167}{T'+273}\,dT' = \frac{\dot{q}\, r_0^2}{4}$

$\boxed{\dot{q} = 1.64 \times 10^8 \ W/m^3}$

surface heat flux

$q_s'' = \frac{\dot{q} V}{A}$

↳ becos of SS.

$\dot{q} V$ = amount of heat generated in the body.

$q_s'' = \frac{\dot{q}\, \pi r_0^2 L}{2\pi r_0 L} = 8.2 \times 10^6 \ W/m^2$

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Heat conduction equation in Dimensionless form

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k}$$ → valid on the solid body → In the region γ

$$-k\frac{\partial T}{\partial n} = h(T - T_\infty)$$ → in Bounding surface

∂n → gradient along the outward drawn normal.

(x, y, z) select $T(t=0) = T_0$ → in Region γ

L = reference length

① $\varphi = \frac{x}{L}$, $\eta = \frac{y}{L}$, $\psi = \frac{z}{L}$

zeta

(φ, η, ψ) coordinate System ⇒ $\frac{\partial}{\partial N}$

② T = ?

~~choose~~ (a) select a reference temp (T_∞)

(b) " " " temp difference ($T_0 - T_\infty$)

Define $\theta = \dfrac{T - T_\infty}{T_0 - T_\infty}$

Dimensionless :- ① to analyze the dimensionless no.

② Will be able to ~~simply~~ write equation in the most simplified form.

choose proper scaling to simplify the eqn and reduce the no of terms

$$\frac{1}{\alpha}\frac{\partial \theta}{\partial t}(T_0 - T_\infty) = \frac{(T_0 - T_\infty)}{L^2}\left(\frac{\partial^2\theta}{\partial \varphi^2} + \frac{\partial^2\theta}{\partial \eta^2} + \frac{\partial^2\theta}{\partial \psi^2}\right) + \frac{\dot{q}}{k}$$

$$\boxed{\frac{\partial \theta}{\partial \tau} = \frac{\partial^2\theta}{\partial \varphi^2} + \frac{\partial^2\theta}{\partial \eta^2} + \frac{\partial^2\theta}{\partial \psi^2} + \frac{\dot{q}L^2}{(T_0 - T_\infty)k}}$$ → valid for all $\tau > 0$.

$\frac{\dot{q}L^2}{(T_0 - T_\infty)k}$ → dimensionless heat source

$\tau = \frac{\alpha t}{L^2}$ → dimensionless time

→ fourier number

$$\tau = \frac{\frac{k}{\rho c_p}t}{L^2}$$

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$$-\frac{k}{L}\cancel{(T_0 - T_\infty)}\frac{\partial \theta}{\partial N} = h\theta\cancel{(T_0 - T_\infty)}$$

$$\boxed{-\frac{k}{L}\frac{\partial \theta}{\partial N} = h\theta} \longrightarrow \text{Bounding surface.}$$

$$\boxed{\theta = 1} \rightarrow \text{initial condition} \longrightarrow \tau = 0$$

$$\cancel{\theta(T_0 - T_\infty) + T_\infty = T_0}.$$

$$\theta =$$

$$\boxed{\frac{\partial \theta}{\partial N} + \frac{hL}{k}\theta = 0.} \longrightarrow \tau > 0.$$

$$\boxed{\theta = 1} \longrightarrow \tau = 0$$

$$\Rightarrow \tau = \frac{kt}{\rho c_p L^2} = \frac{\left(\frac{k}{L}\right)L^2}{\left(\frac{\rho c_p}{t}\right)L^3}$$

$\left(\frac{k}{L}\right)L^2$ → heat conductance in the same reference volume.

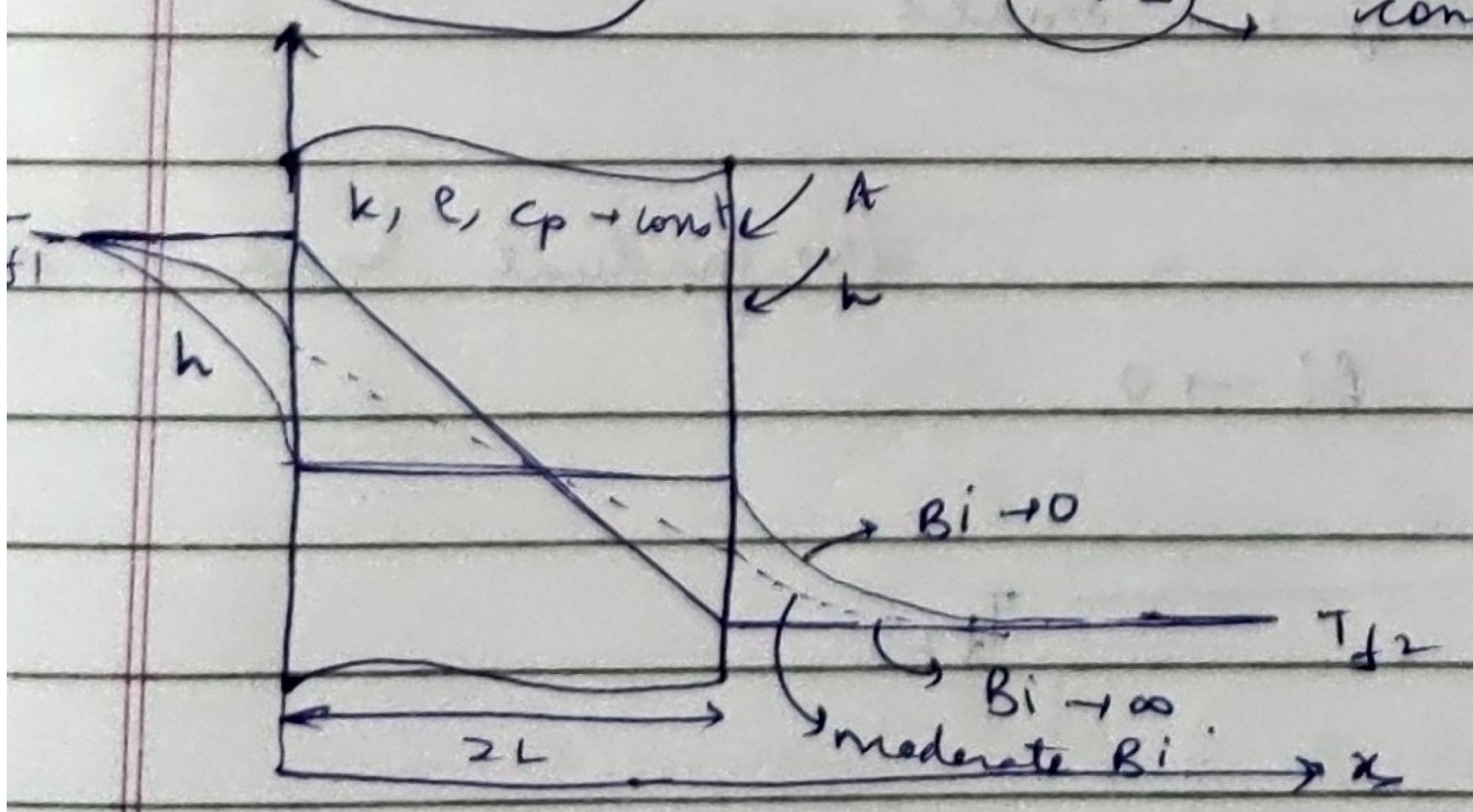
$\left(\frac{\rho c_p}{t}\right)L^3$ → rate of energy storage in the reference volume L^3.

if τ is large — heat conduction >>> rate of energy storage.
→ heat will penetrate much deeper
→ At a given depth, temp changes much faster rate

$$\rightarrow Bi = \frac{hL}{k} = \frac{h}{k/L}$$

h → convective heat transfer coeff at the boundary

k/L → conduction



$T_{f1} > T_{f2}$

1D slab
(steady state)

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At steady state, $q = \dfrac{T_{f1} - T_{f2}}{\Sigma R_t}$

$$q = \frac{T_{f1} - T_{f2}}{\frac{2}{hA} + \frac{2L}{kA}}$$

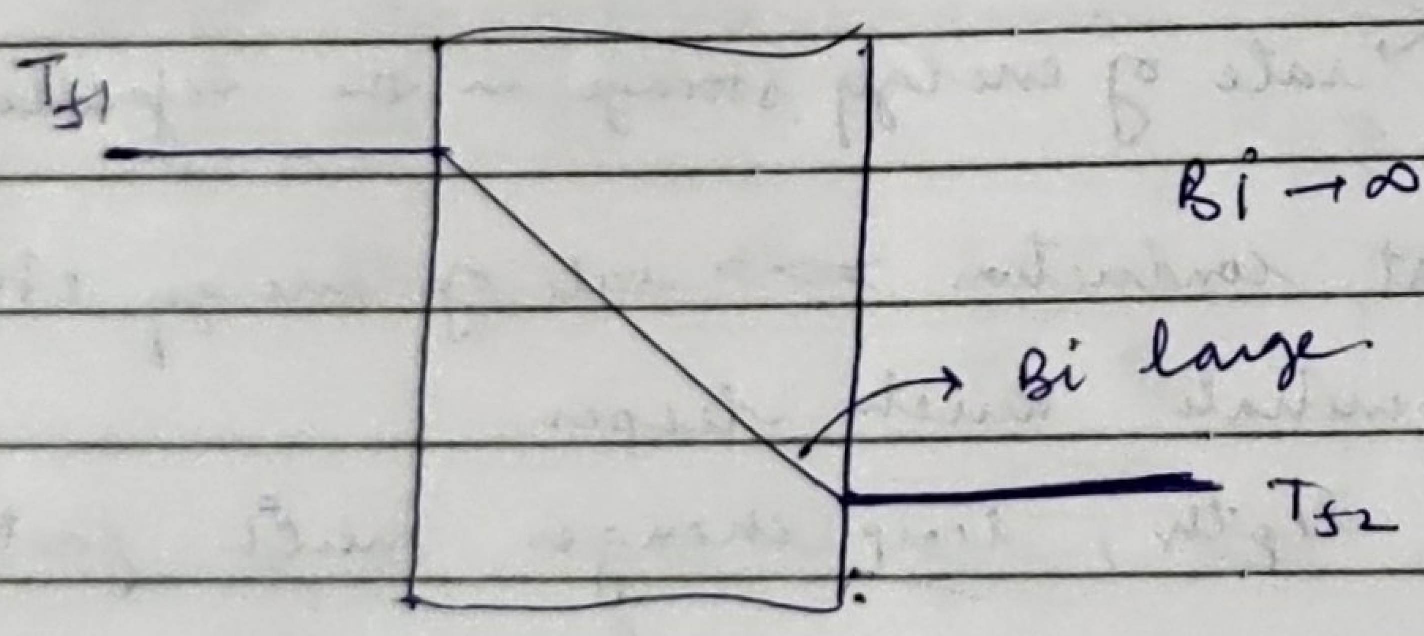
internal resistance

surface resistance

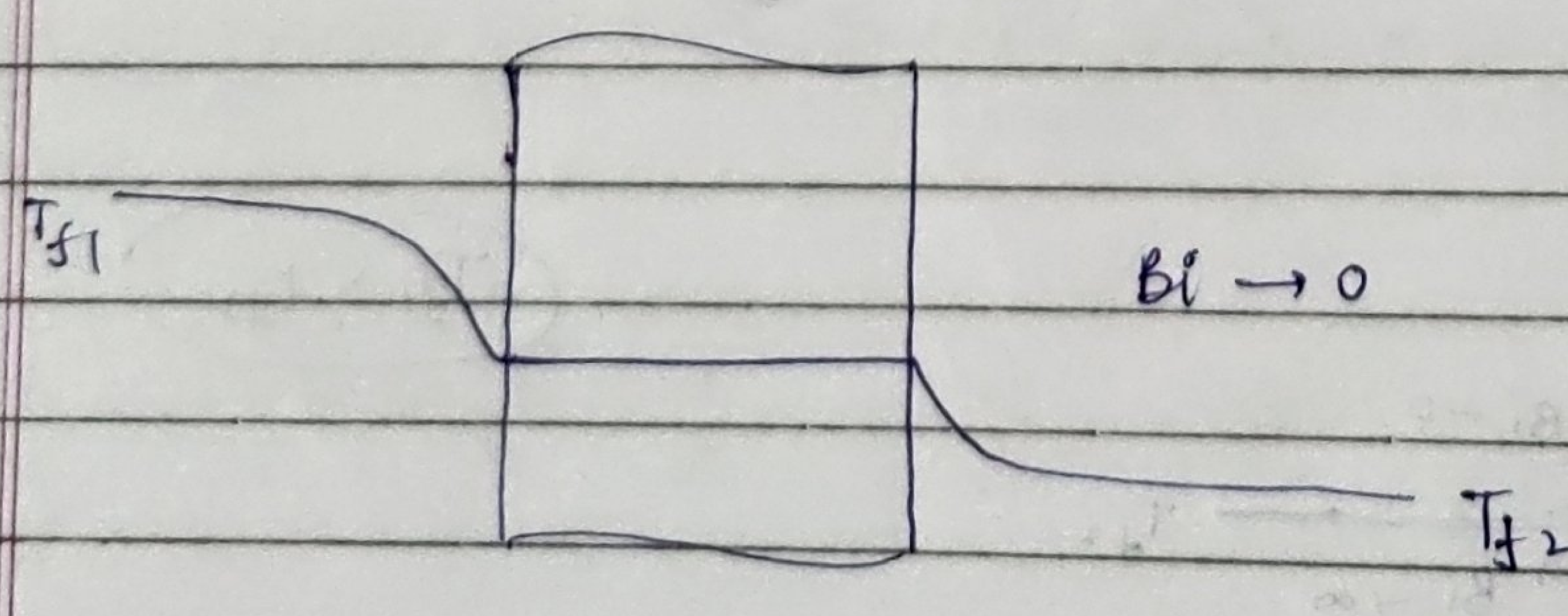
$$Bi = \frac{\text{Internal resistance}}{\text{Surface R}} = \frac{\frac{2L}{kA}}{\frac{2}{hA}} = \frac{hL}{k}$$

of solid

$\Rightarrow$ Bi is large, $\rightarrow$ internal R >> surface R
h large, k small

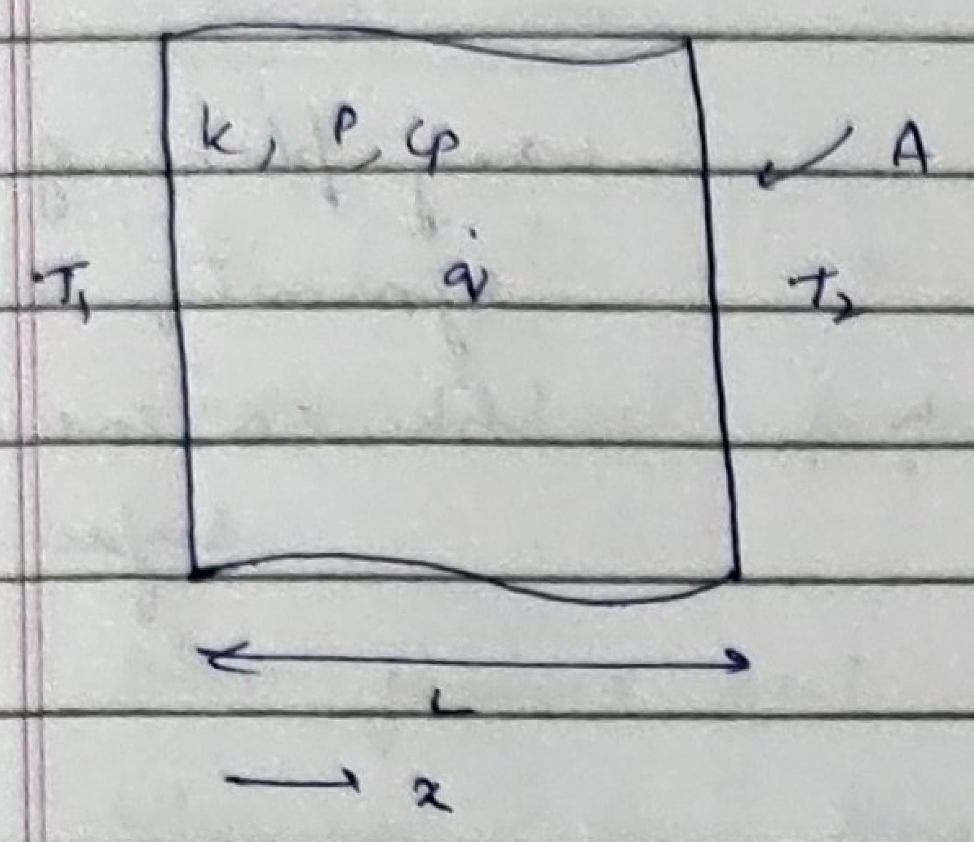


$\Rightarrow$ Bi is small, $\rightarrow$ surface R >> internal R
k large, h small



(No gradient inside the solid

Q)



$$0 = k\frac{d^2T}{dx^2} + \dot{q}$$

$$T\Big|_{x=0} = T_1$$

$$T\Big|_{x=L} = T_2$$

~~θ = 1~~ $$\bar{x} = \frac{x}{L}$$

$$\bar{T} = \frac{T - T_1}{T_2 - T_1}$$

$$0 = k\frac{(T_2 - T_1)}{L^2}\frac{d^2\bar{T}}{d\bar{x}^2} + \dot{q}$$

$$\cancel{d^2T} \quad \frac{d^2\bar{T}}{d\bar{x}^2} + \frac{\dot{q}L^2}{\underbrace{k(T_2 - T_1)}_{S = const}} = 0$$

$$\frac{d^2\bar{T}}{d\bar{x}^2} = -S$$

$$\bar{T}\Big|_{\bar{x}=0} = 0$$

$$\bar{T}\Big|_{\bar{x}=1} = 1$$

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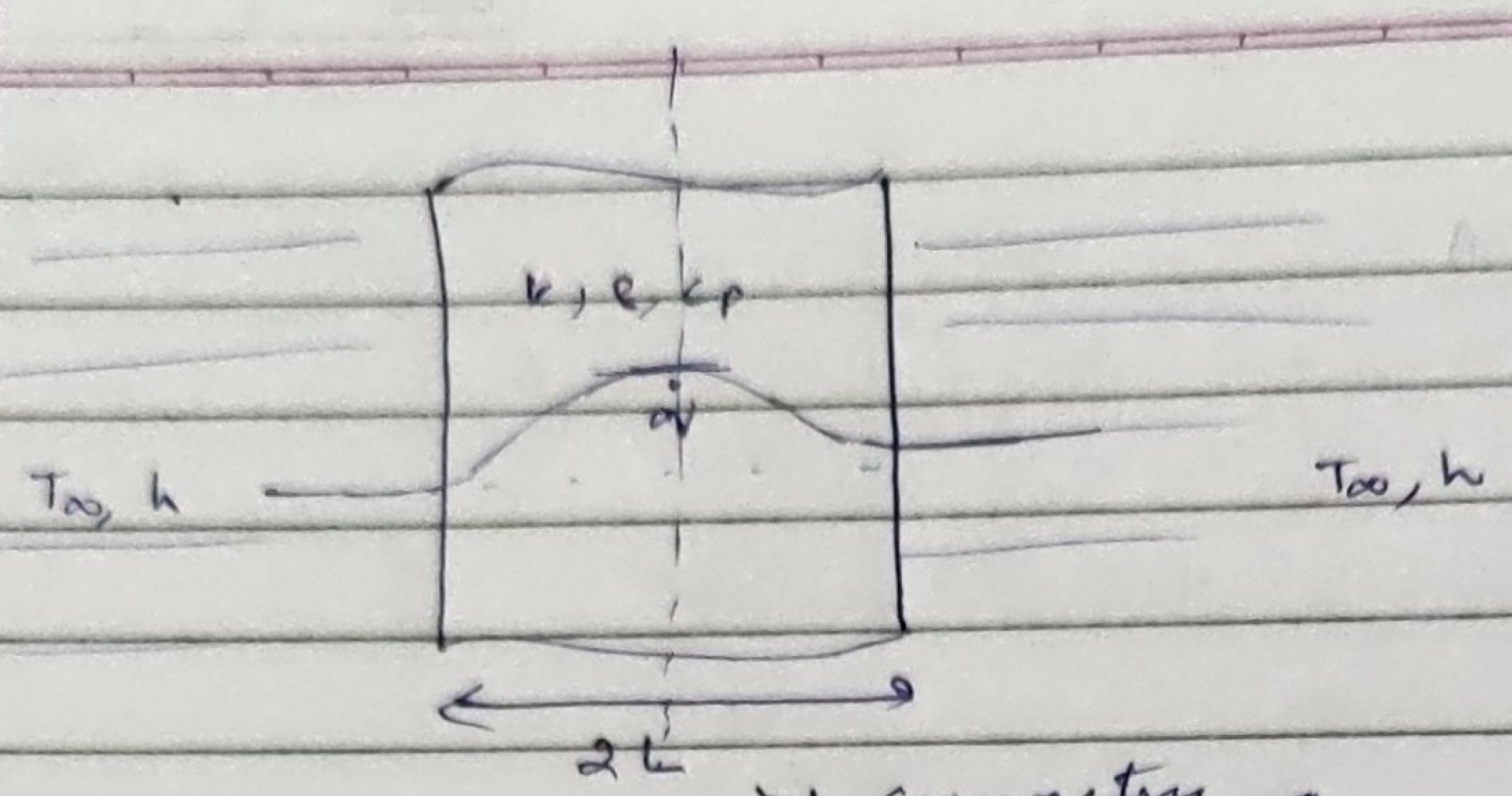
2)



any plane of symmetry is like an insulated surface

↳ symmetry

$\cancel{d^2\bar{T}}$ $\quad \frac{d^2 T}{dx^2} + \frac{\dot{q}}{k} = 0$

$\bar{x} = \frac{x}{L}$

$\bar{T} = \frac{T}{T_\infty}$ and find the equation (wrong) ✗

(reference temp difference is missing)

$\frac{d^2\bar{T}}{d\bar{x}^2} = -\frac{\dot{q}L^2}{k\,T_\infty}$

$\left.\frac{dT}{dx}\right|_{x=0} = 0$

$\frac{\dot{q}L^2}{k}$, temp difference

$-k \left.\frac{dT}{dx}\right|_{x=L} = h\left(T|_{x=L} - T_\infty\right)$

$-k\,\frac{d\bar{T}}{d\bar{x}} \times \frac{\dot{q}L^2}{k \cdot L} = h\,\frac{\dot{q}L^2}{k}\bar{T}$

$\bar{T} = \frac{T - T_\infty}{\frac{\dot{q}L^2}{k}}$

$\left.\frac{d\bar{T}}{d\bar{x}}\right|_{\bar{x}=1} = -Bi\,\bar{T}$

$\left(\frac{d^2\bar{T}}{d\bar{x}^2}\right)\frac{\dot{q}L^2}{kL^2} + \frac{\dot{q}}{k} = 0.$

$\frac{d^2\bar{T}}{d\bar{x}^2} + 1 = 0$

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Fourier law

$$\underline{q}''(\underline{r},t) = -k \, \nabla T(\underline{r},t)$$

Implies heat propogates with infinite speed

Non-fourier heat conduction → considers relaxation time which describes the time lag in the response of the heat flux to a temp gradient.

→ Non fourier heat conduction model considers finite speed of heat propogation

↳ high unsteadiness, high heat flow

Cattaneo & Vernotte eqn → $\underline{q}''(\underline{r}, t+\tau_0) = -k \, \nabla T(\underline{r},t)$

τ_0 → relaxation time for fast transient effect of thermal inertia.

↳ material property

↳ $\tau_0 > 1$

single phase lagging

~~for~~ most solid materials, $\tau_0 \simeq 10^{-10}$ s to 10^{-14} s

gases, $\tau_0 \simeq 10^{8}$ s to 10^{-10} s

tissue, some non-homogeneous material, $\tau_0 \approx 10^{2}$ s

$$\left.\begin{aligned} \rho c_p \frac{\partial T}{\partial t} &= -\nabla \cdot \underline{q}'' + \dot{q} \\ \underline{q}''(\underline{r},t) &= -k \, \nabla T(\underline{r},t) \end{aligned}\right\} \Rightarrow \frac{\rho c_p}{k} \frac{\partial T}{\partial t} = \nabla^2 T + \frac{\dot{q}}{k}$$

→ parabolic PDE

In case of high unsteadiness, high heat flow

$$\underline{q}''(\underline{r}, t+\tau_0) = -k \, \nabla T(\underline{r},t)$$ → Cattaneo ~~~~ vernotte relation

1st order approximation

$$\underline{q}''(\underline{r},t) + \tau_0 \left(\frac{\partial q_0''(\underline{r},t)}{\partial t} \right) = -k \, \nabla T(\underline{r},t)$$

$$\Downarrow$$

$$\nabla^2 T + \frac{\tau_0}{k} \frac{\partial q''}{\partial t} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} + \frac{\tau_0}{\alpha} \frac{\partial^2 T}{\partial t^2}$$ → hyperbolic PDE

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Dual phase lagging

$$\underline{q}''(\underline{r}, t+\tau_q) = -k\,\nabla T(\underline{r}, t+\tau_T)$$

$\tau_T \rightarrow$ delay associated with ∇T

$\rightarrow$ Associated with microstructured interactions (Phonon – electron interactions)

	space	time
Fourier :-	Macroscopic	Macroscopic
Single phase lagging	Macroscopic	Microscopic
Dual phase	Microscopic	Microscopic

Uniqueness of solution of heat conduction eqn

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \nabla^2 T + \frac{\dot{q}}{k} \quad \text{in } R,\ t>0.$$

BC :- $T(\underline{r}, t) = \phi(\underline{r}, t)$ on boundary surface $t>0$

IC :- $T(\underline{r}, 0) = f(\underline{r})$ in R, $t=0$.

If possible, consider two solutions $T_1(\underline{r}, t)$, $T_2(\underline{r}, t)$

linear problem $\Rightarrow \theta = (T_1 - T_2)$ is also a soln.

$$\left.\begin{aligned} &\frac{1}{\alpha}\frac{\partial \theta}{\partial t} = \nabla^2 \theta \\ &\theta(\underline{r}, t) = 0 \\ &\theta(\underline{r}, 0) = 0 \end{aligned}\right\}$$

Define an energy function, $E(t) = \frac{1}{2}\int_V \vartheta^2(\underline{r},t)\, dV \geq 0$

$\searrow E(0) = 0$.

$$\frac{dE(t)}{dt} = \int_V \vartheta \frac{\partial \vartheta}{\partial t} dV = \alpha \int_V \vartheta \nabla^2 \vartheta \, dV$$

use $\int_S \vartheta \nabla \vartheta \cdot \hat{n}\, ds = \int_V \left(\vartheta \nabla^2 \vartheta + |\nabla \vartheta|^2 \right) dV$

$$\frac{dE(t)}{dt} = \alpha \iint_S \vartheta \cdot \nabla \vartheta \cdot \hat{n}\, ds - \int_V |\nabla \vartheta|^2 dV$$

$$\Rightarrow \frac{dE(t)}{dt} \leq 0$$

$\Rightarrow E(t) \geq 0$

$\frac{dE}{dt} \leq 0$

$E(t=0) = 0$

$\Rightarrow E = 0$.

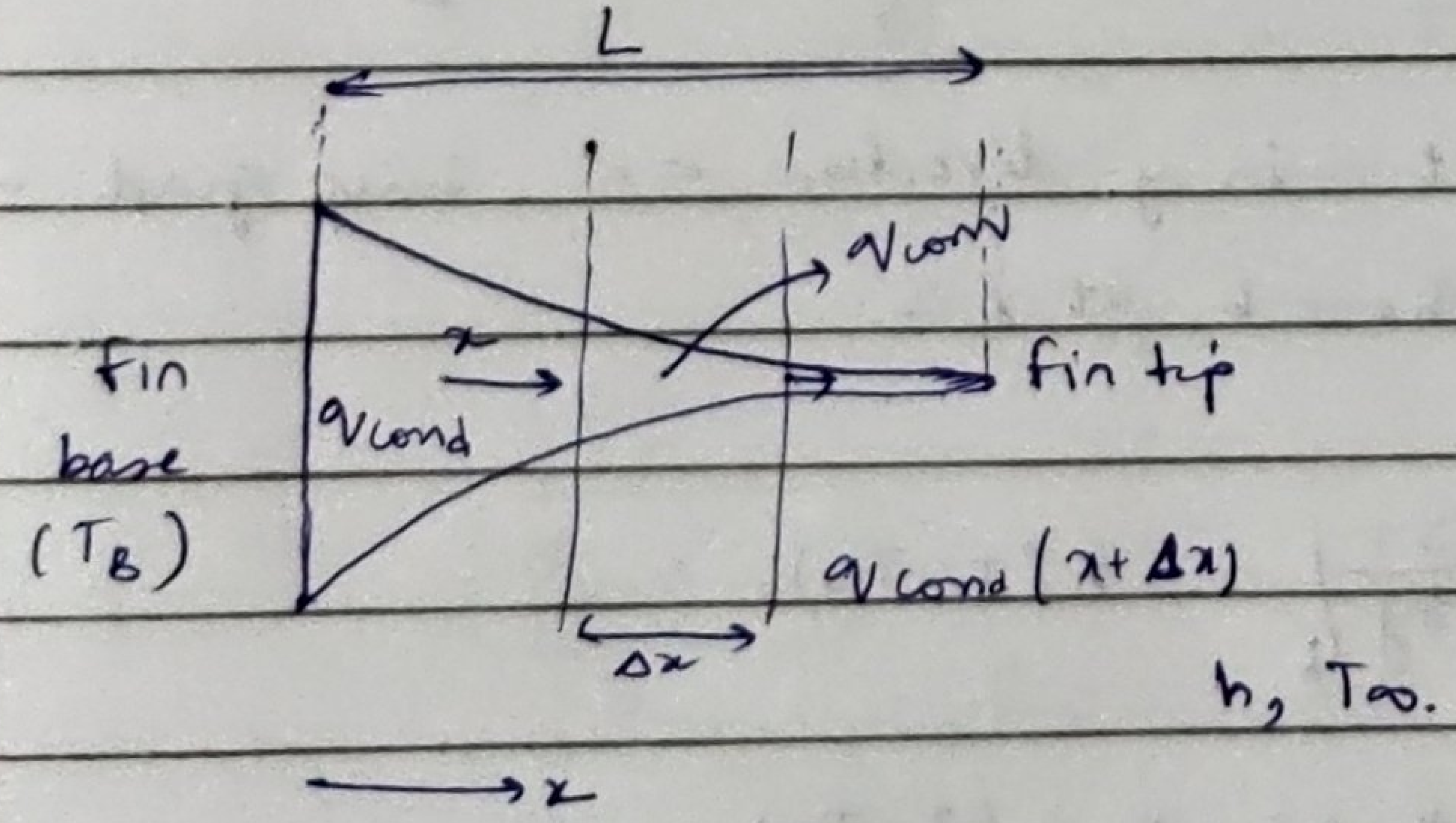
$\vartheta = 0$

$\Rightarrow T_1(\underline{r}, t) = T_2(\underline{r}, t)$

$\Rightarrow$ unique solution

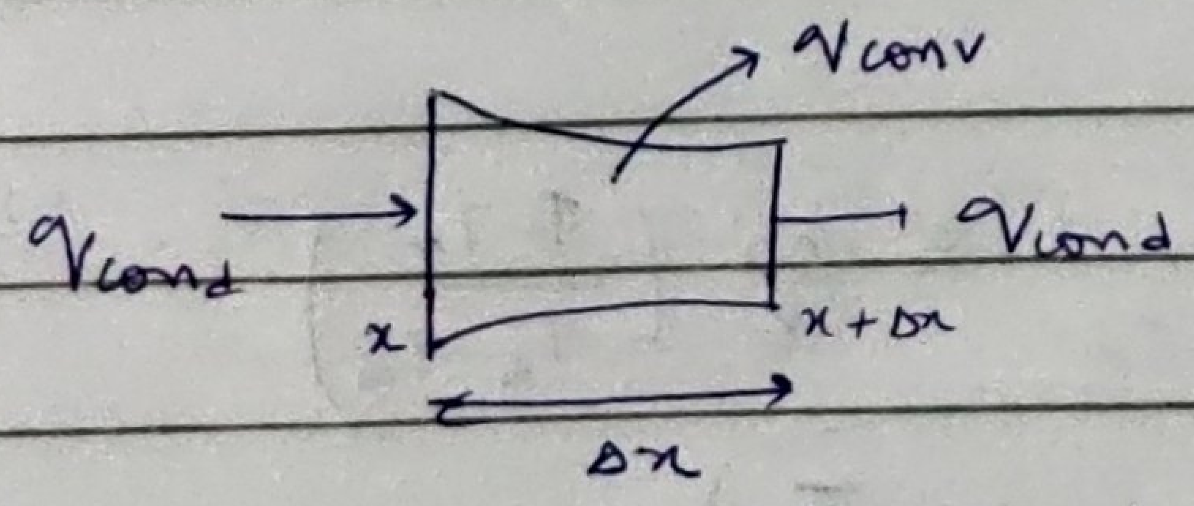
FINS/ EXTENDED SURFACES

→ ↑ heat transfer area, ↑ heat transfer coefficient



- steady state
- 1D
- no heat source
- const k, ρ, C_p

$\bar{x} = \frac{x}{L}$



$\bar{T} = \frac{T - T_\infty}{T_B - T_\infty}$

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$$q_{cond}(x) - q_{cond}(x+\Delta x) - q_{conv} = 0.$$

$$-k\, A_c(x) \frac{dT}{dx}\Big|_x + k\, A_c(s) \frac{dT}{dx}\Big|_{x+\Delta x} - h\, \underbrace{P \Delta x}_{A_s} (T - T_\infty) = 0$$

P ↳ perimeter

$$k \frac{d}{dx}\left(A_c(x) \frac{dT}{dx}\right) - h P (T - T_\infty) = 0$$

fin eq : $$\frac{d}{d\bar{x}}\left[A_c(x) \frac{d\bar{T}}{dx}\right] - \frac{hPL^2}{k}\bar{T} = 0$$

BC :- ① fin base :- $T|_{x=0} = T_B$

$\bar{T}|_{\bar{x}=0} = 1$

② fin Tip :- 3 types.

(i) for long fin :- $T|_{x=L} = T_\infty$

$\bar{T}|_{\bar{x}=1} = \bar{T}_t = 0.$

(ii) Small tip surface ⇒ $\frac{dT}{dx}\Big|_{x=L} = 0$ ⇒ $\frac{d\bar{T}}{d\bar{x}}\Big|_{\bar{x}=1} = 0$

insulated fin tip

(iii) Convective BC at fin tip → $-\frac{d\bar{T}}{d\bar{x}}\Big|_{\bar{x}=1} = Bi_t\, \bar{T}(\bar{x}=1)$

Bi_t ↳ $\frac{h_t L}{k}$ (at fin tip)

⇒ 1D → temp gradient in y-direction << temp grad in x

let the thickness be b at x :-

$$-k \frac{\partial T}{\partial y}\Big|_b = h(T - T_\infty)$$

approx $$\sim -k \frac{\Delta T}{b} \simeq h(T - T_\infty)$$

$$\frac{\Delta T}{T_B - T_\infty} \simeq \frac{hb}{k}\left(\frac{T - T_\infty}{T_B - T_\infty}\right)$$

$$\overline{\Delta T} = \frac{hb}{k}\bar{T}$$

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→ for $\overline{\Delta T}$ to be small, $\frac{hb}{k} << 1$

Q) A_c

$k = 400$ W/mk

$b = 1$ cm

$h = 10$ W/m²k

$Bi, t = 0.00025$ → very small

1D assumption is valid

$$\rightarrow \frac{d}{dx}\left[A_c(x)\frac{d\theta}{dx}\right] = \frac{hP}{k}\theta$$

$\underline{\underline{A_c = \text{const}}}$

$$\frac{d^2\theta}{dx^2} - \frac{hP\theta}{kA_c} = 0 \qquad m^2 = \frac{hP}{kA_c}$$

$$\theta = C_1e^{mx} + C_2e^{-mx}$$

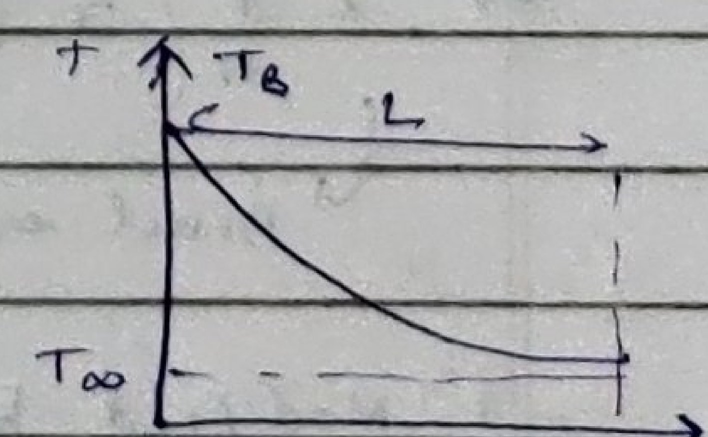
$$\theta(x) = C_3\sinh mx + C_4\cosh mx$$

→ Case(i) :- BC → $\theta|_{x=0} = \theta_b$ $\quad (\theta_b = T_B - T_\infty)$

$$\theta|_{x=L} = T|_{x=L} - T_\infty.$$

$$\theta|_{x=L} = 0 \rightarrow \text{for long fin } (T|_{x=L} = T_\infty)$$

$$\boxed{\theta_b = C_4}$$



$$\theta(x=L) = 0 = C_3\sinh mL + \theta_b\cosh mL$$

$$\boxed{\theta(x) = \theta_b e^{-mx}} \qquad \frac{T - T_\infty}{T_B - T_\infty} = e^{-mx} = e^{-\sqrt{\frac{hP}{A_ck}}x}$$

Heat removed by fin (at fin base) $\quad Q|_{\text{long fin}} = -kA_c\frac{dT}{dx}\Big|_{x=0} = \sqrt{hPkA_c}\,(T - T_\infty)$

$$\Rightarrow Q_{LF} = \int_0^L hP(T - T_\infty)\,dx$$

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Case ② : Insulated BC

$$\left.\frac{\partial T}{\partial x}\right|_{x=L} = 0 \quad \text{or} \quad \left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh(m(L-x))}{\cosh mL}$$

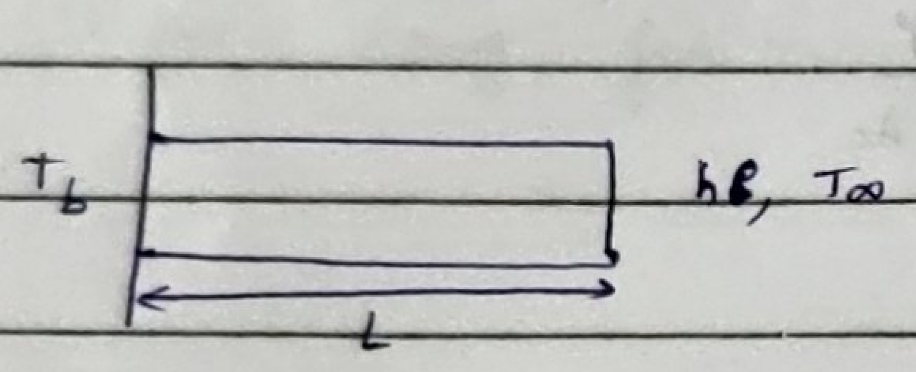
rate of heat transfer $= Q_{\text{Insulated fin}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0}$

$$Q_{IF} = \sqrt{hPkA_c}\,(T - T_\infty)\,(\tanh mL)$$

for $m = 1 \longrightarrow \tanh(L) \approx 1 \rightarrow L > 3$

long fin for practical purpose

Case-3 Convection from fin tip



$$\left.-k\frac{dT}{dx}\right|_{x=L} = h_e\left[T|_L - T_\infty\right]$$

$$\frac{\theta(x)}{\theta(b)} = \frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh[m(L-x)] + N\sinh[m(L-x)]}{\cosh mL + \sinh mL}$$

$$N = \frac{h_e}{mk}$$

rate of heat transfer from extended surface, $Q = \sqrt{hPkA_c}\,\theta_b\left(\frac{\sinh mL + \cosh mL}{\cosh mL + N\sinh mL}\right.$

$\Rightarrow$ $x\frac{d}{dx}\left(x\frac{dy}{dx}\right) + (m^2x^2 - \nu^2)y = 0$ $\quad$ $m =$ parameter

Bessel eqn. $\quad$ order ν $\quad$ 1st kind $\quad$ 2nd kind $\quad$ $\nu =$ real const (0, integer, fraction)

$$y = C_1 J_\nu(mx) + C_2 Y_\nu(mx)$$

$\Rightarrow$ Modified bessel eqn: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - (m^2x^2 + \nu^2)y = 0.$

$$y = C_1 I_\nu(mx) + C_2 K_\nu(mx)$$

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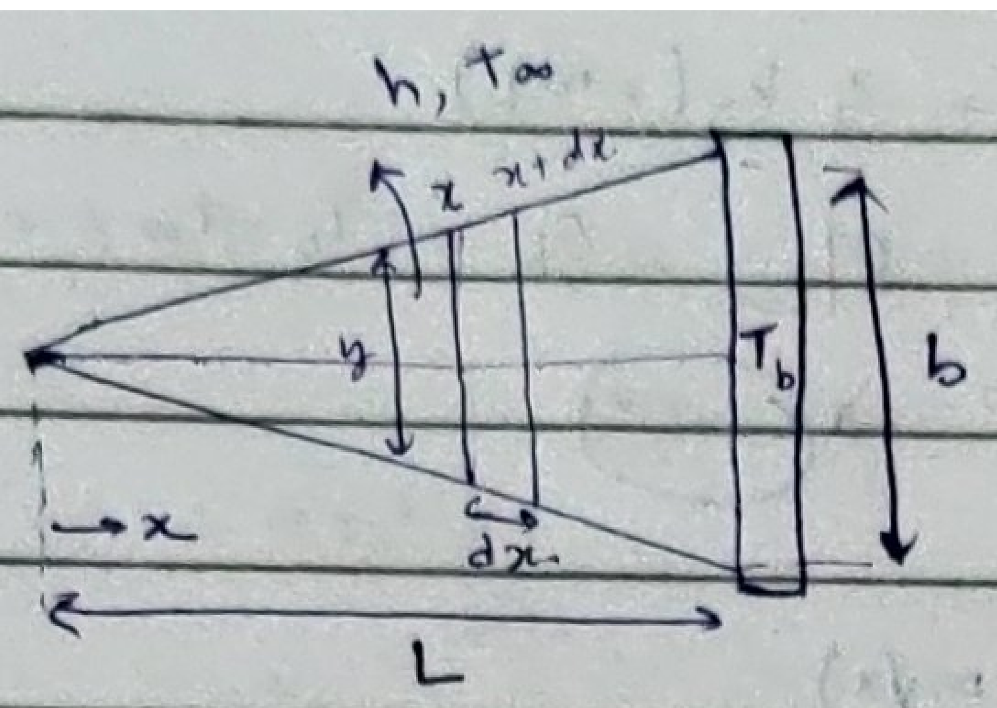
Q)



$$\frac{x}{y} = \frac{L}{b}$$

$W \times y$

consider const thermophysical properties, steady state
(k, ρ, c_p)

width of the beam = W

$b << W$

$$\frac{d}{dx}\left(A_c(x)\frac{dT}{dx}\right) - \frac{h\,P(x)}{k}(T-T_\infty) = 0.$$

$$A_c(x) = W(y) = W\frac{xb}{L}$$

$$P(x) = (y+W)2$$

$$\boxed{P(x) = \left(\frac{xb}{L} + W\right)2} \approx 2W.$$

$$\frac{d}{dx}\left(W\frac{xb}{L}\frac{dT}{dx}\right) - \frac{h(2W)}{k}(T-T_\infty) = 0.$$

$$\frac{d}{dx}\left(x\frac{dT}{dx}\right) - \frac{2hL}{bk}(T-T_\infty) = 0.$$

$$x\frac{d^2T}{dx^2} + \frac{dT}{dx} - \frac{2hL}{bk}(T-T_\infty) = 0.$$

$\theta = T - T_\infty$

$$x\frac{d^2\theta}{dx^2} + \frac{d\theta}{dx} - \frac{2hL}{bk}\theta = 0.$$

$$m^2 = \frac{2hL}{kb}.$$

$$x^2\frac{d^2\theta}{dx^2} + x\frac{d\theta}{dx} - m^2 x\,\theta = 0.$$

$$d\eta = \frac{dx}{2\sqrt{x}}$$

define $\eta \to \sqrt{x}$

$\eta = \sqrt{x}$

$$\frac{d\eta}{dx} = \frac{1}{2}\frac{1}{\sqrt{x}}$$

$$\frac{d}{dx}\left(\frac{d\theta}{dx}\right)$$

$$\frac{d\theta}{dx} = \frac{d\theta}{d\eta}\times\frac{d\eta}{dx}.$$

$$\eta^2\frac{d^2\theta}{d\eta^2} + \eta\frac{d\theta}{d\eta} - 4m^2\eta^2\theta = 0$$

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$$\theta = c_1 I_0(2m\eta) + c_2 k_0(2m\eta)$$

$\theta\big|_{x=0}$ = finite , $\theta\big|_{x=L} = \theta_b = T_b - T_\infty$

$K_0(0) \to \infty \to c_2 = 0$

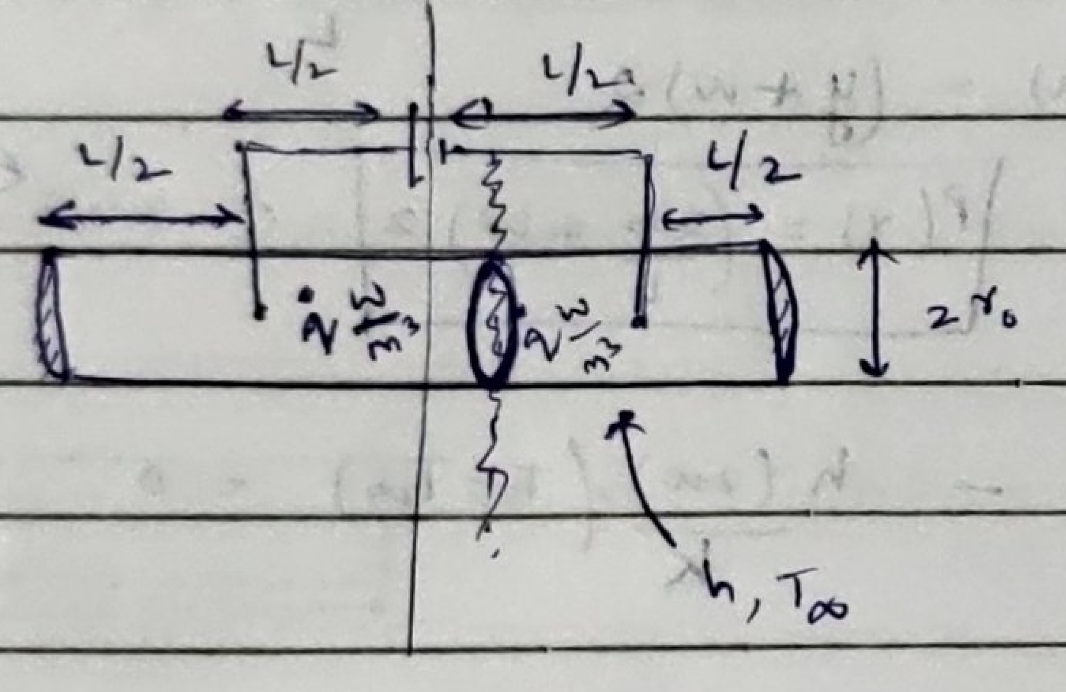
$$\frac{\theta(x)}{\theta_b} = \frac{I_0(2m\sqrt{x})}{I_0(2m\sqrt{L})}$$

rate of heat transfer from fin, $Q = kA\frac{d\theta}{dx}\Big|_{x=L}$

$\boxed{\text{no -ve}}$

$$Q = w\sqrt{2hkb}\,\theta_b \frac{I_1(2m\sqrt{L})}{I_0(2m\sqrt{L})}$$

Q)



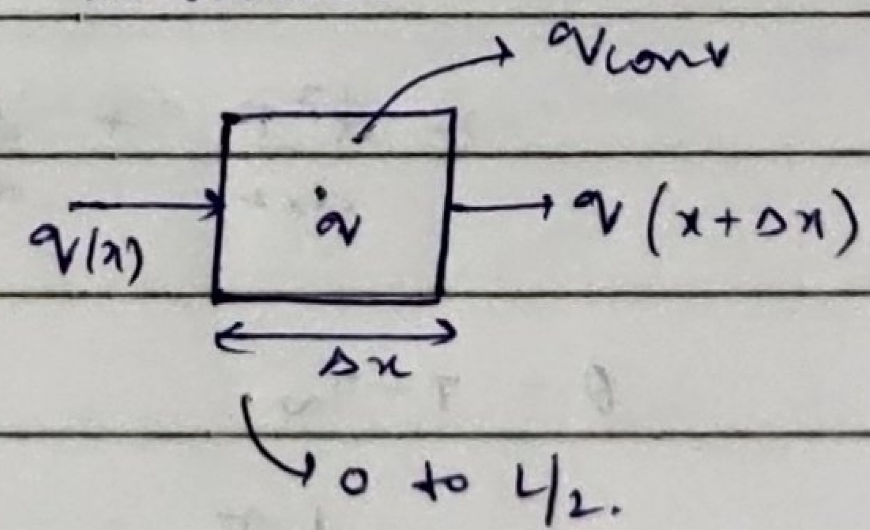
symmetry

$\overset{r}{\underset{}{\text{I}}}\!\to z$

write the SS energy balance equation.

2 ends of the rod are insulated

$\Rightarrow$ ~~[illegible]~~



$$q''(\pi r_0^2)\Big|_z - q''(\pi r_0^2)\Big|_{z+dz} - h(2\pi r_0)(T-T_\infty) + \dot{q}(\pi r_0^2)\,dz = 0$$

$$\left[q(z) - \left[q(z) + \frac{dq}{dz}\Delta z\right]\right]\pi r_0^2 - h(2\pi r_0)\Delta z(T-T_\infty) + \dot{q}(\pi r_0^2)\Delta z$$

$$q(x) + \pi r_0^2\,\Delta x\,\dot{q} = q(x+\Delta x) + q_{conv}.$$

$$= q(x) + \frac{dq}{dx}\Delta x + 2h\pi r_0\Delta x\,(T(x) - T_\infty)$$

$\Delta x \to 0$

$$\frac{dq}{dx} + 2\pi r_0 h (T - T_\infty) = \pi r_0^2 \dot{q}$$

$$q = -k\pi r_0^2 \frac{dT}{dx}$$

$$-k\pi r_0^2 \frac{d^2T}{dx^2} + 2\pi r_0 h (T - T_\infty) - \pi r_0^2 \dot{q} = 0$$

$$k\cancel{\pi} r_0^2 \frac{d^2T}{dx^2} - 2\cancel{\pi}\cancel{r_0} h (T - T_\infty) + \cancel{\pi} r_0^2 \dot{q} = 0$$

$$\boxed{k r_0 \frac{d^2T}{dx^2} - 2h(T - T_\infty) + r_0 \dot{q} = 0}.$$

$$k r_0 \frac{d^2\theta}{dx^2} - 2h\theta + r_0 \dot{q} = 0.$$

$$\boxed{\frac{d^2\theta}{dx^2} - \underbrace{\frac{2h}{k r_0}}_{m^2} \theta + \frac{\dot{q}}{k} = 0} \longrightarrow 0 \le x \le L/2$$

θ_1 $\qquad \dot{q} \neq 0$.

$D^2 - m^2 = 0$

$D = \pm m$

$$\cancel{D^2 - m^2}. \boxed{\frac{d^2\theta}{dx^2} - m^2\theta = 0} \to \frac{L}{2} \le x \le L$$

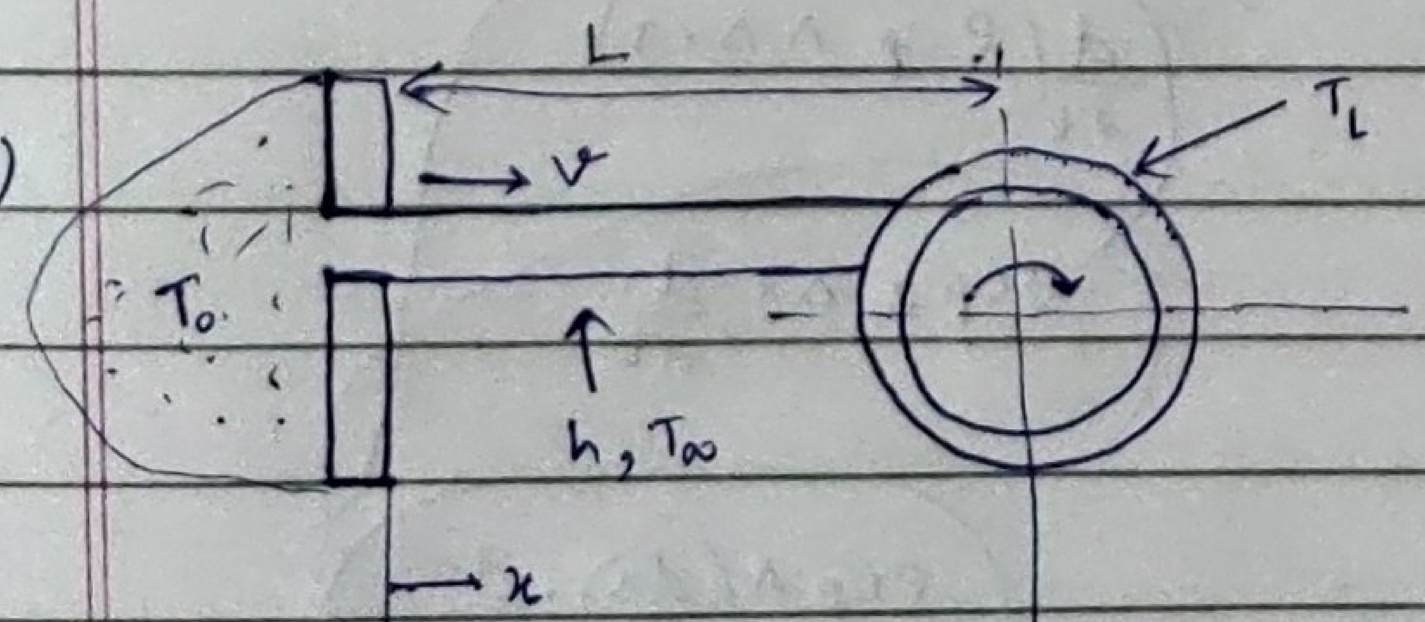
θ_2. $\qquad \dot{q} = 0$

$x = 0$, ~~...~~, $\frac{d\theta_1}{dx} = 0$ – symmetry

$x = L$, $\frac{d\theta_2}{dx} = 0$

$x = L/2$, $\theta_1\big|_{x=L/2} = \theta_2\big|_{x=L/2}$, $\frac{d\theta_1}{dx} = \frac{d\theta_2}{dx}$.

Q)



thin wires of cross sectional area A, perimeter P.

extruded with velocity v.

(Consider it as a moving fin)

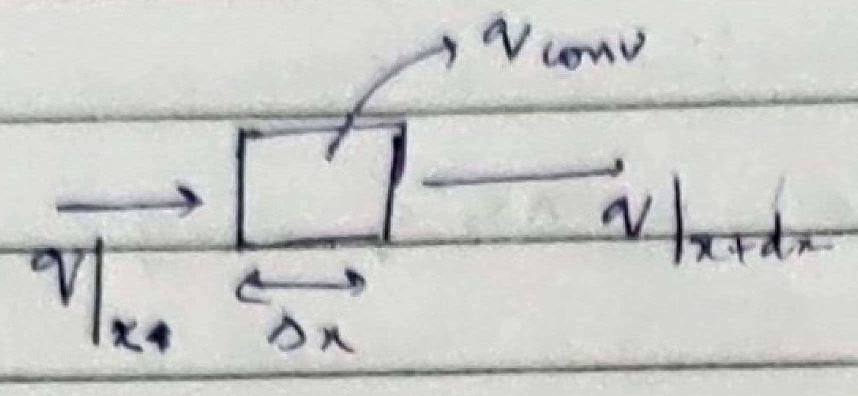
Establish the energy balance eqn?

for moving wire.

extrusion of metal

h = const

k, ρ, c_p = const



$$q|_x A - q|_{x+dx} A - q_{conv} P \Delta x = 0 \quad (SS)$$

$$q|_x = -k\frac{dT}{dx} + \rho v c_p T$$

$$-\frac{dq}{dx}A - q_{conv} P = 0.$$

$$-\frac{d}{dx}\left(-k\frac{dT}{dx} + \rho v c_p T\right) - hP(T-T_\infty) = 0.$$

$$k\frac{d^2T}{dx^2} - \left(\frac{dT}{dx}\right)(\rho v c_p) - hP(T-T_\infty) = 0.$$

$$\boxed{k\frac{d^2T}{dx^2} - hP(T-T_\infty) = \frac{dT}{dx}(\rho v c_p)}$$

$x=0, \quad T=T_0$

$x=L, \quad T=T_L$

$$q(x) = q(x+\Delta x) + hP\Delta x(T-T_\infty) + \rho c_p A \Delta x\, v \frac{dT}{dx}$$

$$q(x) + \frac{dq}{dx}\Delta x$$

$$\frac{d}{dt}(\rho c_p A \Delta x T)$$

$$\rho c_p A \Delta x \frac{dT}{dt}$$

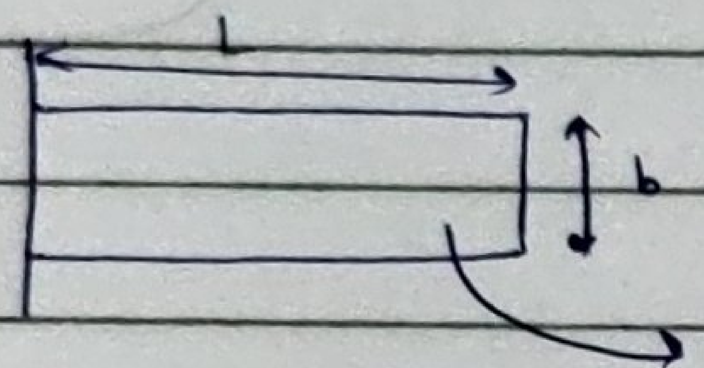
$$\rho c_p A \left(\frac{dx}{dt}\right)\cdot\frac{dT}{dx}$$

v

d) <u>Fin optimization</u>

For a given shape, fin material, convection condition, there exists an optimized design that transfers max amount of heat per unit mass of the fin.

$W >> L$



A_P = profile area = bL.

width = w

mass of the fin $\propto (A_P)\,W$

what L, b maximizes $\left(\frac{q}{W}\right)$? for a given area

$q' = \frac{q}{W}$

→ <u>Adiabatic case</u> :-

rate of heat transfer from fin,

$$q = \sqrt{hPkA_c}\,(T_b - T_\infty)\tanh mL \qquad m^2 = \frac{hP}{kA_c}$$

$$q = \sqrt{hPkA_c}\,(T_b - T_\infty)\tanh(N) \qquad \text{define, } N^2 = \frac{hPL^2}{kA_c}$$

$A_c = bW$

$P = 2(b+W) \simeq 2W$

$b << W$

$$q = \sqrt{h(2W)k\,bW}\,(T_b - T_\infty)\tanh(N)$$

$$\boxed{\frac{q}{W} = \sqrt{2hkb}\,(T_b - T_\infty)\tanh(N)}$$

$$N^2 = \frac{h\,2WL^2}{k(bW)}$$

$$N^2 = \frac{2hL^2}{bk}$$

$$\boxed{A_p = bL}$$

$$\frac{q}{W} = \sqrt{2hkb}\,(T_b - T_\infty)\tanh\left(\frac{2hA_p^2}{b^3k}\right)$$

$$N^2 = \frac{2h\,A_p^2}{b^3k}$$

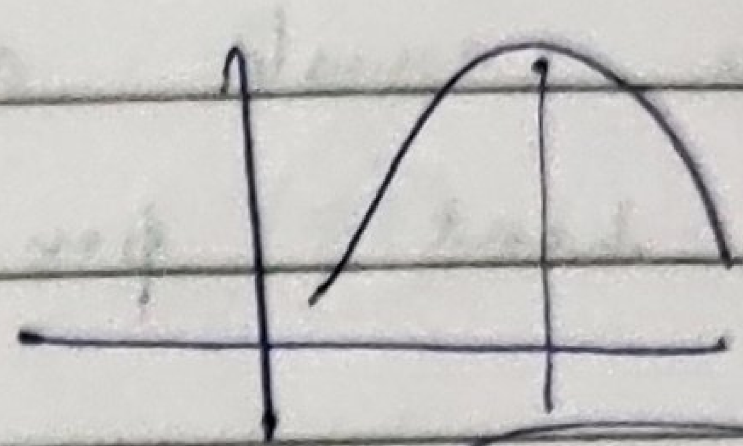
$$b = \left(\frac{2hA_p^2}{N^2k}\right)^{1/3}$$

$\cancel{\frac{q}{W} = \sqrt{2hk}}$

$$q' = (4 h^2 k A_p)^{1/3} (T_b - T_\infty) \left(N^{-1/3} \tanh(N)\right)$$

$$q' \longrightarrow N^{-1/3} \tanh(N) = f(N)$$

$$\frac{df}{dN} = 0 \quad \text{(or) plot}$$



N = 1.419

$N_{opt} = 1.419$