stream function
potential flow v/s laminar
potential function

Potential flow
Frictionless irrotational flows
__/__/____

FLUID MECHANICS

Continuum :- fluid is a continuous medium
↳ Invalid if smallest dimension is comparable/less than the mean free path

→ Pathline :- Path of trajectory traced out by a moving fluid particles.

→ Streamline :- lines drawn to the flow field so that at a given instant they are tangent to the direction of flow at every point in the flow field

→ streakline :- The line joining all the fluid particles which are passed through a fixed point at same line

→ Viscosity :- measure of a fluid's resistance to flow.

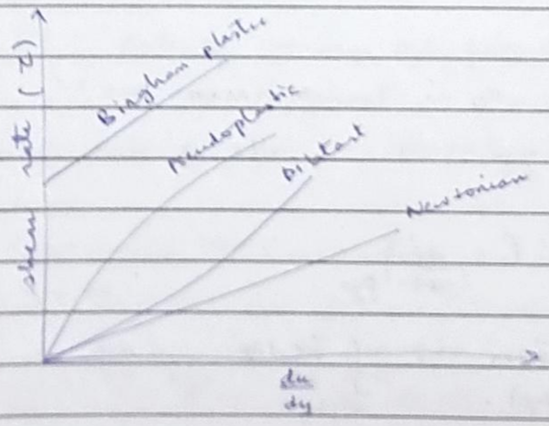
→ shear stress arises due to viscous flow

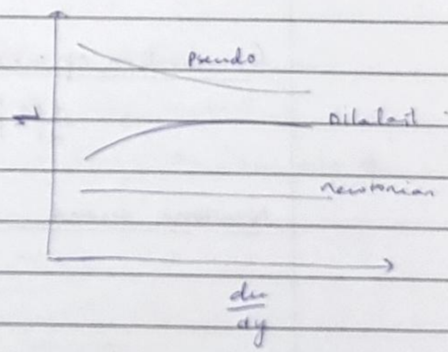
Newtonian fluids :- $\tau_{yx} \propto \frac{du}{dy}$ | $\tau_{yx} = \mu \frac{du}{dy}$ |
↳ shear stress ∝ rate of deformation

Non-newtonian fluids

$$\tau_{yx} = k\left(\frac{du}{dy}\right)^n \qquad k = \text{consistency index}$$

$$\eta = k\left(\frac{du}{dy}\right)^{n-1} \rightarrow \text{apparent viscosity.}$$





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→ $n<1$ → pseudoplastic → shear thinning fluids
↳ apparent viscosity decreases with ↑ in deformation rate
↳ polymer soln, colloids, ketchup, paint

→ $n>1$ → dilatant (shear thickening)
↳ apparent viscosity ↑ with deformation rate
↳ suspension of starch and sand

→ Bingham plastic :- fluid behaves like solid until min yield stress is exceeded

toothpaste
drilling muds
clay suspension

$$\tau_{yx} = \tau_y + \mu_p \frac{du}{dy}$$

$\frac{\text{force in x-direction}}{\text{area y direction}}$ (x momentum is being transmitted to y direction)

Surface Tension : energy required for creating unit surface area

→ Inviscid → $\mu = 0$

→ compressible → varying density

Conservation of Mass

$$\left.\frac{dM}{dt}\right|_{sys} = 0 \qquad M = \text{Mass}$$

Newtons second law

$$\vec{F} = \left.\frac{d\vec{P}}{dt}\right|_{sys}$$

$$\vec{P}_{sys} = \int_{M(sys)} \vec{V}\,dm = \int_{\forall(sys)} \vec{V}\rho\,d\forall$$

__/__/__

Energy

$$\boxed{\delta Q - \delta W = dE}$$

heat added; work done; change in internal energy

$$\dot{Q} - \dot{W} = \left.\frac{dE}{dt}\right|_{sys}$$

$$E_{sys} = \int_M e\,dM = \int_{\forall(sys)} e\rho\,d\forall$$

$$e = u + \frac{v^2}{2} + gz$$

→ $N_{sys} = \int_M \eta\,dm$ $\qquad \eta$ = intensive property, N = extensive

$$\underbrace{\left.\frac{dN}{dt}\right|_{sys}}_{①} = \underbrace{\frac{\partial}{\partial t}\int_{CV} \eta\rho\,d\forall}_{②} + \underbrace{\int_{CS} \eta\rho\vec{V}\cdot d\vec{A}}_{③}$$

→ integral eqn

(Reynolds transport theorem)

relation of system derivatives to the control volume formulation

$\frac{dN}{dt}$ = rate of change in extensive property

② = rate of change of property N in control volume
↳ amt of N present in the cv at any instant of time

③ = rate of efflux → property N exiting the surface

CONTINUITY EQN (conservation of mass) $\qquad \eta = N/M$

$N = M$

$$\left.\frac{dM}{dt}\right|_{sys} = \frac{\partial}{\partial t}\int_{CV} \frac{M}{M}\rho\,d\forall + \int_{CS} \rho\vec{V}\cdot d\vec{A}$$

$$\frac{dm}{dt} = 0 \checkmark \qquad \boxed{\frac{\partial}{\partial t}\int \rho\,d\forall + \int_{CS} \rho\vec{V}\cdot d\vec{A} = 0}$$

__/__/__

Momentum equation for inertial CV

$N = P$

$\eta = \frac{P}{m} = \vec{V}$

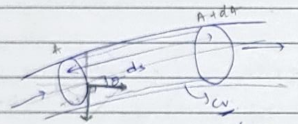
$$\left.\frac{d\vec{P}}{dt}\right)_{sys} = \frac{\partial}{\partial t}\int_{CV} \vec{V}\rho\, d\forall + \int_{CS} \vec{V}(\rho\vec{V}\cdot d\vec{A}) \rightarrow \text{total force acting on the CV leads to rate of change of momentum with CV}$$

non-accelerating CV

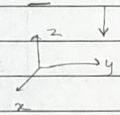
$$\frac{d\vec{P}}{dt} = \vec{F} = \underbrace{\vec{F}_S + \vec{F}_B}_{\text{net force acting of the CV}}$$

Bernoulli's eqn (conservation of energy)

steady flow $\rightarrow \int_{CS} \rho\vec{V}\cdot d\vec{A} = 0$



surface area = dA



$$-\rho V_s A + (\rho(V_s + dV_s)(A + dA)) = 0$$

Cannot be used across pumps, propellers, turbines, windmills

when friction cannot be neglected

$$F_S = \underbrace{PA - (P+dP)(A+dA)}_{\text{at the ends}} + \underbrace{\left(P + \frac{dP}{2}\right)}_{\text{avg pressure}} dA \quad \text{on the surface}$$

(s direction) $\sin\theta = \frac{dz}{ds}$

$$F_B = -\rho g\left(A + \frac{dA}{2}\right) dz \rightarrow ds \sin\theta$$

$$\rightarrow -\rho g \sin\theta \left(A + \frac{dA}{2}\right) ds$$

$$\int \vec{V}(\rho\vec{V}\cdot d\vec{A}) = -V_s(\rho V_s A) + (V_s + dV_s)(\underbrace{\rho(V_s + dV_s)(A + dA)}_{\rho V_s A})$$

$$= \rho V_s A\, dV_s$$

$$-A\,dP - \frac{1}{2}dP\,dA - \rho g A\,dz - \frac{1}{2}\rho g\,dA\,dz = \rho V_s A\, dV_s$$

$$-\frac{dP}{\rho} - g\,dz = \rho V_s dV_s$$

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{const}$$

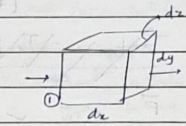
__/__/__

restrictions

1. steady flow
2. No friction losses
3. incompressible
4. flow along a streamline
5. inviscid flow

Rectangular co-ordinate

$$\rho|_{x+dx/2} = \rho + \left(\frac{\partial\rho}{\partial x}\right)\frac{dx}{2}$$



$$u|_{x+dx/2} = u + \left(\frac{\partial u}{\partial x}\right)\left(\frac{dx}{2}\right)$$

$$① \rightarrow \int \rho\vec{V}\cdot d\vec{A} = \left(\rho - \left(\frac{\partial\rho}{\partial x}\right)\frac{dx}{2}\right)\left(u - \left(\frac{\partial u}{\partial x}\right)\frac{dx}{2}\right)dy\,dz = -\rho u\,dy\,dz + \frac{1}{2}\left[u\left(\frac{\partial\rho}{\partial x}\right) + \rho\frac{\partial u}{\partial x}\right]dx\,dy\,dz$$

Write for all the surfaces:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial\rho v}{\partial y} + \frac{\partial\rho w}{\partial z} + \frac{\partial\rho}{\partial t} = 0$$

$\cancel{\nabla\cdot i\partial}$

→ acceleration :- total derivative $\rightarrow \frac{D\vec{V}}{Dt} = u\frac{\partial\vec{V}}{\partial x} + v\frac{\partial\vec{V}}{\partial y} + w\frac{\partial\vec{V}}{\partial z} + \frac{\partial\vec{V}}{\partial t}$

Substantial derivative

→ free vortex

→ Euler equation :- $\rho\frac{D\vec{V}}{Dt} = \rho\vec{g} - \nabla p$ without viscosity

→ [Inertial force resists the change in velocity]

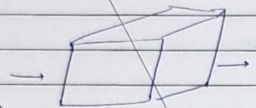
Pumps
~~Navier~~
~~restrictions~~
~~Bernoulli restrictions~~
~~Boundary conditions~~

Navier stokes equation

Turbulent flow $\rightarrow$ SDs (TP)
~~Tensor~~ (~~stress~~)
~~direction~~
~~Drag~~ rotation

flowmeters
static pressure measurement

instruments
venturi flow



~~Tensor~~ ~~gas~~ ~~representation~~

[Force balance] $\rightarrow$ reynolds transport

~~$\frac{T_{xx}|_{x+\Delta x} - T_{xx}|_x}{\Delta x}$~~

$\rightarrow \vec{F} = m\frac{D\vec{v}}{Dt}$ $\quad \vec{f} = \rho\frac{D\vec{v}}{Dt} \rightarrow$ force per unit volume.

Net force acting on the control volume =

$$[T_{xx}|_{x+\Delta x} - T_{xx}|_x]\Delta y\Delta z + [T_{yx}|_{y+\Delta y} - T_{yx}|_y]\Delta x\Delta z$$

$$+ [T_{zx}|_{z+\Delta z} - T_{zx}|_z]\Delta x\Delta y$$

$\rightarrow T_{xx} - P = \tau_{xx}$ $\qquad \tau_{xy} = T_{xy}$

$T_{yy} - P = \tau_{yy}$ $\qquad \tau_{yx} = T_{xy}$

$T_{zz} - P = \tau_{zz}$

slides in

$$\rho\frac{Du}{Dt} = \rho g_x + \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{yx}}{\partial y} + \frac{\partial T_{zx}}{\partial z}$$

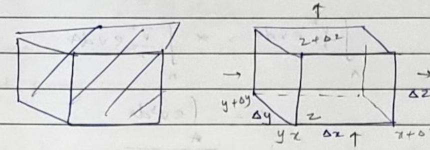
$$\rho\frac{Dv}{Dt} = \rho g_y + \frac{\partial T_{xy}}{\partial x} + \frac{\partial T_{yy}}{\partial y} + \frac{\partial T_{zy}}{\partial z}$$

$$\rho\frac{Dw}{Dt} = \rho g_z + \frac{\partial T_{xz}}{\partial x} + \frac{\partial T_{yz}}{\partial z} + \frac{\partial T_{zz}}{\partial z}$$

BL separation $\rightarrow$ (Turbulent flow)

Navier stokes equation

momentum $\rightarrow$ quantity of motion



rate of accumulation = IN − OUT + net force acting on the system

$m\vec{v}\cdot\vec{A}$ = flowrate

x component : $\Delta y\Delta z(\rho v_x v_x|_x) - \Delta y\Delta z(\rho v_x v_x|_{x+\Delta x}) +$

rate of momentum in x direction

Mass entering through the face at x

Mass leaving through the phase $x+\Delta x$

$\Delta x\Delta z(\rho v_y v_x|_y - \rho v_y v_x|_{y+\Delta y}) +$

x-momentum enters and leaves the face y and $y+\Delta y$

$\Delta x\Delta y(\rho v_z v_x|_z - \rho v_z v_x|_{z+\Delta z})$

conduction : $\Delta y\Delta z(\tau_{xx}|_x - \tau_{xx}|_{x+\Delta x}) + \Delta x\Delta z(\tau_{yx}|_y - \tau_{yx}|_{y+\Delta y})$

due to mass in

$+ \Delta x\Delta y(\tau_{zx}|_z - \tau_{zx}|_{z+\Delta z})$ direction

$-\mu\frac{du}{dz}$ $\rightarrow$ shear stress in the x-direction due to transport of x-momentum in z

(Newtonian fluid) $\rightarrow$ incompressible $\rightarrow$ const μ

IN OUT

$\Delta y\Delta z(P|_x - P|_{x+\Delta x})$

$\rho g_x \Delta x\Delta y\Delta z$

accumulation : $\Delta x\Delta y\Delta z\frac{\partial(\rho v_x)}{\partial t}$

$$\rho\frac{Dv}{Dt} = -\nabla p - (\nabla\cdot\tau) + \rho g \rightarrow \text{const } \rho, \text{ const } \mu$$

BL : No shear at liq-vapour interface

No slip at liq-solid interface

$$\frac{\partial(\rho v_x)}{\partial t} + \frac{\partial(\rho v_x^2)}{\partial x} + \frac{\partial(\rho v_x v_y)}{\partial y} + \frac{\partial(\rho v_x v_z)}{\partial z} = -\frac{\partial p}{\partial x}$$

$$\mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right]$$

$$+ \rho g_x$$

$W = \frac{F \times ds}{P (A\ ds)}\ \frac{m}{\rho}$ $\quad v = \frac{m}{\rho}$

__/__/____

kinetic energy coefficient

$$\int_A \frac{v^2}{2} \rho v\, dA = \alpha \int_A \frac{\bar{v}^2}{2} \rho v\, dA$$

point velocity is replaced by avg velocity $\rightarrow \alpha$ = correction factor

$$\alpha = \frac{\int_A \rho v^3 dA}{\dot{m} \bar{v}^2}$$

→ Ideal Bernoulli

$\alpha = 1,\ h_{LT} = 0$

frictionless flow.

→ Head loss = $h_{LT} = h_L + h_m$

h_L → major losses (due to friction) (in const area)

h_m → due to entrance, fittings, area change etc

Bernoulli

Work done = 0

$$\left(\frac{P_1}{\rho} + \frac{\alpha_1 \bar{v}_1^2}{2} + g z_1\right) - \left(\frac{P_2}{\rho} + \frac{\alpha_2 \bar{v}_2^2}{2} + g z_2\right) = h_{LT}.$$

$$h_L = f \frac{L}{D} \frac{v^2}{2}$$

f = friction factor

$f = f(Re, \frac{e}{D})$ → Moody chart

roughness.

$h_m = K \frac{\bar{v}^2}{2}$ (or) $h_m = f \frac{L_e}{D} \frac{\bar{v}^2}{2}$

K → sudden change in area

L_e → equi length (bends)

$e = \frac{\epsilon}{D}$ (ϵ → roughness, D → dia)

Pump

$$\dot{W}_{pump} = \dot{m}\left[\left(\frac{P}{\rho} + \frac{\bar{v}^2}{2} + gz\right)_{discharge} - \left(\frac{P}{\rho} + \frac{\bar{v}^2}{2} + gz\right)_{suction}\right]$$

$$W_{pump} = \dot{m}\left(\frac{\Delta P}{\rho}\right)$$

__/__/____

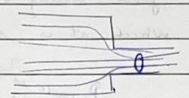
pump adds energy to the fluid in the form of gain in pressure

→ Pump, fan, blowers.

Centrifugal pumps:

→ Vena contracta → fluid streamline cannot change its direction abruptly (sharp edges)

(max velocity) (P = min)



→ When the fluid passes through the orifice, gradual contraction takes place

notes

study

fluidization

~~(NPSH)~~ (NPSH)

Cavitation

liq to vapour

unsteadyness

(Flow pattern will change)

Priming → pumps.

Priming is done by filling with water

→ removal of air from pump and suction line to cause liq to flow.

decline in flow → low pressure → impeller will damage

Types of pump

① +ve displacement

②

impeller → is a rotor used to increase the pressure and flow of a fluid.

Boundary layers ?

→ pump / turbine

pump → absorbs energy and converts into KE

turbine → extracts energy and reduces pressure from fluid and does work

capacity, head, Power input → parameters

head

→ Pump head is the capacity. to what height it will raise the fluid

Pump capacity

flow rate through the pump → vol of liq allowed to travel through the pump in a given time → L/min

⇒ head → energy imparted to the medium

__/__/__

Tensor → Magnitude, direction and plane

9 components — Tensors are simply mathematical objects that can be used to describe physical properties just like scalar and vectors.

→ Manometer → to measure pressure

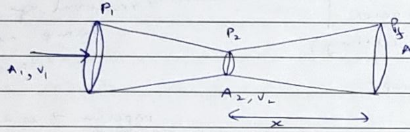
→ Buoyancy : Net vertical force acting on the body due the liq in which it is immersed.

Flow measuring instruments

(Pressure is due to gravity the fluid upward will try to push you down)

① Venturimeter

$Q = A_1V_1 = A_2V_2$

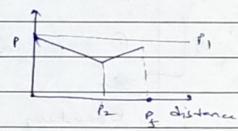


$V_2 > V_1$

$\therefore P_2 < P_1$

$A_1V_1 = A_2V_2$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2}$$



large eddies → turbulent

$P_f < P_1$ → bcoz of eddies.

length x must be large to avoid eddies (↓ decreases P)

pressure coeff (c_p)

relative pressure throughout the flow field

$$c_p = \frac{P_s - P_\infty}{\frac{1}{2}\rho u_\infty^2} \rightarrow \frac{\text{pressure forces}}{\text{inertial forces}}$$

V_s = velocity at the surface

$$P_s - P_\infty = \frac{1}{2}\rho u_\infty^2 - \frac{1}{2}\rho V_s^2$$

Dynamic P = kinetic energy per unit volume

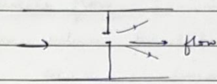
__/__/__

C_V = coeff of venturi meter $= \dfrac{\text{actual flow}}{\text{theoretical flow}}$

↳ velocity rate is getting converted to pressure rate

stream contraction and ~~...~~ frictional effects eddies

Orificemeter

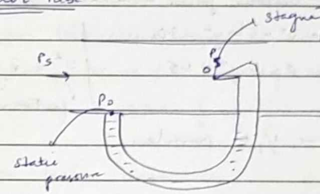


$$V = \frac{C_0}{\sqrt{1-\beta^4}}\sqrt{\frac{2(\Delta P)}{\rho}}$$

→ venturimeter and orificemeter both will measure avg flow rate (not local)

$Q \propto \sqrt{h}$

→ Pitot tube (local velocity)



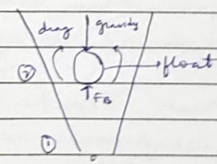
$$u_0 = \sqrt{\frac{2\Delta P}{\rho}}$$

$\Delta P = P_s - P_0$

→ Bourdon tube → Pressure measurement → static P ↳ measuring ...

Rotameter $(Q \propto h) \rightarrow (Q \propto \Delta P)$

variable area.



$u_2 < u_1$ $F_D = 6\pi\eta r u$

$F_D \downarrow$ as float moves up.

$u \downarrow$ → hence float attains equilibrium

NPSHR → required min pressure

NPSHA → available → calculated

__/__/__

$C_D = \frac{\text{drag force} / \text{Projected area}}{\frac{1}{2}\rho u^2}$

rotameter coeff

drag force $= C_D \times A \times \frac{1}{2}\rho u^2$

drag coeff: used to quantify resistance of an object in fluid environment

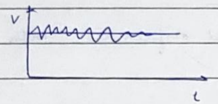
→ Turbinometer → Hot wire anemometer

→ Fluidization → notes.

Turbulent flow

notes SD4

$V = \bar{V} + V_t$



→ Viscous sub layer → viscous forces dominate → $u^+ = \frac{\bar{u}}{u^*} = y^+$

→ buffer layer / transition → $u^+ = 5\ln y^+ - 3.05$

→ turbulent core → $u^+ = 2.5 \ln y^+ + 5.5$

$u^* = \bar{V}\sqrt{f/2}$ friction velocity $y^+ = \frac{y u^* \rho}{\mu}$

$\left(\frac{\bar{u}}{U}\right) = \left(\frac{y}{R}\right)^{1/7}$ → 1/7th power law.

at the boundaries, stress cannot be calculated. (undeterminate)

can be calculated

actual head available → NPSH-A → system property | min req to avoid cavitation → NPSH-R → pump property

NPSHA > NPSHR

Cavitation: Bubbles forms. These bubbles collapse in the form of implosion (violent explosion inward). It causes shock waves to travel through the liquid which hit the impeller and cause damage.

NPSH → measure of pressure at the suction $\left(\frac{P_i}{\rho g} + \frac{v_i^2}{2g}\right) - \frac{P_v}{\rho g}$

shock wave - strong pressure waves produced by object moving faster than speed of sound

Eg: bad plan

__/__/__

Surface forces:- exerted on a surface of the body

Body forces:- acts throughout the volume of the body

Non-Dimensionalize:- to convert PDE to ODE

No of variables can be reduced

$\boxed{\text{h v/s laminar (or) turbulence}}$ → HT

Couette flow

→ flow between parallel plates → one plate is stationary and other is moving

$\Delta P = 0$ → linear

$\Delta P \neq 0$ → parabolic

Creeping flow

Convection = 0

→ Hagen Poiseuille eqn → used to find ΔP.

→ friction factor → f → moody diagram.

$\Delta P = \frac{f}{2}\frac{L}{D}\frac{\bar{V}^2}{2}$

⟹ Cone-plate viscometer.

Boundary layer separation (why??)

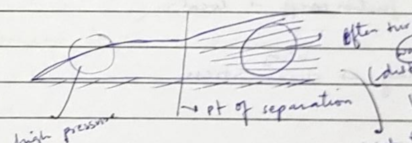
→ flow no longer follows the profile of the body around which it flows, starts forming eddies and vortex

detachment of boundary layer from a surface into a wake (forming eddies)

Magnitude or direction of velocity changes

high pressure gradient

drag force will increase.



high pressure

pt of separation

often turbulent wake (disturbed flow)

high Re

because of the viscous losses, the stagnation pressure in the wake is low

wake low pressure

∴ drag direction (not preferable) so shape of the object is important to avoid BL separation

__/__/__

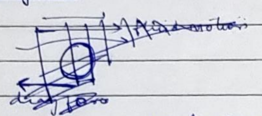
→ static pressure :- pressure when there is no movement of fluid

→ Dynamic → movement of fluid

→ to avoid BL separation, golf ball has dimples.

→ Drag force :- opposite to the relative motion of any object moving wrt surrounding fluid.

immersed bodies.



mechanical force generated by solid object moving through a fluid.

→ acts opposite to the motion of the solid body.

→ source of drag depends on shape of solid.

Drag force → Pressure pressure drag = ΔP.A
→ or (shear).

→ lift force ⊥er to the motion
↳ component of pressure force ⊥er

→ flowing fluid around the surface of an object exerts force on it. ⊥er direction → lift
opp to motion → drag

Drag = avg force = $C_D \frac{\rho V^2}{2} A$
↓
drag coeff

C_f = friction coeff → local.

→ friction drag → shear stress.
because

__/__/__

Turbine (gas turbine)

Turbine —connected— generator → electricity
↳ chemical energy to mechanical
generator → produce magnetic field

→ Fuel must be burnt in a combustion chamber.
~~Beca~~ The energy produced during combustion is used to rotate the shaft fixed to the generator → and then it is used to produce electricity.

Water turbine : uses energy from the fluid to do work.

→ Re → characterizes the type of flow

how to derive reynolds no.?
height of BL becomes const after travelling certain distance
see few problems with Pressure gradient
continuum;

→ Industrial applications of fluid mechanics ?- pumps n all
↳ airfoil n all ↳ find more
airplane wing, tail etc propeller blade
(dimples in golf ball)

dimensionless numbers

→ lagrangian / eulerian approach