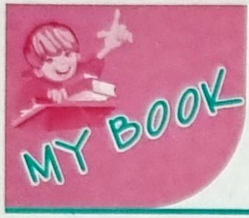


SIGN UP

I am always with you

First Name
Last Name
Address
E-mail
I am
Birthday

MORE REASONS TO EAT FRUIT

STRAWBERRIES

Madan

Strawberries can Potentially fight against Cancer and aging

BANANAS

Bananas are great for Athletes because they give you Energy

PINEAPPLE

Pineapples help Fight arthritis

SOURCE: INTERNET

Fox and McDonald

→ Monday - tutorial & assign

INDEX

Name: Madhuri Sarda .... Class: .... Sec.: .... Roll No.: ....

Sechool Name : ....

Sl. No.	Date	Particular	Page	Sign.
		Transport phenomena → [class test → Jan 27]		
		Mass transfer - 2 - RM → test → [without reflux]		
		test - March 10 (Wednesday)		
		[BL] ~~heat transfer, mass transfer~~] test ✓	[Watch TP lec] [watch cancel] today	
		31th march → 3rd test TP → heat and mass transfer		
		12th April - test → MT - rm		
		$$\frac{k\,\partial^2 T}{\partial x^2} = \rho C_p \frac{\partial T}{\partial t}$$		

TRANSPORT PHENOMENA

→ Don't remember any formula

→ SDG – 80% , Manish – 20%

→ Bird , stewart, lightfoot – book ✓

→ Transport phenomena by KS Gandhi

→ J·L Plawsky – T P fundamentals ✓

→ 4 tests (min) → 1 hr

→ disturbance thickness of boundary layer thickness → difficult to measure accurately

→ three boundary layers → hydrodynamic
→ thermal
→ concentration / mass transfer

→ Main goal → to find drag , factors depending → momentum part

→ if Re is same , pressure drop will be same

→ Continuity equation

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \rightarrow \text{incompressible flow.}$$

→ Navier stoke's eqn

$$\underbrace{\rho D\bar{v}}_{\text{substantial derivative}} / Dt = -\nabla p - [\nabla \cdot \bar{\tau}] + \rho g$$

substantial derivative (derivative following the motion)

$$\underbrace{z\text{-component}}_{\text{newtonian fluid, const } \rho} :- \rho\left(\underbrace{\frac{\partial v_z}{\partial t}}_{\text{transient term}} + \underbrace{v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}}_{\text{convective}}\right) = \overbrace{-\frac{\partial p}{\partial z}}^{\text{pressure gradient}} +$$

$$\mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \underbrace{\rho g_z}_{\text{body force}}$$

viscous stress

shear stress ↓ diffusion

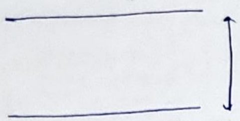
→ Boundary condition

① No slip at liq-solid interface

② No shear at the liq-vapour interface

→ surface forces will become imp in microscale which must be added to navier-stokes eqn. Body force can be neglected.

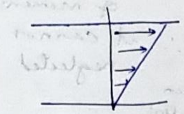
Q1)



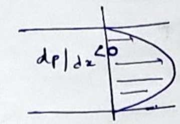
0.5 micron

$\rho = 1.23\ kg/m^3$

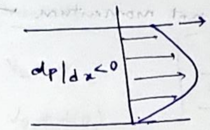
→ 3600 rpm : $\frac{3600 \times 2\pi}{60} = 376.99 = 120\pi$

→  → u

couette flow → no pressure gradient

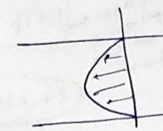
→ 

$dp/dx < 0$ v

→  → u

$dp/dx < 0$ → coutte and pressure gradient

→ mirror image if $\frac{dp}{dx} > 0$.

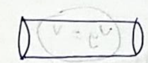
→ 

$dp/dx > 0$.

x is direction of flow.

→ hydrodynamically fully developed → velocity is not varying wrt x.
→ $v = f(r)$

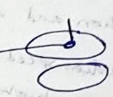
→ Thermally fully developed → temp might be a function of x
→ $T(r, x)$



$\hat{T} = \frac{T_w(x) - T(r,x)}{T_w(x) - T_m(x)}$

$\frac{\partial \hat{T}(x,r)}{\partial x} = 0.$

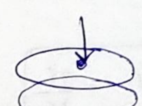
→ Order of analysis

→ disturbance will not percolate ← 

$V_r \neq 0$

$V_r = f(z, r)$

f(r) also bcoz area of cross section is changing in r direction

 → to maintain separation between two plates

→ flow between two plates

industrial application:- top plate rotates → bearing
liq → lubricant → viscous

low flow rate (to prevent the loss)

$$\frac{-\rho\phi^2}{r^3} = -\frac{dp}{dr} + \frac{\mu}{r}\frac{d^2\phi}{dz^2}$$

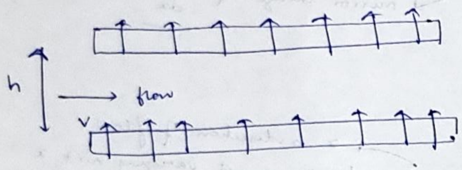
$\frac{-\rho\phi^2}{r^3}$ → velocity is low ∴ convective term can be neglected → [creeping flow condition]

$\frac{\mu}{r}\frac{d^2\phi}{dz^2}$ → viscosity ↑ ∴ it cannot be neglected — of momentum

$dz \downarrow$ → gap between two plates is small

→ $Q \neq f(\rho)$ → as μ plays a major role not momentum.

Q)



→ fully developed flow

$$\frac{\partial v_x}{\partial x} = 0$$

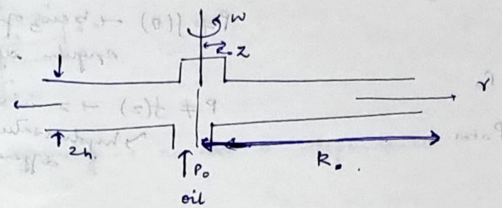
$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \Rightarrow \frac{\partial v_y}{dy} = 0 \qquad \boxed{v_y = \text{const}} \qquad v_y = v$$

No slip condition

exception → suction and ~~industrial injection~~ injection speed is too high

Eg:- rapid boiling, evaporation, condensation

→



$$\boxed{v_\theta = \frac{\omega r}{2}\left(1 + \frac{z}{h}\right)}$$

Continuity

$v_z = 0$

$$\boxed{\frac{\partial}{\partial r}(r v_r) = 0}$$

$v_\theta \neq f(\theta)$

$\boxed{v_\theta = f(z)}$ → upper plate is moving and lower is fixed

$r v_r = \text{const}$

$$\boxed{v_r = \frac{\phi(z)}{r}}$$

$v_r = f(r, z)$

$\boxed{v_r \neq f(\theta)}$

→ $v_\theta = \frac{\omega r}{2}\left(1 + \frac{z}{h}\right)$ ✓ Neglect convective and body force

$$\cancel{\rho\left(\frac{v_r v_\theta}{r}\right)} = \frac{-\partial p}{\partial r}$$

$$\cancel{\rho\left(\frac{v_r v_\theta}{r}\right)} \qquad \frac{\partial p}{\partial \theta} = 0 \qquad \boxed{\cancel{\frac{\partial^2 v_\theta}{\partial z^2}} = 0}$$

$\boxed{p = f(r)}$ angular symmetry

$$\boxed{\frac{\partial^2 v_\theta}{\partial z^2} = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right)}$$ ✓

→ $$\boxed{\frac{\partial p}{\partial r} = \mu\frac{\partial^2 v_r}{\partial z^2}}$$

$$r\underbrace{\frac{\partial p}{\partial r}}_{\text{function of } r} = \mu\underbrace{\frac{\partial^2 r v_r}{\partial z^2}}_{\text{function of } z} = \underline{\text{const}}$$ ✓

if they have to be equal both should be equal to const

$$r\frac{\partial p}{\partial r} = k$$

$P = f(r_0)$

$P \neq f(\theta) \rightarrow$ bcoz of angular symmetry

$r = R_0, \quad P = P_0$

$r = R, \quad P = P_{atm}$

$P \neq f(z) \rightarrow z$ is small $\rightarrow$ hydrostatic difference

→ (v) r component

$z = h, \; V_r = 0, \; V_\theta \neq 0, \; V_z = 0$

$z = -h, \; V_r = 0, \; V_\theta = 0, \; V_z = 0$

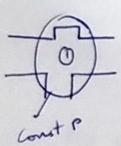
$$V_r = \frac{k}{2\mu r}(z^2 - h^2)$$

$$\dot{Q} = \text{volumetric flowrate} = \int_{-h}^{h} (2\pi r) V_r \, dz$$

$\cancel{\dot{Q} = P_0 - P_{atm}}$

→ vertical load : = ① + ②

① const pressure (given)



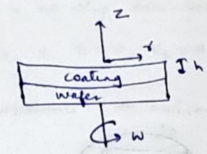
$$= (P_0 - P_{atm})\pi R_0^2 + \int_{R_0}^{R} (P - P_{atm})\, 2\pi r \, dr$$

$$\text{LOAD} = \frac{\pi (P_0 - P_{atm})}{2 \ln R/R_0}(R^2 - R_0^2)$$

→ If it is laminar → no dependence of w

beyond laminar → w dependent

Q)



→ angular symmetry → no θ dependence.

→ Continuity

→ V_θ = const , $g_r = g_\theta = g_z = 0$

$$\frac{\partial}{\partial \theta} = 0, \quad \frac{\partial}{\partial t} = 0, \quad \frac{\partial p}{\partial r} = \frac{\partial p}{\partial \theta} = \frac{\partial p}{\partial z} = 0$$

→ Continuity → $\frac{1}{r}\left(\frac{\partial}{\partial r}(r V_r)\right) + \frac{\partial V_z}{\partial z} = 0$

$V_r = f(r, z)$

$V_\theta, V_r \gg V_z$

film is coming down in the z direction so that we have uniform thickness of the film is decreasing

$\boxed{V_z \text{ is there}}$

→ $V_\theta \neq f(z)$ → as film is very thin → given

→ V_z is very small

→ neglect acceleration

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r V_r)\right) << \frac{\partial^2 V_r}{\partial z^2}$$

→ as z is very small when compared to r.

→ rV_r varies slightly wrt r

$z = 0, \; V_r = 0, \; V_\theta \neq 0.$

$z = h; \; \frac{\partial V_r}{\partial z} = 0$

→ $\boxed{V_\theta = r w}$

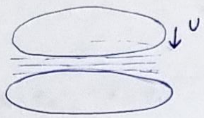
→ h → function of time [avg value of velocity $= \frac{dh}{dt}$]

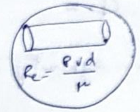
$$\boxed{V_z \rightarrow z = h = \frac{dh}{dt}}$$

→ Q)

no rotation

displacing fluid





(d) length scale is ⊥ to direction of flow

→ ① $R >>> h$

② v_r is small (leakage is small)

h ~~for~~ flow :- $\frac{h \rho v_r}{\mu} = Re$

③ $\frac{dp}{dz}$ is small

④ $v_z << v_r$

⑤ $\frac{\partial v_z}{\partial z}$ may not be small

⑥ $\frac{\partial}{\partial z}\left(\frac{\partial v_z}{\partial z}\right)$ can be appreciable

⑦ $v_\theta = 0$

⑧ $\frac{\partial v_r}{\partial r} < \frac{\partial v_r}{\partial z}$

Order of magnitude calculated w.r.t. final value

→ Continuity :-

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{\partial v_z}{\partial z} = 0$$

$$\boxed{\frac{v_r}{r} + \frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} = 0}$$

→ Navier stokes

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\underbrace{\frac{1}{r}\frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} + \frac{\partial^2 v_r}{\partial r^2}}_{\text{very small w.r.t. } \frac{\partial^2 v_r}{\partial z^2}} + \frac{\partial^2 v_r}{\partial z^2}\right)$$

$$\frac{\partial v_r}{\partial r} \approx \frac{v_r - 0}{R - 0} \rightarrow \boxed{\frac{1}{r}\frac{\partial v_r}{\partial r} \approx \frac{1}{R}\frac{v_r}{R}}$$

$\frac{1}{R} \cdot \frac{v_r}{R}$, $\frac{v_r}{R^2}$

→ v_r = no slip Boundary condition

→ v_z → $z = 0,\ v_z = 0$; $z = h,\ v_z = -U$

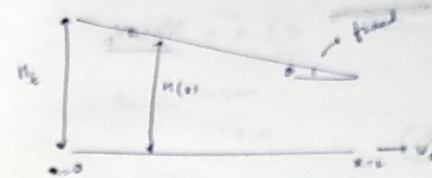
→ resistance to motion = force exerted by the fluid (pressure force).

$\frac{dp}{dr}$ ↓ radial direction

Q) neglect entire ~~LHS~~ LHS.

velocity is in x, y

gravity can be neglected



$$\frac{\partial p}{\partial x} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \rightarrow ① \text{ assume} \rightarrow \frac{\partial p}{\partial x} >> \frac{\partial p}{\partial y}$$

mistake in question [pressure gradient]

$$\frac{\partial p}{\partial y} = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \rightarrow ② \text{ — no need to consider}$$ [for values of H as small]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} \rightarrow \text{as } \frac{\partial^2 u}{\partial x^2} << \frac{\partial^2 u}{\partial y^2}$$

as x is quite large

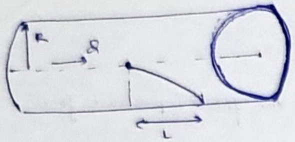
as y is small

$y = 0,\ u = v_0$

$y = H,\ u = 0$

$$u = v_0 + \frac{1}{2\mu}\frac{\partial p}{\partial x} y^2 - \frac{v_0}{H(x)} y - \frac{H(x)}{2\mu}\frac{\partial p}{\partial x} y$$

$$V = \int_0^{H(x)} u\,dy$$

→  ↓g

assume cells to be spherical

→ density is similar so they will travel for sometime with the fluid and then settle

→ cells will have same velocity as fluid

→ cell forces :

(net force due to gravity)

~~with buoyancy~~

$F_g = M_{cell}\, g$

$= \frac{4}{3}\pi a^3 (\rho + \Delta\rho) g$

Buoyancy force $= \frac{4}{3}\pi a^3 \rho g$ F_b

Stokes's eqn $= f_d = 6\pi\mu a V_T$ → velocity of cells going down towards the bottom of the tubes.

drag force

[Slow flow, laminar

$$\boxed{f_g = f_d + f_b}$$

$V_T = $ ✓

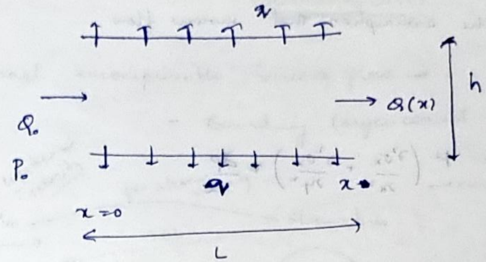
$T = R/V_T$

→ $\langle V_x \rangle = \frac{\Delta P R^2}{8\mu L}$ $Q = \pi R^2 \langle V_x \rangle = \frac{\Delta P \pi R^4}{8\mu L}$

$$\boxed{\frac{2Q}{\pi R^2} = \frac{\Delta P R^2}{4\mu L}} \checkmark$$

→ $L = \int_0^T V_x\, dt = $ ✓

Q)



$P(x)$ ✓ $\frac{P_0 - P_L}{L}$ ✓

(a) $P(x)$ if $q = 0$.

$$0 = -\frac{dp}{dx} + \mu \frac{\partial^2 v_x}{\partial y^2}$$

$v_x = 0$ → $y = 0$
→ $y = h$

$\langle v_x \rangle = -\frac{1}{12\mu}\frac{dp}{dx} h^2$

$Q_0 = \langle v_x \rangle (h \times W)$

$h <<< L$

(b) $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$

$v_x = f(x, y)$

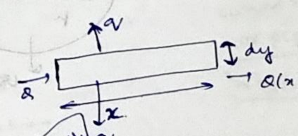
$$\rho\left(v_y \frac{\partial v_x}{\partial y} + v_x \frac{\partial v_x}{\partial x}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial x^2}\right)$$

neglect

nearly hagen poiseuille flow

neglected as it is viscous dominated

neglect convective terms



$$\boxed{Q(x) = Q_0 - 2qWx}$$

$$\boxed{\frac{dp}{dx} = \frac{-12\mu}{Wh^3} Q_0}$$

$q << Q$

$$\frac{dp}{dx} = \mu \frac{\partial^2 v_x}{\partial y^2}$$

Criteria necessary for the ... flow dominates

$$\rho\left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y}\right) = \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}\right) \ast \frac{\partial p}{\partial x}$$

$$\frac{\rho\left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y}\right)}{\mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}\right)} << 1$$

$\frac{\partial^2 v_x}{\partial x^2} \approx \frac{V_x}{L^2} \approx 0$ $\qquad$ $\frac{\partial^2 v_x}{\partial y^2} \rightarrow \frac{V_x}{h^2}$

$v_y \approx v$

$$\rightarrow \frac{\rho\left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y}\right)}{\mu\left(\frac{\partial^2 v_x}{\partial y^2}\right)}$$

$$\frac{\rho q h}{\mu} << 1$$

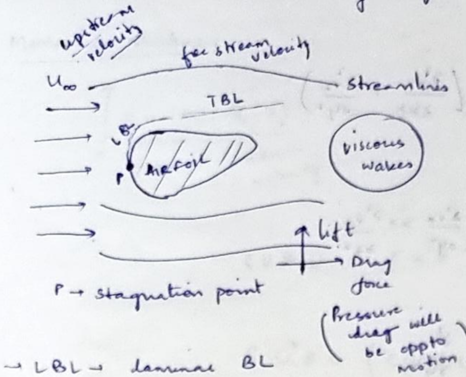
criteria

BOUNDARY LAYER

(Fox and Mc Donald)

→ External incompressible viscous flow →

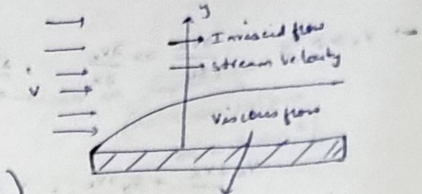
- Boundary layer concept



P → stagnation point

(Pressure drag will be oppo to motion)

→ viscosity = 0 → inviscid flow outside the BL



Navier stokes is applicable

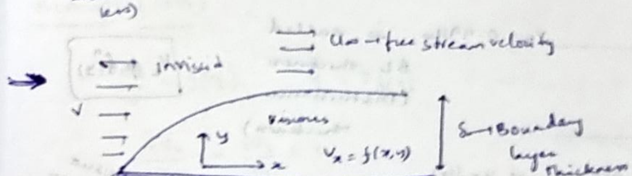
$v = v_\infty$ free stream

→ LBL → laminar BL

→ TBL → turbulent BL

→ Drag force
- Pressure drag (which will be less)
- Shear

→ for a flat plate free stream velocity = approach velocity
need not be true for curved surface or pipe
$v_\infty = f(x)$



$V_x = U_\infty$ → Inviscid flow (outside BL)

$V_x = 0.99\,U_\infty \rightarrow y = \delta$

δ → difficult to measure

→ differential analysis

→ integral analysis

→ Differential :- meaning at every instant ~~and~~ of time.

→ reynold's stresses → due to fluctuating components.

→ flow inside a boundary layer

$$V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} = \gamma \left(\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial y^2} \right)$$

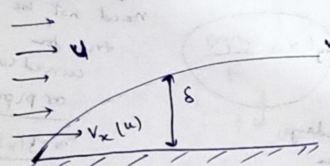
→ $V_x >> V_y$ $\quad \frac{\partial V_x}{\partial y} >> \frac{\partial V_x}{\partial x}$, $\quad \frac{\partial^2 V_x}{\partial y^2} >> \frac{\partial^2 V_x}{\partial x^2}$

$$V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} = \gamma \frac{\partial^2 V_x}{\partial y^2}$$

Exact method → Blasius numerical soln → to solve

Approximate → Momentum Integral equation

BOUNDARY LAYER THICKNESS



$V_x = 0.99 u_\infty$ is called BL thickness (Disturbance thickness)

$\boxed{V_x \neq f^n(y)}$

$V_x = f(x,y)$ $\qquad \frac{\partial V_x}{\partial y} = 0 \rightarrow y = \delta(x)$

velocity near the plate < outside the boundary layer

Displacement thickness (δ^*)

(viscous flow) { Amount of reduction in mass flow rate in a BL = $\int_0^\infty \rho(U-u)(dy \times 1)$

Same redn in the case of inviscid flow

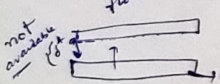
inviscid flow

$\rho U \delta^* = \int_0^\infty \rho(U-u)\,dy$ redn

$\delta^* \neq \delta$

In inviscid flow, mass flow rate can be reduced by restricting the flow area. So move the plate by δ^*

$$\boxed{\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy}$$



when solid plate is moved upward (redn of mass flow rate)

→ Blocking the ~~flo~~ flow path → $\delta^* \times 1$ area.

→ integral vanishes near the boundary layer

→ reduction in momentum

Momentum thickness

$\int_0^\infty \underbrace{\rho u (dy)}_{\text{Mass}} \underbrace{(u_\infty - u)}_{\text{redn in velocity}}$ = redn in momentum of the actual mass in the BL

$\rho U \theta U = \rho U^2 \theta$ → redn in inviscid flow (raise the platform by distance θ)

$$\rho U^2 \theta = \int_0^\infty \rho(u)(u_\infty - U)\,dy$$

$$\boxed{\theta = \int_0^\infty \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy \approx \int_0^\theta \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy}$$

BLASIUS (soln)

$\frac{V_x}{U} = g(\eta)$ $\qquad \eta = y/\delta(x)$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0$$

$$\boxed{V_x = \frac{\partial \psi}{\partial y}, \quad V_y = -\frac{\partial \psi}{\partial x}}$$

$$\boxed{V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} = \nu \frac{\partial^2 V_x}{\partial y^2}}$$

ψ = stream function

$\frac{\partial \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial y \partial x} = 0$ $\qquad \psi$ = exact differential

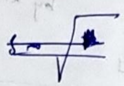
$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \nu \frac{\partial^2 v_x}{\partial y^2}$$

$v_x \approx U$, $y \approx \delta$ → at boundary layer

$$\frac{\partial v_x}{\partial y} \approx 0 .$$

$$U \frac{U}{x} \sim \nu \cdot \frac{U}{\delta^2}$$

$$\delta \sim \sqrt{\frac{\nu x}{U}} \qquad \eta = \frac{y}{\delta} = y\sqrt{\frac{U}{\nu x}}$$



$$\delta \sim \sqrt{\frac{\mu x x}{\rho U x}}$$

$$\delta \sim x\sqrt{\frac{\mu}{\rho U x}}$$

$$\boxed{\delta \sim x\, Re_x^{-1/2}}$$

→ $$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \nu \frac{\partial^2 v_x}{\partial y^2}$$

① introducing stream function ψ

②

dimensionless stream function

$$f = \frac{\psi}{\sqrt{\nu x U}} \to f(\eta)$$

$$g(\eta) = \frac{v_x}{U}$$

$$v_x = \frac{\partial \psi}{\partial y} \qquad v_y = -\frac{\partial \psi}{\partial x}$$

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}}$$

$$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0$$

$$v_x = U\frac{\partial f}{\partial \eta}$$

$$f' = \frac{\partial f}{\partial \eta}$$

→ $$\delta = \frac{5x}{\sqrt{Re_x}}$$

→ τ_w = wall shear stress

$$\tau_w = \frac{0.332\,\rho U^2}{\sqrt{Re_x}}$$ → shear stress coeff

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.664}{\sqrt{Re_x}} \qquad c_f = f(x)$$

kinetic energy

$$\delta = \frac{5.0x}{\sqrt{Re_x}}$$

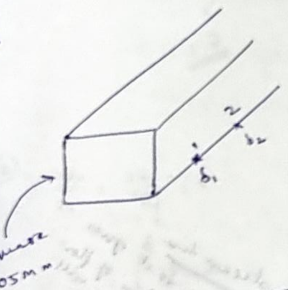
⇒ total τ is integration over whole x including c_f

→ $$\tau_w = \frac{1}{L}\int_0^L c_f \frac{1}{2}\rho U^2\, dx$$

✓ average shear stress

→ drag force : $C_D = 2c_f$ → check??

⇒



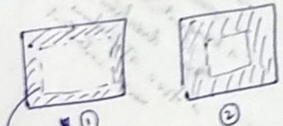
Square 305 mm

$\delta_1^* = 1.5$ mm, $\delta_2^* = 2.1$ mm } displacement thickness

Find the change in static pressure between 1 and 2? as a function of free stream dynamic pressure at ①

by pitot tube

→ dynamic pressure $= \frac{1}{2}\rho U^2$



BL thickness

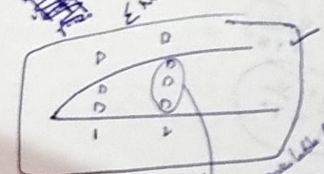
flowrate 1 & 2 equal?

$U_2 > U_1$ ✓

mass flowrate should be equal

At ① : $\frac{1}{2}\rho U_1^2$ → dynamic pressure

free stream dynamic p → $\frac{1}{2}\rho U_2^2$ ②



area available for free flow is reducing

$U_2 \neq U_1$

Bernoulli's equation
↓
inviscid flow valid

$$\frac{P_1 - P_2}{\frac{1}{2}\rho U_\infty^2} = ?$$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2}$$

$$P_1 - P_2 = \frac{1}{2}(\rho)(V_2^2 - V_1^2)$$

$$P_1 - P_2 = \frac{1}{2}\rho(U_2^2 - U_1^2)$$

$$\boxed{P_1 - P_2 = \frac{1}{2}\rho U_1^2\left(\left(\frac{U_2}{U_1}\right)^2 - 1\right)}$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(\left(\frac{U_2}{U_1}\right)^2 - 1\right)U_1^2$$

$$\frac{P_1 - P_2}{\frac{1}{2}\rho U_1^2} = \left(\frac{U_2}{U_1}\right)^2 - 1$$

$$\Rightarrow \frac{U_2}{U_1} = \frac{A_1}{A_2} = \frac{(L - 2\delta_1^*)^2}{(L - 2\delta_2^*)^2}$$

Q) use the numerical results

① to obtain δ^*/δ at $\eta = 5$, $\eta \to \infty$

② $\frac{v_y}{U}$ at BL edge

$\Rightarrow v_y \neq 0$, $v_x = V_\infty \rightarrow$ BL is not streamline

$$\delta^* = \int_0^\delta \left(1 - \frac{v_x}{U}\right)dy \qquad \eta = y\sqrt{\frac{U}{\nu x}}$$

$$\frac{\partial \eta}{\partial y} = \sqrt{\frac{U}{\nu x}}$$

stream line

→ tangent to it gives the direction of flow

→ two streamlines will never intersect

$$f' = V_x/U$$

$$\delta^* = \int_0^{\eta} \left(1 - \frac{v_x}{U}\right)\left(\frac{\partial \eta}{\sqrt{\frac{U}{\nu x}}}\right).$$

$$\cancel{\delta^* = \sqrt{\frac{\nu x}{U}}\, t}$$

$$\cancel{\delta^* = \sqrt{\frac{\nu x}{U}}\left[\eta - [f]\right]}$$

$$\delta^* = \sqrt{\frac{\nu x}{U}}\left[\eta - f(\eta)\right]_0^{\eta}$$

$\delta/5$

$$\boxed{\frac{\delta^*}{\delta} = \frac{1}{5}\left[\eta - f(\eta)\right]_0^{\eta}}$$

$$\frac{\delta^*}{\delta} = \frac{1}{5}(5 - 3.28)$$

from the table

$\eta = 5$
$y = \delta$

$\eta \to \infty \rightarrow \eta - f(\eta) \rightarrow$ will be converging

after 8.4 diff will be const

$\eta - f(\eta) \neq f(\eta)$

1.7

$\Rightarrow 0.34$

②
$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0$$

$$V_y = \int_0^\delta \frac{\partial V_x}{\partial x} dy$$

$$V_y = \frac{1}{2}\left[\sqrt{\frac{\nu U}{x}}\right]\left[\eta f' - f\right]$$

$$\frac{V_y}{U} = \frac{0.84}{\sqrt{Re_x}}$$

$\Rightarrow$ for any point x, δ is fixed → and η, f are defined at all the points in the fluid domain

$$\frac{1}{2}\left(\sqrt{\frac{\nu U}{x}}\right)\left(5 - 3.2 \ldots\right)$$

MI equation

Macroscopic balance

$N \rightarrow$ extensive

$\eta \rightarrow$ intensive

$\eta = N/M$

$$\boxed{\left.\frac{dN}{dt}\right|_{sys} = \frac{\partial}{\partial t}\int_{cv} \eta \rho \, d\forall + \int_{cs} \eta \rho \vec{V}\cdot d\vec{A}}$$

$\frac{dN}{dt}$ = total rate of change of an arbitrary extensive property of the system

$\int \eta \rho \, d\forall$ = amt of extensive property present in the control volume at any instant of time

$\rho \vec{V}\cdot d\vec{A}$ = efflux – net addition of extensive property of the CV through CS

$\rightarrow \left.\frac{dm}{dt}\right|_{sys} = 0 \rightarrow$ mass is always conserved.

$N = m$

$$0 = \frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \vec{V} d\vec{A} \rightarrow \text{integral form of continuity equation}$$

steady state $\int_{cs} \rho \vec{V} dA = 0 \rightarrow \Sigma V_i A_i = 0$

avg velocity

$\rightarrow$ in = –ve
out = +ve $\vec{V}\cdot\vec{A}$

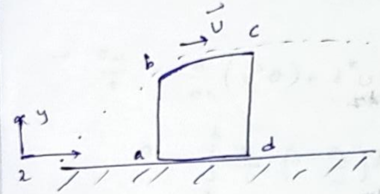
$\rightarrow$ momentum : $N = \vec{P}$ $\quad \frac{d\vec{P}}{dt}$ = forces on the CV

$$F = \frac{\partial}{\partial t}\int_{cv} \vec{V} \rho \, d\forall + \int_{cs} \vec{V}\rho(\vec{V}\cdot d\vec{A})$$

$F_S + F_B$.

$\rightarrow$ all velocities are measured relative to CV.

valid for laminar, turbulent steady 2D flow.



$\delta = f(x)$

$$\int_{cs} \rho \vec{V}\cdot d\vec{A} = 0$$

$$\dot{m}_{ab} + \dot{m}_{bc} + \dot{m}_{cd} + \dot{m}_{ad} = 0$$

$\dot{m}_{ad} \approx 0$

$$\dot{m}_{bc} = -\dot{m}_{ab} - \dot{m}_{cd}.$$

$$\dot{m}_{ab} = -\int_0^\delta \left\{ \int \rho V_x \, dy \right\} dz$$

$$\dot{m}_{x+dx} = \dot{m}_x + \left.\frac{\partial \dot{m}}{\partial x}\right|_x dx$$

cd

$$\dot m_{cd} = \left\{ \int_0^\delta \rho v_x \, dy + \frac{\partial}{\partial x}\left[\int_0^\delta \rho v_x \, dy \right] dx \right\} dz$$

$$\dot m_{bc} = -\dot m_{ab} - \dot m_{cd}$$

$$f_{Sx} + \cancel{f_{Bx}} = \frac{\partial}{\partial t}\int_{cv} \cancel{v_x} \rho \, d\forall + \int_{cs} v_x \rho \vec v \cdot d\vec A$$

0 body force

0 steady state

$$f_{Sx} = m_{f\,ab} + m_{f\,bc} + m_{f\,cd}$$

momentum through surface ab ..

$m_{f\,ad} = 0$ → no flow

$$m_{f\,ab} = -\left\{ \int_0^\delta v_x \rho v_x \, dy \right\} dz$$

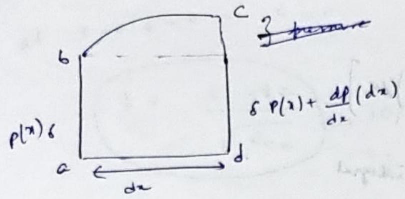
$$m_{f\,cd} = m_{f\,ab} + \frac{\partial}{\partial x}(m_{f\,ab})\,dx$$

$m_{f\,cd} =$ ↓

⇒ $m_{f\,bc} = ?$ → outer edge of BL

$$m_{f\,bc} = U \dot m_{bc}$$

→ f_{Sx} → pressure, → shear } macroscale flow — micro additional forces must be considered



$F_{ab} = p \delta \, dz$

$$F_{cd} = \left(p + \left(\frac{dp}{dx}\Big|_2\right) dx\right)(\delta + d\delta)\, dz$$

→ pressure force must be considered in curved surface bc

→ shear force [x direction]

ab = 0

cd = 0

bc = 0 → $\frac{dv_x}{dy} = 0$

ad = $-(\tau_w)\, dx\, dz$ — wall shear stress

→ $$\frac{\tau_w}{\rho} = \frac{d}{dx}(U^2\theta) + \delta^* U \frac{dU}{dx}$$ → ODE → wall shear stress → turbulent, → laminar; → momentum integral equation

$$\theta = \int_0^\delta \frac{v_x}{U}\left(1 - \frac{v_x}{U}\right) dy$$

$$\delta^* = \int_0^\delta \left(1 - \frac{v_x}{U}\right) dy$$

→ no pressure gradient to compare with Blasius —

→ and $U = const$ → for flow over a flat plate

$\frac{dU}{dx} = 0$

$$\frac{\tau_w}{\rho} = U^2 \frac{d\theta}{dx}$$

$$\frac{y}{\delta} = \eta$$

$$\rightarrow \tau_w = \rho U^2 \frac{d\delta}{dx} \left[\underbrace{\int_0^1 \frac{v_x}{U}\left(1-\frac{v_x}{U}\right) d\eta}_{\text{constant definite integral}} \right]$$

$$\frac{v_x}{U} = f(\eta)$$

$$\tau_w = \rho U^2 \frac{d\delta}{dx} \cdot \beta$$

$$\frac{v_x}{U} = a + b\eta + c\eta^2$$

$\eta = 1 \quad v_x = 1.$

$\eta = 0 \quad v_x = 0$

$\eta = 1 \quad \frac{\partial v_x}{\partial y} = 0. \longrightarrow \frac{\partial v_x}{\partial \eta} = 0$

$$\frac{v_x}{U} = 2\eta - \eta^2$$

$$\tau_w = +\mu \frac{\partial v_x}{\partial y}$$

wall shear stress.

$$\frac{2\mu}{\delta \rho U} = \frac{2}{15}\frac{d\delta}{dx}$$

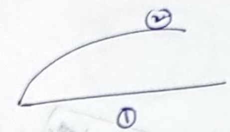
at $x=0$ $\delta = 0 \rightarrow c = 0.$

$$\delta = \frac{5.48x}{\sqrt{Re_x}}$$ → 10% error when compared to $\frac{5.0x}{\sqrt{Re_x}}$ → laminar

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.73}{\sqrt{Re_x}}$$

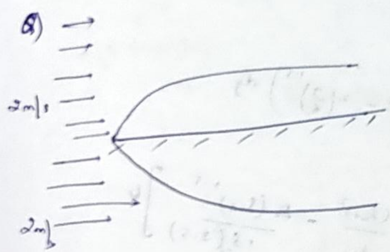
exact soln :- $\frac{0.664}{\sqrt{Re_x}}$

→ will assumption in $\frac{v_x}{U}$ → will δ change?



we know pt 1 and 2 exactly
what happens between 1 and 2
will not affect the final result as bl is thin

Q)



$$\frac{v_x}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \qquad U = 2\,m/s$$

$$\delta = \frac{5.48}{\sqrt{Re_x}}$$

$\nu = 1\times10^{-6}\,m^2/s$

$$F = 2\int_0^{0.1} \tau_w \underbrace{b}_{\text{width}}\, dx$$

$$\tau_w = \mu \frac{dv_x}{dy}$$

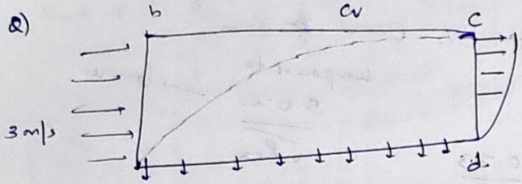
$$\frac{dv_x}{dy} = \left(\frac{2}{\delta} - \frac{2y}{\delta^2}\right)U$$

$$\tau_w = \mu\left(\frac{2}{\delta} - \frac{2y}{\delta^2}\right)U$$

$$F = 2b\int_0^{0.3} \mu\left(\frac{2}{\delta} - \frac{2y}{\delta^2}\right) U\, dx$$

$F_D = 0.26N$

Q)



$$\boxed{\frac{u}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^{1.5}}$$

$\dot{m}_{bc} = ?$

$w = 1.5\,m$

$\delta_{cd} = 1.5\,mm$

$\int \rho \vec{V}\cdot d\vec{A} = 0$

$\dot{m}_{ab} + \dot{m}_{bc} + \dot{m}_{cd} + \dot{m}_{ad} = 0$

$\dot{m}_{ad} \rightarrow + \rho(0.2)(ad)(w)$

$\dot{m}_{bc}$: +ve

$\dot{m}_{ab} \rightarrow - \rho U(\delta_{cd})(w)$

$\dot{m}_{cd} \rightarrow \left[\int_0^{\delta_{cd}} \rho V_x\, dy\right] w$

$$U w \rho \int_0^{\delta_{cd}} \left(\frac{3}{2}\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^{1.5}\right) dy$$

$$\rho w \left[\frac{3}{2}\frac{(\delta_{cd})^2}{\delta_{cd}} - \frac{2(\delta_{cd})^{2.5}}{(\delta_{cd})^{1.5}(2.5)}\right] U$$

$$\rho w \left[\frac{3}{2}(\delta_{cd}) - \frac{5}{4}(\delta_{cd})^{1}\right] U$$

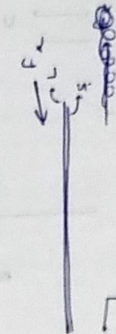
$~~\dot{m}_{bc} = 1.4\,kg/s~~$

$$-\rho U \delta_{cd} w + \dot{m}_{bc} + \rho U\left[\frac{3}{4}\delta_{cd} - \frac{5}{2}(\delta_{cd})\right] w + \left(\rho(0.2)(2)\times 10^{-3}\right) w$$

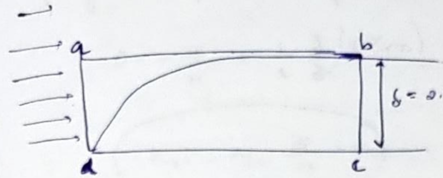
$$\dot{m}_{bc} = \left[(1000)(3)(1.5\times 10^{-3}) - (1000)(3)\left[\frac{3}{4}1.5\times 10^{-3} - \frac{5}{4}(1.5\times 10^{-3})\right] - 1000(0.2)(2\times 10^{-3})\right] w$$

$$\boxed{\dot{m}_{bc} = \rho w\left(0.3 U_\infty \delta - v_0 L\right)}$$

$\dot{m}_{bc} = 1.4\,kg/s$



Q)



$$\frac{V_x}{U} = \frac{y}{\delta}$$

$U = 30\,m/s$

$$F_{sx} = \dot{M}_{f\,ad} + \dot{M}_{f\,bc} + \dot{M}_{f\,cd} + \dot{M}_{f\,ab}$$

convective momentum.

$\dot{M}_{f\,ad} \rightarrow -(\rho U b\delta)(U)$

$\dot{M}_{f\,cd} \rightarrow 0$

$\dot{M}_{f\,ab} \rightarrow U\,\frac{1}{2}\rho U b\delta$

$$b\int_0^{\delta} V_x(\rho V_x\, dy)$$

$$b\int_0^{\delta} U\frac{y}{\delta}\,\rho\, U\frac{y}{\delta}\, dy$$

$$\frac{U^2}{\delta^2} b\rho \int_0^{\delta} y^2\, dy$$

$$\boxed{\frac{U^2}{\delta^2} b\rho \frac{\delta^3}{3}}$$

$$-\rho U^2 b\delta + \frac{\rho U^2 b\delta}{3} + \frac{1}{2}\rho U^2 b\delta$$

$$\underline{F_x} \Rightarrow -\frac{1}{6}\rho U^2 b\delta = -0.133\,N$$

$\boxed{F_x = \text{total force acting on the fluid}}$

on the fluid

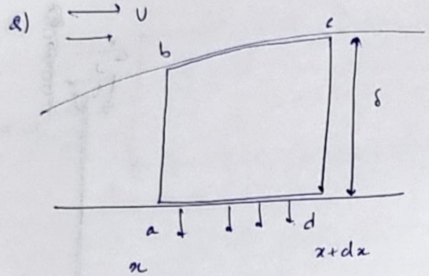
on solidity surface

by the fluid force on the solid will be on $\rightarrow$ direction $(-F_x)$

force should be applied to hold the plate cont. $(+F_x)$ -0.133

drag force

2)



$$0 = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$\dot m_{ab} + \dot m_{bc} + \dot m_{ad} + \dot m_{cd} = 0$$

$-\int_0^\delta \rho u w\, dy$ $w\rho v_0\, dx$ $w\int_0^\delta \rho u\, dy + \frac{\partial}{\partial x}\left[\left(\int_0^\delta \rho u\, dy\right) dx\right] w$

$$\dot m_{bc} = -w\rho v_0 dx - w\frac{\partial}{\partial x}\left[\left(\int_0^\delta \rho u\, dy\right) dx\right]$$

τ_w = force on the wall.

→ τ_w = wall shear stress ✓

$$-(\tau_w)\, w\, dx = m_{f\,ab} + m_{f\,ad} + m_{f\,cd} + m_{f\,bc}$$

force: out +ve, in −ve

$-\int_0^\delta (\rho u w\, dy) u$ 0 ($v_x = 0$) $\cancel{w\rho v_0 dx}\, v_0$ no x component of velocity $\dot m_{bc} U$

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U^2} - \frac{v_0}{U}$$

TURBULENT FLOW

→ fully developed steady flow.

$$0 = -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{d}{dr}(r\tau_{rz}) \quad (z\text{-direction})$$

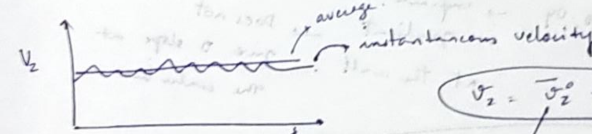
$$\tau_{rz} = r/2\, \frac{\partial p}{\partial z} + c$$

At $r = 0$, $\tau_{rz} = 0 \Rightarrow c = 0$.

$$\tau_{rz} = \frac{r}{2}\frac{\partial p}{\partial z}.$$

$$\tau_w = -\tau_{rz}\big|_{r=R} = -\frac{R}{2}\frac{\partial p}{\partial z}.$$

for both laminar and Turbulent



$$v_z = \bar v_z^0 + v_z'$$

fluctuation comp of velocity

time smooth velocity

$\overline{v_z'} = 0$ $\overline{(v_z')^2} \neq 0$

$$\frac{\sqrt{\overline{(v_z')^2}}}{\langle \bar v_z \rangle} = \text{measure of turbulence}$$

→ viscous core sublayer: $v_z^+ = \frac{\overline{v_z}}{v_*}$ $\frac{y v_*}{\nu} = y^+$ → friction velocity $v_* = \sqrt{\frac{\tau_w}{\rho}}$

→ transition :- $\frac{\overline{v_z}}{v_*} = v_z^+ = 2.5 \ln \frac{y v_*}{\nu} + 5.0$

$5 < y^+ < 26$

→ turbulent dominance:- $v_z^+ = \frac{1}{0.36} \ln y^+ + 3.8$ $y^+ \geq 26$

→ power law eqn $\frac{\overline{v_z}}{U} = \left(\frac{y}{R}\right)^{1/7}$ → distance from pipe

radius

centerline velocity

$Re \sim 10^4 - 10^5$

plug flow $\frac{\overline{V}}{U} \approx 0$

very high turbulence

→ $1/7^{th}$ power law

$\frac{\overline{V}}{U} = \frac{2n^2}{(n+1)(2n+1)}$ $n \sim 1/7$ $\frac{\overline{V}}{U} = 0.8$ | 0.5

centerline velocity

limitations

$\frac{y}{R} < 0.04$ → infinite vel. gradient at the wall → Does not give 0 slope at the centerline.

$$\frac{\tau_w}{R} = U^2 \frac{d\delta}{dx} \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) dn$$

can be used inside the boundaries

$y=0$, $y=R$ are problematic

→ MI equation is valid for both laminar and turbulent

$$\tau_w = \rho U^2 \frac{d}{dx} \int_0^\delta \frac{v_x}{U}\left(1 - \frac{v_x}{U}\right) dy.$$

non-zero slope limitation will affect the integral but it will be minimized becoz of integration.

∞ gradient will affect this

→ 1/7 power law can be used for RHS, LHS we need something else becoz of the limitation of power law.

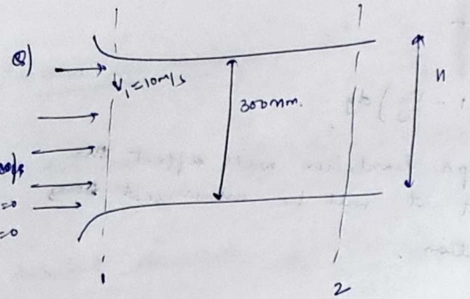
we need $-\frac{R}{2}\frac{\partial P}{\partial z}$

this can be evaluated by calculating friction factor using moody diagram

→ end result will still be the same

W - width

Q)



$\delta^* = \frac{\delta}{8}$??

→ Pressure gradient is required for the flow from ① to ②

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$$\delta^* = \int_0^\delta \left(1 - \left(\frac{y}{\delta}\right)^{1/7}\right) dy$$

$\eta = \frac{y}{\delta}$

$\frac{d\eta}{dy} = \frac{1}{\delta}$

$d\eta = \frac{1}{\delta} dy$

$\cancel{\delta^* = \delta\int_0^1 (1 - \eta^{1/7})\, d\eta}$

$\cancel{\delta^* = \delta}$

$$\delta^* = \int_0^\delta$$

$$\delta^* = \left[y - \frac{y^{1/7+1}}{\delta^{1/7}\left(\frac{8}{7}\right)}\right]_0^\delta$$

$$\delta^* = \delta - \frac{\delta^{\frac{1}{7}+1}}{\delta^{\frac{1}{7}}\,\frac{8}{7}}$$

$$\delta^* = \delta\left(1 - \frac{7}{8}\right)$$

$$\delta^* = \frac{\delta}{8}$$

→ $\delta_2^* = \delta_2/8$

→ at pt 1 :- $A = W\,H/2$, V_1

at pt 2 - A =

$\rho V_1 A_1 = \rho V_2 A_2$

$V_1(WH) = V_2 W(H - 2\delta^*)$

top and bottom

inviscid flow situation

case of inviscid

[converting the entire situation to an inviscid flow]

[δ^* is used to transform the flow into inviscid flow]

[δ^* is used just to get free stream velocity]

$V_2 = 10.9$ m/s

Velocity in an inviscid flow situation

→ Use bernoulli eqn to find pressure

→ use Bernoulli at 0, 1 and 1, 2

$P_1 < P_0$

$$\frac{V_0^2}{2} + \frac{P_0}{\rho} = \frac{V_1^2}{2} + \frac{P_1}{\rho}$$

$P_1 = -\frac{1}{2}\rho V_1^2 = -61.5$ Pa

$P_2 = -73.1$ Pa

→ Find P_1, P_2 ✓

$$F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho\, dV + \int_{CS} u\rho \vec{V}\cdot d\vec{A}$$

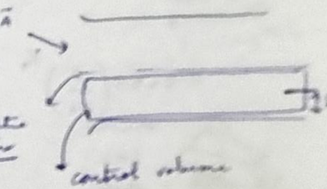
$F_{Bx} = 0$

Shear — pressure

$$-\bar{\tau}(LW) + (P_1 - P_2)W\frac{H}{2} = -\left[\rho V_1 \frac{H}{2} W\right]V_1 + \left[\left(\int_0^\delta \rho u\, dy\right) W u\right] + \rho V_2^2 W\left(\frac{H}{2} - \delta\right)$$

avg wall shear stress

$\bar{\tau} = 0.3$ N/m²



inviscid has free stream velocity which is V_2

BOUNDARY LAYERS IN TURBULENT FLOW

$$\boxed{\frac{v_x}{U} = \left(\frac{y}{\delta}\right)^{1/7} = \eta^{1/7}}$$ → derivative blows up at solid liq interface.

$$\frac{\tau_w}{\rho} = U^2 \frac{d\theta}{dx}$$ → M I eqn can be used on RHS
→ not valid at LHS

→ LHS → $$\boxed{\tau_w = -\frac{R}{2}\frac{dP}{\partial z}}$$

$P_1 - P_2$ (upstream P_1, downstream P_2)

$$\frac{\Delta P}{\rho} = \frac{P_1 - P_2}{\rho} = h$$ → head loss = h for const area conduit kept in horizontal position

$$h = f\frac{L}{D}\frac{\bar{v}^2}{2}$$ $\bar{v}$ = avg flow velocity

→ Blasius correlation: $$f = \frac{0.3164}{Re^{0.25}}$$ → for smooth pipe with very high accuracy.

$$\tau_w = -\frac{R}{2}\frac{\partial P}{\partial z} = \frac{R}{2}\frac{\Delta P}{L} = \frac{R}{2L}\rho h.$$

$$\tau_w = \frac{R}{2L}\rho f \frac{L}{D}\frac{\bar{v}^2}{2}$$

~~$\tau_w = 0.0225 \left(\frac{\nu}{U\delta}\right)^{1/4}$~~

MI Eqn

$$\underbrace{0.0225\left(\frac{\nu}{U\delta}\right)^{1/4}}_{\text{LHS shear stress}} = \frac{d\delta}{dx}\underbrace{\int_0^1 \eta^{1/7}(1-\eta^{1/7})\,d\eta}_{\text{RHS 1/7th power law}} = \frac{7}{72}\frac{d\delta}{dx}$$

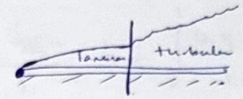
$$\frac{4}{5}\delta^{5/4} = 0.23\left(\frac{\nu}{U}\right)^{1/4} x + c$$ → valid for turbulent flow (2)

$x=0,\ \delta=0 \longrightarrow c=0 \longrightarrow$ at $x=0$

↳ this is by assuming that flow is turbulent from the begining

$$\boxed{\delta = \frac{0.37x}{(Re_x)^{1/5}}}$$

$$\delta^{5/4} = \frac{x^{4/5} \times x^{1/5}}{x^{1/5}}$$



→ mixed flow

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0577}{(Re_x)^{1/5}}$$ $\boxed{5\times10^5 < Re_x < 10^7}$ (1)

Assumption: From the begining the flow is turbulent

→ These equations (1), (2) are not valid for mixed flows.

FLUID FLOW ABOUT IMMERSED BODIES

$$\bar{F} = \int_{\text{body surface}} dF_{shear} + \int_{\text{body surface}} dF_{pressure}$$

body surface: area ll to flow of fluid.
dF_{shear}: friction drag

C_D = drag coeff = $\frac{F_D}{\frac{1}{2}\rho v^2 A}$ $C_D = f(Re)$

b = breadth
L = length

→ friction drag $F_D = \int \tau_w dA$

$$C_D = \frac{\int_{s} \tau_w\, dA}{\frac{1}{2}\rho U^2 A}$$

for laminar flow $C_f = \frac{0.664}{\sqrt{Re_x}}$

$$C_D = \frac{1.328}{\sqrt{Re_x}}$$

$$C_f = \frac{0.0577}{(Re_x)^{1/5}}$$ → pt value laminar

C_D = avg value → over the entire length.

⇒ Turbulent flow

$$C_D = \frac{0.072}{(Re_L)^{1/5}} \qquad Re_L < 10^7$$

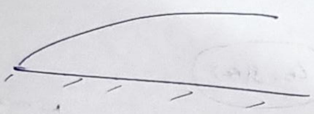
$$C_D = \frac{0.455}{(\log Re)^{2.58}} \rightarrow 10^7 < Re < 10^9$$

empirical

→ Mixed flow

Q) momentum thickness remains const

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$



$$-\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} = 0$$

$$\mu \frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x}$$

$$u = \frac{\partial p}{\partial x}\frac{y^2}{\mu} + C_1 y + C_2$$

$$\mu \frac{\partial u}{\partial y} =$$

$$\frac{v_x}{U} = 2\eta - \eta^2$$

$$\frac{v_x}{U} = \eta^{1/7}$$

$$\theta = \delta_{lam} \int_0^1 (2\eta - \eta^2)(1 - (2\eta - \eta^2))\, d\eta = \delta_{tur} \int_0^1 (\eta^{1/7})(1 - \eta^{1/7})\, d\eta$$

$$\frac{\delta_{tur}}{\delta_l} \quad \delta_{lam} \int_0^1 (2\eta - \eta^2) - (2\eta - \eta^2)^2\, d\eta = \delta_{tur} \int_0^1 (\eta^{1/7} - \eta^{2/7})\, d\eta$$

$$2\eta - \eta^2 - (4\eta^2 + \eta^4 - 4\eta^3)$$

$$\frac{\eta^{\frac{1}{7}+1}}{\frac{8}{7}} - \frac{\eta^{\frac{2}{7}+1}}{\frac{9}{7}}$$

$$\left[\frac{2\eta^2}{2} - \frac{\eta^3}{3} - \frac{4\eta^3}{3} - \frac{\eta^5}{5} + \frac{4\eta^4}{4}\right]_0^1 \qquad \frac{7}{8} - \frac{7}{9}$$

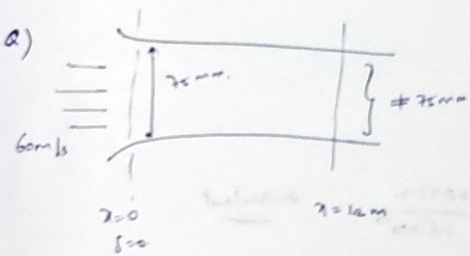
$$1 - \frac{1}{3} - \frac{4}{3} - \frac{1}{5} + 1$$

$$2 - \frac{5}{3} - \frac{1}{5} \quad \frac{2}{15}$$

$$\delta_{lam}\left(\frac{2}{15}\right) = \frac{7}{72}\delta_{turb}$$

$$\frac{\delta_t}{\delta_L} = 1.37$$

Q)



$$\frac{\partial p}{\partial x} \neq 0$$

A = changes

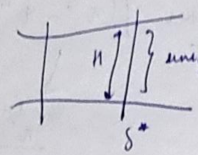
→ $\nu = 1.5 \times 10^{-5}\, m^2/s$

$$Re_x = \frac{\rho V x}{\mu/\rho} = \frac{Vx}{\nu}$$

$$\delta = \frac{5.48x}{\sqrt{Re_x}}$$

Bernoulli's eqn is applicable outside the BL

→ $P_0 = P_{1,2}$ = same.



→ $\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2}$ } outside the BL

$V_1 = V_2$

$\cancel{\rho} V_1 A_1 = \cancel{\rho} V_2 A_2$ change viscous flow to inviscid

V_2 is same

use displacement thickness.

$75 = H - 2\delta^*$

$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$ — turbulent / laminar ?? at $x = 1.2$

use Parabolic distribution if Laminar

or 1/7 th power law

$Re_x = \frac{\rho V x}{\mu} = \frac{V x}{\nu} = \frac{60 \times 1.2}{1.5 \times 10^{-5}}$ turbulent

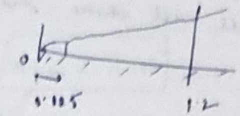
$\delta^* = \frac{\delta}{8}$

$\delta^* = 2.56$ mm

$H = 75 + 2\delta^*$

$\frac{5.48x}{\sqrt{Re_x}} = \cancel{\delta}$

$\frac{5.48\sqrt{x}}{\sqrt{\frac{V}{\nu}}} = \cancel{\delta}$



$Re_x = 10^5$

$\frac{V x}{\nu} = 10^5$

$x = 0.025$ M

$\delta = 0.85$ MM

$x > 0.025$

$\frac{4}{5}(0.85 \times 10^{-3})^{5/4} = (0.23)\left(\frac{1.5 \times 10^{-5}}{60}\right)^{1/4} 0.025 + C$

4.49×10^{-7} 1.28×10^{-7}

$\boxed{C = -1.23 \times 10^{-7}}$

$\frac{4}{5}\delta^{5/4} = 0.23\left(\frac{\nu}{U}\right)^{1/4} x - 1.23 \times 10^{-7}$

$\boxed{\delta = 20.08 \text{ MM}}$

Seam → uneven flowing of two halves of the ball

Q) Ball

Why does the ball spin?

→ One side of the ball will shine and other side will be rough.

→ Bcoz of the seam, air is distributed evenly.

→ Flow pattern on both sides of the seam will be different

→ flow on one side will encounter seam and other side will not encounter

→ BL on one side of the ball will be turbulent and other side will be laminar.

→ As a result there will be unequal pressure on both the sides

→ One side should be laminar and other should be turbulent so that the ball spins in air

$$\frac{\rho V D}{\mu} = Re_D$$

$$\frac{V(7.2)}{1.5\times10^{-5}} = Re_D$$

does not encounter the seam

$$V_1 = \frac{(1.4\times10^5)(1.5\times10^{-5})}{7.2\times10^{-2}} = 0.2916\ m/s \times 10^2 = 105\ km/hr \rightarrow \text{upper}\ \text{T - T}$$

~~$V_2 = \frac{(1.5\times10^4)(1.5\times10^{-5})}{7.2} = 0.1979\ m/s$~~ → T – T

(a) L – L
(b) T – T
(c) L – T

ball doesn't move (a), (b); ball move (c)

$Re_D < 1.5\times10^4 \longrightarrow$ L – L

→ $V_{Lower} = (Re_D)_L \frac{\nu}{D} = 71.2$ km/hr

→ for swing to take place $\boxed{71.2 < V < 105}$ effective range.

(b) late swing (starts spinning after sometime)

release the ball at speed > upper critical speed.

starting velocity > 105 km/hr

Velocity should decrease after 15m

→ Drag to slow the ball.

$V_{final}|_{15m} = 105$ km/hr → upper critical

$V_1 = 1.03 V_f$

$$C_D = \frac{F_D}{\left(\frac{1}{2}\rho V^2 A\right)}$$

A → frontal area $\frac{\pi D^2}{4}$

$$F_D = (C_D)\left(\frac{1}{2}\rho V^2\right)\left(\frac{\pi D^2}{4}\right) = -m\frac{dv}{dt}$$

C_D: 0.15 = given

$$\int_0^x \frac{\left(C_D \frac{1}{2}\rho \frac{\pi D^2}{4}\right)}{m}\,dx = \int_{V_i}^{V_f} -\frac{dv}{v}$$

$a = \frac{dv}{dt}\cdot\frac{dx}{dx} = v\frac{dv}{dx}$

$$\frac{C_D \frac{1}{2}\rho \frac{\pi D^2}{4}}{m}x = -\ln\left(\frac{V_f}{V_i}\right)$$

$$\boxed{\frac{C_D \frac{1}{2}\rho \frac{\pi D^2}{4}(15)}{m} = +\ln\left(\frac{V_i}{V_f}\right)}$$ → for late swing initial velocity.

$\rho = 1.22$ kg/m³
$m = 0.152$ kg
$C_D = 0.15$
$D = 7.2$ cm

$(0.15)\frac{1}{2}\times1.22\times\frac{\pi}{4}(7.2\times10^{-2})^2\ldots$ = 0.152

HEAT TRANSFER FUNDAMENTALS

→ Heat in − out + Generation = 0 (SS)

= rate of accumulation (unsteady state)

CV — control volume

→ $\dot{q}$ = amt of energy generation per unit vol

$$\rho \hat{C_p} \frac{\partial T}{\partial t} = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \dot{q}$$

for a system experiencing only conduction and no viscous heat dissipation

work that need to be done over frictional forces that try to impede the motion

→ BC :-

① temp specified at point T(x) is known

② temp at the interface is equal ($T_{S_1} = T_{S_2}$)

③ adiabatic (or) insulated → $\frac{\partial T}{\partial x}\Big|_{x=x_0} = 0$. at $x = x_0$ at or specified location

④ Conduction flux to solid-fluid interface is equal to convective flux away from the surface

$$\boxed{-k\frac{dT}{dx}\Big|_{x=L} = h(T_S - T_\infty)}$$

→ how to reduce contact resistance → filling up the interfacial spaces → remove air and add liq with high thermal conductivity

→ or by polishing

→ h → depends on geometry, laminar or turbulent, fluid properties

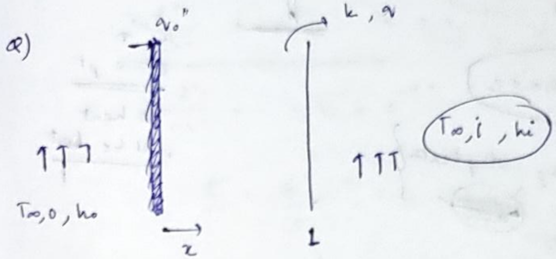
→ $h = f(\rho, \mu, k, v, x, C_p)$

x — characteristic length

$h = f(Re, Pr)$

$$\boxed{Nu_x = \frac{hx}{k} = f(Re, Pr)}$$

Q)



strip heater :- supply q_0'' } amount of $\dot{q}$ will go outside (h_o) and inside (h_i)

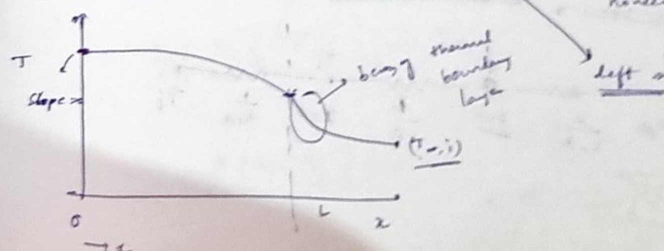
↳ to minimize loss of heat ($\dot{q}$) outside the chamber

$$h_o(T_{\infty,o} - T_0) = q_0'' = -k\frac{dT}{dx}\Big|_{x=0}$$

$$\boxed{\cancel{-k\frac{dT}{dx}\Big|_{x=L} = h(T_L - T_{\infty,i})}}$$

—(a) no heat generation is lost outside chamber — left side

because of strip heater



$\frac{d^2T}{dx^2} = 0$

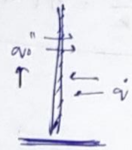
$\frac{dT}{dx} = C_1$

$T = C_1x + C_2$

$A h_o (T_{Li} - T_{\infty i}) = \dot{q} A L$ ✓

$T_L = ?$

at some value of q_o'' → there will be no heat loss



q_o'' at a pt

no heat will be lost

$\frac{d^2T}{dx^2} = -\frac{\dot{q}}{k}$

$\frac{dT}{dx} = -\frac{\dot{q}}{k}x + C_1$

$T = -\frac{\dot{q}}{k}\frac{x^2}{2} + C_1x + C_2$

{all the heat generated by strip heater will go outside → at SS $\cancel{q_o''}$}

① $\frac{dT}{dx}\Big|_{x=0} = 0$ (BC) → 0

② $-k\frac{dT}{dx}\Big|_{x=L} = h_i(T_L - T_i)$

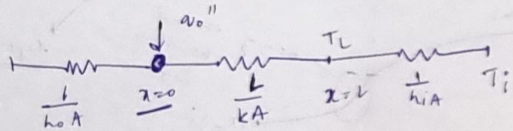
$T_L = 60$

$T_0 = 65$

$\cancel{q_o'' A = A h_o (T_\infty - T_0)}$ (b) $q_o'' A = h_o A (T_0 - T_{\infty,o})$ ✓

⇒ no heat is going to the wall at this state

⇒ (c) $\cancel{q_o'' = 200}$ $\cancel{(T_0 - T_{\infty,o})}$



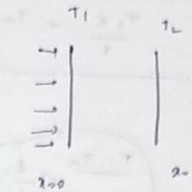
$\boxed{T(x=0) = 55C}$

⇒ Q) $q(x) = q_o'' \alpha e^{-\alpha x}$

heat generated per unit volume ✓; flux ✓

(a) $x=0$, $T = T_1$

$x=L$, $T = T_2$.



$k\frac{d^2T}{dx^2} + q(x) = 0.$

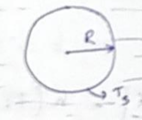
$k\frac{d^2T}{dx^2} + q_o''(e^{-\alpha x})\alpha = 0$

$k\frac{d^2T}{dx^2} = -q_o''\alpha e^{-\alpha x}$

$\frac{dT}{dx} = \frac{A}{-\alpha}e^{-\alpha x} + C_1$ $\quad A = \frac{-q_o''\alpha}{k}$

$T = T_1 - (T_1 - T_2)\frac{x}{L} + \frac{q_o''}{\alpha k}\left(1 - \frac{x}{L}\right) + \frac{q_o''}{\alpha k}\left(\frac{x}{L}e^{-\alpha L} - e^{-\alpha x}\right)$

Q)



→ large motionless fluid

→ no convection

→ conduction

(k)

T_∞

fluid

$\frac{1}{r^2}\left(\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)\right) = 0.$

$\frac{d}{dr}\left(kr^2\frac{dT}{dr}\right) = 0.$

$kr^2\frac{dT}{dr} = C_1$

$dT = \frac{C_1}{kr^2}dr$

$T = -\frac{C_1}{k}\frac{1}{r}$ + C_2

$r \to \infty$: $T_\infty = C_2$

$r = R$, $T = T_s$

$T_s = -\frac{C_1}{kR} + T_\infty$

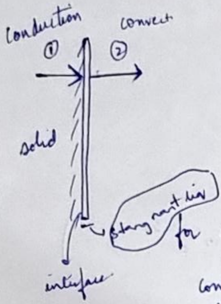
$C_1 = (T_\infty - T_s) kR$

$$T = \frac{-(T_\infty - T_s)R}{r} + T_\infty$$

k = fluid

$$\boxed{T = \frac{T_s R}{r} + T_\infty\left(1 - \frac{R}{r}\right)}$$

$$\frac{T - T_\infty}{T_s - T_\infty} = \frac{R}{r}$$ → temp distribution around the fluid



conduction = convection at the interface

for CS → Heat in = Heat out
control surface

$$-k\left(\frac{dT}{dr}\right)_{r=R} = h(T_s - T_\infty)$$
liquid

between stagnant liv and moving liv

no convection
hypothetical situation

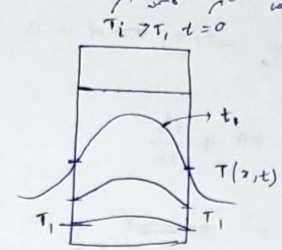
$$-k\left(\frac{(T_\infty - T_s)kR}{kR^2}\right) = h(T_s - T_\infty)$$

Solid	stagnant liv	moving liv

$$\frac{hR}{k} = 1$$

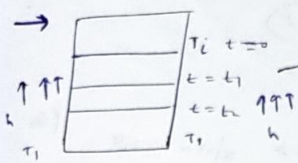
TRANSIENT CONDUCTION

$T_i > T_1,\ t = 0$ — solid temp uniform — exposed to convection at $t = 0$



$$\boxed{k\frac{d^2T}{dx^2} = \rho C_p \frac{\partial T}{\partial t}}$$

→



↑↑T h → T = f(t)

$T \neq f(x)$

[k high, Thin bar] h → depends

low h, high k, low L → more uniform distribution in the bar.

$$Bi = \frac{hL}{k_s}$$ = ratio of resistance = $\frac{\text{Conductive}}{\text{convective}}$

Biot number — solid

(LC)

LUMP CAPACITANCE MODEL → resistance to heat transfer inside the solid is very less when compared to outside

$R_{cond} << R_{conv}$

→ $Bi << 1$ ✓

→ Assumption : temp is space-wise uniform inside the solid at any instant of time

$\dot{In} - \dot{Out} + generation = \dot{stored}$

Eg: A hot solid being cooled by convective process
~~Eg~~ Putting in a cold liv and no heat generation

$-\dot{Out} = \dot{Stored}$

$-hA_s(T-T_\infty) = \rho V c \frac{dT}{dt}$ → for quenching process

$\frac{\rho V c}{hA_s}\frac{d\theta}{dt} = -\theta$ $\qquad \theta = T - T_\infty$

$\frac{\rho V c}{hA_s} \ln \theta_i/\theta = t$

$\frac{\theta}{\theta_i} = \frac{T-T_\infty}{T_i - T_\infty} = \exp\left(-\frac{hA_s}{\rho V c}\right)t$ → $\underline{Bi < 0.1}$

Alternative to L C method
→ Analytical
→ Numerical
→ graphical (Heisler charts)

Q) - $\dot{E}_{out}$ + generated = stored.

$-hA_s(T-T_\infty) + (I^2R')L = \cancel{\rho V c \frac{dT}{dt}}$ (solid → 0, steady state)

$A_s = \pi D L$

$Bi < 0.1 = \frac{h r_o}{k_s}$

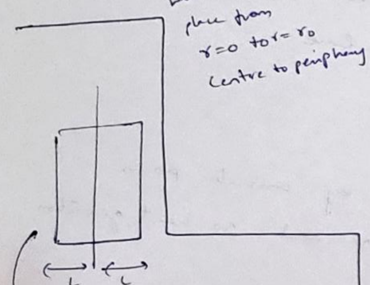
$Bi = \frac{h r_o}{k_s} = 0.012 < 0.1$

~~$h(T - T_\infty) = I^2R'$~~

Max drop in temp will take place from $r=0$ to $r=r_o$ centre to periphery

$T = T_\infty + \frac{I^2 R_e'}{\pi D h}$

$\underline{T = 88.7°C}$



$\frac{hL/2}{k}$

$\frac{V}{A_s}$ = length scale for Bi

→ Transient

$0 - h\pi D(T - T_\infty) + I^2 R_e' = \rho c \frac{\pi D^2}{4}\frac{dT}{dt}$

$\frac{d\theta}{dt} + A\theta = B.$

$\frac{T - T_\infty - B/A}{T_i - T_\infty - B/A} = \exp\left(-\frac{4h}{\rho c D}t\right)$

T_i ~~T~~ $\underline{T = 87.7°C}$

$t = 8.3s$

Q) $Bi = hr_o/k < \underline{0.1}$

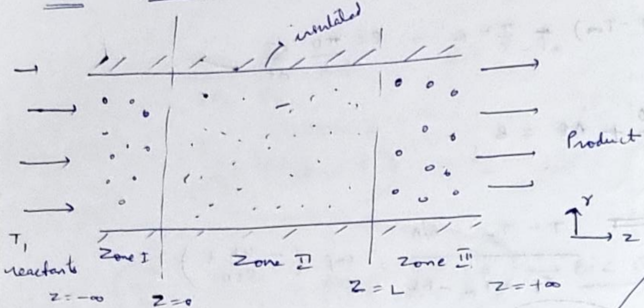
$-hA_s(T - T_\infty) = \frac{d}{dt}(\rho c_p V dT)$

$\theta = T - T_\infty$

$\rho_{air} = \rho_{polymer}$ → it will fall

$\underline{\theta_o = 350K}$

HEAT CONDUCTION WITH CHEMICAL HEAT SOURCE



Conduction + convection + heat generation

Superficial velocity $v_1 = \frac{w}{\pi R^2 \rho_1}$ ← flow rate

Volumetric heat generation S_c

Zone - I → inert particles
Zone - II → catalyst → reaction ($z = 0$ to L)
Zone - III → no reaction → inert

} static particles

gases will flow → reaction

$T \neq f(r)$ → as walls are insulated (R is not large) ← radius

$T = f(z)$ → becoz heat is generated becos of reaction

→ Consider → exothermic

→ no heat generation in I, III

→ heat content will ↑ from $z = 0$ to L becoz of exothermic nature of reaction

IN - OUT + Gen = 0

IN → cond, conv

OUT → $\pi R^2 q_z|_{z+\Delta z}$

Cond $= \pi R^2 q_z|_z$ → conductive heat flux entering at z

Conv :- $\dot{m} c_p \Delta T$

$= \dot{m} c_p (T - T_0)|_z$ (T_0 → reference temp)

$\dot{m} = \pi R^2 \rho_1 v_1$

$= w\, c_p (T - T_0)|_z$

Out :- $(\dot{m} c_p (T - T_0))_{z+\Delta z}$

Gen :- $S_c (\pi R^2 \Delta z)$

$$\pi R^2 \left(q_z|_z - q_z|_{z+\Delta z} \right) + \dot{m} c_p \left(\Delta T|_z - \Delta T|_{z+\Delta z} \right) + S_c \pi R^2 \Delta z = 0$$

$\dot{m} = \pi R^2 \rho_1 V_1$

$\Delta z \to 0$

$$\frac{q_z|_z - q_z|_{z+\Delta z}}{\Delta z} + \rho_1 v_1 c_p \frac{\Delta T|_z - \Delta T|_{z+\Delta z}}{\Delta z} + S_c = 0$$

$$\boxed{\frac{dq_z}{dz} + \rho\, v_1 c_p \frac{dT}{dz} = S_c}$$

$$\boxed{-k_{z_{eff}} \frac{d^2T}{dz^2} + \rho v_1 C_p \frac{dT}{dz} = S_c}$$

cond — conv — generation

effective thermal conductivity for gas and solid mix } assumption

$k_{z_{eff}}$

→ Zone-I :- $S_c = 0$ $\quad -k_z \frac{d^2T'}{dz^2} + \rho C_p v_1 \frac{dT'}{dz} = 0$

Zone-II :- $\quad -k_z \frac{d^2T''}{dz^2} + \rho C_p v_1 \frac{dT''}{dz} = S_c$

Zone-III ; $\quad -k_z \frac{d^2T'''}{dz^2} + \rho C_p v_1 \frac{dT'''}{dz} = 0$

6 BC required :- $z = -\infty,\ T' = T_1$

$T' = T'' \rightarrow$ at $z = 0$

$T'' = T''' \rightarrow$ at $z = L$

$k \frac{dT'}{dz}\Big|_{z=0} = k \frac{dT''}{dz}\Big|_{z=0}$

$k \frac{dT''}{dz}\Big|_{z=L} = k \frac{dT'''}{dz}\Big|_{z=L}$

} continuity of temp and heat flux

$z = +\infty \quad T''' =$

$$\boxed{\theta^{I} = C_1 + C_2 e^{Bz}}$$

$$\boxed{\theta^{II} = C_3 e^{m_3 z} + C_4 e^{m_4 z}}$$

non-dimentionalize (check bird stewart lightfoot)

$$\boxed{\theta^{III} = C_5 + C_6 e^{Bz}}$$

const

θ^{III} = finite

as $z = \infty \rightarrow T''' \neq \infty$

$C_6 = 0$

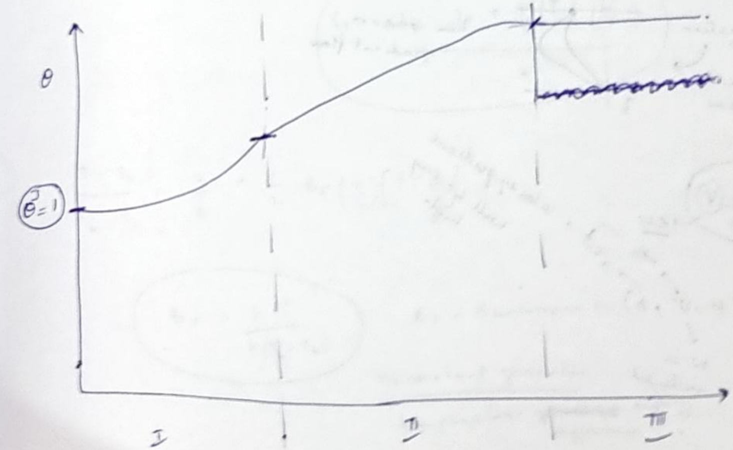
$\theta^{III}(T^{III}) = C_5$

$\theta^{I} = C_1 \rightarrow z = -\infty \rightarrow$ dimensionless form

$\cancel{C_1 = T_1}$ $\quad C_1 = 1$

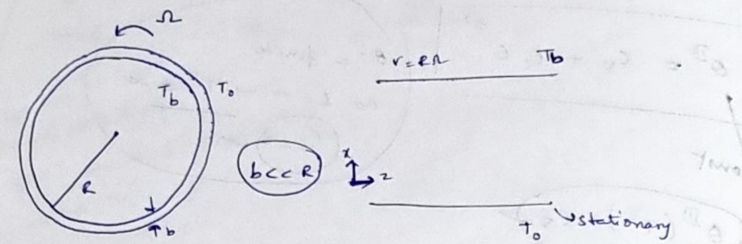
→ $\frac{d\theta'''}{dz} = 0 \rightarrow \frac{d\theta''}{dz} = 0$ at $z = L$

$$\theta = \frac{T - T^0}{T_1 - T^0}$$



Viscous heat generation

→ heat conduction with viscous heat source

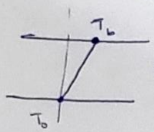


→ high viscosity lubricant

→ lubricant should be such that its viscosity will be stable at temp T_b, T_0 → and see temp the lubricant can experience

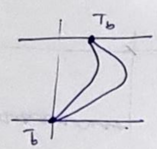
$T_b > T_0$

→ frictional heat will be generated . i.e with time the fluid will get heated .



temp profile } similar to couette flow.

→ if there is heat generation → Viscous heat generation



like pressure gradient flow.

→ $V_z = \frac{x}{b} V$ → RW

$\dot{Q} = f(\mu, v, b)$ — $\dot{Q}$ → heat generation; v → walls; b → 10^{-6} → velocity gradient will be very high; W is included in this

$$\dot{q} = \mu\left(\frac{\partial v_z}{\partial x}\right)^2$$

given

$$k\frac{d^2T}{dx^2} + \dot{q} = 0$$ → heat diffusion equation

$T = f(x)$

$$k\frac{d^2T}{dx^2} + \mu\left(\frac{\partial v_z}{\partial x}\right)^2 = 0$$ → from navier stokes

$$k\frac{d^2T}{dx^2} + \mu\left(\frac{V}{b}\right)^2 = 0$$

$$\boxed{T = -\frac{\mu}{k}\left(\frac{V}{b}\right)^2\frac{x^2}{2} + c_1x + c_2}$$

$T = T_0 \rightarrow x = 0$

$T = T_b \rightarrow x = b$

$T_0 = c_2$

$$T_b = -\frac{\mu}{k}\left(\frac{V}{b}\right)^2\left(\frac{b^2}{2}\right) + c_1 b + T_0$$

$$c_1 = \frac{T_b - T_0 + \frac{\mu}{k}\left(\frac{V}{b}\right)^2\left(\frac{b^2}{2}\right)}{b}$$

$$\frac{T - T_0}{T_b - T_0} = \frac{x}{b} + \frac{1}{2} Br\left(\frac{x}{b}\right)\left[1 - \frac{x}{b}\right]$$

$$Br = \frac{\mu V^2}{k(T_b - T_0)}$$

Br = Brinkmann no:

$$\frac{\text{Viscous heat generation}}{\text{temperature difference}}$$

Condition for max temp intermediate the 2 walls

$$\left(\frac{dT}{dx}\right)\frac{1}{T_b - T_0} = \frac{1}{b} + \frac{1}{2}\frac{Br}{b}\left[\frac{1}{b} - \frac{2x}{b}\right] = 0$$

$$1 + \frac{Br}{2}\left[1 - \frac{2x}{b}\right] = 0$$

$$-\frac{2}{Br} = 1 - \frac{2x}{b}$$

$$\frac{2x}{b} = 1 + \frac{2}{Br}$$

$$x = b\left(\frac{1}{2} + \frac{1}{Br}\right)$$

$0 < x < b$

$0 < b\left(\frac{1}{2} + \frac{1}{Br}\right) < b$

$\cancel{\frac{1}{2} + \frac{1}{Br}} \neq 0$

$\frac{1}{2} > \frac{1}{Br} > 0$ $Br > -2$ just the mag $Br > 2$

⇒ $Br > 2$ → to have max in between the plates

→ $T > T_b$ → so temp will have max

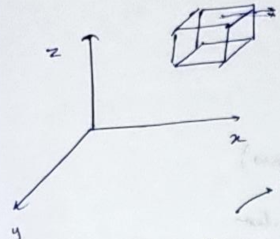
→ Viscous heat → dissipative

① high velocity gradient → $\left(\frac{dv_z}{dx}\right)^2$

②

⇒ we cannot calculate T explicitly as T depends on V also it should be solved — using shell heat balance

Equation of change for non-isothermal system (like navier stokes)



fluid in through x, y, z faces

out from $x+\Delta x$, $y+\Delta y$, $z+\Delta z$

1st law of thermo for open system

$$\begin{bmatrix}\text{rate of accumulation of internal and kinetic energy}\end{bmatrix} = \begin{Bmatrix}\text{rate of IE and KE in by convection}\end{Bmatrix} - \begin{Bmatrix}\text{Rate out by conv}\end{Bmatrix}$$

$$+ \begin{Bmatrix}\text{Net rate of heat addn by cond}\end{Bmatrix} - \begin{Bmatrix}\text{Work done by the system on the surroundings}\end{Bmatrix}$$

$+ \{$ on

→ rate of accumulation of IE + KE $= \Delta x \Delta y \Delta z \frac{\partial}{\partial t}\left(\rho U + \frac{1}{2}\rho v^2\right)$

U = IE per unit mass of the fluid

v = mag of local fluid velocity

→ rate of convection of IE & KE into the element

x-face $\Delta y \Delta z \left\{ v_x\underbrace{\left(\rho U + \frac{1}{2}\rho v^2\right)}_{\text{per unit volume}}\Big|_x - v_x\left(\rho U + \frac{1}{2}\rho v^2\right)\Big|_{x+\Delta x}\right\}$ → Energy / unit time

+ y face

+ z-face

conduction

$$\Delta y \Delta z \left\{ q_x|_x - q|_{x+\Delta x} \right\} + y \text{ face} + z \text{ face}$$

flux

Work done
- against volumetric force (body)(gravity)
- against surface forces (pressure)(viscous)

rate of work done = force $\times$ velocity in the direction of force

rate of work done ~~by~~ against gravity $= -\rho(\Delta x \Delta y \Delta z)(v_x g_x + v_y g_y + v_z g_z)$

rate of work done ~~by~~ against static pressure $= \Delta y \Delta z \left\{ P v_x|_{x+\Delta x} - P v_x|_x \right\} +$

$\Delta x \Delta z \left\{ P v_y|_{y+\Delta y} - P v_y|_y \right\} +$

$\Delta y \Delta x \left\{ P v_z|_{z+\Delta z} - P v_z|_z \right\}$

against viscous forces $= \Delta y \Delta z \left\{ \begin{matrix} \tau_{xx} v_x \\ \tau_{xy} v_y \\ \tau_{xz} v_z \end{matrix} \right\} \underbrace{\begin{matrix} \tau_{yx} v_x \\ \tau_{yy} v_y \\ \tau_{yz} v_z \end{matrix}}_{\Delta x \Delta z} \underbrace{\begin{matrix} \tau_{zx} v_x \\ \tau_{zy} v_y \\ \tau_{zz} v_z \end{matrix}}_{\Delta x \Delta y}$

τ_{yx}: y → direction in which it gets transported; x → momentum, force in x direction

$$\rho C \frac{DT}{Dt} = k \nabla^2 T - \underbrace{T\left(\frac{\partial P}{\partial T}\right)(\nabla v)}_{\text{expansion effects}} + \underbrace{\mu \phi_v}_{\text{viscous dissipation}}$$

~~$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$~~

Simplifications

$$\rho C \frac{DT}{Dt} = k \nabla^2 T + \dot{q}$$

solid → ρ = const

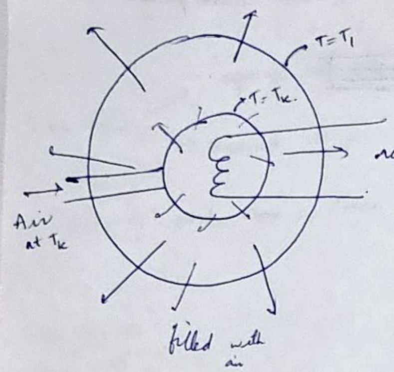
$$\rho C_p \frac{\partial T}{\partial t} + k \nabla^2 T + \dot{q}$$

$$\rightarrow \rho C_p \left[\underbrace{\frac{\partial T}{\partial t}}_{\text{transient}} + \underbrace{v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z}}_{\text{convective}} \right] = \overbrace{k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right]}^{\text{conduction}} + \underbrace{\mu \phi_v}_{\text{viscous effects}} + \underbrace{\dot{Q}}_{\text{heat generation per unit volume}}$$

~~convective~~

Transpiration cooling

(two concentric porous spherical shell)



refrigeration coil (to maintain inner sphere at T_k)

→ conduction + convection

⇒ $T_1 > T_k$

→ if the air is switched off → we will have only conduction

→ $V_\theta = 0$, $V_\phi = 0$

$$\frac{1}{r^2}\frac{d}{dr}(r^2 \rho V_r) = 0 \quad \text{→ continuity eqn.}$$

$$r^2 \rho V_r = C = \frac{w_r}{4\pi}$$

$$V_r = \frac{w_r}{4\pi r^2 \rho} \qquad w_r \text{ - mass flow rate}$$

$T = f(r)$

→ $$\rho \bar{C}_p \left(V_r \frac{\partial T}{\partial r} \right) = k \left[\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)\right] + 0 + 0 \right] + 0 + \cancel{\phi}^{\,0}$$

viscous generation $= 0$

$$\frac{dT}{dr} = \frac{4\pi k}{w_r \bar{C}_p} \frac{d}{dr}\left(r^2 \frac{dT}{dr} \right)$$

$T_1 = \frac{4\pi k}{w_r \bar{C}_p} r^2 \frac{dT}{dr} + C_1$

$\frac{dT}{T - T_k} = \frac{dr}{r^2}$

$\int_{T_k}^{T_1} \ln(T - C_1) = -\frac{R_0}{r} + C_2$

Conductive flux → out to in

convective → in to out

→ If air would not have used, load on refrigeration coil will be higher

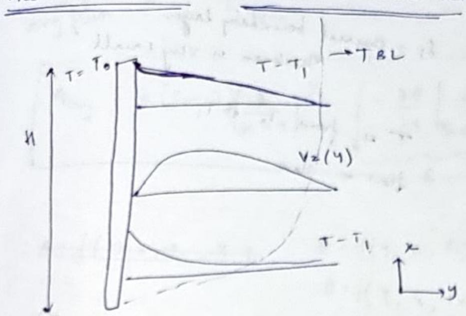
$\ln\left(\frac{T - C_1}{T_k - C_1}\right) = -\frac{R_0}{r} + C_2$

~~$T = T_k$~~

$\frac{T - C_1}{T_k - C_1} = e^{-\frac{R_0}{kr}}$

FREE CONVECTION FROM A VERTICAL PLATE

$T_0 > T_1$



temp profile is changing cos we are looking in entry length

$\frac{T_1 - C_1}{T_k - C_1} = e^{-\frac{R_0}{r} + \frac{R_0}{kr}}$

Assumption :- ① Dimension in the x-direction is very large, thus V_y, V_z are functions of y and z only

② V_y is small; fluid almost moves directly upward and therefore, y component of the equation of motion can be neglected

$$\frac{T - C_1}{T_k - C_1} = e^{-\frac{R_0}{r}} C_2$$

$\beta = \frac{1}{V}\frac{\Delta V}{\Delta T}$ → coeff of Thermal expan

Buoyant force $= \rho g \Delta V = \rho g \beta V \Delta T$

$r = kR$

$C_2 = e^{R_0/kR}$

$\frac{\text{Buoyant force}}{V} = \rho g \beta \Delta T$

$$\frac{T_1 - C_1}{T_k - C_1} = e^{-\frac{R_0}{R}} e^{\frac{R_0}{kR}}$$

~~$T = C_1 + (T_k - C_1)$~~

$T_1 = C_1 + T_k - C_1($ $)$

$$C_1 = \frac{T_1 - T_k}{1 - e^{-\frac{R_0}{R}} e^{\frac{R_0}{kR}}}$$

equation of motion : $\rho\left(v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}\right) = \mu\left(\frac{\partial^2 v_z}{\partial y^2} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right) + \rho g \beta (T - T_1)$

neglect

Continuity : $\frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$

Energy : $\rho \hat{C}_p \left(v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z}\right)(T - T_1) = k\left(\frac{\partial^2}{\partial y^2} + \cancel{\frac{\partial^2}{\partial z^2}}\right)(T - T_1)$

neglect

conductive fluxes

$\frac{\partial v_z}{\partial z} << \frac{\partial v_z}{\partial y}$

δ_T = thermal boundary layer thickness is very small

$v_y << v_z$

T is from T_0 to T_1

flow is slow

→ v_z does not vary much with z

$\frac{\partial T}{\partial z} << \frac{\partial T}{\partial y}$

→ y direction temp gradient will be more → conduction in y not z

[liq is moving upward becoz of buoyancy plate is stationary]

[T_0 is maintained at the wall]

B C

① AT y = 0 → $v_z = 0$, $v_y = 0$, $T = T_0$

② At $y = \infty$, $v_z = 0$, $v_y = 0$, $T = T_1$

(iii) at $z = -\infty$, $T = T_1$, $v_z = 0$ $v_y = 0$ → far away from plate

→ dimensionless variable

$\theta = \frac{T - T_1}{T_0 - T_1}$ $\xi = z/H$ $\eta = \left(\frac{\beta}{\mu \alpha H}\right)^{1/4} y$

ϕ_z, ϕ_y = dimensionless velocity

$\frac{\partial \phi_y}{\partial \eta} + \frac{\partial \phi_z}{\partial \xi} = 0$

$\frac{1}{Pr}\left[\phi_y \frac{\partial}{\partial \eta} + \phi_z \frac{\partial}{\partial \xi}\right)\phi_z = \frac{\partial^2 \phi_z}{\partial \eta^2} + \theta$

$\phi_y \frac{\partial \theta}{\partial \eta} + \phi_z \frac{\partial \theta}{\partial \xi} = \frac{\partial^2 \theta}{\partial \eta^2}$

$q_{avg} = \frac{k}{H} \int_0^H -\frac{\partial T}{\partial y}\Big|_{y=0} dz$

avg flux

$$\boxed{q_{avg} = k(T_0 - T_1)\left(\frac{\beta}{\mu \alpha H}\right)^{1/4} \underbrace{\int_0^1 -\frac{\partial \theta}{\partial \eta}\Big|_{\eta=0} d\xi}_{C}}$$

→ f(Pr)

C is → weak function of Pr

~~$\theta = f(y, z, Pr, \eta, \xi)$~~, $\theta = f(y, z, Pr)$

$\theta = f(\eta, \xi, Pr)$

$\frac{\partial \theta}{\partial \eta}\Big|_{\eta=0} = f(\xi, Pr)$

$-\int_0^1 \frac{\partial \theta}{\partial \eta}\Big|_{\eta=0} d\xi = f(Pr)$

→ resembling creeping flow

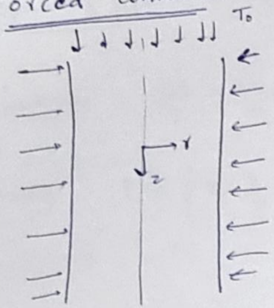
→ more conductive heat transfer than convective

→ Pr will small

Pr	0.73	10.	10^2	10^3
C	0.517	0.612	0.652	0.653

Forced convection

(from Bird, stewart)



$T = f(r, z)$

const heat flux through the wall

$T_z \neq f(z) \Rightarrow$ wrong because const flux is being added

$$v_z = v_{z\,max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

$$v_{z\,max} = \frac{(P_0 - P_L)R^2}{4\mu L}$$

$\boxed{T_z = f(r, z)}$

$v_r = 0$

$$\rho \hat{C_p}\left(\underbrace{\cancel{\frac{\partial T}{\partial t}}}_{0} + \underbrace{\cancel{v_r \frac{\partial T}{\partial r}}}_{0} + \underbrace{\cancel{\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}}}_{0} + v_z \frac{\partial T}{\partial z}\right) = k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \underbrace{\cancel{\frac{1}{r^2}\left(\frac{\partial^2 T}{\partial \theta^2}\right)}}_{0} + \frac{\partial^2 T}{\partial z^2}\right) + \underbrace{\cancel{\mu \phi_v}}_{0} + \underbrace{\cancel{\phi}}_{0}$$

$$\underbrace{\rho \hat{C_p}\, v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]\frac{\partial T}{\partial z}}_{\text{sensible heat}} = k\left[\underbrace{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)}_{\text{conduction}} + \frac{\partial^2 T}{\partial z^2}\right]$$

r direction >> z direction

Convection is going to dominate in z direction

→ Thermally fully developed → dimensionless temp profile will not change

$$\rightarrow \theta = C_0\,\xi + \underbrace{\psi(\xi)}$$

$\theta \to T$, $\xi \to z$, $\xi \to r$

— Incropera

$$\rightarrow q'' = h(T_s - T_\infty)$$

$$q = \int q'' \, dA_s$$

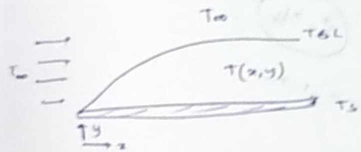
$$q = \bar{h} A_s (T_s - T_\infty)$$

$$\bar{h} = \frac{1}{A_s}\int h \, dA_s = \frac{1}{L}\int_0^L h \, dx$$

$$h = f(\rho, \mu, C_p, k, L, v)$$

$$\boxed{\frac{hL}{k} = f(Re, Pr)}$$

THERMAL BOUNDARY LAYER



$y = 0,\ v_x = 0,\ T = T_s$

$$\rho \hat{C_p}\left(v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y}\right) = k\left[\underbrace{\cancel{\frac{\partial^2 T}{\partial x^2}}}_{\text{small}} + \frac{\partial^2 T}{\partial y^2}\right]$$

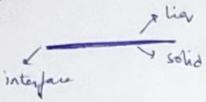
≈ 0

$$\boxed{v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\left(\frac{\partial^2 T}{\partial y^2}\right)}$$

$\theta = \frac{T_s - T}{T_s - T_\infty} \approx 0.99$ reached the edge of boundary layer

→ $q_s'' = -k_s \frac{dT}{dy}\Big|_{y=0}$ → solid side of interface.

lost by solid



$q_s'' = -k_f \frac{dT}{dy}\Big|_{y=0}$.

$$v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \frac{k}{c_p \rho} \frac{\partial^2 T}{\partial y^2}$$

$y = 0 \mid T = T_s)$

$$\boxed{-k_s \frac{dT}{dy}\Big|_{y=0} = h(T_s - T_\infty)}$$

$$h = \frac{-k_f \frac{dT}{dy}\big|_{y=0}}{(T_s - T_\infty)}$$ → conductive heat flux at the interface

$T^* = \frac{T - T_s}{T_\infty - T_s}$ ~~$x^* = x/L$~~ ~~$y^* = y/L$~~ $y^* = y/L$

$x^* = x/L$

$$\frac{hL}{k_f} = \frac{\partial T^*}{\partial y^*}\Big|_{y=0}.$$

nusselt number

dimensionless temp gradient at the interface

→ $v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 v_x}{\partial y^2}$

$$\rho \hat{c}_p \left(v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \dot{q} + \mu \phi_v$$

$f(\eta) = \frac{\psi}{\sqrt{\nu x U_\infty}}$ $\eta = y\sqrt{\frac{U_\infty}{\nu x}}$ $T^* = \frac{T - T_s}{T_\infty - T_s}$

$$\boxed{\frac{d^2 T^*}{d\eta^2} + \frac{P_r}{2} f \frac{dT^*}{d\eta} = 0}$$

$T^* = 0, \ \eta = 0$

$T^* = 1, \ \eta \to \infty$

$P_r = \frac{c_p \mu}{k}$ → should be specified before we start writing the eqn

$\boxed{0.6 < P_r < 50}$

$$\boxed{\frac{dT^*}{d\eta}\Big|_{\eta=0} = 0.332 P_r^{1/3}}$$

laminar

$$h_x = \frac{q_x''}{T_s - T_\infty} = \frac{1}{T_s - T_\infty} \frac{\partial T}{\partial y}\Big|_{y=0} = \frac{k}{L} \frac{\partial T^*}{\partial y^*}\Big|_{y=0} = k\left(\frac{U_\infty}{\nu x}\right)^{1/2} \frac{dT^*}{d\eta}\Big|_{\eta=0}$$

$$\boxed{Nu_x = \frac{h_x x}{k} = 0.332 Re_x^{1/2} P_r^{1/3}}$$

$0.6 < P_r < 50$

⇒ $\frac{\delta}{\delta_t} \approx P_r^{1/3}$ → $Re_x < 5 \times 10^5$

$$\bar{C}_{f,x} = \frac{\bar{\tau}_{s,x}}{\rho U_\infty^2/2} = 1.328\, Re_x^{-1/2} = 2\, C_{f,x}$$

$$\boxed{\bar{h}_x = 2 h_x}$$

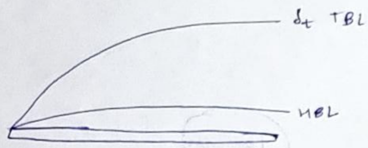
$$\boxed{\bar{Nu}_x = \frac{\bar{h}_x x}{k}} = 0.664\, Re_x^{1/2}\, Pr^{1/3} \qquad 0.6 < Pr < 50$$

Avg coefficients

→ Q) $Pr << 1$ → liq metals $k\uparrow$

$\frac{\text{momentum}}{\text{thermal}} < 1$

$$v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \alpha \left[\cancel{\frac{\partial^2 T}{\partial x^2}} + \frac{\partial^2 T}{\partial y^2} \right]$$



laminar

$\frac{\partial T}{\partial y} \to 0$. $\frac{\partial^2 T}{\partial x^2} \to 0$ → conduction is less.

$$v_x \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2}$$

v_x → U_∞ as MBL << TBL

$$T^* = \frac{T - T_0}{T_s - T_0}$$

$y = 0 \quad T^* = 1$

$y = \infty \quad T^* = 0$

$x = 0 \quad T^* = 0$

$$\to \frac{d}{dx}\left\{ \operatorname{erf}\left(\frac{x}{\sqrt{4\alpha t}}\right) \right\}\Bigg|_{x=0} = \frac{1}{(\pi \alpha t)^{1/2}}$$

$$T = T_s + (T_\infty - T_s)\, \operatorname{erf}\left(\frac{y}{\sqrt{4(\alpha/U)x}} \right) \to mk$$

$$-k_f \frac{\partial T}{\partial y}\Bigg|_{y=0} = -k(T_s - T_\infty) \frac{\partial T^*}{\partial y}\Bigg|_{y=0}$$

$$\boxed{T^* = 1 - \operatorname{erf}\left(\frac{y}{\sqrt{\ }} \right)}$$

$$-k \frac{\partial T}{\partial y}\Bigg|_{y=0} = k_f (T_s - T_\infty) \frac{1}{\sqrt{\pi \frac{\alpha}{U} x}} = h(T_s - T_\infty)$$

$\alpha = \frac{k}{U}$

$$h_x = \frac{k_f \sqrt{U}}{\sqrt{\pi \alpha x}}$$

$$\frac{hx}{k} = Nu_x = \frac{1}{\sqrt{\pi}} \sqrt{\frac{U x \rho C_p}{k}}$$

$$Nu_x = (0.565)(Pe)^{1/2} \qquad Pe = Re\, Pr$$

Fick's law :- $N_A = x_A(N_A + N_B) - D_{AB} \nabla C_A$ $\quad D_{AB}\left(\frac{\partial C_A}{\partial x} + \frac{\partial C_A}{\partial y} + \frac{\partial C_A}{\partial z}\right)$

N_A → in terms mole (molar flux)

$x_A(N_A+N_B)$ → bulk flow

$D_{AB}\nabla C_A$ → diffusion superimposed

→ homogeneous reaction + uniform throughout the vol
↳ reaction occurs everywhere inside the domain.

BC

$C_{A,S1} = C_{A,S2}$ → at interface

Impervious surface :- $\frac{dC_A}{dx} = 0$

$$\rightarrow \left(\underbrace{\frac{\partial C_A}{\partial t}}_{\text{transient}} + \underbrace{v_x \frac{\partial C_A}{\partial x} + v_y \frac{\partial C_A}{\partial y} + v_z \frac{\partial C_A}{\partial z}}_{\text{convection}}\right) = \overbrace{D_{AB}\left(\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2}\right)}^{\text{conduction / diffusion}} + \underbrace{R_A}_{\text{reaction rate}}$$

→ R_A = generation / depletion → will come in the governing equation if it occurs throughout the control volume
↓
homogeneous reaction

→ If Reaction is occurring on the catalyst surface → it will come under BC
↓
boundary conditions

$$-D_{AB}\frac{dC_A}{dz}\Big|_{z = \text{cat surface}} = (-R_A'') \rightarrow \text{heterogeneous rxn}$$

heterogeneous

→ BC

① Catalyst : $-D_{AB}\frac{dC_A}{dz}\Big|_{z_{cat}} = -R_A''$

② at interface $C_{A,S1} = C_{A,S2}$

③ Impervious surface $dC/dz = 0$ at specified location
like adiabatic

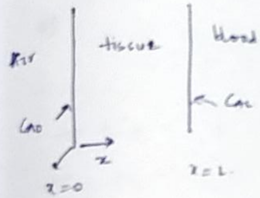
④ $-D_A \frac{dC_A}{dz}\Big|_{z=L} = h_m(C_{AS} - C_{A\infty})$
h_m → Similar to convective heat transfer coeff

Q) $z=0$, $C_A(0)$
$z = L$, $C_A(L)$ } const

[Not the case of equimolar counter diffusion]

k_0''' → homogeneous

k_0'' → heterogeneous



[O_2 moves from tissue to blood]

(some amt of O_2 is going to be consumed by tissue)

tissue consumes O_2
reaction is 0 order

R_A → has to be there in the governing eqn.

→ no convection
→ homogeneous
→ only diffusion
→ steady state
→ $C_A = f(x)$

$-R_A''' = k_0'''$

$$0 = D_{AB}\frac{\partial^2 C_A}{\partial x^2} + R_A$$

$$0 = D_{AB}\frac{\partial^2 C_A}{\partial x^2} - k_0'''$$

$$\frac{d^2 C_A}{dx^2} = +\frac{k_0'''}{D_{AB}}$$

$$C_A = +\frac{k_0'''}{D_{AB}}\frac{x^2}{2} + C_1 x + C_2$$

$N_A = -D_{AB}\dfrac{dC_A}{dz}$

Q) even after some time
the individual containers can be assumed to be pure O_2, pure N_2

→ $A = O_2$
$B = N_2$ ✓

No reaction
large volume
$C_A = f(z)$
steady state
$V_x, V_y = 0$, $V_z \neq 0$ → because of V

$z = L$ ——— $z = 0$

$C_A = C_{A0}$ at $z = L$

$C_A = 0$ at $z = 0$

$$V_z \frac{\partial C_A}{\partial z} = D_{AB}\frac{\partial^2 C_A}{\partial z^2}$$

$$\frac{C_A}{C_{A0}} = \frac{e^{Vz/D_{AB}} - 1}{e^{VL/D_{AB}} - 1}$$

$\dfrac{Vz}{D_{AB}}$ = small → expand

$$\frac{C_A}{C_{A0}} = \frac{Vz/D_{AB}}{VL/D_{AB}} = \frac{z}{L}$$ ✓

Q) $\left.\dfrac{dC_A}{dx}\right|_{x=0} = -\dfrac{N_{A0}''}{D_{AB}}$

$C_A(\infty) = 0$ ✓

Q) Sublimation of Naphthalene ball.

○ → ball (control vol)

CV
Mass in + out − generation = Accumulation
0   0 (no reaction)   ✓ (-ve) → $\dfrac{d}{dt}(\rho V)$

$Sh = \dfrac{h_m L}{D_{AB}} = 2$ → no imposed velocity (calculated before for temp)

$h_m A \Delta C = -\dfrac{d}{dt}\left(\frac{4}{3}\pi r^3 \rho\right)$

Q) $\dfrac{d}{dr}\left(r\dfrac{dx_A}{dr}\right) = 0$

(i) known conc at $r = r_i$
(ii) equality of diffusion and convection at $r = r_0$

Q) $r_A'' = -k_1'' C_A$

$NO + CO \Rightarrow \frac{1}{2}N_2 + CO_2$

$k_1'' = 0.05\ m/s$

$X_{A1} = 0.15$

k_1 is very small

more C_A will be present on the surface (as conversion is low)

→ fast reaction → mole fraction of NO on the catalyst ≈ 0

→ diffusion = rate of reaction → Steady state

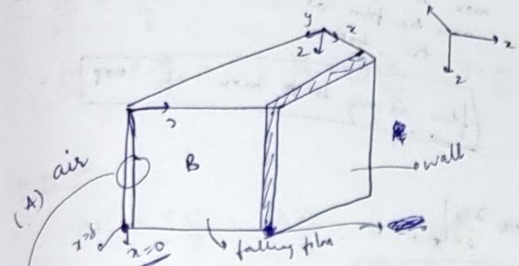
$$\frac{d^2 C_A}{dz^2} = 0$$

→ $-D_{AB}\, c \frac{d x_A}{dz} = -k_1'' c\, x_{AS}$ ✓

Q) instantly → conc of O_2 inside will be 0 at any instant. as reaction is instantaneous

linear profile

→ Diffusion into a falling film



diffusion $\left(\cancel{\frac{\partial C_A}{\partial t}} + \cancel{v_x}\frac{\partial C_A}{\partial x} + \cancel{v_y}\frac{\partial C_A}{\partial y} + v_z\frac{\partial C_A}{\partial z}\right) = D_{AB}\left[\frac{\partial^2 C_A}{\partial x^2} + \cancel{\frac{\partial^2 C_A}{\partial y^2}} + \cancel{\frac{\partial^2 C_A}{\partial z^2}}\right] + R_A$

SS: $\frac{\partial C_A}{\partial t} = 0$, $v_x = 0$, $v_y = 0$, $v_z \neq 0$, $\frac{\partial C_A}{\partial z} \neq 0$, $\frac{\partial^2 C_A}{\partial y^2} = 0$, $\frac{\partial^2 C_A}{\partial z^2} \approx 0$ because transfer of C_A in z is more due to convection and not diffusion, $R_A = 0$

$$v_z \frac{\partial C_A}{\partial z} = D_{AB}\frac{\partial^2 C_A}{\partial x^2}$$

$z = 0$ $C_A = 0$ → pure liquid film

$x = 0$ $C_A = C_{A0}$ for all z
interface conc; C_{A0} known → const

$x = \delta$ $\frac{dC_A}{dx} = 0$ ∀ all z

$x = \infty \rightarrow C_A = 0$ ✓

Velocity will be ~~v_{max}~~ v_{max} throughout → as diffusion thickness is less

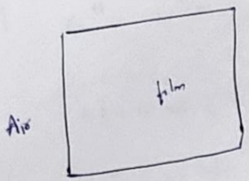
$$v_{max}\frac{\partial C_A}{\partial z} = D_{AB}\frac{\partial^2 C_A}{\partial x^2}$$

$z = 0$ $C_A = 0$ ∀ x

$x = 0$ $C_A = C_{A0}$ ∀ z

$x = \infty$ $C_A = 0$ ∀ z

$$\frac{C_A}{C_{A0}} = 1 - \mathrm{erf}\frac{x}{\sqrt{4D_{AB}z/v_{max}}}$$



more A
near the film at
the bottom

$\boxed{\text{flux is going to be more at top}}$

$$N_{Ax}(z)\big|_{x=0} = -D_{AB}\frac{\partial C_A}{\partial x}\Big|_{x=0}$$

$$\boxed{N_{Ax}(z)_{x=0} = C_{A0}\sqrt{\frac{D_{AB}V_{max}}{\pi z}}}$$

→ A is absorbed by a laminar film of liq B.

→ conc of A increases from $z=0$ to $z=L$

↓ $\therefore \frac{\partial C_A}{\partial z} \neq 0$

$x>0$

(at $x=0$ $C_A = \text{const}$ (C_{A0}))

↓ interface

ANALOGY

Continuity, momentum

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \rightarrow (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \qquad \nu = \frac{\mu}{\rho} \rightarrow (2)$$

Energy eqn

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \dot{q} + \underbrace{\cancel{\mu\phi}}_{\text{neglect}} \qquad \alpha = \frac{k}{\rho c_p} \rightarrow (3)$$

Conc. BL

$$\underbrace{u\frac{\partial C_A}{\partial x} + v\frac{\partial C_A}{\partial y}}_{\text{advection}} = \underbrace{D_{AB}\frac{\partial^2 C_A}{\partial y^2}}_{\text{diffusion}} + \dot{R}_A \qquad D_{AB} \rightarrow (4)$$

$\frac{v_x}{V}$, $\frac{T_s - T}{T_s - T_\infty}$, $\frac{C_{As} - C_A}{C_{As} - C_{A\infty}}$

$\frac{\partial u}{\partial y} >> \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x}$] velocity BL

$\frac{\partial T}{\partial y} >> \frac{\partial T}{\partial x}$] Thermal BL

$\frac{\partial C}{\partial y} >> \frac{\partial C}{\partial x}$] conc BL

incompressible
no heat generation
no reaction.

Equations	BC's (solid plate) Wall	Free stream	Similarity parameter
$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{\nu}{VL}\frac{\partial^2 u^*}{\partial y^{*2}}$	$u^*(x^*,0) = 0$ $v^*(x^*,0) = 0$	$u^*(x^*,\infty) = \frac{u_\infty(x^*)}{V}$	Re_L
$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{\alpha}{VL}\frac{\partial^2 T^*}{\partial y^{*2}}$ $1/Re\,Pr$	$T^*(x^*,0) = 0$ $T^* = \frac{T_s - T}{T_s - T_\infty}$	$T^*(x^*,\infty) = 1$	Re_L, Pr
$u^*\frac{\partial C_A^*}{\partial x^*} + v^*\frac{\partial C_A^*}{\partial y^*} = \frac{D_{AB}}{VL}\frac{\partial^2 C_A^*}{\partial y^{*2}}$ $1/Re\,Sc$	$C_A^*(x^*,0) = 0$	$C_A^*(x^*,\infty) = 1$	Re_L, Sc

$\rightarrow u^* = f(x^*, y^*, \frac{\partial p^*}{\partial x^*}, Re_L)$

$\rightarrow \frac{\partial u^*}{\partial y^*}\Big|_{y^*=0} = f_2(x^*, \frac{\partial p^*}{\partial x^*}, Re_L)$

wall shear stress

$\frac{dp^*}{dx^*} \rightarrow$ can be found outside the BL.

Bernoulli's eqn [for a specified geometry]

$\frac{\partial p^*}{\partial x^*} \rightarrow$ known.

$\tau_w = \mu \frac{\partial u^*}{\partial y^*}\Big|_{y^*=0} = f_5(x^*, Re_L)$

$\bar{\tau}_w = f_6(Re_L)$

length avg

$T^* = f_3(x^*, y^*, Re_L, Pr, \frac{dp^*}{dx^*})$

$q_s = -k_f \frac{\partial T}{\partial y}\Big|_{y=0}$

$Nu = f_4(x^*, Re, Pr) \rightarrow$ geometry is specified

$\overline{Nu} = f_5(Re, Pr)$

$\rightarrow C_A^* = f_6(x^*, y^*, Re_L, Sc, \frac{dp^*}{dx^*})$

$N_A'' = -D_{AB} \frac{\partial C_A}{\partial y}\Big|_{y=0}$

$Sh = f_7(x^*, Re_L, Sc)$

$\overline{Sh} = f_8(Re_L, Sc) = \frac{\bar{h}_m L}{D_{AB}}$

$Pr = \frac{\nu}{\alpha}$

$Sc = \frac{\nu}{D_{AB}}$

momentum BL

$\frac{\delta}{\delta_t} = Pr^n$

thermal

$\nu = \alpha \rightarrow$ Both BL will grow at same rate

momentum, thermal

$\frac{\delta_t}{\delta_c} = Le^n \rightarrow$ Lewis

conc

dependence on geometry?? $\rightarrow$ for any geometrically similar question

Reynold's analogy

$\Rightarrow \frac{dp^*}{dx^*} = 0$, flat plate $\rightarrow$ then $\boxed{u_\infty(x^*) = V}$ ✓

so, $\rightarrow u^*(x^*, \infty) = \frac{u_\infty(x^*)}{V} = 1$

then all the BL's will be identical

then the governing eqns are dynamically similar

$\rightarrow$ solutions will be equivalent

$\rightarrow$ if $Pr, Sc = 1 \rightarrow$ equations will be identical.

$\rightarrow$ dynamically similar system } solution of 1 will be similar to other

$\rightarrow \frac{C_f Re_L}{2} = Nu = Sh$

$\frac{Nu}{Re\, Pr}$ = stanton number for HT

$\frac{C_f}{2} = St = St_m$

$\frac{Sh}{Re\, Sc}$ = mass transfer st.

1

Modified reynolds

$\frac{C_f}{2} = St \cdot Pr^{2/3} = j_H$

j = colburn j factor

$\frac{C_f}{2} = St_m\, Sc^{2/3} = j_m$

$\frac{C_f}{2} = j_H = j_m$

$\rightarrow$ strictly valid for laminar but can extend it to Turbulent

Reynold's analogy - valid for turbulent also (experimentally)

$\hookrightarrow$ when change in pressure is not too severe.

not valid for flow through orifice

Q) $Re_1 = \frac{V_1 L_1}{\nu} = 6.59 \times 10^6$

$Re_2 = \frac{V_2 L_2}{\nu} = 6.59 \times 10^6$

→ Re, Pr, Sc are same

$Pr_1 = 0.703$ | $Sc_2 = \frac{\nu}{D_{AB}} = 0.7$

$T^*(x^*, y^*) = C_A^*(x^*, y^*)$

$\boxed{\overline{Nu_L} = \overline{Sh_L}}$

Q) Where the thickness of thermal BL is max heat transfer will be more ∴ that strip heater must supply more heat

→ heat to be supplied by the heater will be more in turbulent zone

→ h - depends on 2 things

① laminar / turbulent

② $h \propto \frac{1}{\delta} \rightarrow$ BL

ANALOGY

$\frac{\mu}{\rho}$, α , D_{AB}
$\quad\quad\ \frac{k}{\rho c_p}$

BL approximation

$$\frac{\partial u}{\partial y} >> \frac{\partial u}{\partial x} , \quad \frac{\partial T}{\partial y} >> \frac{\partial T}{\partial x} , \quad \frac{\partial C_A}{\partial y} >> \frac{\partial C_A}{\partial x}$$

$$\left| \frac{\partial^2 u}{\partial x^2} << \frac{\partial^2 u}{\partial y^2} \right|$$

→ $\frac{\nu}{VL} = \frac{1}{Re}$ → momentum transport $\quad Re = \frac{\rho V L}{\mu}$ ($\nu \to \mu/\rho$)

$$\frac{\alpha}{VL} = \frac{1}{Re} \cdot \frac{1}{Pr} \quad\quad Pr = \frac{\mu c_p}{k}$$

$$\frac{D_{AB}}{VL} = \frac{\nu}{VL} \cdot \frac{D_{AB}}{\nu} = \frac{1}{Re} \cdot \frac{1}{Sc} \quad\quad \cancel{Sc = \mu D_{AB}} \quad \cancel{Sc = \rho D}$$

$$Sc = \frac{\mu}{\rho D_{AB}}$$

Smidth number

→ Nu = nusselt number = $\frac{\text{convective heat}}{\text{conductive heat}} = \frac{hL}{k_f}$

→ $Bi = \frac{hL}{k_{solid}} = \frac{\text{conductive resistance}}{\text{convective resistance}}$ > in the solid

→ $Le = \frac{\alpha}{D_{AB}} = \frac{Sc}{Pr}$

Reynold's analogy (dynamically similar systems)

$$Pr = Sc = 1 , \quad \frac{dp^*}{dx^*} = 0$$

$$\boxed{C_f \frac{Re_L}{2} = Nu = Sh} \quad\quad \frac{C_f}{2} = St = St_m$$

$$St = \frac{Nu}{Re_L Pr} \quad\quad St_m = \frac{Sh}{Re_L Sc}$$

COLBURN ANALOGY (modified)

[j = coulburn 'j' factor]

$$\frac{C_f}{2} = St_m \, Sc^{2/3} = j_m \quad\quad 0.6 < Pr < 60$$

$$\frac{C_f}{2} = St \, Pr^{2/3} = j_H \quad\quad 0.6 < Sc < 300$$

$$\boxed{\frac{C_f}{2} = St \cdot Pr^{2/3} = St_m \cdot Sc^{2/3}}$$ → valid for laminar but can be extended to turbulent

→ $C_f = \frac{\tau_s}{\frac{1}{2}\rho V^2}$

→ easy to calculate for momentum

Boundary layers

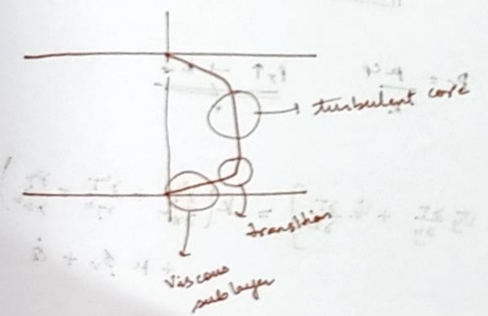
→ $v_x > v_y$ $\quad \Big| \quad \frac{\partial v_x}{\partial y} > \frac{\partial v_x}{\partial x} \quad \Big| \quad \frac{\partial^2 v_x}{\partial x^2} \simeq 0 \Big|$

→ δ, δ^* $\quad\quad \delta = \frac{5x}{\sqrt{Re_x}}$

→ $\tau_w = \frac{0.332 \rho U^2}{\sqrt{Re_x}}$

C_f = shear stress coeff $= \frac{0.664}{\sqrt{Re_x}}$
local value

C_D = drag coeff
avg value
$$\frac{F_D}{\frac{1}{2}\rho V^2 A} = C_D$$



→ Momentum integral → laminar → turbulent

→ $Nu = \frac{hL}{k}$ $\quad Sh = \frac{h_m L}{D_{AB}}$

→ Euler equation :-

$$\rho \frac{D\bar{V}}{Dt} = -\nabla p - [\nabla \cdot \bar{\tau}] + \rho g$$

$$\rightarrow \rho \left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right] + \rho g_z$$

→ $Bi << 1 \rightarrow \frac{hL}{k} << 1$

→ lump capacitance model :- gradient inside the solid is negligible at any instant of time.

$Bi >> 1 \rightarrow$ $\boxed{k \frac{\partial^2 T}{\partial x^2} = \rho C_p \frac{\partial T}{\partial t}}$

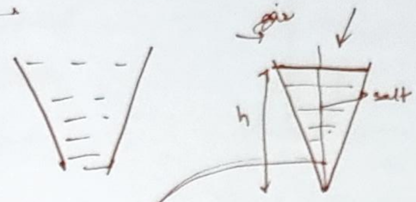
$Pr = \frac{\mu C_p}{k}$ $\quad Pr \uparrow \rightarrow k \downarrow$

$$\rightarrow \rho \hat{C_p} \left[\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right] = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \mu \phi_v + \dot{Q}$$

$$\rightarrow \frac{dC_A}{dt} + v_x \frac{dC_A}{dx} + v_y \frac{dC_A}{dy} + v_z \frac{dC_A}{dz} = D_{AB} \left[\frac{\partial^2 C_A}{\partial x^2} \cdot \text{---} \right] + R_A.$$

$$N_A = x_A (N_A + N_B) - D_{AB} \nabla C_A$$

→



cone of salt goes beyond certain level it will get corroded.

y_{salt} $\quad \frac{dm}{dt} = -h_m (C_A - C_{A\infty}) A$ $\quad$ C wrt t

m_{salt}

$y_{H_2O} = \frac{m_{H_2O}}{m_{H_2O} + m_{salt}}$

$\frac{1}{2} b \times h$