16.02

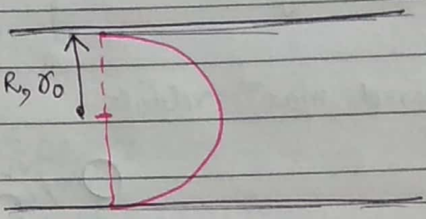
Internal Forced Convection

CONSIDERATIONS : / RESTRICTIONS

1. Pipe flow
2. Constant cross section
3. Mass flow rate is constant
4. Laminar flow

FEATURES OF FLOW:

$$\boxed{u = u_{max}\left(1-\left(\frac{r}{r_0}\right)^2\right)}$$



$$\boxed{u_{max} = 2\bar{u}}$$

$$\boxed{\bar{u} \times \pi r_0^2 \times \rho = \dot{m}}$$

$\boxed{Re \leq 2100}$ LAMINAR

$\boxed{Re \geq 10000}$ TURBULENT

Key feature of turbulence
- formation of eddies

Size of largest eddy is of the size of the size of the substance where it is flowing.

For bank of tubes, it is tube D.

Fully developed laminar flow

occurs after $0.05 \times Re_D$

$$\left(\frac{L}{D}\right)_{fd,l} = 0.05\, Re_D$$

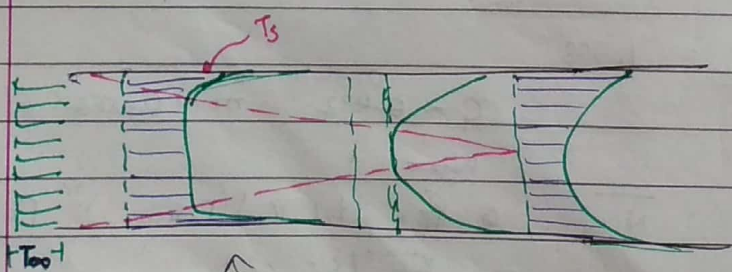
$$\Rightarrow \frac{\partial}{\partial x}\left[u(r,x)\right] = 0$$

— no change in u_r w.r.t x

$$\left(\frac{L}{D}\right)_{fd,t} > 10$$

Thermal $\quad L = 0.05\, Pr$

TEMPERATURE PROFILE



Fluid in contact with pipe reach Ts just after they enter.

$$\boxed{\frac{\partial T(r,x)}{\partial x} \neq 0}$$ ✗ not the defn

We do not have fully developed flow in heat transfer because any temp^r diff will cause heat flow until T becomes same everywhere.

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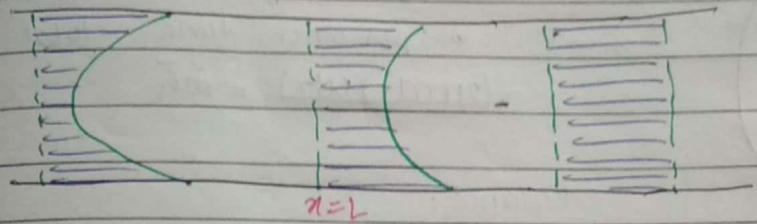


So, for heat transfer, same definition of fully developed flow will not work.
But no one is interested in this last profile because there is no heat transfer.

$T_s - T_\infty$ = ~~total~~ highest possible amount of temp^r change
$T - T_\infty$ = difference from lowest temp.

$\hat{T} = \dfrac{T_0 - T_\infty}{T_s - T_\infty}$ fraction of unaccomplished Temp change

$\dfrac{\partial}{\partial x}[\hat{T}] = 0$ — fully developed flow in heat transfer

$\underbrace{q = h(T_s - T_\infty)}_{\text{External flow}}$

$q = h(T_s - \overline{T_m})$

$\overline{T_m}$ = average temp^r at any cross section
- bulk temp^r
- Average temp^r
- Mixing cup temp^r

imagine the fluid as a jelly ~~mixing~~ mixed in cup
Avg temp^r = $\overline{T_m}$

$\therefore \boxed{\hat{T} = \dfrac{T_s(x) - T(r,x)}{T_s(x) - \overline{T_m}(x)}}$

T_s is a function of x because if we have a constant wall heat flux it should vary.

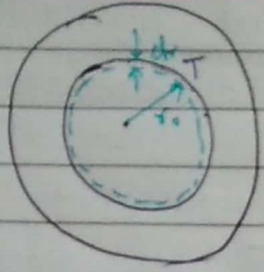
$\overline{T_m}$, T_b, T_m, $\overline{T_b}$] - different notations.

Temp diff at diff r, so h different!

$\bar{h}$ = avg enthalpy per unit mass

so, per unit time, total enthalpy

$\rho(2\pi r dr) u(r) = \dot{m}\bar{h}$



Balance:

$$\dot{m}\bar{h} = \int_{r_0=0}^{r_0} \rho\, 2\pi r(dr)\, u(r)\, C_p (T - T_{ref})$$

defining $\boxed{\bar{h} = C_p(\bar{T}_m - T_{ref})}$

If I say a stream at any cross sect^n $= \bar{T}_m$, it carries the same energy as original stream

For now, ① $T_{ref} = 0$

(ii) substitute u as a f^n of r, $\bar{u}$

$$\dot{m}(C_p)(\bar{T}_m - 0) = \int_{r=0}^{r_0} \rho(2\pi r)\, u_{max}\left(1 - \frac{r^2}{r_0^2}\right) T\, dr$$

can't be integrated now as $T = T(r, x)$

$$= \rho(2\pi) \overset{(2\bar{u})}{\cancel{u_{max}}}\, T\, C_p \int_{r=0}^{r_0} \left(r - \frac{r^3}{r_0^2}\right) dr$$

$$\dot{m}\bar{T}_m = 4\pi\bar{u}\rho T \frac{r_0^2}{4}$$

$$\bar{T}_m(x) = \frac{\pi \bar{u} \rho\, r_0^2}{\dot{m}} . T(r, x)$$

$\frac{r^2}{2} - \frac{r^4}{4r_0^2}$

$\frac{r_0^2}{2} - \frac{r_0^2}{4}$

$\frac{r_0^2}{4}$

$\dot{m} = \rho\pi$

$$\bar{T}_m = \frac{\rho(4\pi\bar{u})}{\dot{m}} \int_{r=0}^{r_0} r\left(1 - \left(\frac{r}{r_0}\right)^2\right) T(r, x)\, dr$$

$$= \frac{\cancel{\rho}(4\cancel{\pi})(\cancel{\dot{m}})}{\cancel{\dot{m}}\, \cancel{\pi} r_0^2 \cancel{\rho}} \int_{r=0}^{r_0} r\left(1 - \left(\frac{r}{r_0}\right)^2\right) T(r, x)\, dr$$

$$\boxed{\bar{T}_m(x) = \frac{4}{r_0^2} \int_0^{r_0} r\left(1 - \left[\frac{r}{r_0}\right]^2\right) T(r, x)\, dr}$$

$$\frac{\partial \hat{T}}{\partial x} = 0$$

$\hat{T} \neq f(x)$ only a f^n of r at fully developed flow

$$\Rightarrow \frac{\partial \hat{T}}{\partial r} \neq f(x)$$

$$\Rightarrow \frac{1}{(T_s(x) - T_m(x))} \cdot \left[-\frac{\partial T(r,x)}{\partial r} \right] \Bigg|_{r=r_0} \neq f(x)$$

(restricting to wall)

At wall, the only way of heat transfer is by conduction

$$q = -k \frac{\partial T(r,x)}{\partial r} \Bigg|_{r=r_0}$$

$$\frac{q}{k} = -\frac{\partial T(r,x)}{\partial r} \Bigg|_{r=r_0}$$

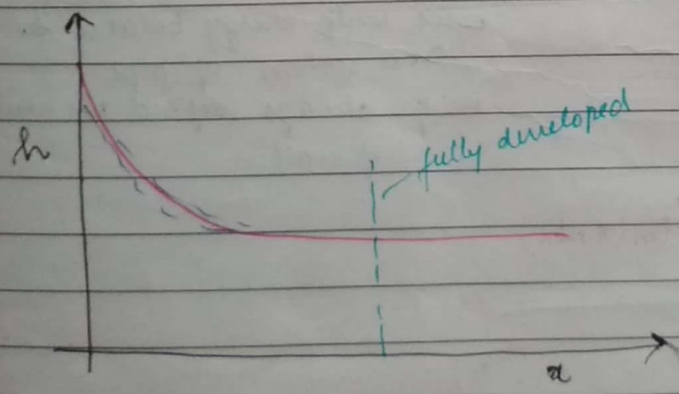
$$\frac{1}{T_s(x) - T_m(x)} \left(\frac{q}{k} \right) \neq f(x)$$

$$\frac{1}{T_s(x) - T_m(x)} \frac{h\,(T_s(x) - T_m(x))}{k} \neq f(x)$$

$$\boxed{\frac{h}{k} \neq f(x)} \;\therefore$$

Here, ~~no const~~ $h \neq f(x)$ for a fully develepod flow because no convective transfer in fully develop temp^r profile

FULLY DEVELOPED FLOW AND TEMPERATURE PROFILE



Before thermally fd, it is similar to that in external flow. After fd, it is constant w.r.t 'x'.

~~Therm~~ Hydrodynamic BL also plays a role

$$\frac{\partial}{\partial x} \left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)} \right] = 0$$

(A) $$\left\{ \left[\frac{\partial T_s(x)}{\partial x} - \frac{\partial T(r,x)}{\partial x} \right] (T_s(x) - T_m(x)) - \left[\frac{\partial T_s(x)}{\partial x} - \frac{\partial T_m(x)}{\partial x} \right] (T_s(x) - T(r,x)) = 0 \right.$$

a) constant wall tempr :

$$\frac{\partial T_s(x)}{\partial x} = 0 \Rightarrow -\frac{\partial T(r,x)}{\partial x}\left[T_s - \bar{T}_m(x)\right] = -\frac{\partial \bar{T}_m(x)}{\partial x}\left(T_s - T(r,x)\right)$$

$$\Rightarrow \boxed{-\frac{\partial T(r,x)}{\partial x}\left(T_s - \bar{T}_m(x)\right) = -\frac{\partial \bar{T}_m(x)}{\partial x}\left[T_s - T(r,x)\right]}$$

b) constant wall flux :

$$\underset{\text{const}}{q} = \underset{\text{const for fully developed.}}{h}\left(T_s(x) - \bar{T}_m(x)\right) \longrightarrow \text{(B)}$$

$$\Rightarrow \boxed{\frac{d\,T_s(x)}{dx} = \frac{d\,\bar{T}_m(x)}{dx}}$$

From (A)

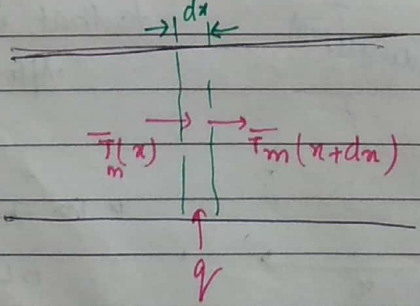
$$\left[\frac{\partial T_s(x)}{\partial x} - \frac{\partial T(r,x)}{\partial x}\right] = 0$$

replaced partial with total derivative

$$\therefore \boxed{\frac{dT_s(x)}{dx} = \frac{d\,\bar{T}_m(x)}{dx} = \frac{\partial T(r,x)}{\partial x}}$$

How $\bar{T}_m(x)$ changes with x?

- we write energy balance for different cross section of pipe
- q always defined per unit area of wall.



Writing energy balance :

$$\underbrace{\dot{m}\,C_p\,\bar{T}_m(x) + (2\pi r_0)\,dx\,q}_{E_{in}} = \underbrace{\dot{m}\,C_p\,\bar{T}_m(x+dx)}_{E_{out}}$$

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$$2\pi r_0 q = \dot{m} C_p \frac{d\bar{T}_m}{dx}$$

$$\boxed{\frac{d\bar{T}_m}{dx} = \frac{2\pi r_0 q}{\dot{m} C_p}} \quad \text{—— (C)}$$

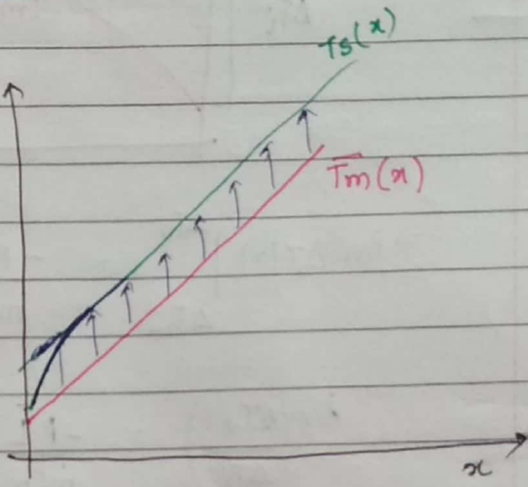
Case A: Constant wall flux

from (B)

q const

h const

$T_s - \bar{T}_m$ const diff



Near the entrance

h is more, q const.

so $(T_s(x) - \bar{T}_m(x))$ is less.

$\bar{T}_m(x)$ is const at entrance also because it has been derived from energy conservat^n which is valid everywhere. $T_s(x)$ adjusts accordingly.

For const wall temp^r:

$$q = h(T_s - \bar{T}_m(x))$$

from (C)

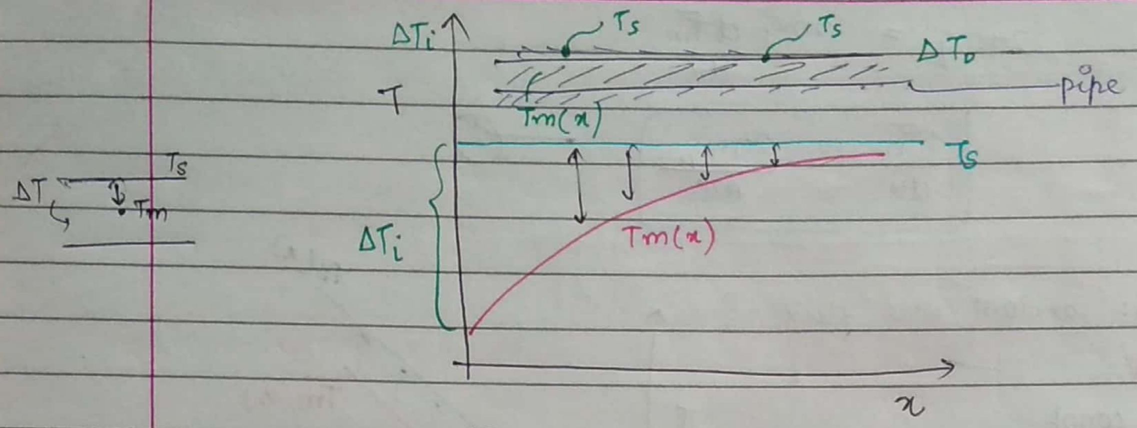
$$\frac{d\bar{T}_m}{dx} = \frac{P}{\dot{m} C_p} h(T_s - \bar{T}_m(x)) \qquad P: \text{Perimeter}$$

$$-\frac{d(T_s - \bar{T}_m(x))}{dx} = \frac{P}{\dot{m} C_p} h(x)(T_s - \bar{T}_m(x))$$

$$\Delta T(x) = T_s - \bar{T}_m(x) \Rightarrow -\frac{d(\Delta T(x))}{dx} = \frac{P}{\dot{m} C_p} h(x)\,\Delta T(x)$$

$$\frac{d\,\Delta T(x)}{\Delta T(x)} = -\frac{P}{\dot{m} C_p} h(x)\,dx$$

$$\ln|\Delta T(x)|\Big|_{x=0}^{x=L} = -\frac{P}{\dot{m} C_p} \int_{x=0}^{x=L} h(x)\,dx$$



$$\ln \frac{\Delta T_i}{\Delta T_o} = \frac{Ah}{\dot m C_p}$$

$$\Rightarrow \ln \Delta T(x) \Big|_{\Delta T_i}^{\Delta T_o} = \frac{-P}{\dot m C_p} \int_{x=0}^{x=L} h(x)\,dx$$

$$\ln\left(\frac{\Delta T_o}{\Delta T_i}\right) = \frac{-PL}{\dot m C_p}\left[\frac{1}{L}\int_{x=0}^{L} h(x)\,dx\right]$$

$$\ln\left(\frac{\Delta T_o}{\Delta T_i}\right) = \frac{-PL\bar h}{\dot m C_p}$$

PL is the surface area

$$\ln\left(\frac{\Delta T_o}{\Delta T_i}\right) = -\frac{A\bar h}{\dot m C_p}$$

$$\Delta T_o = \Delta T_i \exp\left(\frac{-A\bar h}{\dot m C_p}\right)$$

$$\Rightarrow \dot m C_p = \ln\left(\frac{\Delta T_i}{\Delta T_o}\right) \times Ah \quad \text{(x)}$$

temperature funct^n at any x is an exponential ~~function~~ decay

RATE OF HEAT TRANSFER

a) constant wall flux

$$Q_{total} = q_w \times \text{area}$$

b) constant wall temp^r

$$Q_{total} = \dot m C_p (T_{out} - T_{in})$$
$$= \dot m C_p ((T_{out} - T_s) - (T_{in} - T_s))$$
$$= \dot m C_p [\Delta T_{in} - \Delta T_{out}]$$

from (*) $Q_{total} = \frac{-A\bar{h}}{\ln\left(\frac{\Delta T_L}{\Delta T_i}\right)} = -\frac{PL\bar{h}_L}{\ln\left(\frac{\Delta T_L}{\Delta T_i}\right)}$

$$= \frac{-PL\bar{h}_L(\Delta T_{in} - \Delta T_{out})}{\ln\left(\frac{\Delta T_{out}}{\Delta T_i}\right)}$$

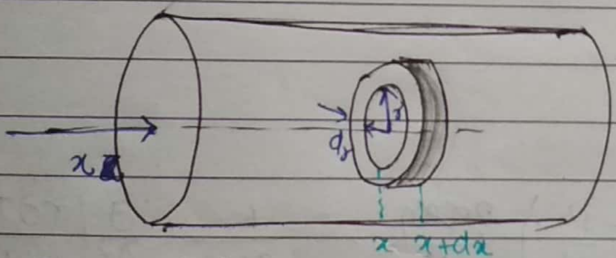
Logarithmic mean $= \frac{\text{diff}}{\ln(\text{ratio})}$

$$Q_{total} = \frac{PL\bar{h}_L(\Delta T_{in} - \Delta T_{out})}{\ln\left(\frac{\Delta T_{in}}{\Delta T_{out}}\right)} = \boxed{PL\bar{h}_L \Delta T_{lm} = Q_{total}}$$

$\bar{h}_L$ = averaged over length L.

What is $\bar{h}$?

we derive for simplest case (laminar flow, straight pipe)



Assumptions :
1. Thermodynamically and hydrodynamically FD flow
2. steady state (no T change)
3. $v = v(r)$, $T = T(r, x)$
4. constant wall flux

Energy balance

Z direction, we say only convection bcoz it is dominant.

Convective heat transfer balances conduction

Convective $E_{in}|_x = \rho[2\pi r\,dr\,u(r)]\,C_p\,T(x, r)$

$E_{out}|_{x+dx} = \rho[2\pi r\,dr\,u(r)]\,C_p\,T(x+dx, r)$

Conductive

$$\underbrace{-k\frac{\partial T}{\partial r}2\pi r\,dx\Big|_r}_{E_{in}} \underbrace{-\left[k\frac{\partial T}{\partial r}2\pi r\,dx\Big|_{r+dr}\right]}_{-E_{out}} = 0$$

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$$-\cancel{\rho} 2\pi r\, u(r) C_p \frac{\partial T(r,x)}{\partial x} + \frac{2\pi k \partial}{\partial r}\left[r \frac{\partial T(r,x)}{\partial r}\right] = 0$$

$$u(r) \frac{\partial T(r,x)}{\partial x} = \alpha \frac{1}{r} \frac{\partial}{\partial r}\left[\frac{r\partial T}{\partial r}\right]$$

$$u(r) = u_{max}\left(1-\left(\frac{r}{R}\right)^2\right) = 2\bar{U}\left(1-\left(\frac{r}{R}\right)^2\right)$$

$$\boxed{\dot{m} = \rho \pi R^2 \bar{u}} \qquad u(r) = \frac{2\dot{m}}{\rho \pi R^2}\left(1-\left(\frac{r}{R}\right)^2\right)$$

to calculate $\bar{h}$, we need this

$$\frac{2\dot{m}}{\rho \pi R^2}\left(1-\frac{r^2}{R^2}\right)\frac{\partial T(r,x)}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left[\frac{r\partial T}{\partial r}\right] \quad \text{—} (1)$$

this needs to be simplified

from assumptⁿ 4, $\dfrac{\partial T(r,x)}{\partial x} = \cancel{\dfrac{\partial \bar{T}_m}{\partial x}}\ \dfrac{d\bar{T}_m(x)}{dx}$

from (C)

$$\frac{d\bar{T}_m(x)}{dx} = \frac{2\pi R q}{\dot{m} C_p}$$

(1) can be written as

$$\frac{2\cancel{\dot{m}}}{\cancel{\rho}\cancel{\pi} R^{\cancel{2}}}\left(1-\frac{r^2}{R^2}\right)\frac{2\cancel{\pi} R q}{\cancel{\dot{m}}\cancel{C_p}} = \frac{k}{\cancel{\rho}\cancel{C_p} r}\frac{\partial}{\partial r}\left(\frac{r\partial T}{\partial r}\right)$$

$$\frac{4q_w}{KR}\left(1-\frac{r^2}{R^2}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\partial T}{\partial r}\right)$$

$$\frac{\partial}{\partial r}\left[\frac{r\partial T}{\partial r}\right] = \frac{4q_w}{KR}\left(r-\frac{r^3}{R^2}\right)$$

$$\frac{r\partial T}{\partial r} = \frac{4q_w}{KR}\int r-\frac{r^3}{R^2}\,dr$$

$$\frac{r\partial T}{\partial r} = \frac{4q_w}{KR}\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right] + C_1$$

$$\frac{\partial T}{\partial r} = \frac{4q_w}{KR}\left[\frac{r}{2}-\frac{r^3}{4R^2}\right] + \frac{C_1}{r}$$

$$T = \frac{4q_w}{KR}\left[\frac{r^2}{4} - \frac{r^4}{16R^2}\right] + C_1 \ln r + C_2$$

Boundary conditions:

i) $r = 0, \quad \frac{\partial T}{\partial r} = 0$ — typical condit^n for all cylindrical co-ordinates

ii) ~~r = R~~

$\therefore C_1 = 0$

$$T(x,r) = \frac{4q_w}{KR}\left[\frac{r^2}{4} - \frac{r^4}{16R^2}\right] + C_2(x)$$ — all x dependence on C_2

~~$q = h(T_s - T_m)$~~

Using $$\overline{T_m}(x) = \frac{4}{R^2}\int_0^R r\left(1 - \frac{r^2}{R^2}\right) T(r,x)\,dr$$

$$\overline{T_m}(x) = \frac{4}{R^2}\int_0^R r\left(1 - \frac{r^2}{R^2}\right) \cdot \frac{4q_w}{KR}\left\{\left[\frac{r^2}{4} - \frac{r^4}{16R^2}\right] + C_2(x)\right\}dr$$

$$\cancel{= \frac{4}{R^2}\left[C_2(x)\int_0^R \left(r - \frac{r^3}{R^2}\right)dr + \frac{4q_w}{KR}\int_0^R \cdots\right]}$$

$$\overline{T_m}(x) = \frac{4}{R^2}\left[C_2(x)\int_0^R\left(r - \frac{r^3}{R^2}\right)dr + \frac{4q_w}{KR}\int_0^R r\left(1 - \frac{r^2}{R^2}\right)\left(\frac{r^2}{4} - \frac{r^4}{16R^2}\right)dr\right]$$

$$= \frac{4}{R^2}\left[C_2(x)\cdot\frac{R^2}{4} + \frac{4q_w}{KR} \times \frac{7}{384}R^4\right]$$

$$\overline{T_m}(x) = C_2(x) + \frac{7}{24}\frac{Rq_w}{K}$$

$$C_2(x) = \overline{T_m}(x) - \frac{7}{24}\frac{q_w R}{K}$$ ~~= T(r,x)~~

$$\boxed{T(r,x) = \frac{4q_w}{KR}\left[\frac{r^2}{4} - \frac{r^4}{16R^2}\right] + \overline{T_m}(x) - \frac{7}{24}\frac{q_w R}{K}}$$

$$T(R,x) = \frac{4q_w}{KR}\left[\frac{R^2}{4} - \frac{R^2}{16}\right] + \overline{T_m}(x) - \frac{7}{24}\frac{q_w R}{K}$$

$$T(R,x) \cdot \frac{3q_w}{4} = \frac{11}{24}\frac{q_w R}{K} + \overline{T_m}(x)$$

$$q_w = \frac{24}{11K}\left(T(R,x) - \overline{T_m}(x)\right)R$$

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$h_x\left(\bar{T}_m(x) - \bar{T}(x,R)\right) = q_w$

$\left(T_m(x) - T(x,R)\right) h_x = \frac{24}{11}\left[T(x,R) - T_m(x)\right] \frac{k}{R}$

$$h_x = \frac{24}{11} \frac{k}{R}$$

$$= \frac{48}{11} \frac{K}{D}$$

$$Nu = \frac{h_x D}{K} = 4.3$$

$\boxed{Nu = 4.3}$ constant

Heat Transfer Correlations for internal forced convection

1.

Turbulent flow : Momentum - Heat - transfer analogy.

↓

the major vehicle for transport is eddies
so, if one rate is ~~same~~ fast, other can't be slow.
So, transport rate of one can be used to conclude the rate of other.

flux ∝ gradient

ν (mom. diffusivity)

when only molecular transport

τ Momentum flux $= -\mu \frac{\partial u}{\partial y} = -\frac{\mu}{\rho} \frac{\partial (u\rho)}{\partial y}$

q Heat flux $= -k \frac{\partial T}{\partial y} = -\frac{k}{\rho C_p} \frac{\partial (T \rho C_p)}{\partial y}$

α

$$\tau = -(\nu + \varepsilon_m) \frac{\partial (u\rho)}{\partial y}$$

$$q = -(\alpha + \varepsilon_h) \frac{\partial (T\rho C_p)}{\partial y}$$

ε : eddy diffusivity
↓
m - of momentum
h - of heat

$$\frac{\tau}{q} = \frac{\partial (u\rho)}{\partial (T\rho C_p)}$$

$[\varepsilon_m + \nu \simeq \alpha + \varepsilon_h]$

~~but~~ in eddies, $\varepsilon_m, \varepsilon_h$ are major contributors.

because rates same $(\nu \simeq \alpha)$

ρ: constant

$$\frac{\tau_w}{q_w} = \frac{\bar{u}}{C_p(\bar{T}_w - \bar{T}_m)}$$

q_w ↓ $\bar{h}(T_w - \bar{T}_m)$

$\bar{U} = du$ (0 to $\bar{u}$)
$T_w - \bar{T}_m = dT$ (T_w to $\bar{T}_m$)
↓
these are approx.

$\tau_w = f \cdot \frac{1}{2} \rho \bar{v}^2$ (wall shear stress can be expressed in terms of friction factor and avg. velocity head)

$$\frac{\frac{f \rho \bar{u}^2}{2}}{\bar{h}(T_w - \bar{T}_m)} = \frac{\bar{u}}{C_p(T_w - \bar{T}_m)}$$

$$\boxed{\bar{h} = \frac{f \rho C_p \bar{u}}{2}}$$

shows how $\bar{h}$ & f are related

coefficients for both transfer

$$\bar{Nu} = \frac{\bar{h}D}{k} = \frac{\frac{1}{2} f C_p \rho \bar{u} D \mu}{k \mu}$$

$f = 0.046\ Re^{-0.2}$ (smooth pipe)

$$\bar{Nu} = \frac{1}{2}(0.046) Re^{-0.2} \cdot Pr \cdot Re$$

$$\boxed{\overline{Nu} = 0.023\, Re^{0.8}\, Pr}$$

$Pr \approx 1$
works only for this

For turbulent flow, entry region is very small - we can ignore.
This is valid for both constant wall temp^r and constant heat flux

Ditlus - Boelter equation

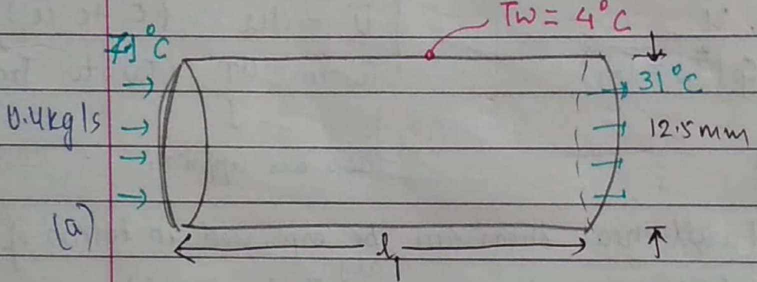
$$\boxed{Nu = 0.023\, Re^{0.8}\, P_r^{1/3}}$$

works over large range of P_r

Problems :

(1) Water @ 0.4 kg/s is to be cooled from 71 to 31°C. Which would result in less pressure drop - to run the water through a 12.5 mm dia pipe with wall temp 4°C or through a 15 mm dia pipe with wall temp^r 27°C.

(a)

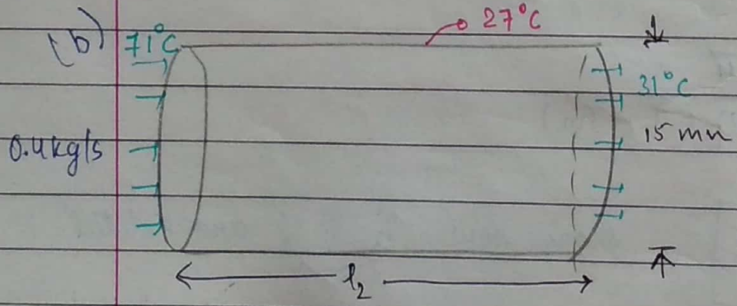


Analysis

1. Internal flow
2. Const. wall temp^r

$\dot{m} = 0.4\,kg/s$

(b)



Correlation used:

laminar HD, TD. flow

$A\,\bar{h}_L\,\Delta T_{lm} = Q_{total}$

$A\,\bar{h}_L\,\Delta T_{lm} = \dot{m} C_p \Delta T$

(1) $\Delta T_{in} = 67°C$ $\quad \Delta T_{out} = 27°C$

$$\Delta P = 4f\left(\frac{L}{D}\right)\frac{\rho \bar{v}^2}{2}$$

In FM, more dia. ~~mo~~ less ΔP

~~Here~~, we will check length of tube because $\Delta P \propto l$ (we 've heat transfer also)

→ length

means total amt. of heat transfer - given

$\bar{h} = ?$ through Nu

To find this, we need to ~~define~~ find entry length & on not take as FD. without proving.

$$\Delta T_{lm_1} = \frac{\Delta T_{in} - \Delta T_{out}}{\ln\left(\frac{\Delta T_{in}}{\Delta T_{out}}\right)} = 44.7\ ^\circ C = 317\ K$$

Tavg for property evaluation

$$Q_{total} = A\bar{h}_L \Delta T_{lm}$$

$$\dot{m} C_p \Delta T = A \bar{h}_L \Delta T_{lm}$$

for $\bar{h}_L$ we need to know reynolds no.

$$Re = \frac{D\bar{V}\rho}{\mu} = \frac{\frac{\pi D}{4} D\bar{V}\rho}{\pi D/4\,\mu} = \frac{4\dot{m}}{\pi D \mu}$$

$$T_f = \frac{T_{in} + T_{out}}{2} = \frac{10\,8}{2} = 324\ K$$

At 324 K

(a)

μ = ~~0.014036 $\times 10^{-2}$~~ 528×10^{-6}

ρ = ~~1097.72~~ 987 kg/m^3

C_p = ~~1.5766~~ ~~KJ/kg.K~~ 4.182×10^3

k = ~~70.08~~ ~~$\times 10^{-3}$~~ 645×10^{-3}

P_r = ~~3.16~~ 3.42

(b)

$T_f = 324$ K

$\Delta T_{lm} = 16.68$

k =

$Re = 7.72 \times 10^4$ | $Re = 6.43 \times 10^4$

entry length 12.5×10^{-2} m | 15×10^{-2} m

For turbulent flow, Le = 10x diameter

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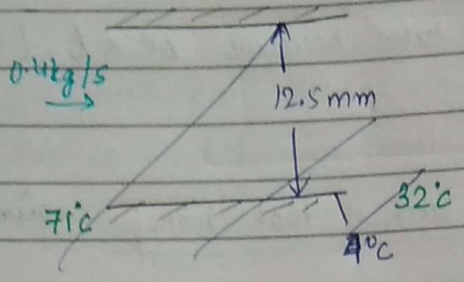
Problem-1



Hence, assuming Le less than length required, hence FD flow

FD turbulent pipe flow

Correlation: Dittus Boelter equat^n

Pr other values than

Dittus eq^n $\boxed{Nu = 0.023\, Re^{0.8} Pr^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14}}$

$\frac{\mu_{bulk}}{\mu_{wall\ temp^r}}$

viscosity correction for highly viscous fluids.

Co eff.

FD flow, h doesn't change with length

$Nu = 281.73$

$\bar{h} = 1.45 \times 10^4$

$q_{tot} = \bar{h} A (\Delta T_{lm})$

66912

$A = 0.10485$

$2\pi D L_1 = 2.7$ m

$\boxed{f = 0.046 \times Re^{-0.2}}$ $f = \frac{2\bar{h}}{\rho C_p \bar{u}}$

$\Delta P = 4f\left(\frac{L}{D}\right)\frac{\rho \bar{v}^2}{2}$ $= \frac{2\bar{h}\,\pi D^2 \rho}{\rho C_p \dot{m}\, 4}$

$= 4.84 \times 10^{-3}$

$\bar{u} = \frac{4\dot{m}}{\pi D^2 \rho} = 3.302$

$f = 4.84 \times 10^{-3}$

$\Delta P = 22.47$ KPa

$Nu = 243.4$

$\bar{h} = 1.046 \times 10^4$

$66912 = \bar{h} A (\Delta T_{lm})$

$A = 0.38348$

$\pi D L_2 = 8.14$ m

$f = 5.02 \times 10^{-3}$

$\Delta P =$

$\bar{u} = \frac{4\dot{m}}{\pi D^2 \rho} =$

$\Delta P = 28.143$ KPa

Why higher?
Fluid friction is over compensated by long length.
[friction dominates]

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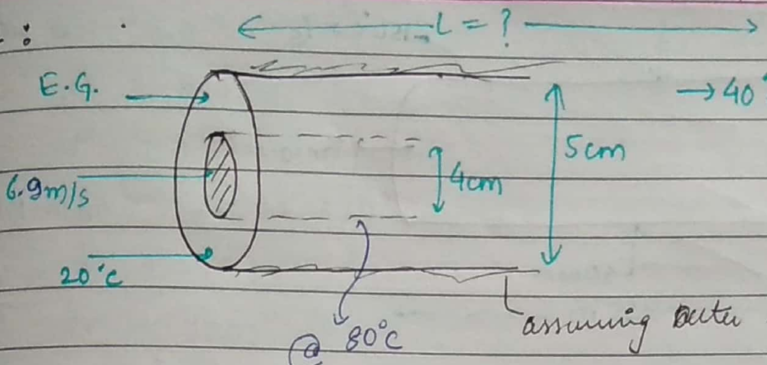
Problem 2 :



$T_f = 30°c$
$= 303\,K$

$D = D_h = \dfrac{4\times\pi(5^2-4^2)\times10^{-4}}{4\times\pi\,\frac{(5+4)}{2}\times10^{-2}}$

$= 1\times10^{-2}\,m$

$\boxed{L_e = 10\times D_e} = 0.1\,m$

$Re = \dfrac{u\rho D_h}{\mu} = 4897$

$\rho = 1114.4\ kg/m^3$

$C_p = 2.415\times10^3$

$\mu = 1.57\times10^{-2}$

$k = 252\times10^{-3}$

$Pr = 151$

$(T=353K)\ \mu_w = 0.323\times10^{-2}$

Not a laminar flow, not a FD flow.

Transition region.

Case I : with viscosity factor

II (without)

$Nu = 109.65$

$h = \cancel{3447.75}\ \boxed{2763.18}$

$Nu = 136.815$

$h = \cancel{3134}\ \boxed{3447.75}$

$\dot m = \rho\bar u\,\dfrac{\pi D^2}{4}$

$Q = \dot m C_p\Delta T = \rho\bar u\,\dfrac{\pi(D_1^2-D_2^2)}{4}C_p(20) = 29169.38$

$\dfrac{40}{\dfrac{T_o-T_i}{\ln\frac{40}{60}}}$

$Q = \bar h A\Delta T_{lm}$

$\Delta T_{lm} = \dfrac{(20-80)-(40-80)}{\ln\left(\dfrac{20-80}{40-80}\right)}$

$29169.38 = 27363.18\times4\pi(10^{-2})\times L_1\times(+49.33)$

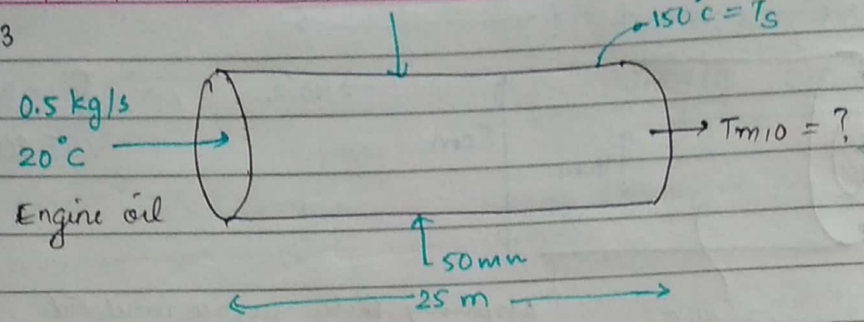
$L_1 = \cancel{30.6\,m}\ 15.8\ m$

$L_2 = \cancel{24.5\,m}\ 12.6\ m$

There is significant difference b/w lengths due to viscosity correction

Hence, entry length insignificant

Problem-3



$\dot m = \frac{\pi D^2}{4} \bar u \rho$

$\bar u = \frac{4\dot m}{\pi D^2 \rho}$

$= 0.292$ m/s

$q_{tot} = \dot m C_p \Delta T$

$h A \Delta T_{lm} =$

(i) T_f = Let $T_{m,o} = 80°C$

$T_f = 323$ K $\simeq 320$ K

(423 K) $\mu_w = 0.564 \times 10^{-2}$

$\mu = 14.1 \times 10^{-2}$

$\rho = 871.8$

$C_p = 1.993 \times 10^3$

$k = 143 \times 10^{-3}$

$Pr = 1965$

$L_e = 0.5$ m

$Re = \frac{\rho \bar u D}{\mu} = 90$

$\boxed{x_{fd,h} \quad L_e = 0.05 \times Re \times D}$ $= \frac{x}{D}$

$\boxed{x_{fd,t} \quad L_e = 0.05 \times Re \times Pr \times D}$

$= 442$ m

$L_e = 0.225$

flow is hd fully developed but not thermally

Kay's correlation :

$$\overline{Nu}_D = 3.66 + \frac{0.0668\, Gz_D}{1 + 0.04\, Gz_D^{2/3}}$$

$= 11.53$

$Gz_D = \left(\frac{D}{L}\right) Re_D Pr$

$= 353.7$

$h = 32.98$

$$\Delta T_L = \Delta T_i \exp\left(-\frac{P \times L \times \bar h_L}{\dot m\, C_p}\right)$$

$$(T_o - 150) = (20 - 150) \exp\left(-\frac{\pi (50 \times 10^{-3}) \times 25 \times 33}{0.5 \times 1.993 \times 10^3}\right)$$

$T_o = 35.85$ °C

(ii) Let $T_{m,o} = \frac{80 + 36}{2} = 331 \simeq 330$ K

150
293

(423K)

$\mu_w = 0.504 \times 10^{-2}$

$\mu = 8.36 \times 10^{-2}$

$\rho = 865.8 \ kg/m^3$

$C_p = 2.035 \times 10^3 \ J/kg.K$

$k = 141 \times 10^{-3}$

$Pr = 1205$

$Re = \frac{\rho \bar{u} D}{\mu} = 151.2$

$G_{zD} = \left(\frac{D}{L}\right) Re_D . Pr$

$= 364.4$

$\overline{Nu}_D = 11.665$

$h = 32.896$

$\Delta T_L \Rightarrow T_o - 150 = (20-150) \exp\left(- \pi (50 \times 10^{-3}) \times 25 \times 32.896 \right.$

$T_o = 35^\circ C$

$\therefore \underline{T_o = 36^\circ C}$

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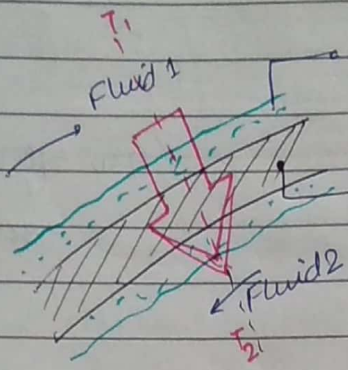
Process Heat Transfer
D. Q. Kern

23.03.18

Heat Exchanger



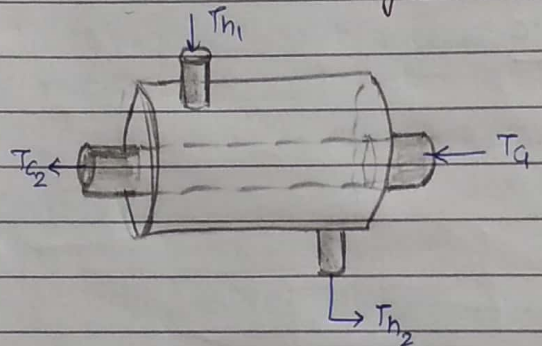
deposit^n at both side of wall (can be ignored at some places but not always)

metal surface

It is a like a composite solid wall with one side one fluid flowing and other side, other fluid.

Types

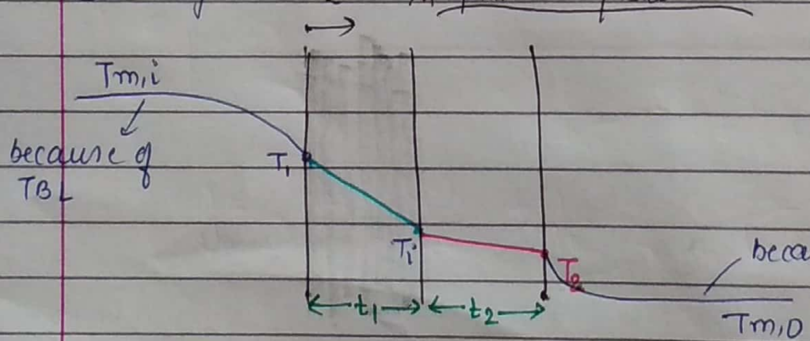
1. Double pipe heat exchanger
2. Shell and tube heat exchanger.



generally, corroding liquids are sent through inner pipe. so that it corrodes only one pipe

@ steady state

A plane composite wall



T_{mi}, T_{mo} are mixing cup temp^r, average temp^r.

(i) heat flux will be same at all z

$$q_z = h_i(T_{mi} - T_1)$$

$$= \frac{-k_1}{t_1}(T_i - T_1)$$

$$= \frac{-k_2}{t_2}(T_2 - T_i)$$

$$= h_o(T_2 - T_{mo})$$

$T_{mi} - T_1 = \frac{q_z}{h_i}$

$T_1 - T_i = \frac{q_z t_1}{k_1} = \frac{q_z}{(k_1/t_1)}$

$T_i - T_2 = \frac{q_z}{(k_2/t_2)}$

$T_2 - T_{mo} = \frac{q_z}{h_o}$

$$T_{mi} - T_{mo} = q_z \left[\frac{1}{h_i} + \frac{1}{\left(\frac{k_1}{t_1}\right)} + \frac{1}{\left(\frac{k_2}{t_2}\right)} + \frac{1}{h_o} \right]$$

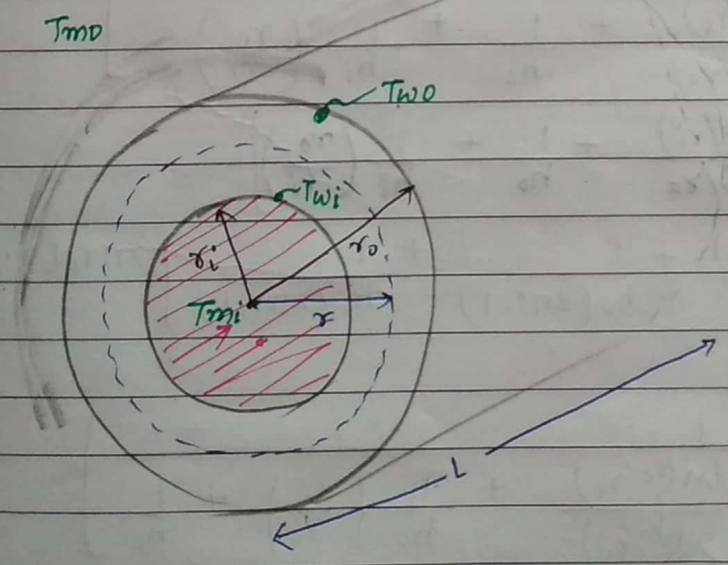
$$\boxed{q_z = \frac{(T_{mi} - T_{mo})}{\frac{1}{h_i} + \frac{1}{\left(\frac{k_1}{t_1}\right)} + \frac{1}{\left(\frac{k_2}{t_2}\right)} + \frac{1}{h_o}}}$$

$$\boxed{q_z = \underline{U}(T_{mi} - T_{mo})}$$

Defining eq^n for U — U : overall heat transfer coeff.

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{\left(\frac{k_1}{t_1}\right)} + \frac{1}{(k_2/t_2)} + \frac{1}{h_o}$$

For curved wall (pipe), we cannot keep same flux, but total heat transfer is known.



$$\boxed{Q = h_i (T_{mi} - T_{wi})(2\pi r_i L)} \quad \text{— (A)}$$

$$\frac{Q}{A} = -k\frac{dT}{dr}$$

$$Q = (2\pi r L)(-k)\frac{dT}{dr}$$

$$\int_{r_i}^{r_o} \frac{1}{r} dr = -\frac{2\pi k L}{Q} \int_{T_{wi}}^{T_{wo}} dT$$

$$Q \ln\frac{r_o}{r_i} = -(2\pi k L)(T_{wo} - T_{wi})$$

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(A) contd..

$$Q = \frac{1}{\ln(r_o/r_i)} - (T_{wo} - T_{wi})\,(2\pi K L)$$

$$\text{or}\quad Q = \frac{1}{\ln(r_o/r_i)}\,(T_{wi} - T_{wo})\,(2\pi K L) \quad \text{—— (B)}$$

$$\text{or}\quad Q = h_o\,(2\pi r_o L)\,(T_{wo} - T_{mo}) \quad \text{—— (C)}$$

$$\text{or}\quad Q = h_i\,(T_{mi} - T_{wi})\,(2\pi r_i L) \quad \text{—— (A)}$$

We need to refer it to some area.
Which area to choose?

$$Q = U_o\,(T_{mi} - T_{mo})\,A_o \quad = U_i\,(T_{mi} - T_{mo})\,A_i$$

$$\cancel{3Q} = Q\left[\frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi K L} + \frac{1}{h_o(2\pi r_o L)} + \frac{1}{h_i(2\pi r_i L)}\right] = (T_{mi} - T_{mo})$$

$$Q = \cancel{Q}\left[\frac{\ln(r_o/r_i)}{2\pi K L} + \frac{1}{h_o(2\pi r_o L)} + \frac{1}{h_i(2\pi r_i L)}\right] \times 2\pi r_o L\; U_o$$

$$U_o = \left[\frac{1}{\frac{\ln(r_o/r_i)}{(K/r_o)} + \frac{1}{h_o} + \frac{1}{h_i}\left(\frac{r_o}{r_i}\right)}\right]$$

$$\boxed{\frac{1}{U_o} = \frac{\ln(r_o/r_i)}{K/r_o} + \frac{1}{h_o} + \frac{1}{h_i}\left(\frac{r_o}{r_i}\right)}$$

$$\cancel{Q} = \cancel{Q}\,U_i\left[\frac{\ln(r_o/r_i)}{2\pi K L} + \frac{1}{h_o(2\pi r_o L)} + \frac{1}{h_i(2\pi r_i L)}\right] 2\pi r_i L$$

$$U_i = \left[\frac{1}{\frac{\ln(r_o/r_i)}{(K/r_i)} + \frac{1}{h_o}\left(\frac{r_i}{r_o}\right) + \frac{1}{h_i}}\right]$$

$$\boxed{\frac{1}{U_i} = \frac{\ln(r_o/r_i)}{(K/r_i)} + \frac{1}{h_o}\left(\frac{r_i}{r_o}\right) + \frac{1}{h_i}}$$

This Applies to a diffrential area as well all $L \to dL$

$$dQ = U_o(T_{mi} - T_{mo})\,dA_o = U_i(T_{mi} - T_{mo})\,dA_i$$

If very thin wall, $r_o \approx r_i$

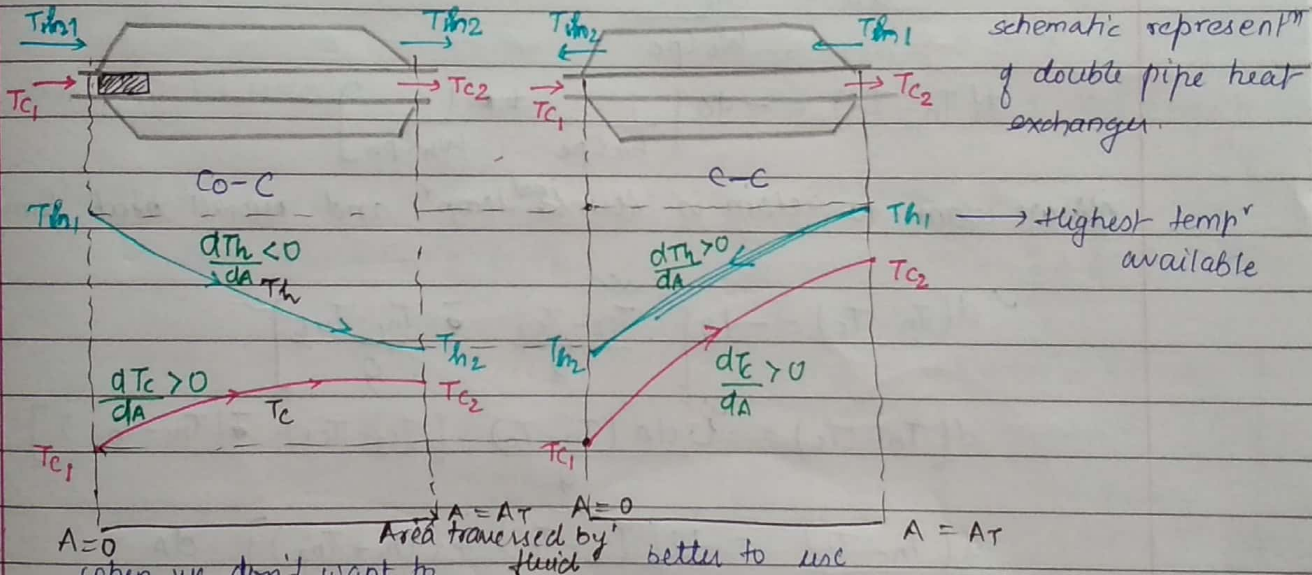
$$\therefore \frac{1}{U_o} = \frac{1}{U_i} = \frac{1}{h_o} + \frac{1}{h_i}$$

If wall is not thin, but highly conducting $(k \to \infty, \frac{1}{k} \to 0)$

$$\frac{1}{U_o} = \frac{1}{h_i(r_i/r_o)} + \frac{1}{h_o}$$

Temperature gradient in CC & CO-C heat exchanger

counter current — co-current



when we don't want to heat it much

better to use when we want to reach highest possible temp^r

can be length also

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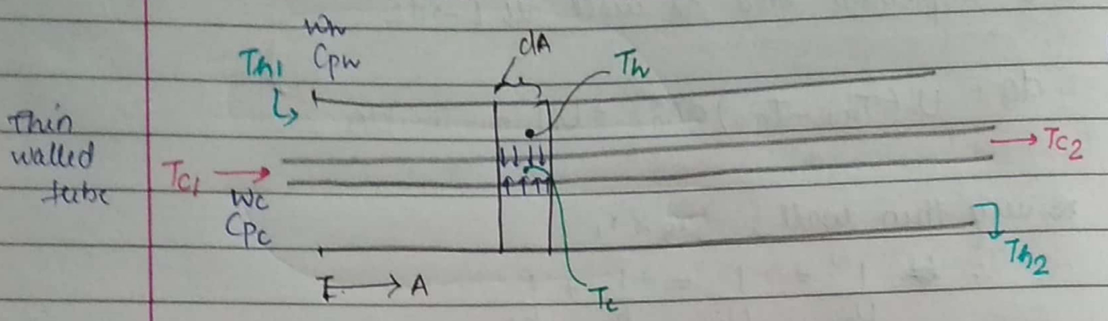
Properties & U are evaluated at some avg. temp & can be treated as constant

Assume
(i) C_{ph}, C_{pc} to be constant
(ii) we take some avg temp where properties remain const.
$\Rightarrow$ U constant



$$\boxed{dQ = W_c C_{pc} dT_c = -W_h C_{ph} dT_h} \boxed{= U(T_h - T_c)\,dA}$$

$$\boxed{Q = W_c C_{pc}(T_{c2} - T_{c1}) = W_h C_{ph}(T_{h1} - T_{h2})}$$

$$dT_h = \frac{-dQ}{W_h C_{ph}}$$

$$dT_c = \frac{dQ}{W_c C_{pc}}$$

$$d(T_h - T_c) = -dQ\left[\frac{1}{W_c C_{pc}} + \frac{1}{W_h C_{ph}}\right]$$

All we want to retain is terminal temp^r and overall heat transfer

$$d(T_h - T_c) = -dQ\left[\frac{T_{c2} - T_{c1}}{Q} + \frac{T_{h1} - T_{h2}}{Q}\right]$$

$$d(T_h - T_c) = -\frac{U\,dA\,(T_h - T_c)}{Q}\cdot\left[(T_{c2} - T_{c1}) + (T_{h1} - T_{h2})\right]$$

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = -U\left[(T_{c2} - T_{c1}) + (T_{h1} - T_{h2})\right]\cdot\frac{dA}{Q}$$

$$Q\int_{\Delta T_A = 0}^{\Delta T_{AT}} \frac{d(T_h - T_c)}{(T_h - T_c)} = -U\int_{A=0}^{A_T}\left[(T_{h1} - T_{c1}) - (T_{h2} - T_{c2})\right]dA$$

$$Q \ln\frac{\Delta T_{AT}}{\Delta T_{A0}} = UA_T(\Delta T_{AT} - \Delta T_{A0})$$

$$Q = UA_T\left(\frac{\Delta T_{AT} - \Delta T_{A0}}{\ln\left(\frac{\Delta T_{AT}}{\Delta T_{A0}}\right)}\right)$$

$$\boxed{Q = UA(LMTD)}$$

Log. mean temp diff.

valid for CC & Co-C exchanger

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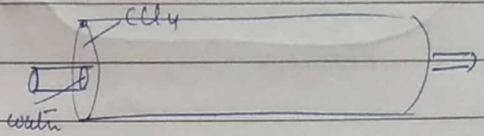
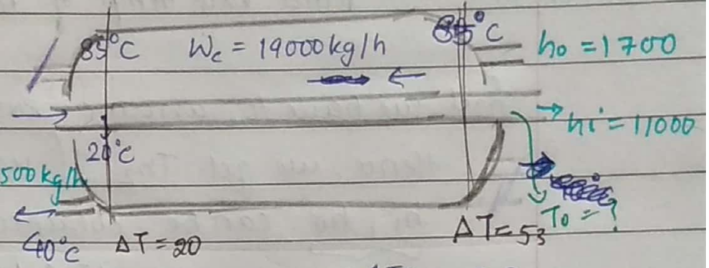
Q. CCl_4 flowing @ 19000 kg/h is to be cooled from 85°C to 40°C using 13,500 kg/h of CW available @ 20°C. CCl_4 flows outside of a ~~doo~~ double pipe $h_o = 1700$ W/m² °C. Wall resistance may be neglected, $h_i = 11000$ W/m².K. What is the area required? C-C

$C_p\ CCl_4 = 0.837 \frac{KJ}{kg \cdot K}$



$C_{pw} = \cancel{1} \ 4.18 \frac{KJ}{kg \cdot K}$



Assuming thin pipe $\left\{ U = \left(\frac{1}{h_o} + \frac{1}{h_i}\right)^{-1} \right.$

$\Delta T_{lm} = \frac{\Delta T_{in} - \Delta T_o}{\ln\left(\frac{\Delta T_{in}}{\Delta T_o}\right)}$

$\Delta T_{in} = 65$

$\Delta T_o = 20$

$C_{p,CCl_4} = 0.837$ KJ/kg.K

$C_{p,H_2O} = 4.2$ KJ/kg.K

$Q = UA\ (LMTD)$

$\cancel{19000}\ W_{CCl_4} * C_p CCl_4 * (85-40) = Q = 715635 \times 10^3$

$\cancel{18}\ W_w * C_w * (T_o - 20) = Q \Rightarrow T_o = 32.68$ °C

$U_i = \left(\frac{1}{h_i} + \frac{1}{h_o}\right)^{-1} = \cancel{667.857}\ 1472.441$ (Thin tube, no metal res.)

$LMTD = \frac{85-40}{\ln\frac{65}{20}} = \cancel{38.18}\ 33.6$

$A_i = 2\pi r_i L = \pi D_i L$

$\frac{10^3 \times 715635}{3600} = \cancel{667.857}^{1472.44} (\cancel{38.18}^{33.6}) A$

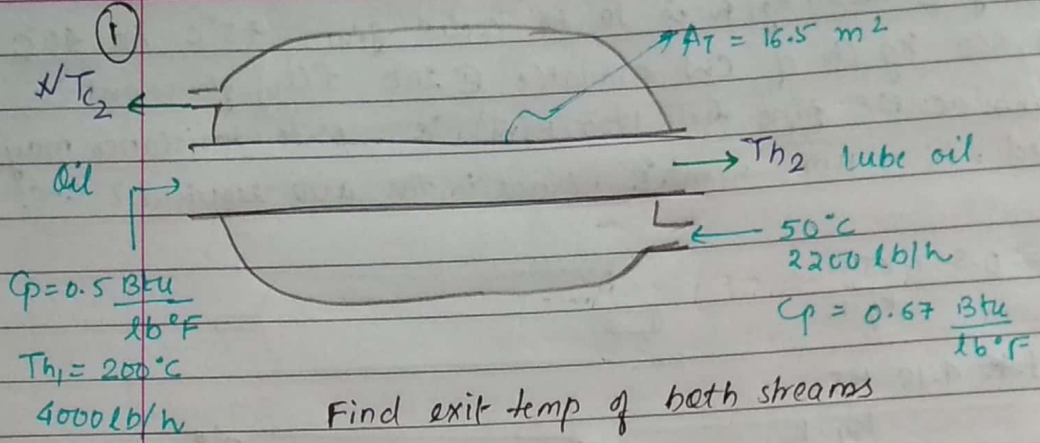
$A = \cancel{14.465} \times 10^3$ m²

$A = \boxed{4.018 \text{ m}^2}$

Q.

1 Btu = 1056 Joules

(1)



Find exit temp of both streams

first we have to assume one of exit temp's (let us assume T_{c_2})

Hence, we get T_{h_2} (using $Q = mc_p \Delta T$)

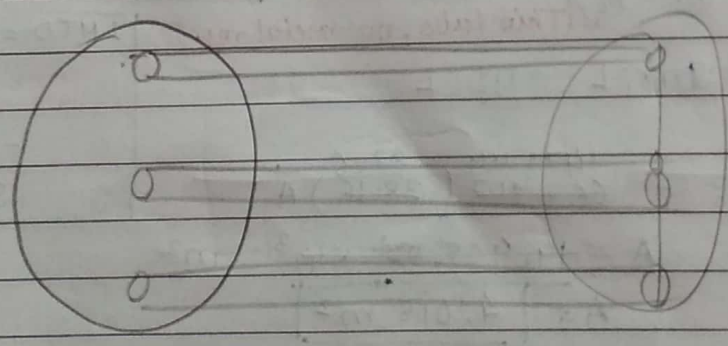
h_i, h_o can be obtained (using ext, internal flow concept)

$\underbrace{h_i, h_o}_{U}$ $Q = U A_T LMTD$.

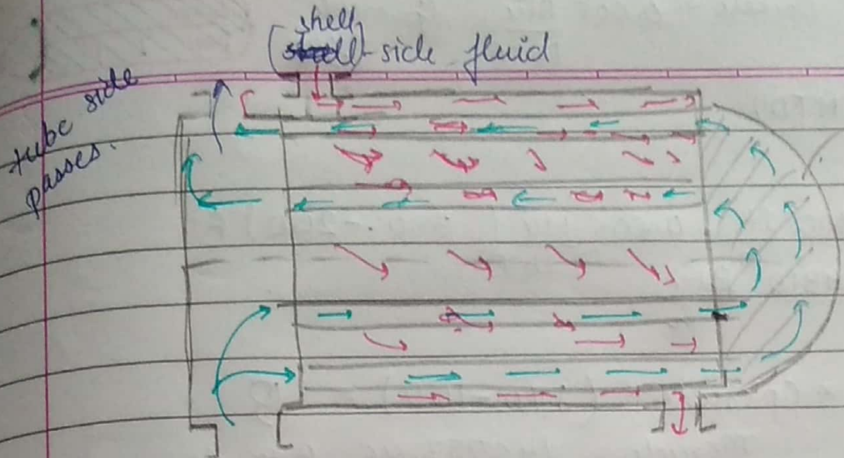
h_i → internal flow $\quad$ if known → T_{mo}, T_{mi} known

we get T_f → all properties

Longer pipe – sagging } drawbacks.
bends – leakage

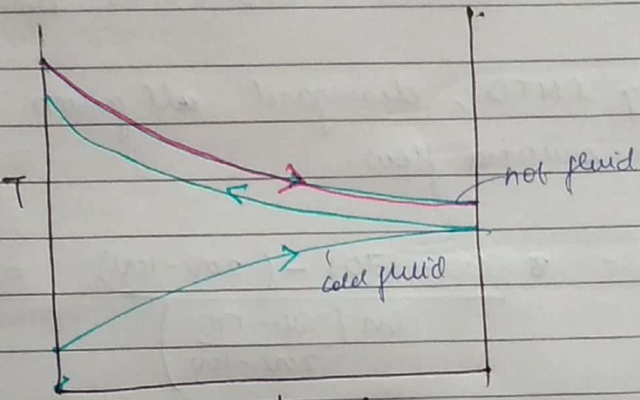


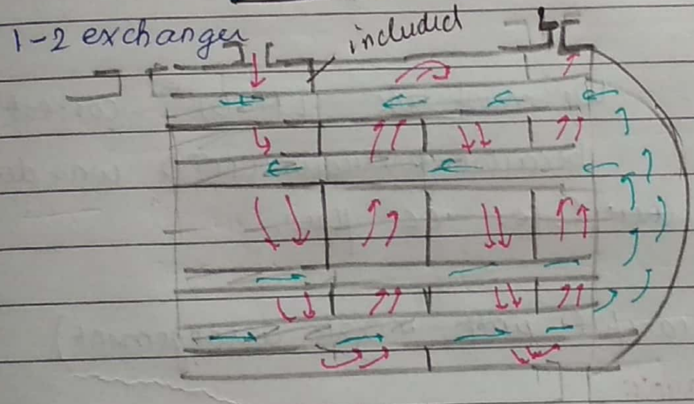
Velocity becomes less if all tubes are used for entire Liquid.
to avoid this, we ~~use~~ do not use all tubes.



Here, we have both co c & cc flow

Multipass exchangers
- we have passes
- we've mixture of parallel & counter flow.





turbulence increases
heat exchange increases

nozzle arrangements also included with multipass exchanges

(2) 43,800 lb/h of 42° API Kerosene leaves the distillation column at 390°F. This stream is to be cooled at 200°F using 34°API crude available at 100°F. Crude is to be heated to 170°F. What is the area required?

i) Shell side - Kerosene

ii) Nozzle arrangement as shown

We need shell & tube exchanger.

°API – API gravity

$C_{p,kero} = 0.605 \frac{Btu}{lb°F}$, $C_{p,crude} = .490$

(1) $Q = UA(LMTD)$

$$Q = 43800 \frac{lb}{h} \cdot 0.605 \frac{Btu}{lb°F} (390 - 200)°F$$

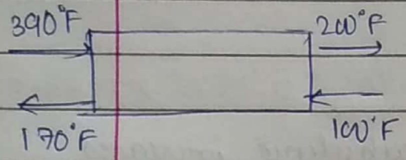
$$Q = 5034810 \frac{Btu}{h}$$

$$\dot{m}_{crude} * C_{p\,crude} * (170 - 100) = Q$$

$$\dot{m}_{crude} = 146787.46 \ \frac{lb}{h}$$

While taking LMTD, disregard all given info. Assume it to be due to counter flow.

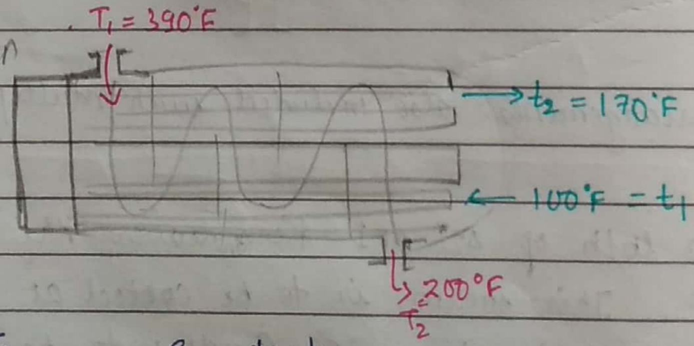
$$LMTD = \frac{(390-170) - (200-100)}{\ln\left(\frac{390-170}{200-100}\right)} = 152.196\ °F$$



We must use $(LMTD)^*$ – correctⁿ factor because previous LMTD was derived using ~~cou~~ Co/Cor CC flow.

See chart (be careful with nozzle arrangement)

~~Chan~~ given



$$R = \frac{T_1 - T_2}{t_2 - t_1} = 2.7143 \qquad S = \frac{t_2 - t_1}{T_1 - t_1} = 0.2414$$

F_T (Tempʳ diff factor) = 0.925

$$(LMTD)_{actual} = (LMTD) * F_T$$

$h_i \rightarrow$ internal flow

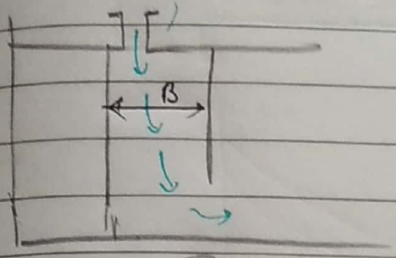
$h_o \rightarrow$ neither Co-C nor C-C

so, we use diff. correlation.

j_H = coulbour J factor

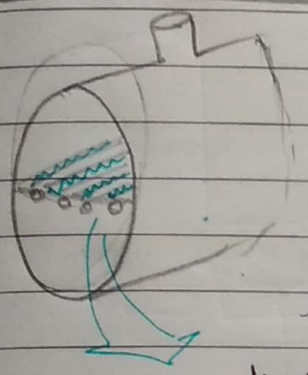
$\Downarrow$

we need to obtain Re



$$Re = \frac{D \bar{V} \rho}{\mu} = \frac{D_{eq} * \bar{V} \rho * (\text{Flow cross sect}^n)}{\mu * (\text{Flow cross-sect}^n)}$$

$$= \frac{D_{eq} * \dot{m}}{\mu * (\text{Flow cross sect}^n)}$$

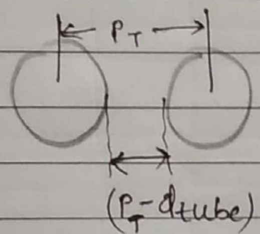


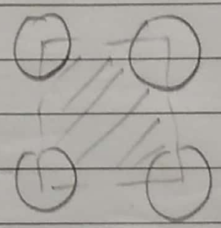
Assuming cross flow

$$\text{Flow area} = (P_T - d_{tube}) * \#\text{no. of tube}$$

$$= (P_T - d_{tube}) * \left(\frac{\text{shell dia}}{P_T}\right)$$

To compensate for this assumpt^n we include correct^n factor in Deq





$$D_{eq} = \frac{4\left[(P_T)^2 - \pi \frac{(d_t)^2}{4}\right]}{\pi d_t}$$