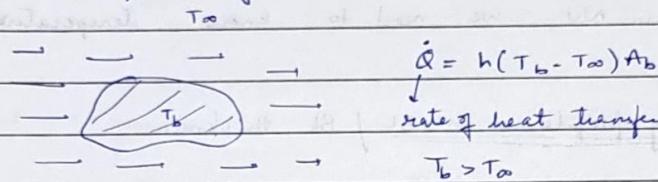


HEAT TRANSFERConvective heat transfer

- Newton's law of cooling



h = heat transfer coefficient

- h = function of flow conditions i.e. if fluid flows at higher velocity, cooling is faster

Fluid flow

external flow

internal flow

BL grows indefinitely

BL grows from the surfaces and merge to form fully developed flow

- pump, fan, blower can be used to generate fluid flow.
↳ forced convection

- If the flow is generated by density difference \rightarrow free convection (natural)

- If the fluid is stationary, heat transfer occurs due to conduction.

- at S-T interface, conduction = convection at steady state.

$$-k_f \frac{dT}{dy} \Big|_{y=0} = h(T_w - T_\infty) \quad \text{wall}$$

Non-dimensionalize

$$\hat{T} = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\hat{y}_t = \frac{y_t}{S_T}$$

$$\frac{d\hat{T}}{d\hat{y}_t} \Big|_{\hat{y}_t=0} = \frac{h \delta_T}{k_f} \quad \text{Nusselt number}$$

intensive \rightarrow independent of mass

Nusselt number:

dimensionless temperature gradient at the solid wall

dimensionless heat transfer coefficient

To obtain Nu , we need to know temperature profile

Velocity profile in the BL / BL thickness

Boundary integral method is used

dimensionless form \rightarrow decrease the no. of variables, ~~PDE to ODE~~
PDE to ODE

$\delta(x) = \text{momentum BL thickness} = f(Re_x)$ $Re_x = \frac{U_{\infty} x}{\nu}$

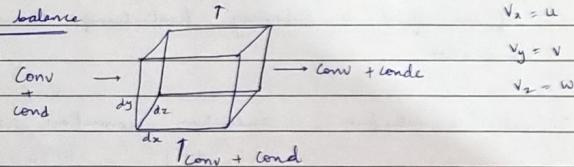
continuity, momentum eqns were used

Thermal boundary layer

continuity, momentum, energy eqns

Neglect viscous dissipation \rightarrow energy generation

Energy balance



$$\rightarrow \text{In: } \left[\rho V_x c_p T/x + \left(-k \frac{\partial T}{\partial x} \right)_x \right] dy dz + \left(\rho V_y c_p T/y + \left(-k \frac{\partial T}{\partial y} \right)_y \right) dx dz + \left(\rho V_z c_p T/z + \left(-k \frac{\partial T}{\partial z} \right)_z \right) dy dx$$

Out: same $|_{x+d_x} -$

generation: $\dot{q} dx dy dz$

accumulation: $\frac{\partial}{\partial t} (\rho dx dy dz c_p T)$

in - out + gen = accu

$\frac{\partial T}{\partial x} \neq 0$

$$\frac{dT}{dx} = \frac{U T|_x - U T|_{x+dx}}{dx} + \frac{V T|_y - V T|_{y+dy}}{dy} + \frac{W T|_z - W T|_{z+dz}}{dz} + \frac{\dot{q}}{c_p}$$

$$\frac{1}{c_p} \left(\frac{k \frac{\partial T}{\partial x}|_x - k \frac{\partial T}{\partial x}|_{x+dx}}{dx} \right) + \frac{\dot{q}}{c_p} = \frac{\partial T}{\partial t} + \frac{U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} + W \frac{\partial T}{\partial z}}{\nu} = \frac{k}{c_p} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{\dot{q}}{c_p}$$

$$\frac{\partial T}{\partial t} + \frac{U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} + W \frac{\partial T}{\partial z}}{\nu} = \frac{k}{c_p} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{\dot{q}}{c_p}$$

$$\lambda = \frac{k}{c_p}$$

(heat diffusion cap) $\rightarrow \frac{k}{c_p}$

Volume generation

$\Pr = \gamma = \frac{\text{concentration diffusivity}}{\text{thermal diffusivity}}$

$$\rightarrow b_r(x) = \frac{b(x)}{(f_r)^{1/2}}$$

thermal BL thickness

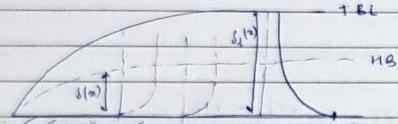
flow w/o longitudinal

$$\rightarrow Nu = C Re^m Pr^n$$

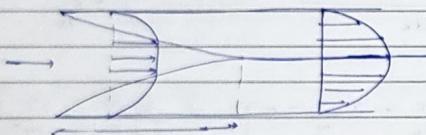
b) correlation depends on laminar or turbulent.

Internal forced convection

Flat plate



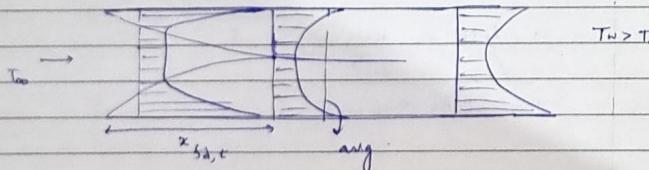
Internal forced convection



$x_{\text{hyd},h}$ = hydrodynamic entry length

concept of T_{∞} is not valid. mean temperature at a given mass section is used to replace T_{∞} .

T_m = mean temperature or mixing cup temperature



$$\rightarrow q = h(T_s(x) - T_m(x)) \rightarrow \text{Newton's law for internal flow}$$

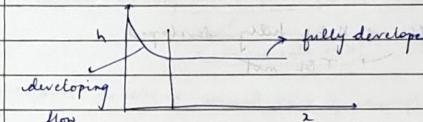
\rightarrow Avg temp T along the length of the tube $\therefore \frac{dT_m(x)}{dx} \neq 0$
 $\frac{dT(x,x)}{dx} \rightarrow \text{changes}$

$T(x,y)$ and $T_m(x)$ must be there while calculating fully developed condition.

$$\hat{T} = \frac{T_s(x) - T(x,x)}{T_s(x) - T_m(x)}$$

$$\left(\frac{\partial \hat{T}}{\partial x} \right) = 0 \rightarrow \text{fully developed condition}$$

\rightarrow Thermally fully developed $\rightarrow h = f(x)$



$$h \propto \frac{1}{\delta(x)}$$

\rightarrow fully developed flow (for const heat flux)

$$(T_w(x) - T_m(x)) = \frac{q}{h}$$

$$\frac{dT_w(x)}{dx} = \frac{dT_m(x)}{dx} = \frac{\partial T(x,x)}{\partial x}$$

\rightarrow Energy balance

valid for all

$$2\pi x dx h (T_w(x) - T_m(x)) = -mc_p T_m'(x) + mc_p T_m'(x+dx)$$

$$dT_m(x) = 2\pi x v_n$$

$$\rightarrow Q_{\text{dot}} = \dot{m} c_p (T_{m,0} - T_{m,i})$$

$$\Delta T = T_w - T_m$$

$$\Delta t_{\text{dot}} = \dot{m} c_p (\Delta T_i - \Delta T_o)$$

$$\frac{\Delta T_o}{\Delta T_i} = \exp \left(- \frac{A + h_L}{m c_p} \right)$$

$$\Delta t_{\text{dot}} = A \bar{h}_L \Delta T_{\text{LM}}$$

$$A = 2\pi r_0 l$$

$$\rightarrow \text{hydraulic dia} = \frac{\text{wetted area}}{\text{wetted perimeter}}$$

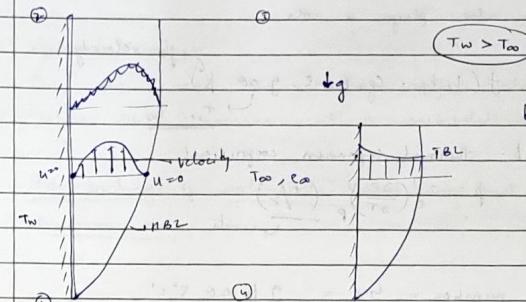
\rightarrow Thermal entry length \rightarrow NBL fully developed
 \rightarrow TBL not

$$h_2 = \text{heat transfer coefficient} = \frac{D}{x} \text{ Re}^{0.8}$$

\rightarrow Dittus Boelter eqn \rightarrow smooth circular tube \rightarrow turbulent flow
 \downarrow calculate Nu.
 for both const surface temp and heat flux

\rightarrow HTC :- larger the coeff, more heat will be transferred.
 4 cases of heat transfer

NATURAL CONVECTION (Bouyancy is imp. driving force)



because density gradient, internally there is some pressure gradient \therefore flow occurs

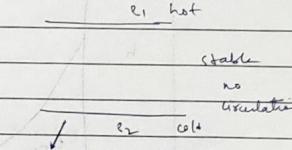
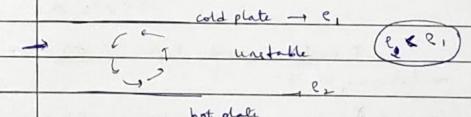
\rightarrow fluid gets heated up and rises.
 \downarrow gravity.
 \rightarrow Variable density incompressible flow
 (volumetric strain rate is 0)
 \checkmark circulation
 \rightarrow At (2) the fluid has no more space to rise
 \rightarrow it goes to (3)
 when the heating is over at (3) gravity pulls it down.

Boussinesq approximation approx

density variation is only for body them only and not other

force has to satisfy continuity

quiescent - inactive period



\rightarrow Because of density diff in will rise.
 of fluid.

→ fluid moves due to buoyancy force created by density gradient

→ body force also plays a role

$$h = f(l, b_f, \rho_f, \mu_f, c, g, \alpha_f, k) \quad \text{for velocity}$$

$\downarrow T_w - T_\infty$

β = thermal expansion coefficient

$$\beta = -\frac{1}{c} \left(\frac{\partial c}{\partial T} \right)_P \quad \frac{1/k}{\text{write}}$$

$$\text{Grashof number} = \frac{G_f}{l} = \frac{g \beta \Delta \theta \cdot l^2}{\nu^2}$$

ratio of buoyancy to viscous force

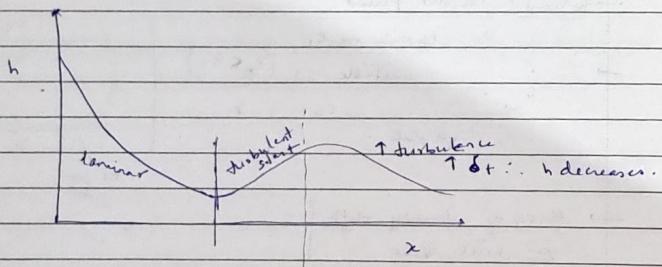
$$Nu = f(P_r, G_f)$$

$$Ra = G_f \cdot P_r \cdot l$$

Rayleigh number

part of Ra
to determine whether laminar or turbulent

$$\text{Forced conv} \quad h \propto \frac{1}{\delta t}$$



forced boiling
circulation aid is used

Pool boiling

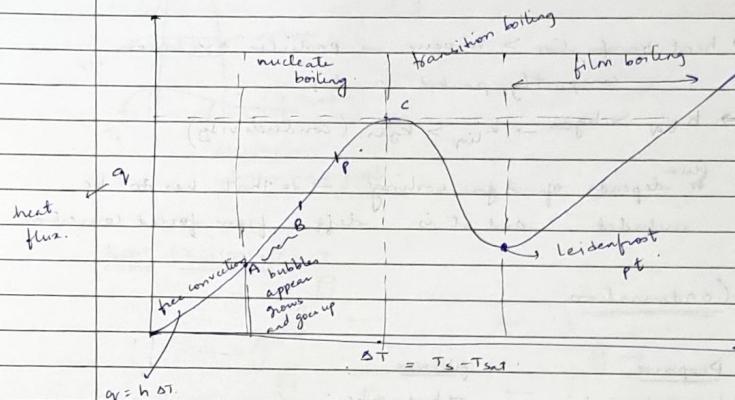
No circulation aid is used

Subcooled boiling

Only liquid near the heating surface reaches the boiling pt while bulk remains at a subcooled state.

Pool Boiling of saturated liquid (i.e. just about to boil)

Nicholson wire



$\rightarrow AB \vdash q_f \text{ bcoz motion } \uparrow$
fluid circulation

$\rightarrow BP \rightarrow \text{no. of sites } \uparrow \rightarrow \text{vapour bubble form rapidly}$
nucleation $\therefore q_f \uparrow$

$\rightarrow PC \rightarrow \text{vapour fraction on the surface } \uparrow$
 $\therefore h \uparrow$ but $q_f \uparrow$ as $\Delta T \uparrow$

$\Rightarrow \text{vapour } h < \text{ liquid}$

→ After certain excess temp., h reduces and is significant.
at & after reaching pt c.

→ Beyond c → surface remains partially covered by vapour.
↳ transition boiling
between nucleate and film boiling (v +)

→ heat flux decreases until vapour film is formed
↳ heat heat h_{film} is transferred to the liquid by conduction and radiation.

→ heat transfer \dot{m} > vapour → particles are more compactly packed in liquid

→ $h_{\text{film}} > h_{\text{gas}} \rightarrow k_{\text{liquid}} > k_{\text{gas}}$ (conductivity)

→ flux depends of λ for boiling → so that has to be included and it is diff from forced convection

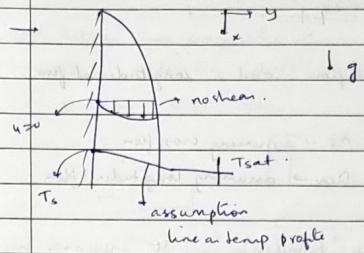
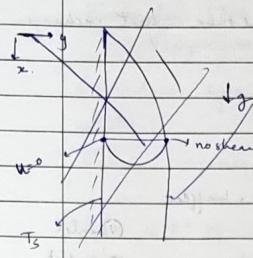
Condensation

Dropwise

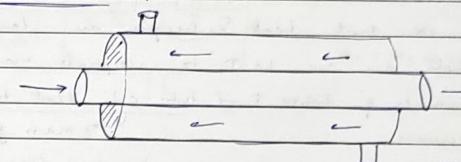
→ hydrophobic surface
→ higher heat transfer
Coef : high heat transfer
→ surfaces are difficult to maintain → polished surface

filenwise

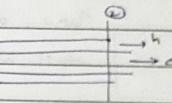
→ hydrophilic
→ nearly rotates less.
→ thickness of liquid τ in the direction of flow.



HEAT EXCHANGERS



Co-current : $\frac{dT_h}{dL} < 0, \frac{dT_c}{dL} > 0$



Counter-current : $\frac{dT_h}{dL} > 0, \frac{dT_c}{dL} > 0$

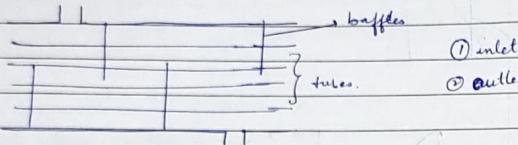
$$\text{Total heat } Q = U A_f (\Delta T_{\text{MTD}}) + U A_c (\Delta T_{\text{MTD}})$$

$U = \frac{h_1 h_2}{h_1 + h_2 + h_{inter}}$

design eqn $\frac{\Delta T_2 - \Delta T_1}{\ln(\frac{\Delta T_2}{\Delta T_1})}$ heat transfer across the surface

When area becomes large, shell and tube heat exchanger is preferred.

Shell and tube exchanger



Something in between cross flow and longitudinal flow.

$$Re = \frac{Dv \rho}{\mu}$$

$A_f \rightarrow$ assuming cross flow.

$Dv \rightarrow$ assuming longitudinal flow.

→ Baffles are used to provide turbulence. It increases h , heat transfer rate.

→ We can see that heat exchangers are long, to avoid this shell dia can be T to incorporate more tubes.

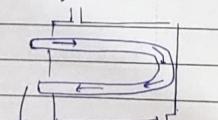
If number of tubes T → tube side heat transfer coeff will decrease

\downarrow less flowrate per tube +

Here, one can bend the tubes and make them per the shell once again → more area, same length

multiple pass :- 1-2 exchange

shell (dome)
tubes 2 times



(J_n is used to calculate Nu)

(shell n tube PED read) ??

Evaporator (PED) slides see vapourise \rightarrow if not steam.
Vapour is steam

Calandria type evaporators are used for concentrating solutions.

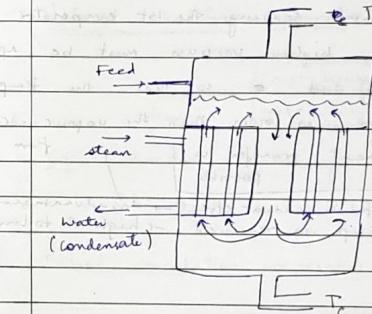
→ Obtain capacity of the evaporator and economy
(kg H₂O evaporated / kg of steam fed)

→ T_{sat} or BP of soln is needed.
 \downarrow can be obtained from Duhings plot
BP H₂O v/s soln system specific

→ kettle type evaporator is

→ steam will be shell side

→ solution tube side



→ No LMTD required.

$$m_s A_T = \dot{V} A_T (T_s - T_c)$$

steam \rightarrow concentrate

→ enthalpy

$$m_s h_s + m_f h_f = m_{cond} h_{cond} + m_c h_c + m_u h_u$$

\downarrow feed \downarrow condensate

$$m_s h_s + m_f h_f = m_c h_c + m_u h_u$$

$$m_f = m_c + m_u$$

$$m_f x_f = m_c x_c$$

→ Negligible BP-elevation $\rightarrow T_c = T_v$ → all the properties have properties close to 130

$$i_s m_s = i_f c_p (T_c - T_f) + n_v \Delta v \rightarrow \text{Plots are not required}$$

Evaporation

2 plots are required $\rightarrow H \text{ vs } x$, Duhring's plot
many solute

Multiple effect evaporator

Vapour generated from ① is used as heating steam for other.

→ BPT is negligible

→ Since vapour and concentrate leaving the 1st evaporator are at the same temp, higher vacuum must be applied in the vapour space of end \rightarrow so that the temp.

if (core) in 2nd evaporator is lower than the vapour stream \rightarrow feed and heat transfer is possible

→ forward feed \rightarrow thick disc is fed to coldest effect \rightarrow disadvantage can use no pump is required \rightarrow high P to low Backward effect

→ Backward feed \rightarrow pump

→ mixed feed

→ Net feed

→ flashing occurs \rightarrow vaporization due to decrease in P.
 \downarrow loses heat

Evaporation

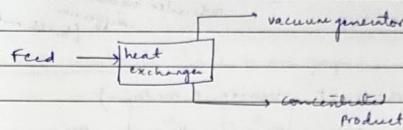
Application:

To concentrate the solution

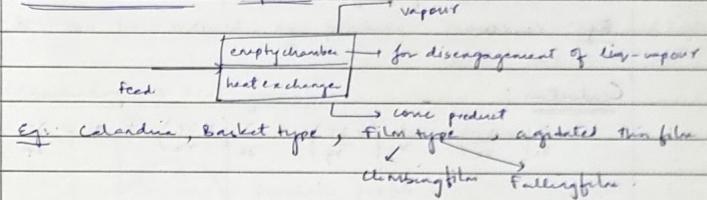
Produce crystals (evaporative crystallization)

→ how to remove water?

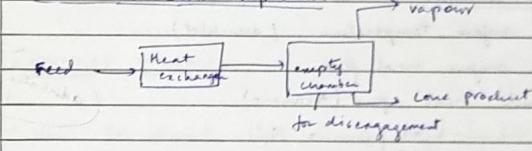
① By heating ② by applying vacuum.
 \downarrow Both are used



Natural circulation evaporator



Forced circulation evaporation



Conduction

In solids, due to lattice vibration of molecules.

or stationary fluid

$$q_x = -k \frac{\partial T}{\partial x} \quad \text{by Fourier's law}$$

(convection (surface to moving fluid))

heat transfer due to bulk motion.

diffusion + advection = convection

movement of

molecules without
diffusion (bulk motion)

$$q = h(T - T_{ao})$$

Radiation (does not require a medium)

→ energy travels from source through space.

→ has wave like property

Eg: candle

Conduction:

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + q = \dot{\sigma} T$$

Boundary conditions

(1) const surface temperature (Dirichlet)

(2) const surface heat flux

(i) finite heat flux (Neumann)

$$-\frac{k}{\partial x} \frac{\partial T}{\partial x} \Big|_{x=x_0} = q_s$$

adibatic or
insulated

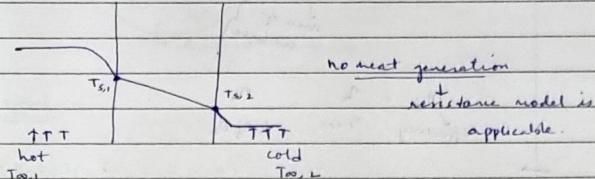
(ii) adiabatic or insulated

$$\frac{\partial T}{\partial x} \Big|_{x=0} = 0$$

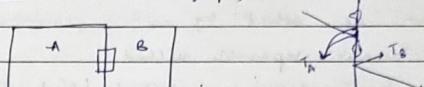
(3) convection (Robin BC) (mixed)

$$-\frac{k}{\partial x} \frac{\partial T}{\partial x} \Big|_{x=0} = h(T_{ao} - T)$$

→ 1D steady state conde

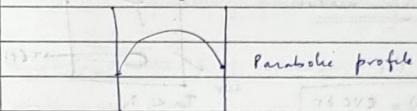


Contact resistance (due to air gaps)



It can be reduced by polishing, remove air and add high thermal conductivity layer

Conduction with heat generation



Parabolic profile

Extended Surface

→ used to depict heat transfer by conduction within the solid and heat transfer by convection from boundaries.
See to heat transfer in the solid

→ used to enhance heat transfer between solid and fluid
by ~~area~~ & increases heat transfer area.

fin performance

$$\epsilon_f = \frac{\eta_f}{\eta_{max}} = \frac{hA_f}{hA_s B_b}$$

ratio of the fin heat transfer rate to heat transfer that would exist without one fin
(area of fin base)

$$\eta_f = \frac{\eta_f}{\eta_{max}} = \frac{h_f}{h A_f B_b}$$

efficiency. $\eta_{max} = h A_s B_b$

→ 1D steady state conduction (notes)

→ Can be solved by

① Variable separable method

② Finite difference method (find temperature at each grid intersection)

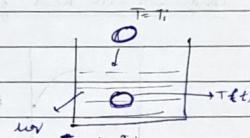
Transient heat conduction

Lump capacitance model

Temperature variation inside the solid is negligible.

$$E_{out} = E_{in}$$

$$-hA(t-T_{in}) = \rho C V \frac{dT}{dt}$$



$$\frac{\partial}{\partial t} = \frac{T - T_{in}}{T_i - T_{in}} = \exp\left(-\left(\frac{hA_s}{\rho C V}\right)t\right)$$

$$\alpha = \int_T dt = \frac{hA_s}{\rho C V} t$$

$$Bi = \frac{hL}{k_s}$$

$$Bi << 1$$

∴ i.e. $R_{cond} \ll R_{conv}$

$$L = \sqrt{A_s}$$

$$Bi = \frac{\text{conductive resistance}}{\text{convective}}$$

$$\frac{hA_s t}{\rho C V} = \frac{ht}{\rho C L c} = \frac{hL}{\rho C} \frac{k t}{L c} = \frac{hL}{\rho C k} \frac{dt}{L c} = Bi \cdot fo$$

$$fo = \frac{dt}{L c}$$

Fourier number.

Dimensionless time.

Spatial effect

$$T = T(x, t, T_i, T_o, L, k, x, h)$$

→ use finite difference to solve.

→ find the stability criteria → fo

→ Stability criteria: It is determined by requiring that the coefficient associated with the node of interest at previous time is greater than or equal to 0.

→ Implicit: generally unconditionally stable.

shell and tube heat exchanger

shell side fluid

more viscous fluid → becos it will cause turbulence → greater undergoing phase change

heat generation

large temp. difference

low pressure

tube side

more fouling → easy to clean

less viscous

hotter fluid

down flow rate

high pressure fluid

$$w_{in} (T_{c1} - T_{c2})$$

1/1

Radiation

$$\lambda = \frac{c}{f} \rightarrow \text{m/s}$$

wavelength \downarrow frequency f

→ EM radiations

$$E = h\nu$$

$$h = \text{Planck const}$$

$$\rightarrow E_b = \sigma T^4 \rightarrow \text{energy radiated per unit time and per unit area}$$

$$\sigma = \text{Stefan-Boltzmann const}$$

→ Black body :- absorb all the radiation incident
appear black to eye. (no reflection)

→ notes

→ Design of heat exchanger (back)

LMTD

logarithmic mean temperature difference

↳ used to determine the temp driving force for heat exchanger.

$$LMTD = \frac{\Delta T_2 - \Delta T_1}{\ln(\frac{\Delta T_2}{\Delta T_1})}$$

LMTD for counter current is much higher than co-current

lower shell
matrix
upper elem
20

②

③

④