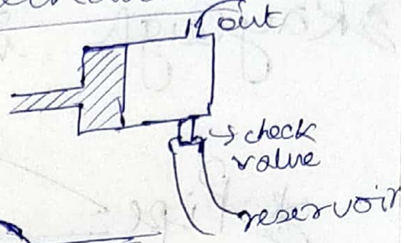
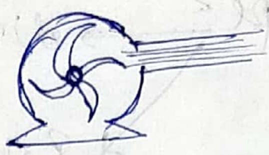
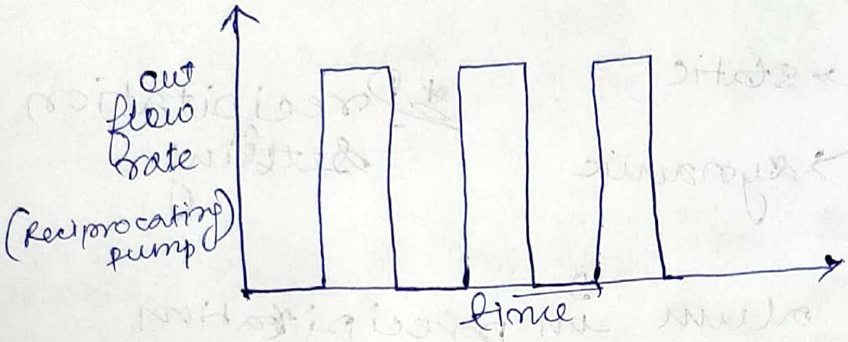
Introduction to Fluid Mechanics

Pumps → Reciprocating
Pumps → Centrifugal

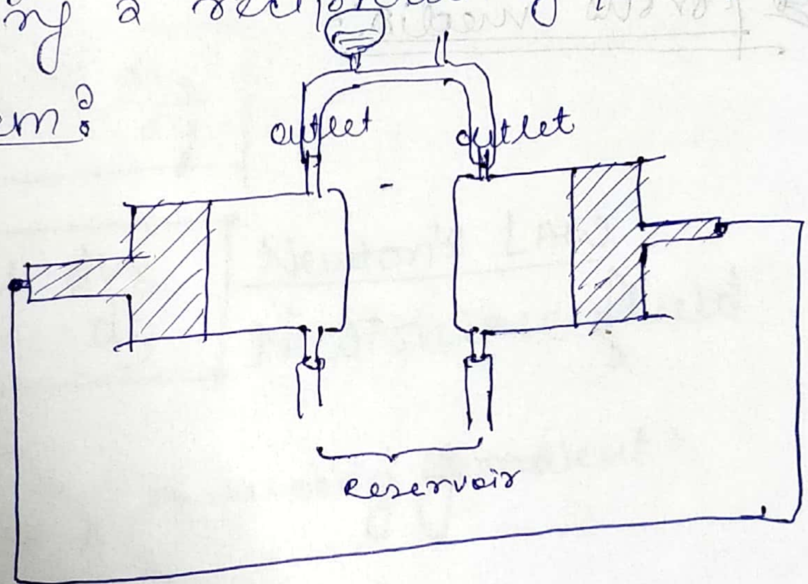


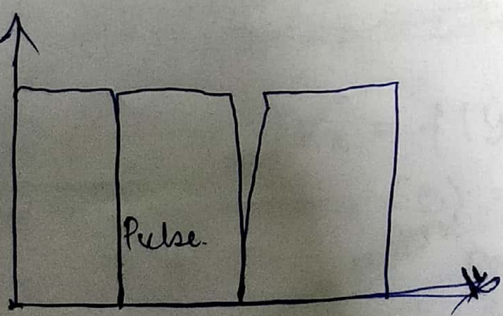




→ We can go upto higher pressures while using a reciprocating pump.

Better system:





* Compressor
* Blowers
* Valves

* Rayleigh instability

* Pipe
  - → length
  - → diameter
  - → Sch No.

* Nozzle

* Mixing
  - → static
  - → dynamic

* Precipitation settling

role of alum in precipitation

* Porous media:

Viscosity

[FRIDAY]

SHEAR STRESS → τ_{yx} ↳ direction of momentum
↓ transferred to which dir.

τ has 9 ~~values~~ components and hence it is called "Tensor."

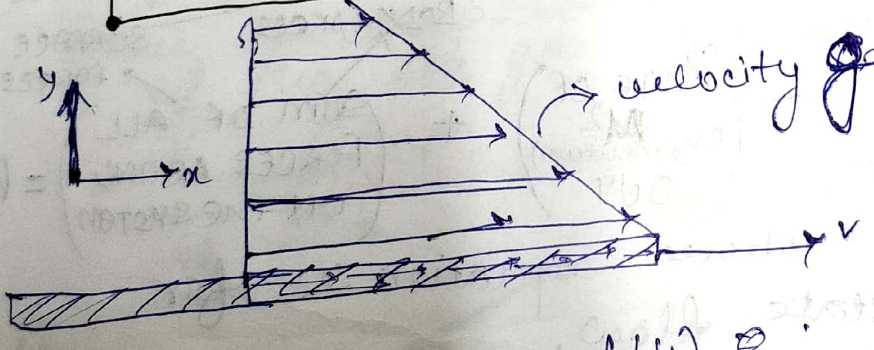
τ_{xy} τ_{xz}
τ_{zx} τ_{yy} τ_{yz}
τ_{xx} τ_{zy} τ_{zz}

NORMAL STRESS → $\tau_{xx}, \tau_{yy}, \tau_{zz}$ are written as $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ and are (normal stresses).

$$\boxed{\tau_{yx} \propto \frac{dv_x}{dy}}$$

$$\boxed{\tau_{yx} = -\mu \frac{dv_x}{dy}}$$

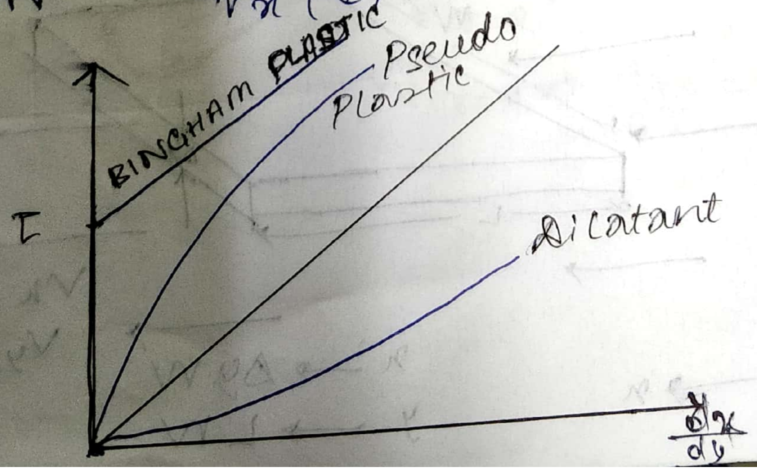
Newton's LAW
Newtonian fluid



NO SLIP CONDITION

$v_x = f(y)$

$v_x(0) = V$



$$\tau_{yx} = k\left(\frac{du}{dy}\right)^n$$

$$\tau_{yx} = k\left(\frac{du}{dy}\right)^{n-1}\frac{\partial u}{\partial y}$$

$$* \boxed{\tau_{yx} = \eta \frac{du}{dy}}$$

$\eta \Rightarrow$ Apparent viscosity.

Example: BINGHAM PLASTIC ? Toothpaste

~~* Velocity:~~

* Velocity distribution in Laminar flow:-

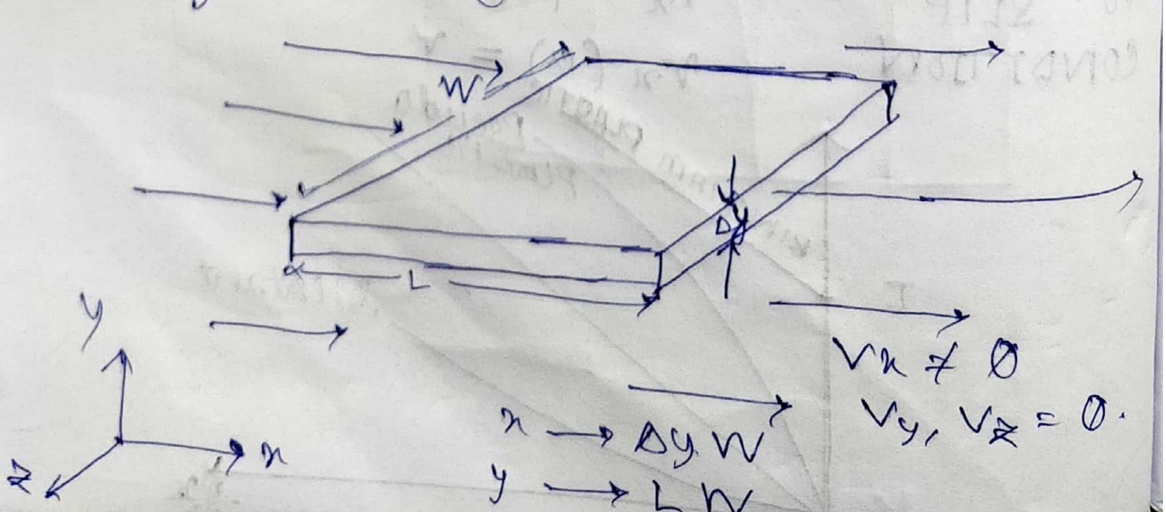
* SHELL MOMENTUM BALANCE:-

$$\begin{pmatrix}\text{RATE OF} \\ M^2 \\ \text{(momentum)} \\ \text{IN}\end{pmatrix} - \begin{pmatrix}\text{RATE OF} \\ M^2 \\ \text{(momentum)} \\ \text{OUT}\end{pmatrix} + \begin{pmatrix}\text{SUM OF ALL} \\ \text{FORCES ACTING} \\ \text{ON THE SYSTEM}\end{pmatrix} = 0$$

BODY FORCES

SURFACE FORCES

i.e. the above equation is for steady state flow.



$V_x \neq 0$

$V_y, V_z = 0.$

$x \rightarrow \Delta y\, W$

$y \rightarrow L\, W$

→ momentum enters at Δy and leaves at $y + \Delta y$.

* Ways of momentum transport

- **Convective transport** :-

→ ~~molecule~~ medium moves through (enters and leaves) the volume.

- **Conductive transport** :-

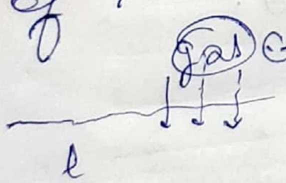
→ depends upon gradient (molecular transport).

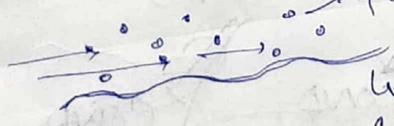
BOUNDARY CONDITIONS:

1. No relative velocity at solid-fluid interface : NO SLIP.

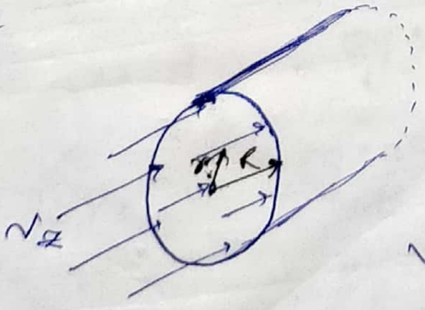
$\tau = 0$ (NO SHEAR).

2. Liq.-gas interface. (due to comparatively less viscosity of the gas medium).





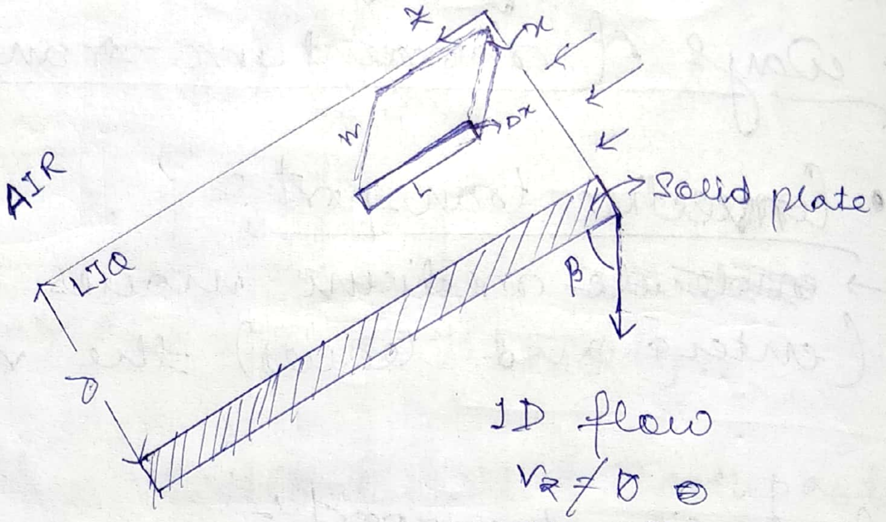
3.



$v_z|_{r=R} = 0.$

$v_z|_{r=0} \Rightarrow \frac{dv_z}{dr} = 0$

FLOW OF A FALLING FILM



1D flow
$V_z \neq 0$
$V_x = 0$.

2. STEADY FLOW $\quad$ CONST PROP.
$V_z \neq f(t)$

$V_z = f(x) \qquad V_z \neq f(\cancel{z}, \cancel{y})$

function form of V_z.

$$M^2_{IN} - M^2_{OUT} + \sum F = 0.$$

IN → CONV, COND
OUT → ~~COND~~

CONV: $(\rho \Delta x\, w\, V_z|_{z=0}) \cdot V_z|_{z=0}$

$m \qquad v$

$(\rho \Delta x\, w\, V_z|_{z=L}) \cdot V_z|_{z=L}$

$$\Rightarrow (\rho \Delta x\, w V_z|_{z=0}) \cdot V_z|_{z=0} - (\rho \Delta x\, w V_z|_{z=L})\, V_z|_{z=L} =$$

Since $V_z|_{z=0} = V_z|_{z=L}$

⇒ Incompressible fluids: Fluids for which the density remains constant.

for conductive transfer:

$$LW.\tau_{xz}|_{x} = - LW.\tau_{xz}|_{x+\Delta x}.$$

hence:

$$LW.\tau_{xz}|_{x} - LW.\tau_{xz}|_{x+\Delta x} + LW\Delta x\,\rho g\cos\theta = 0$$

[i.e. the final steady state eq^n].

Dividing by Δx and at $\Delta x \to 0$,

$$\lim_{\Delta x \to 0} \frac{\tau_{xz}|_{x+\Delta x} - \tau_{xz}|_{x}}{\Delta x} = \rho g\cos\theta$$

$$\frac{d}{dx}(\tau_{xz}) = \rho g\cos\theta$$

$$\tau_{xz} = \rho g\cos\theta . x + c$$

$\boxed{\text{NO SHEAR AT LIQ-AIR INTERFACE:}}$

$\tau_{xz} = 0$ at $x = 0 \Rightarrow c_1 = 0$.

$$\boxed{\tau_{xz} = \rho g\cos\theta . x}$$

[Assuming it be a Newtonian fluid]

$$-\mu\frac{dv_z}{dx} = \rho g\cos\theta . x$$

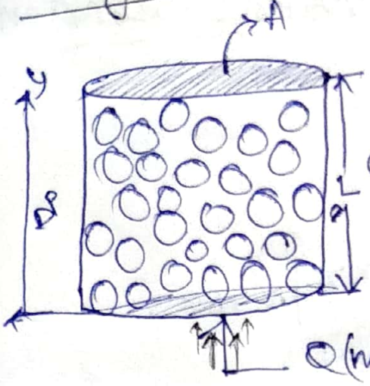
$$\int_{v_z}^{0} dv_z = -\frac{\rho g\cos\theta}{\mu}\int_{x}^{\delta} x\,dx$$

$$v_z = \frac{\rho g\cos\theta}{\mu}\left[\frac{\delta^2 - x^2}{2}\right].$$

$$\boxed{v_z = \frac{\rho g\delta^2\cos\theta}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right]}$$

Scanned by CamScanner

1st Aug, 2018



Cause & Effect

$$v = \left(\frac{k}{\mu}\right)\frac{\Delta P}{L}$$

$$= -\left(\frac{k}{\mu}\right)\frac{\partial P}{\partial y}.$$

k = permeability (unit m^2).

μ = viscosity.

Porosity:

$$\phi = \frac{\text{void volume}}{\text{total volume}}$$

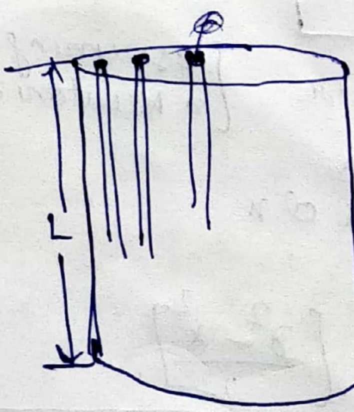
$$v = \frac{Q}{A} \qquad \left[q = -k\frac{\partial P}{\partial x}\right]$$

$$1-\phi = \frac{\text{solid volume}}{\text{total volume}}$$

total vol = AL

total void vol = ϕAL

$A_{open} \times L = \phi AL \Rightarrow A_{open} = \phi A.$

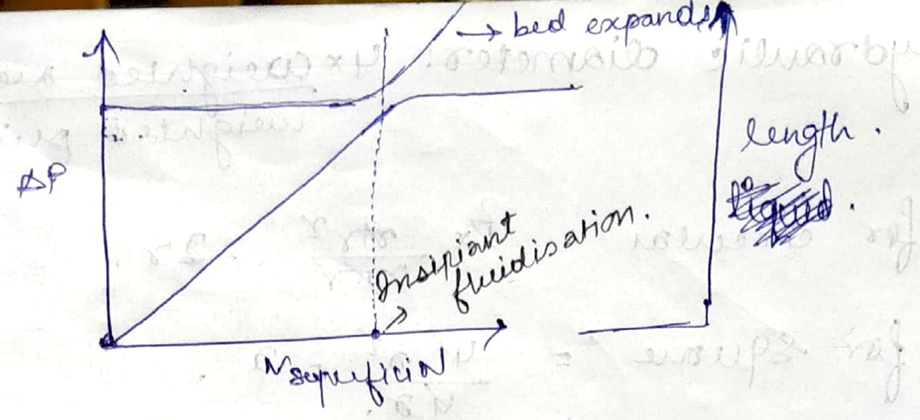


$$V_{superficial} = \frac{Q}{A}.$$

$$V_{interstitial} = \frac{Q}{A_{open}} = \frac{Q}{\phi A}$$

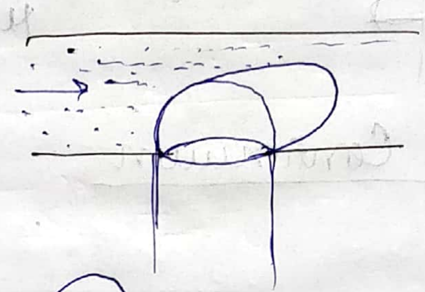
$$V_{interstitial} = \frac{V_{superficial}}{\phi}$$

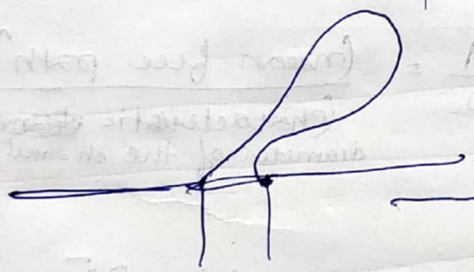
$$\left[V_{superficial} = \phi \cdot V_{interstitial}\right]$$

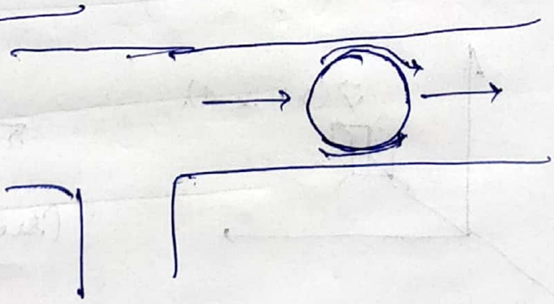


⇒ This process is known as fluidisation (particles gets suspended and the bed starts expanded, fluidised).

⇒ Fluidised bed reactors: Example FCC unit.







$\frac{\rho v d}{\eta}$ = Reynolds no.

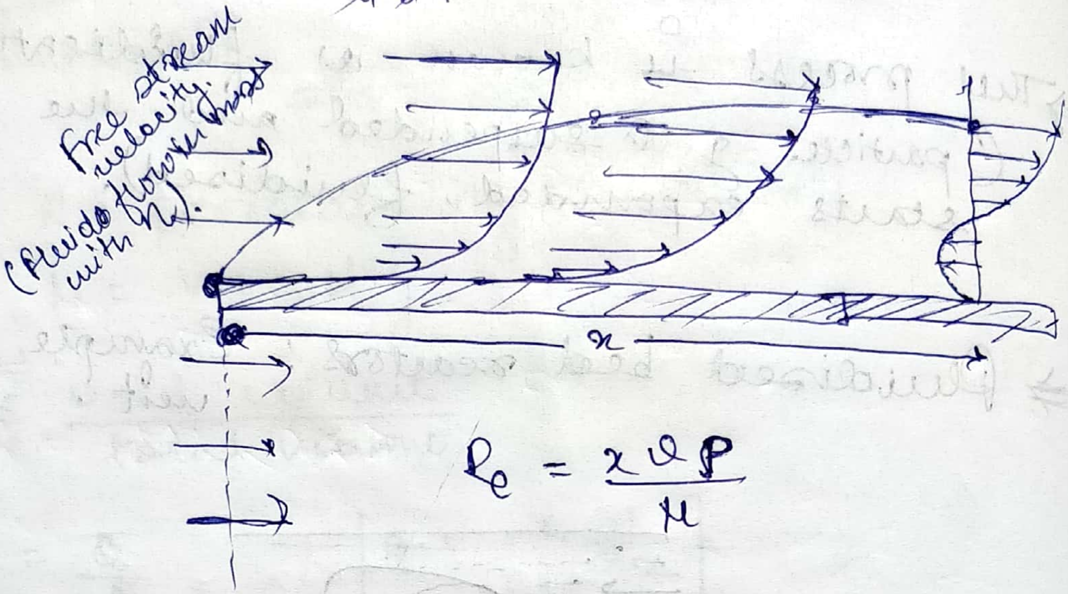
ρ → density, v → velocity, d → diameter, η → viscosity.

$$\frac{\mu U}{\sigma} = \frac{\text{viscous force}}{\text{interfacial forces}} = \text{Cappilary number.}$$

* hydraulic diameter: $4 \times \frac{\text{weighted area}}{\text{weighted perimeter}} = D_h$

for circular $= 4 \times \frac{\pi r^2}{2\pi r} = 2r$.

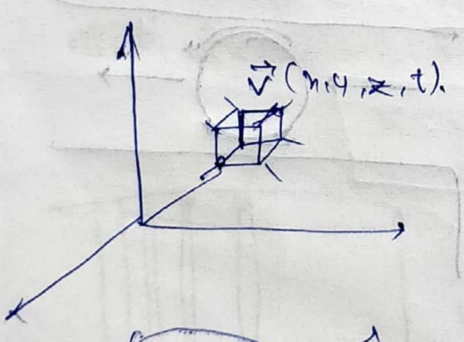
for square $= \frac{4 a^2}{4a} = a$.



$$R_e = \frac{x v \rho}{\mu}$$

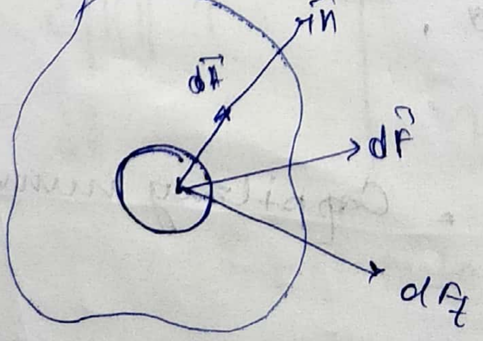
* Fluid as a Continuum

Knudsen number $= \frac{\lambda}{D} = \frac{\text{(mean free path)}}{\text{(characteristic diameter of the channel)}}$



$\bar{a}(x,y,z,t) = \frac{D\vec{V}}{Dt}$

(substantial derivative).



Normal stress

$$\sigma_n = \lim_{\delta A_n \to 0} \frac{\delta F_n}{\delta A_n}$$

Shear stress:

$$\tau_n = \lim_{\delta A_n \to 0} \frac{\delta F_t}{\delta A_n}$$

$$\sigma_{xx} = \lim_{\partial A_x \to 0} = \frac{\partial F_x}{\delta A_x}$$

$$\tau_{xy} = \lim_{\partial A_x \to 0} \frac{\partial F_y}{\partial A_x}$$

cause direcⁿ

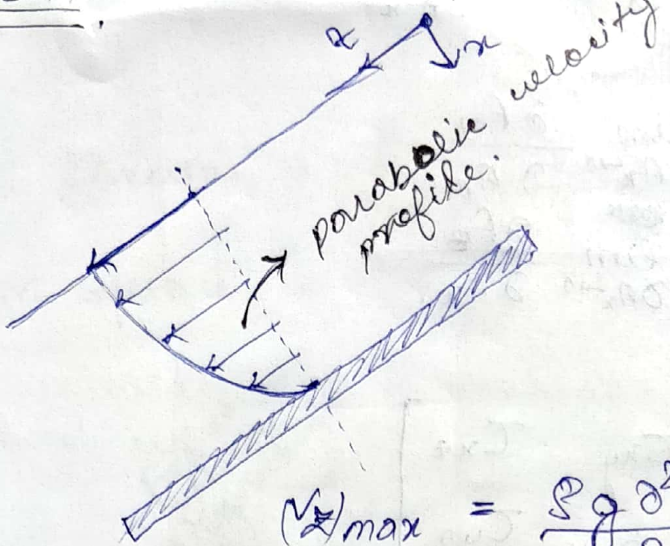
direcⁿ of ~~force~~ arrestd

$$\tau_{xz} = \lim_{\partial A_x \to 0} \frac{\partial F_z}{\partial A_x}.$$

$$\overline{\overline{\overline{\tau}}} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

$$P = -\frac{1}{3}\left[\sigma_{xx} + \sigma_{yy} + \sigma_{zz}\right] = -\sigma$$

FRIDAY



$$(v_z)_{max} = \frac{\rho g \delta^2 \cos\theta}{2\mu}$$

$$v_z = (v_z)_{max}\left[1 - \frac{x^2}{\delta^2}\right]$$

$$Q = \delta W \langle v_z \rangle$$

Any type of velocity is always average over the direction of flow ($\perp r$).

$$\langle v_z \rangle = \frac{\int_0^W \int_0^\delta v_z \, dx \, dy}{\int_0^W \int_0^\delta dx \, dy}$$

Since velocity is only a function of x here:

$$\langle v_z(x) \rangle = \frac{\cancel{\left(\int_0^W dy\right)} \int_0^\delta v_z \, dx}{\cancel{\left(\int_0^W dy\right)} \int_0^\delta dx}$$

$$\langle v_z \rangle = \frac{v_{z\,max} \int_0^\delta \left[1 - \left(\frac{x}{\delta}\right)^2\right] dx}{\delta}$$

$$= \frac{(v_z)_{max}}{\delta}\left[\delta - \frac{\delta^3}{3\delta^2}\right]$$

$$= \frac{v_{z\,max}}{\cancel{\delta}} \cdot \frac{2\cancel{\delta}}{3}$$

$$\left[\langle v_z \rangle = \frac{\rho g \delta^2 \cos\theta}{3\mu} \right] \rightarrow v_z \text{ average}$$

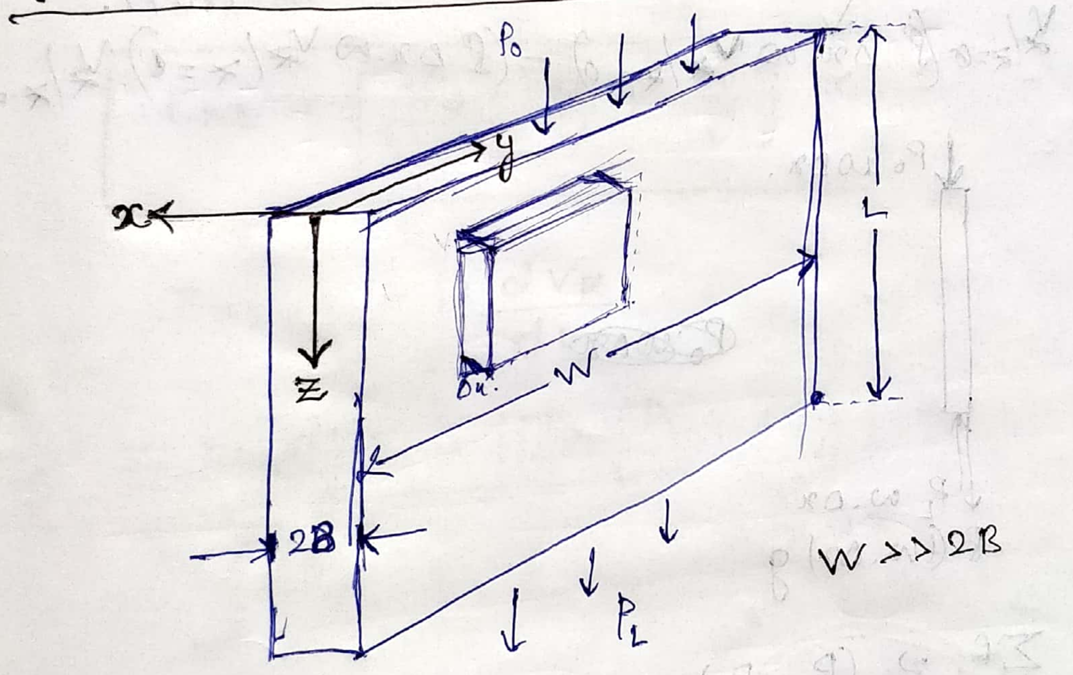
$$\text{Drag on the plate} = \tau_{xz}\big|_{x=\delta} \cdot LW$$

$$= \rho g \delta \cos\theta . LW = \underbrace{(LW\delta)}_{\text{volume}} \rho g \cos\theta$$

$$= \rho g (Vol) \cos\theta = \text{component of gravity pulling it down.}$$

$$\text{Reynolds No. } (Re) \equiv \frac{DV\rho}{\mu}$$

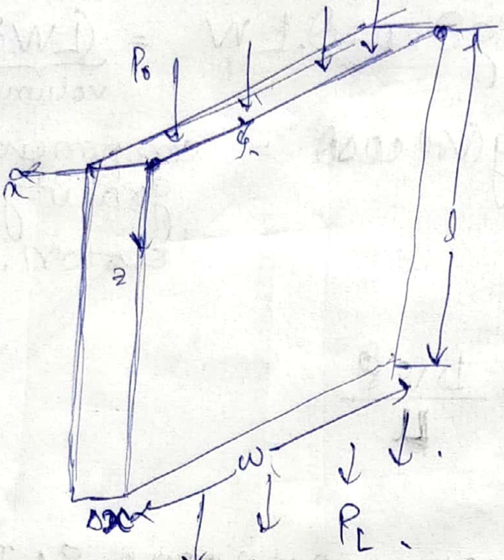
LAMINAR FLOW IN NARROW SLIT :-



$W >> 2B$

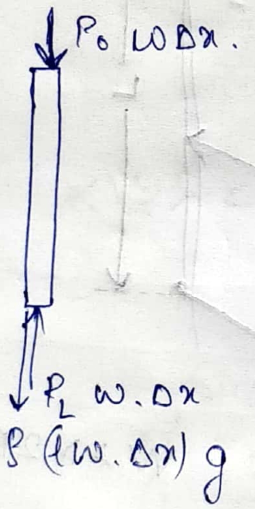
1) Velocity profile.
2) Average velocity.
3) Relation b/w $\langle v \rangle$ of v_{max}.
4) Vol^m rate of flow.

$v_z = f(x)$ only



M^2 (IN) $\qquad\qquad\qquad$ M^2 (OUT).

$$V_z|_{z=0}\left(\rho\,\Delta x.w\,V_z|_{z=0}\right)-\left(\rho\,\Delta x.w\,V_z|_{(z=l)}\right)V_z|_{z=l}$$



$$\Sigma F \Rightarrow (P_0-P_L)\,w.\Delta x+\rho(lw\Delta x)\,g.$$

$$\Rightarrow \Delta x.w\left[(P_0-P_L)+\rho gl\right]$$

for conductive transfer: $lw\,\tau_{xz}|_{x}-lw\,\tau_{xz}|_{x+\Delta x}$

$$M^2(\text{IN})-M^2(\text{OUT})+\Sigma F=0.$$

$$\Rightarrow \Delta x.\cancel{w}\left[\rho\,(\cancel{V_z|_{z=0}})^2-\rho\,(\cancel{V_z|_{z=l}})^2\right]+\cancel{wl}\left[\tau_{xz}|_x-\tau_{xz}|_{x+\Delta x}\right]$$
$$+\Delta x.\cancel{w}\left[(P_L-P_0)-\rho gl\right]=0.$$

[since $(V_z|_{z=0} = V_z|_{z=l})$]

$$\left[\tau_{xz}|_x - \tau_{xz}|_{x+\Delta x}\right] = \left[\frac{(P_0 - P_L) + \rho g l}{l}\right] \Delta x$$

dividing by Δx and as $\Delta x \to 0$.

$$\frac{\tau_{xz}|_x - \tau_{xz}|_{x+\Delta x}}{\Delta x} = \left[\frac{(P_0 - P_L) + \rho g l}{l}\right]$$

$$\frac{d}{dx}(\tau_{xz}) = \left[\left(\frac{P_0 - P_L}{l}\right) + \rho g\right]$$

$$\tau_{xz} = \left[\left(\frac{P_0 - P_L}{L}\right) + \rho g\right] x + c_1$$

using at $x = 0$. $\tau_{xz} = 0 \Rightarrow c_1 = 0$.

$$\boxed{\tau_{xz} = \left[\left(\frac{P_0 - P_L}{L}\right) + \rho g\right] x}$$

$$\tau_{xz} = -\mu \frac{dv_z}{dx}.$$

$$-\mu \frac{dv_z}{dx} = \left[\left(\frac{P_0 - P_L}{L}\right) + \rho g\right] x.$$

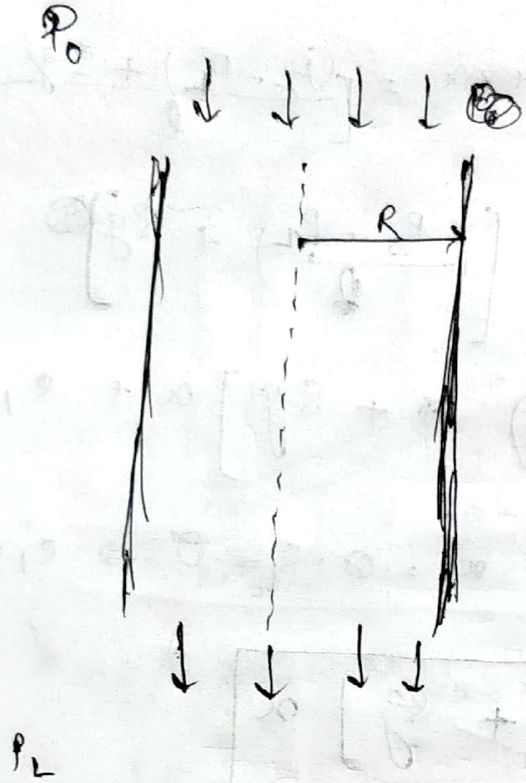
$$\int_0^{v_z} dv_z = \frac{(\quad)}{-\mu} \int_B^x x\,dx.$$

$$v_z = \frac{\left[\left(\frac{P_0 - P_L}{L}\right) + \rho g\right] B^2 \left(1 - \left(\frac{x}{B}\right)^2\right)}{2\mu}$$

$$v_z = \boxed{\frac{B^2}{2\mu}\left(\left(\frac{P_0 - P_L}{L}\right) + \rho g\right)\left[1 - \left(\frac{x}{B}\right)^2\right]}$$

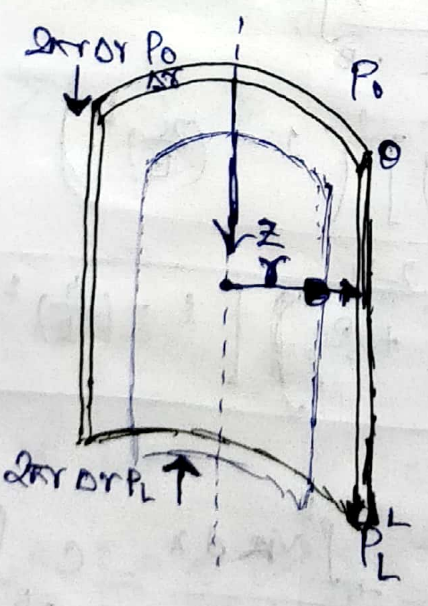
$$\langle v_z \rangle = \frac{\int_0^W \int_{-B}^{B} v_z \,dx\,dy}{\int_0^W \int_{-B}^{B} dx\,dy} = \frac{\int_{-B}^{B} v_z\,dx}{2B} = C \int_{-B}^{B} 2B -$$

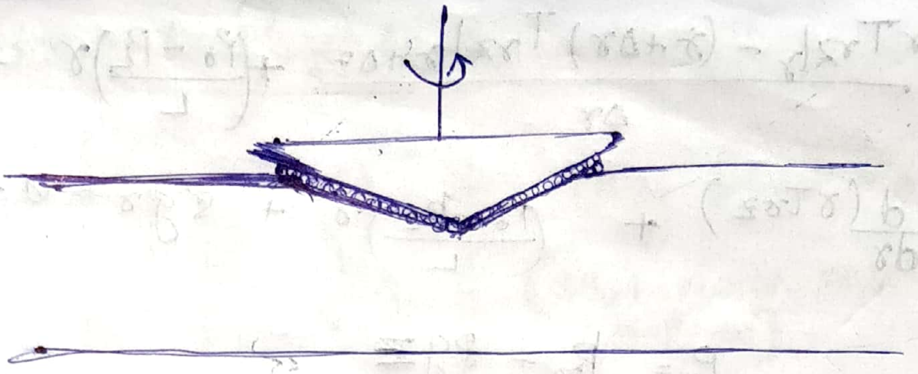
* FLOW THROUGH A CIRCULAR TUBE:-



⇒ Stokes law is valid for cases only in which Reynold's no. is less than

⇒ Reynold's number can be thought of as the ratio of inertial force (ρv) and the viscous force (η).





used to measure viscosity.

⇒ back to circular tube:-

$v_z = f(r)$ $\quad (v_z \neq f(z, \theta))$

$v_\theta, v_r = 0.$

Boundary conditions:

① NO SLIP $v_z|_{r=R} = 0$

② $(v_z)_{max}$ at $r = 0$

$$\left.\begin{aligned} -\mu \frac{dv_z}{dr}\Big|_{r=0} &= 0 \\ \tau_{rz}|_{r=0} &= 0 \end{aligned}\right] \text{(symmetric)}$$

$\rho(2\pi r \Delta r\, v_z|_z)\, v_z|_z - (\quad)|_{z=L}$ —①

IN $\qquad$ OUT

⇒ Incompressible steady flow hence eq ① $= 0$ (no accumulation).

IN $\qquad$ OUT

$\rightarrow 2\pi r L\, \tau_{rz}|_r - 2\pi(r+\Delta r) L\, \tau_{rz}|_{r+\Delta r}$ (shear)

$+ 2\pi r \Delta r P_0 - 2\pi r \Delta r P_L$ (surface)

$+ \rho(2\pi r \Delta r L) g$ (body force)

$= 0.$

$$\Rightarrow \frac{r\tau_{rz}|_r - (r+\Delta r)\tau_{rz}|_{r+\Delta r}}{\Delta r} + \left(\frac{P_0 - P_L}{L}\right) r + \rho g r = 0$$

$$-\frac{d}{dr}(r\tau_{rz}) + \left(\frac{p_0 - p_L}{L}\right) r + \rho g r = 0.$$

$$P = \frac{p - \rho g z}{}$$

$$P_0 = p_0$$

$$P_L = p_L - \rho g L$$

$$P_0 - P_L = p_0 - p_L + \rho g L$$

$$\frac{-d}{dr}(r\tau_{rz}) = \left(\frac{P_0 - P_L}{L}\right) r + C_1$$

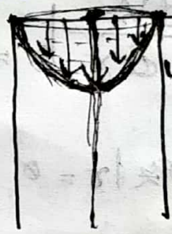
$$\cancel{r}\,\tau_{rz} = \left(\frac{P_0 - P_L}{L}\right)\frac{r^{\cancel{2}}}{2} + C_1$$

using $\tau_{rz} = 0$ at $r = 0$.
$C_1 = 0$

$$\tau_{rz} = \left(\frac{P_0 - P_L}{L}\right)\frac{r}{2}$$

similarly using $-\mu \frac{dv_z}{dr} = \tau_{rz}$, we get:

$$* \quad v_z = \left(\frac{P_0 - P_L}{4\mu L}\right) R^2 \left[1 - \left(\frac{r}{R}\right)^2\right]$$

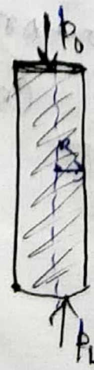

velocity profile.

$$* \quad \langle v_z \rangle = \frac{\int_0^{2\pi}\int_0^R v_z\, r\, dr\, d\theta}{\int_0^{2\pi}\int_0^R r\, dr\, d\theta} = \frac{(P_0 - P_L)R^2}{8\mu L}$$

$$* \quad \pi r^2 \langle v_z \rangle = \dot{Q} = \frac{\pi (P_0 - P_L) R^4}{8\mu L}$$ (HAGEN POISEUILLE EQ^n)

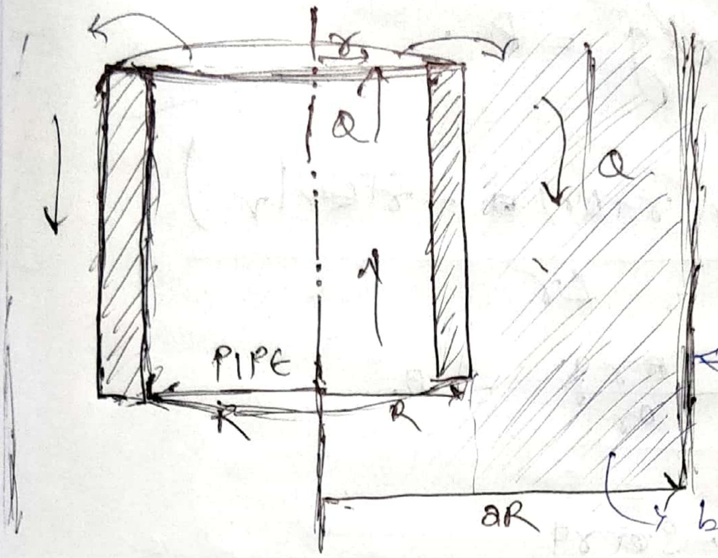
$$* \quad \pi r^2 \langle v_z \rangle \rho = \dot{m}$$

$$* \quad \mu = \mu(T).$$



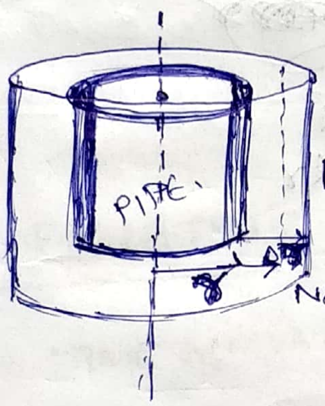
$$\pi R^2 (p_0 - p_L) + \pi R^2 L \rho g = \tau_{rz}|_{r=R}\, 2\pi R L$$

Tutorial Problem



~~Find~~ Find v_z.

(SS, NEWTONIAN, LAMINA, 1D flow).



→ had this been a pressure difference flow, the length of the shell would have been important.

→ rate of convective momentum IN = rate of convective momentum OUT

RATE OF M^2

IN − OUT + Σf = 0.

IN: ~~CONV~~, COND; OUT: ~~COND~~; Σf: ~~SURF~~, BODY

RATE OF M^2 (COND).

IN − OUT + gravity = 0

$$= \tau_{rz}|_{r} \, \cancel{2\pi r L} - \tau_{rz}|_{r+\Delta r} \, 2\pi(r+\Delta r)\cancel{L}$$

$$+ \rho \, \cancel{2\pi} r \, \cancel{\Delta r L} g = 0 .$$

$$= -\frac{\left(\tau_{rz}|_{r+\Delta r}(r+\Delta r) - r\tau_{rz}|_{r}\right)}{\Delta r}$$

$$+ \frac{\rho r g}{\cancel{\Delta r}} = 0.$$

$$\frac{d(r\tau_{rz})}{dr} = \rho r g$$

$$r\tau_{rz} = \rho g \frac{r^2}{2}$$

$$r\tau_{rz} \Big| = \rho g \int_{R} r \, dr$$

* THE EQUATIONS OF CHANGE (ISO THERMAL SYSTEM)

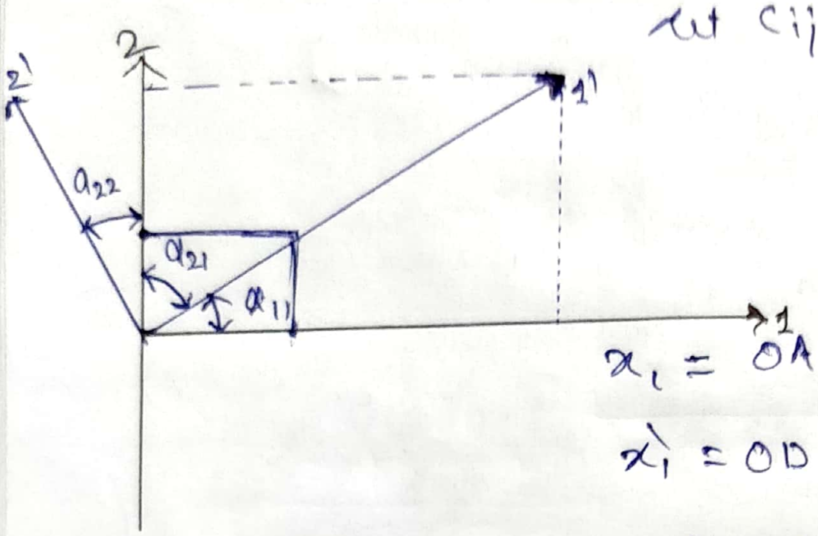
* PARTIAL TIME DERIVATIVE $\rightarrow \frac{\partial c}{\partial t}$
* TOTAL TIME DERIVATIVE
* SUBSTANTIAL TIME DERIVATIVE $\rightarrow \frac{Dc}{Dt}$.

$$\frac{Dc}{Dt} = \frac{\partial c}{\partial t} + v_x \frac{\partial c}{\partial x} + v_y \frac{\partial c}{\partial y} + v_z \frac{\partial c}{\partial z}$$

Components of position vector :-

in the original system x_i

in the rotated system x_i'



let $C_{ij} = \cos$ { angle b/w the old axis and new axis }

i = old axis

j = new axis

$x_1 = OA$

$x_1' = OD = OC + OD = OC + AB$

$= x_1 \cos a_{11} + x_2 \sin a_{11}$

$= x_1 C_{11} + x_2 C_{21} = \sum_{i=1}^{n} x_i C_{i1}$

$a_{11} = 90° - a_{21}$

$\sin a_{11} = \cos a_{21} = C_{21}$

$a_{11} = a_{22} = a_{12} - 90°$

$\sin a_{11} = -\cos a_{12}$

$x_2' = PD = PB - DB = PB - CA$

$= x_2 \cos a_{11} - x_1 \sin a_{11}$

$= x_2 C_{22} + x_1 C_{12}$

$= \sum_{i=1}^{n} x_i C_{i2}$

Ashutosh Singh

$x_j' = x_1 C_{1j} + x_2 C_{2j} + x_3 C_{3j}$

$= \sum_{i=1}^{3} x_i C_{ij} \qquad x_j = x_i C_{ij}$

j = free index

i = dummy index

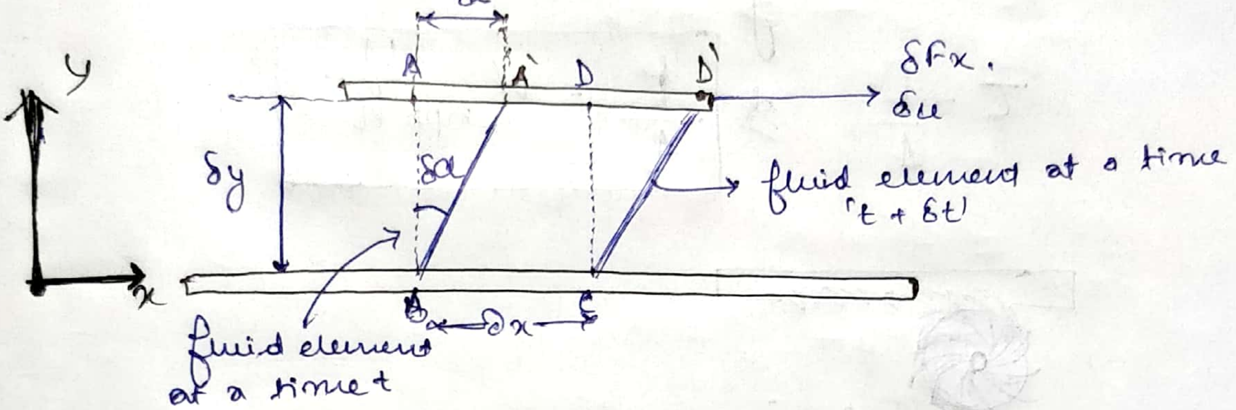
Scanned by CamScanner

* u is a vector if its components transform

$u_j = u_i c_{ij}$

* The elements of a matrix represents the components of a tensor ~~component~~ if they obey transformation rule

$$\tau' = c^T . \tau . c$$



$$\tau_{yx} = \lim_{\delta y \to 0} \frac{\delta F_x}{\delta A_y} = \frac{dF_x}{dA_y}$$

Angular deformation rate:- $= \lim_{\delta t \to 0} \frac{\delta a}{\delta t}$

$AA' = \delta l = \delta u \, \partial t$

if da is very small

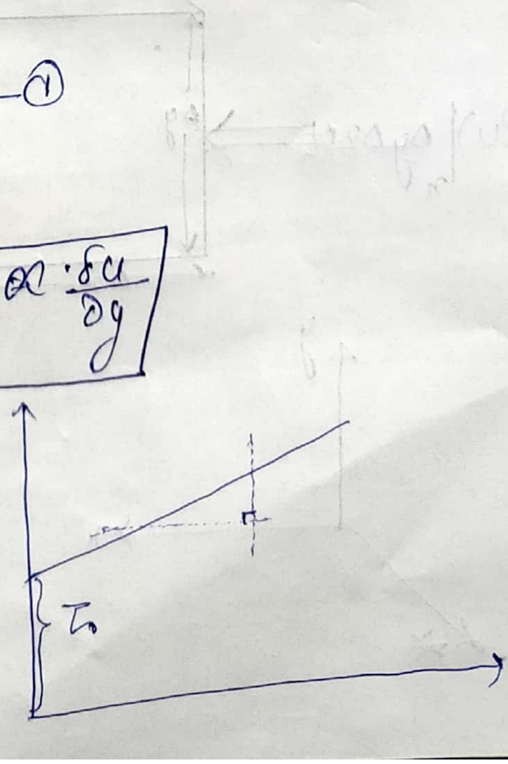
$\delta l = \delta a \, \delta y$

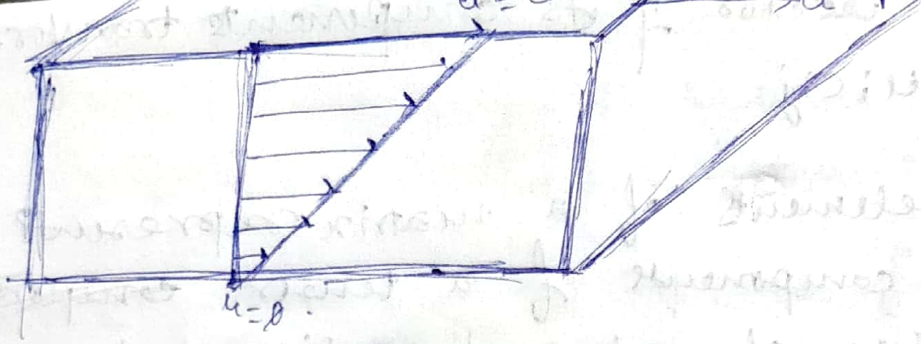
$\delta u \, \delta t = \delta a \, \partial y$

$\frac{\delta a}{\delta t} = \frac{\delta u}{\delta y}$ — ①

$\boxed{\tau_{yx} \propto \frac{\delta a}{\delta t}}$ ① implies that $\boxed{\tau_{yx} \propto \frac{\delta u}{\partial y}}$

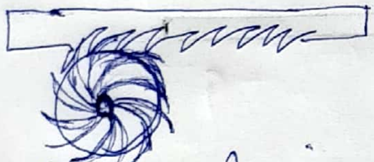
$\tau = \tau_0 + \mu \frac{\partial u}{\partial y}$





$$\frac{\partial u}{\partial y} = \frac{u-0}{h} = \frac{u}{h}$$

$$\tau = \frac{F}{A} \quad \boxed{\frac{F}{A} = \mu_{app}\frac{U}{h}}$$

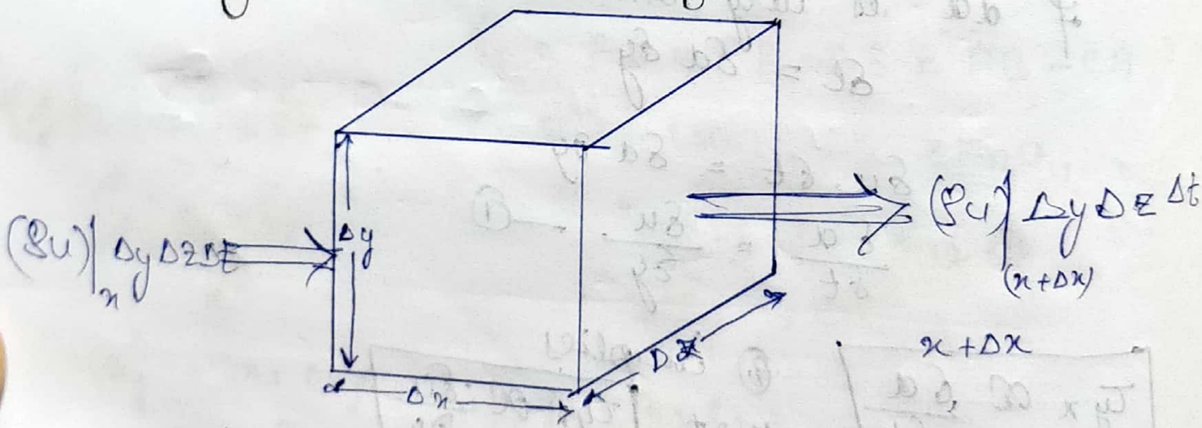


Rack and Pinion

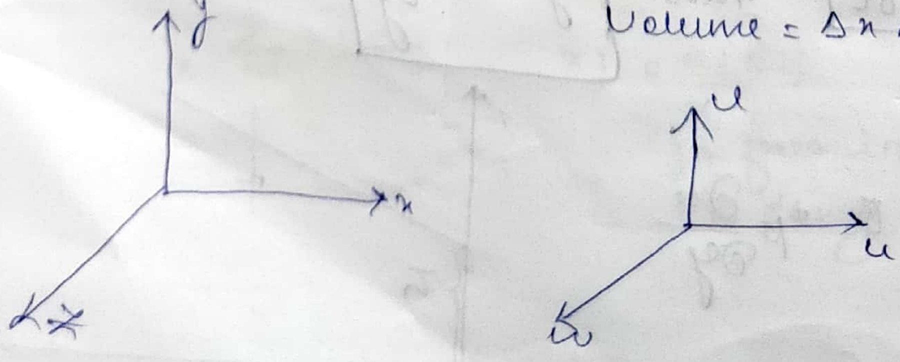
Parallel plate
cone and plate } viscometer.

Conservation of Mass:

* Rectangular co-ordinate system:



Volume = $\Delta x \Delta y \Delta z$



$$\left.\begin{aligned}&\frac{\cancel{\Delta y}\,\cancel{\Delta z}\,\cancel{\Delta t}}{\Delta x\,\cancel{\Delta y}\,\cancel{\Delta z}\,\cancel{\Delta t}}\left[(\rho u)|_x-(\rho u)|_{x+\Delta x}\right]\\&+\frac{\cancel{\Delta x}\,\cancel{\Delta z}\,\cancel{\Delta t}}{\cancel{\Delta x}\,\Delta y\,\cancel{\Delta z}\,\cancel{\Delta t}}\left[(\rho v)_y-(\rho v)_{y+\Delta y}\right]\end{aligned}\right\}=\frac{\cancel{\Delta x}\,\cancel{\Delta y}\,\cancel{\Delta z}\,\Delta\rho}{\cancel{\Delta x}\,\cancel{\Delta y}\,\cancel{\Delta z}\,\Delta t}$$

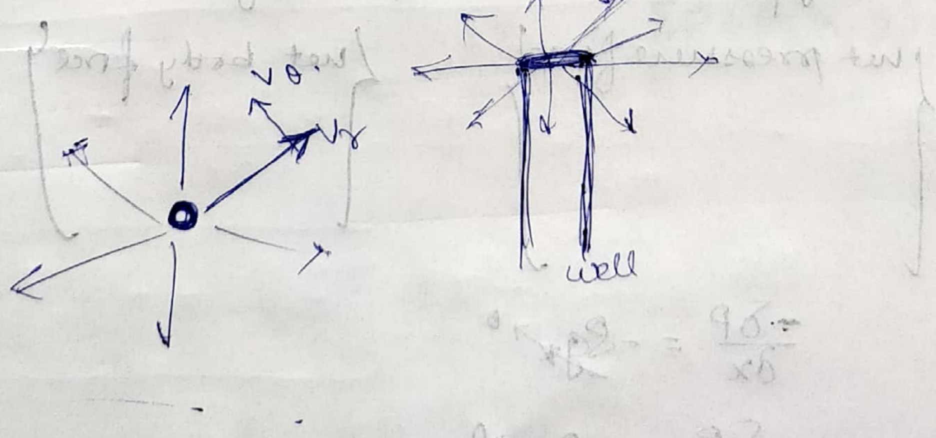
mass accumulation $= \Delta x \Delta y \Delta z \Delta \rho$

$$\left.\begin{aligned}&\underbrace{\frac{(\rho u)|_x-(\rho u)|_{x+\Delta x}}{\Delta x}}_{\to\,-\frac{\delta(\rho u)}{\partial x}}\\&+\underbrace{\frac{(\rho v)_y-(\rho v)_{y+\Delta y}}{y}}_{\to\,-\frac{\delta(\rho v)}{\partial y}}\end{aligned}\right]=\frac{\Delta\rho}{\Delta t}$$

$$\to -\frac{\delta(\rho w)}{\delta z}.$$

$$\left[\frac{\delta(\rho u)}{\delta x}+\frac{\delta(\rho v)}{\partial y}+\frac{\delta(\rho w)}{\partial z}=-\frac{\delta\rho}{\delta t}\right]=-\frac{\delta\rho}{\delta t}$$

$$\left[\frac{\delta(\rho u)}{\delta x}+\frac{\delta(\rho v)}{\delta y}+\frac{\delta(\rho w)}{\delta z}=-\frac{\delta\rho}{\delta t}\right]$$



The Basic Eqn(s) of Fluid Mechanics

✓ Total force:

$$d\vec{F} = \underbrace{d\vec{F}_S}_{\text{surface force}} + \underbrace{d\vec{F}_B}_{\text{body force}} = (-\nabla p + \rho\vec{g})\, d\forall$$

$$\frac{d\vec{F}}{d\forall} = -\nabla p + \rho\vec{g}$$

✓ Newton's Second Law:

$$\frac{d\vec{F}}{d\forall} = \rho\vec{a} = 0$$

$$\underbrace{-\nabla p}_{\text{net pressure force}} + \rho\vec{g} = 0$$

$$\boxed{-\nabla p + \rho\vec{g} = \rho\vec{a}}$$

when $\vec{a} = \vec{0}$; case of fluid statics.

$$-\nabla p + \rho\vec{g} = \vec{0}.$$

$$\underbrace{\nabla p}_{\text{vector}} = \rho\overbrace{\vec{g}}^{\text{vector}}$$

$$\underbrace{-\nabla p}_{\text{net pressure force}} + \underbrace{\rho\vec{g}}_{\text{net body force}}$$

$$-\frac{\partial p}{\partial x} = -\rho \cancel{g_x}^{0}$$

$$-\frac{\partial p}{\partial y} = -\rho \cancel{g_y}^{0}$$

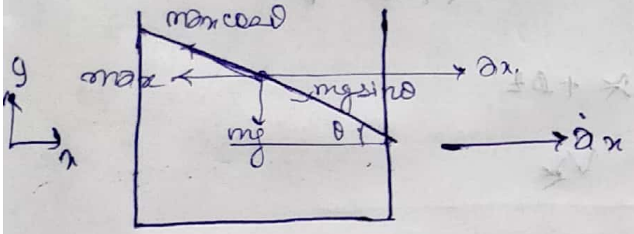
$$-\frac{\partial p}{[illegible]} = -\rho g$$

$$-\frac{\partial P}{\partial z} = \rho g \qquad \rho \leftarrow \frac{PM}{ZRT}, \quad T \leftarrow (T_0 - mz)$$

$$\boxed{\frac{dP}{P} = \frac{Mg\,dz}{R(T_0 - mz)}} \quad \left(\text{considering } \rho = \frac{PM}{RT}\right)$$

$$\frac{dP}{P} = \frac{Mg\,dz}{R(T_0 - mZ)}$$

$$\boxed{-\nabla P + \rho\vec{g} = \rho\frac{D\vec{V}}{Dt}}$$



→ Pressure is constant along the free surface.

$$ma_x\cos\theta = mg\sin\theta.$$

$$\boxed{\tan\theta = \frac{a_x}{g}}$$

in

$$-\left(\hat{i}\frac{\delta P}{\delta x} + \hat{j}\frac{\delta P}{\delta y} + \hat{k}\frac{\delta P}{\delta z}\right) + \rho\left(\hat{i}\cancel{g_x}^{\,0} + \hat{j}(-g) + \hat{k}\cancel{g_z}^{\,0}\right) = \rho\left(\hat{i}a_x + \hat{j}\cancel{a_y}^{\,0} + \cancel{\hat{k}a_z}^{\,0}\right)$$

$$\frac{\partial P}{\partial x} = -\rho a_x. \qquad \frac{\delta P}{\delta z} = 0. \quad [P = P(x, y)]$$

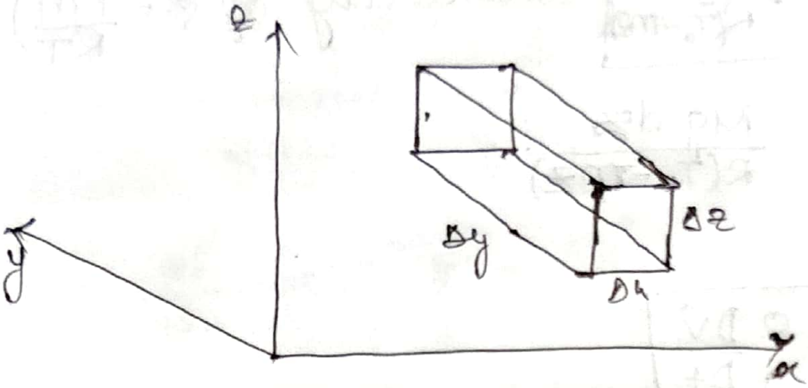
$$\frac{\delta P}{\delta y} = -\rho g$$

$$dP = \frac{\delta P}{\partial x}dx + \frac{\delta P}{\delta y}dy$$

$$dP = -\rho a_x\,dx - \rho g\,dy$$

$$0 \quad -\rho a_x\,dx = -\rho g\,dy \Rightarrow \boxed{\frac{dy}{dx} = \frac{-a_x}{g}} \text{ free surface.}$$

$$\tan\theta = \left.\frac{dy}{dx}\right|_{\text{free space}} = \frac{-\partial x}{g}$$



Rate of mass in at $x = (\rho v_x)|_x \Delta y \Delta t$

" " " out at $x+\Delta x = (\rho v_x)|_{x+\Delta x} \Delta y \Delta z$

y	$y+\Delta y$	z	$z+\Delta z$
v_y	v_y	v_z	v_z

Rate of mass accumulation

$$\Delta x \Delta y \Delta z \frac{\partial \rho}{\partial t} = \Delta y \Delta z [\rho v_x|_x - \rho v_{x+\Delta x}] + \Delta x \Delta y [\rho v_z - \rho v_{z+\Delta z}] + \Delta z \Delta x [\rho v_y - \rho v_{y+\Delta y}]$$

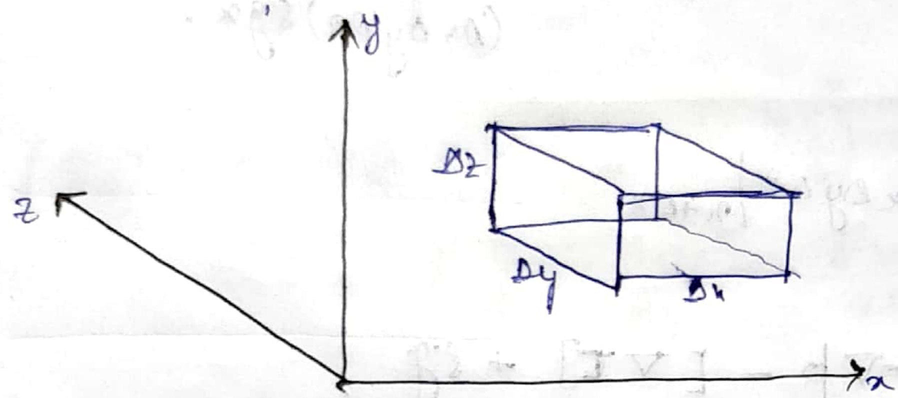
$$\frac{\partial \rho}{\delta t} = -\left(\frac{\partial [\rho v_x]}{\partial x} + \frac{\delta [\rho v_y]}{\delta y} + \frac{\delta [\rho v_z]}{\delta z}\right)$$

$$\frac{\delta \rho}{\delta t} + v_x \frac{\delta \rho}{\delta x} + v_y \frac{\delta \rho}{\delta y} + v_z \frac{\delta \rho}{\delta z} = -\rho \left[\frac{\delta v_x}{\delta x} + \frac{\delta v_y}{\delta y} + \right.$$

$$\boxed{\frac{D\rho}{Dt} = -\rho(\nabla v)}$$

→ Anything coming into the system will always be negative.

The eq^n of motion



Rate of M^2 accumula^n = Rate of M^2 IN − OUT + ΣF

→ x component of M^2

$$= \underbrace{\Delta y \Delta z [v_x \rho v_x|_x}_{\dot{m}} - \Delta y \Delta z [v_x \rho v_x|_{x+\Delta x}$$

y-force $\Delta x \Delta z$

$$\overbrace{\Delta x \Delta z\, v_y \rho}^{\dot{m}} v_x$$

$\tau_{xx} A_x|_x - (\ \ ) A_x|_{x+\Delta x}$

$\tau_{yx} A_y|$

$\tau_{zx} A_z|$

FORCES

SURFACE (P) → PA

BODY (gravity) mg $(\Delta x\, \Delta y\, \Delta z)\, \rho g_x$.

$$P_x\, \Delta y \Delta z\big|_x - P_x\, \Delta y\, \Delta z\big|_{x+\Delta x}$$

$$\rho \frac{Dv}{Dt} = -\nabla p - [\nabla \cdot \tau] + \rho g$$

mass. acc / vol — press. force / vol — shear / vol — grav / vol.

⇓ const μ, NEWTONIAN FLUID.

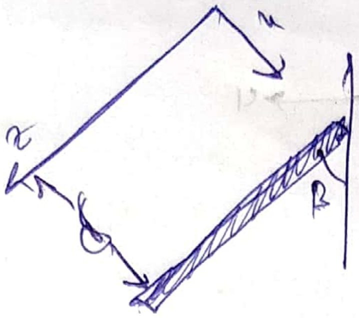
$$\boxed{\rho \frac{Dv}{Dt} = -\nabla p + \mu \nabla^2 v + \rho g}$$

NAVIER STOKES LAW.

IDEAL FLUID

$$\boxed{\rho \frac{Dv}{Dt} = -\nabla p + \rho g}$$

EULER EQUATION.



freely falling

$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}} + \cancel{v_x}\frac{\partial v_z}{\partial x} + \cancel{v_y}\frac{\partial v_y}{\partial x} + v_z\cancel{\frac{\partial v_z}{\partial z}}\right) = -\cancel{\frac{dP}{dz}}$$

$$+ \mu\left[\frac{d^2 v_z}{dx^2} + \cancel{\frac{d^2 v_z}{dy^2}} + \cancel{\frac{d^2 v_z}{dz^2}}\right] + \rho g\cos\beta$$

$$\mu\frac{d^2 v_z}{dx^2} + \rho g\cos\beta = 0.$$

$$\boxed{\frac{\partial \tau_{xz}}{\partial x} = \rho g\cos\beta}$$

$\Rightarrow$ For cylindral case

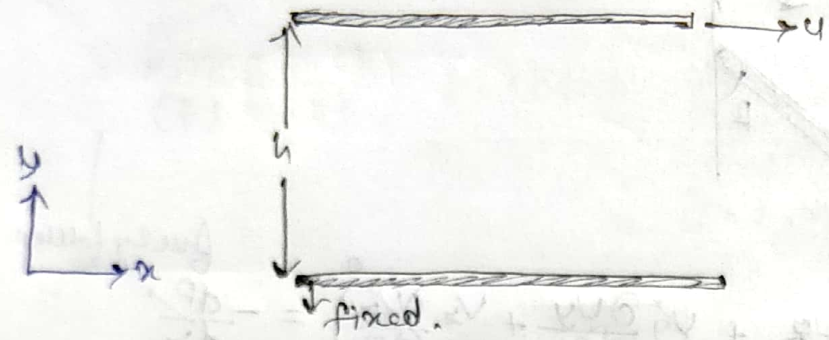
$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}} + v_r\frac{\partial v_z}{\partial r} + \cancel{\frac{v_\theta}{r}}\frac{\partial v_z}{\partial \theta} + v_z\cancel{\frac{\partial v_z}{\partial z}}\right)$$

$$= -\frac{\partial P}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\cancel{\frac{\partial^2 v_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_x}{\partial z^2}}\right]$$

$$+ \rho g_z.$$

$$\rho v_r\frac{\partial v_z}{\partial r} = -\frac{\partial P}{\partial z} + \frac{\mu}{r}\frac{\partial}{\partial r}\left[r\frac{\partial v_z}{\partial r}\right] + \rho g$$

Ques



(a) $\rho\left(\cancel{\frac{\partial v_x}{\partial t}} + \cancel{v_x \frac{\partial v_x}{\partial x}} + \cancel{v_y \frac{\partial v_x}{\partial y}} + \cancel{v_z \frac{\partial v_x}{\partial z}}\right)$

$$= -\cancel{\frac{\partial p}{\partial x}} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right]$$

$+ \cancel{\rho g_x}$

$\cancel{\rho v_x \frac{\partial v_x}{\partial x} = \mu \frac{\partial^2 v_x}{\partial y^2}}$

$\mu \frac{\partial^2 v_x}{\partial y^2} = 0.$ $\qquad -\frac{\partial}{\partial y}(\tau_{yx}) = 0$

$\cancel{\mu \frac{\partial v_x}{\partial y} = C_1}$ $\qquad \tau_{yx} = C$

$\cancel{\mu v_x = C_1 x + C_2}$

$\cancel{v_x = 0 \text{ at } x}$

$-\mu \frac{\partial v_x}{\partial y} = C$

$v_x = -\frac{C_1}{\mu} y + C_2$

v_x at $y = 0$ $= 0$ ✓

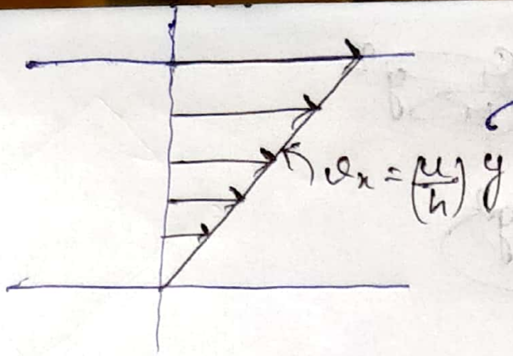
$\Rightarrow C_2 = 0$

$v_x|_h = u$

$u_x = \frac{-C_1 h}{\mu}$

$C_1 = -\frac{\mu u}{h}$ $\Rightarrow v_x = \frac{\cancel{\mu} u}{\cancel{\mu} h} y$

$\Rightarrow \boxed{v_x = \frac{u}{h} y}$

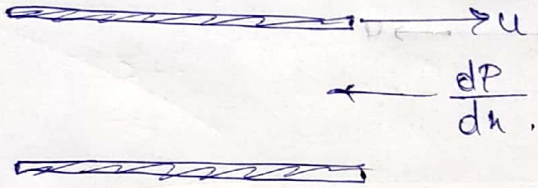


→ COUETTE-FORC

$$\frac{P_L - P_0}{L - 0}$$

$\frac{dP}{dx}$ = −ve favourable

= +ve unfavourable



$$-\frac{dP}{dx} + \mu \frac{\partial^2 v_x}{\partial y^2} = 0$$

$$\mu \frac{\partial^2 v_x}{\partial y^2} - \frac{dP}{dx} = 0$$

↳ (assume a +ve const c)

$$\mu \frac{\partial^2 v_x}{\partial y^2} = c$$

$$-\frac{\partial}{\partial y}(\tau_{yx}) = c$$

$$\tau_{yx} = -cy$$

$$\cancel{-}\mu \frac{\partial v_x}{\partial y} = \cancel{-}cy$$

$$v_x = \frac{c_1}{\mu}\frac{y^2}{2} + c_2 y + c_3$$

$$\cancel{u = \frac{c_1}{2\mu} h^2 \Rightarrow c_1 = \frac{2\mu u}{h^2}}$$

$$V_x = \frac{2\mu U}{h^2} \times \frac{c_1}{2\mu} y^2$$

$$V_x = \left(\frac{c_1 U}{h^2}\right) y^2$$