$\theta_a(0) = \int_0^{x} \exp$ $\theta_a(0) \sim \sqrt{Pr}\,.\,1$

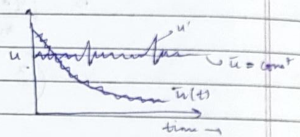
Sound velocity → $a = \sqrt{\gamma R T}$

$T_w = T_1\left[1 + 0.167\, Ma^2\right] = 302.178$
$Ma = 0.8$

TURBULENT BOUNDARY LAYER (chap 6)

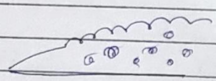
$u = \bar{u} + u'$

$\bar{u} = \frac{1}{t_p}\int_0^{t_p} u \, dt_p$



take time frame such that avg of fluctuating component is 0

$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0$.



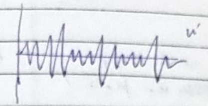
$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \cancel{\bar{w}\frac{\partial \bar{u}}{\partial z}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + \nu \nabla^2 \bar{V}_i - \cancel{\frac{\partial}{\partial x}\overline{u'^2}} - \cancel{\frac{\partial}{\partial z}(\overline{u'w'})}$

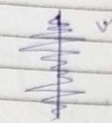
0 z symmetry ~0 0 z - symmetry

$o(u') \sim o(v')$

$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{d\bar{p}}{dx} + \nu\frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\partial}{\partial y}(\overline{u'v'})$

$u' > 0, \ v' < 0$





$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0$.

$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{d\bar{p}}{dx} + \nu\frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\partial}{\partial y}(\overline{u'v'})$

$\bar{u}\frac{\partial \bar{T}}{\partial x} + \bar{v}\frac{\partial \bar{T}}{\partial y} = \alpha\frac{\partial^2 \bar{T}}{\partial y^2} - \frac{\partial}{\partial y}(\overline{v'T'})$ = −ve why?

$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0$

$\frac{\partial u'}{\partial x} = -\frac{\partial v'}{\partial y}$

(if one is +ve, other is −ve)

$-\overline{u'v'} = \epsilon_M \frac{\partial \bar{u}}{\partial y}$ — Reynolds stress

ϵ_M — eddy momentum diffusivity; 'analogous to ν'

$-\overline{v'T'} = \epsilon_H \frac{\partial \bar{T}}{\partial y}$ — analogous to α

ϵ_H — eddy thermal diffusivity

due to eddies

ν – system property
ϵ_M – not system property → depends on the eddies

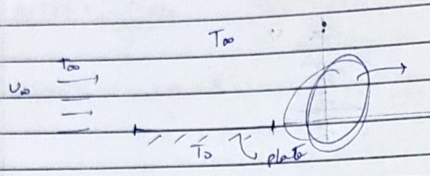
Momentum eqn – – – – = $-\frac{1}{\rho}\frac{d\bar{p}}{dx} + \nu\frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial}{\partial y}\left(\epsilon_M\frac{\partial \bar{u}}{\partial y}\right)$

$= \frac{\partial}{\partial y}\left(\underbrace{(\nu + \epsilon_M)}_{\epsilon_{app}\ \text{apparent}}\frac{\partial \bar{u}}{\partial y}\right)$

Energy eqn :- – – – – $\frac{\partial}{\partial y}\left((\alpha + \epsilon_H)\frac{\partial \bar{T}}{\partial y}\right)$

$\epsilon_M, \epsilon_H, \bar{u}, \bar{v}$, [illegible], $\bar{p}$, $\bar{T}$ → unknown

[illegible] – const



→ what will happen away from the plate?

$$\bar u \frac{\partial \bar u}{\partial x} + \bar v \frac{\partial \bar u}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar P_\infty}{\partial x} + \left\{\nu \frac{\partial^2 \bar u}{\partial y^2} - \frac{\partial(\overline{u'v'})}{\partial y}\right\} \;\; \frac{1}{\rho}\left[\frac{\partial}{\partial y}\left\{\mu \frac{\partial \bar u}{\partial x} - \rho\overline{u'v'}\right\}\right]$$

τ_{app}

reynolds stress

$$+ \left[\frac{\partial}{\partial y}\left\{(\nu+\epsilon_M)\frac{\partial \bar u}{\partial y}\right\}\right] \qquad = \rho\epsilon_M \frac{\partial \bar u}{\partial y}$$

→ $\bar u \frac{\partial \bar u}{\partial x} + \bar v\frac{\partial \bar u}{\partial y} = -\frac{1}{\rho}\cancel{\frac{dP_\infty}{dx}} + \left[\frac{\partial}{\partial y}\left\{(\mu+\epsilon_M)\frac{\partial \bar u}{\partial y}\right\}\right]$

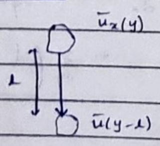
≈ 0

→ $\bar u \frac{\partial \bar T}{\partial x} + \bar v\frac{\partial \bar T}{\partial y} = \left[\frac{\partial}{\partial y}\left\{(\alpha+\epsilon_H)\frac{\partial \bar T}{\partial y}\right\}\right]$

→ $\frac{\partial \bar u}{\partial x} + \frac{\partial \bar v}{\partial y} = 0$

Unknowns: $\bar u, \bar v, \bar T, \epsilon_M, \epsilon_H$ (5) (3 equations)

Mixing length model



$\sim (\bar u_x(y) - \bar u_x(y-l)) \sim O'(u') \sim l \frac{\partial \bar u}{\partial y}$

$O(u') \sim O(v')$

$-\overline{u'v'} \approx l^2 \left(\frac{\partial \bar u}{\partial y}\right)^2$

$-\rho\overline{u'v'} = \rho\epsilon_M \frac{\partial \bar u}{\partial y}$

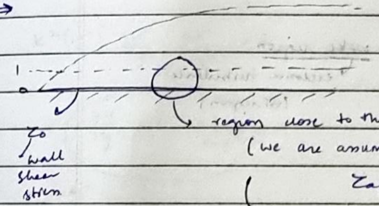
$$\epsilon_M = l^2 \left|\frac{\partial \bar u}{\partial y}\right|$$

→ $l \propto y \sim \kappa y$

kappa const

→ We need the velocity profile to find the wall shear stress.

⇒



(we are assuming that the apparent shear stress is const)

$\tau_{app} \approx$ const

we will define wall coordinate system

$\tau_{app} = \cancel{\nu}\frac{\partial\bar u}{\partial y} - \cancel{\overline{u'v'}} = \nu\frac{\partial \bar u}{\partial y} + \epsilon_M \frac{\partial\bar u}{\partial y}$

$\boxed{\tau_{app} = \rho(\nu+\epsilon_M)\frac{\partial \bar u}{\partial y}}$ $\qquad \epsilon_M = l^2\left|\frac{\partial\bar u}{\partial y}\right|$

$\tau_{app} = \rho(\nu+\epsilon_M)\frac{\partial\bar u}{\partial y} = \tau_0$

⇒ Why do we use wall coordinate system?

$u^+ = \frac{\bar u}{u^*}$, $v^+ = \frac{\bar v}{u^*}$

$y^+ = \frac{y u^*}{\nu}$ $\qquad x^+ = \frac{x u^*}{\nu}$

$u^* = \left(\frac{\tau_0}{\rho}\right)^{1/2}$

$$(\nu + \epsilon_M)\frac{\partial \bar{u}}{\partial y} = \frac{\tau_0}{\rho}$$

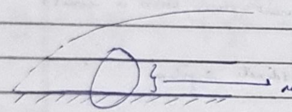
$$(\nu + \epsilon_M)\frac{(u^*)^2}{\nu}\frac{\partial u^+}{\partial y^+} = (u^*)^2$$

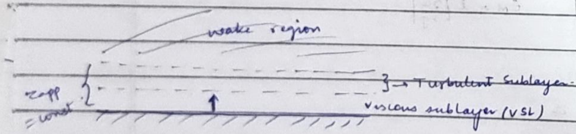
$$\frac{\partial u^+}{\partial y^+} = \frac{1}{\left(1 + \frac{\epsilon_M}{\nu}\right)}$$

$$\left(1 + \frac{\epsilon_M}{\nu}\right)\frac{\partial u^+}{\partial y^+} = 1$$

} valid only where τ_{app} is const

– region outside wall coordinate system is wake region → excessive turbulence (BT region)





$\frac{\epsilon_M}{\nu}$ → will determine which layer will dominate (TSL or VSL)

$\nu = \frac{\mu}{\rho}$ → molecular diffusivity

$\nu >> \epsilon_M \to$ VSL

$$\frac{\partial u^+}{\partial y^+} = 1$$

$$\int_0^{u^+} \partial u^+ = \int_0^{y^+} \partial y^+$$

$u^+ = y^+ \to$ $0 < y^+ < y^+_{VSL}$

⇒ $\epsilon_M >> \nu$

$$\frac{\epsilon_M}{\nu}\frac{\partial u^+}{\partial y^+} = 1$$

$$\epsilon_M = l^2 \frac{\partial \bar{u}}{\partial y}$$

$l \sim k y$

$$k^2 (y^+)^2 \frac{\partial u^+}{\partial y^+}\frac{\partial u^+}{\partial y^+} = 1$$

$$\epsilon_M = k^2 y^2 \frac{\partial \bar{u}}{\partial y}$$

$$\frac{\partial u^+}{\partial y^+} = \frac{1}{k y^+}$$

$$\epsilon_M = k^2 \left(\frac{y^+ \nu}{u^*}\right)^2 \frac{\partial u^+}{\partial y^+} \times \frac{u^*}{\nu / u^*}$$

$$\epsilon_M = k^2 \frac{\nu u^*}{u^*}(y^+)^2 \frac{\partial u^+}{\partial y^+}$$

$$\frac{\epsilon_M}{\nu} = k^2 \frac{u^*}{u^*}(y^+)^2 \frac{\partial u^+}{\partial y^+}$$

$$\int_{u^+_{VSL}}^{u^+} du^+ = \int_{y^+_{VSL}}^{y^+} \frac{dy^+}{k y^+}$$

$$\boxed{u^+ - \underbrace{u^+_{VSL}}_{y^+_{VSL}} = \frac{1}{k}\ln\left(\frac{y^+}{y^+_{VSL}}\right)}$$

$$u^+ = y^+_{VSL} + \frac{1}{k}\ln\left(\frac{y^+}{y^+_{VSL}}\right)$$

$y^+_{VSL} = 11.6$, $K = 0.4$

$$u^+ = \underbrace{y^+_{VSL} - \frac{1}{k}\ln(y^+_{VSL})}_{const} + \underbrace{\left(\frac{1}{k}\right)}_{const}\ln y^+$$

$$u^+ = \underbrace{A}_{\sim 5.5} + \underbrace{B}_{\sim 2.5}\ln y^+$$

$u^+ = \frac{\bar{u}}{u^*}$

Turbulent BL

$u^+ = y^+ \qquad 0 < y^+ \le y^+_{VSL}$

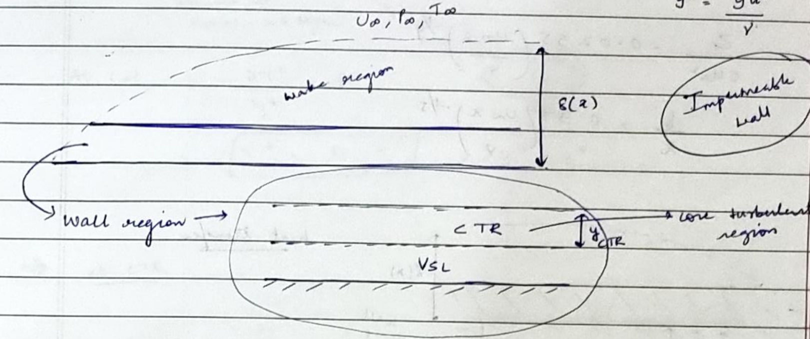
$u^+ = A + B\ln y^+ \qquad y^+_{VSL} \le y^+ \le y^+_{CTR}$

friction velocity

$u^* = \left(\frac{\tau_0}{\rho}\right)^{1/2}$ (τ_0 is unknown)

$u^+ = \frac{\bar{u}}{u^*}$

$y^+ = \frac{y u^*}{\nu}$



τ_{app} = constant

How to check whether the function is correct within the domain?
① experimental values ② check the BC.

→ assume $u^+ = f_u(y^+)$, $\frac{\bar{u}}{u^*} = f_u\left(\frac{y u^*}{\nu}\right)$ → inside the Boundary layer

at the edge of BL → $\frac{U_\infty}{(\tau_0/\rho)^{1/2}} = f_u\left(\frac{\delta(\tau_0/\rho)^{1/2}}{\nu}\right)$ → ①

Prandtl's 1/7th law → $u^+ = 8.7(y^+)^{1/7}$ τ_0, δ are unknowns

Use integral approach → $\bar{u}\frac{\partial \bar{u}}{\partial y} + \bar{v}\frac{\partial \bar{u}}{\partial x} = -\frac{1}{\rho}\cancel{\frac{dP_\infty}{dx}} + \nu\frac{\partial^2 \bar{u}}{\partial y^2}$

Pr ~ 1 → for Turbulent BL
↳ assumption

$$\int_0^\delta \bar{u}\frac{\partial \bar{u}}{\partial y}dy + \int_0^\delta \bar{v}\frac{\partial \bar{u}}{\partial y}dy = \int_0^\delta \nu\frac{\partial^2 \bar{u}}{\partial y^2}dy$$

$$\frac{d}{dx}\int_0^\delta \bar{u}^2 dy + [\bar{v}\bar{u}]_0^\delta = \left[\nu\frac{\partial \bar{u}}{\partial y}\right]_0^\delta$$

$$\frac{d}{dx}\int_0^\delta \bar{u}^2 dy + U_\infty \bar{v}|_\delta = -\frac{1}{\rho}(\tau_0)$$

$$\frac{d}{dx}\int_0^\delta \bar{u}^2 dy + U_\infty\left[-\frac{d}{dx}\int_0^\delta \bar{u}\,dy\right] = -\frac{\tau_0}{\rho}$$

$$\frac{d}{dx}\int_0^{\delta} \bar{u}(u_\infty - \bar{u})\,dy = \tau_0/\rho \longrightarrow (2)$$

use $u^+ = 8.7(y^+)^{1/7}$

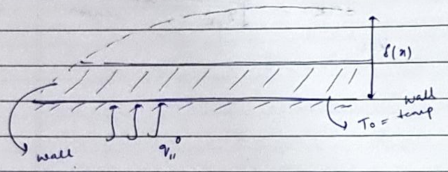
After integrating $\quad u_\infty = f(\delta, \tau_0) \longrightarrow (3)$

Compare eqn (3) & (1)

$$\frac{\tau_0}{\rho u_\infty^2} = 0.0255\left(\frac{u_\infty \delta}{\nu}\right)^{-1/4}$$

$$\frac{\delta}{x} = 0.37\left(\frac{u_\infty x}{\nu}\right)^{-1/5}$$

heat transfer



$$(\alpha+\epsilon_H)\frac{\partial \bar T}{\partial y} = -\frac{q''_{0,app}}{\rho c_p}$$

$q_{app} = \text{const} \ne f(y)$

$$q''_{0,app} = q''_0 = -\rho c_p(\alpha+\epsilon_H)\frac{\partial \bar T}{\partial y}$$

$$y^+ = \frac{y u^*}{\nu} \qquad \frac{\nu\,\partial y^+}{u^*} = \partial y$$

$$\frac{\partial \bar T}{\partial y^+}\left(\frac{\rho c_p u^*}{-q''_0}\right) = \frac{1}{\left(\frac{\alpha}{\nu}+\frac{\epsilon_H}{\nu}\right)}$$

$$\frac{\partial(T_0-\bar T)}{\partial y^+}\,\frac{\rho c_p u^*}{q''_0} =$$

→ $T^+(y^+, x^+)$

$$\frac{\partial T^+}{\partial y^+} = \frac{1}{\left(\frac{1}{Pr}+\frac{\epsilon_M}{\nu}\times\frac{\epsilon_H}{\epsilon_M}\right)}$$

eddy thermal diffusivity

$\frac{\epsilon_H}{\epsilon_M} = \frac{1}{Pr_t}$ → not a const (depends on velocity)

Pr_t → turbulent Pr

eddy momentum diffusivity

eddies are transferring the energy & momentum

In the absence of eddies at α, ν resemblance (ϵ_H, ϵ_M)

Pr_t → due to eddies

~~Pr~~ Pr → due to molecular

$$\frac{\partial T^+}{\partial y^+} = \left(\frac{1}{\frac{1}{Pr}+\frac{\epsilon_M}{\nu}\frac{1}{Pr_t}}\right)$$

close to the wall → VSL → due to molecular movement we have momentum transfer

At conduction sub layer → $\frac{\epsilon_M}{\nu}\frac{1}{Pr_t} << \frac{1}{Pr}$

similar to viscous sub layer

ratio is very small (?) some assumptions basic

At CSL, $\frac{\partial T^+}{\partial y^+} = Pr$

$$\boxed{T^+ = Pr\,y^+} \qquad 0 < y^+ < y^+_{CSL}$$

→ at CTR

$$\frac{\partial T^+}{\partial y^+} = \frac{1}{\frac{\epsilon_M}{\nu}\times\frac{1}{Pr_t}}$$

use prandtl mixing length $\quad l = Ky$

$$\frac{\epsilon_M}{\nu} = K^2 y_+^2\,\frac{du^+}{dy^+} = K y_+$$

$$u^+ = A + B\ln y^+$$

$$\frac{du^+}{dy^+} = \frac{B}{y^+} \qquad B = \frac{1}{K}$$

(at a particular $y^+ \to Pr_t$ is taken to be const)

$$\frac{\partial T^+}{\partial y^+} = \frac{Pr_t}{K y^+}$$

Amt of energy & momentum transferred by eddies is same wrt y

$$\int_{T_{CSL}}^{T^+} dT^+ = \frac{Pr_t}{K}\int_{y^+_{CSL}}^{y^+}\frac{dy^+}{y^+}$$

$$T^+ = Pr\,y^+ + \frac{Pr_t}{K}\ln\left(\frac{y^+}{y^+_{CSL}}\right)$$

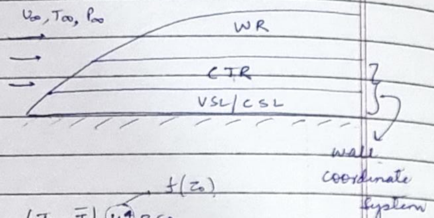
TURBULENT BL

$T^+ = Pr\, y^+ \quad 0 < y^+ < y^+_{CSL}$

$T^+ = Pr\, y^+_{CSL} + \frac{Pr_t}{k} \ln\left(\frac{y^+}{y^+_{CSL}}\right) \quad y^+_{CSL} < y^+ < y^+$

$U^+ = y^+ \quad 0 < y^+ < y^+_{VSL}$

$U^+ = A + B \ln y^+ \quad y^+_{VSL} < y^+$



$u^* = (\tau_0/\rho)^{1/2}$ — friction velocity

velocity based on the wall shear stress

$T^+ = \frac{(T_0 - \bar{T})\, u^* \rho c_p}{q''}$ — $f(\tau_0)$

→ apply this at the edge of BL becoz we know the boundary conditions at the edge of the BL (CTR) (~~[illegible]~~)

$T^+ = Pr\, y^+_{CSL} + \frac{Pr_t}{k}\left[\ln y^+ - \ln y^+_{CSL}\right]$

$\frac{(T_0 - \bar{T})\, u^* \rho c_p}{q''} = Pr\, y^+_{CSL} + \frac{Pr_t}{k}\left[\ln \frac{y u^*}{\nu} - \ln y^+_{CSL}\right]$

at BL $\bar{T} = T_\infty$

extend this at the edge of BL

$\frac{(T_0 - T_\infty)\, u^* \rho c_p}{q''} = Pr\, y^+_{CSL} + \frac{Pr_t}{k}\left[\ln \frac{\delta u^*}{\nu} - \ln y^+_{CSL}\right] \longrightarrow (1)$

$\frac{1}{h}$ —

$\frac{\delta}{x} = 0.37 \left(\frac{u_\infty x}{\nu}\right)^{-1/5}$ → cannot use this ?? — why?

we have assumed a velocity & got this δ (now we won't assume any velocity profile)

$u^+ = A + B \ln y^+$

$u^+ = y^+_{VSL} + \frac{1}{k} \ln\left(\frac{y^+}{y^+_{VSL}}\right)$

~~[illegible]~~

$u^+ = y^+_{VSL} - \frac{1}{k} \ln y^+_{VSL} + \frac{1}{k} \ln y^+$

$u^+ = B + \frac{1}{k} \ln y^+$

$\frac{\bar{u}}{u^*} = \frac{1}{k} \ln \frac{y u^*}{\nu} + B$

extend this at the edge of BL

$\boxed{\frac{U_\infty}{u^*} = \frac{1}{k} \ln\left(\frac{\delta u^*}{\nu}\right) + B} \longrightarrow (2)$

$\left(\frac{U_\infty}{u^*}\right)^2 = \frac{U_\infty^2}{\tau_0/\rho} = \frac{2}{\left(\tau_0 / \frac{1}{2}\rho U_\infty^2\right)} = \frac{2}{C_{f,x}}$

$\boxed{\left(\frac{U_\infty}{u^*}\right) = \left(\frac{2}{C_{f,x}}\right)^{1/2}}$

$\ln\left(\frac{\delta u^*}{\nu}\right) = k\left[\frac{U_\infty}{u^*} - B\right]$

$\frac{h}{\rho c_p U_\infty} = St_x = \frac{Nu}{Pe} = \frac{Nu}{Re \cdot Pr}$

$Pr \sim 1$

Pr should be close to 1.

$St_x \propto \frac{1}{2} C_{f,x}$

$= f(Pr)\, C_{f,x}$

$\frac{C_{f,x}}{2} = St_x (Pr)^{2/3}$

$St(Pr)^{2/3} = \frac{C_f}{2}$

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Von karman assumed 3 layers → Buffer layer

→ $Nu_x = \frac{1}{2} C_f P_r^{1/3}$ $0 < Pr < 0.5$

$\approx 0.0296\, Re_x^{4/5}\, P_r^{1/3}$.

$\overline{Nu} = 0.037\, Re_L^{4/5}\, P_r^{1/3}$ $Pr \gtrsim 0.5$

$\frac{d\bar{u}}{dy}(\nu + \epsilon_M) = \tau_0/\rho$

$\frac{d\bar{T}}{dy}(\alpha + \epsilon_H) = \frac{-q_0''}{\rho c_p}$.

assume:- $Pr_t = 1 \Rightarrow \epsilon_H = \epsilon_M$ if $Pr \approx 1$. $\alpha = \nu$

$\frac{d\bar{T}}{d\bar{u}} = \frac{-q_0''}{\tau_0 c_p}$

$\int_{T_0}^{T_\infty} d\bar{T} = -\frac{1}{c_p}\frac{q_0''}{\tau_0}\int_0^{U_\infty} d\bar{u}$

$\frac{T_\infty - T_0}{-q_0''} = \frac{+1}{c_p \tau_0} U_\infty$

$c_p\left(\frac{T_0 - T_\infty}{q_0''}\right) = \frac{U_\infty}{\tau_0}$

$\frac{\rho U_\infty c_p (T_0 - T_\infty)}{q_0''} = \frac{\rho U_\infty^2}{\tau_0}$

$\frac{\rho U_\infty c_p}{h} = \frac{\frac{1}{2}\rho U_\infty^2}{\frac{1}{2}\tau_0}$

$\frac{h}{\rho U_\infty c_p} = \frac{C_{f,x}}{2}$

$St_x = \frac{1}{2} C_{f,x}$ — reynolds analogy assumption ($Pr_t = 1$)

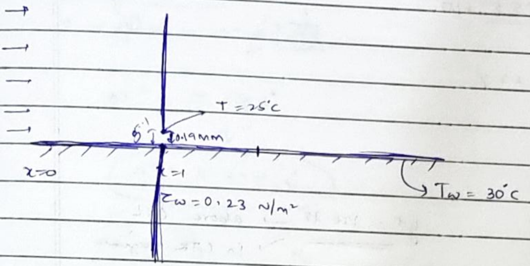
$Pr \approx 1$ — assume this for many equation

$Pr_t = 1$.

amount of momentum transferred by eddies = amt of energy transferred by eddies

$\epsilon_M \approx \epsilon_H$.

Q) $U_\infty = 10\,m/s$, $T_\infty = 10°C$



$q_0'' = h\,\Delta T$

h

$y^+_{CSL} = 13.2$

~~19mm → CTR region~~

$y^+ = \frac{y u^*}{\nu}$ $u^* = \left(\frac{\tau_0}{\rho}\right)^{1/2}$

$y^+ = 5.1$ — within CSL

$u^* = \left(\frac{0.23}{1.13}\right)^{1/2}$

$\nu = 16.7 \times 10^{-6}$

$q_0'' = -k\left(\frac{d\bar{T}}{dy}\right)_{y=0 = y = 0.19mm}$

$\boxed{q_0'' = -k\left[\frac{5}{0.19 \times 10^{-3}}\right]}$ = 15.67%

$u^+ < y^+$

$\frac{dT}{dy}$

$\frac{y u^*}{\nu}$

$\frac{dT}{dy} = const$ $u^* = \left(\frac{\tau_0}{\rho}\right)^{1/2}$

$T^+ = Pr\, y^+$

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(b) $St = Pr^{-2/3} \frac{C_{f,x}}{2}$

$C_{f,x} = \frac{\tau_0}{\frac{1}{2}\rho u_\infty^2} = \frac{0.23}{\frac{1}{2} \times 1.13 \times 16^2}$ → $\underline{St = 2.53 \times 10^{-3}}$

$St = \frac{h}{\rho C_p V_\infty} = \frac{q_w''}{\rho C_p V_\infty (T_w - T_\infty)}$

find h

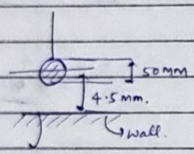
$h(T_w - T_\infty) = q_w''$

$q_w'' = 3 \times 10^{-3}$

error $= \frac{3 - 2.53}{3} \times 100$

$= 15.67\%$

q''



$T_w = 30°C$

$y^+ = 120.78$ → above CSL
→ in CTR regime

$T^+ = Pr\, y^+_{csl} + \frac{Pr_t}{K} \ln\left(\frac{y^+}{y^+_{csl}}\right)$

$Pr_t = 0.9$

$K = 0.41$

$y^+_{csl} = 13.2$

$T^+ = 2.195 y^+ + 358$

$T^+ = 14.10$

$T^+ = \frac{(T_w - \bar{T})\rho C_p u^*}{q''} = 14.10$

$\bar{T} = 11°C$

$\frac{dT^+}{dy^+} = 2.195 \frac{1}{y^+}$

$\left.\frac{dT^+}{dy^+}\right|_{y^+ = 121} = \frac{2.195}{121} = 0.018$

distance is small → assume linear variation

$\frac{\Delta T^+}{d^+} = 0.018$ $\qquad d^+ = \frac{d \times u^*}{\nu} = \underline{1.35}$

$\Delta T^+ = 0.024$

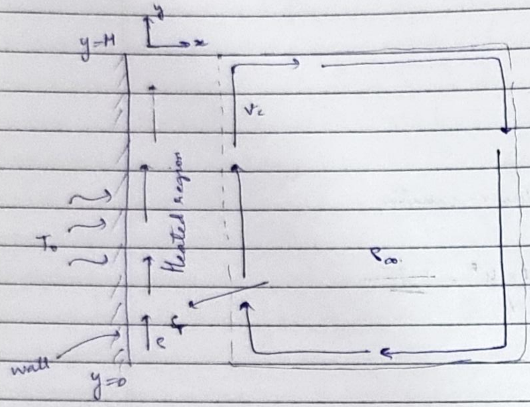
→ difference → ΔT^+

$\Delta T^+ = \frac{\Delta T \rho C_p u^*}{q''}$

$\Delta T = 3.2 \times 10^{-2}\,°C$

(e) Yes or No boom
(Pr_t → will change)

NATURAL CONVECTION



temperature difference is the main cause of velocity

$\delta_T << H$

$Q = h(T_0 - T_\infty)$ from solid to liq

$h \to h_{avg}$ average heat transfer coeff.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \Rightarrow \frac{\partial p}{\partial y} = \frac{dp_\infty}{dy}$$

not required

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \text{Additional term}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \cancel{\frac{\partial^2 v}{\partial y^2}}\right) - \rho g$$

0

$= \rho_\infty$

$$\frac{dp_\infty}{dy} = -\rho_\infty g$$

being Boussinesq approxn

$$\Rightarrow \rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \mu\frac{\partial^2 v}{\partial x^2} + g(\rho_\infty - \rho)$$

bouyancy

density on bouyancy force

Assumption: Effect of temp on density >> effect of temp on any other physical property (ρ, α, μ)

Boussinesq Approx

$$\frac{1}{\vartheta}\frac{D\vartheta}{Dt} = 0 \longrightarrow \text{incompressible fluid} \Rightarrow \frac{D\rho}{Dt} = 0$$

rate of change of volume of packet of fluid with respect to single packet

$$\beta_\infty = \frac{1}{\vartheta}\frac{\partial \vartheta}{\partial T} = -\frac{1}{\rho_\infty}\left.\frac{d\rho_\infty}{dT}\right|_\infty$$

$$\rho = \rho_\infty + \left.\frac{\partial \rho}{\partial T}\right|_\infty (T - T_\infty) + \text{higher order term}$$

$$\rho = \rho_\infty - \rho_\infty\beta_\infty(T - T_\infty)$$

$$(\rho_\infty - \rho) = \rho_\infty\beta_\infty(T - T_\infty)$$

$$\rightarrow \rho_\infty\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \mu\frac{\partial^2 v}{\partial x^2} + \rho_\infty\beta_\infty(T - T_\infty)g$$

$$\underbrace{u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}}_{\text{Inertia}} = \underbrace{\nu}_{\mu_\infty/\rho_\infty}\frac{\partial^2 v}{\partial x^2} + \underbrace{\beta_\infty(T - T_\infty)g}_{\text{bouyancy}}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \underbrace{\alpha}_{\text{const}}\frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

$$u^* = \frac{u}{v_c} \qquad v^* = \frac{v}{v_c} \qquad x^* = \frac{x}{H} \qquad y^* = \frac{y}{H}$$

$$\theta = \frac{T - T_\infty}{T_0 - T_\infty}$$

$$\left[\frac{v_c^2 u^*}{H}\frac{\partial u^*}{\partial x^*} + \frac{v_c^2 v}{H}\frac{\partial v^*}{\partial y^*}\right] = \left[\frac{v_c \nu}{H^2}\frac{\partial^2 v^*}{\partial x^{*2}} + \beta_\infty g(T_0 - T_\infty)\right]\frac{H}{v_c^2}$$

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*} = \frac{H}{v_c^2}\times\frac{v_c}{H^2}\,\nu\frac{\partial^2 v^*}{\partial x^2} + \underbrace{\left[\frac{\beta_\infty g(T_0 - T_\infty)H}{v_c^2}\right]}\theta$$

will tell you whether natural convection will dominate or not

natural v/s forced

$$\frac{g\beta_\infty q\,\Delta T H}{\nu^2} \times \frac{\nu^2}{v_c^2 H^2}$$

$\frac{g\beta_\infty q\,\Delta T H}{\nu^2} \to Gr_H$, $\frac{\nu^2}{v_c^2 H^2} \to \frac{1}{Re^2}$

$\frac{Gr_H}{Re^2} = Ri$ (richardson's number)

Ri → will tell you whether natural or forced convection dominates

Scaling analysis

$x \sim \delta_T$

$y \sim H$

$v \sim v_c$ (characteristic velocity)

$u \sim u$

$\Delta T \sim T_0 - T_\infty$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\sim \frac{u}{\delta_T} \sim \frac{v_c}{H} \qquad u \sim \frac{v_c \delta_T}{H}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right)$$

$$\sim u\frac{\Delta T}{\delta_T} \sim v_c\frac{\Delta T}{H} \sim \alpha\frac{\Delta T}{\delta_T^2}$$

$$\frac{v_c \Delta T}{H} \sim \frac{v_c \Delta T}{H}$$

$$\frac{v_c \Delta T}{H} \sim \alpha\frac{\Delta T}{\delta_T^2}$$

$$\delta_T \sim \sqrt{\frac{\alpha H}{v_c}} \qquad v_c \sim \frac{\alpha H}{\delta_T^2}$$

δ_T, v_c are unknown

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 v}{\partial x^2} + g\beta(T - T_\infty)$$

$$\sim \frac{u v_c}{\delta_T} \sim \frac{v_c v_c}{H} \sim \frac{\nu v_c}{\delta_T^2} \sim g\beta\,\Delta T$$

$$\sim \frac{v_c^2}{H} \sim \frac{v_c^2}{H} \sim \frac{\nu v_c}{\delta_T^2} \sim g\beta\Delta T$$

Inertia — viscous/friction — Bouyancy → omnipresent

$$\frac{\text{Inertia}}{\text{Bouyancy}} = \frac{\frac{v_c^2}{H}}{g\beta\Delta T} = \frac{\frac{\alpha^2 H^2}{\delta_T^4}\times\frac{1}{H}}{g\beta\Delta T} = \frac{\alpha^2 H}{\delta_T^4 (g\beta\Delta T)} \times \frac{H^3\nu}{H^3\nu}$$

$$= \frac{\alpha/\nu}{\frac{g\beta\Delta T H^3}{\alpha\nu}}\left(\frac{H}{\delta_T}\right)^4$$

$\alpha/\nu \to \frac{1}{Pr}$, $\frac{g\beta\Delta T H^3}{\alpha\nu} \to \frac{1}{Ra_H}$

$$= Ra_H^{-1}\,Pr^{-1}\left(\frac{H}{\delta_T}\right)^4$$

$$\frac{\text{Friction}}{\text{Bouyancy}} = \frac{\nu\frac{v_c}{\delta_T^2}}{g\beta\Delta T}$$

$$= \frac{\nu\frac{\alpha H}{\delta_T^4}\times\frac{H^3}{H^3}}{g\beta\Delta T}$$

$$= \frac{1}{\frac{g\beta\Delta T H^3}{\nu\alpha}} \times \left(\frac{H}{\delta_T}\right)^4 = Ra_H^{-1}\left(\frac{H}{\delta_T}\right)^4$$

$$\Rightarrow \sim Ra_H^{-1}Pr^{-1}\left(\frac{H}{\delta_T}\right)^4 \sim Ra_H^{-1}\left(\frac{H}{\delta_T}\right)^4 \sim 1$$

high Pr. → $Pr >> 1$ — Inertia has very low magnitude.

$$Ra_H^{-1}\left(\frac{H}{\delta_T}\right)^4 \sim 1$$

$$\boxed{\delta_T \sim H \times Ra_H^{-1/4}} \qquad h \sim \frac{k}{\delta_T}$$

$$Nu \sim \frac{hH}{k} \sim Ra_H^{1/4}$$

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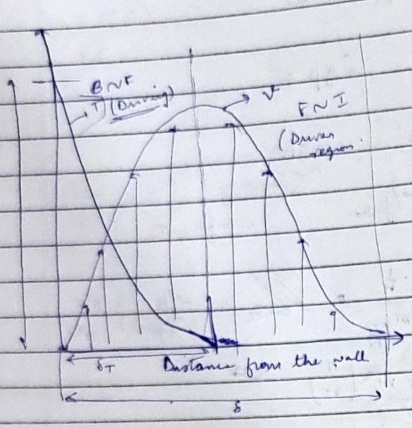


Buoyancy → B

Friction → F

Inertia → I

Friction ~ Inertia

$\sim \nu \frac{v_c}{\delta^2} \sim \frac{v_c^2}{H}$

$\delta^2 \sim \frac{\nu H}{\frac{\alpha H}{\delta_T^2}}$

$\delta^2 \sim \left(\frac{\nu}{\alpha}\right) \times \delta_T^2$

$\delta^2 \sim Pr\, Ra_H^{-1/2} H^2$

$\delta \sim Pr^{1/2} Ra_H^{-1/4} H$

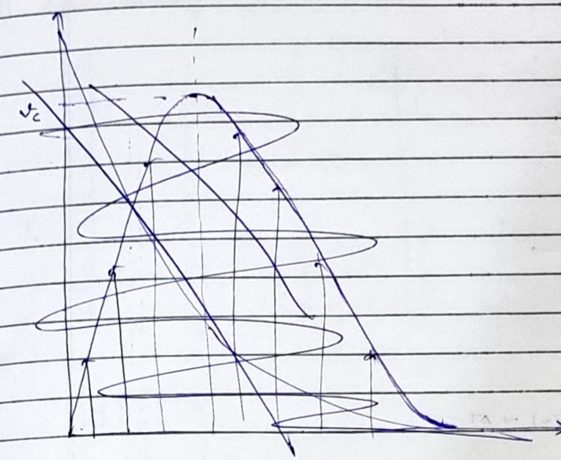
$\frac{\delta}{\delta_T} \sim \frac{Pr^{1/2} H Ra_H^{-1/4}}{H Ra_H^{-1/4}} \sim Pr^{1/2}$

Low Pr (Pr <<< 1)

$Ra_H^{-1} Pr^{-1} \left(\frac{H}{\delta_T}\right)^4 \sim 1$

$\delta_T \sim H (Bo)^{-1/4}$ Bo → Boussinesq number.

$Nu \sim Bo^{1/4}$



Gr → idea about scale of viscous sub layer.

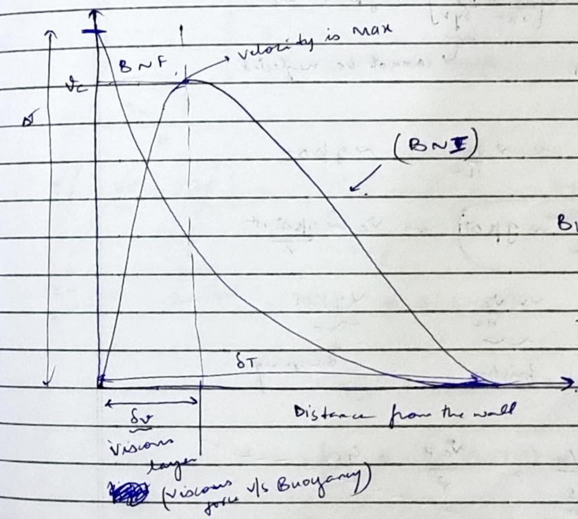
~~Ra~~

$Gr \approx \frac{B}{F} \sim 10^8 - 10^{10}$

$Gr^{1/4} = \frac{H}{\delta_v}$

$Ra_H^{1/4} = \frac{H}{\delta_T}$ Pr > 1

$Bo^{1/4} = \frac{H}{\delta_T}$ Pr < 1



$v_c \sim \frac{\alpha H}{\delta_T^2}$

Buoyancy ~ Friction

$g\beta\Delta T \sim \nu \frac{v_c}{\delta_v^2}$

$\delta_v^2 \sim \frac{\nu \frac{\alpha H}{\delta_T^2}}{g\beta\Delta T}$

$\delta_v^2 \sim \frac{H^4}{\frac{g\beta\Delta T H^3}{\nu\alpha} \times \delta_T^2}$

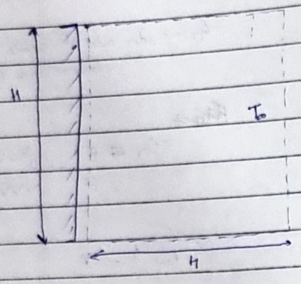
$\delta_v^2 \sim H^2 Pr^{1/2} Ra_H^{-1/2}$

$\delta_v^2 \sim H^2 \left(\frac{Pr}{Ra_H}\right)^{1/2}$

$\delta_v^2 \sim H^2 (Gr)^{-1/2}$

$\delta_v \sim H (Gr)^{-1/4}$

8)



$u \sim u$

$v \sim v_c$

$(T-T_\infty) \sim (T_0 - T_\infty) \sim \Delta T$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \nu\left[\frac{\partial^2 v}{\partial x^2} + \underbrace{\frac{\partial^2 v}{\partial y^2}}_{\text{cannot be neglected}}\right] + g\beta(T-T_\infty)$$

$x \sim H, \quad y \sim H$

$$\sim u\frac{v_c}{H} \qquad \sim \frac{v_c^2}{H} \qquad \sim \nu\frac{v_c}{H^2} \qquad \sim g\beta\Delta T$$

$$\nu\frac{v_c}{H^2} \sim g\beta\Delta T \rightarrow v_c \sim \frac{g\beta\Delta T H^2}{\nu}$$

$$\underbrace{\sim \frac{v_c^2}{H}}_{\text{inertia}} \qquad \underbrace{\sim \frac{\nu v_c}{H^2}}_{\text{friction}} \qquad \underbrace{\sim g\beta\Delta T}_{\text{Buoyancy}}$$

$$\frac{v_c^2/H}{\nu v_c/H^2} = \frac{v_c H}{\nu} = Gr.$$

$$\Rightarrow u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2}$$

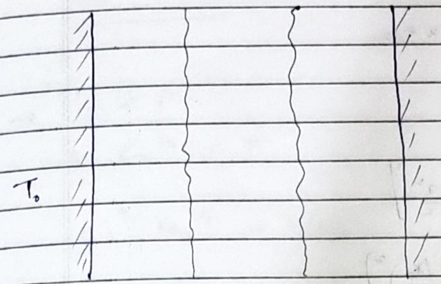
$$\sim u\frac{\Delta T}{H}, \quad \sim v_c\frac{\Delta T}{H} \quad \sim \alpha\frac{\Delta T}{H^2}$$

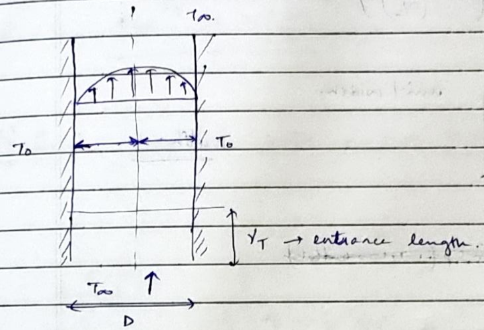
$$\sim v_c\frac{\Delta T}{H} \qquad \sim \alpha\frac{\Delta T}{H^2}$$

$$v_c \sim \frac{\alpha}{H}$$

18th April end sem (24)

Q) Solar chimney.





$$\rho\left(\cancel{u\frac{\partial v}{\partial x}} + \cancel{v\frac{\partial v}{\partial y}}\right) = -\frac{dP_\infty}{dy} + \nu\frac{\partial^2 v}{\partial x^2} - \rho g$$

≈ 0, $u << v$; 0 fully developed. ; $-\rho_\infty g$

$$= \nu\frac{\partial^2 v}{\partial x^2} + g(\rho - \rho_\infty)$$

$$= \nu\frac{\partial^2 v}{\partial x^2} + g\beta_\infty(T-T_\infty)$$

$$0 = \nu\frac{d^2 v}{dx^2} + g\beta_\infty(T-T_\infty)$$

$$\nu\frac{d^2 v}{dx^2} = -g\beta_\infty(T-T_\infty)$$ — we need energy eqn to solve this eqn.

BC $\rightarrow x = \frac{D}{2}$ $\quad v = 0$

$x = 0, \quad \frac{\partial v}{\partial x} = 0$

$(T - T_\infty) << (T_0 - T_\infty) \rightarrow$ assumption

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$$\nu \frac{\partial^2 v}{\partial x^2} = -g\beta_\infty(T_0 - T_\infty)$$

$$v(x) = -\frac{g\beta_\infty(T_0 - T_\infty)}{2\nu}x^2 + C_1 x + C_2$$

$C_1 = 0$

$$v(x) = +\frac{g\beta_\infty(T_0 - T_\infty)}{\nu}\left[\frac{D^2}{8} - \frac{x^2}{2}\right]$$

$$v(x) = \frac{g\beta_\infty(T_0 - T_\infty)D^2}{8\nu}\left(1 - \left(\frac{x}{D/2}\right)^2\right)$$

Mass flowrate = $\rho V_{avg} A$ unit width.

$$\dot m = 2\rho \int_0^{D/2} v(x)\,dx$$

width = w

$$\dot q = \dot m c_p \underbrace{\Delta T}_{(\text{out-in.})}$$

average heat flux → $q''_{0-H} = \frac{\dot q}{2\times(w\times H)}$

$w = 1$.

$$Nu_{0-H} = \frac{h_{0-H}\times H}{k} = \frac{\rho_\infty g\beta_\infty(\Delta T)^2 D^3 c_p}{24\times H\times k\,\Delta T} = \frac{Ra_D}{24}.$$

$y_T < H$.

→ $Pr > 1$, $\delta_T \sim H Ra_H^{-1/4}$

$\frac{D}{2} \sim y_T Ra_H^{-1/4}$. ($y_T \sim \frac{D}{2} Ra_H^{1/4}$)

→ $y_T < H$ → $\frac{D}{2} Ra_H^{1/4} < H$.

$$\frac{Ra_H}{Ra_D} = \left(\frac{H}{D}\right)^3.$$

Problem 4.13

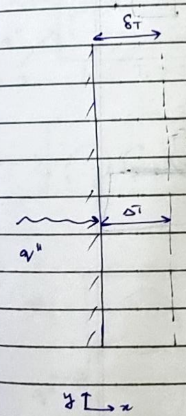
→ $y_T < H$

$\frac{D}{2} Ra_H^{1/4} < H$

$\left(\frac{H}{D}\right)^{3/4} Ra_D^{1/4} < 2(H/D)$

$$Ra_D^{1/4} < 2\left(\frac{H}{D}\right)^{1/4}.$$

Natural convection with uniform heat flux



$q'' \sim k\frac{\Delta T}{\delta_T}$ → from scaling

$\Delta T \sim \frac{q''\delta_T}{k}$

($\Delta T \sim f(y)$ → at the wall)

$Pr > 1$

$\delta_T \sim H(Ra_H)^{-1/4}$

$\sim H\left(\frac{g\beta\Delta T H^3}{\nu\alpha}\right)^{-1/4}$

$\delta_T^4 \sim H^4\left[\frac{g\beta q''\delta_T H^3}{k}\right]^{-1}$

$\delta_T^5 \sim H^5\left[\frac{\nu\alpha k}{\underbrace{g\beta\Delta T q'' H^4}_{Ra_H^*}}\right]$

Ra_H^* → Rayleigh number based on heat flux

$$\boxed{\delta_T \sim H(Ra_H^*)^{-1/5}}$$

$Pr<1$

$$\delta_T \sim H\left(Pr\, Ra_H^{*}\right)^{-1/5}$$

Problem 3 → Problem 4.17

Assumption

1. resistance of bottle is negligible
2. Natural convection only
3. lump capacitance model ($T = f(t)$ → only)

$$\dot{Q} = m_{cD} c_{p,cD} \left.\frac{dT}{dt}\right|_{cD}$$ (cD → cold drink)

rate at which cold drink is losing heat

$$\dot{Q} = h A (T_{cD} - T_{air})$$

A → area of bottle

$$m_{cD} c_{p,cD} \left.\frac{dT}{dt}\right|_{cD} = hA(T_{cD} - T_{air}) \rightarrow \text{at SS}$$

h → will change if the orientation of bottle changes.

$$\frac{1}{h} = \frac{1}{h_{air}} + \frac{1}{h_{cD}}$$

$$\frac{mc_p \Delta T}{t} \sim hA\Delta T \rightarrow \text{scaling}$$

$$t \sim \frac{mc_p}{hA}$$

t_1 → time taken to cool → vertical

t_2 → horizontal

$$\frac{t_1}{t_2} = \frac{h_2}{h_1}$$

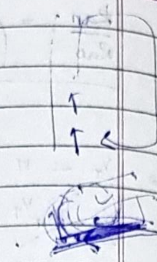
Nu|air

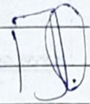
Nu|cD

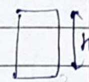
$$\frac{h_{cD}}{h_{air}} = \frac{k(Ra_H)^{1/4}}{k(Ra_H)^{1/4}} \sim 58.1$$ ✓

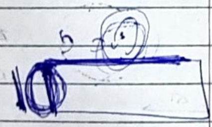
$$\frac{1}{h} = \frac{1}{h_{air}} + \frac{1}{h_{cD}}^{\approx 0}$$

$h \approx h_{air}$









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Problem - 4

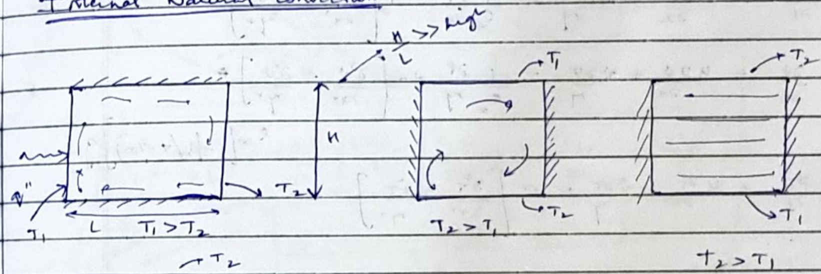
→ Similarity solution

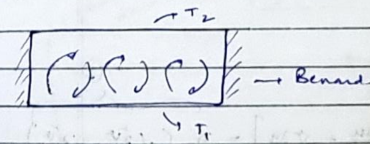
→ heat transfer results including the effect of turbulence

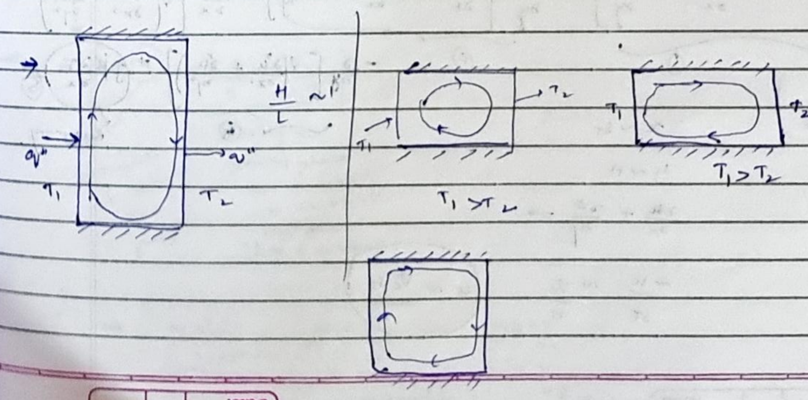
→ Inclined walls

→ Horizontal walls

Internal Natural convection







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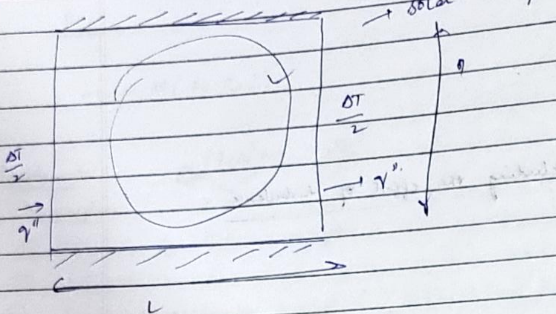
* Internal natural convection



L

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_\infty}\frac{\partial P}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right]$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_\infty}\frac{\partial P}{\partial y} + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] + \rho_\infty\left[1-\beta(T-T_\infty)\right]g$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right]$$

→ From momentum balance

$$\underbrace{\frac{\partial}{\partial x}\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right]}_{1} - \underbrace{\frac{\partial}{\partial y}\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right]}_{2} = \underbrace{\frac{\partial}{\partial x}\left[\nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)\right]}_{3} - \underbrace{\frac{\partial}{\partial y}\left[\nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\right]}_{4} + \underbrace{g\beta\frac{\partial T}{\partial x}}_{5 \text{ Buoyancy}}$$

(benard convection)

$x \sim \delta_T$

$y \sim H$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{u}{\delta_T} \sim \frac{v_c}{H}$$

$$u \sim \frac{v_c \delta_T}{H}$$

$$\underbrace{\sim \frac{1}{\delta_T}\left[\frac{v_c}{t} \sim \frac{u v_c}{\delta_T} \sim \frac{v_c v_c}{H}\right]}_{1} \qquad \underbrace{\sim \frac{1}{H}\left[\frac{u}{t} \sim \frac{u^2}{\delta_T} \sim \frac{v_c u}{H}\right]}_{2}$$

$$\underbrace{\sim \frac{1}{\delta_T}\left[\nu\left(\frac{v_c}{\delta_T^2} \sim \frac{v_c}{H^2}\right)\right]}_{3} \qquad \underbrace{\sim \frac{1}{H}\left[\nu\left(\frac{u}{\delta_T^2} \sim \frac{u}{H^2}\right)\right]}_{4}$$

$$\underbrace{\sim g\beta\frac{\Delta T}{\delta_T}}_{5}$$

$$\rightarrow \sim \left[\frac{v_c}{\delta_T t} \sim \frac{v_c^2}{H\delta_T} \sim \frac{v_c^2}{H\delta_T}\right] \sim \left[\frac{v_c\delta_T}{H^2 t} \sim \frac{v_c^2\delta_T}{H^2}\right]$$

$>>$ $>>$

$$\rightarrow \sim \left[\frac{v_c}{\delta_T t} \sim \frac{v_c^2}{H\delta_T} \sim \frac{v_c^2}{H\delta_T}\right] \times \frac{t\delta_T}{t} \sim \left[\cancel{\frac{v_c\delta_T}{H^2 t}} \sim \cancel{\frac{v_c^2\delta_T}{H^2}} \sim \cancel{\frac{v_c^2\delta_T}{H^3}}\right]$$

inertia term

$>>>$

$$\sim \left[\frac{\nu v_c}{\delta_T^3} \sim \cancel{\frac{\nu v_c}{H^2\delta_T}}\right] \sim \left[\cancel{\frac{\nu v_c}{\delta_T H^2}} \sim \cancel{\frac{\nu v_c \delta_T}{H^4}}\right] \sim g\beta\frac{\Delta T}{\delta_T}$$

viscous

$$\Rightarrow \sim \left[\sim 1 \quad \underbrace{\sim \frac{v_c t}{H} \quad \sim \frac{v_c t}{H}}\right]$$

will dominate at initial flow phase.

during the growing phase ~~these terms~~ will be small. ~~[illegible]~~ (being t is small)

$$\Rightarrow \underbrace{\sim \frac{\partial^2 v}{\partial x \partial t}}_{\text{inertia}} \qquad \underbrace{\sim \nu\frac{\partial^2 v}{\partial x^3}}_{\text{viscous / friction}} \qquad \underbrace{\sim g\beta\frac{\partial T}{\partial x}}_{\text{Buoyancy}}$$

$$\sim \frac{v_c}{\delta_T t} \qquad \sim \nu\frac{v_c}{\delta_T^3} \qquad \sim g\beta\frac{\Delta T}{\delta_T}$$

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$$N_A\big|_r - N_A\big|_{r+dr} = \dot{\Phi}_1$$

$$\frac{d}{dt}(\dot{m}) = -D\frac{dc}{dx}$$

Buoyancy → driving force

$$\frac{\text{Inertia}}{\text{friction}} = \frac{v_c/\delta_T t}{\nu \frac{v_c}{\delta_T^3}}$$

$$\sim \frac{\delta_T^2}{\nu t} \quad \left[\frac{\alpha t}{\nu t} = \frac{1}{Pr}\right]$$

$$\rightarrow \sim \frac{1}{Pr} \qquad \sim 1 \qquad \sim \frac{g\beta\Delta T\,\delta_T^2}{\nu v_c}$$

→ Viscous ~ Buoyancy.

$$\frac{g\beta\Delta T\,\delta_T^2}{\nu v_c} \sim 1$$

$$v_c \sim \frac{g\beta\Delta T(\alpha t)}{\nu} \qquad \delta_T \sim (\alpha t)^{1/2}$$

$$\rightarrow \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

(thermal inertia)

$$\sim \frac{\Delta T}{t} \sim \left[\frac{v_c\,\Delta T}{H}\right] \sim \left[\frac{\alpha\,\Delta T}{\delta_T^2}\right]$$

thermal inertia with time at wall decrease

at $t = t_f$

convection ~ conduction

$$\frac{v_c\,\Delta T}{H} \sim \frac{\alpha\,\Delta T}{\delta_T^2}$$

$$\delta_T \sim (\alpha t)^{1/2}$$

$$\delta_{T_f} \sim (\alpha t_f)^{1/2}$$

final

$$v_c \sim \frac{\alpha H}{\delta_f^2}$$

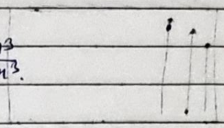
$$\frac{g\beta\Delta T(\alpha t_f)}{\nu} \sim \frac{\alpha H}{(\delta_T)^2} \rightarrow \alpha t_f$$

$$t_f \sim \frac{H\nu}{g\beta\Delta T\alpha}$$

$$\delta_{T,f} \sim (\alpha t_f)^{1/2}$$

$$(\delta_{T_f})^4 \sim (\alpha t_f)^2$$

$$\sim \alpha^2\left(\frac{H\nu}{g\beta\Delta T\alpha}\right) \times \frac{H^3}{H^3}$$



$$\sim H^4 \cdot \frac{1}{\frac{g\beta\Delta T H^3}{\alpha\nu}} \sim \frac{H^4}{Ra_H}$$

$$\delta_{T_f} \sim H(Ra_H)^{-1/4}$$

⇒ Viscous ~ Inertia

$$\frac{\nu v_c}{(\delta_v)^3} \sim \frac{v_c}{(\delta_v)t}$$

$$(\delta_v)^2 \sim (\nu t)$$

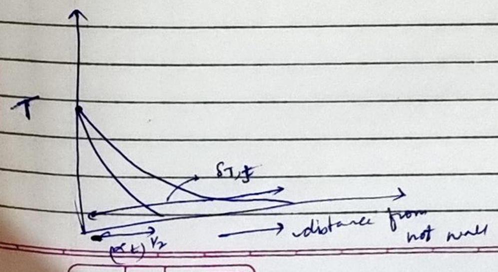
$$\rightarrow \delta_{v,f} \sim (\nu t_f)^{1/2}$$

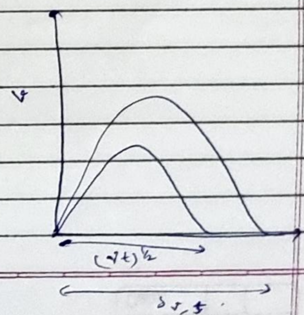
$$(\delta_{v,f})^2 \sim \frac{\nu}{\alpha}(\alpha t_f)$$

$$\left(\frac{\delta_{v,t}}{\delta_{T,f}}\right)^2 \sim Pr$$

$$\boxed{\delta_{v,t} \sim Pr^{1/2} H Ra^{-1/4}}$$

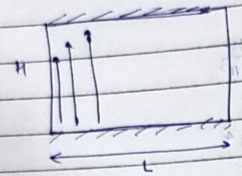
(final jet thickness)





~~① distinct wall~~

① distinct vertical jet moving upward



$$L < \delta_{T,j}$$

$$L > H \, Ra_H^{-1/4}$$

$$\frac{H}{L} < Ra_H^{1/4}$$

② distinct layer of fluid

$$\delta_{v,j} < L$$

$$(Pr)^{1/2} H \, Ra_H^{-1/4} < L$$

$$\frac{H}{L} < Pr^{-1/2} \, Ra_H^{1/4}$$