JC

Heat - kinetic energy of molecules

3 modes of heat transfer :-

a) Conduction - Transfer of heat between 2 bodies in contact having some temp. gradient. (like in solids)

b) Convection - Involves conduction as well as flow of matter. (like in fluids)

c) Radiation - Direct transmission of heat. No matter movement required.

Rate Laws :-

a) conduction - Fourier's law : $\vec{q} = -k \vec{\nabla} T$ (assum. : k is const.)

b) Convection - Newton's law of cooling : $q = h(T_{body} - T_{\infty})$

c) Radiation - Stefan-Boltzmann Law : $q \propto T^4$

Conduction :-

Governed by : Fourier's Law - $\vec{q} = -k \vec{\nabla} T$

In 1D :- (in steady state) $\longrightarrow$ No accum. of heat flux

$$q_x = -k \frac{dT}{dx}$$

T_1 T_2

$x = 0$ $x = L$

$$q = -k\left(\frac{T_2 - T_1}{L - 0}\right)$$

$$= \frac{-k(T_2 - T_1)}{L} = \frac{k(T_1 - T_2)}{L}$$

ex : 60°F

1/8 inch

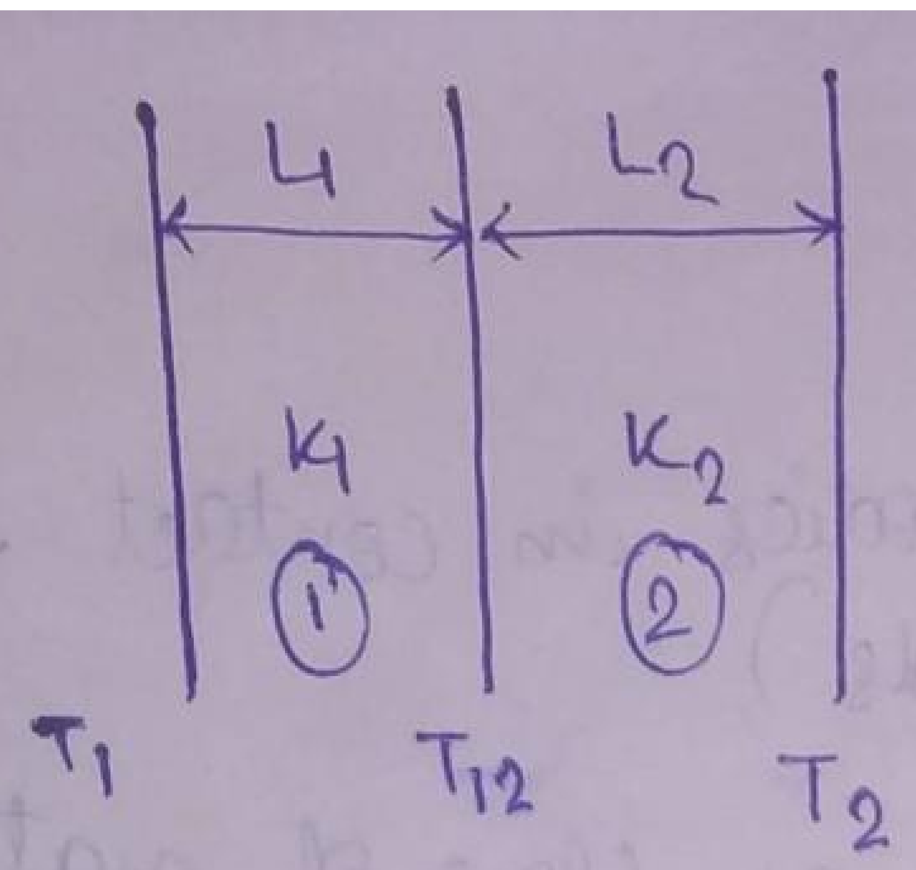
cold box (20°F)

$$k = 0.41 \frac{Btu}{h\ ft\ °F}$$

from out to inside box.

$$q = 0.41 \times \frac{(60-20)}{1/96\ ft} = 1574.4 \frac{Btu}{ft^2 hr}$$

c:



$T_1 > T_{12} > T_2$

q_x through 1 $= \dfrac{k_1 (T_1 - T_{12})}{L_1}$

assum : linear temp. profile.

q_x through 2 $= \dfrac{k_2 (T_{12} - T_2)}{L_2}$

$\because q_x$ is not accumulated (assum. : steady state)

$$\therefore \frac{k_1 (T_1 - T_{12})}{L_1} = \frac{k_2 (T_{12} - T_2)}{L_2}$$

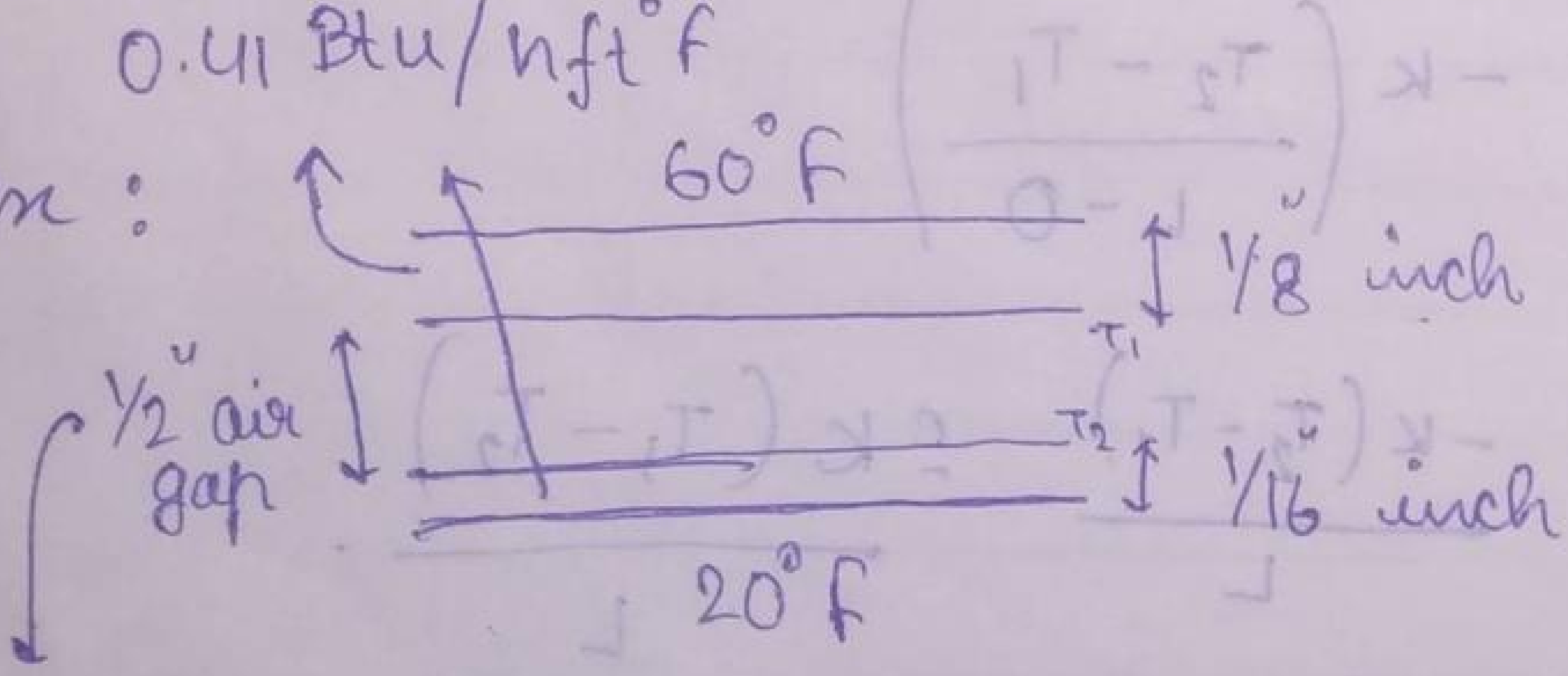
$$\Rightarrow \boxed{T_{12} = \frac{k_1 T_1 L_2 + k_2 T_2 L_1}{k_1 L_2 + k_2 L_1}}$$

$$\therefore q_x = \frac{k_1}{L_1}\left(T_1 - \frac{k_1 T_1 L_2 + k_2 T_2 L_1}{k_1 L_2 + k_2 L_1}\right)$$

$$= \frac{k_1}{L_1}\left(\frac{\cancel{k_1 T_1 L_2} + k_2 L_1 T_1 - \cancel{k_1 T_1 L_2} - k_2 T_2 L_1}{k_1 L_2 + k_2 L_1}\right)$$

$$\boxed{q_x = \frac{k_1 k_2 (T_1 - T_2)}{k_2 L_1 + k_1 L_2}} = \frac{T_1 - T_2}{\dfrac{L_1}{k_1} + \dfrac{L_2}{k_2}}$$

ex :



* No convection occuring in the air gap since top temp. is high and gap is very narrow.

$q_x = 22$

$k = 0.014 \dfrac{Btu}{h\,°F\,ft}$

$$q_x = \frac{60 - 20}{\dfrac{1}{96 \times 0.41} + \dfrac{1}{24 \times 0.014} + \dfrac{1}{192 \times 0.41}}$$

$$= 13.27 \ Btu/ft^2 h$$

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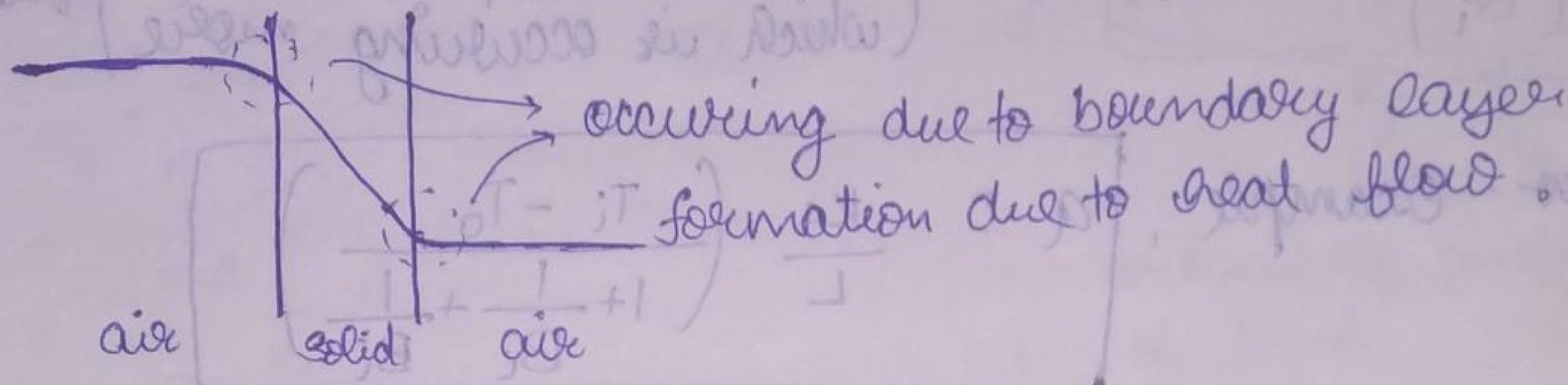
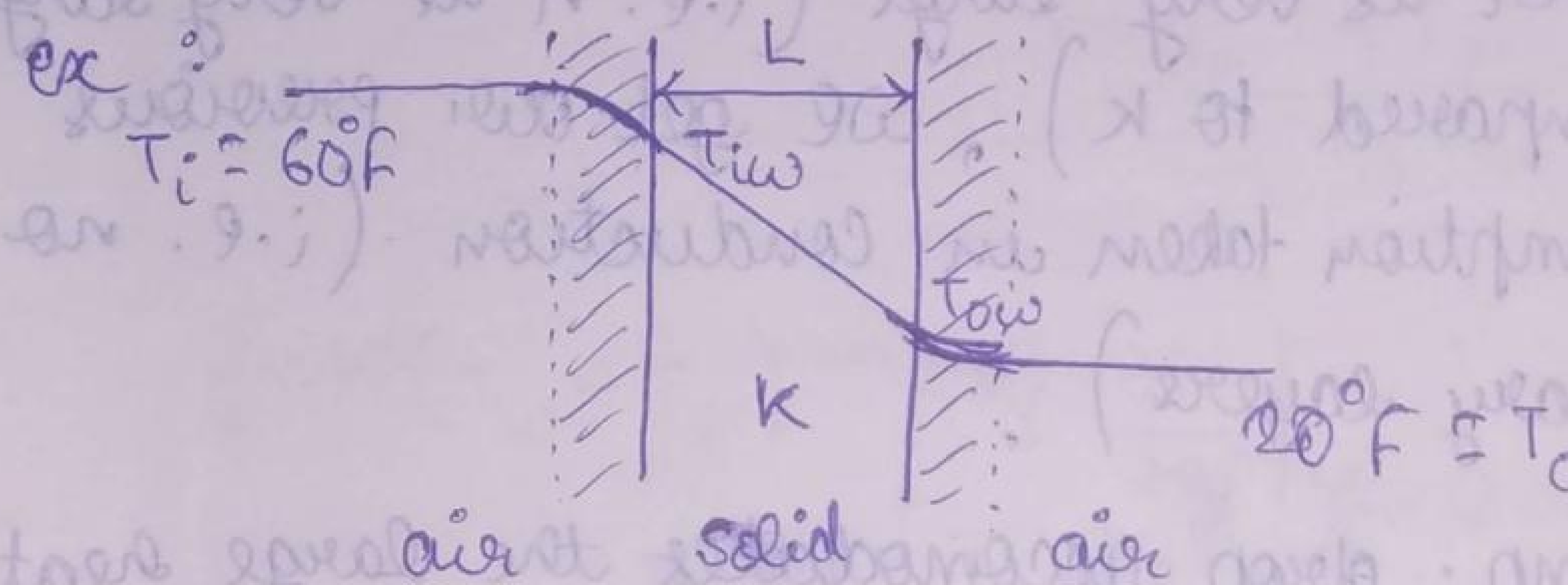
Temp. profile near the surface :-



ex :



* For shaded region since we have air, there is air flow as well hence, heat transfer is by convection.

$\therefore q_x = h(T_i - T_{iw}) \rightarrow$ from air to solid $= h(60 - T_{iw})$

$q_x = \frac{k}{L}(T_{iw} - T_{ow}) \rightarrow$ ~~from~~ inside solid $= \frac{k}{L}(T_{iw} - T_{ow})$

$q_x = h(T_{ow} - T_0) \rightarrow$ from solid to air $= h(T_{ow} - 20)$

$$h(60 - T_{iw}) = \frac{k}{L}(T_{iw} - T_{ow})$$

$$q_x = \frac{60 - 20}{\frac{L}{k} + \frac{1}{h} + \frac{1}{h}}$$

$$\frac{60h + \frac{kT_{ow}}{L}}{\frac{k}{L} + h} = T_{iw}$$

$$\frac{k}{L}\left(\frac{60h + \frac{kT_{ow}}{L}}{\frac{k}{L} + h} - T_{ow}\right) = h(T_{ow} - 20)$$

$$\frac{k}{L}\left(\frac{T_{ow}(60 - h)}{\frac{k}{L} + h}\right) = h(T_{ow} - 20)$$

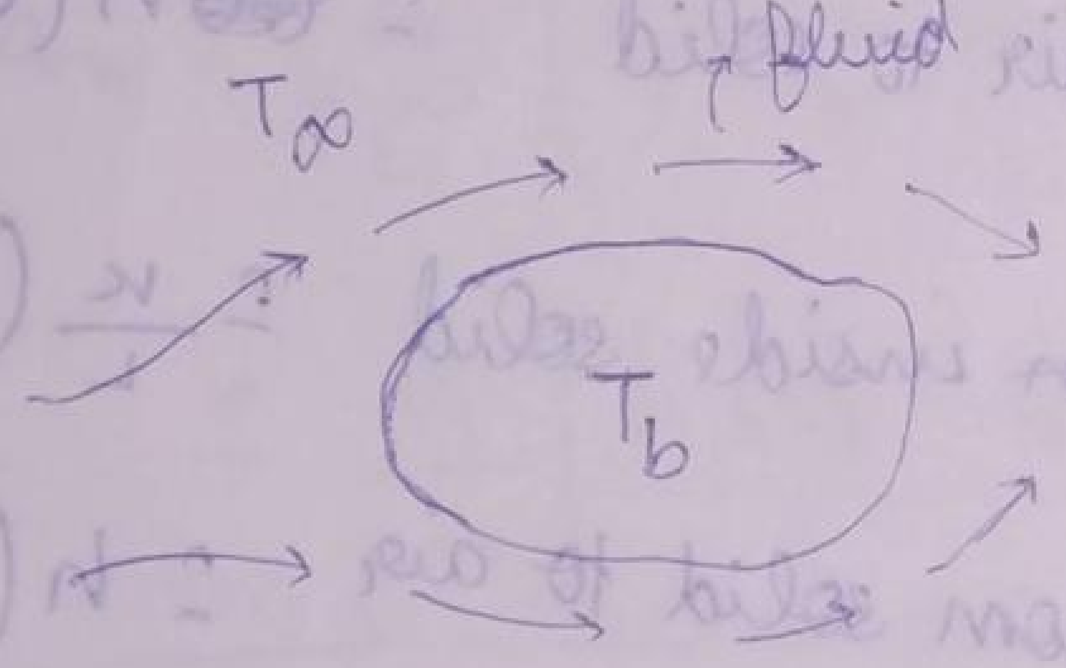
$\boxed{\text{Biot number } (B_i) = \frac{Lh}{k}}$: compares conduction (in solid phase) and convection (in fluid phase) (which is occuring more)

For prev. example, $\boxed{q = \frac{k_s}{L}\left(\frac{T_i - T_0}{1 + \frac{1}{B_i} + \frac{1}{B_i}}\right)}$

when B_i is very large (i.e. h is very large as compared to k), we get our previous assumption taken in conduction (i.e. no boundary layers).

* When B_i is high, small temp. drop accomodates the large heat flux easily (i.e. bulk temp. dominates).

~~Convections~~ Convective Heat Transfer :-

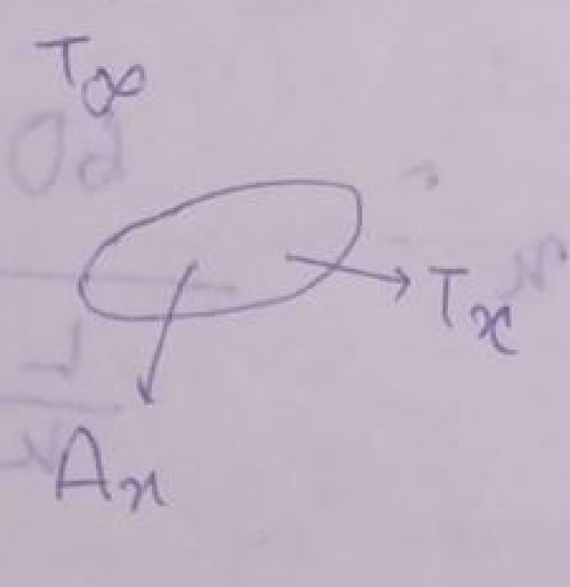


$\dot{Q} = A_b h (T_b - T_\infty)$

h → heat transfer co-eff. (is a fn. of the properties of the medium causing heat transfer)

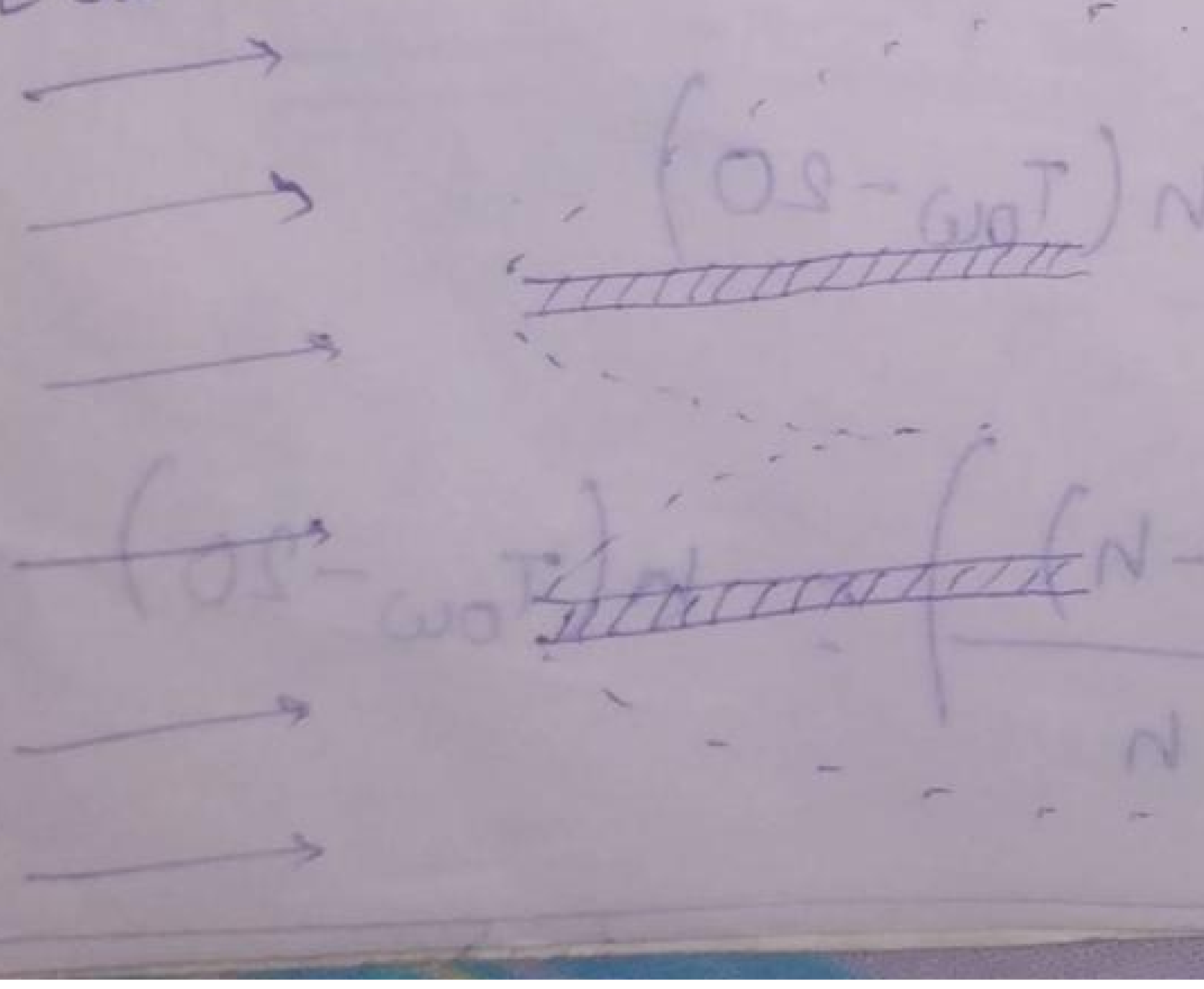
A_b → area of the body.

For a differential area :-



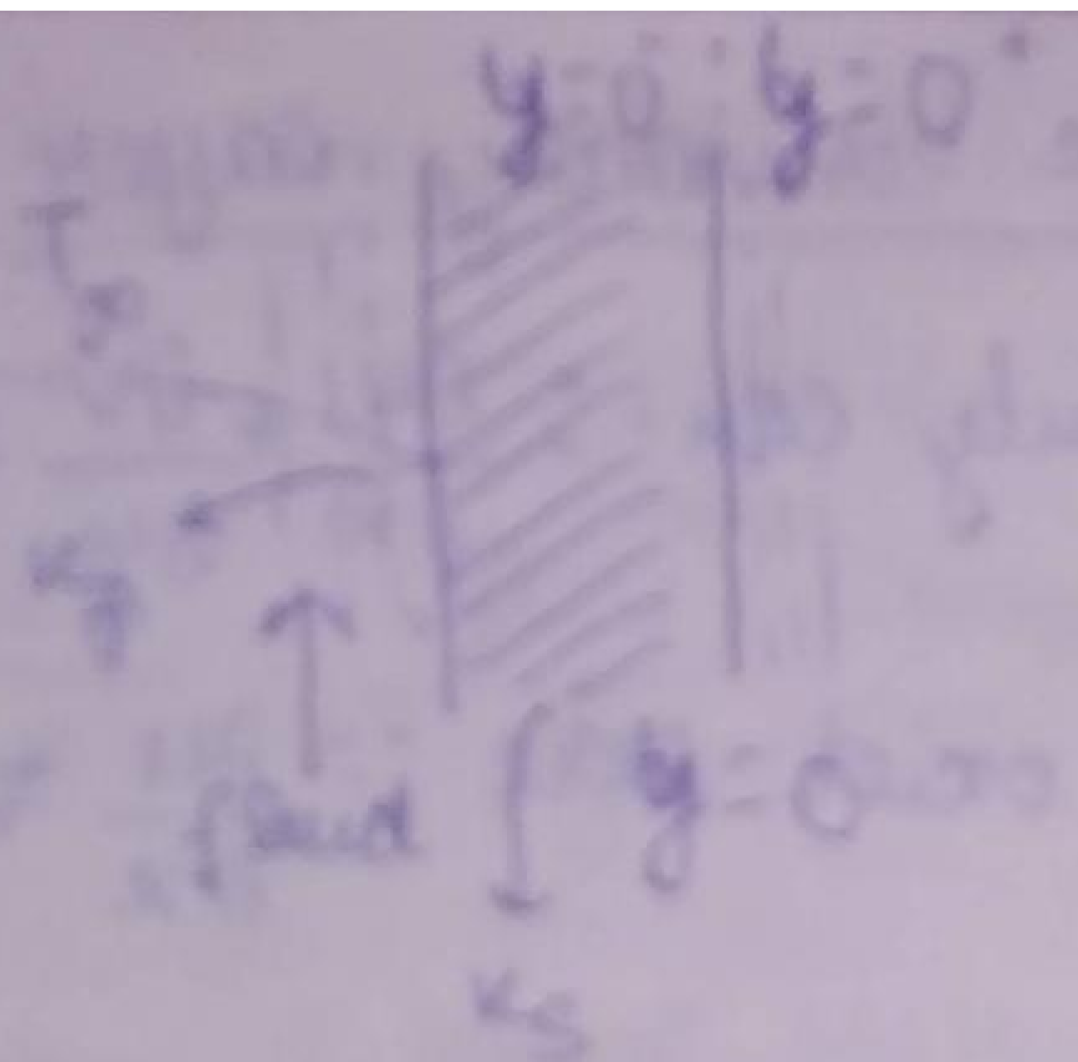
$d\dot{Q}_x = (dA_x)\, h\, (T_x - T_\infty)$

External and Internal Flow :-



For external flow, we have forever expanding boundary layer flow.

For internal flow, 2 boundary layers eventually merge into a fully-developed flow.



$$-k_s \cdot \frac{\partial T}{\partial y}\Big|_{y=0^-} = h(T_w - T_\infty)$$

$$= -k_f \cdot \frac{\partial T}{\partial y}\Big|_{x=0^+}$$

conductivity co-eff of fluid

$$\hat{T} = \frac{T - T_\infty}{T_w - T_\infty}, \quad \hat{y} = \frac{y}{\delta_t}$$

thermal boundary layer thickness.

$$h(T_w - T_\infty) = -k_f \cdot \frac{\partial T}{\partial y}\Big|_{y=0}$$

$$\frac{\partial \hat{y}}{\partial y} = \frac{1}{\delta_t}$$

$$\cancel{T_w - T_\infty = \frac{T - T_\infty}{\hat{T}}}$$

$$\cancel{h\left(\frac{T - T_\infty}{\hat{T}}\right) = -k_f \frac{\partial T}{\partial y}}$$

$$\frac{\partial \hat{T}}{\partial y} = \frac{\partial T}{\partial y}\left(\frac{1}{T_w - T_\infty}\right)$$

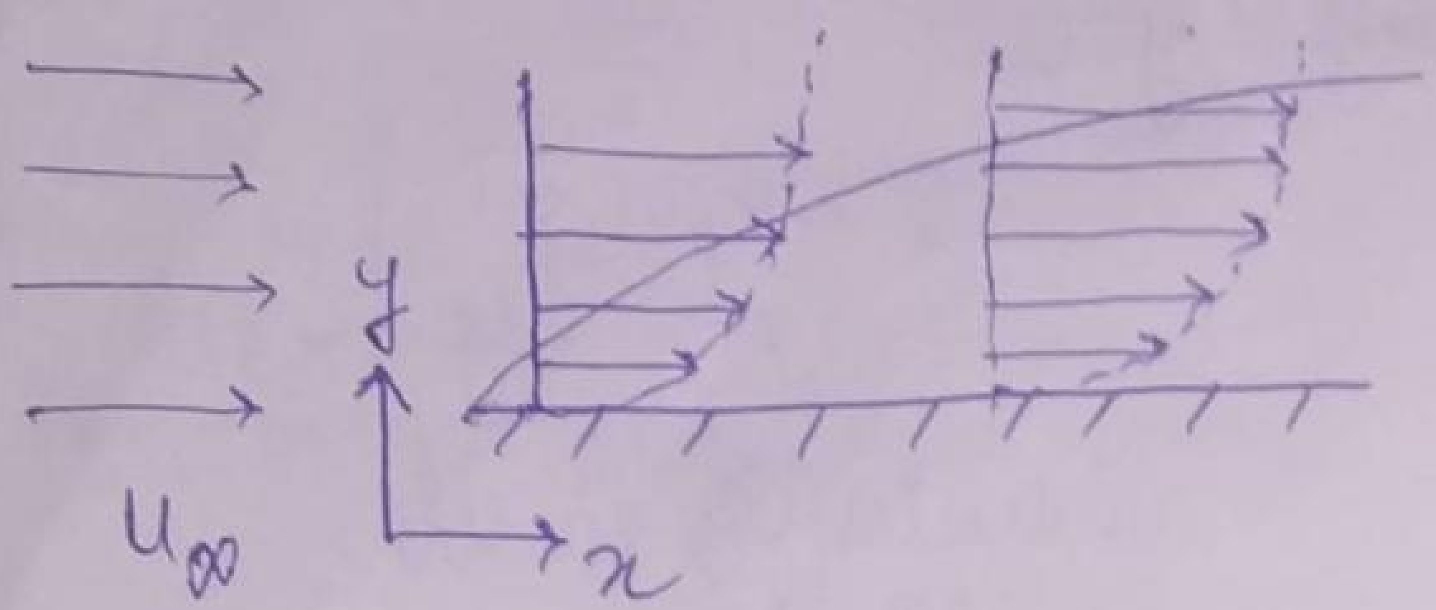
$$\frac{\partial \hat{T}}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial y} = \frac{\partial T}{\partial y}\left(\frac{1}{T_w - T_\infty}\right)$$

$$\Rightarrow \frac{\partial \hat{T}}{\partial \hat{y}}\left(\frac{1}{\delta_t}\right) = \frac{\partial T}{\partial y}\cdot\left(\frac{1}{T_w - T_\infty}\right)$$

$$h(\cancel{T_w - T_\infty}) = -k_f \frac{\partial \hat{T}}{\partial \hat{y}}\left(\frac{1}{\delta_t}\right)(\cancel{T_w - T_\infty})$$

$$\Rightarrow \boxed{\frac{\partial \hat{T}}{\partial \hat{y}}\Big|_{\hat{y}=0} = \frac{-h\delta_t}{k_f}} \longrightarrow \text{Nusselt Number}$$

Obtaining velo. profile in hydrodynamic boundary layer :-



continuity eqn. : $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

v has to exist here $\because \frac{\partial u}{\partial x} \neq 0$

Navier-stokes eqn. :-

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\cdot \cancelto{0}{\frac{dp}{dx}} + \underbrace{\frac{\mu}{\rho}}_{\text{Kinematic viscosity }(\nu)}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

(Press. is same (1 atm) everywhere).

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\cdot \cancelto{0}{\frac{dp}{dy}} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

order of magnitude estimate :-

$$\frac{\partial^2 u}{\partial x^2} \quad v/s \quad \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) \qquad \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)$$

$$\left.\begin{aligned} O\left(\frac{\partial u}{\partial x}\right) &= \frac{u_\infty}{x} \\ O\left(\frac{\partial u}{\partial y}\right) &= \frac{u_\infty}{\delta} \end{aligned}\right\} \text{order of magnitude estimates,}$$

$$O\left(\frac{\partial^2 u}{\partial x^2}\right) = \frac{u_\infty}{x^2}, \quad O\left(\frac{\partial^2 u}{\partial y^2}\right) = \frac{u_\infty}{\delta^2}$$

This is much greater than

$$\therefore\ u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\left(\cancelto{0}{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\Rightarrow u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\cdot\frac{\partial^2 u}{\partial y^2}$$

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$$\Rightarrow \frac{\partial(u^2)}{\partial x} - u\frac{\partial u}{\partial x} + \frac{\partial(uv)}{\partial y} - u\frac{\partial v}{\partial y} = \nu \cdot \frac{\partial^2 u}{\partial y^2}$$

$$\Rightarrow \frac{\partial(u^2)}{\partial x} + \frac{\partial(uv)}{\partial y} = \nu \cdot \frac{\partial^2 u}{\partial y^2} \qquad \because \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\right)$$

continuity

Boundary Integral Method :-

$$\int_0^{\delta(x)} \frac{\partial(u^2)}{\partial x}\,dy + \int_0^{\delta(x)} \frac{\partial(uv)}{\partial y}\,dy = \nu\int_0^{\delta(x)} \frac{\partial^2 u}{\partial y^2}\,dy$$

$$\Rightarrow \int_0^{\infty} \frac{\partial(u^2)}{\partial x}\,dy + \Big[uv\Big]_0^{\delta(x)} = \nu\left[\frac{\partial u}{\partial y}\right]_0^{\delta(x)}$$

$$\Rightarrow \int_0^{\infty} \frac{\partial(u^2)}{\partial x}\,dy + (uv)_{\delta(x)} = \nu\left[\overset{\nearrow 0}{\frac{\partial u}{\partial y}\Big|_{\delta(x)}} - \frac{\partial u}{\partial y}\Big|_0\right]$$

$$\Rightarrow \int_0^{\infty} \frac{\partial(u^2)}{\partial x}\,dy + u_\infty V_\infty = -\nu \cdot \frac{\partial u}{\partial y}\Big|_0$$

$$\int_0^{\delta(x)} \frac{\partial u}{\partial x}\,dy + \int_0^{\delta(x)} \frac{\partial v}{\partial y}\,dy = 0$$

$$\Rightarrow V_\infty = -\int_0^{\delta(x)} \frac{\partial u}{\partial x}\,dy$$

$$\Rightarrow \int_0^{\delta(x)} \frac{\partial(u^2)}{\partial x}\,dy - u_\infty\int_0^{\delta(x)} \frac{\partial u}{\partial x}\,dy = -\nu \cdot \frac{\partial u}{\partial y}\Big|_0$$

$$\Rightarrow \int_0^{\delta(x)} \frac{\partial}{\partial x}\Big[u(u-u_\infty)\Big]\,dy = -\nu \cdot \frac{\partial u}{\partial y}\Big|_0, \quad \hat{u} = \frac{u}{u_\infty}, \quad \hat{y} = \frac{y}{\delta(x)}$$

Leibnitz rule :-

$$\frac{d}{dt}\int_{\alpha(t)}^{\beta(t)} f(x,t)\,dt = \int_{\alpha(t)}^{\beta(t)} \frac{\partial}{\partial t}\Big[f(x,t)\,dx\Big] + f(\beta,t)\frac{d\beta}{dt} - f(\alpha,t)\frac{d\alpha}{dt}$$

IC

$$\frac{d}{dx}\int_0^{\delta(x)} u(u-u_\infty)\,dy = -\nu \cdot \frac{\partial u}{\partial y}\Big|_{y=0}$$

$$\hat{u} = \frac{u}{u_\infty}, \quad \hat{y} = \frac{y}{\delta(x)}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial y} \qquad dy = \delta(x)\,d\hat{y}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \hat{y}} \cdot \frac{1}{\delta(x)}$$

$$\cancel{\frac{d}{dx}\int_0^{\delta(x)} u(u-u_\infty)\,dy = -\nu \cdot \frac{1}{\delta(x)} \cdot \frac{\partial}{\partial \hat{y}}(\hat{u} \cdot u_\infty)\Big|_{\hat{y}=0}}$$

$$\Rightarrow \frac{d}{dx}\int_0^1 \hat{u}u_\infty(\hat{u}u_\infty - u_\infty)\,\delta(x)\,d\hat{y} = -\nu \cdot \frac{1}{\delta(x)} \cdot \frac{\partial}{\partial \hat{y}}\left[\hat{u}u_\infty\right]\Big|_{\hat{y}=0}$$

$$u_\infty \cdot u_\infty \frac{d}{dx}\left[\delta(x)\int_0^1 \hat{u}(\hat{u}-1)\,d\hat{y}\right] = -\nu \frac{u_\infty}{\delta(x)} \cdot \frac{\partial \hat{u}}{\partial \hat{y}}\Big|_{\hat{y}=0}$$

$\hat{u} = 1$ @ $\hat{y} = 1$ $\Rightarrow \hat{u} = f(\hat{y})$

* Funda: while approximating a fn. by a polynomial don't go to very high degree polynomial otherwise fn. will experience lot of oscillation.

$$\hat{u} = a + b\hat{y} + c\hat{y}^2 + d\hat{y}^3$$

$\hat{u} = 0$ @ $\hat{y} = 0$

@ $\hat{y} = 1$, $\dfrac{d\hat{u}}{d\hat{y}} = 0$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \quad \left[\text{From Navier stoke's eq}\right]$$

at $\hat{y} = 0$, $u = v = 0$

hence $\dfrac{\partial^2 \hat{u}}{\partial \hat{y}^2} = 0$

$a = 0$ (i)

$b + c + d = 1$ (ii)

$$\frac{d\hat{u}}{d\hat{y}} = 3d\hat{y}^2 + 2c\hat{y} + b \Rightarrow 3d + 2c + b = 0 \text{ (iii)}$$

$$\frac{d^2\hat{u}}{d\hat{y}^2} = 6d\hat{y} + 2c \Rightarrow 2c = 0 \Rightarrow c = 0 \text{ (iv)}$$

$b + d = 1$

$3d + b = 0$

$d = -1/2$, $b = 3/2$

$$\boxed{\hat{u} = \frac{3\hat{y}}{2} - \frac{\hat{y}^3}{2}} \longrightarrow \text{Putting this in integral eqn.:-}$$

$$u_\infty \cdot \frac{d}{dx}\left[\delta(x)\int_0^1\left(\frac{3\hat{y}}{2} - \frac{\hat{y}^3}{2}\right)\left(\frac{3\hat{y}}{2} - \frac{\hat{y}^3}{2} - 1\right)d\hat{y}\right] = -\nu \cdot \frac{1}{\delta(x)} \cdot \frac{3}{2}$$

$$= u_\infty \cdot \frac{d}{dx}\left[\delta(x) \cdot \int_0^1 \frac{9\hat{y}^2}{4} - \frac{3\hat{y}^4}{4} - \frac{3\hat{y}}{2} - \frac{3\hat{y}^4}{4} + \frac{\hat{y}^6}{4} + \frac{\hat{y}^3}{2}\, d\hat{y}\right] = -\nu \cdot \frac{1}{\delta(x)} \cdot \frac{3}{2}$$

$$: u_\infty \cdot \frac{d}{dx}\left[\delta(x) \cdot \left(-\frac{39}{280}\right)\right] = -\frac{\nu}{\delta(x)} \cdot \frac{3}{2}$$

$$\Rightarrow \int_0^{\delta(x)} \delta(x) \cdot d(\delta(x)) = \frac{\nu}{u_\infty} \cdot \frac{\cancel{3}}{\cancel{2}} \times \frac{\cancel{280}^{140}}{\cancel{39}_{13}} dx = \int_0^x \frac{140\nu}{13u_\infty} dx$$

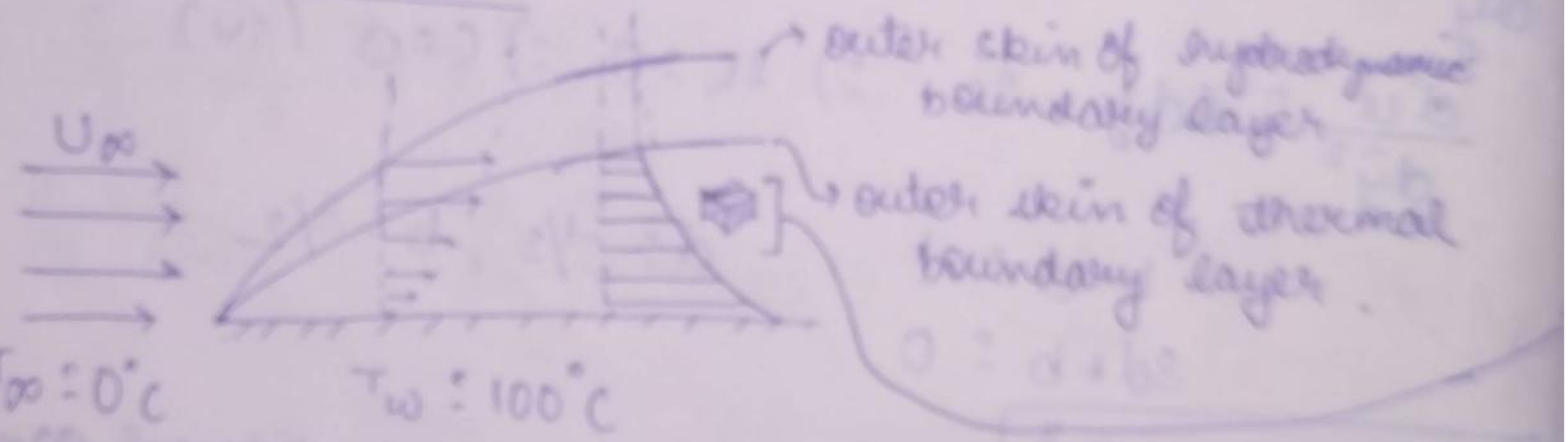
$$\Rightarrow \frac{\delta^2(x)}{2} = \frac{140\nu}{13u_\infty} \cdot x$$

$$\Rightarrow \delta^2(x) = \frac{280\nu}{13u_\infty} \cdot x \Rightarrow \delta(x) = \sqrt{\frac{280\nu}{13u_\infty}} \cdot \sqrt{x}$$

$$\boxed{\delta(x) = 4.64\, x\, (Re_x)^{-1/2}} \qquad = \sqrt{\frac{280\,\nu}{13\,u_\infty x}} \cdot x$$

(Re_x)

Thermal Boundary layer :-



Thickness of each boundary layer depends on diffusion co-efficient.

* In case of hydrodynamic, it is diffusion of momentum and governed by kinematic viscosity (ν).

* In thermal case, it is diffusion co-efficient $\frac{k}{\rho c_p} \cong \alpha$.

$$\frac{\nu}{\alpha} = \text{Prandtl No} = \frac{c_p \mu}{k}$$
(P_r)

(see Atta's part)

if $P_r \geq 1$: hydrodynamic boundary layer is thicker

if $P_r < 1$: thermal boundary layer is thicker.

$$\frac{\delta_t(x)}{\delta(x)} = \psi(P_r)$$ ratio of thermal BL thickness and hydrodynamic BL thickness is a function of P_r.

Flowing fluid has 2 types of energy

a) Internal energy
b) Flow energy

together constitute enthalpy

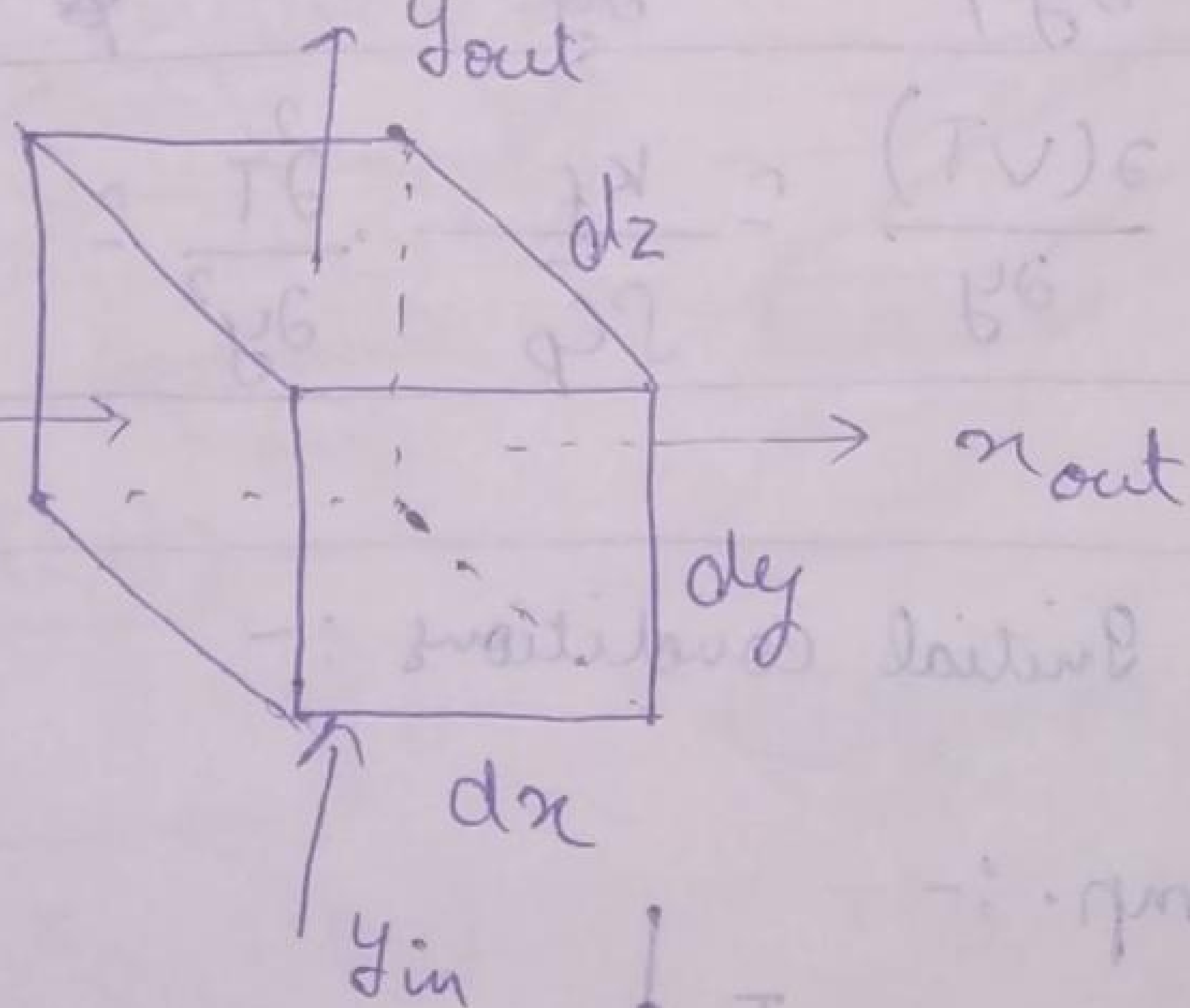
Enthalpy / ~~per~~ unit vol : $\rho \underbrace{\hat{C}_p (T - T_{ref})}_{\text{enthalpy/unit mass}}$

Flux of any quantity through a surface = $\rho \times V$

ρ : density relating to that quantity

V : velocity

Neglecting conduction in x-direc. and convection in y-direc. :-



~~x_{in} = ... $k(dy\,dz)\frac{dT}{dx}$, x_{out}~~

Flux :-

$x_{in} = u \cdot \rho \hat{C}_p T \big|_x (dy\,dz)$, $x_{out} = u \cdot \rho \hat{C}_p T \big|_{x+dx} (dy\,dz)$

$$\dot{y}_{in}^{cond} = -k_f \frac{\partial T}{\partial y}\bigg|_y (dx\,dz) \quad , \quad y_{out}^{cond} = -k_f \frac{\partial T}{\partial y}\bigg|_{y+dy} (dx\,dz)$$

$$y_{in}^{conv} = v.\rho\hat{C}_p T\Big|_y (dx\,dz) \quad , \quad y_{out}^{conv} = v.\rho\hat{C}_p T\Big|_{y+dy} (dx\,dz)$$

$$u.\rho\hat{C}_p T\Big|_x (dy\,dz) - k_f \frac{\partial T}{\partial y}\bigg|_y (dx\,dz) + v.\rho\hat{C}_p T\Big|_y (dx\,dz) =$$

$$u.\rho\hat{C}_p T\Big|_{x+dx} (dy\,dz) - k_f \frac{\partial T}{\partial y}\bigg|_{y+dy} (dx\,dz) +$$

$$v.\rho\hat{C}_p T\Big|_{y+dy} (dx\,dz)$$

dividing by $(dx\,dy\,dz)$ on LHS and RHS :-

$$u\rho\hat{C}_p \cdot \frac{\partial T}{\partial x} - k_f \frac{\partial^2 T}{\partial y^2} + v\rho\hat{C}_p \cdot \frac{\partial T}{\partial y} = 0$$

$$\cancel{\rho\hat{C}_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k_f \frac{\partial^2 T}{\partial y^2} = 0}$$

$$\boxed{\frac{\partial(uT)}{\partial x} + \frac{\partial(vT)}{\partial y} = \frac{k_f}{\rho\hat{C}_p} \cdot \frac{\partial^2 T}{\partial y^2} = \alpha \cdot \frac{\partial^2 T}{\partial y^2}}$$

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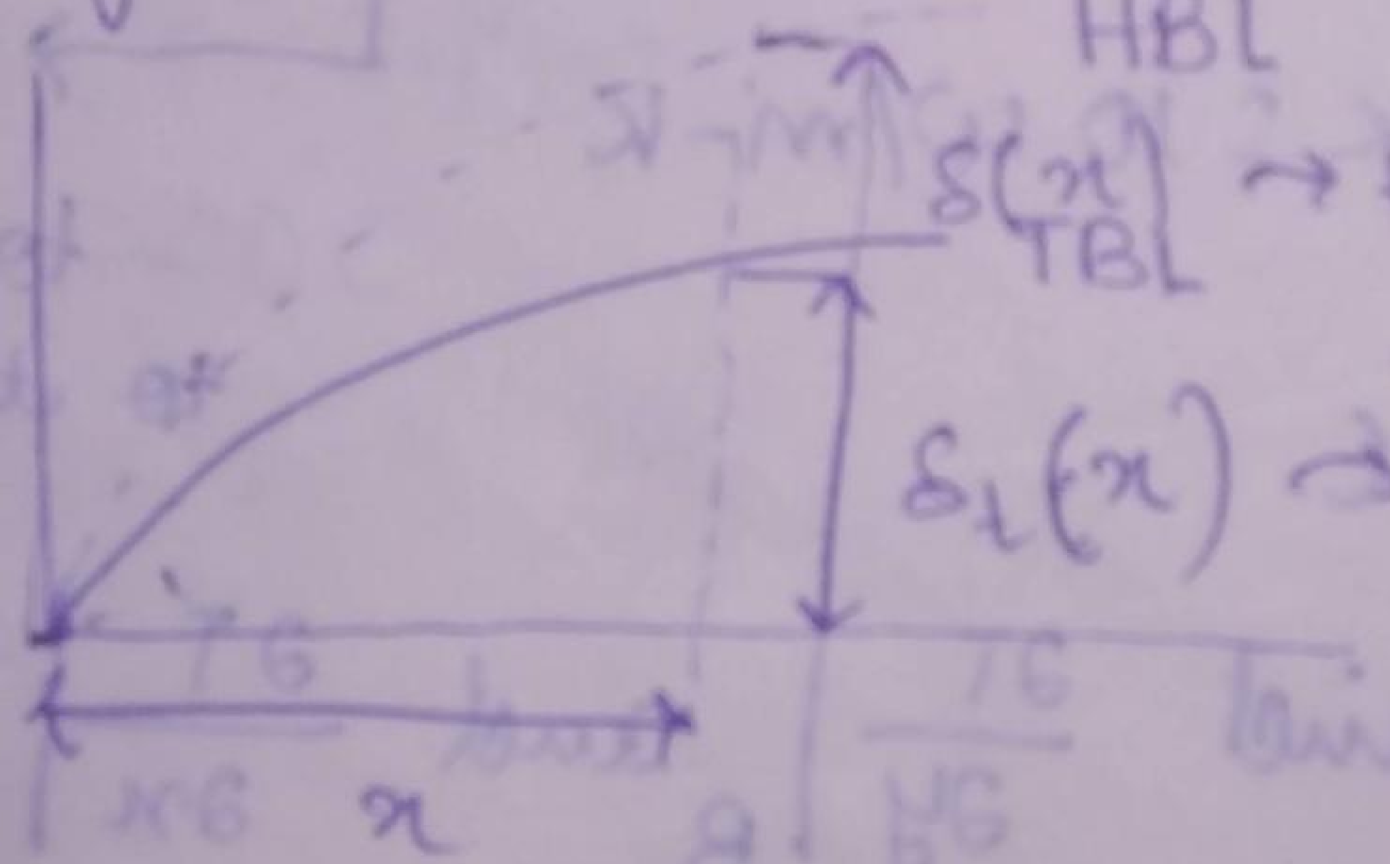
TC] $\frac{\partial(uT)}{\partial x} + \frac{\partial(vT)}{\partial y} = \frac{k_f}{\rho \hat{C}_p} \cdot \frac{\partial^2 T}{\partial y^2} = \alpha \cdot \frac{\partial^2 T}{\partial y^2}$ → Energy Balance eqn. in thermal boundary layer

Assum : hydrodynamic BL is thicker than thermal BL. advantage - velo. expr in HBL holds everywhere in TBL.

Boundary integral method :-



$\delta(x)$ → thickness of HBL

$\delta_t(x)$ → thickness of TBL.

$$\int_0^{\delta_t(x)} \frac{\partial(uT)}{\partial x}dy + \int_0^{\delta_t(x)} \frac{\partial(vT)}{\partial y}dy = \alpha\int_0^{\delta_t(x)} \frac{\partial^2 T}{\partial y^2}dy$$

$$\int_0^{\delta_t(x)} \frac{\partial(Tu)}{\partial x}dy + T_\infty V\Big|_{\delta_t(x)} = \alpha\int_0^{\delta_t(x)} \frac{\partial^2 T}{\partial y^2}dy$$

$$\int_0^{\delta_t(x)} \frac{\partial(Tu)}{\partial x}dy + T_\infty \underbrace{\int_0^{\delta_t(x)} -\frac{\partial u}{\partial x}dy}_{\text{obtained from continuity eqn. integration.}} = \alpha\int_0^{\delta_t(x)} \frac{\partial^2 T}{\partial y^2}dy$$

$$\Rightarrow \frac{d}{dx}\int_0^{\delta_t(x)} u(T-T_\infty)\,dy = -\alpha\frac{\partial T}{\partial y}\Big|_{y=0}$$

$$\hat{T} = \frac{T-T_\infty}{T_w-T_\infty}, \quad \hat{y}_t = \frac{y}{\delta_t(x)}, \quad \hat{u} = \frac{u}{u_\infty}, \quad \hat{y} = \frac{y}{\delta(x)}$$

$$\Rightarrow \frac{d}{dx}\int_0^{\delta_t(x)} \hat{u}\cdot u_\infty(T_w-T_\infty)\hat{T}\,dy = -\alpha\frac{\partial T}{\partial y}\Big|_{y=0}$$

$$\Rightarrow \frac{d}{dx}\left[\delta_t(x)\int_0^1 \hat{u}\,\hat{T}\,d\hat{y}_t\right] = \frac{-\alpha}{u_\infty\delta_t(x)}\cdot\frac{d\hat{T}}{d\hat{y}}\Big|_{\hat{y}=0}$$

$$\hat{u} = \frac{3}{2}\hat{y} - \frac{1}{2}\hat{y}^3$$

$$\hat{T} = a + b\hat{y}_t + c\hat{y}_t^2 + d\hat{y}_t^3$$

$$\boxed{\begin{array}{l} @\ \hat{y}_t = 0,\ \hat{T} = 1 \\ @\ \hat{y}_t = 1,\ \hat{T} = 0 \end{array}}$$

$@\ \hat{y}_t = 1,\ \dfrac{\partial\hat{T}}{\partial\hat{y}} = 0$

$@\ \hat{y}_t = 0,\ \dfrac{\partial^2\hat{T}}{\partial\hat{y}^2} = 0$

$\therefore a = 1,\ d = 1/2,\ b = -3/2$

$b + c + d = -1$

$b + 2c + 3d = 0$

$2c = 0 \Rightarrow c = 0$

$$\hat{T} = 1 - \frac{3}{2}\hat{y}_t + \frac{1}{2}\hat{y}_t^{3}$$

$$\frac{d}{dx}\left[\delta_t(x)\int_0^1 \left(\frac{3\hat{y}}{2} - \frac{1}{2}\hat{y}^{3}\right)\left(1-\frac{3\hat{y}_t}{2}+\frac{1}{2}\hat{y}_t^{3}\right) d\hat{y}_t\right] = \frac{3\alpha}{2u_\infty \delta_t(x)}$$

$$\frac{\hat{y}}{\hat{y}_t} = \psi(\nu, \alpha)$$

$$\frac{d}{dx}\left[\delta_t(x)\int_0^1 \left(\frac{3\hat{y}_t\psi(\nu,\alpha)}{2} - \frac{1}{2}\hat{y}_t^{3}\psi^{3}(\nu,\alpha)\right)\left(1-\frac{3\hat{y}_t}{2}+\frac{1}{2}\hat{y}_t^{3}\right) d\hat{y}_t\right] = \frac{3\alpha}{2u_\infty \delta_t(x)}$$

on solving integral,

$$\frac{d}{dx}\left[\delta_t(x)\underbrace{\left(\frac{3\psi}{20} - \frac{3\psi^{3}}{280}\right)}_{\xi}\right] = \frac{3}{2}\cdot\frac{\alpha}{u_\infty \delta_t(x)}$$

$$\int_0^{\delta_t(x)} \delta_t(x)\cdot \xi \, d\delta_t(x) = \int_0^x \frac{3\alpha}{2u_\infty} dx$$

$$\xi \cdot \frac{\delta_t^2(x)}{\cancel{2}} = \frac{3\alpha x}{\cancel{2}u_\infty}$$

$$\Rightarrow \boxed{\delta_t(x) = \sqrt{\frac{3\alpha x}{\xi u_\infty}}}$$

$$\psi = \frac{\hat{y}}{\hat{y}_t} = \frac{\delta_t(x)}{\delta(x)}$$

$$\delta(x) = 4.64\sqrt{\frac{\nu x}{u_\infty}}$$

$$\psi = \sqrt{\frac{3\alpha x}{\xi u_\infty}} \times \sqrt{\frac{u_\infty}{\nu x}} \times \frac{1}{4.64}$$

$$\psi = 0.37\sqrt{\frac{\alpha}{\nu\xi_f}}$$

$$= 0.37\sqrt{\frac{\alpha}{\nu}} \cdot \frac{1}{\sqrt{\frac{3\psi}{20} - \underbrace{\frac{3\psi^3}{280}}_{\text{neglected } (\because \psi < 1)}}}$$

$$\psi = 0.37\sqrt{\frac{\alpha}{\nu}} \cdot \sqrt{\frac{20}{3\psi}}$$

$$= 0.96\sqrt{\frac{\alpha}{\nu\psi}}$$

$$\psi^{3/2} = 0.96\sqrt{\frac{\alpha}{\nu}} \Rightarrow \psi = 0.97\left(\frac{\alpha}{\nu}\right)^{1/3}$$

$$\boxed{\psi = 0.97\,P_r^{-1/3}} \approx \boxed{\psi = P_r^{-1/3}}$$

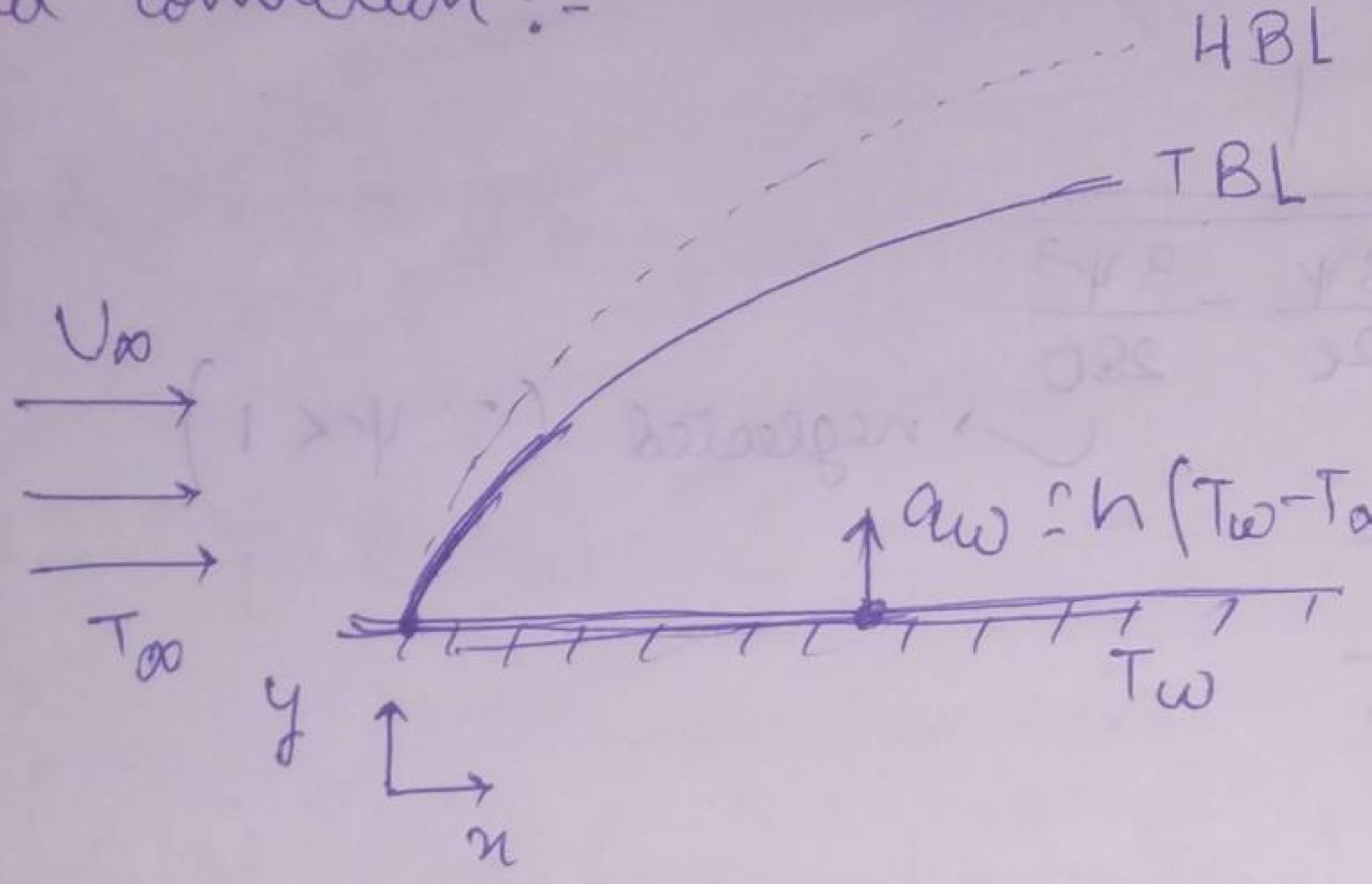
$$\cancel{\delta_t(x) = \sqrt{\frac{8\alpha x}{u_\infty}} \cdot \frac{1}{\sqrt{\frac{3}{20}P_r^{-1/3} - \frac{3}{280}P_r^{-1}}}}$$

$$\delta(x) = 4.64\,x\,(Re_x)^{-1/2}$$

$$P_r^{-1/3} = \frac{\delta_t(x)}{4.64\,x\,(Re_x)^{-1/2}}$$

$$\boxed{\delta_t(x) = 4.64\,x\,(P_r)^{-1/3}(Re_x)^{-1/2}}$$

Obtaining the heat transfer co-efficient of for external forced convection :-



$$q_w = h(T_w - T_\infty) = -k_f \frac{\partial T}{\partial y}\bigg|_{y=0}$$

$$\hat{T} = \frac{T-T_\infty}{T_w-T_\infty}, \quad \frac{d\hat{T}}{dy} = \frac{1}{T_w-T_\infty}\cdot\frac{dT}{dy}$$

$$\hat{y} = \frac{y}{\delta_t(x)} \Rightarrow \frac{d\hat{y}}{dy} = \frac{1}{\delta_t(x)}$$

$$\frac{d\hat{T}}{dy} = \frac{d\hat{T}}{d\hat{y}} \times \frac{d\hat{y}}{dy} = \frac{d\hat{T}}{d\hat{y}} \times \frac{1}{\delta_t(x)} = \frac{1}{T_w-T_\infty}\cdot\frac{dT}{dy}$$

$$q_w = -k_f\cdot\frac{(T_w-T_\infty)}{\delta_t(x)}\cdot\frac{d\hat{T}}{d\hat{y}}\bigg|_{\hat{y}=0}$$

$$= \cancel{-}k_f\cdot\frac{(T_w-T_\infty)}{\delta_t(x)}\cdot\left(\cancel{-}\frac{3}{2}\right)$$

from $\hat{T} = 1 - \frac{3}{2}\hat{y}_t + \frac{1}{2}\hat{y}_t^3$ obtained in previous lecture.

$$h\cancel{(T_w-T_\infty)} = \frac{k_f}{\delta_t(x)}\cdot\frac{3}{2}\cancel{(T_w-T_\infty)}$$

$$h = \frac{3k_f\, Re_x^{1/2}\, Pr^{1/3}}{2\times 4.64\, x} = \frac{0.323 k_f\, Re_x^{1/2}\, Pr^{1/3}}{x}$$

$$\frac{h_x x}{k_f} = \boxed{Nu_x = 0.328\, Re_x^{1/2}\, Pr_x^{1/3}}$$ → Nusselt number co-relation at a given x

$$Re_x = \frac{x u_\infty}{\nu}$$

Restrictions :- (for above case).

a) Ext. forced convection
b) Constant wall temp.
c) Constant props.
d) Flat Plate geometry
e) $P_{x} > 1$
f) Laminar flow

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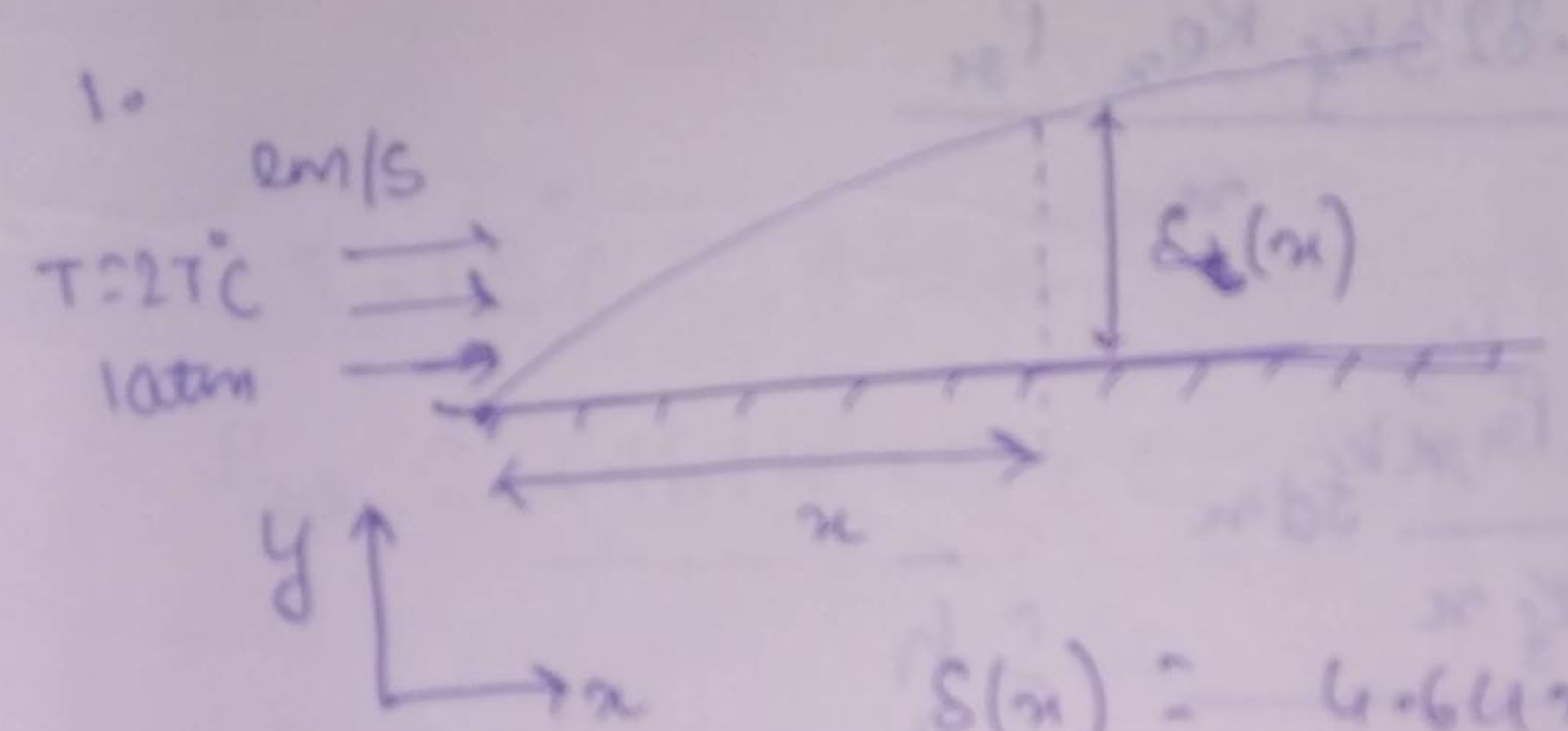
JC

Tutorial :- (PS 1)

1.



$\mu = 1.85 \times 10^{-5}$ kg/ms

$\rho = 1.177$ kg/m³.

$$\delta(x) = 4.64x\,(Re_x)^{-1/2}$$

$$Re_x = \frac{u\rho x}{\mu}$$

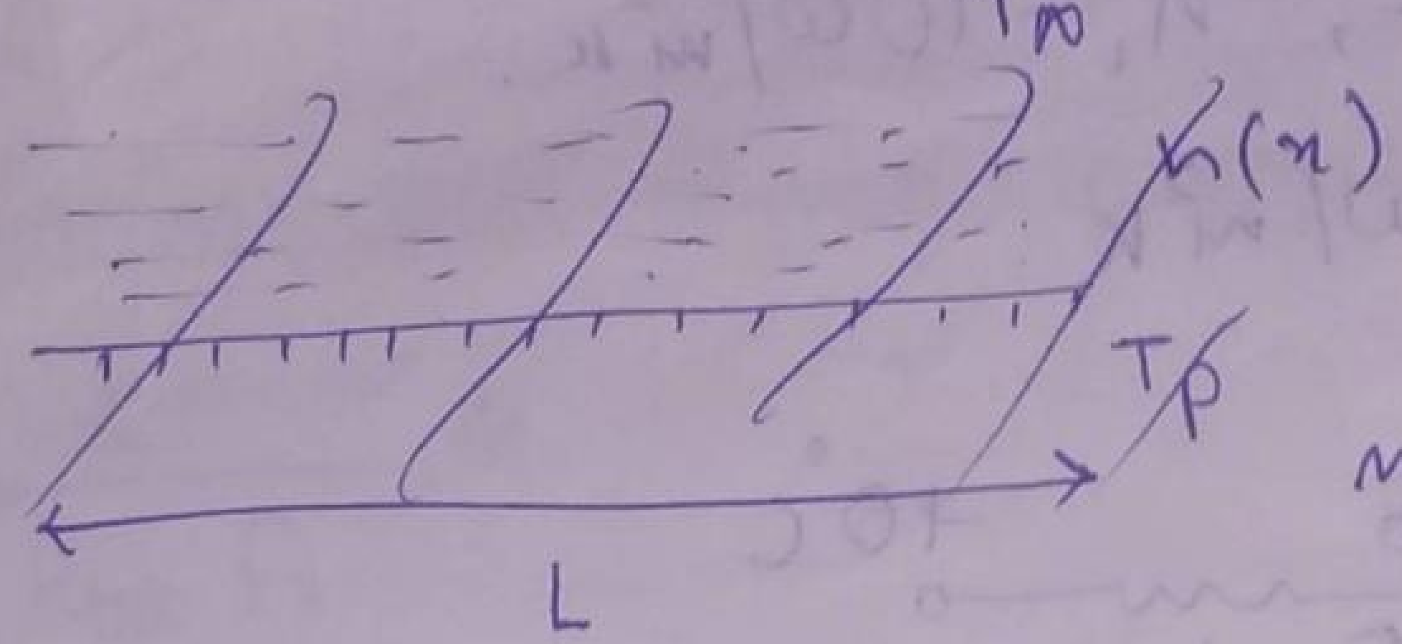
at $x = 20$cm :-

$$Re_x = \frac{2 \times 1.177 \times 0.2}{1.85 \times 10^{-5}} = \cancel{38270.27}\ 25448.65$$

$$\delta(x) = 4.64 \times 0.2 \times (\cancel{38270.27}\ 25448.65)^{-1/2} = 4.64 \times 0.2 \times 0.00\cancel{\ldots}6268$$

$$= 0.00\cancel{\ldots}582 = \cancel{\ldots}\ 5.27 \text{ mm}$$ Ans

1.



* For flat plate, transition Reynold's number is 4×10^6.

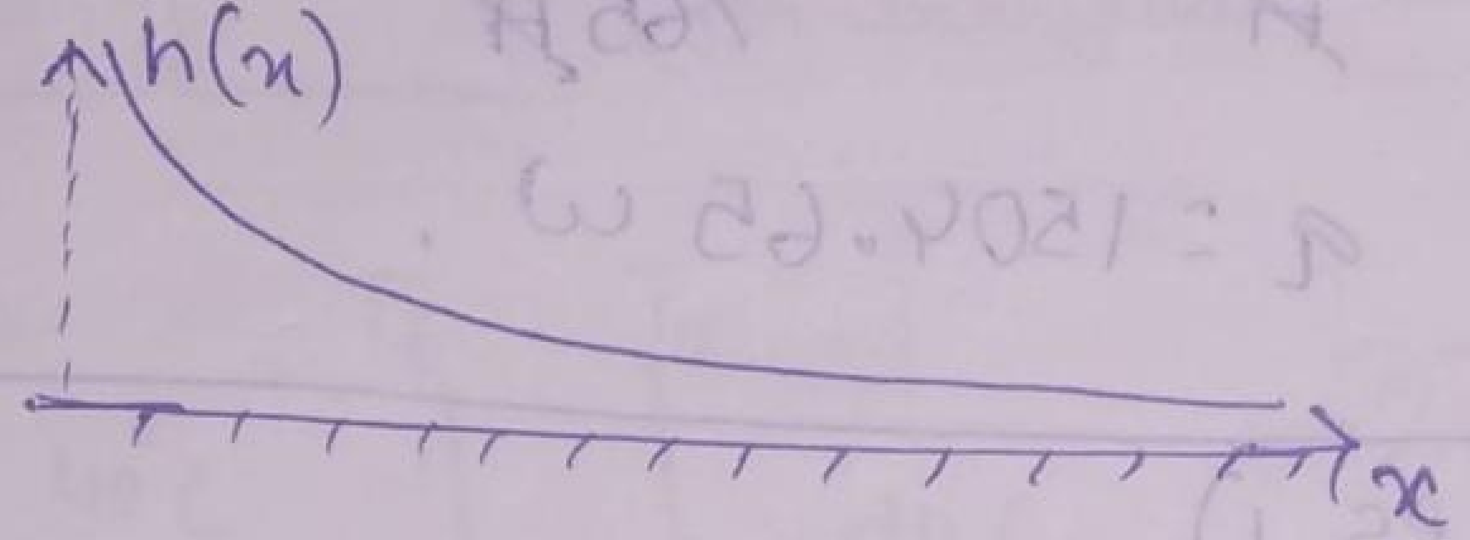
at $x = 400$ cm :-

$$Re_x = \frac{2\times1.177\times4}{1.85\times10^{-5}} = 508972.973$$

$$\delta(x) = 4.64\times4\times(508972.973)^{-1/2} = 0.02602$$

$= 26.02$ mm Ans.

2.



$h(x) \propto 1/x$ $\quad h = \frac{0.323 k_f Re_x^{1/2} Pr^{1/3}}{x}$

$$\frac{\int_0^L \frac{0.323\, k_f\, Re_x^{1/2}\, Pr^{1/3}\, x\, k_f}{x\, k_f\, x}\, dx}{\int_0^L dx} = \bar{h}$$

↳ average h.

$$\bar{h} = \frac{\int_0^L 0.323\, Re_x^{1/2}\, Pr^{1/3} \cdot \frac{k_f}{x}\, dx}{\int_0^L dx}$$

$$= \frac{1}{L}\int_0^L 0.323\, (u_\infty/\nu)^{1/2} \cdot Pr^{1/3} \cdot \frac{k_f}{\sqrt{x}}\, dx$$

$$= \frac{1}{L} h_x \times 0.323 \left(\frac{u_\infty}{\nu}\right)^{1/2} \cdot P_r^{1/3} \cdot k_f \int_0^L \frac{1}{\sqrt{x}}\, dx$$

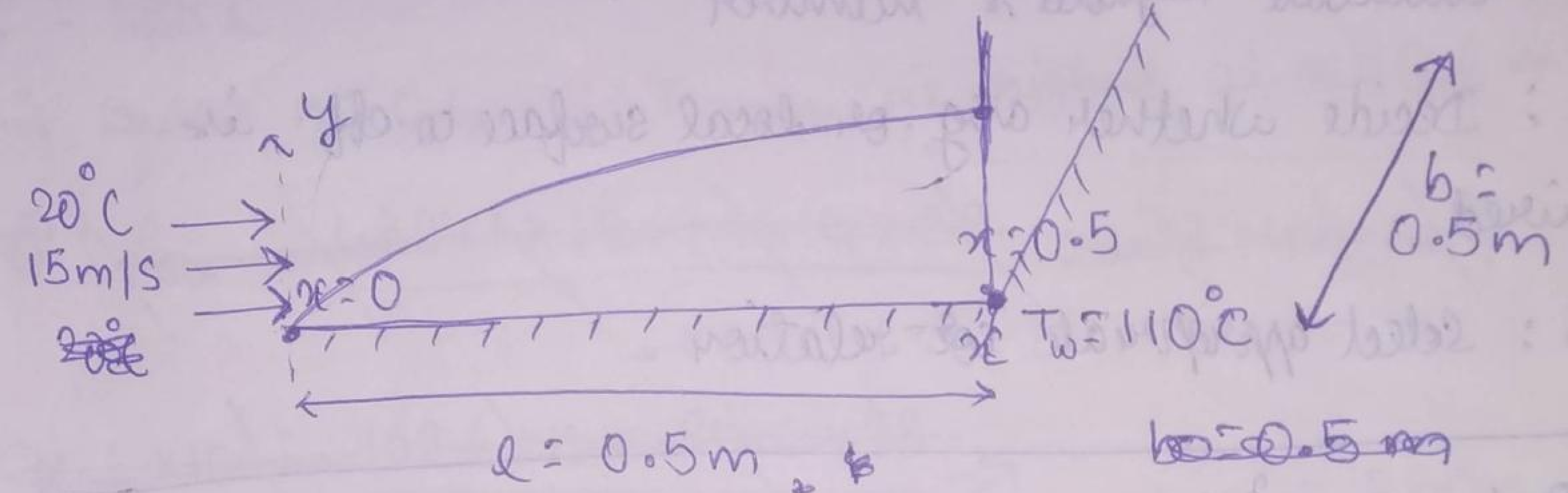
$$= \frac{1}{L} \times 0.323 \times \left(\frac{u_\infty L}{\nu}\right)^{1/2} \cdot P_r^{1/3} \cdot k_f \times 2 \Rightarrow \frac{\bar{h} L}{k_f} = 2 \times 0.323 \times Re_L^{1/2} \times P_r^{1/3}$$

$$= 2 \times Nu_L$$

$$\boxed{\bar{h} = 2h|_{x=L}}$$

Hence, proved.

3.



~~b = 0.5 m~~

Reference temp. $= \frac{20+110}{2}$

$= 65°C$ (temp. of film).

$T = 65°C = 338K$

$$\rho_{air} = \frac{1.1614 \times 12 + 0.995 \times 38}{50} = 1.0349 \text{ kg/m}^3.$$

$$\mu_{air} 10^7 = \frac{184.6 \times 12 + 208.2 \times 38}{50} = 202.536 \text{ Ns/m}^2$$

$$\mu_{air} = 2.0254 \times 10^{-5} \text{ Ns/m}^2$$

$$P_{r,air} = \frac{0.707 \times 12 + 0.7 \times 38}{50} = 0.702$$

$$Re_{,air} = \frac{1.0349 \times 15 \times 0.5}{2.02536 \times 10^{-5}} = 383228.167$$

Problem solving approach :-

Step 1 : Become clear about flow geometry (flat plate, sphere, pipe).

Step 2 : Specify appropriate reference temp. and eval. corresponding fluid properties at that temp.

Step 3 : Calculate Reynold's number

Step 4 : Decide whether avg. or local surface co-eff. is required.

Step 5 : Select appropriate co-relation.

$$k_f \times 10^3 = \frac{26.3 \times 12 + 30 \times 38}{50} = 29.112$$

$$k_f = 2.9112 \times 10^{-2} \ W/m.K$$

$$h = \frac{0.323 \times 2.9112 \times 10^{-2} \times (383228.167)^{1/2} (0.702)^{1/3}}{0.5}$$

$$= 10.35 \ W/m^2.K = h|_{x=L}$$

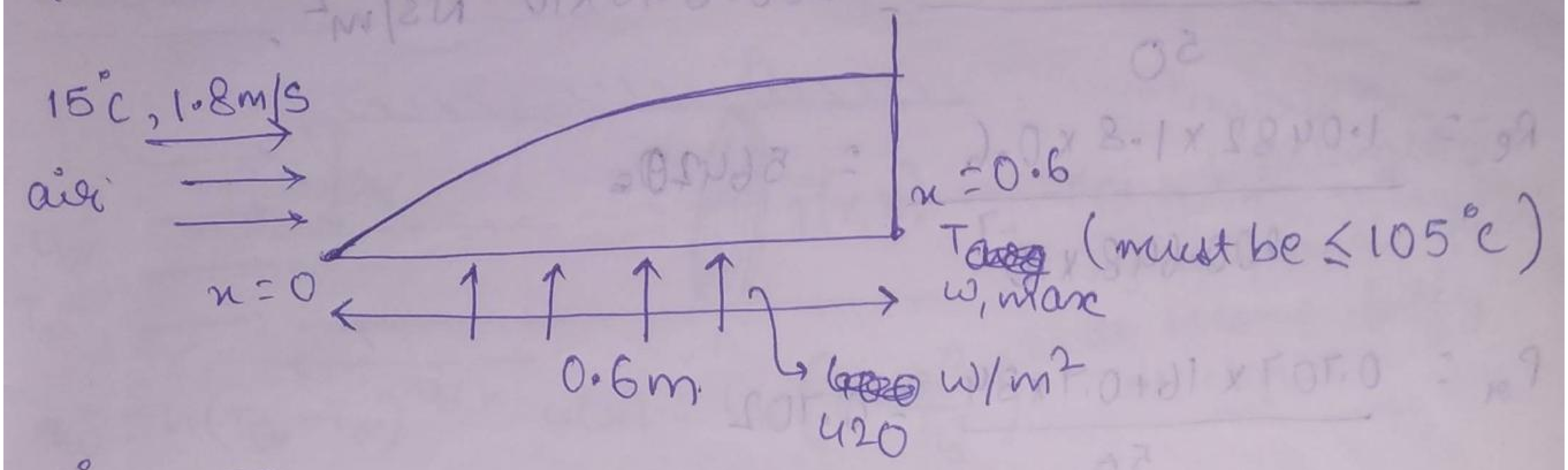
$$Nu|_{x=L} = \frac{10.35 \times 0.5}{2.9112 \times 10^{-2}} = 177.71$$

$$\delta|_{x=L} = 4.64 \times 0.5 \times (383228.167)^{-1/2} = 3.75 \ mm$$

$$\delta_t|_{x=L} = 4.64 \times 0.5 \times (383228.167)^{-1/2} \times (0.702)^{-1/3} = 3.33 \ mm$$

$\overline{Nu}|_{L} =$, $\overline{h}_L =$

4.



$15^\circ C = 288\ K$

h is lowest $x = 0.6$, $\therefore T_{w,max}$ is highest at $x = 0.6\ m$.

~~$\rho_{air} = \frac{1.3947 \times 12 + 1.1614 \times 38}{50} = 1.2174\ kg/m^3$~~

~~$\mu_{air} \times 10^7 = \frac{159.6 \times 12 + 184.6 \times 38}{50}$~~ $\rightarrow \therefore T_f = 350K$

412

assume $T_{fw} =$ ~~300~~ $350\ K$, back-calculate T_w.

$T_w = 2 \times 350 - 288 = 412K$.

$\rho = 0.995\ kg/m^3$, $\mu = 2.082 \times 10^{-5}\ Ns/m^2$,

$Re = \frac{0.995 \times 1.8 \times 0.6}{2.082 \times 10^{-5}} = 51613.83$

$P_r = 0.700$, $k_f = 30 \times 10^{-3}\ W/m.K$

$$h = \frac{0.323 \times (51613.83)^{1/2} \times 0.7^{1/3} \times 30 \times 10^{-3}}{0.6}$$

$= 4.56\ W/m^2.K$.

$4.56 \times \Delta T = 420 \Rightarrow \Delta T = 92.105K$.

$T_{w,obt.} = 380\ K$ (new assump.)

$T_f = 334\ K$, $\rho = \frac{1.1614 \times 16 + 0.995 \times 34}{50} = 1.0482\ kg/m^3$

$$\mu = \frac{184.6 \times 16 + 208.2 \times 34}{50} = 200.648 \times 10^{-7} \ Ns/m^2 .$$

$$Re = \frac{1.0482 \times 1.8 \times 0.6}{200.648 \times 10^{-7}} = 56420$$

$$P_r = \frac{0.707 \times 16 + 0.7 \times 34}{50} = 0.702$$

$$k_f = \frac{26.3 \times 16 + 30 \times 34}{50} \times 10^{-3} = 28.816 \times 10^{-3} \ W/m.K$$

$$h = \frac{0.323 \times 28.816 \times 10^{-3} \times (56420)^{1/2} \times (0.702)^{1/3}}{0.6}$$

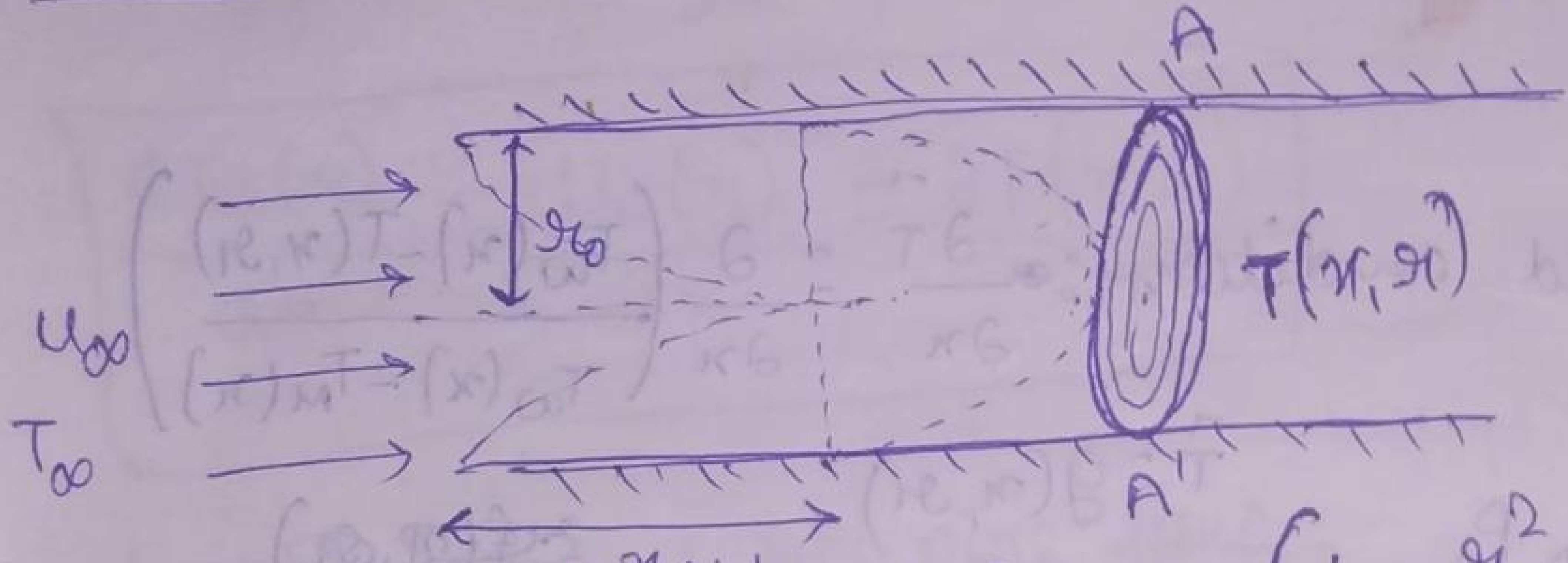
$$= 4.58 \ W/m^2.K$$

$4.58 \times \Delta T = 420 \Rightarrow \Delta T = 91.7 \approx 92$

$T_w = 288 + 91.7 \approx 379.7 \approx 380K$. (same as initial guess)

Hence, $T_w = 380K$. It is not safe.

Internal Forced Convection :-



$x_{fd,h}$: entry length for hydrodynamic flow to become fully developed.

$q_w = h(T_s - T_\infty)$ $\quad u(r) = u_\infty\left(1 - \frac{r^2}{r_0^2}\right)$

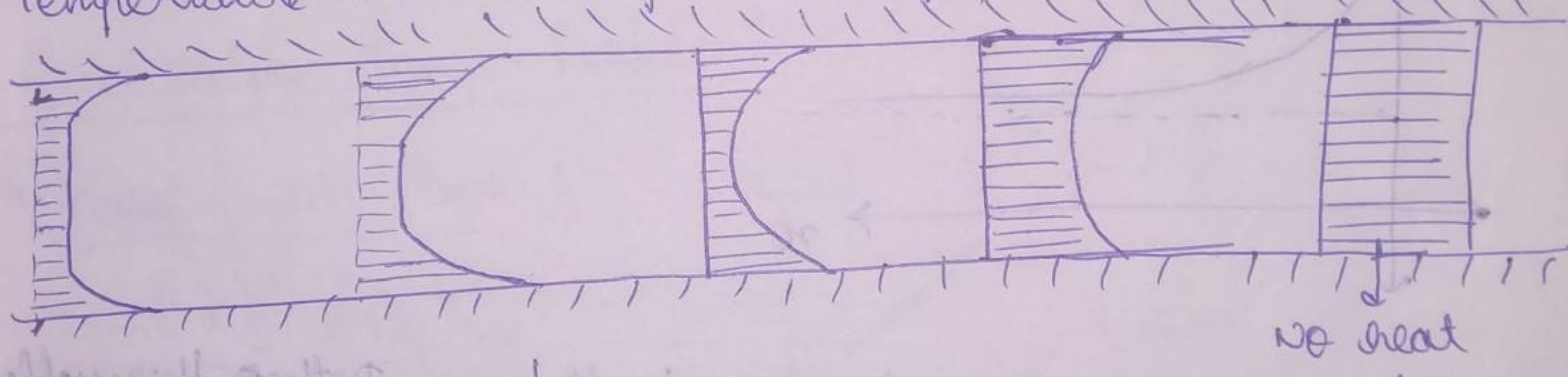
$u_\infty = 2u_{avg}$ $\quad (u_{avg} = \bar{u})$

$\pi r_0^2 \bar{u} \rho = \dot{m}$ = constant flow rate

$\left(\frac{x_{fd,h}}{D}\right)_{lam} = 0.05 Re_D$, $\left(\frac{x_{fd,h}}{D}\right)_{turb} \approx 10$

D : diameter

Temperature variation as flow develops :-



For cross-section AA' :-

$T_{ref} = 0.$

$\hat{h}_{in} = \hat{C}_p (T(x,r))$

$dH = (2\pi r\, dr)\, u(r) \rho \,.\, \hat{C}_p T(x,r)$

$H = \int_0^{r_0} 2\pi\rho \hat{C}_p r\, u(r) . T(x,r)\, dr$

enthalpy coming out of AA' $\Rightarrow \frac{H}{\dot{m} C_p} = T_m$ = mean temp. or mixing cup temp. $= \frac{2\pi\rho}{\dot{m}} \int_0^{r} r\, u(r) . T(x,r)\, dr$

func. of x.

$$\hat{T} = \frac{T_{\omega}(x) - T(x,r)}{T_{\omega}(x) - T_{M}(x)}$$

Thermally fully-developed condition : $\dfrac{\partial \hat{T}}{\partial x} = \dfrac{\partial}{\partial x}\left(\dfrac{T_{\omega}(x) - T(x,r)}{T_{\omega}(x) - T_{M}(x)}\right)$

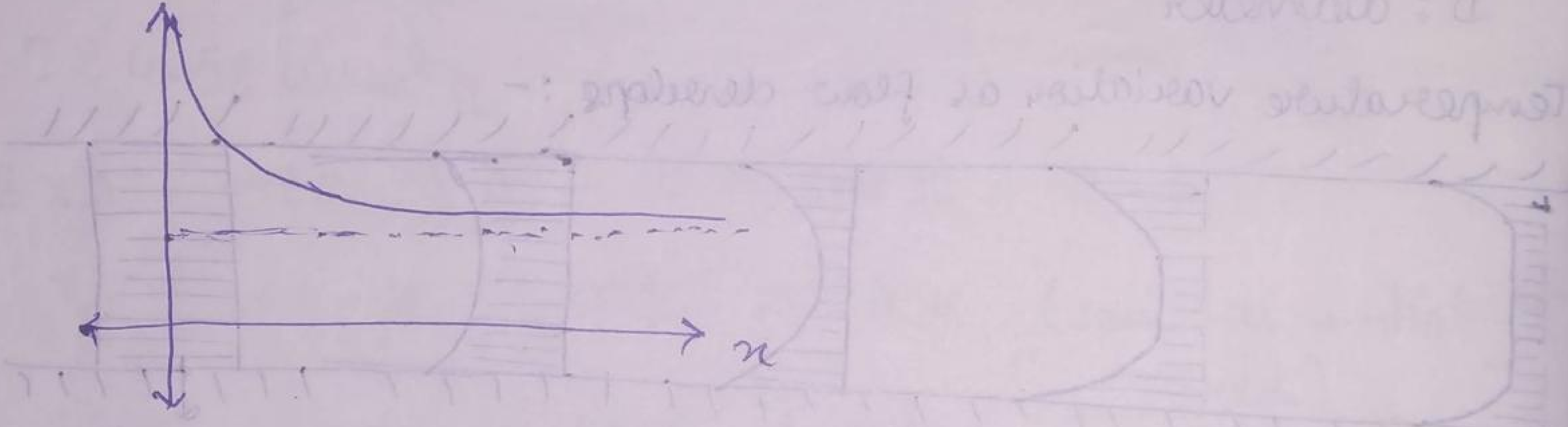
$\hat{T} = g(x, r)$ $\quad \cancel{\neq f(x,r)}$

$$\left(\frac{x_{fd,t}}{D}\right)_{lam} = 0.05 Re_D P_r$$

$$\frac{\partial \hat{T}}{\partial r} = \frac{-\partial T(x,r)/\partial r}{T_{\omega}(x) - T_{M}(x)} \neq f(x)$$

$$-k\frac{\partial T(x,r)}{\partial r} = h\left(T_{\omega}(x) - T_{M}(x)\right) \longrightarrow$$ ↳ Thermally fully developed condition hai sirf !!

$$\Rightarrow \frac{\partial \hat{T}}{\partial r} = \frac{h}{k} \neq f(x)$$



Internal flow, constant surface heat flux. Both thermally & HDly FD :-

$$q_{\omega} = h\left(T_{\omega}(x) - T_{M}(x)\right) \Rightarrow T_{\omega}(x) - T_{M}(x) = \frac{q_{\omega}}{h} = \text{Const.}$$

$$\Rightarrow \frac{dT_{\omega}(x)}{dx} = \frac{dT_{M}(x)}{dx}$$

$$\hat{T} = \frac{T_{\omega}(x) - T(x,r)}{T_{\omega}(x) - T_{M}(x)} \Rightarrow \frac{\partial \hat{T}}{\partial x} = \frac{dT_{\omega}(x)/dx - \partial T(x,r)/\partial x}{T_{\omega}(x) - T_{M}(x)}$$

For fully developed condition. $\longrightarrow 0 = \dfrac{dT_{\omega}(x)/dx - \partial T(x,r)/\partial x}{q_{\omega}/h}$

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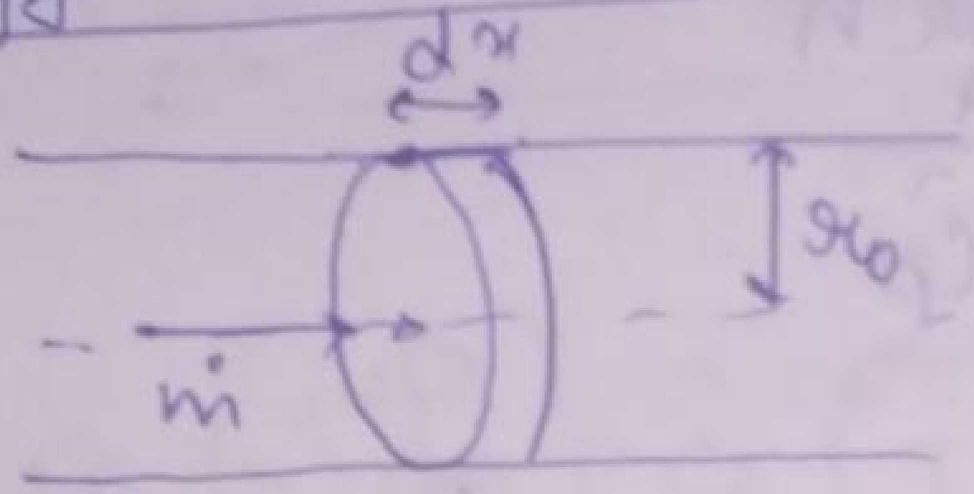
$$\frac{dT_w(x)}{dx} = \frac{\partial T(x,r)}{\partial x}$$

$$\boxed{\frac{dT_w(x)}{dx} = \frac{dT_M(x)}{dx} = \frac{\partial T(x,r)}{\partial x}}$$

Energy Balance for pipe flow :-



$$(2\pi r_0 dx)\, h(T_w(x) - T_M(x)) = \dot{m}\hat{C}_p T_m(x)\big|_{x+dx} - \dot{m}\hat{C}_p T_m(x)\big|_x$$

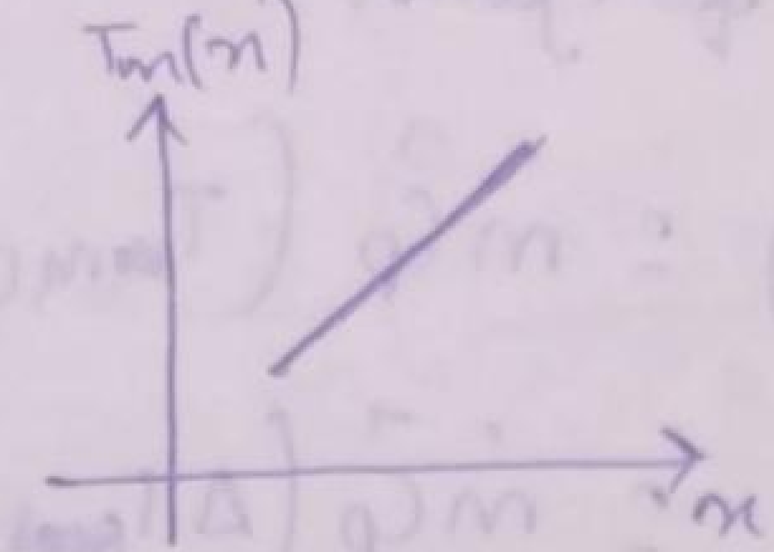
$$\boxed{\frac{dT_m(x)}{dx} = \frac{2\pi r_0 q_w}{\dot{m}\hat{C}_p}} \qquad q_w = h(T_w(x) - T_m(x)).$$

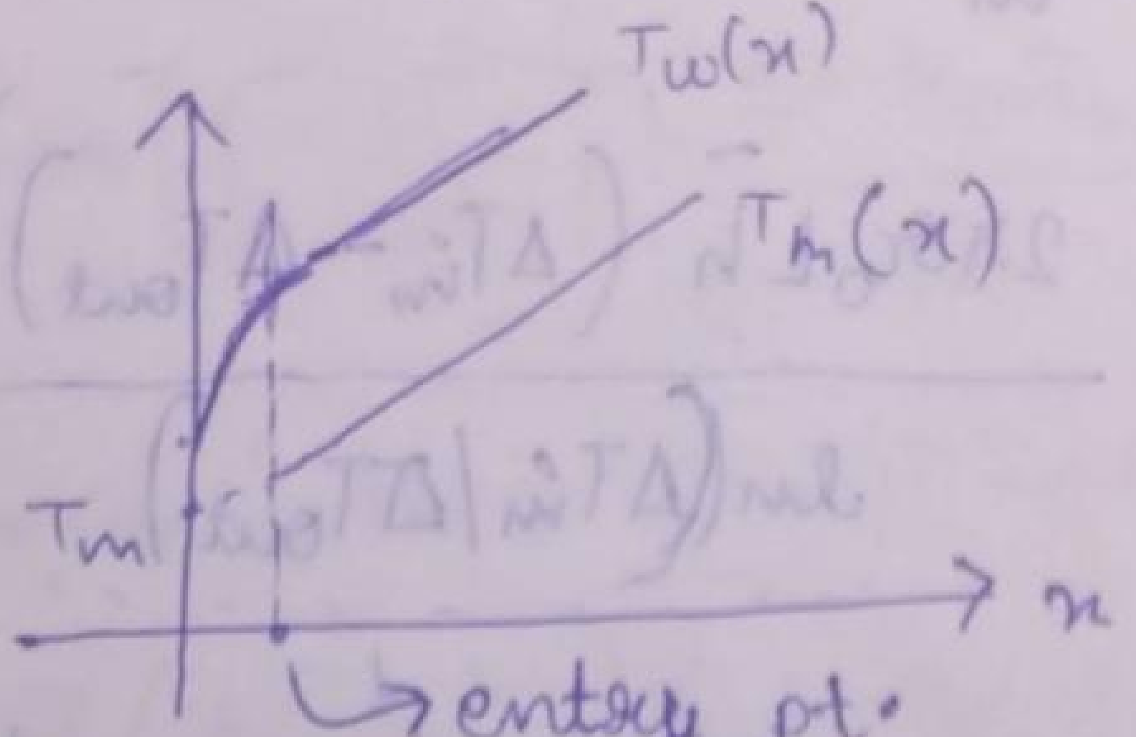
q_w ↓ const.

Qualitative Temp. Profile :-

a) Const. wall flux :-

$$\frac{dT_m(x)}{dx} = \frac{2\pi r_0 q_w}{\dot{m}\hat{C}_p}$$





b) Const. wall temp. :-

$$\Delta T = T_w - T_m(x) \Rightarrow \frac{d(\Delta T)}{dx} = -\frac{d(T_m(x))}{dx} \Rightarrow -\frac{d(\Delta T)}{dx} = \frac{2\pi r_0 h(\Delta T)}{\dot{m}\hat{C}_p}$$

$$\Rightarrow \int_{\Delta T_{in}}^{\Delta T_{out}} \frac{d(\Delta T)}{\Delta T} = \int_0^L \frac{2\pi r_0 h}{\dot{m}\hat{C}_p}\, dx \Rightarrow \ln\left(\frac{\Delta T_{out}}{\Delta T_{in}}\right) = -\frac{2\pi r_0 hL}{\dot{m}\hat{C}_p}$$

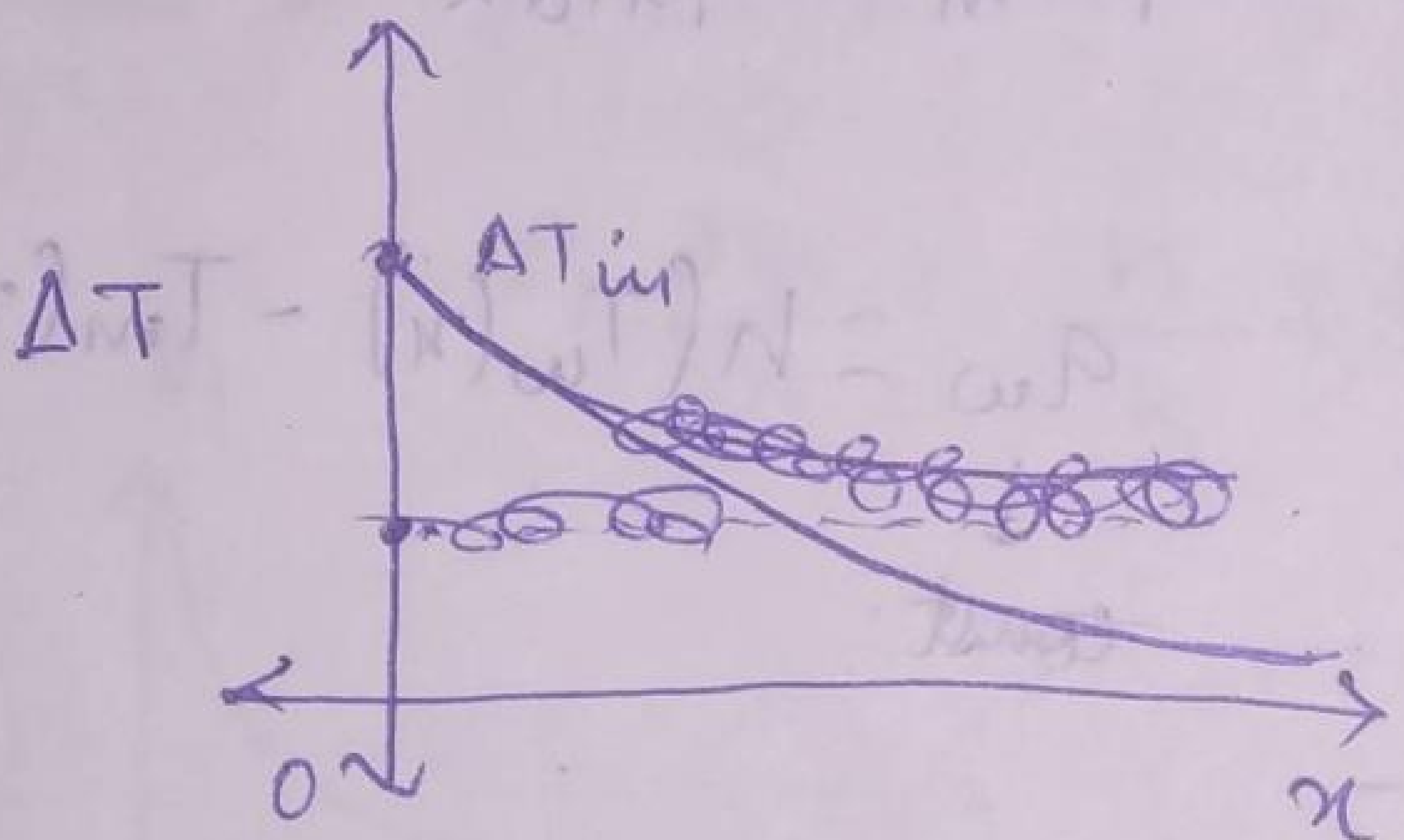
$$\Rightarrow \ln\left(\frac{\Delta T_{out}}{\Delta T_{in}}\right) = -\frac{2\pi r_0 L}{\dot{m}\hat{C_p}} \cdot \frac{1}{L}\int_0^L h\,dx$$

$$= -\frac{2\pi r_0 L}{\dot{m}\hat{C_p}} \cdot \bar{h}$$

$$\Rightarrow \ln\left(\frac{\Delta T_{in}}{\Delta T_{out}}\right) = \frac{2\pi r_0 L \bar{h}}{\dot{m}\hat{C_p}} = \frac{A_{curve\ SA}\,\bar{h}}{\dot{m}\hat{C_p}}$$

$$\boxed{\Delta T_{out} = \Delta T_{in}\, e^{-2\pi r_0 L \bar{h}/\dot{m}\hat{C_p}}}$$

$\Delta T = T_w - T_m(x)$



Rate of heat transfer for const. wall temp.:-

$$\dot{Q} = \dot{m}\hat{C_p}\,(T_{m,o} - T_{m,i}) = \dot{m}\hat{C_p}\left(T_{m,o} - T_w + (T_w - T_{m,i})\right)$$

$$= \dot{m}\hat{C_p}\,(\Delta T_{in} - \Delta T_{out})$$

$$\dot{m}\hat{C_p} = \frac{2\pi r_0 L \bar{h}}{\ln(\Delta T_{in}/\Delta T_{out})} \Rightarrow \dot{Q} = \frac{2\pi r_0 L \bar{h}\,(\Delta T_{in} - \Delta T_{out})}{\ln(\Delta T_{in}/\Delta T_{out})}$$

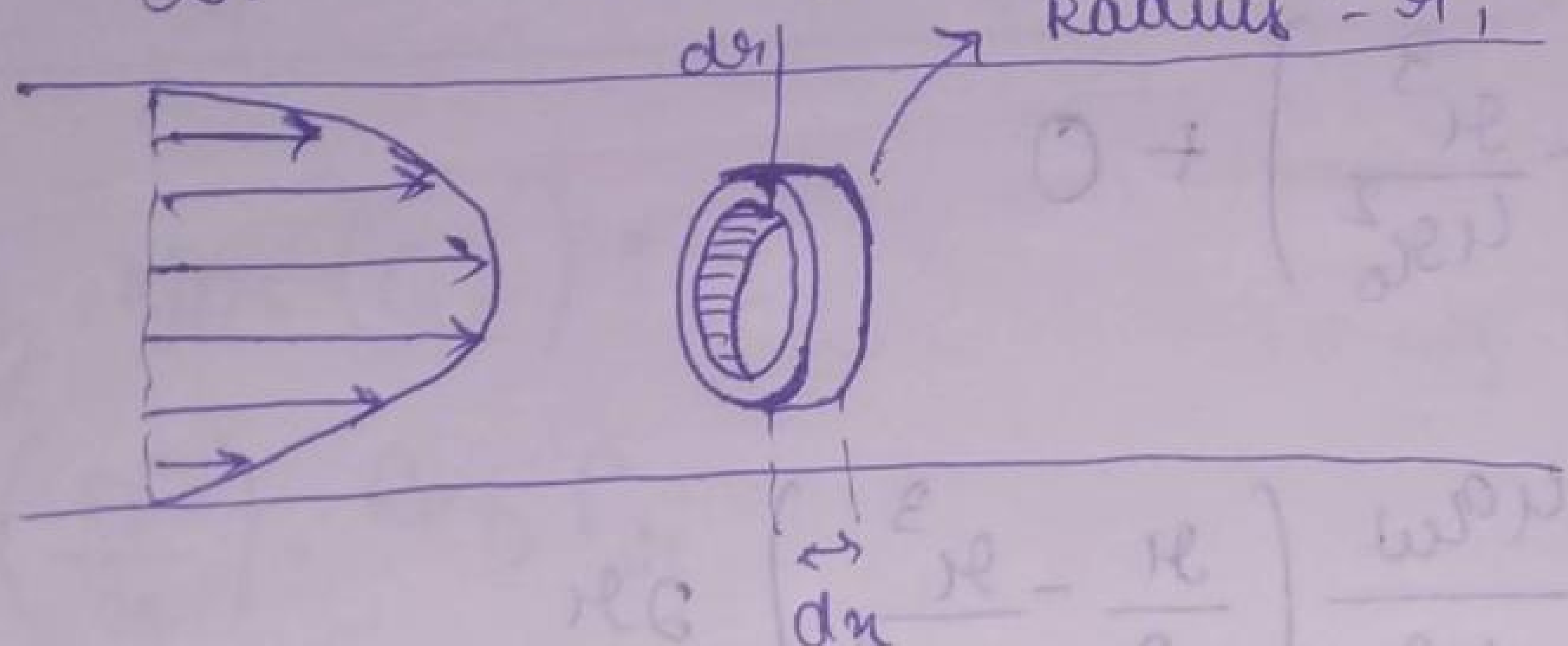
$$\boxed{\dot{Q} = A_T \bar{h}\,(\Delta T_{ln})}$$

→ logarithmic mean temp. diff

Laminar flow, fully developed, constant wall flux :-

$q_w = h(T_w(x) - T_M(x))$

$= -k \frac{\partial T}{\partial r}(x,r)\Big|_{r=r_0}$



$(2\pi r\, dr) \overset{u(r)}{\cancel{dx}} \rho \hat{C}_p \left(T(r,x)\big|_x - T(r,x)\big|_{x+dx} \right) + \left(\cancel{-k \frac{\partial T(x,r)}{\partial r} 2\pi} \right.$

$\left. \left(-k \frac{\partial T(x,r)}{\partial r} . 2\pi r dx\Big|_r - \left(-k \frac{\partial T(x,r)}{\partial r} . 2\pi r dx\Big|_{r+dr}\right)\right)\right) = 0$

$\Rightarrow \cancel{2\pi} \rho \hat{C}_p r u(r) . \frac{\partial T(x,r)}{\partial x} = \cancel{2\pi} k . \frac{\partial}{\partial r}\left(r \frac{\partial T(x,r)}{\partial r}\right)$

$\Rightarrow \rho \hat{C}_p r u(r) . \frac{dT_M(x)}{dx} = k . \frac{\partial}{\partial r}\left(r \frac{\partial T(x,r)}{\partial r}\right)$

$\Rightarrow \rho \cancel{\hat{C}_p} r u(r) . \frac{2\pi r_0 q_w}{\dot{m} \cancel{\hat{C}_p}} = k . \frac{\partial}{\partial r}\left(\frac{r \partial T(x,r)}{\partial r}\right)$

$\Rightarrow \rho r u_{max}\left(1 - \frac{r^2}{r_0^2}\right) . \frac{2\pi r_0 q_w}{\dot{m}} = k . \frac{\partial}{\partial r}\left(\frac{r \partial T(x,r)}{\partial r}\right)$

Now, $u_{max} = 2\bar{u}$, $\dot{m} = \pi r_0^2 \bar{u} \rho$

$\Rightarrow \cancel{\rho} r . 2\cancel{\bar{u}}\left(1 - \frac{r^2}{r_0^2}\right) \frac{. 2\cancel{\pi} \cancel{r_0} q_w}{\cancel{\pi} \overset{r_0}{\cancel{r_0^2}} \cancel{\bar{u}} \cancel{\rho}} = k \frac{\partial}{\partial r}\left(\frac{r \partial T(x,r)}{\partial r}\right)$

$\cancel{r \partial T(x,r)/\partial r}$

$\Rightarrow \int_0^r \frac{4 r q_w}{r_0}\left(1 - \frac{r^2}{r_0^2}\right) \partial r = \int k\, \partial \left(\frac{r \partial T(x,r)}{\partial r}\right)$

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$$\Rightarrow \frac{4q_w}{k r_0}\left(\frac{r^2}{2}-\frac{r^4}{4r_0^2}\right) = r\,\frac{\partial T(x,r)}{\partial r} + C_1$$

$$\Rightarrow \frac{\partial T(x,r)}{\partial r} = \frac{4q_w}{k r_0}\left(\frac{r}{2}-\frac{r^3}{4r_0^2}\right) + 0$$

$$\Rightarrow \cancel{T(x,r)} \int \partial T(x,r) = \int \frac{4q_w}{k r_0}\left(\frac{r}{2}-\frac{r^3}{4r_0^2}\right)\partial r$$

$$\Rightarrow T(x,r) = \frac{4q_w}{k r_0}\left(\frac{r^2}{4}-\frac{r^4}{16r_0^2}\right) + C_2(x)$$

$$T_m(x) = \frac{2\pi\rho}{\dot m}\int_0^{r_0} r u(r) \cdot T(x,r)\,dr$$

$$\Rightarrow T_m(x) = \frac{4}{r_0^2}\left[\frac{4q_w}{r_0 k}\cdot\frac{7}{384} r_0^4 + C_2(x)\cdot\frac{r_0^2}{4}\right]$$

$$\cancel{\Rightarrow C_2(x) = \frac{T_m(x)}{\dots} - \frac{7q_w}{96 r_0 k}} \Rightarrow T_m(x) = \frac{16 q_w r_0}{k}\cdot\frac{7}{384} + C_2(x)$$

$$= \frac{7q_w r_0}{24k} + C_2(x)$$

$$T(x,r) = \frac{4q_w}{k r_0}\left(\frac{r^2}{4}-\frac{r^4}{16r_0^2}\right) + \left(T_m(x) - \frac{7q_w r_0}{24k}\right)$$

At wall, $$T_w(x) = \frac{4q_w}{k r_0}\left(\frac{r_0^2}{4}-\frac{r_0^4}{16r_0^2}\right) + T_m(x) - \frac{7q_w r_0}{24k}$$

$$\frac{q_w r_0}{k} - \frac{r_0 q_w}{4k} + T_m(x) - \frac{7q_w r_0}{24k}$$

$$\Rightarrow T_w(x) - T_m(x) = \frac{11 q_w r_0}{24\dots}$$

$$\Rightarrow \left(\cancel{T_w(x)} - T_m(x)\right) = \frac{11h\left(\cancel{T_w(x)} - T_m(x)\right) r_0}{24k}$$

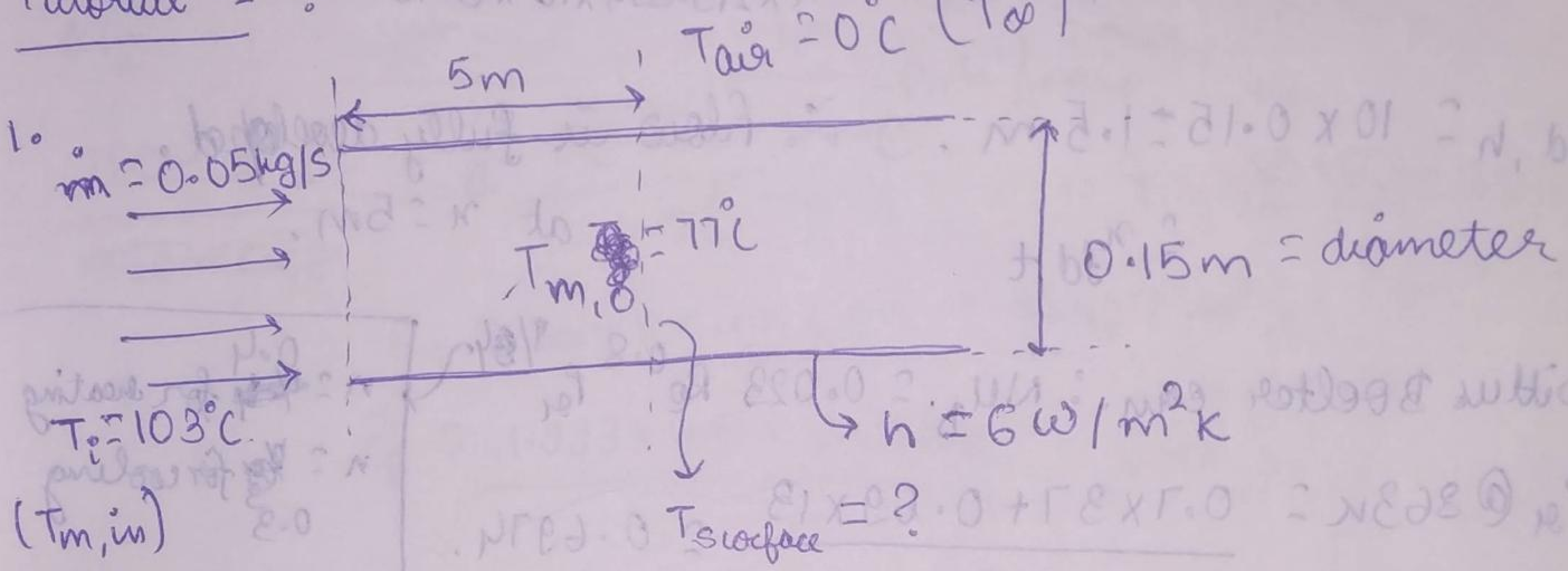
$$\Rightarrow \boxed{\frac{h(2r_0)}{k} = \frac{48}{11} = Nu_D = 4.36}$$ ~~for Thermal Entry length problem.~~

Graetz No. (G_z) :-

$$G_z = \left(\frac{D}{x}\right) \cdot Re_D P_r$$

Tutorial - 2 :-

1.



$\dot{Q} = ?$

$$\dot{Q} = \dot{m}\hat{C}_p\left(T_{m,o} - T_{m,in}\right) \quad (\hat{C}_p \text{ calculated at } \bar{T} = \frac{T_{m,o} + T_{m,in}}{2}$$

$= 90°C$

$= 363K$

$$\hat{C}_p \text{ at } 363K. \quad \hat{C}_p = \frac{1.009 \times 37 + 1.014 \times 13}{50} \times 10^3$$

$$= 1010.3\ J/kg.K.$$

$$\dot{Q} = 0.05 \times 1010.3(77 - 103) = -1313.39\ J/s \quad \underline{Ans}$$

Rate of heat loss for entire length of pipe.

$\dot{q}$: flux $= h_i(T_{m,0} - T_{S,L}) = h_0(T_{S,L} - T_\infty)$

μ at 363K $= \frac{208.2 \times 37 + 230.1 \times 13}{50} \times 10^{-7} = 213.894 \times 10^{-7}\ NS/m^2$

$Re = \frac{D\bar{V}\rho}{\mu} = \frac{4\dot{m}}{\pi\mu D} = \frac{4 \times 0.05}{\pi \times 213.894 \times 10^{-7} \times 0.15} = 19842.22$

$x_{fd,h} = 10 \times 0.15 = 1.5\ m$. $\therefore$ flow is fully developed at $x = 5m$.

$\approx x_{fd,t}$

Dittus Boelter eqn: $Nu_D = 0.023\ Re^{0.8}\ P_r^{n}$

$n = 0.4$ for heat

$n = 0.3$ for cooling

P_r @ 363K $= \frac{0.7 \times 37 + 0.69 \times 13}{50} = 0.6974$.

$Nu_D = 0.023(19842.22)^{0.8}(0.6974)^{0.3} = 56.60$

k at 363K $= \frac{30 \times 37 + 33.8 \times 13}{50} \times 10^{-3} = 30.988 \times 10^{-3}\ W/mK$

$h_i = \frac{56.6 \times 30.988 \times 10^{-3}}{0.15} = 11.69\ W/m^2K$.

$11.69(77 - T_{S,L}) = 6(T_{S,L} - 0)$

$\Rightarrow T_{S,L} = 50.88\ °C$ Ans.

$\dot{q} = 6 \times 50.88 = 305.28\ W/m^2$ Ans.

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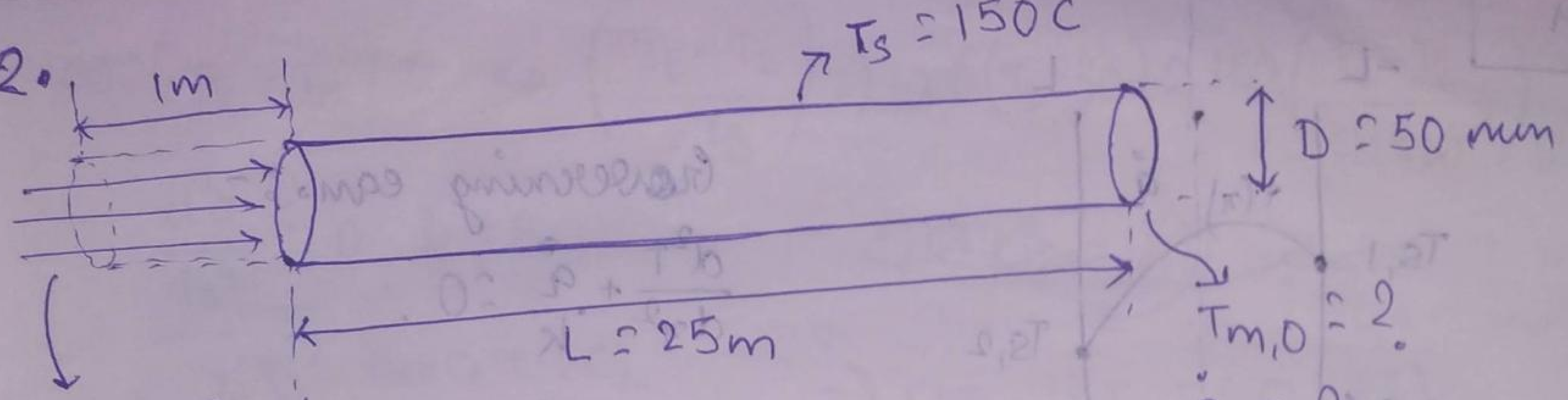
2.



$\dot{m} = 0.5$ kg/s

$T_{m,i} = 20°C$

Total rate of heat transfer.

$$\frac{\Delta T_0}{\Delta T_i} = e^{-A_T \bar{h} / \dot{m}\hat{C}_p}$$

First guess, $T_{m,0} = 85°C$.

$$\cancel{\frac{150 - T_{m,0}}{150 - T_{m,i}}} = \frac{150-85}{150-20} = e^{-A_T\bar{h}/\dot{m}\hat{C}_p}$$

$$\bar{T} = \frac{85+20}{2} = 52.5°C = 325.5 K$$

$$A_T = \pi \times 0.05 \times 25 = 3.927 m^2,$$

$$\hat{C}_p = \frac{1.993 \times 4.5 + 2.035 \times 5.5}{10} \times 10^3$$

$$= 2016.1 \text{ J/kgK}.$$

$$1/2 = e^{-3.927 \times \bar{h}/0.5 \times 2016.1}$$

$$0.693 = \frac{3.927 \times \bar{h}}{0.5 \times 2016.1} \Rightarrow \bar{h} = 177.89 \text{ W/m}^2\text{K}$$

~~$\dot{Q} = 177.89 \times 3.927 \times \left(\frac{130-65}{\ln(2)}\right) = 65508.90$ J/s~~

$$\mu = \frac{14.1 \times 4.5 + 8.36 \times 5.5}{10} \times 10^{-2} = 0.10943 \text{ NS/m}^2$$

$$Re = \frac{4 \times 0.5 \cancel{10}}{\pi \times 0.10943 \times 0.05} = 116.352, \quad Pr = \frac{1965 \times 4.5 + 1205 \times 5.5}{10}$$

$$= 1547$$

$$x_{fd,h} = 0.05 \times 0.05 \times 116.352 = 0.29 m$$

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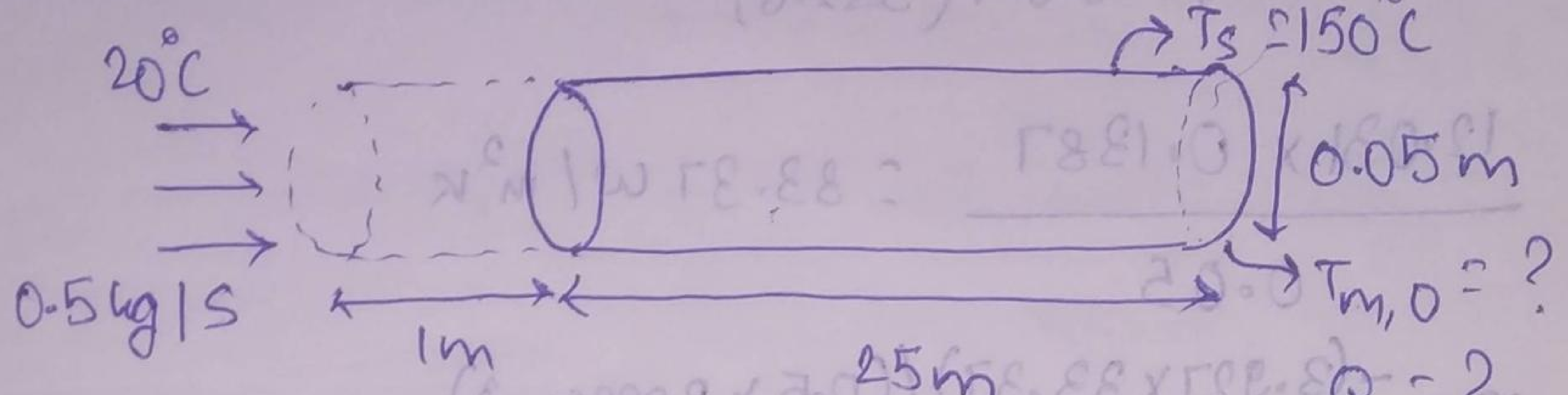
JC

2.



First guess, $T_{m,o} = 120°C$.

$\bar{T} = \frac{120+20}{2} = 70°C = 343K$.

$\mu = \frac{5.31\times 7 + 3.56\times 3}{10}\times 10^{-2} = 0.04785\ Ns/m^2$

$P_r = \frac{793\times 7 + 546\times 3}{10} = 718.9$

$\hat{C}_p = \frac{2.076\times 7 + 2.118\times 3}{10} = 2.0886\ kJ/kgK$

$k = \frac{139\times 7 + 138\times 3}{10}\times 10^{-3} = 0.1387\ W/mK$

$\rho = \frac{859.9\times 7 + 853.9\times 3}{10} = 858.1\ kg/m^3$

$N_{Re} = \frac{\frac{\cancel{858.1}\times 0.5\times 4}{\cancel{858.1}\times \pi\times 0.05^{\cancel{2}}}\times \cancel{0.05}}{0.04785} = 266.1$

$x_{fd,h} = 0.05\times 266.1\times 0.05 = 0.66\ m$

$x_{fd,t} = 0.05\times 266.1\times 718.9\times 0.05 = 478.25\ m$.

$\overline{Nu}_D = 3.66 + \dfrac{0.068\, G_z}{1 + 0.04\, G_z^{2/3}}$, $G_z = \dfrac{D}{L} \cdot Re_D \cdot Pr$

$G_z = \dfrac{0.05}{25} \times 718.9 \times 266.1 = 382.6$

$\overline{Nu}_D = 3.66 + \dfrac{0.068 \times 382.6}{1 + 0.04(382.6)^{2/3}} = 12.031$

$\bar{h} = \dfrac{12.031 \times 0.1387}{0.05} = 33.37\ W/m^2K$

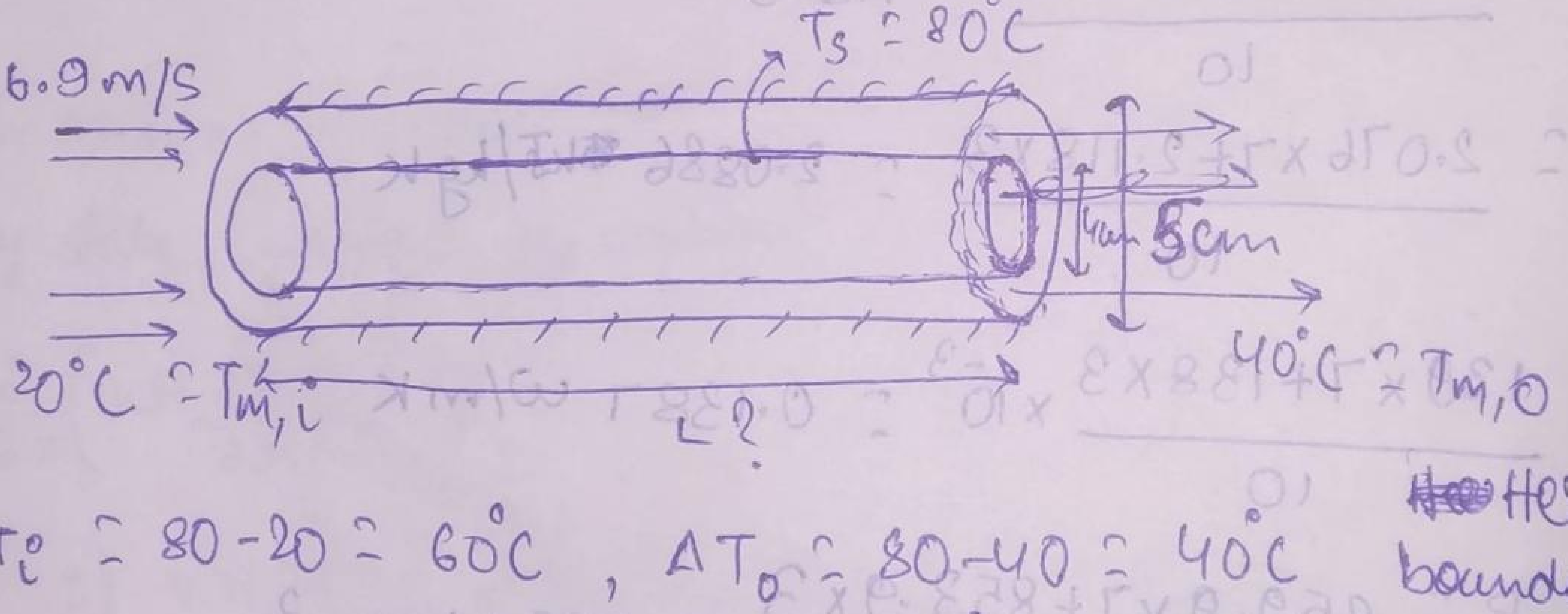
$\Delta T_o = 130\, e^{-(3.927 \times 33.37)/(0.5 \times 2.0886 \times 10^3)}$

$= 114.67\ ^\circ C$

$T_{m,o} = 150 - 114.67 = 35.33^\circ C$

Next guess = $35.33^\circ C$ and proceed with same steps ...

3.



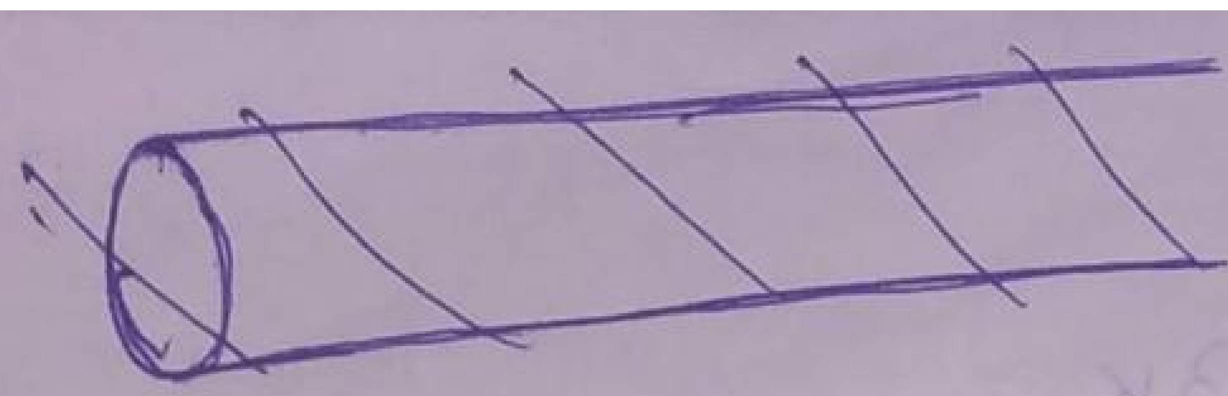
$\Delta T_i = 80 - 20 = 60^\circ C$, $\Delta T_o = 80 - 40 = 40^\circ C$

$A_T = \pi \times 0.04 \times L = 0.126L\ m^2$

Here, the boundary lay[er] merge eventuall[y] ∴ it will be treat[ed] as forced internal convecti[on]

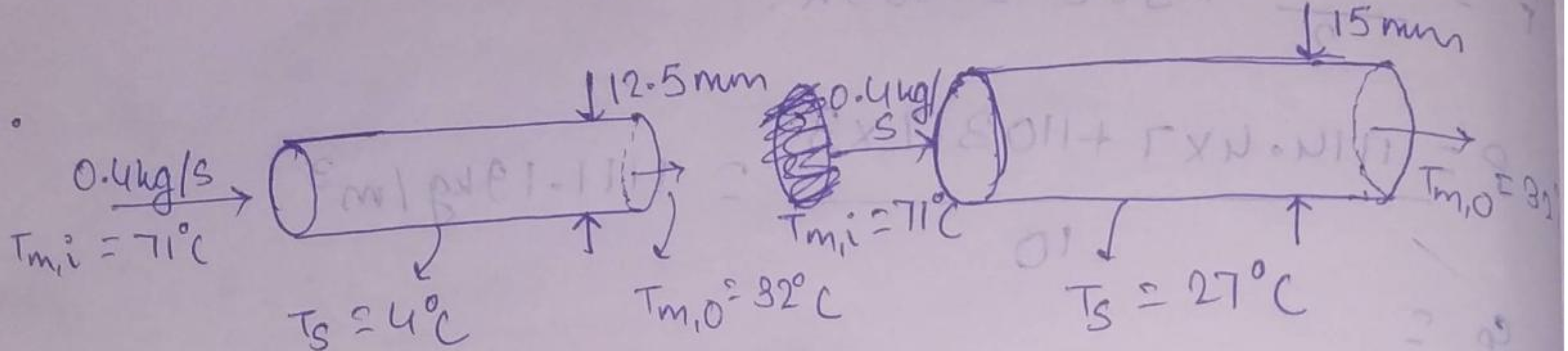
$\dfrac{4 \times \text{wetted area}}{\text{wetted perimeter}}$ = equ. diameter (hydraulic diameter).

$= \dfrac{4 \times \frac{1}{4}\pi(D_o^2 - D_i^2)}{\pi(D_o + D_i)} = D_o - D_i = 1cm$



Press. drop for Turbulent flow $= 4f\frac{L}{D}\cdot\frac{\rho v^2}{2}$

4.



$$\Delta T_{out} = \Delta T_{in}\, e^{-A_T \bar{h}/\dot{m}\hat{C}_p}$$

$$\bar{T} = \frac{71+32}{2} = 51.5°C = 324.5K$$

$$\cancel{c_p = \frac{1.007\times25.5 + 1.009\times24.5}{50} = 1.008\ kJ/kg\,K}$$

$$\cancel{\mu = \frac{184.6\times25.5 + 208.2\times24.5}{50}\times10^{-7}} =$$

$$\hat{C}_p = \frac{4.18\times0.5 + 4.182\times4.5}{5} = 4.1818\ kJ/kg\,K$$

$$\mu = \frac{577\times0.5 + 528\times4.5}{5}\times10^{-6} = 532.9\times10^{-6}\ Ns/m^2$$

$$r = \frac{3.77\times0.5 + 3.42\times4.5}{5} = \cancel{3.293}\ 3.45$$

$$= \frac{640\times0.5 + 645\times4.5}{5}\times10^{-3} = 644.5\times10^{-3}\ W/mK$$

* When entry length < 10% of total length, the average heat transfer co-eff. same as that at fully developed condition

$$Re_1 = \frac{4 \times 0.84}{\pi \times 532.9 \times 10^{-6}} \times \frac{10^3}{12.5}$$

$$= 7.65 \times 10^4$$

$$Re_2 = 7.65 \times 10^4 \times \frac{\cancel{12.5}}{\cancel{15.6}}$$

$$= 6.37 \times 10^4.$$

$$x_{fd,h} = x_{fd,t} = 10 \times 12.5 \times 10^{-3}$$

$$= 0.125\,m \quad (1)$$

$$x_{fd,h} = x_{fd,t} = 10 \times 15 \times 10^{-3}$$

$$= 0.15\,m \quad (2).$$

$Nu_D = 0.023 Re^{0.8} Pr^{0.3}$ ↳ $n = 0.3$ for cooling.

$$Nu_{D,1} = 0.023(7.65 \times 10^4)^{0.8} (3.46)^{0.3}$$

$$= 269.39$$

$$Nu_{D,2} = 0.023(6.37 \times 10^4)^{0.8} (3.46)^{0.3}$$

$$= 232.68$$

$$h_1 = \frac{269.39 \times 644.5 \times \cancel{10^{-3}}}{12.5 \times \cancel{10^{-3}}}$$

$$= 13.89\,kW/m^2K$$

$$h_2 = \frac{232.68 \times 644.5 \times \cancel{10^{-3}}}{15 \times \cancel{10^{-3}}}$$

$$= 10\,kW/m^2K.$$

$$\Delta T_{ln_1} = \frac{(4-71)-(4-32)}{\ln(-67/-28)} = -44.67\,°C$$

$$\Delta T_{ln_2} = \frac{(27-71)-(27-32)}{\ln(-44/-5)} = -17.93\,°C$$

$$0.4 \times 4181.8(71-32) = \pi \times 0.0125 \times L \times 13.89 \times 10^3 \times 44.7$$

$$L = 2.68\ m$$

$$0.4 \times 4181.8(71-32) = \pi \times 0.015 \times L_2 \times 10 \times 10^3 \times 17.93$$

$$L_2 = 7.72\ m.$$

~~fq~~ $f = \dfrac{0.046}{Re^{0.2}}$

$$f_1 = \frac{0.023 \times 2}{(76500)^{0.2}} = 0.0048$$

$$f_2 = \frac{0.023 \times 2}{(63700)^{0.2}} = 0.005$$

$$\frac{dQ}{dt} = \frac{\Delta P \pi r^4}{8 \mu L} \quad \ldots (i)$$

$$(ii) \ldots \Delta P = 4f\frac{L}{D} \cdot \frac{\rho V^2}{2}$$

$$\frac{0.4}{\cancel{1000}\ 987.36} = \frac{\Delta P_1 \times \pi \times \left(\frac{0.0125}{2}\right)^4}{8 \times 532.9 \times 10^{-6} \times 2.68}$$

~~$\Delta P_1 = 0.35\ Pa$~~ ~~$\Delta P_1 = 965.5$ Pa~~

$$\Delta P_2 = \cancel{4 \times 0.0025 \times \frac{7.72}{0.015} \times \frac{987.36}{1000}}$$

$$\Delta P_2 = 4 \times 0.0085 \times \frac{7.72}{0.015} \times \left(\frac{0.4}{987.36} \times \frac{10^4}{1.77}\right)$$

$$= \cancel{0.0864\ kPa}\ \cancel{106.48\ kPa}$$

$$26.88\ kPa.$$

$$\bar{v} = \frac{1.011 \times 0.5 + 1.013 \times 4.5}{5} \times 10^{-3}$$

$$= 1.0128\ m^3/kg \times 10^{-3}$$

$$\rho = 987.36\ kg/m^3$$

$$\Delta P_1 = 4 \times 0.0048 \times \frac{2.68}{0.0125} \times 987.36 \times \left(\frac{0.4}{987.36} \times \frac{10^4}{1.23}\right) \times \frac{1}{2}$$

$$= \cancel{0.028}\ 22.046\ kPa.$$