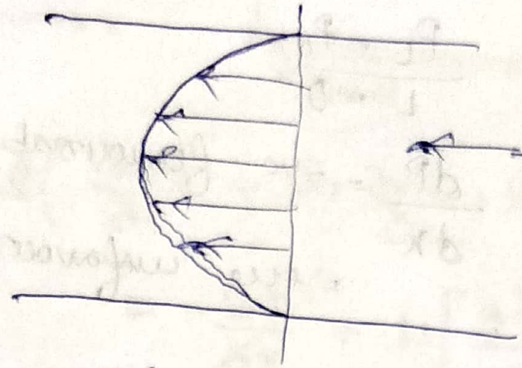
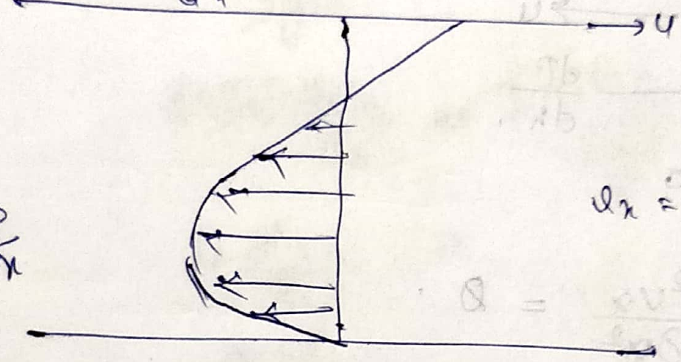
$v_x = \left(\frac{c_1 u}{h^2}\right) y^2$



$\leftarrow \frac{dP}{dx}$ +ve

$u_x = 0$.

+ve $\frac{dP}{dx}$ $\quad U_x = U \longrightarrow$.

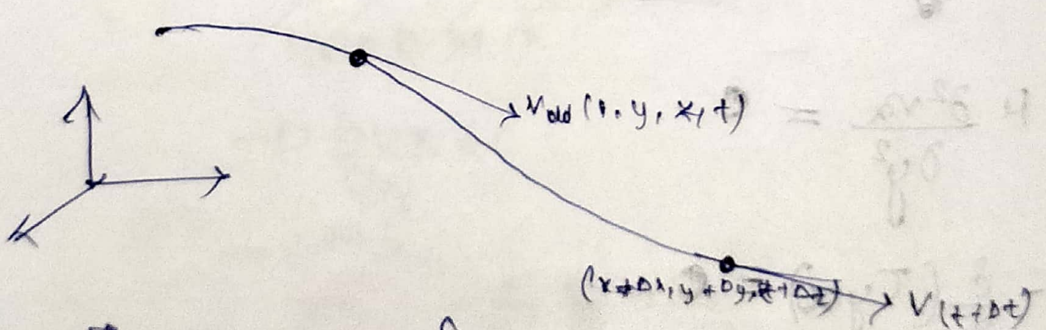



$v_x = \frac{Uy}{h} + \frac{1}{2\mu} A y^2 \left[1 - \right.$

$A = \frac{dP}{dx}$

[Ganguly]

* $V(x, y, z, t)$




$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$

$$u(x+\Delta x, y+\Delta y, z+\Delta z, t+\Delta t) = u(x, y, z, t) + \left(\frac{\partial u}{\partial x}\underbrace{(\Delta x)}_{u\Delta t} + \frac{\partial u}{\partial y}\underbrace{(\Delta y)}_{v\Delta t} + \frac{\partial u}{\partial z}\underbrace{(\Delta z)}_{w\Delta t} + \frac{\partial u}{\partial t}(\Delta t)\right)$$

$$\lim_{\Delta t \to 0} \frac{u(x+\Delta x, y+\Delta y, z+\Delta z, t+\Delta t) - u(x,y,z,t)}{\Delta t}$$

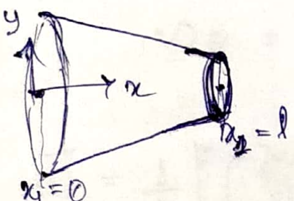
$$\lim_{\Delta t \to 0} \frac{\frac{\partial u}{\partial x}(\Delta x) + \frac{\partial u}{\partial y}(\Delta y) + \frac{\partial u}{\partial z}(\Delta z) + \frac{\partial u}{\partial t}(\Delta t)}{\Delta t}$$

$$= \underbrace{\frac{\partial u}{\partial x}(u) + \frac{\partial u}{\partial y}(v) + \frac{\partial u}{\partial z}(w)}_{\text{convective acceleration}} + \underbrace{\frac{\partial u}{\partial t}}_{\text{local acceleration}}$$

$$= \frac{Du}{Dt}$$

$$= \vec{a}^x$$

Ques $\vec{V} = v_1\left(1 + \frac{x}{L}\right)\hat{i}$ Find i) acceleration
ii) position of particle $x_p$ as a function of time.



i) $$\frac{Du}{Dt} = u\left(\frac{\partial u}{\partial x}\right)$$

$$\vec{a} = v_1\left(1 + \frac{x}{L}\right)\left(\frac{v_1}{L}\right)\hat{i} = \frac{v_1^2}{L}\left(1 + \frac{x}{L}\right)\hat{i}$$

ii) $$\frac{dx_p}{dt} = v_1\left(1 + \frac{x}{L}\right)$$

$$\cancel{x_p = v_1\left(x + \frac{x^2}{2L}\right) + c}$$

$$\frac{df}{dt} = v_1\left[1 + \frac{f}{L}\right]$$

$$1 + \frac{f}{L} = e^{v_1 t/L}$$

$$f = L\left[e^{v_1 t/L} - 1\right]$$

$$\frac{df}{dt} = v_1\left[1 + \frac{f}{L}\right]$$

$$\frac{d^2 f}{dt^2} = \frac{v_1^2}{L}\left[e^{v_1 t/L}\right]$$

Lagrangian Eulerian

$x = 0 \qquad t = 0 \qquad \frac{v_1^2}{L}$

$x = L/2 \qquad t = t_1 \qquad 1.5\frac{v_1^2}{L}$

$x = L \qquad t = t_2 \qquad 2\frac{v_1^2}{L}$

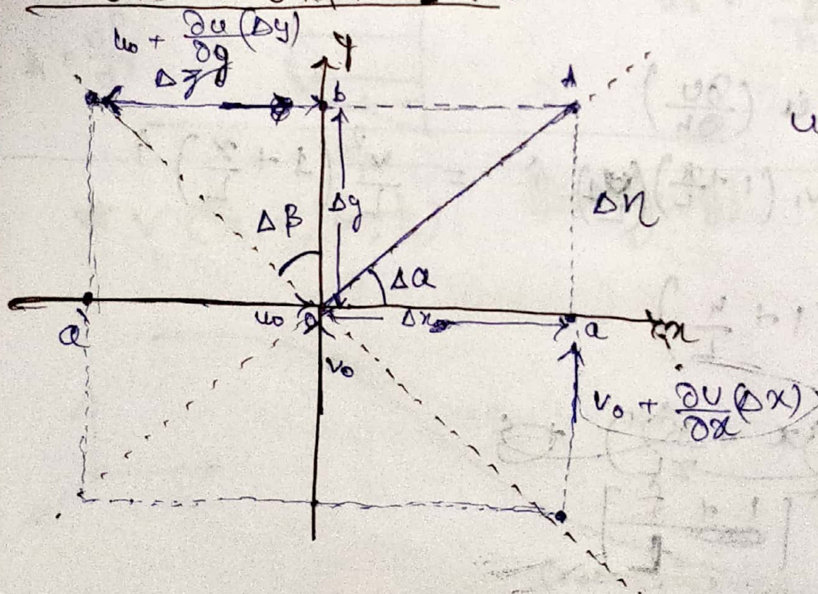
$$\frac{L}{2} = L\left[e^{\frac{v_1 t_1}{L}} - 1\right]$$

$$\frac{3}{2} = e^{\frac{v_1 t_1}{L}}$$

$$\frac{L}{v_1}\ln(3/2) = t_1$$

$$\frac{v_1^2}{L} e^{\frac{v_1}{L}\times\frac{L}{v_1}\ln(3/2)} = 1.5\frac{v_1^2}{L}$$

## FLUID ROTATION:-




$u\hat{i} + v\hat{j}$

$$\Delta \eta = \frac{\partial v}{\partial x}\Delta x \Delta t$$

$$\Delta \alpha = \frac{\Delta \eta}{\Delta x}$$

Angular velocity of OA = $\boxed{\omega_{oa} = \lim_{\Delta t \to 0}\frac{\Delta \alpha}{\Delta t} = \frac{\partial v}{\partial x}}$

$$\omega_{ob} = \lim_{\Delta t \to 0}\frac{\Delta \beta}{\Delta t}$$

$$\Delta \xi = -\frac{\partial u}{\partial y}(\Delta y)(\Delta t)$$

$$\Delta\beta = \frac{\Delta z}{\Delta y} = -\frac{\partial u}{\partial y}(\Delta t).$$

$$\left[\omega_{ob} = -\frac{\partial u}{\partial y}\right]$$

$$\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$$

here the axis of rotation is in the z-direction.

$$\omega_z = \frac{[\omega_{oa} + \omega_{ob}]}{2}$$

$$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$$

$$\omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$\vec{\omega} = \frac{1}{2}\left[\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}\right]$$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

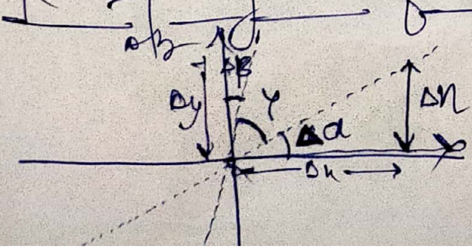
$= \frac{1}{2}\underbrace{\left[\nabla \times \vec{V}\right]}_{[\text{vorticity}](\vec{\zeta})}$ i.e. half of the curl of velocity field.

Flow in which the vorticity is zero is known as the irrotation flow.

Rate of angular deformation: $\Delta\phi = \phi - 90^\circ$

$$= -(\Delta\alpha + \Delta\beta)$$

$$= \frac{d\alpha}{dt} + \frac{d\beta}{dt}$$



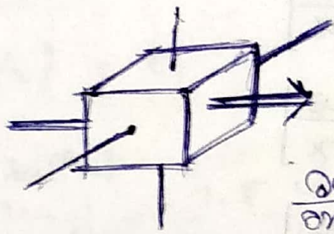
$$\frac{d\alpha}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \alpha}{\Delta t} = \frac{\Delta h/\Delta x}{\Delta t} = \frac{\partial v}{\partial x}$$

$$\frac{d\beta}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \beta}{\Delta t} = \frac{\Delta v/\Delta y}{\Delta t} = \frac{\partial u}{\partial y}$$

Rate of angular deform$^n$ $= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$

## * Vol$^m$ dilation Rate:

$$VDR = \nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial z}$$



$= \Delta \rho [\Delta x \Delta y \Delta z]$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial z} = 0.$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho \omega)}{\partial z} = -\frac{d\rho}{\partial t}$$

$\vec{V} = Ax\hat{i} - Ay\hat{j}$

$\vec{a} = \cancel{\left[u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}\right]}$

$\vec{\gamma} = 0$ (implies irrotational)

$\vec{\omega} = 0$

$-\frac{\delta \gamma}{\delta t} =$

$\vec{V} = u\left(\frac{y}{h}\right)\hat{i}$

$\vec{a} = 0.$

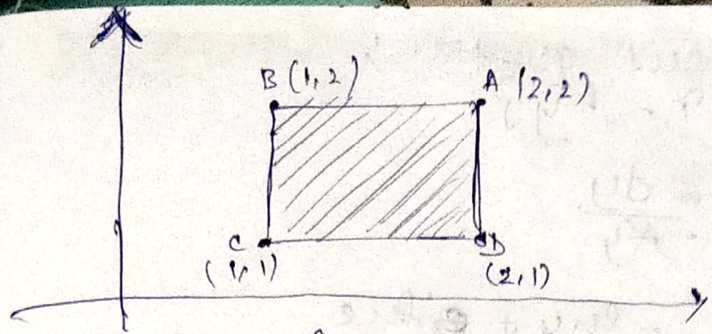
$\vec{\gamma} = -\frac{u}{h}\hat{k}$

$\vec{\omega} =$

$$\vec{a} = \left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right]\hat{i} + \left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right]\hat{j}$$

$$= A^2 x\,\hat{i} + A^2 y\,\hat{j}$$

Rate of vol$^m$ dilation $= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$




$v = Ax\hat{i} - Ay\hat{j}$

$A = 0.5\,s^{-1}$

Point A moves to $x = 1.5\,m$

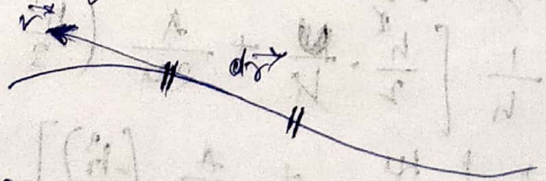
$A x_p = \frac{dx_p}{dt}$

$\boxed{\frac{dx_p}{x_p} = A\,dt}$

$\ln\left(\frac{x_{p_2}}{x_{p_1}}\right) = At$

| point | $t = 0$ | $t = 1.39\,s$ |
| --- | --- | --- |
| a | (1, 1) | (3/2, 2/3) |
| b | (1, 2) | (3/2, 4/3) |
| c | (2, 2) | (3/2, 4/3) |
| d | (2, 1) | |

Lines drawn in the flow field such at a given instant, they are tangent to the direction of the flow at every point in the flow field is known as streamline.




$\vec{V} \times d\vec{r} = 0$

$(\hat{i}u + \hat{j}v) \times (\hat{i}dx + \hat{j}dy) = 0$

$(u\,dy - v\,dx)\hat{k} = 0$

$u\,dy = v\,dx$

$\left[\frac{dx}{u} = \frac{dy}{v}\right] \rightarrow$ eq$^n$ of streamline.

In the previous question,

$\vec{V} = Ax\hat{i} - Ay\hat{j}$

$$\frac{dx}{Ax} = \frac{dy}{-Ay}$$

$$\ln x = -\ln y + \ln c$$

$$\ln xy = \ln c$$

$$\boxed{xy = c}$$

$\psi(x, y, \cancel{t}) = \text{const}$ along a streamline.

$$\frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = 0$$

$$u\,dx - v\,dy = 0$$

[SDG]

$$\langle V_x \rangle = \frac{1}{A} \int_0^{\omega}\int_0^{h} V_x \, dy\, dz \quad \text{where } A = \omega h$$

$$V_x = f(y)$$

$$= \frac{1}{\text{Area}} \int_0^{\omega} dz \int_0^{h} \left( \frac{Uy}{h} + \frac{1}{2\mu} A y^2 \left[1 - \frac{h}{y}\right] \right) dy$$

$$\frac{\cancel{\omega}}{\cancel{A\omega} h} \left[ \frac{U}{h} \int_0^{h} y\,dy + \frac{A}{2\mu} \int_0^{h} (y^2 - yh)\, dy \right]$$

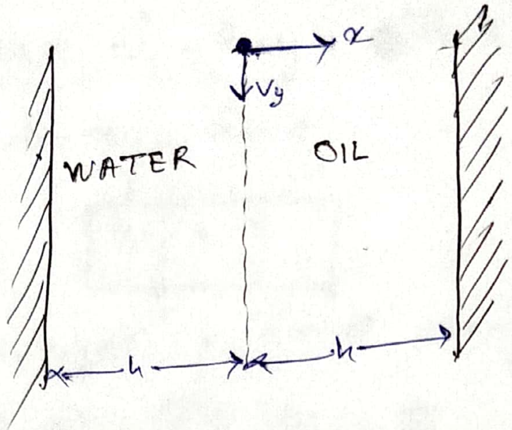
$$= \frac{1}{h} \left[ \frac{h^2}{2} \cdot \frac{U}{h} + \frac{A}{2\mu} \left( \frac{h^3}{3} - \frac{h^3}{2} \right) \right]$$

$$= \frac{1}{h} \left[ \frac{hU}{2} + \frac{A}{2\mu} \left( \frac{-h^3}{6} \right) \right]$$

$$\frac{dP}{dx} = \frac{6\mu U}{h^2}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{dp}{dz}$$

$$+ \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$




$v_y$ oil
$v_y$ water.

$$\rho\left(\cancel{\frac{\partial v_y}{\partial t}}^{0} + \cancel{v_x}^{0}\frac{\partial v_y}{\partial x} + v_y\cancel{\frac{\partial v_y}{\partial y}}^{0} + \cancel{v_z}\frac{\partial v_y}{\partial z}\right) = -\cancel{\frac{dp}{dy}}^{0}$$

$$+ \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_y$$

$$\mu\frac{\partial^2 v_y}{\partial x^2} = -\rho g$$

$$\frac{\partial^2 (v_y)}{\partial x^2} = -\frac{\rho g}{\mu}$$

$$\frac{\partial v_y}{\partial x} = -\frac{\rho g}{\mu}x + c$$

$$v_y = -\frac{\rho g x^2}{2\mu} + c_1 x + d_1$$

where c and d are unknown.

$v_y = 0$ at $x = h$ and $x = -h$.
but $\mu = \mu_1$ for $x \in [-h, 0]$ and $\mu = \mu_2$ for $x \in [0, h]$

$$\cancel{-\frac{\rho g h^2}{2\mu} + ch + d = 0} \quad \text{(I)}$$

$$-\frac{\rho g h^2}{2\mu} - ch + d = 0 \quad \text{(II)}$$

(I) + (II)

$$d = \rho g h^2 \qquad c = 0$$

$$\cancel{-\frac{\rho g x^2}{2\mu} + \frac{\rho g}{2\mu}}$$

$$\cancel{v_y = 0 = -\frac{\rho g h^2}{2\mu_1} - ch + d = 0}$$

$$\cancel{v_y = 0 = -\frac{\rho g h^2}{2\mu_2} + ch + d = 0}$$

$$\cancel{= 2ch + \frac{\rho g h^2}{2}\left(\frac{1}{\mu_1} - \frac{1}{\mu_2}\right)}$$

$$\boxed{\begin{aligned} v_y \text{ water} &= -\frac{\rho_w g x^2}{2\mu_1} + c_1 x + d_1 \\ v_y \text{ oil} &= -\frac{\rho_o g x^2}{2\mu_2} + c_2 x + d_2 \end{aligned}}$$

at the water oil interface

$$v_y \text{ water}\Big|_{x=0} = v_y \text{ oil}\Big|_{x=0}$$

$$\boxed{d_1 = d_2 = d}$$

$$v_y \text{ water}\Big|_{x=h} = -\frac{\rho_w g h^2}{2\mu_1} - c_1 h + d = 0$$

$$v_y \text{ oil}\Big|_{x=-h} = -\frac{\rho_o g h^2}{2\mu_2} + c_2 h + d = 0.$$

$$\tau_{yx \text{ water}} = -\mu_1 \frac{\partial v_y}{\partial x} = -\mu_1\left(\frac{-\rho g x}{\mu_1} + c_1\right)$$

$$\tau_{yx \text{ oil}} = -\mu_1 \frac{\partial v_y}{\partial x} = -\mu_2\left(\frac{-\rho g x}{\mu_2} + c_2\right)$$

at $x = 0$ $\tau_{yx}|_{\text{water}} = \tau_{yx}|_{\text{oil}}$

$$\cancel{-}\mu_1 c_1 = \cancel{-}\mu_2 c_2$$

$$\mu_1 c_1 = \mu_2 c_2 \quad \text{——— (1)}$$

$$-\frac{\rho_w g h^2}{2\mu_1} - c_1 h + d = 0 \quad \text{——— (2)}$$

$$-\frac{\rho_o g h^2}{2\mu_2} + c_2 h + d = 0. \quad \text{——— (3)}$$

$$(c_1+c_2)\cancel{h} + \left(\frac{\rho_w g h^{\cancel{2}}}{2\mu_1} - \frac{\rho_o g h^{\cancel{2}}}{2\mu_2}\right) = 0.$$

$$c_1 + c_2 = \frac{gh}{2}\left(\frac{\rho_o}{\mu_2} - \frac{\rho_w}{\mu_1}\right)$$

$$c_2 = \frac{c_1 \mu_1}{\mu_2}.$$

$$c_1 + \frac{c_1\mu_1}{\mu_2} = \frac{gh}{2}\left(\frac{\rho_o}{\mu_2} - \frac{\rho_w}{\mu_1}\right)$$

$$c_1\left(\frac{\mu_2+\mu_1}{\cancel{\mu_2}}\right) = \frac{gh(\rho_o\mu_1 - \rho_w\mu_2)}{2\cancel{\mu_2}\mu_1}$$

$$c_1 = \frac{gh(\rho_o\mu_1 - \rho_w\mu_2)}{2\mu_1(\mu_2+\mu_1)} \quad \text{— (a)}$$

$$c_2 = \frac{gh(\rho_o\mu_1 - \rho_w\mu_2)}{2\mu_2(\mu_2+\mu_1)} \quad \text{— (b)}$$

$$V_{y\,water} = -\frac{\rho_w g x^2}{2\mu_1} + \frac{gh(\rho_o\mu_1 - \rho_o\mu_2)x}{2\mu_1(\mu_2+\mu_1)} + d$$

$V_{y\,water} = 0$ at $x = -h$.

$$V_{y\,water} = -\frac{\rho_w g h^2}{2\mu_1} - \frac{gh^2(\rho_o\mu_1 - \rho_w\mu_2)}{2\mu_1(\mu_1+\mu_2)} + d = 0.$$

$$d = \frac{gh^2}{2\mu_1}\left(\rho_w + \frac{\rho_o\mu_1 - \rho_w\mu_2}{\mu_1+\mu_2}\right)$$

$$d = \frac{gh^2}{2\mu_1}\left(\frac{\rho_w\mu_1 + \cancel{\rho_w\mu_2} + \rho_o\mu_1 - \cancel{\rho_w\mu_2}}{\mu_1+\mu_2}\right)$$

$$d = \frac{gh^2\cancel{\mu_1}}{2\cancel{\mu_1}}\left(\frac{(\rho_o+\rho_w)}{\mu_1+\mu_2}\right) = \frac{gh^2(\rho_o+\rho_w)}{2(\mu_1+\mu_2)} \quad \text{— (c)}$$

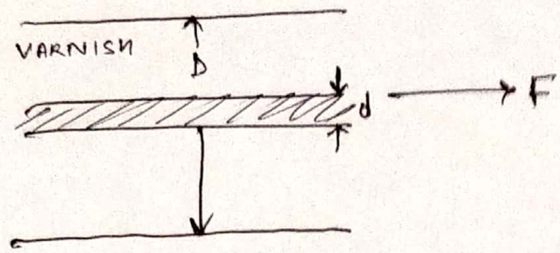
Scanned by CamScanner

$$V_y\,(\text{water}) = \frac{-\rho_w g x^2}{2\mu_1} + \frac{gh}{2\mu_1}\frac{(\rho_o\mu_1 - \rho_w\mu_2)}{(\mu_1 + \mu_2)}x + \frac{gh^2}{2}\frac{(\rho_w + \rho_o)}{(\mu_1+\mu_2)}$$

$$V_y\,(\text{oil}) = -\frac{\rho_o g}{2\mu_2}x^2 + \frac{gh}{2\mu_2}\frac{(\rho_o\mu_1 - \rho_w\mu_2)}{(\mu_1 + \mu_2)}x + \frac{gh^2(\rho_w + \rho_o)}{2(\mu_1+\mu_2)}$$

Sol

Ques




Magnet wire being coated with varnish in a tube wire assembly.

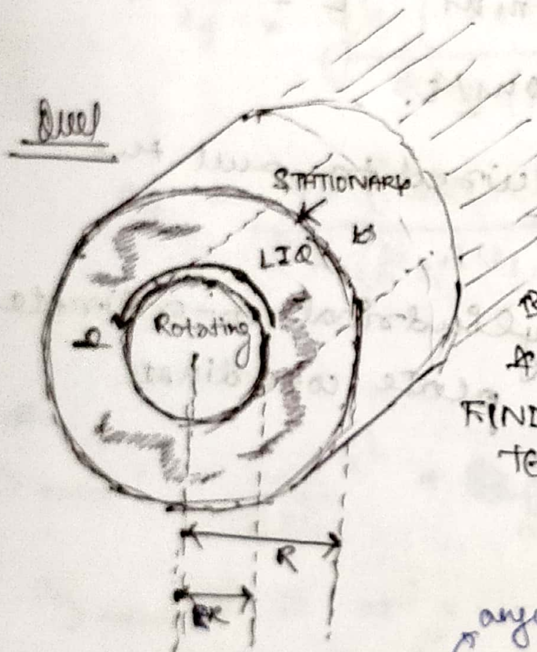
$d = 0.8\,mm$ $D = 0.9\,mm$ $L = 20\,mm$

$\mu = 20\,cp$ , $v = 50\,m/s$.

Find the force F required to pull the wire.

We can simplify this cylindrical co-ordinate ~~sys~~ system to two plate co-ordinate system.

Ques




TWO CYLINDERS
INNER ONE ROTATES
BY THE APPLICATION OF
A KNOWN TORQUE T.
FIND EXPESSION FOR VEL. IN
TERMS OF T.

angular symm.

$$\rho\left(\cancel{\frac{\partial v_\theta}{\partial t}} + \cancel{v_r\frac{\partial v_\theta}{\partial r}} + \frac{v_\theta}{r}\cancel{\frac{\partial v_\theta}{\partial \theta}} + v_z\cancel{\frac{\partial v_\theta}{\partial z}} + \cancel{\frac{v_r v_\theta}{r}}\right) = -\frac{1}{r}\cancel{\frac{\partial p}{\partial \theta}}$$

$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\cancel{\frac{\partial^2 v_\theta}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_\theta}{\partial z^2}} + \frac{2}{r^2}\cancel{\frac{\partial v_r}{\partial \theta}}\right]$$

$$0 = \mu\frac{d}{dr}\left[\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right]$$

$$\Rightarrow \frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(r v_\theta)\right] = 0$$

$$\frac{1}{r}\frac{d}{dr}(rv_\theta) = c$$

$$\frac{d}{dr}(rv_\theta) = cr$$

$$rv_\theta = \frac{cr^2}{2} + d$$

<u>Boundary Conditions</u>

$r = KR, \quad v_\theta = \Omega_0 KR$

$r = R, \quad v_\theta = 0$

$$v_\theta = \frac{cr}{2} + \frac{d}{r}$$

$$\cancel{\Omega_0 K \; v_\theta = \frac{c\Omega_0 KR}{2}} + \frac{d}{KR}.$$

$$\Omega_0 KR = \frac{cKR}{2} + \frac{d}{KR} \quad \text{--- (I)}$$

$$0 = \frac{cR}{2} + \frac{d}{R}.$$

$$c = -\frac{2d}{R^2} \quad \text{--- (I)}.$$

$$c = \left(\Omega_0 KR - \frac{d}{KR}\right)\frac{2}{KR}.$$

$$\frac{\cancel{2}}{KR}\left(\Omega_0 KR - \frac{d}{KR}\right) = -\frac{\cancel{2}d}{R^2}.$$

$$\Omega_0 - \frac{d}{K^2R^2} = -\frac{d}{R^2}.$$

$$\Omega_0 = \frac{d}{R^2}\left(\frac{1}{K^2} - 1\right)$$

$$d = \frac{\Omega_0 R^2}{\left(\frac{1}{K^2} - 1\right)} \quad \text{--- (III)}.$$

$$c = -\frac{2d}{\cancel{R^2}} \cdot \frac{\Omega_0 \cancel{R^2}}{(1/K^2 - 1)} = \frac{-2\Omega_0}{(1/K^2 - 1)} \quad \text{--- (IV)}$$

$$v_\theta = \frac{-\Omega_0 r}{\left(\frac{1}{k^2}-1\right)} + \frac{\Omega_0 R^2}{\left(\frac{1}{k^2}-1\right) r}$$

$$v_\theta = \frac{\Omega_0 R}{\left(1 - \frac{1}{k^2}\right)} \left(\frac{r}{R} - \frac{R}{r}\right) \qquad (2)$$

Expression of T.

$$\tau_{r\theta} = -\mu\left[r \frac{d}{dr}\left(\frac{v_\theta}{r}\right)\right]$$

$$T = \left(\underbrace{2\pi kRL}_{\text{AREA}} \; \underbrace{(-\tau_{r\theta})\Big|_{r=kR}}_{\text{STRESS}} \; \underbrace{(kR)}_{\text{LEVER ARM}}\right)$$

$$T = \frac{4\pi\mu\Omega_0 L R^2}{(1 - 1/k^2)}$$

[GANGULY]

$\vec{V} = Ax\hat{i} - Ay\hat{j}$

$\psi = \psi(x,y)$

$$\left.\begin{aligned} u &= \frac{\partial\psi}{\partial y} \\ v &= -\frac{\partial\psi}{\partial x} \end{aligned}\right\} \text{in general.}$$

for the above case,

$$\frac{\partial\psi}{\partial y} = Ax$$

$$\psi = Axy + c_1(x)$$

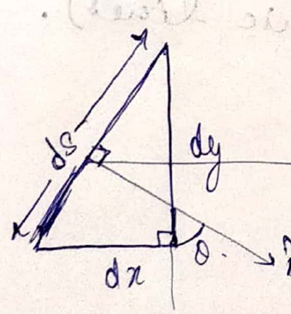
$$-\frac{\partial\psi}{\partial x} = -Ay - c_1'(x) = -Ay$$

$$\cancel{\psi = Axy + c(x,y)}$$

$$-c_1'(x) = 0.$$

$$c_1(x) = \text{const} = c.$$

$$\psi = Axy + c$$



$\vec{V} = u\hat{i} + v\hat{j}$

$\hat{n} = \frac{dy}{ds}\hat{i} - \frac{dx}{ds}\hat{j}$

$\hat{n} = \sin\theta\,\hat{i} - \cos\theta\,\hat{j}$

$dQ = (\vec{V}\cdot\hat{n})\,dA =$

$\frac{dx}{dt} = u = Ax,$

$\int \frac{dx}{x} = \int A\,dt$

$x = x_0 e^{At}$

$y = y_0 e^{-At}$

$\Rightarrow \boxed{xy = x_0 y_0}$

In cylindrical co-ordinates:

$\Rightarrow \frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) = 0.$

$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$

$v_\theta = -\frac{\partial \psi}{\partial r}$

Potential lines:

Lines along which the potential func$^n$ is const. (for all practical purpose, It is analogous to isobaric lines).

$u = \frac{\partial}{\partial x}(\phi)$

$\frac{\partial \psi}{\partial x}$

$v = \frac{\partial}{\partial y}(\phi)$

$-\frac{\partial \psi}{\partial y}.$

Line of const ($\psi$)

$\Rightarrow \left.\frac{dy}{dx}\right|_{\psi = const} = \frac{v}{u}$

$d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy = 0$

$\frac{dy}{dx} = \frac{-\left(\frac{\partial \psi}{\partial x}\right)}{\left(\frac{\partial \psi}{\partial y}\right)} = \frac{v}{u}$ — (i)

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Line of const ($\phi$)

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\left(\frac{\partial \phi}{\partial x}\right)}{\left(\frac{\partial \phi}{\partial y}\right)} = -\frac{u}{v} \quad \text{—(11)}$$

(1) $\times$ (11)

$= -1$.

Hence proved that they are $\perp r$.

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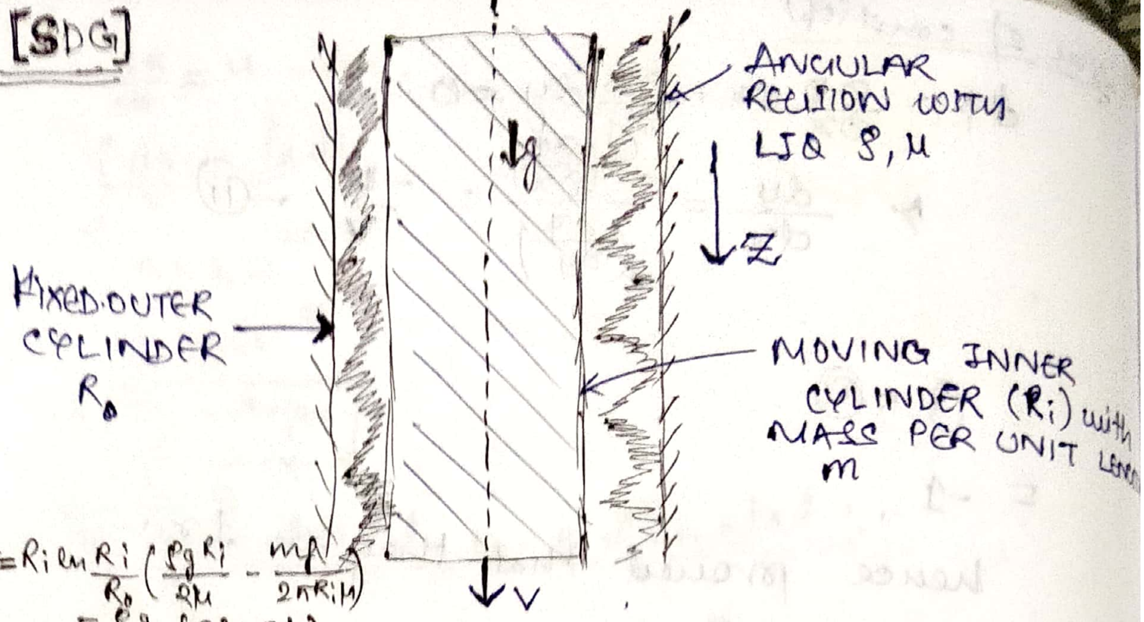
[SDG]




$$V = R_i \ln \frac{R_i}{R_o}\left(\frac{\rho g R_i}{2\mu} - \frac{mg}{2\pi R_i \mu}\right) - \frac{\rho g}{4\mu}\left(R_i^2 - R_o^2\right)$$

$$\Rightarrow \rho\left(\cancel{\frac{\partial v_z}{\partial t}} + \cancel{v_r}\frac{dv_z}{dr} + \frac{\cancel{v_\theta}}{r}\frac{\partial v_z}{\partial \theta} + v_z \cancel{\frac{\partial v_z}{\partial z}}\right) = \frac{dP}{dz}$$

$$+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{dv_z}{dr}\right) + \frac{1}{r^2}\cancel{\frac{\partial^2 v_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right]$$

$$+\rho g_z$$

$$g_z = g$$

$$\Rightarrow 0 = \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right] + \rho g.$$

$$-\frac{\rho g r}{\mu} = \frac{\partial}{\partial r}\left[r\frac{\partial v_z}{\partial r}\right]$$

$$r\frac{\partial v_z}{\partial r} = -\frac{\rho g r^2}{2\mu} + c_1$$

$$\frac{\partial v_z}{\partial r} = -\frac{\rho g r}{2\mu} + \frac{c_1}{r}$$

$$v_z = -\frac{\rho g r^2}{4\mu} + c_1 \ln r + c_2$$

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$V_z|_{r=R_i} = V$

$V_z|_{r=R_o} = 0$

Boundary conditions.

force on the moving cylinder = $mLg$

$2\pi R_i L \cdot \tau_{rz}|_{r=R_i} = mLg$.

$\longrightarrow 2\pi R_i \tau_{rz}|_{r=R_i} = mg$.

$\boxed{\tau_{rz} = -\mu \frac{\partial v_z}{\partial r}}$ → for the force on the fluid.

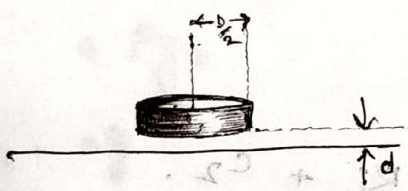
$$-\frac{\tau_{rz}}{\mu} = -\frac{\rho g r}{2\mu} + \frac{C_1}{r}.$$

$$\tau_{rz} = \frac{\rho g r}{2} - \frac{C_1 \mu}{r}$$

$$V = -\frac{\rho g R_i^2}{4\mu} + C_1 \ln R_i + C_2.$$

$$0 = -\frac{\rho g R_o^2}{4\mu} + C_1 \ln R_o + C_2$$

Ques




$m = 30g$, $D = 100mm$

$\mu_{air} = 1.75 \times 10^{-5}\ N/m^2$

$d = 0.1mm$

CALCULATE THE TIME FOR THE PUCK TO LOOSE 10% OF IT'S INITIAL SPEED.

[Ans: $t = 2.3s$]

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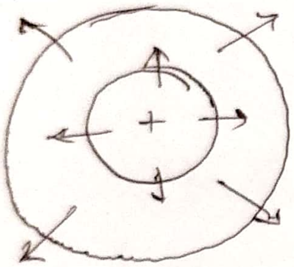


FLOW DUE TO PRESSURE DIFF :
VEL. NOT ZERO AT THE SOLID SURFACES.

CONCENTRIC SPHERICAL POROUS SHELLS.

eqⁿ of continuity:

$$\Rightarrow \cancel{\frac{\partial \rho}{\partial t}}^{0} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\rho \cancel{v_\theta}^{0}\sin\theta)$$

$$+\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(\rho \cancel{v_\phi}^{0}) = 0.$$

$$\rho\frac{\partial}{\partial r}(r^2 v_r) = 0$$

$$v_r = \frac{c}{r^2} \quad \text{—(1)}$$

spherical coordinates:

$$\rho\left(\cancel{\frac{\partial v_r}{\partial t}}^{0} + v_r\frac{\partial v_r}{\partial r} + \frac{\cancel{v_\theta}^{0}}{r}\frac{\partial v_r}{\partial\theta} + \frac{\cancel{v_\phi}^{0}}{r\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\cancel{v_\theta^2}^{0}+\cancel{v_\phi^2}^{0}}{r}\right)$$

$$= -\frac{\partial p}{\partial r} + \mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\cancel{\frac{\partial v_r}{\partial\theta}}^{0}\right)\right.$$

$$\left. + \frac{1}{r^2\sin^2\theta}\cancel{\frac{\partial^2 v_r}{\partial\phi^2}}\right] + \cancel{\rho g_r}^{0}.$$

$$\rho v_r\frac{\partial v_r}{\partial r} = -\frac{\partial p}{\partial r} + \frac{\mu}{r^2}\cancel{\frac{\partial^2}{\partial r^2}(r^2 v_r)}^{0}$$

↳ $r^2 v_r$ const for eqⁿ of continuity. hence this term is const.
$\frac{\partial^2}{\partial r^2}(r^2 v_r) = 0.$

$$\rho v_r \frac{\partial v_r}{\partial r} = -\frac{\partial P}{\partial r}$$

P can only be a function of $r, \theta, \phi$ for the case of steady state flow.

using the eq$^n$ for other two cases ($v_\theta$ and $v_\phi$) produce that $\frac{\partial P}{\partial \theta} = \frac{\partial P}{\partial \phi} = 0$.

hence $P = P(r)$.

$$\frac{\partial P}{\partial r} = \frac{dP}{dr}$$

OBTAIN THE PR. PROFILE $P(r)$ IN TERMS OF $P_R$ and $V_R$, THE PR and VEL AT THE SPHERE OF RAD. R,

$$\rho v_r \frac{dv_r}{dr} = -\frac{dP}{dr}$$

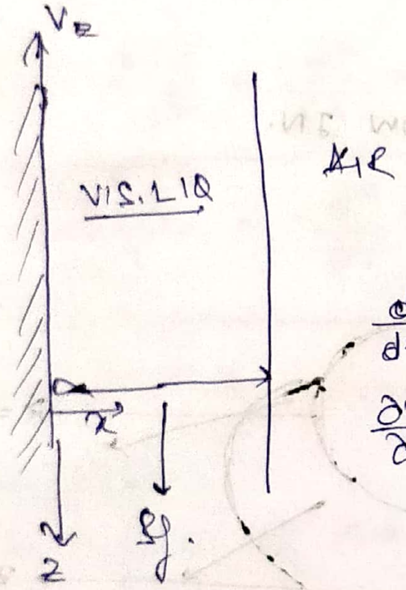
$$\frac{\rho C_0}{r^2} \frac{d}{dr}\left(\frac{C}{r^2}\right) = -\frac{dP}{dr}$$

$$+\frac{\rho C}{r^2} \frac{C}{r^3} = +\frac{dP}{dr}$$

$$\text{(1)} \quad \frac{dP}{dr} = \frac{\rho C^2}{r^5}$$

**Que** Airways of lungs are coated with a thin lining of highly viscous fluid (of $\mu$ and $\rho$). The thickness of liquid lining ($h$) is much less than radius of airways ($r$). Pump upwards towards mouth, opposing the pull of gravity.

velocity of airway wall $\rightarrow v_0$.




$v_z = f(x)$ only.

$\frac{dP}{dz} = 0.$ $\quad g_z = g.$

$\frac{\partial v_z}{\partial t} = 0.$

$$0 = 0 + \mu \frac{\partial^2 v_z}{\partial x^2} + \rho g_z$$

$$v_z = c_1 x + c_2 .$$

**BOUNDARY CONDITIONS**

at $x = 0$, $v_z = -V_0$.

at $x = h$, $\frac{dv_z}{dx} = 0.$

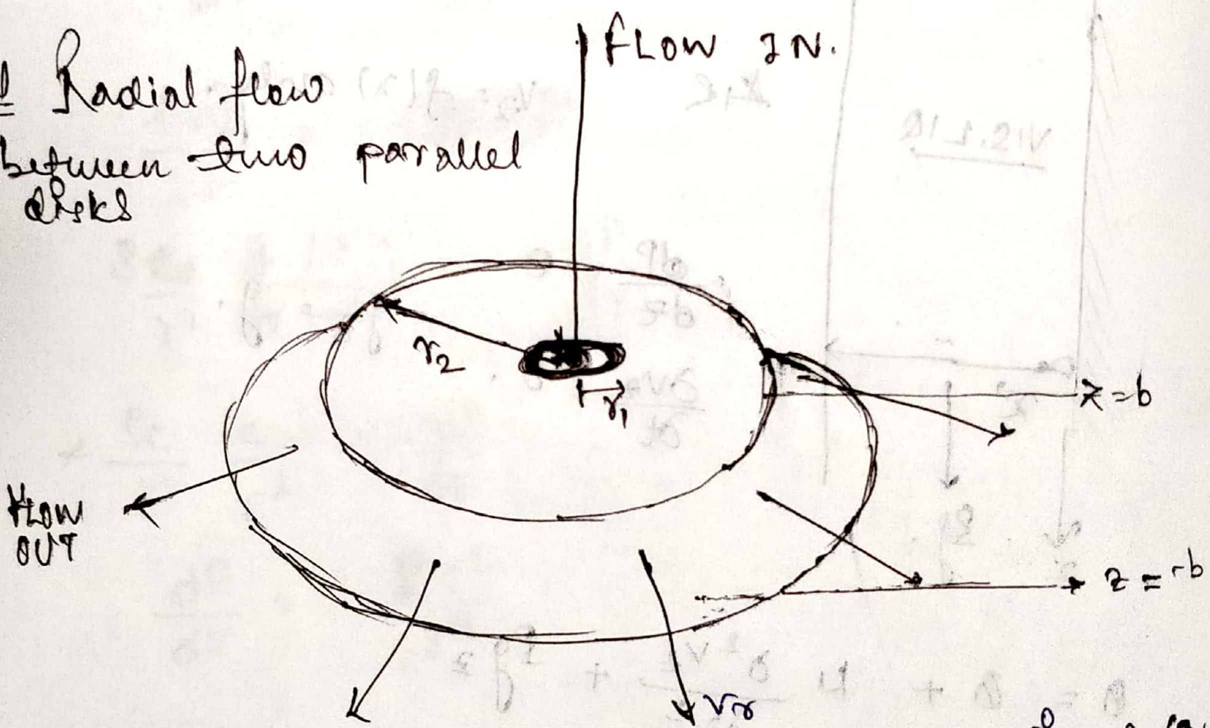
$v_z = f(x)$

$\langle v_z \rangle = -A \ldots -B$

$\langle v_z \rangle = -ve$

A B $\Rightarrow v_0 > \frac{\rho g h^2}{3\mu}$

Radial flow between two parallel disks




$$\text{Eq}^n: \cancel{\frac{\partial \rho}{\partial t}}^{0} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho \cancel{v_\theta})^{0} + \frac{\partial}{\partial z}(\cancel{\rho v_z})^{0} = 0$$

steady state

$$\rho \frac{\partial}{\partial r}(r v_r) = 0.$$

$$r v_r = c$$

$$v_r = \frac{c}{r}$$

$\Rightarrow c$ is a const. wrt $(r, \theta)$

$r_1 \le r_{eq} \le r_2$

* $v_r = V(r, z)$

Eq^n of motion :-

$$\Rightarrow \rho\left(\cancel{\frac{\partial v_r}{\partial t}} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right)$$

$$= -\frac{\partial P}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]$$

$$+ \rho g_r$$

$$\Rightarrow \rho v_r \frac{\partial v_r}{\partial r} = -\frac{\partial P}{\partial r} + \mu \frac{\partial^2 v_r}{\partial z^2}$$

$$-\frac{\rho\phi(z)}{r} \cdot \frac{\phi(z)}{r^2} = -\frac{dP}{dr} + \mu\frac{\partial^2 v_r}{\partial z^2}$$

$$= \boxed{-\frac{\rho(\phi(z))^2}{r^3} = -\frac{dP}{dr} + \frac{\mu}{r}\frac{\partial^2 \phi(z)}{\partial z^2}}$$

* $\phi(z)$ represents convective flow (moving).

## Assumption

⇒ For slow flow of the in between the two plates,

CREEPING FLOW → characterised by $Re < 1$.

$$R = \frac{DV\rho}{\mu} = \frac{\text{inertial force}}{\text{viscous force}}$$

$F = 6\pi\mu R v$ (an example of creeping flow)

* $\phi^2 = 0$.

$$\frac{\mu}{r}\frac{\partial^2 \phi}{\partial z^2} = \frac{dP}{dr}$$

$$v_r = \frac{\phi}{r}$$

$\phi = f$

$$v_r = \frac{\Delta P\, b^2}{2\mu r \ln\left(\frac{r_2}{r_1}\right)}\left[1 - \left(\frac{z}{b}\right)^2\right]$$

BC

$v_r(r, z) = 0$
at $z = +b$
and at $z = -b$.

$\Delta P = p_1 - p_2$

$$Q = \frac{4\pi (\Delta P)\, b^3}{3\mu \ln\left(\frac{r_2}{r_1}\right)}$$

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