Quantum Field Theory I

Homework 6

Due 23/06/2020

1. **Scalar QED (4 pts.)**: Consider the theory of a complex (charged) scalar field, invariant under local $U(1)$ transformations given by

$$\mathcal{L} = (D_\mu \phi)^\dagger D^\mu \phi - m^2 \phi^\dagger \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

where

$$D_\mu \phi = (\partial_\mu + ieA_\mu)\phi \ ,$$

and the $U(1)$ gauge field strength tensor is $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$.

   (a) Derive the Feynman rules for the theory.

   (b) Draw all the Feynman diagrams and compute the amplitude for the process $\gamma\gamma \to \phi^+\phi^-$.

   (c) Choose one of the diagrams and compute its contribution to the differential cross section $d\sigma/d\cos\theta$.

2. **Bhabha Scattering (5 pts.)** : Compute the differential cross section $d\sigma/d\cos\theta$ for the process $e^+e^- \to e^+e^-$. Draw all the contributing Feynman diagrams, compute the amplitude and its square. Be careful about the interference terms. To simplify, take $m_e = 0$. Verify that using Mandelstam variables it can be written as

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{s}\left[u^2\left(\frac{1}{s}+\frac{1}{t}\right)^2 + \left(\frac{t}{s}\right)^2 + \left(\frac{s}{t}\right)^2\right] ,$$

where we defined

$$\alpha \equiv \frac{e^2}{4\pi}$$