UNIVERSIDADE DE SÃO PAULO
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# Supersymmetric models with non-decoupling D-terms

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# Modelos supersimetricos com D-termos não desacoplados

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In this thesis we will work with natural units $\hbar = c = 1$.

The metric used is

$$g_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1).$$

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# Abstract

One of the most puzzling theoretical problems of the Standard Model (SM) is the famous hierarchy problem, *i.e.* the sensitivity of the electroweak scale to the details of what happens at much larger energy scales. The most studied solution to solve this problem is an extension of the SM that is based on the incorporation of a symmetry relating fermions and bosons. The symmetry is the so-called supersymmetry, and its simplest phenomenologically viable realization is the Minimal Supersymmetric Standard Model (MSSM). The experimental discovery of a scalar particle closely resembling a Higgs boson with a mass of 125 GeV has however put the MSSM at odds. This is because the tuning needed to accommodate the experimentally observed Higgs mass turns out to be worst than 1%. While this level of tuning is possible, it would be desirable to have supersymmetric theories with a lower tuning.
In this thesis we study how to solve the tuning problem of the MSSM increasing the tree level mass of the Higgs boson via additional D-terms in the scalar potential. In particular, we extend the SM gauge group adding an $SU(2)$ group. Given that two breaking patterns of the extended symmetry can lead to the SM gauge group, we study their consequences at the level of gauge boson and Higgs masses.



*Keywords* : Supersymmetry; Higgs boson mass; D-terms.

# Resumo

Um dos problemas teóricos mais desconcertantes do Modelo Padrão é o famoso problema da hierarquia, *i.e.* a sensibilidade da escala eletrofraca aos detalhes do que acontece em escalas de energia muito maiores . A solução mais estudada para resolver este problema é uma extensão do Modelo Padrão que se baseia na incorporação de uma simetria que relaciona fermiões e bosons. A simetria é a chamada supersimetria, e sua realização fenomenologicamente viável mais simples é o Modelo Padrão Supersimétrico Minimal. A descoberta experimental de uma partícula escalar muito parecida com um bóson de Higgs com uma massa de 125 GeV, no entanto, colocou o Modelo Padrão Supersimétrico Minimal em desacordo. Isso ocorre porque o ajuste necessário para acomodar a massa do Higgs observada experimentalmente é pior do que 1%. Embora este nível de ajuste seja possível, seria desejável ter teorias supersimétricas com um ajuste mais baixo. Nesta tese, estudamos como resolver o problema de ajuste do MSSM aumentando a massa no nível da árvore do bóson de Higgs através de D-termos adicionais no potencial escalar. Em particular, estendemos o grupo de gauge do Modelo Padrao adicionando um grupo $SU(2)$. Dado que dois padrões de ruptura da simetria estendida podem levar ao grupo de gauge do Modelo Padrao, estudamos suas conseqüências ao nível de bosons de gauge e das massas de Higgs.

$palavras - chaves$ : Supersimmetría; Massa do bóson de Higgs; D-termos.

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# Chapter 1

# Introduction

The discovery of the Higgs Boson in 2012 at the LHC was one of the most important confirmations that the Standard Model (SM) of particle physics is a theory which goes in the correct direction of a full description of the basic laws of nature. At this point it is well known that the Standard Model describes remarkably well almost all the known phenomena at the particle level. However, there are experimental and theoretical reasons that call for its extension. To illustrate this, consider the following list with some of the issues that are not answered within the SM

- Neutrino Masses;
- Baryon asymmetry of the universe;
- Dark Matter;
- Strong CP Problem;
- Charge quantization;
- Hierarchy Problem;

The first three problems are related with fundamental physical observations that are not addressed in the SM, while the other three are 'theoretical problems'. By theoretical problems I mean that the theory works fine if one add certain features to the SM in a appropriate way, but they imply a lack of understanding. In this thesis I will focus on models whose motivation is to solve the last problem of the previous list, namely the Hierarchy Problem. This solution is based on the incorporation of a new symmetry, the so-called Supersymmetry (SUSY). This symmetry relates fermions with bosons and vice versa.

## 1.1 A fine-tuning problem in the SM and the Hierarchy problem

In the Standard Model of particle physics, the particles acquire mass via the Higgs mechanism. Doing the math we will find that the masses of the SM particles depend in a very simple way on the vacuum expectation value (VEV) of the Higgs doublet. The squared masses of the fermions, the massive gauge bosons and the physical Higgs Boson are given by

$$\begin{aligned} m_f^2 &= y_f^2\, v^2, \\ m_W^2 &= \frac{g^2}{2}\, v^2, \\ m_Z^2 &= \left(\frac{g^2 + g'^2}{2}\right) v^2, \\ m_h^2 &= \frac{\lambda}{2}\, v^2. \end{aligned} \tag{1.1}$$

Here $v$ is the VEV and $y_f$ corresponds to the Yukawa coupling constant for the fermion $f$[1]. The vacuum expectation value is related with the mass parameter $\mu^2$ of the Higgs potential given in equation (9). The minimum of the Higgs potential, interpreted as a classical potential, is at the non zero value given by

$$\frac{2}{\lambda}\,\mu^2 \equiv \frac{v^2}{2}. \tag{1.2}$$

The simple formula for the masses of the SM particles given above does not take into account the effects of loop corrections. It can be shown that the VEV given in the last equation receives enormous quantum corrections from every particle that couples directly or indirectly to the Higgs field. Consider for example the loop correction to the Higgs mass term given by a top fermion (see the right side of Figure 1.1 [2]).

Using a sharp momentum cut-off as regulator, it is possible to show that this diagram gives a contribution proportional to

$$\delta\mu^2 \propto -y_t^2\,\Lambda^2, \tag{1.3}$$

where the negative sign of this equation comes from the fermion loop. The cut-off $\Lambda$ is physical and represents the energy scale at which new physics emerges. At the very least, we expect gravity to be important at the Planck scale: $M_P \approx 10^{19}$ GeV. The last equation implies that the bare $-\mu^2\,(H^\dagger H)$ term in the Higgs potential receives a

[1] The rest of the notation in equation (1.1) is fixed at the appendix in the section of the Standard Model Lagrangian.

[2] Such correction comes from the direct Higgs-Top coupling that can be found from the $-\overline{q}_L\, Y_u\, \tilde{H}\, u_R +$ $h.c.$ term in the Yukawa Lagrangian given by equation (8) of the Appendix.

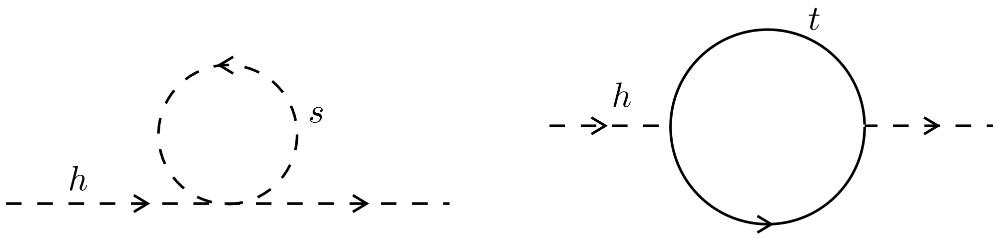


**Figure 1.1:** Loop diagrams obtained from the coupling of the Higgs with an scalar particle ($s$) in the left side, and a Top fermion in the right side.

correction proportional to $-y_t^2\,\Lambda^2\,(H^\dagger\,H)$. Therefore the $-\mu^2$ coefficient must be replaced by its physical value:

$$\mu^2_{phys}\;=\;\mu^2\;+\;\delta\mu^2. \tag{1.4}$$

Going back to the Higgs potential and minimizing it again, it is obtained that the minimum is now given by

$$\frac{2}{\lambda}\,\mu^2_{phy}\;=\;\frac{v^2}{2}. \tag{1.5}$$

Since $v$ is measured to have a value around 246 GeV[36], the last equation implies that

$$\mu_{phys}\approx 123\sqrt{\lambda}\text{ GeV} \tag{1.6}$$

It follows that in order to treat the Higgs coupling $\lambda$ perturbatively, $\mu_{phys}\;\approx\;100$ GeV is needed. On the other hand if we set the cut-off $\Lambda$ to be of the order of the Planck scale, then equation (1.4) implies that to arrive at a value $\mu^2_{phys}\;\approx\;(10^2\,\text{GeV})^2$ would require that we start with an equally huge value of the Lagrangian parameter $\mu^2$, relying on a remarkable cancellation, or fine tuning, to get us from $\mu^2_{phys}\;\approx\;(\,10^{19}\,\text{GeV}\,)^2$ to $\mu^2_{phys}\;\approx\;(\,10^2\,\text{GeV}\,)^2$. This is the so-called fine tuning problem of the SM of particle physics. It is important to call the attention that only the scalar particles are quadratic sensitive to the physical cut-off $\Lambda$, so this is not a problem for fermions and gauge bosons.

The problem of how to get the electroweak scale (which is fixed by the value of the VEV $v$), to be much smaller than a more fundamental scale like the Planck scale is called the Hierarchy Problem.
Now consider the case in which the Higgs boson couples to a complex scalar particle $s$ by a $y_s\,|H|^2\,|s|^2$ term. This kind of coupling will yield to a loop diagram like the one in the left side of Figure 1.1 . Using the sharp momentum cut-off as regulator, it can be shown that this loop diagram gives a mass contribution proportional to

$$\delta\mu^2\;\propto\;+y_s\,\Lambda^2. \tag{1.7}$$

If each of the quarks and leptons of the Standard Model is accompanied by two complex scalars fields with $y_s\;=\;y_f^2$ , then the UV contributions of equations (1.3) and (1.7) can cancel against each other and there is no necessity of tuning the parameters related to the Higgs mass. Of course, it would probably be needed to add more restrictions on the theory to ensure that this success persists to higher loop orders, but the idea of eliminating the tuning on $\mu^2$ by relating bosons and fermions reflects how Supersymmetry could solve the fine tuning problem of the Standard Model. Notice that as a consequence of this cancellation the electroweak scale $M_{EW}$ will not be pushed to the cut-off scale $\Lambda$, this allows a hierarchy $M_{EW} \ll M_P$ and solves the Hierarchy problem.

## 1.2 The Minimal Supersymmetric Standard Model (MSSM) and beyond

Supersymmetry is a proposed invariance under generalized space-time transformations connecting fermions and bosons. The incorporation of this symmetry implies to extend our notion of space-time and it enables to transform a fermion into a boson and vice versa, schematically

$$Q\,|fermion\rangle \;=\; |boson\rangle\,, \qquad\qquad Q\,|boson\rangle \;=\; |fermion\rangle\,.$$

The operator $Q$ that generates such transformations is an anticommuting spinor. As spinors are intrinsically complex objects, then the hermitian conjugate $Q^\dagger$ is also a symmetry generator. If $P^\mu$ is the four-momentum generator of space-time translations and $M^{\mu\nu}$ is the generator of the Lorentz transformations, then it is possible to prove that the simplest supersymmetric extension of the Poincaré algebra (called Super-Poincaré algebra) is given by the anticommuting relations

$$\begin{aligned} \{Q_\alpha\,,\,Q^\dagger_{\dot\alpha}\} \;&=\; -2\sigma^\mu_{\alpha\dot\alpha}\,P_\mu,\\ \{Q_\alpha\,,\,Q^\dagger_{\beta}\} \;&=\; \{Q_{\dot\alpha}\,,\,Q^\dagger_{\dot\beta}\} \;=0, \end{aligned} \tag{1.8}$$

together with the commuting relations

$$\begin{aligned} [Q_\alpha\,,\,P_\mu] \;&=\; [Q^\dagger_{\dot\alpha}\,,\,P_\mu] \;=\; 0,\\ [Q_\alpha\,,\,M^{\mu\nu}] \;&=\; i(\sigma^{\mu\nu})_\alpha^{\;\beta}\;\,Q_\beta,\\ [Q^\dagger_{\dot\alpha}\,,\,M^{\mu\nu}] \;&=\; i(\bar\sigma^{\mu\nu})_{\dot\alpha}^{\;\dot\beta}\;\,Q^\dagger_{\dot\beta}, \end{aligned} \tag{1.9}$$

and the Poincaré algebra

$$\begin{aligned} &[P_\mu\,,\,P_\nu] = 0,\\ &[M_{\mu\nu}\,,\,M_{\rho\sigma}] \;=\; i\,g_{\nu\rho}\,M_{\mu\sigma} \;-\; i\,g_{\mu\rho}\,M_{\nu\sigma} \;-\; i\,g_{\nu\sigma}\,M_{\mu\rho} \;+\; i\,g_{\mu\sigma}\,M_{\nu\rho},\\ &[M_{\mu\nu}\,,\,P_\rho] \;=\; i\,g_{\rho\nu}\,P_\mu - i\,g_{\rho\mu}\,P_\nu, \end{aligned} \tag{1.10}$$

where $Q_\alpha$ and $Q^\dagger_{\dot\alpha}$ are the generators of supersymmetry transformations[1], while $\sigma^{\mu\nu}$ and $\bar\sigma^{\mu\nu}$ are given in terms of the commutator between $\sigma^\mu$ and $\bar\sigma^\mu$ according to equation

[1] The $\alpha$, $\dot\alpha$ indices runs from 1 to 2, see the appendix for a complete explanation on this issue.

(29)[1].

The single-particle states of a supersymmetric theory fall into irreducible representations of the supersymmetry algebra. This is what is called a supermultiplet. Each supermultiplet contains both fermion and boson states, which are commonly known as superpartners of each other. As we will see in Chapter 2, we will distinguish between two kind of supermultiplets; the chiral (or matter) supermultiplet, which will be formed by a combination of two-component Weyl fermion and a complex scalar field, and the vector (or gauge) supermultiplet, which will be formed by a combination of spin-1/2 'gauginos' and the usual spin-1 gauge bosons.

Since the SUSY generators $Q$, $Q^\dagger$ commute with the gauge transformations, particles in the same supermultiplet must also be in the same representation of the gauge group, so they must share the same gauge quantum numbers. Also notice that $P^2$ commutes with all the generators of the Super-Poincaré algebra, which implies that all of the particles within the same supermultiplet have the same eigenvalue of $-P^2$, *i.e.* equal masses. This last statement has important phenomenological consequences since this would imply that several sparticles[2] should have been already discovered, for they share the same mass and gauge coupling constant that the SM particles. Up to now, there have been no experimental signals of supersymmetric partners, therefore if we want to describe a realistic model of particle physics we must have that supersymmetry is broken. The question of how the mechanism that breaks SUSY works is still an open problem, but it can be circumvented writing all possible soft[3] supersymmetry breaking terms.

The supersymmetric model of particle physics with the minimum number of new particles and interactions that is consistent with phenomenology is called the Minimal Supersymmetric Standard Model (MSSM) [3]. An immediate feature of the MSSM is that we know the gauge transformation properties for each of the superpartners, since we know the supermultiplet they belong to. The full particle content and gauge properties of this model are summarized in Table 3.1 and 3.2 of Chapter 3.

The MSSM is an appealing model for many reasons. First of all, as already sketched it proposes an elegant solution to the Hierarchy problem. Moreover, it is compatible with Electroweak Precision Tests (EWPT) and it is connected with other very interesting ideas like the inclusion of gravity and models of particle physics where there is a manifest unification of the forces of nature (GUT theories). However, as any other beyond the SM theory known up to now, the model has some drawback, probably the most important being that at tree level the mass of the lightest[4] Higgs boson cannot exceed the mass of the $Z$ gauge boson, with an upper bound

$$m_{h^0} \leq m_Z \,|\, \cos(2\beta) \,|, \tag{1.11}$$

[1] $\sigma^\mu$ and $\bar{\sigma}^\mu$ are four-vectors composed by the Pauli matrices and the identity matrix (see equation (16) of the appendix).

[2] Sparticles refers to the supersymmetry partners: squarks, sleptons, gauginos and higgsinos.

[3] Here 'soft' means that the supersymmetry breaking terms does not introduce quadratic divergences.

[4] Anomaly cancellation requires two chiral Higgs supermultiplet, so in the MSSM there are two Higgs doublets.

where $\beta$ corresponds to a new parameter that relates the VEVs of the two neutral components of the Higgs doublets. For large values of $\tan(\beta)$ we have that the Higgs boson mass is equal to the mass of the $Z$ boson. Nowadays we know that the $Z$ boson is lighter than the Higgs Boson, so clearly equation (1.11) should be corrected. The obvious guess is that the mass of the light Higgs boson is increased through large radiative corrections from the top-stop particles. However, to obtain a 125 GeV Higgs boson in the MSSM we need masses of the stops around 10 TeV, and this reintroduces fine tuning problems [5, 27, 28]. This motivates the study of models with extra tree level contributions to the masses of the Higgs sector.

In the next chapter we will show that in supersymmetric theories, besides the soft SUSY breaking terms, the scalar potential is composed by F-terms, whose origin is related to the interacting matter content of the theory, and by D-terms, whose origin is related not only with the matter content, but also with the gauge symmetry structure of the theory.
For example, suppose we introduce an additional $U(1)_X$ gauge symmetry group suitably broken at a high energy scale to the MSSM gauge group. If the breaking is driven by two scalars $\phi_1$ and $\phi_2$, with $U(1)_X$ charges $q_1$ and $q_2$ respectively, then we will have a D-term contribution to the scalar potential given by

$$V_{D_X} = -\frac{g_X^2}{2}\left(q_1\,|\phi_1|^2 + q_2\,|\phi_2|^2 + q_u\,|h_u|^2 + q_d\,|h_d|^2\right)^2, \tag{1.12}$$

where $g_X$ is the $U(1)_X$ gauge coupling constant. As this equation shows, one could obtain tree level contributions to the masses of the Higgs sector by adding extra D-terms, as long as the higgses shares the new gauge interaction[1].

Models with extra D-terms have been explored in references [2, 15]. Another way of increasing the Higgs mass at tree level is by adding extra F-terms like in references [8, 11]. There is also the possibility of combining these two ingredients (see for example reference [7]).

In this work I will study the possibility of increasing the tree level Higgs Boson mass via D-terms, focusing in particular on a simple non-abelian gauge extension of the MSSM. The full gauge group that we will consider in this thesis is

$$G = SU(2) \times SU(2) \times SU(3) \times U(1). \tag{1.13}$$

Notice that the full spontaneous symmetry breaking process can occur in two different ways:

$$\begin{aligned} &SU(3)_c \times SU(2)_I \times SU(2)_{II} \times U(1)_Y \to SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_c \times U(1)_{em}, \\ &SU(3)_c \times SU(2)_L \times SU(2)_Z \times U(1)_X \to SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_c \times U(1)_{em}. \end{aligned} \tag{1.14}$$

[1] In equation (1.12) it has been assumed that both higgses are charged under the new interaction with charges $q_u$ and $q_d$.

So far in the literature only the first symmetry breaking channel has been studied.

This thesis is organized as follows: in Chapter 2 I will develop the supersymmetry formalism necessary to construct supersymmetric gauge theories. In Chapter 3 I will present the MSSM and then I will comment on some of the most important features of this model, paying special attention to the Higgs sector and the Higgs mass problem. In Chapter 4 I will introduce the idea of using extra D-terms to increase the tree level Higgs mass, and in particular I will explore the possibility of increasing the Higgs mass in the two symmetry breaking patterns given at equation (1.14). Finally, in Chapter 5 I will stress the main results of this study and I will also comment on the future work that could be done.

# Chapter 2

# Introduction to supersymmetry

The history of supersymmetry dates back to the beginning of the 1970's within the context of strong interactions. As several hadrons had been discovered and successfully organized in multiplets of $SU(3)$, questions about bigger multiplets including particles with different spins emerged. A more systematic study of supersymmetry begins in 1974 when J. Wess and B. Zumino wrote the first supersymmetric field theory in 4 dimensions [13]. By that time there was knowledge about a series of No-go theorems, in particular the Coleman-Mandula theorem states that a consistent 4-dimensional quantum field theory can only have a Lie group symmetry which is always a direct product of the Poincaré group and an internal group. However in 1975 the Haag - Lopuszanski -Sohnius theorem generalized the Coleman-Mandula theorem, stating that the possible symmetries of a consistent 4-dimensional quantum field theory can also include Supersymmetry as a nontrivial extension of the Poincaré group.

The simpler way of building 4-dimensional supersymmetric quantum field theories in a systematic way is by introducing the so-called Superspace formalism. In this formalism, instead of using ordinary fields, one deals with superfields. The superfields are functions of anti-commuting coordinates, as well as of commuting coordinates. Superfields were first introduced by A. Salam and J.A. Strathdee in their paper on supergauge transformations [6]. In this chapter we will see that the virtue of the Superspace formalism lies in the fact that allows the construction of actions that are automatically supersymmetric.

In what follows we will consider only $\mathcal{N} = 1$ supersymmetry , *i.e.* we will work with only one copy of the supersymmetry generators $Q_\alpha$ , $Q^\dagger_{\dot\alpha}$. In this chapter, we will follow the presentation of references [26, 42].

## 2.1 Superspace formalism

The superspace formalism extends the Minkowski space and allows the construction of supersymmetric invariant actions in a formal and straightforward way. The superspace is a supermanifold with 4 anti-commuting coordinates denoted by $\theta^\alpha$ and $\theta^\dagger_{\dot\alpha}$ [1], and 4 (commuting) spacetime coordinates $x^\mu$. Therefore a point in superspace is labeled by the coordinates

$$(x^\mu, \theta^\alpha, \theta^\dagger_{\dot\alpha})\,.$$

[1] Here $\alpha$ and $\dot\alpha$ are spinorial indices that runs from 1 to 2 according to the dotted/undotted notation of the appendix.

The anti-commuting coordinates $\theta^\alpha$ and $\theta^\dagger_{\dot\alpha}$ are constant, complex, two-component Weyl spinors. For these anti-commuting coordinates the rules of Grassmann variables apply. For example, if $f$ is a general function of a single anti-commuting variable $\theta$, then the function must be at most linear in the anti-commuting coordinate

$$f(\theta) \,=\, f_0 \,+\, \theta\, f_1\,, \tag{2.1}$$

with $f_0$ and $f_1$ being, in general, functions of other variables.

The basic object used to represent a supermultiplet is a superfield $S(x,\theta,\theta^\dagger)$, which is a function in superspace of both the commuting and anti-commuting coordinates. Any superfield can be expanded in a power series in the anti-commuting variables with components that are ordinary fields depending only on $x^\mu$. Since there are two independent components of $\theta_\alpha$ and $\theta^\dagger_{\dot\alpha}$ the expansion of the superfield always finishes with terms containing at most two $\theta$'s and two $\theta^\dagger$'s. A general superfield is given by

$$\begin{aligned} S(x,\theta,\theta^\dagger) =& \, a(x) \,+\, \theta\,\xi(x) \,+\, \theta^\dagger\,\chi^\dagger(x) \,+\, \theta\theta\, b(x) \,+\, \theta^\dagger\theta^\dagger\, c(x) \,+\, \theta^\dagger\bar\sigma^\mu\theta\, v_\mu(x) \\ &+\, \theta^\dagger\theta^\dagger\theta\,\eta(x) \,+\, \theta\theta\theta^\dagger\,\zeta^\dagger(x) + \theta\theta\theta^\dagger\theta^\dagger\, d(x)\,. \end{aligned} \tag{2.2}$$

In this equation, both bosonic ($a\,,\,b\,,\,c\,,d$ and $v_\mu$ ) and fermionic ($\xi\,,\,\chi^\dagger\,,\,\eta$ and $\zeta^\dagger$) fields appear. Each of these fields is a complex function of $x^\mu$ and the number of bosonic and fermionic degrees of freedom is the same (16 for each set). Notice that since a complex scalar field has dimension $[mass]^1$ and a fermion field has dimension $[mass]^{3/2}$, then the anti-commuting coordinates $\theta$ and $\theta^\dagger$ have dimension $[mass]^{-1/2}$.

The terms given in the last equation exhaust all non-vanishing combinations of powers of $\theta$ and $\theta^\dagger$, any other possible term can be reduced using equations (25)-(28) to one of the terms present in equation (2.2). The general superfield $S(x,\theta,\theta^\dagger)$ can carry additional Lorentz or spinor indices, and can be regarded either as an anti-commuting or commuting object.

To integrate over the anti-commuting variables in superspace, define the measures

$$d^2\theta \,\equiv\, \frac{1}{4}\, d\theta^\alpha d\theta^\beta \epsilon_{\alpha\beta}\,, \qquad\qquad d^2\theta^\dagger \,=\, -\frac{1}{4}\, d\theta^\dagger_{\dot\alpha}\, d\theta^\dagger_{\dot\beta}\, \epsilon^{\dot\alpha\dot\beta}\,. \tag{2.3}$$

This definition, together with equation (2.2), suggests an immediate way of obtaining a Lagrangian contribution from a generic superfield.
Notice that since the anti-commuting variables follow the usual rules of Grasmann variables, the integration of a superfield just picks out the coefficients accompanied by $\theta\theta$ and/or $\theta^\dagger\theta^\dagger$. For example

$$\int d^2\theta d^2\theta^\dagger\, S(x,\theta,\theta^\dagger) \,=\, d(x)\,. \tag{2.4}$$

Thus Lagrangian contributions can be obtained by integrating superfields over the anti-commuting variables.

Supersymmetry transformations are generated by introducing differential operators $Q\,,\ Q^\dagger$

defined by

$$
\begin{aligned}
Q_\alpha &\equiv i\frac{\partial}{\partial\theta^\alpha} - (\sigma^\mu\theta^\dagger)_\alpha\,\partial_\mu, & Q^\alpha &\equiv -i\frac{\partial}{\partial\theta_\alpha} + (\theta^\dagger\bar{\sigma}^\mu)^\alpha\,\partial_\mu, \\
Q^{\dagger\dot{\alpha}} &\equiv i\frac{\partial}{\partial\theta^\dagger_{\dot{\alpha}}} - (\bar{\sigma}^\mu\theta)^{\dot{\alpha}}\,\partial_\mu, & Q^\dagger{}_{\dot{\alpha}} &\equiv -i\frac{\partial}{\partial\theta^{\dagger\dot{\alpha}}} + (\theta\sigma^\mu)_{\dot{\alpha}}\,\partial_\mu\,.
\end{aligned}
\tag{2.5}
$$

Using these equations one can show that they satisfy the following anti-commuting relations

$$
\begin{aligned}
&\{\,\hat{Q}_\alpha\,,\,\hat{Q}^\dagger_{\dot{\beta}}\,\} = -2\,\sigma^\mu_{\alpha\dot{\beta}}\hat{P}_\mu, \\
&\{\,\hat{Q}_\alpha\,,\,\hat{Q}^\dagger_{\beta}\,\} = 0\,, \qquad\qquad \{\,\hat{Q}^\dagger_{\dot{\alpha}}\,,\,\hat{Q}^\dagger_{\dot{\beta}}\,\} = 0\,.
\end{aligned}
\tag{2.6}
$$

These results agree with equation (1.8) given at the introduction.

A general supersymmetry transformation is defined by

$$
\begin{aligned}
\delta_\epsilon &= \frac{-i}{\sqrt{2}}\left(\epsilon\hat{Q} + \epsilon^\dagger\hat{Q}^\dagger\right) \\
&= \frac{1}{\sqrt{2}}\left(\epsilon^\alpha\frac{\partial}{\partial\theta^\alpha} + \epsilon^\dagger_{\dot{\alpha}}\frac{\partial}{\partial\theta^\dagger_{\dot{\alpha}}} + i(\epsilon\sigma^\mu\theta^\dagger + \epsilon^\dagger\bar{\sigma}^\mu\theta)\partial_\mu\right),
\end{aligned}
\tag{2.7}
$$

where $\epsilon$ and $\epsilon^\dagger$ are constant, anti-commuting, two-component Weyl fermionic objects that parametrizes the supersymmetry transformation.

Applying a supersymmetry transformation to a general superfield shows that a supersymmetry transformation can be interpreted as a translation in superspace with

$$
\begin{aligned}
x^\mu &\to x^\mu + i\,\epsilon\sigma^\mu\theta^\dagger + i\epsilon^\dagger\bar{\sigma}^\mu\theta, \\
\theta^\alpha &\to \theta^\alpha + \epsilon^\alpha, \\
\theta^\dagger_{\dot{\alpha}} &\to \theta^\dagger_{\dot{\alpha}} + \epsilon^\dagger_{\dot{\alpha}}.
\end{aligned}
\tag{2.8}
$$

In the future it will be useful to have the variations of the components of the general superfield $S(x^\mu,\theta,\theta^\dagger)$ under the supersymmetry transformation of equation (2.7).

The variations of the boson fields are

$$
\begin{aligned}
\sqrt{2}\,\delta_\epsilon\,a &= \epsilon\xi + \epsilon^\dagger\chi^\dagger\,, \\
\sqrt{2}\,\delta_\epsilon\,b &= \epsilon^\dagger\zeta^\dagger - \frac{i}{2}\epsilon^\dagger\bar{\sigma}^\mu(\partial_\mu\xi), \\
\sqrt{2}\,\delta_\epsilon\,c &= \epsilon\eta - \frac{i}{2}\epsilon\sigma^\mu(\partial_\mu\chi^\dagger), \\
\sqrt{2}\,\delta_\epsilon\,d &= -\frac{i}{2}\epsilon^\dagger\bar{\sigma}^\mu\partial_\mu\eta - \frac{i}{2}\epsilon\sigma^\mu(\partial_\mu\zeta^\dagger), \\
\sqrt{2}\,\delta_\epsilon\,v^\mu &= \epsilon\sigma^\mu\zeta^\dagger - \epsilon^\dagger\bar{\sigma}^\mu\eta - \frac{i}{2}\epsilon\sigma^\nu\bar{\sigma}^\mu(\partial_\nu\xi) + \frac{i}{2}\epsilon^\dagger\bar{\sigma}^\nu\sigma^\mu(\partial_\nu\chi),
\end{aligned}
\tag{2.9}
$$

while the variations of the fermion fields are

$$
\begin{aligned}
\sqrt{2}\,\delta_\epsilon\,\xi_\alpha &= 2\epsilon_\alpha b - (\sigma^\mu\epsilon^\dagger)_\alpha(v^u + i\partial_\mu a),\\
\sqrt{2}\,\delta_\epsilon\,\chi^{\dagger\dot\alpha} &= 2\epsilon^{\dagger\dot\alpha} c + (\bar\sigma^\mu\epsilon)^{\dot\alpha}(v^u - i\partial_\mu a),\\
\sqrt{2}\,\delta_\epsilon\,\eta_\alpha &= 2\epsilon_\alpha d - i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu c - \frac{i}{2}(\sigma^\nu\bar\sigma^\mu\epsilon)_\alpha\partial_\mu v_\nu,\\
\sqrt{2}\,\delta_\epsilon\,\zeta^{\dagger\dot\alpha} &= 2\epsilon^{\dagger\dot\alpha} d - i(\bar\sigma^\mu\epsilon)^{\dot\alpha}\partial_\mu b + \frac{i}{2}(\bar\sigma^\nu\sigma^\mu\epsilon^\dagger)^{\dot\alpha}\partial_\mu v_\nu.
\end{aligned}
\tag{2.10}
$$

Notice from these equations that as anticipated in the introduction, a supersymmetry transformation always transform a fermion into a boson and vice versa.

A very important observation that can be made at this point comes from considering the integral over all the superspace of a general superfield

$$\mathcal{I} = \int d^4x \int d^2\theta\, d^2\theta^\dagger\; S(x,\theta,\theta^\dagger). \tag{2.11}$$

Using equations (2.4) and (2.7) we can show that this object is automatically supersymmetry invariant

$$\delta_\epsilon\,\mathcal{I} = 0. \tag{2.12}$$

This is because the supersymmetry transformation of equation (2.7), when applied to equation (2.11), will produce a total space-time derivative that gives a null contribution to the action $\mathcal{I}$.
A final useful observation comes from the fact that the anti-commutative derivative is not supersymmetry covariant, since

$$\left[\delta_\epsilon\,,\,\frac{\partial}{\partial\theta^\alpha}\right]\; S(x,\theta,\theta^\dagger) \neq 0\,. \tag{2.13}$$

This implies that if one wishes to construct supersymmetric Lagrangian using anti-commuting derivatives, then proper supersymmetry covariant derivatives are needed. The 'chiral' covariant derivative that commutes with supersymmetry transformations are defined by

$$D_\alpha \equiv \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu\theta^\dagger)_\alpha\,\partial_\mu\,, \qquad\qquad D^\alpha \equiv -\frac{\partial}{\partial\theta_\alpha} - i(\theta^\dagger\bar\sigma^\mu)^\alpha\,\partial_\mu\,. \tag{2.14}$$

It is also possible to define an anti-chiral covariant derivative for a Grassmann-even superfield

$$\overline{D}_{\dot\alpha} S^* \equiv (D_\alpha S)^*\,, \tag{2.15}$$

which implies that

$$\overline{D}^{\dot\alpha} = \frac{\partial}{\partial\theta^\dagger_{\dot\alpha}} - i(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu\,, \qquad\qquad \overline{D}_{\dot\alpha} = -\frac{\partial}{\partial\theta^{\dagger\dot\alpha}} + i(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu\,. \tag{2.16}$$

A general superfield $S(x,\theta,\theta^\dagger)$ is in general a reducible representation of the Super-Poincaré algebra. If one wishes to use a superfield to represent a supermultiplet in which we embed single particle states, then further restrictions must be applied to the general superfield of equation (2.2). There are different kinds of supermultiplets, of which two will be used to construct the MSSM, these are the chiral and vector supermultiplets.

### 2.1.1 Chiral superfields

Chiral supermultiplets, represented by chiral superfields, are used to construct supersymmetric Lagrangian for free propagating fields and (non gauge) interactions between bosons and fermions. A superfield $\Phi(x^\mu, \theta, \theta^\dagger)$ that satisfies the condition

$$\overline{D}_{\dot\alpha}\, \Phi(x^\mu, \theta, \theta^\dagger) \,=\, 0\,, \tag{2.17}$$

is said to be a chiral superfield. The complex conjugate of this superfield is called anti-chiral superfield and satisfies

$$D_\alpha\, \Phi^*(x^\mu, \theta, \theta^\dagger) \,=\, 0\,. \tag{2.18}$$

A way to observe what these conditions mean is to perform a change of coordinates in the superspace to the set

$$(y^\mu\,,\, \theta^\alpha\,,\, \theta^\dagger_{\dot\alpha}), \qquad\qquad y^\mu \,\equiv\, x^\mu \,+\, i\,\theta^\dagger\, \bar\sigma^\mu\, \theta\,. \tag{2.19}$$

Using these variables, the chiral covariant derivatives are represented by

$$D_\alpha \,=\, \frac{\partial}{\partial\theta^\alpha} \,-\, 2i(\sigma^\mu\theta^\dagger)_\alpha\, \frac{\partial}{\partial y^\mu}\,, \qquad\qquad \overline{D}_{\dot\alpha} \,=\, -\frac{\partial}{\partial\theta^{\dagger\dot\alpha}}. \tag{2.20}$$

From the last equation it is clear that the constraint given by equation (2.17) is solved by any function of $y^\mu$ and $\theta$ but no $\theta^\dagger$. Therefore the chiral superfield has a component expansion given by

$$\Phi \,=\, \phi(y) \,+\, \sqrt{2}\theta\, \psi(y) \,+\, \theta\theta\, F(y). \tag{2.21}$$

Similarly, the constraint for the anti-chiral superfield is solved by any function of $y^\mu$ and $\theta^\dagger$ but no $\theta$. The component expansion of the anti-chiral superfield is

$$\Phi^* \,=\, \phi^*(y^*) \,+\, \sqrt{2}\theta^\dagger\, \psi^\dagger(y^*) \,+\, \theta^\dagger\, \theta^\dagger\, F^*(y^*). \tag{2.22}$$

The degrees of freedom of a chiral supermultiplet are a complex scalar field $\phi$ of dimension $[mass]^1$, a complex 2-component Weyl fermion $\psi$ of dimension $[mass]^{3/2}$ and a complex scalar field $F$ of dimension $[mass]^2$. By replacing back the $y^\mu$ coordinate in these equations and expanding in the anti-commuting coordinate, we find the field component expansion in the original coordinates

$$\begin{aligned}\Phi \,=&\,\phi(x) \,+\, i\theta^\dagger\bar\sigma^\mu\theta\, \partial_\mu\phi(x) \,+\, \frac{1}{4}\theta\theta\, \theta^\dagger\theta^\dagger\, \partial_\mu\partial^\mu\phi(x) \\ &+\, \sqrt{2}\theta\psi(x) \,-\, \frac{i}{\sqrt{2}}\,\theta\theta\theta^\dagger\bar\sigma^\mu\partial_\mu\psi(x) \,+\, \theta\theta\, F(x)\,,\end{aligned} \tag{2.23}$$

$$\begin{aligned}\Phi^* \,=&\,\phi^*(x) \,-\, i\theta^\dagger\bar\sigma^\mu\theta\, \partial_\mu\phi^*(x) \,+\, \frac{1}{4}\theta\theta\, \theta^\dagger\theta^\dagger\, \partial_\mu\partial^\mu\phi^*(x) \\ &+\, \sqrt{2}\theta^\dagger\psi(x)^\dagger \,-\, \frac{i}{\sqrt{2}}\,\theta^\dagger\theta^\dagger\theta\sigma^\mu\partial_\mu\psi^\dagger(x) \,+\, \theta^\dagger\theta^\dagger\, F^*(x)\,.\end{aligned} \tag{2.24}$$

By comparing with the general superfield of equation (2.2) we see that the chiral superfield $\Phi$ can be obtained from the general superfield $S$ by identifying

$$\begin{aligned}&a \,=\, \phi\,, \qquad b \,=\, F\,, \qquad c \,=\, 0\,, \qquad d \,=\, \frac{1}{4}\partial_\mu\partial^\mu\phi\,, \qquad v_\mu \,=\, i\partial_\mu\phi\,, \\ &\xi \,=\, \sqrt{2}\psi_\alpha\,, \qquad \chi^{\dagger\dot\alpha} \,=\, 0\,, \qquad \eta_\alpha \,=\, 0\,, \qquad \zeta^{\dagger\dot\alpha} \,=\, -\frac{i}{\sqrt{2}}(\bar\sigma^\mu\partial_\mu\psi)^{\dot\alpha}.\end{aligned} \tag{2.25}$$

| DoF | $\phi$ | $\psi$ | $F$ |
|---|---|---|---|
| off-shell | 2 | 4 | 2 |
| on-shell | 2 | 2 | 0 |

**Table 2.1:** Degrees of freedom (Dof) of the fields involved in the free Wess-Zumino model.

This is useful since we already know the variation of the components of a general superfield. For a chiral superfield we thus have

$$\begin{aligned} \delta_\epsilon \phi &= \epsilon \psi, \\ \delta_\epsilon \psi_\alpha &= -i(\sigma^\mu \epsilon^\dagger)_\alpha \partial_\mu \phi + \epsilon_\alpha F, \\ \delta_\epsilon F &= -i\epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \psi. \end{aligned} \tag{2.26}$$

As shown in equation (2.11), a Lagrangian is obtained by integrating over the fermionic coordinates. There are two type of contributions; the D-term contributions, which are obtained by integrating over all the fermionic coordinates $\theta$ and $\theta^\dagger$, and the F-term contributions that are obtained by integrating only over $\theta$ or $\theta^\dagger$.

A chiral superfield can contribute to the Lagrangian with an F-term given by

$$[\Phi]_F + c.c \equiv \int d^2\theta \, \Phi\big|_{\theta^\dagger=0} + c.c. = \int d^2\theta \, d^2\theta^\dagger \, \delta^{(2)}(\theta^\dagger)\Phi + c.c. \tag{2.27}$$

In the last equation we have added the complex conjugate to ensure the action to be real.

Notice from equation (2.23) that taking the D-term contribution of a chiral superfield is also possible, but it is pointless since it will yield to a total derivative contribution to the action. Nevertheless, an important result can be found by taking the D-term contribution of the product of a single chiral superfield and its complex conjugate

$$\begin{aligned} [\Phi^* \Phi]_D &= \int d^2\theta \, d^2\theta^\dagger \, \Phi^*\Phi \\ &= \partial^\mu \phi^* \partial_\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + F^*F + \text{total derivative}, \end{aligned} \tag{2.28}$$

this will lead to the simplest 4-dimensional supersymmetric field theory, known as the free Wess-Zumino model:

$$S_{wz}^{free} = \int d^4x \, \left( \partial^\mu \phi^* \partial_\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + F^*F \right). \tag{2.29}$$

The action given by the last equation is composed by kinetic terms only, except for the term containing the $F$ fields which have trivial equations of motion: $F = F^* = 0$. These fields are not physical fields (they are called auxiliary fields), and their purpose is to match the off-shell degrees of freedom between bosonic and fermionic fields as shown in Table 2.1

By construction, the action of the Wess-Zumino model is invariant under the supersymmetry transformations. One can also check that the supersymmetry algebra closes. We find that the commutator between two distinct supersymmetry transformations parametrized by different spinors $\epsilon_1$ and $\epsilon_2$, applied to any of the fields of equation (2.29), will be proportional to a space-time derivative. This derivative is associated with the generator of space-time translations *i.e.* another element of the Super-Poincaré algebra.

In order to incorporate interactions between fermions and bosons in a SUSY invariant way, an holomorphic function $W(\Phi)$ of only chiral superfields $\Phi_i$ is introduced

$$W(\Phi) \equiv \frac{1}{2}M^{ij}\,\Phi_i\Phi_j + \frac{1}{6}y^{ijk}\,\Phi_i\Phi_j\Phi_k, \tag{2.30}$$

where the labels are used to distinguish different chiral superfields $\Phi_i$, while $M^{ij}$ and $y^{ijk}$ are objects that must be totally symmetric under the interchange of the indices. The function given in equation (2.30) is called superpotential, and its particular form comes from the fact that the possible interacting terms that one can write down are highly constrained by supersymmetry invariance and renormalizability[1]. Since products of chiral superfields are chiral, the superpotential is itself a chiral superfield, and a SUSY Lagrangian can be obtained taking

$$[W(\Phi)]_F + c.c. = -\frac{1}{2}W^{ij}\,\psi_i\psi_j + W^i\,F_i + c.c. \tag{2.31}$$

with $W^i$ and $W^{ij}$ given by

$$W^i \equiv \left.\frac{\delta\, W}{\delta\Phi_i}\right|_{\Phi\to\phi}, \qquad W^{ij} \equiv \left.\frac{\delta^2\, W}{\delta\Phi_i\delta\Phi_j}\right|_{\Phi\to\phi}. \tag{2.32}$$

Adding equations (2.28) and (2.31), and replacing the auxiliary fields by their new equations of motion: $F_i = -W_i^*$, $F_i^* = -W_i$, one obtains the action for the interacting Wess-Zumino model

$$\begin{aligned} S_{wz} = \int d^4x\ & \partial^\mu\phi^{j*}\,\partial_\mu\phi_j + i\,\psi^{j\dagger}\bar{\sigma}^\mu\partial_\mu\psi_j - V(\phi\,,\phi^*) \\ & -\frac{1}{2}\left(M^{ij}\,\psi_i\psi_j + M^*_{ij}\,\psi^{i\dagger}\psi^{j\dagger}\right) \\ & +\frac{1}{2}\left(y^{ikj}\,\phi_i\psi_j\psi_k + y^*_{ijk}\,\phi^{i*}\psi^{j\dagger}\psi^{k\dagger}\right). \end{aligned} \tag{2.33}$$

This action contains kinetic terms for both the complex scalar and the 2-component Weyl fermion, plus a mass term for the fermions and Yukawa type interactions. The scalar potential is given by

$$\begin{aligned} V(\phi\,,\phi^*) &\equiv F^j\,F_j^* \\ &= W^j\,W_j^* \\ &= M^*_{ik}\,M^{kj}\,\phi^{i*}\phi_j + \frac{1}{2}\left(M^{in}\,y^*_{kjn}\,\phi_i\phi^{j*}\phi^{k*} + M^*_{in}\,y^{jkn}\,\phi^{i*}\phi_j\phi_k\right) \\ &\quad + \frac{1}{4}\,y^{ijn}\,y^*_{kln}\,\phi_i\phi_j\phi^{k*}\phi^{l*}\,. \end{aligned} \tag{2.34}$$

[1]There is an extra term $E^i\Phi^i$ allowed, which is gauge invariant only if $\Phi_i$ is a gauge singlet. Since there is no gauge singlet in the MSSM, this term has been omitted.

Notice that the scalar potential is completely determined by the auxiliary fields, and it is positive definite because is formed by the sum of squares of absolute values. Both properties are general features of scalar potentials in supersymmetric theories.

### 2.1.2 Vector superfields

A vector superfield is used to represent a gauge supermultiplet, thus we expect that it contains the usual spin-1 gauge boson, and a fermion field called gaugino. A vector superfield $V(x,\theta,\theta^\dagger)$ is defined as a superfield satisfying the reality condition

$$V(x,\theta,\theta^\dagger) = V^*(x,\theta,\theta^\dagger). \tag{2.35}$$

From equation (2.2) we see that it is possible to obtain a vector superfield from a general superfield via the identification

$$a = a^*, \qquad c = b^*, \qquad d = d^*, \qquad v_\mu = v_\mu^*, \qquad \xi^\dagger = \chi^\dagger, \qquad \zeta^\dagger = \eta^\dagger\,. \tag{2.36}$$

Defining

$$\eta_\alpha \equiv \lambda_\alpha - \frac{i}{2}\left(\sigma^\mu \partial_\mu \xi^\dagger\right)_\alpha, \qquad v_\mu \equiv A_\mu, \qquad d \equiv \frac{1}{2}D + \frac{1}{4}\partial_\mu\partial^\mu a\,, \tag{2.37}$$

it is possible to find, using equations (2.2) and (2.36), the component field expansion of the vector superfield

$$\begin{aligned} V(x,\theta,\theta^\dagger) &= a + \theta\xi + \theta^\dagger\xi^\dagger + \theta\theta\, b + \theta^\dagger\theta^\dagger\, b^* + \theta^\dagger\bar{\sigma}^\mu\theta\, A_\mu + \theta^\dagger\theta^\dagger\theta\left(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\xi^\dagger\right) \\ &\quad + \theta\theta\theta^\dagger\left(\lambda^\dagger - \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\xi\right) + \theta\theta\theta^\dagger\theta^\dagger\left(\frac{1}{2}D + \frac{1}{4}\partial_\mu\partial^\mu a\right). \end{aligned} \tag{2.38}$$

Clearly there are too many fields in equation (2.38) if we wish to use a vector superfield to represent a gauge supermultiplet. The extra fields can be eliminated by applying a 'supergauge transformation'. Consider for example the vector superfield $V$ for a $U(1)$ gauge symmetry and consider a supergauge transformation given by

$$V \to V + i\,(\Omega^* - \Omega), \tag{2.39}$$

where $\Omega$ and $\Omega^*$ are chiral and anti-chiral superfields with component fields given by equation (2.23). The previous transformation implies that the transformations for the component fields given in equation (2.38), are

$$\begin{aligned} &a \to a + i(\phi^* - \phi), \\ &\xi_\alpha \to \xi_\alpha - i\sqrt{2}\psi_\alpha, \\ &b \to b - iF, \\ &A_\mu \to A_\mu + \partial_\mu(\phi^* + \phi), \\ &\lambda_\alpha \to \lambda_\alpha, \\ &D \to D\,. \end{aligned} \tag{2.40}$$

| DoF | $A_\mu$ | $\lambda$ | $D$ |
|---|---|---|---|
| off-shell | 2 | 4 | 2 |
| on-shell | 2 | 2 | 0 |

**Table 2.2:** Degrees of freedom of the fields involved in equation (2.41).

It follows from this equation that an appropriated choice of $\text{Im}(\phi)$, $\psi$ and $F$ allows to get rid of the extra fields $a$, $\xi$ and $b$. This choice is known as the Wess-Zumino gauge. The vector superfield in this gauge is

$$V_{wz_{gauge}} = \theta^\dagger\bar{\sigma}^\mu\theta\, A_\mu + \theta^\dagger\theta^\dagger\theta\,\lambda + \theta\theta\theta^\dagger\lambda^\dagger + \frac{1}{2}\theta\theta\theta^\dagger\theta^\dagger\, D\,. \tag{2.41}$$

The fields involved in this equation are a vector boson $A_\mu$ of dimension $[mass]^1$, a 2-component Weyl fermion $\lambda$ of dimension $[mass]^{3/2}$, and a real scalar field $D$ with dimension $[mass]^2$. In analogy with the case studied in the previous subsection, the extra scalar field $D$ is an auxiliary field used to match the off-shell degrees of freedom, as shown in Table 2.2.

Notice that equation (2.39) implies the usual gauge transformation for the vector gauge boson with parameter $2\text{Re}(\phi)$, therefore the supergauge transformation contains the usual gauge transformation as a special case, and working in the Wess-Zumino gauge is equivalent to partially fixing the supergauge while maintaining the freedom to perform the usual gauge transformations.

Taking the D-term component of a vector superfield leads to a supersymmetry invariant Lagrangian, a fact that has already been shown in equation (2.28)[1]. On the other hand, notice from equation (2.40) that $D(x)$ is supergauge invariant by itself, taking the D-term component of $V(x,\theta,\theta^\dagger)$ leads to a supergauge invariant contribution to the Lagrangian. The term obtained, called a Fayet-Iliopoulos term, may play a role in supersymmetry breaking and is allowed only in the abelian case. The Fayet-Iliopoulos term is

$$\begin{aligned}\mathcal{L} &= -2\kappa\,[V]_D,\\ &= -\kappa\, D,\end{aligned} \tag{2.42}$$

where $\kappa$ is a positive constant of dimension $[mass]^2$. The role of this term in SUSY breaking will be discussed later.

Supergauge invariant kinetic terms for the gauge fields can be constructed using a vector superfield. Consider a vector superfield whose supergauge transformation is given by equation (2.39), and define the abelian strength superfields by

$$\mathcal{F}_\alpha \equiv -\frac{1}{4}\overline{DD}\, D_\alpha V\,, \qquad \mathcal{F}^\dagger_{\dot\alpha} \equiv -\frac{1}{4}DD\,\overline{D}_{\dot\alpha} V\,, \tag{2.43}$$

where $DD \equiv D^\alpha D_\alpha$ and $\overline{DD} \equiv \bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha}$. Notice that $\mathcal{F}_\alpha$ is a chiral superfield by construction and similarly $\mathcal{F}^\dagger_{\dot\alpha}$ is an anti-chiral superfield. One can verify that the strength superfield is supergauge invariant by applying the supergauge transformation and using that $\Omega$ ($\Omega^*$) is a chiral (anti-chiral) superfield

$$\mathcal{F}_\alpha \to -\frac{1}{4}\overline{DD}\, D_\alpha\,(V + i(\Omega^* - \Omega)) = \mathcal{F}_\alpha. \tag{2.44}$$

[1]Notice that $\Phi^*\Phi$ is a vector superfield since it satisfy the constraint given in equation (2.35).

The same result can be found for the anti-chiral superfield $\mathcal{F}^{\dagger}_{\dot{\alpha}}$. The component field of the strength superfields can be obtained by a direct computation of equation (2.43). The algebraic work can be simplified by taking the vector superfield in the Wess-Zumino gauge, since the result will be valid in any gauge given that the strength superfields are supergauge invariant. The result is

$$\begin{aligned}
\mathcal{F}_{\alpha} &= \lambda_{\alpha} + \theta_{\alpha} D + \frac{i}{2}\,(\sigma^{\mu}\bar{\sigma}^{\nu}\theta)_{\alpha}\, F_{\mu\nu} + i\theta\theta\left(\sigma^{\mu}\partial_{\mu}\lambda^{\dagger}\right)_{\alpha}, \\
\mathcal{F}^{\dot{\alpha}\dagger} &= \lambda^{\dot{\alpha}\dagger} + \theta^{\dot{\alpha}\dagger} D - \frac{i}{2}\left(\bar{\sigma}^{\mu}\sigma^{\nu}\theta^{\dagger}\right)^{\dot{\alpha}} F_{\mu\nu} + i\theta^{\dagger}\theta^{\dagger}\,(\bar{\sigma}^{\mu}\partial_{\mu}\lambda)^{\dot{\alpha}},
\end{aligned} \tag{2.45}$$

where $F_{\mu\nu}$ is the usual abelian strength tensor

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}. \tag{2.46}$$

The kinetic terms are obtained from the F-term contributions of the strength superfields

$$\begin{aligned}
S_{gauge} &= \int d^4x \left( \frac{1}{4}[\mathcal{F}^{\alpha}\mathcal{F}_{\alpha}]_F + c.c. \right) \\
&= \int d^4x \left( -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\lambda^{\dagger}\bar{\sigma}^{\mu}\partial_{\mu}\lambda + \frac{1}{2}D^2 \right).
\end{aligned} \tag{2.47}$$

This action contains the usual abelian kinetic term for the gauge bosons together with a kinetic term for the gauginos. The term containing the $D$ field gives the trivial equation of motion $D = 0$, and it will be integrated out just as the $F$ auxiliary field. To obtain a full supersymmetric $U(1)$ gauge theory, consider a set of chiral superfields $\Phi_i$ carrying $U(1)$ charges $q_i$. These chiral superfields transforms under supergauge transformation as

$$\Phi_i \to e^{2i\,g\,q_i\Omega}\,\Phi_i\,, \qquad \Phi^{*i} \to e^{-2i\,g\,q_i\Omega^*}\,\Phi^{*i}, \tag{2.48}$$

where $g$ is the gauge coupling constant of the group, and $\Omega$, $\Omega^*$ are chiral and anti-chiral superfields that parametrizes the supergauge transformation. Since a vector superfield transforms according to equation (2.39), then the supergauge invariant term that couples matter to gauge fields is

$$\Phi^{*i}\,e^{2\,g\,q_i V}\,\Phi_i. \tag{2.49}$$

The supergauge invariant Lagrangian is obtained from the D-term contribution of this term. The exponential of the vector superfield can be expanded. Working in the Wess-Zumino gauge we find that the series terminates with the $V^2$ term, since

$$V^2_{wz} = -\frac{1}{2}\theta\theta\theta^{\dagger}\theta^{\dagger}\,A_{\mu}A^{\mu}, \qquad V^n_{wz} = 0\,, \quad \text{for n} \geq 3. \tag{2.50}$$

Adopting the Wess-Zumino gauge to simplify the computation, we find that the action is given by

$$\begin{aligned}
S &= \int d^4x\, [\Phi^{*i}\,e^{2\,g\,q_i V}\,\Phi_i]_D \\
&= \int d^4x\;\; D_{\mu}\phi^{*i}\,D^{\mu}\phi_i + i\,\psi^{\dagger i}\,\bar{\sigma}^{\mu}\,D_{\mu}\psi_i + F^{*i}F_i \\
&\quad + g\,q_i\,\phi^{*i}\phi_i D - \sqrt{2}g\,q_i\left(\phi^{*i}\psi_i\lambda + \lambda^{\dagger}\psi^{\dagger i}\phi_i\right),
\end{aligned} \tag{2.51}$$

where the covariant derivates for the fields are given by

$$D_\mu \phi_i \,=\, \partial_\mu \phi_i \,-\, ig\, q_i\, A_\mu \phi_i, \qquad\qquad D_\mu \psi_i \,=\, \partial_\mu \psi_i \,-\, ig\, q_i\, A_\mu \psi_i. \tag{2.52}$$

The full supergauge invariant abelian theory is then constructed by adding the contributions of the actions given in equations (2.47) and (2.51), and replacing the auxiliary field $D$ by its equation of motion:

$$D \,=\, -g\, q_i\, |\phi_i|^2\,. \tag{2.53}$$

We are now in position to generalize this construction for non-abelian supersymmetric gauge theories.

## 2.2 Supersymmetric gauge theories

Consider a simple non-abelian gauge group with a universal gauge coupling constant $g$ [1], and Lie algebra generators $T^a$, where $a$ is an index that runs over the adjoint representation of the group. As already pointed out, the generators and the field component of a vector superfield are in the same representation of the gauge group. There will be one vector superfield $V^a$ for every Lie algebra generator $T^a$.

In the non abelian case, the basic object to construct gauge kinetic terms is the exponential of $V \,\equiv\, 2g\, V^a\, T^a$ . The non abelian generalization of the supergauge transformation is given by

$$e^V \,\rightarrow\, e^{i\Omega^\dagger}\, e^V\, e^{-i\Omega}, \tag{2.54}$$

where $\Omega \,\equiv\, 2g\, T^a\, \Omega^a$ with $\Omega^a$ a chiral superfield that is used as parameter of the transformation.

By expanding this equation and keeping terms linear in $\Omega$ and $\Omega^*$, we obtain

$$V^a \,\rightarrow\, V^a \,+\, i(\Omega^{*a} \,-\, \Omega^a) \,+\, g\, f^{abc}\, V^b(\Omega^{*c} \,+\, \Omega^c) \,+\, \dots\ , \tag{2.55}$$

where the dots represent terms of higher order in $V$. The parameters of the transformation $\Omega^a$ and $\Omega^{*a}$ can be chosen appropriately to obtain the Wess-Zumino gauge, in which the vector superfield $V^a$ is given by the usual expression of equation (2.41) with proper gauge indices.

In analogy with equation (2.43), the non-abelian field strength tensor superfield and its conjugate, are defined by

$$\mathcal{F}_\alpha \,\equiv\, -\frac{1}{4}\overline{DD}\left(e^{-V}\, D_\alpha\, e^V\right), \qquad\qquad \mathcal{F}^\dagger_{\dot\alpha} \,\equiv\, \frac{1}{4}DD\left(e^V\, \overline{D}_{\dot\alpha}\, e^{-V}\right). \tag{2.56}$$

[1] The generalization for groups given by the direct product of other groups, like the MSSM case, is straightforward.

Using equation (2.54) and the chiral properties of $\Omega$ and $\Omega^\dagger$, one can show that they transform as

$$\mathcal{F}_\alpha \to e^{i\Omega}\,\mathcal{F}_\alpha\, e^{-i\Omega}, \qquad\qquad \mathcal{F}^\dagger_{\dot\alpha} \to e^{i\Omega^\dagger}\,\mathcal{F}^\dagger_{\dot\alpha}\, e^{-i\Omega^\dagger}. \tag{2.57}$$

It follows that a supergauge invariant object is constructed by taking the trace $tr(\mathcal{F}^\alpha\mathcal{F}_\alpha)$. To obtain the component field expansion of the non-abelian field strength tensor, the exponential of $V$ can be expanded adopting the Wess-Zumino gauge to simplify the calculation. Defining $\mathcal{F}_\alpha \equiv 2g\,T^a\mathcal{F}^a_\alpha$, we find

$$\mathcal{F}^a_\alpha = \lambda^a_\alpha + \theta_\alpha D^a + \frac{i}{2}\,(\sigma^\mu\bar\sigma^\nu\theta)_\alpha\, F^a_{\mu\nu} + i\theta\theta\left(\sigma^\mu D_\mu\lambda^{\dagger a}\right)_\alpha, \tag{2.58}$$

where $F^a_{\mu\nu}$ is the usual Yang-Mills strength tensor

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,f^{abc}\,A^b_\mu\,A^c_\nu, \tag{2.59}$$

and the covariant derivative for the gaugino is given by

$$D_\mu\lambda^a = \partial_\mu\lambda^a + g\,f^{abc}\,A^b_\mu\,\lambda^c. \tag{2.60}$$

In analogy with the abelian case, we see that from equation (2.58) it is possible to obtain an F-term contribution to the Lagrangian. This will lead to the non abelian generalization of the kinetic terms for the gauge fields

$$\begin{aligned} S_{gauge} &= \int d^4x\ \frac{1}{4}[\mathcal{F}^{\alpha a}\mathcal{F}^a_\alpha]_F + c.c.\\ &= \int d^4x\ -\frac{1}{4}F^{\mu\nu a}F^a_{\mu\nu} + i\lambda^{\dagger a}\bar\sigma^\mu D_\mu\lambda^a + \frac{1}{2}D^aD^a. \end{aligned} \tag{2.61}$$

The coupling of the gauge fields to matter is similar to the abelian case. The generalization of the supergauge transformation for the set of chiral fields $\Phi_i$ is given by

$$\Phi_i \to \left(e^{2ig\,\Omega^aT^a}\right)_i^{\ j}\,\Phi_j, \qquad\qquad \Phi^{*i} \to \Phi^{*j}\left(e^{-2ig\,\Omega^{*a}T^a}\right)_j^{\ i}. \tag{2.62}$$

In view of this equation, it is clear from equation (2.54) that the supergauge invariant term is

$$\Phi^{*i}\,\left(e^V\right)_i^{\ j}\,\Phi_j. \tag{2.63}$$

A D-term contribution can be obtained from here. As usual, the exponential is expanded in the Wess-Zumino gauge, and the result will be valid in any gauge because the term is supergauge invariant. The non-abelian generalization of equation (2.51) is

$$\begin{aligned} S &= \int d^4x\ \left[\Phi^{*i}\,\left(e^V\right)_i^{\ j}\,\Phi_j\right]_D\\ &= \int d^4x\ -D_\mu\phi^{*i}\,D^\mu\phi_i + i\,\psi^{\dagger i}\,\bar\sigma^\mu\,D_\mu\psi_i + F^{*i}F_i + g\,(\phi^{*i}\,T^a\,\phi_i)D^a\\ &\quad - \sqrt{2}g\left((\phi^{*i}\,T^a\psi_i)\lambda^a + \lambda^{\dagger a}(\psi^{\dagger i}\,T^a\,\phi_i)\right), \end{aligned} \tag{2.64}$$

with covariant derivatives given by

$$D_\mu \phi_i \;=\; \partial_\mu \phi_i \;-\; ig\, A^a_\mu T^a \phi_i, \qquad\qquad D_\mu \psi_i \;=\; \partial_\mu \psi_i \;-\; ig\, A^a_\mu T^a \psi_i. \tag{2.65}$$

As in the abelian case, the full supergauge Lagrangian is obtained by adding the contributions of equations (2.61) and (2.64), followed by the replacement of the auxiliary field $D^a$ by its equation of motion

$$D^a \;=\; -g\,(\phi^* T^a \phi). \tag{2.66}$$

This replacement will lead to additional contribution to the scalar potential. The scalar potential of a supersymmetric theory is completely determined by the contribution of the auxiliary fields

$$\begin{aligned} V(\phi\,,\phi^*) \;&=\; F^{*i} F_i \;+\; \frac{1}{2}\, D^a D^a, \\ &=\; W^*_i W^i \;+\; \frac{1}{2}\, g^2\,(\phi^* T^a \phi)^2. \end{aligned} \tag{2.67}$$

As can be seen from this equation, the two type of contributions to the scalar potential are F-terms and D-terms. This result will be important for the future, since we wish to study how to increase the tree level Higgs mass that is found in the MSSM by adding extra D-term contributions to the scalar potential.

# Chapter 3

# The Minimal Supersymmetric Standard Model

This chapter begins with a few remarks on some aspects of supersymmetry breaking. After showing two different mechanism for the breaking, the possible SUSY breaking terms that one can write are presented. Next, the results encountered in the study of supergauge Lagrangians will be applied to the Standard Model gauge group and particle content, leading to the Minimal Supersymmetric Standard Model. Then electroweak symmetry breaking is analyzed and the mass spectrum for the Higgs states is computed. After that, we comment on the tuning situation of the MSSM. Finally, we will comment on some of the most important problems of this model, yielding to the conclusion that the MSSM should be extended.

## 3.1 Supersymmetry breaking

As already pointed out, if SUSY were an exact symmetry of nature it would imply that several superpartners should have been discovered by now. Any realistic supersymmetric theory must therefore exhibit broken supersymmetry. It makes sense to think that if that is the case, then superymmetry should be spontaneously broken, *i.e* the Lagrangian that describes the theory is SUSY invariant but there is a vacuum state $|0\rangle$ that is not

$$Q_\alpha\,|0\rangle \neq 0, \qquad Q^\dagger_{\dot\alpha}\,|0\rangle \neq 0.$$

From the supersymmetry algebra relation of equation (1.8), it is possible to show that the Hamiltonian is given by

$$H = P_0 = \frac{1}{4}\{Q_1\,,\,Q_1^\dagger\} + \frac{1}{4}\{Q_2\,,\,Q_2^\dagger\}. \tag{3.1}$$

As $Q^\dagger$ is the hermitian conjugate of $Q$, then the right side of equation (3.1) is positive semi-definite, and any eigenstate of the Hamiltonian has energy value $E \geq 0$. This implies that if supersymmetry is spontaneously broken then the vacuum state must have positive energy.

Assuming that just as in the Standard Model, the spontaneous breaking takes place when some of the scalar fields develop non trivial VEVs, and considering that the scalar potential in supersymmetry is described by equation (2.67), one can conclude that supersymmetry will be spontaneously broken if the vacuum expectation value of some of the auxiliary fields $F_i$, $D^a$ is different from zero. This can be guaranteed if the equations $F^i = 0$ and $D^a = 0$ cannot all be simultaneously satisfied for any possible value of the fields.

### 3.1.1 F-term supersymmetry breaking

Models in which supersymmetry is spontaneously broken due to a non-zero vacuum expectation value of an F-term are called O'Raifeartaigh models [16]. In these models one has a set of chiral supermultiplets $\Phi_i$, and a superpotential $W(\Phi)$ such that the equations of motion of the auxiliary fields $F_i = -W_i^* = 0$ cannot all be simultaneously satisfied. The simplest example is given by the O'Raifeartaigh superpotential

$$W = -k\Phi_1 + m\,\Phi_2\Phi_3 + \frac{y}{2}\,\Phi_1\Phi_2^3, \tag{3.2}$$

where the constants $k$, $m$ and $y$ can all be chosen to be real and positive through field phase rotations. The F-term contribution to the scalar potential will be given by

$$\begin{aligned} V_F &= |F_1|^2 + |F_2|^2 + |F_3|^2 \\ &= |k - \frac{y}{2}\phi_3^2|^2 + m^2\,|\phi_3|^2 + |m\phi_2 + y\,\phi_1\phi_3|^2 . \end{aligned} \tag{3.3}$$

From the last equation we see that $F_1 = F_2 = 0$ is not possible, then the potential will have to be positive at its minimum and supersymmetry will be spontaneously broken.
The first term in equation (3.2) is necessary to achieve tree level F-term breaking in renormalizable superpotentials, but this term is allowed only when the corresponding chiral supermultiplet is a gauge singlet which does not exist in the MSSM. This may be viewed as a call to extend the MSSM with an extra gauge singlet.

### 3.1.2 D-term supersymmetry breaking

Spontaneous supersymmetry breaking can also occur if some D-term gets a non-zero VEV, via the Fayet-Iliopoulos mechanism [17]. As mentioned before in subsection 2.1.2, if the gauge symmetry includes a $U(1)$ gauge group then it is possible to include an extra term given by

$$\mathcal{L} = -\kappa\,D. \tag{3.4}$$

This term modifies equation (2.53), in such a way that the new equation of motion for the auxiliary field is given by

$$D = \kappa - g\sum_i q_i\,|\phi_i|^2 . \tag{3.5}$$

Therefore, the D-term contribution to the scalar potential will be

$$V_D = \frac{1}{2}\left(\kappa - g\sum_i q_i\,|\phi_i|^2\right)^2 . \tag{3.6}$$

To see what this implies, consider the simple case in which only one of the scalars $\phi$ is charged under the gauge group. Then one can see from the last equation that the sign of $qg$ decides whether supersymmetry can be spontaneously broken or not. When the product $qg$ is positive there will be a SUSY preserving solution with $\langle 0|\,D\,|0\rangle = 0$ and $\langle 0|\,\phi\,|0\rangle \neq 0$. On the other hand, when the product $qg$ is negative, supersymmetry is spontaneously broken since the auxiliary field can get a non trivial VEV with $\langle 0|\,\phi\,|0\rangle = 0$ and $\langle 0|\,D\,|0\rangle = \kappa$. This situation could

| Matter Content | | | | |
|---|---|---|---|---|
| Name | | Spin 0 | Spin 1/2 | $SU(3)_C$ , $SU(2)_L$ , $U(1)_Y$ |
| Quarks, Squarks ($\times$3 families) | $Q$ | $\tilde{q}_L = (\tilde{u}_L\ \tilde{d}_L)$ | $q_L = (u_L\ d_L)$ | ($\mathbf{3}$, $\mathbf{2}$, 1/6) |
| | $\overline{u}$ | $\tilde{u}^*_R$ | $u^\dagger_R$ | ($\mathbf{3}$, $\mathbf{1}$, -2/3) |
| | $\overline{d}$ | $\tilde{d}^*_R$ | $d^\dagger_R$ | ($\mathbf{3}$, $\mathbf{1}$, 1/3) |
| Leptons, Sleptons ($\times$3 families) | L | $\tilde{l}_L = (\tilde{\nu}\ \tilde{e}_L)$ | $l_L = (\nu\ e_L)$ | ($\mathbf{1}$, $\mathbf{2}$, -1/2) |
| | $\overline{e}$ | $\tilde{e}^*_R$ | $e^\dagger_R$ | ($\mathbf{1}$, $\mathbf{1}$, 1) |
| Higgs, higgsinos | $H_u$ | $h_u = (H_u^+\ H_u^0)$ | $\tilde{h}_u = (\tilde{H}_u^+\ \tilde{H}_u^0)$ | ($\mathbf{1}$, $\mathbf{2}$, 1/2) |
| | $H_d$ | $h_d = (H_d^0\ H_d^-)$ | $\tilde{h}_d = (\tilde{H}_d^0\ \tilde{H}_d^-)$ | ($\mathbf{1}$, $\mathbf{2}$, -1/2) |

**Table 3.1:** Chiral Supermultiplets in the Minimal Supersymmetric Standard Model. The spin-0 fields are complex scalar bosons, while the spin-1/2 fields are left-handed two-component Weyl fermions.

take place in the MSSM if the $U(1)$ gauge group is identified with the hypercharge group $U(1)_Y$. However, this cannot be since this will imply that some of the sparticles can gain VEVs, which is precisely what we need to avoid in order for the SUSY breaking to take place. One obvious alternative is to consider $U(1)$ as an extra gauge group of the MSSM.

As we can see from these discussions, even the simpler realizations of spontaneously broken supersymmetry require an extension of the MSSM. Some of the most successful approaches to achieve supersymmetry breaking are dynamical, and consist of adding a Hidden sector[1] in which supersymmetry breaking takes place. This information is communicated to the MSSM via messengers which can be regarded as gravitational interactions or ordinary gauge interactions. Useful information about this subject can be found in [1, 4] .

### 3.1.3 Soft supersymmetry breaking terms

If supersymmetry is the solution to the hierarchy problem, then the possible supersymmetry breaking terms that one can write down are limited to mass terms and coupling parameters with positive mass dimension. These are called soft terms, since they do not introduce quadratically divergent loop corrections that could spoil the cancellation between fermionic and bosonic parts. The breaking of supersymmetry is introduced explicitly with an effective Lagrangian formed by the terms

$$\mathcal{L}_{soft} = -\frac{1}{2}\left(M_a\lambda^a\lambda^a + \frac{1}{3}a^{ijk}\phi_i\phi_j\phi_k + b^{ij}\phi_i\phi_j + t^i\phi_i\right) + c.c. - (m^2)^i_j\,\phi^{*j}\,\phi_i. \tag{3.7}$$

The first term in equation (3.7) is a Majorana mass term for the gauginos. The other terms are either scalar masses or trilinear scalar interactions, except for the b-term $b^{ij}\phi_i\phi_j + c.c.$ which will play an important role in electroweak symmetry breaking, and the linear term $t^i\phi^i + c.c.$ which does not exist in the MSSM since it requires $\phi_i$ to be a gauge singlet.

[1] *i.e* to assume that there exist a completely new sector which is not charged under the MSSM gauge group.

| Gauge Content | | | |
|---|---|---|---|
| Name | Spin 1 | Spin 1/2 | $SU(3)_C\,,\,SU(2)_L\,,\,U(1)_Y$ |
| Gluino, gluon | $G$ | $\tilde{G}$ | $(\mathbf{8},\,\mathbf{1},\,0)$ |
| Winos, $W$ bosons | $W^{\pm}\ \ W^0$ | $\tilde{W}^{\pm}\ \ \tilde{W}^0$ | $(\mathbf{1},\,\mathbf{3},\,0)$ |
| Bino, $B$ boson | $B$ | $\tilde{B}$ | $(\mathbf{1}\ ,\mathbf{1},0)$ |

**Table 3.2:** Gauge supermultiplets in the the Minimal Supersymmetric Standard Model. The spin-1 fields are vector bosons, while the spin-1/2 fields are left-handed two-component Weyl fermions.

## 3.2 MSSM Lagrangian

The simplest realization of spontaneously-broken supersymmetry is the so-called Minimal Supersymmetric Standard Model. We know from the last chapter that the coupling between matter and gauge particles is described by equation (2.64). On the other hand, interactions between fermions and bosons are introduced by means of the superpotential (see equation (2.30)), so including the kinetic terms for the gauge fields and computing the scalar potential from the auxiliary fields will give all the information that is needed to write the full MSSM Lagrangian. Since the Standard model gauge group will be used, then to describe the MSSM it is needed to specify only three ingredients: the particle content with the gauge transformation properties, the superpotential and the SUSY breaking terms.

The particle content of the MSSM with the gauge transformations properties are summarized in Tables 3.1 and 3.2. We notice that aside from the fact that we need to add one more scalar boson Higgs, the MSSM doubles the number of particles that we encounter in the SM.

To specify the MSSM superpotential, we must take into account that it must be composed only by chiral superfields, and it must be renormalizable and supergauge invariant. We find that the terms that satisfy these conditions, and involve the chiral supermultiplet of Table 3.1, are given by

$$W_{MSSM} \,=\, Y^u_{ij}\,\bar{u}^i Q^j H_u \,+\, Y^d_{ij}\,\bar{d}^i Q^j H_d \,+\, Y^e_{ij}\,\bar{e}^i L^j H_d \,+\, \mu\, H_u H_d. \tag{3.8}$$

Here $Y^f_{ij}$ are Yukawa couplings with $i\,,j$ matrix indices in family space, $\mu$ corresponds to a $[mass]^1$ parameter.
Nevertheless, there are other terms that could have been written in equation (3.8), like for example

$$\alpha^{ijk}\,Q_i\,L_j\,\overline{d}_k, \qquad \beta^i\,L^i\,H_u. \tag{3.9}$$

However all of these terms, and any other possible term that could have been written in the MSSM superpotential, violates either lepton or baryon number, or both. This could lead to a disaster for the proton, since there are terms that one can write down in equation (3.8), that would imply an extremely short proton lifetime compared with the current lower bound $\tau_p \,>\, 10^{31}$ years [25]. To avoid this difficulty we introduce a $Z_2$ discrete symmetry called R-parity (or matter parity). This symmetry forbids terms like the ones in the last equation. R-parity is implemented by assigning to each particle the following quantum number

$$P_R \,=\, (-1)^{3(B-L)\,+\,2s}, \tag{3.10}$$

where $s$ is the spin of the particle and $B$, $L$ are the baryon and lepton number respectively. With this definition, all the Standard Model particles and the Higgs bosons have even R-parity $P_R = +1$, while the sparticles have odd R-parity $P_R = -1$. R-parity is an extra ingredient that is part of the MSSM and has three important experimental consequences

- The lightest sparticle (called LSP) must be stable. If the LSP is electrically neutral, then it interacts only weakly with ordinary matter and can be an attractive dark matter candidate[29] ;

- Each sparticle different of the LSP must eventually decay into a state that contains an odd number of LSPs;

- In collider experiments, sparticles can only be produced in pairs.

As final step, to obtain the MSSM we must specify the soft SUSY breaking terms. Applying the result of equation (3.7) to the MSSM gauge group and particle content we find

$$\begin{aligned}
\mathcal{L}_{soft}^{MSSM} = & -\frac{1}{2}\left(M_1\,\tilde{B}\tilde{B} + M_2\,\tilde{W}\tilde{W} + M_3\,\tilde{G}\tilde{G} + c.c.\right) \\
& - m_{h_u}^2\,h_u^\dagger h_u - m_{h_d}^2\,h_d^\dagger h_d - (b\,h_u h_d + c.c\,) \\
& - \left(\tilde{u}_R^*\,\boldsymbol{A_u}\,\tilde{q}_L h_u + \tilde{d}_R^*\,\boldsymbol{A_d}\,\tilde{q}_L h_d + \tilde{e}_R^*\,\boldsymbol{A_e}\,\tilde{l}_L h_d + c.c\right) \\
& - \left(\tilde{q}_L^\dagger\,\boldsymbol{M_Q^2}\,\tilde{q}_L + \tilde{l}_L^\dagger\,\boldsymbol{M_L^2}\,\tilde{l}_L + \tilde{u}_R^*\,\boldsymbol{M_u^2}\,\tilde{u}_R + \tilde{d}_R^*\,\boldsymbol{M_d^2}\,\tilde{d}_R + \tilde{e}_R^*\,\boldsymbol{M_e^2}\,\tilde{e}_R\right).
\end{aligned} \tag{3.11}$$

In the previous equation $M_1$, $M_2$ and $M_3$ are Majorana gauginos masses ( notice that the gauge index has been suppressed), $\boldsymbol{A_u}$, $\boldsymbol{A_d}$ and $\boldsymbol{A_e}$ are $3\times 3$ complex matrices in family space, while $\boldsymbol{M_Q^2}$, $\boldsymbol{M_L^2}$, $\boldsymbol{M_u^2}$, $\boldsymbol{M_d^2}$ and $\boldsymbol{M_e^2}$ are also $3\times 3$ complex matrices in family space, with the additional condition that they must be hermitian for the Lagrangian to be real.

At this point it is important call the attention on the fact that that the soft SUSY breaking terms in equation (3.11) could spoil the effectiveness of SUSY as a solution to the Hierarchy problem, since they tend to push the EW scale up to the scale of SUSY breaking [5]. In order to avoid this, the typical scale $m_{soft}$ of soft SUSY breaking should not be greater than a few TeV [30]. The parameters of equation (3.11) are related to $m_{soft}$ according to

$$\begin{aligned}
& M_1\,,\,M_2\,,\,M_3\,,\,\boldsymbol{A_u}\,,\,\boldsymbol{A_d}\,,\,\boldsymbol{A_e}\,\sim\,m_{soft}, \\
& \boldsymbol{M_Q^2}\,,\,\boldsymbol{M_L^2}\,,\,\boldsymbol{M_u^2}\,,\,\boldsymbol{M_d^2}\,,\,\boldsymbol{M_e^2}\,,\,m_{h_u}^2\,,\,m_{h_d}^2\,,\,b\,\sim\,m_{soft}^2.
\end{aligned} \tag{3.12}$$

Notice that the soft SUSY breaking terms introduce many new parameters that were not present in the ordinary Standard Model. As it will be discussed later, this will originate problems related to new flavor mixing and additional CP-violating effects.

Now that we know which are the ingredients necessary to obtain the minimal supersymmetric version of the Standard Model, the complete expression of the MSSM Lagrangian can be given. First notice that the Lagrangian can be separated in two pieces

$$\mathcal{L}_{MSSM} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft}, \tag{3.13}$$

where $\mathcal{L}_{soft}$ is given by equation (3.11) and $\mathcal{L}_{SUSY}$ represents the supersymmetry preserving Lagrangian that can be written as

$$\mathcal{L}_{SUSY} = \mathcal{L}_{matter} + \mathcal{L}_{gauge} + \mathcal{L}_{int} + V_{scalar}. \tag{3.14}$$

We must now specify what are these contributions. We begin with the Lagrangian that couples matter to the gauge particles, $\mathcal{L}_{matter}$. This Lagrangian is composed by three pieces: a kinetic Lagrangian for the quarks and squarks, together with trilinear interactions involving the gauginos

$$\begin{aligned}\mathcal{L}_q =& + D_\mu \tilde{u}_R^* D^\mu \tilde{u}_R - i\, u_R \bar{\sigma}^\mu D_\mu u_R^\dagger - \sqrt{2}\left(g_c\, \tilde{u}_R^* t^a u_R^\dagger \tilde{G}^a + g' y_u\, \tilde{u}_R^* u_R^\dagger \tilde{B} + h.c.\right)\\ &+ D_\mu \tilde{d}_R^* D^\mu \tilde{d}_R - i\, d_R \bar{\sigma}^\mu D_\mu d_R^\dagger - \sqrt{2}\left(g_c\, \tilde{d}_R^* t^a d_R^\dagger \tilde{G}^a + g' y_d\, \tilde{d}_R^* d_R^\dagger \tilde{B} + h.c.\right)\\ &+ (D_\mu \tilde{q}_L)^\dagger\, (D^\mu \tilde{q}_L) - i\, q_L^\dagger \bar{\sigma}^\mu D_\mu q_L\\ &- \sqrt{2}\left(g_c\, \tilde{q}_L^\dagger t^a q_L \tilde{G}^a + g\, \tilde{q}_L^\dagger \tau^a q_L \tilde{W}^a + g' y_q\, \tilde{q}_L^\dagger q_L \tilde{B} + h.c.\right),\end{aligned} \tag{3.15}$$

a kinetic Lagrangian for the leptons and sleptons, together with trilinear interactions involving only the $SU(2)_L$ and $U(1)_Y$ gauginos

$$\begin{aligned}\mathcal{L}_l =& + D_\mu \tilde{e}_R^* D^\mu \tilde{e}_R - i\, e_R \bar{\sigma}^\mu D_\mu e_R^\dagger - \sqrt{2}\left(g' y_e\, \tilde{e}_R^* e_R^\dagger \tilde{B} + h.c.\right)\\ &+ \left(D_\mu \tilde{l}_L\right)^\dagger \left(D^\mu \tilde{l}_L\right) - i\, l_L^\dagger \bar{\sigma}^\mu D_\mu l_L\\ &- \sqrt{2}\left(g\, \tilde{l}_L^\dagger \tau^a l_L \tilde{W}^a + g' y_l\, \tilde{l}_L^\dagger l_L \tilde{B} + h.c.\right),\end{aligned} \tag{3.16}$$

and a kinetic Lagrangian for the higgses and higgsinos, together with trilinear interactions involving only the $SU(2)_L$ and $U(1)_Y$ gauginos

$$\begin{aligned}\mathcal{L}_h =& + (D_\mu h_u)^\dagger\, (D^\mu h_u) - i\, \tilde{h}_u^\dagger \bar{\sigma}^\mu D_\mu \tilde{h}_u\\ &- \sqrt{2}\left(g\, h_u^\dagger \tau^a \tilde{h}_u \tilde{W}^a + g' y_{h_u}\, h_u^\dagger \tilde{h}_u \tilde{B} + h.c.\right)\\ &+ (D_\mu h_d)^\dagger\, (D^\mu h_d) - i\, \tilde{h}_d^\dagger \bar{\sigma}^\mu D_\mu \tilde{h}_d\\ &- \sqrt{2}\left(g\, h_d^\dagger \tau^a \tilde{h}_d \tilde{W}^a + g' y_{h_d}\, h_d^\dagger \tilde{h}_d \tilde{B} + h.c.\right).\end{aligned} \tag{3.17}$$

In equations (3.15)-(3.17), $D_\mu$ corresponds to the covatiant derivative that must be specified according to the different gauge transformation properties of the fields, $y_f$ is the hypercharge associated to the fermion $f$, and the notation for the gauge coupling constant is the same that the one used in the Standard Model Lagrangian given in the appendix. Notice that the Lagrangians given above are for only one generation, adding the other generations imply to write two more copies of each of the terms given in equations (3.15)-(3.17) with proper names/indices.

The next contribution for the MSSM Lagrangian, comes from the kinetic terms for the gauge bosons and their respective gauginos

$$\begin{aligned}\mathcal{L}_{gauge} =& -\frac{1}{4} G^{\mu\nu a} G^a_{\mu\nu} - \frac{1}{4} W^{\mu\nu a} W^a_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}\\ &+ i\tilde{G}^{\dagger a} \bar{\sigma}^\mu D_\mu \tilde{G}^a + i\tilde{W}^{\dagger a} \bar{\sigma}^\mu D_\mu \tilde{W}^a + i\tilde{B}^\dagger \bar{\sigma}^\mu D_\mu \tilde{B}.\end{aligned} \tag{3.18}$$

This Lagrangian contains the usual abelian and non-abelian strength fields of the SM, together with kinetic terms for the gauginos. Consider next the interaction terms that are computed from the superpotential of equation (3.8). They are given by

$$\begin{aligned}\mathcal{L}_{int} =& \mu\, \tilde{h}_u\, \tilde{h}_d \,+\, u_R^\dagger\, Y_u\, h_u\, q_L \,+\, u_R^\dagger\, Y_u\, \tilde{h}_u\, \tilde{q}_L \,+\, \tilde{u}_R^*\, Y_u\, \tilde{h}_u\, q_L \,+\, h.c. \\ &+\, d_R^\dagger\, Y_d\, h_d\, q_L \,+\, d_R^\dagger\, Y_d\, \tilde{h}_d\, \tilde{q}_L \,+\, \tilde{d}_R^*\, Y_d\, \tilde{h}_d\, q_L \,+\, h.c. \\ &+\, e_R^\dagger\, Y_e\, h_d\, l_L \,+\, e_R^\dagger\, Y_e\, \tilde{h}_d\, \tilde{l}_L \,+\, \tilde{e}_R^*\, Y_e\, \tilde{h}_d\, l_L \,+\, h.c.\end{aligned} \tag{3.19}$$

Aside from the first term, the rest of the interactions correspond to Yukawa type interactions. Finally, as pointed out before, the scalar potential is composed by the contributions coming from the auxiliary fields

$$V_{scalar} \,=\, V_F \,+\, V_D. \tag{3.20}$$

The F-term contributions are obtained from the MSSM superpotential

$$\begin{aligned}V_F =& \mu^2 \left(h_u^\dagger h_u \,+\, h_d^\dagger h_d\right) \,+\, Y_{ij}^{u*}\, Y_{ik}^{u}\, \tilde{u}_R^{j}\, \tilde{u}_R^{*\,k}\, h_u^\dagger h_u \,+\, Y_{ij}^{u*}\, Y_{ik}^{u}\, \tilde{q}_L^{j\,\dagger}\, \tilde{q}_L^{k}\, h_u^\dagger\, h_u \\ &+\, Y_{ij}^{u*}\, Y_{km}^{u}\, \tilde{u}_R^{i}\, \tilde{u}_R^{*\,k}\, \tilde{q}_L^{\dagger\,j}\, \tilde{q}_L^{\dagger\,m} \,+\, Y_{ij}^{d*}\, Y_{ik}^{d}\, \tilde{d}_R^{j}\, \tilde{d}_R^{*\,k}\, h_d^\dagger h_d \,+\, Y_{ij}^{d*}\, Y_{ik}^{d}\, \tilde{q}_L^{j\,\dagger}\, \tilde{q}_L^{k}\, h_d^\dagger\, h_d \\ &+\, Y_{ij}^{d*}\, Y_{km}^{d}\, \tilde{d}_R^{i}\, \tilde{d}_R^{*\,k}\, \tilde{q}_L^{\dagger\,j}\, \tilde{q}_L^{\dagger\,m} \,+\, Y_{ij}^{e*}\, Y_{ik}^{e}\, \tilde{e}_R^{j}\, \tilde{e}_R^{*\,k}\, h_d^\dagger h_d \,+\, Y_{ij}^{e*}\, Y_{ik}^{e}\, \tilde{q}_L^{j\,\dagger}\, \tilde{q}_L^{k}\, h_d^\dagger\, h_d \\ &+\, Y_{ij}^{e*}\, Y_{km}^{e}\, \tilde{e}_R^{i}\, \tilde{e}_R^{*\,k}\, \tilde{q}_L^{\dagger\,j}\, \tilde{q}_L^{\dagger\,m},\end{aligned} \tag{3.21}$$

where all the family indices have been written. Notice that the second and third terms gives precisely the type of coupling needed to cancel top loop contributions to the Higgs mass, as pointed out in the introduction.

The D-terms can be obtained from equation (2.66). Since the contributions will be given according to the gauge structure of the theory, then the D-term contribution to the scalar potential can be written as

$$V_D \,=\, \frac{1}{2}\left(D_Y^2 \,+\, D_L^a D_L^a \,+\, D_c^a D_c^a\right), \tag{3.22}$$

where the different contributions are encountered from the general result of equation (2.66)

$$\begin{aligned}D_Y \,&=\, g' \left(y_u\, \tilde{u}_R^*\, \tilde{u}_R \,+\, y_d\, \tilde{d}_R^*\, \tilde{d}_R \,+\, y_e\, \tilde{e}_R^*\, \tilde{e}_R \,+\, y_q\, \tilde{q}_L^\dagger\, \tilde{q}_L \,+\, y_{h_u}\, h_u^\dagger\, h_u \,+\, y_{h_d}\, h_d^\dagger\, h_d\right), \\ D_L^a \,&=\, -g\, \left(\tilde{q}_L^\dagger\, \tau^a\, \tilde{q}_L \,+\, \tilde{l}_L^\dagger\, \tau^a\, \tilde{l}_L \,+\, h_u^\dagger\, \tau^a\, h_u \,+\, h_d^\dagger\, \tau^a\, h_d\right), \\ D_c^a \,&=\, -g_c\, \left(\tilde{q}_L^\dagger\, t^a\, \tilde{q}_L \,+\, \tilde{u}_R^*\, t^a\, \tilde{u}_R \,+\, \tilde{d}_R^*\, t^a\, \tilde{d}_R\right).\end{aligned} \tag{3.23}$$

Now that we know the complete MSSM Lagrangian, we can study the scalar potential that will lead to the spontaneous breaking of the electroweak symmetry.

## 3.3 Electroweak symmetry breaking in the MSSM

In the the MSSM electroweak symmetry breaking is driven by the two Higgs doublets $h_u$ and $h_d$. It will be shown that the neutral components of these doublets can get non trivial VEVs, denoted by $v_u$ and $v_d$ respectively. In this section we will derive the Higgs scalar potential, and then we will find the conditions necessary for the doublets to gain VEVs. Then we will obtain the masses of the scalar particles, and we will show that the lightest Higgs boson satisfies a severe bound regarding its tree level mass.

### 3.3.1 Higgs scalar potential

It will be useful for the rest of the thesis to divide the contributions for the Higgs potential in three terms; F-terms , D-terms and SUSY breaking terms.

The F-term contribution can be computed from the last term of equation (3.8), resulting in

$$V_{Higgs}^{F} = |\mu|^2 \left( |h_u|^2 + |h_d|^2 \right). \tag{3.24}$$

The D-term contribution to the Higgs potential will come from the first two terms of equation (3.23), however we just need to keep the terms that contains products of the Higgs doublets. This is because when computing the square of the first two terms of equation (3.23), there will be terms that contains the product of some of the higgses with other fields, like for example

$$g^2 \, (h_u^\dagger \, \tau^a \, h_u) \, (\tilde{l}_L^\dagger \, \tau^a \, \tilde{l}_L). \tag{3.25}$$

This type of term will give no contribution to the minimum equations, since any other field different from the Higgses will not get a VEV[1]. We will thus discard them from the beginning. The D-term contribution can be written as

$$\begin{aligned} V_{Higgs}^{D} &= V_{U(1)}^{D} + V_{SU(2)}^{D} \\ &= \frac{g'^2}{2} \left( \frac{1}{2} \, |h_u|^2 - \frac{1}{2} \, |h_d|^2 \right)^2 + \frac{g^2}{2} \left( \frac{1}{2} \, h_u^\dagger \sigma^a \, h_u + \frac{1}{2} \, h_d^\dagger \sigma^a \, h_d \right)^2 . \end{aligned} \tag{3.26}$$

Finally, from equation (3.11) we can see that there are mass terms, and a $b$ term contribution coming from the SUSY breaking sector

$$V_{Higgs}^{soft} = (b \, h_u h_d + h.c.) + m_{h_u}^2 \, |h_u|^2 + m_{h_d}^2 \, |h_d|^2 . \tag{3.27}$$

Adding all the contributions of the previous equations, we will obtain that the Higgs scalar potential in terms of the component fields is given by

$$\begin{aligned} V_{Higgs} &= \left( m_{h_u}^2 + |\mu|^2 \right) \left( \left|H_u^+\right|^2 + \left|H_u^0\right|^2 \right) + \left( m_{h_d}^2 + |\mu|^2 \right) \left( \left|H_d^-\right|^2 + \left|H_d^0\right|^2 \right) \\ &+ b \, (H_u^+ H_d^- - H_u^0 H_d^0) + h.c. + \frac{g^2}{2} \left| H_u^+ (H_d^0)^* + H_u^0 (H_d^-)^* \right|^2 \\ &+ \frac{g'^2 + g^2}{8} \left( \left|H_u^0\right|^2 + \left|H_u^+\right|^2 - \left|H_d^0\right|^2 - \left|H_d^-\right|^2 \right)^2 . \end{aligned} \tag{3.28}$$

Let us analyze the conditions that are necessary so that the fields $H_u^0$ and $H_d^0$ get non trivial VEVs. First notice that we can reduce a possible VEV of one component of either $h_u$ or $h_d$

[1]Possible sneutrino VEVs are forbidden by R-parity.

to zero by an $SU(2)_L$ gauge transformation. We can take $H_u^+ = 0$ at the minimum of the potential and then notice that

$$\left.\frac{\partial V_{Higgs}}{\partial H_u^+}\right|_{min} = b\, H_d^- + \frac{g^2}{2}\,(H_d^0)^*\,(H_u^0)^*\,H_d^- . \tag{3.29}$$

This will vanish if either $H_d^-$ or $b + \frac{g^2}{2}(H_d^0)^*(H_u^0)^*$ is null. The last option is unfavorable for symmetry breaking, so we will have that at the minimum both charged part of the fields are zero. Therefore, we can focus our attention in the neutral part of the Higgs potential, which is given by

$$\begin{aligned} V_{neu} = &\left(m_{h_u}^2 + |\mu|^2\right)\left|H_u^0\right|^2 + \left(m_{h_d}^2 + |\mu|^2\right)\left|H_d^0\right|^2 \\ &- b\,H_u^0 H_d^0 + h.c. + \frac{g'^2 + g^2}{8}\left(\left|H_u^0\right|^2 - \left|H_d^0\right|^2\right)^2 . \end{aligned} \tag{3.30}$$

The only term that depends on the phases of the fields is $b\,H_u^0 H_d^0$, so we can take $b$ to be real and positive, because any possible phase of $b$ can be absorbed into the relative phase of the neutral fields.

It is possible to check that a minimum requires $H_u^0 H_d^0$ to be real and positive; this will imply that the VEVs $\langle H_u^0\rangle$ and $\langle H_d^0\rangle$ must have phases with equal magnitude and opposite sign. Since these fields have opposite hypercharges, we can exploit our freedom to make a $U(1)_Y$ gauge transformation and get rid of the phases. In addition, if we want a Higgs component with non null VEV we have to guarantee that the origin is not a stable minimum. From the matrix of second derivatives $\left(\partial^2 V/\partial\phi_i\partial\phi_j\right)$, we see that this can be achieved by imposing that the matrix must have a negative eigenvalue at the origin. This will lead to the condition

$$b^2 > \left(m_{h_u}^2 + |\mu|^2\right)\left(m_{h_d}^2 + |\mu|^2\right). \tag{3.31}$$

Also notice from equation (3.30) that for $H_u^0 = H_d^0$, the $b$ term grows arbitrarily negative. Thus the condition for the potential to be bounded from below is

$$2\mu^2 + m_{h_u}^2 + m_{h_d}^2 > 2b. \tag{3.32}$$

Assuming that the Higgs potential parameters satisfy these previous conditions, we can expect that the neutral components $H_u^0$ and $H_d^0$ get non-zero VEVs $v_u$ and $v_d$. These VEVs are related to the Higgs potential parameters via the minimum conditions. If the potential has a minimum at $v_u$ and $v_d$, then it must be that

$$\begin{aligned} (m_{h_u}^2 + |\mu|^2)\,v_u &= b\,v_d + \frac{g^2 + g'^2}{4}\,(v_d^2 - v_u^2)\,v_u, \\ (m_{h_d}^2 + |\mu|^2)\,v_d &= b\,v_u + \frac{g^2 + g'^2}{4}\,(v_u^2 - v_d^2)\,v_d. \end{aligned} \tag{3.33}$$

Notice that there is one combination of $v_u$ and $v_d$ that is fixed by experiments, since it determines the masses of the electroweak gauge bosons. The masses of the $W$ and $Z$ bosons are provided by the terms

$$\mathcal{L} \supset (D_\mu h_u)^\dagger (D^\mu h_u) + (D_\mu h_d)^\dagger (D^\mu h_d), \tag{3.34}$$

when the fields are evaluated at their VEVs. The masses of the electroweak gauge bosons are

$$m_W^2 = \frac{g^2}{2}({v_u}^2 + {v_d}^2), \qquad m_Z^2 = \frac{g^2 + g'^2}{2}(v_u^2 + v_d^2). \tag{3.35}$$

This will imply that the VEVs of the neutral higgses satisfy

$$(v_u^2 + v_d^2)^{1/2} = \left(\frac{2\, m_W^2}{g^2}\right)^{1/2} = 174\,\mathrm{GeV}. \tag{3.36}$$

The ratio of the VEVs is usually written as

$$\tan(\beta) \equiv \frac{v_u}{v_d}, \tag{3.37}$$

so that equation (3.33) can be written as

$$\begin{aligned} m_{h_u}^2 + |\mu|^2 &= b\cot(\beta) + \frac{m_Z^2}{2}\cos(2\beta), \\ m_{h_d}^2 + |\mu|^2 &= b\tan(\beta) - \frac{m_Z^2}{2}\cos(2\beta). \end{aligned} \tag{3.38}$$

These equations can be used to eliminate $b$ and $|\mu|$ in favor of $m_Z^2$ and $\tan(\beta)$. This last parameter is not fixed by any experiment, but since both VEVs are positive and real, then $\beta$ must lie between 0 and $\pi/2$.

### 3.3.2 Masses of the Higgs states

Let us begin by counting the relevant scalar degrees of freedom during EWSB. There are two scalar doublets that participates in electroweak symmetry breaking, and each of their components is a complex scalar field. This gives a total of 8 degrees of freedom . On the other hand, EWSB implies that there are three broken generators

$$\underbrace{SU(2)_L}_{3\,\text{generators}} \times \underbrace{U(1)_Y}_{1\,\text{generator}} \to \underbrace{U(1)_{em}}_{1\,\text{generator}},$$

therefore by Goldstone's theorem there should be three massless states. This means that in the MSSM we will find five massive Higgs scalar particles: two CP-even neutral scalars $h^0$ and $H^0$, one CP-odd neutral scalar $A^0$ and two charged scalars $H^+$ and $H^-$. To obtain the mass term of the fields we look at the quadratic part of the Higgs potential when the fields are shifted by their VEVs (*i.e.* when we expand the potential around its minimum). The imaginary and real part of the neutral fields are decoupled because the quadratic part of the neutral Higgs potential is invariant under CP transformations.

We start with the imaginary part of the neutral fields. In the $\left(\mathrm{Im}H_u^0, \mathrm{Im}H_d^0\right)$ basis the mass matrix is

$$\mathcal{M}^2_{CP-odd} = \begin{pmatrix} b\cot(\beta) & b \\ b & b\tan(\beta) \end{pmatrix}. \tag{3.39}$$

The diagonalization of this matrix will lead to a massless mode which will be 'eaten' by the $Z$ gauge boson, and to a CP-odd neutral scalar $A^0$ with mass

$$m_{A^0}^2 = \frac{2b}{\sin(2\beta)}. \tag{3.40}$$

Next, consider the mass term for the charged components

$$V_{Higgs} \supset \begin{pmatrix} H_u^{+*} & H_d^- \end{pmatrix} \begin{pmatrix} b\cot(\beta) + m_W^2\cos^2(\beta) & b + m_W^2\cos(\beta)\sin(\beta) \\ b + m_W^2\cos(\beta)\sin(\beta) & b\tan(\beta) + m_W^2\sin^2(\beta) \end{pmatrix} \begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix}. \tag{3.41}$$

After diagonalizing this matrix, we find one eigenstate with null mass that will provide the longitudinal mode of the $W^+$ gauge boson[1], and one massive eigenstate that will correspond to $H^+$, with mass given by

$$m_{H^+}^2 = m_W^2 + m_{A^0}^2. \tag{3.42}$$

The conjugate of $H^+$ will give the other charged state $H^-$, with the same mass.

Finally, from the mass term of the real component of the neutral higgses we obtain two massive CP-even neutral Higgs states, denoted by $h^0$ and $H^0$. A useful basis to study the behavior of the CP-even states is the so-called 'Higgs basis'

$$\begin{aligned} h_{SM} &= \sin(\beta)\, h_u + \cos(\beta)\, \tilde{h}_d, \\ H_{SM} &= \cos(\beta)\, h_u - \sin(\beta)\, \tilde{h}_d, \end{aligned} \tag{3.43}$$

designed in such a way that $\langle h_{SM} \rangle = v$, while $\langle H_{SM} \rangle = 0$. In the previous equation, $\tilde{h}_d = i\sigma_2\, h_d^*$.

The CP-even mass matrix in this basis results

$$\mathcal{M}_{CP-even}^2 = \begin{pmatrix} m_Z^2\cos^2(2\beta) & \frac{m_Z^2}{2}\sin(4\beta) \\ \frac{m_Z^2}{2}\sin(4\beta) & m_{A^0}^2 + m_Z^2\sin^2(2\beta) \end{pmatrix}. \tag{3.44}$$

Experimental data from the LHC requires $h^0 \sim h_{SM}$ to better than $10 - 15\%$ and $m_h^0 =$ 125.36 GeV [36]. While it is not a problem to accommodate $h^0 \sim h_{SM}$ ( it is sufficient to take the $\tan(\beta) \gg 1$ limit), we see from the first entry of equation (3.44) that there is an upper bound $m_{h^0} \leq m_Z|\cos(2\beta)| \leq m_Z$, clearly excluded by data. However this is a tree level relation, and it is possible to enhance the Higgs mass bound by 1-loop quantum corrections coming from the top-stop particles [37] [2]

$$m_{h^0}^2 \leq m_Z^2\cos^2(2\beta) + \frac{3\, m_t^4\, G_f}{\sqrt{2}\pi^2}\ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right). \tag{3.45}$$

Here $G_f$ is the Fermi coupling constant, while $m_t$ and $m_{\tilde{t}}$ are the masses of the top quark and the average mass of the stop squarks. As we already pointed out, to obtain the observed Higgs mass we need heavy stops which leads to fine tuning problems. Therefore there are two

[1] The hermitian conjugate of the massless eigenstate will provide the longitudinal mode of the $W^-$ gauge boson.

[2] This formula is the simplified version in which the mixing of the stops is neglected, the complete 1-loop correction can be found in [18].

options: either we accept the tuning, or we search for extensions of the MSSM in which we can accommodate a light Higgs without fine tuning problems.

## 3.4 Fine tuning in the MSSM

Now that we have presented the MSSM, we can turn again our attention to the fine tuning problem encountered in the SM. Earlier in this document it was mentioned that the most important quantum correction to the Higgs mass comes from the top quark. Now that we have introduced the stop superpartners we can see from the second and third term of equation (3.21), that we have both the quartic type of coupling and the relation between the bosonic and fermionic couplings to the Higgs, needed to perform the 1-loop cancellation of the diagrams in Figure 1.1 given at the introduction. Therefore we would have no fine tuning problem when supersymmetry is an exact symmetry. We know however that this is not the case, since SUSY must be broken and the soft terms (like for example $A_t\, \tilde{t}_R^*\, \tilde{t}_L\, h_u$) introduce additional 1-loop corrections to the Higgs mass parameter [1].

Instead of taking a diagrammatic approach, we prefer to consider renormalization group methods. In the Standard Model, the beta function for the Higgs mass parameter has a contribution which is proportional to the mass of any state that couples directly or indirectly to the Higgs. This implies that no matter what the initial value of the Higgs mass parameter is, it will always grow considerably in the UV when heavy states are coupled to the Higgs. This is the source of the fine tuning problem in the SM, since it is not possible to understand why the EW scale (governed by the physical Higgs mass parameter) is protected from the physics that we expect to find at more high energy scales like $M_{Planck}$. In exact SUSY theories, the contributions from any fermionic/bosonic state that couples to the Higgs is canceled by its respective bosonic/fermionic superpartner. On the other hand, in softly broken SUSY we encounter that the running of the mass parameter is proportional to $m_{soft}^2$. As a useful and illustrative example, consider the beta function for the soft Higgs mass parameter $m_{h_u}^2$, which is given by [24]

$$\begin{aligned} \frac{d\, m_{h_u}^2}{d\, \log \mu} =& \frac{3}{8\pi^2} tr[(m_{h_u}^2 + M_Q^2) Y_u^\dagger Y_u + Y_u^\dagger M_u^2 Y_u + A_u^\dagger A_u] \\ &+ \frac{3}{80\pi^2} g'^2 \left(m_{h_u}^2 - m_{h_d}^2 + tr[M_Q^2 - M_L^2 - 2M_u^2 + M_d^2 + M_e^2]\right) \\ &- \frac{3}{8\pi^2} g^2 |M_2|^2 - \frac{3}{40\pi^2} g'^2 |M_1|^2, \end{aligned} \tag{3.46}$$

where $\mu$ is the renormalization scale. This equation confirms what we have stated above about the relation of the Higgs mass parameter with $m_{soft}$, and implies that in the MSSM the soft scale $m_{soft}$ may be a new source of tuning.
We can introduce a quantitative measure for the level of fine tuning of a given observable $\hat{O}$ that depends on several parameters $p_i$. The standard measure is given by [23]

$$\Delta_{p_i}^{\hat{O}} = max_{p_i} \left| \frac{\delta \log \hat{O}}{\delta \log p_i} \right|. \tag{3.47}$$

[1] At two loop level there is also an important contribution to the Higgs mass parameter due to the gluino.

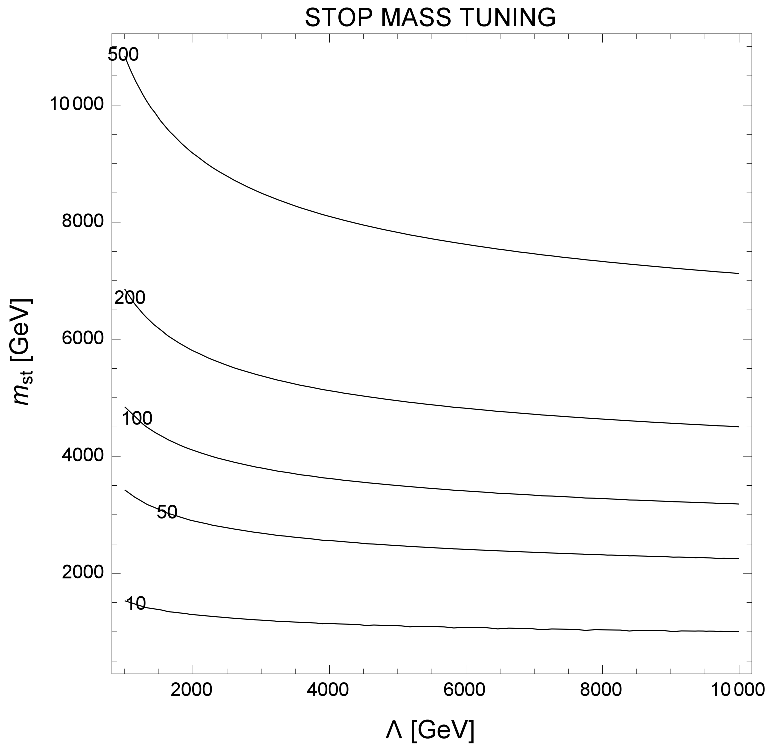


**Figure 3.1:** The values for the stop mass and the physical cut-off are shown for different amounts of tuning. We show $\Delta = 10, 50, 100, 200,$ and $500$.

In this equation $\Delta^{\hat{O}}_{p_i}$ is the sensitivity of the observable $\hat{O}$ with respect to the parameter $p_i$ at high energy. The sensitivity tell us the percentage of change of a given observable when the parameter changes of 1%. This definition is connected to a naturalness criterion, since it is expected that in a natural theory any observable should not be too sensitive to the details of what happens at high energy. Thus we take as a convention that 10 is the natural upper bound, and if a given observables has $\Delta > 10$ we say that the theory suffers of a naturalness problem.

We can use equations (3.46) and (3.47) to estimate the tuning on the Higgs mass. The most important sources of tuning comes from stop and gluino masses. Solving the RGE's of equation (3.46) at the leading order, we get

$$\Delta^{m_h^2}_{m_{\tilde{t}}^2} = \frac{3y_t^2\, m_{\tilde{t}}^2}{8\pi^2\, m_{h_u}^2} \log\frac{\Lambda}{\Lambda_{EW}}, \tag{3.48}$$

$$\Delta^{m_h^2}_{M_3} = \frac{y_t^2\, g_c^2\, M_3^2}{2\pi^4\, m_{h_u}^2} \log\frac{\Lambda}{\Lambda_{EW}}, \tag{3.49}$$

where $\Lambda$ is the physical cut-off energy scale ($i.e$ the scale at which the soft terms are generated) and $m_{\tilde{t}}$ is the mass of the stops[1]. Considering a 125 GeV Higgs, we can use these equations to see the different possibles values of the masses and the cut-off, for a given amount of tuning. Figures 3.1 and 3.2 show this information for tunings that goes from one part in ten up to one part in five hundred.

[1]The tuning computation yields to the same result for both complex scalar fields $\tilde{t}_L$ and $\tilde{t}^*_R$.

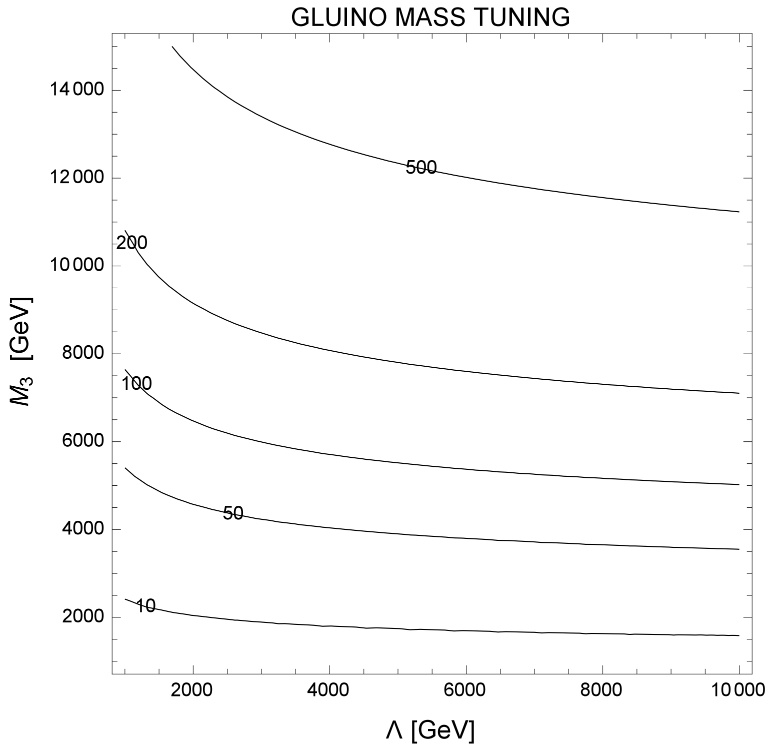


**Figure 3.2:** The values for the gluino mass and the physical cut-off are shown for different amounts of tuning. We show $\Delta = 10, 50, 100, 200$, and $500$.

We can see from Figure 3.1 that for $m_{\tilde{t}} > 2$ TeV the tuning becomes considerably. However, notice that we can have a low amount of tuning with relatively light stops (below 2 TeV) and a high energy cut-off. A similar situation holds for the gluino, for which we need to consider the stringent experimental lower bound $m_{\tilde{g}} \gtrsim 2$ TeV [43]. From this estimate we could clearly expect that for a heavier sparticle mass spectrum the naturalness problem of the MSSM becomes worse.

## 3.5 Open problems in the MSSM

Up to now, we have discussed only two problems that we encounter in the MSSM which indicates that if supersymmetry is realized in nature, and is the solution to the Hierarchy problem, then it should be described by physics beyond the Minimal Supersymmetric Standard Model. These two problems were to find an adequate description of the mechanism necessary to generate the soft SUSY breaking terms, and to achieve a Higgs boson with a mass in agreement with the experimental observations without introducing too much tuning. However, there are other problems in the MSSM whose solution could also be source of physics beyond the minimal model, the most important drawbacks comes from extra flavor mixing and CP-violating effects, and also from the so-called $\mu$ problem.

### 3.5.1 Flavor and CP problems

The soft SUSY breaking terms given in equation (3.11) do not only introduce a new energy scale $m_{soft}$, they also introduce a big amount of new parameters. In the MSSM it is found that, after rotating all the possible phases and mixing angles, there are a total of 105 parameters[10]. Most of the new parameters in the soft Lagrangian will lead to flavor mixing or CP violating processes in addition to those predicted by the Standard Model which have not been observed by experiments.

In the Standard Model we find that flavor changing processes, like for example Kaon oscillations, are strongly suppressed due to the GIM mechanism. Some of the soft SUSY breaking and interaction terms in the MSSM Lagrangian can violate flavor and give raise to possible additional contributions to the Kaon system that are severely constrained [41]. A supersymmetry analog of the GIM mechanism is needed and possible solutions within the MSSM leads to fine tuning problem [40]. Common solutions to explain the suppression are related with the specific mechanism by which SUSY breaks, and are thus naturally related to physics beyond the MSSM [38, 39].

In addition, in the MSSM there are several extra CP-violating phases with respect to the SM. This leads to extra contributions to CP-violating results that are also severely constrained by experiments. Consider, for example, the electric dipole moment (EDM) of the neutron, whose experimental upper bound is $d_N < 1.1 \times 10^{-24}$ e cm [12]. In the MSSM, the EDM of the neutron can be estimated to be [14]

$$d_N \simeq \left(\frac{100\text{GeV}}{m}\right)^2 \sin(\phi_a - \phi_b) \times 10^{-23}\,\text{e cm}, \tag{3.50}$$

where $m$ stands for gluino and squark masses, while $\phi_a$ and $\phi_b$ are physical phases related to soft term parameters. The last equation implies that to avoid experimental conflicts, either the CP-violating phases must be really small or close to each other, or the sparticle mass spectrum must be in the multi-TeV range [33]. Both of these alternatives lead to fine tuning problems[44].

All of the dangerous flavor changing and CP-violation effects that are encountered in the MSSM can be avoided, if the following conditions are satisfied

$$\boldsymbol{M_Q^2} = M_Q^2\,\mathbf{I}, \quad \boldsymbol{M_L^2} = M_L^2\,\mathbf{I}, \quad \boldsymbol{M_u^2} = M_u^2\,\mathbf{I}, \quad \boldsymbol{M_d^2} = M_d^2\,\mathbf{I}, \quad \boldsymbol{M_e^2} = M_e^2\,\mathbf{I}, \tag{3.51}$$

$$\boldsymbol{A_u} = A_{u\,0}\,\boldsymbol{Y_u}, \qquad \boldsymbol{A_d} = A_{d\,0}\,\boldsymbol{Y_d}, \qquad \boldsymbol{A_e} = A_{e\,0}\,\boldsymbol{Y_e}, \tag{3.52}$$

$$\text{Im}(M_1),\, \text{Im}(M_2),\, \text{Im}(M_3),\, \text{Im}(A_{u\,0}),\, \text{Im}(A_{d\,0}),\, \text{Im}(A_{e\,0}), = 0\,. \tag{3.53}$$

The first two equations ensure that supersymmetry contributions to FCNC processes will be very small, while the last equation ensures that the only CP-violating phases present in the MSSM will be the usual CKM phases encountered in the Yukawa couplings. Together, these conditions form what is usually called 'soft supersymmetry breaking universality'. These are very strong assumptions, and it is not difficult to imagine that the possible explanations for these conditions lie within a bigger picture than the one presented in the MSSM.

### 3.5.2 The $\mu$ problem

In the MSSM superpotential we have a unique term containing the two higgses. This is the so-called $\mu$ term

$$W_\mu = \mu H_u H_d. \tag{3.54}$$

Since this is a SUSY preserving term, there is no good reason to not think that $\mu$ can be very large, even of the order of a high energy scale like the Planck or GUT scale. On the other hand, consider the minimum conditions given in equation (3.33). From these equations it is possible to obtain the mass of the $Z$ gauge boson in terms of the $b$, $m_{H_d}^2$, $m_{H_u}^2$ and $\mu$ parameters

$$m_Z^2 = \frac{\left|m_{H_d}^2 - m_{H_u}^2\right|}{\left(1 - \frac{4b^2}{[m_{H_d}^2 + m_{H_u}^2 + 2|\mu|^2]^2}\right)^{1/2}} - m_{H_d}^2 - m_{H_u}^2 - 2|\mu|^2. \tag{3.55}$$

Except for $\mu$, all the parameters of the right hand side of this equation are soft supersymmetry breaking parameters, expected to be $\mathcal{O}(m_{soft})$. In order to get the correct value for $m_Z^2$, we need also $\mu \sim \mathcal{O}(m_{soft})$. This is puzzling, since there is no dynamical explanation in the MSSM.

A possible solution for this problem postulates the absence of an explicit $\mu$ term in the Lagrangian of the MSSM, with the $\mu$ term generated from non-renormalizable Lagrangian terms. Examples of this are the Kim-Nilles mechanism [34] or the Giudice-Masiero mechanism [35]. Another solution is given by Next to Minimal Supersymmetric Standard Model (NMSSM) [32], where one postulates the existence of an additional gauge singlet $S$ that couples to the MSSM higgses by a term in the superpotential given by

$$W = \lambda_s S H_u H_d. \tag{3.56}$$

Such a term is not forbidden by any symmetry of the MSSM and can generate a $\mu$ term if the gauge singlet gets a non zero VEV. Notice also that after integrating out the heavy singlet $S$, there will be an extra F-term contribution to the Higgs potential that could improve the Higgs mass bound encountered in the MSSM, without invoking loop corrections. The important point is that in any of the cases mentioned above, the solution to the $\mu$ problem implies to extend the MSSM with additional particles and/or symmetries.

# Chapter 4

# Gauge extensions of the MSSM and the Higgs mass

In this chapter we will study the possibility of increasing the tree level mass of the SM-like Higgs boson in supersymmetric models in which we add an extra $SU(2)$ gauge group to the MSSM gauge group. The idea is to obtain a 125 GeV Higgs boson without relying on radiative corrections coming from heavy states. One key observation is that in the MSSM, the quartic Higgs couplings are related to the Higgs mass term after EWSB takes place, and since in supersymmetry this type of interactions are generated also from D-term contributions to the scalar potential, we expect that adding extra D-terms we could enhance the upper bound on the Higgs mass.

Consider as a simple example the case in which we add an extra abelian $U(1)_X$ gauge group[2]. If the Higgs doublets of the MSSM are charged under this group, then there will be an extra D-term contribution in addition to the usual D-terms of the MSSM, which is given by

$$D_X = -g_x \left( q_h \, |h_u|^2 - q_h \, |h_d|^2 + q_\phi \, |\phi_1|^2 - q_\phi |\phi_2|^2 \right) . \tag{4.1}$$

In this equation $g_x$ is the gauge coupling constant of the group, the fields $\phi_1$ and $\phi_2$ are new scalar particles responsible for breaking spontaneously the $U(1)_X$ gauge symmetry[1] at a sufficiently high energy scale, and $q_{field}$ are the $U(1)_X$ charges. After the $U(1)_X$ spontaneous symmetry breaking, we are left with the MSSM gauge group and massive $\phi_1$ and $\phi_2$. By integrating out these particles we generate additional contribution to the MSSM Higgs potential, in such a way that after EWSB we obtain an improved Higgs mass bound

$$m_{h^0}^2 \leq m_Z^2 \cos^2(2\beta) + \frac{g_x^2 \, v^2}{2} \cos^2(2\beta) \left( 1 + \frac{4 q_\phi^2 \, g_x^2 \, \langle \phi \rangle^2}{2 m_\phi^2} \right)^{-1} . \tag{4.2}$$

Here $v$ and $\langle \phi \rangle$ are the Standard Model and heavy scalar VEVs respectively. In this type of models we expect that the new contributions to the Higgs potential do not decouple after the heavy scalars are integrated out, therefore the non-decoupling D-terms will leave a 'footprint' on the Higgs mass.

This kind of simple abelian gauge extension of the MSSM have been widely studied in the literature, see for example references [19, 20, 21]. However, it has been shown that radiative corrections and the presence of gauge kinetic mixing terms can reduce the positive impact of the

[1] Of course, for this to happen we must also specify both the potential for the new scalars and the conditions so that some of the scalar components gain non-zero VEVs.

| Superfield | Spin-0 | $SU(2)_I\,,\,SU(2)_{II}\,,\,U(1)_Y$ |
|---|---|---|
| $H_u$ | $h_u$ | $(\mathbf{2},\,\mathbf{1},\,1/2)$ |
| $H_d$ | $h_d$ | $(\mathbf{2},\,\mathbf{1},\,\text{-}1/2)$ |
| $\Sigma$ | $\Sigma$ | $(\mathbf{2},\,\overline{\mathbf{2}},\,0)$ |

**Table 4.1:** Fields and charges of the Higgs sector for the first symmetry breaking pattern considered.

non-decoupling D-terms in raising the Higgs mass [22]. This motivates us to explore other gauge extensions of the MSSM. In particular we will study the effects of the extra D-terms in a $SU(2)$ gauge extension. As mentioned before, this allows for two distinct symmetry breaking patterns, which we will analyze in turn.

## 4.1 Symmetry breaking pattern I

This model was studied in [2]. At high energies, the extended gauge group of the MSSM is $G = SU(3)_c \times SU(2)_I \times SU(2)_{II} \times U(1)_Y$. In this model the symmetry breaking pattern is

$$SU(3)_c \times SU(2)_I \times SU(2)_{II} \times U(1)_Y \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{em}.$$

The MSSM particles will have their usual $SU(3)_c$ and $U(1)_Y$ charges, and to simplify our analysis we will take them charged only under $SU(2)_I$ [1]. To accomplish the breaking to $SU(2)_L$, we will add a $SU(2)$ bidoublet chiral superfield $\Sigma$. The relevant particle content for the Higgs sector with their gauge charge assignment is given in Table 4.1.

We will also need to add a gauge singlet $S$, in order to guarantee the breaking of the extended gauge symmetry also in the limit in which supersymmetry is not broken. The relevant superpotential for the Higgs sector will be

$$W_{Higgs} \,=\, \mu\, H_u H_d \,+\, \lambda\, S \left(\frac{1}{2}\Sigma\Sigma \,-\, w^2\right) \,+\, \lambda_S\, H_u H_d\, S, \tag{4.3}$$

where $\Sigma\Sigma$ is contracted with two epsilon indices. Since the gauge singlet $S$ gives no D-term contribution to the scalar potential, we will simplify our analysis by neglecting the last term of this equation[2]. The corresponding soft supersymmetry breaking terms are

$$\begin{aligned} \mathcal{L}_{soft} \,=&\, -\,\frac{1}{2}\,(M_I \tilde{W}_I \tilde{W}_I \,+\, M_{II} \tilde{W}_{II} \tilde{W}_{II} \,+\, h.c.) \,-\, m_s^2\,|S|^2 \\ &-\, m_\Sigma^2\, tr(\Sigma^\dagger \Sigma) \,-\, \frac{B_\Sigma}{2}\,(\Sigma\Sigma \,+\, h.c.)\,. \end{aligned} \tag{4.4}$$

The terms given above are Majorana masses for the gauginos of $SU(2)$, a $b$ term for the scalar $\Sigma$, and mass terms for $\Sigma$ and $S$.

[1] In the original paper, the charge assignment was such that it allows to make extra gauge group asymptotically free. However this is an unnecessary complication for our purpose.

[2] We will also neglect the other two possible superpotential terms for the gauge singlet: $M_s\, S^2$ and $k\, S^3$. Strictly speaking, this model can give also an additional contribution to the F-term Higgs potential (analogous to what happens in the NMSSM [32]). Here we will suppose this contribution to be negligible and focus on the additional D-terms only.

Considering the $\Sigma$ field, the total D-term contribution to the Higgs potential is

$$\begin{aligned} V^D &= \frac{g_I^2}{8}\left(tr[\Sigma^\dagger\,\sigma^a\,\Sigma] + h_u^\dagger\,\sigma^a\,h_u + h_d^\dagger\sigma^a h_d\right)^2 \\ &+ \frac{g_{II}^2}{8}\left(tr[\Sigma\,\sigma^a\,\Sigma^\dagger]\right)^2 + \frac{g'^2}{8}\left(|h_u|^2 - |h_d|^2\right)^2 . \end{aligned} \tag{4.5}$$

Adding these terms to the F-term contribution coming from the superpotential of equation (4.3) with the relevant soft terms of equation (4.4), we find a scalar potential for $\Sigma$ that will lead to the spontaneous breaking $SU(2)_I \times SU(2)_{II} \to SU(2)_L$. The bidoublet can be parametrized by a $2\times 2$ matrix containing a $SU(2)_L$ singlet $\sigma$, and a triplet $T^a$

$$\Sigma = \frac{1}{\sqrt{2}}\left(\sigma\,\boldsymbol{I} + T^a\sigma^a\right). \tag{4.6}$$

The fields are normalized to have canonical kinetic terms. The first stage of the symmetry breaking will be driven by the singlet VEV $\langle\sigma\rangle = u$, which will take place for large values of $B_\Sigma$. The minimum lies in a D-flat direction and the MSSM higgses remain massless. Electroweak symmetry breaking will be driven by the usual MSSM VEVs $\langle h_u\rangle = v_u$, $\langle h_d\rangle = v_d$, together with an additional VEV of the remaining triplet $\langle T^3\rangle = v_T$.

The breaking of the extended gauge group will cause the $SU(2)$ gauge bosons $W_I$ and $W_{II}$ to mix and form massive eigenstates $W'$ and (for now) massless eigenstates $W$. The relation between mass and gauge eigenstates is

$$\begin{pmatrix} W \\ W' \end{pmatrix} = \begin{pmatrix} \frac{g_{II}}{\sqrt{g_I^2+g_{II}^2}} & \frac{g_I}{\sqrt{g_I^2+g_{II}^2}} \\ -\frac{g_I}{\sqrt{g_I^2+g_{II}^2}} & \frac{g_{II}}{\sqrt{g_I^2+g_{II}^2}} \end{pmatrix}\begin{pmatrix} W_I \\ W_{II} \end{pmatrix}. \tag{4.7}$$

It is possible to use this information to connect the gauge coupling constants $g_I$ and $g_{II}$, to the $SU(2)_L$ gauge coupling constant $g$. To achieve this, we notice that the covariant derivative for any of the MSSM particles charged under $SU(2)_I$ contains the term

$$D_\mu \supset i g_I\,W^a_{\mu\,I}\,\tau^a + \ldots \tag{4.8}$$

Using equation (4.7), we can express the gauge eigenstates $W_I$ in terms of the mass eigenstates. By identifying the coefficient that accompanies the $SU(2)_L$ gauge bosons with the MSSM gauge coupling, we obtain the relation

$$g = \frac{g_I\,g_{II}}{\sqrt{g_I^2 + g_{II}^2}}. \tag{4.9}$$

Thus $g$ connects to $g_I$ and $g_{II}$ in a very simple way.

### 4.1.1 Gauge Bosons

Once all the scalars of the theory develop their respective VEVs, the masses of the gauge bosons will come from the kinetic terms

$$\mathcal{L} \supset tr(D_\mu\Sigma)^\dagger(D^\mu\Sigma) + (D_\mu H_u)^\dagger(D^\mu H_u) + (D_\mu H_d)^\dagger(D^\mu H_d), \tag{4.10}$$

where the covariant derivative for the bidoublet is given by

$$D_\mu\Sigma = \partial_\mu\Sigma - i g_I\,W^a_{\mu\,I}\,\tau^a\,\Sigma + i g_{II}\,\Sigma\,W^a_{\mu\,II}\,\tau^a. \tag{4.11}$$

From the charged states we can find the masses of the usual SM $W$ gauge bosons and the new heavy gauge bosons $W'$. In the $(W_I^{\pm}, W_{II}^{\pm})$ basis, the mass matrix of the charged sector is given by

$$\mathcal{M}_{ch}^2 = \frac{1}{2}\begin{pmatrix} g_I^2\,(u^2 + v^2 + v_T^2) & -\,g_I g_{II}\,(u^2 - v_T^2) \\ -\,g_I g_{II}\,(u^2 - v_T^2) & g_{II}^2\,(u^2 + v_T^2) \end{pmatrix}. \tag{4.12}$$

The exact eigenvalues of the matrix given by this equation are complicated and little illuminating. However in the $u \gg v \gg v_T$ limit the masses of the $W$ and $W'$ gauge bosons have simple expressions

$$\begin{aligned} m_W^2 &\simeq \frac{g^2}{2}\,v^2\left(1 - \frac{g_I^4}{(g_I^2 + g_{II}^2)^2}\frac{v^2}{u^2} + 4\frac{v_T^2}{v^2}\right), \\ m_{W'}^2 &\simeq \frac{g_I^2 + g_{II}^2}{2}\,u^2\left(1 + \frac{g_I^4}{(g_I^2 + g_{II}^2)^2}\frac{v^2}{u^2} + \left(\frac{g_I^2 - g_{II}^2}{g_I^2 + g_{II}^2}\right)^2\frac{v_T^2}{u^2}\right). \end{aligned} \tag{4.13}$$

As expected, the $W$ MSSM mass is recovered, but there are small corrections due to the additional VEVs $u$ and $v_T$. On the other hand, we can find the masses of the neutral gauge bosons $Z$ and $Z'$ by considering the neutral states of (4.10). In the $(W_I^3, W_{II}^3, B)$ basis, the mass matrix of the neutral sector is given by

$$\mathcal{M}_{neu}^2 = \frac{1}{2}\begin{pmatrix} g_I^2\,(u^2 + v^2 + v_T^2) & -\,g_I g_{II}\,(u^2 + v_T^2) & -\,g_I g'\,v^2 \\ -\,g_I g_{II}\,(u^2 + v_T^2) & g_{II}^2\,(u^2 + v_T^2) & 0 \\ -\,g_I g'\,v^2 & 0 & g'^2\,v^2 \end{pmatrix}. \tag{4.14}$$

The matrix given in this last equation has a null eigenvalue which corresponds to the photon, while the other two combinations are identified with the $Z$ and $Z'$ gauge boson. In the $u \gg v \gg v_T$ limit, the masses of the neutral gauge bosons are given by

$$\begin{aligned} m_Z^2 &\simeq \frac{g^2 + g'^2}{2}\,v^2\left(1 - \frac{g_I^4}{(g_I^2 + g_{II}^2)^2}\frac{v^2}{u^2}\right), \\ m_{Z'}^2 &\simeq \frac{g_I^2 + g_{II}^2}{2}\,u^2\left(1 + \frac{g_I^4}{(g_I^2 + g_{II}^2)^2}\frac{v^2}{u^2} + \frac{v_T^2}{u^2}\right). \end{aligned} \tag{4.15}$$

This shows that just as in the case of the $W$, the $Z$ gauge boson mass is corrected by the additional VEVs. The additional contributions to the gauge bosons masses are constrained by EWPM, in particular by the $\rho$ parameter

$$\rho = \frac{m_W^2}{m_Z^2\,cos^2(\theta_W)}. \tag{4.16}$$

In the limit $u \gg v \gg v_T$ we obtain

$$\rho \simeq 1 + 4\frac{v_T^2}{v^2}. \tag{4.17}$$

The value of $\rho$ is measured to be $\rho = 1.01032 \pm 0.00009$ at $m_Z$ [36], and implies that the triplet VEV is constrained to satisfy $v_T \lesssim 9$ GeV.

### 4.1.2 Scalar potential and the Higgs bound

As usual, the Higgs scalar potential will be composed by contributions coming from D-terms and F-terms, as well as the soft SUSY breaking terms. We already know what are the contributions that come from the usual MSSM Higgs sector, so we focus our attention on the contributions given by the extra particle content. We have already shown the D-terms and soft SUSY breaking contributions to the scalar potential, therefore we just need to specify the F-term contribution. This contribution is computed from the second term of the superpotential given by equation (4.3), and it is given by

$$V_{\Sigma}^{F} = \lambda^2 \left( \frac{1}{4}\, |\Sigma\Sigma|^2 - \frac{w^2}{2}(\Sigma\Sigma + h.c.) + w^4 \right). \tag{4.18}$$

Adding this contribution to equations (4.4) and (4.5), we get the full Higgs potential

$$\begin{aligned} V =& \,(m_{h_u}^2 + |\mu|^2)\, |h_u|^2 + (m_{h_d}^2 + |\mu|^2)\, |h_d|^2 + (b\, h_u h_d + h.c.) + m_{\Sigma}^2\, tr(\Sigma^\dagger \Sigma) \\ & - \frac{B_\Sigma}{2}(\Sigma\Sigma + h.c.) + \lambda^2 \left( \frac{1}{4}\, |\Sigma\Sigma|^2 - \frac{w^2}{2}(\Sigma\Sigma + h.c.) + w^4 \right) + \frac{g'^2}{8}\left( |h_u|^2 - |h_d|^2 \right)^2 \\ & + \frac{g_I^2}{8}\left( tr[\Sigma^\dagger\, \sigma^a\, \Sigma] + h_u^\dagger\, \sigma^a\, h_u + h_d^\dagger\, \sigma^a h_d \right)^2 + \frac{g_{II}^2}{8}\left( tr[\Sigma\, \sigma^a\, \Sigma^\dagger] \right)^2 . \end{aligned} \tag{4.19}$$

After electroweak symmetry breaking the particles of the theory will gain mass and it is possible to estimate the masses of the CP-even Higgs states by using quantum mechanics perturbation theory.

By expanding the potential around the minimum and computing the matrix of second derivatives, we can obtain the full mass matrix of the Higgs sector. The CP-even mass matrix will now be a $4 \times 4$ matrix, with the form

$$\mathcal{M}_{CP-even}^2 = \begin{pmatrix} \mathcal{M}_h^2 & \mathcal{M}_{h\,\Sigma}^2 \\ \mathcal{M}_{h\,\Sigma}^{2\;T} & \mathcal{M}_\Sigma^2 \end{pmatrix}, \tag{4.20}$$

where $\mathcal{M}_h^2$ is a $2 \times 2$ mass matrix containing only the CP-even Higgs states of the MSSM. Similarly, $\mathcal{M}_\Sigma^2$ contains only the heavy Higgs associated to the bidoublet $\Sigma$, while $\mathcal{M}_{h\,\Sigma}^2$ contains mixed terms between the two type of higgses. In the Higgs basis of equation (3.43), the matrices are

$$\begin{aligned} \mathcal{M}_h^2 &= \begin{pmatrix} \frac{(g_I^2 + g'^2)}{2}\, v^2\, c_{2\beta}^2 & \frac{(g_I^2 + g'^2)}{4}\, v^2\, s_{4\beta} \\ \frac{(g_I^2 + g'^2)}{4}\, v^2\, s_{4\beta} & \frac{b}{s_{2\beta}}\,(3 + c_{4\beta}) + \frac{s_{2\beta}}{2}\left( 4b + (g_I^2 + g'^2) v^2\, s_{2\beta} \right) \end{pmatrix}, \\ \mathcal{M}_{h\,\Sigma}^2 &= \begin{pmatrix} \frac{g_I^2 uv\, c_{2\beta}(16 m_\Sigma^2 + 2(g_I^2 + g_{II}^2) u^2 - g_I^2 v^2 c_{2\beta})}{4\sqrt{2}(8 m_\Sigma^2 + (g_I^2 + g_{II}^2) u^2)} & -\frac{g_I^2 uv\, c_{2\beta}(16 m_\Sigma^2 + 2(g_I^2 + g_{II}^2) u^2 - g_I^2 v^2 c_{2\beta})}{4\sqrt{2}(8 m_\Sigma^2 + (g_I^2 + g_{II}^2) u^2)} \\ \frac{g_I^2 uv}{8\sqrt{2}}\left( -4 s_{2\beta} - \frac{g_I^2 v^2\, s_{4\beta}}{8 m_\Sigma^2 + (g_I^2 + g_{II}^2) u^2} \right) & \frac{g_I^2 uv}{8\sqrt{2}}\left( 4 s_{2\beta} - \frac{g_I^2 v^2\, s_{4\beta}}{8 m_\Sigma^2 + (g_I^2 + g_{II}^2) u^2} \right) \end{pmatrix}, \\ \mathcal{M}_\Sigma^2 &= \begin{pmatrix} \mathcal{M}_\Sigma^{11} & \mathcal{M}_\Sigma^{12} \\ \mathcal{M}_\Sigma^{21} & \mathcal{M}_\Sigma^{22} \end{pmatrix}, \end{aligned} \tag{4.21}$$

with

$$
\begin{aligned}
\mathcal{M}_{\Sigma}^{11} &= \frac{4(8m_{\Sigma}^2 + (g_I^2 + g_{II}^2)u^2)^3 \,+\, g_I^2 v^2 c_{2\beta}(256m_{\Sigma}^2 - 4(g_I^2 + g_{II}^2)^2 u^4 + g_I^2(g_I^2 + g_{II}^2)u^2 v^2 c_{2\beta})}{16(8m_{\Sigma}^2 + (g_I^2 + g_{II}^2)u^2)^2},\\
\mathcal{M}_{\Sigma}^{22} &= \frac{8(8m_{\Sigma}^2 + (g_I^2 + g_{II}^2)u^2)^3 \,+\, 2g_I^2 v^2 c_{2\beta}(-256m_{\Sigma}^2 + 4(g_I^2 + g_{II}^2)^2 u^4 + g_I^2(g_I^2 + g_{II}^2)u^2 v^2 c_{2\beta})}{32(8m_{\Sigma}^2 + (g_I^2 + g_{II}^2)u^2)^2},\\
\mathcal{M}_{\Sigma}^{12} &= \mathcal{M}_{\Sigma}^{21} \;=\; \frac{g_I^4(g_I^2 + g_{II}^2)u^2 v^4 c_{2\beta}^2}{(8m_{\Sigma}^2 + (g_I^2 + g_{II}^2)u^2)^2} - m_{\Sigma}^2 - \frac{(g_I^2 + g_{II}^2)}{4}u^2.
\end{aligned}
\tag{4.22}
$$

In these equations $s_{2\beta} = \sin(2\beta)$, $c_{2\beta} = \cos(2\beta)$ and so on.
In order to estimate the upper bound on the Higgs boson mass, we integrate out the heavy states and use a seesaw-like formula

$$
\mathcal{M}_h^{2\,eff} = \mathcal{M}_h^2 \,-\, (\mathcal{M}_{h\Sigma}^2)^T\,(\mathcal{M}_{\Sigma}^2)^{-1}\,\mathcal{M}_{h\Sigma}^2. \tag{4.23}
$$

From the first entry of the CP-even mass matrix we can read the Higgs mass bound

$$
m_{h^0}^2 \,\leq\, \left(\frac{\eta\, g^2 \,+\, g'^2}{2}\right) v^2\, \cos^2(2\beta), \tag{4.24}
$$

where $\eta$ is given in the large $\tan(\beta)$ limit by

$$
\eta = \frac{g_I^2 + g_{II}^2}{g_{II}^2} + \frac{u^2(g_I^2 + g_{II}^2)\,\left(8m_{\Sigma}^2 + u^2(g_I^2 + g_{II}^2)\right)\left[(8m_{\Sigma}^2 + u^2(g_I^2 + g_{II}^2))^2 + 16g_I^2 m_{\Sigma}^2 v^2\right]}{8v^4\, m_{\Sigma}^2\, g_{II}^2 g_I^2\,\left(8m_{\Sigma}^2 - 3u^2(g_I^2 + g_{II}^2)\right)}. \tag{4.25}
$$

As we can see from these equations, the maximum possible value of the Higgs mass will now depend also on the parameters related to the heavy states $u$ and $m_{\Sigma}$, and on the $SU(2)$ gauge coupling constants $g_I$ and $g_{II}$. We see that we could obtain an improved Higgs mass bound if $\eta$ is bigger than one. In reference [15] it has been shown that there is a large region in the parameter space from which we can have $\eta$ to be big enough to achieve a 125 GeV Higgs without introducing any significant amount of tuning. The good chances of this model to solve the Higgs mass bound problem has lead to further studies. The phenomenological implications for this model have been studied in references [9, 31].

## 4.2 Symmetry breaking pattern II

This breaking pattern has not been studied before. We will consider that at a sufficiently high energy scale the extended gauge group of the MSSM is $G = SU(3)_c \times SU(2)_L \times SU(2)_Z \times U(1)_X$. The two stages to break spontaneously the full gauge group down to color and electromagnetism, are

$$
SU(3)_c \times SU(2)_L \times SU(2)_Z \times U(1)_X \;\rightarrow\; SU(3)_c \times SU(2)_L \times U(1)_Y \;\rightarrow\; SU(3)_c \times U(1)_{em}.
$$

To achieve the first stage of the breaking we will add two chiral supermultiplets denoted by $\Phi$ and $\bar{\Phi}$. These superfields will be $SU(2)_Z$ doublets with $X$ charges $+1/2$ and $-1/2$ respectively. They will be singlets of $SU(3)_c$ and $SU(2)_L$.

| Superfield | Spin-0 | $SU(2)_L\,,\ SU(2)_Z\,,\ U(1)_X$ |
|---|---|---|
| $H_u$ | $h_u$ | $(\mathbf{2},\ \mathbf{1},\ 1/2)$ |
| $H_d$ | $h_d$ | $(\mathbf{2},\ \mathbf{1},\ \text{-}1/2)$ |
| $\Phi$ | $\phi$ | $(\mathbf{1},\ \mathbf{2},\ 1/2)$ |
| $\bar{\Phi}$ | $\bar{\phi}$ | $(\mathbf{1},\ \mathbf{2},\ \text{-}1/2)$ |

**Table 4.2:** Fields and charges of the Higgs sector for the second symmetry breaking pattern considered.

The rest of the MSSM particles will be uncharged under $SU(2)_Z$, and will remain with the same $SU(2)_L$ charge assignment of the MSSM. On the other hand, their $U(1)_X$ charges will have the same value of their respective MSSM $U(1)_Y$ charges. In Table 4.2 we have listed the relevant fields for the Higgs sector with their respective charges. The charge assignment is anomaly free.

The most general, supergauge invariant, and renormalizable superpotential for the new chiral supermultiplets is given by

$$W \,=\, \mu'\,\Phi\,\bar{\Phi} \,+\, \lambda_s\, S(\,\Phi\,\bar{\Phi} - w^2) \,+\, M_s S^2 \,+\, kS^3. \tag{4.26}$$

In this equation we have included a gauge singlet $S$ to achieve the breaking of the extended gauge symmetry also in the exact supersymmetric limit. Once again, we will neglect the contributions from the gauge singlet because we are only interested in the D-term contributions. We will therefore keep only the first term of the last equation. The relevant superpotential for the Higgs sector is

$$W_{Higgs} \,=\, \mu\, H_u\, H_d \,+\, \mu'\,\Phi\,\bar{\Phi}. \tag{4.27}$$

We must also add extra soft terms, however notice that we already know what the possible soft terms for the scalar components $\phi$ and $\bar{\phi}$ of the new chiral supermultiplets are, because their charge gauge assignment implies that they are totally similar to the case of the MSSM higgses. We will have mass terms for the scalars and a $b_\phi$ soft term, giving for the total potential

$$V_{\phi\,\bar{\phi}} \,=\, \left(m_\phi^2 \,+\, |\mu'|^2\right)\,|\phi|^2 \,+\, \left(m_{\bar{\phi}}^2 \,+\, |\mu'|^2\right)\,|\bar{\phi}|^2 \,+\, b_\phi\,\left(\phi\bar{\phi} \,+\, h.c.\right) \,+\, \text{D-terms}. \tag{4.28}$$

This potential will drive the first stage of the symmetry breaking $SU(2)_Z\times U(1)_X \,\rightarrow\, U(1)_Y$. The necessary conditions for the breaking to take place are completely similar to those already encountered in the MSSM (see equations (3.32) - (3.31)). For example the scalar potential will be bounded from below if $2\mu'^2 \,+\, m_{h_\phi}^2 \,+\, m_{h_{\bar{\phi}}}^2 \,>\, 2b_\phi$. The new scalars will gain VEVs according to

$$\langle\phi\rangle \,=\, \begin{pmatrix} 0 \\ u_\phi \end{pmatrix}, \qquad\qquad \langle\bar{\phi}\rangle \,=\, \begin{pmatrix} u_{\bar{\phi}} \\ 0 \end{pmatrix}. \tag{4.29}$$

From this equation we can check that the generator that leaves invariant the vacuum is

$$Y \,=\, T_3^z \,+\, X, \tag{4.30}$$

where $T_3^z$ corresponds to the third component of the $SU(2)_Z$ generator, while $X$ is the generator associated to $U(1)_X$. Since this spontaneous symmetry breaking will lead to a $U(1)_Y$ unbroken symmetry, we have identified with $Y$ the generator that leaves invariant the vacuum.

We can use this information to connect the gauge coupling constants $g_z$ and $g_x$ to $g'$. Notice that the covariant derivative for any of the MSSM particles contains a term

$$D_\mu \supset i\, g_x\, B'_\mu\, X + \dots \tag{4.31}$$

We thus identify $g'$ by replacing equation (4.30) into the covariant derivative, and then writing the gauge boson $B'$ associated to $U(1)_X$, in terms of the gauge mass eigenstates in the limit $v_u = v_d = 0$. After that, the $U(1)_Y$ gauge coupling constant $g'$, should be identified with the constant factor that accompanies the $B$ gauge boson.

From the mass term for the neutral gauge bosons $B'$ and $W'$, we will find that after the first breaking there will be a massless eigenstate $B$, and a massive eigenstate $Z'$. The relation between mass and gauge eigenstates is

$$\begin{pmatrix} B_\mu \\ Z'_\mu \end{pmatrix} = \begin{pmatrix} \frac{g_z}{\sqrt{g_z^2+g_x^2}} & \frac{g_x}{\sqrt{g_z^2+g_x^2}} \\ -\frac{g_x}{\sqrt{g_z^2+g_x^2}} & \frac{g_z}{\sqrt{g_z^2+g_x^2}} \end{pmatrix} \begin{pmatrix} B'_\mu \\ W'^{3}_\mu \end{pmatrix}, \tag{4.32}$$

from which we get

$$g' = \frac{g_z\, g_x}{\sqrt{g_z^2 + g_x^2}}. \tag{4.33}$$

This relation is quite intuitive due to the fact that the symmetry breaking pattern described above is analog to the EWSB encountered in the MSSM.

### 4.2.1 Gauge Bosons

To obtain the physical gauge boson and their respective masses, we need to consider the following covariant derivative terms

$$\mathcal{L} \supset (D_\mu h_u)^\dagger (D^\mu h_u) + (D_\mu h_d)^\dagger (D^\mu h_d) + (D_\mu \phi)^\dagger (D^\mu \phi) + \left(D_\mu \bar{\phi}\right)^\dagger \left(D^\mu \bar{\phi}\right). \tag{4.34}$$

After the scalar fields acquire VEVs, we can obtain the full mass matrix of the gauge sector and then we can diagonalize it. We distinguish between the neutral and charged sector as usual. For the charged sector we find that the diagonalization process is particularly simple since there is no mixing between the $W_\mu$ and $W'_\mu$ gauge bosons.

The charged gauge bosons are defined by

$$W^\pm_\mu = \frac{W^1_\mu \pm \mathrm{i}\, W^2_\mu}{\sqrt{2}} \qquad W'^\pm_\mu = \frac{W'^1_\mu \pm \mathrm{i}\, W'^2_\mu}{\sqrt{2}}, \tag{4.35}$$

and their masses result

$$m_W^2 = \frac{g^2}{2}\, v^2, \qquad\qquad m_{W'}^2 = \frac{g_z^2}{2}\, u^2. \tag{4.36}$$

We see that the mass of the $W$ boson is the same as in the MSSM.

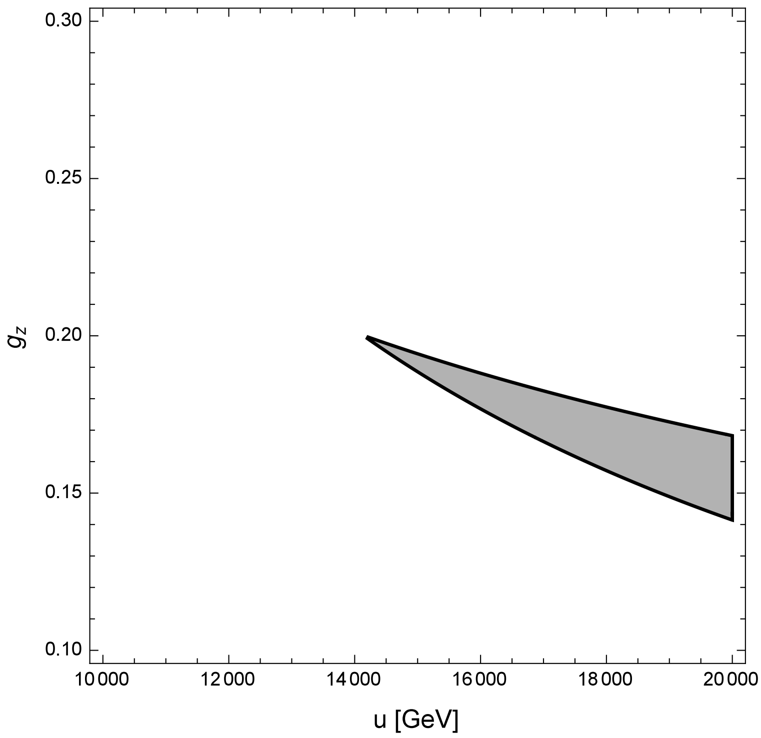


**Figure 4.1:** Possible values for the gauge coupling constant $g_z$ and the VEV $u$ associated to the first stage of the breaking.

For the neutral sector the diagonalization is more complicated since there is mixing between the $B'_\mu$, the $W^3_\mu$ and the $W'^3_\mu$ gauge bosons. In the limit $\frac{v}{u} \ll 1$, the masses of the $Z$ and $Z'$ gauge bosons result

$$\begin{aligned} m_Z^2 &\simeq \frac{(g^2 + g'^2)}{2} v^2 \left(1 - \frac{g'^4}{g_z^4} \frac{v^2}{u^2}\right), \\ m_{Z'}^2 &\simeq \frac{g_z^4 u^2 + g'^4 v^2}{2(g_z^2 - g'^2)}, \end{aligned} \tag{4.37}$$

and, as expected, we get a massless photon. The $\rho$ parameter in the same limit is given by

$$\rho \simeq 1 + \frac{g'^4}{2g_z^4} \frac{v^2}{u^2}. \tag{4.38}$$

We can combine the measured value of $\rho$ with the bounds on searches of the $W'$ gauge boson $m_{W'} > 1.49$ TeV [36], to obtain the allowed region in the parameter space for the $g_z$ and $u$ parameters.

As we can see from Figure 4.1, experimental data tell us that the allowed values for these parameters are a small gauge coupling constant and a heavy VEV. The maximum possible value for the gauge coupling constant $g_z$ is around 0.2, while the minimum possible value for the VEV $u$ is around 14 TeV.

### 4.2.2 Scalar potential and the Higgs bound

Using the gauge charge assignment of Table 4.2 we find the following D-term contribution to the scalar Higgs potential

$$
\begin{aligned}
D_x &= -\frac{g_x}{2}\left(|\phi|^2 - |\bar{\phi}|^2 + |h_u|^2 - |h_d|^2\right), \\
D_z^a &= -\frac{g_z}{2}\left(\phi^\dagger \sigma^a \phi + \bar{\phi}^\dagger \sigma^a \bar{\phi}\right), \\
D_L^a &= -\frac{g}{2}\left(h_u^\dagger \sigma^a h_u + h_d^\dagger \sigma^a h_d\right).
\end{aligned}
\tag{4.39}
$$

This allow us to write the complete scalar potential for the Higgs sector. In addition to the D-term contribution given by the previous equation, the scalar potential will also include the terms given in equation (4.28) and the usual MSSM Higgs contribution. The total scalar potential is therefore given by

$$
\begin{aligned}
V =& \, b\,(h_u h_d + h.c.) + b_\phi\,(\phi\bar{\phi} + h.c.) + \left(m_{h_u}^2 + |\mu|^2\right)|h_u|^2 + \left(m_{h_d}^2 + |\mu|^2\right)|h_d|^2 \\
&+ \left(m_\phi^2 + |\mu'|^2\right)|\phi|^2 + \left(m_{\bar{\phi}}^2 + |\mu'|^2\right)|\bar{\phi}|^2 + \frac{g_x^2}{8}\left(|\phi|^2 - |\bar{\phi}|^2 + |h_u|^2 - |h_d|^2\right)^2 \\
&+ \frac{g_z^2}{8}\left(\phi^\dagger \sigma^a \phi + \bar{\phi}^\dagger \sigma^a \bar{\phi}\right)^2 + \frac{g^2}{8}\left(h_u^\dagger \sigma^a h_u + h_d^\dagger \sigma^a h_d\right)^2.
\end{aligned}
\tag{4.40}
$$

We can now compute the mass matrix for the CP-even Higgs states just as we did in the previous subsection. Notice that the following conditions must hold at the minimum of the scalar potential

$$
\begin{aligned}
8\left(m_{h_u}^2 + |\mu|^2\right) &= g_x^2\, u^2 \cos(2\alpha) + (g^2 + g_x^2)\, v^2 \cos(2\beta) + 8b\cot(\beta), \\
8\left(m_{h_d}^2 + |\mu|^2\right) &= -g_x^2\, u^2 \cos(2\alpha) - (g^2 + g_x^2)\, v^2 \cos(2\beta) + 8b\tan(\beta), \\
8\left(m_\phi^2 + |\mu'|^2\right) &= (g_x^2 + g_z^2)\, u^2 \cos(2\alpha) + g_x^2\, v^2 \cos(2\beta) + 8b_\phi \cot(\alpha), \\
8\left(m_{\bar{\phi}}^2 + |\mu'|^2\right) &= -(g_x^2 + g_z^2)\, u^2 \cos(2\alpha) - g_x^2\, v^2 \cos(2\beta) + 8b_\phi \tan(\alpha),
\end{aligned}
\tag{4.41}
$$

where we have introduced a new angle

$$
\tan(\alpha) \equiv \frac{u_\phi}{u_{\bar{\phi}}}. \tag{4.42}
$$

The $4 \times 4$ CP-even mass matrix can be written in the Higgs basis as

$$
\mathcal{M}_{CP-even}^2 = \begin{pmatrix} \mathcal{M}_h^2 & \mathcal{M}_{h\,\phi}^2 \\ \mathcal{M}_{h\,\phi}^2{}^T & \mathcal{M}_\phi^2 \end{pmatrix}, \tag{4.43}
$$

where

$$
\mathcal{M}_h^2 = \begin{pmatrix} \frac{v^2}{2}(g^2 + g_x^2)\cos^2(2\beta) & -\frac{v^2}{4}(g^2 + g_x^2)\sin(4\beta) \\ \frac{v^2}{4}(g^2 + g_x^2)\sin(4\beta) & \frac{b}{\sin(2\beta)} + \frac{v^2}{2}(g^2 + g_x^2)\sin^2(\beta)\cos^2(\beta) \end{pmatrix}, \tag{4.44}
$$

$$
\mathcal{M}_{h\,\phi}^2 = \begin{pmatrix} -\frac{g_x^2}{2}\, uv \sin(\alpha)\cos(2\beta) & \frac{g_x^2}{2}\, uv \cos(\alpha)\cos(2\beta) \\ -\, g_x^2\, uv \sin(\alpha)\sin(\beta)\cos(\beta) & g_x^2\, uv \cos(\alpha)\cos(\beta)\sin(\beta) \end{pmatrix}, \tag{4.45}
$$

and

$$\mathcal{M}_\phi^2 = \begin{pmatrix} 2b_\phi \cot(\alpha) + \frac{u^2}{2}(g_z^2 + g_x^2)\sin^2(\alpha) & -2b_\phi - \frac{u^2}{2}(g_z^2 + g_x^2)\sin(\alpha)\cos(\alpha) \\ -2b_\phi - \frac{u^2}{2}(g_z^2 + g_x^2)\sin(\alpha)\cos(\alpha) & 2b_\phi \tan(\alpha) + \frac{u^2}{2}(g_z^2 + g_x^2)\cos^2(\alpha) \end{pmatrix}. \tag{4.46}$$

In analogy with the previous subsection, we integrate out the heavy $\phi$ and $\bar{\phi}$ fields and use a seesaw-like formula for the light Higgs mass matrix:

$$\mathcal{M}_h^{2\,eff} = \mathcal{M}_h^2 - (\mathcal{M}_{h\phi}^2)^T (\mathcal{M}_\phi^2)^{-1} \mathcal{M}_{h\phi}^2. \tag{4.47}$$

The expressions in this case are much simpler, and we find

$$\mathcal{M}_h^{2\,eff} = \begin{pmatrix} \frac{v^2}{2}(g^2 + g'^2)\cos^2(2\beta) & \frac{v^2}{2}(g^2 + g'^2)\sin(4\beta) \\ \frac{v^2}{2}(g^2 + g'^2)\sin(4\beta) & \frac{2b}{\sin(2\beta)} + \frac{v^2}{2}(g^2 + g'^2)\sin^2(\beta)\cos^2(\beta) \end{pmatrix}. \tag{4.48}$$

This is a rather surprising result, since what we have obtained is that the extra D-term contribution of equation (4.39) does not leave any mark on the CP-even mass matrix after we integrate out the heavy states. Preliminary computations in which we use additional scalar triplets to drive the $SU(2)_Z \times U(1)_X \to U(1)_Y$ breaking are showing the same conclusion. We suspect that the result can be generalized to a generic representation, and we are currently working to prove this result in full generality.

# Chapter 5

# Conclusions

The discovery of a 125 GeV Higgs complicates the outlook for natural supersymmetry. In the simplest phenomenologically viable realization of supersymmetry, the MSSM, we find a problematic tree level upper bound on the Higgs boson mass $m_h \leq m_Z$, in such a way that large radiative corrections are needed to push the Higgs mass to its observed value of 125 GeV. This requires very heavy stops, with a tuning worse than the percent level.

Due to structure of the scalar potential in supersymmetry, we can address the Higgs mass problem either introducing new trilinear terms in the superpotential (like in the NMSSM) or adding extra gauge groups to the MSSM. If the MSSM higgses are charged under the new forces, then there will be additional D-term contributions to the scalar potential.
Both the new F-terms and D-terms introduce additional quartic Higgs couplings that may in principle increase the Higgs boson mass at tree level, softening the role of radiative corrections and diminishing the tuning.

In this thesis we have studied an extended gauge group $SU(2) \times SU(2) \times U(1)$. While in the literature the symmetry breaking pattern

$$SU(2)_I \times SU(2)_{II} \times U(1)_Y \rightarrow SU(2)_L \times U(1)_Y, \tag{5.1}$$

has been analyzed and shown to increase the tree level Higgs boson mass, we have focused on the pattern

$$SU(2)_L \times SU(2)_Z \times U(1)_X \rightarrow SU(2)_L \times U(1)_Y, \tag{5.2}$$

with breaking driven by two $SU(2)_Z$ doublets. Surprisingly, in this case there is no increase in the tree level Higgs boson mass, once the effect of the heavy doublets are taken into account.

Turning to the future, we can extend the work in two directions. The first one is to prove that the result encountered for the Higgs mass bound is independent of the representation chosen for the scalars $\phi$ and $\bar{\phi}$ that drives the first stage of the symmetry breaking. Preliminary results using two $SU(2)_Z$ triplets point to the same outcome.

Finally, we should study the effects of including the gauge singlet $S$, which was introduced with the only purpose of guarantee the first stage of the symmetry breaking in the supersymmetry limit $m_{soft} \rightarrow 0$. This could have a positive impact on the Higgs mass, since there will be an additional term $\lambda_s \, S \, H_u \, H_d$ in the superpotential that can give an F-term contribution that is similar to a term encountered in the NMSSM ( see equation (3.56)).

After performing these analysis, we will have additional information that should lead us to a more solid conclusion on the idea of increasing the Higgs mass using D-terms.

# Appendix

## .1 The Standard Model Lagrangian

The Standard Model of particle physics is a quantum non-abelian gauge theory, whose gauge symmetry group $G_{SM}$ is composed by the direct product of three simple groups

$$G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y,$$

with $c$ standing for Color, $L$ standing for Left isospin and $Y$ by Hipercharge. The matter content of the theory and the gauge transformations are summarized in Table 1.

The dynamics of the Standard Model is described by a Lagrangian accounting for several sectors, all gauge invariant:

$$\mathcal{L}_{SM} = \mathcal{L}_{matter} + \mathcal{L}_{gauge} + \mathcal{L}_{int} + \mathcal{L}_{Higgs}. \tag{3}$$

We begin by specifying $\mathcal{L}_{matter}$, which corresponds to the Lagrangian that couples the matter content of the theory to the gauge bosons

$$\mathcal{L}_{matter} = (D^\mu H)^\dagger (D_\mu H) + i\bar{\ell}_L \not{D} \ell_L + i\bar{e}_R \not{D} e_R + i\bar{q}_L \not{D} q_L + i\bar{u}_R \not{D} u_R + i\bar{d}_R \not{D} d_R, \tag{4}$$

where $\not{D} = \gamma^\mu D_\mu$, and $D_\mu$ the appropriate covariant derivative according to the gauge transformation properties summarized in Table 1. For example, since the quark doublet $q_L$ transform as $\sim (3, 2, 1/6)$, the covariant derivative for this field is given by

$$D_\mu q_L = \partial_\mu q_L + i\, g_c \lambda^a G^a_\mu q_L + i\, g\, \tau^a W^a_\mu q_L + i\, \frac{g'}{6} B_\mu q_L, \tag{5}$$

where $g_c$ , $g$ and $g'$ are the gauge coupling constant of $SU(3)_c$ , $SU(2)_L$ and $U(1)_Y$ respectively. The Lagrangian given by the last equation is given only for one generation, adding the other generations only involves adding two more copies of the Lagrangian with proper names for the field.

The kinetic term for the gauge bosons are given by the Lorentz invariant combination of abelian and non-abelian strength tensors. It follows that $\mathcal{L}_{gauge}$ is given by

$$\mathcal{L}_{gauge} = -\frac{1}{4} G^{\mu\nu a} G^a_{\mu\nu} - \frac{1}{4} W^{\mu\nu a} W^a_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}, \tag{6}$$

with

$$\begin{aligned} G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu, \\ W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu. \end{aligned} \tag{7}$$

| Type | Notation | $SU(3)_C$ , $SU(2)_L$ , $U(1)_Y$ |
|---|---|---|
| Quarks (×3 families) | $q_L$ | (**3**, **2**, 1/6) |
| | $u_R$ | (**3**, **1**, 2/3) |
| | $d_R$ | (**3**, **1**, -1/3) |
| Leptons (×3 families) | $\ell_L$ | (**1**, **2**, -1/2) |
| | $e_R$ | (**1**, **1**, -1) |
| Higgs | $H$ | (**1**, **2**, 1/2) |
| Gauge bosons | $B$ | (**1**, **1**, 0) |
| | $W$ | (**1**, **3**, 0) |
| | $G$ | (**8**, **1**, 0) |

**Table 1:** Fields of the Standard Model with their gauge transformation properties. The bosons of the SM are a spin-0 field (the Higgs) and three spin-1 fields, while the fermions are all spin 1/2 fields represented by 4-component Dirac spinors. The subscripts $L$ and $R$ are to distinguish between left and right handed fields.

The $a$ index runs over the adjoint representation.

The possible gauge invariant interactions between SM particles are contained in $\mathcal{L}_{int}$, which corresponds to the Yukawa interactions

$$\mathcal{L}_{int} = -\bar{\ell}_L\, Y_e\, H\, e_R - \bar{q}_L\, Y_d\, H\, d_R - \bar{q}_L\, Y_u\, \tilde{H}\, u_R, \tag{8}$$

where $Y_e$ , $Y_d$ and $Y_u$ are the Yukawa matrices in flavor space[1], and $\tilde{H} \equiv i\sigma_2\, H^*$ .

Finally, the Higgs sector consist of the potential responsible for breaking spontaneously the electroweak symmetry down to electromagnetism

$$\mathcal{L}_{Higgs} = -\mu^2\, (H^\dagger\, H) + \frac{\lambda}{4}\, (H^\dagger\, H)^2, \tag{9}$$

where $\mu^2$ and $\lambda$ are positive constants of mass dimension $[mass]^2$ and $[mass]^0$ respectively.

## .2 Dotted/undotted notation and Weyl spinors

The restricted Lorentz group $SO(1,3)^+$ is a subgroup of the full Lorentz group and it consists of all Lorentz transformations that can be connected to the identity by a continuous curve lying in the group, therefore it does not contain time or space reflection. An important fact is that there is an homomorphism[2] between the restricted Lorentz group and the special linear group $SL(2,\mathbf{C})$, which consist of the group of all of the $2\times 2$ complex matrices with determinant equal to one.

[1] The family indices are suppressed.

[2] Homomorphism is an structure-preserving map between the two groups.

Spinors are usually defined as objects transforming under the Lorentz group but they can as well be defined as the objects transforming under the basic representations of $SL(2,\mathbf{C})$. Therefore we will define a spinor as a two complex component object transforming under an element $M$ of $SL(2,\mathbf{C})$

$$\psi_\alpha \to {\psi_\alpha}' = M_\alpha^{\ \beta}\, \psi_\beta, \tag{10}$$

with $\alpha\,,\,\beta = 1\,,\,2$ labeling the two components, $\psi_\alpha$ is called a left-handed Weyl spinor. Since for $SL(2,\mathbf{C})$ a representation and its complex conjugate are not equivalent, then $M$ and $M^*$ gives different representations. A two-component object $\psi^\dagger$ transforming as

$$\psi^\dagger_{\dot\alpha} \to {\psi^\dagger_{\dot\alpha}}' = M^{*\ \dot\beta}_{\dot\alpha}\, \psi^\dagger_{\dot\beta}, \tag{11}$$

is called a right-handed Weyl spinor[1]. Dotted and undotted spinor indices are used to refering to these two different representations. These are called the fundamental and conjugate representation of $SL(2,\mathbf{C})$. It is possible to raise and lower the dotted and undotted indices by using the antysimmetric matrix

$$\epsilon_{\alpha\,\beta} = \epsilon_{\dot\alpha\,\dot\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{12}$$

$$\epsilon^{\alpha\,\beta} = \epsilon^{\dot\alpha\,\dot\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{13}$$

They act on the spinors as

$$\psi^\alpha = \epsilon^{\alpha\beta}\,\psi_\beta \qquad \psi_\alpha = \epsilon_{\alpha\beta}\,\psi^\beta \qquad \psi^{\dagger\dot\alpha} = \epsilon^{\dot\alpha\,\dot\beta}\,\psi^\dagger_{\dot\beta} \qquad \psi^\dagger_{\dot\alpha} = \epsilon_{\dot\alpha\,\dot\beta}\,\psi^{\dagger\dot\beta}. \tag{14}$$

Let us introduce the Weyl (or chiral) basis of the gamma matrices

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}, \tag{15}$$

where

$$\sigma^\mu = (I\,,\,\sigma_i) \qquad\qquad \bar\sigma^\mu = (I\,,\,-\sigma_i). \tag{16}$$

Here $\sigma^i$ are the usual Pauli matrices. A Dirac spinor is given in terms of Weyl spinors according to

$$\Psi_D = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot\alpha} \end{pmatrix} \qquad\qquad \overline{\Psi}_D = \begin{pmatrix} \chi^\alpha & \xi^\dagger_{\dot\alpha} \end{pmatrix}. \tag{17}$$

From the kinetic Dirac Lagrangian term

$$\bar\psi_D\,\gamma^\mu\,\partial_\mu\,\psi_D, \tag{18}$$

it is possible to show that the correct dotted and undotted structure for the $\sigma$ matrices is $(\,\sigma^\mu\,)_{\alpha\,\dot\alpha}$ and $(\,\bar\sigma^\mu\,)^{\dot\alpha\,\alpha}$. Therefore the heights of the spinor indices are important.

[1]Here $\dot\alpha, \dot\beta = 1, 2$ are use to label the two components of $\psi^\dagger$.

Because of the fact that $\epsilon^{\alpha\beta}$ is antisymmetric, if we want to suppress indices and write an expression like $\xi\,\psi$ we must specify what do we mean, therefore we define

$$\xi\,\psi \;\equiv\; \xi^{\alpha}\,\psi_{\beta} \qquad\qquad \xi^{\dagger}\,\psi^{\dagger} \;\equiv\; \xi^{\dagger}_{\dot{\alpha}}\;\psi^{\dagger\dot{\alpha}}, \tag{19}$$

also notice that

$$\begin{aligned} \xi^{\alpha}\,\psi_{\alpha} \;&=\; \epsilon^{\alpha\beta}\,\xi_{\beta}\;\epsilon_{\alpha\lambda}\,\psi^{\lambda}, \\ &=\; -\xi_{\beta}\,(\epsilon^{\beta\alpha}\,\epsilon_{\alpha\lambda})\,\psi^{\lambda}, \\ &=\; -\xi_{\beta}\,\psi^{\beta}. \end{aligned} \tag{20}$$

Here we have used that $\epsilon^{\alpha\beta}\,\epsilon_{\beta\gamma} \;=\; \delta^{\alpha}_{\gamma}$ in the second line, we therefore postulate that the components of spinors are Grassmann variables and demand

$$\begin{aligned} \{\psi_{\alpha}\,,\,\psi^{\beta}\} \;=\; \{\psi_{\alpha}\,,\,\psi_{\beta}\} \;=\; \{\psi^{\alpha}\,,\,\psi^{\beta}\} \;=\; 0, \\ \{\xi^{\dagger}_{\dot{\alpha}}\,,\,\xi^{\dagger\dot{\beta}}\} \;=\; \{\xi^{\dagger}_{\dot{\alpha}}\,,\,\xi^{\dagger}_{\dot{\beta}}\} \;=\; \{\xi^{\dagger\dot{\alpha}}\,,\,\xi^{\dagger\dot{\beta}}\} \;=\; 0, \end{aligned} \tag{21}$$

and all mixed anticommutators vanish too

$$\{\,\psi_{\alpha}\,,\,\xi^{\dagger}_{\dot{\beta}}\,\} \;=\; 0\,,\;\; \text{etc.} \tag{22}$$

These equations implies that $\psi\,\xi \;=\; \xi\,\psi$ , $\psi^{\dagger}\,\xi^{\dagger} \;=\; \xi^{\dagger}\,\psi^{\dagger}$ and that

$$(\xi\,\sigma^{\mu}\,\psi^{\dagger})^{\dagger} \;=\; \psi\,\sigma^{\mu}\,\xi^{\dagger}, \tag{23}$$

$$(\psi^{\dagger}\,\bar{\sigma}^{\mu}\,\xi)^{\dagger} \;=\; \xi^{\dagger}\,\bar{\sigma}^{\mu}\,\psi, \tag{24}$$

which are useful equations when computing the complex conjugate of fermionic bilinears.

## .2.1 Spinor identities

It is possible to exchange the order of spinors in a bilinear by means of the identities

$$\psi\,\sigma^{\mu}\,\xi^{\dagger} \;=\; -\,\xi^{\dagger}\,\bar{\sigma}^{\mu}\,\psi, \tag{25}$$

$$\xi\,\sigma^{\mu}\,\bar{\sigma}^{\nu}\,\psi \;=\; \psi\,\sigma^{\nu}\,\bar{\sigma}^{\mu}\,\xi, \tag{26}$$

$$\xi\,\sigma^{\mu\nu}\,\psi \;=\; -\,\psi\,\sigma^{\mu\nu}\,\xi, \tag{27}$$

$$\xi^{\dagger}\,\bar{\sigma}^{\mu\nu}\,\psi^{\dagger} \;=\; -\,\psi^{\dagger}\,\bar{\sigma}^{\mu\nu}\,\xi^{\dagger}, \tag{28}$$

where

$$(\sigma^{\mu\nu})_{\alpha}^{\;\;\beta} \;\equiv\; \frac{1}{4}\,(\sigma^{\mu}\bar{\sigma}^{\nu} \,-\, \sigma^{\nu}\,\bar{\sigma}^{\mu})_{\alpha}^{\;\;\beta}\,, \qquad\qquad (\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\;\;\dot{\beta}} \;\equiv\; \frac{1}{4}\,(\bar{\sigma}^{\mu}\,\sigma^{\mu} \,-\, \bar{\sigma}^{\nu}\,\sigma^{\mu})^{\dot{\alpha}}_{\;\;\dot{\beta}}\,. \tag{29}$$

All these identities are useful when working with the superspace formalism introduced in Chapter 2.

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