Results of the GICS Experiments

19 March 2024

1 Global Industrial Classification Standard (GICS) Experiment Results

The dataset analysed here is that discussed in Kocuk and Córnuejols (2020).

The following is the pre-processing conducted by the authors:

"We first collected historical stock returns and market capitalization information from the Wharton Research Database Services (WRDS). Since working with tens of thousands of different stocks is not appropriate for this study, we focused on stocks in the S&P 500 index. Following Bertsimas, Gupta, Paschalidis (2012), we further simplified our analysis by focusing on 11 sectors according to the Global Industrial Classification Standard (GICS). As a consequence of this simplification, we do not need to keep track of the assets that enter or leave the S&P 500, rather we concentrate on the overall performance of each sector as the average performance of its constituents. Using WRDS, we collected the return and market capitalization information for all the stocks that have been in the S&P 500 between January 1987- December 2016, spanning a 30-year period. We then computed the return of a sector for each month as a weighted average of the returns of the companies in S&P 500 in that particular time period, where the weights are taken as the market capitalization of each stock in that sector. This procedure gave us 360 sector return vectors of size 11, denoted by $R_t, t = 1, \ldots, 360$. We also recorded the percentage market capitalization of each sector j in month t, denoted by $M_j^t,\ j = 1, \ldots, 11, t = 1, \ldots, 360$.

We use an R package called MVN (Korkmaz and Goksuluk and Zararsiz (2014)) to formally test the multivariate normality of the sector return vectors. We also test whether the returns of individual sectors are normally distributed. According to our extensive tests, we conclude that there is significant evidence that the neither the sector return vector nor the returns of individual sectors are normally distributed as expected."

1.1 Evaluating the effects of estimating mean return using historic time-series data with the 2-norm

We take the average of the the past N datapoints to estimate the mean return each sector. That is, given today's index i, we estimate the mean return $\hat{\mu}^i(N) = \frac{1}{N}\sum_{j=i-N}^{i-1} \mu^j$. We vary $N \leq 120$ and take the 2-norm between the estimated return for $\hat{\mu}^i(N)$ and the true return for $\mu = \frac{1}{360}\sum_{j=1}^{360} \mu^j$,

$||\hat{\mu}^i(N) - \mu||_2$. We then report the mean of the norms over $i = 121, \ldots, 360$, that is $\frac{1}{240}\sum_{i=121}^{360} ||\hat{\mu}^i(N) - \mu||_2$ for each N. The results of this experiment are given in Figure 1 ("2-norm.pdf") and Figure 2 ("`2-norm_stdev.pdf`"), where the latter is includes the standard deviation of the norms over $i = 121, \ldots, 360$ as error bars.

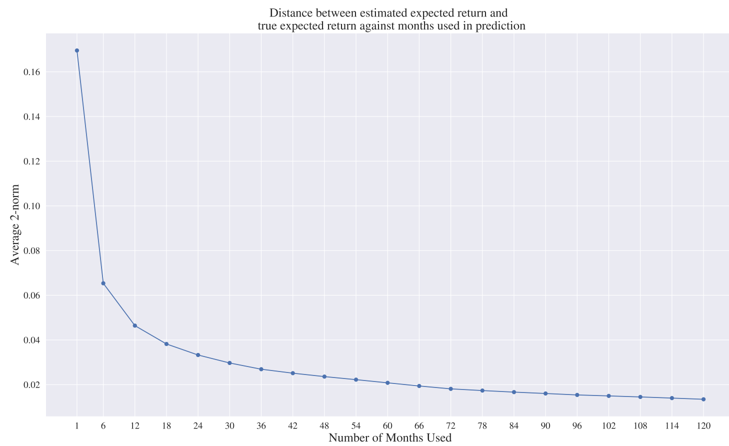

Figure 1: Mean 2-norm over the number of months used to estimate the true mean.

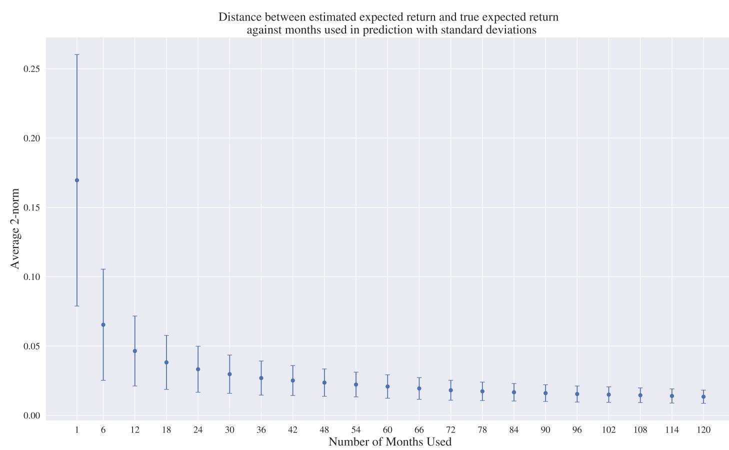

Figure 2: Mean 2-norm over the number of months used to estimate the true mean with standard deviations.

We observe from the figures that using more months to estimate the true mean μ does indeed result in a better estimate (we no longer see a significant plateau as we saw previously, where we were considering $\mu = \mu^{i+6}$).

1.2 Evaluating the effects of estimating mean return using historic time-series data with mean-variance portfolio optimization

We take the average of the the past N datapoints to estimate the mean return each sector. That is, given today's index i, we estimate the mean return $\hat{\mu}^i(N) = \frac{1}{N}\sum_{j=1}^{N}\mu^j$. We vary $N \leq 120$. We find the portfolio that maximizes the return with estimated mean return $\hat{\mu}$ and the true estimated return $\mu = \frac{1}{360}\sum_{j=1}^{360}\mu^j$.

The frontiers are defined as follows:

- True Frontier: $\mu^\top x^*$ for different risk tolerance levels v, where $x^* = \underset{x\in\mathcal{X}}{\operatorname{argmax}} \mu^\top x$.
- Estimated Frontier: $\frac{1}{240}\sum_{i=121}^{360}(\hat{\mu}^i(N)^\top \hat{x}^*)$ for different risk tolerance levels v, where $\hat{x}^* = \underset{x\in\mathcal{X}}{\operatorname{argmax}} \hat{\mu}^i(N)^\top x$ for each $i = 121, \ldots, 360$ and N.
- Actual Frontier: $\frac{1}{240}\sum_{i=121}^{360}(\mu^\top \hat{x}^*)$ for different risk tolerance levels v, where $\hat{x}^* = \underset{x\in\mathcal{X}}{\operatorname{argmax}} \hat{\mu}^i(N)^\top x$ for each $i = 121, \ldots, 360$ and N.

All portfolios were optimized using Gurobi Optimizer version 11.0.1 and programmed on Python version 3.12.2.

The results can be found in Figure 3 ("frontier_plot.pdf"). Figure 4 ("frontier_plot_zoomed.pdf") shows the same plot focused on the actual frontiers.

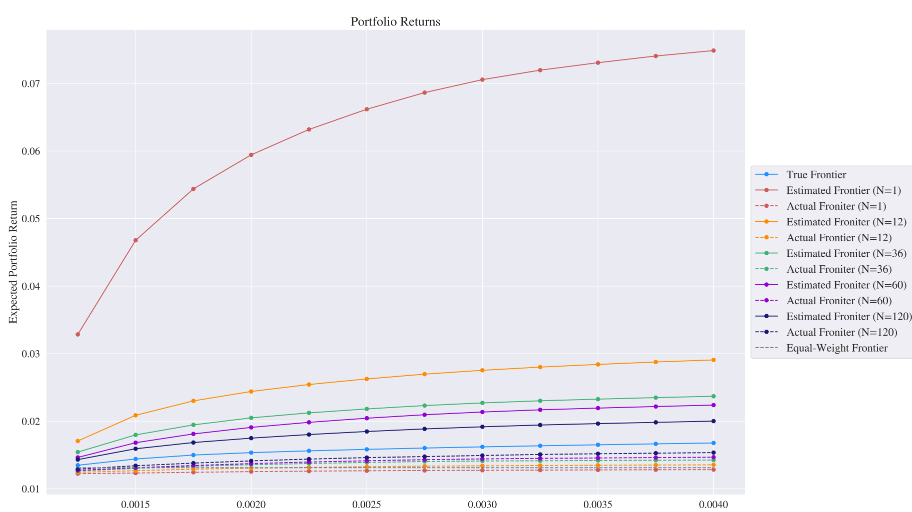

Figure 3: Mean 2-norm over the number of months used to estimate the true mean.

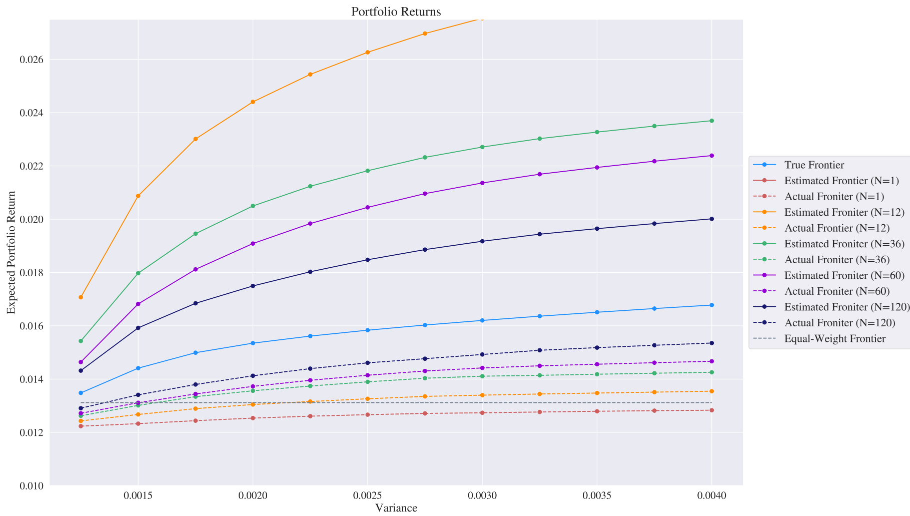

Figure 4: Mean 2-norm over the number of months used to estimate the true mean with standard deviations.

Again, we see that using more historical data actually improves the portfolio constructed. The gap between the estimated frontier and actual frontier is smaller, and the perfomance of the actual frontier is better when N is larger.