Ecuaciones de movimiento en forma covariante

$$\boxed{\frac{d}{d\tau}\left(\frac{\partial L}{\partial u_\mu}\right) - \frac{\partial L}{\partial x_\mu} = 0} \quad \text{Ecs. E-L}$$

$A^\nu = A^\nu(x_\mu)$ depende de x_μ

$u_\nu = u_\nu(\tau)$ depende de τ

Lagrangiano completo covariante

$$L = -\frac{1}{2} m \cdot \underbrace{u^\mu u_\mu}_{\text{índices mudos}} - q \underbrace{A^\mu u_\mu}$$

$$L = -\frac{1}{2} m g^{\mu\sigma} u_\sigma u_\mu - q A^\mu u_\mu$$ μ, σ: mudos

$$\boxed{L = -\frac{1}{2} m g^{\nu\sigma} u_\sigma u_\nu - q A^\lambda u_\lambda}$$

⊛

$$\underline{\underline{\frac{\partial L}{\partial u_\mu}}} = -\frac{1}{2} m g^{\nu\sigma} \frac{\partial}{\partial u_\mu}(u_\sigma u_\nu) - qA^\lambda \underbrace{\frac{\partial u_\lambda}{\partial u_\mu}}_{\delta^\mu_\lambda \text{ Kronecker}} =$$

$$= -\frac{1}{2} m g^{\nu\sigma} \overbrace{\frac{\partial u_\sigma}{\partial u_\mu}}^{\delta^\mu_\sigma} u_\nu - \frac{1}{2} m g^{\nu\sigma} u_\sigma \overbrace{\frac{\partial u_\nu}{\partial u_\mu}}^{\delta^\mu_\nu} - q \underbrace{A^\lambda \delta^\mu_\lambda}_{A^\mu}$$

$$= -\frac{1}{2} m \underbrace{g^{\nu\sigma} \delta^\mu_\sigma}_{g^{\nu\mu}} u_\nu - \frac{1}{2} m \overbrace{g^{\nu\sigma} u_\sigma \delta^\mu_\nu}^{g^{\mu\sigma}} - qA^\mu =$$

$$= -\frac{1}{2} m \underbrace{g^{\nu\mu} u_\nu}_{u^\mu} - \frac{1}{2} m \underbrace{g^{\mu\sigma} u_\sigma}_{u^\mu} - qA^\mu =$$

$$= -\frac{1}{2} m u^\mu - \frac{1}{2} m u^\mu - qA^\mu = \underline{\underline{-mu^\mu - qA^\mu}}$$

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$$\frac{\partial L}{\partial x_\mu} = -q \frac{\partial A^\lambda}{\partial x_\mu} u_\lambda$$

u_ν no depende de x_μ

$$\boxed{\frac{d}{d\tau}\left(\frac{\partial L}{\partial u_\mu}\right) - \frac{\partial L}{\partial x_\mu} = 0}$$

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$$\frac{d}{d\tau}\left(-mu^\mu - qA^\mu\right) + q\frac{\partial A^\lambda}{\partial x_\mu} u_\lambda = 0$$

$$-m\frac{du^\mu}{d\tau} - q\frac{dA^\mu}{d\tau} + q\frac{\partial A^\lambda}{\partial x_\mu} u_\lambda = 0$$

$$\frac{\partial A^\mu}{\partial x_\rho}\frac{dx_\rho}{d\tau} \quad \frac{dx_\rho}{d\tau} = u_\rho$$

regla de la cadena

$$-m\frac{du^\mu}{d\tau} - q\frac{\partial A^\mu}{\partial x_\rho} u_\rho + q\frac{\partial A^\lambda}{\partial x_\mu} u_\lambda = 0$$

mudo

$$m\frac{du^\mu}{d\tau} = q\left(\frac{\partial A^\lambda}{\partial x_\mu}u_\lambda - \frac{\partial A^\mu}{\partial x_\rho}u_\rho\right)$$

mudo

$$m\frac{du^\mu}{d\tau} = q\left(\frac{\partial A^\nu}{\partial x_\mu}u_\nu - \frac{\partial A^\mu}{\partial x_\nu}u_\nu\right)$$

$$m\underbrace{\frac{du^\mu}{d\tau}}_{\frac{d^2x^\mu}{d\tau^2}} = q\underbrace{\left(\frac{\partial A^\nu}{\partial x_\mu} - \frac{\partial A^\mu}{\partial x_\nu}\right)}_{F^{\mu\nu}}u_\nu$$

$$\boxed{m\frac{d^2x^\mu}{d\tau^2} = qF^{\mu\nu}u_\nu}$$