# Stochastic Models
# Lecture notes

2024

Apuntes de las clases de *Stochastic Models* dadas por *Dave Kaplan* y transcritos a LaTeX por *Víctor Mira Ramírez* durante el curso 2023-2024 del grado en Física de la *Southern Illinois University*.

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# Chapter 1

# Introduction

What is Operations Research? A way to use existing resources more efficiently. Efficience is the key of this lectures. We are going to talk about mathematical, technical and everyday models, but what is a model? A model is a selective representation of reality. We are going to talk from models ranging from simulation ones (aerodynamics) to economical models or industrial ones.

The reasons why we use models may seem obvious, some of them may be time efficiency, safety... In the end the objective is to predict outcomes and optimize how we act.

## 1.1 Types of models

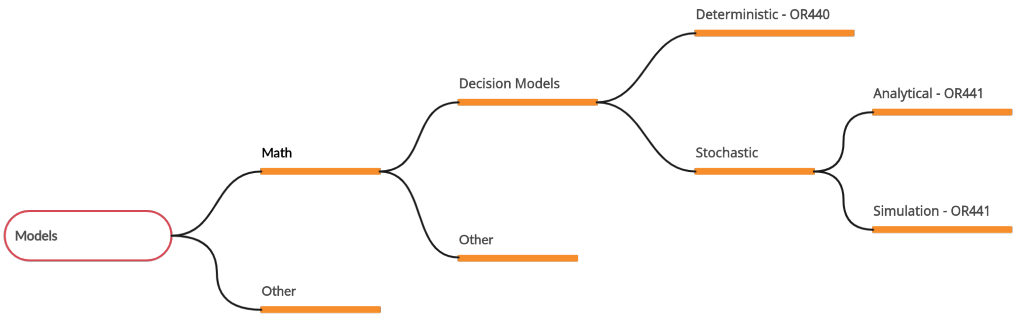
According to the type of data we have, we talk about two types of models

- **Deterministic**: All relevant data is assumed to be known with certainty.
- **Stochastic**: Some of the data are uncertaint

Models could be considered as deterministic and stochastic according to the variables that you are considering.

## 1.2 Modeling approach

1. Formulate the problem
2. Construct a mathematical model
3. Derive a solution from the model
4. Estsablish control over the solution
5. Implement the solution



# Chapter 2

# Decision Making Under Uncertainty

## 2.1 News Vendor Problem

How much production should you allocate in advance before knowing how much demand there's going to be? We should make a model.

### 2.1.1 Introduction

Nature is not against you, the outcome is indifferent. The problem structure will be:

1. Decision: You select an action $a_i$ from the set of possible actions $A = \{a_1, a_2, \ldots, a_k\}$.
2. A state of nature $s_j$ occurs from a set of possible states $S = \{s_1, s_2, \ldots, s_n\}$.
3. $p_j$ = probability that state $s_j$ is observed.
4. You recieve a reward $r_{ij}$ based on $a_i$ and $s_j$.

Well our next step should be to build a Reward Table for the problem, in which in return you recieve for each possible combination of decission and states of nature.

| Decision | State of Nature | | | | Minumum Return | Maximum Return |
|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | -10 | 15 | 15 | 15 | -10 | 15 |
| 2 | -20 | 5 | 30 | 30 | -20 | 30 |
| 3 | -30 | -5 | 20 | 45 | -30 | 45 |

Table 2.1: Reward Table

### 2.1.2 The Maximin Criterion

This criterion chooses the action that minimizes the worst outcome. We see that the minimum risk option is buying no newspapers, as the minimum return for all the others is negative (if you can't sell a single newspaper). This is extremely risk averse.

### 2.1.3 The Maximax Criterion

On the other hand, this criterion maximizes the best outcome. In this case we see that the maximum outcome would come from buying three newspapers (and selling all of them). This is extremelly optimistic.

### 2.1.4 The Minimax Regret Criterion

This criterion minimizes the maximum regret, being regre the extra amount you could have obtained if you had known the state of nature beforehand. The table is computed by substracting actual returns from best returns.

| Decision | State of Nature | | | | Maximum Regret |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | |
| 0 | 0 | 15 | 30 | 45 | 45 |
| 1 | 10 | 0 | 15 | 30 | 30 |
| 2 | 20 | 10 | 0 | 15 | 20 |
| 3 | 30 | 20 | 10 | 0 | 30 |

Table 2.2: Regret Table

In this case we see that the minimum regret would be obtained if you bought 2 newspapers. This criterion is way more reasonable and approaches a more sensitive solution.

### 2.1.5 The Expected Value Criterion

This criterion chooses the action that maximizes the expected return. Expected return if we choose action $a_i$ is:

$$ER_i = \sum_{j=1}^{p} r_{ij}p_j = r_{i1}p_1 + r_{i2}p_2 + \cdots + r_{in}p_n$$

For our case:

$$ER_0 = 0(0.1) + 0(0.3) + 0(0.4) + 0(0.2) = 0$$
$$ER_1 = -10(0.1) + 15(0.3) + 15(0.4) + 15(0.2) = 12.5$$
$$ER_2 = -20(0.1) + 5(0.3) + 30(0.4) + 30(0.2) = 17.5$$
$$ER_3 = -30(0.1) - 5(0.3) + 20(0.4) + 45(0.2) = 12.5$$

| Decision | State of Nature | | | | Expected Return |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | -10 | 15 | 15 | 15 | 12.5 |
| 2 | -20 | 5 | 30 | 30 | 17.5 |
| 3 | -30 | -5 | 20 | 45 | 12.6 |

Table 2.3: Expected Return Table

**Ejemplo 2.1.1** (Real Estate Developement)

| Decision | State of Nature | | | Minimum Return | Maximum Return | Expected Return | Minimum Regret |
|---|---|---|---|---|---|---|---|
| | None | Med | Large | | | | |
| Residential | 4 | 16 | 12 | 4 | 16 | 12.4 | 3 |
| Comertial 1 | 5 | 6 | 10 | 5 | 10 | 7 | 10 |
| Comertial 2 | -1 | 4 | 15 | -1 | 15 | 6.3 | 12 |
| Probability | 0.2 | 0.5 | 0.3 | | | | |

## 2.2 Decision Trees

Decision Trees are a graphical representation of a decision problem. They are useful for a sequence of decisions. Legend:

- **Decision Fork** Decision point (square nodes)
- **Event Fork** Uncertain outcomes (circle nodes)
- **Square Lines** Possible decisions
- **Circle Lines** Possible outcomes

**Ejemplo 2.2.1** (Decision Tree fo the News Vendor Problem)
*tree*

But trees are not giving us more information than tables. Then why use them? well let's see. We'll use a