Experiment 3:

Laser Beam Caracterization

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Abstract

Contents

1 Experiments

1.1 Power transmission through an iris

Task:

Show that the laser power transmitted (P_{trans}) through an iris (pinhole) of radius a is given by

$$P_{\text{trans}} = P_0 \left[1 - e^{-2(a/w)^2}\right]$$

Where P_0 is the total laser power incident on the iris (or pinhole) and w is the radius of the Gaussian laser beam at the location of the iris. Focus the laser beam using a short focal length lens (10 cm). Place the iris and the detector on a translational stage. Place the iris at an appropriate distance from the lens to obtain at least 1 cm beam diameter at the iris plane. This is also necessary for task 2. Record the transmitted power as a function of iris radius. Plot P_{trans} vs. a then fit the data using the relation given above. From the fit you should be able to find the radius of the beam at the iris location.

To derive the expression of the power transmitted through an iris of radius a, we will start by recalling the expression for the intensity profile of a Gaussian laser beam:

$$I(\rho) = I_0 e^{-2(\rho/w)^2} \tag{1}$$

Moreover, the total power of the beam is related to the maximum irradiance at $\rho = 0$ by the expression:

$$P_0 = \frac{1}{2} I_0 \pi w^2 \tag{2}$$

We know that the power is intensity over area, then by integrating this expression we should get the power.

$$P(a) = \int_S I(\rho) dA = \int_0^a I(\rho)\, \hat{\mathbf{n}} dr = \int_0^a I(\rho) 2\pi r\, d\rho = \int_0^a 2\pi r I_0 e^{-2(\rho/w)^2}\, d\rho = 2\pi I_0 \int_0^a \rho e^{-2(\rho/w)^2}\, d\rho$$

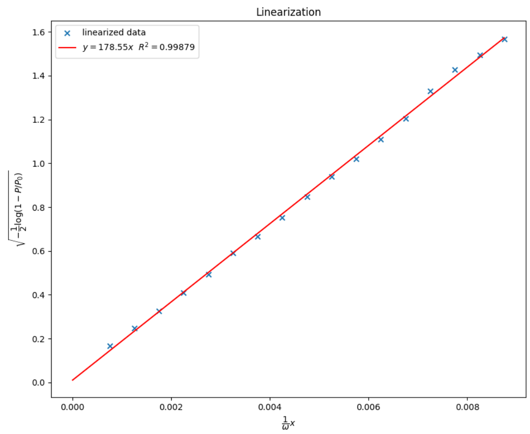
We'll use the change of variable $\quad u = -2\left(\rho/w\right)^2 \longrightarrow du = -{}^{4\rho}/_{w^2}\, dr \Longrightarrow d\rho = -{}^{w^2}/_{4\rho}\, du$

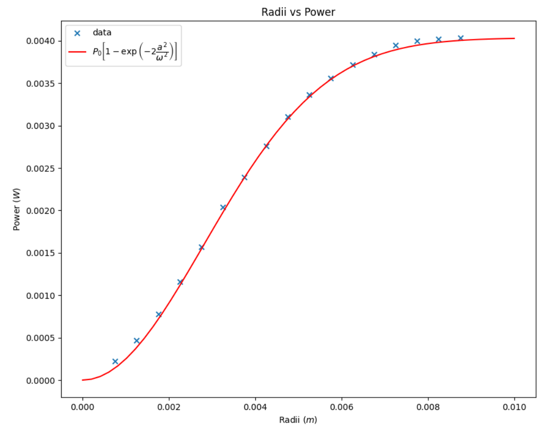
$$2\pi I_0 \int_0^{-2(a/w)^2} -\frac{w^2}{4} e^u\, du = -\pi I_0 \frac{w^2}{2} \left[e^u\right]_0^{-2(a/w)^2} = -P_0 \left(e^{-2(a/w)^2} - e^0\right) = P_0 \left(1 - e^{-2(a/w)^2}\right)$$

Finally, we obtained the expression that we were looking for the power transmitted through an iris of radius a:

$$P_{\text{trans}} = P(a) = P_0 \left(1 - e^{-2(a/w)^2}\right) \tag{3}$$

The data recorded during the procedure described by the task generated the left plot after it was linearized and fitted with a slope of $1/\omega$ to obtain a beam radius of $\omega = 0.0056m$. With that knowledge, the original data was fitted to obtain the right plot.





1.2 Measure the profile of a diverging laser beam using iris or pinhole

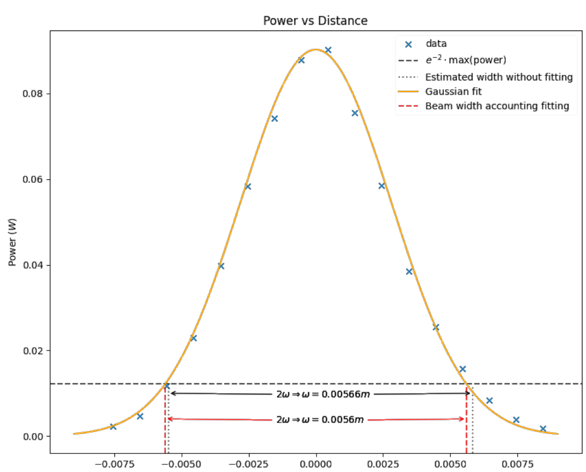
Use the same configuration in task 1. Use an iris to scan across the laser beam at the same location you used for task 1. Make the iris opining as small as possible. Make sure to move the iris and the detector at the same time (on the same stage). This is why you need to put the detector and the iris on the same translational stage. Plot the power as a function of the transverse distance cross the beam. Discuss the profile.

Task:

From the profile determine the beam radius at the location of the iris. Recall that the Gaussian beam radius is half the width of the profile measured at $1/e^2$ of the maximum power. Compare this result to that obtained in task 1.

Task:

Now fit your profile (experimental data) to a Gaussian and determine w at the location of the iris.



First of all, we drew a horizontal line at P_0/e^2 to be able to obtain the beam radius. In the plot, the black lines represent the first part of the task, at which the data is still not fitted and the limits at which the data cross this horizontal line are discrete. With this method we obtained a beam radius of $\omega = 0.00566m$, which is very close to the obtained in task 1 without using any fitting.

After fitting the data with a gaussian, we can clearly see where the horizontal line intersects the gaussian and obtain a value of ω that is exactly the same as in task 1 $\omega = 0.0056m$.

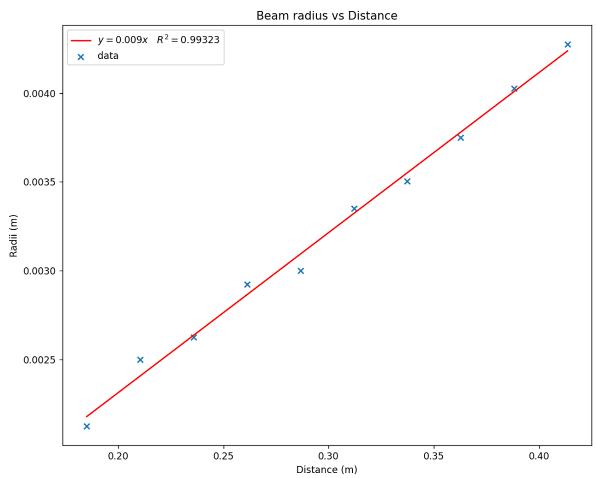
1.3 Measure the laser beam divergence

Task:

Measure the laser radius of the focused He-Ne laser that you used in tasks 1 and 2 at 5 locations from the laser focus. Use the method of task 1 (use only one iris diameter appropriate for the size of the beam at location of the iris). From your data determine the beam radius for each location. Plot the beam radius vs. distance **from the beam waist**. Determine the beam divergence angle from your plot.

We took 10 data points instead of 5, giving us hopefully a more sensible result once we fit the data. In the plot we can see the beam radius of each location by looking at its corresponding y-axis value. However they are presented in the following table.

To obtain a value for the divergence angle, we will use the formula $\theta = \lambda / \pi\omega_0$, where ω_0 is the slope of the fitting and λ is the wavelength of our laser, $\lambda = 632.8nm$. For our fitting, the value of the beam divergence angle in our setup was $\theta = 2.368 \cdot 10^{-5}$ rad $\iff \theta = 0.0013567641°$



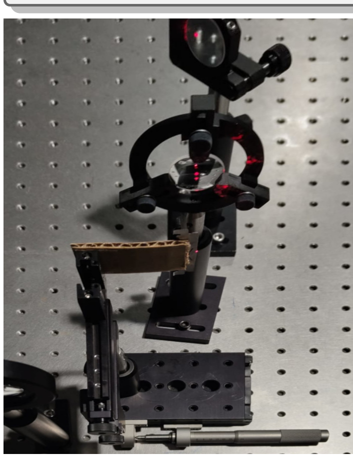
Distance (m)	0.1850	0.2104	0.2358	0.2612	0.2866	0.3120	0.3374	0.3628	0.3882	0.4136
Radii (m)	0.002125	0.002500	0.002625	0.002925	0.003000	0.003350	0.003505	0.003750	0.004025	0.004275

Table 1: Data table

1.4 Determine the waist location for a focused laser beam using a razor blade

Task:

Keep the lens of the previous tasks in place and move the detector and everything else. Place a razor blade on two translation stages such that you can move them perpendicular to each other. Put the razor blade as close as possible to the focus. Translate the razor blade across and along the laser beam and observe the diffraction pattern. You should be able to determine the location of the focus relative to the center of the lens. Report your observations and explain.



As we can see in the picture, a razor blade was placed on a cardboard piece and glued to two translational stages so that the focus could be located with the maximum accuracy possible.

This was achieved by looking at its diffraction pattern and trying to get the pattern as symmetric as possible, so as to ensure that the razor blade was placed perfectly on the focus.

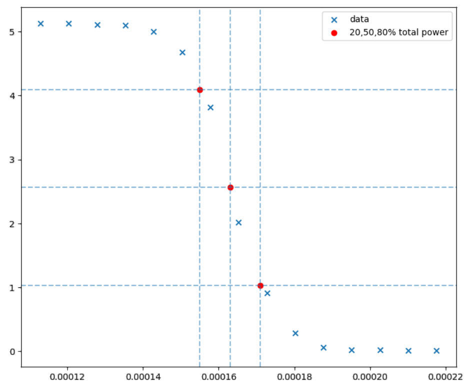
Once the focus was found, the distance between the lens and the razor blade was measured, by measuring the distance from the razor blade to the lens frame and then adding the distance from the frame to the center of the lens itself, obtaining a final value of $d = 5.66\ cm$

1.5 Measure Gaussian beam profile using knife-edge scan

Mount the razor blade on the micrometer stage and place it exactly at the focus as in Task 4. Orient the blade such that it can be translated across the laser beam. Place a power meter in the path of the laser beam. You may need to place a lens immediately after the blade to collect all the laser light by the power meter detector.

Task:

Measure the beam power as a function of the razor blade position for at least 20 positions and plot your data. Be sure to record the total power and the positions of the blade when the transmitted power is equal to 20%, 50%, and 80% of the total beam power. Be sure to normalize the data (power/total power) when you plot power vs. position.



The recorded total power found was 5.12 mW, and the power at 80,50 and 20% were 15.5, 16.3 and 17.1 mW respectively. With this information and multiplying the normalized data with the total power we get this plot, with miliwatts on the OY axis and meters ond the OX.

Task:

The data can be fitted to the following function which represents the normalized power as function of the blade position.

$$P_{\text{normalized}}(x) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{x - x_0}{w}\right)\right]$$

In the above equation, erf is the error function, x_0 is the position at which the power is 50% of the total power of the beam.

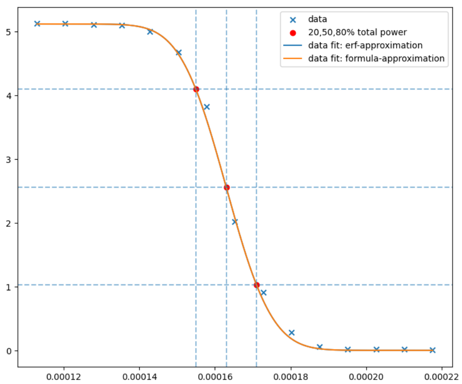
The erf is defined as

$$erf(s) = \frac{2}{\sqrt{\pi}} \int_0^s e^{-t^2}\, dt$$

As you can see, evaluating this integral for each of the x values is not simple and must be done numerically.

Another way to obtain the beam radius is by taking the derivative of power with respect to the blade position. *Plot the derivative of the power vs. position data* and determine the laser beam radius from this plot. You need to fit the data to derivative of the above equation which is given by

$$\frac{dP_{\text{normalized}}}{dx}(x) = \frac{1}{w\sqrt{2}} \cdot \exp\left[-\left(\frac{x - x_0}{w}\right)^2\right]$$



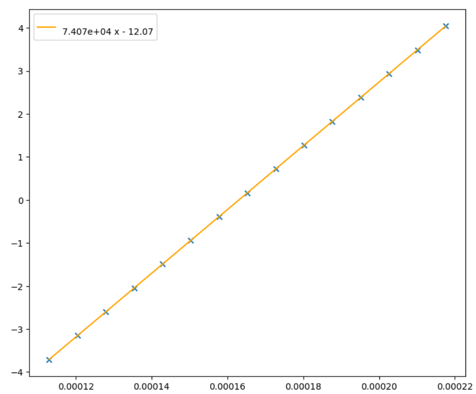
By using *python* libraries such as *numpy* and *scipy* we can approximate the error function either by its definition and numerical approximation of the integral, or either by using the *erf()* method on the library *math*. Both of these ways of approximating the error function gives us the same fitting curve, which are superposed on this image.

By linearization and plotting the following expression we obtain:

$$P = 0.5\left(1 + \text{erf}\left(\frac{x - x_0}{\omega}\right)\right) \iff 2P - 1 = \text{erf}\left(\frac{x - x_0}{\omega}\right) \iff$$

$$\text{erf}^{-1}(2P - 1) = \frac{1}{\omega}x - \frac{x_0}{\omega} = 1.35 \cdot 10^{-5}x - 12.07$$

So that we get $\boxed{\omega = 1.35 \cdot 10^{-5} m}$



I didn't need to use the derivative method because of the *python* libraries mentioned above, which allowed me to calculate the *erf* function itself.

Task:

Also, the radius is related to the position at 20% and 80% of the total beam power by $w = 1.188(x_2 - x_1)$. Where x_2 and x_1 are the positions corresponding to the 20% and 80% of the beam power, respectively.

By doing so we get: $\omega = 1.188(17.1 - 15.5) \cdot 10^{-5} = 1.188 \cdot 1.6 \cdot 10^{-5} \iff \boxed{\omega = 1.9 \cdot 10^{-5}\ m}$

Task:

An excellent fit to the actual normalized power vs position can be obtained using the following approximation.

$$P_{\text{normalized}}(s) = \frac{1}{1 + e^{[a_1 s + a_3 s^3 + a_5 s^5]}}$$

where

$$s = \frac{\sqrt{2}(x - x_0)}{w}$$

and $a_1 = -1.5954086$, $a_3 = -7.3638857 \cdot 10^{-2}$, $a_5 = 6.4121343 \cdot 10^{-4}$

Determine the radius of the beam by fitting your data to the above expression.

By doing this approximation we got the same approximation as on the second part, and thus we got the same value for the beam radius $\omega = 1.35 \cdot 10^{-5}$. We did this by fitting the data to a 5 degree polynomial and extracting the value of omega from there.

1.6 Measure Gaussian beam radius using optical chopper

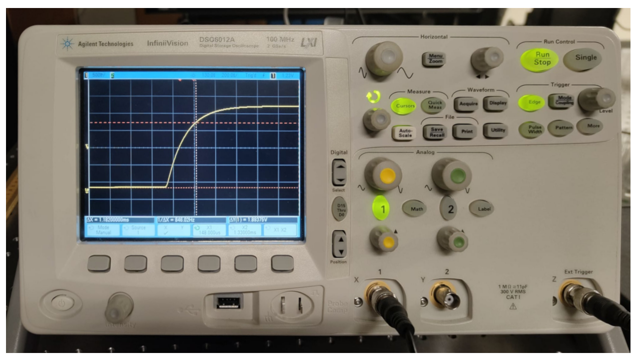
The optical chopper is a rotating toothed wheel that alternately blocks and let's pass a light beam. Place the optical chopper exactly at the focus that you determined with the razor blade. Make sure that the outer set of teeth intercepts the laser beam. You need to use a lens to collect all the light and focus it to the detector. Place a photodetector in the path of the laser beam and connect output of the photodetector to one input of the oscilloscope. Set the oscilloscope to trigger on the rising edge of the detector signal.

Task:

Measure the time it takes the photodetector signal to rise from 20% to 80%. The distance traveled by the chopper blade during this time is equal to the blade speed times the measured 20% to 80% time. The radius is related to the times at 20% and 80% of the total beam power by $w = 1.188v(t_2 - t_1)$. Where t_2 and t_1 are the times corresponding to the 20% and 80% points, respectively. Do this experiment twice, once with the blade crossing the beam horizontally and once crossing vertically

For this task, a oscilloscope was used to visualize the function of time. In the picture we can see how we measured the time it took the signal to get from 20% to 80% of the total signal value.

Doing this experiment vertically or horizontally did not change the results obtained, which were $\Delta t = 0.00024s$



Task:

You need to measure the speed of the point on the chopper blade where the laser beam hits. Think about the quantities that you need to measure to obtain the speed, v.

The linear velocity of a spinning disc is $v = 2\pi f R$, and since the frequency of the motor was was $f = 80\ hz$ we only need the value of the radius to obtain the velocity. We measured the radius at which the laser was hitting when the data was taken, $R = 3.85\ cm$. Thus, the velocity was $v = 19.35\ {}^{m}/_{s}$. With this theta, we obtain a beam radius of $\omega = 0.0055\ m$

1.7 Determine the waist radius and the Rayleigh range of a focused laser beam

If you know the *location of the waist* then the Rayleigh range, z_o , can be determined from the plot of the beam radius vs. the distance from the waist according to the equation of $w(z)$. In many applications, we focus the laser beams using short focal length lenses. Then determining the waist radius and the Rayleigh range precisely become challenging. However, it is possible to determine z_o and w_o if you know the radius of the beam at two locations separated by a distance $\Delta z = z_2 - z_1$. You can use the equation of $w(z)$ to show that z_o is given by

$$z_0 = \frac{\lambda \Delta z^2 \left(w_1^2 + w_2^2 + 2\sqrt{w_1^2 w_2^2 - \left(\frac{\lambda \Delta z}{\pi}\right)^2} \right)}{\pi \left(w_1^4 + w_2^4 - 2w_1^2 w_2^2 + 4\left(\frac{\lambda \Delta z}{\pi}\right)^2 \right)} \tag{4}$$

Task:

Use the same focused beam in task 1. Measure the beam radius at two locations using the method of your choice. Be sure to measure the distance between the two positions. Use the above result to find z_o. Find the radius of the waist w_o. Find the location of the waist. **Be sure to record your distances relative to the center of the lens** (your reference).

For this task, I used two data points from task3, $(\text{Distance (m)}, \text{Radii (m)}) = (0.3628, 0.003750)$ and $(0.3882, 0.004025)$ obtaining a value of $\Delta z = 0.0254m$, $\omega_1 = 0.003750$ and $\omega_2 = 0.004025$. By substitution on the above formula we obtain a value of $z_0 = 0.0017\ m$

Since $z_0 = \frac{\pi \omega_0^2}{\lambda} \iff \omega_0 = \sqrt{\frac{z_0 \lambda}{\pi}} = 0.000018\ m$

Task:

Derive the above relation. Start with the expression for $w(z)$ at z_1 and z_2, the two locations that you measure w_1 and w_2 at. This derivation is not easy. It needs careful consideration of how to collect terms and what to solve for.

The derivation is quite long and may be difficult to follow. We need first to point out two expressions that we will need for the mathematical derivation of the given expression (4).

$$\begin{cases} \omega_1 = \omega_0 \sqrt{1 + \left(\frac{z_1}{z_0}\right)^2} \\ \\ \omega_2 = \omega_0 \sqrt{1 + \left(\frac{z_2}{z_0}\right)^2} \end{cases} \iff \begin{cases} \left(\frac{\omega_1}{\omega_0}\right)^2 = 1 + \left(\frac{z_1}{z_0}\right)^2 \\ \\ \left(\frac{\omega_2}{\omega_0}\right)^2 = 1 + \left(\frac{z_2}{z_0}\right)^2 \end{cases} \iff \begin{cases} z_1 = z_0 \sqrt{\left(\frac{\omega_1}{\omega_0}\right)^2 - 1} \\ \\ z_2 = z_0 \sqrt{\left(\frac{\omega_2}{\omega_0}\right)^2 - 1} \end{cases} \tag{5}$$

$$z_0 = \frac{\pi \omega_0^2}{\lambda} \iff \omega_0^2 = \frac{\lambda}{\pi} z_0 \tag{6}$$

$$\Delta z = z_2 - z_1 = z_0\sqrt{\left(\frac{\omega_2}{\omega_0}\right)^2 - 1} - z_0\sqrt{\left(\frac{\omega_1}{\omega_0}\right)^2 - 1} \quad \Longleftrightarrow \Delta z = z_0\sqrt{\frac{\omega_2^2\pi}{\lambda z_0} - 1} - z_0\sqrt{\frac{\omega_1^2\pi}{\lambda z_0} - 1}$$

$$\Longleftrightarrow \Delta z + z_0\sqrt{\frac{\omega_1\pi}{\lambda z_0} - 1} = z_0\sqrt{\frac{\omega_2^2\pi}{\lambda z_0} - 1} \qquad \Longleftrightarrow \left(\Delta z + z_0\sqrt{\frac{\omega_1^2\pi}{\lambda z_0} - 1}\right)^2 = z_0^2\frac{\omega_2^2\pi}{\lambda z_0} - z_0^2$$

$$\Longleftrightarrow \Delta z^2 + z_0\frac{\omega_1^2\pi}{\lambda} - z_0^2 + 2\Delta z z_0\sqrt{\frac{\omega_1^2\pi}{\lambda z_0} - 1} = z_0\frac{\omega_2^2\pi}{\lambda} - z_0^2 \quad \Longleftrightarrow 2\Delta z z_0\sqrt{\frac{\omega_1^2}{\lambda z_0} - 1} = z_0\left(\frac{\omega_2^2\pi}{\lambda} - \frac{\omega_1^2\pi}{\lambda}\right) - \Delta z^2$$

$$\Longleftrightarrow 4\Delta z^2 z_0^2\left(\frac{\omega_1^2\pi}{\lambda z_0} - 1\right) = \left(z_0\left(\frac{\omega_2^2\pi}{\lambda} - \frac{\omega_1^2\pi}{\lambda}\right) - \Delta z^2\right)^2 = z_0^2\left(\frac{\omega_2^2\pi}{\lambda} - \frac{\omega_1^2\pi}{\lambda}\right)^2 - 2z_0\Delta z^2\left(\frac{\omega_2^2\pi}{\lambda} - \frac{\omega_1^2\pi}{\lambda}\right) + \Delta z^4$$

$$\Longleftrightarrow 4\Delta z^2 z_0\frac{\omega_1^2\pi}{\lambda} - 4\Delta z^2 z_0^2 = z_0^2\left(\frac{\omega_2^2\pi}{\lambda} - \frac{\omega_1^2\pi}{\lambda}\right)^2 - 2z_0\Delta z^2\left(\frac{\omega_2^2\pi}{\lambda} - \frac{\omega_1^2\pi}{\lambda}\right) + \Delta z^4$$

$$\Longleftrightarrow z_0^2\left[-\left(\frac{\omega_2^2\pi}{\lambda} - \frac{\omega_1^2\pi}{\lambda}\right)^2 - 4\Delta z^2\right] + z_0\left[4\Delta z^2\frac{\omega_1^2\pi}{\lambda} + 2\Delta z^2\left(\frac{\omega_2^2\pi}{\lambda} - \frac{\omega_1^2\pi}{\lambda}\right)\right] - \Delta z^4 = 0$$

$$\Longleftrightarrow z_0^2\left[-\left(\frac{\pi}{\lambda}\left(\omega_1^2 - \omega_2^2\right)\right)^2 - 4\Delta z^2\right] + z_0\left[2\Delta z^2\frac{\pi}{\lambda}\left(\omega_1^2 + \omega_2^2\right)\right] - \Delta z^4 = 0$$

$$\Longleftrightarrow z_0 = \frac{-2\Delta z^2\frac{\pi}{\lambda}\left(\omega_1^2 - \omega_2^2\right) \pm \sqrt{\left(2\Delta z^2\frac{\pi}{\lambda}\left(\omega_1^2 + \omega_2^2\right)\right)^2 + 4\Delta z^4\left(-\left[\frac{\pi}{\lambda}\left(\omega_1^2 - \omega_2^2\right)\right]^2 - 4\Delta z^2\right)}}{2\left[-\left(\frac{\pi}{\lambda}\left(\omega_1^2 - \omega_2^2\right)\right)^2 - 4\Delta z^2\right]}$$

$$\Longleftrightarrow z_0 = \frac{\left(-\frac{2\omega_1^2\Delta z^2\pi}{\lambda} - \frac{2\omega_2^2\Delta z^2\pi}{\lambda}\right) \pm \sqrt{4\Delta z^4\frac{\pi^2}{\lambda^2}\left(\omega_1^2 + \omega_2^2\right)^2 - 4\Delta z^4\frac{\pi^2}{\lambda^2}\left(\omega_1^2 - \omega_2^2\right) - 4\Delta z^4\frac{\pi^2}{\lambda^2}4\Delta z^2\frac{\lambda^2}{\pi^2}}}{2\left(-4\Delta z^2 - \frac{\pi^2}{\lambda^2}\left(\omega_1^2 - \omega_2^2\right)^2\right)}$$

$$\Longleftrightarrow z_0 = \frac{+\frac{\Delta z^2\pi}{\lambda}\left(\omega_1^2 + \omega_2^2 \pm \sqrt{\omega_1^4 + \omega_2^4 + 2\omega_1^2\omega_2^2 - \omega_1^4 - \omega_2^4 + 2\omega_1^2\omega_2^2 - 4\left(\frac{\Delta z\lambda}{\pi}\right)^2}\right)}{-\frac{\pi^2}{\lambda^2}\left(-\left(\omega_1^2 - \omega_2^2\right)^2 - 4\left(\frac{\Delta z\lambda}{\pi}\right)^2\right)}$$

$$\Longleftrightarrow z_0 = \frac{\Delta z^2\left(\omega_1^2 + \omega_2^2 \pm \sqrt{4\omega_1^2\omega_2^2 - 4\left(\frac{\Delta z\lambda}{\pi}\right)^2}\right)}{\frac{\pi}{\lambda}\left(\left(\omega_1^2 - \omega_2^2\right)^2 + 4\left(\frac{\Delta z\lambda}{\pi}\right)^2\right)} \Longleftrightarrow \boxed{z_0 = \frac{\Delta z^2\lambda\left(\omega_1^2 + \omega_2^2 \pm 2\sqrt{\omega_1^2\omega_2^2 - 4\left(\frac{\Delta z\lambda}{\pi}\right)^2}\right)}{\pi\left(4\left(\frac{\Delta z\lambda}{\pi}\right)^2 + \omega_1^4 + \omega_2^4 - 2\omega_1^2\omega_2^2\right)}}$$

2 Addenda

I am not sure why the results from task 5 are way off of the results from other tasks, which seem to be more accurate with each other. There has been a misunderstanding or miscalculation of the task that lead to the inconsistent results even from the different methods inside the task.

LaTeX code that generates this document

PHOTONICS-LabRep3:polarization.tex

Python code that generates the plots and contains the data

PHOT-E3.py