$$\text{Show that } 2\Delta d = \Delta m \lambda \iff \Delta d = \frac{\Delta m \lambda}{2} = \frac{M\lambda}{2}$$

$$OPD = \Delta d = n\,\overline{AD} + \overline{DE} - n\overline{AB} - \overline{BC} = \frac{nt}{\cos\theta_j} + t\tan\theta_i \sin(\theta_i - \theta_j) - nt - t\tan\theta_i \sin(\theta_i - \theta_j) - \frac{t\cos(\theta_i - \theta_j)}{\cos\theta_j}$$

$$= t\left[1 - n + \frac{1}{\cos\theta_j}\left(n - \cos\theta_i \cos\theta_j - \sin\theta_i \sin\theta_j\right)\right] + \bigstar$$

- $\overline{AB} = t$

- $\cos\theta_j = \dfrac{t}{\overline{AD}} \iff \overline{AD} = \dfrac{t}{\cos\theta_j}$

- $\tan\theta_i = \dfrac{\overline{DE}}{x} \iff x = \dfrac{\overline{DE}}{\tan\theta_i} = t\sin(\theta_i - \theta_j) \iff \overline{DE} = t\tan\theta_i \sin(\theta_i - \theta_j)$

- $\overline{BC} = \overline{DE} - \overline{BF} = t\left[\tan\theta_i \sin(\theta_i - \theta_j) + \dfrac{\cos(\theta_i - \theta_j)}{\cos\theta_j} - 1\right]$

  $-\overline{DE} = t\tan\theta_i \sin(\theta_i - \theta_j)$

  $-\overline{BF} = t - w = t - \dfrac{t\cos(\theta_i - \theta_j)}{\cos\theta_j}$

  - $\cos(\theta_i - \theta_j) = \dfrac{w}{L} \iff w = L\cos(\theta_i - \theta_j) = \dfrac{t\cos(\theta_i - \theta_j)}{\cos\theta_j}$

  - $\cos(\theta_j) = \dfrac{t}{L} \iff L = t/\cos\theta_j$

Snell's Law

$$n_i \sin\theta_i = n_j \sin\theta_j \quad \text{with } n_i = 1$$

$$\sin\theta_j = \frac{\sin\theta_i}{n_j} = \frac{\sin\theta_i}{n}$$

$$\sin\theta_i \iff \sin\theta_i \iff \Delta d = t\left[1 - n + \frac{n}{\sqrt{n^2 - \sin^2\theta_i}}\left(n - \cos\theta_i \frac{\sqrt{n^2 - \sin^2\theta_i}}{n} - \frac{\sin^2\theta_i}{n}\right)\right] =$$

$$\implies \cos\theta_j = \sqrt{n^2 - \sin^2\theta_i}/n$$

$$= t\left[1 - n + \frac{n^2}{\sqrt{n^2 - \sin^2\theta_i}} - \cos\theta_i + \frac{-\sin^2\theta_i}{\sqrt{n^2 - \sin^2\theta_i}}\right] = \frac{M\lambda}{2} \iff$$

$$\iff \frac{M\lambda}{2t} = 1 - n - \cos\theta_i + \frac{n^2 - \sin^2\theta_i}{\sqrt{n^2 - \sin^2\theta_i}} = 1 - n - \cos\theta_i + \sqrt{n^2 - \sin^2\theta_i} \iff \left(\sqrt{n^2 - \sin^2\theta_i}\right)^2 = \left(\frac{M\lambda}{2t} - 1 + n + \cos\theta_i\right)^2$$

$$\iff n^2 - \sin^2\theta_i = \frac{M^2\lambda^2}{4t^2} - \frac{M\lambda}{8t} + \frac{nM\lambda}{2t} + \frac{M\lambda}{2t}\cos\theta_i - \frac{M\lambda}{2t} + 1 - n - \cos\theta_i + \frac{nM\lambda}{8t} - n + n^2 + n\cos\theta_i + \frac{M\lambda}{8t}\cos\theta_i - \cos\theta_i + n\cos\theta_i + \cos^2\theta_i \iff$$

$$\lambda \ll 1 \implies \lambda^2 \ll\ll 1 \implies \frac{M^2\lambda^2}{4t^2} \approx 0$$

$$\iff n^2 - \sin^2\theta_i = \frac{M\lambda}{t} + n\frac{M\lambda}{t} + \frac{M\lambda}{t}\cos\theta_i + 1 - 2n - 2\cos\theta_i + n^2 + 2n\cos\theta_i + \cos^2\theta_i \iff$$

$$\iff 1 - 2n - 2\cos\theta_i + 2n\cos\theta_i + \frac{M\lambda}{t}\left(n + \cos\theta_i - 1\right) = 0 \iff \left(-2n - 2\cos\theta_i + 2n\cos\theta_i\right)t = -M\lambda\left(n + \cos\theta_i - 1\right) \iff$$

$$\iff -2nt + 2nt\cos\theta_i + M\lambda n = -M\lambda\cos\theta_i + M\lambda + 2t\cos\theta_i \iff 2nt - 2nt\cos\theta_i - M\lambda n = M\lambda\cos\theta_i - M\lambda - 2t\cos\theta_i \iff$$

$$\iff n\left(2t(1 - \cos\theta_i) - M\lambda\right) = (2t - M\lambda)(1 - \cos\theta_i) \iff \boxed{n = \frac{(2t - M\lambda)(1 - \cos\theta_i)}{2t[1 - \cos\theta_i] - M\lambda}} \quad \square$$