$$\textcircled{H}^{\mu\nu}_{s} = \varepsilon_0 c^2 \left(g^{\mu\lambda} F_{\lambda\sigma} F^{\sigma\nu} + \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} \right)$$

Demostración $\partial_\mu \textcircled{H}^{\mu\nu}_{s} = -F^{\nu\sigma} J_\sigma$.

$$\partial_\mu \left(g^{\mu\lambda} F_{\lambda\sigma} F^{\sigma\nu} + \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} \right) =$$

$$= g^{\mu\lambda} \partial_\mu (F_{\lambda\sigma}) F^{\sigma\nu} + g^{\mu\lambda} F_{\lambda\sigma} \partial_\mu (F^{\sigma\nu}) +$$

$$+ \frac{1}{4} g^{\mu\nu} \partial_\mu (F^{\rho\sigma}) F_{\rho\sigma} + \frac{1}{4} g^{\mu\nu} \underbrace{\partial_\mu (F_{\rho\sigma}) F^{\rho\sigma}}_{\partial_\mu (F^{\rho\sigma}) F_{\rho\sigma}} =$$

$$= g^{\mu\lambda} \partial_\mu (F_{\lambda\sigma}) F^{\sigma\nu} + g^{\mu\lambda} F_{\lambda\sigma} \partial_\mu (F^{\sigma\nu}) +$$

$$+ \frac{1}{2} g^{\mu\nu} \partial_\mu (F^{\rho\sigma}) F_{\rho\sigma} =$$

$$= \underset{(*)}{\partial^\lambda (F_{\lambda\sigma})} F^{\sigma\nu} + g^{\mu\lambda} F_{\lambda\sigma} \partial_\mu (F^{\sigma\nu}) +$$

$$+ \frac{1}{2} \underset{(**)}{g^{\mu\nu} \partial_\mu (F^{\rho\sigma})} F_{\rho\sigma} = \downarrow$$

$$(*) \quad \partial^\lambda (F_{\lambda\sigma}) = \mu_0 J_\sigma$$

$$(**) \quad g^{\mu\nu} \partial_\mu (F^{\rho\sigma}) = \partial^\nu (F^{\rho\sigma}) =$$

$$= -\partial^\rho (F^{\sigma\nu}) - \partial^\sigma (F^{\nu\rho}) \uparrow$$

$$\partial_\mu \mathcal{F}^{\mu\nu} = 0 \rightarrow \partial^\nu (F^{\rho\sigma}) + \partial^\rho (F^{\sigma\nu}) + \partial^\sigma (F^{\nu\rho}) = 0$$

$$\downarrow = \mu_0 F^{\sigma\nu} \partial_\sigma + g^{\mu\lambda} F_{\lambda\sigma} \partial_\mu (F^{\sigma\nu}) -$$

$$- \frac{1}{2} F_{\rho\sigma} \partial^\rho (F^{\sigma\nu}) - \frac{1}{2} F_{\rho\sigma} \partial^\sigma (F^{\nu\rho}) =$$

$$= - \mu_0 F^{\nu\sigma} J_\sigma + F_{\lambda\sigma} \partial^\lambda (F^{\sigma\nu}) -$$

$F^{\sigma\nu} = - F^{\nu\sigma}$ ↑

$- \frac{1}{2} F_{\rho\sigma} \partial^\rho (F^{\sigma\nu}) - \frac{1}{2} F_{\rho\sigma} \partial^\sigma (F^{\nu\rho}) =$

ponemos λ mudo

ponemos λ mudo

ponemos λ (antisimétrico) mudo $= - F^{\rho\nu}$

$= - F_{\sigma\rho}$ (antisimétrico)

$$= - \mu_0 F^{\nu\sigma} J_\sigma + F_{\lambda\sigma} \partial^\lambda (F^{\sigma\nu}) - \frac{1}{2} F_{\lambda\sigma} \partial^\lambda (F^{\sigma\nu}) -$$

$$- \frac{1}{2} F_{\lambda\rho} \partial^\lambda (F^{\rho\nu}) = - \mu_0 F^{\nu\sigma} J_\sigma + F_{\lambda\sigma} \partial^\lambda (F^{\sigma\nu}) -$$

mudo ponemos σ

$$- \frac{1}{2} F_{\lambda\sigma} \partial^\lambda (F^{\sigma\nu}) - \frac{1}{2} F_{\lambda\sigma} \partial^\lambda (F^{\sigma\nu}) =$$

$$- F_{\lambda\sigma} \partial^\lambda (F^{\sigma\nu})$$

$$= - \mu_0 F^{\nu\sigma} J_\sigma$$

$$\rightarrow \boxed{\partial \textcircled{H}^{\mu\nu}_S = - \underbrace{\varepsilon_0 c^2}_{1/\mu_0} \mu_0 F^{\nu\sigma} J_\sigma = - F^{\nu\sigma} J_\sigma}$$