$$\left.\begin{array}{l}\vec{E}^{\parallel}=0\\ B^{\perp}=0\end{array}\right\}(1)$$

$$\left.\begin{array}{l}\vec{E}(x,y,z,t)=\vec{E}_0(x,y)\,e^{i(kz-\omega t)}\\ \vec{B}(x,y,z,t)=\vec{B}_0(x,y)\,e^{i(kz-\omega t)}\end{array}\right\}(2)$$

$$\left.\begin{array}{ll}(3a) & \vec{\nabla}\cdot\vec{E}=0\\ (3b) & \vec{\nabla}\cdot\vec{B}=0\\ (3c) & \vec{\nabla}\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t}\\ (3d) & \vec{\nabla}\times\vec{B}=\dfrac{1}{c^2}\dfrac{\partial\vec{E}}{\partial t}\end{array}\right\}(3) \qquad c^2=\frac{1}{\varepsilon_0\mu_0}$$

$$\left.\begin{array}{l}\vec{E}_0(x,y)=E_x(x,y)\,\hat{u}_x+E_y(x,y)\,\hat{u}_y+E_z(x,y)\,\hat{u}_z\\ \vec{B}_0(x,y)=B_x(x,y)\,\hat{u}_x+B_y(x,y)\,\hat{u}_y+B_z(x,y)\,\hat{u}_z\end{array}\right\}(4)$$

Sustituyendo (2) y (4) en (3c) y (3d):

$$\vec{\nabla}\times\vec{E}=\begin{vmatrix}\hat{u}_x & \hat{u}_y & \hat{u}_z\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ E_x e^{i(kz-\omega t)} & E_y e^{i(kz-\omega t)} & E_z e^{i(kz-\omega t)}\end{vmatrix}=$$

$$=\left[\frac{\partial E_z}{\partial y}e^{i(kz-\omega t)}-ikE_y e^{i(kz-\omega t)}\right]\hat{u}_x+$$

$$+\left[ikE_x e^{i(kz-\omega t)}-\frac{\partial E_z}{\partial x}e^{i(kz-\omega t)}\right]\hat{u}_y+$$

$$+\left[\frac{\partial E_y}{\partial x}e^{i(kz-\omega t)}-\frac{\partial E_x}{\partial y}e^{i(kz-\omega t)}\right]\hat{u}_z$$

$$-\frac{\partial \vec{\mathbb{B}}}{\partial t}=\left(i\omega B_x\hat{u}_x+i\omega B_y\hat{u}_y+i\omega B_z\hat{u}_z\right)e^{i(kz-\omega t)}$$

(3c) $\vec{\nabla}\times\vec{\mathbb{E}}=-\frac{\partial\vec{\mathbb{B}}}{\partial t}$

$$\left.\begin{aligned}&\frac{\partial E_z}{\partial y}-ikE_y=i\omega B_x \quad (ii)\\ &ikE_x-\frac{\partial E_z}{\partial x}=i\omega B_y \quad (iii)\\ &\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}=i\omega B_z \quad (i)\end{aligned}\right\}$$

$$\vec{\nabla}\times\vec{\mathbb{B}}=\begin{vmatrix}\hat{u}_x & \hat{u}_y & \hat{u}_z\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ B_x e^{i(kz-\omega t)} & B_y e^{i(kz-\omega t)} & B_z e^{i(kz-\omega t)}\end{vmatrix}=$$

$$=\left[\frac{\partial B_z}{\partial y}e^{i(kz-\omega t)}-ikB_y e^{i(kz-\omega t)}\right]\hat{u}_x+$$

$$+\left[ikB_x e^{i(kz-\omega t)}-\frac{\partial B_z}{\partial x}e^{i(kz-\omega t)}\right]\hat{u}_y+$$

$$+\left[\frac{\partial B_y}{\partial x}e^{i(kz-\omega t)}-\frac{\partial B_x}{\partial y}e^{i(kz-\omega t)}\right]\hat{u}_z$$

$$\frac{1}{c^2}\frac{\partial\vec{\mathbb{E}}}{\partial t}=\left(-\frac{i\omega}{c^2}E_x\hat{u}_x-\frac{i\omega}{c^2}E_y\hat{u}_y-\frac{i\omega}{c^2}E_z\hat{u}_z\right)e^{i(kz-\omega t)}$$

(3d) $\vec{\nabla}\times\vec{\mathbb{B}}=\frac{1}{c^2}\frac{\partial\vec{\mathbb{E}}}{\partial t}$

$$\left.\begin{aligned}
&\frac{\partial B_z}{\partial y} - ikB_y = -\frac{i\omega}{c^2}E_x && (v)\\
&ikB_x - \frac{\partial B_z}{\partial x} = -\frac{i\omega}{c^2}E_y && (vi)\\
&\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = -\frac{i\omega}{c^2}E_z && (iv)
\end{aligned}\right\}$$

Multiplicamos (iii) por "k" y (v) por "ω" y restamos

$$\left.\begin{aligned}
&(iii) && ik^2E_x - k\frac{\partial E_z}{\partial x} = ik\omega B_y\\
&(v) && \omega\frac{\partial B_z}{\partial y} - ik\omega B_y = -\frac{i\omega^2}{c^2}E_x
\end{aligned}\right\}$$

$$ik^2E_x - i\frac{\omega^2}{c^2}E_x = k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y} \qquad \left(\frac{1}{i} = -i\right)$$

$$\boxed{E_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}\right)} \qquad (5a)$$

Multiplicamos (ii) por "k" y (vi) por "ω" y sumamos:

$$\left.\begin{aligned}
&(ii) && k\frac{\partial E_z}{\partial y} - ik^2E_y = i\omega kB_x\\
&(vi) && i\omega kB_x - \omega\frac{\partial B_z}{\partial x} = -i\frac{\omega^2}{c^2}E_y
\end{aligned}\right\}$$

$$-ik^2E_y + i\frac{\omega^2}{c^2}E_y = -k\frac{\partial E_z}{\partial y} + \omega\frac{\partial B_z}{\partial x} \qquad \left(\frac{1}{i} = -i\right)$$

$$\boxed{E_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right)} \qquad (5b)$$

Multiplicamos (ii) por "$\frac{\omega}{c^2}$" y (vi) por "k" y sumamos:

$$\left.\begin{aligned}
&(ii)\quad \frac{\omega}{c^2}\frac{\partial E_z}{\partial y} - ik\frac{\omega}{c^2}E_y = i\frac{\omega^2}{c^2}B_x \\
&(vi)\quad ik^2 B_x - k\frac{\partial B_z}{\partial x} = -ik\frac{\omega}{c^2}E_y
\end{aligned}\right\}$$

$$-i\frac{\omega^2}{c^2}B_x + ik^2 B_x = k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}$$

$$\boxed{B_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right)} \quad (5c)$$

Multiplicamos (iii) por "$\frac{\omega}{c^2}$" y (v) por "k" y restamos

$$\left.\begin{aligned}
&(iii)\quad ik\frac{\omega}{c^2}E_x - \frac{\omega}{c^2}\frac{\partial E_z}{\partial x} = i\frac{\omega^2}{c^2}B_y \\
&(v)\quad k\frac{\partial B_z}{\partial y} - ik^2 B_y = -ik\frac{\omega}{c^2}E_x
\end{aligned}\right\}$$

$$-i\frac{\omega^2}{c^2}B_y + ik^2 B_y = \frac{\omega}{c^2}\frac{\partial E_z}{\partial x} + k\frac{\partial B_z}{\partial y}$$

$$\boxed{B_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial y} + \frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right)} \quad (5d)$$

Sustituimos (5) en (3a) y (3b):

$$\left.\begin{aligned}&(5a)\\&(5b)\end{aligned}\right] \longrightarrow (3a)\quad \vec{\nabla}\cdot\vec{E} = 0$$

Teniendo en cuenta (2) y (4): $(\vec{\nabla}\cdot\vec{E}=0)$

$$\vec{\nabla}\cdot\left[\left(E_x\hat{u}_x+E_y\hat{u}_y+E_z\hat{u}_z\right)e^{i(kz-\omega t)}\right]=0$$

$$\frac{\partial}{\partial x}\left(E_x e^{i(kz-\omega t)}\right)+\frac{\partial}{\partial y}\left(E_y e^{i(kz-\omega t)}\right)+$$

$$+\frac{\partial}{\partial z}\left(E_z e^{i(kz-\omega t)}\right)=0$$

$$\frac{\partial E_x}{\partial x}e^{i(kz-\omega t)}+\frac{\partial E_y}{\partial y}e^{i(kz-\omega t)}+ikE_z e^{i(kz-\omega t)}=0$$

$$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+ikE_z=0$$

Usamos (5a) y (5b):

$$\frac{ik}{(\omega/c)^2-k^2}\frac{\partial^2 E_z}{\partial x^2}+\cancel{\frac{i\omega}{(\omega/c)^2-k^2}\frac{\partial^2 B_z}{\partial x\partial y}}+$$

$$+\frac{ik}{(\omega/c)^2-k^2}\frac{\partial^2 E_z}{\partial y^2}-\cancel{\frac{i\omega}{(\omega/c)^2-k^2}\frac{\partial^2 B_z}{\partial x\partial y}}+ikE_z=0$$

Dividimos por "ik" en ambos lados de la ecuación:

$$\frac{1}{(\omega/c)^2-k^2}\left(\frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}\right)+E_z=0$$

Multiplicamos por $\left(\frac{\omega}{c}\right)^2-k^2$ a ambos lados:

$$\boxed{\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\left(\frac{w}{c}\right)^2-k^2\right]E_z=0} \qquad (6a)$$

Teniendo en cuenta (2) y (4): $(\vec{\nabla}\cdot\vec{B}=0)$

$$\vec{\nabla}\cdot\left[\left(B_x\hat{u}_x+B_y\hat{u}_y+B_z\hat{u}_z\right)e^{i(kz-wt)}\right]=0$$

$$\frac{\partial}{\partial x}\left(B_x e^{i(kz-wt)}\right)+\frac{\partial}{\partial y}\left(B_y e^{i(kz-wt)}\right)+$$

$$+\frac{\partial}{\partial z}\left(B_z e^{i(kz-wt)}\right)=0$$

$$\frac{\partial B_x}{\partial x}e^{i(kz-wt)}+\frac{\partial B_y}{\partial y}e^{i(kz-wt)}+ikB_z e^{i(kz-wt)}=0$$

$$\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+ikB_z=0$$

Usamos (5c) y (5d):

$$\frac{ik}{(w/c)^2-k^2}\frac{\partial^2 B_z}{\partial x^2}-\cancel{\frac{i}{(w/c)^2-k^2}\frac{w}{c^2}\frac{\partial^2 E_z}{\partial x\partial y}}+$$

$$+\frac{ik}{(w/c)^2-k^2}\frac{\partial^2 B_z}{\partial y^2}+\cancel{\frac{i}{(w/c)^2-k^2}\frac{w}{c^2}\frac{\partial^2 E_z}{\partial x\partial y}}+$$

$$+ikB_z=0$$

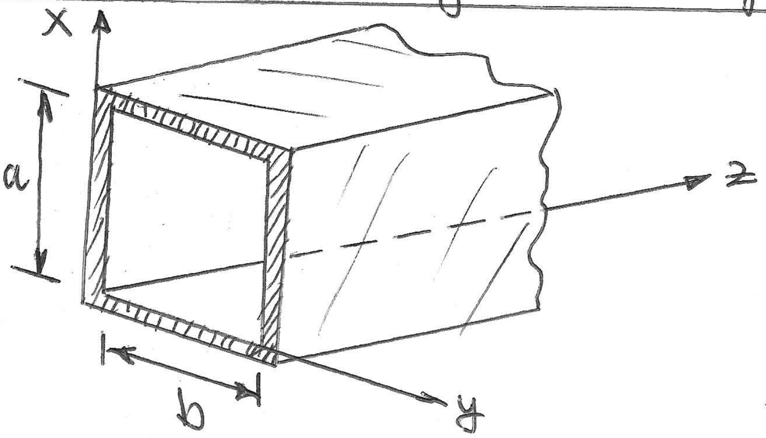
Dividimos por "ik" en ambos lados de la ecuación:

$$\frac{1}{(w/c)^2-k^2}\left(\frac{\partial^2 B_z}{\partial x^2}+\frac{\partial^2 B_z}{\partial y^2}\right)+B_z=0$$

Multiplicamos por $\left(\frac{w}{c}\right)^2 - k^2$ a ambos lados:

$$\boxed{\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \left(\frac{w}{c}\right)^2 - k^2\right] B_z = 0} \quad (6b)$$

Modos TE en una guía rectangular



$E_z = 0$

Hay que resolver (6b).

Separación de variables:

$$\boxed{B_z(x,y) = X(x)\,Y(y)}$$

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + \left[\left(\frac{w}{c}\right)^2 - k^2\right] X Y = 0$$

Dividimos por XY y tenemos en cuenta $\left\{\begin{array}{l} X = X(x) \\ Y = Y(y) \end{array}\right.$

$$\frac{1}{X}\frac{d^2 X}{dx^2} + \frac{1}{Y}\frac{d^2 Y}{dy^2} = k^2 - \left(\frac{w}{c}\right)^2$$

$$\left.\begin{array}{l} \dfrac{1}{X}\dfrac{d^2 X}{dx^2} = -k_x^2 \longrightarrow \dfrac{d^2 X}{dx^2} + k_x^2 X = 0 \\[2ex] \dfrac{1}{Y}\dfrac{d^2 Y}{dy^2} = -k_y^2 \longrightarrow \dfrac{d^2 Y}{dy^2} + k_y^2 Y = 0 \end{array}\right\} \text{MAS}$$

Condición:

$$-k_x^2 - k_y^2 = k^2 - \left(\frac{w}{c}\right)^2 \rightarrow \boxed{-k_x^2 - k_y^2 + \left(\frac{w}{c}\right)^2 - k^2 = 0}$$

Solución general para $X(x)$:

$$\boxed{X(x) = A\,\mathrm{sen}(k_x x) + B\cos(k_x x)}$$

Condición de contorno (1)

$$B^{\perp} = 0 \rightarrow B_x = 0 \text{ para } \begin{cases} x = 0 \\ x = a \end{cases}$$

Usando la ecuación (5c) teniendo en cuenta que $E_z = 0$ (TE):

$$B_x = \frac{i}{(w/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial x} - \frac{w}{c^2}\underbrace{\cancel{\frac{\partial E_z}{\partial y}}}_{=0\ (E_z=0,\,TE)}\right)$$

$$B_x = \frac{ik}{(w/c)^2 - k^2}\frac{\partial B_z}{\partial x}$$

Para $x = 0$:

$$(B_x)_{x=0} = \frac{ik}{(w/c)^2 - k^2}\left(\frac{\partial B_z}{\partial x}\right)_{x=0} = 0$$

$$\left(\frac{\partial B_z}{\partial x}\right)_{x=0} = Y\left(\frac{dX}{dx}\right)_{x=0} = 0 \rightarrow \left(\frac{dX}{dx}\right)_{x=0} = 0$$

Para $x = a$:

$$B_x(a) = \frac{ik}{(w/c)^2 - k^2}\left(\frac{\partial B_z}{\partial x}\right)_{x=a} = 0$$

$$\left(\frac{\partial B_z}{\partial x}\right)_{x=a} = Y\left(\frac{dX}{dx}\right)_{x=a} = 0 \rightarrow \left(\frac{dX}{dx}\right)_{x=a} = 0$$

$$\frac{dX}{dx} = A k_x \cos(k_x x) - B k_x \operatorname{sen}(k_x x)$$

$$\left(\frac{dX}{dx}\right)_{x=0} = 0 \rightarrow A = 0$$

$$\left(\frac{dX}{dx}\right)_{x=a} = 0 \rightarrow \operatorname{sen}(k_x a) = 0 \rightarrow k_x a = m\pi$$

$$m = 0, 1, 2, \ldots$$

$$\boxed{k_x = \frac{m\pi}{a} \qquad m = 0, 1, 2, \ldots}$$

Haciendo lo mismo para B_y:

$$\boxed{k_y = \frac{n\pi}{b} \qquad n = 0, 1, 2, \ldots}$$

$$\boxed{B_z(x,y) = B_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)} \quad TE_{mn}$$

Número de onda:

$$\left.\begin{array}{l} k_x = \dfrac{m\pi}{a} \\ k_y = \dfrac{n\pi}{b} \end{array}\right\} \quad -k_x^2 - k_y^2 + \frac{\omega^2}{c^2} - k^2 = 0$$

$$\boxed{k = \sqrt{\frac{\omega^2}{c^2} - \pi^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)}}$$