① $y\,(x+y-1)\,dx + (x+2y)\,dy = 0$

1. Exacta?

$$\left.\begin{aligned}\frac{\partial P}{\partial y} &= x+2y+1 \\ \frac{\partial Q}{x} &= 1\end{aligned}\right\}\text{ NO exacta}$$

2. Factor integrante $\frac{\partial P}{\partial y}$

$$\frac{\mu'(x)}{\mu(x)} = \frac{(x+2y+1)-1}{\underbrace{x+2y}_{Q}} = 1$$

$$\boxed{\mu(x) = e^x}$$

Multiplico por $\mu(x)$

$e^x y\,(x+y+1)\,dx + e^x\,(x+2y)\,dy = 0$ Exacta

*OJO!
si me equivoco con el F.I. $g(y)$ no queda en función de y.

$$F(x,y) = y\int xe^x\,dx + (y^2+y)e^x\,dx + g(y) =$$

$u = x \quad du = dx$
$dv = e^x dx \quad v = e^x$

$$= \underbrace{yxe^x - \cancel{ye^x} + y^2e^x + \cancel{ye^x}}\ + g(y)$$

$$e^x(x+2y) = \frac{\partial F}{\partial y}(x,y) = xe^x + 2ye^x + g'(y)$$

$g'(y) = 0 \Rightarrow g(y) = c$

$$\boxed{F(x,y) = e^x\,(yx + y^2) = K}$$

② $(6ty)\,dt + (4y + 9t^2)\,dy = 0$

$$\frac{\partial P}{\partial y} = 6t \neq \frac{\partial Q}{\partial t} = 18t \qquad \text{NO EXACTA.}$$

FACTOR INTEGRANTE.

$$e^{\int \frac{2}{y} dy}$$

$$\frac{\mu'(y)}{\mu(y)} = \frac{6t - 18t}{6ty} = \frac{2}{y} \Rightarrow \mu(y) = y^2$$

Multiplico por $\mu(x)$:

$$(6ty^3)\,dt + (4y^3 + 9y^2t^2)\,dy = 0$$

$$F(t,y) = 3t^2y^3 + g(y)$$

$$4y^3 + 9y^2t^2 = \frac{\partial F}{\partial y}(x,y) = 9t^2y^2 + g'(y) \Rightarrow g(y) = y^4$$

$$F(t,y) = 3t^2y^3 + y^4 = K$$

$$y^3(3t^2 + y) = K$$

③ $(4t - 2y)\,dt + (2t - 4y)\,dy = 0$

$$\frac{\partial P}{\partial y} = -2 \neq \frac{\partial Q}{\partial t} = 2 \qquad \text{No exacta}$$

$$\frac{\mu'(x)}{\mu(x)} = \frac{-4}{-2t - 2y}$$

Depende de t y de y

$\mu(t+y) = \mu(z)$ con $z = t + y$

$$\frac{\mu'(z)}{\mu(z)} = \frac{\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial t}}{Q(t,y) - P(t,y)} = \frac{-4}{-2t - 2y} = \frac{2}{t+y} = \frac{2}{z}$$

$$\mu(z) = e^{\int \frac{2}{z} dz} = z^2$$

$$\mu(t+y) = (t+y)^2$$

Ahora con $\mu(t-y) = \mu(z)$ con $z = t-y$

$$\frac{\mu'(z)}{\mu(z)} = \frac{-4}{6t-6y} = -\frac{2}{3}\frac{1}{t-y} = -\frac{2}{3}\frac{1}{z}$$

$$\mu(z) = e^{\int -\frac{2}{3z}dz} = z^{-2/3}$$

$$\mu(t-y) = (t-y)^{-2/3}$$

OTRA FACTOR INTEGRANTE:

$$\mu(t,y) = \frac{1}{t^2-y^2}$$ La edo es homogénea

- Si P y Q son homogéneas con el mismo orden (ecuación diferecial homogénea) entonces

$$\mu(t,y) = \frac{1}{tP(t,y) + yQ(t,y)}$$

④ $(3t - y - 3ty + 3y^2)dt + (5ty - t^2 - 4y^2 - 2t)\,dy = 0$

¿Exacta? NO

$$\frac{\partial P}{\partial y} = -1 - 3t + 6y \neq \frac{\partial Q}{\partial t} = 5y - 2t - 2$$

Factor integrante: $\mu(t-y) = \mu(z)$ con $\boxed{z = t-y}$

$$\frac{\mu'(z)}{\mu(z)} = \frac{-1-3t+6y-5y+2t+2}{5ty-t^2-4y^2-2t+(3t-y-3ty+3y^2)} =$$

$$= \frac{-t+y+1}{2ty-t^2-y^2+t-y} = \frac{1-(t-y)}{(t-y)-(t-y)^2} = \frac{1-z}{z-z^2} =$$

$$= \frac{\cancel{1-z}}{z(\cancel{1-z})} = \frac{1}{z}$$

$$\mu(t,y) = t-y$$

⑤ $(2y^2 - 3ty)\,dt + (3ty - 2t^2)\,dy = 0$

$$\frac{\partial P}{\partial y} = 4y - 3t \neq \frac{\partial Q}{\partial t} = 3y - 4t$$

$\mu(t\cdot y) = \mu(z)$ con $z = t\cdot y$

$$\frac{\mu'(z)}{\mu(z)} = \frac{\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial t}}{yQ - tP} = \frac{t+y}{3ty^2 - 2t^2y - 2ty^2 + 3t^2y} =$$

$$= \frac{y+t}{ty^2 + t^2y} = \frac{1}{ty} = \frac{1}{z} \Rightarrow \mu(z) = e^{\int \frac{1}{z}dz} = z$$

$$\boxed{\mu(t,y) = t\cdot y}$$

Teorema 2.29. *[Euler] Si tenemos una función diferenciable en un abierto de* $\mathbb{R}^2$ *y homogénea de orden* α *entonces*

$$\boxed{t\frac{\partial f}{\partial t}(t,y) + y\frac{\partial f}{\partial y}(t,y) = \alpha f(t,y).}$$

demo

$f(\lambda t, \lambda y) = \lambda^\alpha f(t,y)$

Regla cadena. (varias variables)

$$\frac{\partial}{\partial \lambda}(f(\lambda t, \lambda y)) = \underbrace{t}_{=\frac{\partial t}{\partial \lambda}}\frac{\partial f}{\partial t} + \underbrace{y}_{=\frac{\partial y}{\partial \lambda}}\frac{\partial f}{\partial y}$$

$$\frac{\partial}{\partial \lambda}(\lambda^\alpha f(t,y)) = \alpha \lambda^{\alpha-1} f(t,y)$$

tomando $\boxed{\lambda = 1}$ FIN

P y Q son funciones homogéneas del mismo orden

$$\Rightarrow \mu(t,y) = \frac{1}{tP + yQ}$$

es un factor integrante de $P(t,y)dt + Q(t,y)dy = 0$

¿ $\frac{\partial}{\partial y}\left(\frac{P}{tP+yQ}\right) = \frac{\partial}{\partial t}\left(\frac{Q}{tP+yQ}\right)$?

A

$$\frac{\cancel{P_y}(tP+yQ) - \cancel{P}(tP_y + Q + yQ_y)}{(tP+yQ)^2} = \frac{yP_yQ - PQ - yPQ_y}{(tP+yQ)^2}$$

$$\frac{Q_t(tP + \cancel{yQ}) - Q(P + tP_t + \cancel{yQ_t})}{(tP+yQ)^2} = \frac{tQ_tP - PQ - tQP_t}{(tP+yQ)^2}$$

B

¿cuándo $A = B$?

$$A = B \iff yP_yQ - yPQ_y = tQ_TP - tQP_t$$

$$\iff Q(yP_y + TP_t) = P(yQ_y + tQ_T)$$

$$\alpha PQ = \alpha PQ$$

ejemplo 2.30

$$\left(1+e^{t/y}\right)dt + 2e^{t/y}\left(1-\frac{t}{y}\right)dy = 0$$

Producto de funciones homogéneas

$$\mu(t,y) = \frac{1}{t\left(1+e^{t/y}\right)+2ye^{t/y}\left(1-\frac{t}{y}\right)}$$