Physics 251
Spring 2024

Homework Set 4
Due Thursday March 14 2024 by 5:00 PM

1. Text, exercise 6.4, pages 6-23 and 6-24 modified as follows:

(Note that, if $k' \neq k$, the equilibrium spacing between the two masses will not be the same as the distance from either mass to the closest wall – that is not shown in the diagram, but it will not affect the use of this method.)

a. *Derive* the governing differential equations for the "arbitrary configuration" (i.e., not for a normal mode). Show all steps of your reasoning.

Hint: Do that by finding, in the arbitrary configuration, the total force exerted by the springs on each mass. Note, e.g., that only the left and center springs exert forces on the left mass.

If you have a question about whether or not you have the correct results for this part, please see me.

b. Use the "method of searching for normal coordinates" to derive:

i. the normal coordinates. As part of this, be sure to prove that the coordinates that you so identify are indeed normal coordinates. Show all steps of your reasoning.

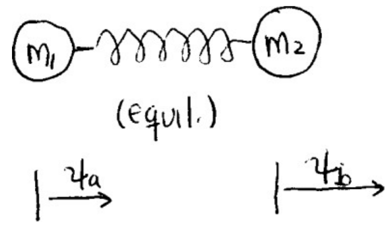
ii. the mode frequencies. Show all steps of your reasoning.

c. Derive the mode configurations: Using your results from part b, in particular, from the general solution for each of the two normal coordinates as a function of time, derive final results for each of $\psi_a(t)$ and $\psi_b(t)$ as explicit functions of time assuming unspecified initial conditions. Show all steps of your reasoning.

2. **Normal Modes of Diatomic Molecule - Equal mass case**

As we saw in class some time ago, it is possible to predict the frequency for vibrational radiation from diatomic molecules by mentally replacing the chemical bond by a spring whose equivalent spring constant can, at least roughly, be deduced from only simple considerations. In this problem we will reconsider that situation from the point of view of normal modes.

As a first step, we recall our model for the system. For this, refer to the figure below.



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Let $\psi_a(t)$ and $\psi_b(t)$ be displacements from equilibrium. Note that both are measured positive to the right, though either or both can be displaced to the left of its equilibrium position.

Take $m_1 = m_2$.

a. Before getting involved in math: Based on work we did a few weeks ago, what do you expect the (nonzero) mode frequency to be? Justify your answer in a few or so, English sentences.

b. Derive the governing differential equations for the "arbitrary" configuration (*i.e.,* not a normal mode) for the system in the (ψ_a, ψ_b) coordinate system defined above. Show all steps of your derivation.

   If you have doubts that you have arrived at the correct governing differential equations for the "arbitrary" configuration, see me.

c. Now find the mode frequencies using the "method of finding normal coordinates." <u>Show all steps of the work</u>. As part of your work/answers, the following must be included:

   i. Derive the normal coordinates (in terms of $\psi_a(t)$ and $\psi_b(t)$). Clearly identify them and prove that they are indeed normal coordinates.

   ii. Show that the method of finding normal coordinates implies that one of the normal modes has "zero frequency." What sort of motion of the molecule does that imply? Why?

   iii. From the method of normal coordinates, is the other mode frequency what you expected? What is the physical meaning of the higher frequency normal coordinate? (I.e., what kind of motion internal to the molecule does it represent?)

3. **Normal Modes of Diatomic Molecule via method of normal coordinates – Unequal mass case**

   Consider the situation of Prob. 2 above, but this time, take ***m*₂ not equal to *m*₁**. Let $\psi_a(t)$ and $\psi_b(t)$ be displacements from equilibrium. Note that both are measured positive to the right.

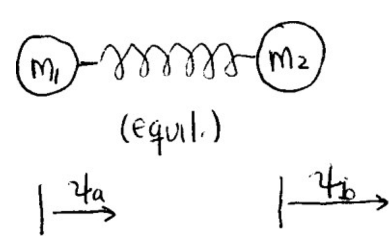
   a. Derive the governing differential equations for the "arbitrary" configuration (*i.e.,* not a normal mode) for the system in the (ψ_a, ψ_b) coordinate system defined above. Show all steps of your derivation.

   b. Derive the normal coordinates for this system, showing all necessary steps of the derivation. Clearly identify the normal coordinates and prove that they are indeed normal coordinates.

   c. Derive the mode frequencies from that and show that they are as you would expect from previous (i.e., before we embarked on material of chapter 6) work in this course.

4. **Normal Modes of Diatomic Molecule – Unequal mass case, revisited via determinant ("systematic") Method**

   We recall our model for the system:



Let $\psi_a(t)$ and $\psi_b(t)$ be displacements from equilibrium. Note that both are measured positive to the right. Take $\boldsymbol{m_2}$ **not equal to** $\boldsymbol{m_1}$. Recall that the systematic (determinant) method is discussed in section 6.14 of the K-text and was also discussed in the posted Class Notes set for our 11th Class session (Tuesday February 13).

a. In the previous problem, you derived the governing differential equations for the "arbitrary" configuration for the system. Rewrite those results as the answer to this part.

b. Now use the systematic (determinant) method to find the mode frequencies. Show all steps of the necessary work/logic. Are the frequencies you found are what you would expect based on previous work in this course?

c. Continue the systematic method to find the mode shape (configuration) of the higher frequency normal mode. Show that it is what one expects from earlier in our course.