$y'y'' = y'''$

Sea $t(x) = y'(x) \rightarrow t t' = t'' \Leftrightarrow \int t t' \, dx = \int t'' \, dx \Leftrightarrow \frac{1}{2} t^2 + C_1 = t' \overset{vs}{\Leftrightarrow} \frac{dt}{dx} = \frac{1}{2} t^2 + C_1 \Leftrightarrow$

$\Leftrightarrow \frac{dt}{dx} = \frac{1}{2}(t^2 + C_2) \Leftrightarrow \int \frac{1}{t^2 + C_2} dt = \frac{1}{2} \int dx \longrightarrow$ <u>3 casos, $C_2 < 0$, $C_2 = 0$, $C_2 > 0$</u>

$\boxed{C_2 = 0}$

$\int \frac{1}{t^2} dt = \frac{1}{2} \int dx \Leftrightarrow -\frac{1}{t} = \frac{1}{2} x + C_3 \Leftrightarrow \frac{1}{t} = -\frac{C_4 + x}{2} \Leftrightarrow t(x) = \frac{-2}{(x + C_4)} \Leftrightarrow$

$\Leftrightarrow \frac{dy}{dx} = \frac{-2}{x + C_4} \Leftrightarrow y = \int \frac{-2}{x + C_4} dx \Leftrightarrow \boxed{y = -2 \ln|x + C_4| + C_5}$

$y' = -\frac{2}{x+a}$
$y'' = \frac{2}{(x+a)^2}$
$y''' = -\frac{4}{(x+a)^3}$

$-\frac{2}{x+a} \cdot \frac{2}{(x+a)^2} = -\frac{4}{(x+a)^3}$ ✓

• Si $t^2 = 0 \Leftrightarrow t = 0 \Leftrightarrow y' = 0 \Leftrightarrow \boxed{y = x + C_6}$

$y' = 1$
$y'' = 0$
$y''' = 0$

$1 \cdot 0 = 0$ ✓

$\boxed{a = C_2 > 0}$ sea $a \in \mathbb{R} : a > 0$

$\frac{-2}{(x+C_8)} \cdot \frac{2}{(x+C_8)^2} = \frac{-4}{(x+C_8)^3}$ ✓

$y' = \frac{-2}{x + C_8}$ $y''' = -\frac{4}{(x+C_8)^3}$
$y'' = \frac{2}{(x+C_8)^2}$

$\int \frac{1}{t^2 + a} dt = \frac{1}{2} \int dx \Leftrightarrow \sqrt{a} \int \frac{\sec^2(u)}{a \tan^2(u) + a} du = \frac{\sqrt{a}}{a} \int \frac{\sec^2(u) \, du}{\sec^2(u)} = \frac{1}{\sqrt{a}} \arctan\left(\frac{t}{\sqrt{a}}\right) = \frac{1}{2} x + C_7$

$\sec^2(x) - 1 = \tan^2(x)$
$\sec^2(x) = \tan^2(x) + 1$
$s \cdot \sec^2(x) = s \cdot (\tan^2(x) + 1)$

$\Leftrightarrow \sqrt{a} \tan\left(\frac{\sqrt{a}(x + C_8)}{2}\right) = t(x) \Leftrightarrow \int dy = \sqrt{a} \int \tan\left(\frac{x\sqrt{a} + C_8}{2}\right) dx = \boxed{-2 \ln\left|\cos\left(\frac{\sqrt{a}(x + C_8)}{2}\right)\right| + C_9 = y(x)}$

$\{a > 0\}$

$t = \sqrt{a} \tan(u)$ $\Rightarrow u = \arctan\left(\frac{t}{a}\right)$
$dt = \sqrt{a} \sec^2(u) \, du$

• Si $t^2 + a = 0 \rightarrow t^2 = -a$ sin sol. en $\mathbb{R}$

$\boxed{b = C_2 < 0}$ sea $b \in \mathbb{R} : b < 0$ tq $b = -a$

$\boxed{\left(-\frac{1}{\sqrt{a}} \text{arctanh}\left(\frac{t}{\sqrt{a}}\right)\right)}$

$\int \frac{1}{t^2 + b} dt \Leftrightarrow y = -2 \ln\left|\cos\left(\frac{\sqrt{-a}(x + C_8)}{2}\right)\right| + C_9 \longrightarrow$ sin sol en $\mathbb{R}$

• Si $t^2 + b = 0 \Leftrightarrow t^2 - a = 0 \Leftrightarrow t^2 = a \Leftrightarrow t = \pm a \Leftrightarrow$

$\frac{dy}{dx} = a \Leftrightarrow \int dy = \int a \, dx \Leftrightarrow \boxed{y = ax + C_{10}}$

$\frac{dy}{dx} = -a \Leftrightarrow \int dy = -\int a \, dx \Leftrightarrow \boxed{y = -ax + C_{11}}$

$y' = a$
$y'' = 0$
$y''' = 0$

$a \cdot 0 = 0$ ✓
$-a \cdot 0 = 0$ ✓