$$\boxed{\vec{E}(\vec{r},t) = \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r}-wt)}}$$

$$\frac{\partial \vec{E}}{\partial t} = \vec{E}_0\,(-iw)\, e^{-i(\vec{k}\cdot\vec{r}-wt)} = -iw\,\vec{E}$$

$$\boxed{\frac{\partial}{\partial t} \longmapsto -iw}$$

$$\vec{\nabla}\cdot\vec{E} = \vec{\nabla}\cdot\left(\vec{E}_0\, e^{-i\vec{k}\cdot\vec{r}}\, e^{-iwt}\right) =$$

$$= e^{-iwt}\,\vec{\nabla}\Big(\underset{\substack{\uparrow\\ \text{vector}\\ \vec{a}}}{\vec{E}_0}\; \underset{\substack{\uparrow\\ \text{escalar}\\ \psi}}{e^{i\vec{k}\cdot\vec{r}}}\Big) = \downarrow$$

Fórmula (7):

$$\vec{\nabla}\cdot(\psi\,\vec{a}) = (\vec{\nabla}\psi)\cdot\vec{a} + \psi(\vec{\nabla}\cdot\vec{a})$$

$$\downarrow = \vec{\nabla}(e^{i\vec{k}\cdot\vec{r}})\,\vec{E}_0\, e^{-iwt} + e^{i\vec{k}\cdot\vec{r}}\,\underbrace{\cancel{(\vec{\nabla}\cdot\vec{E}_0)}}_{=0\ (\vec{E}_0 = \text{cte})}\,e^{-iwt} =$$

$$= \vec{E}_0\, e^{-iwt}\, \underbrace{\vec{\nabla}(e^{i\vec{k}\cdot\vec{r}})}_{\text{calculamos esto}}$$

$$\vec{\nabla}(e^{i\vec{k}\cdot\vec{r}}) = e^{i\vec{k}\cdot\vec{r}}\,\vec{\nabla}(i\vec{k}\cdot\vec{r}) =$$

$$= i e^{i\vec{k}\cdot\vec{r}}\,\underbrace{\vec{\nabla}(\vec{k}\cdot\vec{r})}_{\circledast}$$

Fórmula (9):

$$\vec{\nabla}(\vec{a}\cdot\vec{b}) = (\vec{a}\cdot\vec{\nabla})\vec{b} + (\vec{b}\cdot\vec{\nabla})\vec{a} +$$

$$+ \vec{a}\times(\vec{\nabla}\times\vec{b}) + \vec{b}\times(\vec{\nabla}\times\vec{a})$$

$$\circledast = \vec{\nabla}(\vec{k}\cdot\vec{r}) = \underbrace{\cancel{(\vec{r}\cdot\vec{\nabla})\vec{k}}}_{=0\ (\vec{k}\text{ no depende de }\vec{r})} + (\vec{k}\cdot\vec{\nabla})\vec{r} +$$

$$+ \underbrace{\vec{r}\times\cancel{(\vec{\nabla}\times\vec{k})}}_{=0\ (\vec{k}\text{ no depende de }\vec{r})} + \vec{k}\times\underbrace{\cancel{(\vec{\nabla}\times\vec{r})}}_{=0} = (\vec{k}\cdot\vec{\nabla})\vec{r} =$$

$$\vec{\nabla}\times\vec{r} = \begin{vmatrix} \hat{u}_x & \hat{u}_y & \hat{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0$$

$$= \left(k_x\frac{\partial}{\partial x} + k_y\frac{\partial}{\partial y} + k_z\frac{\partial}{\partial z}\right)(x\hat{u}_x + y\hat{u}_y + z\hat{u}_z) =$$

$$= k_x\frac{\partial x}{\partial x}\hat{u}_x + k_y\frac{\partial y}{\partial y}\hat{u}_y + k_z\frac{\partial z}{\partial z}\hat{u}_z =$$

$$= k_x\hat{u}_x + k_y\hat{u}_y + k_z\hat{u}_z = \vec{k}$$

de donde:

$$\vec{\nabla}\left(e^{i\vec{k}\cdot\vec{r}}\right) = i\, e^{i\vec{k}\cdot\vec{r}}\, \vec{\nabla}\left(\vec{k}\cdot\vec{r}\right) = i\vec{k}\, e^{i\vec{k}\cdot\vec{r}}$$

y queda:

$$\vec{\nabla}\cdot\vec{\mathbb{E}} = \vec{\mathbb{E}}_0\, e^{-i\omega t}\, \vec{\nabla}\left(e^{i\vec{k}\cdot\vec{r}}\right) = i\vec{k}\,\vec{\mathbb{E}}_0\, e^{i\vec{k}\cdot\vec{r}}\, e^{-i\omega t} =$$

$$= i\vec{k}\,\vec{\mathbb{E}}$$

$$\boxed{\vec{\nabla} \longmapsto i\vec{k}}$$