Símbolo de Levi-Civita (símbolo de permutación)

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} \epsilon^{0123} = +1 \\ \operatorname{sgn}\begin{pmatrix} 0 & 1 & 2 & 3 \\ \mu & \nu & \rho & \sigma \end{pmatrix} \end{cases}$$

Tensor dual del campo electromagnético

$$\boxed{\mathcal{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}}$$

$$F_{\rho\sigma} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{pmatrix}$$

$$\mathcal{F}^{\mu\nu} = \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & -E_z/c & -E_y/c \\ B_y & B_z & 0 & E_x/c \\ B_z & E_y/c & -E_x/c & 0 \end{pmatrix}$$

$$0123 \rightarrow +1 \} \; 0\,(+1)$$

$$\left.\begin{array}{l} 0132 \\ \downarrow \quad 1 \text{ perm.} \\ 0123 \end{array}\right\} 1\,(-1)$$

$$\left.\begin{array}{l} 0213 \\ \downarrow \quad 1 \text{ perm.} \\ 0123 \end{array}\right\} 1\,(-1)$$

$$\left.\begin{array}{l} 0231 \\ \downarrow \quad 1 \text{ perm.} \\ 0132 \\ \downarrow \quad 1 \text{ perm.} \\ 0123 \end{array}\right\} 2\,(+1)$$

$$\left.\begin{array}{l} 0312 \\ \downarrow \quad 1 \text{ perm.} \\ 0132 \\ \downarrow \quad 1 \text{ perm.} \\ 0123 \end{array}\right\} 2\,(+1)$$

$$\left.\begin{array}{l} 0321 \\ \downarrow \quad 1 \text{ perm.} \\ 0123 \end{array}\right\} 1\,(-1)$$

$$\left.\begin{array}{l} 2301 \\ \downarrow \quad 1 \text{ perm.} \\ 0321 \\ \downarrow \quad 1 \text{ perm.} \\ 0123 \end{array}\right\} 2\,(+1)$$

$$\left.\begin{array}{l} 1203 \\ \downarrow \quad 1 \text{ perm.} \\ 0213 \\ \downarrow \quad 1 \text{ perm.} \\ 0123 \end{array}\right\} 2\,(+1)$$

$$\left.\begin{array}{l} 1230 \\ \downarrow \quad 1 \text{ perm.} \\ 1203 \\ \downarrow \quad 1 \text{ perm.} \\ 1023 \\ \downarrow \quad 1 \text{ perm.} \\ 0123 \end{array}\right\} 3\,(-1)$$

$$\left.\begin{array}{l} 1302 \\ \downarrow \quad 1 \text{ perm.} \\ 1203 \\ \downarrow \quad 1 \text{ perm.} \\ 1023 \\ \downarrow \quad 1 \text{ perm.} \\ 0123 \end{array}\right\} 3\,(-1)$$

$$\left.\begin{array}{l} 1320 \\ \downarrow \quad 1 \text{ perm.} \\ 1023 \\ \downarrow \quad 1 \text{ perm.} \\ 0123 \end{array}\right\} 2\,(+1)$$

$$\left.\begin{array}{l} 2310 \\ \downarrow \quad 1 \text{ perm.} \\ 2013 \\ \downarrow \quad 1 \text{ perm.} \\ 0213 \\ \downarrow \quad 1 \text{ perm.} \\ 0123 \end{array}\right\} 3\,(-1)$$

Componentes del tensor dual $\mathcal{F}^{\mu\nu}$ del tensor campo electromagnético $F^{\mu\nu}$

$$\boxed{\mathcal{F}^{\mu\nu} \equiv \frac{1}{2} \in^{\mu\nu\rho\sigma} F_{\rho\sigma}} \qquad \in^{\mu\nu\rho\sigma} = \begin{cases} \in^{0123} = +1 \\ \operatorname{sgn}\begin{pmatrix} 0 & 1 & 2 & 3 \\ \mu & \nu & \rho & \sigma \end{pmatrix} \end{cases}$$

$$F_{\rho\rho} = 0 \quad \Rightarrow \quad \mathcal{F}^{\mu\mu} = 0 \quad \Rightarrow \quad \mathcal{F}^{00} = \mathcal{F}^{11} = \mathcal{F}^{22} = \mathcal{F}^{33} = 0$$ (diagonal nula)

$$\left.\begin{array}{l} \mathcal{F}^{i0} = \frac{1}{2} \in^{i0\rho\sigma} F_{\rho\sigma} = -\frac{1}{2} \in^{0i\rho\sigma} F_{\rho\sigma} = -\mathcal{F}^{0i} \\ \mathcal{F}^{ji} = \frac{1}{2} \in^{ji\rho\sigma} F_{\rho\sigma} = -\frac{1}{2} \in^{ij\rho\sigma} F_{\rho\sigma} = -\mathcal{F}^{ij} \end{array}\right\}$$ (tensor antisimétrico)

$$\mathcal{F}^{01} = \frac{1}{2} \in^{01\rho\sigma} F_{\rho\sigma} = \frac{1}{2} \in^{0123} F_{23} + \frac{1}{2} \in^{0132} F_{32} = \frac{1}{2}(+1)(-B_x) + \frac{1}{2}(-1)(B_x) = -\frac{1}{2} B_x - \frac{1}{2} B_x = -B_x$$

$$\mathcal{F}^{02} = \frac{1}{2} \in^{02\rho\sigma} F_{\rho\sigma} = \frac{1}{2} \in^{0213} F_{13} + \frac{1}{2} \in^{0231} F_{31} = \frac{1}{2}(-1)(B_y) + \frac{1}{2}(+1)(-B_y) = -\frac{1}{2} B_y - \frac{1}{2} B_y = -B_y$$

$$\mathcal{F}^{03} = \frac{1}{2} \in^{03\rho\sigma} F_{\rho\sigma} = \frac{1}{2} \in^{0312} F_{12} + \frac{1}{2} \in^{0321} F_{21} = \frac{1}{2}(+1)(-B_z) + \frac{1}{2}(-1)(B_z) = -\frac{1}{2} B_z - \frac{1}{2} B_z = -B_z$$

$$\mathcal{F}^{12} = \frac{1}{2} \in^{12\rho\sigma} F_{\rho\sigma} = \frac{1}{2} \in^{1203} F_{03} + \frac{1}{2} \in^{1230} F_{30} = \frac{1}{2}(+1)\left(\frac{E_z}{c}\right) + \frac{1}{2}(-1)\left(-\frac{E_z}{c}\right) = \frac{1}{2}\frac{E_z}{c} + \frac{1}{2}\frac{E_z}{c} = \frac{E_z}{c}$$

$$\mathcal{F}^{13} = \frac{1}{2} \in^{13\rho\sigma} F_{\rho\sigma} = \frac{1}{2} \in^{1302} F_{02} + \frac{1}{2} \in^{1320} F_{20} = \frac{1}{2}(-1)\left(\frac{E_y}{c}\right) + \frac{1}{2}(+1)\left(-\frac{E_y}{c}\right) = -\frac{1}{2}\frac{E_y}{c} - \frac{1}{2}\frac{E_y}{c} = -\frac{E_y}{c}$$

$$\mathcal{F}^{23} = \frac{1}{2} \in^{23\rho\sigma} F_{\rho\sigma} = \frac{1}{2} \in^{2301} F_{01} + \frac{1}{2} \in^{2310} F_{10} = \frac{1}{2}(+1)\left(\frac{E_x}{c}\right) + \frac{1}{2}(-1)\left(-\frac{E_x}{c}\right) = \frac{1}{2}\frac{E_x}{c} + \frac{1}{2}\frac{E_x}{c} = \frac{E_x}{c}$$