① Determinar las ecuaciones implícitas de la variedad lineal de $\mathbb{R}^4$ generada por los puntos

$P=(1\ 1\ 0\ {-2})$ $\quad Q=(2\ 0\ 0\ 3)$ $\quad R=(0\ {-1}\ 1\ 1)$

$(x,y,z,t) = P + \langle \overline{PQ}, \overline{PR} \rangle = (1,1,0,-2) + \langle (1,-1,0,5), (-1,-2,1,3) \rangle$

$$\begin{aligned} x &= 1+\alpha-\beta \\ y &= 1-\alpha-2\beta \\ z &= \beta \\ t &= -2+5\alpha+3\beta \end{aligned} \qquad \left\{ \begin{aligned} x &= 1+\alpha-z \\ y &= 1-\alpha-2z \\ t &= -2+5\alpha+3z \end{aligned} \right.$$

$x+z-1=\alpha$

$y+2z-1=-\alpha \qquad -y-2z+1=\alpha$

$x+z-1=-y-2z+1$

$t=-2+5(x+z-1)+3z$

$t=-2+5x+5z+5+3z$

$-5x-8z+t=-7$

$$\left. \begin{aligned} x+y+3z &= 2 \\ 5x \quad +8z-t &= 7 \end{aligned} \right\} \text{ecuaciones implícitas}$$

② Encuentra las ecuaciones paramétricas e implícitas de las siguientes variedades lineales de $\mathbb{R}^3$

A) $r := \begin{cases} x+2y=3 \\ 3y+z=2 \end{cases}$ $\quad s := \begin{cases} x+z=0 \\ y+2z=-1 \end{cases}$ $\quad$ B) r y $t := \begin{cases} 2x-z=0 \\ x+y+3z=-1 \end{cases}$ $\quad$ C) s y $\pi := y-z=5$

a)
$$\begin{aligned} x+2y \quad\quad &= 3 \\ 3y+z &= 2 \\ x \quad\quad +z &= 0 \\ y+2z &= -1 \end{aligned}$$

$A=\begin{pmatrix} 1&2&0 \\ 0&3&1 \\ 1&0&1 \\ 0&1&2 \end{pmatrix}$ $\quad A^*=\begin{pmatrix} 1&2&0&3 \\ 0&3&1&2 \\ \cancel{1}&\cancel{0}&\cancel{1}&\cancel{0} \\ 0&1&2&1 \end{pmatrix}$

$rg(A)=3$

$\begin{vmatrix} 1&3&0 \\ 0&3&1 \\ 1&0&1 \end{vmatrix} = 5 \neq 0$

$z=-x$

$x-3=-2y \qquad y=-\frac{x}{2}+\frac{3}{2}$

$\det(A^*) = \begin{vmatrix} 3&1&2 \\ 0&1&0 \\ 1&2&-1 \end{vmatrix} + \begin{vmatrix} 2&0&3 \\ 3&1&2 \\ 1&2&-1 \end{vmatrix} = -5+5=0 \longrightarrow$ SCD $\Rightarrow$ Solución única $(1,1,-1)$

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b) $A=\begin{pmatrix} 1&2&0 \\ 0&3&2 \\ 2&0&-1 \\ 1&1&3 \end{pmatrix}$ $\quad A^*=\begin{pmatrix} 1&2&0&3 \\ 0&3&-2&2 \\ \cancel{2}&\cancel{0}&\cancel{-1}&\cancel{0} \\ 1&1&3&-1 \end{pmatrix}$

$\begin{vmatrix} 1&2&0 \\ 0&3&2 \\ 2&0&-1 \end{vmatrix} = 5 \neq 0 \quad rg(A)=3 \Rightarrow rg(A^*) \geq 3$

$\det(A^*) -2\begin{vmatrix} 2&0&3 \\ 3&2&2 \\ 1&3&-1 \end{vmatrix} - \begin{vmatrix} 1&2&3 \\ 0&3&2 \\ 1&1&-1 \end{vmatrix} = 10-10=0$

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c)
$$\begin{aligned} x+z &= 0 \\ y+2z &= -1 \\ y-z &= 5 \end{aligned}$$

$A=\begin{pmatrix} 1&0&1 \\ 0&1&2 \\ 0&1&-1 \end{pmatrix}$ $\quad A^*=\begin{pmatrix} 1&0&1&0 \\ 0&1&2&-1 \\ 0&1&-1&5 \end{pmatrix}$ $\quad \det(A^*)=0 \quad \det(A) \neq 0$

$rg\,A=3$

$(2,3,-2) \qquad rg\,A^*=3$

Sea el espacio afín standard $\mathbb{R}^4$

Consideremos $\quad L := \begin{cases} x-y=0 \\ z+t=2 \end{cases} \quad M := (0,0,0,0)+\langle (2,2,-3,3),(0,0,1,-1) \rangle$

(4ptos) ¿Cuál es la posición relativa de L y M?

(4ptos) Determinar $L+M$ y $L \cap M$, Justifica las respuestas

$L=a+F \qquad F \subset \mathbb{R}^4 \qquad \begin{aligned} x-y&=0 \\ z+t&=0 \end{aligned} \qquad \begin{aligned} x&=y \\ z&=-z \end{aligned} \qquad \begin{aligned} &(1,1,0,0) \\ &(0,0,1,-1) \end{aligned}$ $\quad$ Quita el término independiente para que pase por $(0,0,0,0)$

$L := (0,0,1,1)+\langle (1,1,0,0),(0,0,1,-1) \rangle \qquad (0,0,1,1)=\alpha(2,2,-3,3)+\beta(0,0,1,-1)$

$(2,2,-3,3)=2(1,1,0,0)+(-3)(0,0,1,-1)$

$$\left\{ \begin{aligned} 0&=2\alpha \\ 0&=2\alpha \\ 1&=-3\alpha+\beta \\ 1&=-3\alpha-\beta \end{aligned} \right. \qquad \begin{aligned} \alpha&=0 \\ \beta&=1 \\ \beta&=-1 \end{aligned}$$

$F+G=F \qquad F=G \Rightarrow L$ y M son paralelas

$L \cap M = \emptyset$

$\dim(L+M) = \overset{2}{\overset{\shortparallel}{\dim(L)}} + \overset{2}{\overset{\shortparallel}{\dim(M)}} - \overset{2}{\overset{\shortparallel}{\dim(F \cap G)}} + 1$ 3-plano

$(0,0,1,1) + \langle (1,1,0,0), (0,0,1,-1), (0,0,1,1) \rangle$

$\vec{ab}$ $\quad \vec{ba} = (0,0,1,1)$

$0 = 2\alpha \quad \alpha = 0$
$0 = 2\alpha$
$1 = -3\alpha + \beta \quad \beta = 1$
$1 = -3\alpha - \beta \quad \beta = -1$

En $\mathbb{R}_3[x] = \{ax^3 + bx^2 + cx + d\} \cong \mathbb{R}^4$

$H: = \{p(x) \in \mathbb{R}_3[x] : p(1) = 0, \ p'(0) = -1\}$

$L: = x^3 + \langle 1, x, x^2 \rangle$

$H: \{(a, b, -1, 1-a-b)\}$

$H: = (0,0,1,-1) + \langle (1,0,0,-1), (0,1,0,-1) \rangle$

$H: = (-x+1) + \langle x^3 +$

$L: x^3 + \langle x^2, x, 1 \rangle$

$p(x) = ax^3 + bx^2 + cx + d$

$p(1) = 0$

$a + b + c + d = 0 \qquad a + b - 1 = -d$

$p'(x) - 3ax^2 + 2bx + c \qquad d = 1 - a - b$

$p'(0) = -1 \qquad c = -1$