Cálculo del flujo de $\vec{F}$ (PROBLEMA 8)

$$\vec{F} = \frac{d\vec{P}_{mec}}{dt} = \oint_S \overleftrightarrow{T} \cdot d\vec{s} = \oint_S (\overleftrightarrow{T} \cdot \hat{n})\, ds$$

$$T_{xx} = \frac{1}{2} \varepsilon_0 E^2$$

$$T_{yy} = -\frac{1}{2} \varepsilon_0 E^2$$

$$T_{zz} = -\frac{1}{2} \varepsilon_0 E^2$$

$$T_{xy} = T_{xz} = T_{yz} = 0$$

$$\vec{T}_x = (T_{xx}, T_{xy}, T_{xz}) = (T_{xx}, 0, 0) = T_{xx}\, \hat{u}_x$$

$$\vec{T}_y = (T_{xy}, T_{yy}, T_{yz}) = (0, T_{yy}, 0) = T_{yy}\, \hat{u}_y$$

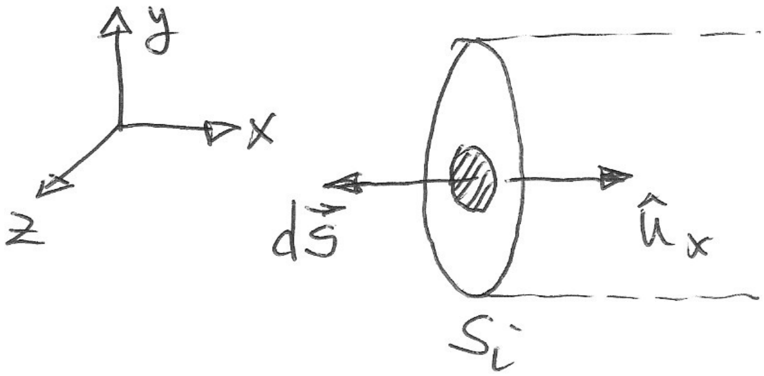
$$\vec{T}_z = (T_{xz}, T_{yz}, T_{zz}) = (0, 0, T_{zz}) = T_{zz}\, \hat{u}_z$$

El flujo a través de la superficie cerrada es la suma de tres flujos:

$$\oint_S (\overleftrightarrow{T} \cdot \hat{n})\, ds = \int_{S_i} (\overleftrightarrow{T} \cdot \hat{n})\, ds + \int_{S_e} (\overleftrightarrow{T} \cdot \hat{n})\, ds +$$

$$+ \int_{S_L} (\overleftrightarrow{T} \cdot \hat{n})\, ds$$

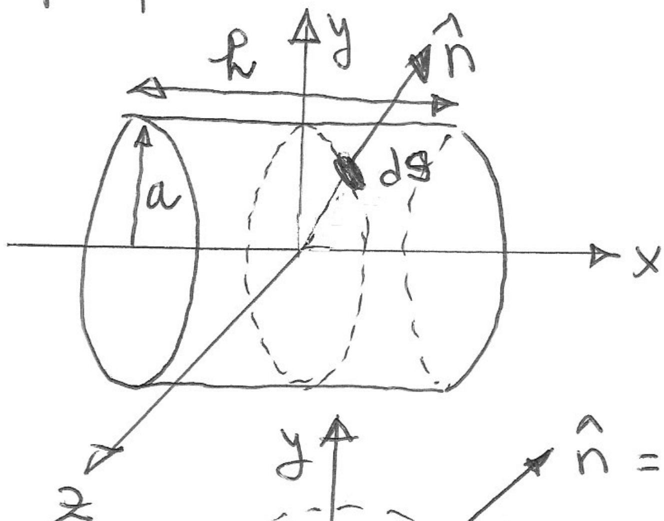
S_i : base del cilindro situada entre las placas del condensador

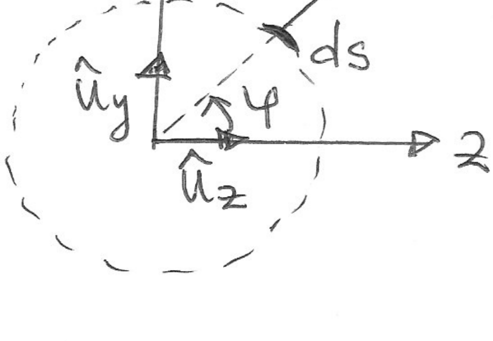


$$d\vec{S} = \hat{n}\, dS = -dS\,\hat{u}_x$$

S_e : base del cilindro situada fuera del condensador (el flujo será nulo porque fuera del condensador $\overleftrightarrow{T} = 0$)

S_L : superficie lateral del cilindro.





$$\hat{n} = \operatorname{sen}\varphi\,\hat{u}_y + \cos\varphi\,\hat{u}_z$$

$$\hat{n}\,ds = -\hat{u}_x\,ds$$

$$\int_{S_i} (\overleftrightarrow{T}\cdot\hat{n})\,ds = \int_{S_i} (\vec{T}_x\cdot\hat{n}, \vec{T}_y\cdot\hat{n}, \vec{T}_z\cdot\hat{n})\,ds =$$

$$= -\int_{S_i} (T_{xx}\underbrace{\hat{u}_x\cdot\hat{u}_x}_{=1}, T_{yy}\underbrace{\hat{u}_y\cdot\hat{u}_x}_{=0}, T_{zz}\underbrace{\hat{u}_z\cdot\hat{u}_x}_{=0})\,ds =$$

$$= -\int_{S_i} (T_{xx}, 0, 0)\,ds = -\hat{u}_x \int_{S_i} T_{xx}\,ds =$$

$$= -\hat{u}_x \int_{S_i} \frac{1}{2}\varepsilon_0 E^2\,ds = -\hat{u}_x \frac{\varepsilon_0 E^2}{2} S$$

área de la placa del condensador

$$\int_{S_e} (\overleftrightarrow{T}\cdot\hat{n})\,ds \overset{\overleftrightarrow{T}=0 \text{ fuera}}{=} 0$$

$$\int_{S_L} (\overleftrightarrow{T}\cdot\hat{n})\,ds = \int_{S_L} (\vec{T}_x\cdot\hat{n}, \vec{T}_y\cdot\hat{n}, \vec{T}_z\cdot\hat{n})\,ds =$$

$$= \int_{S_L} \left(-T_{xx}\hat{u}_x(\operatorname{sen}\varphi\,\hat{u}_y + \cos\varphi\,\hat{u}_z), T_{yy}\hat{u}_y(\operatorname{sen}\varphi\,\hat{u}_y + \cos\varphi\,\hat{u}_z), T_{zz}\hat{u}_z(\operatorname{sen}\varphi\,\hat{u}_y + \cos\varphi\,\hat{u}_z)\right)\underbrace{h\,a\,d\varphi}_{ds} =$$

$$= ha \int_0^{2\pi} (0, T_{yy} \operatorname{sen}\varphi, T_{zz} \cos\varphi)\, d\varphi =$$

$$= ha \left[\hat{u}_y \int_0^{2\pi} T_{yy} \operatorname{sen}\varphi\, d\varphi + \hat{u}_z \int_0^{2\pi} T_{zz} \cos\varphi\, d\varphi \right] =$$

$$= ha \left[\hat{u}_y T_{yy} \underbrace{\int_0^{2\pi} \operatorname{sen}\varphi\, d\varphi}_{= -\cos\varphi \big|_0^{2\pi} = 0} + \hat{u}_z T_{zz} \underbrace{\int_0^{2\pi} \cos\varphi\, d\varphi}_{= \operatorname{sen}\varphi \big|_0^{2\pi} = 0} \right] = 0$$

T_{yy} y T_{zz} no dependen de φ (son ctes).

luego:

$$\oint_S (\overleftrightarrow{T} \cdot \hat{n})\, ds = \int_{S_i} (\overleftrightarrow{T} \cdot \hat{n})\, ds + \int_{S_e} (\overleftrightarrow{T} \cdot \hat{n})\, ds +$$

$$+ \int_{S_L} (\overleftrightarrow{T} \cdot \hat{n})\, ds = -\hat{u}_x \frac{\varepsilon_0 E^2}{2} S + 0 + 0 =$$

$$= -\hat{u}_x \frac{\varepsilon_0 E^2}{2} S$$