Entregable 9

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Calcular $\sum_{s=1}^{2} \nu^s(p)\bar{\nu}^s(p)$

Por notación, voy a tomar $\mathbb{I}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ y $\mathbb{I}_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$\sum_{s=1}^{2} \nu^s(p)\bar{\nu}^s(p) = \sum_{s=1}^{2} \begin{pmatrix} \sqrt{p\cdot\sigma}\,\eta^s \\ -\sqrt{p\cdot\bar{\sigma}}\,\eta^s \end{pmatrix} \left(\sqrt{p\cdot\sigma}\,\eta^{s\dagger} \quad -\sqrt{p\cdot\bar{\sigma}}\,\eta^{s\dagger}\right)\gamma^0$$

$$= \sum_{s=1}^{2} \begin{pmatrix} \sqrt{p\cdot\sigma}\,\eta^s \\ -\sqrt{p\cdot\bar{\sigma}}\,\eta^s \end{pmatrix} \left(\sqrt{p\cdot\sigma}\,\eta^{s\dagger} \quad -\sqrt{p\cdot\bar{\sigma}}\,\eta^{s\dagger}\right)\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} =$$

$$= \sum_{s=1}^{2} \begin{pmatrix} \sqrt{p\cdot\sigma}\,\eta^s \\ -\sqrt{p\cdot\bar{\sigma}}\,\eta^s \end{pmatrix} \left(-\sqrt{p\cdot\bar{\sigma}}\,\eta^{s\dagger} \quad \sqrt{p\cdot\sigma}\,\eta^{s\dagger}\right)$$

Si ahora sumamos sabiendo que $\eta^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\eta^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$= \begin{pmatrix} \sqrt{p\cdot\sigma} \\ 0 \\ -\sqrt{p\cdot\bar{\sigma}} \\ 0 \end{pmatrix} \left(-\sqrt{p\cdot\bar{\sigma}} \quad 0 \quad \sqrt{p\cdot\sigma} \quad 0\right) + \begin{pmatrix} 0 \\ \sqrt{p\cdot\sigma} \\ 0 \\ -\sqrt{p\cdot\bar{\sigma}} \end{pmatrix} \left(0 \quad -\sqrt{p\cdot\bar{\sigma}} \quad 0 \quad \sqrt{p\cdot\sigma}\right) =$$

$$= \begin{pmatrix} -\sqrt{p\cdot\bar{\sigma}}\sqrt{p\cdot\sigma} & 0 & (\sqrt{p\cdot\sigma})^2 & 0 \\ 0 & 0 & 0 & 0 \\ (\sqrt{p\cdot\bar{\sigma}})^2 & 0 & -\sqrt{p\cdot\bar{\sigma}}\sqrt{p\cdot\sigma} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\sqrt{p\cdot\sigma}\sqrt{p\cdot\bar{\sigma}} & 0 & (\sqrt{p\cdot\sigma})^2 \\ 0 & 0 & 0 & 0 \\ 0 & (\sqrt{p\cdot\bar{\sigma}})^2 & 0 & -\sqrt{p\cdot\bar{\sigma}}\sqrt{p\cdot\sigma} \end{pmatrix} =$$

$$= \begin{pmatrix} -\sqrt{p\cdot\bar{\sigma}}\sqrt{p\cdot\sigma}\,\mathbb{I}_2 & p\cdot\sigma \\ p\cdot\bar{\sigma} & -\sqrt{p\cdot\bar{\sigma}}\sqrt{p\cdot\sigma}\,\mathbb{I}_2 \end{pmatrix} =$$

Sabiendo que $\sqrt{p\cdot\bar{\sigma}}\sqrt{p\cdot\sigma} = m$, obtenemos

$$= \begin{pmatrix} -m\,\mathbb{I}_2 & p\cdot\sigma \\ p\cdot\bar{\sigma} & -m\,\mathbb{I}_2 \end{pmatrix} = p^\nu \begin{pmatrix} 0 & \sigma_\nu \\ \bar{\sigma}_\nu & 0 \end{pmatrix} - m\,\mathbb{I}_4 = \gamma_\nu p^\nu - m\,\mathbb{I}_4 = \not{p} - m\,\mathbb{I}_4$$

Por tanto,

$$\boxed{\sum_{s=1}^{2} v^s(p)\,\bar{v}^s(p) = \not{p} - m}$$