Photonics Laboratory
Physics 472
Spring 2024

Experiment#3 Laser Beam Characterization

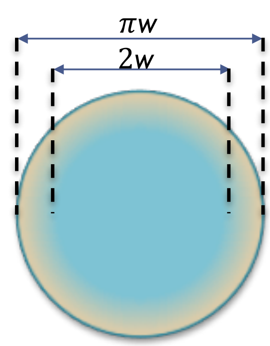
BACKGROUND

The electric field of a Gaussian beam given by the expression below is a solution to the wave equation.

$$E(\rho, z) = E_o\left(\frac{w_o}{w(z)}\right) e^{-\left(\frac{\rho^2}{w^2(z)}\right)}\ e^{-ikz-ik\left(\frac{x^2+y^2}{2R(z)}\right)+i\varphi(z)} = E_o\left(\frac{w_o}{w(z)}\right) e^{-\left(\frac{\rho^2}{w^2(z)}\right)} e^{-i\phi(z)}$$

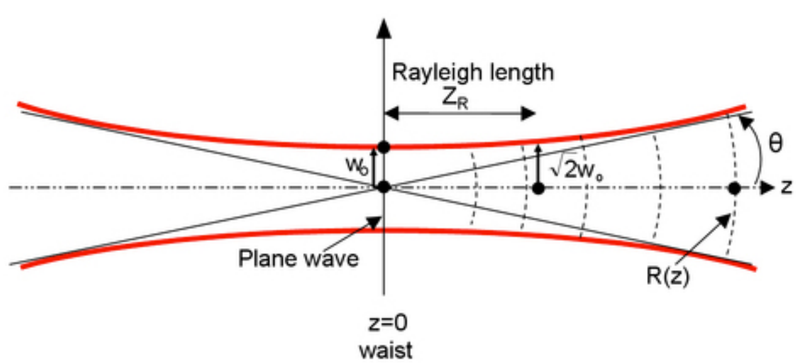
where $\rho^2 = x^2 + y^2$ is the radial distance from the axis of the beam. w_0 is the beam radius at the waist, which is the minimum beam diameter. The beam waist is usually located within the laser cavity or in the external cases at the focus of a lens. Most lasers have their waist at the center of the cavity or at the output coupler. The total phase has three terms. The first term kz is the plane wave phase, the second term $\frac{\rho^2}{w^2(z)}$ is due to the change in the radius of curvature of the wavefront, and the third term $\varphi(z)$ represents the deviation of the Gaussian beam wavefronts from that of a plane wave behavior and is called the **Gouy phase** retardation.

The beam radius at a distance z from the waist is given below. This radius is approximately equal to the total diameter of the beam divided by π. About 99% of the beam power is contained within the πw diameter and 86% of the beam power is contained within the circle of radius w.



$$w(z) = w_o\sqrt{1+\left(\frac{z}{z_o}\right)^2}$$

In the above expression z_o is the **Rayleigh range**. It is the distance from the beam waist at which the beam area doubles, $w(z_o) = \sqrt{2}w_o$.



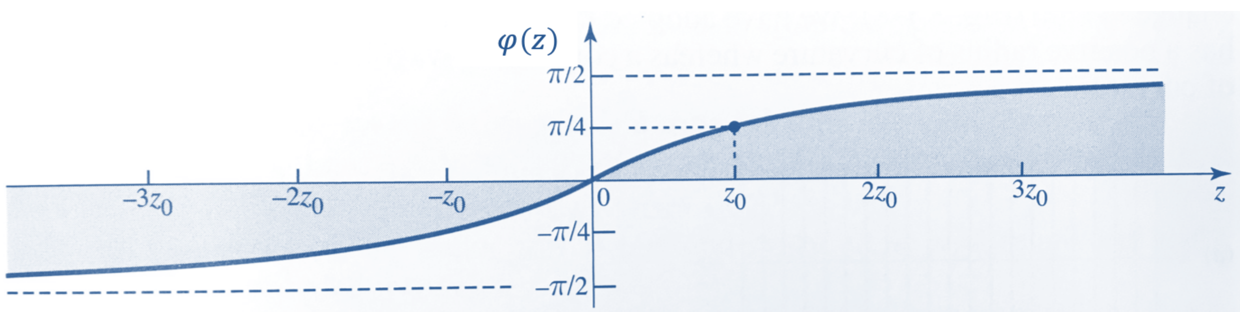
Rayleigh range is related to the radius at the waist by $z_o = \frac{\pi w_o^2}{\lambda}$. The radius of curvature at a distance z from the waist is given by

$$R(z) = z\left[1 + \left(\frac{z_o}{z}\right)^2\right]$$

Note that at z=0 the wavefronts are planer *i.e.*, $R(z) = \infty$ and at z=z_o the radius of curvature becomes $2z_o$ and this is the smallest value R can have. Wavefront radius of curvature at a distance z from the waist. $R(z) \sim z \text{ as } z \to \infty$.

The Gouy phase retardation is important as the beam goes through a focus. The total phase shift that a laser beam experience by going from the far field before the focus to the far field after the focus is π. The Gouy phase is given by

$$\varphi(z) = tan^{-1}\left(\frac{z}{z_o}\right)$$



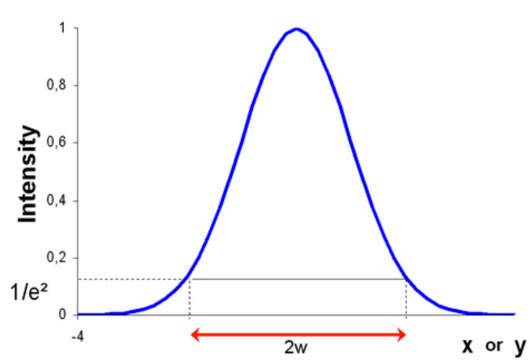
The irradiance of the Gaussian beam can be found using $I(\vec{r})^2 \propto |E(\vec{r})|^2$

$$I(\rho, z) = I_o\left(\frac{W_o}{W(z)}\right)^2 e^{-2\frac{\rho^2}{w^2(z)}}$$

Where I_o is the maximum irradiance at $\rho = z = 0$. I_o is related to the total power P of the beam by

$$P = \frac{1}{2} I_o\left(\pi w_o^2\right)$$

This result is independent of z. The divergence angle of the beam is related to the waist radius by



$$\theta_o = \frac{\lambda}{\pi w_o}$$

Here λ is the laser wavelength. For $z >> z_o$ the divergence angle is the slope of the $w(z)$ vs. z plot.

EXPERMENTS

Note: task#1 and Task#2 must be done for the same beam size.

Task#1: Power transmission through an iris.

Show that the laser power transmitted (P_{trans}) through an iris (pinhole) of radius a is given by

$$P_{trans} = P_o\left[1 - e^{-2\left(\frac{a}{w}\right)^2}\right]$$

Where P_o is the total laser power incident on the iris (or pinhole) and w is the radius of the Gaussian laser beam at the location of the iris. Focus the laser beam using a short focal length lens (10 cm). Place the iris and the detector on a translational stage. Place the iris at an appropriate distance from the lens to obtain at least 1 cm beam diameter at the iris plane. This is also necessary for task #2. Record the transmitted power as a function of iris radius. Plot P_{trans} vs. a then fit the data using the relation given above. From the fit you should be able to find the radius of the beam at the iris location.

Task#2: Measure the profile of a diverging laser beam using iris or pinhole:

Use the same configuration in task#1. Use an iris to scan across the laser beam at the same location you used for task#1. Make the iris opining as small as possible. Make sure to move the iris and the detector at the same time (on the same stage). This is why you need to put the detector and the iris on the same translational stage. Plot the power as a function of the transverse distance cross the beam. Discuss the profile.

(a) From the profile determine the beam radius at the location of the iris. Recall that the Gaussian beam radius is half the width of the profile measured at $1/e^2$ of the maximum power. Compare this result to that obtained in task#1.

(b) Now fit your profile (experimental data) to a Gaussian and determine w at the location of the iris.

Task#3: Measure the laser beam divergence:

Measure the laser radius of the focused He-Ne laser that you used in tasks#1,2 at 5 locations from the laser focus. Use the method of task#1 (use only one iris diameter appropriate for the size of the beam at location of the iris). From your data determine the beam radius for each location. Plot the beam radius vs. distance ***from the beam waist***. Determine the beam divergence angle from your plot.

Task#4: Determine the waist location for a focused laser beam using a razor blade.

Keep the lens of the previous tasks in place and move the detector and everything else. Place a razorblade on two translation stages such that you can move them perpendicular to each other. Put the razorblade as close as possible to the focus. Translate the razorblade across and along the laser beam and observe the diffraction pattern. You should be able to determine the location of the focus relative to the center of the lens. Report your observations and explain.

Task#5: Measure Gaussian beam profile using knife-edge scan:

Mount the razorblade on the micrometer stage and place it exactly at the focus as in Task#4. Orient the blade such that it can be translated across the laser beam. Place a power meter in the path of the laser beam. You may need to place a lens immediately after the blade to collect all the laser light by the power meter detector.

(a) Measure the beam power as a function of the razor blade position for at least 20 positions and plot your data. Be sure to record the total power and the positions of the blade when the transmitted power is equal to 20%, 50%, and 80% of the total beam power. Be sure to normalize the data (power/total power) when you plot power vs. position.

(b) The data can be fitted to the following function which represents the normalized power as function of the blade position.

$$P_{normalized}(x) = \frac{1}{2}\left[1 + erf\left(\frac{x - x_o}{w}\right)\right]$$

In the above equation, *erf* is the error function, x_o is the position at which the power is 50% of the total power of the beam.

The erf is defined as

$$erf(s) = \frac{2}{\sqrt{\pi}}\int_0^s e^{-t^2}dt$$

As you can see, evaluating this integral for each of the x values is not simple and must be done numerically.

Another way to obtain the beam radius is by taking the derivative of power with respect to the blade position. *Plot the derivative of the power vs. position data* and determine the laser beam radius from this plot. You need to fit the data to derivative of the above equation which is given by

$$\frac{dP_{normalized}(x)}{dx} = \frac{1}{w\sqrt{2}}\left\{exp\left[-\left(\frac{x - x_o}{w}\right)^2\right]\right\}$$

(c) Also, the radius is related to the position at 20% and 80% of the total beam power by $w = 1.188(x_2 - x_1)$. Where x_2 and x_1 are the positions corresponding to the 20% and 80% of the beam power, respectively.

(d) An excellent fit to the actual normalized power vs position can be obtained using the following approximation.

$$P_{normalized}(s) = \frac{1}{1 + e^{[a_1 s + a_3 s^3 + a_5 s^5]}}$$

where

$$s = \frac{\sqrt{2}(x - x_o)}{w}$$

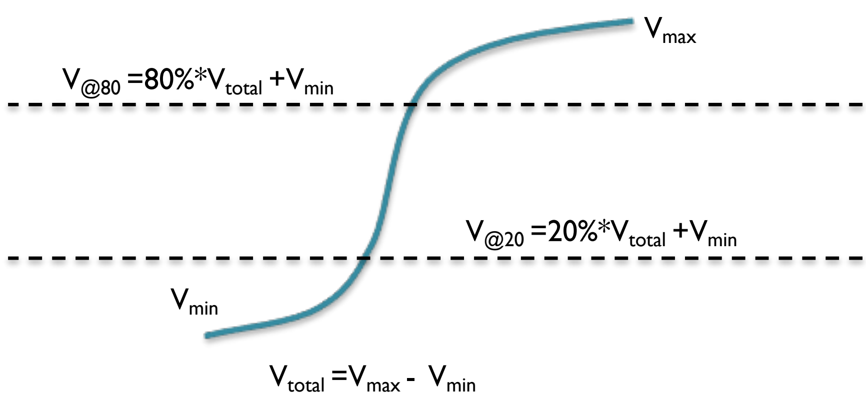
and $a_1 = -1.5954086, \;\; a_3 = -7.3638857 \times 10^{-2}, \;\;\; a_5 = 6.4121343 \times 10^{-4}$.

Determine the radius of the beam by fitting your data to the above expression.

Task#6: Measure Gaussian beam radius using optical chopper:

The optical chopper is a rotating toothed wheel that alternately blocks and let's pass a light beam. Place the optical chopper exactly at the focus that you determined with the razor blade. Make sure that the outer set of teeth intercepts the laser beam. You need to use a lens to collect all the light and focus it to the detector. Place a photodetector in the path of the laser beam and connect output of the photodetector to one input of the oscilloscope. Set the oscilloscope to trigger on the rising edge of the detector signal.

(a) Measure the time it takes the photodetector signal to rise from 20% to 80%. The distance traveled by the chopper blade during this time is equal to the blade speed times the measured 20% to 80% time. The radius is related to the times at 20% and 80% of the total beam power by $w = 1.188v(t_2 - t_2)$. Where t_2 and t_1 are the times corresponding to the 20% and 80% points, respectively. Do this experiment twice, once with the blade crossing the beam horizontally and once crossing vertically.



(b) You need to measure the speed of the point on the chopper blade where the laser beam hits. Think about the quantities that you need to measure to obtain the speed, v.

Task#7: Determine the waist radius and the Rayleigh range of a focused laser beam:

If you know the *location of the waist* then the Rayleigh range, z_o, can be determined from the plot of the beam radius vs. the distance from the waist according to the equation of $w(z)$. In many applications, we focus the laser beams using short focal length lenses. Then determining the waist radius and the Rayleigh range precisely become challenging. However, it is possible to determine z_o and w_o if you know the radius of the beam at two locations separated by a distance $\Delta z = z_2 - z_1$. You can use the equation of $w(z)$ to show that z_o is given by

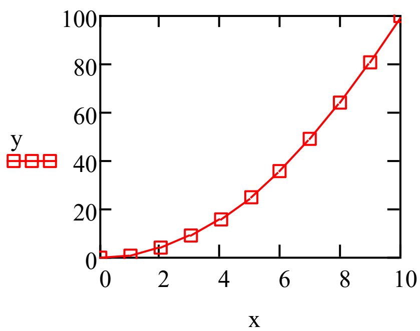
$$z_o = \frac{\lambda \Delta z^2 \left(w_1^2 + w_2^2 + 2\sqrt{w_1^2 w_2^2 - \left(\frac{\lambda \Delta z}{\pi} \right)^2} \right)}{\pi \left(w_1^4 + w_2^4 - 2 w_1^2 w_2^2 + 4 \left(\frac{\lambda \Delta z}{\pi} \right)^2 \right)}$$

(a) Use the same focused beam in task#1. Measure the beam radius at two locations using the method of your choice. Be sure to measure the distance between the two positions. Use the above result to find z_o. Find the radius of the waist w_o. Find the location of the waist. ***Be sure to record your distances relative to the center of the lens*** (your reference).

(b) Derive the above relation. Start with the expression for $w(z)$ at z_1 and z_2, the two locations that you measure w_1 and w_2 at. this derivation is not easy. It needs careful consideration of how to collect terms and what to solve for.

How to take the derivative of data in MathCad

$$F := \begin{array}{c} \begin{array}{cc} x & y := x^2 \end{array} \\ \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 2 & 4 \\ 3 & 9 \\ 4 & 16 \\ 5 & 25 \\ 6 & 36 \\ 7 & 49 \\ 8 & 64 \\ 9 & 81 \\ 10 & 100 \end{pmatrix} \end{array} \qquad \begin{array}{l} y := F^{\langle 1 \rangle} \\ x := F^{\langle 0 \rangle} \end{array}$$

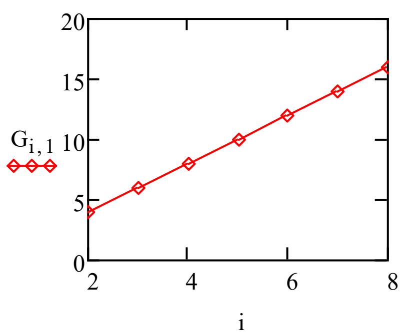


To take the derivative of y numerically you need to have a constant step size (h) for x and the values of y. Note that you can't find the derivative of y for the first two values and the last two values. Therefore, the index *i* below goes from 2-8.

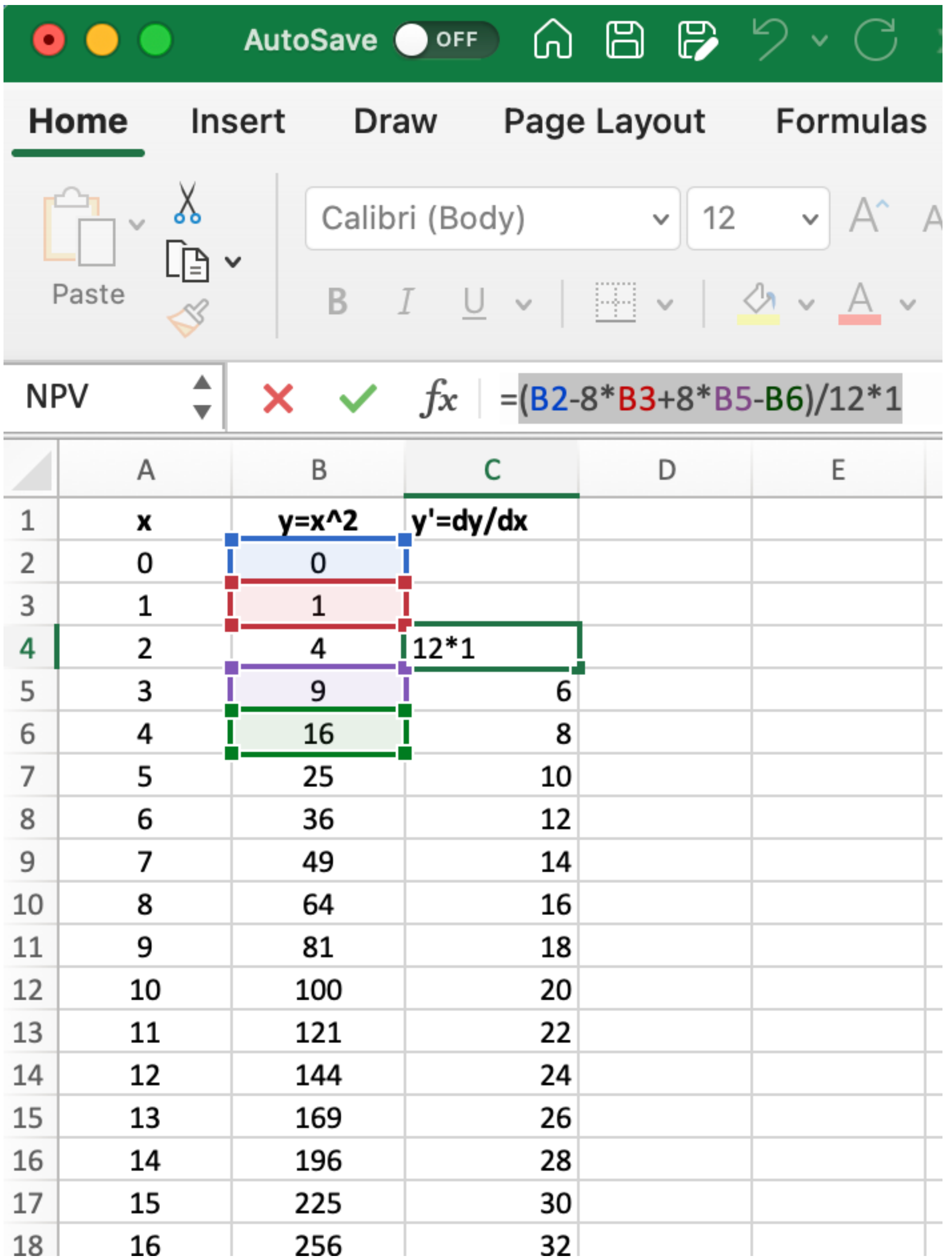
$$i := 2..8 \qquad h := 1$$

$$G_{i,1} := \frac{-F_{i+2,1} + 8 \cdot F_{i+1,1} - 8 \cdot F_{i-1,1} + F_{i-2,1}}{12 \cdot h}$$

This is y'



How to take the derivative of data in Excel



	A	B	C	D	E
1	x	y=x^2	y'=dy/dx		
2	0	0			
3	1	1			
4	2	4	12*1		
5	3	9	6		
6	4	16	8		
7	5	25	10		
8	6	36	12		
9	7	49	14		
10	8	64	16		
11	9	81	18		
12	10	100	20		
13	11	121	22		
14	12	144	24		
15	13	169	26		
16	14	196	28		
17	15	225	30		
18	16	256	32		

Note: The step size was 1. Therefore, in the denominator 12 is multiplied by 1.