Radioactive Decay and Stability of Uranium-235 and Uranium-238

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1 TASKS

Task 1

Write a python code to calculate the remaining quantity of a sample containing $N_0 = 1 \times 10^6$ nuclei of Uranium-235 (^{235}U) and Uranium-238 (^{238}U) nuclei after a period of time t, given their respective half-life $T_{1/2}$:

- ^{238}U: $T_{1/2} = 4{,}468$ billion years
- ^{235}U: $T_{1/2} = 703{,}8$ million years

To do so, we store the values of the half-lives from both of the isotopes along with the initial nuclei count and the time span (in years because of the magnitude of the half-live of Uranium). The script calculates the corresponding disintegration constant λ for each isotope, and uses the known solution of the decay ODE to calculate the final number of nuclei after a given time lapse.

$$\lambda_i = \frac{\log 2}{T_{1/2,i}} \tag{1a}$$

$$N_i(t) = N_0 \cdot \exp -\lambda_i t \tag{1b}$$

For $t = 1 \times 10^{10}$ years and $N_0 = 1 \times 10^6$ the output was:

```
Given a sample of Uranium isotopes of N0 = 1E+06 nuclei:
Remaining quantity of Uranium-235 after 1E+10 years: 5.282E+01 nuclei
Remaining quantity of Uranium-238 after 1E+10 years: 2.120E+05 nuclei
```

Indicating that after 10 billion years, there will be 53 nuclei of ^{235}U and $2 \cdot 10^5$ nuclei of ^{238}U from the original 10^6 nuclei of both samples.

Task 2

Plot the decay curves of ^{235}U and ^{238}U vs time and compare how the two nuclei decay differently given their vastly different half-lives.

Now we are going to plot both of this values for a series of points in the time span that we selected, and see how the curves differ.

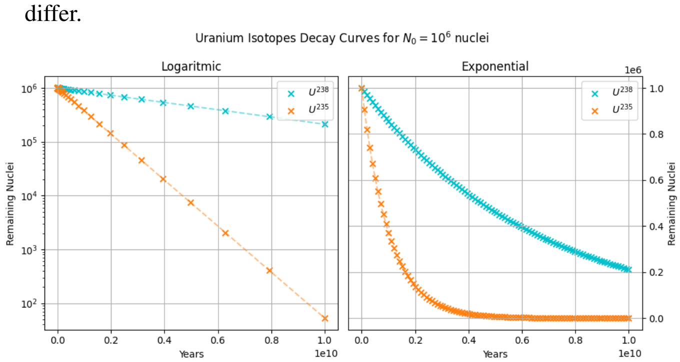

Figure 1: Decay curves of ^{235}U and ^{238} vs time

To the left, we plot the results in a natural logarithmic way to clearly see the exponential nature of the solutions, making the plots linear and easy to compare.

To the right, we plot the curves without taking the logarithm and clearly observe how the exponential function for ^{235}U is much stronger than that of ^{238}U, which is consistent with the results we obtained previously in Task 1, as their half-lives differ in 3 orders of magnitude.

```
def landa(a):
    return log(2)/a

def Decay(N0,T12,t):
    l = landa(T12)
    return N0 * exp(-l*t)

tmax =  10 # 1e10
Nt =    100
t_log = np.logspace(0, tmax,     Nt)
t =     np.linspace(0, 10**tmax, Nt)

decayU238_log = np.array(Decay(N0,T12_U238,t_log))
decayU235_log = np.array(Decay(N0,T12_U235,t_log))
decayU238     = np.array(Decay(N0,T12_U238,t))
decayU235     = np.array(Decay(N0,T12_U235,t))
```

Task 3

Calculate and plot the activity $A(t)$ over time for both Uranium isotopes, and interpret the results.

Similarly to the sample nuclii quantity over time, we will plot now the activity, which is related to the quantity $N(t)$ by (3).

$$A_i(t) = \lambda_i N_i(t) \tag{3}$$

Notice that the y-axis for the logarithmic plot is in Bq whereas the exponential is in kBq for more clarity in the axis visualization.

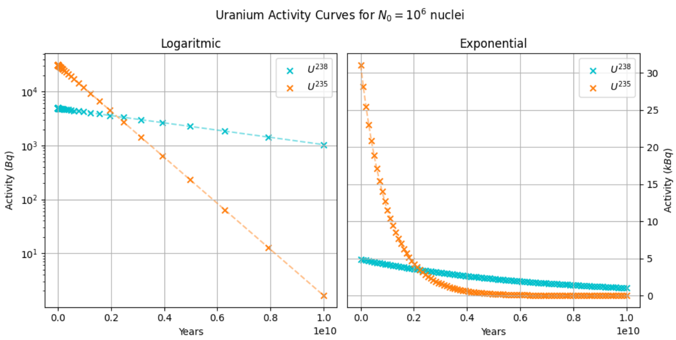

Figure 2: Decay curves of ^{235}U and ^{238} vs time

On this plots we can clearly see how the activity of ^{238}U is way more constant than that of ^{235}U, indicating once again the faster decay of this second isotope.

Although the activity of ^{235}U may be lower that that of ^{238}U after a given time, this happens because mostly all of the original sample of ^{235}U has already decayed.

```
def Activity(T12,N):
    return landa(T12) * N

actvtyU235 = Activity(T12_U235,
          Decay(N0,T12_U235,t))*365*24*60*60/1e3
actvtyU238 = Activity(T12_U238,
          Decay(N0,T12_U238,t))*365*24*60*60/1e3
actvtyU235_log = Activity(T12_U235,
              Decay(N0,T12_U235,t_log))*365*24*60*60
actvtyU238_log = Activity(T12_U238,
              Decay(N0,T12_U238,t_log))*365*24*60*60
```

Task 4

First steps of decay chains: Simulate the decay of ^{238}U into ^{234}Th and ^{235}U into ^{231}Th, then plot the number of nuclei and the activity of these daughter isotopes to analyze their contributions to the overall decay process.

This code block simulates the decay of ^{238}U into ^{234}Th and ^{235}U into ^{231}Th, which then gets plotted to obtain the figure 3.

```
def task4(N0, isotopos):
    isotopos_seg = [(name,
                    convertir_a_segundos(T12, unidad))
                    for name, T12, unidad in isotopos]
    t_max = convertir_a_segundos(1e10, 'y')
    t_points = 1000
    t = np.linspace(0, t_max, t_points)

    decays = {}
    activities = {}
    for i, (name, T12) in enumerate(isotopos_seg):
        if name == 'Uranium 238':
            daughter_name = 'Torium 234'
        elif name == 'Uranium 235':
            daughter_name = 'Torium 231'
        else:
            continue

        N_parent = Decay(N0, T12, t)
        N_daughter = N0 - N_parent
        A_parent = Activity(T12, N_parent)
        A_daughter = Activity(T12, N_daughter)

        decays[name] = N_parent
        decays[daughter_name] = N_daughter
        activities[name] = A_parent
        activities[daughter_name] = A_daughter
```

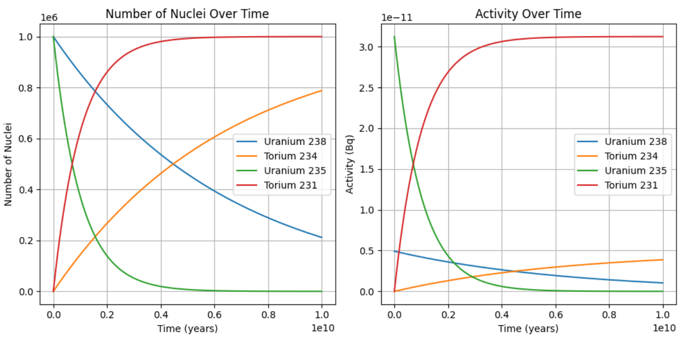

Figure 3: First steps of decay chains

The long-lived parents ^{238}U and ^{235}U, dominate the overall decay activity over geological timescales due to their slow decay rates and high half-lifes. On the other side, the short-lived daughters, ^{234}Th and ^{231}Th, show spikes in activity shortly after production, driven by their rapid decay and low half-lifes. While the daughters contribute significantly to the total activity in the short term, the parents provide a sustained contribution over millions of years.

Task 5

Decay percentage: Calculate how long it takes for each isotope to decay to 90 %, 50 %, and 10 % of its original quantity, and relate this to their half-lives.

```
porcentages = [0.9, 0.5, 0.1]
unidades = {'y':'años', 'h':'horas',
            'd':'días', 's':'segundos'}

for nombre, T12, unidad in T:
    print(f"\nDecaimiento del {nombre}")
    for p in porcentages:
        t, N_new = 1, N0*p + 1
        while N_new > N0*p:
            N_new = Decay(N0, T12, t)
            t *= 1.00006
        print(f"  al {p*100}%:
            {Decimal(t):.3E}{unidades[unidad]}")
```

This code snippet runs through each isotope name and half-life with its corresponding temporal unit. For each isotope it calculates the time it takes for the isotope to decay to a previously stored value (0.9, 0.5 and 0.1).

The time step increases with each iteration of the loop as a product instead of as a sum, so that it works regardless of the time unit that we aim to probe (years, hours, days or seconds). Note that the variable `N_new` starts as `N0*p+1` so that it satisfies the condition of the while loop the first time it gets there for each percentage for each isotope.

The output was:

```
Decay of Uranium 238
  to 90.0%: 6.792E+8 years
  to 50.0%: 4.468E+9 years
  to 10.0%: 1.484E+10 years

Decay of Torium 234
  to 90.0%: 3.664E+0 days
  to 50.0%: 2.410E+1 days
  to 10.0%: 8.007E+1 days

Decay of Uranium 235
  to 90.0%: 1.070E+8 years
  to 50.0%: 7.039E+8 years
  to 10.0%: 2.338E+9 years

Decay of Torium 231
  to 90.0%: 3.880E+0 hours
  to 50.0%: 2.552E+1 hours
  to 10.0%: 8.478E+1 hours
```

Task 6

Link with Binding Energy:

- Discuss how the radioactive decay behaviour of ^{235}U and ^{238}U compares with the binding energy insights from the previous exercise.
- Explain why ^{238}U is more stable (due to a longer half-life) while ^{235}U decays faster despite its slightly higher binding energy per nucleon (as seen in the previous exercise).

The decay behavior of ^{235}U and ^{238}U stems from the energetics and stability of each isotope's nucleus. In the previous exercise, we observed that ^{235}U has a slightly higher binding energy per nucleon than ^{238}U, implying it is more tightly bound per nucleon.

A higher binding energy per nucleon suggests greater stability against spontaneous decay. However, this relationship doesn't determine the isotope's half-life by itself. Decay is also influenced by factors such as nuclear structure, quantum tunneling, and the available decay pathways, so that despite having a higher binding energy per nucleon, ^{235}U ends up decaying faster than ^{238}U.

The greater stability of ^{238}U compared to ^{235}U can be explained by the energy potential barriers from their alpha decay processes. This barrier is the one that an alpha particle has to overcome in order to escape the strong nuclear force that balances the nuclear integrity with the Coulombian repulsion of the electrically charged particles in the nucleus.

^{238}U has a longer half-life because it has a higher energy barrier for alpha decay and thus a lower probability of spontaneous emission of an alpha particle. ^{235}U has a lower energy barrier for decay, which makes alpha decay more probable. This fact, ^{235}U having a shorter half-life despite slightly higher binding energy, demonstrates that binding energy per nucleon is not the only factor that determines an isotope's stability.

Task 7

Explain how the decay of ^{238}U and ^{235}U is used in radiometric dating (e.g., Uranium-Lead dating) to estimate the age of rocks and the Earth.

The U-Pb dating method is based on measuring the ratios of ^{206}Pb to ^{238}U and ^{207}Pb to ^{235}U from a sample and using the decay law to know how old a sample is. Given the long half-lives and distinct decay constants (λ_{238} and λ_{235}), both decay pathways provide two independent estimations for the age of the sample. The age is derived from the decay equation:

$$\text{age} = \frac{1}{\lambda} \ln\left(1 + \frac{\text{daughter}}{\text{parent}}\right) \tag{4}$$

The dual decay pathways allow for cross-check, as having two isotopes allows for a more precise method to rely on. Knowing the ratio we don't need a initial quantity of nucleids, we only need the decay constant for each isotope on top of the ratios. Afterwards, both results can be averaged to obtain a result.

2 ADDITIONAL QUESTIONS

Task 8

Discuss the role of radioactivity in nuclear power generation as a sustainable energy source, contrasting its use with conventional fossil fuels.

Nuclear power, driven by radioactive decay, offers an efficient and low-carbon energy alternative to fossil fuels. It relies on nuclear fission, where isotopes like ^{235}U split upon neutron absorption, releasing significant energy and sustaining a chain reaction. With a far greater energy density than fossil fuels, 1 gram of uranium can produce energy equivalent to several tons of coal that would otherwise contaminate the atmosphere when burned.

Meanwhile, nuclear energy produces minimal greenhouse gases, avoiding the CO_2 emissions linked to global warming from fuel combustion. While radioactive waste requires secure storage due to its long half-life, advancements in waste management enhance safety and sustainability. Moreover, unlike fossil fuels, uranium is abundant and less environmentally damaging to extract.

Nuclear plants operate with low pollutant emissions, preserving air quality and reducing public health risks. As a reliable, high-yield energy source, nuclear power can complement renewable energy in a sustainable energy mix. However, it is difficult for governments to justify them taking into account their high building and maintaining costs, and how the population feels about them. Moreover, they take lots of years to pay for themselves compared to coal plants.

I see as real challenge to communicate to the general public its benefits and balance out its inconveniences.

Task 9

Discuss the role of ^{238}U's slow decay in the production of Plutonium-239 (^{239}Pu) in breeder reactors, and why ^{235}U is more suitable for nuclear weapons and reactors.

^{238}U is not directly fissile, so it is mostly used for producing ^{239}Pu in breeder reactors, while ^{235}U is a naturally fissile isotope, and thus is used in direct applications in nuclear reactors and weapons.

In breeder reactors, ^{238}U goes through a process of neutron capture when exposed to a high neutron flux, producing ^{239}U which, through beta emission, decays into ^{239}Np and then to ^{239}Pu. This process converts non-fissile ^{238}U into fissile ^{239}Pu, that can sustain chain reactions. The abundant ^{238}U gets converted into usable nuclear fuel, significantly extending the energy yield from Earth's uranium resources.

On the other side, ^{235}U is already capable of sustaining chain reactions when struck by thermal neutrons. This enables its use in both nuclear reactors and weapons. Its shorter half life, makes it capable of reaching critical mass more easily than ^{238}U, enabling the rapid chain reaction needed for explosive energy release in nuclear weapons.