NUCLEAR AND PARTICLE PHYSICS

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PRACTICE 1. BINDING ENERGY AND NUCLEAR FISSION OF URANIUM-238

PART I | BINDING ENERGY CALCULATION USING THE SEMI-EMPIRICAL MASS FORMULA

The first step we'll take is define a python function that, given the Z and A numbers of a certain nuclii, it will return the binding energy of said nuclii in MeV. This is the Semi-Empirical Mass Function:

$$B(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + \delta(A,Z) \qquad (1.1)$$

Where a_i are constants defined experimentally which we will take as:

$a_v = 15'8$ MeV $\quad a_s = 18'3$ MeV $\quad a_c = 0'714$ MeV $\quad a_a = 23'2$ MeV

And $\delta(A,Z)$ is the pairing term: $\begin{cases} 0 & \text{if } A \text{ is odd} \\ 11'2A^{-1/2} & \text{if } A \text{ is even and } Z \text{ is even} \\ -11'2A^{-1/2} & \text{if } A \text{ is even and } Z \text{ is odd} \end{cases}$

1. Calculate the binding energy per nucleon for Uranium-235 and Uranium-238

To do so we'll take as input the atomic number of Uranium (Z_U) which is 92. The atomic massic number of each isotope is the number that follows the element for each of the isotopes (235 and 238)

As we are asked about the binding energy per nucleon, we'll divide the output of our function for the Semi-Empirical Mass Formula (SEMF) by its corresponding nucleon total mass, in other words, the massic number of the isotope.

As an output on console we got:
'Binding energy per nucleon of Uranium-235: 7'575 MeV'
'Binding energy per nucleon of Uranium-238: 7'558 MeV'

$\Rightarrow \frac{B(A,Z)}{A}(U^{235}) = 7'575$ MeV
$\frac{B(A,Z)}{A}(U^{238}) = 7'558$ MeV

2. Plot the binding energy per nucleon for Uranium isotopes, including Uranium-235 and Uranium-238. Analyze which isotope among these two exibits greater stability.

According to literature, Uranium has naturally or lab occuring isotopes ranging from A=218 to 242 with their respective massive or minuscule half lifes. If we calculate the binding energy per nucleon for each of these isotopes with the SEMF we observe that as we calculated earlier, U^{235} has a slightly higher binding energy per nucleon than that of U^{238}, indicating that U^{235} is slightly more stable based on the criterion that the higher the binding energy per nucleon, the more stable the nucleus.

If we also plot the literature* experimental values we observe the difference that reality has compared to the SEMF approximation, although the general pattern remains, showing that in fact U^{235} has a higher B/A compared to U^{238} and that the difference is more or less the same $\sim 0'02$ MeV

The stability from having a higher B/A can be explained energetically, because systems in nature tend to lower energy states and a higher B/A implies is in a lower energy state and is less likely to undergo fission or radioactive decay in general. Moreover, a higher B/A means that the nucleons are more tightly bound and thus the forces that keep the nucleus together are higher making it more difficult for de nuclii to spontaneously decay, leading to greater stability.

* the literature used was: https://barwinski.net/isotopes/query-select.php → 92-Uranium - Binding energy per nucleon – Query ALL Isotopes of selected element

PART II Fission of Uranium-238

1. Calculate the total binding energy for the following: Uranium-238, Krypton-92, Barium-141

For this part, we will agian use the SEMF, but this time we'll use the output of the function as is, without dividing by the massic number. The output was:

'Binding energy of Uranium-238: 1798'898 MeV' $\Longrightarrow$ B(92,238) = 1798'9 MeV
'Binding energy of Krypton-92: 775'977 MeV' B(36,92) = 776'0 MeV
'Binding energy of Barium-141: 1163'474 MeV' B(56,141) = 1163'5 MeV

2. Explain wether the fission process of Uranium-238 is energetically favourable. How does the energy released during fission compare to the binding energy of U^{238}?

To ascertain wether the fission process is energeticaly favourable, we will check if the process is spontaneous. To do so, we calculate the binding energy of the products and add it up (Kr-92 and Ba-141) and the binding energy of the fission fuel (U-238). If the binding energy of the products is greater than the binding energy of the fuel, the reaction is energetically favourable.

If we calculate $B(36,92) + B(56,141) - B(92,238)$ we obtain 140'553 MeV which is greater than zero, meaning that in fact the process is favourable. Moreover we obtained the magnitude of the energy release, 140'6 MeV, which is $\simeq$ 7'81% of the original mass of U^{238}.

Heavier nuclii such as U^{238} tend to undergo fission because of their binding energy per nucleon. If we recall the plot with Z on the x-axis and B/A on the y-axis, we saw that binding energy per nucleon had a peak around mid-sized nuclii such as Fe, and then it started to decay as Z augmented.

This means that when heavy nuclii divide into mid-sized ones, the greater binding energy per nucleon of the products results in total binding energy being greater on them than the original binding energy of the fuel despite having the same raw number of particles. Consequently, fission implies a release of energy. This release of energy makes the process energetically favourable as we discussed before, and is the reason behind heavy nuclii being prone to undergo fission spontaneously.

Moreover, the bigger the nuclii, the more protons it contains and as Coulomb repulsion grows faster than the nuclear force can keep up with, making the nucleus further unstable.

We should also consider that although Kr92 and Ba141 are possible fission products, they are not the only ones and in reality fission leads to a range of products due to its probabilistic nature.

Relate this to the concept of nuclear energy production in reactors or atomic bombs.

In nuclear reactors, this energy release of 141 MeV is released in a controlled manner into water by heating it. Water gets converted into steam and generate electricity by moving turbines. The fission process keeps going thanks to chain reactions, where neutrons emmited from a fission event start new events on other fuel atoms.

In atomic bombs, the process is the same but in an uncontrolled manner. This implies that a very large energy release happens in a very short lapse of time, provoking the well known explosion.

Although only 8% of the binding energy gets released, the process emits such high quantities of energy because of the trillions of nuclii that undergo fission in both fission nuclear energy generation plants and fission nuclear bomb explosions.