HOW LATE CAN YOU SLEEP IN THE MORNING?

David H. Ahl

Probabilities and expected values are a vital part to writing almost any game or simulation. Here are two real-life problem situations (I face them practically daily) which can be solved with simple statistics. Be warned: the second is considerably more difficult than the first.

GETTING TO WORK

Driving to work, you can take one of two routes. Route 1 is 5 miles long and has 4 traffic lights. Each light is on a 1-minute cycle but with different intervals. Light 1 is green in your direction for 30 sec., red for 30 sec. Light 2 is 20 sec. green, 40 sec. red. Light 3 is 25 sec. green, 35 sec. red. Light 4 is 40 sec. green, 20 sec. red.

Route 2 is 5.2 miles long with only 1 traffic light which is green your way 20 sec., red 40 sec.

Speed limit in the town is 35 mph.



- 1. Which is the best route and what is the expected time difference between the two?
- 2. Route 2 also takes you by a factory loading dock. If a truck is just arriving (occurs 1 day out of 30) you will be held up an average of 3 minutes. Does this change your answer?

AND ONCE THERE

Parking your car, you rush into the 10-story Morristown AT&T Building for your 8:30 am appointment with the vice president whose office is on the 9th floor. As you reach the elevators you glance at your watch and see that it is 8:29. From past experience you know that there is a 40% chance of an elevator stopping at any given floor; a stop takes an average of 10 seconds. The elevator passes from one floor to another in 6 seconds. There are 3 elevators all of which have an indicator on the 1st floor (where you are) that shows the location and direction of that elevator. At the point of your arrival, each elevator is equally likely to be on any of the 10 floors going on either direction.

- 1. Assuming you take the elevator, what is the probability that you will make your appointment on time? What is the probability that you will be less than 1 minute late? Less than 2 minutes late?
- 2. Under the conditions stated, exactly when would you expect to arrive on the 9th floor?

- 3. You also know from past experience that you can run up the stairs to the 2nd floor in 10 seconds. The 2nd to 3rd takes you 10% more time (11 sec.) and 3rd to 4th 10% more time (12.1 sec.) and so on. What position of elevators upon your arrival would cause you to run up all 9 floors? (Many answers are possible select one "break even" combination).
- 4. Assuming your office is on the 3rd floor and you are faced with the same situation as above (time 8:29 with 8:30 appointment on the 9th floor), but with no elevator indicators, what is your best strategy to be on time for the meeting or as little late as possible? That is, do you run up or wait for the elevator?

PARTIAL ANSWERS

"Getting to Work." 1. Route 2 is approximately 0.47 sec. faster. 2. The arriving truck has an expected value of -6 sec. per day on Route 2, hence Route 1 now has the edge by 5.53 sec.

"And Once There." Once you get on an elevator, it is a fairly simple matter to determine how long it will take to get to the ninth floor (8 floors x 6 sec. = 48) plus (40% chance of stopping x 10 sec. per stop x 8 floors = 32) equals 80 sec. But figuring when the elevator will arrive on the first floor is something else again. Any elevator can be at any floor going in either direction. Hence, Elevator 1 has a 0.056 probability of being at, say, Floor 3 going up*. How long until it returns to Floor 1? Well, if we know it goes to the 10th floor, that's easy, but it has only a 0.4 chance of going to the 10th floor, 0.4 chance of going to 9 and so on. Multiplied by 18 different possible starting positions and by 3 elevators, this is a nasty problem. In a situation like this you have to ask yourself whether a heuristic, or rule of thumb, or best guess wouldn't provide an adaquate answer. For example, you might want to make the assumption that at least one elevator is at the 4th floor (or below) and heading down.

It is sometimes easier to come up with a solution if you think of the problem in entirely different terms. For example, think of the elevator as a one-way trolley on a circular track — station 1 is Floor 1, station 2 is Floor 2 going up, and so on. Station 18 is Floor 2 coming down and then back to station 1 again. Using this approach may make it easier to work the problem.

By the way, it should be apparent that a computer isn't much help in solving this particular problem. However, if this were part of a much larger simulation in which the output of one part provided the input to the next (a very typical situation), a computer would be almost vital to the solution.

If you're still with me and want to read a fun little book on the subject, get "Flaws and Fallacies in Statistical Thinking" by Stephen Campbell published by Prentice-Hall.

*If there are 10 floors and the elevator has an equal chance of being at any floor going in either direction, why isn't the probability of being at Floor 3 going up 0.1 x 0.5 = 0.05? Simply because Floor 1 and 10 do not have a direction associated with them, hence there are really only 18 locations. Actually, that's over-simplified because the elevator may not even reach Floor 10, or 9 or 8 etc.

Elevator Problem

- · 4-story building with I elevator
- · You are on the first floor
- · Meeting on 4th floor starts in 30 seconds
- · Elevator takes 5 seconds to move one floor
- · A stop at a floor takes 10 seconds
- · 40% chance that elevator will stop at a floor (includes loading and unloading)
- · Elevator automatically returns to the 1st floor when not in use
- · Questions:

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21

- 1. What is the probability that you arrive on time?
- 2. At what time do you expect to arrive?

0

have stopped at either the 3rd on 4th flow

17 MARKOV CHAINS

I. Overview

- 17.1 What is a Stochastic Process?
- 17.2 What is a Markov Chain?
- 17.3 n-Step Transition Probabilities
- 17.4 Classification of States in a Markov Chain
- 17.5 Steady-State Probabilities and Mean First Passage Times
- 17.6 Absorbing States

17.1 WHAT IS A STOCHASTIC PROCESS?

I. Notation

- (A) A discrete-time stochastic process is an indexed collection of random variables $\{X_t\}$, where the time index t runs through a given set T. Sounds complicated, Usa Gamble's Problem to illustrate
- (B) States $1, 2, \ldots, s$: mutually exclusive categories for values of X_t .

II. Ex. Gambler's Problem

- (A) You have \$1.
- (B) You need \$3.
- (C) Attempt to win \$3 by placing \$1 bets.
 - 1. Win \$1 with probability 0.4.
 - 2. Lose \$1 with probability 0.6.
- (D) Stop gambling when you have either
 - 1. \$0. You do not have any money to bet.
 - 2. \$3. You reached your goal.
- (E) Questions
 - 1. What is the probability of going broke?
- Can we develop formulas to answer these questions?
- 2. What is the probability of accumulating \$3?
- (F) Let $X_t =$ number of dollars you have after t bets.
- (G) Possible states: 0, 1, 2, 3.

III. Ex. Stock Price

(A) Let X_t be the price of a particular stock at the beginning of the tth trading day. Random variable.

17.2 WHAT IS A MARKOV CHAIN?

I. Ex. Gambler's Problem

- (A) p = probability of winning a single bet.
- (B) States 0, 1, 2, 3 correspond to having \$0, \$1, \$2, \$3.
- (C) $X_t = \text{state after } t \text{ bets.}$
- (D) W_t = winnings on tth bet, which equals
 - 1. -1 if lose bet.
 - 2. 0 if no bet is placed. If you already have to or \$3, no bet is placed
 - 3. 1 if wins bet.
- (E) $X_{t+1} = X_t + W_t$.
- (F) p_{ij} = probability of moving from state i to state j on single bet $= P(X_{t+1} = j | X_t = i)$.
- (G) Transition Matrix

 O 1 2 3

 O 1 2 3

 O 1 2 3

 O 1 2 3

 O 1 2 3

 O 1 0 0 0 1 0 0 0 $\beta = 1 \quad \beta_{10} \quad \beta_{11} \quad \beta_{12} \quad \beta_{13} = 1 \quad 1-p \quad 0 \quad p \quad 0 = 1 \quad 6 \quad 0 \quad 4 \quad 0$ 2 $P_{2e} \quad P_{21} \quad P_{22} \quad P_{23} \quad 2 \quad 0 \quad 1-p \quad 0 \quad p \quad 2 \quad 0 \quad 6 \quad 0 \quad 4$ 3 $P_{3o} \quad P_{31} \quad P_{32} \quad P_{33} \quad 3 \quad 0 \quad 0 \quad 1 \quad 3 \quad 0 \quad 0 \quad 1$ Note: Winsten starts states at s = 1. In this example it is more natural to start at s = 0.

 (H) $p_{11}(p_1) = p_1 = p_2 = p_3$
- (H) $p_{ij}(n)$ = probability of moving from state i to state j in n bets. $= P(X_{t+n} = j | X_t = i). \quad \text{have it to start place in bets (if not reach <math>^{\delta_{O_{co}}} i_3$ first) and With $^{\delta_{O_{co}}} j_3$.
 - (I) n-step Transition Matrix

$$p^{(n)} = \begin{bmatrix} p_{oo}(n) & p_{o_1}(n) & p_{o_2}(n) & p_{o_3}(n) \\ p_{1o}(n) & p_{1i}(n) & p_{12}(n) & p_{13}(n) \end{bmatrix} = ?$$

$$\vdots$$

$$p_{3o}(n) & p_{3i}(n) & p_{32}(n) & p_{33}(n) \end{bmatrix}$$

II. Notational Answers to Our Questions

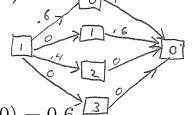
We can answer our quostion in terms of our notation If we ever get to

(A) Probability of going broke is $\lim_{n\to\infty} p_{10}(n)$, if it exists. State of then we remain there. So proloo) includes

(B) Probability of accumulating \$3 is $\lim_{n\to\infty} p_{13}(n)$, if it exists.

(C) But how do we calculate $p_{ij}(n)$ and $\lim_{n\to\infty} p_{ij}(n)$?

III. Formulas for $p_{ij}(2)$



$$p_{10}(2) = p_{10}p_{00} + p_{11}p_{10} + p_{12}p_{20} + p_{13}p_{30}$$

$$\rho^{(2)} = (.6)(1) + (0)(.6) + (.4)(0) + (0)(0) = 0.6$$

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$$P^{(2)} = P^{2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ .6 & 0 & .4 & 0 \\ 0 & .6 & 0 & .4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ .6 & 0 & .4 & 0 \\ 0 & .6 & 0 & .4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ .6 & .24 & 0 & .66 \\ .36 & 0 & .24 & .4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

IV. Formulas for
$$p_{ij}(n)$$

$$p_{10}(n) \neq p_{10}p_{00}(n-1) + p_{11}p_{10}(n-1) + p_{12}p_{20}(n-1) + p_{13}p_{30}(n-1).$$

$$P^{(n)} = PP^{(n-1)} = PP^{n-1} = P^{n}.$$
by an induction proof.

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.6 & 0 & 0.4 & 0 \\ 0 & 0.6 & 0 & 0.4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^{2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.60 & 0.24 & 0 & 0.16 \\ 0.36 & 0 & 0.24 & 0.40 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^{4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.7440 & 0.0576 & 0 & 0.1984 \\ 0.4464 & 0 & 0.0576 & 0.4960 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^{8} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.7869 & 0.0033 & 0 & 0.2098 \\ 0.4721 & 0 & 0.0033 & 0.5246 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^{16} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.7895 & 0.0000 & 0 & 0.2105 \\ 0.4737 & 0 & 0.0000 & 0.5263 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^{32} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.7895 & 0.0000 & 0 & 0.2105 \\ 0.4737 & 0 & 0.0000 & 0.5263 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
So the know probability of going broke if the start with \$1\$ instead of \$1\$.

* Absorbten states - states 0 and 3.

Transient states - states 1 and 2.

* Absorbtion states - states 0 and 3.

Transient states - states 1 and 2.

j is reachable from i if I h & Pi; (n) > 0.

i and i communicate if j is reachable from i and i is reachable from j.

V. Markov Chains

7ambler's

Problem(A) Markovian property

 $\chi_{t} = \int_{b \in \mathbb{R}} \int_{t}^{t} 1.$ {X_t} has the Markovian property if

 $\left(X_{t+1} = j \mid X_{t+1} = j \mid X_{t+1} = j \mid X_{t+1} = j \mid X_{t+1} = k_{t+1}, X_{t+1} = k_{t+1}, X_{t+1} = i\right) = P(X_{t+1} = j \mid X_{t} = i)$ regardless how you got to \$1. for $t = 0, 1, 2, \dots$ and every $i, j, k_0, k_1, \dots, k_{t-1}$.

> 2. Conditional probability depends only on the current state, not the past states. Memoryless property,

the past states. Memoryless property,

(B) Stationary probabilities

Probability of winning a bet remains the same. Contract with between time (shooting basket)

1. $p_{ij} = P(X_{t+1} = j | X_t = i) = P(X_1 = j | X_0 = i)$ The past states of skill where you might improve over time (shooting basket) $v_{ij} = v_{ij} = v_{i$

- (C) Finite-state Markov chain a stochastic process with the following properties
- 1. Finite number of states. 21,2,3
 - 2. Markovian property.
 - 3. Stationary transition probabilities.
 - 4. A set of initial probabilities $q_i = P(X_0 = i)$. In our example $P(X_0 = i) = P(X_0 = i)$. Transition matrix (D) Transition matrix

Reminder: states 1,2,75

Reminder: For Gamblers
$$p_{11} p_{12} \cdots p_{1s}$$

Problem, we used $p_{21} p_{22} \cdots p_{2s}$

state o for $p_{31} p_{32} \cdots p_{3s}$.

 $p_{31} p_{32} \cdots p_{3s}$.

VI. Ex. Inventory Problem

(A) Camera store stocks a particular model.

(B) Store can place order for camera on Sat. night and receives order

- on Mon. morning. Closed on Sunday Independence ensure Markovian property (C) D_i = demand for camera in week i. (dependent sales high one week low next week would violate Markovian property) (D) D_i are i.i.d. random variables. Identical ensures stationary probabilities.

 Identical ensures stationary probabilities.

 Independence ensure Markovian property (dependent sales would violate Markovian property)

 Identical ensures stationary probabilities.

 Independence ensure Markovian property

 Identical ensures stationary probabilities.
 - (F) Ordering Policy
 - 1. If no cameras are in stock, then order 3.
 - 2. Otherwise, do not order.
 - (G) Sales are lost when demand exceeds inventory.

No absorbing states. You can always return to a given state.

VII. Ex. Stock Model I

- (A) Probability that price of stock increases tomorrow equals
 - 1. 0.7 if price increased today.
 - 2. 0.5 if price decreased today.

/III. Ex. Stock Model II

- (A) Probability that price of stock increases tomorrow equals
 - 1. 0.9 if price increased today and yesterday.
 - 2. 0.6 if price increased today but decreased yesterday.
 - 3. 0.5 if price decreased today but increased yesterday.
 - 4. 0.3 if price decreased today and yesterday.

Might try to formulate as above, but don't have Markov property.

5tate.

1 +t .9 0 6.1 0

2 -t .6 0 .4 0

3 +- 0 .5 0 .5

4 -- 0 .3 0 .7

Note that probabilities depend on two days - need more states to achieve Markovian property,

Do stock prices only depend on price on previous day, or do they depend on a longer history.
This question has been debated.
It appears that for most stocks, price depends only on previous day.

17.3 *n*-STEP TRANSITION PROBABILITIES CHAPMAN-KOLMOGOROV EQUATIONS

I. 2-Step Equation We already used this equation for the gambler's proble

(A)
$$p_{ij}(2) = \sum_{k=1}^{s} p_{ik} p_{kj}$$
. $\beta_{i,j} \rho_{i,j} + \beta_{i,j} \rho_{i,j} + \cdots + \beta_{i,s} \rho_{s,j}$

(B)
$$P^{(2)} = PP = P^2$$
.

II. n-Step Equation

$$(A) \ p_{ij}(n) = \sum_{k=1}^{s} p_{ik}(v) \ p_{kj}(n-v) \quad \forall i,j,n \text{ and } 0 \leq v \leq n.$$

$$(B) \ p_{ij}(n) = \sum_{k=1}^{s} p_{ik} p_{kj}(n-1). \quad \text{ Let } V = 1.$$

$$(C) P^{(n)} = PP^{(n-1)} = PP^{n-1} = P^n.$$

$$\rho^{(2)} = \rho^2 = \begin{bmatrix} .9 & 0 & .1 & 0 \\ .6 & 0 & .4 & 0 \\ 0 & .5 & 0 & .5 \\ 0 & .3 & 0 & .7 \end{bmatrix} \begin{bmatrix} .9 & 0 & .1 & 0 \\ .9 & 0 & .1 & 0 \\ .6 & 0 & .4 & 0 \\ .6 & 0 & .4 & 0 \\ .6 & 0 & .4 & 0 \\ .6 & 0 & .4 & 0 \\ .7 & 0 & .5 & 0 & .5 \\ .30 & .15 & .20 & .35 \\ .18 & .21 & .12 & .49 \end{bmatrix}$$

III. Unconditional Probabilities

(A) q_i = probability that process starts in state i.

(B)
$$P(X_n = j) = q_1 p_{1j}(n) + q_2 p_{1j}(n) + \dots + q_s p_{sj}(n)$$
.

Gambler example $q_1 = 1$ $q_0 = q_2 = q_3 = 0$

$$P(X_{n}=0) = 90 Poo(n) + 91 P_{10}(n) + 92 P_{20}(n) + 93 P_{30}(n)$$

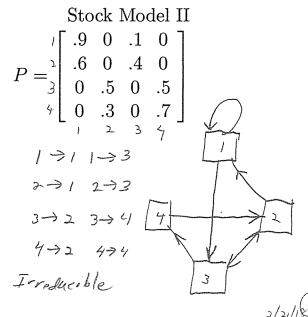
$$= 0 () + 1 P_{10}(n) + 0 () + 0 ()$$

$$= P_{10}(n)$$

17.4 CLASSIFICATION OF STATES IN A MARKOV CHAIN

I. Reachability

- (A) A path from state i to j is a sequence of transitions that begins in i and ends in j, such that each transition in the sequence has a positive probability.
- (B) State j is reachable $(i \to j)$ from i if there is a path from i to j. 1. Equivalently $\exists n \ni p_{ij}(n) > 0$.
- (C) States i and j communicate $(i \leftrightarrow j)$ if $i \to j$ and $j \to i$.
- (D) Properties
 - 1. $i \leftrightarrow i$.
 - $2. \ i \leftrightarrow j \Rightarrow j \leftrightarrow i.$
 - 3. $i \leftrightarrow j$ and $j \leftrightarrow k \Rightarrow i \leftrightarrow k$.
- (E) States can be partitioned into disjoint classes, where states that communicate belong to the same class.
- (F) A Markov chain is *irreducible* if all the states communicate with each other.
- (G) A set of states S in a Markov chain is a *closed set* if no state outside of S is reachable from any state in S.



II. Recurrence

- (A) f_{ii} = probability that process will ever return to state i given it starts in i.
- (B) State i is an absorbing state if $p_{ii} = 1$.
- (C) State i is transient if $f_{ii} < 1$.
 - 1. Equivalently, there is a state j such that $i \to j$ but $j \not\to i$.
- (D) State i recurrent if $f_{ii} = 1$.
 - 1. Equivalently, i is recurrent if it is not transient.
- (E) Properties
 - 1. State i is recurrent \Rightarrow expected number of times that the process is in state i is infinite.
 - 2. State i is transient \Rightarrow expected number of times that the process is in state i is finite.
 - 3. State *i* is recurrent $\Leftrightarrow \sum_{n=1}^{\infty} p_{ii}(n) = \infty$.
- (F) Every state in a class is either recurrent or transient.
- (G) All states in an irreducible finite-state Markov chain are recurrent.

Stock Model II

$$P = \begin{bmatrix} .9 & 0 & .1 & 0 \\ .6 & 0 & .4 & 0 \\ 0 & .5 & 0 & .5 \\ 0 & .3 & 0 & .7 \end{bmatrix}$$

How do you tell which states are recurrent for this problem? Might leave I and bounce back and forth between 2 and 3 forever Every state communicates with every other state, so all states are recurrent

III. Periodicity

(A) The period of state i is the largest integer $t \ni p_{ii}(n) = 0 \quad \forall$ values of n other than $t, 2t, 3t, \ldots$ Complicated definition but the concept is simple values is aperiodic if it has period 1. Suppose $t \rightleftharpoons j$ and shortest path from $t \Rightarrow j \Rightarrow i$ (B) A state is aperiodic if it has period 1. Suppose $t \rightleftharpoons j$ and shortest path from $t \Rightarrow j \Rightarrow i$ has length t $t \Rightarrow j \Rightarrow i$ length is not a multiple of t

(D) A chain is ergodic if all the states are recurrent, aperiodic, and communicate with each other.

(E) Ex. Gambler's Problem

$$P^{1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.600 & 0 & 0.400 & 0 \\ 0 & 0.600 & 0 & 0.400 \\ 0 & 0 & 0.600 & 0 & 0.400 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad P^{2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.600 & 0.240 & 0 & 0.160 \\ 0.360 & 0 & 0.240 & 0.400 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^{3} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.744 & 0 & 0.096 & 0.160 \\ 0.360 & 0.144 & 0 & 0.496 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad P^{4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.744 & 0.058 & 0 & 0.198 \\ 0.446 & 0 & 0.058 & 0.496 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^{5} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.779 & 0 & 0.023 & 0.198 \\ 0.446 & 0.035 & 0 & 0.519 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad P^{6} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.779 & 0.014 & 0 & 0.208 \\ 0.467 & 0 & 0.014 & 0.519 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^{7} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.787 & 0 & 0.006 & 0.208 \\ 0.467 & 0.008 & 0 & 0.525 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad P^{8} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.787 & 0.003 & 0 & 0.210 \\ 0.472 & 0 & 0.003 & 0.525 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^{9} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.789 & 0 & 0.001 & 0.210 \\ 0.472 & 0.002 & 0 & 0.526 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad P^{10} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.789 & 0.001 & 0 & 0.210 \\ 0.473 & 0 & 0.001 & 0.526 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

p, (n) = 0 it n is odd

p_1, (n) = 0 if n is even

State 1 has period 2

States 03 are aperiodic transient states

The chain is not ergodic periodic states

not all states communicate

(F) Ex. Stock Model II

$$P^{1} = \begin{bmatrix} 0.900 & 0 & 0.100 & 0 \\ 0.600 & 0 & 0.400 & 0 \\ 0 & 0.500 & 0 & 0.500 \\ 0.000 & 0.300 & 0 & 0.700 \end{bmatrix} \qquad P^{2} = \begin{bmatrix} 0.810 & 0.050 & 0.090 & 0.050 \\ 0.540 & 0.200 & 0.060 & 0.200 \\ 0.300 & 0.150 & 0.200 & 0.350 \\ 0.180 & 0.210 & 0.120 & 0.490 \end{bmatrix}$$

$$P^{3} = \begin{bmatrix} 0.759 & 0.060 & 0.101 & 0.080 \\ 0.606 & 0.090 & 0.134 & 0.170 \\ 0.360 & 0.205 & 0.090 & 0.345 \\ 0.288 & 0.207 & 0.102 & 0.403 \end{bmatrix} \qquad P^{4} = \begin{bmatrix} 0.719 & 0.075 & 0.100 & 0.107 \\ 0.599 & 0.118 & 0.097 & 0.186 \\ 0.447 & 0.148 & 0.118 & 0.286 \\ 0.383 & 0.172 & 0.112 & 0.333 \end{bmatrix}$$

$$P^{5} = \begin{bmatrix} 0.692 & 0.082 & 0.102 & 0.125 \\ 0.599 & 0.104 & 0.107 & 0.178 \\ 0.491 & 0.145 & 0.104 & 0.260 \\ 0.448 & 0.156 & 0.107 & 0.289 \end{bmatrix} \qquad P^{6} = \begin{bmatrix} 0.672 & 0.088 & 0.102 & 0.138 \\ 0.612 & 0.107 & 0.103 & 0.179 \\ 0.529 & 0.130 & 0.107 & 0.234 \\ 0.497 & 0.140 & 0.107 & 0.256 \end{bmatrix}$$

$$P^{7} = \begin{bmatrix} 0.658 & 0.092 & 0.102 & 0.148 \\ 0.615 & 0.105 & 0.104 & 0.176 \\ 0.554 & 0.124 & 0.105 & 0.217 \\ 0.531 & 0.130 & 0.106 & 0.233 \end{bmatrix} \qquad P^{8} = \begin{bmatrix} 0.647 & 0.096 & 0.103 & 0.155 \\ 0.616 & 0.105 & 0.103 & 0.175 \\ 0.573 & 0.118 & 0.105 & 0.204 \\ 0.556 & 0.123 & 0.105 & 0.216 \end{bmatrix}$$

$$P^{10} = \begin{bmatrix} 0.634 & 0.099 & 0.103 & 0.163 \\ 0.618 & 0.104 & 0.103 & 0.174 \\ 0.596 & 0.111 & 0.104 & 0.189 \\ 0.587 & 0.113 & 0.104 & 0.195 \\ 0.586 & 0.114 & 0.103 & 0.174 \\ 0.603 & 0.109 & 0.104 & 0.184 \\ 0.603 & 0.109 & 0.104 & 0.184 \\ 0.597 & 0.111 & 0.104 & 0.189 \end{bmatrix} \qquad P^{12} = \begin{bmatrix} 0.628 & 0.101 & 0.103 & 0.168 \\ 0.620 & 0.104 & 0.103 & 0.173 \\ 0.608 & 0.107 & 0.104 & 0.184 \\ 0.603 & 0.109 & 0.104 & 0.184 \end{bmatrix}$$

 $P_{33}(2) = 2 \qquad p_{35}(n) = 0 \quad \forall n = 3 \Rightarrow \text{state 3 is aperiodic} \\ P_{34}(1) = 7 \Rightarrow \text{state 4 is a periodic} \\ P_{34}(1) = 7 \Rightarrow \text{state 4 is a periodic} \\ P_{35}(1) = 7 \Rightarrow \text{state 4 is a periodic} \\ P_{35}(1) = 7 \Rightarrow \text{state 4 is a periodic} \\ P_{35}(1) = 1 \Rightarrow \text{state 4 is a periodic}$ P22(2) = ,2 P22(n)>0 Vn33 => state 2 is aperiodic

that you can go directly from state i All states are recurrent aperiodic, to i in one step. E.g. P2=0.

and one communication class.

Thair is ergodic.

All states are recurrent aperiodic,

The interpolation class.

17.5 STEADY-STATE PROBABILITIES AND MEAN

FIRST PASSAGE TIMES

- Look at $\rho^{(32)}$ for Gambler's Problem and Stock Model II.

 Note that $\rho^{(32)}$ has essentially identical rows for Stock Model II, which means probability of being in I. Goal: Analyze long-run properties of Markov Chains. State I after 32 stops

 Answer questions (what is the probability of going broke? is .103 regardless of starting state.

 (A) $\lim_{n\to\infty} p_{ij}(n)$.

 "accumulating \$3?

 - (B) $\lim_{n\to\infty} P\left(X_n=j\right)$. Independent of starting state
 - (C) Expected number of transitions to move from i to j.

II. Steady-State Probabilities for Ergodic Markov Chains

(A) Th: If $\{X_t\}$ is an ergodic Markov chain, then

It is one thing to value a most engage power and it looks like it converges. It is another
$$\lim_{n\to\infty} P^{(n)} = \lim_{n\to\infty} P^n = \begin{bmatrix} \pi_1 & \pi_2 & \cdots & \pi_s \\ \pi_1 & \pi_2 & \cdots & \pi_s \\ \pi_1 & \pi_2 & \cdots & \pi_s \\ \vdots & \vdots & & \vdots \\ \pi_1 & \pi_2 & \cdots & \pi_s \end{bmatrix}.$$

Prove convergence
$$\lim_{n\to\infty} P^{(n)} = \lim_{n\to\infty} P^n = \begin{bmatrix} \pi_1 & \pi_2 & \cdots & \pi_s \\ \pi_1 & \pi_2 & \cdots & \pi_s \\ \vdots & \vdots & & \vdots \\ \pi_1 & \pi_2 & \cdots & \pi_s \end{bmatrix}.$$

It may not exist if not ergodic.

Ex.
$$p=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

$$p^{3}=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

$$p^{4}=\begin{bmatrix}0 & 1\\ 0 & 1\end{bmatrix}$$

- 2. $\lim_{n\to\infty} p_{ij}(n) = \lim_{n\to\infty} P(X_n = j) = \pi_j$.
- (B) $\pi_1, \pi_2, \ldots, \pi_s$ found by solving the *steady-state* equations:

Probability of being in
$$\pi_1 = \overline{\pi_1 p_{11}} + \pi_2 p_{21} + \cdots + \pi_s p_{s1}$$
 state 1 after next step.
$$\pi_2 = \pi_1 p_{12} + \pi_2 p_{22} + \cdots + \pi_s p_{s2}$$

$$\vdots$$

$$\pi_s = \pi_1 p_{1s} + \pi_2 p_{2s} + \cdots + \pi_s p_{ss}$$

$$\pi_s=\pi_1p_{1s}+\pi_2p_{2s}+\cdots+\pi_sp_{ss}$$
 $1=\pi_1+\pi_2+\cdots+\pi_s.$) π_s represent probabilities, so must sum to 1. Need this equation, o.w. π_s [00:0] works

- 1. In matrix form, $\pi = \pi P$.
- (C) π_j 's are called the steady-state probabilities (distribution).
- (D) π_i is the long-run probability of finding the process in state j.

(E) Ex. Steady-State Probabilities for Stock Model II

$$P = \begin{bmatrix} .9 & 0 & .1 & 0 \\ .6 & 0 & .4 & 0 \\ 0 & .5 & 0 & .5 \\ 0 & .3 & 0 & .7 \end{bmatrix}$$

(1)
$$\pi_{1} = \pi_{1} p_{11} + \pi_{2} p_{21} + \pi_{3} p_{31} + \pi_{4} p_{41} = .9\pi_{1} + .6\pi_{2}$$

(2)
$$\pi_2 = \pi$$
, $\rho_{12} + \pi_{12} p_{32} + \pi_{3} p_{32} + \pi_{4} \rho_{42} = .5 \pi_{3} + .3 \pi_{4}$

(3)
$$\pi_3 = \pi$$
, $\rho_{13} + \pi_2 \rho_{23} + \pi_3 \rho_{33} + \pi_4 \rho_{43} = 1 \pi_1 + 1 \pi_2 \rho_{23}$

(4)
$$\pi_4 = \pi_1, \rho_{14} + \pi_2 \rho_{24} + \pi_3 \rho_{34} + \pi_4 \rho_{44} = ,5\pi_3 + ,7\pi_4$$

(5)
$$I = \pi_1 + \pi_2 + \pi_3 + \pi_4$$

(2')
$$\Pi_{2} = ,5\Pi_{3} + ,3\Pi_{4} = ,3\Pi_{4} + ,3\Pi_{4} = .6\Pi_{4}$$

 $\therefore \Pi_{2} = \Pi_{3}$

(5')
$$I = \Pi_1 + \Pi_2 + \Pi_3 + \Pi_4 = G\Pi_2 + \Pi_2 + \Pi_2 + \Pi_2 + \Pi_4$$

 $I = \Pi_2 \left(8 + \frac{1}{.6} \right)$
 $\Pi_1 = .103448276$

$$TT_1 = .630689655$$

$$TT_2 = .103448276$$

$$TT_3 = .103448276$$

III. Matrix Approach to Solving Steady-State Probabilities

(A) Method

1.
$$\pi = \pi P$$

$$[0, \cdots 0] = \pi P - \pi$$

 $[0, \cdots 0] = \pi (P - I)$

- 2. Must replace one of the constraints with $1 = \pi_1 + \pi_2 + \cdots + \pi_s$.
 - a. Corresponds to replacing a column in P-I with $\begin{bmatrix} 1\\ \vdots\\ 1 \end{bmatrix}$.
 - b. Can replace any column.
 - c. For consistency, replace the last column.
 - d. Let \overline{P} be the modified matrix.

3.
$$\begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix} = \pi \overline{P}$$

 $\begin{bmatrix} \overline{0} & \cdots & 0 & \overline{D} \end{bmatrix} \overline{P}' = \pi \overline{P} \overline{P}' = \pi \overline{I} = \pi$
 $\therefore \pi = last \ row \ of \overline{P}'$

(B) Ex. Steady-State Probabilities for Stock Model II

$$\pi = \pi P \qquad P = \begin{bmatrix} .9 & 0 & .1 & 0 \\ .6 & 0 & .4 & 0 \\ 0 & .5 & 0 & .5 \\ 0 & .3 & 0 & .7 \end{bmatrix}$$

$$P-T = \begin{bmatrix} -.1 & 0 & .1 & 0 \\ -.6 & -1 & .4 & 0 \\ 0 & .5 & -1 & .5 \\ 0 & .3 & 0 & -.3 \end{bmatrix} \qquad \begin{bmatrix} -.1 & 0 & .1 & 1 \\ -.6 & -1 & .4 & 1 \\ 0 & .5 & -1 & 1 \\ 0 & .3 & 0 & 1 \end{bmatrix}$$

$$\vec{p}^{-1} = \begin{bmatrix}
-4.2069, 9655 - .0345 & 3.2759 \\
-2.0690 - .3448 - .3448 & 2.7586 \\
-.4138 - .0690 - 1.0690 & 1.5517
\end{bmatrix}$$

$$.6207 .1034 .1034 .1724$$

IV. Transient Analysis

- (A) The behavior of a Markox chain before steady state is reached is the *transient* (*short-run*) behavior.
- (B) There is no general rule for how quickly steady state is reached.
 - 1. Often 10 to 30 steps is sufficient. Ex. P(32) for Gambler's Problem and 5 took Model I.

V. Intuitive Interpretation of Steady-State Probabilities

(A) Steady-state equation for state j

$$\pi_{j} = \pi_{1}p_{1j} + \pi_{2}p_{2j} + \cdots + \pi_{j}p_{jj} + \cdots + \pi_{s}p_{sj}$$

$$\pi_{j} (1 - p_{jj}) = \pi_{1}p_{1j} + \pi_{2}p_{2j} + \cdots + \pi_{j-1}p_{j-1,j} + \pi_{j+1}p_{j+1,j} + \cdots + \pi_{s}p_{sj}$$

$$Probability that you leave j given that you are in it.$$

$$RHS = probability that you enter j given that you are not in it.$$

(B) "Flow out" = "Flow in".



VI. Expectations for Irreducible Finite-State Markov

- (A) $\lim_{n\to\infty} p_{ij}(n)$ may not exist for non aperiodic Markov chains.
- (B) Ex.

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad P^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad P^3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad P^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P_{r_{i}}(n) = \begin{cases} 0 & \text{if } n \text{ odd} \\ 1 & \text{if } n \text{ even} \end{cases}$$
 ; $\lim_{n \to \infty} P_{n}(n) \text{ does not exist}$

Ask stevento what % of time is spent in state !

(C) If X_t is an irreducible finite-state Markov chain, then

$$\frac{\lim_{n\to\infty}(\sum_{k=1}^n p_{ij}(k))}{n} = \pi_j,$$

where the π_j 's satisfy the steady-state equations.

(D) Ex. (cont.)

(E) $\pi_j = \text{long-run expected fraction of time the system is in state } j$.

The probability of being in state j. Use above example. $p_n(n) = \begin{cases} 0 & \text{if } n \text{ is the } n \text{ is th$

VII. Expected Average Cost Per Unit Time

- (A) $C(X_t) = \text{cost incurred when the process is in state } X_t \text{ at time } t$.
- (B) Long-run expected average cost per unit time = $\sum_{j=1}^{s} \pi_{j} C(j)$.

- (C) Ex. Stock Model II Investment Strategy
 - 1. Investment Strategy A
 - a. If price increased today, use option to bet price will increase tomorrow.

\$4 if price increases tomorrow.

\$-5 if price decreases tomorrow.

- b. If price decreased today, use option to bet price will decrease tomorrow.
 - \$-4 if price increases tomorrow.

\$3 if price decreases tomorrow.

c. What is the long-run expected payoff per day?

$$= (.621)(4) + (.103)(-4) + (.103)(-5) + (.172)(3)$$

$$= 2.073$$

- 2. Investment Strategy B
 - a. What if we only bet when the price increased today?

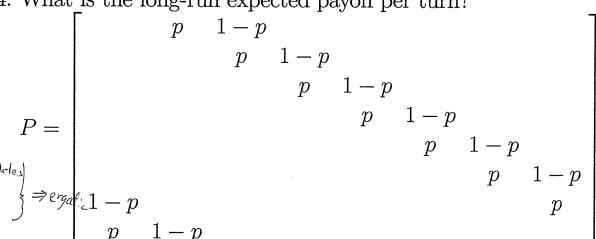
long-run expected payoff per day =
$$\overline{n}$$
, $\dot{C}(1) + \overline{n}$, $C(2) + \overline{n}$, $C(3) + \overline{n}$, $C(4)$

$$= (.621)(4) + (.03)(0) + (.03)(-5) + (.172)(0)$$

$$= /.969$$

(D) Ex. Simplified Monopoly

- 1. Board consists of 8 squares.
- 2. Piece moves clockwise.
 - a. One square: probability p.
 - b. Two squares: probability 1 p.
- 3. Each square is worth a certain amount of \$.
- 4. What is the long-run expected payoff per turn?



\$100

-100

4 -100

100

\$200

-100

ble p = 1-p $\Rightarrow steady-state probabilities$ $\pi_1 = (1-p)\pi_7 + p\pi_8$ $\pi_2 = (1-p)\pi_8 + p\pi_1 \quad \pi_7 = \pi_2 = \pi_8 = \pi_8 = \pi_8$ $\pi_3 = (1-p)\pi_1 + p\pi_2 \quad \text{This makes in furtive sense}$ $\pi_4 = (1-p)\pi_2 + p\pi_3 \quad \text{because ell squares are alike}$ $\pi_5 = (1-p)\pi_3 + p\pi_4 \quad \text{being hid}$ $\pi_6 = (1-p)\pi_4 + p\pi_5$

$$\pi_7 = (1 - p) \pi_5 + p \pi_6$$

$$\pi_8 = (1 - p) \pi_6 + p \pi_7$$

$$1 = \pi_1 + \pi_2 + \dots + \pi_s$$

long-run expected payoff per turn = \$\frac{8}{5} T; C(j)\$ = \frac{1}{5} (200) + \frac{1}{5} (100) + \frac{1}{5} (-100) + \frac{1}{5} (100) + \frac{1}{5} (-200) + \frac{1}{5} (300) + \frac{1}{5} (-100)\$ = \frac{1}{5} (200) = 25

(Can almost evaluate real Monopoly game after all properties have been purchased and all development has taken place. Exceptions: \$200 for crossing Go, chance cards, jail). Table 3

TABLE **3**Steady-State Probabilities for Monopoly

	<i>n</i> Position	Steady-State Probability
0	Go	.0346
1	Mediterranean Ave.	.0237
2	Community Chest 1	.0218
3	Baltic Ave.	.0241
4	Income tax	.0261
5	Reading RR	.0332
6	Oriental Ave.	.0253
7	Chance 1	.0096
8	Vermont Ave.	.0258
9	Connecticut Ave.	.0237
10	Visiting jail	.0254
11	St. Charles Place	.0304
12	Electric Co.	.0311
13	State Ave.	.0258
14	Virginia Ave.	.0288
15	Pennsylvania RR	.0313
16	St. James Place	.0318
17	Community Chest 2	.0272
18	Tennessee Ave.	.0335
19	New York Ave.	.0334
20	Free parking	.0336
21	Kentucky Ave.	.0310
22	Chance 2	.0125
23	Indiana Ave.	.0305
24	Illinois Ave.	.0355
25	B and O RR	.0344
26	Atlantic Ave.	.0301
27	Ventnor Ave.	.0299
28	Water works	.0315
29	Marvin Gardens	.0289
30	Jail	.1123
31	Pacific Ave.	.0300
32	North Carolina Ave.	.0294
33	Community Chest 3	.0263
34	Pennsylvania Ave.	.0279
35	Short Line RR	.0272
36	Chance 3	.0096
37	Park Place	.0245
38	Luxury tax	.0295
39	Boardwalk	.0295

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✓III. First Passage Times

- (A) First passage time = # of steps from state i to j for first time.
- (B) Recurrence time = # of steps from state i to i for first time.
- (C) $f_{ij}(n) = \text{probability that first passage time from } i \text{ to } j \text{ is } n.$
- (D) Equations

$$f_{ij}(1) = p_{ij}$$

$$f_{ij}(2) = p_{ij}(2) - f_{ij}(1) p_{jj}$$

$$\vdots$$

$$f_{ij}(n) = p_{ij}(n) - f_{ij}(1) p_{jj}(n-1) - f_{ij}(2) p_{jj}(n-2) - \dots - f_{ij}(n-1) p_{jj}$$

IX. Expected First Passage Time

- (A) $\sum_{n=1}^{\infty} f_{ij}(n) \le 1.$
 - 1. $\sum_{n=1}^{\infty} f_{ij}(n) < 1 \Rightarrow \text{may never reach } j \text{ from } i.$
 - 2. $\sum_{n=1}^{\infty} f_{ij}(n) = 1$, then $f_{ij}(1), f_{ij}(2), \ldots$ is probability distribution for first passage time.
- (B) m_{ij} = expected first passage time from i to j.

$$m_{ij} = \begin{cases} \infty & \text{if } \sum_{n=1}^{\infty} f_{ij}(n) < 1\\ \sum_{n=1}^{\infty} n f_{ij}(n) & \text{if } \sum_{n=1}^{\infty} f_{ij}(n) = 1 \end{cases}$$

(C) $m_{1j}, m_{2j}, \ldots, m_{sj}$ found by solving

$$m_{1j} = 1 + \sum_{k \neq j} p_{1j_k} m_{kj} = \rho_{ij} + \sum_{k \neq j} \rho_{ik} (i + m_{kj})$$

$$m_{2j} = 1 + \sum_{k \neq j} p_{2j_k} m_{kj}$$

$$\vdots$$

$$m_{sj} = 1 + \sum_{k \neq j} p_{sj} m_{kj}$$

(D)
$$m_{ii} = 1/\pi_i$$
.

$$\begin{array}{c} w_{o,k} \downarrow_{h,u} \; prob \mid_{em} \; a \; ft_{ex} \; St_{eck} \; Model \; II \\ | Ex. \; Passage Times \; for Gambler's Problem: \; P = \left[\begin{array}{c} 0 & 1 & 2 & 3 \\ 1 & 0 & 0 & 0 \\ 0 & 6 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{array} \right] \\ | Doc (x) = \beta_{10} = 0.6 \\ | F_{0}(x) = \beta_{10}(x) - F_{10}(1) \; \beta_{20} = .6 - (.6)/1 \; \} = 0 \\ | F_{0}(x) = \beta_{10}(x) - F_{10}(1) \; \beta_{20}(x) - F_{10}(x) \; \beta_{20} = .744 - .6(1) - (.0)(1) = .444 \\ | F_{10}(x) = \beta_{10}(x) - F_{10}(1) \; \beta_{20}(x) - F_{10}(x) \; \beta_{20}(x) - F_{10}(x) \; \beta_{20}(x) - F_{20}(x) \; \beta_{20} = .77856 - .6 - .744 \\ | F_{10}(x) = \beta_{10}(x) - F_{10}(1) \; \beta_{20}(x) - F_{10}(x) \; \beta_{20}(x) - F_{10}(x) \; \beta_{20}(x) - F_{20}(x) \; \beta_{20} = .77856 - .6 - .744 \\ | F_{10}(x) = \sum_{n=1}^{\infty} |F_{10}(n)| = \lim_{n \to \infty} |P_{10}(n)| = .789474 \\ | F_{10}(x) = \sum_{n=1}^{\infty} |F_{10}(n)| = \lim_{n \to \infty} |P_{10}(n)| = .789474 \\ | F_{10}(x) = \sum_{n=1}^{\infty} |F_{10}(n)| = \lim_{n \to \infty} |P_{10}(n)| = .789474 \\ | F_{10}(x) = \sum_{n=1}^{\infty} |F_{10}(n)| = \lim_{n \to \infty} |P_{10}(n)| = .789474 \\ | F_{10}(x) = \sum_{n=1}^{\infty} |F_{10}(n)| = \lim_{n \to \infty} |P_{10}(n)| = .789474 \\ | F_{10}(x) = \sum_{n=1}^{\infty} |F_{10}(n)| = \lim_{n \to \infty} |P_{10}(n)| = .789474 \\ | F_{10}(x) = \sum_{n=1}^{\infty} |F_{10}(n)| = \lim_{n \to \infty} |P_{10}(n)| = .789474 \\ | F_{10}(x) = \lim_{n \to \infty} |F_{10}(n)| = \lim_{n \to \infty} |F_{10}(n)| = .789474 \\ | F_{10}(x) = \lim_{n \to \infty} |F_{10}(n)| = \lim_{n \to \infty} |F_{10}(n)| = .789474 \\ | F_{10}(x) = \lim_{n \to \infty} |F_{10}(n)| = \lim_{n \to \infty} |F_{10}(n)| = .789474 \\ | F_{10}(x) = \lim_{n \to \infty} |F_{10}(n)| = \lim_{n \to \infty} |F_{10}(n)| = .789474 \\ | F_{10}(x) = \lim_{n \to \infty} |F_{10}(n)| = \lim_{n \to \infty} |F_{10}(n)| = .789474 \\ | F_{10}(x) = \lim_{n \to \infty} |F_{10}(n)| = \lim_{n \to \infty} |F_{10}(n)| = .789474 \\ | F_{10}(x) = \lim_{n \to \infty} |F_{10}(n)| = \lim_{n \to \infty} |F_{10}(n)| = .789474 \\ | F_{10}(x) = \lim_{n \to \infty} |F_{10}(n)| = \lim_{n \to \infty} |F_$$

17.6 ABSORBING CHAINS

- I. Suppose that a Markov chain has absorbtion states and that eventually one of the absorbing states will be reached.
 - (A) Let k be an absorbing state.
 - (B) Let f_{ik} = probability of being absorbed into k starting from i.
 - (C) $f_{kk} = 1$, since $p_{kk} = 1$.
 - (D) $f_{ik} = 0$, if i is a recurrent state.
 - (E) $f_{1k}, f_{2k}, \ldots, f_{sk}$ found by solving

$$f_{1k} = p_{11}f_{1k} + p_{12}f_{2k} + \dots + p_{1s}f_{sk}$$

$$f_{2k} = p_{21}f_{1k} + p_{22}f_{2k} + \dots + p_{2s}f_{sk}$$

$$\vdots$$

$$f_{sk} = p_{s1}f_{1k} + p_{s2}f_{2k} + \dots + p_{ss}f_{sk}.$$
II. Ex. Gambler's Problem:
$$P = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 0 & 0 \\ 0 & .6 & 0 & .4 & 0 \\ 0 & .6 & 0 & .4 & 0 \\ 0 & .6 & 0 & .4 & 0 \\ 0 & .0 & 0 & 1 \end{bmatrix}$$

 $f_{10} = p_{10} f_{00} + p_{11} f_{10} + p_{12} f_{20} + p_{13} f_{30} = .6(1) + 0 f_{10} + .4 f_{20} + (0)(0) = .6 + .4 f_{20}$ $f_{20} = p_{20} f_{00} + p_{21} f_{10} + p_{22} f_{20} + p_{23} f_{30} = 0(1) + .6 f_{10} + 0 f_{50} + .4(0) = .6 f_{10}$ $f_{30} = 0$

$$f_{20} = .6f_{10}$$
 $f_{10} = .64.4f_{20} = .64.4(.6f_{10}) \rightarrow f_{10} = .789474$
 $f_{20} = .473684$

III. Ex. State College

- (A) Student's progress can be modeled as a Markov chain.
- (B) Student's class status is observed at beginning of fall semester.
- (C) When students quit, they never re-enroll.
- (D) Probabilities given in the transition matrix below.
- (E) Questions:
 - 1. Probability that a freshman graduates?
 - 2. Probability that a freshman graduates in 4 years? $\rho_{is}(4)$
 - 3. Expected number of years for an entering freshman to m_{k} graduate?

		1	2	3	4	5	6
		F.	So.	J.	Sen.	Q.	G.
1	Freshman	.10	.80	0	0	.10	0
2	Sophomore	0	.10	.85	0	.05	0
3	Junior	0	0	.15	.80	.05	0
4	Senior	0	0	0	.10	.05	.85
5	Quits	0	0	0	0	1	0
6	Graduates	0	0	0	0	0	1

Probability that a freshman graduates $f_{66} = 1$ $f_{56} = 0$ $f_{16} = .1f_{16} + .8f_{26} + .1f_{56} \longrightarrow .9f_{16} = .8f_{26} \longrightarrow f_{16} = .746228$ $f_{26} = .1f_{26} + .85f_{36} + .05f_{56} \longrightarrow .9f_{26} = .85f_{36} \longrightarrow f_{26} = .839506$ $f_{36} = .15f_{36} + .8f_{46} + .05f_{56} \longrightarrow .85f_{36} = .8f_{46} \longrightarrow f_{36} = .8f_{86}$ $f_{46} = .1f_{46} + .05f_{56} + .85f_{66} = .1f_{46} + .85 \longrightarrow .9f_{46} = .85$ $f_{46} = .1f_{46} + .05f_{56} + .85f_{66} = .1f_{46} + .85 \longrightarrow .9f_{46} = .85$ $f_{56} = 0$ $f_{56} = 0$

Summary of Notation and Definitions

- A stochastic process is an indexed collection of random variables $\{X_t\}$, where the index t runs through a given set T
- States $1, 2, \ldots, s$ mutually exclusive categories for values of X_t
- $p_{ij} = P(X_{t+1} = j | X_t = i)$
- $p_{ij}(n) = P(X_{t+n} = j | X_t = i)$
- P = transition matrix
- $P^{(n)} = n$ -step transition matrix
- Chapman-Kolmogorov equations: $p_{ij}(n) = \sum_{k=0}^{s} p_{ik}(v) p_{kj}(n-v) \quad \forall i, j, n \text{ and } 0 \leq v \leq n$
- $q_i = P(X_0 = i)$
- State j is reachable $(i \to j)$ from i if there is a path from i to j.
- States i and j communicate $(i \leftrightarrow j)$ if $i \to j$ and $j \to i$.
- A Markov chain is *irreducible* if all the states communicate with each other
- A set of states S in a Markov chain is a *closed set* if no state outside of S is reachable from any state in S.
- f_{ii} = probability that process will ever return to state i given it starts in i
- State i is an absorbing state if $p_{ii} = 1$.
- State i is transient if there is a state j such that $i \to j$ but $j \nrightarrow i$.
- State *i* is *recurrent* if it is not transient.
- The period of state i is the largest integer $t \ni p_{ii}(n) = 0 \quad \forall \text{ values of } n \text{ other than } t, 2t, 3t, \dots$
- A recurrent state is aperiodic if it is not periodic.
- Every state in a class has the same period.
- A chain is ergodic if all the states are recurrent, aperiodic, and communicate with each other.
- A recurrent state *i* is *positive recurrent* if, starting in *i*, the expected time for the process to reenter *i* is finite. Otherwise state *i* is *null recurrent*.
- In a finite-state Markov chain all recurrent states are positive recurrent.
- A positive recurrent state that is aperiodic is ergodic
- First passage time = # of steps in going from state i to j for the first time
- Recurrence time = # of steps in going from state i to i for the first time
- $f_{ij}(n)$ = probability that first passage time from i to j is n
- m_{ij} = expected first passage time from i to j
- π_j = the steady-state probabilities of the Markov chain

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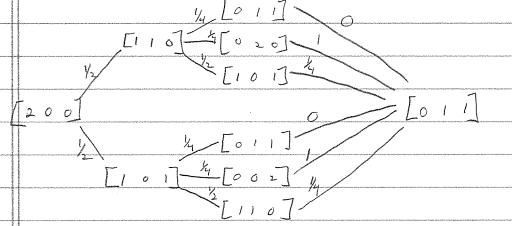
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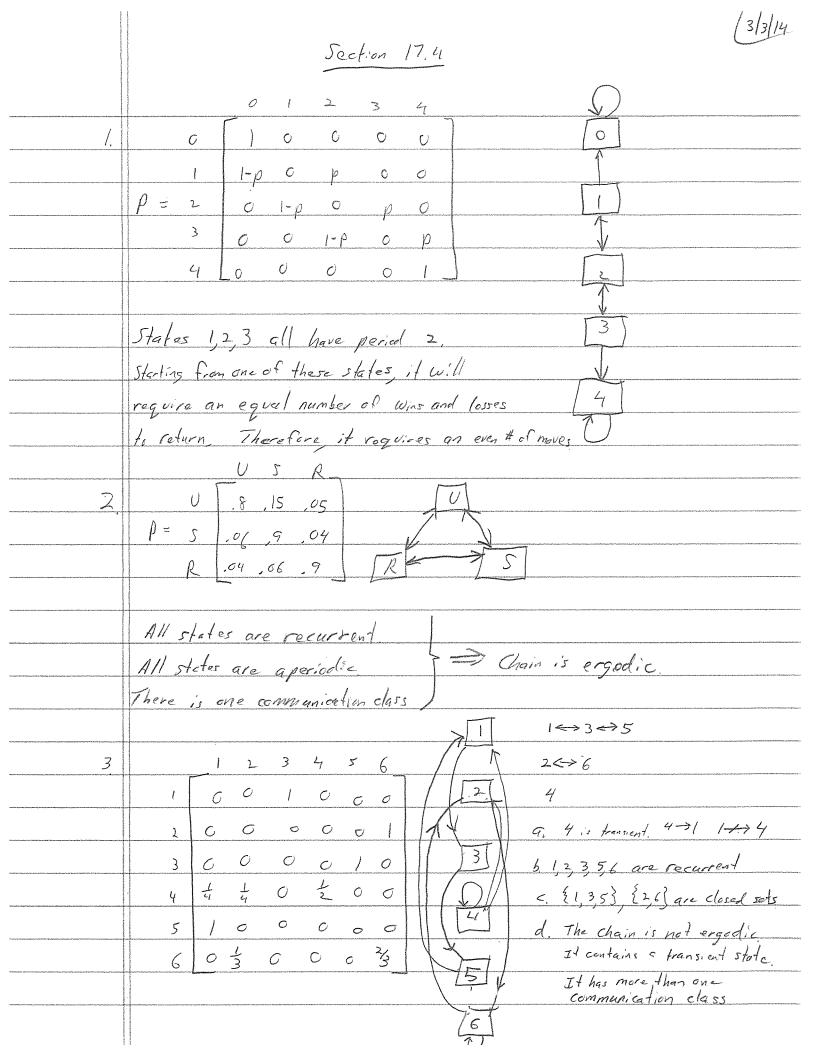
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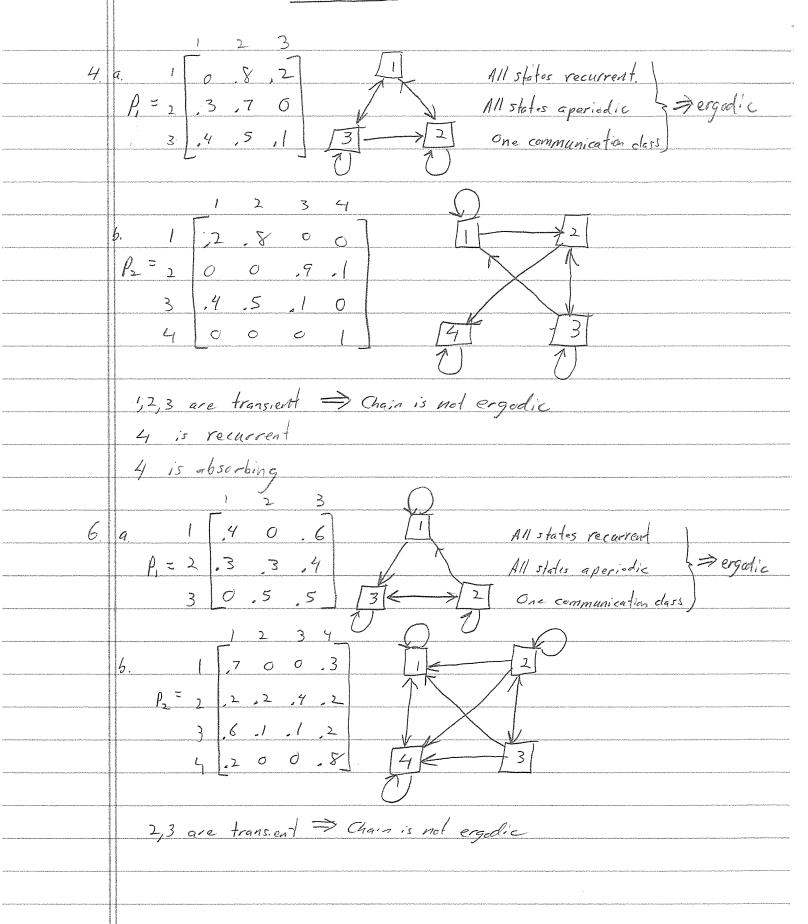
- 9. You can answer this question by constructing $P^{(2)}=P^2$ or by noticing that the only path from [200] to [020]

 in 2 steps is through [110] $P = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = \frac{1}{8}$
- b. You can answer this question by constructing $p^{(3)} = p^3$ or by using the diagram below



$$p(3) = (\frac{1}{2})(\frac{1}{4})(0) + (\frac{1}{2})(\frac{1}{4})(1) + \frac{1}{2}(\frac{1}{2})(\frac{1}{4}) + (\frac{1}{2})(\frac{1}{4})(1) + (\frac{1}{2})(\frac{1}{4}$$





eriore representation of the control	
and the second s	[,80 ,15 ,05] $\overline{\Pi}_1 = \overline{\Pi}_1 \rho_{11} + \overline{\Pi}_2 \rho_{21} + \overline{\Pi}_3 \rho_{21} = .8 \overline{\Pi}_1 + .06 \overline{\Pi}_2 + .04 \overline{\Pi}_3$
### ### ##############################	$P = .06,90.04$ $\Pi_2 = \Pi_1 p_1 + \Pi_2 p_3 + \Pi_3 p_3 = .15 \Pi_1 + .9 \Pi_2 + .06 \Pi_3$
The second of th	[.04 .06 .90] TI3 = TT, P13 + TT2 P23 + TT3 P33 = .05 TT, + .04 TT2 + .9 TT3
The report of the control of the con	$I = \pi_1 + \pi_2 + \pi_3$
distribution of the College of the C	-,2 ,15 1
	$\overline{P} = 06 - 10 \Gamma 0 0 \overline{1} = \overline{\pi} \overline{P}$
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	Recuerent aperiodic, once class.
	-, 2, 06, 04, 0 1 1 .: Standy-state probabilities
	15-10.06,0 ~ 15-10.06 0 exist.
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3_	$a. p = \frac{2}{3} \frac{1}{3} \frac{1}{$
Wykolada Sahahali Mayaka — ayaki kaka ka kama ka	
	$T_{1} = (\frac{6}{5})(\frac{1}{2}) = \frac{3}{5}$
	Recurrent aperiodic one class. The
	: Steady state probabilities exist
	<i>c</i> ,
ONEMIZZENNEZIO ZI SANDANINANA SIBILARI ZI SANDANI ALI GARININANI ZI SANDANI ZI SANDANI ZI SANDANI ZI SANDANI Z	$m_1 = \frac{1}{1} = \frac{5}{3}$
John-Salasharin midding film (Scholana Salasha Salasha Salasha Salasha Salasha Salasha Salasha Salasha Salasha	$m_{12} = 1 + p_{12} m_{21} = 1 + \frac{1}{3} m_{21} \rightarrow m_{12} = 1 + \frac{1}{3} \left(1 + \frac{1}{2} m_{12}\right) \rightarrow m_{12} = \frac{1}{3} + \frac{1}{6} m_{12}$
- Miller Ada (1887) (See Control (1886) (See Ada (1886) (See Control (1886) (See Contr	$m_{21} = 1 + p_{21} m_{12} = 1 + \pm m_{12}$ $m_{12} = \frac{6}{5} \left(\frac{4}{3}\right) = 8/5$
	$m_{22} = \frac{1}{\pi_1} = \frac{5}{2}$ $m_{21} = 1 + \frac{1}{2} \left(\frac{8}{5} \right) = \frac{9}{5}$

	b. [8,20] Recarrent a periodic, one class i. Steady-state probabilities exist. 8,20 3 13 15 16 17 18 18 18 19 10 10 10 10 10 10 10 10 10
5 (2) 5.25 (3) 1,8175 (4) 5 (5) \$\frac{2}{4} (6) 1,5625 (7)	
	Note: M,2, M32 differ from solution manual.

5	A square matrix is doubly stochastic if its entries are nonnegative
nng agaya, iyayan juun bilka ililimingi farah kari rayaa iyayaali uun aasi ilililika ka ka katifan iyo ka kati	and the entries in each row and column sum to 1.
	The If P is an ergodic doubly-stochastic matrix, then all states have same stoods-state probabilities ergodic implies that steady-state probabilities exist. Proof: We must show that $T = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \cdots & \frac{1}{5} \end{bmatrix}$ is a solution of $T = TIP$.
	Pis ergodic implies that steady-state probabaties exist
	Proof: We must show that T= [5 5 s] is a solution of 11=11.
L ,	$ \frac{\pi}{1} = \pi, \rho_1 + \pi_2 \rho_2 + \dots + \pi_s \rho_{s_1} = \frac{1}{5} \rho_1 + \frac{1}{5} \rho_2 + \dots + \frac{1}{5} \rho_{s_1} = (\rho_1 + \rho_2 + \dots + \rho_{s_1}) = \frac{1}{5} = \frac{1}{5} $
	$\pi_{s} = \pi, \rho_{1s} + \Pi_{2} \rho_{2s} + \cdots + \pi_{s} \rho_{ss} = \frac{1}{s} \rho_{1s} + \frac{1}{s} \rho_{2s} + \cdots + \frac{1}{s} \rho_{sr} = (\rho_{1s} + \rho_{2s} + \cdots + \rho_{sc}) = \frac{1}{s} \rho_{1s} + \frac{1}{s} \rho_{2s} + \cdots + \frac{1}{s} \rho_{sr} = (\rho_{1s} + \rho_{2s} + \cdots + \rho_{sc}) = \frac{1}{s} \rho_{1s} + \frac{1}{s} \rho_{2s} + \cdots + \frac{1}{s} \rho_{sr} = (\rho_{1s} + \rho_{2s} + \cdots + \rho_{sc}) = \frac{1}{s} \rho_{1s} + \frac{1}{s} \rho_{2s} + \cdots + \frac{1}{s} \rho_{sr} = (\rho_{1s} + \rho_{2s} + \cdots + \rho_{sc}) = \frac{1}{s} \rho_{1s} + \frac{1}{s} \rho_{2s} + \cdots + \frac{1}{s} \rho_{sr} = (\rho_{1s} + \rho_{2s} + \cdots + \rho_{sc}) = \frac{1}{s} \rho_{1s} + \frac{1}{s} \rho_{2s} + \cdots + \frac{1}{s} \rho_{sr} = \frac{1}{s} \rho_{1s} + \frac{1}{s} \rho_{2s} + \cdots + \frac{1}{s} \rho_{sr} = \frac{1}{s} \rho_{1s} + \frac{1}{s} \rho_{2s} + \cdots + \frac{1}{s} \rho_{sr} = \frac{1}{s} \rho_{1s} + \frac{1}{s} \rho_{2s} + \cdots + \frac{1}{s} \rho_{sr} = \frac{1}{s} \rho_{1s} + \frac{1}{s} \rho_{sr} + \frac{1}$
	$[T = [\frac{1}{5} \cdot - \frac{1}{5}] \text{ is a solution of } T = TP.$
and the second of the second o	10 20 0
non programment and the second	For stock 1 P= 10 .8 .2 It 72 Recurrent aperiodic, one class 20 .1 .9 .: Steady-state probabilities exist.
NOT AND TO THE STATE OF THE SECOND PROCESS O	20 1,9 Steady-state probabilities exist.
mining (Spanish Carp Andrews Spanish and Schreiber and Andrews Andrews Andrews Andrews Andrews Andrews Andrews	$\pi = \pi \rho_1 + \Pi_2 \rho_2 = 8\pi + 1\pi \rightarrow \pi = 8\pi + 1(1-\pi) \rightarrow \pi = 3$
	$T_{1} = T_{1}, p_{1} + T_{1}, p_{2} = -2T_{1}, + .9T_{1}$ $T_{2} = \frac{2}{3}$
	1=T1+T12
	long-run expected price = $\sum_{j=1}^{\frac{1}{2}} \pi_{j} C(j) = \frac{1}{3} (10) + \frac{1}{3} (20) = \frac{50}{3}$
roal to accommodação de la compressão de comencia de c	$m_{ij} = \frac{1}{\pi_i} = 3$
uwangi garani na sila na na sarang pagbangi sa paga ya pangana bahahir nang alamining pamaya kasalahiri bila b	$m_{12} = 1 + p_{11} m_{12} = 1 + 8m_{12} \rightarrow .2m_{12} = 1 m_{12} = 5$
	$m_{21} = 1 + \beta_2 m_{21} = 1 + 9 m_{21} \rightarrow 1 m_{21} = 1 m_{21} = 10$
which is glass from the last one was sixten as a second of the last of the last of the last of the last of the	$ m_{\gamma}=\pm\frac{3}{2}$
	If the price is 10 today then on average it will be 3 days before the price is 10
	If the price is 10 today, then on average it will be 3 days before the price is 10

(cont.) $\rho = \begin{bmatrix} 9 & 1 \\ -9 & 1 \end{bmatrix}$ Recurrent, apparisolic, one class is $51 + 51 = 11$, $\rho_{11} + 11$, $\rho_{21} = -971$, $\phi = -$
$l_{o-g-ran} = xpected price = \sum_{j=1}^{2} \overline{\Pi_{j}} C(j) = \frac{3}{5} (10) + \frac{2}{5} (25) = \frac{80}{5} = 16$
$m_{11} = \frac{1}{11} = \frac{5}{3}$ $m_{12} = \frac{1}{11} + p_{11}m_{12} = \frac{1}{11} + \frac{9}{11} + \frac{9}{11} + \frac{1}{11} + \frac{9}{11} + \frac{1}{11} + \frac{9}{11} + \frac{1}{11} $
If the price is 10 today, then on average ! will be \$\frac{5}{3}\$ days before the price is 10 "" 15 " " 6,67 " " " " 10 Gray Black Both Neither
Gray 7.2,05,05 $P = Black$ 2.6,1,1 $Both$ 1,1,8,0 Neither .05.05,1,8 Solve

	And increase or a constant in the constant in					\bigcap
11,		12	Ŧ	0	0	1 2 9. This chain is not ergodic
	P =	Ŧ	7	0	0	bacause there are
		0	0	1/3	<u>}</u>	1) two communication. classes
		0	0	<u> </u>	<u>1</u> 3	4 > 3
-		defendance.				

b. Th. I fails because the chain is not ergodic.

 $\lim_{n\to\infty} p(n) > 0 \quad \text{but } \lim_{n\to\infty} p_2(n) = 0 \quad \text{(since it is impossible to move from 3 to 2)}.$

: Steady-state probabilities do not exist.

We need to find the stationary probabilities separately for the two communication classes.

Class 1: $\pi_1 = \overline{\Pi}_2$ $\pi_1 + \overline{\Pi}_2 = 1$ \Rightarrow $\pi_1 = \overline{\Pi}_2 = \frac{1}{2}$ Class 2: $\pi_3 = \overline{\Pi}_4$ $\pi_3 + \overline{\Pi}_4 = 1$ \Rightarrow $\pi_3 = \overline{\Pi}_4 = \frac{1}{2}$

 $\lim_{n\to\infty} p_3(n) = 0 \qquad \lim_{n\to\infty} p_3(n) = \frac{1}{2}$

 $\lim_{n\to\infty} \rho_{43}(n) = \frac{1}{2} \qquad \lim_{n\to\infty} \rho_{41}(n) = 0$

13.	State 0 = New machine of beginning of the month.
	State 0 = New machine of beginning of the month. 1 = 1 month old " " " " " " " " " " " " " " " " " " "
	2=2""""""""""""""""""""""""""""""""""""
	3 = 3 " 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
	Policy 1 2 3
	$0 1 ,9 0 0 \overline{\Pi}_{0} = \overline{\Pi}_{0} p_{00} + \overline{\Pi}_{1} p_{10} + \overline{\Pi}_{2} p_{20} + \overline{\Pi}_{3} p_{30} = .1 \Pi_{0} + .2 \Pi_{1} + .5 \overline{\Pi}_{2} + .1 \Pi_{2} + .2 \Pi_{3} + .2 \Pi_{1} + .2 \Pi_{1} + .2 \Pi_{2} + .2 \Pi_{3} + $
	$\rho = 1_{o2} 0_{o8} 0_{i1} = \pi_{o} \rho_{oi} + \pi_{i} \rho_{i1} + \pi_{i2} \rho_{21} + \pi_{3} \rho_{31} = 9 \pi_{o} + 9 \pi_{3}$
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
en de la comptensión de la comptensión de la comptensión de la comp tensión de la comptensión de la c	1= 11, + 11 ₂ + 11 ₃ + 11 ₄
en e	Solve to obtain To=, 244 TT, =, 344 TT, =, 275 TT, =, 137
	long-run expected cost = Tip (1)(1500) + TT, (,2)(1500) + TT, (,5)(1500) + 500 TT3+TT3(1500)(1)
	$= \frac{(100)^{1/2} $

				namangan sementen mediatekan kecikhan kemen kecikhan kecik		2	3	4	5	6
3,	9.		1	0-5	3	2	.4	, 1	O	O
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		P=	33	0 ie	0	0	1	0	0	0
			4	Sold 20	0	0	0	1	O	Ø
			5	Sold 30	0	0	0	0	1	0
			6	Soldso	[0	0	0	0	0	١
	***************************************									***

 $f_{13} = .3 f_{13} + .2 f_{23} + .4 f_{33} + .1 f_{34}^{4}$ $0 \rightarrow .7f_{13} = .2 f_{23} + .4$ $f_{13} = .3 f_{23}$ $+ .2 f_{53} + .5 f_{63} \rightarrow .7 f_{23} = 0$ $f_{33} = 1$

,7 fi3 = 14 -> fi3 = 4/7 = .571

6.	1 Sta	de j		f dollars					
	Management (Company)		0	1 2	3	4	5	6	
kipitang da katulung na katulung na mangangg penghapy sa katulung ng mga		G	-		all for all little for the land of the lan	ora distribution and analysis of probability property in principal controls.			
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	of contraction and a second	5	Action of the Control			,6		,4	
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 $f_{06} = 0$ $f_{16} = .6 f_{06}^{6} + .4 f_{26} \implies f_{16} = .4 f_{26}$ $f_{26} = .6 f_{06}^{6} + .4 f_{46} \implies f_{26} = .4 f_{46}$ $f_{36} = .6 f_{36}^{6} + .4 f_{66}^{6} \implies f_{36} = .4$ $f_{46} = .6 f_{26} + .4 f_{66}^{6} \implies f_{46} = .6 f_{26} + .4 \implies f_{46} = .6 (.4 f_{46}) + .4 \implies f_{46} = \frac{10}{19}$ $f_{56} = .6 f_{46} + .4 f_{66}^{6} \implies f_{56} = .6 f_{46} + .4$ $f_{66} = 1$

Probability of reaching \$6 starting from \$2 if fee = 4/19 = 2105

			OMM 44 CO 2014 OF STREET OF ST	CONTRACTOR				$-e^{i\left(\frac{1}{2}\right)\right)\right)}{\frac{1}{2}}\right)\right)}\right)\right)}\right)}\right)}\right)}\right)}\right)}\right)}}$
/1.	9.		1	2	3	Ц	5	
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		2 1-yr	0	6	,95	,05	0	
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and the second		e Replaced	σ	0	0		0	
To the second	And the second s	5 Warr, Exf	, 0	0	0	0	N. S. CTERNING	
	i i		CHA					

 $f_{14} = .97 f_{24} + .03 f_{44} \longrightarrow f_{14} = .97 f_{24} + .03 = .97 (.1165) + .03 = .143005$ $f_{24} = .95 f_{34} + .05 f_{44} \longrightarrow f_{24} = .95 f_{34} + .05 = .95 (.07) + .05 = .1165$ $f_{34} = .07 f_{44} + .93 f_{54} \qquad f_{34} = .07$ $f_{44} = 1$

F54 = 0

The fraction of all refrigerators that must be replaced is fix = .143005

OR.	441
1 1 1	Z1Z1 1

Name	
Nomo	
1 2 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	

1. Payoff Insurance Company charges a customer according to his or her accident history during the past 3 years. The amount charged is shown in the table below. Payoff has determined that the probability that a customer will have an accident during the current year is dependent upon the customer's accident history during the past 3 years. These probabilities are shown in the table below. For the sake of simplicity, assume that a customer can have at most one accident during a year.

# of Accidents		Probability of
During Past	Annual	Accident During
3 Years	Premium	Current Year
0	\$100	0.01
1	200	0.05
2	400	0.10
3	800	0.20

- (a) Create a Markov chain to model this problem.
 - 1. Carefully define the states.

2. What is the transition matrix P?

(b)	Compute $P^{(8)}$. You are welcome, even encouraged, to use computer software to do this. Please attach some sort of printout to document your work.
(c)	Set up the steady-state equations.
(d)	Solve the steady-state equations. You are welcome, even encouraged, to use computer
(u)	software to do this. Please attach some sort of printout to document your work.
(e)	Determine the long-run expected average premium paid by a Payoff customer.

Name Key

1. Payoff Insurance Company charges a customer according to his or her accident history during the past 3 years. The amount charged is shown in the table below. Payoff has determined that the probability that a customer will have an accident during the current year is dependent upon the customer's accident history during the past 3 years. These probabilities are shown in the table below. For the sake of simplicity, assume that a customer can have at most one accident during a year.

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3	800	0.20

- (a) Create a Markov chain to model this problem.
 - 1. Carefully define the states.

2. What is the transition matrix P?

NNN NNA NAN ANN NAA ANA AAN AAA 1 NNN ,99 .01 2 NNA ,95 ,05 3 NAN ,95 ,05 4 ANN ,95 ,05 5 NAA ,90 ,10 7 AAN ,90 ,10

3 pts.

(b) Compute $P^{(8)}$. You are welcome, even encouraged, to use computer software to do this. Please attach some sort of printout to document your work.

(c) Set up the steady-state equations.

$$\pi_{1} = .99 \, \pi_{1} + .95 \, \pi_{4}$$
 $\pi_{2} = .01 \, \pi_{1} + .95 \, \pi_{4}$
 $\pi_{3} = .95 \, \pi_{2} + .90 \, \pi_{6}$
 $\pi_{5} = .95 \, \pi_{3} + .90 \, \pi_{7}$
 $\pi_{5} = .05 \, \pi_{2} + .10 \, \pi_{6}$
 $\pi_{6} = .05 \, \pi_{3} + .10 \, \pi_{7}$
 $\pi_{7} = .90 \, \pi_{5} + .80 \, \pi_{8}$
 $\pi_{8} = .90 \, \pi_{5} + .80 \, \pi_{8}$
 $\pi_{8} = .90 \, \pi_{5} + .80 \, \pi_{8}$
 $\pi_{8} = .90 \, \pi_{5} + .80 \, \pi_{8}$

(d) Solve the steady-state equations. You are welcome, even encouraged, to use computer software to do this. Please attach some sort of printout to document your work.

(e) Determine the long-run expected average premium paid by a Payoff customer.

$$5p^{35}$$

$$Long-run expected average premium = \sum_{j=1}^{8} T_{j} C(j)$$

$$= 100 (.96767) + 200 (.01019) + 200 (.01019) + 400 (.00057) + 400 (.$$