

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\frac{\partial \vec{E}}{\partial t} = \vec{E}_0 (-i\omega) e^{-i(\vec{k} \cdot \vec{r} - \omega t)} = -i\omega \vec{E}$$

$$\frac{\partial}{\partial t} \mapsto -i\omega$$

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= \vec{\nabla} \cdot (\vec{E}_0 e^{i\vec{k} \cdot \vec{r}} e^{-i\omega t}) = \\ &= e^{-i\omega t} \vec{\nabla} (\vec{E}_0 e^{i\vec{k} \cdot \vec{r}}) = \downarrow \\ &\quad \begin{array}{cc} \uparrow & \uparrow \\ \text{vector} & \text{escalar} \\ \vec{a} & \psi \end{array} \end{aligned}$$

Fórmula (7):

$$\vec{\nabla} \cdot (\psi \vec{a}) = (\vec{\nabla} \psi) \cdot \vec{a} + \psi (\vec{\nabla} \cdot \vec{a})$$

$$\begin{aligned} \downarrow &= \vec{\nabla} (e^{i\vec{k} \cdot \vec{r}}) \vec{E}_0 e^{-i\omega t} + e^{i\vec{k} \cdot \vec{r}} (\vec{\nabla} \cdot \vec{E}_0) e^{-i\omega t} = \\ &= 0 (\vec{E}_0 = \text{cte}) \end{aligned}$$

$$= \vec{E}_0 e^{-i\omega t} \underbrace{\vec{\nabla} (e^{i\vec{k} \cdot \vec{r}})}_{\text{calculamos esto}}$$

$$\begin{aligned}\vec{\nabla}(e^{i\vec{k}\cdot\vec{r}}) &= e^{i\vec{k}\cdot\vec{r}} \vec{\nabla}(i\vec{k}\cdot\vec{r}) = \\ &= i e^{i\vec{k}\cdot\vec{r}} \underbrace{\vec{\nabla}(\vec{k}\cdot\vec{r})}_{\textcircled{\otimes}}\end{aligned}$$

Fórmula (9):

$$\begin{aligned}\vec{\nabla}(\vec{a}\cdot\vec{b}) &= (\vec{a}\cdot\vec{\nabla})\vec{b} + (\vec{b}\cdot\vec{\nabla})\vec{a} + \\ &+ \vec{a}\times(\vec{\nabla}\times\vec{b}) + \vec{b}\times(\vec{\nabla}\times\vec{a})\end{aligned}$$

$$\begin{aligned}\textcircled{\otimes} = \vec{\nabla}(\vec{k}\cdot\vec{r}) &= (\vec{r}\cdot\vec{\nabla})\vec{k} + (\vec{k}\cdot\vec{\nabla})\vec{r} + \\ &= 0 (\vec{k} \text{ no depende de } \vec{r}) \\ &+ \vec{r}\times(\vec{\nabla}\times\vec{k}) + \vec{k}\times(\vec{\nabla}\times\vec{r}) = (\vec{k}\cdot\vec{\nabla})\vec{r} = \\ &= 0 (\vec{k} \text{ no depende de } \vec{r}) = 0 \\ \vec{\nabla}\times\vec{r} &= \begin{vmatrix} \hat{u}_x & \hat{u}_y & \hat{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0\end{aligned}$$

$$= \left(k_x \frac{\partial}{\partial x} + k_y \frac{\partial}{\partial y} + k_z \frac{\partial}{\partial z} \right) (x\hat{u}_x + y\hat{u}_y + z\hat{u}_z) =$$

$$= k_x \frac{\partial x}{\partial x} \hat{u}_x + k_y \frac{\partial y}{\partial y} \hat{u}_y + k_z \frac{\partial z}{\partial z} \hat{u}_z =$$

$$= k_x \hat{u}_x + k_y \hat{u}_y + k_z \hat{u}_z = \vec{k}$$

de donde:

$$\vec{\nabla} (e^{i\vec{k}\cdot\vec{r}}) = i e^{i\vec{k}\cdot\vec{r}} \vec{\nabla} (\vec{k}\cdot\vec{r}) = i\vec{k} e^{i\vec{k}\cdot\vec{r}}$$

y queda:

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= \vec{E}_0 e^{-i\omega t} \vec{\nabla} (e^{i\vec{k}\cdot\vec{r}}) = i\vec{k} \vec{E}_0 e^{i\vec{k}\cdot\vec{r}} e^{-i\omega t} \\ &= i\vec{k} \vec{E} \end{aligned}$$

$$\boxed{\vec{\nabla} \mapsto i\vec{k}}$$