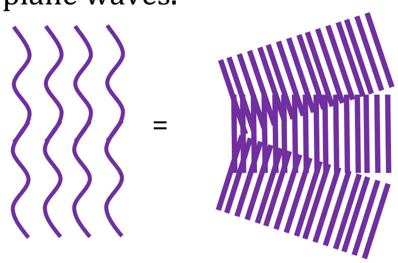
Photonics Laboratory
Physics 472
Spring 2024

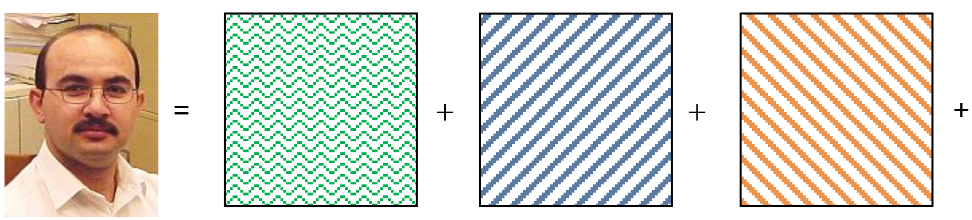
Experiment#2 Fourier Transforms and Spatial Filtering

BACKGROUND

Fourier optics describes the propagation of light using harmonic analysis and linear systems. Harmonic analysis is based on the expansion of any function as a superposition of harmonic functions of different frequencies. In other words any arbitrary wave can be analyzed as a superposition of plane waves.



For example my picture can be constructed of large number of harmonic functions of different spatial frequencies and complex amplitudes.

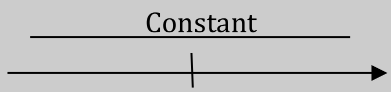
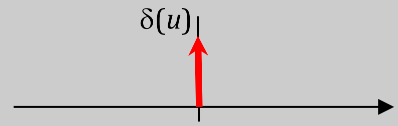
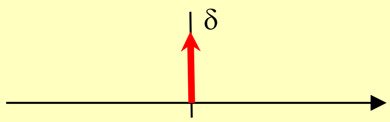
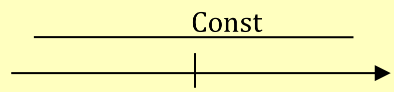
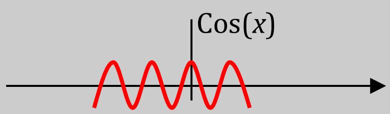
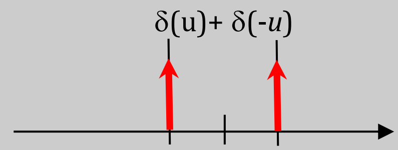
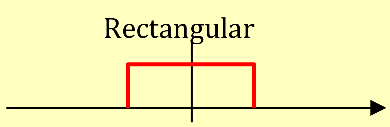
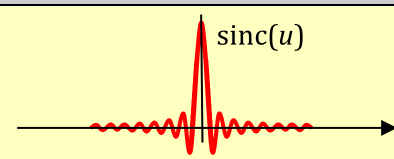
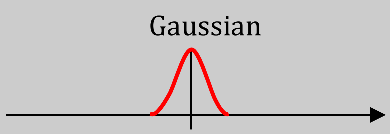
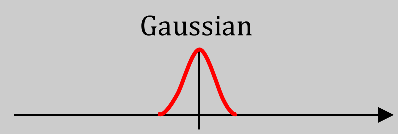


In the spatial frequency domain the Fourier transform and its inverse in 1D and 2D are given in the table below.

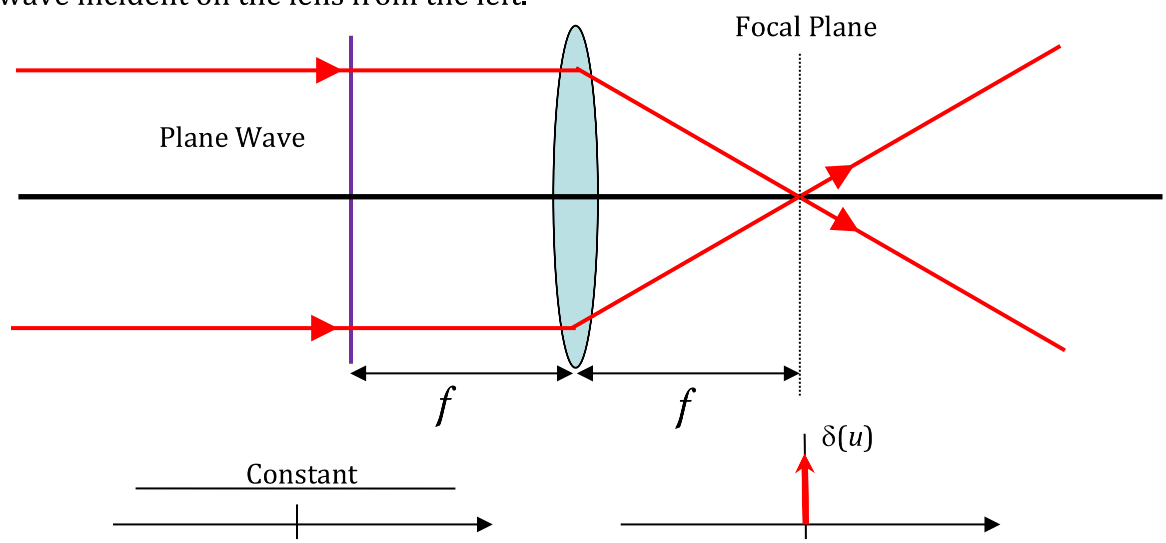
	Fourier Transform	Inverse Fourier Transform
1D	$G(u)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g(x)e^{iUx}\,dx$	$g(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} G(u)e^{-iUx}\,du$
2D	$G(u,v)=\frac{1}{2\pi}\iint g(x,y)e^{i(ux+vy)}dxdy$	$g(x,y)=\frac{1}{2\pi}\iint G(u,v)e^{-i(ux+vy)}dudv$

Where u is the spatial frequency given by $u = \frac{2\pi}{\lambda} \sin\theta$. θ is the angle between the direction of the incident beam and that of the scattered one. *The spatial frequency represents the number of lines per unit length.*

Examples of important Fourier transforms of some functions are given below.

Function	*Fourier Transform*
 Constant	 δ(u)
 δ	 Const
 Cos(x)	 δ(u)+ δ(-u)
 Rectangular	 sinc(u)
 Gaussian	 Gaussian

Fourier transform is equal to the far field (Fraunhaufer) diffraction. As you know the far field diffraction pattern of a slit is a Sinc function and that of a small circular aperture is an Airy pattern. A simple device that performs a Fourier transform is a lens. Consider a plane wave incident on the lens from the left.



The pattern at the focal plane of the lens is a single point. This resembles the delta function in 1D. Recall that the Fourier transform of an extended constant function is the delta function. Therefore, the pattern at the focal plane of a lens is the Fourier transform of input function. This pattern also is the same as that of the far field diffraction pattern.

If the field distribution, $g(x,y)$, at a distance f in front of the lens then $G(X,Y)$ the field distribution at the back focal plane of the lens. In terms of the spatial frequencies, G is the Fourier transform of $g(x,y)$ and given by

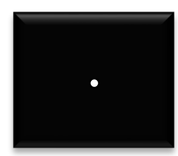
$$G(u,v) = \frac{1}{\lambda f} \iint g(x,y) e^{i(ux+vy)} dxdy$$

Where λ is the light wave length, f is the focal length of the lens, and $u=(2\pi/\lambda f)X$ and $v=(2\pi/\lambda f)Y$ are the spatial frequencies.

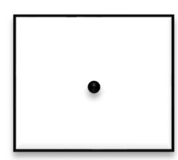
Spatial Filters:

As an application to the Fourier transform by a lens one can perform optical filtering and image processing using appropriate filter(s). The filter must be placed at the focal plane of the lens (Fourier Plane). Important types of spatial filters are listed below. Keep in mind that these filters allow certain spatial frequencies to propagate beyond the focal plane.

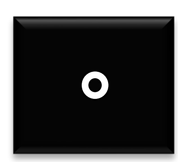
- *Low-Pass filter*: This filter blocks high spatial frequencies that are larger than the cutoff frequency u_s. Here u_s is defined as $u_S = D / 2\lambda f$. Where D is the diameter of the circular aperture, f is the focal length of the lens, and λ is the wavelength of the light used to illuminate the object. It is achieved by using a pinhole at the focal plane of the lens. The high spatial frequencies are responsible for the details, noise, patterns, edges, ...



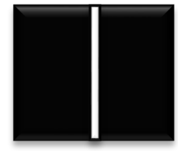
- *High-Pass filter*: it blocks low spatial frequencies that are smaller than the cutoff frequency u_s. It is a complement of the low-pass filter. The low spatial frequencies are responsible for the overall intensity distributions, large smooth areas, and smooth changes in image. As a result a high-pass filter reduces the intensity of the image and allows for the edges to be more pronounced. This is why this type of filter is called *edge enhancement filter*.



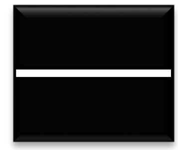
- *Band-pass filter*: is used to enhance certain details in the image. It has a ring-shaped annular aperture. Very large and very small features of the image are blocked. This type of filter is important when used with microscopes.



- *<u>Vertical-pass filter</u>*: it transmits vertical frequencies and blocks the horizontal ones. This mean an object with horizontal and vertical lines will have an image consists of only horizontal lines.



- *<u>Horizontal-pass filter</u>*: it transmits horizontal frequencies and blocks the vertical ones. This mean an object with horizontal and vertical lines will have an image consists of only vertical lines.



EXPERMENTS

Task#1: Align a laser beam parallel to the optical table.

Before setting up any experiment involving lasers it is very helpful to have the laser beam heading toward your setup parallel to the optical table and most of the time parallel to one of the table edges. This can be achieved by using two points far from each other, one near the laser and the other at the far end of the table. You need to make the beam at the same height between the two points. In general you need two degrees of freedom to align the beam in any given direction between two points. The two degrees of freedom can be two reflections (angular motion) by two mirrors or a translation of one mirrors and a reflection by the other. The laser itself can be used as one of the degrees of freedom if it is easily adjustable vertically or horizontally, and or it can be tip/tilted.

1. Set the height of two apertures or spikes to be the same. You can use the height of the laser beam as it exits the laser or set them to any desirable height by a ruler. Now mount the two apertures or spikes at the opposite ends of the optical table along one of it edges.
2. Direct the laser beam toward the apertures by two mirrors or by one mirror with the appropriate orientation of the laser. The mirror closest to the laser is used to adjust the beam at the near aperture and the second mirror is used to adjust the beam at the far aperture. Recall that translational adjustment of the mirror holder can be very helpful. Normally use coarse adjustment by rotating the mirror post by hand and reserve the knops on the mirror mounts for fine adjustments.

Task#2: Align a lens to the beam and the construction of laser beam expander.

If a lens is needed in the path of the beam then the beam must go through the center of the lens at normal incidence. This keeps the beam alignment.

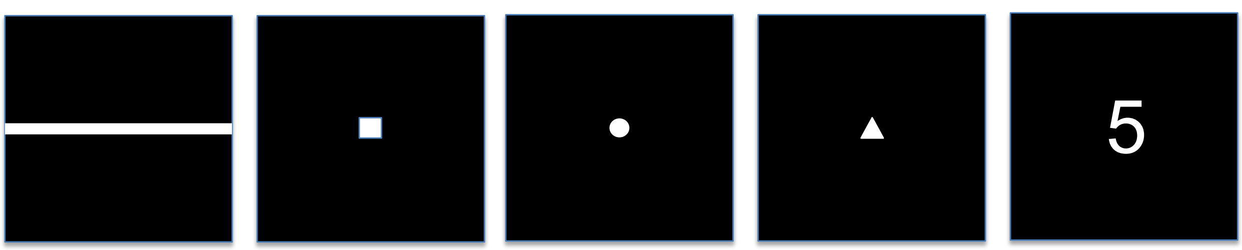
For the beam expander we need to have a short focal length lens in order to get a small beam waist that will diverge quickly. Recall that the beam radius at the focal plane is *approximated* by $\omega = 1.22\lambda f/D$, where D is the input beam diameter, f is the focal length of the lens, and λ is the wavelength.

1. Place an iris after the location, but very close to, where you need the place the lens. Make sure that the iris is well aligned with the beam.
2. Now place the microscope objective ($40\times$ or $60\times$) in the path of the beam and adjust the lens horizontally and vertically until the beam is centered at the iris. You may need to adjust the iris opining to easily see the beam relative to the iris opining.

3. Now take a look at the reflections from the lens (you can use a paper with a hole to see the reflections or another iris placed before the lens) and make sure they are coinciding with beam before the lens. You may need to slightly rotate the post of the lens and use the tip/tilt adjustments to make the beam incident normally at the lens center. Once the reflected (from the front and back surfaces of the lens) and the transmitted beams are centered relative to the irises your lens is aligned.
4. In order to construct a laser beam expander we will use the configuration of an astronomical telescope. Use a *40× microscope objective with a 15cm focal length converging lens or a 60× microscope objective with 10cm focal length converging lens*.
5. Place the 10 cm (15cm) lens after the 60x (40x) lens and adjust the separation between the lenses to obtain a collimated beam (start with a separation equal to the sum of their focal lengths). Use a ruler to measure the size of the expanded beam at different location to make sure that it is collimated. We will use this collimated beam to illuminate several objects (transparencies) and study their Fourier transforms and the effects of spatial filtering.

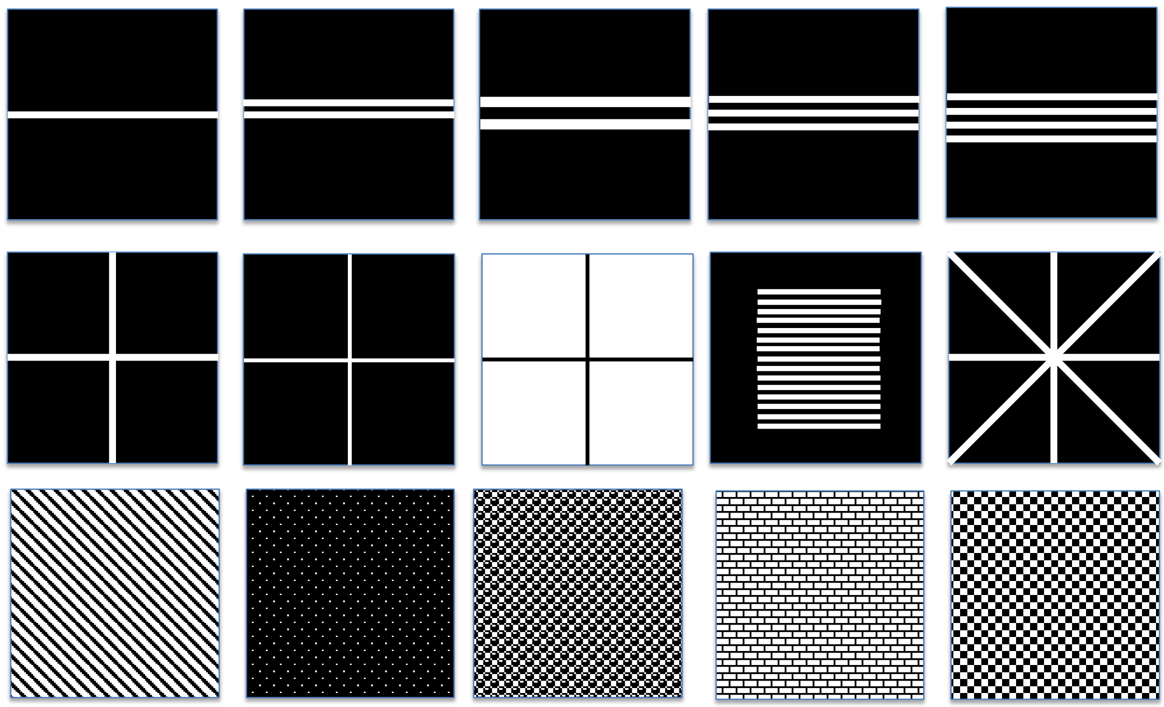
Task#3: Fourier transform of simple functions.

You will be given several transparencies in order to investigate the Fourier transform of the functions represented by these transparencies. Use the expanded collimated laser beam as a source of light to illuminate the transparencies.

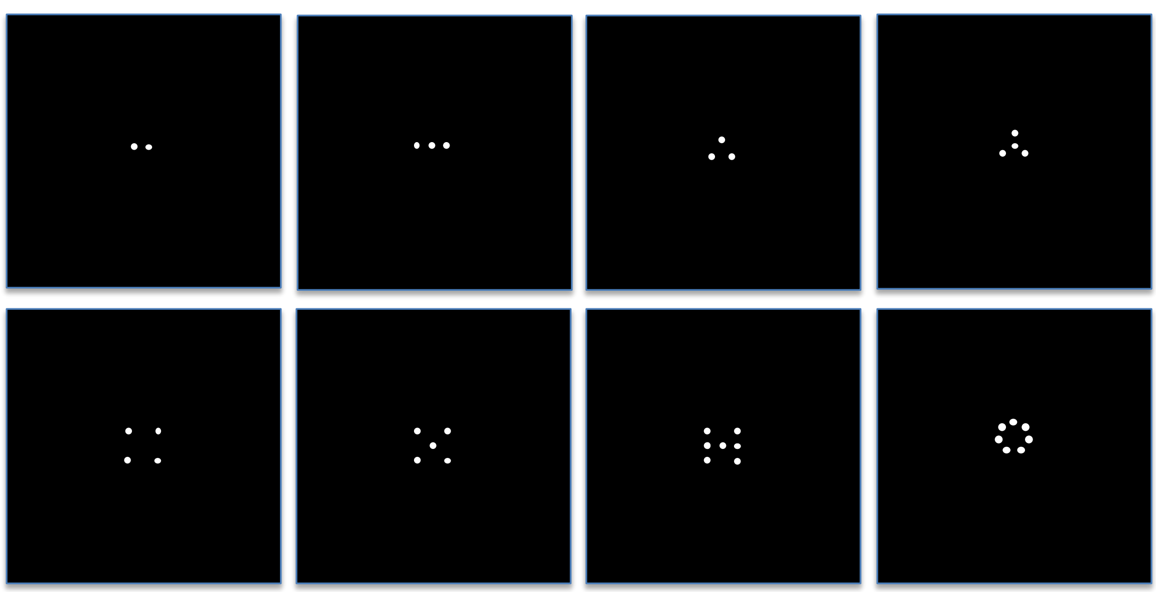


1. Place the transparency that has the rectangular slit in the path of the collimated beam. The transmission of this opining resembles a 1D impulse function. Place a lens after the opining at a distance equal to the focal length of this lens (the Fourier transform lens). You may use any lens. A 20 cm or 25 cm focal length converging lens would be an excellent choice. Remember to keep the height of this lens to be the same as the height of the window (the window is centered to the lens). The image constructed at a far screen should be a square window. Now place a microscope objective at the focus of this imaging lens (the Fourier transform lens) in order to see the pattern at the focal plane. *Remember that the pattern at the focal plane is the Fourier transform of the impulse function*. Record your observations, take a picture of the intensity pattern, and compare them to what you expected to get.

2. Repeat part 1 for these functions on the given below.

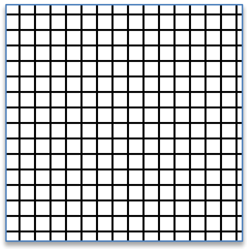


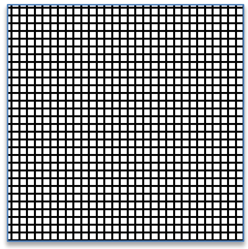
3. Repeat part 1 for the various point sources with configurations given below.



Task#4: Spatial filtering and Fourier transform 1.

Use the expanded collimated laser beam as a source of light for the flowing. Place a mish in the path of the expanded beam and observe its image on a screen located about 0.5 m from the object.





1. Continue with what you did in task#3 part 1. Insert a vertical slit at the location of the focal plane and observe the image on the screen. You may need to use slits of different widths. Take a picture with your phone for the image and explain the results. Remember that in this case the slit is a vertical-pass filter.
2. Insert a horizontal slit at the location of the focus and observe the image on the screen. Take a picture with your phone for the image and explain the results. In this case the slit is a horizontal-pass filter.

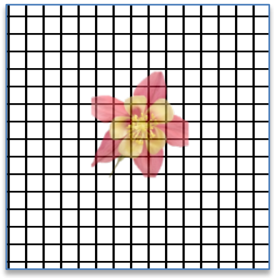
Task#5: Spatial filtering and Fourier transform 2.

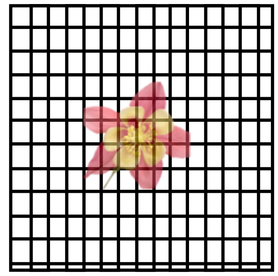
Use the expanded collimated laser beam as a source of light for the flowing. Replace the picture of the mish in task#4 by the picture of the flower and observe its image on a screen located 0.5 m from the object.

1. Insert a microscope objective at the location of the focus to observe the Fourier transform of the flower picture. Take a picture for the image on the screen. Record your observations.
2. Now Insert an iris at the location of the focus and observe the image on the screen. Adjust the size of the iris and observe the changes in the image (take pictures). Record your observations and Explain. Recall that the iris is a low-pass filter.
3. Insert a transparency with a black spot to block the center of the focused image at the location of the focus and observe the image on the screen. Observe the changes in the image (take pictures). Record your observations and Explain. Recall that the black spot on a transparent surface is a high-pass filter
4. Insert a transparency that has a black spot at the center of a transparent ring at the location of the focus and observe the image on the screen. Explain the function of this filter.

Task#6: Spatial filtering and Fourier transform 3.

Use the expanded collimated laser beam as a source of light for the flowing. Use the various filtering methods you employed in task#4 and task#5 to study the effects on the image of a flower with a mish.





Task#7 Laser beam expanders and Spatial Filtering: How to align a pinhole at the focus?

In some experiments, it is necessary to remove fluctuations in the intensity profile of the laser that are acquired when the beam scatters off of dust particles in the air or small defects on the surfaces of various optical components of the system. In Fourier-space these random fluctuations result in higher spatial frequency components added to the system. These components can be filtered out by using a low pass filter (very small circular hole) in the focal plane.

Now place a pinhole at that location of the focus of the short focal length lens that we used to expand the beam. You need to mount the pinhole on translation stages in order to be able to move the pinhole across the beam and along the beam. For best results the pinhole diameter should has approximately the same diameter as the waist of the beam at the focus. A good approximation to the beam diameter at the focus is given by

$$2w_o = \frac{4\lambda}{\pi D} f$$

Where w_o is the beam radius at the focus, D is the beam diameter at the surface of the lens, λ is the wavelength of light, and f is the focal length of the lens.

1. You can locate the best position for the pinhole by monitoring the speckle pattern of the beam in the far field. The number of speckles is approximately constant, so when the pinhole is closest to the focus you will have the biggest speckles in the far field.
2. Adjust the pinhole to maximize the power transmission. This is the best location of the pinhole if it is to be used in beam filtering.