Nuclear and Particle Physics

1 Binding Energy and Nuclear Fission of Uranium-238

1.1 Part 1: Binding Energy Calculation Using the Semi-Empirical Mass Formula

The semi-empirical mass formula is given by:

$$B(Z, A) = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(A - 2Z)^2}{A} + \delta(A, Z)$$

Where $B(Z, A)$ is the total binding energy, A is the mass number (total number of nucleons), Z is the atomic number (number of protons), a_v, a_s, a_c, a_a are constants for the volume, surface, Coulomb, and asymmetry terms respectively, $\delta(A, Z)$ is the pairing term.

Typical values of the constants are:

- $a_v = 15.8\,\mathrm{MeV}$,
- $a_s = 18.3\,\mathrm{MeV}$,
- $a_c = 0.714\,\mathrm{MeV}$,
- $a_a = 23.2\,\mathrm{MeV}$.

The pairing term $\delta(A, Z)$ is given by:

$$\delta(A, Z) = \begin{cases} +11.2/\sqrt{A}\ \ \mathrm{MeV}, & \text{for even-even nuclei – even A,} \\ 0, & \text{for odd A,} \\ -11.2/\sqrt{A}\ \ \mathrm{MeV}, & \text{for odd-odd nuclei – even A.} \end{cases}$$

Tasks

1. Calculate the binding energy per nucleon for Uranium-235 and Uranium-238.

2. Plot the binding energy per nucleon for Uranium isotopes, including Uranium-235 and Uranium-238. Analyze which isotope among these two exhibits greater stability.

1.2 Part 2: Fission of Uranium-238

In nuclear fission, a large nucleus (like Uranium-238) splits into two smaller nuclei (fission fragments). The stability of the fragments can be compared to that of the parent nucleus to determine if fission is energetically favorable.

Tasks

1. Calculate the total binding energy for the following:
   - Uranium-238,
   - Krypton-92,
   - Barium-141.
2. Explain whether the fission process of Uranium-238 is energetically favorable.
3. If so;
   a) calculate and display the energy released during fission,
   b) explain why heavy nuclei such as Uranium-238 tend to undergo fission.
4. Plot a bar graph comparing the total binding energy of Uranium-238, Krypton-92, Barium-141, and the combined energy of the fission fragments.

Additional Questions for Analysis

How does the energy released during fission compare to the binding energy of Uranium-238? Relate this to the concept of nuclear energy production in reactors or atomic bombs.

1.3 Submission Requirements

- A Python script implementing the calculation of binding energies and fission analysis (50%).
- A plot of the binding energy per nucleon for Uranium isotopes (10%).
- A bar chart comparing the binding energies of Uranium-238 and the fission fragments (10%).
- Written answers to the analysis questions (30%).

2 Radioactive Decay and Stability of Uranium-235 and Uranium-238

2.1 Decay Law

The decay law is:

$$N(t) = N_0 e^{-\lambda t}$$

Where $N(t)$ is the number of undecayed nuclei at time t, N_0 is the initial number of nuclei, $\lambda = \frac{\ln 2}{T_{1/2}}$ is the decay constant, t is the elapsed time.

The activity $A(t)$, representing the number of decays per second, is:

$$A(t) = \lambda N(t)$$

Tasks

1. Write a Python code to calculate the remaining quantity of a sample containing $N_0 = 1 \times 10^6$ nuclei of Uranium-235 (U-235) and Uranium-238 (U-238) nuclei after a period of time t, given their respective half-life $T_{1/2}$:
   - U-238: $T_{1/2} = 4.468\,$billion years
   - U-235: $T_{1/2} = 703.8\,$million years
2. Plot the decay curves of U-235 and U-238 vs time and compare how the two nuclei decay differently given their vastly different half-lives.
3. Calculate and plot the activity $A(t)$ over time for both Uranium isotopes, and interpret the results.
4. First steps of decay chains: Simulate the decay of U-238 into Thorium-234 and U-235 into Thorium-231, then plot the number of nuclei and the activity of these daughter isotopes to analyze their contributions to the overall decay process.
5. **Decay percentage:** Calculate how long it takes for each isotope to decay to 90%, 50%, and 10% of its original quantity, and relate this to their half-lives.
6. Link with Binding Energy:
   - Discuss how the radioactive decay behavior of U-235 and U-238 compares with the binding energy insights from the previous exercise.
   - Explain why U-238 is more stable (due to a longer half-life) while U-235 decays faster despite its slightly higher binding energy per nucleon (as seen in the previous exercise).
7. Explain how the decay of U-238 and U-235 is used in radiometric dating (e.g., Uranium-lead dating) to estimate the age of rocks and the Earth.

2.2 Additional Questions

1. Discuss the role of radioactivity in nuclear power generation as a sustainable energy source, contrasting its use with conventional fossil fuels.

2. Discuss the role of U-238's slow decay in the production of Plutonium-239 in breeder reactors, and why U-235 is more suitable for nuclear weapons and reactors.