Experiment 4:

Michelson Interferometer

Student:

Víctor Mira Ramírez

Professor:

Dr. Abdullatif Hamad

SOUTHERN ILLINOIS UNIVERSITY EDWARDSVILLE

College of Arts and Sciences: Department of Physics

PHYS 472 - Photonics Laboratory

Abstract

Contents

1 Experiments

1.1 Calibration of the micrometer and the mirror (M1) movement

Use a short focal length lens to focus a HeNe laser operating at 632.8 nm. This makes the alignment and the calibration of the micrometer movement easier. When the interferometer is well aligned you get the concentric fringes if $d \neq 0$. Move the movable mirror, M_1, by moving the micrometer and *note the fringe movement*.

Task:

Now you need to move the micrometer and at the same time count the fringes. You need to move the micrometer very slowly. Be sure to move the micrometer in one direction to avoid any backlash. You need to count at least 100 fringes as you move the micrometer from the starting point to some other distance. Repeat this 5 times. Record your data and find the average and the uncertainty in the distance measured by the micrometer.

The average number of fringes was $n = 77.25$, the resolution used was $d = 25 \pm 1$ μm, and the wavelength of the laser used was 632.8 nm.

Task:

Knowing the distance that the micromere moved, the wavelength of the laser, and the number of fringes counted you should be able, with the help of the last equation above, to determine the conversion factor that relates the micrometer distance and the actual distance M_1 moved.

$$\text{Conversion Factor } = \frac{2d}{\lambda n} = \frac{2 \cdot 25 \cdot 10^{-6}}{632.8 \cdot 77.25 \cdot 10^{-9}} = 1.023$$

1.2 Determine the wavelength of a laser

Use the same procedure of task#1 to determine the wavelength of the diode lasers provided at your station.

Task:

Repeat the experiment 5 times for each of the lasers and use the average value of the micrometer readings to find the wavelengths of the lasers.

We can calculate the wavelengths of the lasers provided by using the equation $\lambda = \frac{2d}{n}$.

Task:

Determine the uncertainty in the laser wavelength for each case.

Task:

Compare your results to the accepted values for these lasers. Find the %error for each case. Does you results agree within the experimental error.

1.2.1 He-Ne laser

Calculating the wavelength from the data:

$$\lambda = \frac{2 \cdot 25 \cdot 10^{-6}}{77.25} = 6.4725 \cdot 10^{-7} = 647.25\ nm$$

Calculating the uncertainty from the expected value:

$$|632.8 - 647.25| = 14.5\ nm$$

Which corresponds to an error of:

$$\frac{14.5}{632.8} = 0.0229 = 2.3\%\ error$$

1.2.2 Green laser

Calculating the wavelength from the data:

$$\lambda = \frac{2 \cdot 25 \cdot 10^{-6}}{89.8} = 5.5679 \cdot 10^{-7} = 556.79\ nm$$

Calculating the uncertainty from the expected value:

$$|532 - 556.79| = 24.79\ nm$$

Which corresponds to an error of:

$$\frac{24.79}{532} = 0.04659 = 4.7\%\ error$$

1.2.3 Red laser

Calculating the wavelength from the data:

$$\lambda = \frac{2 \cdot 25 \cdot 10^{-6}}{80.4} = 6.2189 \cdot 10^{-7} = 621.89\ nm$$

Calculating the uncertainty from the expected value:

$$|635 - 621.89| = 13.11\ nm$$

Which corresponds to an error of:

$$\frac{13.11}{635} = 0.020645 = 2.1\%\ error$$

1.3 Determine the index of refraction of a glass sample

Use the same laser and configuration in task#1 to determine the index of refraction for the glass sample at your station. In this case you need to mount the sample on a rotating stage and place it in the path of the beam in one of Michelson interferometer arms. To find the index of refraction you need to measure the OPD and the corresponding number of fringes that passes by during the rotation process.

After rotation by an angle θ from normal incidence, the beam travels larger distance than that traveled when the beam was perpendicular to the surface of the sample. Knowing the number of fringes ($N \equiv \Delta m$), the angle (θ), and the thickness of the sample (t) you should be able to find the index of refraction using the following result.

$$n = \frac{(2t - N\lambda)\,[1 - \cos\theta]}{2t\,[1 - \cos\theta] - N\lambda}$$

Task:
Make sure that the sample is perpendicular to the laser beam, normal incidence, how?

To align the laser beam to be perpendicular to the surface of the glass at normal incidence we adjust the stage until there is no lateral shift in the fringe pattern when you slightly move the sample back and forth.

Task:
Rotate the sample slowly and count 300 fringes. Record the rotation angle. Repeat this procedure starting from normal incidence 5 times. Record your data.

Instead of fixing the number of fringes and recording the angle, we recorded the number of fringes every 10°.

Task:
Find the average rotation angle and its uncertainty.

Because of our procedure, we obtained an average number of fringes instead of an average rotation angle. That number was 104.4 fringes.

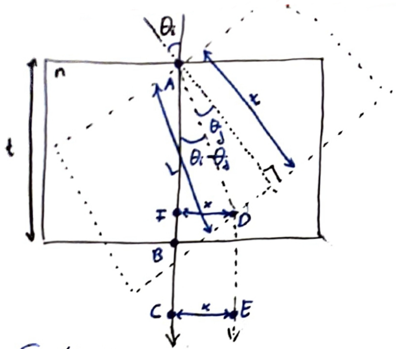
Task:
Determine the index of refraction and its uncertainty.

Taking into account our precision recording angles, that was of 0.1° and that our sample thickness was $t = 0.006$

$$n = \frac{(2t - N\lambda)\,[1 - \cos\theta]}{2t\,[1 - \cos\theta] - N\lambda} = \frac{(2 \cdot 0.006 - 104.4 \cdot 632.8 \cdot 10^{-9}) \cdot [1 - \cos 10^\circ]}{2 \cdot 0.006\,[1 - \cos 10^\circ] - 104.4 \cdot 632.8 \cdot 10^{-9}} = 1.56$$

Task:

Derive the above expression using the equation $2\Delta d = \Delta m\lambda$ and the diagram shown below. Note, deriving this expression is not trivial.



Show that $2\Delta d = \Delta m\lambda \Longleftrightarrow \Delta d = \frac{\Delta m\lambda}{2} = \frac{N\lambda}{2}$

$OPD = \Delta d = n\overline{AD} + \overline{DE} - n\overline{AB} - \overline{BC} = \frac{nt}{\cos\theta_j} + \cancel{t\tan\theta_i\sin(\theta_i-\theta_j)} - nt - \cancel{t\tan\theta_i\sin(\theta_i-\theta_j)} - \frac{t\cos(\theta_i-\theta_j)}{\cos\theta_j}$

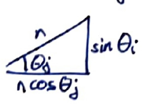
$= \boxed{t\left[1-n+\frac{1}{\cos\theta_j}\left(n - \cos\theta_i\cos\theta_j - \sin\theta_i\sin\theta_j\right)\right]} + t$

- $\boxed{\overline{AB} = t}$
- $\cos\theta_j = \frac{t}{\overline{AD}} \Longleftrightarrow \boxed{\overline{AD} = \frac{t}{\cos\theta_j}}$
- $\tan\theta_i = \frac{\overline{DE}}{x} \Longleftrightarrow x = \frac{\overline{DE}}{\tan\theta_i} = t\sin(\theta_i-\theta_j) \Longleftrightarrow \boxed{\overline{DE} = t\tan\theta_i\sin(\theta_i-\theta_j)}$
- $\overline{BC} = \overline{DE} - \overline{BF} = \boxed{t\left[\tan\theta_i\sin(\theta_i-\theta_j) + \frac{\cos(\theta_i-\theta_j)}{\cos\theta_j} - 1\right]}$
  - $\overline{DE} = t\tan\theta_i\sin(\theta_i-\theta_j)$
  - $\overline{BF} = t - w = t - \frac{t\cos(\theta_i-\theta_j)}{\cos\theta_j}$
    - $\cos(\theta_i-\theta_j) = w/L \Longleftrightarrow w = L\cos(\theta_i-\theta_j) = \frac{t\cos(\theta_i-\theta_j)}{\cos\theta_j}$
    - $\cos(\theta_j) = t/L \Longleftrightarrow L = t/\cos\theta_j$

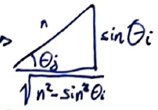
Snell's Law

$n_i\sin\theta_i = n_j\sin\theta_j$ with $n_i = 1$

$\sin\theta_j = \frac{\sin\theta_i}{n_j} = \frac{\sin\theta_i}{n}$



$\Longleftrightarrow$



$\Longleftrightarrow \Delta d = t\left[1-n+\frac{n}{\sqrt{n^2-\sin^2\theta_i}}\left(n - \cos\theta_i\frac{\sqrt{n^2-\sin^2\theta_i}}{n} - \frac{\sin^2\theta_i}{n}\right)\right] =$

$\Longrightarrow \cos\theta_j = \sqrt{n^2-\sin^2\theta_i}/n$

$= t\left[1-n+\frac{n^2}{\sqrt{n^2-\sin^2\theta_i}} - \cos\theta_i + \frac{-\sin^2\theta_i}{\sqrt{n^2-\sin^2\theta_i}}\right] = \frac{N\lambda}{2} \Longleftrightarrow$

$\Longleftrightarrow \frac{N\lambda}{2t} = 1-n-\cos\theta_i + \frac{n^2-\sin^2\theta_i}{\sqrt{n^2-\sin^2\theta_i}} = 1-n-\cos\theta_i + \sqrt{n^2-\sin^2\theta_i} \Longleftrightarrow \left(\sqrt{n^2-\sin^2\theta_i}\right)^2 = \left(\frac{N\lambda}{2t} - 1 + n + \cos\theta_i\right)^2 \Longleftrightarrow$

$\Longleftrightarrow n^2 - \sin^2\theta_i = \cancel{\frac{N^2\lambda^2}{4t^2}} - \frac{N\lambda}{\cancel{2}t} + \frac{nN\lambda}{\cancel{2}t} + \frac{N\lambda}{\cancel{2}t}\cos\theta_i - \frac{N\lambda}{\cancel{2}t} + 1 - n - \cos\theta_i + \frac{N\lambda}{\cancel{2}t} - n + n^2 + n\cos\theta_i + \frac{N\lambda}{\cancel{2}t}\cos\theta_i - \cos\theta_i + n\cos\theta_i + \cos^2\theta_i \Longleftrightarrow$

$\lambda \ll 1 \Rightarrow \lambda^2 \lll 1 \Rightarrow \frac{N^2\lambda^2}{4t^2} \approx 0$

$\Longleftrightarrow \cancel{n^2} - \cancel{\sin^2\theta_i} = -\frac{N\lambda}{t} + n\frac{N\lambda}{t} + \frac{N\lambda}{t}\cos\theta_i + \cancel{1} - 2n - 2\cos\theta_i + \cancel{n^2} + 2n\cos\theta_i + \cancel{\cos^2\theta_i} \Longleftrightarrow$

$\Longleftrightarrow -2n - 2\cos\theta_i + 2n\cos\theta_i + \frac{N\lambda}{t}\left(n + \cos\theta_i - 1\right) = 0 \Longleftrightarrow \left(-2n - 2\cos\theta_i + 2n\cos\theta_i\right)t = -N\lambda\left(n + \cos\theta_i - 1\right) \Longleftrightarrow$

$\Longleftrightarrow -2nt + 2nt\cos\theta_i + N\lambda n = -N\lambda\cos\theta_i + N\lambda + 2t\cos\theta_i \Longleftrightarrow 2nt - 2nt\cos\theta_i - N\lambda n = N\lambda\cos\theta_i - N\lambda - 2t\cos\theta_i \Longleftrightarrow$

$\Longleftrightarrow n\left(2t\left(1-\cos\theta_i\right) - N\lambda\right) = \left(2t - N\lambda\right)\left(1-\cos\theta_i\right) \Longleftrightarrow \boxed{n = \frac{(2t-N\lambda)(1-\cos\theta_i)}{2t\left[1-\cos\theta_i\right] - N\lambda}}$ $\square$

1.4 Twyman-Green interferometer

What do you expect the interference to be if a collimated laser beam is used instead of a diverging one? Use the Astronomical Telescope configuration to produce a collimated enlarged laser beam. In this case the Michelson Interferometer is known as Twyman-Green interferometer.

Task:

Use a microscope objective for the short focal length lens (40x or 60x). Is the interference pattern consistent with what you expected? Describe the interference pattern for parallel and tilted mirrors.

It's important to ensure that all optical components are aligned correctly to observe the expected interference patterns. Misalignment can lead to distorted or unexpected fringe patterns. In a Twyman-Green interferometer, which is essentially a modified version of the Michelson interferometer designed for testing optical components, using a collimated laser beam instead of a diverging one would result in a different interference pattern.

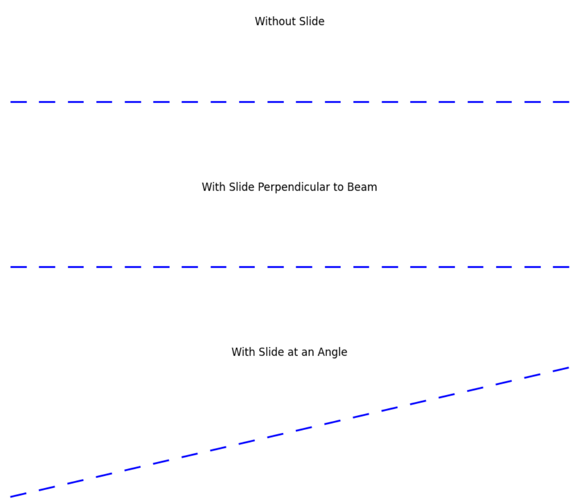
Our case was that of a collimated laser, amplified in size by the microscope objective. The interference fringes were sharp and well-defined, which behaved as expected because a collimated beam maintains the same beam diameter over distance, leading to more uniform wavefronts interacting at the detector.

For parallel mirrors, the pattern was a series of straight, parallel fringes. For the tilted mirror, the fringes were hyperbolic or circular patterns, depending on the degree of tilt and the alignment of the optical components. This behaviour corresponds with the theoretical expectations for collimated laser beams.

Task:

Now insert a microscope slide in the path of the beam in the side of the moving mirror. Describe the resultant fringe pattern (Draw it). Explain the changes in the interference pattern if any.

When you insert a microscope slide into the path of one of the beams in a Twyman-Green interferometer, it introduces an additional OPD due to the change in the speed of light as it passes through the glass slide.

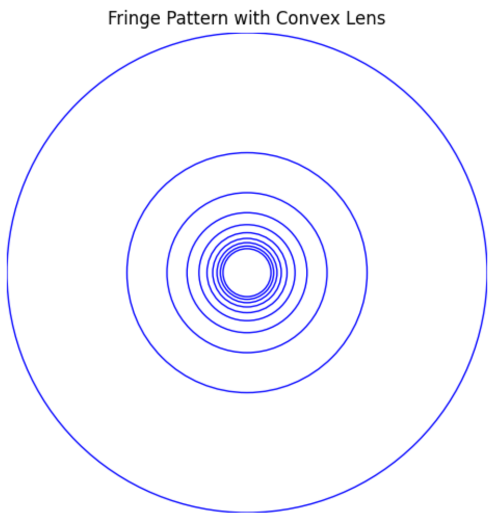


The fringe pattern will shift because the beam passing through the microscope slide will travel slower than the beam in the air, causing a phase shift. The number of fringes that shift corresponds to the additional OPD introduced by the glass slide. If the slide is inserted perpendicular to the beam, the fringes will shift uniformly. However, if it is inserted in an angle, a distortion will appear.

Task:

Insert a convex lens in the path of the beam in the side of the moving mirror. Describe the resultant fringe pattern (Draw it).

Inserting a convex lens into the path of one of the beams in a Twyman-Green interferometer will cause the light waves to converge.



The fringe pattern will change because the convex lens will focus the light to a point, altering the interference. Near the focal point of the lens, the fringes will be circles centered around the lens axis. As we moved away from the focal point, the fringes became more spaced out and eventually reassembled straight lines again when the beams became parallel.

1.5 Determine the difference between the wavelengths of the sodium light doublet

Use the sodium light source to observe fringes on the screen. Since the yellow doublet of sodium has wavelengths very close to each other, 588.995nm and 589.592nm, you should see two sets of fringes. As you move the mirror you will notice that the visibility of the fringes changes. At certain mirror separation you should see sharp or fuzzy fringes. The sharp fringes occur when the bright fringes from each wavelength coincide, and the fuzzy fringes occur when the bright fringes from one wavelength coincide with dark fringes of the other wavelength.

Task:

Use the micrometer to move the mirror and find the distance moved to go from fuzzy fringes to the next set of fuzzy fringes or from sharp bright fringes to the next set of sharp bright fringes. Repeat this procedure 5 times and record your data. find the average distance and its uncertainty.

We recorded the position at which three minimums occured starting from one at zero. Then we find the distance between each position and we average it. From the data we obtained an average distance of $274 \pm 1 \cdot 10^{-6} m$

Task:

Show that the relationship that relates the distance, the average wavelength, and the wavelength difference is given by $\Delta\lambda = \lambda^2/2d$.

The condition for constructive interference (sharp fringes) for both wavelengths can be expressed as:

$$m_1\lambda_1 = m_2\lambda_2$$

But since the wavelengths are very close, m_1 and m_2 will be nearly the same number also. The path difference d for one fringe shift:

$$d = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}$$

The average wavelength can be expressed as:

$$\lambda = \frac{\lambda_1 + \lambda_2}{2} = \frac{588.995 + 589.582}{2} = 589.2885 \; nm$$

Then substituting and taking in account that $\Delta\lambda = \lambda_2 - \lambda_1$:

$$d = \frac{2\lambda^2}{\lambda_2 - \lambda_1} \Longleftrightarrow \Delta\lambda = \frac{\lambda^2}{2d}$$

Task:

From your data find the wavelength difference between the yellow doublet of sodium. Find the uncertainty in the wavelength difference. Compare your experimental result to the known value of the wavelength difference. Find the %error in the wavelength difference and explain.

$$\Delta\lambda = \frac{\lambda^2}{2d} = \frac{(589.2885 \cdot 10^{-9})^2}{2 \cdot (274 \pm 1) \cdot 10^{-6}} = 6.34 \cdot 10^{-10} = 0.634 \; nm$$

Meanwhile, the known value is:

$$|588.995 - 589.592| = 0.597 \; nm$$

Meaning that the error:

$$|0.597 - 0.634| = 0.037 \qquad \frac{0.037}{0.597} = 0.0619 \Longrightarrow 6.2\% \; error$$

2 Addenda

LaTeX code that generates this document

PHOTONICS-LabRep4:interferometer.tex

Python code that generates the plots and contains the data

PHOT-E4.py