$y'y'' = y'''$

Sea $t(x) = y'(x) \to t\,t' = t'' \iff \int t\,t'\,dx = \int t''\,dx \iff \frac{1}{2}t^2 + C_1 = t' \overset{vs}{\iff} \frac{dt}{dx} = \frac{1}{2}t^2 + C_1 \iff$

$\iff \frac{dt}{dx} = \frac{1}{2}(t^2 + C_2) \iff \int \frac{1}{t^2 + C_2}\,dt = \frac{1}{2}\int dx \longrightarrow$ <u>3 casos, $C_2 < 0$ $C_2 = 0$ $C_2 > 0$</u>

1 $\boxed{C_2 = 0}$

$$\int \frac{1}{t^2}\,dt = \frac{1}{2}\int dx \iff -\frac{1}{t} = \frac{1}{2}x + C_3 \iff \frac{1}{t} = -\frac{C_4 + x}{2} \iff t(x) = \frac{-2}{(x + C_4)} \iff$$

$$\iff \frac{dy}{dx} = \frac{-2}{x + C_4} \iff y = \int \frac{-2}{x + C_4}\,dx \iff \boxed{y = -2\log|x + C_4| + C_5}$$

$y' = -\frac{2}{x+a}$ $\quad y'' = \frac{2}{(x+a)^2}$ $\quad y''' = -\frac{4}{(x+a)^3}$ $\qquad -\frac{2}{x+a}\cdot\frac{2}{(x+a)^2} = -\frac{4}{(x+a)^3}$ ✓

• Si $t^2 = 0 \iff t = 0 \iff y' = 0 \iff \boxed{y = x + C_6}$ $\quad y' = 1$ $\quad y'' = 0$ $\quad y''' = 0$ $\qquad 1 \cdot 0 = 0$ ✓

2 $\boxed{a = C_2 > 0}$ sea $a \in \mathbb{R} : a > 0$

$y' = \frac{-2}{x + C_8}$ $\quad y'' = \frac{2}{(x + C_8)^2}$ $\quad y''' = -\frac{4}{(x + C_8)^3}$ $\qquad \frac{-2}{(x+C_8)}\cdot\frac{2}{(x+C_8)^2} = \frac{-4}{(x+C_8)^3}$ ✓

$$\int \frac{1}{t^2 + a}\,dt = \frac{1}{2}\int dx \iff \sqrt{a}\int \frac{\sec^2(u)}{a\tan^2(u) + a}\,du = \frac{\sqrt{a}}{a}\int \frac{\sec^2(u)\,du}{\sec^2(u)} = \frac{1}{\sqrt{a}}\arctan\left(\frac{t}{\sqrt{a}}\right) = \frac{1}{2}x + C_7$$

$\sec^2(x) - 1 = \tan^2(x)$

$\sec^2(x) = \tan^2(x) + 1$

$s \cdot \sec^2(x) = s \cdot (\tan^2(x) + 1)$

$$\iff \sqrt{a}\tan\left(\frac{\sqrt{a}(x + C_8)}{2}\right) = t(x) \iff \int dy = \sqrt{a}\int \tan\left(\frac{x\sqrt{a} + C_8}{2}\right)dx = \boxed{-2\log\left|\cos\left(\frac{\sqrt{a}(x + C_8)}{2}\right)\right| + C_9 = y(x)}$$

$\underbrace{\{a > 0\}}$

$t = \sqrt{a}\tan(u) \Rightarrow u = \arctan\left(\frac{t}{a}\right)$ $\qquad$ • Si $t^2 + a = 0 \longrightarrow t^2 = -a$ sin sol. en $\mathbb{R}$

$dt = \sqrt{a}\sec^2(u)\,du$

3 $\boxed{b = C_2 < 0}$ sea $b \in \mathbb{R} : b < 0$ tq $b = -a$

• Si $t^2 + b = 0 \iff t^2 - a = 0 \iff t^2 = a \iff t = \pm a \iff \frac{dy}{dx} = a \iff \int dy = \int a\,dx \iff \boxed{y = ax + C_{10}}$

$\frac{dy}{dx} = -a \iff \int dy = -\int a\,dx \iff \boxed{y = -ax + C_{11}}$

$y' = a$ $\quad y'' = 0$ $\quad y''' = 0$ $\qquad a \cdot 0 = 0$ ✓ $\quad -a \cdot 0 = 0$ ✓

$$\int \frac{1}{t^2 + b}\,dt = \int \frac{1}{t^2 - a}\,dt = \frac{1}{2}\int dx \iff$$

$\sec^2 x - 1 = \tan^2 x$

$t = \sqrt{a}\sec u \to u = \operatorname{arcsec}\left(\frac{t}{\sqrt{a}}\right)$

$dt = \sqrt{a}\sec u \tan u\,du$

$$\iff \int \frac{a\sec u \tan u}{a(\sec^2 u - 1)}\,du = \int \frac{\sec u \tan u}{\tan^2 u}\,du = \int \frac{\sec u}{\tan u}\,du = \frac{1}{2}\int dx \iff$$

$$\iff \int \frac{1/\cos u}{\sin u/\cos u}\,du = \int \frac{\cos u}{\cos u \sin u}\,du = \int \csc u\,du = -\ln|\csc(u) + \cot(u)| = \frac{x}{2} + C_{12} \iff$$

$$\iff -\log\left|\csc\left(\operatorname{arcsec}\left(\frac{t}{\sqrt{a}}\right)\right) + \cot\left(\operatorname{arcsec}\left(\frac{t}{\sqrt{a}}\right)\right)\right| = \frac{x}{2} + C \iff -\log\left|\frac{\sqrt{a} + t}{t\sqrt{1 - \frac{a}{t^2}}}\right| = C_{12} + \frac{x}{2} \iff \frac{t\sqrt{1 - \frac{a}{t^2}}}{\sqrt{a} + t} = e^{C_{12}}e^{\frac{x}{2}}$$

$$\iff \frac{t\sqrt{1 - \frac{a}{t^2}}}{\sqrt{a} + t} = C_{13}e^{\frac{x}{2}} \iff \frac{t^2\left(1 - \frac{a}{t^2}\right)}{(\sqrt{a} + t)^2} = C_{13}^2 e^x \iff \frac{t^2 - a}{(\sqrt{a} + t)^2} = C_{14}e^x \iff \frac{\cancel{(t + \sqrt{a})}(t - \sqrt{a})}{(\sqrt{a} + t)^{\cancel{2}}} = C_{14}e^x$$

$$\iff \frac{t - \sqrt{a}}{\sqrt{a} + t} = C_{14}e^x \iff t - \sqrt{a} = C_{14}e^x(\sqrt{a} + t) \iff t(1 - C_{14}e^x) = \sqrt{a}(1 + C_{14}e^x) \iff t = \sqrt{a}\,\frac{1 + C_{14}e^x}{1 - C_{14}e^x}$$

$$\longrightarrow \int dy = \sqrt{a}\int \frac{1 + C_{14}e^x}{1 - C_{14}e^x}\,dx \iff \boxed{y = -2\sqrt{a}\log\left|\frac{-C_{14}}{e^x}\right| + C_{15}}$$ Con $C_{14} < 0$

$y' = 2\sqrt{a}$ $\quad y'' = 0$ $\quad y''' = 0$ $\qquad 2\sqrt{a} \cdot 0 = 0$ ✓

$y' = -\frac{\sqrt{a}(ce^x - 1)}{ce^x + 1}$ $\quad y'' = \frac{-2\sqrt{a}ce^x}{(ce^x + 1)^2}$ $\quad y''' = \frac{2\sqrt{a}ce^x(ce^x - 1)}{(ce^x + 1)^3}$

$$\iff \boxed{y = -2\sqrt{a}\log\left|\frac{1}{e^x} + C_{14}\right| - \sqrt{a}\,x + C_{16}}$$ Con $C_{14} < 0$

$\sqrt{a}(ce^x - 1) \cdot 2\sqrt{a}ce^x = 2ace^x(ce^x - 1)$

$$\iff \boxed{y = \sqrt{a}\,x + C_{17}}$$ Con $C_{14} = 0$ ✓ (caso particular de otra)

¿ ~ ✓ ?