1.- $P(-2,-2,3)$ respecto $P'(-3,1,4)$ $\bar{v}$ en dirección PP'. Cosenos directores.

$$\begin{cases} \cos\alpha = \frac{U_x}{|U|} \\ \cos\beta = \frac{U_y}{|U|} \\ \cos\varphi = \frac{U_z}{|U|} \end{cases} \qquad \bar{U} = \frac{1}{\sqrt{11}}(1,-3,-1)$$

$$P'P = P - P' = (-2,-2,3) - (-3,1,4) = (1,-3,-1) \qquad |\bar{U}| = \sqrt{1^2 + (-3)^2 + (-1)^2} = \sqrt{11}$$

$$\cos\alpha = \frac{\pm 1}{\sqrt{11}} \quad \cos\beta = \frac{-3}{\sqrt{11}} \quad \cos\varphi = \frac{-1}{\sqrt{11}} \rightarrow \left(\frac{\pm 1}{\sqrt{11}}\right)^2 = \frac{1}{11} \quad \left(\frac{3}{\sqrt{11}}\right)^2 = \frac{9}{\sqrt{11}} \quad \cos^2\alpha + \cos^2\beta + \cos^2\varphi = \frac{1}{11} + \frac{9}{11} + \frac{1}{11} = 1$$

2.- $\bar{A} = 2\hat{x} + 3\hat{y} - 4\hat{z}$, $\bar{B} = -6\hat{x} - 4\hat{y} + \hat{z}$. Encontrar la componente del vector $\bar{A}\times\bar{B}$ en la dirección $\bar{C} = \hat{x} - \hat{y} + \hat{z}$

$$\bar{C} \equiv (1,-1,1) \equiv \bar{v} \qquad \bar{A} \equiv (2,3,-4) \qquad \bar{B} \equiv (-6,-4,1) \qquad \bar{A}\times\bar{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & 3 & -4 \\ -6 & -4 & -1 \end{vmatrix} = -13\hat{x} + 22\hat{y} + 10\hat{z} \equiv \bar{U} \equiv (-13,22,10)$$

$$\text{Proyección de}_v U = \frac{\bar{U}\cdot\bar{V}}{|\bar{V}|^2}\bar{V} \iff P_{\bar{C}}\bar{A} = \frac{(-13,22,10)\cdot(1,-1,1)}{1^2 + (-1)^2 + 1^2}(-13,22,10) = -\frac{25}{\sqrt{3}}(-13,22,10)$$

3.- Sean dos sistemas de referencia R_1 y R_2, que tienen el mismo origen O y ejes de coordenadas X_1, X_2, X_3 y x_1', x_2', x_3' respectivamente. Sabiendo que el ángulo que forman los ejes X_1 y x_1' es de $30°$, encontrar la matriz de transformación para el cambio de un sistema de coordenadas al otro. Sea un vector, cuyas componentes respecto al sistema R_1 son $\bar{A} = (1,2,-2)$. Encontrar las componentes del vector en R_2. Determinar su módulo y sus cosenos directores respecto R_1 y R_2.

$$\boxed{R_1} \quad \cos\alpha = \frac{A_x}{|A|} = \frac{1}{3} \quad \cos\beta = \frac{A_y}{|A|} = \frac{2}{3} \quad \cos\varphi = \frac{A_z}{|A|} = \frac{-2}{3} \quad \left(|A_{R_1}| = \sqrt{1^2 + 2^2 + (-2)^2} = 3\right)$$

$$\phi_{x_1 x_1'} = 30° \Rightarrow \cos\phi = \frac{\sqrt{3}}{2} \quad \sin\phi = \frac{1}{2}. \text{ Rotamos sobre } OY \text{ ó } OZ, \text{ elijo } OZ: \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} - 1 \\ \sqrt{3} + \frac{1}{2} \\ -2 \end{pmatrix} = \frac{1}{2}\cdot\begin{pmatrix} \sqrt{3} - 2 \\ 2\sqrt{3} + 1 \\ -4 \end{pmatrix}$$

$\rightarrow$ sentido distinto a la solución pero 30° igualmente

$$|A_{R_2}| = \sqrt{\left(\frac{\sqrt{3}+2}{2}\right)^2 + \left(\frac{2\sqrt{3}+1}{2}\right)^2 + \left(\frac{-4}{2}\right)^2} = 3$$

$$\boxed{R_2} \quad \cos\alpha = \frac{\sqrt{3}/2 - 1}{3} \quad \cos\beta = \frac{\sqrt{3} + 1/2}{3} \quad \cos\varphi = -\frac{2}{3}$$

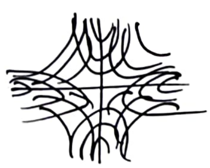
4.- $P(-1,3,2)$ cartesianas ¿Dirección de máximo crecimiento de $\phi(x,y,z) = (x+y)^2 + z^2 - xy + 2z$?

Hemos de evaluar el gradiente en dicho punto y hacer la dirección unitaria.

$$\nabla\phi = \left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}\right) = (2x+y, x+2y, 2(z+1)) \qquad \nabla\phi(-1,3,2) = (1,5,6) \qquad \sqrt{1^2+5^2+6^2} = \sqrt{62} \iff \bar{U}_{\text{máx.crec.}} = \frac{(1,5,6)}{\sqrt{62}}$$

5.- Sea $\phi(x,y) = x^2 - y^2$ un campo, dibuja las líneas de nivel de campo y de gradiente.

$$\nabla\phi = (2x, -2y) \quad \text{Curvas de nivel: } x^2 - y^2 = C \Rightarrow |y| = \sqrt{x^2 + C} \quad \text{Líneas de gradiente: } \frac{dx}{\frac{\partial\phi}{\partial x}} = \frac{dy}{\frac{\partial\phi}{\partial y}} \Rightarrow \frac{dx}{2x} = \frac{dy}{-2y} \Rightarrow |y| = \frac{K}{|x|}$$



6.- Sea la función vectorial $\bar{A} = (3x^2+y)\hat{x} - (\sin x - z)\hat{y} + \alpha\hat{z}$ calcula α para que la divergencia sea nula.

$$\text{Div}\,\bar{A} = \nabla\bar{A} = \frac{\partial A}{\partial x} + \frac{\partial A}{\partial y} + \frac{\partial A}{\partial z} = 6x + \frac{\partial\alpha}{\partial z} = 0 \iff \frac{\partial\alpha}{\partial z} = -6x \iff \int\frac{\partial\alpha}{\partial z}dz = -6\int x\,dz \iff \alpha = -6xz + f(x,y)$$

7.- $r\theta\varphi$

$\bar{A}=(1,1,1)_{esf.} \to ¿\bar{A}_{cart}?$

$\hat{x}=\hat{i}=\sin\theta\cos\varphi\hat{r}+\cos\theta\cos\varphi\hat{\theta}-\sin\varphi\hat{\varphi}$; $\hat{r}=\sin\theta\cos\varphi\hat{x}+\sin\theta\sin\varphi\hat{y}+\cos\theta\hat{z}$

$\hat{y}=\hat{j}=\sin\theta\sin\varphi\hat{r}+\cos\theta\sin\varphi\hat{\theta}+\cos\varphi\hat{\varphi}$; $\hat{\theta}=\cos\theta\cos\varphi\hat{x}+\cos\theta\sin\varphi\hat{y}-\sin\theta\hat{z}$

$\hat{z}=\hat{k}=\cos\theta\hat{r}-\sin\theta\hat{\theta}$; $\hat{\varphi}=-\sin\varphi\hat{x}+\cos\varphi\hat{y}$

$\begin{cases} r=1 \\ \theta=1\text{ rad} \\ \varphi=1\text{ rad}\end{cases}$

$$1\hat{r}+1\hat{\theta}+1\hat{\varphi}=\sin\theta\cos\varphi\hat{x}+\sin\theta\sin\varphi\hat{y}+\cos\theta\hat{z}+\cos\theta\cos\varphi\hat{x}+\cos\theta\sin\varphi\hat{y}-\sin\theta\hat{z}-\sin\varphi\hat{x}+\cos\varphi\hat{y}=$$
$$=(\sin\theta\cos\varphi+\cos\theta\cos\varphi-\sin\varphi)\hat{x}+(\sin\theta\sin\varphi+\cos\theta\sin\varphi+\cos\varphi)\hat{y}+(\cos\theta-\sin\theta)\hat{z}=-0'095\hat{x}+1'7\hat{y}-0'3\hat{z}$$

$\bar{B}=(-1,3,2)_{cil.} \to ¿\bar{B}_{cart}?$

$\hat{\rho}=\cos\varphi\hat{x}+\sin\varphi\hat{y}$

$\hat{\varphi}=-\sin\varphi\hat{x}+\cos\varphi\hat{y}$

$\hat{z}=\hat{z}$

$\begin{cases}\rho=1\\ \varphi=3\\ z=2\end{cases}$

$$-1\hat{\rho}+3\hat{\varphi}+2\hat{z}=-\cos\varphi\hat{x}-\sin\varphi\hat{y}-3\sin\varphi\hat{x}+3\cos\varphi\hat{y}+2\hat{z}=(-\cos\varphi-3\sin\varphi)\hat{x}+(-\sin\varphi+3\cos\varphi)\hat{y}+2\hat{z}=$$
$$=0'567\hat{x}-3'111\hat{y}+2\hat{z}$$

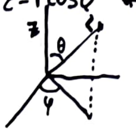
8.- $\Phi(r,\varphi,\theta)=r^3\sin\theta\tan\varphi \to ¿\Phi(\rho,\varphi,z),\ \Phi(x,y,z)?$

$\begin{cases} r=(x^2+y^2+z^2)^{1/2} \\ \tan\theta=\frac{(x^2+y^2)^{1/2}}{z} \\ \tan\varphi=\frac{y}{x}\end{cases}$ $\begin{cases} x=\rho\cos\varphi \\ y=\rho\sin\varphi \\ z=z\end{cases}$

$$\Phi(x,y,z)=(x^2+y^2+z^2)^{3/2}\cdot\frac{z}{(x^2+y^2+z^2)^{1/2}}\cdot\frac{(x^2+y^2)^{1/2}}{z}\cdot\frac{y}{x}=\frac{y}{x}(x^2+y^2+z^2)^3\cdot(x^2+y^2)^{1/2}$$

$$\Phi(\rho,\varphi,z)=\frac{\rho\sin\varphi}{\rho\cos\varphi}(\rho^2\cos^2\varphi+\rho^2\sin^2\varphi+z^2)^3\cdot\rho(\cos^2\varphi+\sin^2\varphi)^{1/2}=\tan\varphi(\rho^2+z^2)^3\rho$$

$z=r\cos\theta \iff \cos\theta=\frac{z}{r}$



$\sin\theta=\cos\theta\cdot\frac{\sin\theta}{\cos\theta}=\cos\theta\tan\theta$

9.- $\Phi(r,\varphi,\theta)=r^2\sin^2\theta\cos^3\varphi$, $¿\bar\nabla\Phi,\ \bar\nabla\cdot(\bar\nabla\Phi)=\Delta\Phi?$

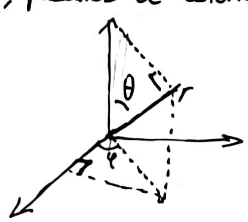
$\begin{cases} r=(x^2+y^2+z^2)^{1/2} \\ \tan\theta=\frac{1}{z}(x^2+y^2)^{1/2} \\ \tan\varphi=\frac{y}{x}\end{cases}$

$z=r\cos\theta \iff \cos\theta=\frac{z}{r} \iff \sin\theta=\cos\theta\tan\theta=\frac{z\tan\theta}{r}=\frac{(x^2+y^2)^{1/2}}{r}$

$\sin\varphi=\frac{y}{r\sin\theta} \iff \sin\varphi=\frac{y}{r\cos\theta\tan\theta}=\frac{y}{z\tan\theta}=\frac{y}{(x^2+y^2)^{1/2}}$

$\cos\varphi=\frac{1}{\frac{r}{\sin\varphi}\cdot\frac{\sin\varphi}{\cos\varphi}}=\frac{1}{\frac{(x^2+y^2)^{1/2}}{y}\cdot\frac{(x^2+y^2)^{1/2}}{z}}=\frac{1}{\frac{x^2+y^2}{yz}}=\frac{yz}{x^2+y^2}$

Primero, pasamos de esféricas a cartesianas:



$$\Phi(x,y,z)=r^2\frac{x^2+y^2}{r^2}\frac{y^3z^3}{(x^2+y^2)^3}=$$
$$=\frac{y^3z^3}{(x^2+y^2)^2}$$

$$\bar\nabla\Phi=\left(\frac{\partial\Phi}{\partial x},\frac{\partial\Phi}{\partial y},\frac{\partial\Phi}{\partial z}\right)=\left(-\frac{4xy^3z^3}{(x^2+y^2)^3},\frac{z^3(3x^2y^2-y^4)}{(x^2+y^2)^3},\frac{y^3z^3}{(x^2+y^2)^2}\right)$$

$$\Delta\Phi=\bar\nabla\cdot(\bar\nabla\Phi)=\bar\nabla\cdot\left(-\frac{4xy^3z^3}{(x^2+y^2)^3},\frac{z^3(3x^2y^2-y^4)}{(x^2+y^2)^3},\frac{y^3z^3}{(x^2+y^2)^2}\right)=\frac{\partial^2\Phi}{\partial x^2}(x,y,z)+\frac{\partial^2\Phi}{\partial y^2}(x,y,z)+\frac{\partial^2\Phi}{\partial z^2}(x,y,z)=\frac{-4y^3(-5x^2+y^2)z^3}{(x^2+y^2)^4}+\frac{2(3x^4y-8x^2y^3+y^5)z^3}{(x^2+y^2)^4}+\frac{6y^3z}{(x^2+y^2)^2}$$

10.- $\bar{A}=(e^\rho\cos\varphi,\ z\sin\varphi,\ \rho^2)$ ¿divergencia y rotacional $(\bar\nabla\cdot\bar{A},\ \bar\nabla\times\bar{A})$?

$$\bar\nabla\cdot\bar{A}=\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho)+\frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi}+\frac{\partial A_z}{\partial z}=\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho e^\rho\cos\varphi)+\frac{1}{\rho}\frac{\partial}{\partial\varphi}(z\sin\varphi)+\frac{\partial}{\partial z}(\rho^2)=\frac{1}{\rho}e^\rho\cos\varphi(\rho+1)+\frac{1}{\rho}z\cos\varphi=\frac{\cos\varphi}{\rho}\left(e^\rho(\rho+1)+z\right)$$

$$\bar\nabla\times\bar{A}=\left(\frac{1}{\rho}\frac{\partial A_z}{\partial\varphi}-\frac{\partial A_\varphi}{\partial z}\right)\hat\rho+\left(\frac{\partial A_\rho}{\partial z}-\frac{\partial A_z}{\partial\rho}\right)\hat\varphi+\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\varphi)-\frac{1}{\rho}\frac{\partial A_\rho}{\partial\varphi}\right)\hat{z}=-\sin\varphi\hat\rho-2\rho\hat\varphi+\left(\frac{1}{\rho}z\sin\varphi+e^\rho\sin\varphi\right)\hat{z}$$

$\begin{cases}\rho=(x^2+y^2)^{1/2}\\ \tan\varphi=\frac{y}{x}\\ z=z\end{cases}$ $\begin{cases}x=\rho\cos\varphi\\ y=\rho\sin\varphi\\ z=z\end{cases}$

$\sin\varphi=\cos\varphi\tan\varphi=\frac{x}{(x^2+y^2)^{1/2}}\cdot\frac{y}{x}=\frac{y}{(x^2+y^2)^{1/2}}$

$\cos\varphi=\frac{x}{\rho}=\frac{x}{(x^2+y^2)^{1/2}}$

$$\bar{A}=\left(e^{(x^2+y^2)^{1/2}}\frac{x}{(x^2+y^2)^{1/2}},\ \frac{zy}{(x^2+y^2)^{1/2}},\ x^2+y^2\right)$$

$$\bar\nabla\cdot\bar{A}=\frac{\partial A_1}{\partial x}+\frac{\partial A_2}{\partial y}+\frac{\partial A_3}{\partial z}=\frac{1}{(x^2+y^2)^{3/2}}\left(e^{(x^2+y^2)^{1/2}}\left(y^2+x^2(x^2+y^2)^{1/2}\right)\right)+\frac{x^2z}{(x^2+y^2)^{3/2}}=\frac{1}{\rho^3}\left(e^\rho(\rho^2\sin^2\varphi+\rho^3\cos^2\varphi)\right)+\frac{\rho^2\cos^2\varphi\, z}{\rho^3}=\frac{1}{\rho}e^\rho(1+\rho)+\frac{1}{\rho}\cos^2\varphi\, z$$

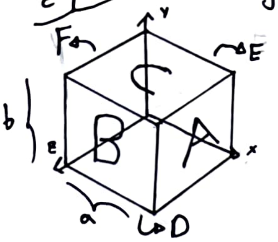
11.- Familia de elipsoides $U=\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}$ ¿Unitario normal en cada punto?

$$\bar{\nabla} U=\frac{\partial U}{\partial x}\hat{x}+\frac{\partial U}{\partial y}\hat{y}+\frac{\partial U}{\partial z}\hat{z}=\frac{2x}{a^2}\hat{x}+\frac{2y}{b^2}\hat{y}+\frac{2z}{c^2}\hat{z}=\left(\frac{2x}{a^2},\frac{2y}{b^2},\frac{2z}{c^2}\right)$$

$$\frac{\bar{\nabla} U}{|\bar{\nabla} U|}=\frac{2\left(\frac{x}{a^2},\frac{y}{b^2},\frac{z}{c^2}\right)}{2\left(\frac{x^2}{a^4}+\frac{y^2}{b^4}+\frac{z^2}{c^4}\right)^{1/2}}=\left(\frac{x}{a^2},\frac{y}{b^2},\frac{z}{c^2}\right)\left(\frac{x^2}{a^4}+\frac{y^2}{b^4}+\frac{z^2}{c^4}\right)^{-1/2}$$

$$|\bar{\nabla} U|=\sqrt{\frac{4x^2}{a^4}+\frac{4y^2}{b^2}+\frac{4z^2}{c^2}}=2\left(\frac{x^2}{a^4}+\frac{y^2}{b^4}+\frac{z^2}{c^4}\right)^{1/2}$$

12.- Dado el campo vectorial $\bar{A}=xy\hat{x}+yz\hat{y}+zx\hat{z}$, evaluar el flujo del vector a través de la superficie de un paralelepípedo rectangular de lados a, b, c con uno de los vértices coincidente con el origen de coordenadas y las aristas paralelas a las direcciones positivas de los ejes rectangulares. Evaluar la integral de volumen de la divergencia del vector. Discute los resultados.



T^{ma} Divergencia $\int_S \bar{A}\,d\bar{a}=\phi=\sum_{i=1}^{6}\phi_i=\frac{a^2bc}{2}+\frac{ab^2c}{2}+\frac{abc^2}{2}=\frac{1}{2}abc(a+b+c)$

$$Div(\bar{A})=\bar{\nabla}\cdot\bar{A}=\frac{\partial \bar{A}\hat{x}}{\partial x}+\frac{\partial \bar{A}\hat{y}}{\partial y}+\frac{\partial \bar{A}\hat{z}}{\partial z}=y+z+x$$

$$\int_V Div(\bar{A})\,dV=\int_V \bar{\nabla}\cdot\bar{A}\,dV=\int_0^a\int_0^b\int_0^c (x+y+z)\,dz\,dy\,dx=\frac{1}{2}abc(a+b+c)$$

$$\phi_1=\int_A \bar{A}\,d\bar{a}=\int_A \bar{A}\hat{x}\,da=\int_A \bar{A}\hat{x}\,dy\,dz=\int_A xy\,dy\,dz=\int_0^c\int_0^b xy\,dy\,dz\left(=x\int_0^c\int_0^b y\,dy\,dz\right)=a\int_0^c\frac{b^2}{2}dz=\frac{ab^2c}{2}$$

$\hookrightarrow d\bar{a}=da\hat{x}=dy\,dz\,\hat{x}$

$$\phi_2=\int_B \bar{A}\,d\bar{a}=\int_B \bar{A}\hat{z}\,da=\int_B \bar{A}\hat{z}\,dx\,dy=\int_B xz\,dx\,dy=\int_0^b\int_0^a xz\,dx\,dy\left(=z\int_0^b\int_0^a x\,dx\,dy\right)=c\int_0^b\frac{a^2}{2}dy=\frac{a^2bc}{2}$$

$\hookrightarrow d\bar{a}=da\hat{z}=dx\,dy\,\hat{z}$

$$\phi_3=\int_C \bar{A}\,d\bar{a}=\int_C \bar{A}\hat{y}\,da=\int_C \bar{A}\hat{y}\,dx\,dz=\int_C yz\,dx\,dz=\int_0^a\int_0^c yz\,dz\,dx\left(=y\int_0^a\frac{c^2}{2}dx\right)=\frac{abc^2}{2}$$

$\hookrightarrow d\bar{a}=da\hat{y}=dx\,dz\,\hat{y}$

$$\phi_4=\int_D \bar{A}\,d\bar{a}=\int_D -\bar{A}\hat{y}\,dx\,dz=-\int_C yz\,dx\,dz=-\int_0^c\int_0^a yz\,dx\,dz\left(=-y\int_0^c\frac{a^2}{2}dx\right)=0$$

$\hookrightarrow d\bar{a}=da(-\hat{y})=-dx\,dz\,\hat{y}$ $\quad y=0$

$$\phi_5=\int_E \bar{A}\,d\bar{a}=\int_E -\bar{A}\hat{z}\,dx\,dy=-\int_D xz\,dx\,dy=-\int_0^b\int_0^a xz\,dx\,dy\left(=-z\int_0^b\frac{a^2}{2}dy\right)=0$$

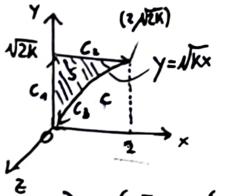
$\hookrightarrow d\bar{a}=da(-\hat{z})=dx\,dy(-\hat{z})$ $\quad z=0$

$$\phi_6=\int_F \bar{A}\,d\bar{a}=\int_F -\bar{A}\hat{x}\,dy\,dz=-\int_F xy\,dy\,dz=-\int_0^c\int_0^b xy\,dy\,dz\left(=-x\int_0^c\frac{b^2}{2}dz\right)=0$$

$\hookrightarrow d\bar{a}=da(-\hat{x})=dy\,dz(-\hat{x})$ $\quad x=0$

$$\int_0^a\int_0^b\int_0^c (x+y+z)\,dz\,dy\,dx=\int_0^a\int_0^b \frac{c^2}{2}+(x+y)c\,dy\,dx=\int_0^a \frac{bc^2}{2}+xbc+\frac{b^2c}{2}\,dx=\frac{abc^2}{2}+\frac{a^2bc}{2}+\frac{ab^2c}{2}=\frac{1}{2}abc(a+b+c)$$

(13.-) Dado el campo vectorial $\bar{A}=x^2y\hat{i}+xy^2\hat{j}+a^3e^{-\beta r}\cos(\alpha x)\hat{k}$ donde a, α y β son constantes. Evaluar la integral de línea del campo vectorial sobre la trayectoria cerrada C en el plano XY de la figura. Los tramos rectos de la trayectoria son paralelos al eje XY y el tramo curvo es la parábola $y^2=Kx$, donde K es una constante. Evaluar la integral de superficie del rotacional del campo vectorial indicado sobre la superficie S encerrada dentro de C. Discutir los resultados.



T^{ma} Stokes $\oint_C \bar{A}\,d\bar{s} = \int_S \bar{\nabla}\times\bar{A}\,d\bar{a}$ $\quad \gamma=\sum_{i=1}^{3}\gamma_i=\oint_c \bar{A}\,d\bar{s} = \frac{8\sqrt{2K}}{3}-\frac{16\sqrt{2K}}{7}-\frac{4K\sqrt{2K}}{5}$ ✓

~~Superficie S = (Área ¼ círculo de radio 2 → ¼π2² = π ; Área cuadrado de lados 2, √2K = 2√2K) S = 2√2K − π~~

$$\gamma_1=\int_{C_1}\bar{A}\,d\bar{s}_1=\int_{C_1}xy^2dy\left(=x\int_0^{\sqrt{2K}}y^2dy\right)=0 \qquad \gamma_2=\int_{C_2}\bar{A}\,d\bar{s}_2=\int_{C_2}x^2y\,dx\left(=y\int_0^{2}x^2dx\right)=\sqrt{2K}\,\frac{8}{3}$$

$\hookrightarrow d\bar{s}_1=dy\hat{y}$ $\qquad \hookrightarrow d\bar{s}_2=dx\hat{x}$

$$\gamma_3=\int_{C_3}\bar{A}\,d\bar{s}_3=\int_{C_3}x^2y\,dx+xy^2dy=\int_{C_3}\left(x^2y+xy^2k\frac{1}{2\sqrt{Kx}}\right)dx=\int_2^0\left(x^2\sqrt{Kx}+\frac{Kx}{2}\sqrt{Kx}\right)dx=\int_2^0\sqrt{Kx^5}\,dx+\frac{K}{2}\int_2^0\sqrt{Kx^3}\,dx=\sqrt{K}\int_2^0x^{\frac{5}{2}}dx+\frac{K\sqrt{K}}{2}\int_2^0x^{\frac{3}{2}}dx=$$

$\hookrightarrow d\bar{s}_3=dx\hat{x}+dy\hat{y}$ $\qquad \hookrightarrow y=\sqrt{Kx}\Leftrightarrow dy=\frac{K}{2\sqrt{Kx}}dx$

$$\left.\frac{2\sqrt{K}x^{\frac{7}{2}}}{7}\right]_2^0+\frac{K\sqrt{K}}{2}\left.x^{\frac{5}{2}}\frac{2}{5}\right]_2^0=-\left(\left.\frac{2\sqrt{K}x^{\frac{7}{2}}}{7}+\frac{K\sqrt{K}}{5}x^{\frac{5}{2}}\right]_0^2\right)=\frac{-16\sqrt{2K}}{7}-\frac{4K\sqrt{2K}}{5}$$

$$\bar{\nabla}\times\bar{A}=\begin{vmatrix}\hat{x}&\hat{y}&\hat{z}\\ \frac{\partial}{\partial x}&\frac{\partial}{\partial y}&\frac{\partial}{\partial z}\\ A_x&A_y&A_z\end{vmatrix}=\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)\hat{x}+\left(\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}\right)\hat{y}+\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)\hat{z}=a^3\alpha e^{-\beta r}\sin(\alpha x)\hat{y}+(y^2-x^2)\hat{z}$$

Hay que integrar primero la y para quitarme los $\sqrt{Kx}$

$$\int_S\bar{\nabla}\times\bar{A}\,d\bar{a}=-\int_S(y^2-x^2)\,dx\,dy=-\int_{\sqrt{Kx}}^{\sqrt{2K}}\int_0^2(y^2-x^2)\,dx\,dy=-\int_{\sqrt{Kx}}^{\sqrt{2K}}2y^2-\frac{8}{3}dy=-2\int_{\sqrt{Kx}}^{\sqrt{2K}}y^2dy+\int_{\sqrt{Kx}}^{\sqrt{2K}}\frac{8}{3}dy=\frac{8\sqrt{2K}}{3}-\frac{8}{3}\sqrt{Kx}-\frac{(2K)^{\frac{3}{2}}}{3}+\frac{2}{3}(Kx)^{\frac{3}{2}}$$

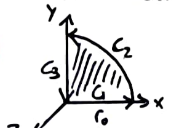
$\hookrightarrow d\bar{a}=da\hat{z}=-dx\,dy\,\hat{z}$

$$\int_S\bar{\nabla}\times\bar{A}\,d\bar{a}=-\int_S(y^2-x^2)\,dx\,dy=\int_S(x^2-y^2)\,dx\,dy=\int_0^2\int_{\sqrt{Kx}}^{\sqrt{2K}}(x^2-y^2)\,dy\,dx=\int_0^2\left[x^2y-\frac{y^3}{3}\right]_{\sqrt{Kx}}^{\sqrt{2K}}dx=\int_0^2\left(x^2\sqrt{2K}-x^2\sqrt{Kx}-\frac{(2K)^{\frac{3}{2}}}{3}+\frac{(Kx)^{\frac{3}{2}}}{3}\right)dx=$$

$\hookrightarrow da\hat{z}=-dx\,dy\,\hat{z}$

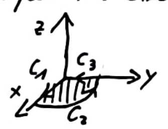
$$=\frac{8\sqrt{2K}}{3}-\frac{16\sqrt{2K}}{7}-\frac{4}{3}\sqrt{2}\,K^{\frac{3}{2}}+\frac{8}{15}\sqrt{2}\,K^{\frac{3}{2}}=\frac{8\sqrt{2K}}{3}-\frac{16\sqrt{2K}}{7}-\frac{4\sqrt{2}\,K^{\frac{3}{2}}}{5}=\frac{8\sqrt{2K}}{3}-\frac{16\sqrt{2K}}{7}-\frac{4K\sqrt{2K}}{5}$$ ✓

(14.-) Dado el vector $\bar{A}=4\hat{r}+3\hat{\theta}-2\hat{\varphi}$ encontrar su integral de línea sobre la trayectoria cerrada de la figura. El tramo curvo es un arco de circunferencia de radio r_0 centrada en el origen. Encontrar también la integral de superficie del rotacional del vector sobre el área encerrada dentro de la trayectoria. Discutir los resultados. Sol: $\gamma=-r_0\pi$



T^{ma} Stokes $\oint_C\bar{A}\,d\bar{s}=\int_S\bar{\nabla}\times\bar{A}\,d\bar{a}$ $\qquad \gamma=\sum_{i=1}^{3}\gamma_i=\oint_c\bar{A}\,d\bar{s}$

$=4r_0-r_0\pi-4r_0=-r_0\pi$ ✓



$$\gamma_1=\int_c\bar{A}\,dr\,\hat{r}=\int_0^{r_0}4\,dr=4r_0 \qquad \gamma_2=\int_c\bar{A}\,r\sin\theta\,d\varphi\,\hat{\varphi}=-\int_0^{\frac{\pi}{2}}2r\sin\theta\,d\varphi=-r_0\pi \qquad \gamma_3=\int_c\bar{A}\,dr\,\hat{r}=\int_{r_0}^{0}4\,dr=-4r_0$$

θ cte., φ cte. $\Rightarrow d\bar{s}=dr\,\hat{r}$ $\qquad \theta$ cte., r cte. $\Rightarrow d\bar{s}=r\sin\theta\,d\varphi\,\hat{\varphi}$ $\qquad \theta$ cte. φ cte. $\Rightarrow d\bar{s}=\hat{r}\,dr$

$$(\bar{\nabla}\times\bar{A})_\theta=\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi}-\frac{\partial}{\partial r}(rA_\varphi)\right)\hat{\theta}=\frac{1}{r}\left(\frac{1}{\sin\theta}0-\left((-2)+r\cdot 0\right)\right)\hat{\theta}=\frac{2}{r}\hat{\theta}$$

$$\int_S\bar{\nabla}\times\bar{A}\,d\bar{a}=-\int_S2\sin\theta\,dr\,d\varphi=-\int_S2\,dr\,d\varphi=-\int_0^{r_0}\int_0^{\pi/2}2\,d\varphi\,dr=-\int_0^{r_0}\pi\,dr=-r_0\pi$$ ✓

$d\bar{a}=-da\,\hat{\theta}=-r\sin\theta\,dr\,d\varphi\,\hat{\theta}$ $\qquad \hookrightarrow \theta=\frac{\pi}{2}$ cte.