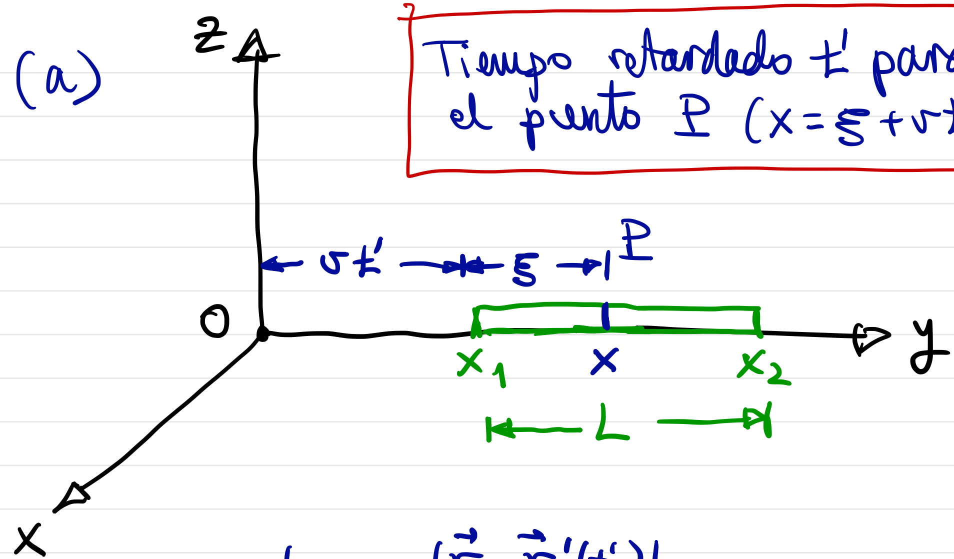
TEMA 6

PROB. 12

(a)

$\boxed{\text{Tiempo retardado } t' \text{ para el punto } P \ (x = \xi + v t')}$



$$t' = t - \frac{|\vec{r} - \vec{r}\,'(t')|}{c}$$

$$\vec{r} = 0 \qquad \vec{r}\,'(t') = (v t' + \xi)\,\hat{u}_x$$

$$t' = t - \frac{v t' + \xi}{c} \qquad 0 \le \xi \le L$$

$$c t' = c t - v t' - \xi$$

$$(c + v)\,t' = c t - \xi \longrightarrow \boxed{t' = \frac{t - \frac{\xi}{c}}{1 + \frac{v}{c}}}$$

$$x = v t' + \xi = \frac{v t - \frac{v \xi}{c}}{1 + \frac{v}{c}} + \xi = \frac{v t - \cancel{\frac{v \xi}{c}} + \cancel{\frac{v \xi}{c}} + \xi}{1 + \frac{v}{c}}$$

$$\boxed{x = \frac{v t + \xi}{1 + \frac{v}{c}}}$$

$$\left.\begin{aligned} \xi = 0 &\rightarrow x_1 = \frac{vt}{1+\frac{v}{c}} \\ \xi = L &\rightarrow x_2 = \frac{vt+L}{1+\frac{v}{c}} \end{aligned}\right\}$$

$$x = \frac{vt+\xi}{1+\frac{v}{c}} \rightarrow dx = \frac{d\xi}{1+\frac{v}{c}}$$

$$\phi(0,t) = \frac{\lambda}{4\pi\varepsilon_0}\int_{x_1}^{x_2}\frac{dx'}{x'} = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{\frac{d\xi}{1+v/c}}{\frac{vt+\xi}{1+v/c}} =$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{d\xi}{vt+\xi} = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{vt+L}{vt}\right) =$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\ln\left(1+\frac{L}{vt}\right)$$

Tiempo retardado

$$\boxed{t' = \frac{t - \frac{\xi}{c}}{1 + \frac{v}{c}}} \qquad 0 \leq \xi \leq L$$

El tiempo retardado es menor para $\xi = L$ (x_2) que para $\xi = 0$ (x_1). Si despejamos t:

$$\left(1 + \frac{v}{c}\right) t' = t - \frac{\xi}{c} \rightarrow \boxed{t = \left(1 + \frac{v}{c}\right) t' + \frac{\xi}{c}}$$

En el instante t contribuye x_1 en t'_1 y x_2 en t'_2:

$$t = \left(1 + \frac{v}{c}\right) t'_1 \qquad [\xi = 0 \ (x_1)]$$

$$t = \left(1 + \frac{v}{c}\right) t'_2 + \frac{L}{c} \quad [\xi = L \ (x_2)]$$

$$\left(1 + \frac{v}{c}\right) t'_1 = \left(1 + \frac{v}{c}\right) t'_2 + \frac{L}{c}$$

$$t'_1 = t'_2 + \frac{L/c}{1 + v/c} = t'_2 + \frac{L}{c + v}$$

$$\boxed{t'_1 = t'_2 + \frac{L}{c + v}}$$