TEMA 12. TEORÍA CLÁSICA DE CAMPOS

Consideremos un campo $\varphi(x)$. Trabajemos con la densidad Lagrangiana, $\mathcal{L}$.

$$L = T - V = \int \mathcal{L} \, d^3x$$

La acción es $\quad S = \int dt \, L = \int_{\mathcal{M}} \mathcal{L} \, d^4x$

Los lagrangianos típicos en QFT dependen de: campos y derivadas primeras

$$\mathcal{L} = \mathcal{L}(\varphi, \partial_\mu \varphi)$$

Consideraremos campos locales ($\mathcal{L}$ depende de $\phi\big|_{x^\mu}$ y $\partial_\mu \varphi\big|_{x^\mu}$, x^μ concreto)

Principio de acción estacionaria

$$0 = \delta S = \delta \int_{\mathcal{M}} d^4x \, \mathcal{L}$$

$$= \delta \int_{\mathcal{M}} d^4x \left[\frac{\partial \mathcal{L}}{\partial \varphi} \delta\varphi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \delta(\partial_\mu \varphi) \right]$$

Observación : $\delta(\partial_\mu \varphi) = \partial_\mu(\delta\varphi)$

Segundo término :

$$\int_{\mathcal{M}} d^4x \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \delta(\partial_\mu \varphi) = \int_{\mathcal{M}} d^4x \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \partial_\mu(\delta\varphi)$$

$$= - \int_{\mathcal{M}} d^4x \, \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right) \delta\varphi \, +$$

$$+ \underbrace{\int_{\mathcal{M}} d^4x \, \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \delta\varphi \right]}_{\text{es una divergencia total}}$$

$$= \int_{\mathcal{M}} d^4x \, \partial_\mu \mathcal{J}^\mu =$$

$$= \int_{\partial\mathcal{M}} d\sigma_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \delta\varphi = \int_{\partial\mathcal{M}} d^3\sigma \, \eta_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \delta\varphi$$

T$^{\underline{a}}$ Stokes

$\delta\varphi|_{\partial\mathcal{M}} = 0$

$$\Rightarrow \boxed{\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi)} \right) = 0}$$

Ecuaciones de Euler-Lagrange.

La densidad de momento canónica es

$$\pi(x) = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}}$$ y la

densidad Hamiltoniana es

$$\mathcal{H} = \pi(x)\,\dot{\varphi}(x) - \mathcal{L}$$

El Hamiltoniano será

$$H = \int d^3x\, \mathcal{H}$$

Veamos algunos ejemplos de teorías clásicas de campos

Campo escalar real (Higgs)

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\varphi\,\partial^\mu\varphi - m^2\varphi^2\right)$$

$$\frac{\partial\mathcal{L}}{\partial(\partial_\alpha\varphi)} = \frac{\partial}{\partial(\partial_\alpha\varphi)}\frac{1}{2}\left(\partial_\mu\varphi\,\partial^\mu\varphi - m^2\varphi^2\right)$$

$$= \frac{1}{2}\frac{\partial}{\partial(\partial_\alpha\varphi)}\left(\partial_\mu\varphi\,\partial^\mu\varphi\right) = \frac{1}{2}\frac{\partial}{\partial(\partial_\alpha\varphi)}\left(g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi\right)$$

$$= \frac{1}{2}\left(g^{\mu\nu}\frac{\partial_\mu\varphi}{\partial_\alpha\varphi}\partial_\nu\varphi + g^{\mu\nu}\partial_\mu\varphi\frac{\partial_\nu\varphi}{\partial_\alpha\varphi}\right)$$

$$= \frac{1}{2}\left(g^{\mu\nu}\delta^\alpha_\mu\,\partial_\nu\varphi + g^{\mu\nu}\partial_\mu\varphi\,\delta^\alpha_\nu\right)$$

$$= \frac{1}{2}\left(g^{\alpha\nu}\partial_\nu\varphi + g^{\mu\alpha}\partial_\mu\varphi\right)$$

$$= \frac{1}{2}\left(g^{\alpha\mu}\partial_\mu\varphi + g^{\mu\alpha}\partial_\mu\varphi\right)$$

$$= g^{\mu\alpha}\partial_\mu\varphi = \partial^\alpha\varphi$$

$$\partial_{\alpha}\left(\frac{\partial \mathcal{L}}{\partial(\partial_{\alpha}\varphi)}\right) = \partial_{\alpha}\partial^{\alpha}\varphi = \Box\varphi$$

$$\frac{\partial \mathcal{L}}{\partial \varphi} = -m^2\varphi$$

$$\Longrightarrow \quad -m^2\varphi - \Box\varphi = 0 \Rightarrow \boxed{(\Box + m^2)\varphi = 0}$$

Klein - Gordon

Otro ejemplos (entregable)

i) $\mathcal{L} = \frac{1}{2}\left[(\partial_t\varphi)^2 - (\partial_x\varphi)^2\right] + \cos\varphi$

$$\Longrightarrow \boxed{\frac{\partial^2\varphi}{\partial t^2} - \frac{\partial^2\varphi}{\partial x^2} + \sin\varphi = 0} \Longleftrightarrow (\Box + \sin)\varphi = 0$$

Sin - Gordon

ii) $\mathcal{L} = -\frac{\hbar}{2i}\left(\psi^*\partial_t\psi - \psi\partial_t\psi^*\right)$

$$-\frac{\hbar^2}{2m}\partial_i\psi\,\partial^i\psi^* - \psi^* V\psi$$

• Obtener la ecuación de Schrödinger

• Hacer el cambio $\psi = N e^{iS/\hbar}$ y

obtener

$$\mathcal{L} = -N^2 \dot{S} - \frac{\hbar^2}{2m} \partial_i N \partial^i N - \frac{N^2}{2m} \partial_i S \partial^i S - V N^2$$

• Las ecuaciones de Euler-Lagrange correspondientes son

$$\partial_t \rho + \frac{1}{m} \nabla_i (\rho \nabla^i S) = 0 \qquad (\rho := N^2)$$

$$\partial_t S + \frac{1}{2m} \partial_i S \partial^i S + V - \frac{\hbar^2}{2m} \frac{\Delta N}{N} \qquad (\Delta := \partial_i \partial^i)$$

(Interpretar dichas ecuaciones. Observad que son ¡completamente equivalentes a la ecuación de Schröd.!!