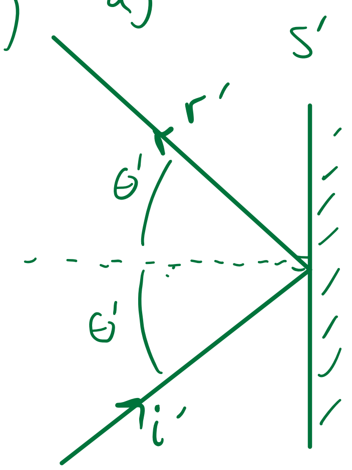
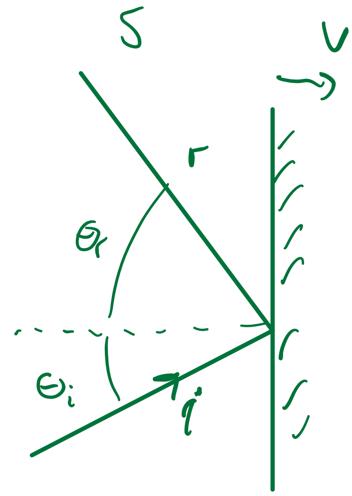
14) a)





(1)

$$c\cos\theta' = \frac{c\cos\theta_i - V}{1-\frac{V}{c}\cos\theta_i}$$

$$c\,\mathrm{sen}\,\theta' = \frac{c\,\mathrm{sen}\,\theta_i\,(1-\beta^2)^{1/2}}{1-\frac{V}{c}\cos\theta_i}$$

i' en términos de i

(2)

$$c\,\mathrm{sen}\,\theta_r = \frac{c\,\mathrm{sen}\,\theta'\,(1-\beta^2)^{1/2}}{\left(1-\frac{V}{c}\cos\theta'\right)}$$

$$-c\cos\theta_r = \frac{-c\cos\theta' + V}{\left(1-\frac{V}{c}\cos\theta'\right)}$$

r en términos de r'

S se mueve con $-V$ respecto de S'.

Reemplazamos (1) en (2)

$$\operatorname{sen}\theta_r = \frac{\operatorname{sen}\theta_i (1-\beta^2)}{\left(1-\frac{V}{c}\cos\theta_i\right)} \frac{1}{1-\frac{V}{c}\frac{\left(\cos\theta_i - \frac{V}{c}\right)}{1-\frac{V}{c}\cos\theta_i}}$$

$$= \frac{\operatorname{sen}\theta_i (1-\beta^2)}{1-\frac{2V}{c}\cos\theta_i + \frac{V^2}{c^2}} \qquad (3)$$

El proceso no es reversible.

Veamos si

$$\frac{\operatorname{sen}\theta_i}{\gamma^2\left(1-2\frac{V}{c}\cos\theta_i + \frac{V^2}{c^2}\right)} \leq 1$$

$$\Updownarrow$$

$$(1-\beta^2)\operatorname{sen}\theta_i \leq \left(1-2\beta\cos\theta_i + \beta^2\right)$$

$$\Updownarrow$$

$$(1-\beta^2)^2\operatorname{sen}^2\theta_i \leq 1 + 4\beta^2\cos^2\theta_i + \beta^4 + 2\beta^2 - 4\beta\cos\theta_i - 4\beta^3\cos\theta_i$$

$$\Updownarrow$$

$$(1-\beta^2)^2\operatorname{sen}^2\theta_i \leq (1+\beta^2)^2 + 4\beta^2\cos^2\theta_i - 4\beta\cos\theta_i(1+\beta^2)$$

$$\Updownarrow$$

$$(1-\beta^2)^2(1-\cos^2\theta_i) \leq (1+\beta^2)^2 + 4\beta^2\cos^2\theta_i - 4\beta\cos\theta_i(1+\beta^2)$$

$$\Updownarrow$$

$$0 \leq \cos^2\theta_i \left(4\beta^2 + (1-\beta^2)^2\right) - 4\beta\cos\theta_i(1+\beta^2) - (1-\beta^2)^2 + (1+\beta^2)^2$$

$$\Updownarrow$$

$$0 \leq \cos^2\theta_i (1+\beta^2)^2 - 4\beta\cos\theta_i(1+\beta^2) + 4\beta^2$$

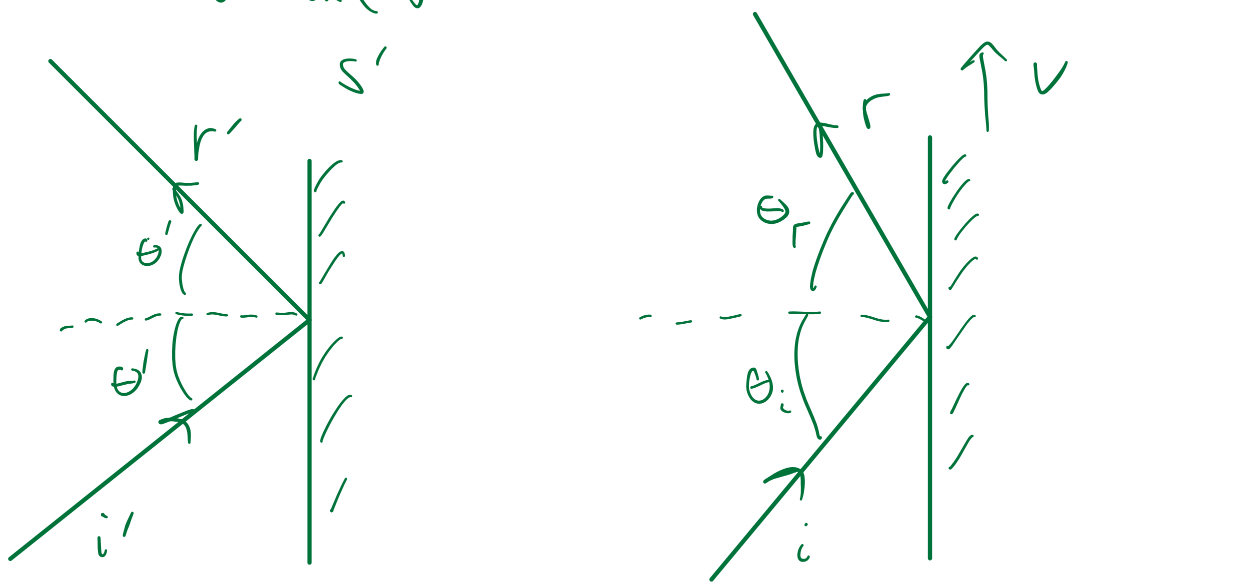
$$\Updownarrow$$

$$0 \leq \left(\cos\theta_i(1+\beta^2) - 2\beta\right)^2 \quad \checkmark$$

Entonces siempre existe rayo reflejado.

Si cambia el signo de V, cambia el signo del denominador en (3)

b)



$$c\cos\theta' = \frac{c\cos\theta_i\,(1-\beta^2)^{1/2}}{1-\frac{V}{c}\sin\theta_i} \; ; \; c\sin\theta' = \frac{c\sin\theta_i - V}{1-\frac{V}{c}\sin\theta_i}$$

i' en función de i

$$c\,\text{sen}\,\theta_r = \frac{c\,\text{sen}\,\theta' + V}{1 + \frac{V}{c}\,\text{sen}\,\theta'}$$

(r en función de r')

(S se mueve con $-V$ respecto a S')

$$\text{sen}\,\theta_r = \frac{\text{sen}\,\theta' + \frac{V}{c}}{1 + \frac{V}{c}\,\text{sen}\,\theta'} \quad ; \quad \text{sen}\,\theta' = \frac{\text{sen}\,\theta_i - \frac{V}{c}}{1 - \frac{V}{c}\,\text{sen}\,\theta_i}$$

$$\text{sen}\,\theta_r = \frac{\left(\text{sen}\,\theta_i - \frac{V}{c}\right) \Big/ \left(1 - \frac{V}{c}\,\text{sen}\,\theta_i\right) + \frac{V}{c}}{1 + \frac{V}{c}\,\dfrac{\left(\text{sen}\,\theta_i - \frac{V}{c}\right)}{1 - \frac{V}{c}\,\text{sen}\,\theta_i}}$$

$$\text{sen}\,\theta_r = \frac{\text{sen}\,\theta_i \;\cancel{\left(1 - V^2/c^2\right)}}{\cancel{\left(1 - \frac{V^2}{c^2}\right)}}$$

En este caso es la ley usual, como debe ser si el espejo es infinitamente largo...
ojo que la solución de la guía para este punto está mal escrita. Subiré la guía corregida hoy.