$$\boxed{\frac{d}{dx^\mu}\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)}\right) - \frac{\partial \mathcal{L}}{\partial A_\nu} = 0}$$ Ecs. Euler-Lagrange

$$\underline{\underline{\mathcal{L} = -\frac{1}{4}\varepsilon_0 c^2 F_{\rho\sigma}F^{\rho\sigma} - J^\nu A_\nu}}$$ (Densidad lagrangiana)

$$\mathcal{L} = -\frac{1}{4}\varepsilon_0 c^2 g^{\rho\xi} g^{\sigma\eta} (\partial_\rho A_\sigma - \partial_\sigma A_\rho)(\partial_\xi A_\eta - \partial_\eta A_\xi) - J^\nu A_\nu$$

$$(*)\ \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = -\frac{1}{4}\varepsilon_0 c^2 \Big[g^{\rho\xi} g^{\sigma\eta} (\delta^\mu_\rho \delta^\nu_\sigma - \delta^\mu_\sigma \delta^\nu_\rho) F_{\xi\eta} +$$

$$+ g^{\rho\xi} g^{\sigma\eta} (\delta^\mu_\xi \delta^\nu_\eta - \delta^\mu_\eta \delta^\nu_\xi) F_{\rho\sigma} \Big] =$$

$$= -\frac{1}{4}\varepsilon_0 c^2 \Big[(g^{\mu\xi} g^{\nu\eta} - g^{\nu\xi} g^{\mu\eta}) F_{\xi\eta} +$$

$$+ (g^{\rho\mu} g^{\sigma\nu} - g^{\mu\sigma} g^{\rho\nu}) F_{\rho\sigma} \Big] =$$

$$= -\frac{1}{4}\varepsilon_0 c^2 \Big[F^{\mu\nu} \underbrace{- F^{\nu\mu}}_{= +F^{\mu\nu}} + F^{\mu\nu} \underbrace{- F^{\nu\mu}}_{= +F^{\mu\nu}} \Big] =$$

$$= -\frac{1}{4}\varepsilon_0 c^2 4 F^{\mu\nu} = -\varepsilon_0 c^2 F^{\mu\nu}$$

$$(*)\ \frac{\partial \mathcal{L}}{\partial A_\nu} = -J^\nu$$

$$\frac{d}{dx^\mu}\left(-\varepsilon_0 c^2 F^{\mu\nu}\right) = -J_\nu$$

$$-\varepsilon_0 c^2 \partial_\mu F^{\mu\nu} = -J^\nu$$

$$\partial_\mu F^{\mu\nu} = \frac{1}{\varepsilon_0 c^2} J^\nu$$

$$\frac{1}{\varepsilon_0 c^2} = \frac{\mu_0 \varepsilon_0}{\varepsilon_0 c^2} = \mu_0$$

$$\boxed{\partial_\mu F^{\mu\nu} = \mu_0 J^\nu}$$

$$\underbrace{\overbrace{\underbrace{\partial_\nu \partial_\mu}_{\text{simétrico}} \overbrace{F^{\mu\nu}}^{\text{antisimétrico}}}}_{=0} = \mu_0 \partial_\nu J^\nu$$

$$\mu_0 \partial_\nu J^\nu = 0 \rightarrow \boxed{\partial_\mu J^\mu = 0}$$ Ecuación de continuidad