PROBLEMA 13
PROBLEMA 15 } Tema 3

En los dos problemas aparece el cálculo del valor medio (en un periodo) del vector de Poyntin

$$\vec{S} = \frac{1}{2\mu} E_0 B_0 e^{-2\beta z} \left\{ \cos\left[2(kz - \omega t + \delta_E) + \phi\right] + \cos\phi \right\} \hat{u}_z$$

Por lo que tenemos que calcular

$$\langle \cos[2(kz - \omega t + \delta_E) + \phi] + \cos\phi \rangle =$$

$$= \langle \cos[2(kz - \omega t + \delta_E) + \phi] \rangle + \underbrace{\langle \cos\phi \rangle}$$

$$= \frac{1}{T}\int_0^T \cos\phi \, dt =$$

$$= \frac{1}{T}\cos\phi \int_0^T dt =$$

$$= \frac{1}{T}\cos\phi \, T = \underline{\underline{\cos\phi}}$$

Para calcular $\langle \cos[2(kz - \omega t + \delta_E) + \phi] \rangle$ tenemos en cuenta:

$$\cos[2(kz - \omega t + \delta_E) + \phi] = \cos(2kz - 2\omega t + 2\delta_E + \phi) =$$

$$= \cos(\underbrace{-2\omega t}_{\alpha} + \underbrace{2kz + 2\delta_E + \phi}_{\beta}) = \cos(\alpha + \beta)$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta$$

$$\cos\left(-2\omega t + 2kz + 2\delta_E + \phi\right) =$$

$$= \underbrace{\cos(-2\omega t)}_{\cos(2\omega t)} \cos\left(2kz + 2\delta_E + \phi\right) -$$

$$- \underbrace{\operatorname{sen}(-2\omega t)}_{= -\operatorname{sen}(2\omega t)} \operatorname{sen}\left(2kz + 2\delta_E + \phi\right) =$$

$$= \cos(2\omega t)\cos\left(2kz + 2\delta_E + \phi\right) +$$

$$+ \operatorname{sen}(2\omega t)\operatorname{sen}\left(2kz + 2\delta_E + \phi\right)$$

Calculando el valor medio en un periodo:

$$\langle \cos[2(kz - \omega t + \delta_E) + \phi] \rangle =$$

$$= \cos\left(2kz + 2\delta_E + \phi\right) \overbrace{\langle \cos(2\omega t) \rangle}^{(1)} +$$

$$+ \operatorname{sen}\left(2kz + 2\delta_E + \phi\right) \overbrace{\langle \operatorname{sen}(2\omega t) \rangle}^{(2)}$$

Calculamos ahora $\langle \cos(2\omega t) \rangle$ y $\langle \operatorname{sen}(2\omega t) \rangle$:

$$\langle \cos(2\omega t)\rangle = \frac{1}{T}\int_0^T \cos(2\omega t)\,dt =$$

$$= \frac{1}{T}\frac{1}{2\omega}\,\text{sen}(2\omega t)\Big|_0^T = \frac{1}{2\omega T}\Big(\text{sen}(2\omega T) -$$

$$-\underbrace{\cancel{\text{sen}(0)}}_{=0}\Big) \underset{\uparrow}{=} \frac{1}{4\pi}\Big[\text{sen}\Big(2\frac{2\pi}{T}T\Big) - 0\Big] =$$

$$\omega = \frac{2\pi}{T}$$

(1)

$$= \frac{1}{4\pi}\,\text{sen}(4\pi) = 0 \longrightarrow \boxed{\langle \cos(2\omega t)\rangle = 0}$$

$$\langle \text{sen}(2\omega t)\rangle = \frac{1}{T}\int_0^T \text{sen}(2\omega t)\,dt =$$

$$= -\frac{1}{T}\frac{1}{2\omega}\cos(2\omega t)\Big|_0^T = -\frac{1}{2\omega t}\Big(\cos(2\omega T) -$$

$$-\underbrace{\cancel{\cos(0)}}_{=1}\Big) \underset{\uparrow}{=} \frac{1}{4\pi}\Big(\cos\Big(2\frac{2\pi}{T}T\Big) - 1\Big) =$$

$$\omega = \frac{2\pi}{T}$$

$$= \frac{1}{4\pi}\Big(\underbrace{\cancel{\cos(4\pi)}}_{=1} - 1\Big) = \frac{1}{4\pi}(1-1) = 0$$

$$\downarrow \quad (2)$$

$$\boxed{\langle \text{sen}(2\omega t)\rangle = 0}$$

Luego:

$$\langle \cos[2(kz-\omega t+\delta_E)+\phi] \rangle =$$

$$= \cos(2kz+2\delta_E+\phi)\underbrace{\langle \cos(2\omega t) \rangle}_{=0} +$$

$$+ \operatorname{sen}(2kz+2\delta_E+\phi)\underbrace{\langle \operatorname{sen}(2\omega t) \rangle}_{=0} = 0$$

$$\boxed{\langle \cos[2(kz-\omega t+\delta_E)+\phi \rangle = 0}$$