09/01/2020

Signals And System:-

> A Signal is any physical phenomena which conveys information.

Example:-

> position (Displacement/distance) $x(t)$
> Velocity $v(t)$
> Accelaration $a(t)$
> force
> Voltage $v(t)$
> Temperature of Ocean as a function of its depth $f(x)$

Note:- From the above example we note that signals can be mathematically modeled using functions

$f(x)$

dependent ← → Independent

Ex:-
> An image is a multi-dimensional signal with two Independent variables characterized as position. $F(x,y)$

Ex:-
> A video signal is a function of 3 Independent variables. $F(x,y,t)$

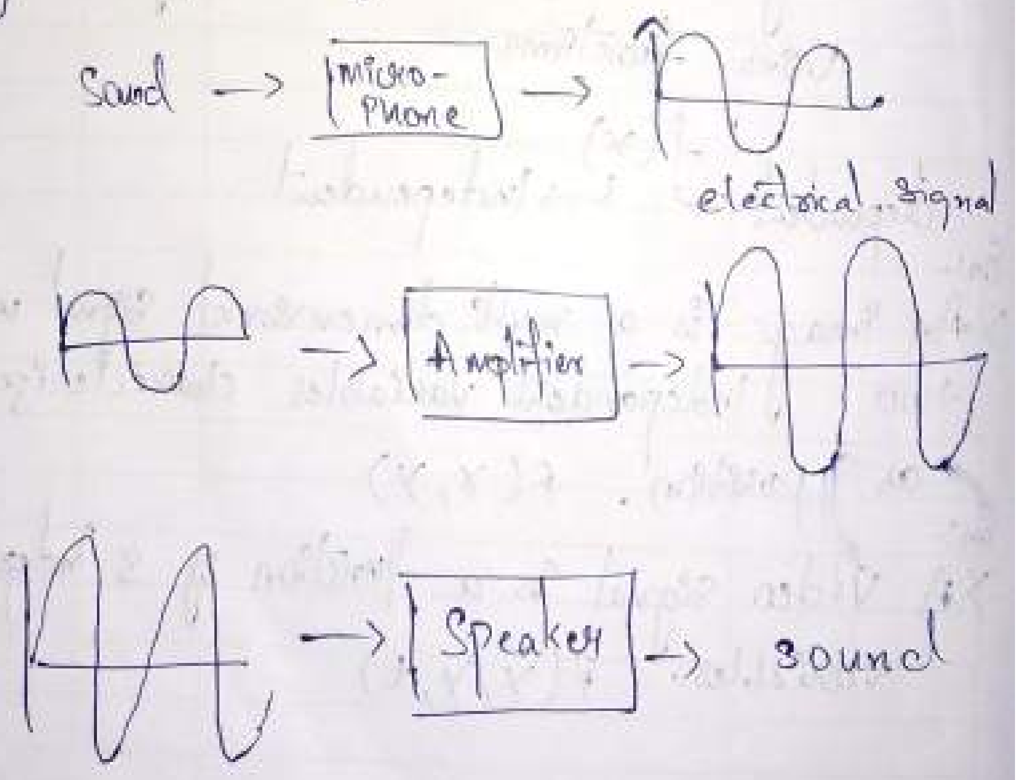
* Signal:-

> A Signal is a function of one or more variables that conveys information about some (usual physical) phenomena.

* System:-

> A System is an entity that processes signal in order to produce a more useful signal may be of the same form or the other form.

1) Eg:- Amplifier system.



2) Integrator, differentiator, clipper, modulator, delayer, filter.

16/01/2020

* Classification of Signals :-

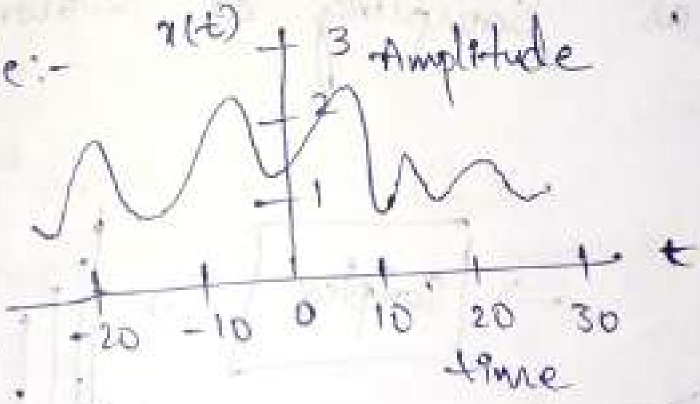
-> Signals can be broadly classified into two types.

i) Continous time signal [CTS]
ii) descrete time signal [DTS]

> CTS :- A Signal that is defined for every real value of time is called C.T.S

Here time is denoted as 't'

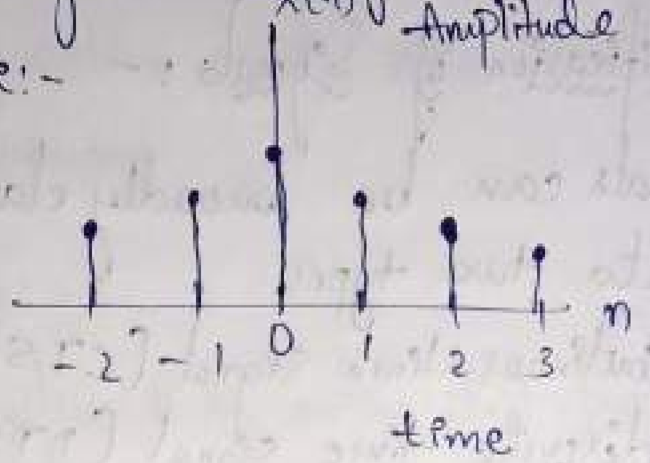
Example :-



Practical Example Includes camera, video, voice, voltage waveform.

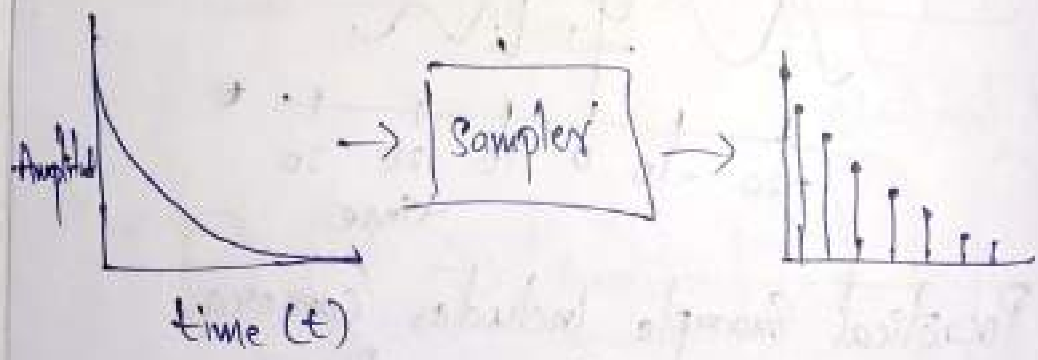
> DTS :- It is a signal that is defined only for descrete instance of time [Integer values of time]

Example:-



Here time is denoted by 'n'.

Note:- A DTS can be generated by a CTS using an operation called as sampling as shown below



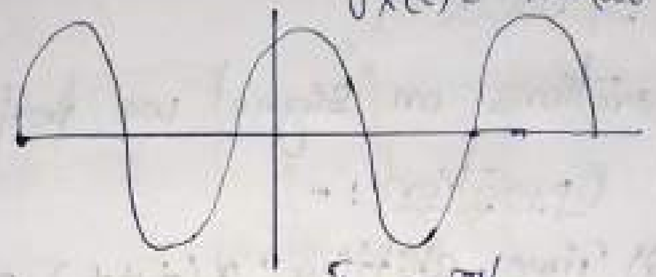
Sampler is a circuit that perform sampling.

* Signal representation:-

> CTS :-

> It can either be represented graphically or mathematically. $x(t) = A\sin(\omega_0 t)$

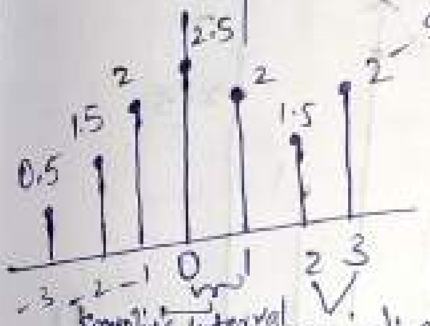
Eg:-



$T_0 = 2\pi/\omega$

> DTS :-

Graphical representation



Sequence representation

$x(n) = \{0.5, 1.5, 2, \underset{\uparrow}{2.5}, 2, 1.5, 2\}$

Functional representation

$$x(n) = \begin{cases} 0.5, & n = -3 \\ 1.5, & n = -2 \\ 2, & n = -1 \\ 2.5, & n = 0 \\ 2, & n = 1 \\ 1.5, & n = 2 \\ 2, & n = 3 \end{cases}$$

Tabular representation.

n	x(n)
-3	0.5
-2	1.5
-1	2
0	2.5
1	2
2	1.5
3	2

Note :- The first sample of a sequence representation will correspond to origin by default, if an arrow mark is not given

* Signal Operation :-

→ In operations on signal we perform

Time Operation :-

a) Time shifting : $x(t \pm t_0)$, $x(n \pm n_0)$

b) Time scaling : $x(at)$, $x(t/a)$, $x(an)$, $x(n/a)$

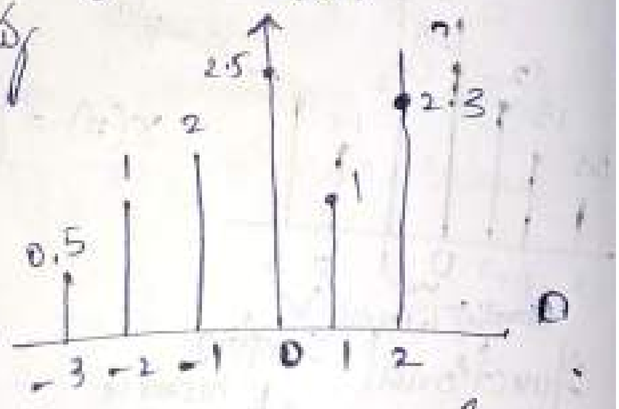
c) Time reversal / folding : $x(-t)$, $x(-n)$

Eg:- Time shifting

Plot (i) $x(n-2)$

(ii) $x(n+2)$

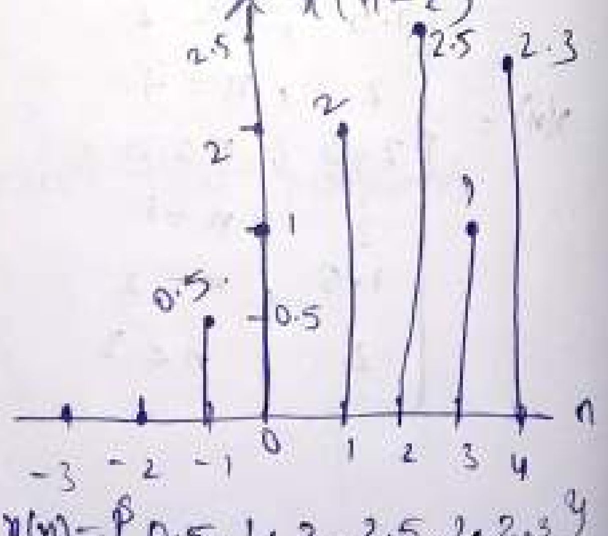
(iii) Sequence representation



(iii) $x(n) = \{0.5, 1, 2, \underset{\uparrow}{2.5}, 1, 2.3\}$

(i)

n	x(n-2)
-3	0
-2	0
-1	0.5
0	1
1	2
2	2.5
3	1
4	2.3

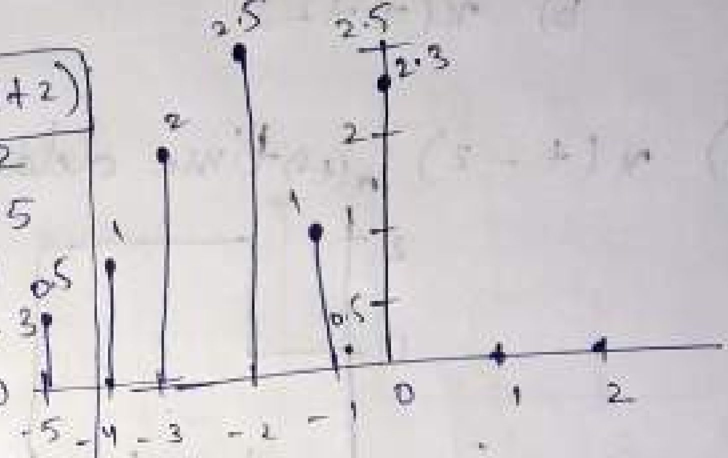


$x(n) = \{0.5, \underset{\uparrow}{1}, 2, 2.5, 1, 2.3\}$

$\therefore$ Note:- $x(n-n_0)$ Indicates the signal should be shifted by n_0 seconds to the right. This is called as time delay operation

(ii) $x(n+2)$

n	$x(n+2)$
-3	2
-2	2.5
-1	1
0	2.3
1	0
2	0
-4	1
-5	0.5
-6	0



$x(n) = \{0.5, 1, 2, 2.5, 1, \underset{\uparrow}{2.3}\}$

Note:- $x(n+n_0)$ Indicates the signal is shifted towards left.

$x(n+n_0)$ is called shifts the signal to the left by n_0 Units and is called as Time advance operation

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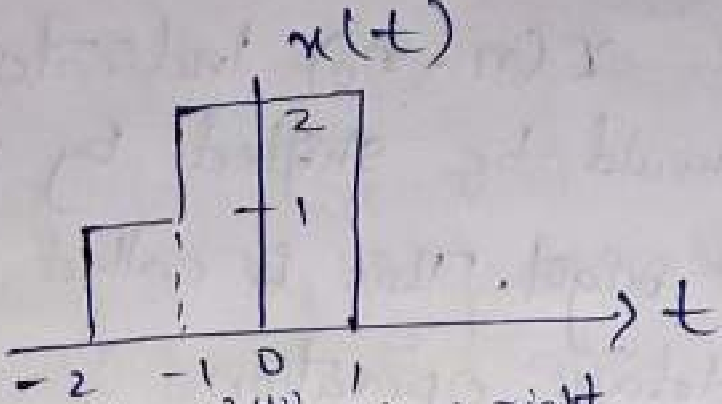
Problem)

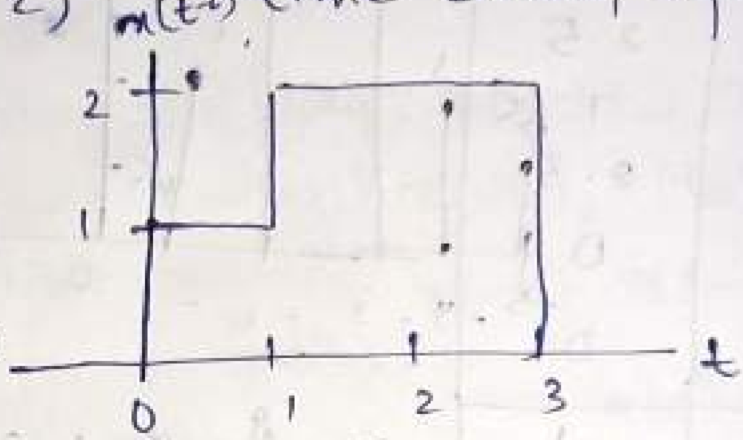


left ← (+2) (-2) (+2) (-2) (+2) (-2) (+2) (-2) → right

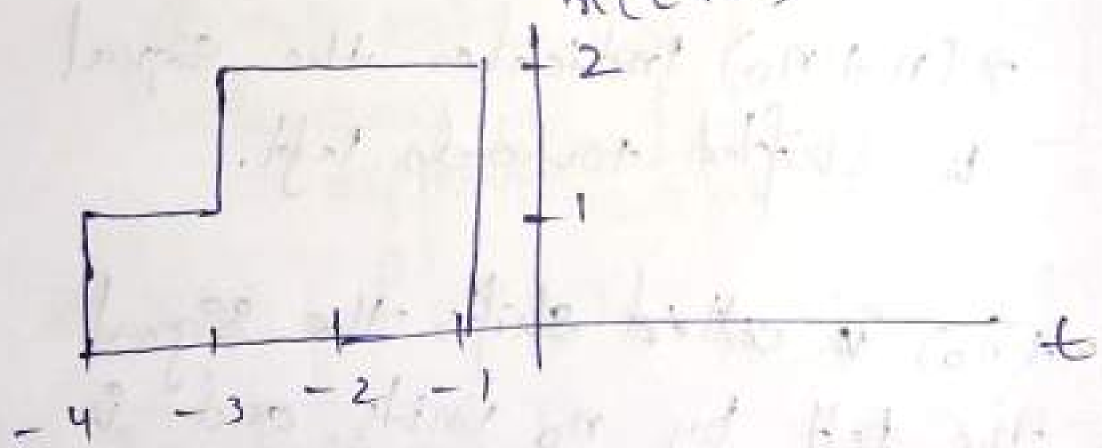
a) x(t-2)

b) x(t+2)

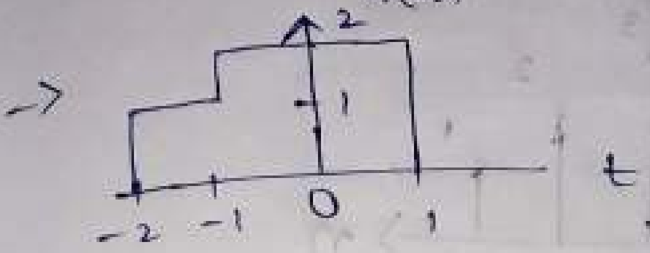
a) x(t-2) time delay operation



(b) x(t+2) time advance operation



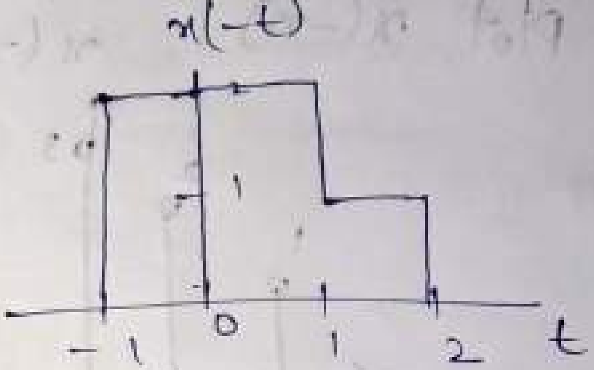
* 2) Time reversal / folding :-



→ plot $x(-t)$

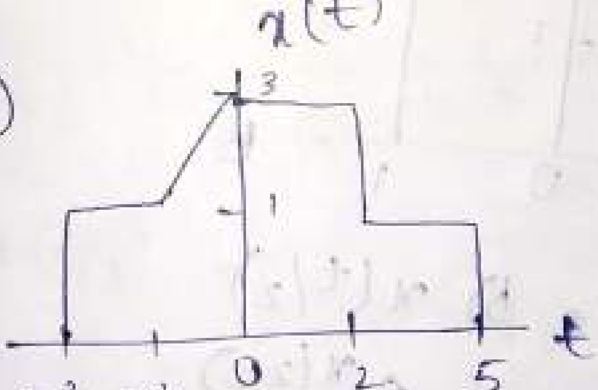
Ans

t	$x(-t)$
-2	0
-1	2
0	2
1	1
2	1

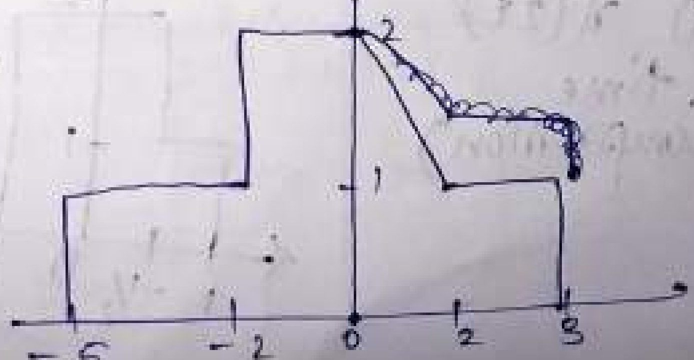


Note :- In time reversal operation, the signals mirror image is taken w.r.t y-axis

→ Q)

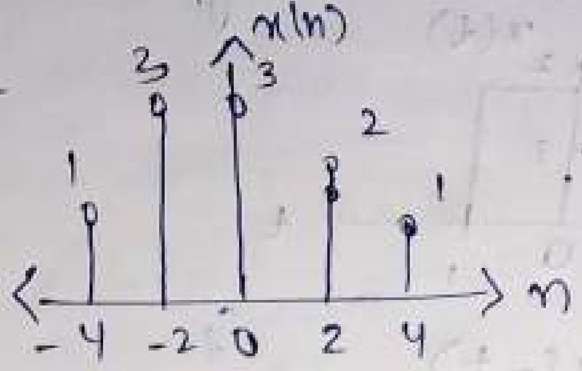


Plot $x(-t)$ →

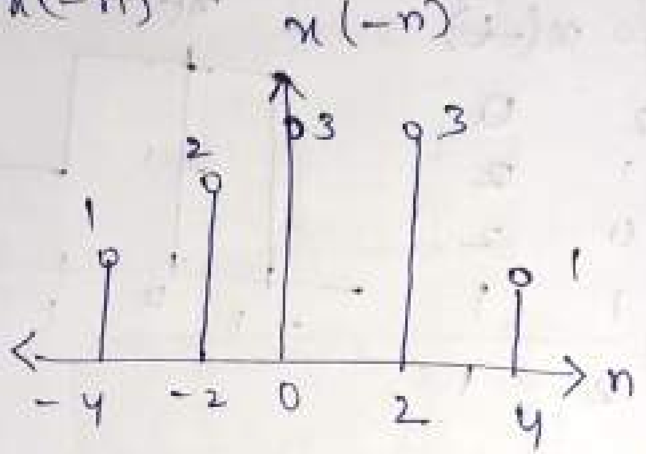


Q) discrete time problem:

Given:-

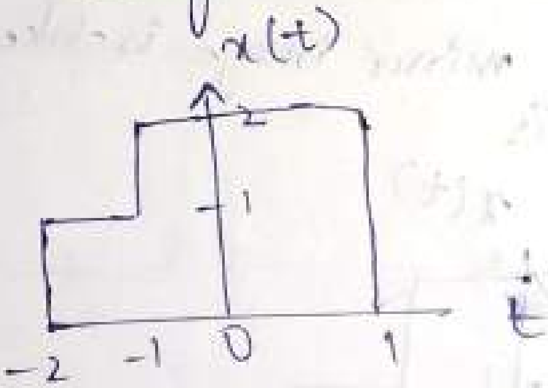


Plot x(-n)



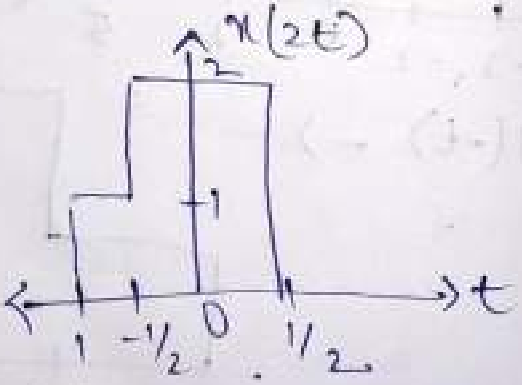
3) Time Scaling:- x(at), x(t/a)

Given:-

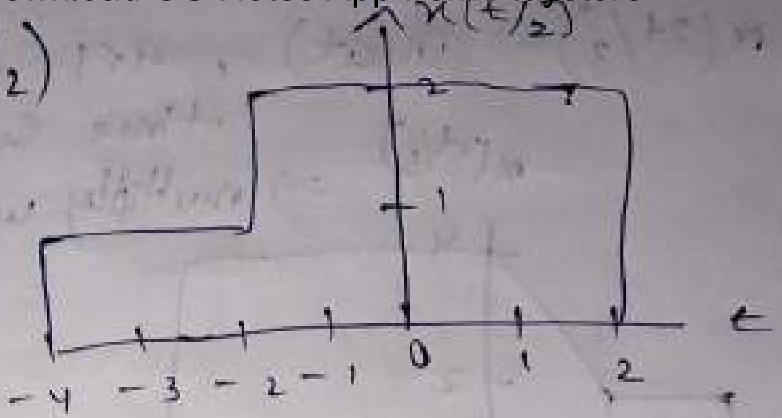


Plot a) x(2t), b) x(t/2)

(a) x(2t)
(time Compression)



(b) $x(t/2)$

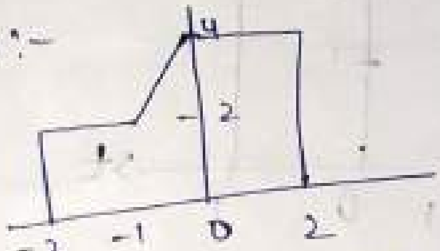


(time expansion)

→ **Time scaling**:-

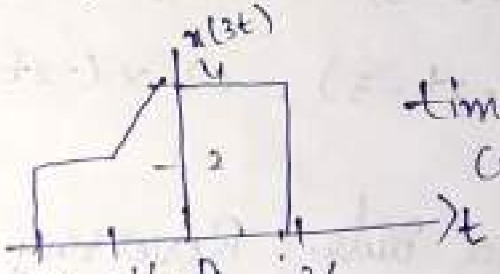
$$x(at) \to \begin{cases} |a| > 1 \to \text{Time Compression} \\ |a| < 1 \to \text{Time Expansion} \end{cases}$$

Q) Given:-



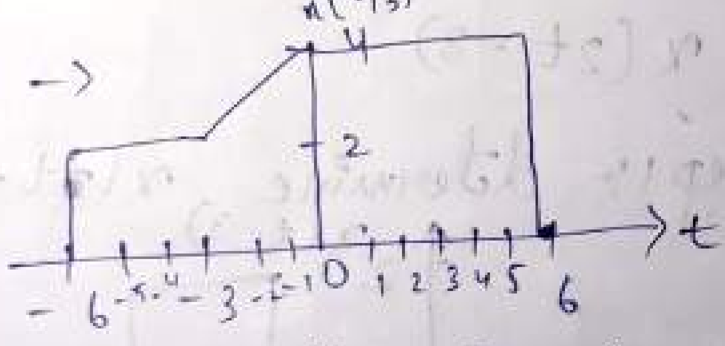
a) $x(3t)$; b) $x(t/3)$

(a) $x(3t) \to$



time compression

b) $x(t/3) \to$

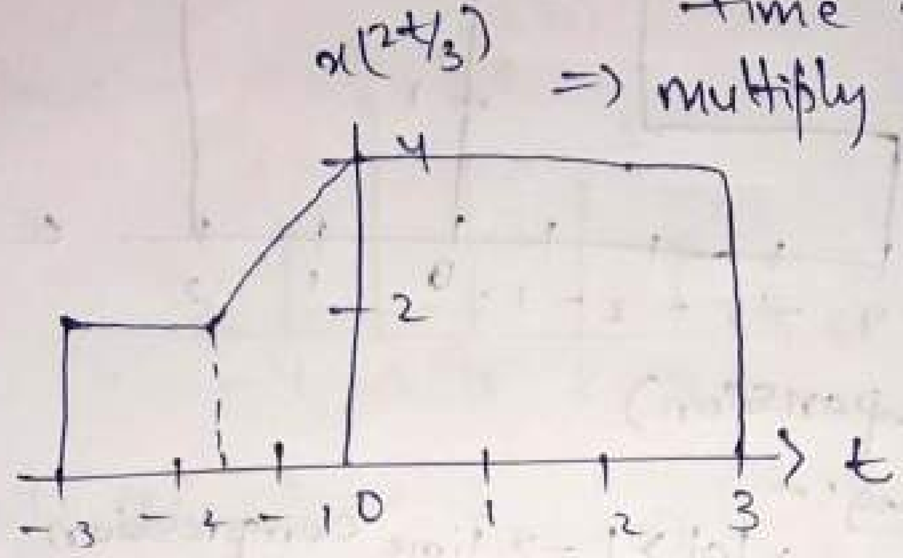


time expansion.

(c) $x(2t/3)$ $\quad x(at)$, $a<1$

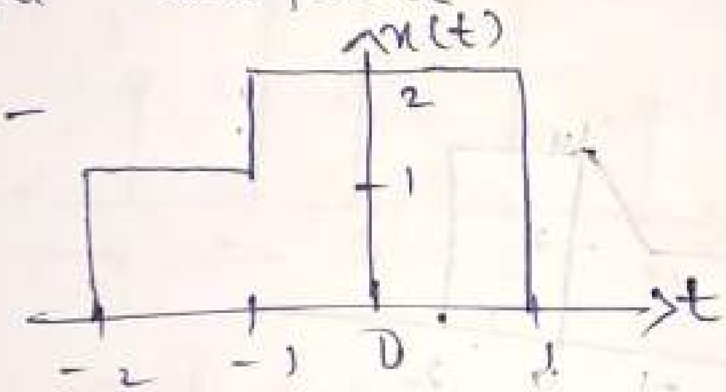
time expansion

$\Rightarrow$ multiply with $3/2$



* multiple transformation:-

Given:-



(a) $x(2t-3)$ b) $x(2t+3)$

(c) $x(-2t-3)$ c) $x(-2t+3)$

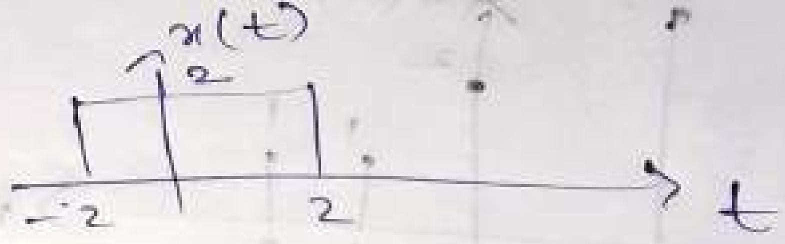
{Precedence rule: ~~Reversing $\rightarrow$ shifting $\rightarrow$ scaling~~}

Shifting $\rightarrow$ Scaling $\rightarrow$ reversing

(a) $x(2t-3)$

22/9/2020

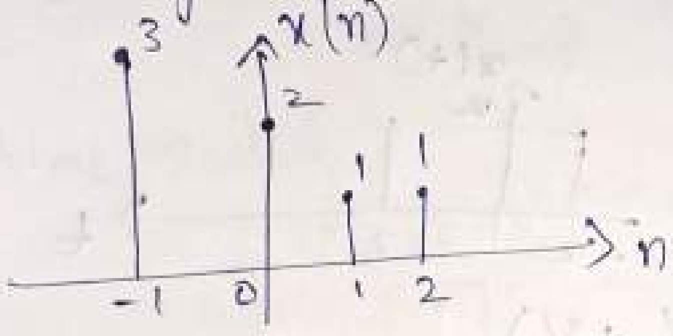
1) Given:-



(a) $x(-t/3 + 3/2)$

Note:- The precedence rule (short cut) method is not valid for signals discrete time signals involving time scaling operation

1)



a) $x(2n)$ b) $x(n/2)$ c) $x(\frac{n}{2}-1)$

d) $x(-\frac{n}{2}-\frac{1}{2})$ e) $x(-n-1)$

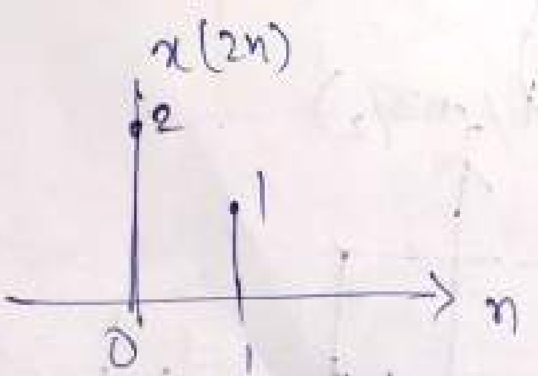
a) $x(2n)$

n	$x(2n)$
0	2
1	1

$-1 \le 2n \le 2$

$-\frac{1}{2} \le n \le 1$

$n = \{0, 1\}$ ∵ Integer



Sequence representation

$x(2n) = \{ \underset{\uparrow}{2}, 1 \}$

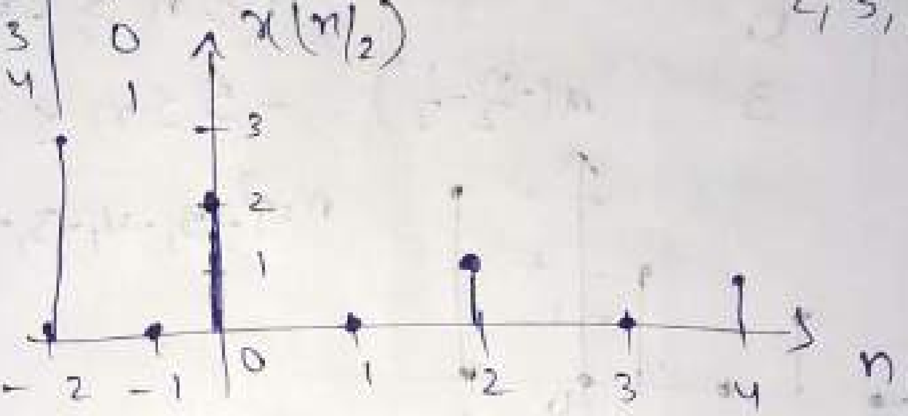
b) $x(n/2)$.

n	$x(n/2)$
-2	3
-1	0
0	2
1	0
2	1
3	0
4	1

$-1 \le n/2 \le 2$ ..

$-2 \le n \le 4$

$n = \{-2, -1, 0, 1, 2, 3, 4\}$



$x\left(\frac{n}{2}\right) = \{3, 0, \underset{\uparrow}{2}, 0, 1, 0, 1\}$

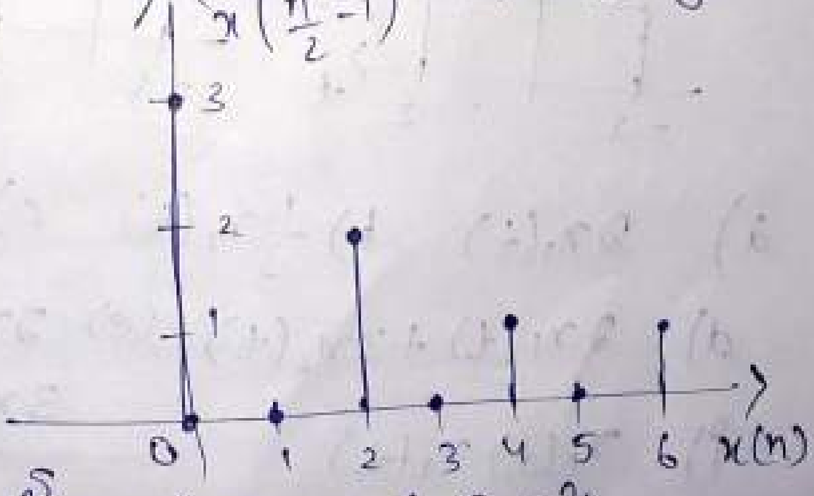
(C) $x\left(\frac{n}{2} - 1\right)$

n	$x(n/2 - 1)$
0	3
1	(3)
2	2
3	0
4	1
5	0
6	1

$-1 \le \frac{n}{2} - 1 \le 2$

$0 \le \frac{n}{2} \le 3$,

$0 \le n \le 6$



$x\left(\frac{n}{2} - 1\right) = \{\underset{\uparrow}{3}, 0, 2, 0, 1, 0, 1\}$

d) $x\left(-\frac{n}{2}-\frac{1}{2}\right)$

n	$x\left(-\frac{n}{2}-\frac{1}{2}\right)$
-5	1
-4	0
-3	1
-2	0
-1	2
0	0
1	3

$-1 \le -\frac{n}{2} - \frac{1}{2} \le 2$

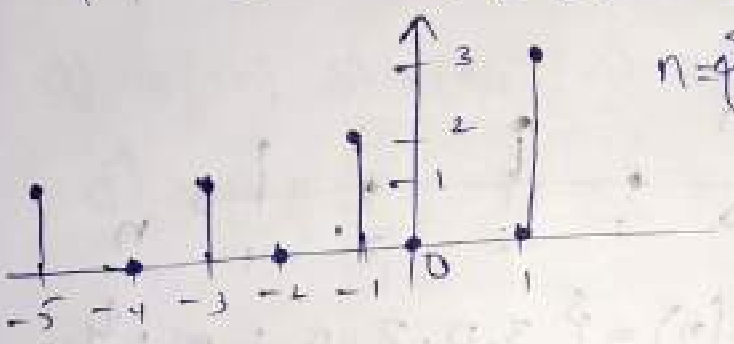
$-\frac{1}{2} \le -\frac{n}{2} \le \frac{5}{2}$

$-1 \le -n \le 5$

$1 \ge n \ge -5$

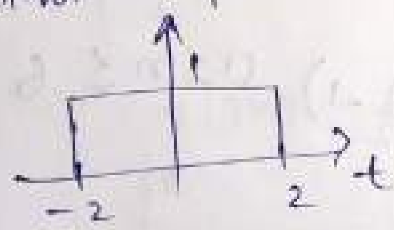
$-5 \le n \le 1$

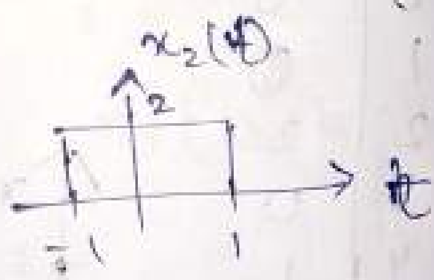
$n = \{-5, -4, -3, -2, -1, 0, 1\}$



* Amplitude Scaling, Addition, Subtraction & multiplication :-

1) Given :- $x_1(t)$





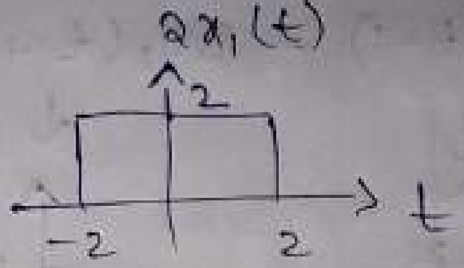
a) $2x_1(t)$ b) $\frac{1}{3}x_2(t)$ c) $-x_1(t)$

d) $2x_1(t) + 3x_2(t)$ (e) $2x_1(t-1) - 3x_2(t+1)$

f) $x_1(t)\, x_2(t)$

Sol (a) $2x_1(t)$

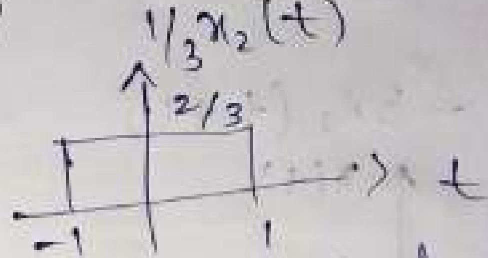
$\Rightarrow$



(∵ This process is called Amplification)

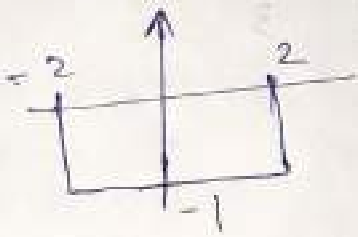
(b) $\frac{1}{3}x_2(t)$

$\Rightarrow$



(∵ This process is called

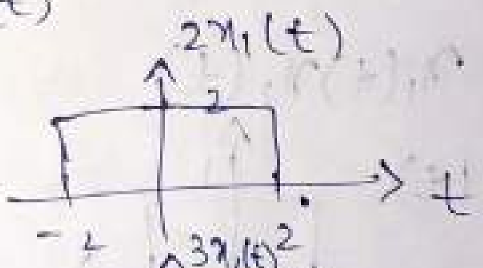
c) $-x_1(t)$



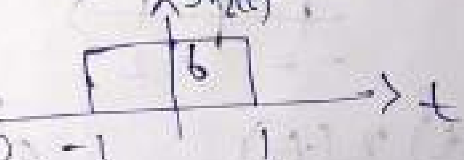
Amplitude reversal

d) $2x_1(t)+3x_2(t)$

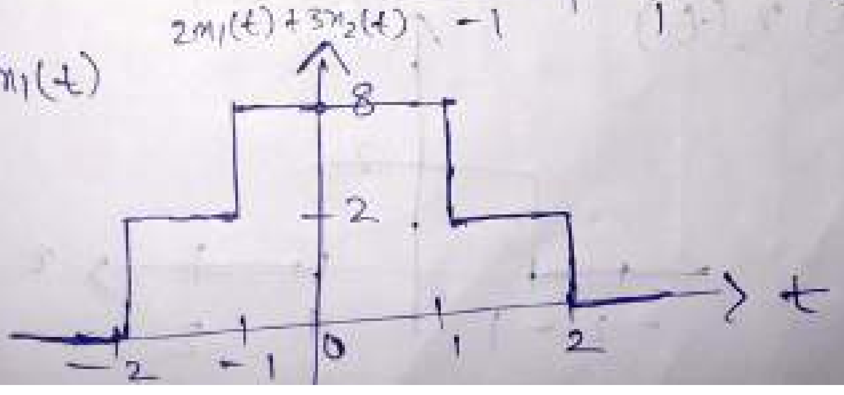
$2x_1(t) \Rightarrow$



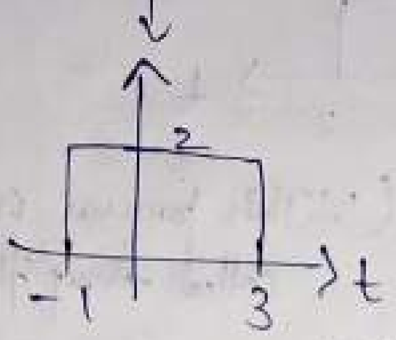
$3x_2(t) \Rightarrow$

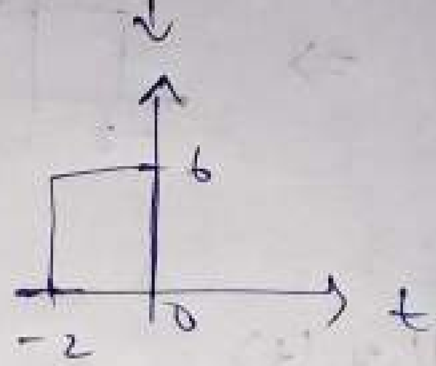


$2x_1(t)$

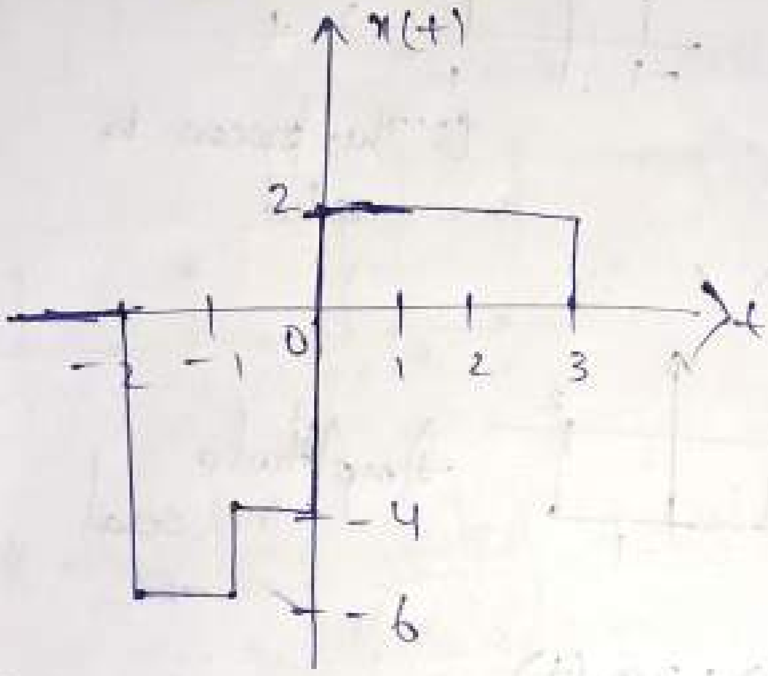


(e) $2x_1(t-1) - 3x_2(t+1)$



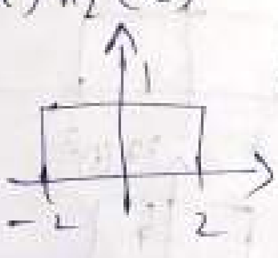


$2x_1(t-1) - 3x_2(t+1)$

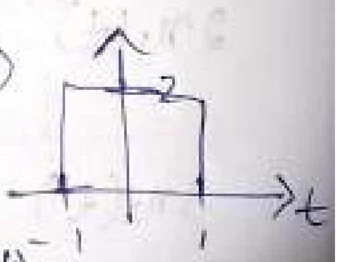


(f) $x_1(t)x_2(t)$

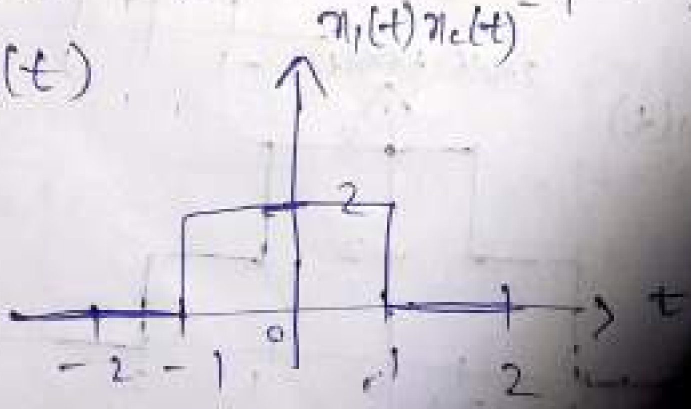
$x_1(t)$



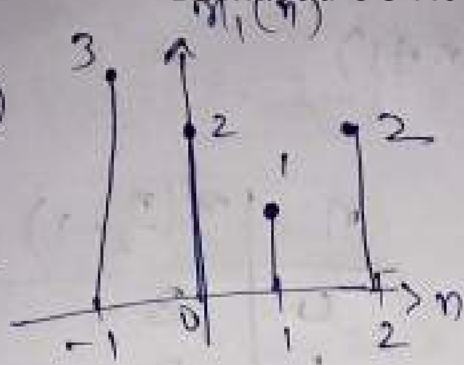
$x_2(t)$

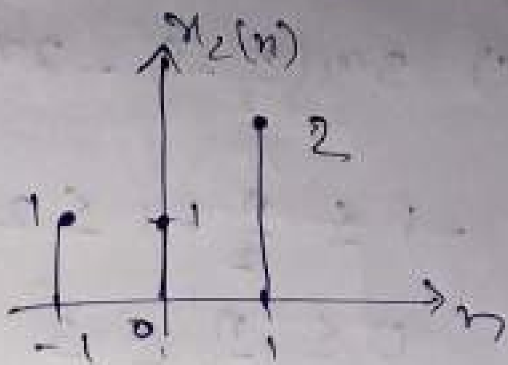


$x_1(t)\,x_2(t)$



a)

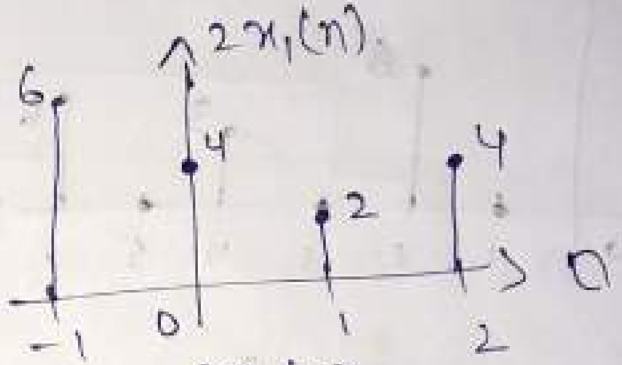




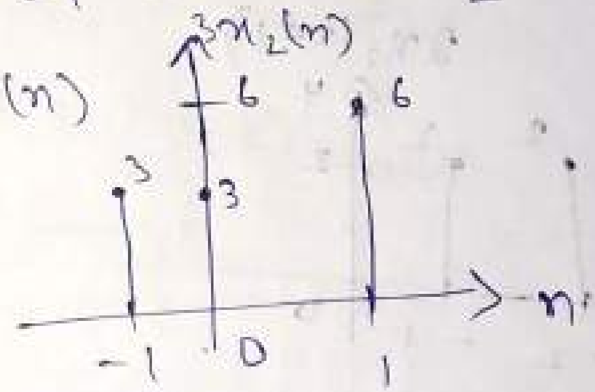
(a) $2x_1(n)$ b) $2x_1(n) + 3x_2(n)$

(c) $3x_1\left(\frac{n}{2}-1\right) - 2x_2(n+1)$ d) $x_1(n)x_2(n)$

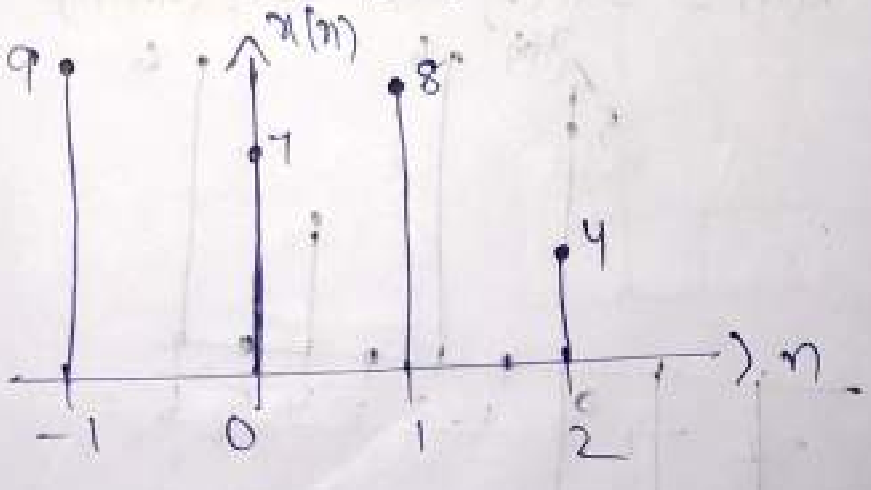
(a) Sol $2x_1(n)$



b) $2x_1(n) + 3x_2(n)$
(previous Problem)



let $x(n) = 2x_1(n) + 3x_2(n)$



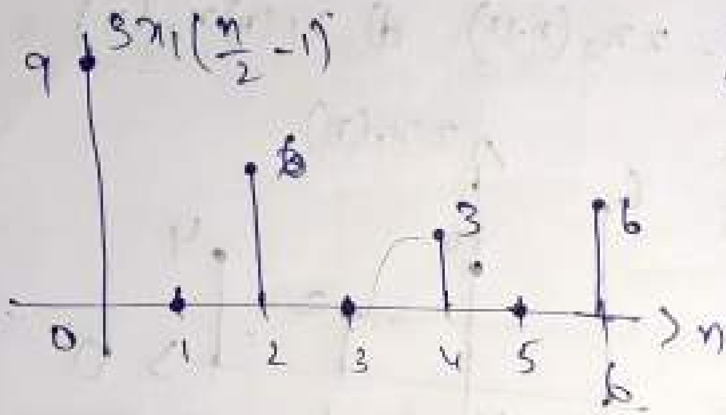
(c) $3x_1\left(\frac{n}{2}-1\right) - 2x_2(n+1)$

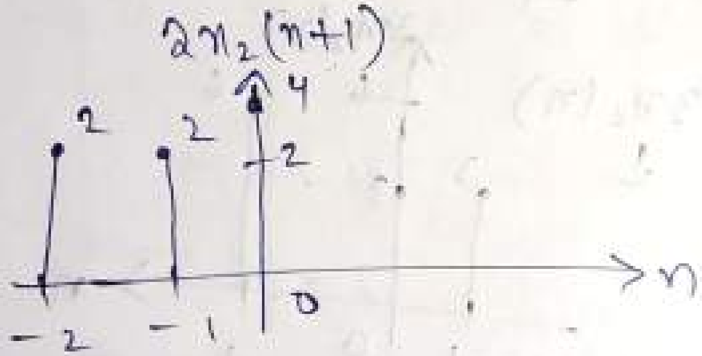
$-1 \le \frac{n}{2} - 1 \le 2$

$0 \le \frac{n}{2} \le 3$

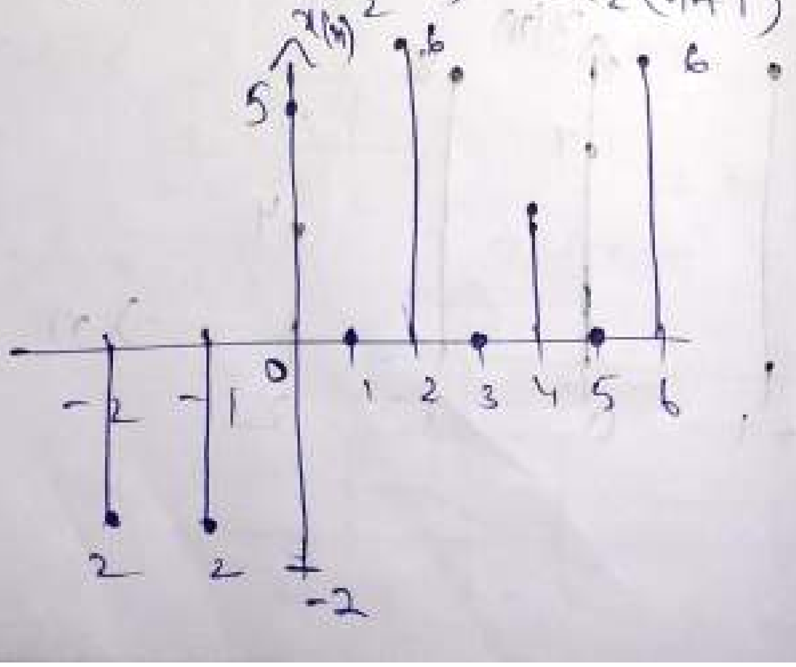
$0 \le n \le 6$

n	$x\left(\frac{n}{2}-1\right)$
0	3
1	0
2	2
3	0
4	1
5	0
6	2





let $x(n) = 3x_1\left(\frac{n}{2}-1\right) - 2x_2(n+1)$



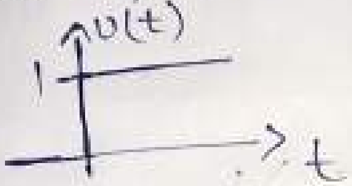
23/01/2020

* Basic / Standard / Elementary signal :-

1) Unit step signal

Heaviside step signal

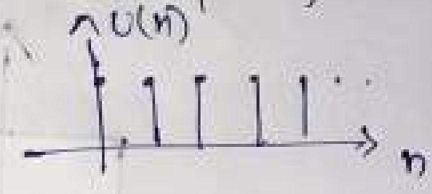
Continuous time unit step signal



$U(t) = 1, \; t \geq 0$

$U(t) = 0, \; t < 0$

Discrete time Unit step signal

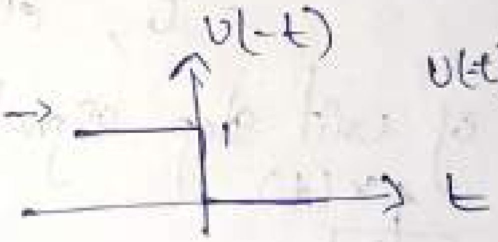


$U(n) = 1, \; n \geq 0$

$U(n) = 0, \; n < 0$

$U(n) = \{1, 1, \ldots\}$

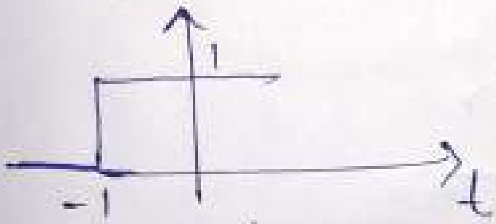
1) Plot $U(-t)$ →



$U(-t) = 1, \; t \leq 0$
$= 0, \; t > 0$

2) Plot $U(t+1)$ & $U(t-1)$

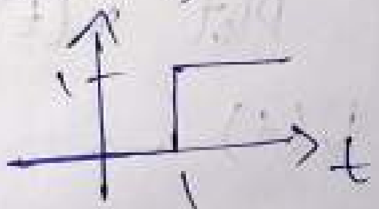
$U(t+1)$



$U(t+1) = 1, \; t \geq -1$

$U(t+1) = 0, \; t < -1$

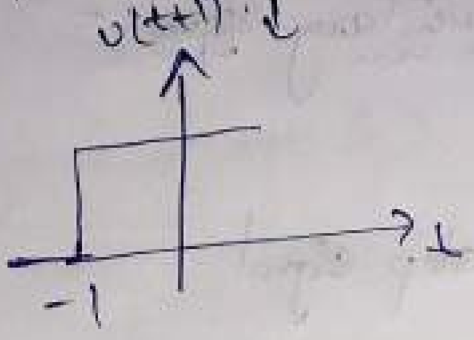
$U(t-1)$

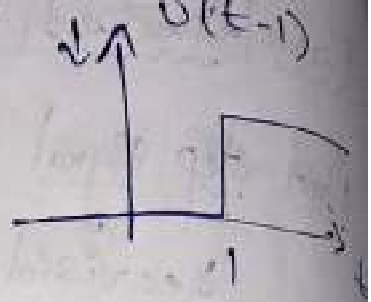


$U(t+1) = 1, \; t \geq 1$

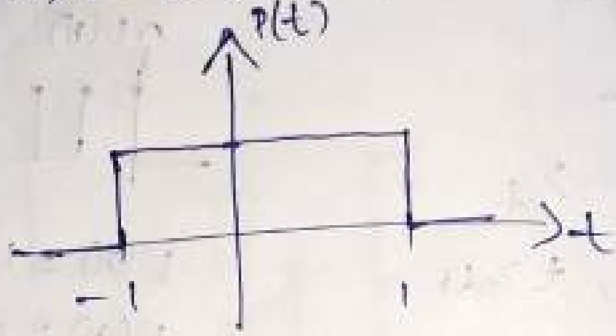
$U(t-1) = 0, \; t < 1$

3) plot $u(t+1) - u(t-1)$





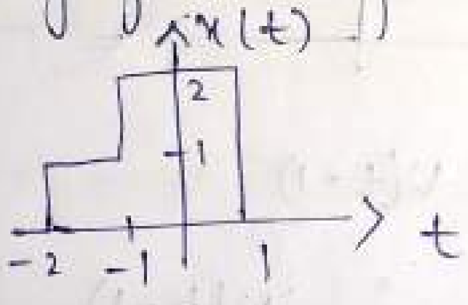
$p(t) = u(t+1) - u(t-1)$



$$p(t) = 1 \; , \; -1 \le t \le 1$$
$$= 0 \; , \; \text{else}$$

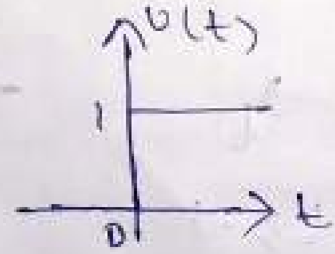
Property of unit step signal:-

1)



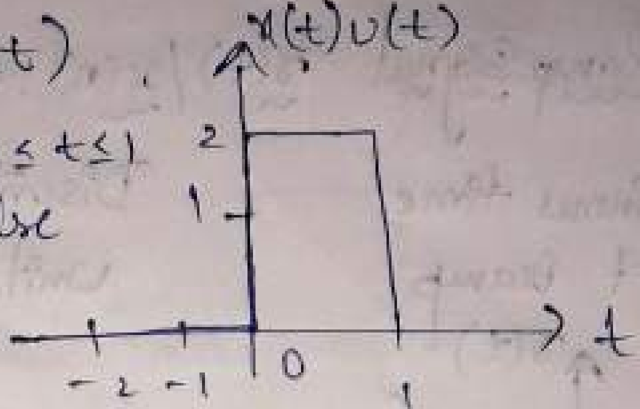
Plot $x(t)\,u(t)$

$u(t) \mapsto$



(a) $x(t)\,u(t)$

$x(t)u(t) = 1, \; 0 \le t \le 1$
$= 0, \; \text{else}$

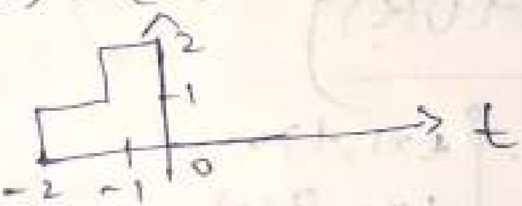


Note:- step signal truncates the ~~positive~~ negative portion of $x(t)$ when multiplied by $u(t)$

III'ly $u(-t)$ truncates positive portion of $x(t)$.

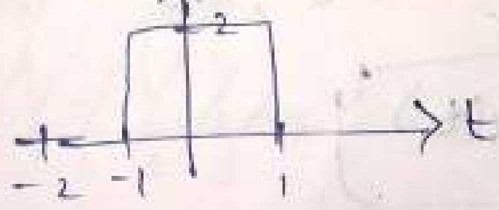
(b) $x(t)\,u(-t)$

$x(t)u(-t) = 1, \; -2 \le t \le -1$
$= 2, \; -1 \le t \le 0$
$= 0, \; \text{else}$



(c) $x(t)\,u(t+1)$

$x(t)u(t+1) = 2, \; -1 \le t \le 1$
$= 0, \; \text{else}$

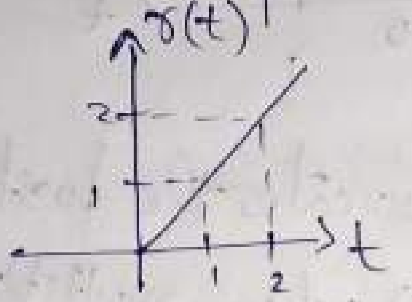


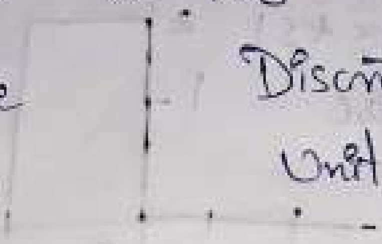
$\therefore$ step signal is used to truncate (cutting of) the portion of a signal.

2) Ramp Signal $r(t) / r(n)$:-

* Continous time unit ramp

Discrete time unit ramp





$y = mx$

$m = 1$

$y = x$

$\Rightarrow r(t) = t, t \geq 0$

$= 0, t < 0$

$\Rightarrow \{r(t) = t\,u(t)\}$

$= r(t) = \begin{cases} t \times 1, & t \geq 0 \\ t \times 0, & (t < 0) \end{cases}$

$= r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$

$\left\{ u(t) = \frac{d}{dt} r(t) \right\}$

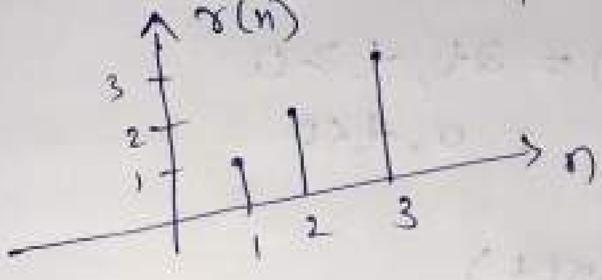
$= \frac{d}{dt} \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$

$= \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

$= u(t)$

$$\boxed{r(t) = \int_{-\infty}^{t} u(t)\, dt}$$

* Discrete time unit ramp :-



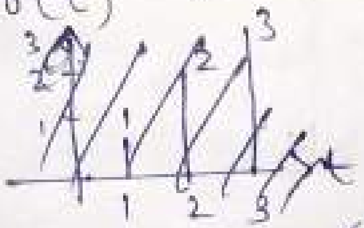
$r(n) = n, \quad n \geq 0$
$\quad\quad\;\; 0, \quad n < 0$

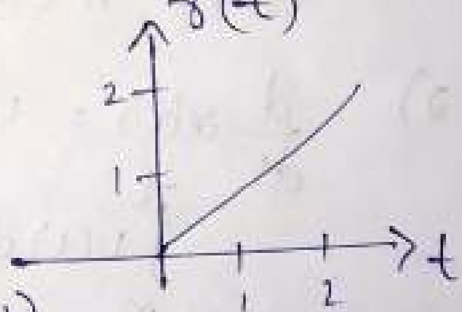
$$\boxed{r(n) = n \cdot u(n)}$$

$$\Rightarrow \quad \boxed{r(n+1) - r(n) = u(n)}$$

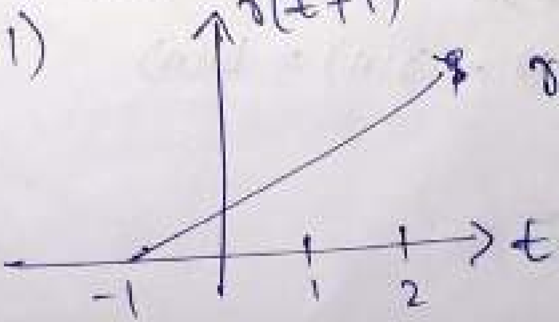
1) Plot $r(t+1)$

Sol $r(t)$



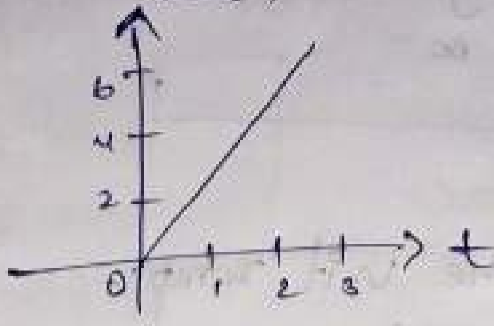


$r(t+1)$



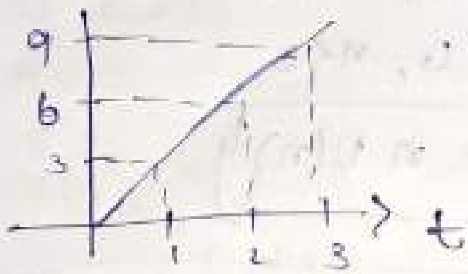
$r(t+1) = t+1, \; t \geq -1$
$\quad\quad\quad = 0, \; t < -1$

2) plot $\cancel{2r(t)}$ $2r(t)$



$$2r(t) = 2t, \; t \geq 0$$
$$= 0, \; t < 0$$

3) plot $3r(t)$



Summary :-

1) $r(t) = t\,u(t)$

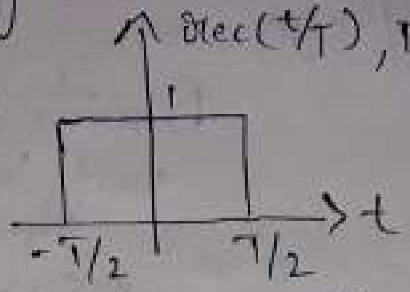
$r(n) = n\,u(n)$

2) $\frac{d}{dt} r(t) = u(t)$

$\int_{-\infty}^{t} u(t)\,dt = r(t)$

3) $r(n+1) - r(n) = u(n)$

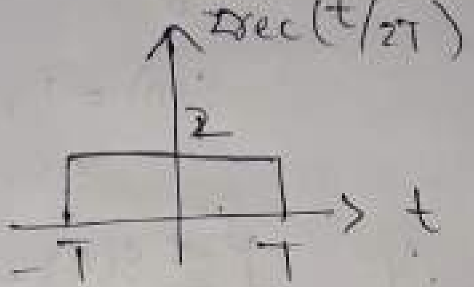
3) Rectangular / Gate / Π / pulse signal :-



$rec(t/T) = 1, \; -T/2 \le t \le T/2$
$= 0$, else

$T \rightarrow$ width of the pulse

* plot $2rec(t/2T)$

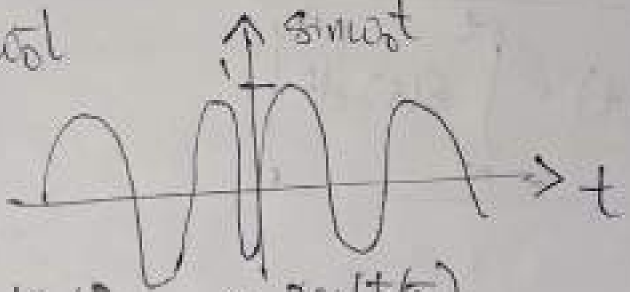


$rec\left(\frac{t}{2T}\right) = 2, \; -T \le t \le T$
0, else

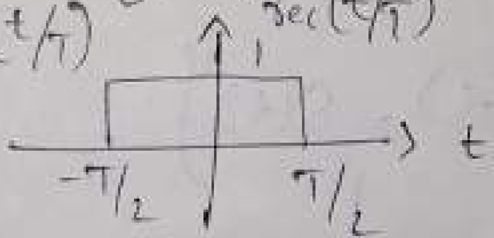
$|t| \le T$

1) Plot $\sin\omega_0 t \, rec(t/T)$

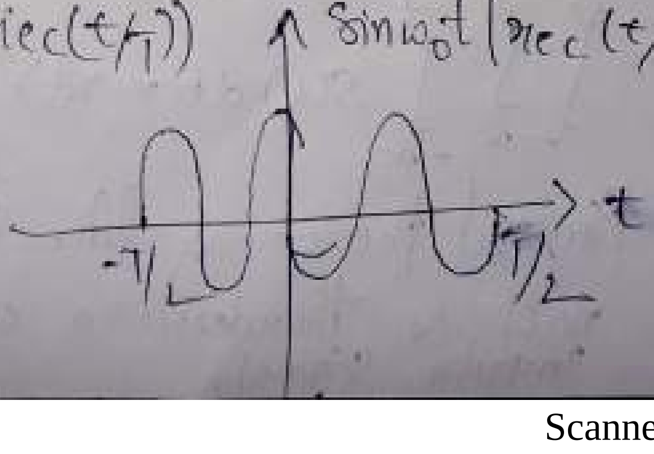
$\sin\omega_0 t$



$rec(t/T)$



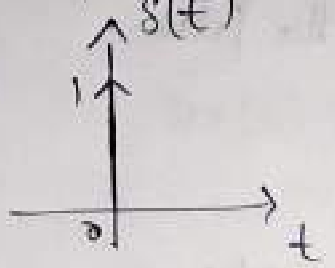
$\sin\omega_0 t \, (rec(t/T))$



4) Impulse signal:-

$\delta(t) / \delta(n)$

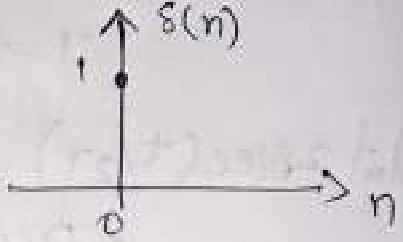
Continuous time unit impulse signal



$$\delta(t) = 1,\ t = 0$$
$$= 0,\ t \neq 0$$

$$\text{Area} = \int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

Discrete time unit impulse signal



$$\delta(n) = 1,\ n = 0$$
$$= 0,\ n \neq 0$$

$$\sum_{-\infty}^{\infty} \delta(n) = 1$$

$$\frac{d}{dt}(u(t)) = \delta(t)$$

$$\Rightarrow \boxed{u(t) = \int_{-\infty}^{t} \delta(t)\, dt}$$

$$\frac{d^2 r(t)}{dt^2} = \delta(t)$$

$$\int_{-\infty}^{t}\int_{-\infty}^{t} \delta(t)\, dt = r(t)$$

Eg:- Physical phenomena like hammering of nail & thunderstorm are examples of impulse signal.

* properties of Impulse function :-

1) $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$ Area is always one

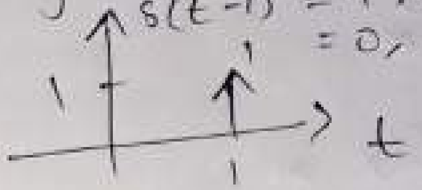
$\sum_{-\infty}^{\infty} \delta(n) = 1$

Area property of Impulse

$\delta(t-1) = 1, \; t = 1$

$= 0, \; t \neq 1$

$\delta(t-1) \rightarrow$



2) Product property

$x(n)\delta(n-n_0) = x(n_0)\delta(n-n_0)$

$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$

Evaluate

1) $(n-3)\delta(n-1)$

Sol we have Product property

$x(n)\delta(n-n_0) = x(n_0)\delta(n-n_0)$

$(n-3)\delta(n-1) = x(1)\delta(n-1)$

$= -2\delta(n-1)$

2) $(t^2+1)\delta(t)$

$x(t_0)$ $x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$

$(t^2+1)\delta(t) = x(0)\delta(t)$

$= 1\delta(t)$

$= \delta(t)$

(3) Sifting property

$$\sum_{n=-\infty}^{\infty} x(n)\,\delta(n-n_0) = x(n_0)$$

$$\int_{t=-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$$

Consider,

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt$$

from product property

$$= \int_{-\infty}^{\infty} x(t_0)\,\delta(t-t_0)\,dt$$

$$= x(t_0)\int_{-\infty}^{\infty} \delta(t-t_0)\,dt$$

$$= x(t_0)$$

//

$$\int_{t_1}^{t_2} x(t)\,\delta(t-t_0)\,dt = x(t_0), \quad t_1 \le t_0 \le t_2$$
$$= 0, \quad \text{else}$$

$$\sum_{n_1}^{n_2} x(n)\,\delta(n-n_0) = x(n_0), \quad n_1 \le n_0 \le n_2$$
$$= 0, \quad \text{else}$$

$$\left\{ \int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0) \right.$$

$$\left. x(t)\,\delta(t) = x(0)\,\delta(t) \right\}$$

1) $\int_0^2 (t^2+1)(\delta(t-1/2))\,dt$

$x(t_0) = x(1/2)$

$= (1/2)^2 + 1 = 5/4$

2) $\int_1^2 (t^2+1)\,\delta(t-3) = 0$

3) Evaluate $\int_{-\infty}^{\infty} (t^2+1/2)\,\delta(t-2)\,dt$

Sol $\int_{-\infty}^{\infty} \underbrace{(t^2+1/2)}_{x(t)}\,\underbrace{\delta(t-2)}_{\delta(t-t_0)}\,dt$

$= x(t_0)$

$= x(2)$

$= 2^2 + 1/2$

$= 4 + 1/2 = 9/2$

4) $\int_{-1000}^{1} (t^2+1/2)\,\delta(t-2)\,dt = 0$

5) $\int_{-\infty}^{\infty} \cos\pi(t-1)\,\delta(t-2)\,dt$

Sol $\Rightarrow x(t) = x(2)$

$= \cos\pi(2-1)$

$= \cos\pi$

$= -1$

(6) $\int_{-\infty}^{\infty} \cos\frac{\pi}{2}(t-5)\,\delta(t-3)\,dt$

Sol $\int_{-\infty}^{\infty} \cos\left(\frac{\pi}{2}(t-5)\right)\delta(t-3)\,dt$

$= x(3)$

$= \cos\frac{\pi}{2}(3-5)$

$= \cos -\pi$

$= \cos\pi = -1$

7) $\int_{-\infty}^{\infty} (t + \sin 2\pi t)\,\delta(t-1)\,dt$

$= x(1)$

$= 1 + \sin 2\pi$

$= 1 + 0$

$= 1$

* (4) Scaling property :-

$$\boxed{\delta(at-b) = \frac{1}{|a|}\,\delta(t-b/a)}$$

Consider $\int_{-\infty}^{\infty} x(t)\,\delta(at-b)$

Put $at-b = u$

$dt = \frac{du}{a}$ $\quad t = \frac{b+u}{a}$

$$\int_{-\infty}^{\infty} x\left(\frac{b+u}{a}\right) \delta(u)\, du$$

$$= \frac{1}{a}\int_{-\infty}^{\infty} x\left(\frac{b+u}{a}\right) \delta(u)\, du = \frac{1}{a}x(0)$$

$$= \frac{1}{a}x(b/a)$$

$$\int_{-\infty}^{\infty} x(t)\,\delta(at-b) = \frac{1}{a}\,x(b/a)$$

$$= \frac{1}{a}\int_{-\infty}^{\infty} x(t)\,\delta(t-b/a)\,dt$$

$$\boxed{\delta(at-b) = \frac{1}{|a|}\,\delta(t-b/a)}$$

$$\left\{\delta(at) = \frac{1}{|a|}\,\delta(t)\right\}$$

$$\left\{\delta(-at-b) = \frac{1}{|a|}\,\delta(t-b/a)\right\}$$

1) $\displaystyle\int_{-\infty}^{\infty} \cos\frac{\pi}{2}(t-5)\,\delta(-2t-3)$

Sol

$$\int_{-\infty}^{\infty} \cos\frac{\pi}{2}(t-5)\,\frac{1}{|-2|}\,\delta(t-3/2)\,dt$$

$$\frac{1}{2}\int_{-\infty}^{\infty} \cos\frac{\pi}{2}(t-5)\,\delta(t-3/2)\,dt$$

$$\frac{1}{2}\left[\cos\frac{\pi}{2}\left(\frac{3}{2}-5\right)\right]$$

$$= \frac{1}{2}\cos\left(\frac{\pi}{2}\right)\left(\frac{7}{2}\right)$$

$$= 1/2 \cos 7\pi/4 = 0.497$$

5) Symmetry property:-

Impulse function exhibits even symmetricity

$\delta(-t) = \delta(t)$, $\delta(-n) = \delta(n)$

$x(-t) = x(t)$

1) Express mathematically the signal $u(2t+1)$

Sol Given:- $u(2t+1)$

We know that

$$u(t) = 1 , \; t \geq 0$$
$$= 0 , \; t < 0$$

$$u(2t+1) = 1 , \; 2t+1 \geq 0$$
$$t \geq -1/2$$

$$= 0 , \; 2t+1 < 0$$
$$t < -1/2$$

2) $u(2t-3)$

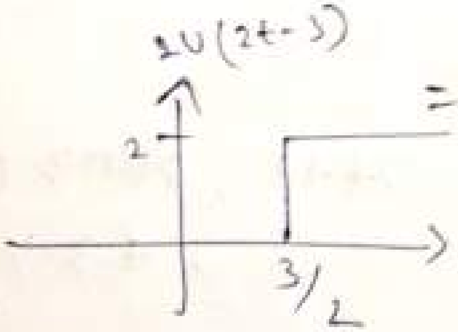
w.k.T $u(t) = 1, \; t \geq 0$
$\qquad = 0, \; t < 0$

$u(2t-3) = 1, \; 2t-3 \geq 0$
$\qquad t \geq 3/2$
$\qquad = 0, \; 2t-3 < 0$
$\qquad t < 3/2$

3) $2u(2t-3)$

$u(t) = 1, \; t \geq 0$
$\qquad = 0, \; t < 0$

$2u(2t-3) = 2, \; 2t-3 \geq 0$
$\qquad t \geq 3/2$
$\qquad = 0, \; t < 3/2$



4) $r(2t)$, ~~scale~~

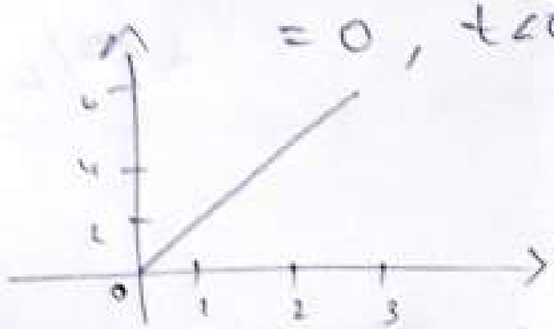
w.k.T $r(t) = t, \; t \geq 0$
$\qquad = 0, \; t < 0$

$r(2t) = 2t, \; 2t \geq 0$
$\qquad = 0, \; 2t < 0$

5) $2r(t)$

$$r(t) = t, \quad t \geq 0$$
$$= 0, \quad t < 0$$

$$2r(t) = 2t, \quad t \geq 0$$
$$= 0, \quad t < 0$$

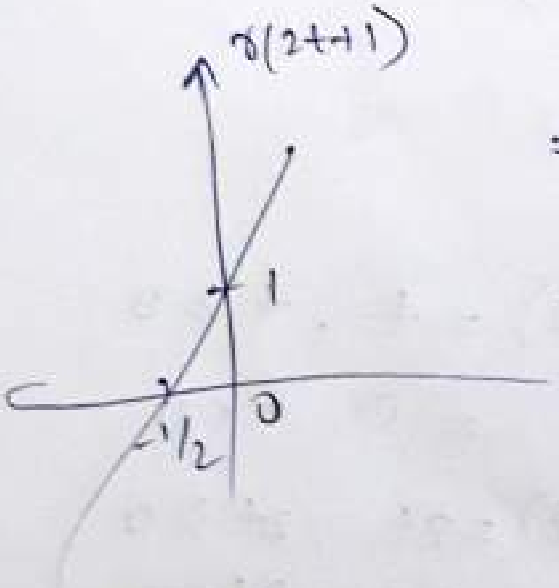


6) $r(2t+1)$

$$r(t) = t, \quad t \geq 0$$
$$= 0, \quad t < 0$$

$$r(2t+1) = 2t+1, \quad 2t+1 \geq 0$$
$$, \quad t \geq -1/2$$

$$= 0 \quad 2t+1 < 0$$
$$t < -1/2$$



* $\delta(at-b) = \frac{1}{|a|}\delta(t-b/a)$

Pb) Evaluate $\delta(2t-3)$

$$\frac{1}{|2|}\delta(t-3/2)$$

Pb) Evaluate $\delta(-2t-3)$

$$\frac{1}{2}\delta(t-3/_{-2})$$

$$\frac{1}{2}\delta(t+3/2)$$

Precedence rule is not applicable for impulse function in time scaled

Pb) $\int_{-\infty}^{\infty}(t^2+1)\,\delta(-2t-3)\,dt$

$$\int_{-\infty}^{\infty}(t^2+1)\frac{1}{2}\delta(t+3/2)\,dt$$

$$\frac{1}{2}\int_{-\infty}^{\infty}(t^2+1)\,\delta(t+3/2)\,dt$$

$$= \frac{1}{2}x(t_0)$$

$$= \frac{1}{2}(t^2+1)_{t=-3/2}$$

$$= \frac{1}{2}\left(\frac{9}{4}+1\right)$$

$$= 13/8$$

Pb1) Evaluate $\sum_{-\infty}^{\infty} n^2 \delta(n-4)$

$x(n) \quad \delta(n-n_0)$

$= (n^2)_{n=4}$

$= (4)^2 = 16$

Pb2) $\sum_{-1}^{2} 4^n \delta(n+3)$

$= 0$ ($\because$ 3 does not lie b/w -1 & 2)

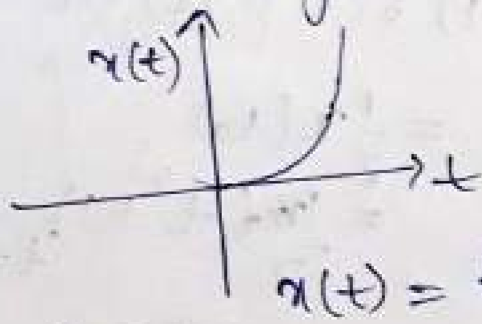
Pb3) $\sum_{-\infty}^{\infty} 4^n \delta(n+3)$

$= x(n_0)$

$= (4^n)_{n=-3}$

$= 4^{-3} = 1/64$

* Parabolic Signal:-



$x(t) = t^2/2, \ t \geq 0$

Mathematically, $= 0, \ t < 0$

$x(t) = \frac{t^2}{2} u(t)$

$$\frac{dx}{dt} = r(t)$$

differentiation of parabolic signal is a ramp

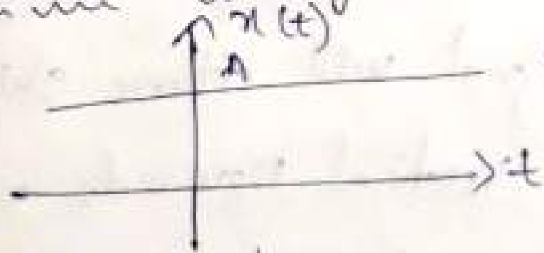
$$u(t) = \frac{d^2}{dt^2} x(t)$$

$$\delta(t) = \frac{d^3}{dt^3} x(t)$$

$$x(t) \xrightarrow{\frac{d}{dt}} r(t) \xrightarrow{d/dt} u(t) \xrightarrow{d/dt} \delta(t)$$

$$\delta(t) \xrightarrow{\int_{-\infty}^{t}} u(t) \xrightarrow{\int_{-\infty}^{t}} r(t) \xrightarrow{\int_{-\infty}^{t}} x(t)$$

* Constant / DC signal :-



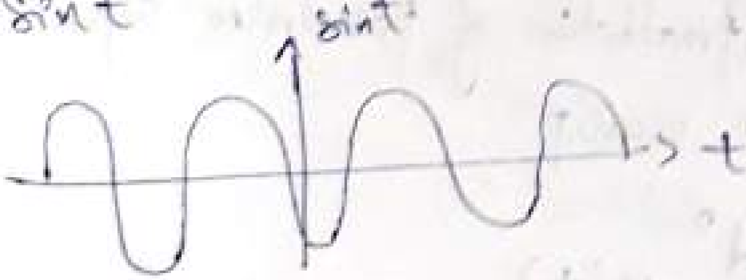
$x(t) = A, \{\forall t\}$

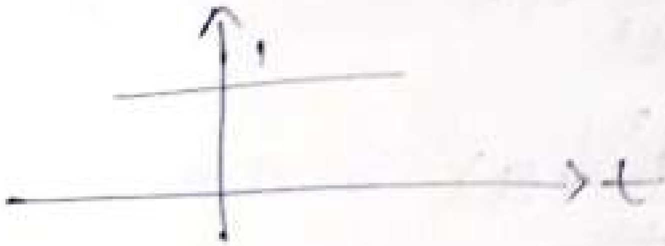
Mathematically $x(t) = A, \{\forall t\}$

$$x(t) = A\left(u(t) + u(-t)\right)$$

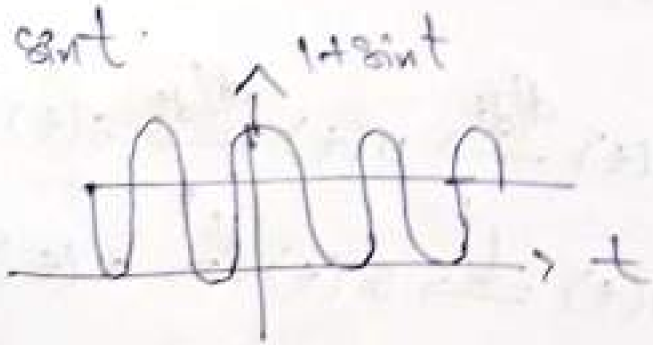
Pb) Plot $1 + \sin t$

$\sin t$



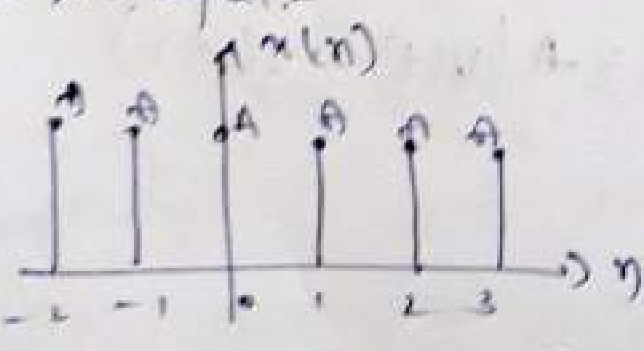


$1 + \sin t$



note:-
A D.C signal when added to any arbitrary signal will either shift the signal vertical upwards or downwards as shown

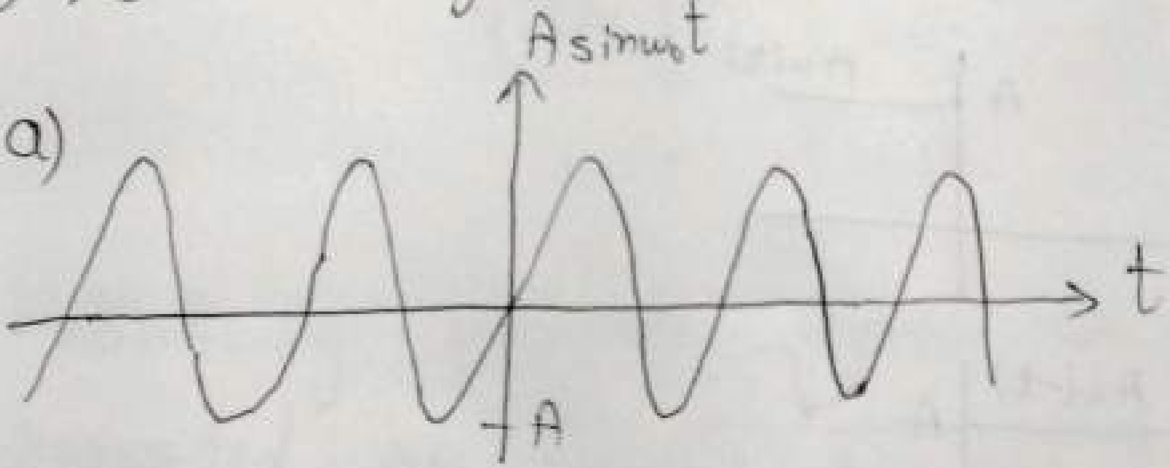
Discrete D.C signal:-



$x(n) = A$

7.) Sinusoidal signal:- $\sin\omega_0 t, \cos\omega_0 t$

a)



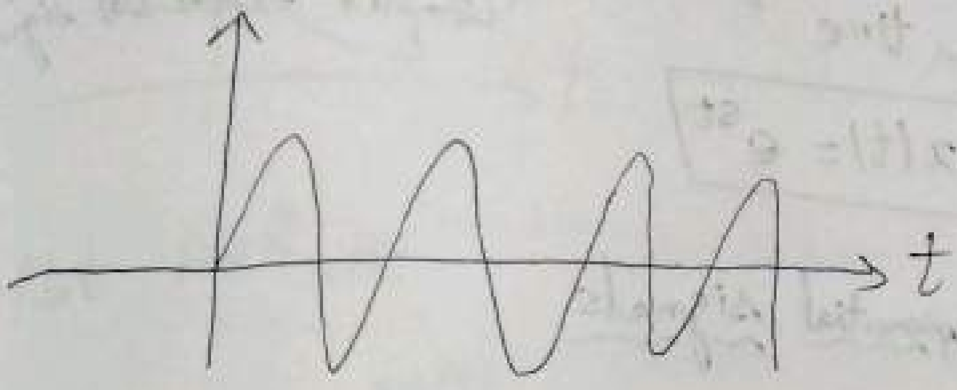
$x(t) = A\sin\omega_0 t$

A - maximum amplitude ω_0 - angular frequency

f_0 - frequency $\omega_0 - 2\pi f_0$

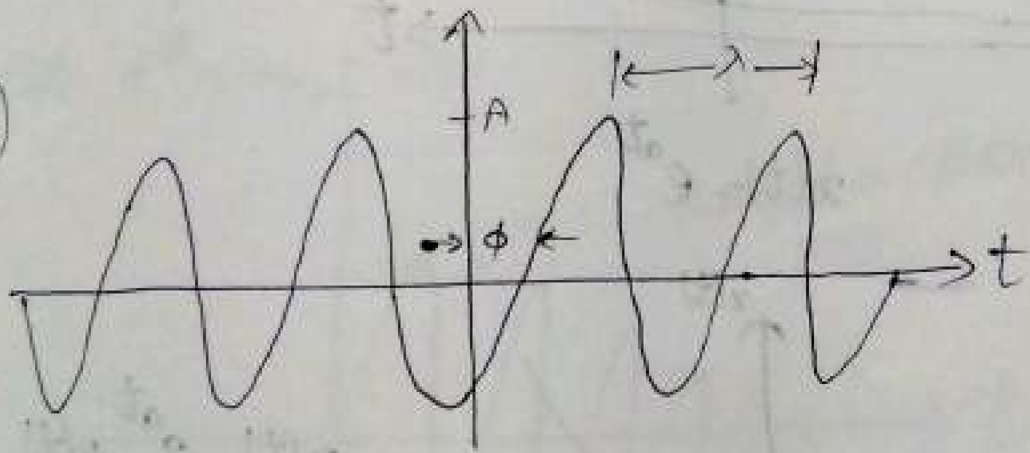
$T_0 = \frac{1}{f_0}$

P6) Plot $\sin \omega_0 t\, u(t)$



$$x(t) = A \sin \omega_0 t, \quad t \geq 0$$
$$= 0 \quad , \quad t < 0$$

b)



$$x(t) = A \sin(\omega_0 t + \phi)$$

$\phi \rightarrow$ phase

$$\phi = \frac{2\pi}{\lambda} \times d$$

The phase of a signal dictates the offset of the signal from its origin.

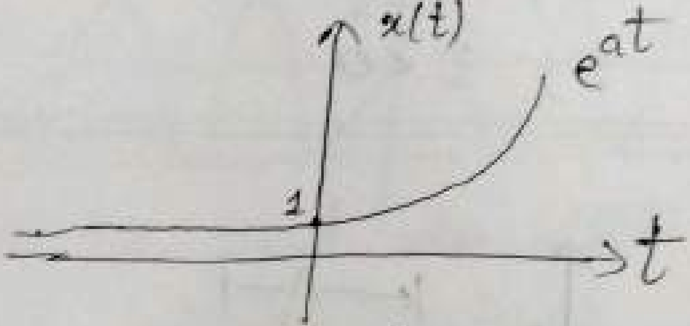
8.) Exponential Signal ← Real exponential signals / Complex exponential signals

(1) Continuous time

$$\boxed{x(t) = e^{st}}$$

→ Real exponential signals

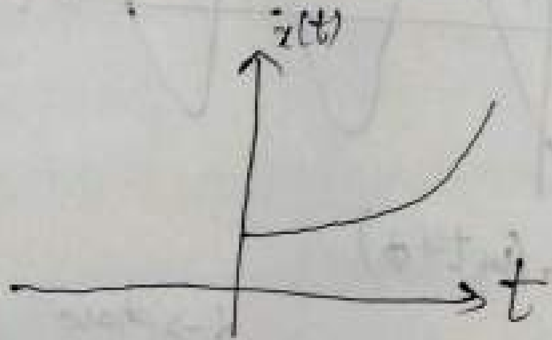
a) Exponentially growing signals



a - real variable

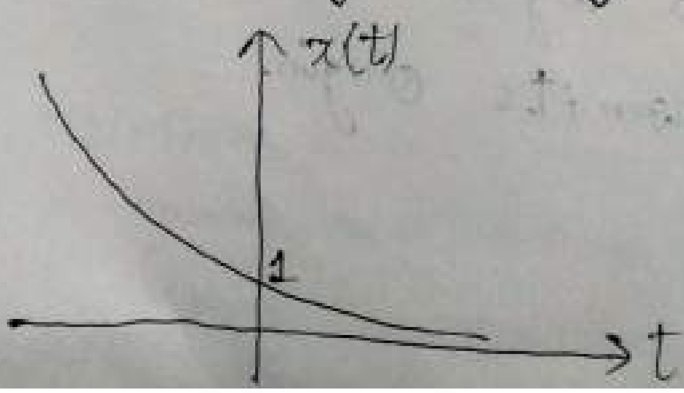
$x(t) = e^{at}$

b) (default)



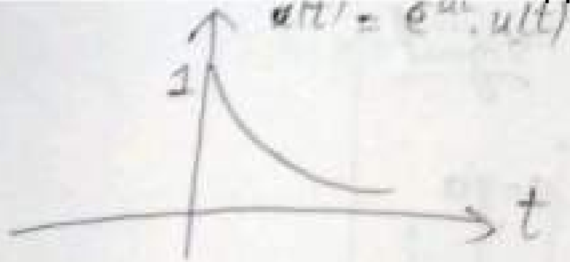
$x(t) = e^{at}.u(t)$

c). Exponentially ~~decreasing~~ decaying signal



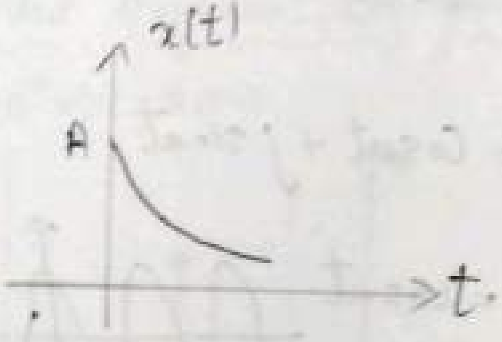
$x(t) = e^{-at}$

d)



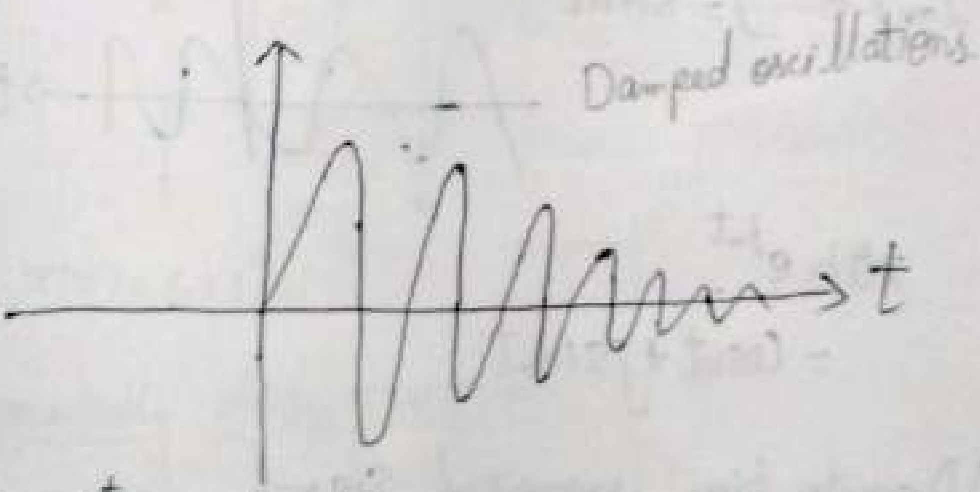
Pb) Plot $Ae^{-at}u(t)$

Sol:-



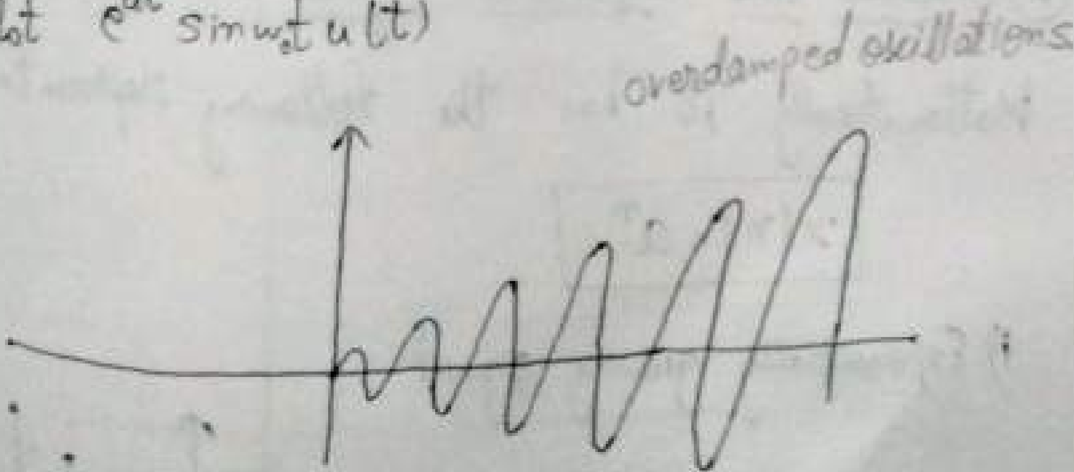
Pb) Plot $e^{-at} \sin w_0 t . u(t)$

Sol:-



Pb) Plot $e^{at} \sin w_0 t \, u(t)$

Sol:-



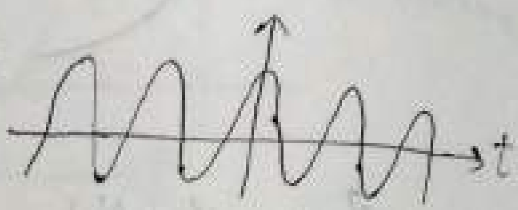
→ Complex exponential signal

$x(t) = e^{jat}$, $s = ja$

Complex exponential function is also a periodic ~~signal~~ function similar to sinusoidal signal

$x(t) = e^{jat} = \cos at + j\sin at$

$Re[e^{jat}] = \cos at$



$Im[e^{jat}] = \sin at$

$x(t) = e^{j\omega t}$

$= \cos\omega t + j\sin\omega t$

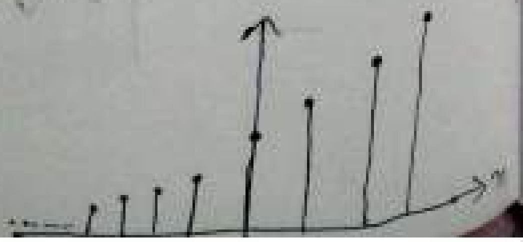
② Discrete time exponential signal

Mathematically it has the following representation

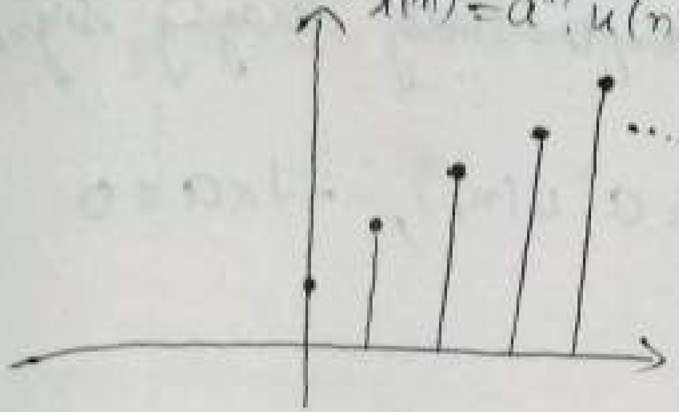
$\boxed{x(n) = a^n}$

i) Exponentially growing signal

$x(n) = a^n$, $a > 1$

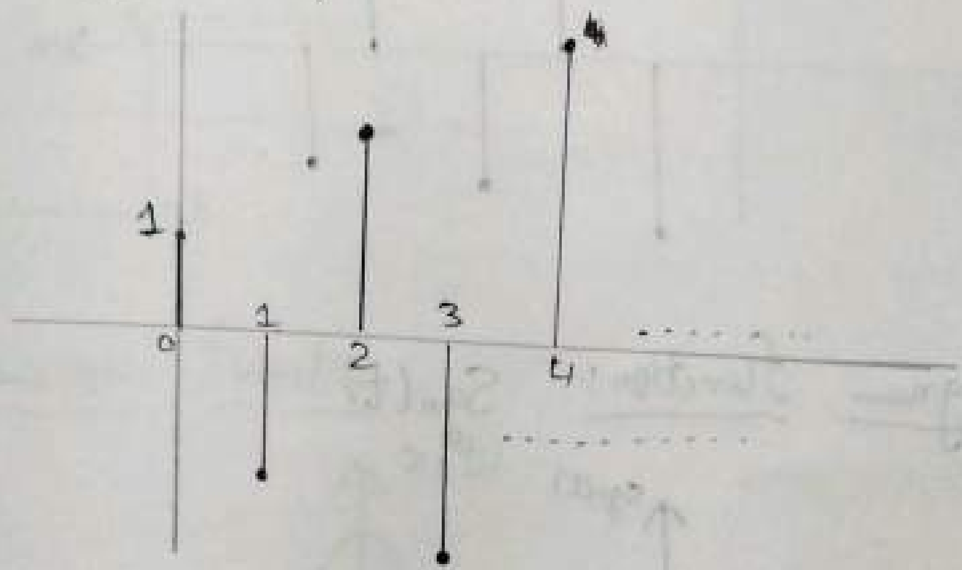


$x(n) = a^n . u(n), \ a > 1$



ii) Double sided exponentially growing signal:-

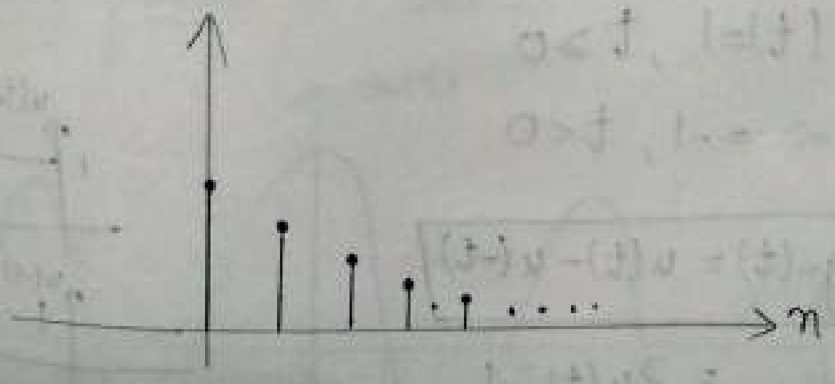
$x(n) = a^n . u(n) \ , a < -1$



$x(n) = (-2)^n u(n)$

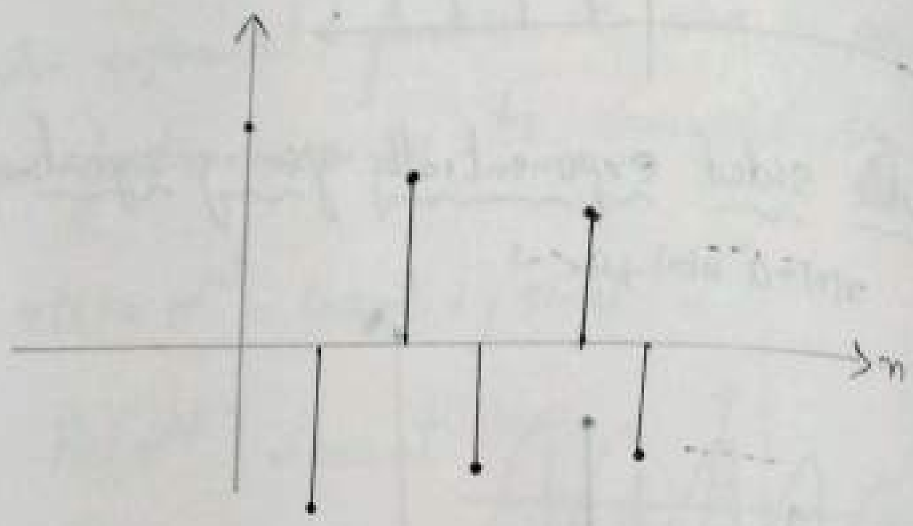
iii) Exponentially decaying signal

$x(n) = a^n u(n), \quad 0 < a < 1$

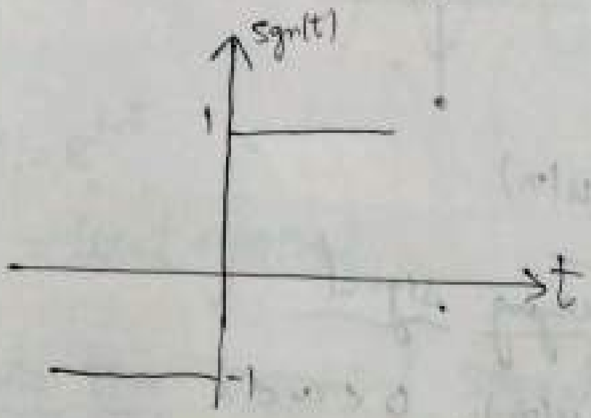


iv) Double sided exponentially decaying signal

$$x(n) = a^n . u(n) \quad , \quad -1 \leq a \leq 0$$



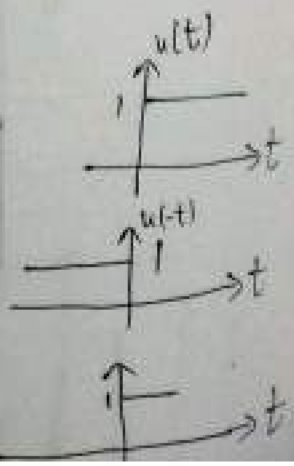
⑨ Signum function :- Sgn(t)

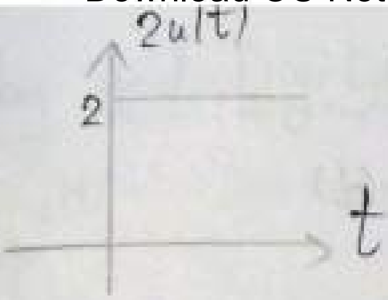


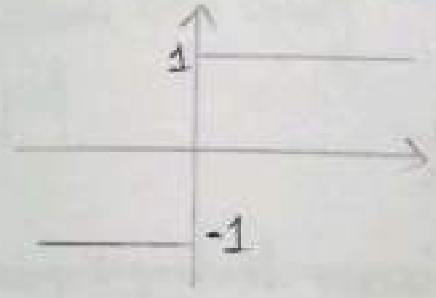
$$Sgn(t) = 1 \;, \; t \geq 0$$
$$= -1 \;, \; t < 0$$

☆ $\boxed{Sgn(t) = u(t) - u(-t)}$

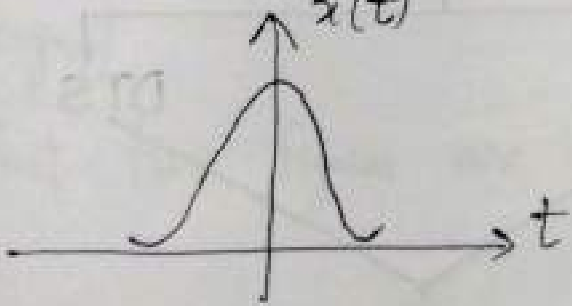
$$= 2u(t) - 1$$







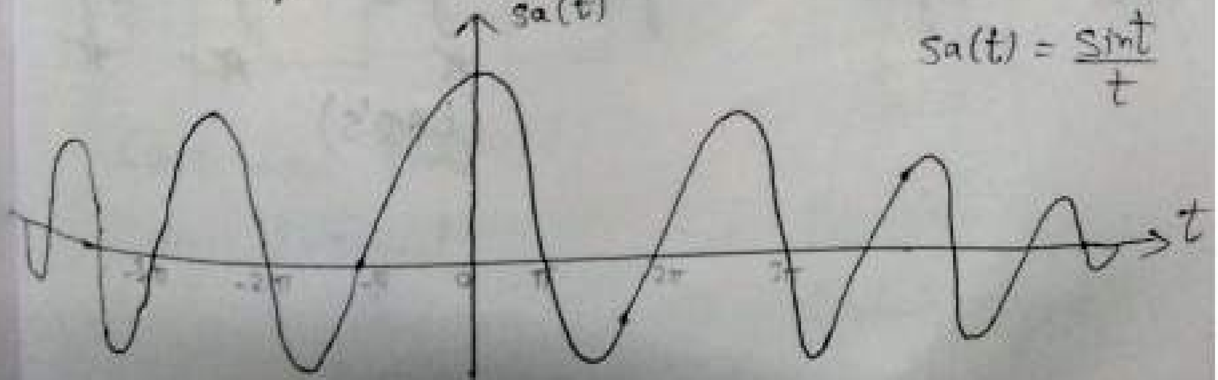
10.) Gaussian function:-



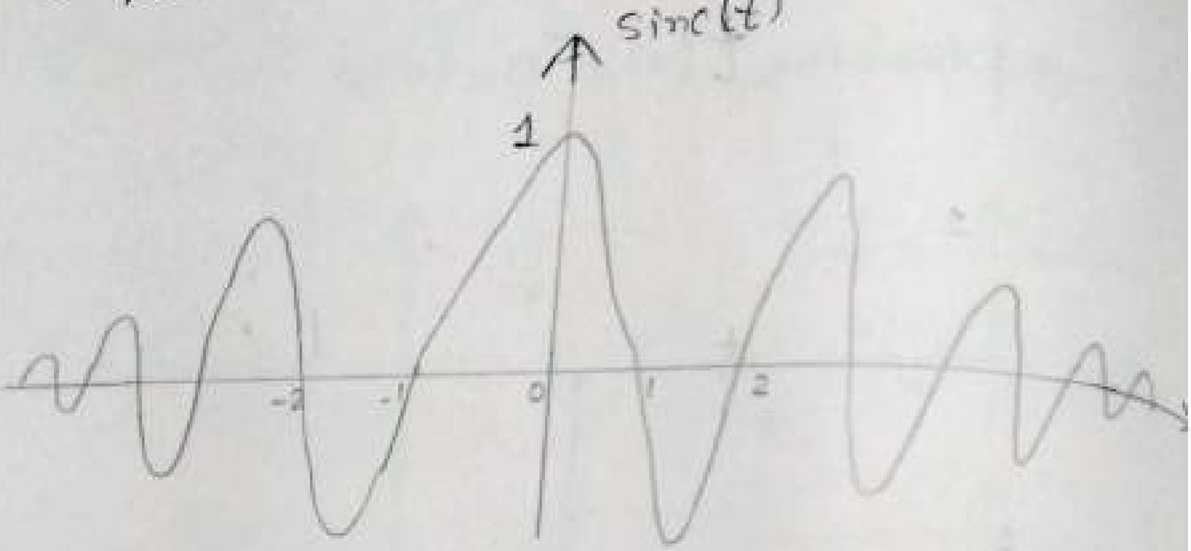
$x(t) = e^{-at^2}$

11.) Sampling function :- Sa(t)

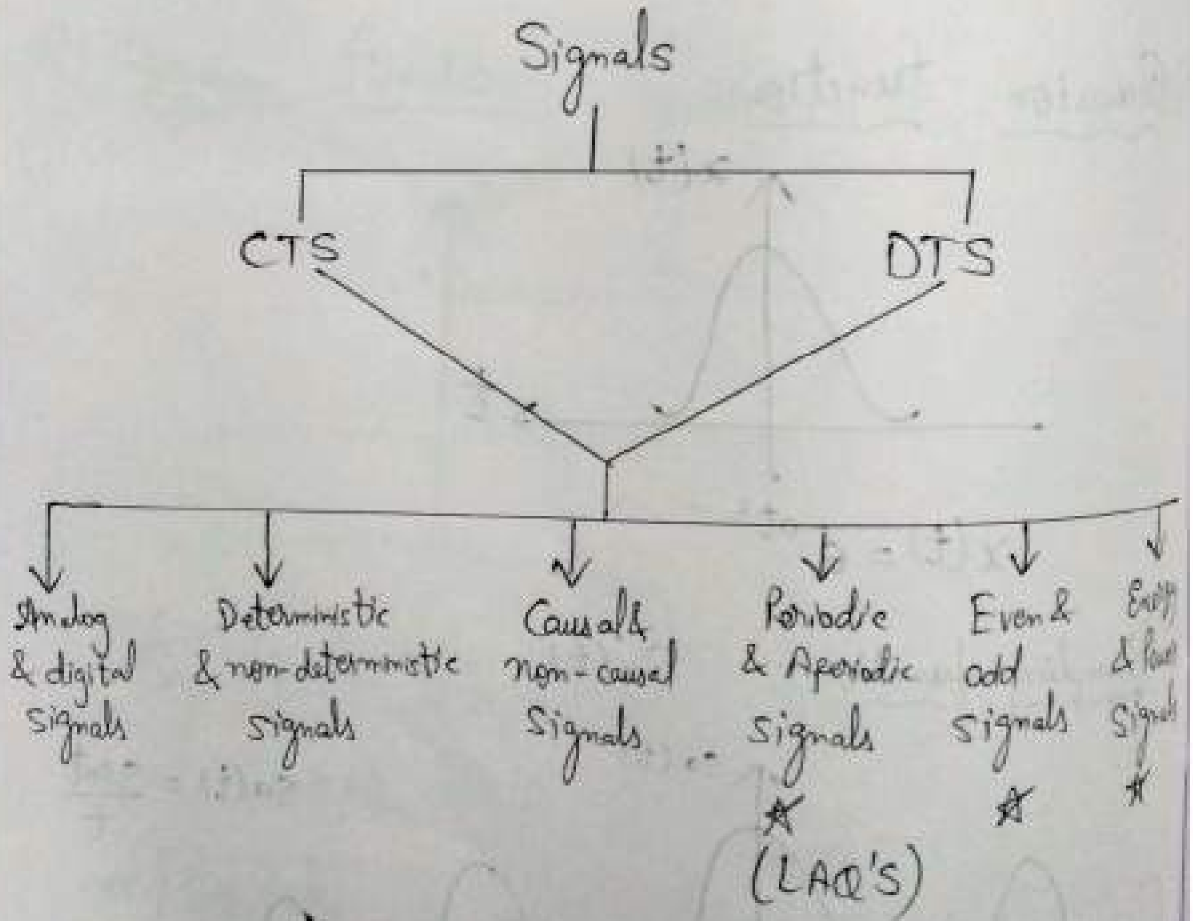
$Sa(t) = \frac{\sin t}{t}$



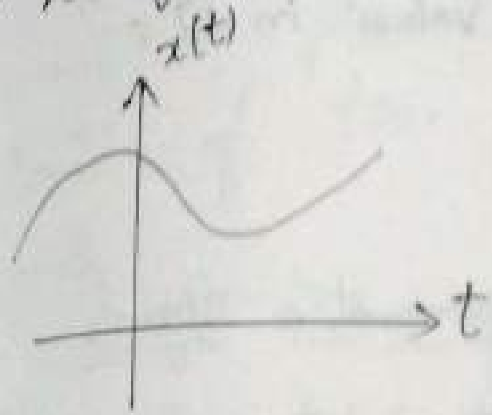
12.) Sinc function sinc(t)



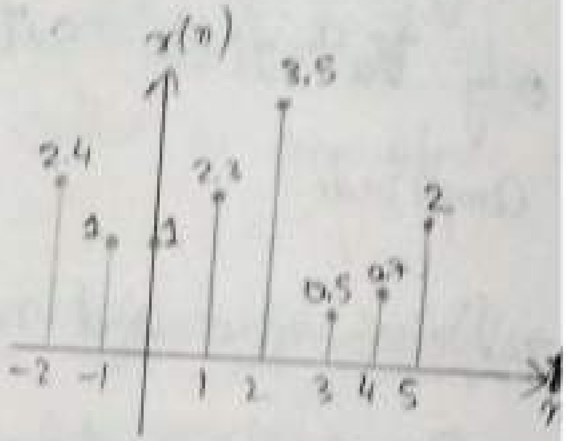
$$\text{Sinc}(t) = \frac{\sin \pi t}{\pi t}$$



1.) Analog and Digital Signals



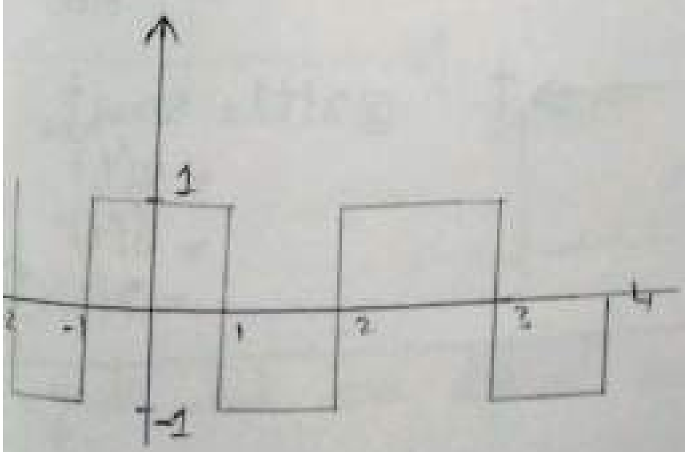
Continuous time analog signal



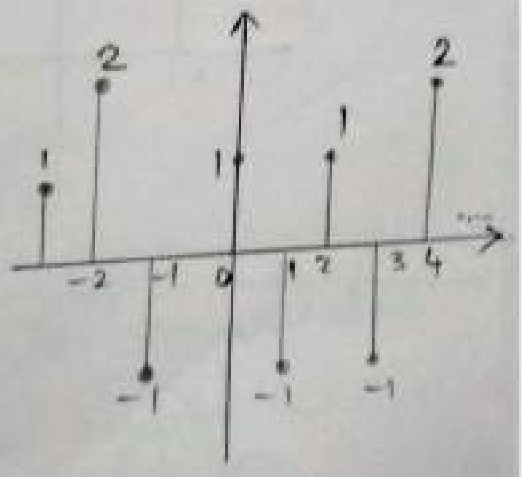
Discrete time analog signal

Analog signal - A signal which can take a continuum range of values (all real values) is called as an analog signal.

The first two figures are analog signals.



Continuous time digital signal.



Discrete time digital signal.

Digital signals are signals which can take only ~~the~~ a finite set of values in its amplitude

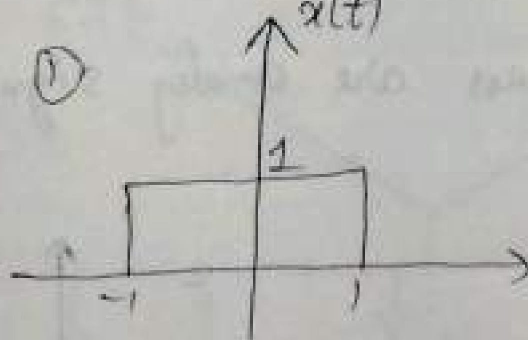
→2) Deterministic and non-deterministic signals

Signal whose value is certain at any instant of time is called deterministic

(or)

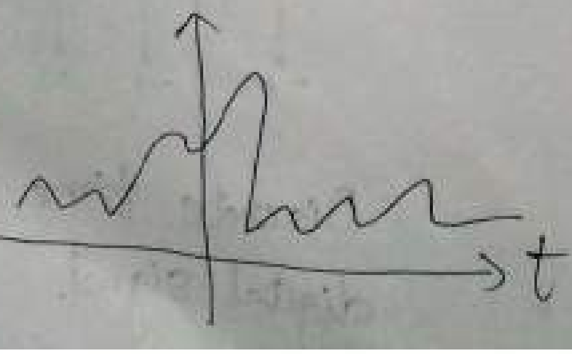
Signal whose value can be determined or predicted at any instant of time are called as deterministic signals

Eg:- ①



$x(t) = 1, \; -1 \leq t \leq 1$
$= 0, \; \text{else}$

② $x(t) = \sin w_0 t$
$= u(t)$
$= \delta(t)$
$= r(t) = \frac{t\,u(t)}{2}$



Non-deterministic signal

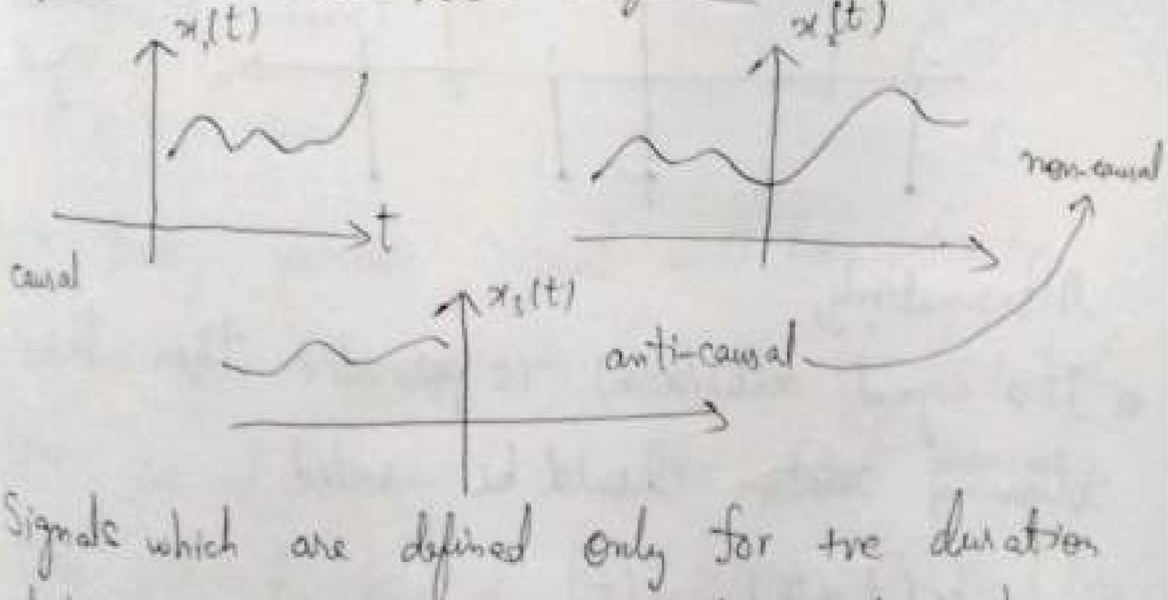
Signals whose values are uncertain or cannot be predicted or determined at any instant of time are called as non-deterministic signals

Since the nature of these signals is random, they are also called as random signals

Eg:- 1.) The noise present in T.V and audio receivers

2.) ECG and EEG

3.) Causal and non-causal signals

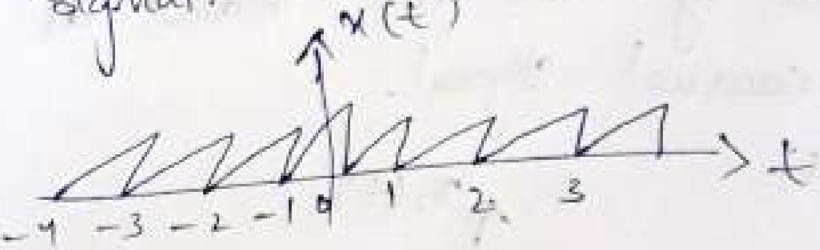


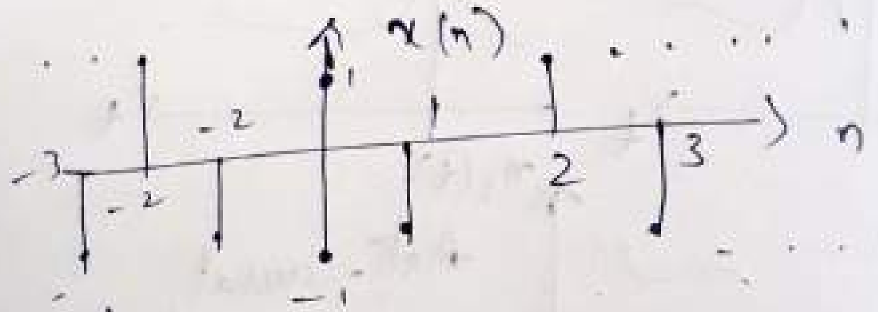
Signals which are defined only for +ve duration of time are called as causal signals whereas signals which have values defined for -ve duration of time are termed as non-causal signals

u(t), r(t) are causal signals. sin $\omega_0 t$, sgn(t) are non-causal

Periodic & Aperiodic Signals:-

→ Signals which have a particular pattern repeating at regular intervals of time are called as periodic signals. Otherwise called Aperiodic signal.





Alternatively,

If a signal x(t) / x(n) is periodic then the following relation should be valid.

$$x(t \pm T) = x(t)$$

$$x(n \pm N) = x(n)$$

T, N – Time periods of CTS & DTS respectively.

Note :- for a signal to be periodic, the neccessary condition is, it should be ever lasting. That is it should exists for $-\infty \le t \le \infty$

$-\infty \le n \le \infty$

Note :- Every periodic signal will have infinite number of time periods, but one fundamental time period, which is the least possible time period. Denoted by letter T_0, N_0 for CTS, DTS respectively.

* check the periodicity & determine it if any :-

1) $x(t) = \sin\omega_0 t$, $T = \frac{2\pi}{\omega_0}$

$$x(t+T_0) = x(t)$$

$$\sin\omega_0(t+T_0) = \sin\omega_0 t$$

$$\sin(\omega_0 t + \omega_0 T_0) = \sin\omega_0 t$$

$$\omega_0 T_0 = m(2\pi)$$

$$m = 1 , T = T_0$$

$$** \boxed{T_0 = \frac{2\pi}{\omega_0}} **$$

2) $x(t) = \sin 2t$

Sol
$$x(t+T_0) = x(t)$$
$$\sin 2(t+T_0) = \sin 2t$$
$$\sin 2t + 2T_0$$
$$\omega_0 = 2 \qquad T_0 = 2\pi/\omega_0$$
$$T_0 = \frac{2\pi}{2}$$
$$\boxed{T_0 = \pi}$$

3) $x(t) = \sin\sqrt{2}t$

Sol
$$\omega_0 = \sqrt{2}$$
$$T_0 = \frac{2\pi}{\sqrt{2}}$$
$$\boxed{T_0 = \sqrt{2}\pi}$$

Note:- All Continuous sinusoids are periodic

4) $x(n) = \sin\omega_0 n$

Sol
$$x(n+N) = x(n)$$
$$\sin\omega_0(n+N) = \sin\omega_0 n$$
$$\sin(n\omega_0 + N\omega_0) = \sin\omega_0 n$$

$\omega_0 N = K(2\pi)$

$$\boxed{N = \frac{2\pi k}{\omega_0}}$$

$$\boxed{\frac{N}{K} = \frac{2\pi}{\omega_0}}$$

→ The above condition states that for a discrete time sinusoid to be periodic, the ratio $2\pi/\omega_0$ must be a rational number.

→ Check the periodicity $x(n) = \sin \omega_0 n$

1) $x(n) = \sin 2n$

Sol $\omega_0 = 2$

$$\frac{2\pi}{\omega_0} = \frac{2\pi}{2}$$

$= \pi$ (irrational)

Its Aperiodic

$N_0 = \infty$

2) $x(n) = \sin 2\pi n$

$\omega_0 = 2\pi$

$$\frac{2\pi}{\omega_0} = \frac{2\pi}{2\pi} = 1 \text{ Periodic}$$

$$N = \frac{2\pi k}{\omega_0}, k = 1, 2, 3 \ldots$$

$$N = \frac{2\pi}{2\pi} k$$

$$N = k$$

$$\boxed{N_0 = 1}$$

3) Given:- $\sin \frac{2\pi}{3} n$

$\omega_0 = 5\pi/3$

$\Rightarrow \cancel{\frac{2\pi}{\omega_0}} = \cancel{\frac{5\pi}{3 \cdot 2\pi/3}} = \cancel{\frac{3/5}{2}}$

$$\frac{2\pi}{\omega_0} = 2\pi / \frac{5\pi}{3} = \frac{6}{5}$$

Periodic

$N = \frac{2\pi}{\omega_0} k$, ~~.......~~ ...

$$N = \frac{2\pi}{5\pi/3} k$$

$N = \frac{6}{5} k$, $k = 5, 10, 15 \ldots$

$$N_0 = \frac{6}{5}(5) = 6$$

$$\boxed{N_0 = 6}$$

__Note__:- The Discrete time sinusoids may or may not be periodic.

* $x(t) = e^{2/3jt}$

1) w.k.t $x(t) = e^{\omega_0 jt}$

$$\omega_0 = 2/3$$

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{2/3} = 3\pi$$

$$(T_0 = 3\pi)$$

(or)

$$e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t$$

$$e^{2/3jt} = \cos\frac{2}{3}t + j\sin\frac{2}{3}t$$

$$= 3\pi + 3\pi$$

$$\Rightarrow \text{l.c.m}(3\pi, 3\pi)$$

$$(T_0 = 3\pi)$$

2) $x(n) = e^{2\pi jn}$

$$\omega_0 = 2\pi$$

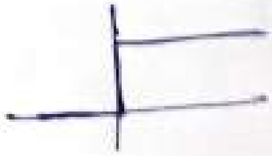
$$\frac{2\pi}{\omega_0} = \frac{2\pi}{2\pi} = 1$$

$$N = \frac{2\pi k}{\omega_0} = \frac{2\pi}{2\pi}k.$$

$$N = k, \quad k = 1, 2, 3 \ldots$$

$$(N_0 = 1)$$

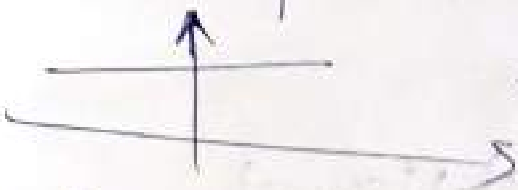
3) $u(t)$



A periodic signal

$x(-t) \rightarrow$ A periodic signal

Constant signal



A periodic signal

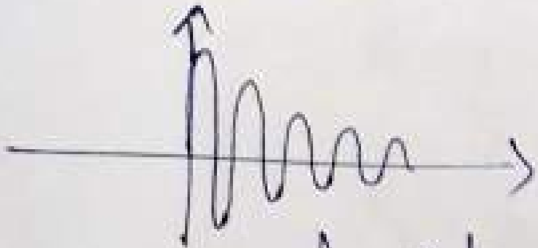
but discrete time constant signal is

$\rightarrow x(n) = 1$

periodic

$\{T = 1\}$

4) $x(t) = e^{-t} \sin t \, u(t)$



Aperiodic

$(T_0 = \infty)$

5) $x(t) = e^{-(2+3j)t}$

Sol Given:- $x(t) = e^{-2t} \cdot e^{-3jt}$

Aperiodic signal   Periodic

$\therefore$ Aperiodic signal.

6) $x(t) = e^{-2+3jt}$

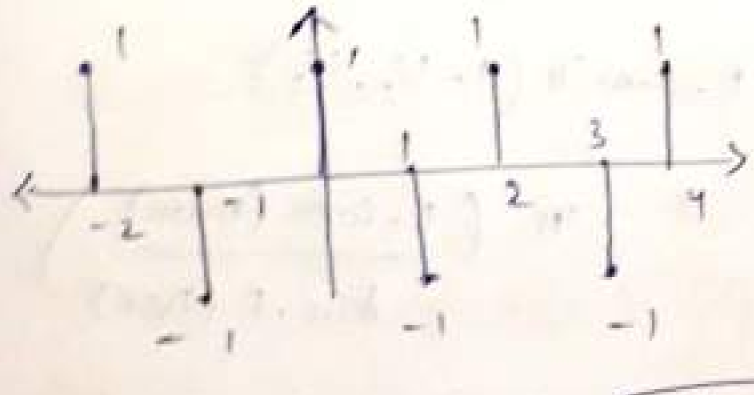
Sol Given:- $x(t) = e^{-2} \cdot e^{3jt}$

Amplitud   Periodic

$\therefore$ Its periodic

$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{3}$

$\rightarrow x(n) = (-1)^n$



Periodic $\Rightarrow \boxed{N = 2}$

(1)

→ $x(t) = \sin 2t + 2\sin 4t + 3\sin 6t$

Sol

S1) $T_1 = \frac{2\pi}{2}$, $T_2 = \frac{2\pi}{4}$ $T_3 = \frac{2\pi}{6}$

$= \pi$ $= \pi/2$ $= \pi/3$

S2) If the ratios of time periods is rational then the signal is periodic otherwise Aperiodic.

$\frac{T_2}{T_1}, \frac{T_3}{T_1}$

$= 1/2$

$= 0.5$ periodic

S3) L.C.M (T_1, T_2, T_3)

LCM $(\pi, \frac{\pi}{2}, \frac{\pi}{3})$

L.C.M $\pi(1, 1/2, 1/3)$

$\pi \left(\frac{\text{L.C.M (num)}}{\text{H.C.F (Den)}} \right)$

$\frac{\pi (\text{L.C.M} (1,1,1))}{\text{H.C.F} (1,2,3)}$

$= \frac{\pi \times 1}{1} = \pi.$

(2) $x(t) = 2\sin 3t + 3\sin 2t + 4\sin 3t$

s1) $T_1 = \frac{2}{3}$, $T_2 = \pi$, $T_3 = \frac{2\pi}{3}$

s2) $\frac{T_1}{T_2} = \frac{2/3}{\pi} = \frac{2}{3\pi}$ irrational

$\frac{T_3}{T_2} = \pi/_{2\pi/3} = 3/_2 =$ rational

Aperiodic

S3) $T = \infty$

Some of the Continous time sinusoid are may or may not be periodic

(3) $x(n) = 2\cos(6\pi n + 90^\circ) + 3\cos(2\pi n + 15^\circ)$
$+ \sin 8\pi n$

sol

$N_1 = \frac{2\pi}{\omega_0}k$ $\qquad$ $N_2 = \frac{2\pi}{\omega_0}k$

$= \frac{2\pi}{6\pi}k$ $\qquad$ $N_2 = \frac{2\pi k}{2\pi} = k$

$N_1 = k/3 \; (k = 3, 6, 9..)$ $\qquad$ $\boxed{N_2 = 1}$ $k = 1, 2, 3..$

$\boxed{N_1 = 1}$ $(k = 3)$

$N_3 = \frac{2\pi}{8\pi}k$

$= \frac{k}{4} \; (k = 4, 8, \cdots)$

$\boxed{N_3 = 1}$ $\boxed{k = 4}$

S2) $\frac{N_2}{N_1} = 1$ $\frac{N_3}{N_1} = 1$

= rational = rational.

S3) $N = L.C.M(N_1, N_2, N_3)$

$= L.C.M(1,1,1)$

$= 1$

$\boxed{N = 1}$

P) $x(t) = \sin 2t \cos 3t$

Sol $x(t) = \frac{1}{2}[\sin 2t \cos 3t]$

$x(t) = \frac{1}{2}[\sin 5t - \sin t]$

$x(t) = \frac{1}{2}\sin 5t - \frac{1}{2}\sin t$.

S2) $T_1 = \frac{2\pi}{5}$ $T_2 = \frac{2\pi}{1}$

S3) $\frac{T_1}{T_2} = 1/5$ = rational = 0.2

∴ Its periodic

S4) $L.C.M(T_1, T_2)$

$L.C.M(\frac{2\pi}{5}, 2\pi)$

$L.C.M.\ 2\pi(1/5, 1)$

$2\pi \cdot L.C.M(1/5, 1) = 2\pi \frac{L.C.M(1,1)}{G.C.D(5,1)}$

$= \frac{1}{1} \times 2\pi$

$\boxed{T = 2\pi}$

(Pb) $x(t) = \cos^2 4t$ $\quad 1-\sin^2 4t$

$1-$

$\cos^2\theta = 1-4$

$2\cos 2\theta - 1$

$x(t) = \frac{1+\cos 8t}{2}$

$x(t) = \frac{1}{2} + \frac{1}{2}\cos 8t$

$\omega_0 = 8$

$T = \frac{2\pi}{\omega_0}$

$T = \frac{2\pi}{8} = \frac{\pi}{4}$

$\boxed{T = \frac{\pi}{4}}$

(Pb) $x(n) = e^{j6\pi n} + e^{j\frac{2\pi n}{3}}$

$\boxed{N = \frac{2\pi}{\omega_0} k}$

S1) $\omega_1 = 6\pi$

$N = \frac{2\pi}{6\pi} k$

$N = k/3$ $(k = 3, 6, 9 .)$

$\boxed{N = 1}$

S2) $\omega_2 = 2\pi/3$

$N_2 = \frac{2\pi}{2\pi/3} k$

$= 3k.$

$\boxed{N_2 = 3}$

S2) $N_1/N_2 = 1/3 = 0.333$

S3) $N = L.C.M(1, 3) = 3$

5*). Energy & Power Signal :-

→ Energy of any arbitrary signal can be expressed using the following formula.

$E = P \times T$

$$\boxed{E = \int_{-\infty}^{\infty} |x(t)|^2 dt}$$

→ For a DTS the energy can be formulated as follows

$$\boxed{E = \sum_{-\infty}^{\infty} |x(n)|^2}$$

→ The power of a signal can be formulated using the following two.

CTS Expressions:

$$\boxed{P_{avg} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt}$$

DTS

$$\boxed{P_{avg} = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} |x(n)|^2}$$

* Determine whether the following signals are energy or power signal & find the same.

(1) $x(t) = e^{-at} u(t)$

Sol Given:- $x(t) = e^{-at} u(t)$

1) $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$E = \int_{-\infty}^{\infty} |e^{-at} \cdot u(t)|^2 dt$

$\Rightarrow E = \int_{-\infty}^{0} |e^{-at} \cdot 0|^2 dt + \int_{0}^{\infty} |e^{-at} \cdot 1|^2 dt$

$\Rightarrow E = \int_{0}^{\infty} e^{-2at} dt$

$\Rightarrow E = \left[\frac{e^{-2at}}{-2a}\right]_0^{\infty}$

$\Rightarrow E = \frac{-1}{2a}\left[e^{-2a(\infty)} - e^{-2(0)}\right]$

$\Rightarrow E = \frac{-1}{2a}\left[0 - 1\right]$

$\Rightarrow \boxed{E = 1/2a}$

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2) $$P_{avg} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$\Rightarrow P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |e^{-at} u(t)|^2 dt$$

$$\Rightarrow P = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T/2} e^{-2at} dt$$

$$\Rightarrow P = \lim_{T \to \infty} \frac{1}{T} \left[\frac{e^{-2at}}{-2a} \right]_0^{T/2}$$

$$\Rightarrow P = \lim_{T \to \infty} \frac{1}{T} \left[\frac{1}{-2a} \left[e^{\frac{-2aT}{2}} - e^{0} \right] \right]$$

$$\Rightarrow P = \lim_{T \to \infty} \frac{1}{T} \left[\frac{1}{-2a} \left[e^{-aT} - 1 \right] \right]$$

$$\Rightarrow P = \lim_{T \to \infty} \frac{1}{-2aT} \left[e^{-aT} - 1 \right]$$

$$\Rightarrow P = \frac{1}{-2a(\infty)} \left[e^{-\infty} - 1 \right]$$

$$\Rightarrow P = -\left(-\frac{1}{\infty} \right)$$

$$\Rightarrow \boxed{P = 0}$$

∴ The signal is an Energy signal

Energy Signal:-

A signal which has a finite Energy is called as an Energy ~~energy~~ signal.

i.e if $E_{x(t)}$, $0 \leq E_{x(t)} \leq \infty$

then $x(t)$ is an Energy Signal

Note:- Power of Energy Signal is always $= 0$

Pb) $x(t) = u(t)$

Sol Given:- $x(t) = u(t)$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$\Rightarrow E = \int_{-\infty}^{0} 0 \, dt + \int_{0}^{\infty} 1 . dt$$

$$\Rightarrow E = [t]_0^{\infty}$$

$$\Rightarrow E = [\infty - 0] = \infty$$

$$\boxed{E = \infty}$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_0^T |x(t)|^2 dt = \lim_{T \to \infty} \frac{1}{T} \int_0^T 1 . dt$$

$$= \lim_{T \to \infty} \frac{1}{T} [t]_0^T$$

$$\boxed{P = 1} \quad \therefore \text{Its Power signal} \quad = \lim_{T \to \infty} 1 = 1$$

→ Power Signal:-

A signal which has finite power is called power signal i.e

If $P_{x(t)}$, $0 < P_{x(t)} < \infty$, then $x(t)$ is a power signal.

Note:- The energy of a power signal is always infinite.

$$E_{x(t)} = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E_{x(n)} = \sum_{-\infty}^{\infty} |x(n)|^2$$

$$P_{avg\, x(t)} = \underset{T\to\infty}{Lt} \frac{1}{T} \int_{\langle T \rangle} |x(t)|^2 dt$$

$$P_{avg\, x(n)} = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{\langle 2N \rangle} |x(n)|^2$$

3.) $x(t) = A\cos(\omega_0 t + \theta)$

Sol:- $E = \int_{-\infty}^{\infty} |A\cos(\omega_0 t + \theta)|^2 dt = A^2 \int_{-\infty}^{\infty} \cos^2(\omega_0 t + \theta) dt$

$$= A^2 \int_{-\infty}^{\infty} \frac{1 + \cos 2(\omega_0 t + \theta)}{2} dt$$

$$E = A^2\int_{-\infty}^{\infty}\frac{1}{2}\,dt + A^2\int_{-\infty}^{\infty}\cos 2(\omega_0 t+\theta)\,dt$$

$$= \infty + \text{''}$$

$$= \infty$$

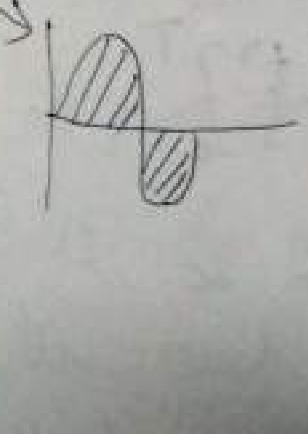
$$P = \underset{T\to\infty}{Lt}\ \frac{1}{T}\int_{\langle T\rangle}|A\cos(\omega_0 t+\theta)|^2\,dt$$

$$= \underset{T\to\infty}{Lt}\ \frac{A^2}{T}\int_{\langle T\rangle}\frac{1+\cos 2(\omega_0 t+\theta)}{2}\,dt$$

$$= \underset{T\to\infty}{Lt}\ \frac{A^2}{2T}\left[\int_{\langle T\rangle}1\,dt + \int_{\langle T\rangle}\cos 2(\omega_0 t+\theta)\,dt\right]$$

$$= \underset{T\to\infty}{Lt}\ \frac{A^2}{2T}\left[\,[t]_0^T + \left(\sin 2(\omega_0 t+\theta)\right)_0^T\right]$$

$$= \underset{T\to\infty}{Lt}\ \frac{A^2}{2T}\left[T+0\right] = \frac{A^2}{2} \qquad \therefore \boxed{P = \frac{A^2}{2}}$$



4.) $x(t) = r(t)$

Sol:- $x(t) = t.u(t)$ or $\begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |t.u(t)|^2 dt$$

$$= \int_{0}^{\infty} |t.1|^2 dt + \int_{-\infty}^{0} |t.0|^2 dt$$

$$= \left[\frac{t^2}{2}\right]_0^{\infty} = \infty$$

$$\therefore \boxed{E = \infty}$$

$$P = \underset{T \to \infty}{Lt} \frac{1}{T} \int_{\langle T \rangle} |x(t)|^2 dt$$

$$= \underset{T \to \infty}{Lt} \frac{1}{T} \int_{\langle T \rangle} |t.u(t)|^2 dt$$

$$= \underset{T \to \infty}{Lt} \frac{1}{T} \int_{0}^{T} |t.u(t)|^2 dt$$

$$= \underset{T \to \infty}{Lt} \frac{1}{T} \int_{0}^{T} |t.1|^2 dt = \underset{T \to \infty}{Lt} \frac{1}{T} \left[\frac{t^3}{3}\right]_0^{T}$$

$$= \underset{T \to \infty}{Lt} \frac{T^3}{3T} = \infty$$

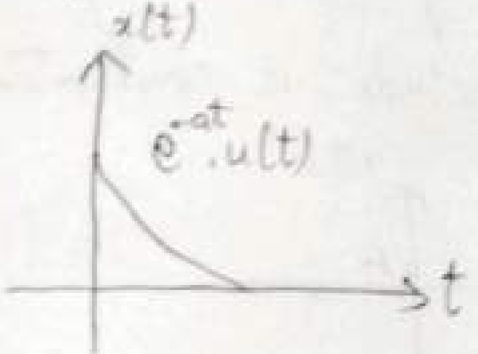
$$\therefore \boxed{P = \infty}$$

Therefore, neither energy nor power signal

Note:-

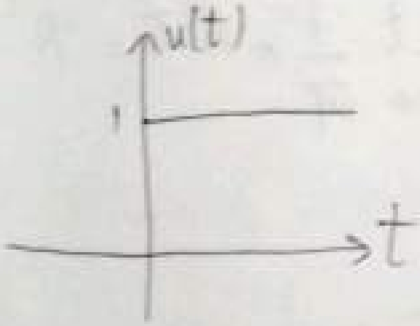
① For energy signals as $t \to \infty$, Amplitude $(-\infty)$ e^{∞}

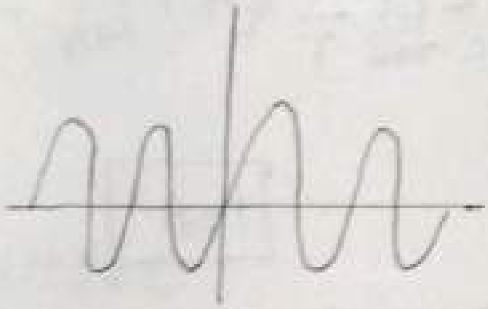
$A \to 0$



Same for DTS, i.e as $n \to \infty$, $A \to 0$

② As $t \to \infty$, if amplitude remains constant then such signals are always power signals

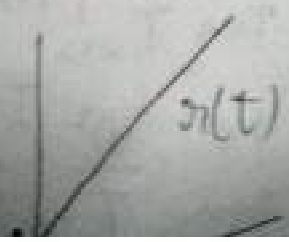




All periodic signals are power signals but all power signals are not periodic signals.

③ If for any signal as $t \to \infty$, if $A \to \infty$ then such signals are neither energy nor power signal

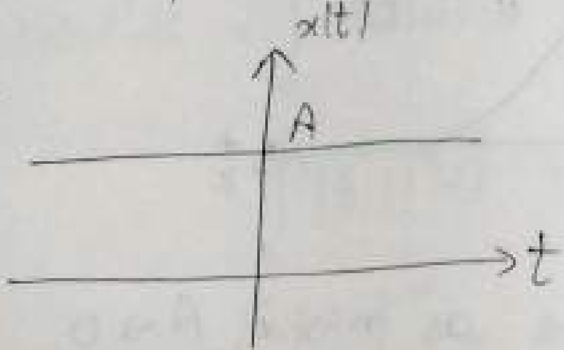
Eg:- Ramp signal
parabolic signal



Pb) $x(t) = A$

Sol:- Just by looking we can say it is a power signal as amplitude is constant.

Method:-



$$P = \underset{T\to\infty}{Lt} \frac{1}{T} \int_{<T>} |x(t)|^2 dt = \underset{T\to\infty}{Lt} \frac{1}{T} \int_{-T/2}^{T/2} |A|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{T} \times [A^2 T]_{-T/2}^{T/2} = \underset{T\to\infty}{Lt} \frac{1}{\cancel{T}} \times A^2 \cancel{T} = A^2$$

$$\therefore \boxed{P = A^2}$$

$\therefore$ Power of D.C signal is A^2

Pb) $x(t) = A e^{j(5t + \frac{\pi}{2})}$

Sol:- It is a power signal

$$P = \underset{T\to\infty}{Lt} \frac{1}{T} \int_{<T>} |A e^{j(5t+\pi/2)}|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{T} \int_0^T 1 \, dt = \underset{T\to\infty}{Lt} \frac{A^2}{T} \times T = A^2$$

Pb) $x(t) = e^{2t}.u(-t)$

as $t \to \infty$

$|x(t)| \to 0$ It is an energy signal

$e^{\infty} u(-\infty) = 0.e^{\infty} = 0$

$$E = \int_{-\infty}^{\infty} |e^{2t}u(-t)|^2 dt$$

$u(-t) = 1, t < 0$
$= 0, t \geq 0$

$$= \int_{-\infty}^{0} |e^{2t}.1|^2 dt + \int_{0}^{\infty} |e^{2t}.0|^2 dt$$

$$= \int_{-\infty}^{0} e^{4t} dt = \left[\frac{e^{4t}}{4}\right]_{-\infty}^{0} = \frac{1}{4}(1-0) = \frac{1}{4}$$

Pb) $x(t) = e^{-2t}.u(t-1)$

[$\because u(t) = 1$]

Sol:- as $t \to \infty$, $|x(t)| = 0$

$\therefore$ Energy signal

$$E = \int_{-\infty}^{\infty} |e^{-2t}.u(t-1)|^2 dt$$

$u(t-1) = 1, t-1 \geq 0$
$t \geq 1$
$= 0, t < 1$

$$E = \int_{-\infty}^{1} |e^{-2t}.0|^2 dt + \int_{1}^{\infty} |e^{-2t}.1|^2 dt = \int_{1}^{\infty} e^{-4t} = \left[\frac{e^{-4t}}{-4}\right]_{1}^{\infty}$$

$$= -\frac{1}{4}\left(e^{-\infty} - e^{-4}\right) = \frac{e^{-4}}{4} = \frac{1}{4e^4}$$

Pb) $x(t) = je^{j5t}$

Sol:- $je^{j5t} = e^{j\pi/2} \cdot e^{j5t}$

$= e^{j(5t+\pi/2)}$

$\left[\because e^{j\pi/2} = \cos\pi/2 + j\sin\pi/2 \right.$
$= 0 + j(1) = j$

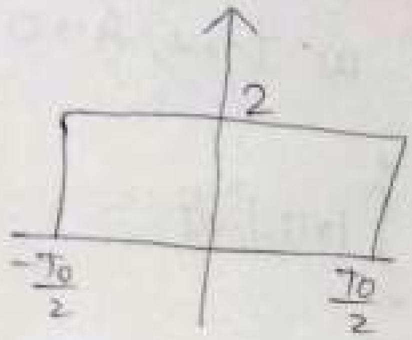
It's a power signal.

Pb) $x(t) = t-2 \quad -2 \le t \le 0$
$= 2-t \quad 0 \le t \le 2$
$= 0 \quad \text{else}$

* All finite duration signals are energy signals

Sol:- $E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-2}^{0} (t-2)^2 dt + \int_{0}^{2} (2-t)^2 dt$

P6) $x(t) = 2\,\mathrm{rec}\left(\frac{t}{T_0}\right)$



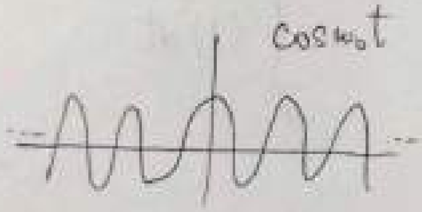
Sol: $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

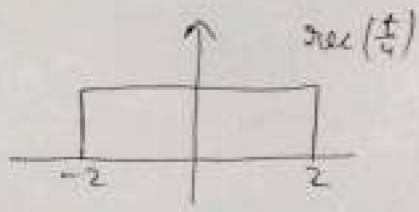
$= \int_{-T_0/2}^{T_0/2} |2|^2 dt$

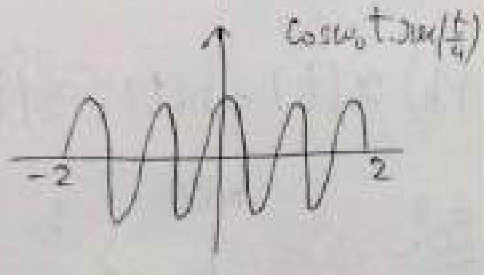
$= \int_{-T_0/2}^{T_0/2} 4\,dt = 4T_0$

$\boxed{P = 0}$

P6) $x(t) = \cos\omega_0 t \cdot \mathrm{rec}\left(\frac{t}{4}\right)$







Sol:- $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$= \int_{-2}^{2} |\cos\omega_0 t|^2 dt$

$= \int_{-2}^{2} \frac{1+\cos 2\omega_0 t}{2}\, dt$

$= \left[\frac{t}{2}\right]_{-2}^{2} + \frac{1}{2}\int_{-2}^{2} \cos 2\omega_0 t\, dt$

$= \frac{4}{2} + \frac{1}{2}\left[\frac{\sin 2\omega_0 t}{2\omega_0}\right]_{-2}^{2} = 2 + \frac{1}{4\omega_0}(2\sin 4\omega_0)$

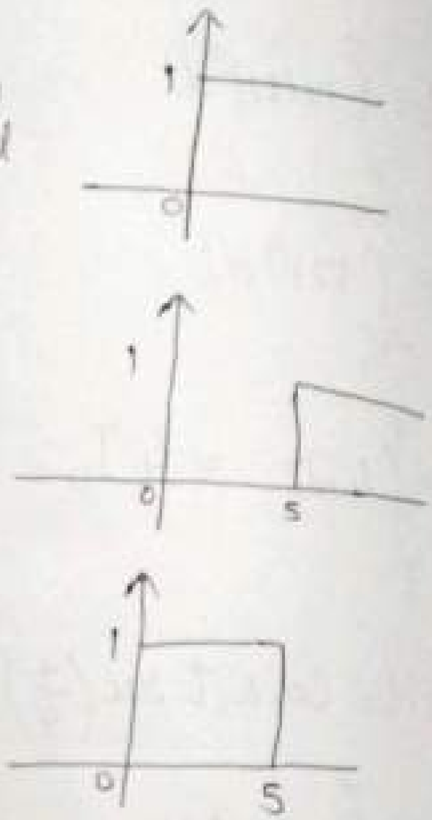
$= 2 + \frac{\sin 4\omega_0}{2\omega_0}$

Pb) $x(t) = u(t) - u(t-5)$

Sol:- as $t \to \infty$, $A \to 0$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

Energy signal



$$= \int_{-\infty}^{\infty} (u(t) - u(t-5))^2 dt$$

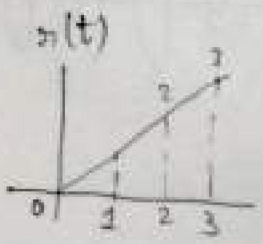
$$= \int_{0}^{5} (1)^2 dt$$

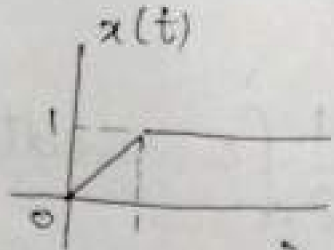
$$= \int_{0}^{5} (1)^2 dt$$

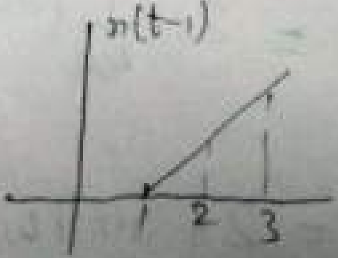
Difference of unit signal is energy signals

Pb) $x(t) = r(t) - r(t-1)$

Sol:-







$x(t) = t, \quad 0 \le t \le 1$
$= 1, \quad t > 1$

It's a power signal [as amplitude const]

$$P = \frac{1}{T} \int_{\langle T \rangle} |x(t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{T} \left[\int_0^1 t^2 dt + \int_1^T (1)^2 dt \right]$$

$$= \underset{T\to\infty}{Lt} \frac{1}{T} \left[\left[\frac{t^3}{3}\right]_0^1 + [t]_1^T \right]$$

$$= \underset{T\to\infty}{Lt} \frac{1}{T} \left[\frac{1}{3} + (T-1) \right]$$

$$= 0 + \underset{T\to\infty}{Lt} \frac{T-1}{T} = 0 + \underset{T\to\infty}{Lt} \left(1 - \frac{1}{T}\right)$$

$$= 1_{//}$$

Pb) $x(n) = A.u(n)$

Sol:- as $n \to \infty$, Amp = A (const)

$\therefore$ Power signal

$$P = \underset{N\to\infty}{Lt} \frac{1}{N+1} \sum_{\langle N \rangle} |x(n)|^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{N+1} \sum_0^N |A.u(n)|^2$$

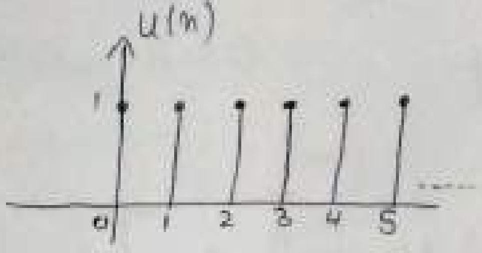
$$= \underset{N\to\infty}{Lt} \frac{A^2}{N+1} \sum_0^N |u(n)|^2 = \underset{N\to\infty}{Lt} \frac{A^2}{N+1} \left[1^2 + 1^2 + 1^2 + \underset{\text{terms}}{- (N+1)} \right]$$

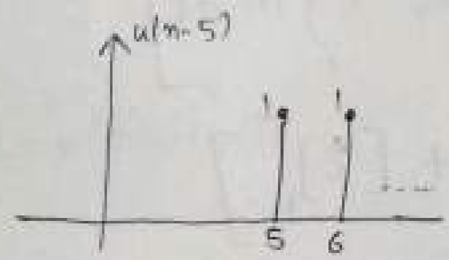
$$= \underset{N\to\infty}{Lt} \frac{A^2}{\cancel{N+1}} \times \cancel{N+1} = A^2$$

$$\boxed{E = \infty}$$

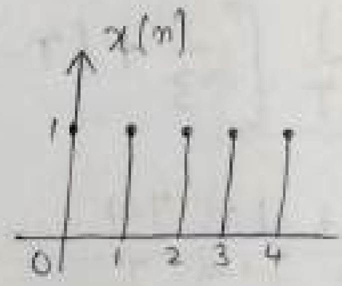
*Pb) $x(n) = u(n) - u(n-5)$

Sol:-





$x(n) =$ Energy signal



$$E = \sum_{0}^{4} |x(n)|^2$$

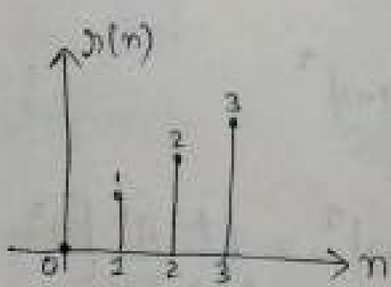
$$= |x(0)|^2 + |x(1)|^2 + |x(2)|^2 + \cdots |x(4)|^2$$

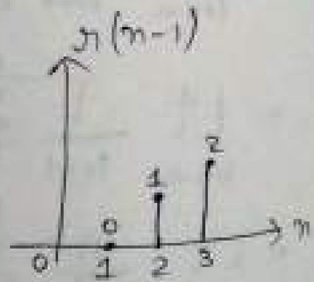
$$= 1+1+1+1+1 = 5$$

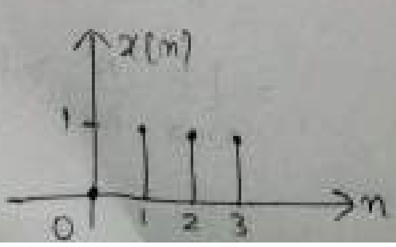
* (IMP)

Pb) $x(n) = r(n) - r(n-1)$

Sol:-







$$P = \underset{N\to\infty}{Lt} \frac{1}{N+1} \sum_{0}^{N} |x(n)|^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{N+1} \left[|x(0)|^2 + |x(1)|^2 + \dots |x(n)|^2 \right]$$

$$= \underset{N\to\infty}{Lt} \frac{1}{N+1} \left[0+1+1+1+\dots \right]$$

$$= \underset{N\to\infty}{Lt} \frac{N}{N+1} = 1$$

$$\therefore \boxed{P=1}$$

Pb) $x(n) = \left(\frac{1}{2}\right)^n u(n)$

Sol:- S1) as $n\to\infty$, $|x(n)| = \left(\frac{1}{2}\right)^\infty \times u(\infty) = 0\times 1 = 0$

Energy signal

S2) $E = \sum_{-\infty}^{\infty} |x(n)|^2 =$ $\qquad x(n) = \left(\frac{1}{2}\right)^n, n \geq 0$

$\qquad = 0$, else

Sol:- $E = \sum_{-\infty}^{0} |0|^2 + \sum_{0}^{\infty} \left(\frac{1}{2}\right)^{2n}$

$$= \sum_{0}^{\infty} \left(\frac{1}{4}\right)^n$$

$$= 1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \dots \infty$$

$$= \frac{1}{1-1/4} = \frac{4}{3} \qquad \therefore \boxed{E = 4/3}$$

Pb) $x(n) = 3^n . u(-n-1)$

Sol:- s1) as $n \to \infty$ $\quad |x(n)| = 3^{\infty} . u(-\infty)$

$= 0$

∴ Energy signal

$x(n) = 3^n . 1 \quad , \quad -n-1 \geq 0 \Rightarrow -1 \geq n$

~~$n \leq 0$~~ $\quad n \leq -1$

$= 3^n . 0 \quad , \quad n > -1$

$$\boxed{\begin{aligned} x(n) &= 3^n, & n \leq -1 \\ &= 0, & n > -1 \end{aligned}}$$

$$E = \sum_{-\infty}^{\infty} |x(n)|^2 = \sum_{-\infty}^{-1} |3^n|^2 + \sum_{-1}^{\infty} |0|^2$$

$$= \sum_{-\infty}^{-1} 3^{2n} = \sum_{1}^{\infty} 3^{-2n}$$

$$= \sum_{1}^{\infty} \left(\frac{1}{9}\right)^n = \frac{1}{9} + \frac{1}{9^2} + \frac{1}{9^3} + \dots$$

$\left[\because S_{\infty} = \frac{a}{1-r}\right]$

$$= \frac{1/9}{1 - 1/9} = \frac{1}{8}$$

3 Pmt, 4 Enady

(17) $x(n) = n, \quad 0 \le n \le 4$

$= 0, \quad \text{else}$

Sol finite duration signal

$\Rightarrow$ Energy signal

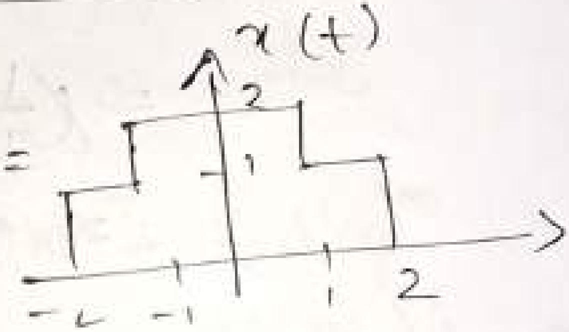
$$E = \sum_{0}^{4} |x(n)|^2$$

$$= \sum_{0}^{4} n^2 = 0^2 + 1^2 + 2^2 + 3^2 + 4^2$$

$$= 1+4+9+16 = 30$$

$$\boxed{E = 30}$$

(18) $x(t) =$



$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$E = \int_{-2}^{2} |x(t)|^2 \, dt$$

$$= \int_{-2}^{-1} 1^2 dt + \int_{-1}^{1} 2^2 dt + \int_{1}^{2} 1^2 dt$$

$$= t\Big]_{-2}^{-1} + 4t\Big]_{-1}^{1} + t\Big]_{1}^{2}$$

$$= 1+8+1$$

$$= 10$$

06/02/2020

(9) $x(n) = A\sin(n\omega_0 + \theta)$

Sol Given:- $x(n) = A\sin(n\omega_0 + \theta)$

$\therefore$ Its Power signal.

$$\left\{ P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} |x(n)|^2 \right\}$$

$$\left\{ P = \lim_{N\to\infty} \frac{1}{N+1} \sum_{0}^{N} |x(n)|^2 \right\}$$

$$\left\{ P = \lim_{N\to\infty} \frac{1}{N} \sum_{1}^{N} |x(n)|^2 \right\}$$

$$\left\{ P = \lim_{N\to\infty} \frac{1}{N+1} \sum_{-N/2}^{N/2} |x(n)|^2 \right\}$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} |A^2 \sin^2(n\omega_0 + \theta)|$$

$$= \lim_{N\to\infty} \frac{A^2}{2N+1} \sum_{-N}^{N} \left[\frac{1 - \cos 2(n\omega_0 + \theta)}{2}\right]$$

$$= \lim_{N\to\infty} \frac{A^2}{2(2N+1)} \left[\sum_{-N}^{N} 1 - \sum_{-N}^{N} \cos 2(\omega_0 n + \theta)\right]$$

$$= \lim_{N\to\infty} \frac{A^2}{2(2N+1)} \sum_{-N}^{N} 1 - \lim_{N\to\infty} \frac{A^2}{2(2N+1)} \sum_{-N}^{N} \cos 2(\omega_0 n + \theta)$$

$$P = \lim_{N\to\infty} \frac{A^2}{2(2N+1)} (2N+1) - 0$$

$$\left\{ P = A^2/2 \right\}$$

20) $x(n) = \{1, -2, \underset{\uparrow}{1}, 2, 3\}$

this is discrete-time energy signal

$$E = \sum_{-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{-2}^{2} |x(n)|^2$$

$$= |x(-2)|^2 + |x(-1)|^2 + |x(0)|^2 + |x(1)|^2 + |x(2)|^2$$

$$= 1^2 + 2^2 + 1^2 + 2^2 + 3^2$$

$$= 1 + 4 + 1 + 4 + 9$$

$$E = 19 \text{ Units}$$

21) $x(t) = \sin t \cos 3t$

Sol Given :- $x(t) = \sin t \cos 3t$

$$x(t) = \frac{1}{2}[\sin 4t - \sin 2t]$$

$$= \frac{1}{2}\sin 4t - \frac{1}{2}\sin 2t$$

we have solved

$$\{A \sin(\omega_0 t + \theta) = A^2/2\}$$

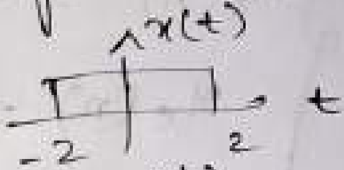
$$= \frac{(1/2)^2}{2} + \frac{(-1/2)^2}{2}$$

$$= 1/8 + 1/8 = 1/4$$

* Even & Odd Signals:-

→ These class of signals is based upon the symmetry property of signals

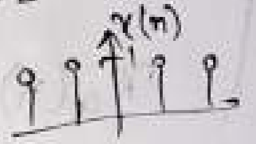
→ A signal $x(t)$ or $x(n)$ is said an even signal if the following condition is satisfied.



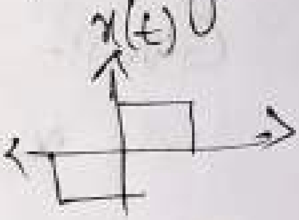
$x(t) \mid x(n)$

$x(-t) = x(t)$

$x(-n) = x(n)$



→ A signal is said to be an odd signal if it satisfied the following condition.



$x(t) \mid x(n)$

$x(-t) = -x(t)$

$x(-n) = -x(n)$

Note:- A signal which is neither even nor odd can be expressed as a sum of even function & odd function as follows.

$$x(t) = x_e(t) + x_o(t)$$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$x(n) = x_e(n) + x_o(n)$$

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

Find the even & odd parts of the following function.

(1) $x(t) = u(t)$

sol Given:- $x(t) = u(t)$

$$x_e(t) = \frac{u(t) + u(-t)}{2}$$

$$x_o(t) = \frac{u(t) - u(-t)}{2}$$

$\therefore$ neither even nor odd

(2) $x(t) = t^2 + t + 1$

$x(-t) = t^2 - t + 1 \neq x(t)$ not even

$\neq -x(t)$ not odd.

$\therefore$ neither even nor odd

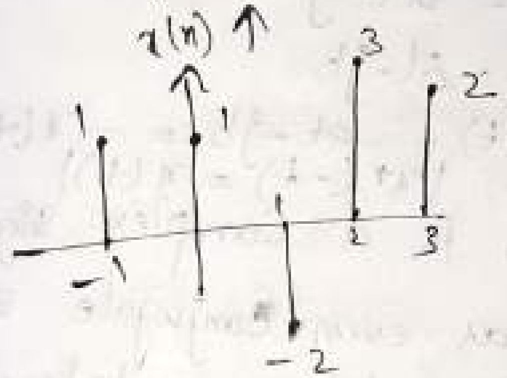
$$x_e(t) = \frac{t^2+t+1+(t^2-t+1)}{2}$$

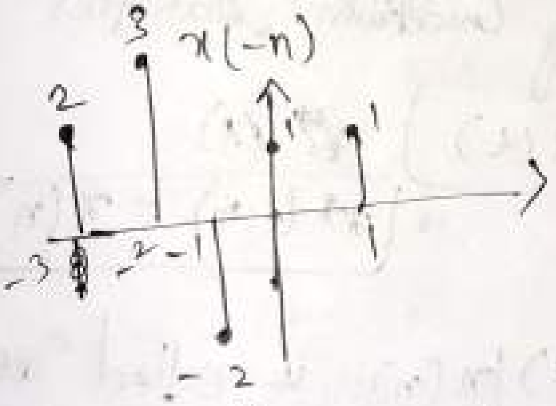
$$x_e(t) = t^2+1$$

$$x_o(t) = \frac{t^2+t+1-(t^2-t+1)}{2}$$

$$x_o(t) = t$$

(3) $x(n) = \{1, 1, -2, 3, 2\}$





$$x_e(n) = \{1, \tfrac{3}{2}, -\tfrac{1}{2}, \underset{\uparrow}{1}, -\tfrac{1}{2}, \tfrac{3}{2}, 1\}$$

$$x_o(n) = \{-1, -\tfrac{3}{2}, \tfrac{3}{2}, \underset{\uparrow}{0}, -\tfrac{3}{2}, \tfrac{3}{2}, 1\}$$

* If $x(t) / x(n)$ is a complex signal.

$$x(t) = a + jbt$$
$$x(-t) = a - jbt$$
$$x^*(-t) = a + jbt$$
$$x^*(-t) = x(t)$$

even conjugate symmetric

$$x(t) = at + jb$$
$$x^*(t) = at - jb$$
$$x^*(-t) = -at - jb = -x(t)$$
$$\{x^*(-t) = x(t)\}$$

> If $x(t) / x(n)$ is a complex signal, then it is called as even conjugate signal if the following condition satisfies

$$\boxed{x^*(-t) = x(t)}$$

$$\boxed{x^*(-n) = x(n)}$$

> A signal $x(t) / x(n)$ is called as odd conjugate signal if the following condition satisfies.

$$\left\{ \begin{array}{l} x^*(-t) = -x(t) \\ x^*(-n) = -x(n) \end{array} \right\}$$

1) $x(t) = e^{jt}$

Sol Given:- $x(t) = e^{jt}$

$x(-t) = e^{-jt}$

$= \cos(-t) + j\sin(-t)$

$= \cos t - j\sin t$

$x^*(-t) = \cos t + j\sin t$

$x^*(-t) = x(t)$

$\therefore$ even conjugate signal

(or) $x^*(t) = e^{-jt}$

$x^*(-t) = e^{jt}$

$x^*(-t) = x(t)$

$\therefore$ even conjugate signal

2) $x(t) = j e^{jt}$

Sol $x^*(t) = -j e^{-jt}$ $\quad (a+jb)^* = a-jb$

$x^*(-t) = -j e^{jt}$ $\quad (jb)^* = -jb$

$x^*(-t) = -x(t)$

$\therefore$ odd conjugate signal

③ $x(t) = x_{ec}(t) + x_{oc}(t)$

If $x(t)\,|\,x(n)$ is neither even nor odd then it can be expressed as a sum of even conjugate function and odd conjugate function

C.T.S

$$x(t) = x_{ec}(t) + x_{oc}(t)$$

$$x_{ec}(t) = \frac{x(t) + x^*(-t)}{2}$$

$$x_{oc}(t) = \frac{x(t) - x^*(-t)}{2}$$

D.T.S

$$x(n) = x_{ec}(n) + x_{oc}(n)$$

$$x_{ec}(n) = \frac{x(n) + x^*(-n)}{2}$$

$$x_{oc}(n) = \frac{x(n) - x^*(-n)}{2}$$

3) $x(t) = a + jbt^2$

Sol

$$x^*(t) = a - jbt^2$$

$$x^*(-t) = a - jbt^2 \neq x(t)$$
$$\neq -x(t)$$

$$x_{ec} = \frac{a + jbt^2 + a - jbt^2}{2} = a$$

$$x_{oc} = \frac{a + jbt^2 - (a - jbt^2)}{2} = jbt^2$$

*) $o + o = o$ $\quad$ $o \to$ odd

$e + e = e$ $\quad$ $e \to$ even

$e \cdot o = o$

$\frac{d}{dt}(e) =$ odd

$\frac{d}{dt}(o) =$ even

→ Prove that sum of two odd function signals is odd and sum of two even signals is even

Proof:- a) In General Consider two odd function $x_1(t)$ & $x_2(t)$

$x_1(-t) = -x_1(t)$ $\quad$ $x(-t) = -x(t)$

$x_2(-t) = -x_2(t)$

let $x(t) = x_1(t) + x_2(t)$

Apply time reversal on both side

$x(-t) = x_1(-t) + x_2(-t)$

$= -x_1(t) - x_2(t)$

$= -(x_1(t) + x_2(t))$

$x(-t) = -x(t)$

$\therefore$ $x(t)$ is odd

$\therefore$ sum of two odd function is odd

b) two even

consider

$x_1(t)$ & $x_2(t)$ be two even functions

$$x_1(-t) = x_1(t)$$
$$x_2(-t) = x_2(t)$$

let $x(t) = x_1(t) + x_2(t)$

Time reversal on both sides.

$$x(-t) = x_1(-t) + x_2(-t)$$
$$x(-t) = x_1(t) + x_2(t)$$
$$x(-t) = x(t)$$

$\therefore$ Sum of two even function is even

c) prove that the product of even & odd is odd function.

Sol Consider $x_1(t)$ as odd function
$x_2(t)$ as even function

$$x_1(-t) = -x_1(t) \quad \text{by def}$$
$$x_2(-t) = x_2(t)$$

let $x(t) = x_1(t) \cdot x_2(t)$

$x(t) = x_1(t) \cdot x_2(t)$

Time reversal on both side

$x(-t) = x_1(-t) \cdot x_2(-t)$

$x(-t) = -x_1(t)\, x_2(t)$

$x(-t) = -x(t)$

$\therefore$ Product of even & odd function is odd

d) Proof product of two even function is even

Sol Consider $x_1(t)$ & $x_2(t)$ be even functions

$x_1(-t) = x_1(t)$ by def.

$x_2(-t) = x_2(-t)$

let $x(t) = x_1(t) \cdot x_2(t)$

Time reversal.

$x(-t) = x_1(-t) \cdot x_2(-t)$

$x(-t) = x_1(t)\, x_2(t)$

$x(-t) = x(t)$

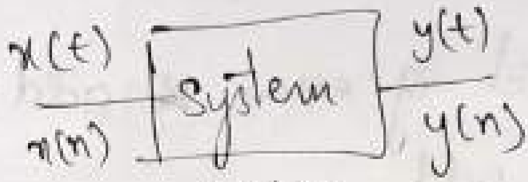
$\therefore$ even function.

* Systems :-

→ In General, A system can be mathematically moduled as follows

$$\{ y(t) = T[x(t)] \}$$

Continous time system



SISO
(single i/p single output)

$$\{ y(t) = \int x(t)\,dt \}$$

$$y(n) = T[x(n)]$$ discrete time system

$$y(n) = x(n-1)$$

→ System can be broadly classified into two types

1) Continous time System
2) Discrete time System

1) Continous time System :-

It is a system which transforms an i/p continous time signal into another o/p continous time signal

$$\underset{i/p}{x(t)} \longrightarrow \boxed{\begin{matrix}CT\\ \text{System}\end{matrix}} \longrightarrow \underset{o/p}{y(t)}$$

$$y(t) = T[x(t)]$$

Eg:- Integrator :- $y(t) = \int_{-\infty}^{t} x(t)\,dt$

Amplifier, differentiator, filters

2) Discrete time signal:-

$$x(n) \longrightarrow \boxed{\begin{matrix}DT\\ \text{System}\end{matrix}} \longrightarrow y(n)$$

It is a system which transforms an i/p discrete time signal into o/p discrete time signal.

$$y(n) = T[x(n)]$$

Eg:- Digital computer, processor, register

CT system / DT system

- lumped parameter & distributed parameter
- causal & non-causal system
- static & dynamic systems
- stable & unstable system
- Invertible & non-invertible system
- linear & non-linear system
- Time invariant & Time variant System

2/02/2020

1) Lumped & Distributed Systems:-

→ Lumped System:- A System in which the physical variables are just functions of time.

Eg:- Resistor, Inductor and Capacitor (R.L.C)

→ Distributed System:- It is a system in which physical variables are functions of space and time.

Eg:- Optical fibre, transmission line, co-axial cable, wave guide

These systems are used at high frequencies

2) Causal & non-Causal Systems:-

→ Causal System:- A System in which the output at any instance of time depends upon the input at the same instance of time or past values of time is called as a causal system.

Such system are physically realizable.

Eg:-

1) $y(t) = x(t-2)$

$t=0$

$y(0) = x(-2)$

$y(-1) = x(-3)$

o/p depends on past i/p

2) $y(t) = x(t)$

$y(0) = x(0)$

$y(1) = x(1)$

o/p depends on present i/p

1) & 2) are causal signals.

→ Non-Causal Systems: — A System in which output at any instance of time depends upon the future values of the input ~~or outputs~~ is called as a non-causal system.

Eg:- 1) $y(t) = 2x(t+2)$

$y(0) = 2x(2)$

non-causal system

$y(-1) = 2x(1)$

o/p depends on future i/p

2) $y(t) = x(t) + x(t+2)$

$y(0) = x(0) + x(2)$

* Investigate the Causality in the following Systems:-

1) $y(t) = x(\sqrt{t})$

Sol Test 1:- $y(2) = x(\sqrt{2}) = x(1.414)$

Test 2:- $y(1/2) = x(0.707)$

$y(0.5) = x(0.707)$

non-causal

2) $y(t) = x(t^2)$

Sol Test 1: $y(2) = x(4)$ non-causal

Test 2: $y(0.5) = x(0.25)$

causal

$\therefore$ Non-Causal

3) $y(t) = x(-t)$

Sol $y(-1) = x(1)$

$y(1) = x(-1)$

non-causal

4) $y(n) = x(n)\, x(n-2)$

Sol Test 1: $y(1) = x(1)\, x(-1)$ Causal

$y(-1) = x(-1)\, x(-3)$ Causal

$\therefore$ Its causal

5) $y(n) = x(n/2)$

Sol $y(2) = x(2/2)$

$y(-2) = x(-2/2)$

$y(-2) = x(-1)$

$\therefore$ non-causal

6) $y(n) = x(\sqrt{n})$

Sol Test!:- $y(4) = x(2)$

~~y(2)~~ $\neq$ Causal

Note:- Donot take fractional values (<1) for Discrete time system Test

3) Static & Dynamic System:-

Static System:- A System in which the o/p at any Instance of time depends upon the i/p at the same instance of time is called as a static system.

Eg:- $y(t) = x(t)$

$y(n) = x(n)$

A purely resistance network is static system.

static system are also called as memoryless system.

Dynamic System:- A system in which the output at any instance of time depends upon i/p and any other instance of time (future or past or both) is called as a Dynamic System

Eg $y(t) = x(t-2)$

$y(t) = x(t+2)$

$y(n) = x(n-2) + x(n+2)$

Dynamic system are memory system

Note:- 1) All static systems are causal, but all causal systems are not static

Eg:- $y(t) = x(t)$ $\quad y(t) = x(t-2)$

2) All non-causal systems are dynamic

$y(n) = x(n-2) + x(n+2)$ but all dynamic systems are non-causal.

Eg:- $y(t) = x(t-2)$

Investigate the memory in following systems

(1) $y(t) = x(\sqrt{t})$

Sol Test 1:- $y(2) = x(1.414)$

Dynamic system

2) $y(t) = x(t^2)$

Sol Test 1:- $y(2) = x(4)$

$\therefore$ Dynamic system

3) $y(t) = x(-t)$

Sol $y(1) = x(-1)$

$\therefore$ Dynamic system

4) $y(n) = x(n/2)$

Sol $y(2) = x(1)$

Dynamic system.

5) $y(t) = \int_{-\infty}^{t} z(\tau)\, d\tau$

Sol Let $\int z(\tau)\, d\tau = x(\tau)$

$y(t) = \left| x(\tau) \right|_{-\infty}^{t}$

$y(t) = \underset{\text{(Present)}}{x(t)} - \underset{\text{(Past)}}{x(-\infty)}$

Dynamic system.

Causal system

* Causality & memory:-

1) $y(n) = \sum_{k=0}^{n} x(k)$

Sol Test 1:- $y(1) = \sum_{0}^{1} x(k)$

$y(1) = x(0) + x(1)$

Test 2:- $y(-1) = \sum_{0}^{-1} x(k)$

$y(-1) = x(-1) + x(0)$

non-causal & dynamic

2) $y(n) = \sum_{-\infty}^{n} x(k)$

Sol Test 1:- $y(1) = \sum_{-\infty}^{1} x(k)$

$= x(1) + x(0) + x(-1) + \dots x(-\infty)$

(causal)

Test 2:- $y(-1) = \sum_{-\infty}^{-1} x(k)$

$y(-1) = x(-1) + x(-2) + \dots x(-\infty)$

Causal and dynamic

3) $y(n) = \sum_{-\infty}^{n+1} x(k)$

Sol $y(1) = \sum_{-\infty}^{2} x(k)$

$y(1) = x(2) + x(1) + x(0) + x(-1) \ldots$

non-causal $x(-\infty)$

$y(-1) = \sum_{-\infty}^{0} x(k)$

$y(-1) = x(0) + x(-1) + x(-2) + \ldots$

$x(-\infty)$

$\therefore$ non-causal & dynamic

4) $y(t) = \frac{d}{dt}(x(t))$

A differentiator is a system which needs two different values present and past to be evaluated so it should be a causal dynamic system.

4) Invertible & Non- Invertible Systems :-

Invertible :- A system whose input can be recovered back by some kind of operation is called as an Invertable System, Otherwise the system is Non-Invertible system

Note:- For a system to be Invertable there must be a one-to-one relationship b/w i/p & o/p

Eg:- Invertable System

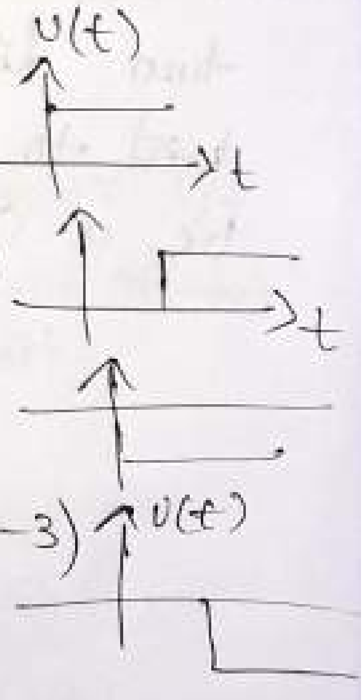
$y(t) = x(t-3)$

Test 1:- $x_1(t) = u(t)$

$y_1(t) = u(t-3)$

Test 2:- $x_2(t) = -u(t)$

$y_2(t) = -u(t-3)$



for $x_1 \neq x_2$

$y_1 \neq y_2$

System is Invertable

The Inverse system for above system is

$y'(t) = x'(t+3)$ $\therefore$ $\boxed{y(t) = x(t+3)}$

Eg:- Non - Inverstable System

$$y(t) = |x(t)| \ , \ y(t) = x^2(t)$$

$\downarrow$ are non - Inverstable system

Test 1:- $x_1(t) = u(t)$
$y_1(t) = u(t)$

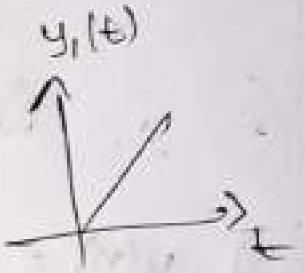
Test 2:- $x_2(t) = -u(t)$
$y_2(t) = u(t)$

1) $y(t) = \int_{-\infty}^{t} x(\tau) d\tau$

Sol

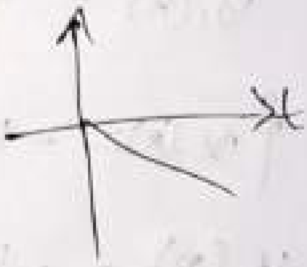
$x(\tau) = u(\tau)$

$y(t) = \int_{-\infty}^{t} u(\tau) d\tau = r(t)$



$x_2(t) = -u(\tau)$

$y_2(t) = \int_{-\infty}^{t} -u(\tau)\, dt = -r(t)$



for $x_1 \neq x_2$
$y_1 \neq y_2$

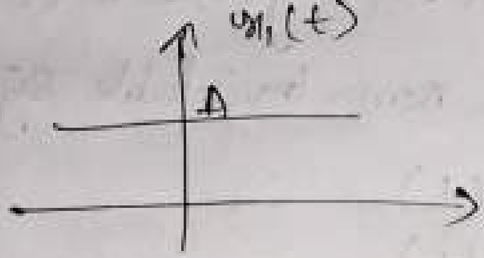
Inverse system is

$$y(t) = \frac{d}{dt} x(t)$$

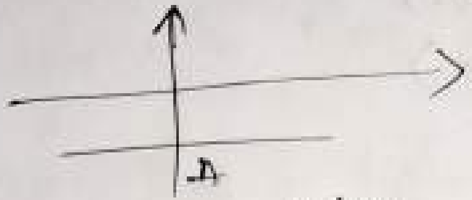
Inverstable

2) $y(t) = \frac{d}{dt}(x(t))$

Sol



$y_1(t) = 0$



$y_2(t) = 0$

$\therefore$ Non-Invertable

3) $y(n) = x(n)\,x(n-1)$

Sol

$\{x_1(n) = u(n)\}$

$y_1(n) = u(n)\,u(n-1)$

$\{x_2(n) = -u(n)\}$

$y_2(n) = -u(n)(-u(n-1))$

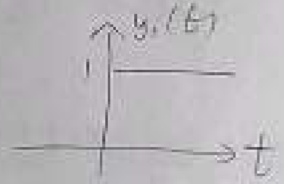
$y_2(n) = u(n)\,u(n-1)$

$\therefore$ Non-Invertable System

4) $y(t) = x^3(t)$

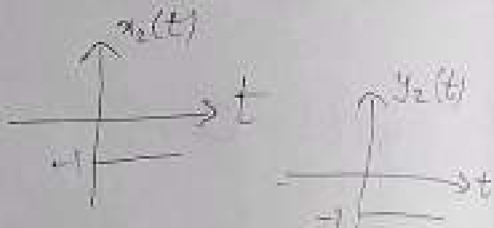
Sol:- Test 1:- $x_1(t) = u(t)$

$y_1(t) = u^3(t)$



Test 2:- $x_2(t) = -u(t)$

$y_2(t) = -u^3(t)$



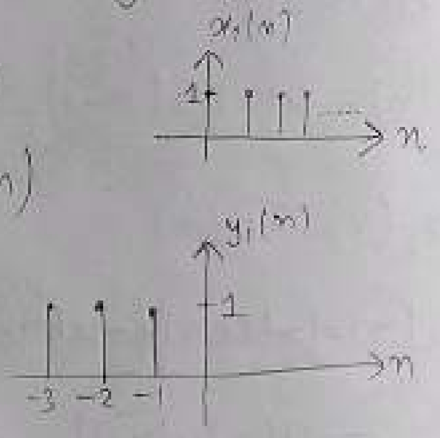
Invertible system

Inverse system is $y = \sqrt[3]{x(t)}$

5) $y(n) = x(1-n) \Rightarrow$ similar to $y(t) = x(1-t)$

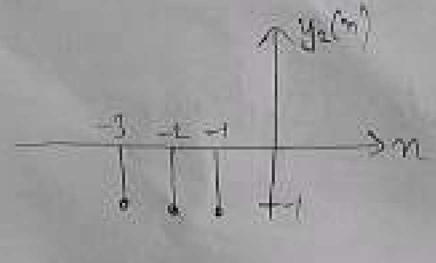
Sol. Test 1:- $x_1(n) = u(n)$

$y_1(n) = u(1-n)$



Test 2:- $x_2(n) = -u(n)$

$y_2(n) = -u(1-n)$



Invertible

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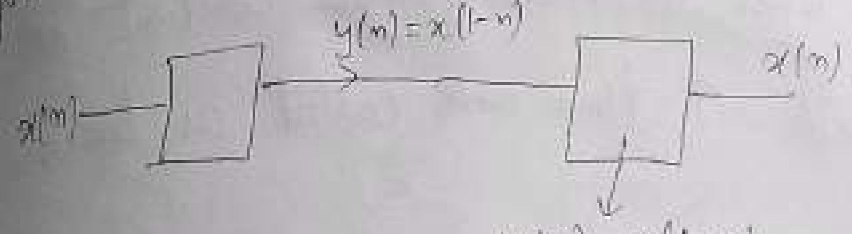
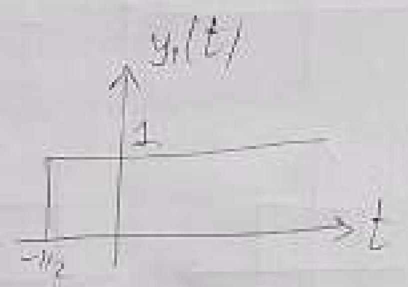
inverse system $y(n) = x(1-n)$



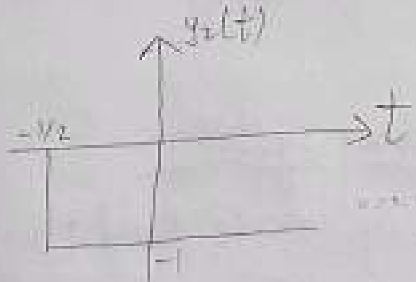
$y(n) = x(1-n)$
$= x(1-(1-n)) = x(n)$

6) $y(t) = x(2t)$

Sol. $x_1(t) = u(t+1)$
$y_1(t) = u(2t+1)$



$x_2(t) = -u(t+1)$
$y_2(t) = -u(2t+1)$



Invertible system

The inverse system is $y(t) = x(t/2)$

7) $y(t) = \cos(x(t))$

Sol. S1) $x_1(t) = 2\pi$
$y_1(t) = \cos 2\pi = 1$

S2) $x_2(t) = 4\pi$ $y_2(t) = \cos 4\pi = 1$

$\therefore$ Non-invertible

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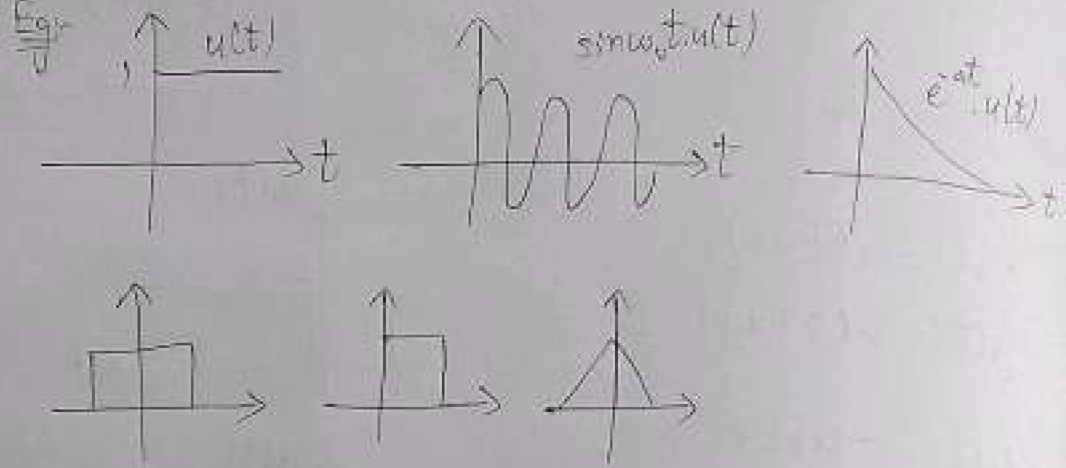
→ Stable and Unstable systems

5) Bounded signals:-

Signals whose amplitude remain constant or die down with ↑ time are called as bounded signals.

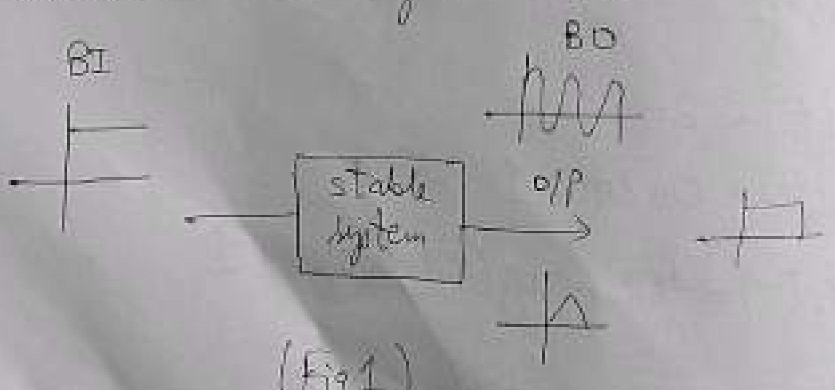
As $t \to \infty$, $A \to 0$, constant

Eg:-



Stable system:-

A system is said to be BIBO (bounded I/P bounded O/P) stable system if it generates a bounded output for every bounded input, as illustrated in the figure below:



(Fig 1)

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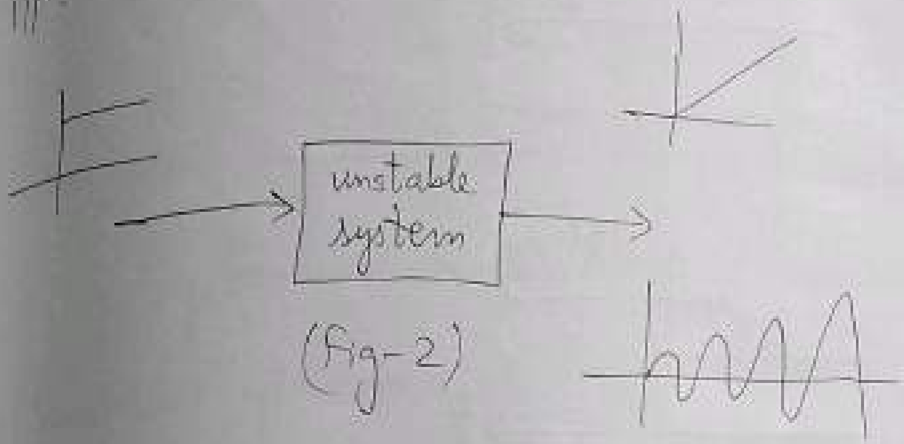
A system is said to be BIBO unstable if it generates an unbounded o/p for an bounded i/p.

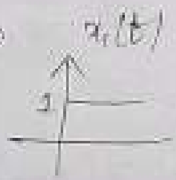


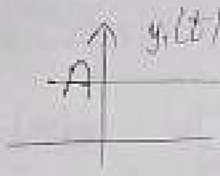
(Fig-2)

Q) 1) $y(t) = A\,x(t)$ → only 1 test can be done in these problems

Sol. Test 1:- $x_1(t) = u(t)$

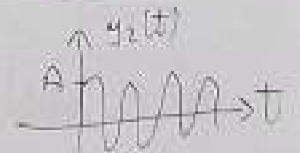
$y_1(t) = A\,u(t)$





Test 2:- $x_2(t) = \sin\omega_0 t . u(t)$

$y_2(t) = A\sin\omega_0 t . u(t)$

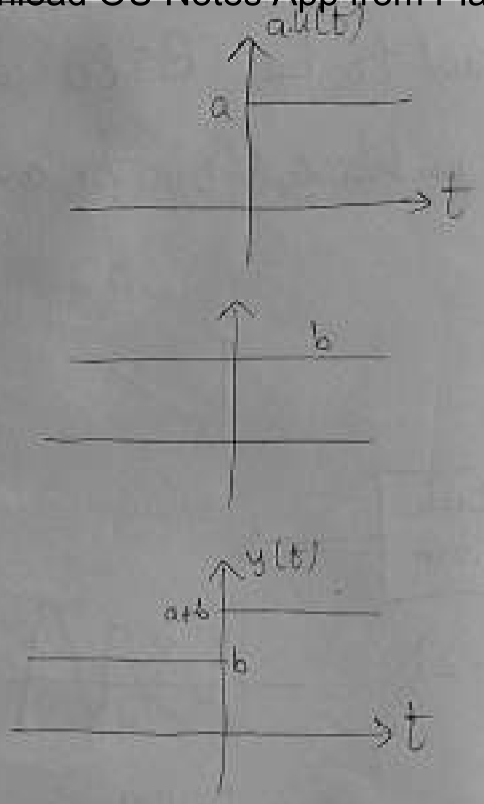


Since the o/p is bounded for every bounded I/P, it is a stable system.

2) $y(t) = ax(t) + b$

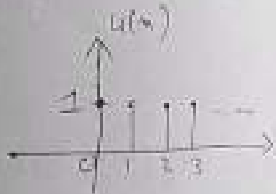
Sol. Test 1:- $x_1(t) = u(t)$

$y_1(t) = a.u(t) + b$



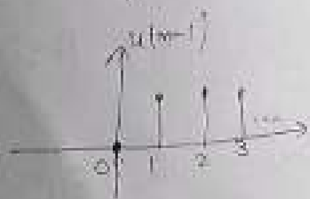
Stable system

3.) $y(n) = x(n-1) + x(n)$

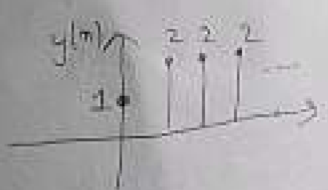


Sol:- Test 1:- $x(n) = u(n)$

$y(n) = u(n) + u(n-1)$



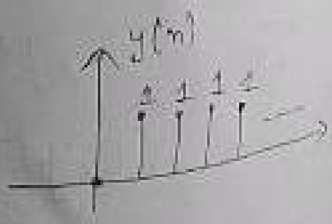
∴ The system is stable



4.) $y(n) = x(n) . x(n-1)$

Sol:- let $x(n) = u(n)$

$y(n) = u(n) . u(n-1)$



∴ Stable system

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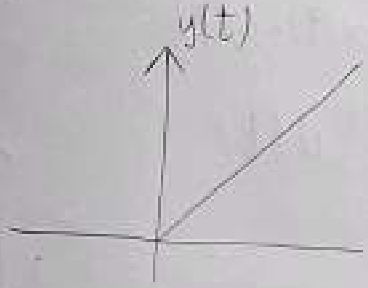
5) $y(t) = t.x(t)$

Sol: let $x(t) = u(t)$

Then $y(t) = t.u(t)$



Here, output $y(t)$ is unbounded (ramp signal)

$\therefore$ System is unstable.

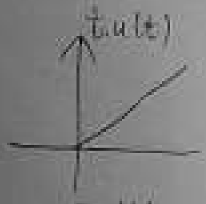
Same for $y(n) = n.x(n)$

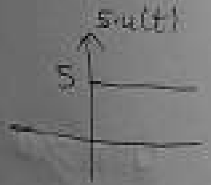
6) $y(t) = (t+5)x(t)$

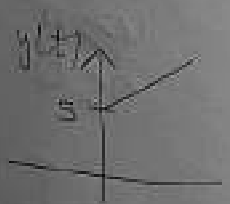
Sol: let $x(t) = u(t)$

$y(t) = (t+5)u(t)$

$y(t) = t.u(t) + 5u(t)$







System is unstable

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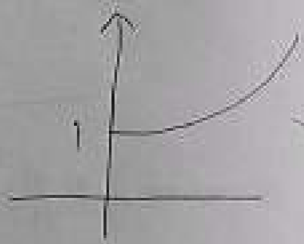
7.) $y(t) = e^{2t} \cdot x(t)$

Sol:- let $x(t) = u(t)$

$y(t) = e^{2t} \cdot u(t)$



$\therefore$ unstable

$\Rightarrow$ exponentially growing system with causal given ... unstable

8.) $y(t) = e^{2t} x(-t)$

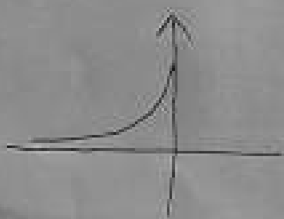
Sol:- $x(t) = u(t)$

$y(t) = e^{2t} \cdot u(-t)$

$= e^{2t}, \; t<0$

$= 0, \; t \geq 0$

$[\because -t \geq 0$
$t \leq 0]$



stable

9.) $y(n) = e^{3n} \cdot x(4-n)$

Sol:- $x(n) = u(n)$

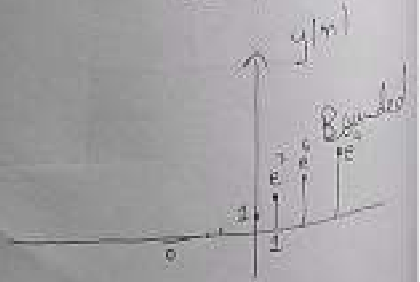
$y(n) = e^{3n} \cdot u(4-n)$

$= e^{3n} \cdot 1, \; n \leq +4$

$= 0, \; n > +4$

Stable

$4-n \geq 0$
$4 \geq n$
$n \leq 4$



$n \to -\infty$
$A \to 0$

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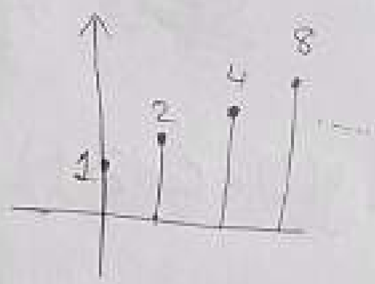
10) $y(n) = 2^n \cdot x(n)$

Sol:- $x(n) = u(n)$

$y(n) = 2^n u(n)$

$= 2^n, \quad n \geq 0$

$= 0, \quad n < 0$



$n \to \infty$

$A \to 0$ Unstable

11) $y(n) = 2^n x(-n)$

Sol:- $x(n) = u(n)$

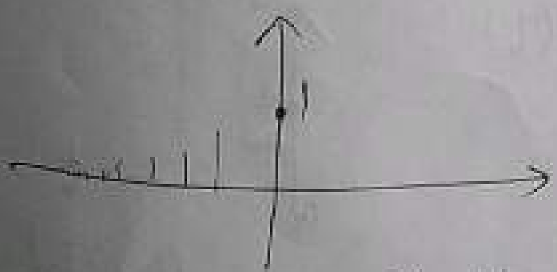
$y(n) = 2^n u(-n)$

$-n \geq 0$

$n \leq 0$

$= 2^n \quad n \leq 0$

$= 0 \quad n > 0$



$n \to \infty$

$A \to 0$

$\therefore$ Stable

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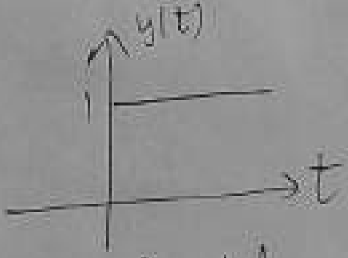
1) $y(t) = x^2(t)$

Sol:- $x(t) = u(t)$

$y(t) = u^2(t) = u(t) \times u(t)$

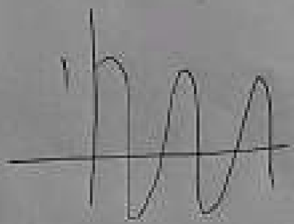


Bounded

Stable system

2) $y(t) = \cos(2\pi t) . x(t)$

Sol:- $y(t) = \cos(2\pi t)\, u(t)$



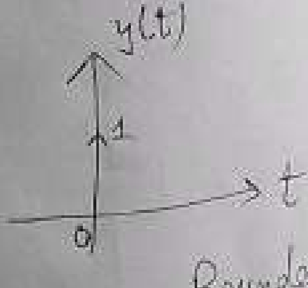
o/p is bounded

$\Rightarrow$ System is stable

3) $y(t) = \frac{d}{dt} x(t)$

Sol:- $x(t) = u(t)$

$y(t) = \frac{d}{dt} u(t) = \delta(t)$



Stable system

Bounded

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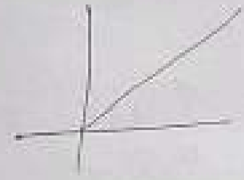
4) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

Sol:- $x(t) = u(t)$

$y(t) = \int_{-\infty}^{t} u(\tau) d\tau$

$= r(t)$



$\therefore$ unstable system.

5) $y(t) = e^{-x(t)}$

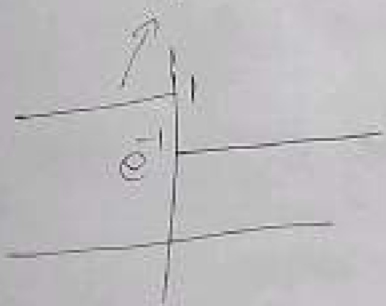
Sol:- $x(t) = u(t)$

$y(t) = e^{-u(t)}$

$e^{-u(t)}$ i.e $e^{-u(0)} = e^{-u(0)}$,

$e^{-u(t)} = e^{-1}$

For -ve time's 1

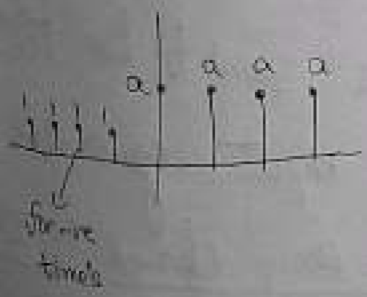


Bounded

Stable.

6) $y(n) = a^{x(n)}$

Sol:- $y(n) = a^{u(n)}$



Stable.

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7.) $y(n) = \sum_{-\infty}^{5} x(k)$

Sol:- $x(k) = u(k)$

$$y(n) = \sum_{-\infty}^{5} u(k)$$

$$= \sum_{0}^{5} u(k) = \sum_{0}^{5} (1) = 5$$

8.) $y(n) = \sum_{n-n_0}^{n+n_0} x(k)$

Sol:- $x(k) = u(k)$

$$y(n) = \sum_{n-n_0}^{n+n_0} u(k)$$

Stable

(3M)

6.) Time invariant & Time variant systems:-

A system is said to be time invariant if its behaviour is independent of time.

Mathematically, a system is said to be time invariant, if a delayed input generates a delayed output without any change in shape as illustrated in the figure below.

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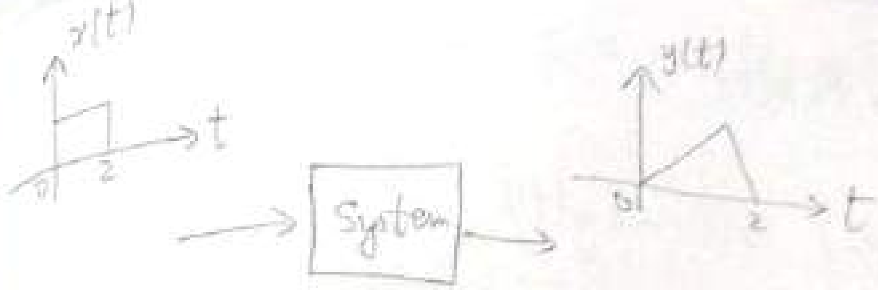
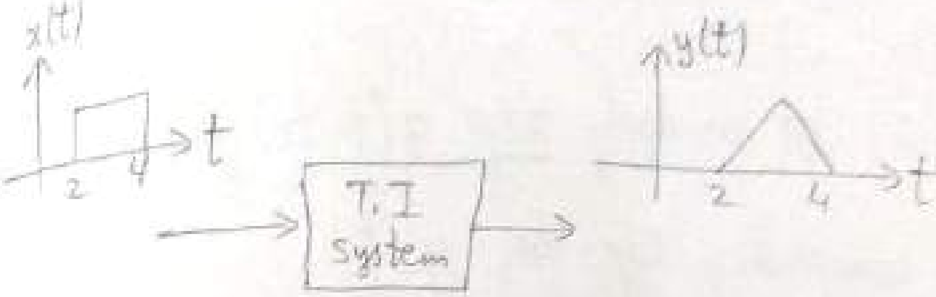




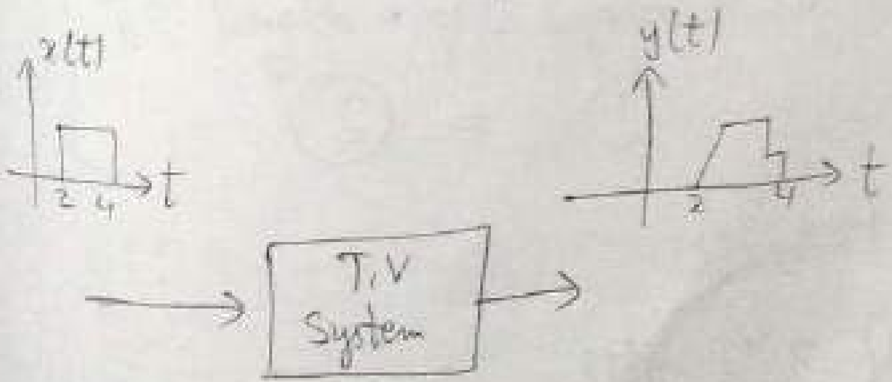
ie if $x(t) \rightarrow y(t)$

then if $x(t-t_0) \rightarrow y'(t) = y(t-t_0)$

then the system is said to be time invariant.

<u>Time variant system:-</u>

If the system doesn't follow the above condition, then it is time dependent & is said to be time variant system which can be illustrated as shown in the figure below.



Pb) $y(t) = A\,x(t)$

Sol- Step1) $x(t) \rightarrow y(t)$

$$y(t) = A\,x(t)$$

Step2) Let us apply I/P $x(t-t_0)$ & let the corresponding O/P be $y'(t)$

$$x(t-t_0) \rightarrow y'(t)$$

$$y'(t) = A\,x(t-t_0) \quad \leftarrow (1)$$

Step3) Let us find $y(t-t_0)$

$$y(t-t_0) = A\,x(t-t_0) \quad \leftarrow (2)$$

Comparing step(2) & step(3)

They are equal.

$\therefore$ Time invariant system.

[i.e O/P due to delayed I/P = delayed O/P

(1) = (2)]

Pb) $y(t) = t.x(t)$

Sol. S1) $x(t) \rightarrow y(t)$

$$y(t) = t.x(t)$$

S2) Let us apply I/P $x(t-t_0)$ & let the corr. o/p be $y'(t)$

$$x(t-t_0) \rightarrow y'(t)$$

$$y'(t) = t.\, x(t-t_0) \quad \text{--- (1)}$$

S3) Let us find $y(t-t_0)$

$$y(t-t_0) = (t-t_0).\, x(t-t_0) \quad \text{--- (2)}$$

$$\because y'(t) \neq y(t-t_0)$$

It is a time variant system.

4) Check whether T.I (or) T.V if $y(t) = x(t) + x(t-2)$ (or) $y(n) = x(n) + x(n-2)$

Sol: S1) $x(t) \rightarrow y(t)$

$$y(t) \rightarrow x(t) + x(t-2)$$

S2) Let us apply I/P $x(t-t_0)$ & let the corr. o/p be $y'(t)$

$$x(t-t_0) \rightarrow y'(t)$$

$$y'(t) = x(t-t_0) + x(t-t_0-2) \quad \text{--- (1)}$$

S3) $$y(t-t_0) = x(t-t_0) + x(t-t_0-2) \quad \text{--- (2)}$$

(1) = (2) i.e $y'(t) = y(t-t_0)$

$\therefore$ Time invariant

15/02/2020

check wheather the following are time variant & time Invariant.

3) $y(t) = x(t) + x(t-2)$

Sol Step 1:- $x(t) \longrightarrow y(t)$

$y(t) = x(t) + x(t-2)$

S2):- let us apply i/p $x(t-t_0)$ & let corresponding o/p be $y'(t)$

$x(t-t_0) \longrightarrow y'(t)$

$y'(t) = x(t-t_0) + x(t-t_0-2)$

S3) let us find $y(t-t_0)$

$y(t-t_0) = x(t-t_0) + x(t-2-t_0)$

S4) $y'(t) = y(t-t_0)$

∴ The above is time Invariant

4) $y(n) = x(n)x(n-5)$

Sol S1) $x(n) \longrightarrow y(n)$

$\Rightarrow y(n) = x(n)\,x(n-5)$

S2) let us Apply i/p $x(n-n_0)$
& let corresponding o/p be $y'(n)$

$x(n-n_0) \rightarrow y'(n)$

$y'(n) = x(n-n_0)\,x(n-5-n_0)$

$y'(n) = x(n-n_0)\,x(n-n_0-5)$

S3) let us find $y(n-n_0)$

$y(n-n_0) = x(n-n_0)\,x(n-5-n_0)$

$y(n-n_0) = y'(n)$

$\therefore$ Time Invariant

5) $y(t) = x(-t)$

Sol S1) $x(t) \longrightarrow y(t)$

$y(t) = x(-t)$

S2) $x(t-t_0) \longrightarrow y'(t)$

$y'(t) = x(-t-t_0)$

S3) $y(t-t_0) = x(-(t-t_0))$

$y(t-t_0) = x(t_0-t)$

$y(t-t_0) \neq y'(t)$.

$\therefore$ Time Variant

6) $\frac{dy(t)}{dt} + y(t) = x(t)$

Sol S1) $x(t) \rightarrow y(t)$

$\frac{dy(t)}{dt} + y(t) = x(t)$ —— (1)

S2) $x(t-t_0) \rightarrow y'(t)$

$\frac{dy(t)}{dt} + y'(t) = x(t-t_0)$ —— (2)

S3) Substitute eq (1) & (2)

$\frac{dy'(t)}{dt} + y'(t) = \frac{dy}{dt}(t-t_0) + y(t-t_0)$

Comparing similar terms

$y'(t) = y(t-t_0)$

$y'(t) = y(t-t_0)$

$\therefore$ time Invariant

7) $t\frac{d^2y}{dt^2} + \frac{dy(t)}{dt} + 2y(t) + 2 = x(t)$

Sol S1) $x(t) \longrightarrow y(t)$

$$t\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + 2y(t) + 2 = x(t) \quad \text{—— (1)}$$

S2) $x(t-t_0) \longrightarrow y'(t)$

$$t\frac{d^2y'(t)}{dt} + \frac{dy'(t)}{dt} + 2y'(t) + 2 = x(t-t_0) \quad \text{—— (2)}$$

S3) substitute (1) in (2)

$$t\frac{d^2y'(t)}{dt} + \frac{dy'(t)}{dt} + 2y'(t) + \cancel{2}$$

$$= (t-t_0)\frac{d^2y(t-t_0)}{dt} + \frac{dy(t-t_0)}{dt} + 2y(t-t_0) + \cancel{2}$$

$$ty'(t) \neq y(t-t_0)(t-t_0)$$

$\therefore$ Time variant.

8) $y(n-1) + 2y(n) = x(n)$

Sol Given:- $x(n) \longrightarrow y(n)$

$$y(n-1) + 2y(n) = x(n)$$

$$x(n-n_0) \longrightarrow y'(n)$$

$$y'(n-1) + 2y'(n) = x(n-n_0)$$

$\cancel{y(n-n_0-1)}$

S3) sub ① in ②

$$y'(n-1) + 2y'(n) = y(n-1-n_0) + 2y(n-n_0)$$

$$y'(n-1) = y(n-n_0-1)$$

$$y(n-1+1) = y(n-n_0-1+1)$$

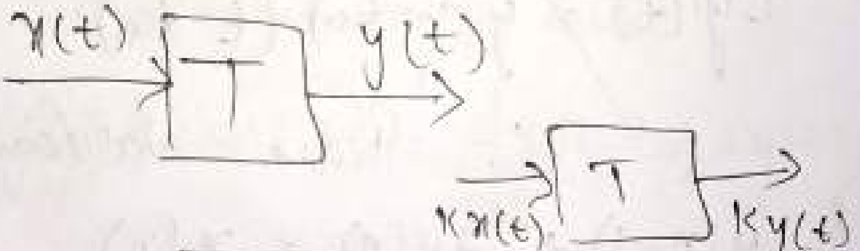
$$y(n) = y(n-n_0)$$

$$y'(n) = y(n-n_0)$$

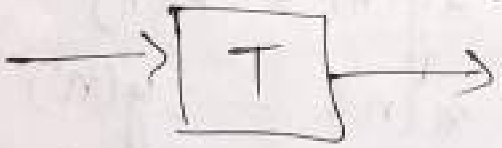
$\therefore$ Time Invariant

*7) Linear and Non-linear System :-

Homogenity

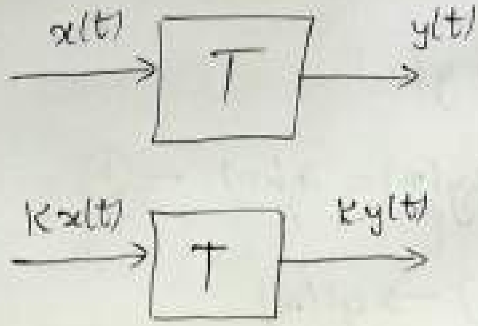


Addivity

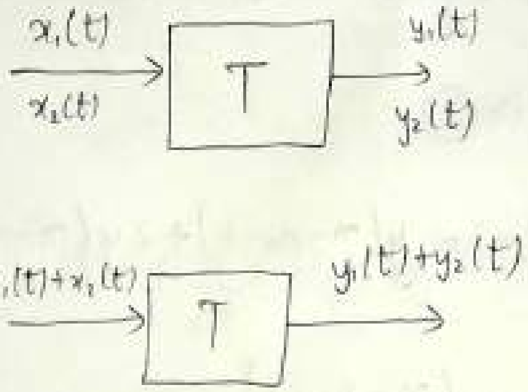


7.) Linear and non-linear Systems

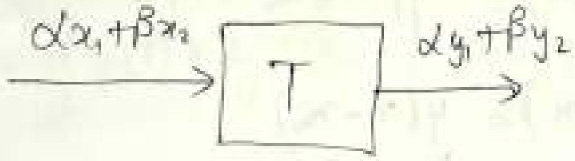
Homogenity



Additivity



Superposition



→ A system is said to be linear if it satisfies the property of superposition, i.e

if $x_1(t) \rightarrow y_1(t)$ and

$x_2(t) \rightarrow y_2(t)$ then

if $\alpha x_1 + \beta x_2 \rightarrow y'(t) = \alpha y_1 + \beta y_2$

otherwise it is a non-linear system.

$y(t) = A\,x(t)$

S1) let $x_1(t)$ generates an o/p $y_1(t)$

$\Rightarrow \quad x_1(t) \rightarrow y_1(t)$

$y_1(t) = A\,x_1(t)$

let $x_2(t)$ generates on o/p $y_2(t)$

$x_2(t) \longrightarrow y_2(t)$

$\Rightarrow \quad y_2(t) = A\,x_2(t)$

S2) let i/p

$\alpha x_1(t) + \beta x_2(t)$ generates an o/p $y_3(t)$

$\alpha x_1(t) + \beta x_2(t) \longrightarrow y_3(t)$

$y_3(t) = A(\alpha x_1(t) + \beta x_2(t))$

S3) let us find $\alpha y_1(t) + \beta y_2(t)$

$\alpha y_1(t) + \beta y_2(t) = \alpha A x_1(t) + \beta A x_2(t)$

$\therefore\ y_3(t) = \alpha y_1(t) + \beta(y_2(t))$

$\therefore$ The System is linear

2) $y(n) = Ax(n) + b$

Sol 1) let $x_1(n) \rightarrow y_1(n)$

$y_1(n) = A x_1(n) + b$

let $x_2(n) \rightarrow y_2(n)$

$y_2(n) = A x_2(n) + b$

S2) let i/p

$\alpha x_1(n) + \beta x_2(n)$ generates $y_3(n)$

$y_3(n) = A(\alpha x_1(n) + \beta(x_2(n))) + b$

S3) let us find $\alpha y_1(n) + \beta y_2(n)$

$$\alpha y_1(n) + \beta y_2(n) = \alpha(Ax_1(n)+b)$$
$$+ \beta(A(x_2(n)+b)$$

$y_3(n) \neq \alpha y_1(n) + \beta y_2(n)$

$\therefore$ Its non-linear

3) $y(t) = x^2(t)$

sol S1) let $x_1(t) \rightarrow y_1(t)$

$y_1(t) \rightarrow x_1^2(t)$

let $x_2(t) \rightarrow y_2(t)$

$y_2(t) \rightarrow x_2^2(t)$

S2) let i/p.

$\alpha x_1(t) + \beta x_2(t)$ generates $y_3(t)$

$y_3(t) = (\alpha x_1(t) + \beta x_2(t))^2$

$= \alpha^2 x_1^2(t) + \beta^2 x_2^2(t) + 2\alpha\beta x_1 x_2$

S3) $\alpha y_1(t) + \beta y_2(t) =$

$\alpha x_1^2(t) + \beta x_2^2(t)$

$y_3(t) \neq \alpha y_1(t) + \beta y_2(t)$

$\therefore$ non-linear

$|a+b| < |a|+|b|$

4) $y(t) = |x(t)|$

Sol S1) $y_1(t) = |x_1(t)|$

$y_2(t) = |x_2(t)|$

S2) $y_3(t) = |\alpha x_1(t) + \beta x_2(t)|$

S3) $\alpha y_1(t) + \beta y_2(t)$

$= \alpha |x_1(t)| + \beta |x_2(t)|$

$\therefore$ non-linear

5) $y(t) = \int_{-\infty}^{t} x(\tau) d\tau$

Sol S1) $y_1(t) = \int_{-\infty}^{t} x_1(\tau) d\tau$

$y_2(t) = \int_{-\infty}^{t} x_2(\tau) d\tau$

S2) $y_3(t) = \int_{-\infty}^{t} x_1(\tau)\alpha + \beta x_2(\tau)\, d\tau$

$\alpha y_1(t) + \beta y_2(t)$

$= \alpha \int_{-\infty}^{\tau} x_1(\tau) d\tau + \beta \int_{-\infty}^{\tau} x_2(\tau) d\tau$

$= y_3(t)$

$\therefore$ Its linear

6) $\frac{dy(t)}{dt} + 2y(t) = x(t)$

sol s1) $\frac{dy_1(t)}{dt} + 2y_1(t) = x_1(-t)$

$\frac{dy_2(t)}{dt} + 2y_2(t) = x_2(-t)$

s2) $y_3(t)$

$\frac{dy_3(t)}{dt} + 2y_3(t) = \alpha x_1(t) + \beta x_2(t)$

s3) $\alpha y_1(t) + \beta y_2(t)$

Substitute s1 in s2

$\frac{dy_3(t)}{dt} + 2y_3(t) = \alpha\left(\frac{dy_1(t)}{dt} + 2y_1(t)\right)$

comparing $+\beta\left(\frac{dy_2(t)}{dt} + 2y_2(t)\right)$

$\frac{dy_3(t)}{dt} = \alpha\frac{dy_1(t)}{dt} + \beta\frac{dy_2(t)}{dt}$

$y_3(t) = \alpha y_1(t) + \beta y_2(t)$

$y_3(t) = \alpha y_1(t) + \beta y_2(t)$

Linear System.

7) $\frac{d^2y(t)}{dt^2} + \frac{2dy(t)}{dt} + 2y(t) + 2 = x(t)$

Sol

S1) $\frac{d^2y_1(t)}{dt^2} + \frac{2dy_1(t)}{dt} + 2y_1(t) + 2 = x_1(t)$

$\frac{d^2y_2(t)}{dt^2} + \frac{2dy_2(t)}{dt} + 2y_2(t) + 2 = x_2(t)$

S2) $\alpha x_1(t) + \beta x_2(t) \longrightarrow y_3(t)$

$$\frac{d^2y_3(t)}{dt^2} + \frac{2dy_3(t)}{dt} + 2y_3(t) + 2 = \alpha x_1(t) + \beta x_2(t)$$

S3) Sub S1 in S2

$$\Rightarrow \frac{d^2y_3(t)}{dt^2} + \frac{2dy_3(t)}{dt} + 2y_3(t) + 2 = \alpha\left[\frac{d^2y_1(t)}{dt^2} + \frac{2dy_1(t)}{dt} + 2y_1(t) + 2\right] + \beta\left[\frac{d^2y_2(t)}{dt^2} + \frac{2dy_2(t)}{dt} + 2y_2(t) + 2\right]$$

Comparing

$$\frac{d^2y_3(t)}{dt^2} = \frac{d^2}{dt^2}(\alpha y_1(t) + \beta y_2(t))$$

$$y_3(t) = \alpha y_1(t) + \beta y_2(t)$$

$$\frac{dy_3(t)}{dt} = \frac{d}{dt}(\alpha y_1(t) + \beta y_2(t))$$

$$y_3(t) = \alpha y_1(t) + \beta y_2(t)$$

$$y_3(t) = \alpha y_1(t) + \beta y_2(t)$$

$$2 = 2\alpha + 2\beta$$
$$\alpha + \beta = 1 \quad (\text{non-linear})$$

$\therefore$ system is not linear

20/01/2020

8) $y(n-1) + 2y(n) = x(n)$

S1) $y_1(n-1) + 2y_1(n) = x_1(n)$
$y_2(n-1) + 2y_2(n) = x_2(n)$

S2) $y_3(n-1) + 2y_3(n) = \alpha x_1(n) + \beta x_2(n)$

S3) $y_3(n-1) + 2y_3(n) = \alpha(y_1(n-1) + 2y_1(n))$
$+ \beta(y_2(n-1) + 2y_2(n))$

Comparing

$y_3(n-1) = \alpha y_1(n-1) + \beta y_2(n-1)$

$y_3(n-1+1) = \alpha y_1(n-1+1) + \beta y_2(n-1+1)$

$y_3(n) = \alpha y_1(n) + \beta(y_2(n)$

$y_3(n) = \alpha y_1(n) + \beta y_2(n)$

$\therefore$ linear,,

(a) $y(n-2) + 2y(n-1) + ny(n) + 2 = x(n)$

Sol S1) $y_1(n-2) + 2y_1(n-1) + ny_1(n) + 2 = x_1(n)$

$y_2(n-2) + 2y_2(n-1) + ny_2(n) + 2 = x_2(n)$

S2) $y_3(n-2) + 2y_3(n-1) + ny_3(n) + 2$

$= \alpha x_1(n) + \beta x_2(n)$

sub ① in ②

$y_3(n-2) + 2y_3(n-1) + ny_3(n) + 2 =$

$\alpha(y_1(n-2) + 2y_1(n-1) + ny_1(n) + 2)$

$+ \beta(y_2(n-2) + 2y_2(n-1) + ny_2(n) + 2)$

$y_3(n-2) = \alpha y_1(n-2) + \beta y_2(n-2)$

$y_3(n) = \alpha y_1(n) + \beta y_2(n)$

$y_3(n-1) + \alpha y_1(n-1) + \beta y_2(n-1)$

$y_3(n) = \alpha y_1(n) + \beta y_2(n)$

$2 = 2\alpha + 2\beta$

$\alpha + \beta = 1$ ×

∴ Non-linear

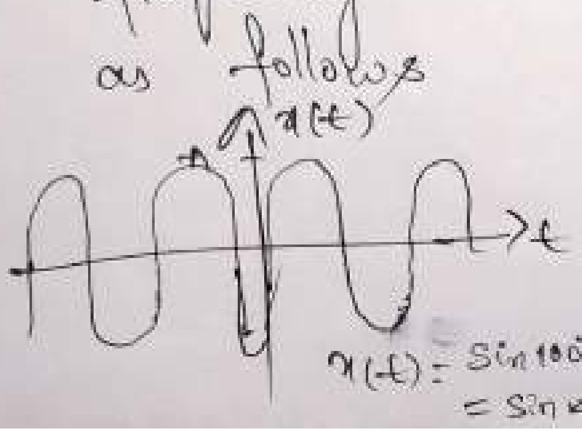
15/02/2020

UNIT:- IV

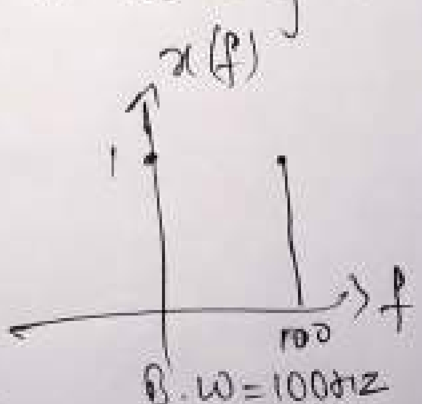
Fourier Series:-

→ So far we have seen the time domain representation of a signal i.e $x(t)$. But very often there are some issues that cannot be tackled using this ~~of~~ time domain like signal bandwidth and noise analysis. Both these issues can be only tackled using. One more important version of a signal called the frequency domain representation of a signal

- It is also called as frequency spectrum of a signal.
- It is a plot of Amplitude vs frequency content in a signal as follows



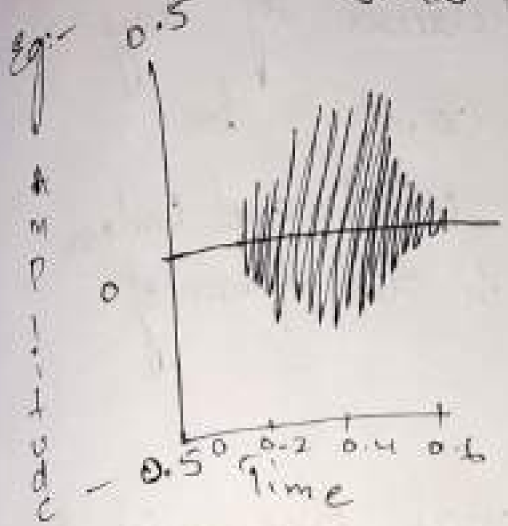
$x(t) = \sin 100t$
$= \sin 200\pi t$



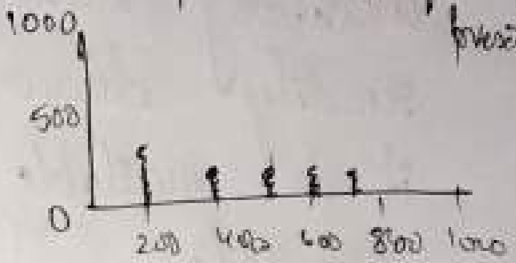
B.W = 100Hz

waveform of Hello

Eg:-



Spectrum (frequency present)

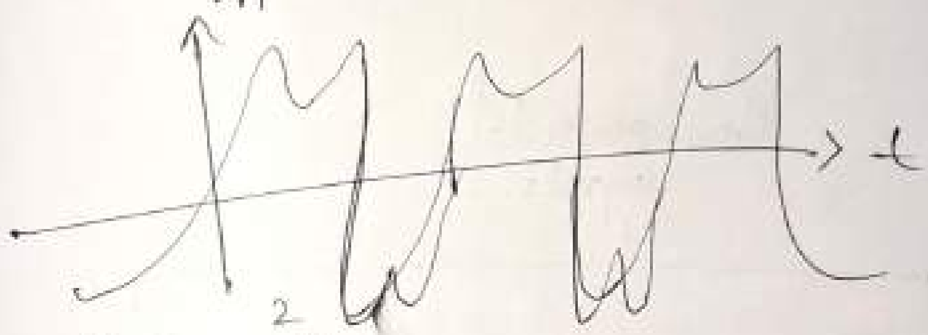


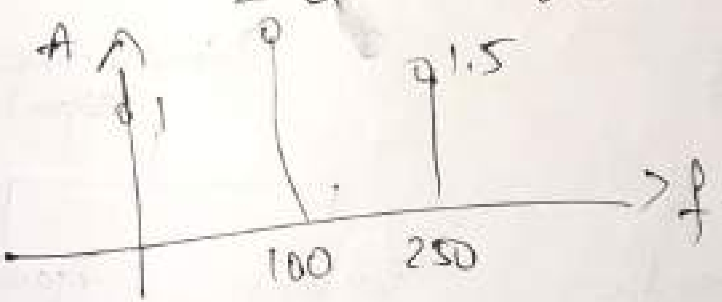
B.W = 700HZ

Q) Draw the frequency spectrum of the following signal

$x(t) = 1 + 2\sin 200\pi t + \frac{3}{2}\sin 500\pi t$

Sol





B.W = 250

Non-sinusoidal signal

Note:- The above example shows that the frequency spectrum of any arbitary signal cannot be readily drawn until and unless it is available in the form of sinusoids

> The above theory was observed by fourier, and he proposed a method of converting any arbitary non-sinusoidal signal into sinusoids, which came to be known as fourier series

Non-sinusoidal signals

- Continous time signal
  - Periodic → CTFS — Unit:-(2)
  - Aperiodic → CTFT (3)
- Discrete-time signal
  - Periodic → DTFS (4)
  - Aperiodic → DTFT (5)

* Discrete time fourier series:- (DTFS)

→ It is a method developed by fourier to convert a time domain discrete time periodic signal into a frequency domain representation

→ The following expression quantifies his theory

$$x(n) \longrightarrow D_K$$

$$\left\{ x(n) = \sum_{K=0}^{N-1} D_K e^{jn\omega_0 k} \right\}$$

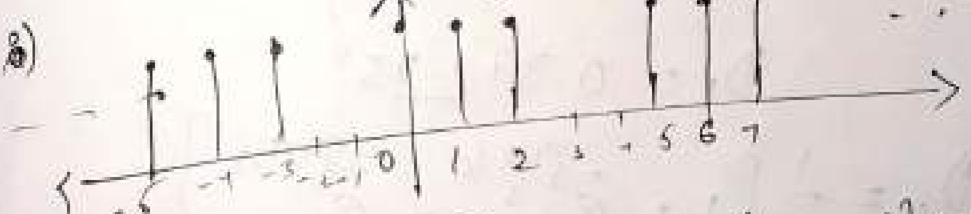
$N \rightarrow$ Time period of $x(n)$

where

$$\left\{ D_K = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-jn\omega_0 k} \right\}, \quad k = 0 \rightarrow N-1$$

1) Find the DTFS of the following D.T signal draw spectrum

a)



(or) $x(n) = \{1,1,1,0,0\}$, $N=5$

Sol s1) $N=5$ (time period)

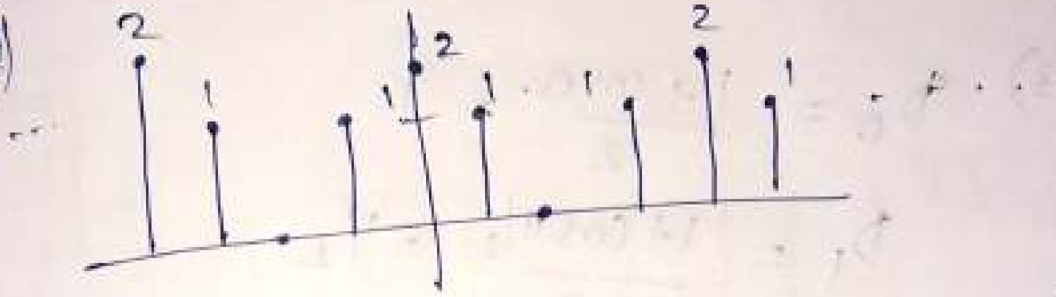
Fourier Series:-

DTFS:-

$$x(n) = \sum_{k=0}^{N_0-1} D_k \, e^{jk\omega_0 n}$$

$$D_k = \frac{1}{N} \sum_{n=0}^{N_0-1} x(n) \cdot e^{-jk\omega_0 n} \quad , k=0 \rightarrow$$

Q)



S1) $N_0 = 4 \quad , \omega_0 = \frac{2\pi}{N_0} = \frac{2\pi}{4} = \pi/2$

S2) $$D_k = \frac{1}{4} \sum_{n=0}^{3} x(n) \, e^{-jk\omega_0 n}$$

$$= \frac{1}{4} \sum_{n=0}^{3} x(n) \, e^{-jk\pi/2 \, n} \quad , \; k = 0 \rightarrow 3$$

$$\Rightarrow D_k = \frac{1}{4} \left[x(0) \, e^{0} + x(1) \, e^{-jk\pi/2} + x(2) \, e^{-jk\pi} + x(3) \, e^{-jk3\pi/2} \right]$$

$$D_k = \frac{1}{4} \left[2 + e^{-jk\pi/2} + 0 + e^{-jk3\pi/2} \right]$$

$$= \frac{1}{4} \left[2 + e^{-jk\pi/2} + e^{-jk(2\pi - \pi/2)} \right] \quad k \rightarrow 0 \rightarrow 3$$

$$= \frac{1}{4} \left[2 + e^{-jk\pi/2} + e^{jk\pi/2} \right]$$

$k = 0 \rightarrow 3$

$$\boxed{e^{-j(2\pi - \theta)} = e^{j\theta}}$$

formula

$$\Rightarrow D_K = \frac{1}{4}\left[2 + \cos K\pi/_2 - j\sin K\pi/_2 + \cos K\pi/_2 + j\sin K\pi/_2\right]$$

$$D_K = \frac{1}{4}\left[2 + 2\cos K\pi/_2\right]$$

$$\Rightarrow D_K = \frac{1}{2}\left[1 + \cos K\pi/_2\right]$$

S3) $D_0 = \frac{1 + \cos 0}{2} = 1$

$$D_1 = \frac{1 + \cos \pi/_2}{2} = 1/_2$$

$$D_2 = \frac{1 + \cos \pi}{2} = 0$$

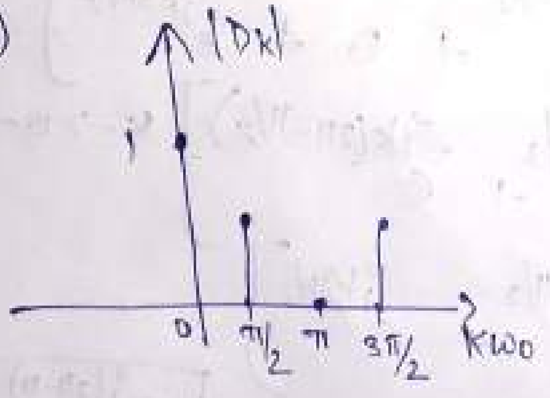
$$D_3 = \frac{1 + \cos 3\pi/_2}{2} = 1/_2$$

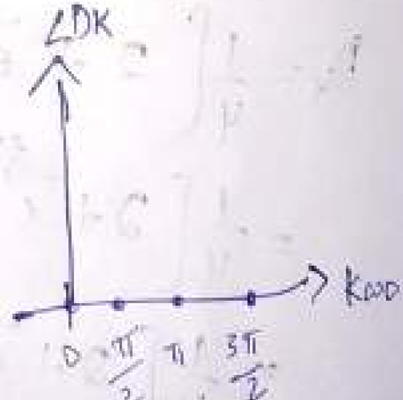
S4) $x(n) = \sum_{K=0}^{3} D_K \, e^{jK\pi/_2 n}$

$$x(n) = D_0 e^0 + D_1 e^{j\pi/_2 n} + D_2 e^{j\pi n} + D_3 e^{j\frac{3\pi}{2}n}$$

$$x(n) = 1 + 1/_2 \, e^{jn\pi/_2} + 0 e^{j\pi n} + \frac{1}{2} e^{j3\pi/_2 n}$$

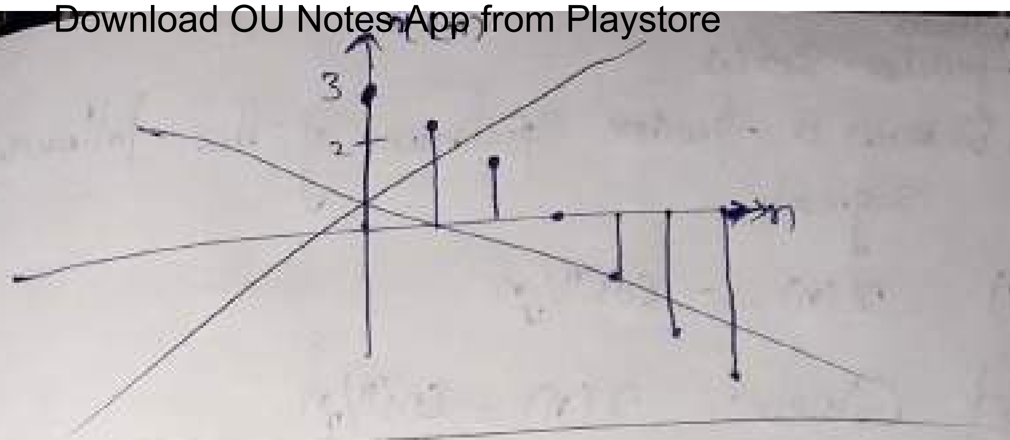
S5)



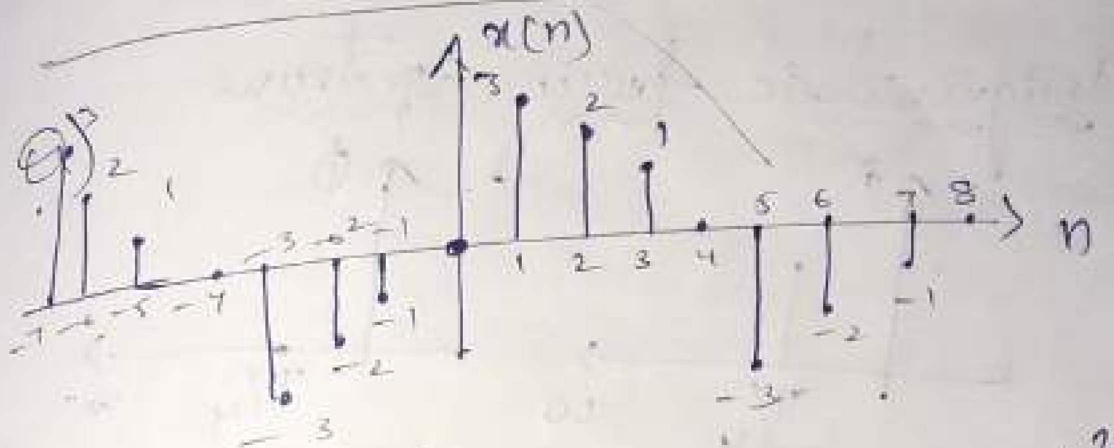


$\cos k\pi/_2$ ...

(a)



(a)



S1) $x(n) = \{0, -3, -2, -1, 0, 3, 2, 1\}$

$N_0 = 8 \quad \omega_0 = 2\pi/8 = \pi/4$

S2) $D_k = \frac{1}{8} \sum_{n=0}^{7} x(n) e^{-jk\pi/4 n} \quad k = 0 \to 7$

$D_k = \frac{1}{8} \{ 0 + 3 e^{-jk\pi/4} + 2 e^{-jk\pi/2} + e^{-jk3\pi/2}$

$+ 0 + (-3) e^{-jk5\pi/4} + (-2) e^{-jk6\pi/4}$

$+ (-1) e^{-jk7\pi/4} \} \quad k = 0 \to 7$

$D_k = \frac{1}{8} \{ 3 e^{-jk\pi/4} + 2 e^{-jk\pi/2} + e^{-jk\frac{3\pi}{4}}$

$- 3 e^{-jk(2\pi - 3\pi/4)} - jk$

29/02/2020

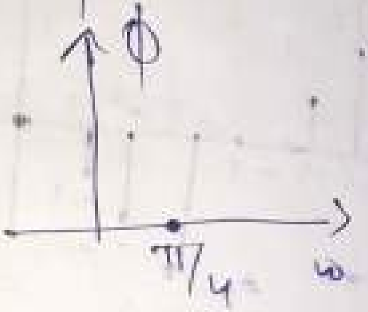
Fourier Series

→ Draw a fourier spectrum of the following signal :-

1) $x(n) = \cos \frac{\pi}{4} n$

Sol Given :- $x(n) = \cos \frac{\pi}{4} n$

Trigonometric fourier spectrum



EFS :-

$$x(n) = \sum_{k=0}^{N_0-1} D_k e^{jk\omega_0 n}$$

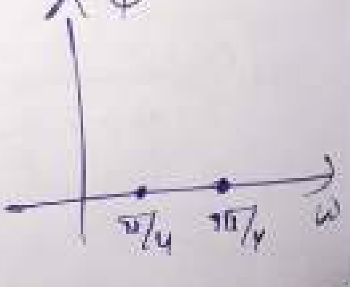
$$x(n) = \frac{e^{j\pi/4 n}}{2} + \frac{e^{-j/4 \pi n}}{2}$$

$$\boxed{e^{-j(2\pi - \theta)} = e^{+j\theta}}$$

$$e^{-j\pi/4 n} = e^{-j(2\pi - \frac{\pi}{4})n}$$

$$x(n) = \frac{e^{j\pi/4 n}}{2} + \frac{e^{j7\pi/4 n}}{2}$$

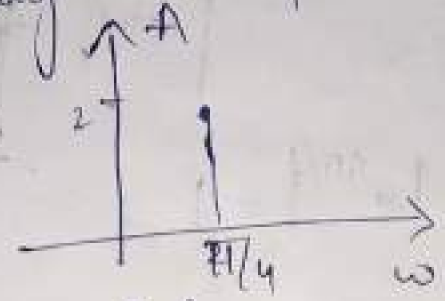
Exponential fourier spectrum :

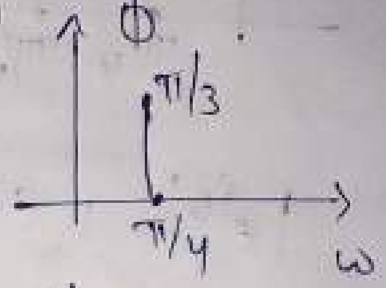


2) $x(n) = 2\cos\left(\frac{\pi}{4}n + \frac{\pi}{3}\right)$

Sol) Given:- $x(n) = 2\cos\left(\frac{\pi}{4}n + \frac{\pi}{3}\right)$

a) Trigonometric fourier spectrum





$\underbrace{\text{magnitude spectrum} \quad \text{Phase spectrum}}_{\text{frequency Spectrum}}$

fourier Spectrum

b) Exponential fourier spectrum

$$x(n) = \sum_{n=0}^{N_0-1} D_k e^{jk\omega_0 n}$$

$$x(n) = \frac{2}{2} e^{j(\pi/4 n + \pi/3)} + \frac{2}{2} e^{-j(\pi/4 n + \pi/3)}$$

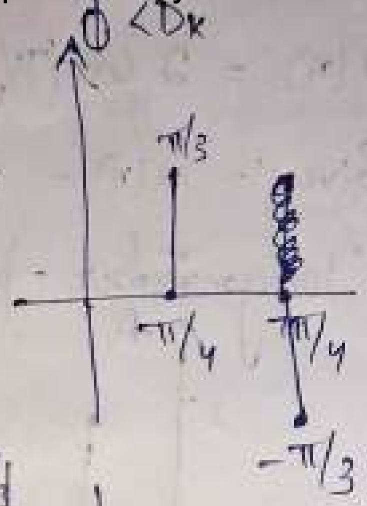
$$= e^{j\pi/3} e^{j\pi/4 n} + e^{-j\pi/3} \cdot e^{-j\pi/4 n}$$

$$= e^{j\pi/3} e^{j\pi/4 n} + e^{-j\pi/3} \cdot e^{j7\pi/4 n}$$

$$\because \boxed{\begin{array}{l} 2\pi - \pi/3 = \pi/4 \\ e^{-j(2\pi - \theta)} = e^{j\theta} \end{array}}$$

$$\boxed{x(n) = e^{j\pi/3} e^{j\pi/4 n} + e^{-j\pi/3} e^{j7\pi/4 n}}$$

exponential fourier series



$|e^{j0}| = 1 \;\therefore\; |e^{j\pi/3}| = 1$

3) $x(n) = 1 + \frac{3}{2}\cos\frac{2\pi}{3}n + 2\sin\frac{2\pi}{3}n$

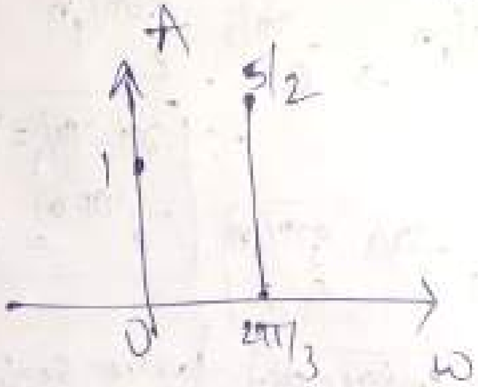
Sol

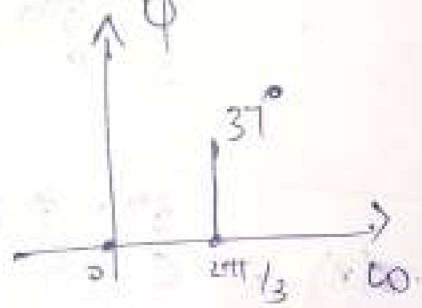
$$\boxed{A\sin\theta + B\cos\theta = \sqrt{A^2+B^2}\,\sin(\theta + \tan^{-1}B/A)}$$

Trigonometric fourier spectrum Polar representation

$x(n) = 1 + \frac{5}{2}\sin\left(\frac{2\pi}{3} + \tan^{-1}3/4\right)$

$x(n) = 1 + \frac{5}{2}\sin(2\pi/3 + 37^\circ)$





Exponential Fourier Series.

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$$x(n) = 1 + \frac{5}{2}\sin\left(\frac{2\pi}{3}n + 37\right)$$

$$\boxed{\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}}$$

$$x(n) = 1 + \frac{5}{4j} e^{j(2\pi/3\, n + 37^\circ)} - \frac{5}{4j} e^{-j(2\pi/3\, n + 37^\circ)}$$

$$x(n) = 1 + \frac{5}{4j} e^{j37^\circ} e^{j2\pi/3\, n} - \frac{5}{4j} e^{-j2\pi/3\, n} e^{-j37^\circ}$$

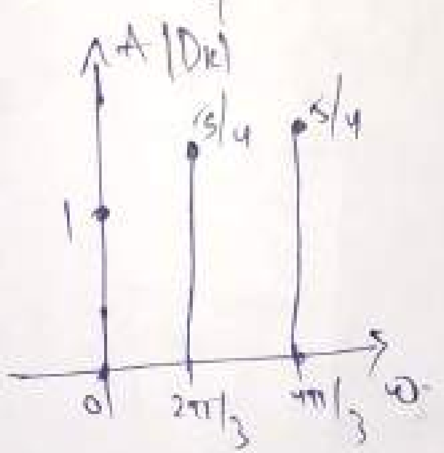
$$x(n) = 1 + \frac{5}{4j} e^{j37^\circ} e^{j2\pi/3\, n} - \frac{5}{4j} e^{-j37^\circ} e^{j\frac{4\pi}{3} n}$$

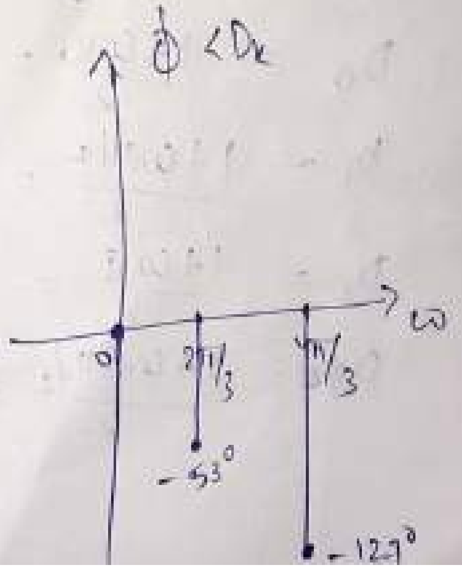
$$\boxed{e^{-j2\pi/3\, n} = e^{j4\pi/3\, n}}$$

$$x(n) = 1 + \frac{5}{4}(-j) e^{j37^\circ} e^{j2\pi/3\, n} - \frac{5}{4}(-j) e^{-j37^\circ} e^{j4\pi/3\, n}$$

$$= 1 + \frac{5}{4} e^{-j90} e^{j37^\circ} e^{j2\pi/3\, n} - \frac{5}{4} e^{-j90^\circ} e^{-j37^\circ} e^{j4\pi/3\, n}$$

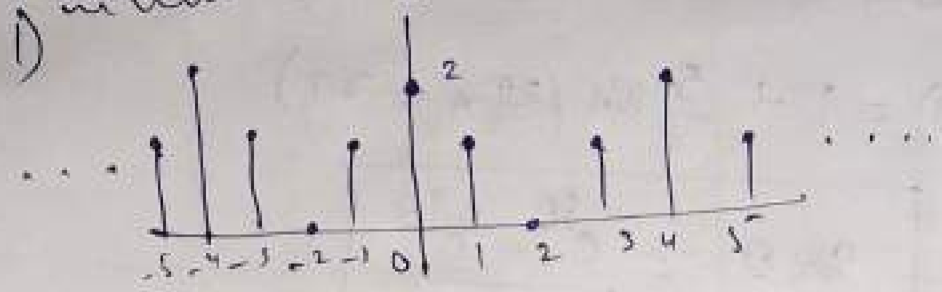
$$\left\{ x(n) = 1 + \frac{5}{4} e^{-j53^\circ} e^{j2\pi/3\, n} - \frac{5}{4} e^{-127^\circ} e^{j4\pi/3\, n} \right\}$$





Problems on DFTS:-

1)



Sol $N_0 = 4$, $\omega_0 = 2\pi/4 = \pi/2$

$$D_k = \frac{1}{N_0}\sum_{n=0}^{N_0-1} x(n)\, e^{-jk\omega_0 n} \quad k = 0-(4-1)$$

$$D_k = \frac{1}{4}\sum_{0}^{3} x(n)\, e^{-jk\omega_0 n} \quad k = 0-3$$

$$D_k = 1/4\left[x(0)\,e^{-0} + x(1)\,e^{-j\pi/2 k} + x(2)\,e^{-j\pi k} + x(3)\,e^{-j\pi/2\, 3k}\right]$$

$$D_k = 1/4\left[2 + 1e^{-jk\pi/2} + e^{-j3k\pi/2}\right]$$

$$D_k = \frac{1}{4}\left[2 + e^{-jk\pi/2} + e^{-jk(2\pi-\pi/2)}\right] \quad k = 0-3$$

$$D_k = 1/4\left[2 + e^{-jk\pi/2} + e^{jk\pi/2}\right]$$

cos(π+π) -cos π/2

$$D_k = \frac{1}{4}\left[2 + 2\cos k\pi/4\right], \quad k = 0 \to 3$$

$$D_k = \frac{1 + \cos k\pi/2}{2}, \quad k = 0 \to 3$$

$$D_0 = \frac{1+\cos 0}{2} = 1$$

$$D_1 = \frac{1+\cos\pi/2}{2} = 1/2$$

$$D_2 = \frac{1+\cos\pi}{2} = 0$$

$$D_3 = \frac{1+\cos 3\pi/2}{2} = 1/2$$

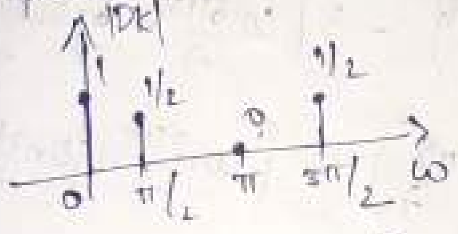
$$x(n) = \sum_{k=0}^{N_0-1} D_k e^{j\Omega_0 nk} = \sum_{0}^{3} D_k e^{jk\pi/2 n}$$

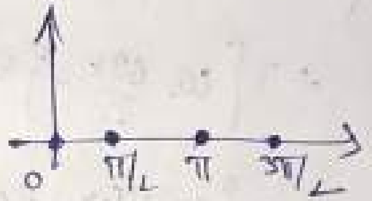
$$x(n) = D_0 e^0 + D_1 e^{j\pi/2 n} + D_2 e^{j\pi n} + D_3 e^{j3\pi/2 n}$$

$$x(n) = 1 + 1/2\, e^{j3\pi/2 n} + 0\, e^{j\pi n} + 1/2\, e^{3\pi/2 jn}$$

The above signal dictates that $x(n)$ contains frequency components $\omega = 0, \pi/2, \pi$ & $3\pi/2$ & has corresponding fourier coefficients as $1, 1/2, 0, 1/2$ respectively

phases are $0, 0, 0$, & 0





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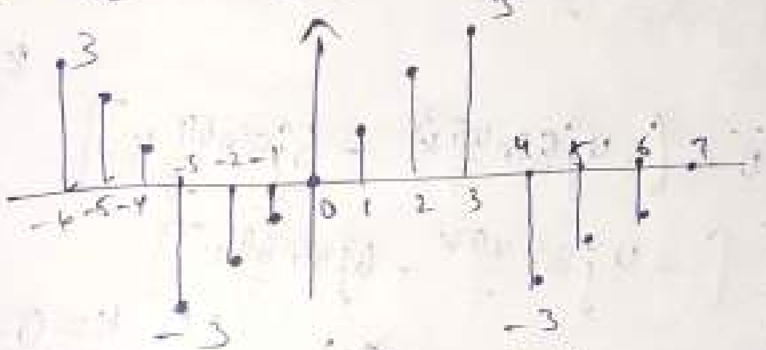


$N_0 = 7$, $\omega_0 = 2\pi/N_0 = 2\pi/7$

$$D_k = \frac{1}{7}\sum_{0}^{6} x(n)\, e^{-jk2\pi/7 n}, \quad k = 0 \to 6$$

$$D_k = \frac{1}{7}\Big[x(0)e^{-0} + x(1)e^{-j2\pi/7 k} + x(2)e^{-j2\pi/7 \times 2k} + x(3)e^{-j\frac{2\pi}{7}3k} + x(4)e^{-j2\pi/7\, 4k} + x(5)e^{-j2\pi/7\, 5k} + x(6)e^{-j2\pi/7\, 6k}\Big]$$

$$D_k = \frac{1}{7}\Big[0 + e^{-jk2\pi/7} + 2e^{-jk4\pi/7} + 3e^{-jk6\pi/7} - 3e^{-j8\pi/7 k} - 2e^{-j\frac{10k\pi}{7}} - e^{-j\frac{12\pi}{7}k}\Big]$$

$$D_k = \frac{1}{7}\Big[0 + e^{-jk\frac{2\pi}{7}} + 2e^{-jk4\pi/7} + 3e^{-jk6\pi/7} - 3e^{-jk(2\pi - \frac{6\pi}{7})} - 2e^{-jk(2\pi - \frac{4\pi}{7})} - 1\cdot e^{-jk(2\pi - \frac{2\pi}{7})}\Big]$$

$$D_k = \frac{1}{7}\Big[0 + e^{-jk\frac{2\pi}{7}} + 2e^{-jk4\pi/7} + 3e^{-jk6\pi/7} - 3e^{jk6\pi/7} - 2e^{jk4\pi/7} - e^{jk2\pi/7}\Big]$$

$$D_k = \frac{1}{7}\Big[0 + \cos\frac{2\pi k}{7} - j\sin\frac{2\pi}{7}k + 2\left[\cos\frac{4\pi k}{7} - j\sin\frac{4\pi k}{7}\right) + 3\left[\cos\frac{6\pi k}{7} - j\sin\frac{6\pi k}{7}\right] - 3\left[\cos\frac{6\pi k}{7} + j\sin\frac{6\pi k}{7}\right] - 2\left[\cos\frac{4\pi k}{7} + j\sin\frac{4\pi k}{7}\right] - \left[\cos\frac{2\pi k}{7} + j\sin\frac{2\pi k}{7}\right]$$

$k = 0 - 6$

$$D_k = \frac{1}{7}\Big[0 + \left(-4j\sin\frac{4\pi}{7}k\right) - 6j\sin\frac{6\pi}{7}k\Big]$$

$$D_k = \frac{1}{7}\Big[-4j\sin\frac{4\pi k}{7} - 6j\sin\frac{6\pi}{7}k\Big]$$

$k = 0 - 6$

$D_0 = 0$

$D_1 = -0.0581j$

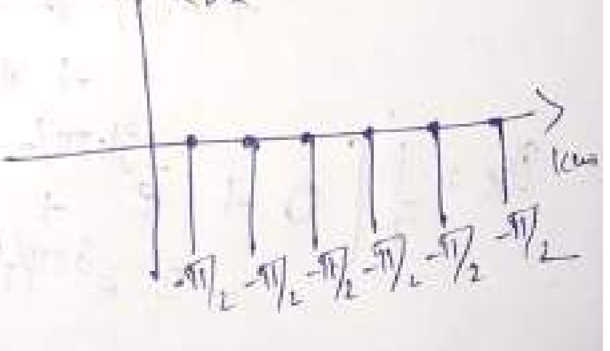
$D_2 = -0.116j$

$D_3 = -0.174j$

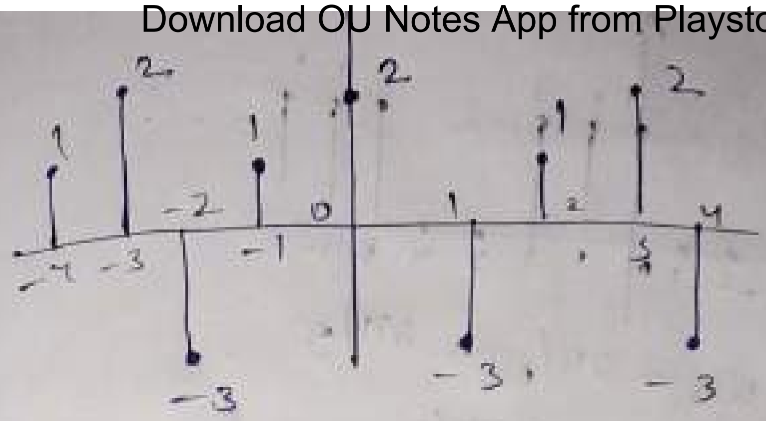
$D_4 = -0.231j$

$D_5 = -0.288j$

$D_6 = -0.345j$



(3)



sol $N_0 = 3, \quad \omega_0 = 2\pi/3 = 2\pi/3$

$$D_k = \frac{1}{N_0} \sum_{n=0}^{N_0-1} x(n) e^{-jk\omega_0 n} \qquad k = 0-2$$

$$D_k = \frac{1}{3} \sum_{0}^{2} x(n) e^{-jkn2\pi/3}$$

$$D_k = \frac{1}{3}\left[x(0) e^{0} + x(1) e^{-jk2\pi/3} + x(2) e^{-jk4\pi/3}\right]$$

$$D_k = \frac{1}{3}\left[2 - 3 e^{-jk2\pi/3} + 1 e^{-jk4\pi/3}\right]$$

$$D_k = \frac{1}{3}\left[2 - 3 e^{-jk2\pi/3} + e^{jk2\pi/3}\right] \qquad k = 0 \to 2$$

$$\boxed{D_k = \frac{1}{3}\left[2 - 3e^{-jk2\pi/3} + e^{jk2\pi/3}\right]}$$

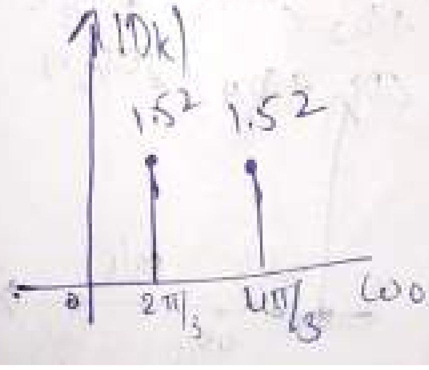
$$D_0 = \frac{1}{3}\left[2 - 3 + 1\right] = 0$$

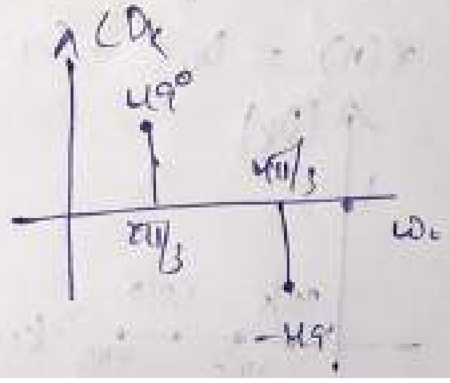
$$D_1 = 1 + 1.5j = 1.52 \angle 49^\circ$$

$$D_2 = \frac{1}{3}\left[2 - 3\left(e^{-j4\pi/3}\right) + e^{j4\pi/3}\right)$$

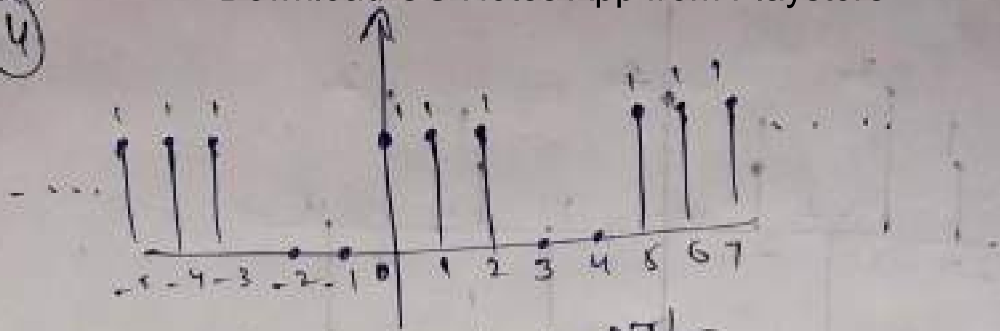
$$= 1 - 1.5j = 1.52 \angle -49^\circ$$

$$x(n) = D_0 e^{0} + D_1 e^{j2\pi/3 n} + D_2 e^{j4\pi/3 n}$$





(4)



$N_0 = 5 \;,\; \omega_0 = 2\pi/N_0 = 2\pi/5$

$D_k = \frac{1}{N_0}\sum_{0}^{4} x(n)\, e^{-jkn2\pi/5} \;,\; k = 0 \to 4$

$D_k = \frac{1}{5}\left[x(0)e^{0} + x(1)e^{-jk2\pi/5} + x(2)e^{-jk4\pi/5} + x(3)e^{-jk6\pi/5} + x(4)e^{-jk8\pi/5}\right]$

$D_k = \frac{1}{5}\left[1 + e^{-j\frac{2\pi}{5}k} + e^{-j\frac{4\pi}{5}k} + e^{j4\pi/5 k} + e^{j\frac{2\pi}{5}k}\right] \quad k = 0 \to 4$

$D_k = \frac{1}{5}\left[1 + 2\cos\frac{2\pi}{5}k + 2\cos\frac{4\pi k}{5}\right] \quad k = 0 \to 4$

$D_0 = \frac{1}{5}[1+2+2] = 1$

$D_1 = \frac{1}{5}\left[1 + 2\cos\frac{2\pi}{5} + 2\cos\frac{4\pi}{5}\right] = 0$

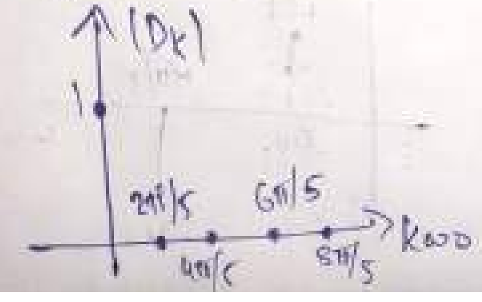
$D_2 = \frac{1}{5}\left[1 + 2\cos 4\pi/5 + 2\cos 8\pi/5\right] = 0$

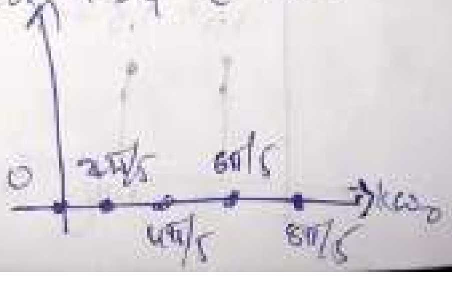
$D_3 = \frac{1}{5}\left[1 + 2\cos\frac{6\pi}{5} + 2\cos\frac{12\pi}{5}\right] = 0$

$D_4 = \frac{1}{5}\left[1 + 2\cos 8\pi/5 + 2\cos\frac{12\pi}{5}\right] = 0$

$x(n) = \sum_{0}^{4} D_k\, e^{jk\omega_0 n} = \sum_{0}^{4} D_k\, e^{jkn2\pi/5}$

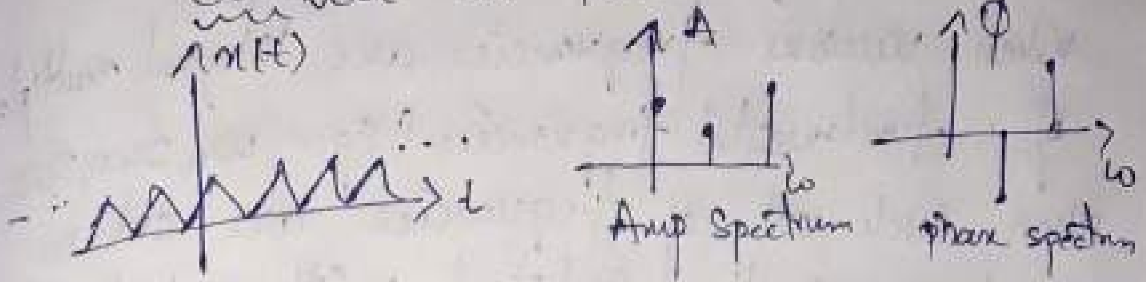
$x(n) = D_0 e^{0} + D_1 e^{j2\pi/5 n} + D_2 e^{j4\pi/5 n} + D_3 e^{j\frac{6\pi}{5}n} + D_4 e^{j8\pi/5 n}$





Unit:- 02

Continous time fourier series (CTFS)



Fourier stated that any arbitrary non-sinusoidal signal can be expressed as a weighted sum of sinusoids, each of which has different amplitudes and frequencies which are harmonically related with each other as shown below

$$x(t) = \sum_{k=0}^{\infty} a_k \cos\omega_0 kt + b_k \sin\omega_0 kt$$

non-sinusoid

Trigonometric fourier series

$$\boxed{x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos\omega_0 kt + b_k \sin\omega_0 kt}$$

a_0 → D.C component of $x(t)$

$$x(t) = a_0 + (a_1 \cos\omega_0 t + b_1 \sin\omega_0 t) + a_2 \cos 2\omega_0 t + b_2 \sin 2\omega_0 t + (a_3 \cos 3\omega_0 t + b_3 \sin 3\omega_0 t) + \ldots$$

Note:-

Harmonic frequencies:-

When various frequencies are integral multiple of fundamental frequencies (Eg:- $\omega_0, 2\omega_0, 3\omega_0, \ldots$) then such set of frequencies are set to be harmonically related with each other

ω_0 = fundamental frequency

ω_0 = 1^{st} harmonic frequency.

$2\omega_0$ = 2^{nd} harmonic freq

$a_1, b_1 \rightarrow$ fourier coefficients of 1^{st} harmonic freq

$a_2, b_2 \rightarrow$ fourier coeff of 2^{nd} harmonic freq.

$a_1 \cos\omega_0 t + b_1 \sin\omega_0 t \rightarrow 1^{st}$ harmonic component

$a_2 \cos 2\omega_0 t + b_2 \sin 2\omega_0 t \rightarrow 2^{nd}$ " "

Trigonometric Identities:-

1) $\int_0^{T_0} \cos k\omega_0 t \, dt = \int_0^{T_0} \sin k\omega_0 t \, dt = 0$

Sol $\int_0^{T_0} \cos\omega_0 t \, dt = \sin\omega_0 t \Big]_0^{T_0}$

$= \sin\omega_0 T_0 - \sin 0$

$= \sin\omega_0 T_0 - 0 = \sin\frac{2\pi}{T_0} T_0$

$= \sin 2\pi = 0$

(2) $\int_{<T_0>} \sin \omega_0 kt \cos \omega_0 mt = 0, \ \forall k \& m$

(3) $\int_{<T_0>} \sin k\omega_0 t \sin m\omega_0 t \, dt = \int_{<T_0>} \cos \omega_0 kt \cos \omega_0 mt \, dt$

$= 0 \quad , k \neq m$

$= T_0/2 \, , \quad k = m$

(4) $\int_{<T_0>} \sin^2 k\omega_0 t \, dt = \int_{<T_0>} \cos^2 k\omega_0 t \, dt = \frac{T_0}{2}$

* Expressions for a_0, a_k, b_k - Trigonometric fourier coefficients.

Sol: consider the fourier series

$$x(t) = a_0 + \sum_{k=1}^{\infty} (a_k \cos \omega_0 kt + b_k \sin \omega_0 kt)$$

Integrate both sides w.r.t t for duration T_0

$$\int_{<T_0>} x(t) \, dt = a_0 \int_{<T_0>} 1 . dt + \sum_{k=1}^{\infty} a_k \int_{<T_0>} \cos k\omega_0 t \, dt + \left. \sum_{k=1}^{\infty} b_k \int_{<T_0>} \sin k\omega_0 t \, dt \right]$$

$$\int_{<T_0>} x(t) \, dt = a_0 (t)_0^{T_0} \qquad T_0 = \frac{2\pi}{100}$$

$$\int_{<T_0>} x(t) \, dt = a_0 T_0$$

$$\Rightarrow \boxed{a_0 = \frac{1}{T_0} \int_{<T_0>} x(t) \, dt}$$

Note:- The expression $a_0' = \frac{1}{T_0} \int_{<T_0>} x(t)\, dt$ dictates that the D.C content in the signal $x(t)$ is simply the average value of the signal

a_k :-

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos k\omega_0 t + b_k \sin k\omega_0 t$$

multiply with $\cos m\omega_0 t$ & integrate w.r.t $\omega_0 t$ / $\cos$ integrate with duration $<T_0>$

$$\int_{<T_0>} x(t) \cos m\omega_0 t\, dt = a_0 \int_{<T_0>} \cos m\omega_0 t\, dt$$

$$+ \sum_{k=1}^{\infty} a_k \int_{<T_0>} \cos k\omega_0 t \cos m\omega_0 t\, dt$$

$$+ \sum_{k=1}^{\infty} b_k \int_{<T_0>} \sin k\omega_0 t \cos m\omega_0 t\, dt.$$

$$\int_{<T_0>} x(t) \cos m\omega_0 t\, dt = a_0[0] +$$

$$\sum_{k=1}^{\infty} a_k \int_{<T_0>} \cos k\omega_0 t \cos m\omega_0 t\, dt + 0$$

$$\int_{<T_0>} x(t) \cos m\omega_0 t\, dt = a_k \int_{<T_0>} \cos \omega_0 k t \cos \omega_0 k t\, dt$$

$$+ \sum_{\substack{k=1 \\ k \neq m}}^{\infty} a_k \int_{<T_0>} \cos \omega_0 k t \cos \omega_0 m t\, dt$$

$$\int_{<T_0>} x(t) \cos m\omega_0 t \, dt = a_k T_0/2$$

$$\Rightarrow \boxed{a_k = \frac{2}{T_0} \int_{<T_0>} x(t) \cos \omega_0 t \, dt}$$

b_k:-

multiply B.S with $\sin m\omega_0 t$ & $\int_{<T_0>}$

$$\int_{<T_0>} x(t) \sin m\omega_0 t \, dt = a_0 \int_{<T_0>} \sin m\omega_0 t \, dt + \sum_{k=1}^{\infty} a_k \int_{<T_0>} \sin m\omega_0 t \cos k\omega_0 t \, dt + \sum_{k=1}^{\infty} b_k \int_{<T_0>} \sin k\omega_0 t \sin m\omega_0 t \, dt$$

$$\int_{<T_0>} x(t) \sin m\omega_0 t = 0 + 0 + \sum_{k=1}^{\infty} b_k \int_{<T_0>} \sin k\omega_0 t \sin m\omega_0 t \, dt$$

$$\int_{<T_0>} x(t) \sin m\omega_0 t = b_k \int_{<T_0>} \sin k\omega_0 t \sin k\omega_0 t \, dt + \sum_{\substack{k=1 \\ k \neq m}}^{\infty} b_k \int_{<T_0>} \sin k\omega_0 t \sin m\omega_0 t \, dt$$

$$= b_k (T_0/2) + 0$$

$$\boxed{b_k = \frac{2}{T_0} \int_{<T_0>} x(t) \sin m\omega_0 t}$$

Polar form of Fourier series / Cosine form of Fourier series

$$x(t) = a_0 + \sum_{k=1}^{\infty} \sqrt{a_k^2+b_k^2}\left[\frac{a_k}{\sqrt{a_k^2+b_k^2}} \cos k\omega_0 t + \frac{b_k}{\sqrt{a_k^2+b_k^2}} \sin k\omega_0 t\right]$$

$$\frac{a_k}{\sqrt{a_k^2+b_k^2}} = \cos\theta_k \qquad \frac{b_k}{\sqrt{a_k^2+b_k^2}} = \sin\theta_k$$

$$\tan\theta_k = \frac{b_k}{a_k}$$

$$\theta_k = \tan^{-1}\left(\frac{b_k}{a_k}\right)$$

$$A_k = \sqrt{a_k^2+b_k^2}$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} A_k\left[\cos\theta_k \cos k\omega_0 t + \sin\theta_k \sin k\omega_0 t\right] \quad \left\{\cos(A-B) = \cos A\cos B + \sin A \sin B\right\}$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} A_k \cos(\omega_0 k t - \theta_k)$$

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(\omega_0 k t - \theta_k)$$

$$A_0 = a_0$$

$$A_k = \sqrt{a_k^2+b_k^2}$$

$$\theta_k = \tan^{-1}\left(\frac{b_k}{a_k}\right)$$

Exponential fourier series (EFS)

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

According to this series any arbitrary signal can be expressed as a weighted sum of complex exponentials as follows, each of which are harmonically related which each other

$$\left[\int_{\langle T_0 \rangle} e^{j(k-m)\omega_0 t} dt = T_0, \; k=m \\ = 0, \; k \neq m \right]$$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

multiply B.S with $e^{-jk\omega_0 t}$ & $\int_{\langle T_0 \rangle} dt$

$$\int_{\langle T_0 \rangle} e^{-jm\omega_0 t} \cancel{dt}\, x(t) = \sum_{k=-\infty}^{\infty} C_k \int_{\langle T_0 \rangle} e^{jk\omega_0 t} e^{-jm\omega_0 t} dt$$

$$\int_{\langle T_0 \rangle} x(t) e^{-jm\omega_0 t} dt = \sum_{k=-\infty}^{\infty} C_k \int_{\langle T_0 \rangle} e^{j\omega_0 t (k-m)} dt$$

$$\int_{\langle T_0 \rangle} x(t) e^{-jm\omega_0 t} = C_k \int_{\langle T_0 \rangle} e^{j\omega_0 t (k-m)} dt + \sum_{\substack{k=-\infty \\ k \neq m}}^{\infty} C_k \int_{\langle T_0 \rangle} e^{j\omega_0 t (k-m)} dt$$

$$\int_{\langle T_0 \rangle} x(t) e^{-jm\omega_0 t} = C_k T_0$$

$$\boxed{C_k = \frac{1}{T_0} \int_{\langle T_0 \rangle} x(t) e^{-jm\omega_0 t} dt}$$

* Relationship b/w TFS & EFS

Consider TFS

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos k\omega_0 t + \sum_{k=1}^{\infty} b_k \sin k\omega_0 t$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \left(\frac{e^{jk\omega_0 t} + e^{-jk\omega_0 t}}{2}\right) + \sum_{k=1}^{\infty} b_k \left(\frac{e^{jk\omega_0 t} - e^{-jk\omega_0 t}}{2j}\right)$$

$$x(t) = a_0 + \sum_{k=1}^{\infty}\left(\frac{a_k}{2} + \frac{b_k}{2j}\right)e^{jk\omega_0 t} + \sum_{k=1}^{\infty}\left(\frac{a_k}{2} - \frac{b_k}{2j}\right)e^{-jk\omega_0 t}$$

$$x(t) = a_0 + \sum_{k=1}^{\infty}\left(\frac{a_k - jb_k}{2}\right)e^{jk\omega_0 t} + \sum_{k=1}^{\infty}\left(\frac{a_k + jb_k}{2}\right)e^{-jk\omega_0 t}$$

$$x(t) = a_0 + \sum_{k=1}^{\infty}\left(\frac{a_k - jb_k}{2}\right)e^{jk\omega_0 t} + \sum_{k=-1}^{-\infty}\left(\frac{a_k + jb_k}{2}\right)e^{jk\omega_0 t}$$

$$x(t) = a_0 + \sum_{k=1}^{\infty}\left(\frac{a_k - jb_k}{2}\right)e^{jk\omega_0 t} + \sum_{k=-1}^{-\infty}\left(\frac{a_{-k} + jb_{-k}}{2}\right)e^{jk\omega_0 t}$$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} = C_0 + \sum_{k=1}^{\infty} C_k e^{jk\omega_0 t} + \sum_{k=-1}^{-\infty} C_{-k} e^{j\omega_0 t}$$

$$\boxed{C_0 = a_0 \qquad C_k = \frac{a_k - jb_k}{2}, \quad C_{-k} = \frac{a_{-k} + jb_{-k}}{2}}$$

$$C_{-k} = \frac{a_k + jb_k}{2}$$

$\therefore \boxed{C_{-k} = C_k^*}$

Notes :-

① $\cos k\pi = (-1)^k = -1, \; n \to \text{odd} \; -1, 3, 5, 7 \ldots$

$= +1, \; n \to \text{even} \; = 2, 4, 6.$

② $\sin k\pi = 0, \; \forall k$

③ $\cos \frac{k\pi}{2} = 0, \; k = 1, 3, 5$

$= -1, \; k = 2, 6, 10$

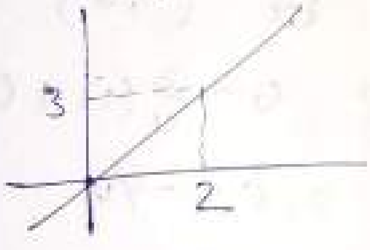
$= 1, \; k = 4, 8, 12$

④ $\sin \frac{k\pi}{2} = 1, \; k = 1, 5, 9, 13$

$= -1, \; k = 3, 7, 11, 15$

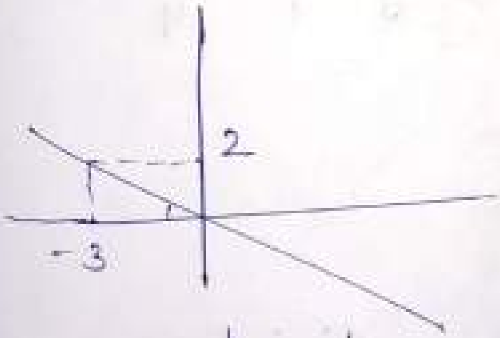
$= 0, \; k = \text{even}$

Co-ordinate Geometry :-

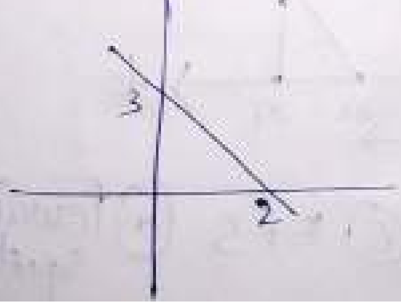


$y = mx$

$y = \frac{3}{2} x$

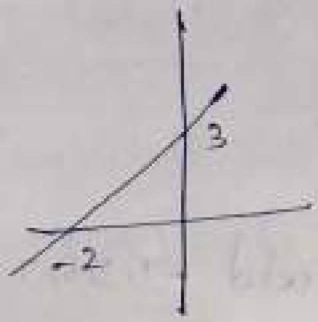


$y = \frac{-2}{3} x$



$y = mx + c$

$y = -\frac{3}{2} x + 3$



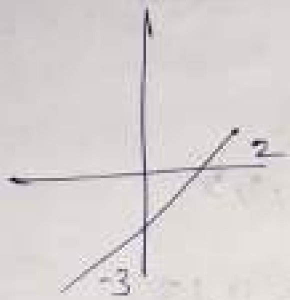
$$y = \frac{3}{2}x + 3$$

$\frac{x}{2} + \frac{y}{3} = 1$

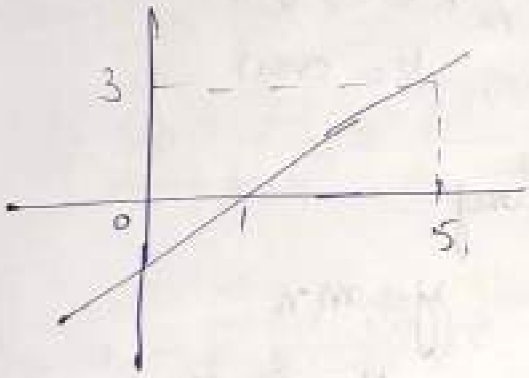
$3x + 2y = 6$

$2y = 6 - 3x$

$y = 3 - \frac{3x}{2}$



$$y = \frac{3}{2}x - 3$$



$$y = mx + c$$

$$m = 3/4$$

$$y = \frac{3}{4}x + c$$

let $(1, 0)$

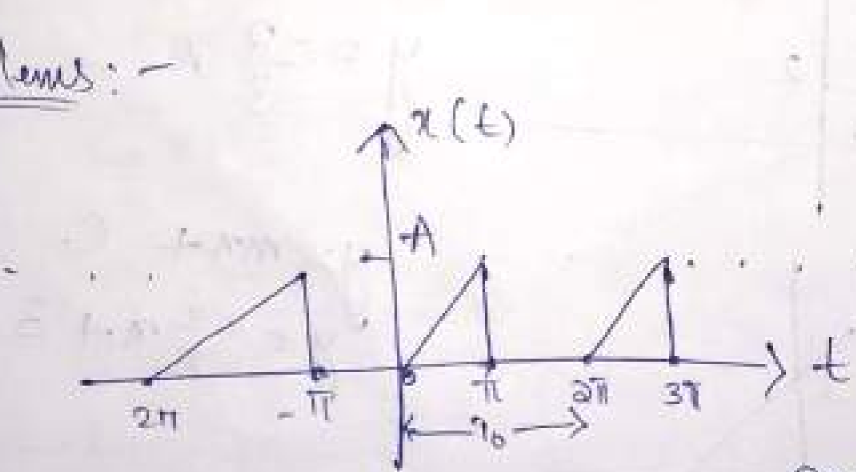
$$0 = \frac{3}{4}(1) + c$$

$$c = -3/4$$

$$\Rightarrow y = \frac{3}{4}x - \frac{3}{4}$$

Problems :-

(1)



(a) TFS (b) PFS (c) EFS (d) Fourier spectrum

$T_0 = 2\pi$, $\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2\pi} = 1$

$\boxed{\omega_0 = 1}$

we have $y = mx$

$x(t) = mt$

$x(t) = \frac{A}{\pi} t$, $0 \le t \le \pi$

$= 0$, $\pi \le t \le 2\pi$

$a_0 = \frac{1}{T_0} \int_{<T_0>} x(t)\, dt$

$a_0 = \frac{1}{2\pi} \int_0^{T_0} x(t)\, dt$

$\Rightarrow \frac{1}{2\pi} \int_0^{2\pi} x(t) = \frac{1}{2\pi} \left[\int_0^{\pi} x(t)\, dt + \int_{\pi}^{2\pi} x(t) dt \right]$

$= \frac{1}{2\pi} \left[\int_0^{\pi} \frac{A}{\pi} t\, dt + 0 \right]$

$= \frac{A}{2\pi^2} \int_0^{\pi} t\, dt$

$= \frac{A}{2\pi^2} \left(\frac{t^2}{2} \right)_0^{\pi}$

$= \frac{A}{2\pi^2} \left(\frac{\pi^2}{2} \right) = A/4$

$\boxed{a_0 = A/4}$

$$a_k = \frac{2}{T_0}\int_{\langle T_0\rangle} x(t)\cos k\omega_0 t\, dt \quad , \omega_0 = 1$$

$$a_k = \frac{2}{2\pi}\int_0^{2\pi} x(t)\cos k\omega_0 t\, dt$$

$$a_k = \frac{1}{\pi}\int_0^{\pi} \frac{A}{\pi} t\cos kt\, dt + \frac{1}{\pi}\int_{\pi}^{2\pi} 0\cos kt\, dt$$

$$a_k = \frac{A}{\pi^2}\int_0^{\pi} t\cos kt\, dt \qquad \text{ILATE}$$

$$a_k = \frac{A}{\pi^2}\left[t\left(\frac{\sin kt}{k}\right)_0^{\pi} - \int_0^{\pi} \frac{\sin kt}{k}\, dt\right]$$

$$a_k = \frac{A}{\pi^2}\left[\frac{t}{k}\left[\sin k\pi - \sin k0\right] - \frac{1}{k}\left[- \cos k\pi\right]_0^{\pi}\right]$$

$$a_k = \frac{A}{\pi^2}\left[t\frac{\sin kt}{k} + \frac{\cos kt}{k^2}\right]_0^{\pi}$$

$$a_k = \frac{A}{\pi^2 k}\left[\frac{\cos \pi k}{k} - \frac{\cos 0}{k}\right]$$

$$a_k = \frac{A}{\pi^2 k^2}\left[-1 - 1\right] \quad , k = \text{odd}$$

$$a_k = A/\pi^2 k^2 \left[+1 - 1\right] \quad , k = \text{even}$$

$$\left\{\begin{array}{l} a_k = \dfrac{-2A}{\pi^2 k^2} \quad , k = \text{odd} \\ a_k = 0, \quad k = \text{even} \end{array}\right\}$$

$$b_k = \frac{A}{\pi^2}\int_0^{\pi} t \sin kt \, dt$$

$$b_k = \frac{A}{\pi^2}\left[t\left[-\frac{\cos kt}{k}\right] - \int_0^{\pi} -\frac{\cos kt}{k}\,dt\right].$$

$$b_k = \frac{A}{\pi^2}\left[-\frac{t\cos kt}{k} + \frac{\sin kt}{k^2}\right]_0^{\pi}$$

$$b_k = \frac{A}{\pi^2}\left[\frac{-\pi\cos\pi k}{k} + 0\right]$$

$$b_k = \frac{-A\cos k\pi}{\pi k}.$$

$$\begin{cases} b_k = \dfrac{A}{\pi k}, & k = \text{odd } 1,3,5,7 \\ b_k = \dfrac{-A}{\pi k}, & k = \text{even } 2,4,6,8 \end{cases}$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos \omega_0 kt + b_k \sin k\omega_0 t \quad , \omega_0 = 1$$

$$x(t) = \frac{A}{4} + \sum_{\substack{k=1,3,5 \\ k=\text{odd}}}^{\infty} -\frac{2A}{\pi^2 k^2}\cos kt + \sum_{\substack{k=1 \\ k=\text{odd}}}^{\infty} \frac{A}{\pi k}\sin kt + \sum_{\substack{k=2 \\ k=\text{even}}}^{\infty} -\frac{A}{\pi k}\sin kt$$

$$x(t) = \frac{A}{4} + \sum_{\substack{k=1 \\ k=\text{odd}}} \sqrt{\frac{4A^2}{\pi^4 k^4} + \frac{A^2}{\pi^2 k^2}} \cos\left(kt - \tan^{-1}\left(\frac{A/\pi k}{-2A/\pi^2 k^2}\right)\right) + \sum_{k=2}^{\infty} \frac{A}{\pi k}\cos\left(kt - \frac{\pi}{2}\right)$$

$$x(t) = A/4 + \sum_{\substack{k=1 \\ k=\text{odd}}} \sqrt{\frac{4A^2}{\pi^4 k^4} + \frac{A^2}{\pi^2 k^2}} \cos\left(kt - \tan^{-1}\left(-\frac{\pi k}{2}\right)\right) + \sum_{\substack{k=\text{even} \\ k=2}} \frac{A}{\pi k}\cos\left(kt - \frac{\pi}{2}\right)$$

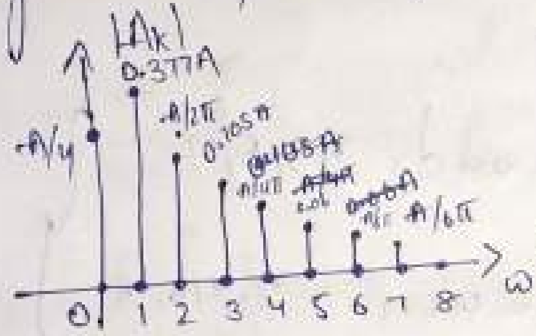
$$\sqrt{\frac{4A^2}{\pi^4k^4}+\frac{A^2}{\pi^2k^2}} = \frac{A}{\pi k}\sqrt{\frac{4}{\pi^2k^2}+1}$$

$$k=1, \quad \frac{A}{\pi}\sqrt{\frac{4}{\pi^2}+1} = 0.377A$$

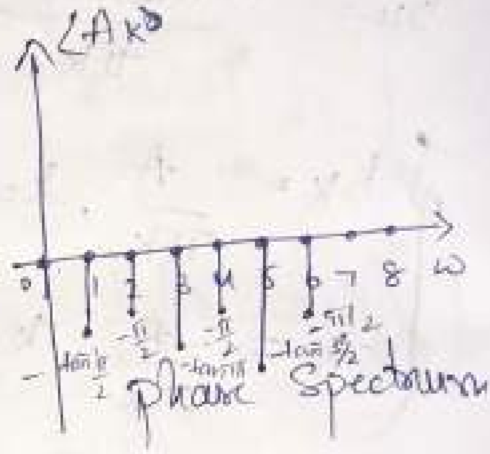
$$k=3, \quad \frac{A}{3\pi}\sqrt{\frac{4}{9\pi^2}+1} = 0.108A$$

$$k=5, \quad \frac{A}{5\pi}\sqrt{\frac{4}{25\pi^2}+1} = 0.06A$$

Trignometric fourier spectrum



Magnitude spectrum



Phase spectrum

(c) EFS

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

$$x(t) = C_0 + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} C_k e^{jk\omega_0 t}$$

$$x(t) = \frac{A}{4} + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} C_k e^{jk\omega_0 t}$$

$$C_k = \frac{a_k - jb_k}{2}$$

$$C_k = \frac{-\frac{2A}{\pi^2k^2} + \frac{jA}{\pi k}}{2}, \quad k = \text{odd}$$

$$C_K = -\frac{A}{\pi^2 k^2} - \frac{jA}{2\pi k}, \quad k = \text{odd}$$

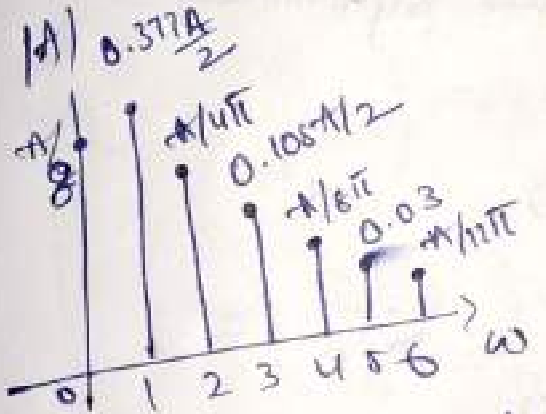
k - even

$$C_K = \frac{0 - j\left(-A/\pi k\right)}{2}$$

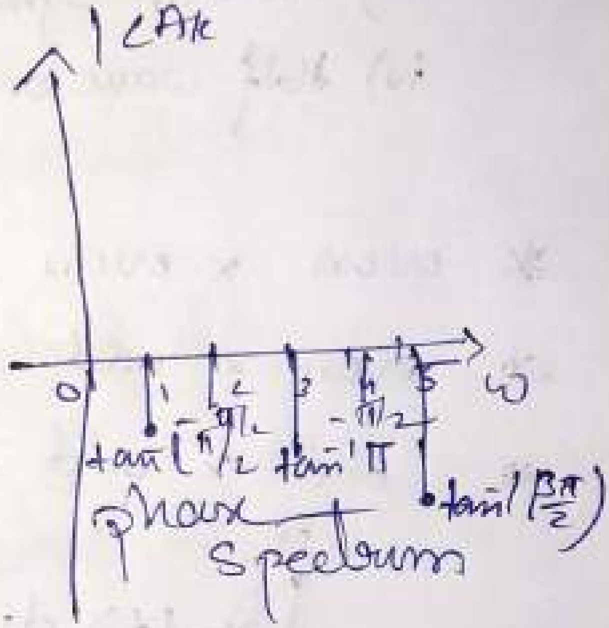
$$C_K = \frac{jA}{2\pi k}, \quad k = \text{even}.$$

$$x(t) = \frac{A}{4} + \sum_{k=1,3,5}^{\infty}\left(\frac{-A}{\pi^2k^2} - \frac{jA}{2\pi k}\right)e^{jkt} + \sum_{k=2,4,6}^{\infty}\frac{jA}{2\pi k}e^{jkt}$$

$$+ \sum_{k=-1,-3,-5}^{-\infty}\left(\frac{-A}{\pi^2k^2} + \frac{jA}{2\pi k}\right)e^{-jkt} + \sum_{k=-2,4,6}^{-\infty}\left(\frac{-jA}{2\pi k}\right)e^{-jkt}$$



magnitude spectrum



phase spectrum

Effect of Symmetricity on fourier series.

→ It is observed that signals which some kind of symmetricity will have a relatively easier fourier series analysis.

→ There are 4 kinds of symmetricity

* i) Even symmetry
* ii) odd symmetry
* iii) Hidden symmetry
* iv) Half wave or Rotational symmetry

* even × even = even
* even × odd = odd
* odd × odd = even

$$\int_0^T x_e(t)\,dt = 2\int_0^{T/2} x_e(t)\,dt$$

$$\int_{-T/2}^{T/2} x_e(t)\,dt = 2\int_0^{T/2} x_e(t)\,dt$$

$$\int_0^T x_o(t)\,dt = 0$$

$$\int_{-T/2}^{T/2} x_o(t)\,dt = 0$$

① Even Symmetricity

let $x(t)$ be even function

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$$

$$= \frac{1}{T_0} 2\int_0^{T_0/2} x(t)\,dt$$

$$\left[a_0 = \frac{2}{T_0}\int_0^{T_0/2} x(t)\,dt\right]$$ even function

$$a_k = \frac{2}{T_0}\int_0^{T_0} x(t)\,\cancel{dt}\cos k\omega_0 t\,dt$$

$$= \frac{2}{T_0} 2\int_0^{T_0/2} x(t)\cos k\omega_0 t\,dt$$

$$\left\{a_k = \frac{4}{T_0}\int_0^{T_0/2} x(t)\cos k\omega_0 t\,dt\right\}$$

$$b_k = \frac{2}{T_0}\int_0^{T_0} x(t)\sin k\omega_0 t\,dt$$

even × odd = odd

$$\boxed{b_k = 0}$$

$$a_0 = \frac{2}{T_0}\int_0^{T_0/2} x(t)\,dt \qquad \left(x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos \omega_0 t k\right)$$

$$a_k = \frac{4}{T_0}\int_0^{T_0/2} x(t)\cos k\omega_0 t\,dt$$

$$b_k = 0$$

∴ For even periodic signal will have only D.C & cosine terms in a, b F.S expression

② odd Symmetry

(a) let $x(t)$ be odd function.

$$a_0 = \frac{1}{T_0}\int_0^{T_0} \underbrace{x(t)}_{\text{odd}}\, dt$$

$$a_0 = \frac{1}{T_0}(0) \Rightarrow \boxed{a_0 = 0}$$

$$a_k = \frac{2}{T_0}\int_0^{T_0} x(t)\cos k\omega_0 t\, dt$$

$$a_k = \frac{2}{T_0}\int_0^{T_0} \text{odd} = 0$$

$$\boxed{a_k = 0}$$

$$b_k = \frac{2}{T_0}\int_0^{T_0} x(t)\sin k\omega_0 t\, dt$$

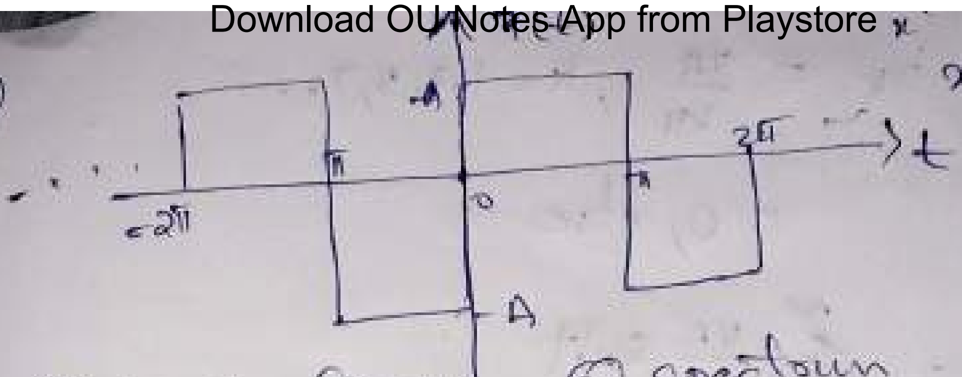
$$= \frac{2}{T_0}\, 2\int_0^{T_0/2} x(t)\sin k\omega_0 t\, dt$$

$$b_k = \frac{4}{T_0}\int_0^{T_0/2} x(t)\sin k\omega_0 t\, dt$$

$$\boxed{x(t) = \sum_{k=1}^{\infty} b_k \sin k\omega_0 t}$$

∴ odd periodic signal will have only b_k term

(13)



$x(t) = t$
$x(-t) = -t$ odd

ⓐ TFS ⓑ EFS ⓒ spectrum

→ As the above signal is odd

then $a_0 = a_k = 0$

$$b_k = \frac{4}{T_0}\int_0^{T_0/2} x(t)\sin k\omega_0 t \, dt$$

here $T_0 = 2\pi$, $\omega_0 = \frac{2\pi}{T_0} = 1$.

$$\boxed{x(t) = A, \quad 0 \le t \le \pi}$$

$$b_k = \frac{4}{2\pi}\int_0^{\pi} A \sin kt \, dt$$

$$= \frac{2A}{\pi}\left[-\frac{\cos kt}{k}\right]_0^{\pi}$$

$$= \frac{2A}{k\pi}\left[-\cos k\pi + \cos 0\right]$$

$$b_k = -\frac{2A}{k\pi}\left[\cos k\pi - 1\right]$$

when k = odd

$$b_k = -\frac{2A}{k\pi}\left[-1-1\right] = \frac{4A}{k\pi}$$

when k = even

$$b_k = -\frac{2A}{k\pi}\left[1-1\right] = 0$$

$$\Rightarrow b_k = \frac{4A}{k\pi}, \quad k = 1,3,5,7\ldots$$

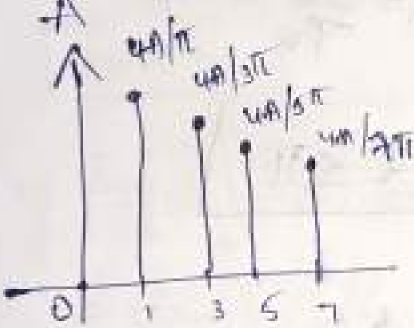
$$= 0, \text{ else}$$

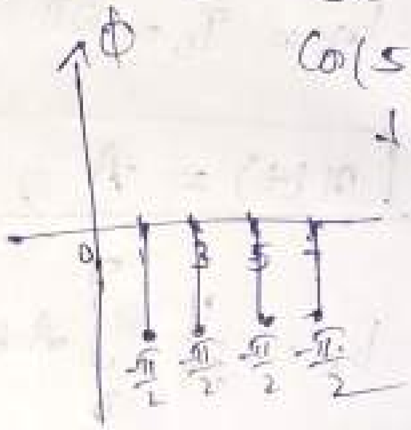
$$x(t) = \sum_{k=1,3,5}^{\infty} \frac{4A}{k\pi} \sin kt$$

$$x(t) = \frac{4A}{\pi}\sin t + \frac{4A}{3\pi}\sin 3t + \frac{4A}{5\pi}\sin 5t + \ldots$$

$$\boxed{\sin kt = \cos(kt - 90)}$$

$$x(t) = \frac{4A}{\pi}\cos\left(t - \frac{\pi}{2}\right) + \frac{4A}{3\pi}\cos\left(3t - \frac{\pi}{2}\right) + \frac{4A}{5\pi}\cos\left(5t - \frac{\pi}{2}\right) + \ldots$$





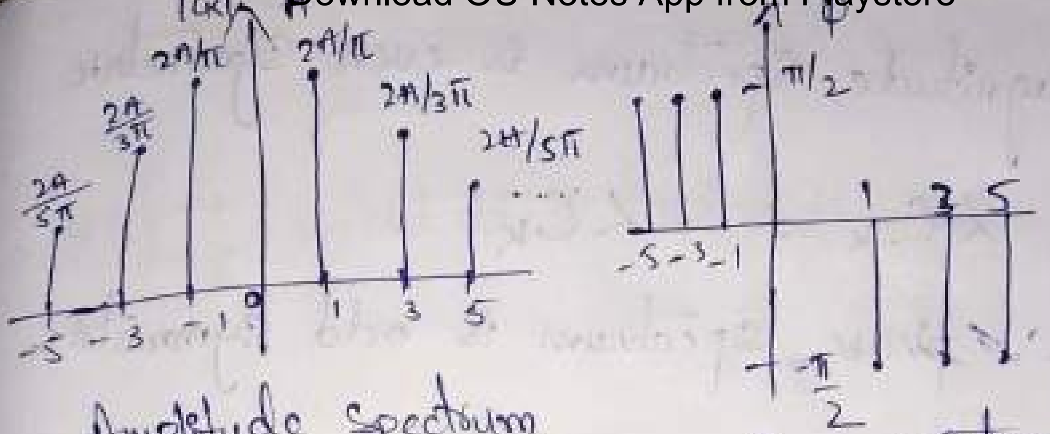
Spectrum

E.F.S :- $x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$

$$C_k = \frac{a_k - jb_k}{2} = -jb_k/2 = -j\frac{4A}{2k\pi}, \quad k = \text{odd}$$

$$= 0, \quad k = \text{even}$$

$$C_{-k} = C_k^* = \frac{j2A}{k\pi} \qquad x(t) = \sum_{k=-1}^{-\infty} \frac{2A}{k\pi} e^{jkt} + \sum_{k=1}^{\infty} \frac{-2Aj}{k\pi} e^{jkt}$$



→ Amplitude Spectrum is always even Symmetric

→ phase spectrum is always odd Symmetric

*) Prove that Amplitude Spectrum is even Symmetric & phase spectrum is odd symmetric for a real signal

Sol As it is known that C_K is in general is complex, can be represented as

$$C_K = a + jb \qquad |C_K| = \sqrt{a^2+b^2}$$

$$\angle C_K = \tan^{-1}(b/a)$$

then

$$C_{-K} = a - jb = C_K^*$$

$$|C_{-K}| = \sqrt{a^2+b^2}$$

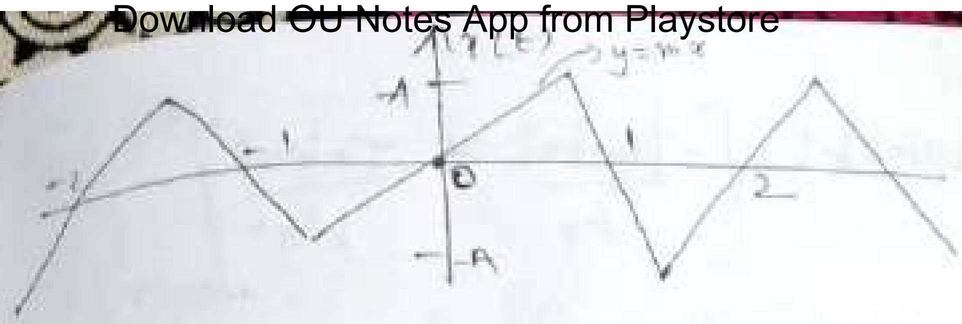
$$\angle C_{-K} = -\tan^{-1}(b/a)$$

$$\therefore \ |C_K| = |C_{-K}|$$

Magnitude spectrum is even symmetric

$$\angle C_{-k} = -\angle C_k$$

$\therefore$ phase spectrum is odd symmetric

2)


sol odd symmetric

$$a_0 = a_k = 0$$

$$b_k = \frac{4}{T_0}\int_0^{T_0/2} x(t)\sin k\omega_0 t \, dt$$

s1) $T_0 = 2 \quad \omega_0 = \frac{2\pi}{2} = \pi$

$x(t) = 2At, \quad 0 \le t \le \frac{1}{2}$

$y = mx$

$y = \frac{A}{1/2} x$

$x(t) = -2At + 2A$

$= 2A(1-t), \quad \frac{1}{2} \le t \le 1$

$y = mx + c$

$y = -2Ax + c$

$0 = -2A(1) + c$

$\boxed{c = 2A}$

s2) $b_k = \frac{4}{2}\int_0^1 x(t).\sin k\pi t \, dt$

$$= 2\left[\int_0^{1/2} 2At \sin k\pi t \, dt + \int_{1/2}^{1} 2A(1-t)\sin k\pi t \, dt\right]$$

$$= 4A\left[\int_0^{1/2} t.\sin k\pi t + \int_{1/2}^{1} \sin k\pi t - \int_{1/2}^{1} t.\sin k\pi t \, dt\right]$$

$$\int_{1/2}^{1} \sin k\pi t = \left[\frac{-\cos k\pi t}{k\pi}\right]_{1/2}^{1} = \frac{+1}{\pi k}\left[\cos\frac{k\pi}{2} - \cos k\pi\right]$$

$$\int_0^{1/2} t.\sin k\pi t = -\frac{t\cos k\pi t}{k\pi} - \int -\frac{\cos k\pi t}{k\pi}(1)$$

$$\int_0^{1/2} t\sin \pi k t = \left[-\frac{t\cos k\pi t}{k\pi} + \frac{\sin k\pi t}{k^2\pi^2}\right]_0^{1/2}$$

$$= \left[\cancel{\frac{1}{2k\pi}[\cos k\pi]} - \frac{\left(\frac{1}{2}\cos\frac{k\pi}{2}+0\right)}{k\pi} + \frac{1}{k^2\pi^2}\left[\sin\frac{k\pi}{2} - \sin 0\right]\right.$$

$$= -\frac{\cos k\pi/2}{2k\pi} + \frac{\sin k\pi/2}{k^2\pi^2}$$

$$\int_{1/2}^{1} t\sin k\pi t = -\frac{t\cos k\pi t}{k\pi} + \frac{\sin k\pi t}{k^2\pi^2}\Bigg|_{1/2}^{1}$$

$$= \frac{-\left(+1\cos k\pi - \frac{1}{2}\cos\frac{k\pi}{2}\right)}{k\pi} + \frac{1}{k^2\pi^2}\left(\sin k\pi - \sin\frac{k\pi}{2}\right)$$

$$= \frac{\cos\frac{k\pi}{2}}{2k\pi} - \frac{\cos k\pi}{k\pi} + \frac{\sin k\pi}{k^2\pi^2} - \frac{\sin\frac{k\pi}{2}}{k^2\pi^2}$$

$$b_k = 4A\left[\cancel{-\frac{\cos k\pi/2}{2k\pi}} + \frac{\sin k\pi/2}{k^2\pi^2} + \cancel{\frac{\cos k\pi/2}{\pi k}} \cancel{-\frac{\cos k\pi}{\pi k}} + \right.$$

$$\left.\cancel{-\frac{\cos k\pi/2}{2k\pi}} + \cancel{\frac{\cos k\pi}{k\pi}} - \frac{\sin k\pi}{k^2\pi^2} + \frac{\sin\frac{k\pi}{2}}{k^2\pi^2}\right]$$

$$b_k = 4A\left[\frac{2\sin k\pi/2}{k^2\pi^2} - \frac{\sin k\pi}{k^2\pi^2}\right]$$

$$b_k = 4A\left(\frac{2\sin k\pi/2}{k^2\pi^2} - 0\right)$$

$$= \frac{8A\sin\frac{k\pi}{2}}{k^2\pi^2} = \frac{8A}{k^2\pi^2}, \quad k = 1, 5, 9, 13, \ldots$$

$$= \frac{-8A}{k^2\pi^2}, \quad k = 3, 7, 11, 15, \ldots$$

$$= 0, \quad k = \text{even}$$

$$x(t) = \sum_{k=1}^{\infty} b_k \sin k\pi t$$

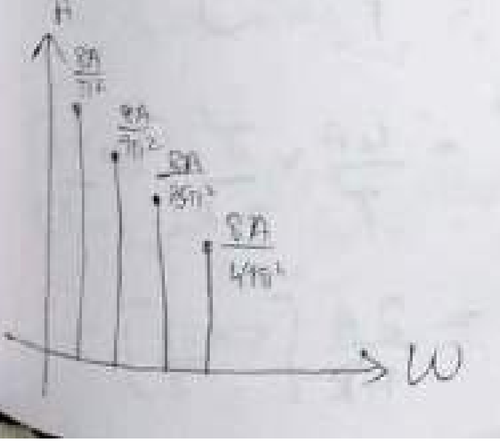
$$= \frac{8A}{\pi^2}\sin\pi t - \frac{8A}{9\pi^2}\sin 3\pi t + \frac{8A}{25\pi^2}\sin 5\pi t -$$

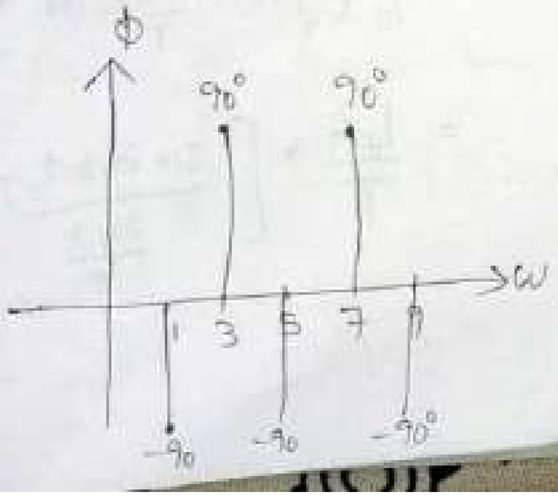
$$\frac{8A}{49\pi^2}\sin 7\pi t + \ldots$$

$$\left[\begin{array}{l} -\sin kt = \cos(kt + \pi/2) \\ \sin kt = \cos(kt - \pi/2) \end{array}\right]$$

$$x(t) = \frac{8A}{\pi^2}\cos(\pi t - 90) + \frac{8A}{9\pi^2}\cos(\pi t + 90)$$

$$+ \frac{8A}{25\pi^2}\cos(\pi t - 90) + \frac{8A}{49\pi^2}\cos(kt + 90) + \ldots$$





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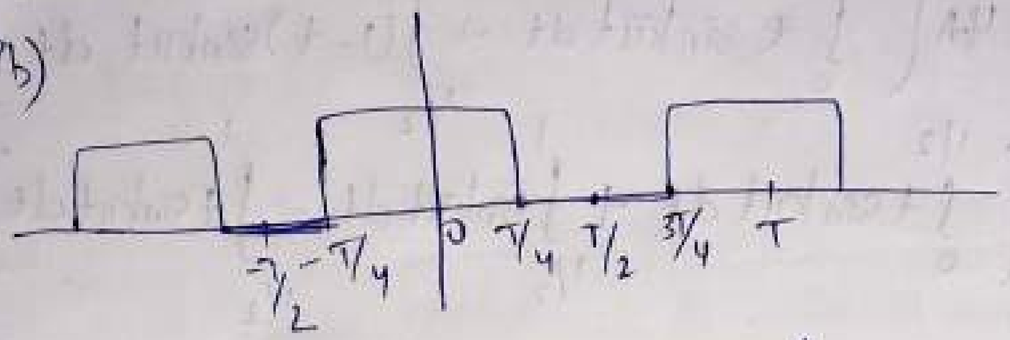


Sol: $\because x(t)$ is even symmetric

$\therefore b_k = 0$

$T_0 = T \qquad \omega_0 = \dfrac{2\pi}{T_0} = \dfrac{2\pi}{T}$

$$x(t) = A \qquad 0 \le t \le T/4$$
$$= 0 \qquad T/4 \le t \le T/2$$

$$a_0 = \frac{2}{T_0}\int_0^{T_0/2} x(t)\,dt$$

$$= \frac{2}{T_0}\int_0^{T/2} x(t)\,dt$$

$$= \frac{2}{T_0}\left[\int_0^{T/4} A\,dt + \int_{T/4}^{T/2} 0.\,dt\right]$$

$$a_0 = \frac{2A}{4} = \frac{A}{2}$$

$$\boxed{a_0 = A/2}$$

$$a_k = \frac{4}{T_0}\int_0^{T/2} x(t)\cos k\omega_0 t\, dt.$$

$$= \frac{4}{T_0}\left[\int_0^{T/4} A\cos\frac{2\pi}{T}t\, dt + \int_{T/4}^{T/2} 0.\cos\frac{2\pi}{T}t\, dt\right]$$

$$= \frac{4A}{T_0}\left[\sin\frac{2\pi k t}{T} \times \frac{T}{2\pi k}\right]_0^{T/4}$$

$$= \frac{\cancel{4}^{2}A}{\cancel{T}} \times \frac{\cancel{T}}{\cancel{2}\pi k}\left[\sin\frac{\pi k}{2}\right]$$

$$= \frac{2A}{\pi k}\left[\sin\frac{\pi k}{2}\right]$$

$$\sin\frac{\pi k}{2} = +1, \quad k = 1, 5, 9, 13$$

$$= -1, \quad k = 3, 7, 11, 15$$

$$a_k = \frac{2A}{\pi k}, \quad k = 1, 5, 9, 13$$

$$= -\frac{2A}{\pi k}, \quad k = 3, 7, 11, 15$$

$$= 0, \quad k = 2, 4, 6 \ldots$$

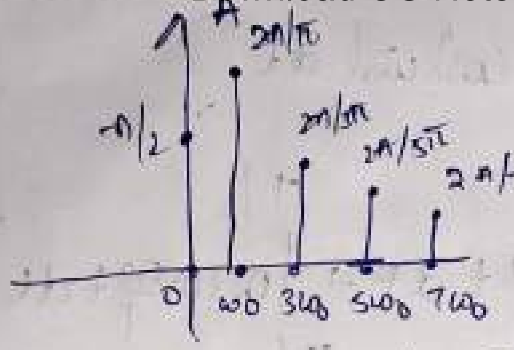
$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos \omega_0 t$$

$$= \frac{A}{2} + \frac{2A}{\pi}\cos\frac{2\pi}{T}t - \frac{2A}{3\pi}\cos\frac{3\pi}{T}t$$

$$+ \frac{2A}{5\pi}\cos 5\frac{2\pi}{T}t - \frac{2A}{7\pi}\cos\frac{7\,2\pi}{T}t + \ldots \infty$$

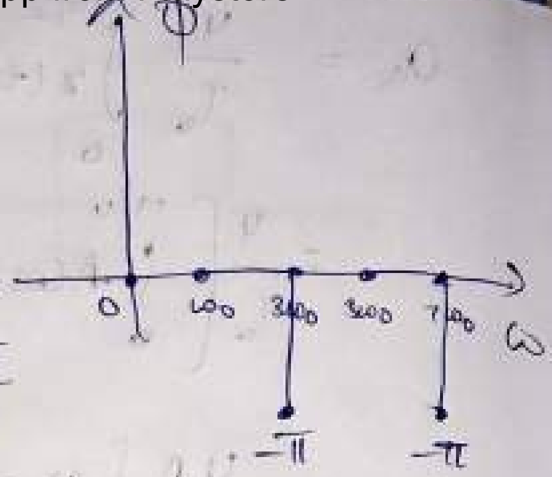
$$x(t) = a_0 + \frac{2A}{\pi}\cos\omega_0 t + \frac{2A}{3\pi}\cos(3\omega_0 t - \pi)$$

$$+ \frac{2A}{5\pi}\cos(5\omega_0 t) + \frac{2A}{7\pi}(\cos(7\omega_0 t - \pi))$$



$\omega_0 = \frac{2\pi}{T}$

magnitude spectrum



Phase Spectrum

E.F.S

$$x(t) = \sum_{-\infty}^{\infty} C_k e^{jk\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T}$$

$$C_0 = a_0 = A/2$$

$$C_k = \frac{a_k - jb_k}{2} = \frac{A}{\pi k}, \quad k = 1, 5, 9 \ldots$$

$$= -\frac{A}{\pi k}, \quad k = 3, 7, 11 \ldots$$

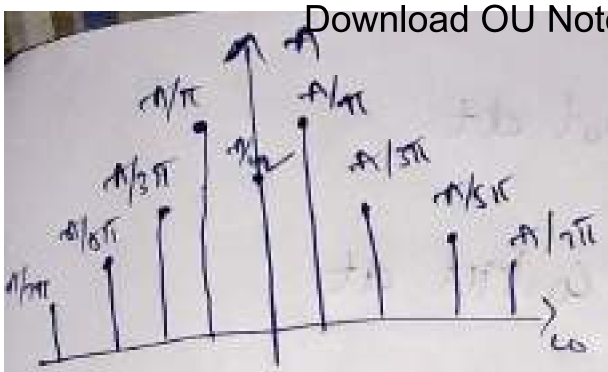
$$x(t) = C_0 + \sum_{k=-\infty}^{-1} C_k e^{jk\omega_0 t} + \sum_{k=1}^{\infty} C_k e^{jk\omega_0 t}$$

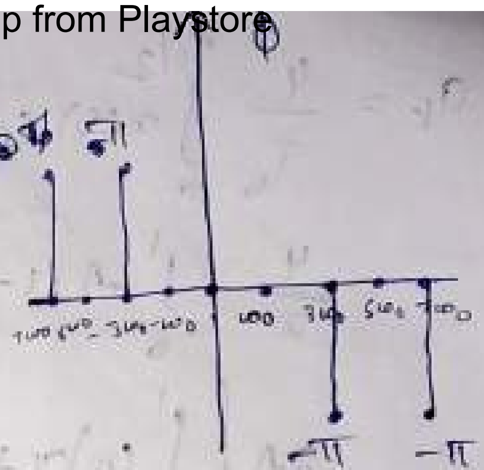
$$x(t) = \frac{A}{2} + \sum_{k=-\infty} C_k e^{jk\omega_0 t} + \sum_{k=1,5,9}^{\infty} \left(\frac{A}{\pi k}\right) e^{jk\omega_0 t}$$

$$= \cancel{\frac{A}{2} + \sum_{\substack{k=-\infty \\ k=3,7,11}}^{-1} \left(\frac{A}{\pi}\right) e^{jk\omega_0 t} + \sum_{k=1,5,9}^{\infty} \left(\frac{A}{\pi}\right) e^{jk\omega_0 t}}, \quad \omega_0 = \frac{2\pi}{T}$$

$$= \frac{A}{2} + \frac{A}{\pi} e^{j\omega_0 t} + \frac{A}{3\pi} e^{j3\omega_0 t} + \frac{A}{5\pi} e^{j5\omega_0 t}$$

$$+ \ldots \ldots + \left(-\frac{A}{\pi} e^{-j\omega_0 t} - \frac{A}{3\pi} e^{-3j\omega_0 t} - \frac{A}{5\pi} e^{-5j\omega_0 t} \right)$$





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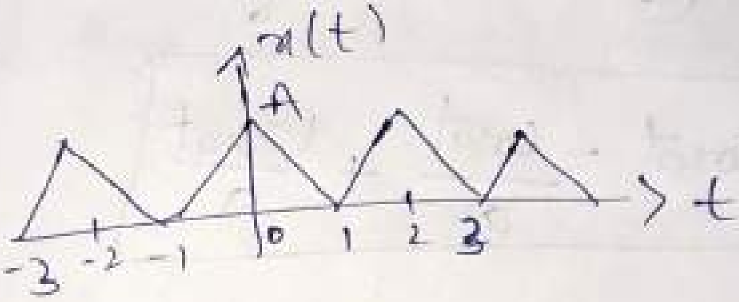


Sol Even Symmetric

$b_k = 0$, $a_0 = ?$, $a_k = ?$

$T_0 = 2$, $\omega_0 = \pi$

$x(t) = -At + A$, $0 \le t \le T_0/2$

$= A(1-t)$, $0 < t < T_0/2$

$0 < t < 1$

$$a_0 = \frac{2}{T_0}\int_0^{T_0/2} x(t)\,dt = \frac{2}{2}\int_0^1 x(t)\,dt$$

$$= A\int_0^1 1-t\,dt$$

$$= A\left[t - \frac{t^2}{2}\right]_0^1$$

$$\boxed{a_0 = A/2}$$

$$a_k = \frac{4}{T_0} \int_0^{T_0/2} x(t) \cos k\omega_0 t \, dt$$

$$\frac{4}{2} \cdot \int_0^1 A(1-t) \cos k\pi t \, dt$$

$$2A \left(\int_0^1 \cos k\pi t \, dt - \int_0^1 t \cos k\pi t \, dt \right)$$

$$\boxed{\int t \cos at = \frac{\cos at}{a^2} + \frac{t \sin at}{a}}$$

$$a_k = 2A \left(\left[\frac{\sin k\pi t}{k\pi}\right]_0^1 - \left[\frac{\cos k\pi t}{k\pi} + \frac{t \sin k\pi t}{k\pi}\right]_0^1 \right)$$

$$= 2A \left[\left(\frac{\sin k\pi}{k\pi}\right) - \left[\frac{\cos\pi - \cos 0}{k^2\pi^2} + \left[\frac{\sin k\pi - 0\sin 0}{k\pi}\right]\right]\right]$$

$$a_k = -\frac{2A}{k^2\pi^2} (\cos k\pi - 1)$$

$k = \text{odd}$

$$a_k = \frac{-2A}{k^2\pi^2} (-1-1) = \frac{4A}{k^2\pi^2}$$

$k = \text{even}$

$$a_k = \frac{-2A}{k^2\pi^2} (1-1) = 0$$

$$a_k = \frac{4A}{k^2\pi^2}, \quad k = 1, 3, 5 \ldots$$

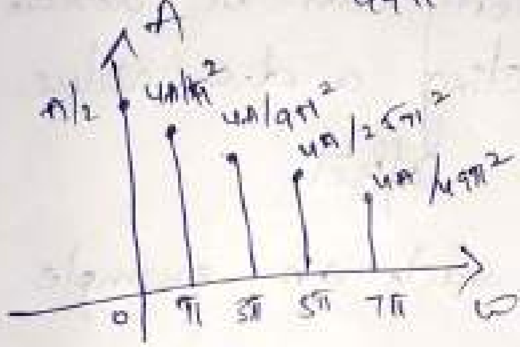
$$= 0, \quad k = \text{even}$$

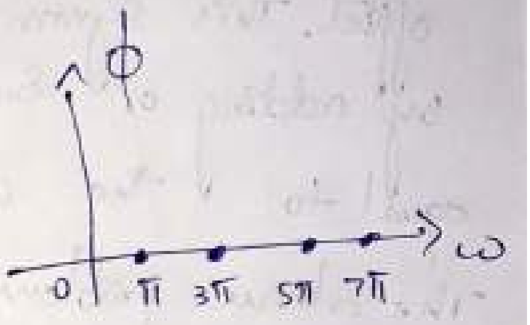
$$a_0 = A/2$$

$$b_k = 0$$

$$x(t) = \frac{A}{2} + \frac{4A}{\pi^2}\cos\pi t + \frac{4A}{9\pi^2}\cos 3\pi t + \frac{4A}{25\pi^2}\cos 5t$$

$$+ \frac{4A}{49\pi^2}\cos 7t$$





E.F.S

$$x(t) = \sum_{-\infty}^{\infty} C_k e^{j\omega_0 kt}, \quad \omega_0 = 2\pi/T$$

$$\omega_0 = \frac{2\pi}{2} = \pi$$

$$C_0 = a_0 = A/2$$

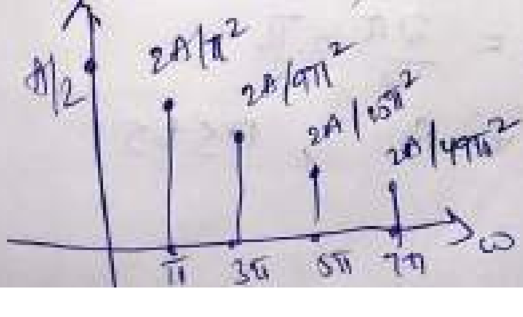
$$C_k = \frac{a_k - jb_k}{2} = \frac{4A}{\pi^2 k^2}, \quad k = \text{odd}$$

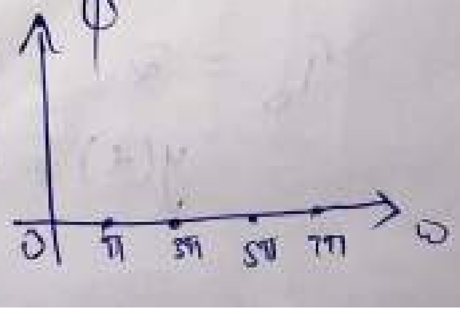
$$= 0, \quad k = \text{even}$$

$$C_{-k} = \frac{4A}{\pi^2 k^2}$$

$$\cancel{x(t) = \sum_{-\infty}^{\infty} \left(\frac{4A}{\pi^2 k^2}\right) e^{j2\pi/2\, kt}}$$

$$x(t) = \sum_{-\infty}^{\infty} \left(4A/\pi^2 k^2\right) e^{j\pi kt}$$





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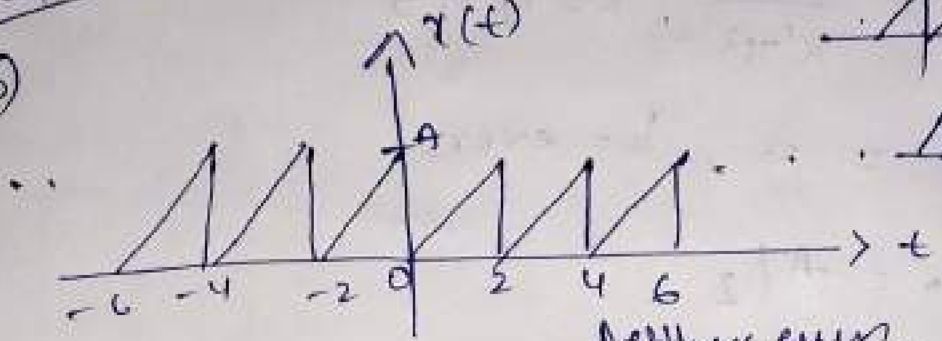
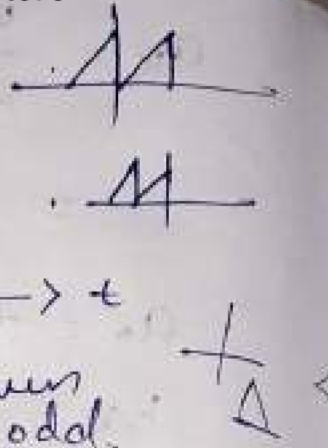




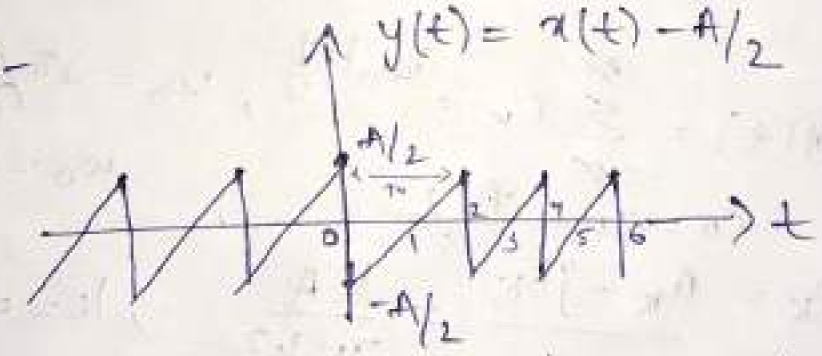
neither even nor odd.

Hidden Symmetricity :-

In some signals some kind of symmetricity exists but is hidden due to a d.c offset. This symmetricity can be revealed by adding or subtracting a d.c. value equal to the d.c offset.

The above problem is such an example.

Step1:-



∴ This is odd symmetric

$a_0 = a_k = 0$

but $b_k = \frac{4}{T_0}\int_0^{T_0/2} x(t) \sin k\omega_0 t \, dt$

$T_0 = 2$, $\omega_0 = \frac{2\pi}{2} = \pi$

$y(t) = \frac{A}{2}(t-1)$, $0 \le t \le 1$

$$b_K = \frac{2}{2}\int_0^1 \frac{A}{2}(t-1)\sin k\pi t\, dt$$

$$= 2\times\frac{A}{2}\int_0^1 (t-1)\sin k\pi t\, dt$$

$$b_K = A\int_0^1 t\sin k\pi t\, dt - A\int_0^1 \sin k\pi t\, dt$$

$$\boxed{\begin{aligned}\int t\sin at\, dt &= \frac{\sin at}{a^2} - \frac{t\cos at}{a}\\ \int t\cos at\, dt &= \frac{\cos at}{a^2} + \frac{t\sin at}{a}\end{aligned}}$$

$$b_K = A\left[\frac{\sin k\pi t}{k^2\pi^2} - \frac{t\cos k\pi t}{k\pi}\right]_0^1 + A\left[\frac{\cos k\pi t}{k\pi}\right]_0^1$$

$$b_K = A\left(\frac{\sin k\pi}{k^2\pi^2} - \frac{\cos k\pi}{k\pi} - [0]\right) + A\left(\frac{\cos k\pi}{k\pi} - \frac{1}{k\pi}\right)$$

$$b_K = A\left(\frac{\sin k\pi}{k^2\pi^2} - \frac{\cos k\pi}{k\pi}\right) + \frac{A\cos k\pi}{k\pi} - \frac{A}{k\pi}$$

$$= \frac{A\sin k\pi}{k^2\pi^2} - \frac{A}{k\pi} \qquad \because \sin k\pi = 0$$

$$= 0 - \frac{A}{k\pi} \quad \forall k$$

$$\boxed{b_K = \frac{-A}{k\pi} \quad \forall k}$$

F.S

$$y(t) = \sum_{k=1}^{\infty} b_k \sin k\omega_0 t \,, \quad \omega_0 = \pi$$

$$y(t) = \sum_{k=1}^{\infty} \left(-\frac{A}{k\pi}\right) \sin k\pi t$$

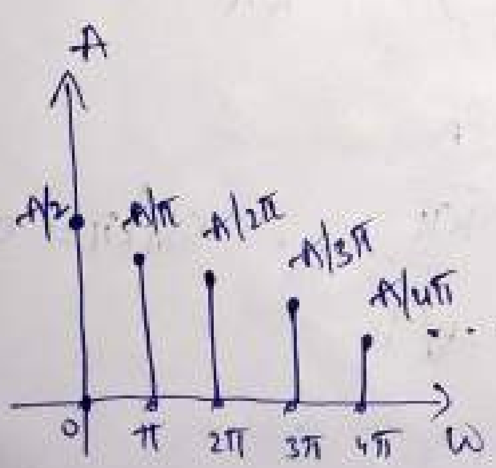
$$x(t) - \frac{A}{2} = \sum_{k=1}^{\infty} \left(-\frac{A}{k\pi}\right) \sin k\pi t \,.$$

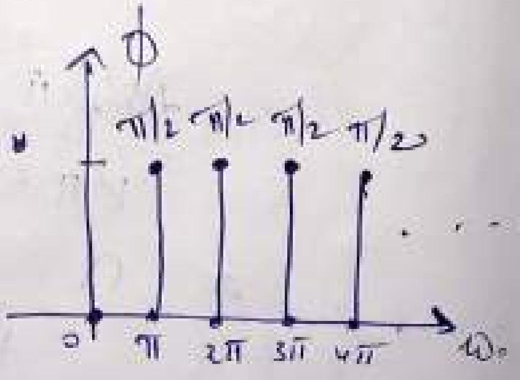
$$\boxed{x(t) = \frac{A}{2} + \sum_{k=1}^{\infty} \left(-\frac{A}{k\pi}\right) \sin k\pi t}$$

$$x(t) = \frac{A}{2} + \left(-\frac{A}{\pi} \sin \pi t - \frac{A}{2\pi} \sin 2\pi t - \frac{A}{3\pi} \sin 3\pi t - \frac{A}{4\pi} \sin 4\pi t + \dots\right)$$

$$\boxed{-\sin kt = \cos(kt + \pi/2)}$$

$$x(t) = \frac{A}{2} + \frac{A}{\pi} \cos\left(\pi t + \pi/2\right) + \frac{A}{2\pi} \cos\left(2\pi t + \frac{\pi}{2}\right) + \frac{A}{3\pi}\left(\cos(3\pi t + \pi/2)\right) + \dots \infty$$





(π,A) (π,0)

$\frac{A-0}{\dots}$

Exponential F.S

$$x(t) = \sum_{-\infty}^{\infty} C_k e^{jk\omega_0 t} \quad , \omega_0 = \pi$$

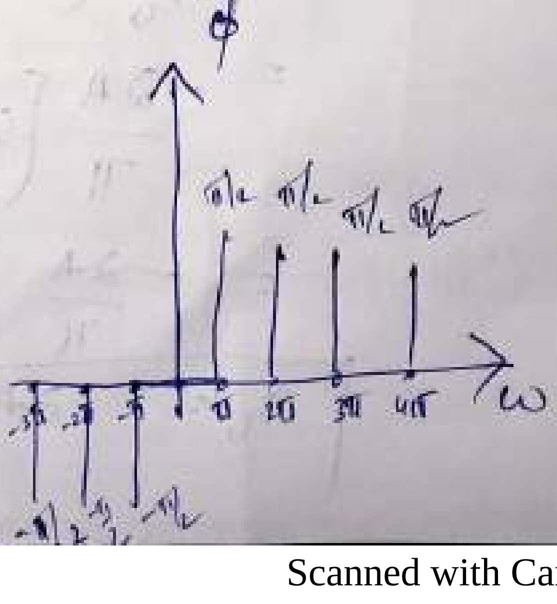
$$C_0 = a_0 = 0$$

$$C_k = \frac{a_k - jb_k}{2} = -j\left(-\frac{A}{k\pi}\right) = \frac{Aj}{k\pi} \quad \forall k$$

$$C_{-k} = C_k^* = -\frac{Aj}{k\pi} \quad \forall k.$$

$$x(t) = \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} C_k e^{jk\omega_0 t} \qquad \boxed{\omega_0 = \pi}$$

$$x(t) = \sum_{k=-\infty}^{\infty} \left(\frac{Aj}{k\pi}\right) e^{jk\pi t}$$



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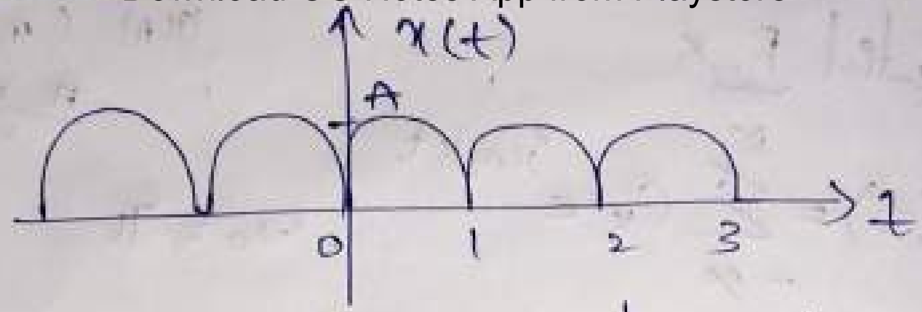


S1) It is even symmetric $b_k = 0$

S2) $a_0 = a_k = ?$

$T_0 = 1$ $\qquad \omega_0 = \frac{2\pi}{T_0} = 2\pi$

$x(t) = A \sin \pi t \quad , \quad 0 \le t \le 1/2$

$T_0 = 2$

$\omega = \frac{2\pi}{2} = \pi$

$$\boxed{a_0 = \frac{2}{T_0} \int_0^{1/2} x(t)\, dt}$$

$$= \frac{2A}{T_0} \int_0^{1/2} \sin \pi t \, dt$$

$$= \frac{2A}{T_0} \left[-\frac{\cos \pi t}{\pi} \right]_0^{1/2} \qquad \boxed{T_0 = 1}$$

$$= -\frac{2A}{\pi} \left[\cos \pi/2 - \cos 0 \right]$$

$$= -\frac{2A}{\pi} \left[0 - 1 \right]$$

$$\boxed{a_0 = 2A/\pi}$$

$$a_k = \frac{4}{T_0}\int_0^{T_0/2} x(t)\cos k\omega_0 t \, dt, \quad \omega_0 T = 2\pi$$

$$a_k = \frac{4A}{1}\int_0^{1/2} \sin\pi t \cos 2\pi k t \, dt$$

$$a_k = 4A\int_0^{1/2} \frac{1}{2}\left\{\sin(1+2k)\pi t + \sin(1-2k)\pi t\right\} dt$$

$$a_k = 2A\int_0^{1/2} \sin(1+2k)\pi t + \sin(1-2k)\pi t \, dt$$

$$a_k = 2A\left\{\left[\frac{-\cos(1+2k)\pi t}{\pi(1+2k)}\right]_0^{1/2} + \left[\frac{-\cos(1-2k)\pi t}{\pi(1-2k)}\right]_0^{1/2}\right\}$$

$$a_k = 2A\left[\frac{-\cos(1+2k)\frac{\pi}{2}}{\pi(1+2k)} + \frac{\cos 0}{\pi(1+2k)} + \left\{\frac{-\cos(1-2k)\frac{\pi}{2}}{\pi(1-2k)} + \frac{\cos 0}{\pi(1-2k)}\right\}\right]$$

$$a_k = \frac{2A}{\pi}\left[\frac{-\cos(1+2k)\frac{\pi}{2}}{1+2k} + \frac{1}{1+2k} - \frac{\cos(1-2k)\frac{\pi}{2}}{1-2k} + \frac{1}{1-2k}\right]$$

$$\because \boxed{\cos(2k+1)\frac{\pi}{2} = \cos(2k-1)\frac{\pi}{2} = 0}$$

$$a_k = \frac{2A}{\pi}\left[\frac{1}{1+2k} + \frac{1}{1-2k}\right]$$

$$a_k = \frac{2A}{\pi}\left[\frac{1-2k+1+2k}{1-4k^2}\right] = \frac{4A}{\pi(1-4k^2)}$$

$$\boxed{a_k = \frac{4A}{\pi(1-4k^2)}} \; \forall k$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos k\omega_0 t$$

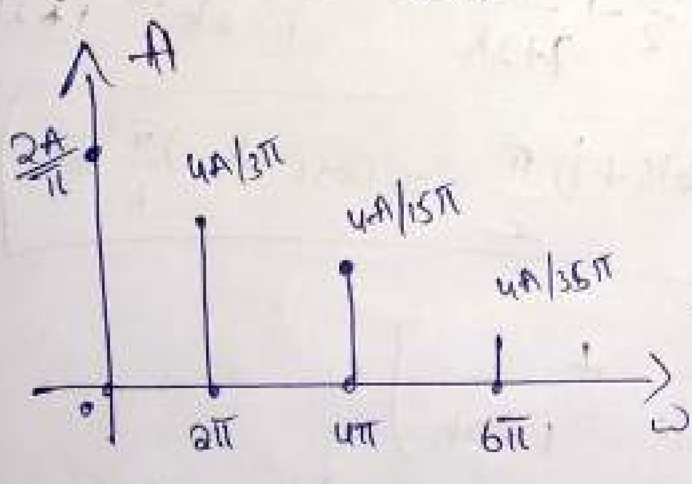
$$x(t) = \frac{2A}{\pi} + \sum_{k=1}^{\infty} \frac{4A}{\pi(1-4k^2)} \cos 2\pi k t$$

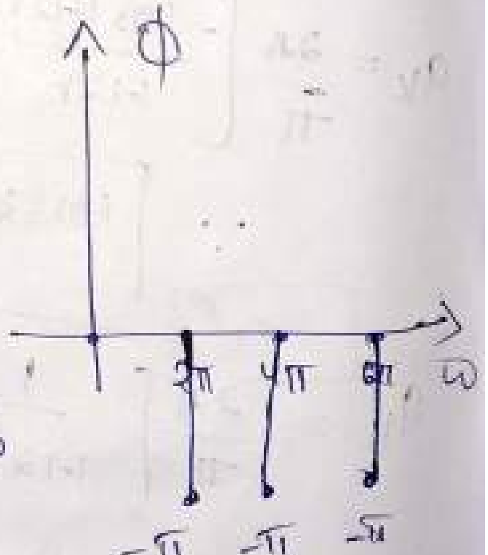
$$x(t) = \frac{2A}{\pi} - \frac{4A}{3\pi}\cos 2\pi t - \frac{4A}{15\pi}\cos 4\pi t - \frac{4A}{35\pi}\cos 6\pi t + \ldots$$

$$x(t) = \frac{2A}{\pi} + \frac{4A}{3\pi}\cos(2\pi t - \pi) + \frac{4A}{15\pi}\cos(4\pi t - \pi) + \frac{4A}{35\pi}\cos(6\pi t - \pi) + \ldots$$

$$\because \boxed{-\cos kt = \cos(kt - \pi)}$$

$$x(t) = \frac{2A}{\pi} + \frac{4A}{3\pi}\cos(2\pi t - \pi) + \frac{4A}{15\pi}\cos(4\pi t - \pi) + \frac{4A}{35\pi}\cos(6\pi t - \pi) + \ldots$$





E.R 8

$$x(t) = \sum_{-\infty}^{\infty} C_k e^{j\omega_0 kt}, \quad \omega_0 = 2\pi$$

$$C_0 = a_0 = 2A/\pi$$

$$C_k = \frac{a_k - jb_k}{2} = \frac{2A}{\pi(1-4k^2)}$$

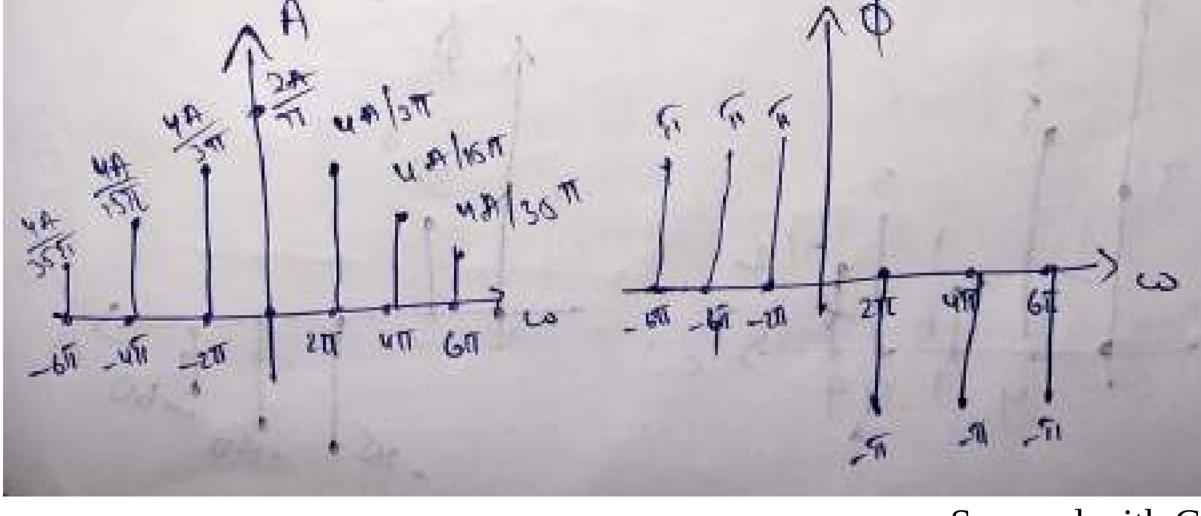
$$C_{-k} = C_k^* = \frac{2A}{\pi(1-4k^2)}$$

$$x(t) = C_0 + \sum_{-\infty}^{\infty} C_k e^{j\omega_0 kt}$$

$$x(t) = \frac{2A}{\pi} + \sum_{k=-\infty}^{\infty} \frac{2A}{\pi(1-4k^2)} e^{j2\pi kt}$$

$$x(t) = \frac{2A}{\pi} + \sum_{k=-\infty}^{\infty} \frac{2A}{\pi(1-4k^2)} e^{j2\pi kt}$$

$$\Rightarrow x(t) = \frac{2A}{\pi} + \sum_{k=-\infty}^{\infty} \frac{2A}{\pi(1-4k^2)} e^{j2\pi kt}$$



3) Draw the frequency spectrum of

$$x(t) = 3 + 4\sin 4t + 3\cos 4t - \cos(7t - 75°) + 3\sin(8t - 100) + \frac{3}{2}\sin(9t - 150)$$

Sol

$$x(t) = 3 + 4\sin 4t + 3\cos 4t - \cos(7t - 75°) + 3\sin(8t - 100) - \frac{3}{2}\sin(9t - 150°)$$

$$\boxed{B\sin\theta + A\cos\theta = \sqrt{A^2 + B^2}\cos\left(\theta - \tan^{-1}\left(\frac{B}{A}\right)\right)}$$

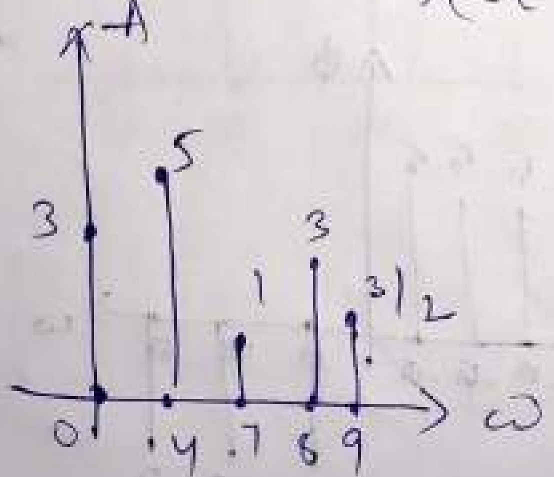
$$x(t) = 3 + 5\cos\left(4t + \tan^{-1}\left(\frac{3}{4}\right)\right) + \cos(7t - 75 - \pi) + 3\cos(8t - 100 - \pi/2) + \frac{3}{2}\cos\left(9t - 150 + \frac{\pi}{2}\right)$$

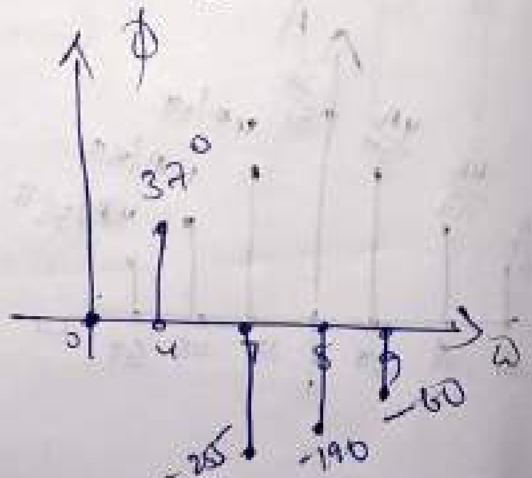
$$-\cos\theta = \cos(\theta - \pi)$$

$$\sin\theta = \cos(\theta - \pi/2)$$

$$-\sin\theta = \cos(\theta + \pi/2)$$

$$x(t) = 3 + 5\cos(4t + 37°) + \cos(7t - 255) + 3\cos(8t - 190) + \frac{3}{2}\cos(9t - 60)$$





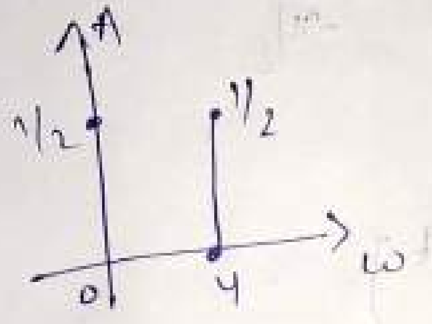
(4) $x(t) = \cos^2 2t$

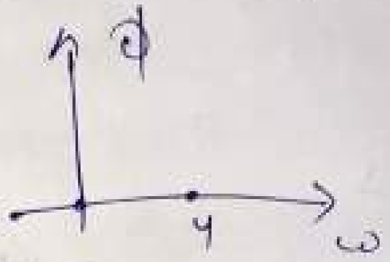
Sol (a) Draw the spectrum

(b) Find the E.F.S & draw the spectrum

Sol $x(t) = \frac{1+\cos 4t}{2}$

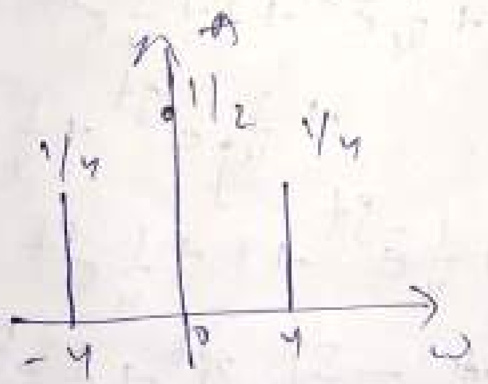
$x(t) = \frac{1}{2} + \frac{1}{2}\cos 4t$

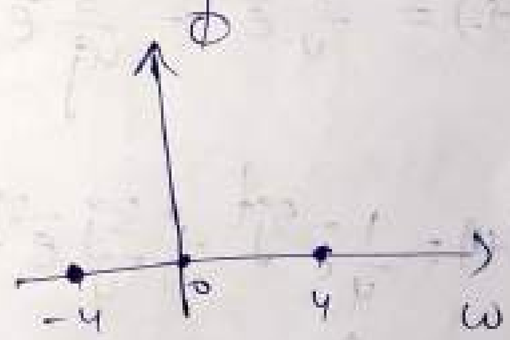




(b) $x(t) = \frac{1}{2} + \frac{1}{2}\left(\frac{e^{j4t} + e^{-j4t}}{2}\right)$

$= \frac{1}{4}e^{j4t} + \frac{1}{2} + \frac{1}{4}e^{j4t}$



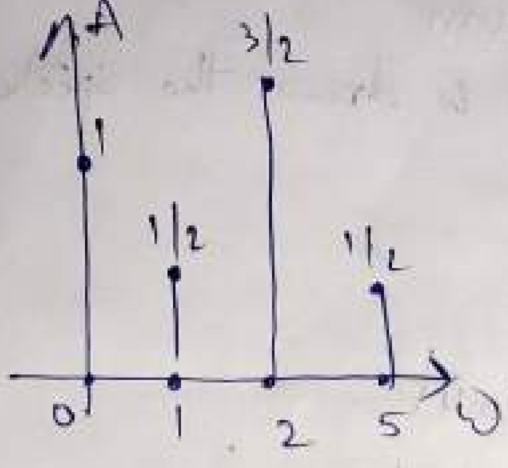


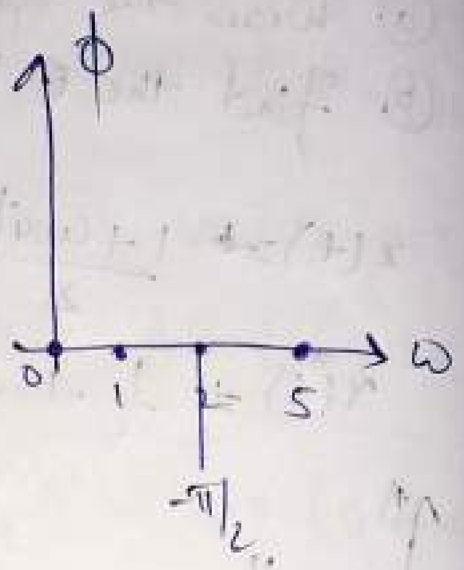
(5) $x(t) = 1 + \sin 2t \cos 3t + \frac{3}{2}\sin 2t$

Sol $x(t) = 1 + \frac{1}{2}[\cos(5t) + \cos t] + \frac{3}{2}\cos(2t - \frac{\pi}{2})$

$= 1 + \frac{1}{2}\cos t + \frac{3}{2}\cos(2t - \frac{\pi}{2}) + \frac{1}{2}\cos 5t$

$$x(t) = -1 + \frac{1}{2}\cos t + \frac{3}{2}\cos(2t - 90) + \frac{1}{2}\cos 5t$$



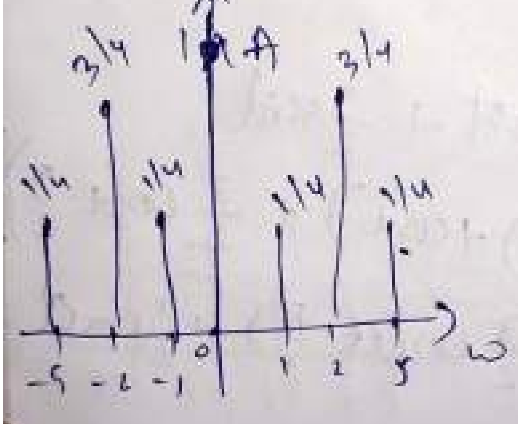


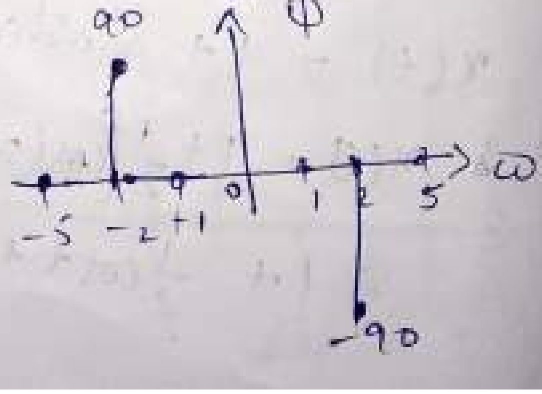
E.F.S

$$x(t) = 1 + \frac{1}{2}\left(\frac{e^{jt} + e^{-jt}}{2}\right) + \frac{3}{2}\left(\frac{e^{2jt} - e^{-2jt}}{2j}\right) + \frac{1}{2}\left(\frac{e^{5jt} + e^{-5jt}}{2}\right)$$

$$x(t) = \frac{1}{4}e^{-5jt} - \frac{3}{4j}e^{-2jt} + \frac{1}{4}e^{-jt} + 1 + \frac{1}{4}e^{jt} + \frac{3}{4j}e^{2jt} + \frac{1}{4}e^{5jt}$$

$$x(t) = \frac{1}{4}e^{-5jt} + \frac{3j}{4}e^{-2jt} + \frac{1}{4}e^{-jt} + 1 + \frac{1}{4}e^{jt} - \frac{3j}{4}e^{2jt} + \frac{1}{4}e^{5jt}$$



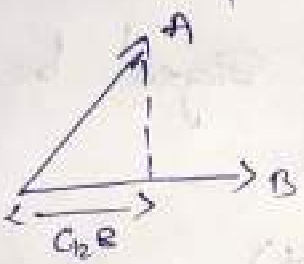


* Analogy b/w Signals & Vectors :-

> Vectors

> Signals are similar to vectors as they also have are dictated by some magnitude and some directions

Component of a vector



$$A \simeq C_{12} B$$

$$C_{12} = \frac{A \cdot B}{|B|^2} = \cos\theta$$

let A & B be the vector as shown in the figure above then the component of A on B can be mathematically expressed as follows

$$A \simeq C_{12}B$$

the best optimum value of C_{12} so that a closely approximates B

$$C_{12} = \frac{A \cdot B}{|B|^2}$$ (perpendicular projection)

Component of a Signal:-

Let $x(t)$ & $y(t)$ be a different real signals

Extending the theory developed in vectors to signals we can write

$$x(t) \simeq C_{12}\, y(t) \quad \{t_1 \le t \le t_2\}$$

Let the error b/w two signal be denoted like as shown

$$e(t) = x(t) - C_{12}\, y(t)$$

To minimize the error between two signals either the energy or Power (mean square value) of the error signal should be minimized

$$P_e(t) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} e^2(t)\, dt$$

$$= \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \left(x(t) - C_{12}\, y(t)\right)^2 dt$$

$$MSE = P_e(t) = \frac{1}{t_2 - t_1}\left[\int_{t_1}^{t_2} x^2(t) - 2C_{12}\int_{t_1}^{t_2} x(t)y(t)dt + \int_{t_1}^{t_2} y^2(t)\,dt\right]$$

In order to minimize the mean square error we need to choose a suitable value of C_{12} such that the error is minimum.

That is $\frac{d}{dC_{12}}(MSE) = 0$

$$\frac{1}{t_2 - t_1}\left[0 - 2\int_{t_1}^{t_2} x(t)y(t)\,dt + 2C_{12}\int_{t_1}^{t_2} y^2(t)\,dt\right] = 0$$

$$C_{12}\int_{t_1}^{t_2} y^2(t)\,dt = \int_{t_1}^{t_2} x(t)y(t)\,dt$$

$$\boxed{C_{12} = \frac{\int_{t_1}^{t_2} x(t)y(t)\,dt}{\int_{t_1}^{t_2} y^2(t)\,dt}}$$

The above Expression looks very much similar to the Expression derived in vectors. Thus bringing out a perfect analogy between signals & vectors

Special Case :-

If $C_{12} = 0$ then $x(t)$ is Independent of $y(t)$ and they are entirely dissimilar to each other such signals are said to be orthogonal to each other

$C_{12} = 0$

i.e $\int_{t_1}^{t_2} x(t)\, y(t)\, dt = 0$

$\therefore$ $x(t)$ & $y(t)$ are orthogonal

If the signals are complex then

$$C_{12} = \frac{\int_{t_1}^{t_2} x(t)\, y^*(t)\, dt}{\int_{t_1}^{t_2} y^2(t)\, dt}$$

and the condition of orthogonality

$$\boxed{\int_{t_1}^{t_2} x(t)\, y^*(t)\, dt = 0}$$

1) Prove that the functions $\sin n\omega_0 t$ & $\cos m\omega_0 t$ are orthogonal for every m & n within the range $\left(t_0, t_0+\frac{2\pi}{\omega_0}\right)$

Sol

$$\int_{t_0}^{t_0+\frac{2\pi}{\omega_0}} \sin n\omega_0 t \cos m\omega_0 t \, dt$$

~~WB~~ $\angle$ ~~2π~~ $\frac{1}{2}\int_0^{2\pi/\omega_0} \sin(n+m)\omega_0 t + \sin(n-m)\omega_0 t \, dt$

$$\Rightarrow \frac{1}{2}\left[\frac{-\cos(n+m)\omega_0 t}{(m+n)\omega_0} - \frac{\cos(n-m)\omega_0 t}{(n-m)\omega_0}\right]_0^{2\pi/\omega_0}$$

$$= \frac{-1}{2\omega_0}\left(\left[\frac{\cos(n+m)2\pi}{(m+n)} + \frac{\cos(n-m)2\pi}{(n-m)}\right] - \left[\frac{\cos 0}{m+n} + \frac{\cos 0}{n-m}\right]\right)$$

$\cos k\pi = 1$

$$= \frac{-1}{2}[0] = 0$$

$\therefore$ orthogonal

2) Prove that the complex exponential functions are orthogonal functions

~~$e^{jn\omega_0 t}$, $e^{jm\omega_0 t}$, $(0, 2\pi/\omega_0)$~~

Sol Let the complex exponential be

$x(t) = e^{jn\omega_0 t}$ & $y(t) = e^{jm\omega_0 t}$, $\left(0, \frac{2\pi}{\omega_0}\right)$

$$\boxed{\int_{t_1}^{t_2} x(t)\, y^*(t)\, dt = 0}$$

$$\int_0^{2\pi/\omega_0} e^{jn\omega_0 t}\, e^{-jm\omega_0 t}\, dt$$

$$\Rightarrow \int_0^{2\pi/\omega_0} e^{j(n-m)\omega_0 t}\, dt$$

$$\Rightarrow \left[\frac{e^{j(n-m)\omega_0 t}}{j(n-m)\omega_0} \right]_0^{2\pi/\omega_0}$$

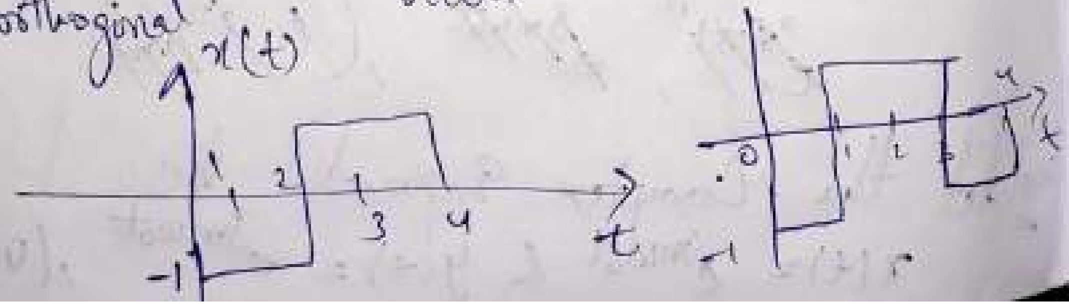
$$= \frac{e^{j(n-m)2\pi} - 1}{j(n-m)\omega_0}$$

$$= \frac{1-1}{j(n-m)\omega_0}$$

$$= 0$$

$\therefore$ orthogonal.

(3) prove that the ~~observe~~ signals are orthoginal below



Sol $\int_{t_1}^{t_2} x(t)\, y(t)\, dt$

$= \int_0^1 +1\, dt + \int_1^2 (-1)\, dt + \int_2^3 +1\, dt + \int_3^4 (-1)\, dt$

$= 0$

$\therefore$ Orthogonal

(58)

A rectangular function $x(t)$ is defined by the following function

$x(t) = 1, \quad 0 \le t \le \pi$

$= -1, \quad \pi \le t \le 2\pi$

approximate the above following by a single sinusoid $\sin t$ over the interval $(0, 2\pi)$ such that the mean square error is minimum. Evaluate the mean square error in this approximation

Sol let $x(t)$ be approximated in terms of $\sin t$ as follows.

$x(t) \simeq C_{12}\, y(t)$

$x(t) = C_{12} \sin t$

C

$$C_{12} = \frac{\int_{t_1} x(t)\, y(t)\, dt}{\int_{t_1}^{t_2} y^2(t)\, dt}$$

$$= \frac{\int_0^{\pi} 1 . \sin t\, dt + \int_{\pi}^{2\pi} (-1) \sin t\, dt}{\int_0^{2\pi} \sin^2 t\, dt}$$

$$= \frac{-\cos t \Big|_0^{\pi} + \cos t \Big|_{\pi}^{2\pi}}{\int_0^{2\pi} \frac{1 - \cos 2t}{2}\, dt}$$

$$C_{12} = \frac{-(\cos\pi - \cos 0) + \cos 2\pi - \cos\pi}{\frac{1}{2}(2\pi) - \frac{1}{2}(\sin 4\pi - \sin 0)}$$

$$\boxed{C_{12} = 4/\pi}$$

$$x(t) \simeq \frac{4}{\pi} \sin t$$

$$MSE = \frac{1}{t_2 - t_1}\left[\int_{t_1}^{t_2} x^2(t)\, dt - 2C_{12}\int_{t_1}^{t_2} x(t)\, y(t)\, dt + C_{12}^2 \int_{t_1}^{t_2} y^2(t)\, dt\right.$$

$$MSE = \frac{1}{2\pi}\left[\int_0^{2\pi} x^2(t)\,dt - 2C_{12}\int_0^{2\pi} x(t)y(t)\,dt + C_{12}^2 \int_0^{2\pi} y^2(t)\,dt\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} 1.dt + \int_{\pi}^{2\pi}(-1)^2 dt - 2\times\frac{4}{\pi}\left[\int_0^{\pi} 1.\sin t + \int_{\pi}^{2\pi} -\sin t\,dt\right] + \frac{16}{\pi^2}\left[\int_0^{2\pi}\sin^2 t\,dt\right]\right]$$

$$= \frac{1}{2\pi}\left[\pi + \pi - \frac{8}{\pi}\left[-\cos t\Big|_0^{\pi} + \cos t\Big|_{\pi}^{2\pi}\right] + \frac{16}{\pi^2}\left[\int_0^{2\pi}\frac{1-\cos 2t}{2}\right]\right]$$

$$= \frac{1}{2\pi}\left[2\pi - \frac{8}{\pi^2}\left[(1-\cos\pi) + (\cos 2\pi - \cos\pi)\right] + \frac{16}{\pi^2}\left[\frac{t}{2} - \frac{1}{2}\frac{\sin 2t}{2}\right]_0^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[2\pi - \frac{8}{\pi}\left[2+2\right] + \frac{16}{\pi^2}\left[\frac{2\pi - 0}{2} - \frac{1}{4}(\sin 4\pi - \sin 0)\right]\right]$$

$$= \frac{1}{2\pi}\left[2\pi - \frac{32}{\pi} + \frac{16}{\pi^2}[\pi]\right]$$

$$= \frac{1}{2\pi}\left[2\pi + \frac{16}{\pi} - \frac{32}{\pi}\right] =$$

$$= \frac{1}{2\pi}\left[2\pi - \frac{16}{\pi}\right] = 1 - \frac{8}{\pi^2} = 0.189$$

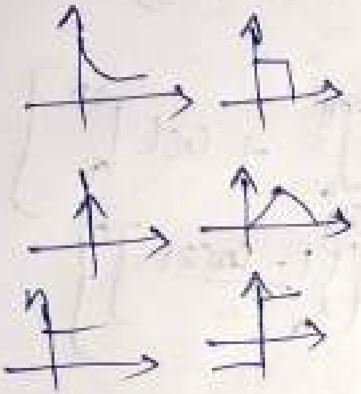
$$MSE = 0.189$$
$$= 18.9\%$$

11/03/2020 UNIT:-03:-

Fourier Transform :-

Aperiodic signal in T.D $x(t)$ $\xrightarrow{F.T}$ frequency domain $X(\omega)$

Eg:-



It is a tool or concept developed by fourier & is available to obtain frequency domain representation of aperiodic signal.

Fourier transform:-

$$x(t) \xrightarrow{F} X(\omega)$$

$$F[x(t)] = X(\omega)$$

$$\Rightarrow X(\omega) = F[x(t)]$$

$$\left[X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \right]$$

$$x(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

→ fourier operator

Inverse fourier transform (IFT) :-

$$X(\omega) \xrightarrow{F^{-1}} x(t)$$

$$x(t) = F^{-1}[x(\omega)]$$

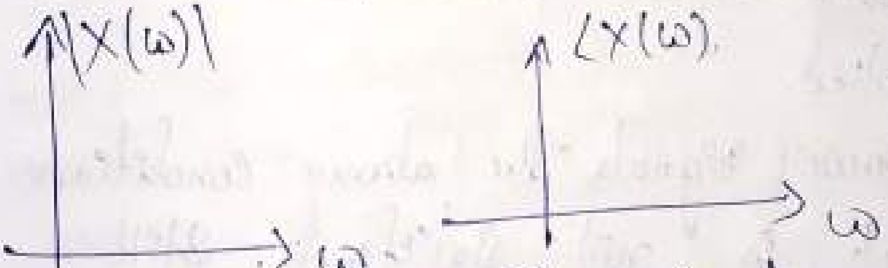
$$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega}$$

Note :-

→ $X(j\omega)$

① $X(\omega)$ is in general Complex

$\therefore X(\omega) = |X(\omega)| \angle X(\omega)$



magnitude spectrum   Phase spectrum

frequency spectrum

② Dirichlet's Conditions:-

① For Fourier a signal $x(t)$ to have fourier transform it should be absolutely integrable

All Energy signal satisfy this Condition

For F.T to exists

$$|X(\omega)| < \infty$$

$$\int_{-\infty}^{\infty} |x(t)|\,|e^{-j\omega t}|\,dt < \infty$$

$$\boxed{\int_{-\infty}^{\infty} |x(t)|\,dt < \infty}$$

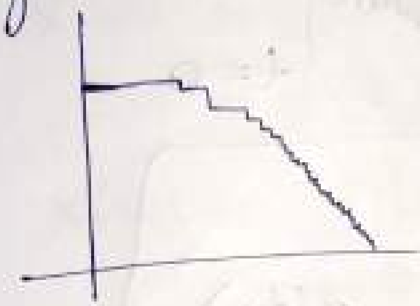
For such signals formula can be directly applied.

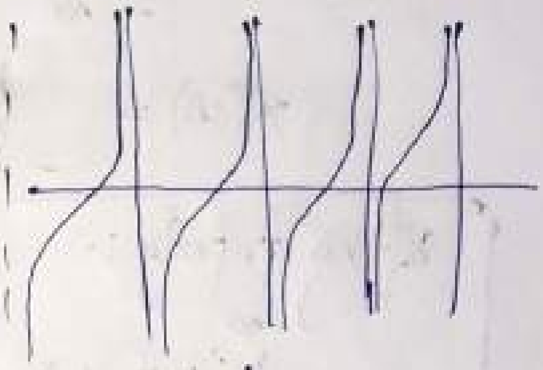
For power signals the above condition though is not satisfied, still have fourier transform which can only be found by using Indirect method i.e. Using Inverse fourier transform formula.

Signals which are neither energy nor Power will not have fourier transform

(2) For fourier transform to exists, signals should have finite number of discontinuities within a finite duration and the number of ~~discon~~ value of discontinuity must be finite

Eg:-

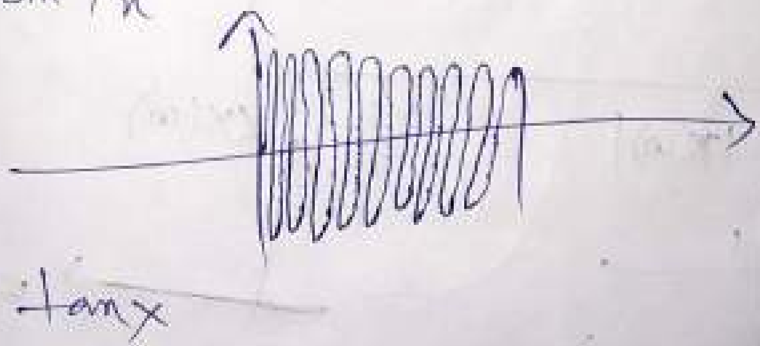




tan x

(3) For a fourier transform to exists the signal should have finite number of maximas within a finite duration and the ~~number~~ value of maximas must also be finite

Eg:- $\sin 1/x$



& tan x

Problem:-

1. $F[\delta(t)]$ find fourier transfer of Impulse signal

Sol $\because$ δt is a finite duration signal

$$x(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{-\infty}^{\infty} \delta(t) \cdot e^{-j\omega t}\, dt$$

$$X(\omega) = e^{-j\omega t}\Big|_{t=0}$$

Sifting property:

$$\int_{-\infty}^{\infty} \delta(t-t_0)\, x(t) = x(t_0)$$

$$\boxed{|X(\omega)| = 1}$$

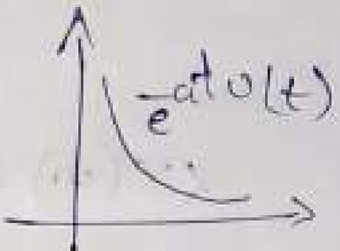
$$\boxed{\delta(t) \xrightarrow{F} 1}$$

$\angle X(\omega) = 0$

④ Note:-
- All Aperiodic signals have continuous spectrum, whereas all periodic signals have discrete spectrum

② $F[e^{-at}u(t)]$:

Sol
$$x(t) = e^{-at}u(t)$$



$$X(\omega) = \int_{-\infty}^{\infty} e^{-at}u(t)\, e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{0}^{\infty} e^{-at}u(t)\, e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{0}^{\infty} e^{-(at+j\omega t)}\, dt$$

$$X(\omega) = \left. \frac{e^{-(a+j\omega)}}{-(a+j\omega)} \right|_{0}^{\infty}$$

$$= \frac{-1}{a+j\omega}[0-1]$$

$$\boxed{X(\omega) = 1/a+j\omega}$$

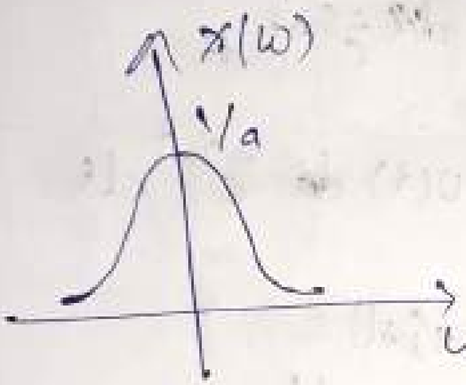
$$\boxed{e^{-at}u(t) \xrightarrow{F} \frac{1}{a+j\omega}}$$

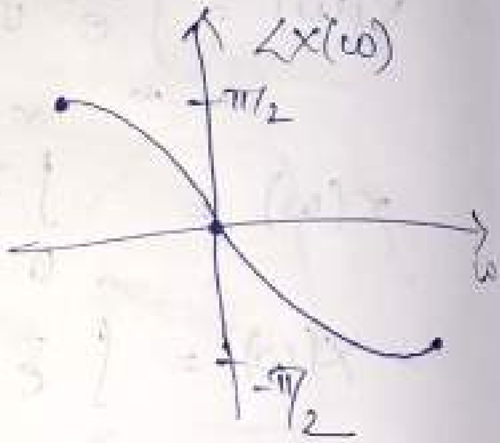
$$X(\omega) = \frac{a - j\omega}{a^2 + \omega^2}$$

$$|X(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

$$\angle X(\omega) = \tan^{-1}\left(\frac{-\omega/\sqrt{a^2+\omega^2}}{a/\sqrt{a^2+\omega^2}}\right)$$

$$\angle X(\omega) = -\tan^{-1}(\omega/a)$$

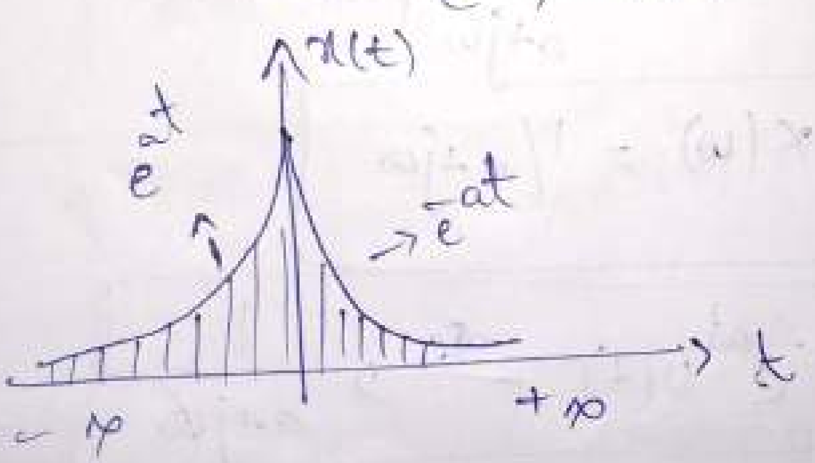




(3) $x(t) = e^{-a|t|}$

Sol $x(t) = e^{at}, \quad t<0$

$\quad = e^{-at}, \quad t>0$



$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \qquad e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-t(a+j\omega)}\, dt$$

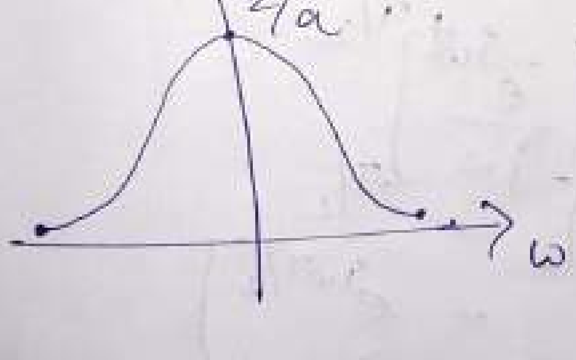
$$= \left.\frac{e^{(a-j\omega)t}}{a-j\omega}\right]_{-\infty}^{0} + \left.\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_{0}^{\infty}$$

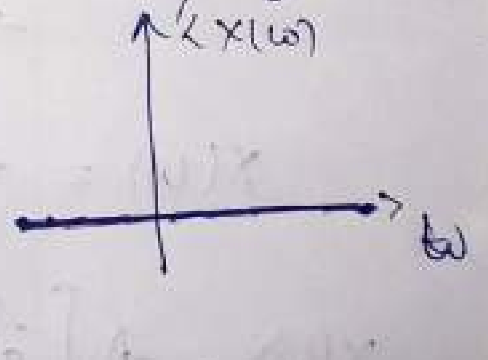
$$= \frac{1-0}{a-j\omega} - \frac{(0-1)}{a+j\omega}$$

$$X(\omega) = \frac{1}{a+j\omega} + \frac{1}{a-j\omega} = \frac{2a}{a^2+\omega^2}$$

$$\boxed{e^{-a|t|} \xrightarrow{F} \frac{2a}{a^2+\omega^2}}$$

$$|X(\omega)| = 2a/(a^2+\omega^2) \qquad \angle X(\omega) = 0^\circ$$

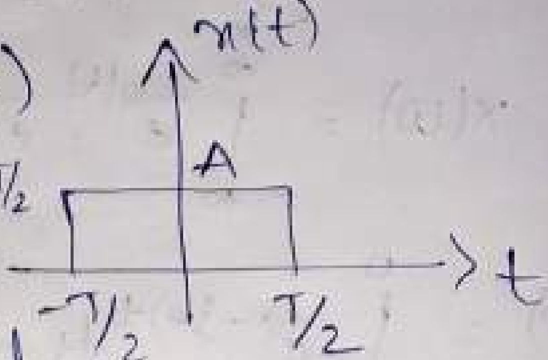




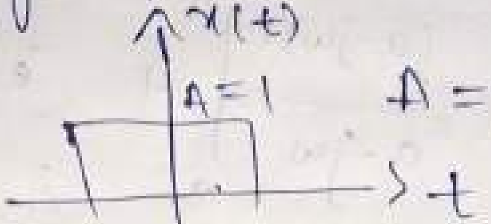
(4) $x(t) = A\,\mathrm{rect}(t/T)$

Sol $A\,\mathrm{rect}(t/T)$

$A\,\mathrm{rect}(t/T) = A, \; -T/2 \le t \le T/2$
$= 0$, else



Unit rectangular function



$A = 1, \; T = 1$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{-\infty}^{\infty} A\,\mathrm{rect}(t/T)\, e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{-T/2}^{T/2} A\,\mathrm{rect}(t/T)\, e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{-T/2}^{T/2} A\, e^{-j\omega t}\, dt$$

$$X(\omega) = A\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-T/2}^{T/2}$$

$$X(\omega) = \frac{A}{-j\omega}\left[e^{-j\omega T/2} - e^{j\omega T/2}\right]$$

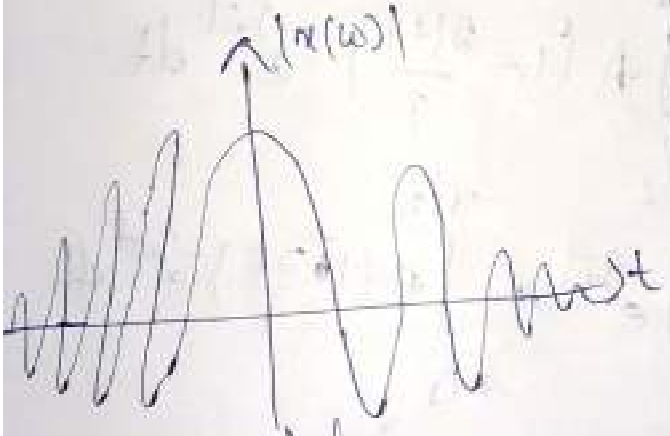
$$X(\omega) = \frac{2A}{\omega}\left[\frac{e^{j\frac{\omega T}{2}} - e^{-j\frac{\omega T}{2}}}{2j}\right]$$

$$X(\omega) = \frac{2A}{\omega}\left[\sin\frac{\omega T}{2}\right)$$

$$X(\omega) = \frac{2A}{\omega} \times \frac{\omega T}{2}\ \frac{\sin(\omega T/2)}{\omega T/2}$$

$$\boxed{X(\omega) = AT\, Sa\left(\frac{\omega T}{2}\right)} \qquad \boxed{Sa(x) = \frac{\sin x}{x}}$$

$$\therefore \boxed{A\,\mathrm{rect}(t/T) \xrightarrow{f} AT\, Sa\left(\frac{\omega T}{2}\right)}$$



magnitude spectrum

(or)

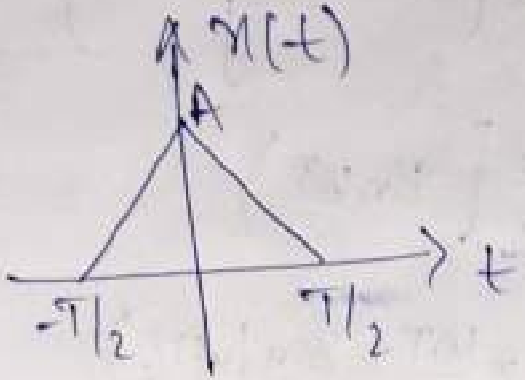
$$X(\omega) = \frac{2A}{\omega}\sin\left(\frac{\omega T}{2}\right)$$

$$\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$$

$$= \frac{2A}{\omega}\,\frac{\sin\left(\frac{\pi\omega T}{2\pi}\right)}{\frac{\pi\omega T}{2\pi}} \times \frac{\pi\omega T}{2\pi}$$

$$\boxed{X(\omega) = AT\,\mathrm{sinc}\left(\frac{\omega T}{2\pi}\right)}$$

(5) $x(t) = A\,\Delta(t/T)$



$$x(t) = A\left(1 - \frac{2t}{T}\right), \quad 0 \le t \le T/2$$

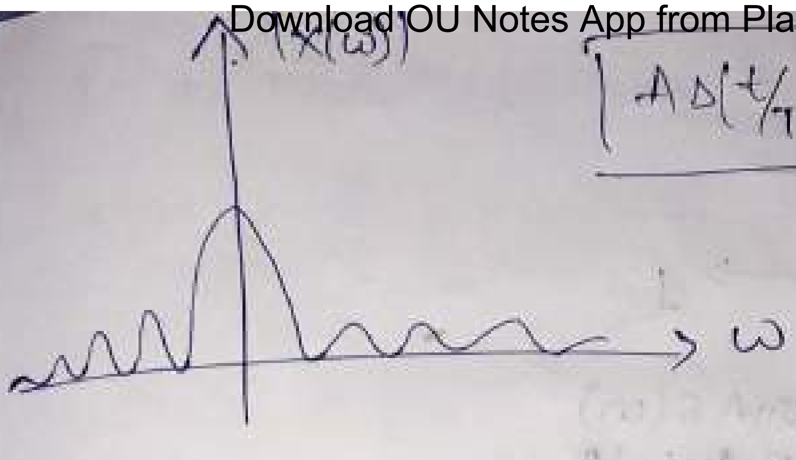
$$= A\left(1 + \frac{2t}{T}\right), \quad -T/2 \le t \le 0$$

$$x(t) = A\left(1 - \frac{2|t|}{T}\right), \quad |t| \le T/2$$

$$X(\omega) = \int_{-T/2}^{T/2} A\left(1 - \frac{2|t|}{T}\right) e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{-T/2}^{0} A\left(1 + \frac{2t}{T}\right) e^{-j\omega t} + \int_{0}^{T/2} A\left(1 - \frac{2t}{T}\right) e^{-j\omega t}\, dt$$

$$X(\omega) = AT\,\text{sinc}^2\left(\frac{\omega T}{2\pi}\right)$$



$$\boxed{A\Delta(t/T) \xrightarrow{F} AT\,\text{sinc}^2\left(\frac{\omega T}{2\pi}\right)}$$

(6) $F^{-1}[\delta(\omega)]$ find the Inverse fourier transform

sol

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega)\, e^{j\omega t}\, d\omega$$

$\because$ sifting property

$$x(t) = \frac{1}{2\pi}(1) = 1/2\pi$$

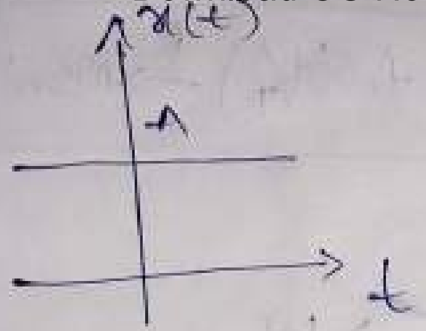
$$F^{-1}[\delta(\omega)] = \frac{1}{2\pi}$$

$$\Rightarrow \boxed{F\left[\frac{1}{2\pi}\right] = \delta(\omega)}$$

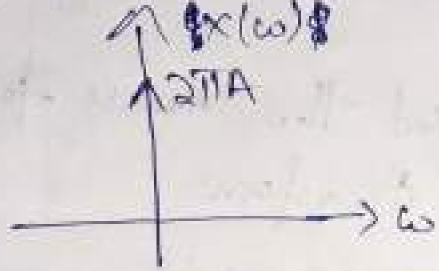
$$\boxed{F[1] = 2\pi\,\delta(\omega)}$$

$$1 \xrightarrow{F} 2\pi\,\delta(\omega)$$

$$A \xrightarrow{F} 2\pi A\,\delta(\omega)$$



$$X(\omega) = 2\pi A\,\delta(\omega)$$



(7) $F^{-1}\left[\delta(\omega-\omega_0)\right]$

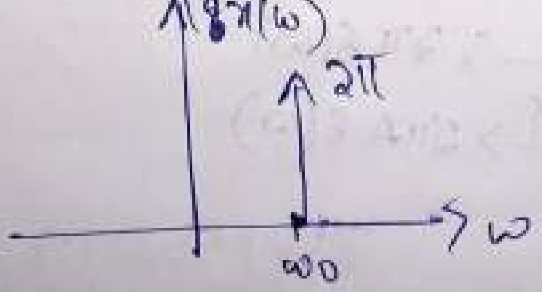
Sol: $$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0)\, e^{j\omega t}\, dt$$

$$x(t) = \frac{1}{2\pi}\left[e^{j\omega_0 t}\right]$$

$$F\left[\frac{e^{j\omega_0 t}}{2\pi}\right] = \delta(\omega-\omega_0)$$

$$\boxed{F\left[e^{j\omega_0 t}\right] = 2\pi\,\delta(\omega-\omega_0)}$$

$$X(\omega) = 2\pi\,\delta(\omega-\omega_0)$$



(8) $F^{-1}\left[\delta(\omega+\omega_0)\right]$

Sol $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\delta(\omega+\omega_0)\,e^{j\omega t}\,dt$

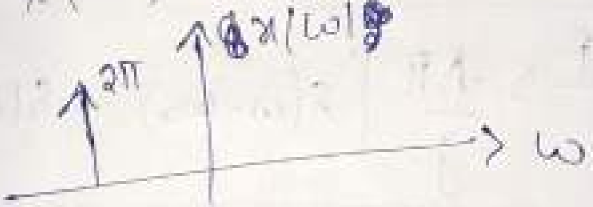
$x(t) = \frac{1}{2\pi}e^{-j\omega_0 t}$

$$\boxed{F\left[\frac{e^{-j\omega_0 t}}{2\pi}\right] = \delta(\omega+\omega_0)}$$

$F\left[e^{-j\omega_0 t}\right] = 2\pi\,\delta(\omega+\omega_0)$

$$\boxed{F\left[e^{-j\omega_0 t}\right] = 2\pi\,\delta(\omega+\omega_0)}$$

$X(\omega) = 2\pi\,\delta(\omega+\omega_0)$



(9) $x(t) = A\cos\omega_0 t$

Sol $F\left[A\cos\omega_0 t\right] = A\,F\left[\cos\omega_0 t\right]$

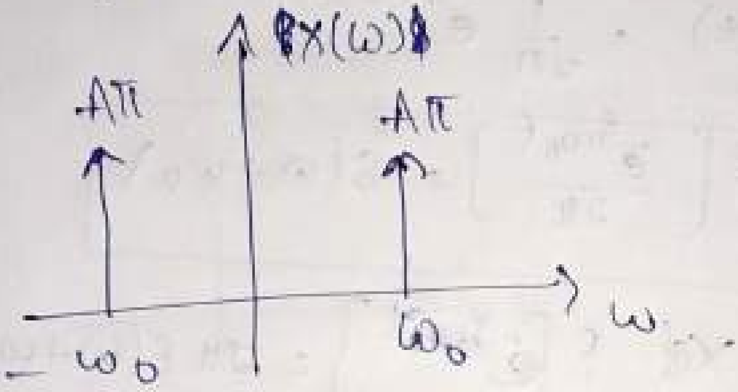
$= A\,F\left[\frac{e^{j\omega_0 t}}{2} + \frac{e^{-j\omega_0 t}}{2}\right]$

$= \frac{A}{2}F\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right]$

$= \frac{A}{2}\left[F\left[e^{j\omega_0 t}\right] + F\left[e^{-j\omega_0 t}\right]\right]$

$$= \frac{A}{2}\left[2\pi\delta(\omega-\omega_0) + 2\pi\delta(\omega+\omega_0)\right]$$

$$F[A\cos\omega_0 t] = A\pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$$



$$\boxed{A\cos\omega_0 t \xrightarrow{F} A\pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]}$$

$$\boxed{A\sin\omega_0 t \xrightarrow{F} \frac{A\pi}{j}\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]}$$

(10) $x(t) = A\sin\omega_0 t$

Sol: $F[A\sin\omega_0 t] = A\, F[\sin\omega_0 t]$

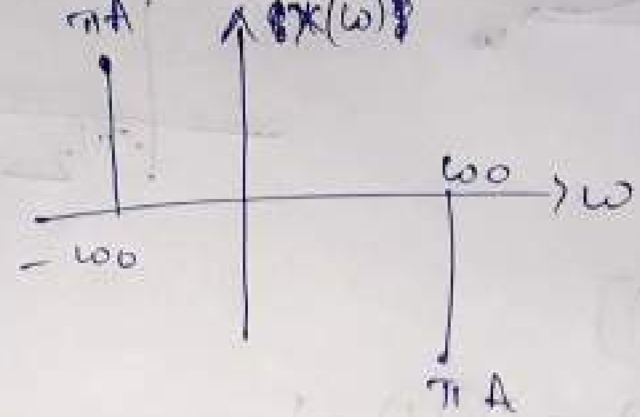
$$= A\left[\frac{e^{j\omega_0 t}}{2j} - \frac{e^{-j\omega_0 t}}{2j}\right]$$

$$= \frac{A}{2j}\left[F[e^{j\omega_0 t}] - F[e^{-j\omega_0 t}]\right]$$

$$= \frac{A}{2j}\left[2\pi\delta(\omega-\omega_0) - 2\pi\delta(\omega+\omega_0)\right]$$

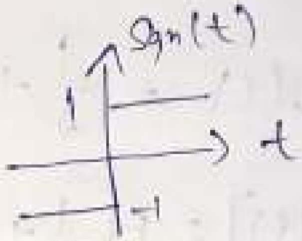
$$= \frac{A\pi}{j}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$$

$$F[A\sin\omega_0 t] = A\pi j\left[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)\right]$$



(11) $F[\mathrm{sgn}(t)]$

Sol



$$\mathrm{sgn}(t) = \lim_{a\to 0}\left[e^{-at}u(t) - e^{at}u(-t)\right]$$

$$= \lim_{a\to 0}\left[\int_{-\infty}^{\infty} e^{-at}u(t)e^{-j\omega t}\,dt - \int_{-\infty}^{\infty} e^{at}u(-t)e^{-j\omega t}\,dt\right]$$

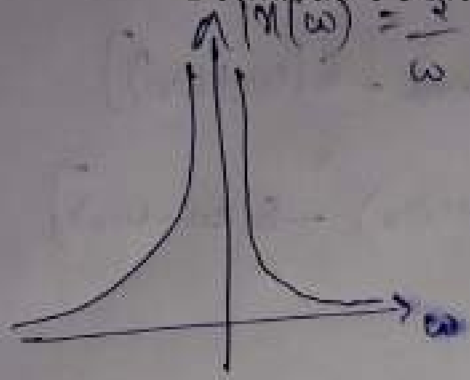
$$= \lim_{a\to 0}\left[\int_{0}^{\infty} e^{-t(a+j\omega)}\,dt - \int_{-\infty}^{0} e^{t(a-j\omega)}\,dt\right]$$

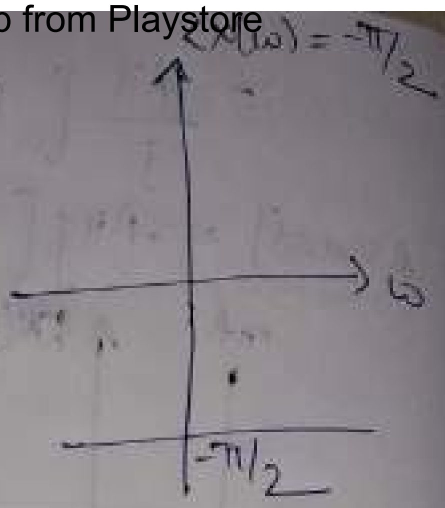
$$= \lim_{a\to 0}\left[\frac{1}{a+j\omega} - \left(\frac{1}{a-j\omega}\right)\right]$$

$$= \lim_{a\to 0}\left[\frac{1}{a+j\omega} - \frac{1}{a-j\omega}\right]$$

$$X(\omega) = \frac{1}{j\omega} - \left(-\frac{1}{j\omega}\right)$$

$$X(\omega) = \frac{2}{j\omega} = \frac{2}{\omega}\angle -90^\circ$$





(12) $F[u(t)]$

Sol

$$\mathrm{Sgn}(t) = 2u(t) - 1$$

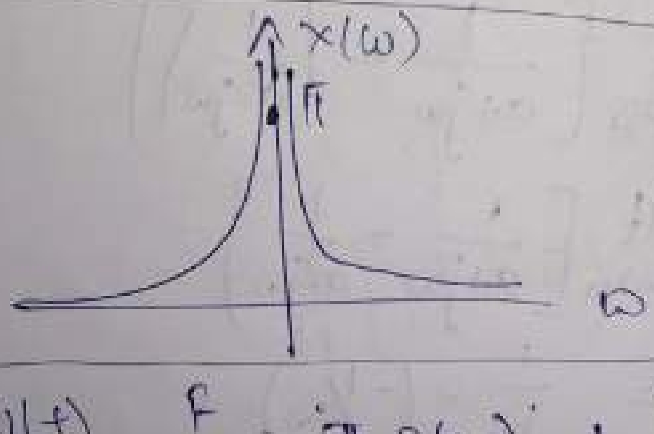
$$u(t) = \frac{1}{2} + \frac{1}{2}\mathrm{sgn}(t)$$

$$F[u(t)] = F\left[\frac{1}{2} + \frac{1}{2}\mathrm{sgn}(t)\right]$$

$$= F\left[\frac{1}{2}\right] + \frac{1}{2}F[\mathrm{sgn}(t)]$$

$$= 2\pi \times \frac{1}{2}\delta(\omega) + \frac{1}{2} \times \frac{2}{j\omega}$$

$$\boxed{X(\omega) = \pi\,\delta(\omega) + \frac{1}{j\omega}}$$



$$\boxed{u(t) \xrightarrow{F} \pi\,\delta(\omega) + \frac{1}{j\omega}}$$

Summary:-

$\boxed{\text{rect}(t) \longleftrightarrow \text{Sinc}\left(\frac{\omega}{2\pi}\right)}$
Unit rectangular

$\boxed{\Delta(t) \xrightarrow{F} \text{sinc}^2\left(\frac{\omega}{2\pi}\right)}$
Unit triangular

$x(t)$ signal	$X(\omega)$ F.T
$\delta(t)$	1
$e^{-at}u(t)$	$1/a+j\omega$
$e^{at}u(-t)$	$1/a-j\omega$
$A\,\text{rect}(t/\tau)$	$A\tau Sa\left(\frac{\omega\tau}{2}\right)$, $A\tau\,\text{sinc}\left(\frac{\omega\tau}{2\pi}\right)$
$A\Delta(t/\tau)$	$A\tau Sa^2\left(\frac{\omega\tau}{2}\right)$, $A\tau\,\text{sinc}^2\left(\frac{\omega\tau}{2\pi}\right)$
$e^{-j\omega_0 t}$	$2\pi\delta(\omega+\omega_0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$\sin\omega_0 t$	$\pi j[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$
$\text{sgn}(t)$	$2/j\omega$
$u(t)$	$\pi\delta(\omega)+1/j\omega$
A	$2\pi A\delta(\omega)$
$e^{-a\lvert t\rvert}$	$2a/a^2+\omega^2$

Properties of F.T

① linearity property

If $x_1(t) \xrightarrow{F} X_1(\omega)$

$x_2(t) \xrightarrow{F} X_2(\omega)$

then $\alpha x_1(t) + \beta x_2(t) \xrightarrow{F} \alpha X_1(\omega) + \beta X_2(\omega)$

Proof:- $F[\alpha x_1(t) + \beta x_2(t)] = \int_{-\infty}^{\infty} (\alpha x_1(t) + \beta x_2(t)) e^{-j\omega t} dt$

$$= \int_{-\infty}^{\infty} \alpha x_1(t) e^{-j\omega t} dt + \int_{-\infty}^{\infty} \beta x_2(t) e^{-j\omega t} dt$$

$$= \alpha \int_{-\infty}^{\infty} x_1(t) e^{-j\omega t} dt + \beta \int_{-\infty}^{\infty} x_2(t) e^{-j\omega t} dt$$

$$= \alpha [X_1(\omega)] + \beta [X_2(\omega)]$$

$$= \alpha X_1(\omega) + \beta X_2(\omega)$$

① $x(t) = 5 - \sin^2 t$

sol Given:- $x(t) = 5 - \sin^2 t$

$$x(t) = 5 - \left[\frac{1}{2} - \frac{1}{2}\cos 2t\right]$$

$$x(t) = \frac{9}{2} + \frac{1}{2}\cos 2t$$

Apply linearity property.

$$F[x(t)] = F[9/2] + \frac{1}{2} F[\cos 2t]$$

$$= 2\pi \times \frac{9}{2}\delta(\omega) + \frac{1}{2}\pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$$

$$= 9\pi\delta(\omega) + \frac{\pi}{2}[\delta(\omega+2) + \delta(\omega-2)$$

$$F[x(t)] = 9\pi\delta(\omega) + \frac{\pi}{2}[\delta(\omega+2) + \delta(\omega-2)]$$

② $x(t) = \sin t + 2\cos 2t$

Sol

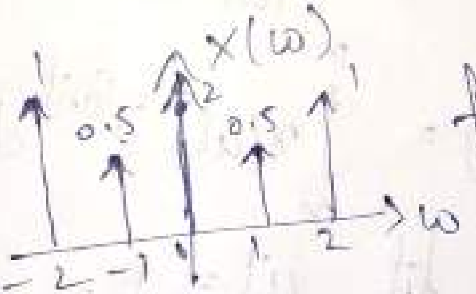
$$F[x(t)] = F[\sin t] + F[2\cos 2t]$$

$$= F[\sin t] + 2F[\cos 2t]$$

$$= j\pi[\delta(\omega+1) - \delta(\omega-1)]$$

$$+ 2\pi[\delta(\omega+2) + \delta(\omega-2)]$$

$$F[x(t)] = \pi[2(\delta(\omega+2) + \delta(\omega-2)) + j(\delta(\omega+1) - \delta(\omega-1))]$$

③ 

find IFT → $x(t)$

Sol

$$X(\omega) = 1.\delta(\omega+2) + \frac{1}{2}\delta(\omega+1) + 2\delta(\omega)$$

$$+ \frac{1}{2}\delta(\omega-1) + \delta(\omega-2)$$

$$F^{-1}[X(\omega)] = F^{-1}\left[\delta(\omega+2) + \frac{1}{2}\delta(\omega+1) + 2\delta(\omega) + \frac{1}{2}\delta(\omega-1) + \delta(\omega-2)\right]$$

$$F^{-1}[X(\omega)] = F^{-1}[\delta(\omega+2)] + \frac{1}{2}F^{-1}[\delta(\omega+1)]$$
$$+ 2F^{-1}[\delta(\omega)] + \frac{1}{2}F^{-1}[\delta(\omega-1)]$$
$$+ F^{-1}[\delta(\omega-2)]$$

$$F^{-1}[X(\omega)] = \frac{e^{-2jt}}{2\pi} + \frac{1}{2}\frac{e^{-jt}}{2\pi} + 2\left(\frac{1}{2\pi}\right)$$
$$+ \frac{1}{2}\frac{e^{jt}}{2\pi} + \frac{e^{2jt}}{2\pi}$$

$$F^{-1}[X(\omega)] = \frac{1}{\pi} + \frac{2\cos 2t}{2\pi} + \frac{1}{2}\frac{(2\cos t)}{2\pi}$$

* **Time Scaling Property:-**

If $x(t) \xrightarrow{F} X(\omega)$ then

$$x(at) \xrightarrow{F} \frac{1}{|a|}X\left(\frac{\omega}{a}\right)$$

Proof:- $F[x(at)] = \int_{-\infty}^{\infty} x(at)\, e^{-j\omega t}\, dt$

Case I :- 'a' be +ve

Let $at = p$

$dt = dp/a$

$$= \int_{-\infty}^{\infty} x(p)\, e^{-j\omega p/a}\, \frac{dp}{a}$$

$$= \frac{1}{a}\int_{-\infty}^{\infty} x(p)\, e^{-j\frac{\omega}{a}p}\, dp$$

$$\boxed{F[x(at)] = \frac{1}{a}X\left(\frac{\omega}{a}\right)}$$

$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

Case ii) $a < 0$

$$F[x(-at)] = \int_{-\infty}^{\infty} x(-at)\, e^{-j\omega t}\, dt$$

let $-at = u \Rightarrow dt = -\frac{1}{a}\, du$

$$F[x(-at)] = \frac{-1}{a} \int_{\infty}^{-\infty} x(u)\, e^{\frac{j\omega}{a} u}\, du$$

$$= \frac{1}{a} \int_{-\infty}^{\infty} x(u)\, e^{-\left(\frac{j\omega}{a}\right) u}\, du$$

$$F[x(-at)] = \frac{1}{a} X(-\omega/a)$$

$$\therefore \boxed{F[x(at)] = \frac{1}{|a|} X\left(\frac{\omega}{a}\right)}$$

① $F[\cos 2t]$

Sol $F[\cos 2t] =$

$\cos t = \pi\left(\delta(\omega+1) + \delta(\omega-1)\right)$

$\cos 2t = \frac{1}{2}\pi\left[\delta\left(\frac{\omega}{2}+1\right) + \delta\left(\frac{\omega}{2}-1\right)\right]$

$$\boxed{\delta(at-b) = \frac{1}{a}\delta(t - b/a)}$$

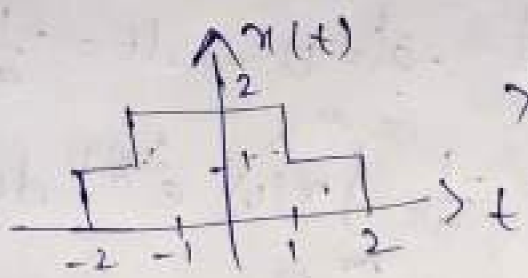
$$\boxed{\delta(at) = \frac{1}{a}\delta(t)}$$

$$\cos 2t = \frac{1}{2}\pi\left[\delta\left(\frac{\omega+2}{2}\right) + \delta\left(\frac{\omega-2}{2}\right)\right]$$

$$= \frac{1}{2}\pi\left[2\,\delta(\omega+2) + 2\,\delta(\omega-2)\right]$$

$$\cos 2t = \pi\left[\delta(\omega+2) + \delta(\omega-2)\right]$$

(2)



$X(\omega) = ?$

Sol method 1:-

$$x(t) = 1 \quad , \quad -2 \le t \le -1$$
$$= 2 \quad , \quad -1 \le t \le 1$$
$$= 1 \quad , \quad 1 \le t \le 2$$

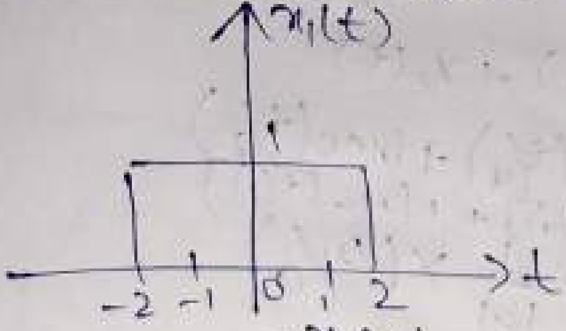
$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

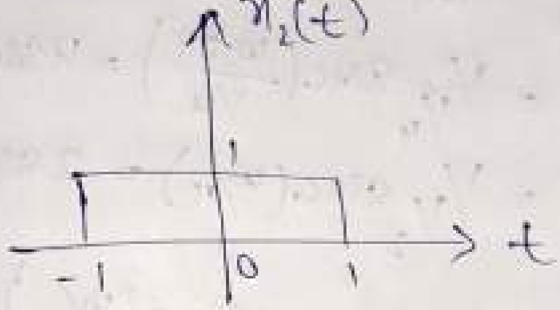
$$= \int_{-2}^{-1} 1 \cdot e^{-j\omega t}\, dt + \int_{-1}^{1} 2 \cdot e^{-j\omega t} + \int_{1}^{2} 1 \cdot e^{-j\omega t}\, dt$$

$=$

method 2:-

$x(t) = x_1(t) + x_2(t)$





$F[x(t)] = F[x_1(t) + x_2(t)]$

$F[x(t)] = F[x_1(t)] + F[x_2(t)]$

$A\,\mathrm{rec}(t/T) \longleftrightarrow AT\,\mathrm{sinc}\left(\frac{\omega T}{2\pi}\right)$

$F[x(t)] = F[x_1(t)] + F[x_2(t)]$

$x_1(t) \Rightarrow A=1,\ T=4,\ \mathrm{rec}(t/T) = \mathrm{rec}(t/4)$

$= 4\,\mathrm{sinc}\left(\frac{4\omega}{2\pi}\right)$

$= 4\,\mathrm{sinc}\left(\frac{2\omega}{\pi}\right)$

$x_2(t) \Rightarrow A=1,\ T=2,\ \mathrm{rec}(t/2) =$

$2\,\mathrm{sinc}\left(\frac{2\omega}{2\pi}\right)$

$= 2\,\mathrm{sinc}(\omega/\pi)$

$$\boxed{F[x(t)] = 4\,\mathrm{sinc}\left(\frac{2\omega}{\pi}\right) + 2\,\mathrm{sinc}(\omega/\pi)}$$

method 3:-

$$rect(t) \longrightarrow sinc\left(\frac{\omega}{2\pi}\right)$$

$$x(t) = x_1(t) + x_2(t)$$

$$= rect(t/4) + rect(t/2)$$

$$F[x(t)] = F[rect(t/4)] + F[rect(t/2)]$$

$$x(at) \rightarrow \frac{1}{|a|} X(\omega/a)$$

$$rect(t/4) = 1/_{1/4}\, sinc\left(\frac{\omega/2\pi}{1/4}\right) = 4\, sinc\left(\frac{2\omega}{\pi}\right)$$

$$rect(t/2) = 1/_{1/2}\, sinc(\omega/\pi) = 2\, sin(\omega/\pi)$$

$$X(\omega) = 4\, sinc\left(\frac{2\omega}{\pi}\right) + 2\, sinc(\omega/\pi)$$

3. Time reversal property:-

If $x(t) \xrightarrow{F} X(\omega)$

then $x(-t) \longrightarrow X(-\omega)$

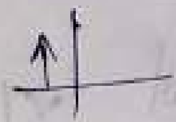
Proof:- From time scaling property

$$x(at) \rightarrow \frac{1}{|a|} X(\omega/a)$$

$$x(-t) \xrightarrow{F} \frac{1}{|-1|} X\left(\frac{\omega}{-1}\right)$$

$$\boxed{x(-t) \longrightarrow X(-\omega)}$$

(1) $F[U(-t)]$



Sol $U(t) \rightarrow \pi\delta(\omega) + \frac{1}{j\omega}$

From time reversed property

$U(-t) \rightarrow \pi\delta(-\omega) - \frac{1}{j\omega}$

$$\boxed{U(-t) \rightarrow \pi\delta(\omega) - 1/j\omega}$$

(2) $F[sgn(t)]$

Sol $Sgn(t) = U(t) - U(-t) = 2U(t) - 1$

$F[Sgn(t)] = F[U(t)] - F[U(-t)]$

$= \cancel{\pi\delta(\omega)} + \frac{1}{j\omega} - \cancel{\pi\delta(-\omega)} + \frac{1}{j\omega}$

$$\boxed{F[Sgn(t)] = \frac{2}{j\omega}}$$

(3) $x(t) = e^{at}U(-t)$ find $F[x(t)]$ (or) $X(\omega)$.

Sol $e^{-at}U(t) \rightarrow \frac{1}{a+j\omega}$

$e^{-a(-t)}U(-t) \rightarrow \frac{1}{a-j\omega}$

$e^{at}U(-t) \rightarrow \frac{1}{a-j\omega}$

(4) $X(\omega) = \dfrac{5j\omega+12}{(j\omega)^2+5j\omega+6}$, $x(t) = ?$

Sol $X(\omega) = \dfrac{5j\omega+12}{(j\omega)^2+5j\omega+6}$

$X(\omega) = \dfrac{5j\omega+12}{(j\omega+3)(j\omega+2)}$

$X(\omega) = \dfrac{(12-10/-2+3)}{j\omega+2} + \dfrac{12-5(3)/-3+2}{j\omega+3}$

$= \dfrac{2}{2+j\omega} + \dfrac{3}{j\omega+3}$

Apply IFT & linearity.

$F^{-1}[X(\omega)] = 2F^{-1}\left[\dfrac{1}{2+j\omega}\right] + 3F^{-1}\left[\dfrac{1}{3+j\omega}\right]$

$$\boxed{x(t) = 2e^{-2t}u(t) + 3e^{-3t}u(t)}$$

(5) $X(\omega) = \dfrac{1}{2-\omega^2+3j\omega}$, $x(t) = ?$

Sol $X(\omega) = \dfrac{1}{(j\omega)^2+3j\omega+2}$

$X(\omega) = \dfrac{1}{(j\omega+1)(j\omega+2)}$

$X(\omega) = \dfrac{1}{j\omega+1} - \dfrac{1}{j\omega+2}$

$$\boxed{x(\omega) = e^{-t} u(t) - e^{-2t} u(t)}$$

4. Property 4 Time Shifting :-

If $x(t) \xrightarrow{F} X(\omega)$

$x(t-t_0) \xrightarrow{F} e^{-j\omega t_0} X(\omega)$

$x(t+t_0) \longrightarrow e^{j\omega t_0} X(\omega)$

Proof:- $x(t) = F^{-1}[X(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$

$F^{-1}[e^{-j\omega t_0} X(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-j\omega t_0} X(\omega) e^{j\omega t} d\omega$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j(t-t_0)\omega} d\omega$

$$\boxed{F^{-1}[e^{-j\omega t_0} X(\omega)] = x(t-t_0)}$$

$F[x(t-t_0)] = e^{-j\omega t_0} X(\omega)$

$$\boxed{x(t-t_0) \xrightarrow{F} e^{-j\omega t_0} X(\omega)}$$

1. $F^{-1}[e^{-j\omega t_0}]$

Sol $X(\omega) = e^{-j\omega t_0} \cdot 1$

$x(t) = ?$

w.k.t $\delta(t) \rightarrow 1$

$\delta(t-t_0) \rightarrow 1 \cdot e^{-j\omega t_0}$

(2) $F^{-1}\left[\dfrac{e^{-j\omega t_0}}{a+j\omega}\right]$

Sol $e^{-at}u(t) \longrightarrow \dfrac{1}{a+j\omega}$

$e^{-a(t-t_0)}u(t-t_0) \rightarrow \dfrac{e^{-j\omega t_0}}{a+j\omega}$

(3) $\dfrac{e^{-2j\omega}}{2-j\omega}$

Sol $e^{2t}u(-t) \longrightarrow \dfrac{1}{2-j\omega}$

$e^{2(t-2)}u(-(t-2)) \longrightarrow \dfrac{e^{-2j\omega}}{2-j\omega}$

$e^{2t-4}u(2-t) \rightarrow \dfrac{e^{-2j\omega}}{2-j\omega}$

$$F^{-1}\left(\frac{e^{-2j\omega}}{2-j\omega}\right) = e^{2t-4}u(2-t)$$

(4) $x(t) = u(t+1) + u(t-1)$

Sol $X(\omega) = ?$

$F[x(t)] = F[u(t+1)] + F[u(t-1)]$

$u(t) \longrightarrow \pi\delta(\omega) + \dfrac{1}{j\omega}$

$$X(\omega) = e^{j\omega}\left[\pi\delta(\omega) + \frac{1}{j\omega}\right] + e^{-j\omega}\left[\pi\delta(\omega) + \frac{1}{j\omega}\right]$$

$$= \frac{e^{j\omega} + e^{-j\omega}}{j\omega} + \pi\left[e^{j\omega}\delta(\omega) + e^{-j\omega}\delta(\omega)\right]$$

$\therefore$ shifting property

$$= \frac{2\cos\omega}{j\omega} + \pi\left[e^{0}\delta(\omega) + e^{0}\delta(\omega)\right]$$

(5) $x(t) = \delta(t-1) - \delta(t+1)$

Sol $X(\omega) = F\{x(t)\} = F\left[\delta(t-1) - \delta(t+1)\right]$

$X(\omega) = F\left[\delta(t-1)\right] - F\left[\delta(t+1)\right]$

$\delta(t) \rightarrow 1$

$\delta(t-1) \rightarrow 1 \cdot e^{-j\omega(1)}$

$\delta(t+1) \rightarrow e^{j\omega}$

$X(\omega) = e^{-j\omega} + e^{j\omega}$

$X(\omega) = 2\cos\omega$

(6) $e^{-2|t-2|} = x(t)$, find $X(\omega)$

Sol $e^{-a|t|} \longleftrightarrow \dfrac{2a}{a^2+\omega^2}$

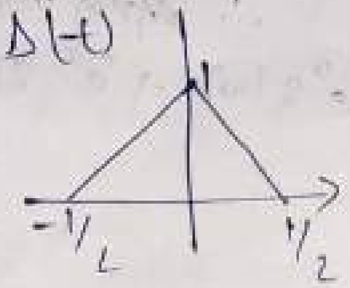
$e^{-2|t|} \longrightarrow \dfrac{4}{4+\omega^2}$

$e^{-2|t-2|} \longrightarrow \dfrac{4}{\omega^2+4} e^{-j\omega(2)} = \dfrac{4e^{-2j\omega}}{\omega^2+4}$

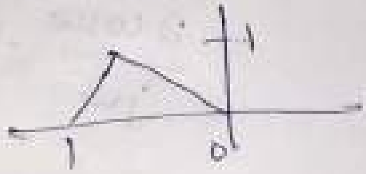
(7) $x(t) = \Delta\left(t - \frac{1}{2}\right)$

Sol

$\Delta(-t)$



$\Delta(t + 1/2)$

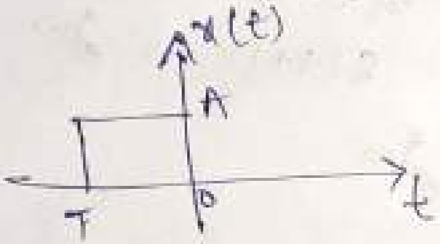


$A\,\Delta(-t/T) \longrightarrow AT\,\mathrm{sinc}^2\left(\frac{\omega T}{2\pi}\right)$

$\Delta(t) \xrightarrow{F} \mathrm{sinc}^2\left(\frac{\omega}{2\pi}\right)$

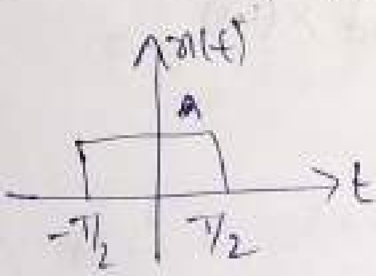
$\Delta\left(t + \frac{1}{2}\right) \longrightarrow \mathrm{sinc}^2\left(\frac{\omega}{2\pi}\right) e^{1/2 j\omega}$

(8)



find $X(\omega)$

Sol $A\,\mathrm{rec}(t/T) \longrightarrow AT\,\mathrm{sinc}\left(\frac{\omega T}{2}\right)$



$x(t) = A\,\mathrm{rec}\left(\frac{t}{T} + \frac{T}{2}\right)$

$A\,\mathrm{rec}\left(\frac{t}{T} + \frac{T}{2}\right) = AT\,\mathrm{sinc}\left(\frac{\omega T}{2\pi}\right) e^{\frac{j\omega}{2}}$

(9) $x(t) = e^{-3t}\,u(t-1)$

Sol $e^{-3t}u(t) \xrightarrow{F} \frac{1}{3 + j\omega}$

$e^{-3(t-1)}u(t-1) \longrightarrow \frac{1}{3 + j\omega}\, e^{-j\omega(1)}$

$$e^{3} \cdot e^{-3t} u(t-1) \rightarrow \frac{e^{-j\omega}}{3+j\omega}$$

$$e^{-3t} u(t-1) \rightarrow \frac{e^{-3-j\omega}}{3+j\omega}$$

$$e^{-3t} u(t-1) \rightarrow \frac{e^{-3-j\omega}}{3+j\omega}$$ //

(10) $F\left[\text{rec}\left(\frac{t+2}{4}\right)\right]$

Sol

$$x(t) = \text{rec}(t/4 + 1/2)$$

$$\text{rec}(t) \rightarrow \text{sinc}\left(\frac{\omega}{2\pi}\right)$$

$$\text{rec}\left(t+\frac{1}{2}\right) \rightarrow e^{j\omega(1/2)} \text{sinc}\left(\frac{\omega}{2\pi}\right)$$

$$\text{rec}\left(\frac{t}{4}+\frac{1}{2}\right) \rightarrow 4\, e^{j(\omega/1/4)1/2} \text{sinc}\left(\frac{\omega/1/4}{2\pi}\right)$$

$$\text{rec}\left(\frac{t}{4}+1/2\right) \rightarrow 4\, e^{\frac{4j\omega}{2}} \text{sinc}\left(\frac{4\omega}{2\pi}\right)$$

$$\text{rec}\left(\frac{t}{4}+1/2\right) \rightarrow 4\, e^{2j\omega} \text{sinc}\left(\frac{2\omega}{\pi}\right)$$

(11) If $x(t) \rightarrow X(\omega)$ find F.T of $x(-2-t) + x(2-t)$

Sol

$F[x(-2-t)]$

$$x(t) \rightarrow X(\omega)$$

$$x(t-2) \rightarrow e^{-2j\omega} X(\omega)$$

$$x(-t-2) \rightarrow e^{2j\omega} X(-\omega) \quad \text{——— (1)}$$

$$F[x(2-t)] = F[x(-t+2)]$$

$$x(t) \longrightarrow X(\omega)$$

$$x(t+2) \longrightarrow e^{2j\omega} X(\omega)$$

$$x(-t+2) \longrightarrow e^{-2j\omega} X(-\omega)$$

$$F[x(-2-t) + x(2-t)] = e^{2j\omega} X(-\omega) + e^{-2j\omega} X(-\omega)$$

$$= X(-\omega)\left[e^{2j\omega} + e^{-2j\omega}\right]$$

$$F[x(-2-t) + x(2-t)] = 2\cos(2\omega)\, X(-\omega)$$