Segment Trees and their Applications

CS5016: Computational Methods and its Applications

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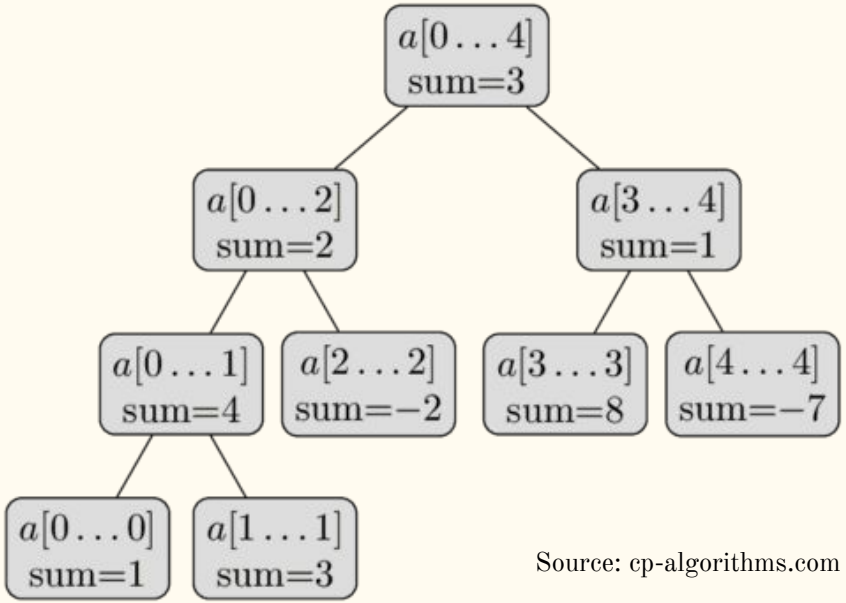
Introduction

- A segment tree is a tree data structure used for storing information about array intervals.
- Allows querying intervals of data in an efficient manner, while still being flexible enough to allow quick modification of the data.

Structure of a Segment Tree

- A segment tree is a binary tree where each node represents a subrange of data.
  - Each leaf node represents a single element of data
  - Each non-leaf node represents the range of data which is the union of its two children's ranges
- The segment tree is built recursively by breaking the range into two equal subranges and building left and right subtrees.
- Requires only linear amount of memory to store.

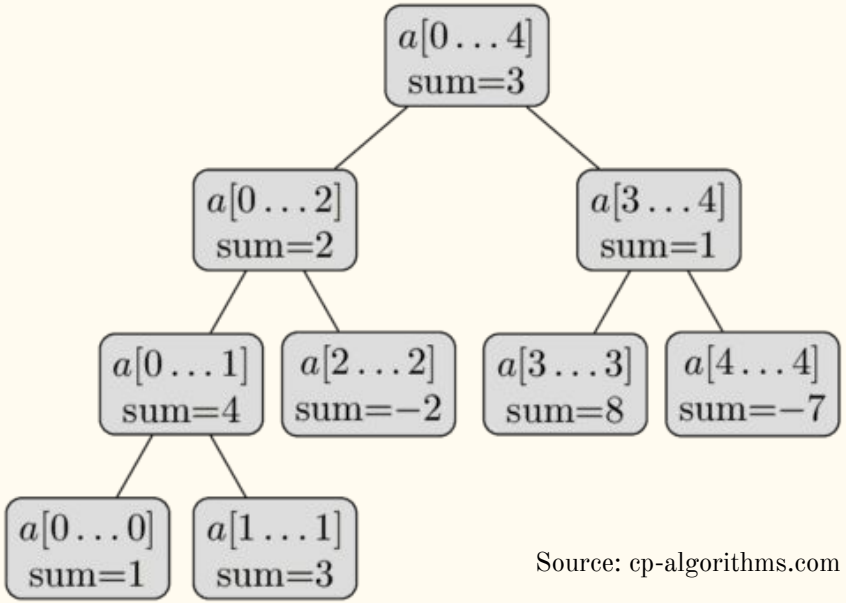


Source: cp-algorithms.com

Sum segment tree for $a = [1, 3, -2, 8, -7]$

Querying in a Segment Tree - O(log n)

- Start at root node and traverse down the tree comparing the query range with node range at each step
- Query range = Node range : Return the pre computed value stored there
- Query range falls completely in the domain of left or right child : Recursively traverse the required subtree
- Query range falls partially in the domains of left and right children : Recursively traverse both subtrees and compute partial answers for both; combine



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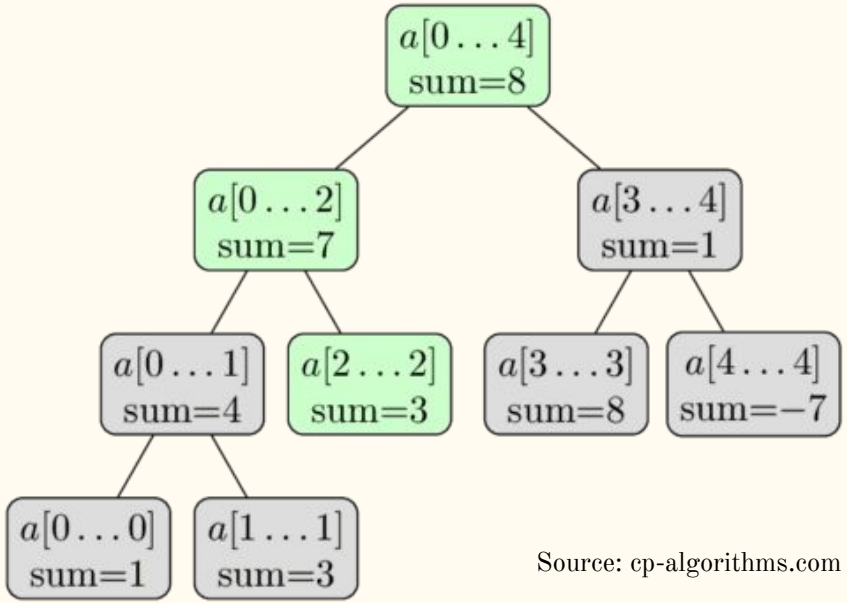
Sum segment tree for $a = [1, 3, -2, 8, -7]$

Updates in a Segment Tree - O(log n)

- Start from the root
- If at leaf node : update it
- If update range ⋂ node range = Φ : do nothing
- Otherwise : Recursively update both left and right subtrees, then update the value of the node



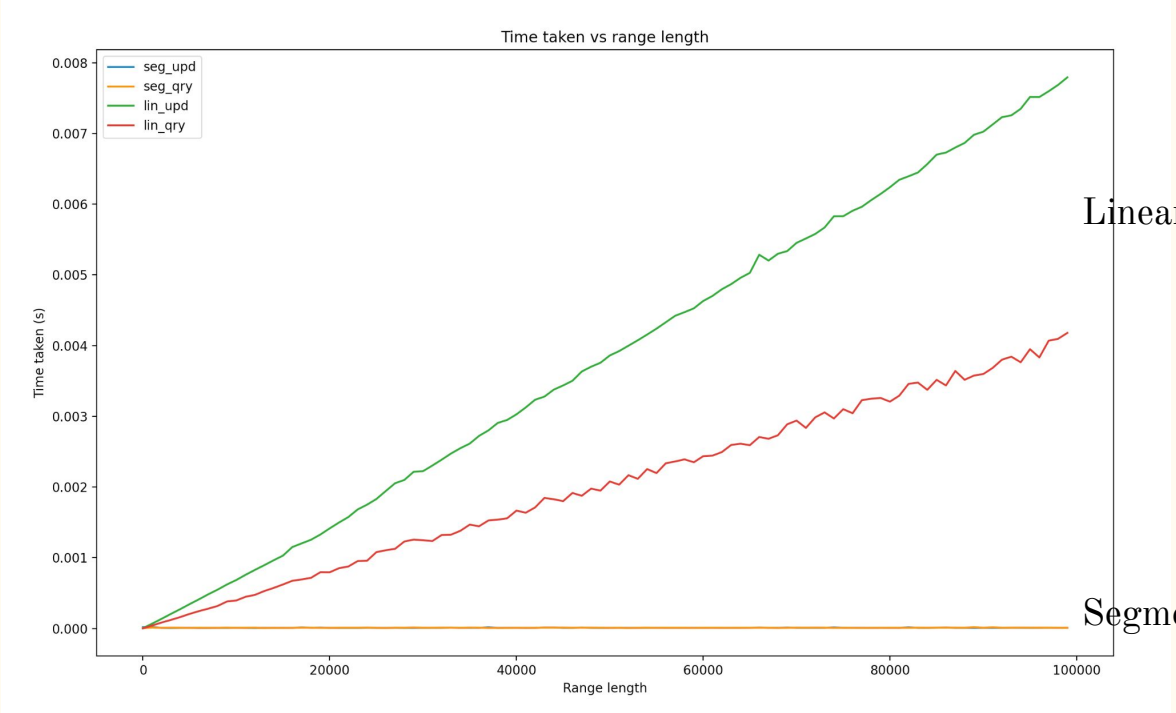
Source: cp-algorithms.com

Updating the segment tree by setting $a[2]=3$

Problem Solving Comparison : Linear Method vs Segment Tree Method

- A list of random integers of length 10^5 was made
- Sum queries were made for the range 0...k for various k using linear method and segment tree method
- Update queries (a += 10) were made for the range 0...k for various k using linear method and segment tree method
- The time taken for each was plotted

Problem Solving Comparison : larger range



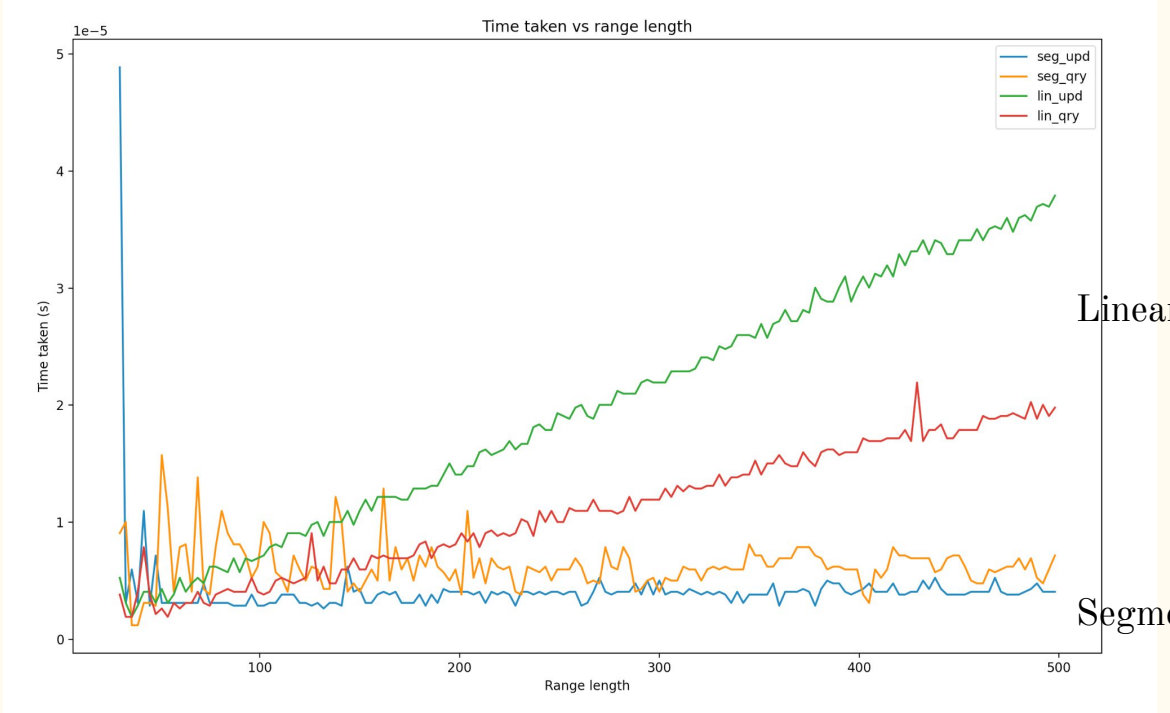
Time taken (s)

Linear method

Segment tree method

Range length

Problem Solving Comparison : smaller range



Time taken (s)

Linear method

Segment tree method

Range length

Applications of Segment Tree

- Highly efficient in querying and updating ranges of data
- Flexibility in supporting various operations
- Some practical applications
  - Computational geometry
  - Pattern recognition and image processing
  - Geographic information systems

Summary

- Segment tree is a powerful data structure that can be used to efficiently solve many problems
- Shortcomings compared linear data structure
  - Although O(n), it require at max 4 times memory compared to storing as array (depending on implementation)
  - Computation time for building the tree
  - Not easily scalable
- Despite the shortcomings, segment trees remain an important tool

References

https://en.wikipedia.org/wiki/Segment_tree

https://cp-algorithms.com/data_structures/segment_tree.html

https://www.geeksforgeeks.org/applications-advantages-and-disadvantages-of-segment-tree/