



In-Hand Pose Estimation Through Purely Tactile Perception and In-Hand Manipulation

A Master Thesis

written by

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The code for this project is available at
https://github.com/vmstavens/in_hand_pose_estimation

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Abstract

Some abstract text explaining the goal, methods and conclusion of the project.

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Acronyms

AEBM analytical elasticity-based models.	MIM Matrix Inversion Method.
CAD Computer Aided Design.	ML machine learning.
cGAN conditional Generative Adversarial Network.	MLP Multi Layered Perceptron.
cobots collaborative robots.	PC point cloud.
CP correspondence problem.	PCR point cloud registration.
CPD Coherent Point Drift.	PE pose estimation.
CSGM Cross-Source Graph Matching.	PNP pick-and-place.
CV computer vision.	PwoF point-contact-without-friction.
DeepGMR Deep Gaussian Mixture Registration.	QAP Quadratic Assignment Problem.
DL deep learning.	QCQP Quadratically Constrained Quadratic Programming.
DOF degrees of freedom.	QP Quadratic Programming.
EE end effector.	RANSAC Random Sample Consensus.
EFM elastic foundation models.	RCQP Relaxed Convex Quadratic Programming.
FEM finite element models.	ROM Range of Motion.
FGM Factorized Graph Matching.	ROS Robot Operating System.
GK grasp kinematics.	RRT Rapidly-exploring Random Tree.
GMM Gaussian Mixture Model.	SDP Semi-Definite Programming.
GNC Graduated Non-Convexity.	SF soft finger.
HF hard finger.	SOCP Second Order Cone Programming.
ICP Iterative Closest Point.	TP Tactile Perception.
IEP Inverse Elasticity Problem.	UR Universal Robots.
IHM In-Hand Manipulation.	
JRMPC Joint Registration of Multiple Point Sets.	

Terms

collaborative robots (cobots) are robots which facilitate human-robot collaboration [1].

computer vision (CV) is a field of artificial intelligence (AI) that enables computers and systems to derive meaningful information from digital images, videos and other visual inputs - and take actions or make recommendations based on that information [2].

correspondence problem (CP) is the problem where one aims at finding correspondences between the pixels in two (or more) images [3].

deep learning (DL) are methods that allow computational models that are composed of multiple processing layers to learn representations of data with multiple levels of abstraction [4].

end effector (EE) is a generic term for all functional units involved in direct interaction of the robot system with the environment or with a given object [5].

manipulator : A serial robot mechanisms. The robot manipulator is represented by a serial chain of rigid bodies, called robot segments, connected by joints [6].

point cloud registration (PCR) is a generic term for all functional units involved in direct interaction of the robot system with the environment or with a given object [5].

pose estimation (PE) A particular instance of feature-based alignment, which occurs very often, is estimating an object's 3D pose from a set of 2D point projections. This pose estimation problem is also known as extrinsic calibration [7].

Robot Operating System (ROS) is a set of open-source software libraries and tools that help build robot applications. [8].

Chapter 1

Introduction

1.1 Context

As of 2022 most of the industrialized world has developed tools for unprecedented growth in wealth and technology on a global scale [9, Chapter 4]. In such times a great deal of consumerism and interconnection is present with people needing products produced faster and more consistently than ever before [9, Chapter 4]. As one would expect, this creates a high demand for manufacturers to reliably and consistently provide products, while also remaining flexible as the demand for different product change rapidly. To accommodate the need for ever-greater volumes of products, consistent, reliable and flexible labor is essential in assembly, transport and manipulation processes in the production pipeline. Due to these types of manual labor being largely done by unskilled workers, automation alternatives are being adopted which provide benefits [9, Chapter 4]. This different approach to manufacturing has been labeled the fourth industrial revolution or i4.0 for short. The beneficiaries are the employer and employee, with the employer having the benefits: Avoid paying monthly salaries to unskilled laborers doing manual tasks, here the automation alternative only requires electrical energy and potential supervision by a few qualified individuals. Potential risks are also involved when hiring humans as the workforce can be inconsistent due to human error [10] or left out due to illness etc. Considerations about workers' rights such as working conditions and wages also need not be considered. Workers furthermore cause production limitations in the form of stand-still hours, such as bathroom and lunch breaks along with after-work hours and holidays. This replacement of manual labor also potentially benefits the employee, as boring and physically wearing work is automated, enabling the employees to take on different and less wearing and potentially dangerous roles. While the issue of labor unemployment becomes apparent solutions that provide support to already hired workers have been developed, such as collaborative robots (cobots) [11] which would negate this problem.

When implementing automation of production lines using robotics, certain categories of problems are revealed. These include assembly, alteration and pick-and-place (PNP), the last being the one of interest in this project.

1.2 Problem Description

Pick-and-place manipulators are used in a wide variety of different fields such as sorting of waste [12] handling of food [13, 14] and factory bin picking [15, 16, 17]. The solutions in these industries are examples of subcategories under the PNP problem, namely sorting and bin picking. Since both of these are subcategories of the PNP problem, they fundamentally follow the same sequential four phases from start to end. These phases are pre-grasping, grasping, transport, and placement [16] for traditional implementations of the PNP pipeline. The pre-grasp phase involves localizing the object(s), potentially estimating their pose and executing the trajectory to move the end effector (EE) grasp, collision-free to said object(s). Here different potential grasps can be considered to determine the best pose for the EE. In the grasping phase, the EE grasps the object in such a manner that the object's entire weight is supported by the EE, and ends when the object no longer is in contact with the environment, which often is the container holding the object. The transportation phase involves the motion of the manipulator to move from the pose achieved after the grasping phase, to a pose ready for placement of the object in the desired placing area or fixture. Here considerations may be needed about how much force and torque the EE's grasp can tolerate while moving without losing the object. Finally, the goal of the placing phase is to place the object within the placing

area or fixture in a desired end pose. Here the constraints on the end pose might differ significantly based on the application, as the pose of greens in a crate might need less precision than if the manipulator hands a bolt to another robotics system in the pipeline.

While these phases make up a traditional PNP system, certain assumptions are made regarding the objects of interest for this pipeline to function. Specifically, the localization and pose estimation (PE) of the pre-grasp phase are assumed possible due to either ensured object poses or estimated poses through computer vision (CV) sensory system. Due to CV being a mature research field a wide range of solution proposals to these problems have been generated [18]. These include classic vision [19, 20], deep learning (DL) based [21] and combinations of these [22]. However, while these may be sufficient for certain tasks they fundamentally suffer from the weaknesses introduced by vision techniques. These are a great number of outliers caused by: occlusions, reflecting, transparent or homogeneous surfaces, and repetitive structures when solving the correspondence problem (CP). Within factory settings, the common ones are transparent and reflective objects, due to metallic, plastic and glass products often being the materials used. While DL solutions have been developed for both reflective [23] and transparent [24] objects, these are use case specific and show limited results in a wider range of applications.

This project suggests a different PNP pipeline for cases where the object's starting pose is unknown. In this PNP pipeline the PE is moved from the pre-grasping phase to a new phase between the grasping and transportation phase, called the PE phase. The specific goal of this project is to develop a solution to this phase using tactile sensors in the EE to determine the object's pose. By using tactile sensing rather than visual the problems presented above will be eliminated. This will be done using a humanoid gripper as the EE with tactile sensors in each finger, more specifically a Shadow Dexterous Hand [25] with 24 degrees of freedom (DOF).

The alternative pipeline this project will enable can be seen in the upper row of Fig. 1.1 compared to the traditional pipeline in the lower row.

In Fig. 1.1(a) the pre-grasping phase can be seen for both pipelines. Here the traditional pipeline in cases of multiple objects often will employ custom fingertips or grippers entirely to facilitate form closure grasps, due to the grippers not having the flexibility to perform reliable force closure. On the contrary force closure can be performed with a humanoid gripper on a wide range of objects with no need for changing gripper equipment.

In Fig. 1.1(b) the grasping phase can be seen which introduces a greater level of complexity when using the suggested pipeline due to the humanoid gripper being a more complex physical system to represent and control. This is compared to the simplicity of executing potential binary commands in the traditional pipeline e.g. open and close.

In Fig. 1.1(c) the transportation phase can be seen, which introduces one of the benefits of using the suggested pipeline. Here tactile sensors in a humanoid gripper can pose and estimate the object and manipulations can be performed to change the object's pose such that easier placement can be performed in the following phase. This form of object manipulation is not feasible for the simple pneumatic grippers used in a traditional pipeline.

In Fig. 1.1(d) the placement phase can be seen, which demonstrates the result of the previous phase, as the traditional pipeline now has to change the grip on the object to properly place it in the socket, while the humanoid gripper simply can insert the part, as it is already oriented properly.

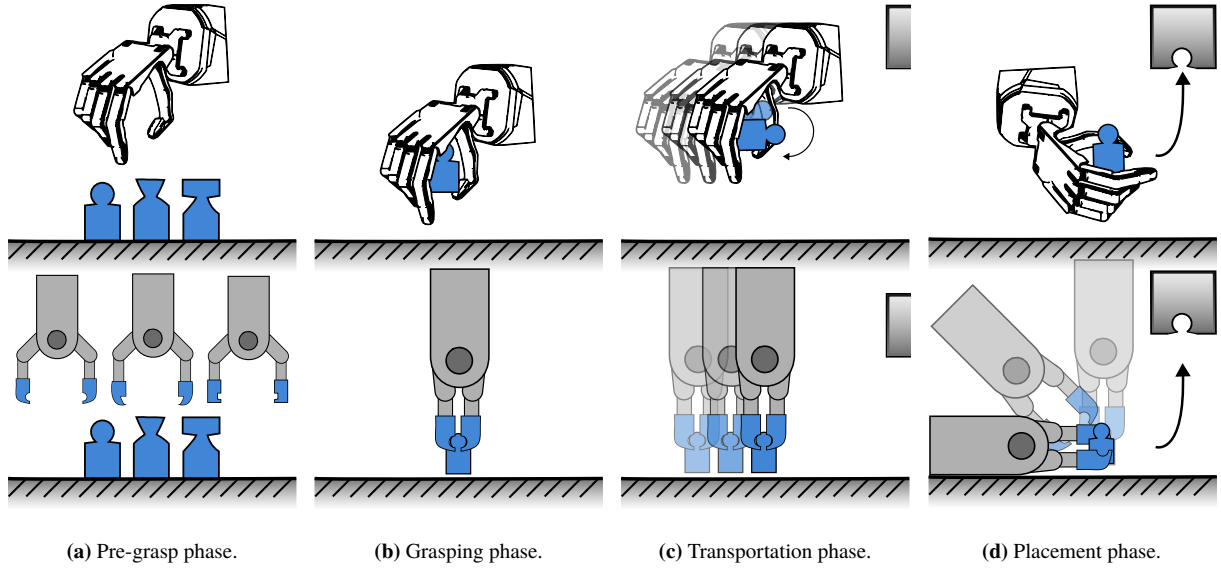


Fig. 1.1: A comparison between the traditional and suggested PNP pipeline.

To solve this PE problem, three sub-problems are identified and labeled problems 1, 2 and 3.

Problem 1 involves modeling the contact between the gripper's tactile sensors and the object, also referred to as tactile perception.

Problem 2 is to convert the collected data from problem 1 to estimated pose candidates.

Problem 3 involves in-hand manipulation. Since the initial grasp of the object might not be oriented in a manner where the recognizable features make context with the tactile sensors, manipulating the object within the EE's grasp will enable further information gathering. Thus the final problem is to control the EE in such a manner that the tactile sensors make context with the object in intelligently decided areas for a better pose estimate.

To test if the developed system successfully solves the PE problem, it is hypothesized that the intelligent probing method provides a statistically significant faster average PE convergence, along with a statistically significant greater success rate when determining the correct pose. A correct pose is here defined as the pose being greater than or equal to 95 % of the ground truth pose, and statistically significant is defined by an α -level of 95 %. This hypothesis will be referred to as H_1 , while the null hypothesis H_0 being that there is no statistically significant difference between intelligent and random probing's PE performance as described above.

1.3 Thesis Overview

This project is composed of 9 chapters, each listed below with an associated description.

Chapter 1: The introduction presents the historical context of the project along with a problem description and thesis overview. The problem description contains the decomposition of the project into sub-problems which will be addressed in the following chapters.

Chapter 2: The system setup presents an overview of the practical details of the project in the form of the system setup along with the code developed to execute the project.

Chapter 3: The literature review addresses the three problems identified in Chapter 1 by providing a thorough literature review on older as well as newer methods for solving each of the identified problems. To end each section, groups are compared and a method is chosen among the ones presented.

Chapter 4: The modeling chapter presents the physical modeling of the system and defines important mathematical notations and quantities as used in the following chapters.

Chapter 5: The Tactile Perception (TP) chapter addresses the method chosen in Chapter 3 to solve the first problem. This chapter expands on the method chosen and its functionality, followed by an evaluation of said method through experiments and their results. Finally, the functionality of the method is concluded.

Chapter 6: The PE chapter addresses the method chosen in Chapter 3 to solve the second problem. This chapter follows the same structure as Chapter 5.

Chapter 7: The In-Hand Manipulation (IHM) chapter addresses the method chosen in Chapter 3 to solve the third problem. This chapter follows the same structure as Chapter 5 and Chapter 6.

Chapter 8: The discussion chapter goes over the combined system and addresses problems, improvements, shortcomings and potential for future development.

Chapter 9: The conclusion chapter addresses the successfulness of the project.

Chapter 2

System Setup

The Shadow Dexterous Hand [25] is a sophisticated robotic hand with 24 DOF and a wide range of sensory feedback capabilities. To develop and test control algorithms for this complex system, simulation is an invaluable tool. In this chapter, the practical setup of our project is presented, which involves simulating the Shadow Dexterous Hand in Gazebo [26] within a Docker container [27].

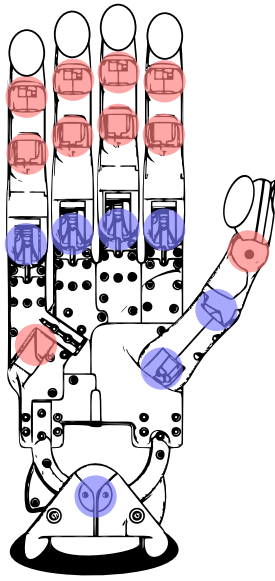
This simulation is based on the Shadow Robot Company's [28] Robot Operating System (ROS) [8] packages [29], which provide a flexible and customizable framework for controlling the hand. This project uses Gazebo, a popular robot simulation environment, to simulate the physics of the hand and its interaction with the environment. To ensure reproducibility and portability, the development framework which encapsulates the simulation environment is built within a Docker container, which allows us to easily distribute our code and dependencies to other researchers.

In this chapter, a detailed description of the hardware and software used in this project's simulation setup is provided, which includes the software architecture of the simulation, including the ROS packages and Gazebo plugins used to simulate the hand and its environment.

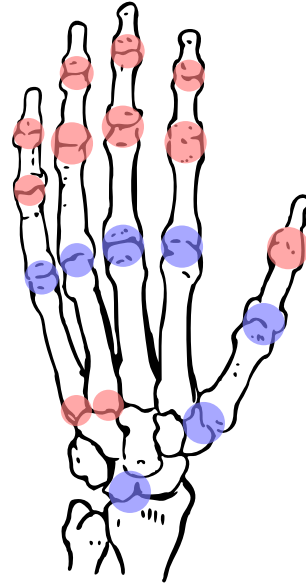
2.1 Simulation Setup

The Computer Aided Design (CAD) model of the Shadow Dexterous Hand is a highly detailed and accurate representation of the physical hand. The model is based on the original design of the real-world hand, which was developed by the Shadow Robot Company. The CAD model includes precise geometry for all of the hand's components, including the finger joints, tendons, and tactile sensors. The model also includes detailed material properties for each component, which are used to simulate the hand's physical behavior in the simulation.

The structure of the hand can be seen in Fig. 2.1(a) compared to a human hand in Fig. 2.1(b). Here each joint is marked with a color, labeling the DOF for that particular joint. The red labels refer to joints with 1 DOF, while blue refers to joints with a DOF equal to 2. As seen here the Shadow Dexterous Hand provides human-like dexterity due to its similar kinematics, see Fig. A.1, and the comparable Range of Motion (ROM) of each joint, see Table A.1 and Table A.2.



(a) Shadow Dexterous hand with joints color coded depending on the degrees of freedom. The total number of degrees of freedom can here be seen as 24. This figure is based on [30].



(b) Shadow Dexterous hand with joints color coded depending on the degrees of freedom. The total number of degrees of freedom can here be seen as 25 [31]. This figure is based on [32].

Fig. 2.1: The Shadow Dexterous hand and a human hand red here marks a joint with 1 degree of freedom, while blue marks a joint with 2.

The hand's geometry is modeled using a combination of standard shapes and custom-designed components. For example, the finger joints are modeled using a series of cylinders and spheres, which are connected by virtual tendons to simulate the motion of the real-world hand. The tactile sensors on the simulated hand, at the writing of this thesis, are purely aesthetical as representative simulated tactile sensors are yet to be supported as a standard component. To generate representative tactile data additional software is therefore required. The tactile sensors can be seen in Fig. 2.1(a) as the ellipsoids mounted at each fingertip.

Multiple hand configurations are available including a left hand, right hand, and both configurations mounted on Universal Robots (UR) manipulators [33]. The configuration chosen for this project is a Shadow Dexterous right hand without being mounted on a manipulator. Fig. 2.2 shows the CAD model of the Shadow Dexterous Hand in simulation.

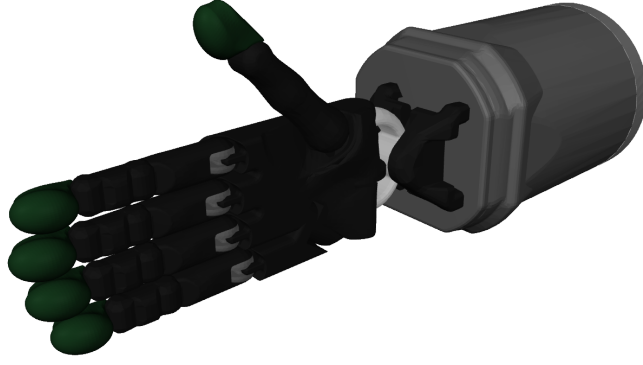


Fig. 2.2: A cutout of the simulated Shadow Dexterous Hand.

2.2 Software Setup

The software setup is shipped in a custom Ubuntu-based docker container [27, 34] with the necessary libraries to communicate and develop applications on the simulated as well as the physical hand, wrapped within a catkin workspace [35]. Additionally, the container comes with common-use libraries for Python and C++ development in ROS including numpy[36], OpenCV [37] and dynamic_reconfiguration [38]. The development environment can be seen illustrated in Fig. 2.3. The communication between the provided ROS packages and the Gazebo simulator is achieved through the ROS-Gazebo framework.

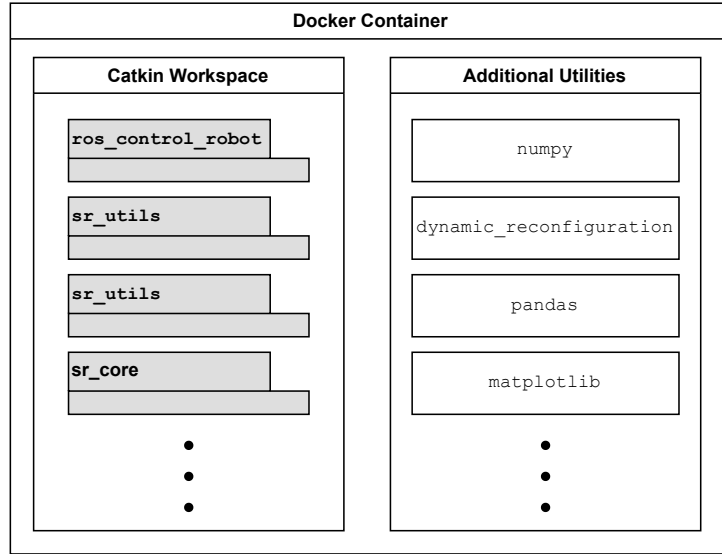


Fig. 2.3: The boxes marked with grey are ROS packages, while the white are python modules.

When communicating with the simulated Shadow Dexterous hand, the primary interface provided is the hand commander i.e. `SrHandCommander` which take provides functionalities such as retrieving the current state of the hand, executing given plans etc. Once a plan \mathbf{Q} is computed as a list joint states $\mathbf{q}_i = [q_0, q_1, \dots, q_n]$ where $i \in \{0, 1, \dots, m\}$ with m being the number of states in the plan \mathbf{Q} , `SrHandCommander` handles the parsing to MoveIt.

`SrHandCommander` which enables the definition of plans to execute and values to be read. Plans are executed using a PID controller [39], which follows an Rapidly-exploring Random Tree (RRT)-Connect sample-based planner by

default to generate paths. Planning is done in the configuration space.

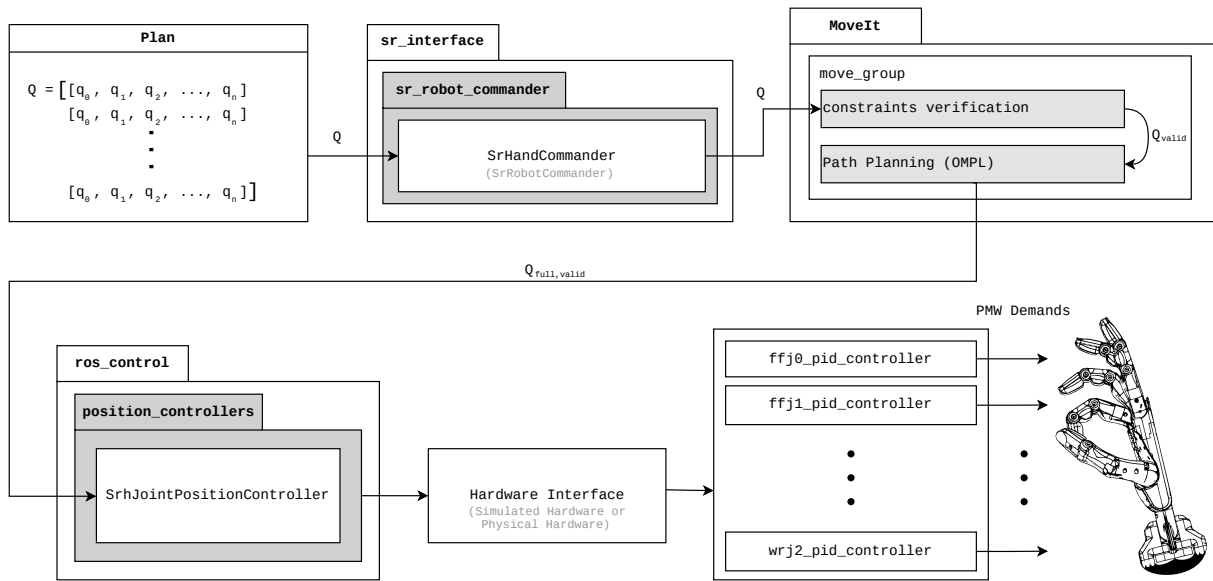


Fig. 2.4: The boxes marked with grey are ROS packages, while the white are python modules.

State of the Art

3.1 Problem 1 - Tactile Perception

Based on the contact model categories described in Chapter 4, the most representative was chosen to be soft finger (SF) models. Within the category of SF models, a method fit for this project's use case is to be chosen to solve problem 1. SF models can furthermore be divided up into three different categories: analytical elasticity-based models (AEBM), elastic foundation models (EFM), finite element models (FEM) [40] and machine learning (ML) models. The different categories can be seen organized in Fig. 3.1

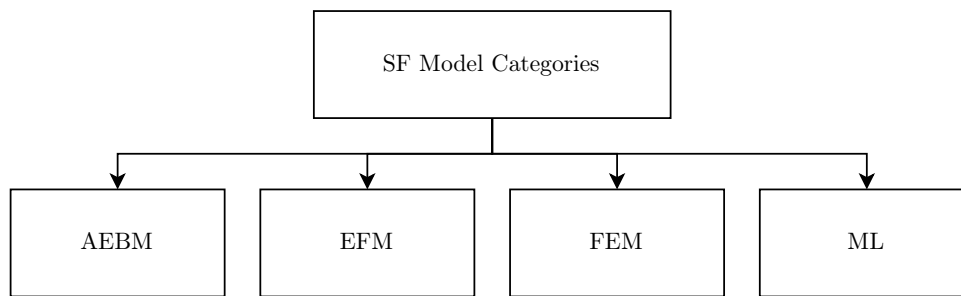


Fig. 3.1: Tree of methods for tactile perception.

AEBM are theoretical formulations of elastic contact areas and the stresses on both the surfaces and the sub-surfaces of the contacting bodies. The first of such models was introduced by Heinrich Rudolf Hertz in 1882 [41] and is still used for simple contact cases. In the formulation of the Hertzian contact model, two assumptions are made: Objects in contact are made of linear elastic materials and only small contact deformations occur compared to the dimension of an object. However, robotic EE fingertips are often made of non-linear elastic materials and for that reason, the Hertzian contact model does not represent the type of contact in this project [42, Chapter 37]. To improve on the Hertzian model, a more general formulation can be made which extends the model from linear to nonlinear elastic contacts [43, 44]. This power-law formulation subsumes the Hertzian contact theory while assuming a circular contact area. Other models have been purposed that combine the descriptions of both friction-contact and the shear-torsion as experienced by the bodies [45].

However, to more accurately describe the contacts involving robot fingers, viscoelastic soft contact model appear more relevant due to such fingers often being made of materials that show viscoelastic properties e.g., rubber, silicone and polymers. Simple models such as Kelvin-Voigt's [46] and Maxwell's [47] models describe the interaction between strain and stress as a spring-damper system in a serial or in a parallel configuration respectively. Models which expand on this idea describe the reacting force as the product of the temporal and the elastic response while incorporating previous stress responses [48]. To simplify this formulation alternatives have been developed to assume no past stress [49, 50, 51]. Upon these, more modern techniques have been developed which have seen use in similar use cases as the ones of interest in this project. One method attempts to expand the description of contacts between rigid indentors and elastic half-spaces, using the Matrix Inversion Method (MIM) as introduced by Kalker [52], to viscoelastic half-spaces as well. Assuming the surfaces are frictionless, the relationship is described in terms of the pressure distribution, the resultant force on the indenter and the penetration [53]. Attempts involving solutions to Boussinesq's problem for polynomial pressures acting over polygonal domains [54]

have also been developed and modernized by combining it with Cerruti's solution [55]. However due to numerical singularities being present, modifications are made to threshold the model. For a more complete description without singularities, Love's formulation has been added leading to a more accurate analytical representation but with the cost of an increased computational complexity [56]. For these Boussinesq-based approaches to be representative two assumptions are made 1) There exists a linear relationship between stress and strain, referred to as deformation, and 2) strains are infinitesimal [57, Chapter 6].

EFM are methods developed to build upon AEBM by allowing a simple discrete contact calculation in more general surface geometries. Here the deformable part of the contact is modeled as a layer over a rigid base with a series of discrete and independent springs in the contact normal. A widely used example of this method is Winkler's elastic foundation model [58], which has been used in structural engineering for modeling different properties of beams such as stability [59], vibrations and buckling [60]. Other EFM methods have shown accurate modeling performance when applied within the field of medical engineering. Here a comparative study between AEBM, EFM and FEM demonstrate the suggested modified EFM performs better than the alternatives in 3D knee models when predicting prosthetic knee performance [40]. A different method attempts to attain vivo contact pressure predictions for improved knee replacement designs [61]. Within the field of robotics EFM have provided solutions to problems such as slip [45], compliance, sliding [62, 63], stiffness and contact mechanics [64] of anthropomorphic grippers. One such method derives friction constraints based on general expressions for non-planar contacts of elastic bodies, where the local geometry and structure of the objects in contact are taken into account. Using these, a linear complementary problem is formulated and solved, resulting in the normal and frictional forces applied at each contact, as well as the relative velocity of the bodies involved [65].

FEM are popular general tools for solving PDE [66] and have seen contact applications in a wide range of engineering disciplines due to the assumptions made in AEBM and EFM not being applicable in these cases. A great number of these cases exist within the manufacturing industry [67] whereas one example is the metal forming processes. Specifically, the estimation of wheel-rail profiles [68] has been addressed using FEM due to the estimation of contacts over a greater surface is needed than what is assumed in AEBM and EFM. Other applications such as quality control through sliding wear estimation [69], analysis of the responses of fully coupled thermo-elasto-plastic solids in contact [70] and performing diagnostics of failures in induction motors [71]. Due to the complexity of modeling the contacts within robotics, FEM have become a popular choice and enabled tactile applications such as cobots tactile skin for ensuring collaborative behavior when in contact [72], performance estimation of new tactile sensor technologies [73] and evaluating complex contact types by extending simulations and analysis systems [74]. The modeling complexity has furthermore inspired using FEM as ground truth results when synthesizing ML data in simulations for deep learning models, which has enabled execution speeds 75 times greater than simply evaluating FEM [75, 76, 77].

The use of these ML models has enabled realistic simulations of tactile sensor data. Current literature applies DL-based approaches to simulate tactile sensor data for various tasks [78, 79]. For instance, simulating realistic tactile images from simulated contact depth to bridging the reality gap for vision-based tactile sensing using a diffusion model [80]. Similarly, a conditional Generative Adversarial Network (cGAN) has been used to simulate realistic tactile sensory data for use in tactile tasks [80]. Solutions using simple Multi Layered Perceptron (MLP) have been applied to enable real-time simulated realistic tactile data [81].

Given the methods presented above, the AEBM Boussinesq-Cerruti approach is considered along with the ML approach with a MLP model.

Although the Boussinesq-Cerruti approach can produce precise tactile data and can be tailored to suit a particular case, it faces certain challenges. The model relies on certain assumptions regarding the materials in contact, including linear deformation and infinitesimal strains. Furthermore, evaluating the model requires complex calculations, such as multidimensional integrals, which significantly increase computation time and hinder real-time

performance. In contrast to the transparency offered by the Boussinesq-Cerruti approach, the MLP approach is limited by the black-box nature of DL models. Despite this drawback, MLPs offer several benefits, such as low execution time and high adaptability to complex systems. Due to the high adaptability and option for real-time performance, the MLP model presented in [81] is chosen to solve the tactile perception problem i.e. problem 1.

3.2 Problem 2 - Pose Estimation

PE, which involves determining the position and orientation of an object in 3D space, has been the subject of many research studies. The literature has identified two main categories of methods for solving this problem: those based on DL, and those based on point cloud registration.

These can along with their subcategories be seen in Fig. 3.2 as inspired by [82].

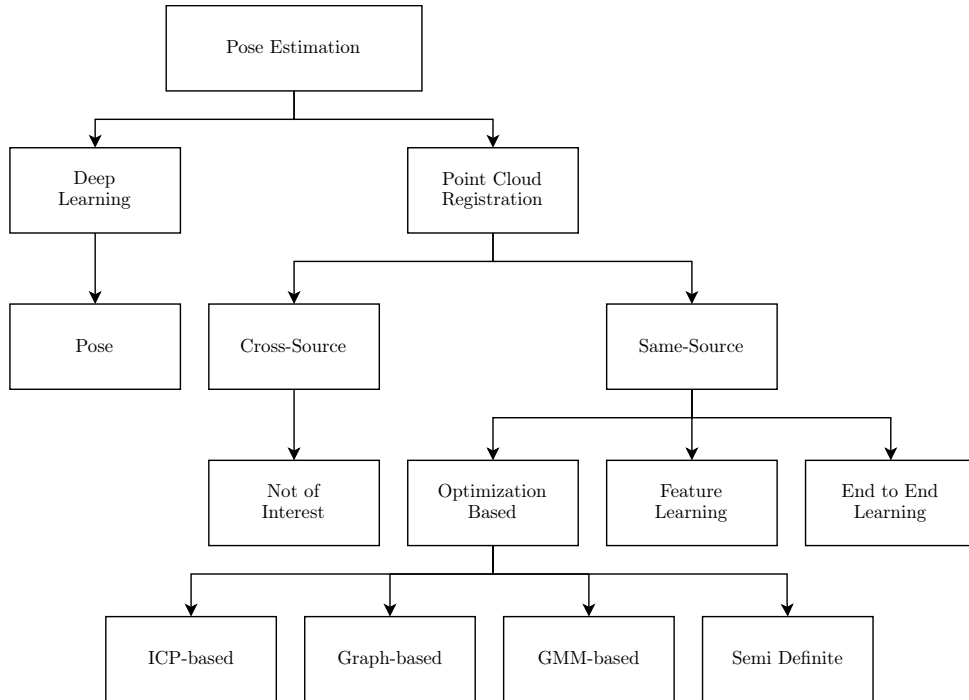


Fig. 3.2: Tree of methods for solving the point cloud registration problem. The categorization is inspired by [82].

Purely DL based methods learn feature representations of input data, often in the form of images, and use them to estimate the subject's pose. This is commonly done in the context of human pose estimation [83, 84, 85]. While these methods have shown extensive use in these cases, their applicability in this project is limited and thus excluded from consideration.

The other method group i.e. point cloud registration (PCR) methods are separated into two subgroups: cross-source and same-source point cloud (PC)s. Here cross-source refers to a PC produced by combining information from sensors of different kinds e.g. visual- and tactile sensors, while same-source methods only produce PCs based on information from the same kind of sensors e.g. only tactile sensors. While cross-source approaches have shown utility in an extensive range of applications [86, 87, 82] their applicability in this project is minimal, as purely tactile pose estimation is the problem of interest as presented in Chapter 1.

PCR methods from the same source data can be categorized into three sub-categories: end-to-end learning, feature-learning, and optimization-based methods. End-to-end learning-based methods use a neural network to estimate the transformation matrix that aligns two point clouds. Proposed solutions include using neural networks for scene

completion to estimate the relative pose between RGB-D scans [88], learning registration patterns as parametric functions through a scan completion module and pairwise matching module [89], and a fast feature-metric point cloud registration framework to minimize the feature-metric projection error without correspondences [90].

In contrast, feature-learning methods use deep neural networks to learn robust feature correspondence searches, which are then used in estimation algorithms such as Random Sample Consensus (RANSAC). In the literature, models have been developed to extract local geometric descriptors from RGB-D reconstructions [91], to learn globally informed 3D local feature descriptors [92], and to use siamese deep learning architectures with convolutional layers through a voxelized smoothed density value (SDV) representation [93].

Lastly, registration methods based on optimization are employed to estimate the transformation matrix through two stages: correspondence searching and transformation estimation. Their goal is to minimize a cost function that gauges the dissimilarity between two point clouds. Within this category, there are four sub-categories identified: Iterative Closest Point (ICP)-based, graph-based, Gaussian Mixture Model (GMM)-based, and semi-definite programming-based methods.

Since the original proposal in 1992 [94] using point-to-point correspondences, ICP-based methods have evolved and incorporated different types of correspondences to improve performance. Examples include point-to-plane [95] and plane-to-plane [96]. Modern approaches also employ complementary methods such as point cloud filtering, adaptive fireworks algorithms, and KD-Trees [97].

The main idea of graph-based registration methods is to use a non-parametric model [98]. In this method, correspondences between two graphs are found by considering both the vertices and edges, making it an optimization problem [98]. To solve this optimization problem, there are two categories of graph-matching methods based on the objective functions' constraints: second-order and high-order methods [99]. Second-order methods include Cross-Source Graph Matching (CSGM) [86], which uses a linear program to solve the graph-matching problem and apply it to solve the cross-source point cloud registration task, Factorized Graph Matching (FGM) [100] factorizes the large pairwise affinity matrix into smaller matrices and solves the graph-matching problem with a simple path-following optimization algorithm. Spectral graph [101] uses a spectral relaxation method to approximate the Quadratic Assignment Problem (QAP), and Semi-Definite Programming (SDP) relaxation is used to relax the non-convex constraint using a convex semi-definite. While higher-order graph matching provide methods for [102] design a probabilistic approach to solve the high-order graph-matching problem, while [103] design a triangle similarity and convert the graph-matching problem into a tensor optimization problem. More recent work, such as [104] suggests an elastic net to control the trade-off between the sparsity and accuracy of the matching results by incorporating the Elastic-Net constraint into the tensor-based graph matching mode.

GMM-based methods commonly tackle the point cloud registration problem by transforming it into a likelihood maximization problem for the input data. This has resulted in the development of several optimization strategies aimed at maximizing the likelihood and optimizing the transformation matrix. For instance, a motion drift idea was introduced into the GMM framework by [105] in the form of Coherent Point Drift (CPD) which imposes constraints on transformation estimation. In another approach, [106] combines GMM with the convex hull to reduce computation complexity. Furthermore, Joint Registration of Multiple Point Sets (JRMPC) [107] cast registration as a clustering problem where the transformation is optimized by solving the GMM. Recently, Deep Gaussian Mixture Registration (DeepGMR) [108] employed DL to learn the correspondences between GMM components and points, enabling the estimation of both the transformation and GMM parameters in a single forward step.

Lastly, within semidefinite programming different optimization groups exist, such as Second Order Cone Programming (SOCP), Quadratic Programming (QP) and Quadratically Constrained Quadratic Programming (QCQP). Due to the subject of interest being a rotation in $SO(3)$, the constraints of the rotation matrix, i.e. quadratic constraints, must be respected. Because of this, the methods of QCQP are of interest. One such example provides estimates

which are insensitive to a large fraction of spurious correspondences through decoupling the scale, rotation, and translation estimation. This decoupling enables the solving of these in cascade for the three transformations. The method is referred to as TEASER (Truncated least squares Estimation And SEmidefinite Relaxation), which solve large SDP relaxations, and additionally comes with a second fast and certifiable algorithm, named TEASER++. To decrease execution time this method uses Graduated Non-Convexity (GNC) to solve the rotation subproblem and applies Douglas-Rachford Splitting to enable efficiently certify global optimality [109]. Secondly, Invariant-based Highly Robust Point Cloud Registration (IRON) applies a similar methodology as to TEASER, but instead applies RANSIC (RANDOM Samples with Invariant Compatibility) to robustly estimates the scale between two sets of point clouds [110]. Finally, Relaxed Convex Quadratic Programming (RCQP) formulates a QCQP problem with a full set of quadratic rotational constraints and obtains a Lagrangian dual relaxation, which empirically recovered a globally optimal solution in 100 % of the tested cases, although why strong relaxation seems to hold has yet to be shown [111].

Among the categories presented above, the ones of particular interest are optimization-based techniques due to their mature mathematical foundation and possible certifiable optimality and outlier rejection capabilities, and DL-based techniques due to their adaptability and low execution time. While DL-based methods can learn feature representations of point clouds and estimate the transformation between two point clouds, optimization-based approaches with outlier rejection can effectively handle noise and outliers in the data, which is a common problem within the PCR problem. Due to this, the chosen method is the optimization based RCQP method with GNC outlier rejection [112].

3.3 Problem 3 - In-Hand Manipulation

References:

resources:

Chapter 4

Modeling

To model the contact between the EE's tactile sensors, eight different categories exist as identified in [113]. The three most common ones within the field of robotics [42, Chapter 37] are point-contact-without-friction (PwoF), hard finger (HF) and the SF model as shown in Fig. 4.1.

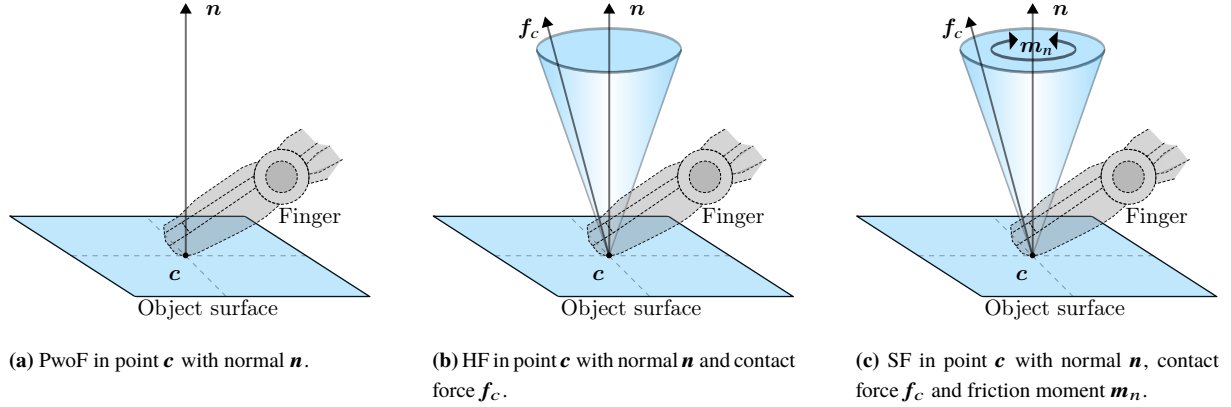


Fig. 4.1: The three most commonly used contact models.

The PwoF model, as shown in Fig. 4.1(a), can only represent forces along the surface normal $n \in \mathbb{R}^3$ at the point of contact $c \in \mathbb{R}^3$ and thus the model does not support surface deformations between the two contacting objects. This model is applied in cases where very little deformation is present, along with the contact having a friction coefficient approximately equal to zero [42, Chapter 38].

The HF model, as shown in Fig. 4.1(b), is representative when the friction between objects is significant, while the contact deformation is small enough to ignore friction moments and deformations [42, Chapter 38]. To model the friction acting on the contact point a great number of methods exist, a common one being the Coulomb friction with different modifications depending on the use case [114]. This model states that the frictional force acting on an object can be formulated as

$$f_f = f_N \mu, \quad (4.1)$$

with f_f being the magnitude of the Coulomb friction, f_N being the magnitude of normal force in the point of contact and $\mu \in [0, 1]$ being the friction coefficient. One visualization of this linear relationship can be seen in the cones illustrated in Fig. 4.1(b) and Fig. 4.1(c). These cones are referred to as friction cones, which for a hard finger model can be formulated as

$$C_{f, \text{HF}} = \{ f_c \mid f_t \leq \mu f_z, \mu f_z \geq 0 \} \quad , \quad f_t = \sqrt{f_x^2 + f_y^2}. \quad (4.2)$$

Here f_c is the magnitude of the contact force, f_t is the magnitude of the tangential force. f_x , f_y and f_z are the magnitudes of the x , y and z components of the contact force ($f_c \in \mathbb{R}^3$) and μ is the friction coefficient [42, Chapter 37]. By applying a contact force that ensures the friction stays greater than the magnitude of the tangential force, neither object slips i.e. f_z must ensure that $f_z \mu$ stays greater than f_t for the objects not to slip. Visually this is the case when the contact force f_c stays within the friction cone, which enables a friction-based grasp type referred to as force closure. Specifically, force closure refers to when the composite wrench cone contains the entire wrench space so that any external wrench w_{ext} on the body can be balanced by contact forces [115]. A force that commonly contributes significantly to the external wrench, and thus to the tangential force, is gravity.

The SF model, as shown in Fig. 4.1(c), is used to represent scenarios where both friction and surface deformations are significant. Due to deformations of the finger, an additional torsional moment about the contact normal will be present [42, Chapter 38]. While an analytical formulation of the SF relation depends on the pressure distribution inside the contact, and can only be derived for a limited number of special cases, the general case can be approximated using

$$\mathcal{E}_{f,\text{SF}} = \left\{ f_c \left| f_t^2 + \frac{m_n^2}{e_n^2} \leq \mu^2 P^2 \right. \right\}, \quad f_t = \sqrt{f_x^2 + f_y^2}. \quad (4.3)$$

This formulation forms a contact ellipsoid $\mathcal{E}_{f,\text{SF}}$ which describes the relationship between the tangential force $\mathbf{f}_t \in \mathbb{R}^3$ and friction moment $\mathbf{m}_n \in \mathbb{R}^3$. The friction parameters in this expression remain the same as for the friction cone, with the additional m_n being the magnitude of the frictional moment, e_n being the eccentricity parameter i.e. the height of the aforementioned ellipsoid and P being the magnitude of the pressure applied from the contact point along the contact normal \mathbf{n} [63, 65].

Based on the model categories described above, the most representative for this project's case, are the SF models since these can provide information about the contact surface's shape, thus enabling the reconstruction of the contact shape from the application of a force distribution [116] i.e. the Inverse Elasticity Problem (IEP). Additionally, these models support descriptions of friction which is crucial to manipulate objects in hand. Illustrations of the system as a SF with friction cone, pressure distribution and the enabling of force closure can be seen in Fig. 4.2 and Fig. 4.3 respectively.

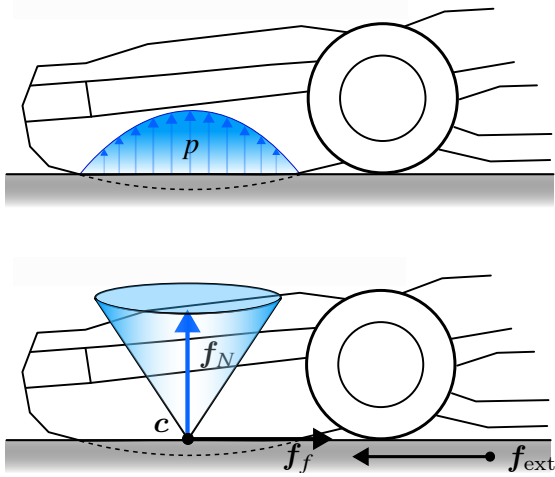


Fig. 4.2: The pressure distribution p and friction cone of a SF model experiencing an external force \mathbf{f}_{ext} .

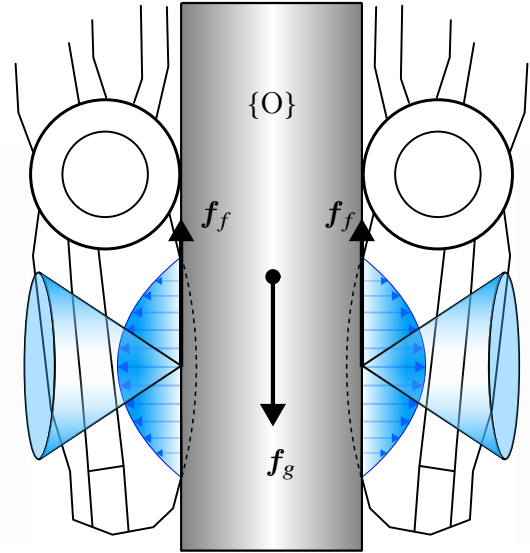


Fig. 4.3: the pressure distribution and friction cone causing force closure to prevent the object $\{O\}$ from falling due to the gravitational force \mathbf{f}_g .

When modeling the kinematics of an anthropomorphic gripper with frame $\{H\} \in \mathbb{R}^{4 \times 4}$ in world frame $\{W\} \in \mathbb{R}^{4 \times 4}$ interacting with an object with frame $\{O\} \in \mathbb{R}^{4 \times 4}$ the relevant parameters must be addressed. In this system the object with position $\mathbf{p} \in \mathbb{R}^3$ and pose $\mathbf{u} \in \mathbb{R}^6$, with the orientation either being represented as a four-dimensional quaternion or a three-dimensional Euler angle, makes contact with the gripper in points $\mathbf{c}_i \in \mathbb{R}^3$. These contact points have frames $\{C\}_i \in \mathbb{R}^{4 \times 4}$ with axes $\{\mathbf{n}_i, \mathbf{t}_i, \mathbf{o}_i\} \subset \mathbb{R}^3$, where $\mathbf{n}_i \in \mathbb{R}^3$ points perpendicular to the contact plane towards the object, while the remaining are contained within the contact plane. For each of these parameters $i = 1, 2, \dots, n_c$, where n_c is the number of contact points. The twist of $\{O\}$ described in $\{W\}$ is denoted $\mathbf{v} = [\mathbf{v}^\top \boldsymbol{\omega}^\top]^\top \in \mathbb{R}^6$ while the non-contact wrench i.e. the wrench caused by external forces such as collisions with the environment and gravity, is $\mathbf{w} = [\mathbf{f}^\top \mathbf{m}^\top]^\top \in \mathbb{R}^6$. The gripper's state is described in terms of its

joints, of which it has n_q , named $\mathbf{q} = [q_1 \ q_2 \ \dots \ q_{n_q}]^\top \in \mathbb{R}^{n_q}$ each of which is revolute and can exert a torque $\boldsymbol{\tau} = [\tau_1 \ \tau_2 \ \dots \ \tau_{n_q}]^\top \in \mathbb{R}^{n_q}$. These parameters can be seen illustrated in Fig. 4.4 showing the system model. While only a single finger here is illustrated the naming conventions and representations simply scale to all the

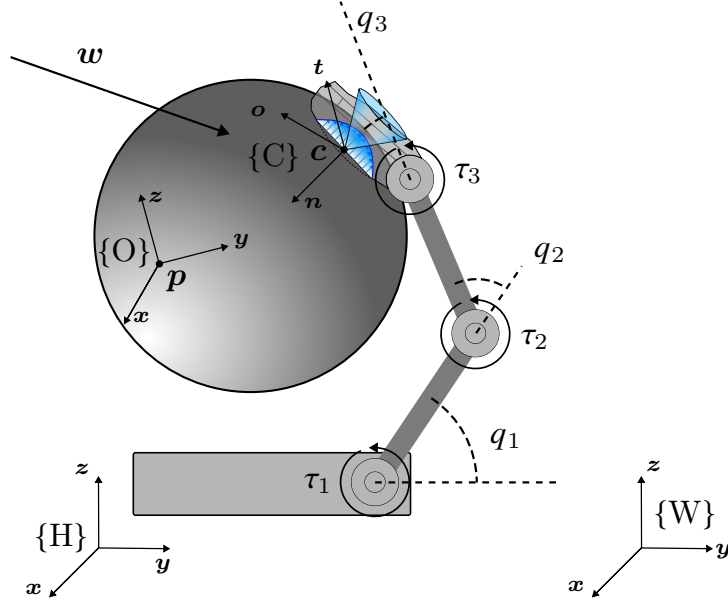


Fig. 4.4: The model of the world representation for this project.

EE's DOFs.

In this system, the twists and wrenches of a contact point c_i on the object and hand, given in contact frame $\{C\}_i$ is referred to as $\mathbf{v}_{i,\xi} \in \mathbb{R}^6$ and $\mathbf{w}_{i,\xi} \in \mathbb{R}^6$, with $\xi = \{\text{obj}, \text{hnd}\}$. Given multiple contact points, complete vectors of twist and wrench can be expressed by appending each contact point's twist and wrench vector. These contain all twists and wrenches of the grasp, one for the object and one for the hand. These vectors are referred to as

$$\mathbf{v}_{c,\xi} = [\mathbf{v}_{1,\xi}^\top \ \mathbf{v}_{2,\xi}^\top \ \dots \ \mathbf{v}_{n_c,\xi}^\top]^\top \in \mathbb{R}^{6 \cdot n_c} \quad (4.4)$$

and

$$\mathbf{w}_{c,\xi} = [\mathbf{w}_{1,\xi}^\top \ \mathbf{w}_{2,\xi}^\top \ \dots \ \mathbf{w}_{n_c,\xi}^\top]^\top \in \mathbb{R}^{6 \cdot n_c} \quad (4.5)$$

respectively.

These definitions are used to describe and analyze the kinematics of grasping and the parameters involved in holding and manipulating objects in hand, also referred to as grasp kinematics (GK). Within GK two matrices are of special interest: the grasping matrix \mathbf{G} and the hand Jacobian \mathbf{J} . The grasping matrix describes the transformation between the twist or wrench of the object in world frame $\{W\}$ to the twists or wrenches of the object in contact frames $\{C\}_i$. The grasp matrix thus can be expressed as

$$\mathbf{G} = [\mathbf{G}_1 \ \mathbf{G}_2 \ \mathbf{G}_3 \ \dots \ \mathbf{G}_{n_c}], \quad (4.6)$$

where $\mathbf{G}_i \in \mathbb{R}^{6 \times 6}$ describes the transformation from $\{W\}$ to the individual $\{C\}_i$, and thus $\mathbf{G} \in \mathbb{R}^{6 \times 6 \cdot n_c}$ describes the transformations for all contact points. Using this grasp matrix, the object wrench and twist can be computed in all contact frames as

$$\mathbf{v}_{c,\text{obj}} = \mathbf{G}^\top \mathbf{v} \quad \text{and} \quad \mathbf{w}_{c,\text{obj}} = \mathbf{G}^\top \mathbf{w}. \quad (4.7)$$

While the grasp matrix describes the transformation from $\{W\}$ to object contact frames, the hand Jacobian relates the joint velocities and torques to the contact twists and wrenches on the hand. The hand Jacobian can thus be

expressed as

$$\mathbf{J} = [\mathbf{J}_1^\top \mathbf{J}_2^\top \mathbf{J}_3^\top \cdots \mathbf{J}_{n_c}^\top]^\top, \quad (4.8)$$

for all contact points. Here $\mathbf{J}_i \in \mathbb{R}^{6 \times n_q}$ for $i = 1, 2, \dots, n_c$ are the individual contact point's hand Jacobians and thus $\mathbf{J} \in \mathbb{R}^{6 \cdot n_c \times n_q}$ is the complete. Using the complete hand Jacobian, the contact twists and wrenches on the hand can be related to the joint velocities and torques as

$$\mathbf{v}_{c,\text{hnd}} = \mathbf{J}\dot{\mathbf{q}} \quad \text{and} \quad \boldsymbol{\tau} = \mathbf{J}^\top \mathbf{w}_{c,\text{hnd}}. \quad (4.9)$$

The modeling described above will enable the use of methods for solving the presented problems. These methods will be determined in Chapter 3.

Chapter 5

Tactile Perception

5.1 Introduction

- start with performance specifications regarding the two problems the tactile perception should be able to solve: can the are the points representative, according to the source yes, but are the normals also representative. This is important for the pose estimation problem. When manipulating the object the finger needs to be able to exert enough friction to be able to hold the object, is this true. compute the contact friction and see if it is stronger than the gravity on the subject. Use numbers to verify if the performance is acceptable
-

5.2 Method

5.3 Experimental Setup

5.4 Results

5.5 Discussion & Conclusion

Chapter 6

Pose Estimation

6.1 Introduction

Here we write the introduction for problem 2.

6.2 Related Work

Chapter 7

In-Hand Manipulation

7.1 Introduction

Here we write the introduction for problem 3.

7.2 Related Work

For history see hand book of robotics chapter 38, the first section.

Chapter 8

System Integration

8.1 Introduction

Here we write the introduction for the system integration.

Chapter 9

Discussion

Chapter 10

Conclusion

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Appendix A

Shadow Dexterous Hand - Technical Specifications

Table A.1 shows the ROM for the Shadow Dexterous Hand and Table A.2 shown the ROM for a human hand. The shorthand abbreviations used in these tables can be seen listed in Table A.3. The joints are numbered from fingertip to base and thus FF1 refers to the first joint after the fingertip on the first finger i.e. the index finger.

ROM - Shadow Dexterous Hand					
Joint(s)	Min deg	Max deg	Min rad	Max rad	Notes
FF1, MF1, RF1, LF1	0	90	0	1.571	Coupled
FF2, MF2, RF2, LF2	0	90	0	1.571	
FF3, MF3, RF3, LF3	-15	90	-0.262	1.571	
FF4, MF4, RF4, LF4	-20	20	-0.349	0.349	
LF5	0	45	0	0.785	
TH1	-15	90	-0.262	1.571	
TH2	-40	40	-0.698	0.698	
TH3	-12	12	-0.209	0.209	
TH4	0	70	0	1.222	
TH5	-60	60	-1.047	1.047	
WR1	-40	28	-0.698	0.489	
WR2	-28	10	-0.489	0.174	

Table A.1: The ranges of motion for each joint in the Shadow Dexterous Hand [117].

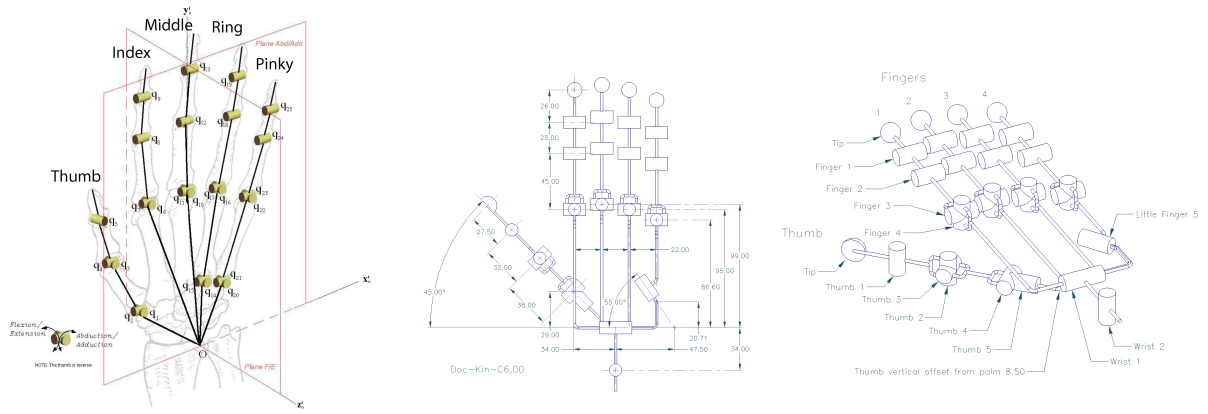
ROM - Human Hand					
Joint(s)	Min deg	Max deg	Min rad	Max rad	Latin Name
TH1	-15	80			Interphalangeal (IP)
TH2 + TH3	-10	55			Metacarpophalangeal (MCP)
TH4 +TH5	-10	55			Carpometacarpal (CMC)
FF1, MF1, RF1, LF1	0	80			Distal interphalangeal (DIP)
FF2, MF2, RF2, LF2	0	100			Proximal interphalangeal (PIP)
FF3, MF3, RF3, LF3	-45	90			Metacarpophalangeal (MCP)
WR1	-80	80			Radiocarpal
WR2	-28	20			Radiocarpal

Table A.2: The theoretical ROM for each finger joint in human hand [118] and found ROM for the wrist joints [119].

To compare the kinematic structure of the Shadow Dexterous Hand and a human hand, seen Fig. A.1.

Joint Name Abbreviation	
Abbreviation	Full Name
FF	First Finger
MF	Middle Finger
RF	Ring Finger
LF	Little Finger

Table A.3: The abbreviations used to reference [120].



(a) The kinematic tree of a human hand according to [121].

(b) The kinematic tree of the Shadow Dexterous Hand according to [122].

Fig. A.1: The kinematic trees of a human hand and the Shadow Dexterous hand.