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Some Placeholder Title

A Master Thesis

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The code for this project is available at
`https://github.com/vmstavens/in_hand_pose_estimation`

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Abstract

Some abstract text explaining the goal, methods and conclusion of the project.

Contents

Acknowledgements

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Acronyms

AEBM analytical elasticity-based models.

cobots collaborative robots.

CP correspondence problem.

CV computer vision.

DL deep learning.

DOF degrees of freedom.

EE end effector.

EFM elastic foundation models.

FEM finite element models.

HF hard finger.

IEP inverse elasticity problem.

MIM Matrix Inversion Method.

PE pose estimation.

PNP pick-and-place.

PwoF point-contact-without-friction.

SF soft finger.

SOTA state of the art.

Terms

collaborative robots (cobots) are robots which facilitate human-robot collaboration[1].

computer vision (CV) is a field of artificial intelligence (AI) that enables computers and systems to derive meaningful information from digital images, videos and other visual inputs - and take actions or make recommendations based on that information[2].

correspondence problem (CP) is the problem where one aims at finding correspondences between the pixels in two (or more) images[3].

deep learning (DL) are methods which allow computational models that are composed of multiple processing layers to learn representations of data with multiple levels of abstraction[4].

end effector (EE) is a generic term for all functional units involved in direct interaction of the robot system with the environment or with a given object[5].

manipulator : A serial robot mechanisms. The robot manipulator is represented by a serial chain of rigid bodies, called robot segments, connected by joints[6].

pose estimation (PE) A particular instance of feature-based alignment, which occurs very often, is estimating an object's 3D pose from a set of 2D point projections. This pose estimation problem is also known as extrinsic calibration[7].

Chapter 1

Introduction

1.1 Context

As of 2022 most of the industrialized world has developed tools for unprecedented growth in wealth and technology on a global scale [8, Chapter 4]. In such times a great deal of consumerism and interconnection is present with people needing products produced faster and more consistently than ever before [8, Chapter 4]. As one would expect, this creates a high demand for manufacturers to reliably and consistently provide products, while also remaining flexible as the demand for different product change rapidly. In order to accommodate the need for ever greater volumes of products, consistent, reliable and flexible labor is essential in assembly, transport and manipulation processes in the production pipeline. Due to these types of manual labor being largely done by unskilled workers, automation alternatives are being adopted which provides benefits [8, Chapter 4]. This different approach to manufacturing has been labeled the fourth industrial revolution or i4.0 for short. The beneficiaries being the employer and employee, with the employer having the benefits: Avoid paying monthly salaries to unskilled laborers doing manual tasks, here the automation alternative only requires electrical energy and potential supervision by few qualified individuals. Potential risks are also involved when hiring humans as the workforce can be inconsistent due to human error [9] or left out due to illness etc. Considerations with regards to workers rights such as working conditions and wage also needs not to be considered. Workers furthermore cause production limitations in the form of stand still hours, such as bathroom and lunch breaks along with after work hours and holidays. This replacement of manual labor also potentially benefits the employee, as boring and physically wearing work is automated, enabling the employees to take on different and less wearing and potentially dangerous roles. While the issue of labor unemployment becomes apparent solutions which provide support to already hired workers have been developed, such as collaborative robots (cobots)[10] which would negate this problem.

When implementing automation of production lines using robotics, certain categories of problems are revealed. These include: Assembly, alteration and pick-and-place (PNP), the last being the one of interest in this project.

1.2 Problem Description

Pick-and-place manipulators are used in a wide variety of different fields such as sorting of waste [11] handling of food [12][13] and factory bin picking [14] [15] [16]. The solutions in these industries are examples of subcategories under the PNP problem, namely sorting and bin picking. Since both of these are subcategories of the PNP problem, they fundamentally follow the same sequential four phases from start to end. These phases being pre-grasping, grasping, transport, and placement [15] for traditional implementations of the PNP pipeline. The pre-grasp phase involves localizing the object(s), potentially estimating their pose and executing the trajectory to move the end effector (EE) grasp, collision free to said object(s). Here different potential grasps can be considered in order to determine the best pose for the EE. In the grasping phase the EE gasps the object in such a manner that the object's entire weight is supported by the EE, and ends when the object no longer is in contact with the environment, which often is the container holding the object. The transportation phase involves the motion of the manipulator to move from the pose achieved after the grasping phase, to a pose ready for placement of the object in the desired placing area or fixture. Here considerations may be needed with regards to how much force and torque the EE's grasp can tolerate while moving without losing the object. Finally the goal of the placing phase is to place the object within

the placing area or fixture in a desired end pose. Here the constraints on the end pose might differ significantly based on the application, as the pose of greens in a crate might need less precision than if the manipulator hands a bolt to the another robotics system in the pipeline.

While these phases make up a traditional PNP systems, certain assumptions are made regarding the objects of interest in order for this pipeline to function. Specifically the localization and pose estimation (PE) of the pre-grasp phase are assumed possible due to either ensured object poses or estimated poses through computer vision (CV) sensory system. Due to CV being a mature research field a wide range of solution proposals to these problems have been generated [17]. These include classic vision [18] [19], deep learning based [20] and combinations of these [21]. However, while these may be sufficient for certain tasks they fundamentally suffer from the weaknesses introduced by vision techniques. These being a great number of outliers caused by: occlusions, reflecting, transparent or homogeneous surfaces, and repetitive structures when solving the correspondence problem (CP). Within factory settings the common ones are transparent and reflective objects, due to metallic, plastic and glass product often being the materials used. While deep learning (DL) solutions have been developed for both for reflective [22] and transparent [23] objects, these are use case specific and show limited results in a wider range of applications.

This project suggests a different PNP pipeline for cases where the object's starting pose is unknown. In this PNP pipeline the PE is moved from the pre-grasping phase to a new phase between the grasping and transportation phase, called the PE phase. The specific goal of this project is to develope a solution to this phase using tactile sensors in the EE to determine the object's pose. By using the tactile sensing rather than visual the problems presented above will be eliminated. This will be done using a humanoid gripper as the EE with tactile sensors in each finger, more specifically a Shadow Dexterous Hand [24] with 20 degrees of freedom (DOF).

In depth explanation of traditional vs. suggested manipulator pipeline
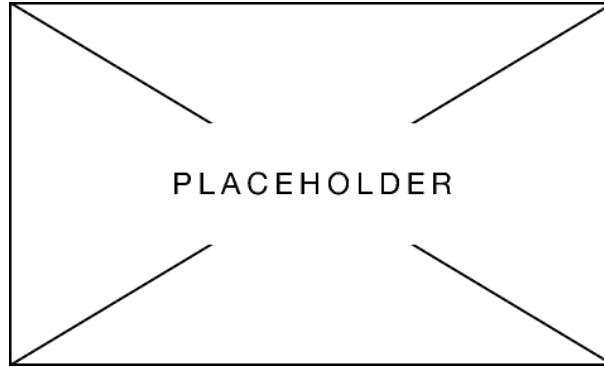

Fig. 1: Cartoon of suggested pnp vs. traditional pnp.

In order to solve this PE problem, three sub problems are identified and labeled problem 1, 2 and 3.

Problem 1 involves modeling the contact between the gripper's tactile sensors and the object, also referred to as tactile perception.

Problem 2 is to convert the collected data from problem 1 to estimated pose candidates.

Problem 3 involves in-hand manipulation. Since the initial grasp of the object might not be oriented in a manner where the recognizable features make context with the tactile sensors, manipulating the object within the EE's grasp will enable further information gathering. Thus the final problem is to control the EE in such a manner that the tactile sensors make context with the object in intelligently decided areas for a better pose estimate.

To test if the developed system successfully solves the PE problem, it is hypothesized that the intelligent probing method provides a statistically significant faster average PE convergence, along with a statistically significant greater

success rate when determining the correct pose. A correct pose is here defined as the pose being greater than or equal to 95% of the ground truth pose, and statistically significant is defined by an α-level of 95%. This hypothesis will be referred to as H_1, while the null hypothesis H_0 being that there is no statistically significant difference between intelligent and random probing's PE performance as described above.

1.3 Thesis Overview

To present the work done in this project, the system modeling is done in Chapter 2 and state of the art (SOTA) is presented in Chapter 3 for each of the problems presented above. Here the solutions best suited for this project's gripper are chosen. Each solution is described in detail, how they are applied, their performance tested and finally evaluated and conclude upon in their respective chapters i.e. chapter Chapter 4, Chapter 5 and Chapter 6. In Chapter 7 the three methods are combined in the final integration and finally the project is discussed and concluded upon in Chapter 8 and Chapter 9 respectively.

Chapter 2

Modeling

In order to model the contact between the EE's tactile sensors, eight different model categories are present[25] whereas three most common ones within the field of robotics[26, Chapter 37] are point-contact-without-friction (PwoF), hard finger (HF) and the soft finger (SF) model as shown in Fig. 1.
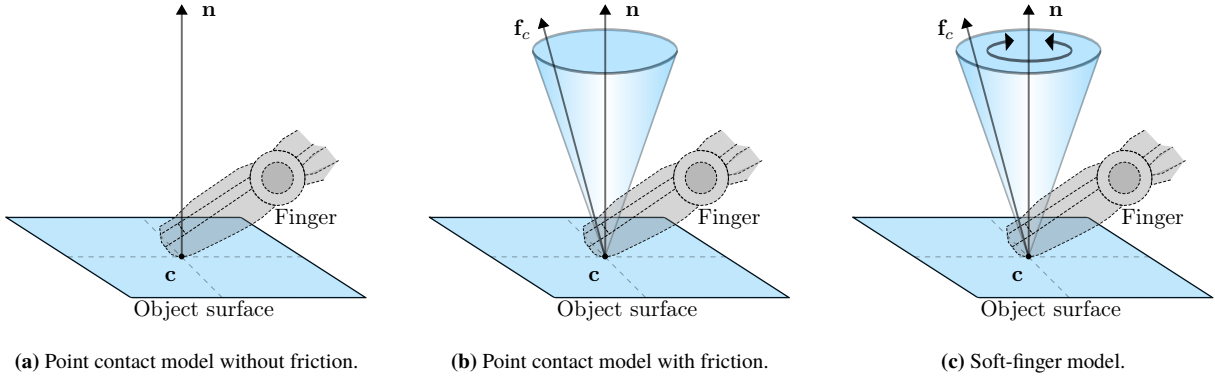

(a) Point contact model without friction.   **(b)** Point contact model with friction.   **(c)** Soft-finger model.

Fig. 1: The three most commonly used contact models.

The PwoF model, as shown in Fig. 1(a), can only represent forces along with the normal of the object's surface at the point of contact and thus the model does not support surface deformations between the two contacting objects. This model is applied in cases where very little deformation is present, along with the contact being slippery[26, Chapter 38].

The HF model, as shown in Fig. 1(b), is representative when the friction between objects is great enough to be significant, while the contact deformation is small enough to ignore friction moments and deformations[26, Chapter 38]. To model the friction acting on the contact point a great number of methods exist, a very common one being the Column friction with different modifications depending on the use case[27].

The SF model, as shown in Fig. 1(c), is used to represent scenarios where both friction and surface deformations are great enough to be impactful in the systems behavior. Due to deformations of the finger an additional torsional moment about the contact normal will be present[26, Chapter 38].

Based on the contact model categories described above, the most representative is SF since these models can provide descriptions of the contact surface topology, and thus enable the solving of the inverse elasticity problem (IEP) by deriving surface features for pose estimation. Furthermore in order to manipulate objects in-hand and ensuring force closure modeling friction is crucial, which is also provided by these models. An illustration of the system as a SF with friction cone and pressure distribution can be seen in Fig. 2, while Fig. 3 shows the model enabling force closure about the object $\{O\}$, where the external force is represented as gravity which is a common occurrence.
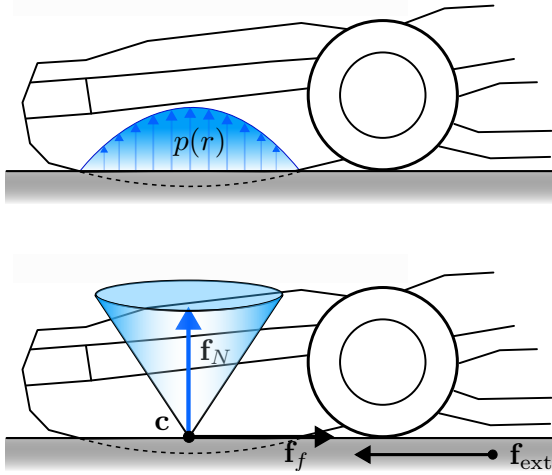
Fig. 2: Contact pressure distribution and friction cone for a SF model in the context of a robotic finger.
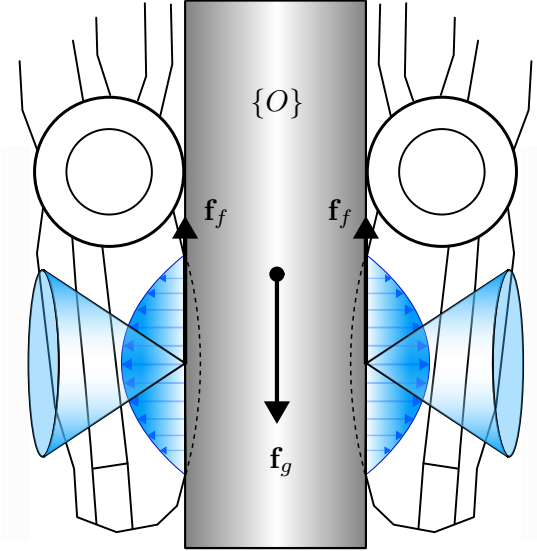

Fig. 3: Contact pressure distribution and friction cone causing force closure to prevent the object $\{O\}$ from falling due to gravity.

Within these figures $p(r)$ is the pressure distribution often as a function of the contact surface's radius r assuming a non-conforming contact case. $\mathbf{f}_N$ is the normal force of the object acting on the EE in the contact point $\mathbf{c}$, $\mathbf{f}_f$ is the friction force opposing the external force $\mathbf{f}_{\text{ext}}$ which often comes in the form of gravity $\mathbf{f}_g$ as see in Fig. 3.
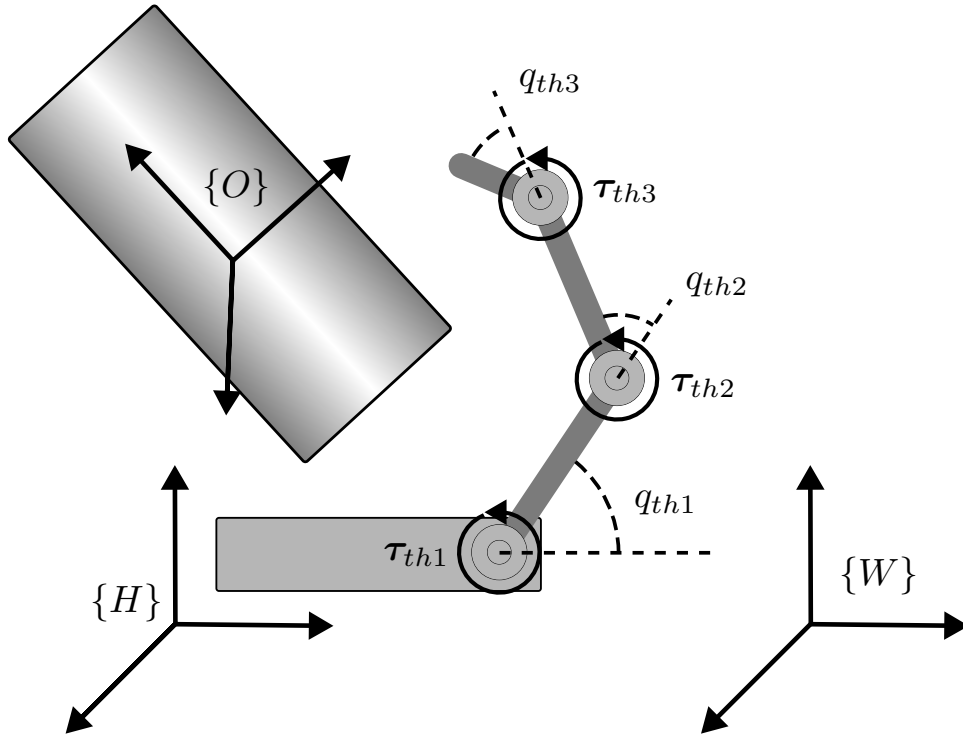

Fig. 4: The model of the world representation for this project.

With the contact model described, a segment of the kinematic tree of the EE can be seen in Fig. 4. Here q_{thi} is the angle of the i'th joint in the thumb which here is the only finger depicted, $\boldsymbol{\tau}_{thi}$ is the torque exerted by said joint, $\{W\}$ is the world frame, $\{O\}$ is the object's frame and $\{H\}$ is the robotic hand's frame. While only a single finger

here is illustrated the naming conventions and representations simply scale to all the EE's DOFs.

To determine which methods best describe the models presented above for this project, the SOTA will be presented in Chapter 3.

Chapter 3

State of the Art

3.1   Problem 1 - Tactile Perception

Based on the contact model categories described above, the most representative is SF since these models can provide descriptions of the contact surface topology, and thus enable the solving of the IEP by deriving surface features for pose estimation. Furthermore in order to manipulate objects in-hand and ensuring force closure the friction Representation is needed. An illustration of the system as a SF can be seen in ..., while .

Within the category of SF models a method fit for this project's use case is to be chosen to solve problem 1. SF models can furthermore be divided up into three different categories: analytical elasticity-based models (AEBM), elastic foundation models (EFM) and finite element models (FEM) [28].

AEBM are theoretical formulations of elastic contact areas and the stresses on both the surfaces and the sub-surfaces of the contacting bodies. These types of formulations however are often restricted to describing simple contact geometries. The first of such models was introduced by Heinrich Rudolf Hertz in 1882[29] and is still used for simple contact cases. In the formulation of the Hertzian contact model two assumptions are made: Objects in contact are made of linear elastic materials and only small contact deformations occur compared to the dimension of object. However, robotic EE fingertips are often made of nonlinear elastic materials and for that reason the Hertzian contact model does not represent the type of contact in this project[26, Chapter 37]. To improve on the Hertzian model, a more generic formulation can be made which extends the model from linear to nonlinear elastic contacts[30][31]. This power-law formulation subsumes the Hertzian contact theory while assuming a circular contact area. Other models have been purposed which combine the descriptions of both friction-contact and the shear-torsion as experienced by the bodies[32]. However, in order to more accurately describe the contacts involving robot fingers, viscoelastic soft contact model appear more relevant due to such fingers often being made of materials which show viscoelastic properties e.g., rubber, silicone and polymers. Simple models such as Kelvin-Voigt's[33] and Maxwell's[34] models describe the interaction between strain and stress as a spring damper system in a serial or in a parallel configuration respectively. Models which expand on this idea describe the reacting force as the product of the temporal and the elastic response, while incorporating previous stress responses[35]. To simplify this formulation alternatives have been developed to assume no past stress [36][37][38]. Upon these, more modern techniques have been developed which has seen use in similar use cases as the ones of interest in this project. One method derives friction constraints based on general expressions for non-planar contacts of elastic bodies, where the local geometry and structure of the objects in contact are taken into account. Using these a linear complementary problem is formulated and solved, resulting in the normal and frictional forces applied at each contact, as well as the relative velocity of the bodies involved. This method is applied on a similar humanoid gripper platform as the one in this project[39]. A different method attempts to expand the description of contacts between rigid indentors and elastic half-spaces, using the Matrix Inversion Method (MIM) as introduced by Kalker[40], to viscoelastic half-spaces as well. Assuming the surfaces are frictionless, the relationship is described in terms of the pressure distribution, the resultant force on the indenter and the penetration[41]. Attempts involving solutions to Boussinesq's problem for polynomial pressures acting over polygonal domains[42] have also been developed and modernized by combining it with Cerruti's solution[43]. However due to numerical singularities being present, modifications are made to threshold the model. For a more complete description without singularities, Love's formulation has been added leading to a more accurate analytical representation but with the cost of an increased computational complexity[44]. In order for these Boussinesq based approaches to be representative two assumptions are made 1)

There exists a linear relation between stress and strain, referred to as deformation, and 2) strains are infinitesimal[45, Chapter 6].

EFM are

3.2   Problem 2 - Pose Estimation

3.3   Problem 3 - In-Hand Manipulation

Chapter 4

Tactile Perception

4.1 Introduction

Here we write the introduction for problem 1.

4.2 Related Work

Here we cite the related work by \cite{source-label} like this [46]

Chapter 5

Pose Estimation

5.1 Introduction

Here we write the introduction for problem 2.

5.2 Related Work

Here we cite the related work by `\cite{source-label}` like this [46]

Chapter 6

In-Hand Manipulation

6.1 Introduction

Here we write the introduction for problem 3.

6.2 Related Work

Here we cite the related work by `\cite{source-label}` like this [46]

Chapter 7

System Integration

7.1 Introduction

Here we write the introduction for the system integration.

Chapter 8

Discussion

Chapter 9

Conclusion

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Appendix A

Appendix A Title