Smoothing

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Key ideas

- Sometimes there are non-linear trends in data
- We can use "smoothing" to try to capture these
- Still a risk of overfitting
- Often hard to interpret

CD4 Data

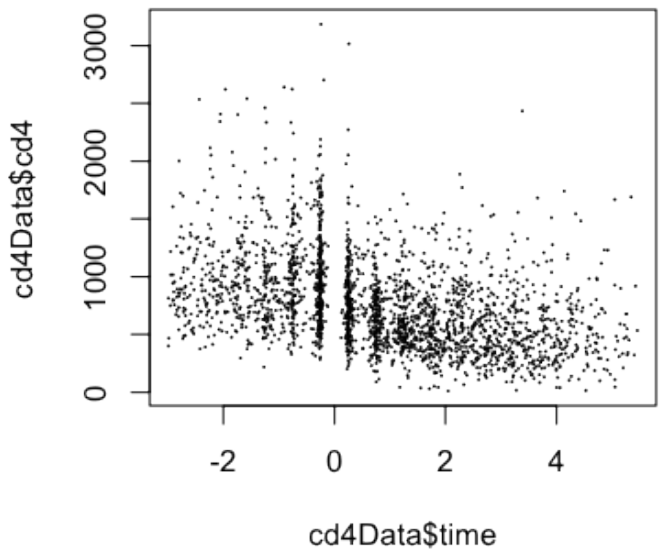
```
download.file("https://spark-public.s3.amazonaws.com/dataanalysis/cd4.data",
              destfile="./data/cd4.data",method="curl")
cd4Data <- read.table("./data/cd4.data",
                      col.names=c("time", "cd4", "age", "packs", "drugs", "sex",
                                  "cesd", "id"))
cd4Data <- cd4Data[order(cd4Data$time),]
head(cd4Data)
```

```
       time  cd4   age packs drugs sex cesd    id
1279 -2.990  814  6.17     3     1   5   -3 30183
2190 -2.990  400 -6.02     0     0   3   -4 41406
1167 -2.984  467 13.94     0     1   1    0 30046
1427 -2.957  749 -4.54     0     1  -1   -7 30498
2032 -2.951 1218  5.57     3     1   5    3 41032
1813 -2.949 1015 -9.15     2     1   0   -7 40375
```

http://www.cbcb.umd.edu/~hcorrada/PracticalML/

CD4 over time

```
plot(cd4Data$time,cd4Data$cd4,pch=19,cex=0.1)
```



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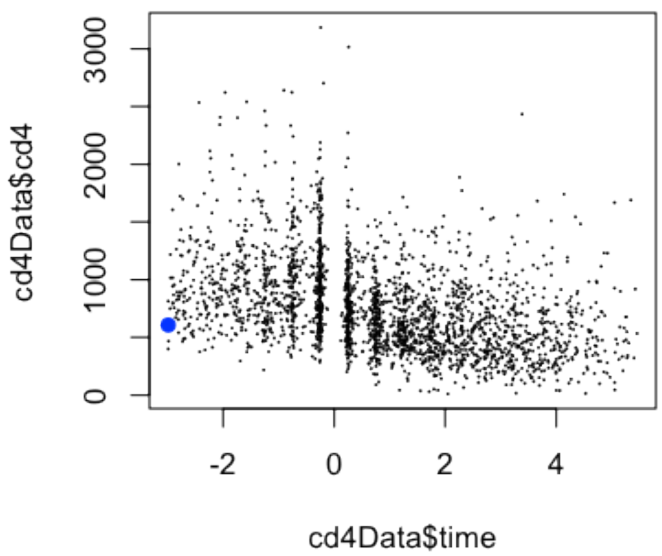
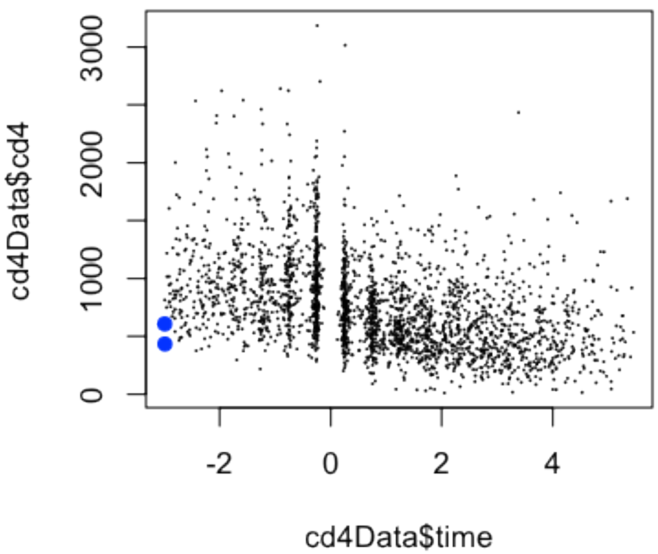
Average first 2 points

```
plot(cd4Data$time,cd4Data$cd4,pch=19,cex=0.1)
points(mean(cd4Data$time[1:2]),mean(cd4Data$cd4[1:2]),col="blue",pch=19)
```



Average second and third points

```
plot(cd4Data$time,cd4Data$cd4,pch=19,cex=0.1)
points(mean(cd4Data$time[1:2]),mean(cd4Data$cd4[1:2]),col="blue",pch=19)
points(mean(cd4Data$time[2:3]),mean(cd4Data$cd4[2:3]),col="blue",pch=19)
```



A moving average

```
plot(cd4Data$time,cd4Data$cd4,pch=19,cex=0.1)
aveTime <- aveCd4 <- rep(NA,length(3:(dim(cd4Data)[1]-2)))
for(i in 3:(dim(cd4Data)[1]-2)){
    aveTime[i] <- mean(cd4Data$time[(i-2):(i+2)])
    aveCd4[i] <- mean(cd4Data$cd4[(i-2):(i+2)])
}
lines(aveTime,aveCd4,col="blue",lwd=3)
```

Average more points

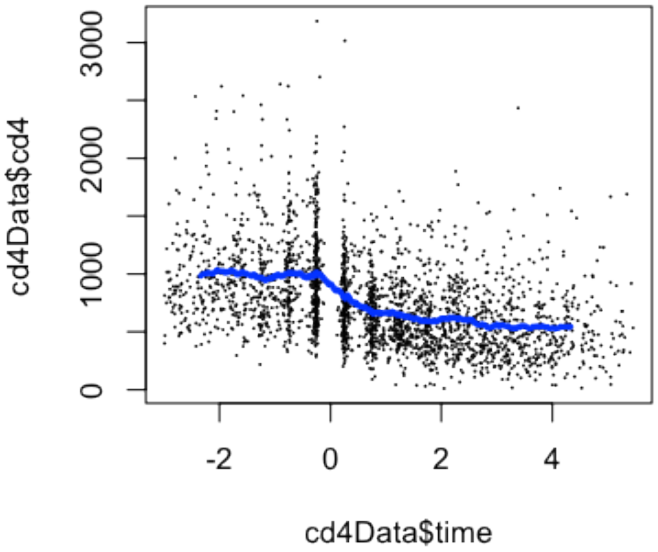
```
plot(cd4Data$time,cd4Data$cd4,pch=19,cex=0.1)
aveTime <- aveCd4 <- rep(NA,length(11:(dim(cd4Data)[1]-10)))
for(i in 11:(dim(cd4Data)[1]-2)){
  aveTime[i] <- mean(cd4Data$time[(i-10):(i+10)])
 aveCd4[i] <- mean(cd4Data$cd4[(i-10):(i+10)])
}
lines(aveTime,aveCd4,col="blue",lwd=3)
```

Average many more

```
plot(cd4Data$time,cd4Data$cd4,pch=19,cex=0.1)
aveTime <- aveCd4 <- rep(NA,length(201:(dim(cd4Data)[1]-200)))
for(i in 201:(dim(cd4Data)[1]-2)){
    aveTime[i] <- mean(cd4Data$time[(i-200):(i+200)])
    aveCd4[i] <- mean(cd4Data$cd4[(i-200):(i+200)])
}
lines(aveTime,aveCd4,col="blue",lwd=3)
```

A faster way

```
filtTime <- as.vector(filter(cd4Data$time,filter=rep(1,200))/200)
filtCd4 <- as.vector(filter(cd4Data$cd4,filter=rep(1,200))/200)
plot(cd4Data$time,cd4Data$cd4,pch=19,cex=0.1); lines(filtTime,filtCd4,col="blue",lwd=3)
```



Averaging = weighted sums

```
filtCd4 <- as.vector(filter(cd4Data$cd4,filter=rep(1,4))/4)
filtCd4[2]
```

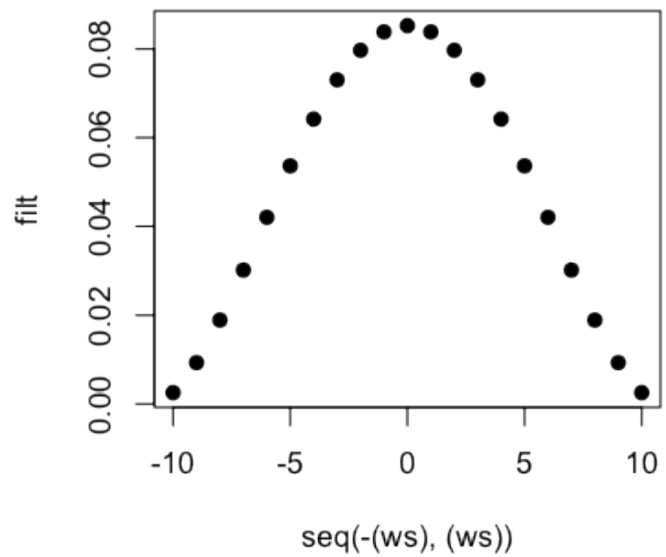
```
[1] 607.5
```

```
sum(cd4Data$cd4[1:4] * rep(1/4,4))
```

```
[1] 607.5
```

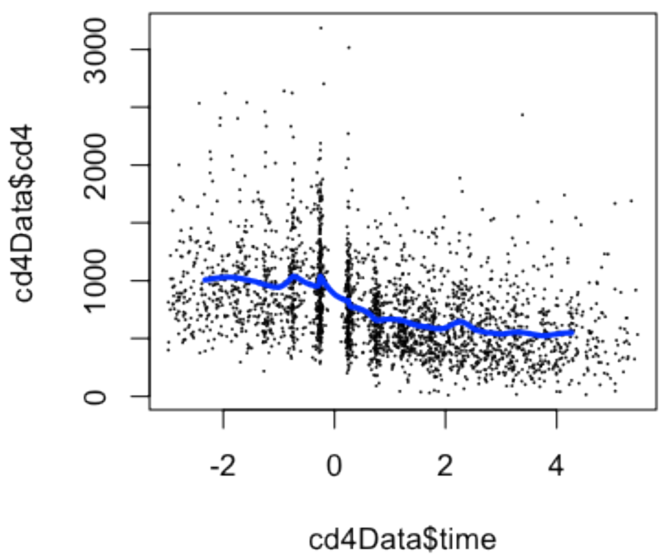
Other weights -> should sum to one

```
ws = 10; tukey = function(x) pmax(1 - x^2,0)^2
filt= tukey(seq(-ws,ws)/(ws+1));filt=filt/sum(filt)
plot(seq(-(ws),(ws)),filt,pch=19)
```



Other weights -> should sum to one

```
ws = 100; tukey = function(x) pmax(1 - x^2,0)^2
filt= tukey(seq(-ws,ws)/(ws+1));filt=filt/sum(filt)
filtTime <- as.vector(filter(cd4Data$time,filter=filt))
filtCd4 <- as.vector(filter(cd4Data$cd4,filter=filt))
plot(cd4Data$time,cd4Data$cd4,pch=19,cex=0.1); lines(filtTime,filtCd4,col="blue",lwd=3)
```



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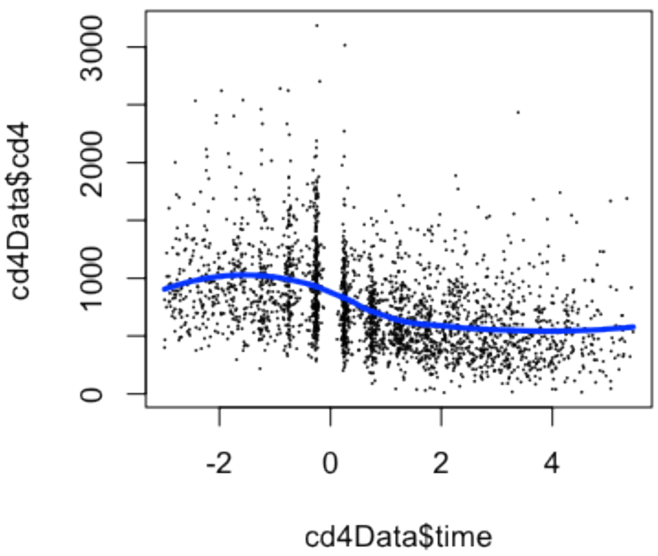
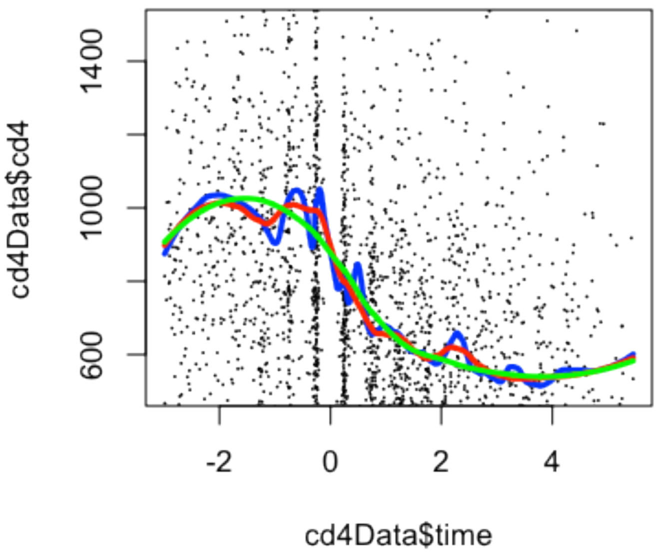
Lowess (loess)

```
lw1 <- loess(cd4 ~ time,data=cd4Data)
plot(cd4Data$time,cd4Data$cd4,pch=19,cex=0.1)
lines(cd4Data$time,lw1$fitted,col="blue",lwd=3)
```



Span

```
plot(cd4Data$time,cd4Data$cd4,pch=19,cex=0.1,ylim=c(500,1500))
lines(cd4Data$time,loess(cd4 ~ time,data=cd4Data,span=0.1)$fitted,col="blue",lwd=3)
lines(cd4Data$time,loess(cd4 ~ time,data=cd4Data,span=0.25)$fitted,col="red",lwd=3)
lines(cd4Data$time,loess(cd4 ~ time,data=cd4Data,span=0.76)$fitted,col="green",lwd=3)
```



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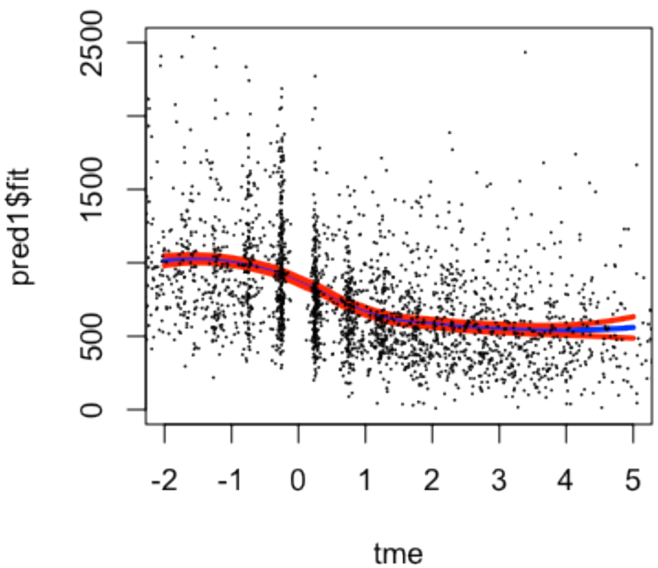
Predicting with loess

```
tme <- seq(-2,5,length=100); pred1 = predict(lw1,newdata=data.frame(time=tme),se=TRUE)
plot(tme,pred1$fit,col="blue",lwd=3,type="l",ylim=c(0,2500))
lines(tme,pred1$fit + 1.96*pred1$se.fit,col="red",lwd=3)
lines(tme,pred1$fit - 1.96*pred1$se.fit,col="red",lwd=3)
points(cd4Data$time,cd4Data$cd4,pch=19,cex=0.1)
```



Splines

$$Y_i = b_0 + \sum_{k=1}^{K} b_k s_k(x_i) + e_i$$

Y_i - outcome for ith observation

b_0 - Intercept term

b_k - Coefficient for kth spline function

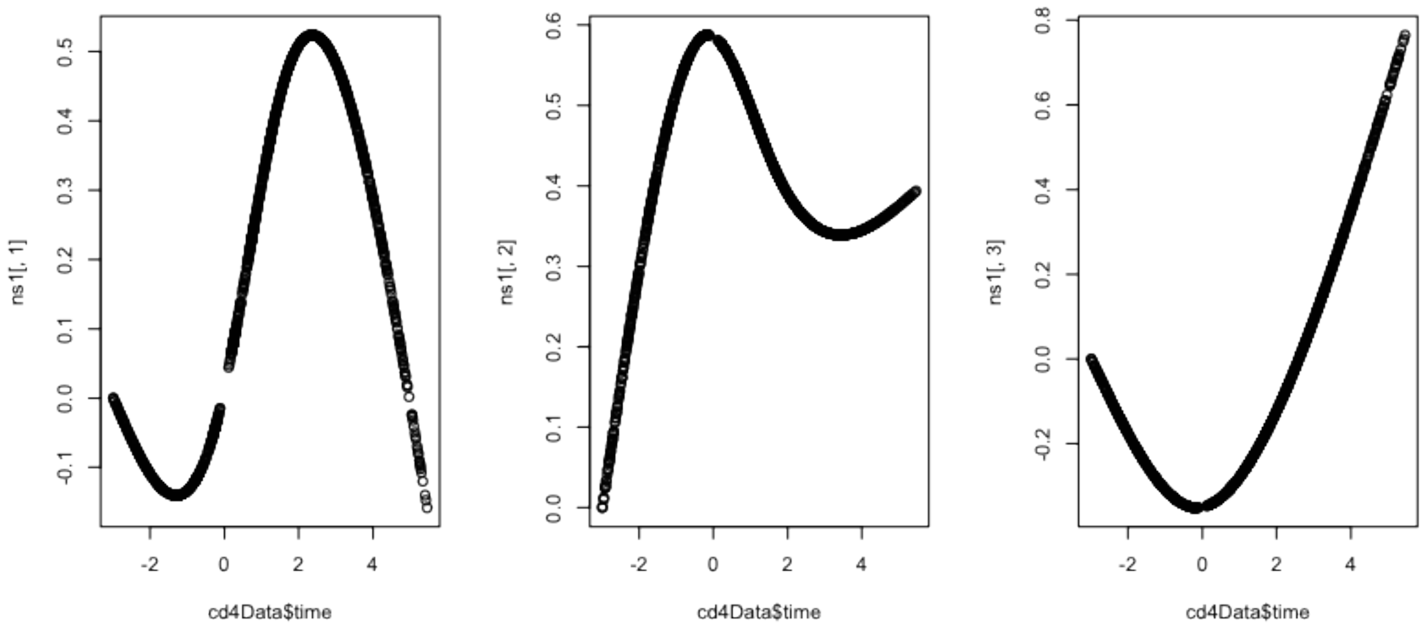
s_k - kth spline function

x_i - covariate for ith observation

e_i - everything we didn't measure/model

Splines in R

```
library(splines)
ns1 <- ns(cd4Data$time,df=3)
par(mfrow=c(1,3))
plot(cd4Data$time,ns1[,1]); plot(cd4Data$time,ns1[,2]); plot(cd4Data$time,ns1[,3])
```



Regression with splines

```
lm1 <- lm(cd4Data$cd4 ~ ns1)
summary(lm1)
```

```

Call:
lm(formula = cd4Data$cd4 ~ ns1)

Residuals:
   Min     1Q Median     3Q    Max 
-780.0 -242.4  -61.3  169.5 2263.7 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)    982.0       33.9   29.01  < 2e-16 ***
ns11          -611.3       32.6  -18.78  < 2e-16 ***
ns12          -373.7       79.4   -4.71  2.6e-06 ***
ns13          -374.8       41.2   -9.09  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

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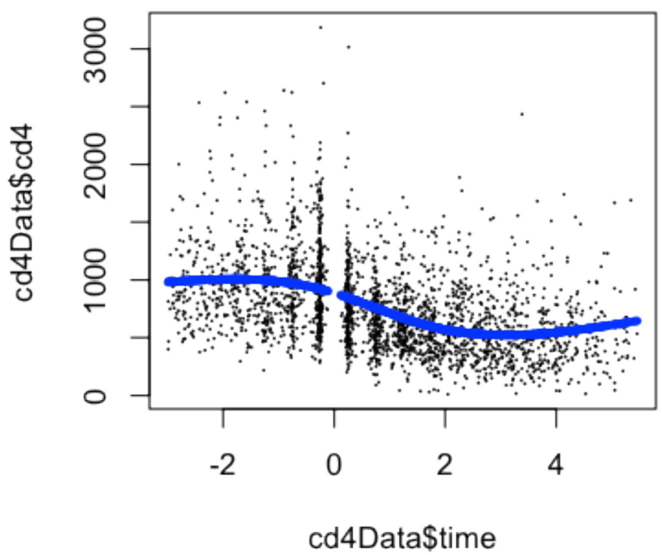
Fitted values

```
plot(cd4Data$time,cd4Data$cd4,pch=19,cex=0.1)
points(cd4Data$time,lm1$fitted,col="blue",pch=19,cex=0.5)
```



Notes and further resources

Notes:

- Cross-validation with splines/smoothing is a good idea
- Do not predict outside the range of observed data

Further resources:

- Hector Corrada Bravo's Lecture Notes
- Rafa Irizarry's Lecture Notes on smoothing, On splines
- Elements of Statistical Learning
- Advanced Data Analysis from An Elementary Point of View