Predicting with trees

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Key ideas

- Iteratively split variables into groups
- Split where maximally predictive
- Evaluate "homogeneity" within each branch
- Fitting multiple trees often works better (forests)

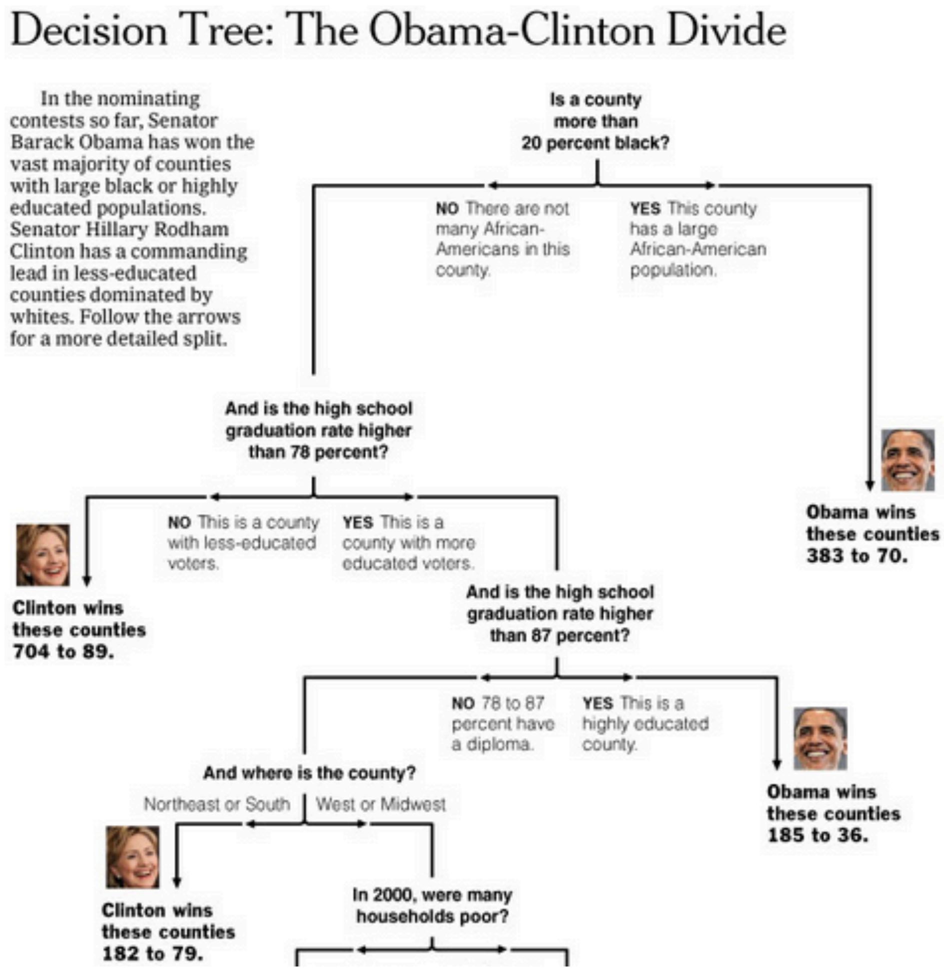
Pros:

- Easy to implement
- Easy to interpret
- Better performance in nonlinear settings

Cons:

- Without pruning/cross-validation can lead to overfitting
- Harder to estimate uncertainty
- Results may be variable

Example Tree



http://graphics8.nytimes.com/images/2008/04/16/us/0416-nat-subOBAMA.jpg

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Basic algorithm

1. Start with all variables in one group
2. Find the variable/split that best separates the outcomes
3. Divide the data into two groups ("leaves") on that split ("node")
4. Within each split, find the best variable/split that separates the outcomes
5. Continue until the groups are too small or sufficiently "pure"

Measures of impurity

$$\hat{p}_{mk} = \frac{1}{N_m} \sum_{x_i \ in \ Leaf \ m} \mathbb{1}(y_i = k)$$

Misclassification Error:

$$1 - \hat{p}_{mk(m)}$$

Gini index:

$$\sum_{k \neq k'} \hat{p}_{mk} \times \hat{p}_{mk'} = \sum_{k=1}^{K} \hat{p}_{mk}(1 - \hat{p}_{mk})$$

Cross-entropy or deviance:

$$-\sum_{k=1}^{K} \hat{p}_{mk} \ln \hat{p}_{mk}$$

Example: Iris Data

```
data(iris)
names(iris)
```

```
[1] "Sepal.Length" "Sepal.Width"  "Petal.Length" "Petal.Width"  "Species"
```

```
table(iris$Species)
```

```

    setosa versicolor  virginica 
        50         50         50 
```

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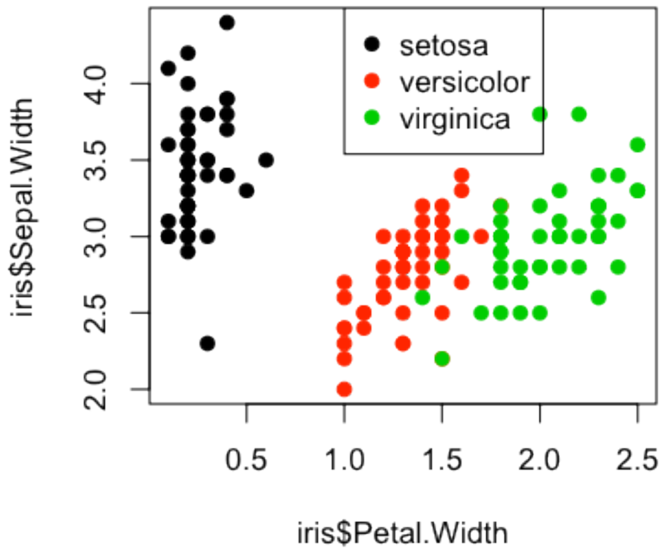
Iris petal widths/sepal width

```
plot(iris$Petal.Width,iris$Sepal.Width,pch=19,col=as.numeric(iris$Species))
legend(1,4.5,legend=unique(iris$Species),col=unique(as.numeric(iris$Species)),pch=19)
```



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Iris petal widths/sepal width

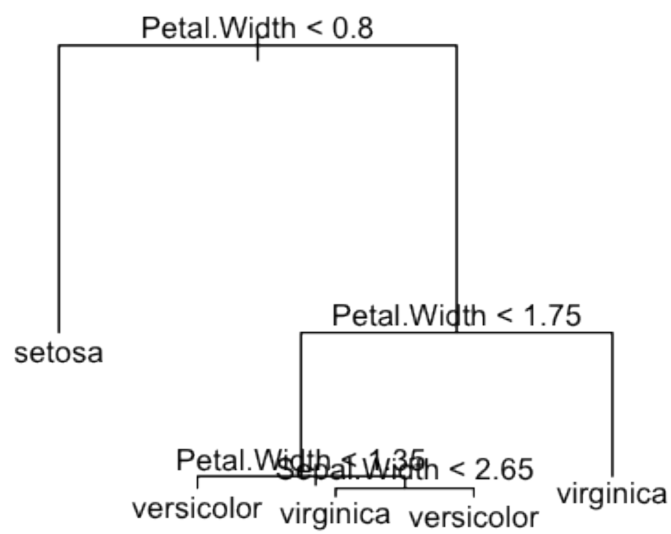
```
# An alternative is library(rpart)
library(tree)
tree1 <- tree(Species ~ Sepal.Width + Petal.Width,data=iris)
summary(tree1)
```

```

Classification tree:
tree(formula = Species ~ Sepal.Width + Petal.Width, data = iris)
Number of terminal nodes:  5
Residual mean deviance:  0.204 = 29.6 / 145
Misclassification error rate: 0.0333 = 5 / 150
```

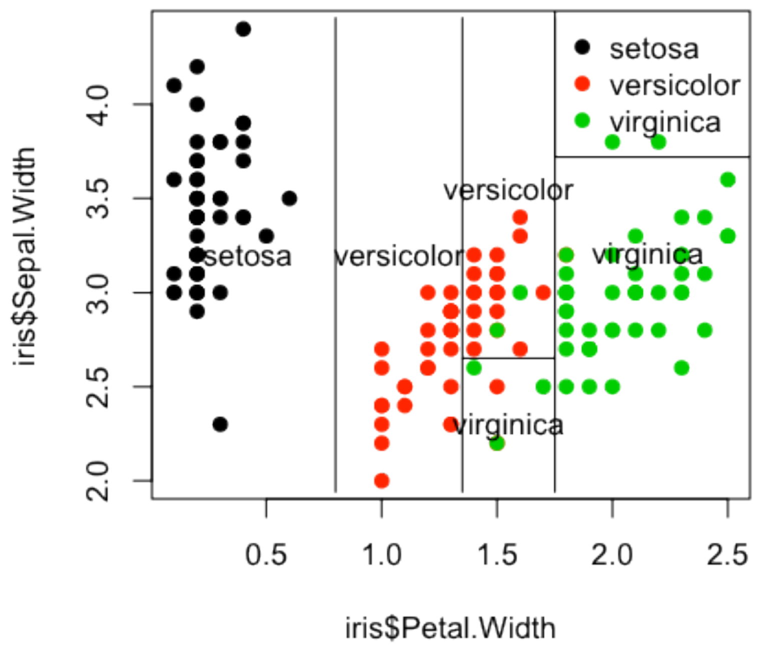
Plot tree

```
plot(tree1)
text(tree1)
```



Another way of looking at a CART model

```
plot(iris$Petal.Width,iris$Sepal.Width,pch=19,col=as.numeric(iris$Species))
partition.tree(tree1,label="Species",add=TRUE)
legend(1.75,4.5,legend=unique(iris$Species),col=unique(as.numeric(iris$Species)),pch=19)
```



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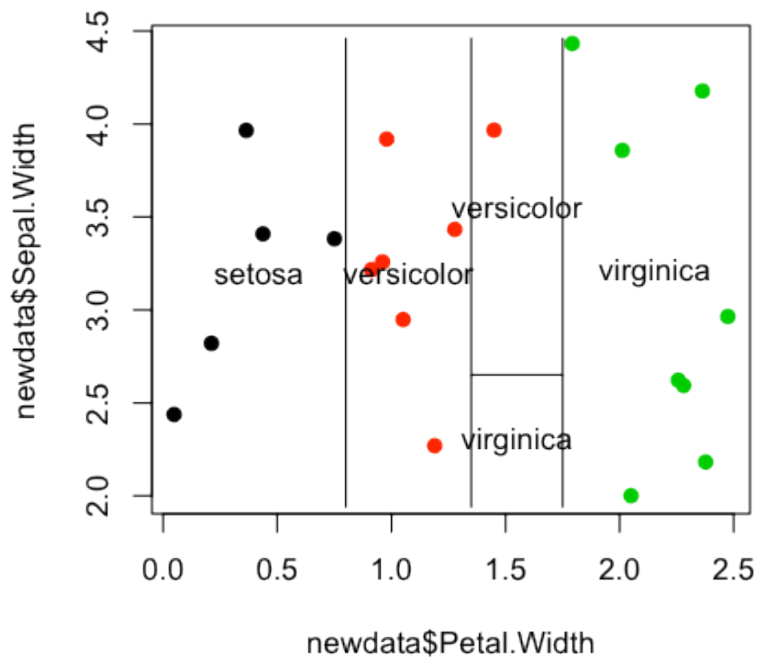
Predicting new values

```
set.seed(32313)
newdata <- data.frame(Petal.Width = runif(20,0,2.5),Sepal.Width = runif(20,2,4.5))
pred1 <- predict(tree1,newdata)
pred1
```

```
   setosa versicolor virginica
1       0    0.02174   0.97826
2       0    0.02174   0.97826
3       1    0.00000   0.00000
4       0    1.00000   0.00000
5       0    0.02174   0.97826
6       0    0.02174   0.97826
7       0    0.02174   0.97826
8       0    0.90476   0.09524
9       0    1.00000   0.00000
10      0    0.02174   0.97826
11      0    1.00000   0.00000
12      1    0.00000   0.00000
13      1    0.00000   0.00000
14      1    0.00000   0.00000
```

Overlaying new values

```
pred1 <- predict(tree1,newdata,type="class")
plot(newdata$Petal.Width,newdata$Sepal.Width,col=as.numeric(pred1),pch=19)
partition.tree(tree1,"Species",add=TRUE)
```



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Pruning trees example: Cars

```
data(Cars93,package="MASS")
head(Cars93)
```

```
  Manufacturer   Model    Type Min.Price Price Max.Price MPG.city MPG.highway            AirBags
1        Acura Integra   Small      12.9  15.9      18.8       25          31               None
2        Acura  Legend Midsize      29.2  33.9      38.7       18          25 Driver & Passenger
3         Audi      90 Compact      25.9  29.1      32.3       20          26        Driver only
4         Audi     100 Midsize      30.8  37.7      44.6       19          26 Driver & Passenger
5          BMW    535i Midsize      23.7  30.0      36.2       22          30        Driver only
6        Buick Century Midsize      14.2  15.7      17.3       22          31        Driver only
  DriveTrain Cylinders EngineSize Horsepower  RPM Rev.per.mile Man.trans.avail Fuel.tank.capacity
1      Front         4        1.8        140 6300         2890             Yes               13.2
2      Front         6        3.2        200 5500         2335             Yes               18.0
3      Front         6        2.8        172 5500         2280             Yes               16.9
4      Front         6        2.8        172 5500         2535             Yes               21.1
5       Rear         4        3.5        208 5700         2545             Yes               21.1
6      Front         4        2.2        110 5200         2565              No               16.4
  Passengers Length Wheelbase Width Turn.circle Rear.seat.room Luggage.room Weight  Origin
1          5    177       102    68          37           26.5           11   2705 non-USA
2          5    195       115    71          38           30.0           15   3560 non-USA
```

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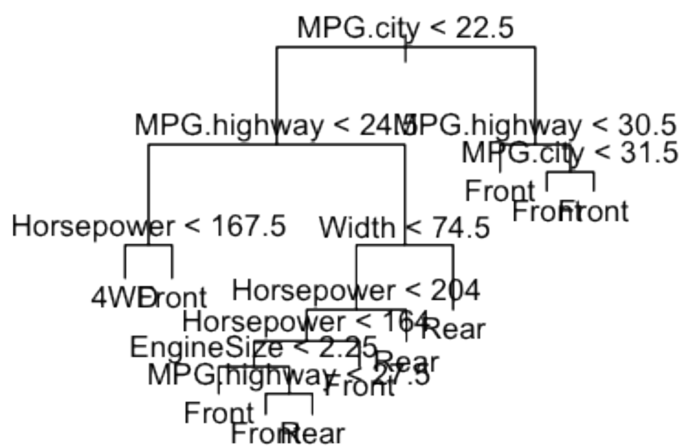
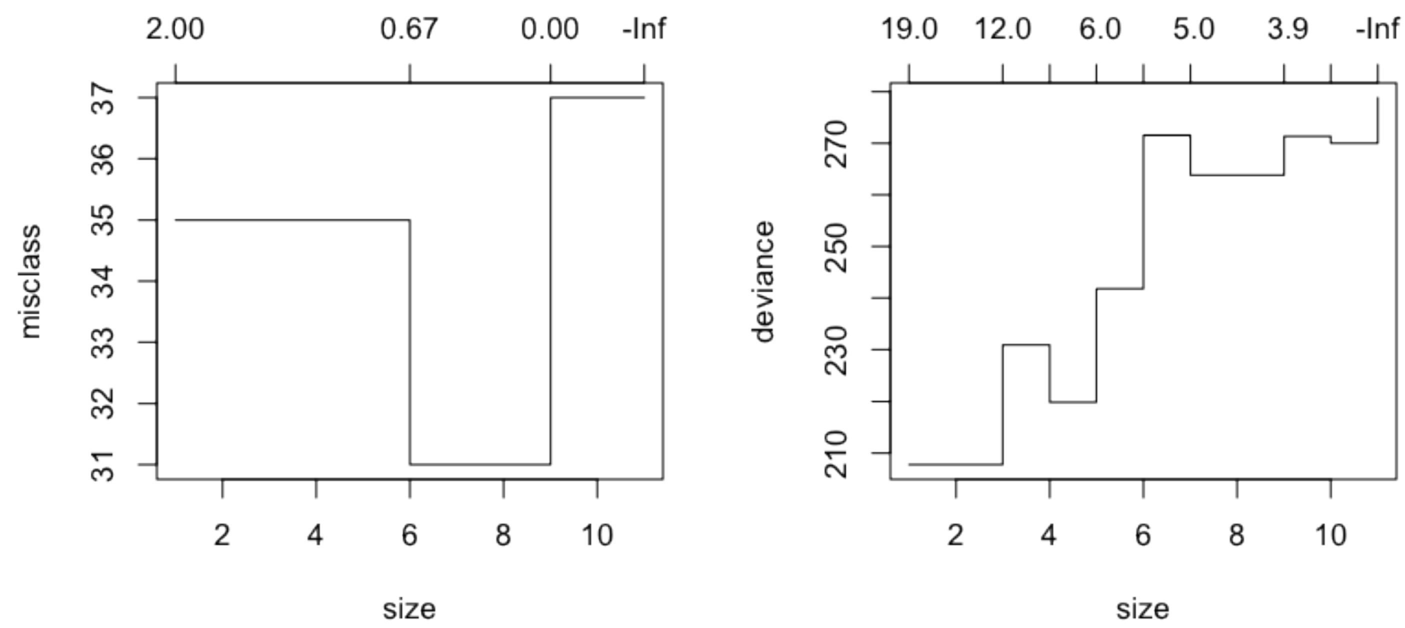
Build a tree

```
treeCars <- tree(DriveTrain ~ MPG.city + MPG.highway + AirBags +
                   EngineSize + Width + Length + Weight + Price + Cylinders +
                   Horsepower + Wheelbase,data=Cars93)
plot(treeCars)
text(treeCars)
```



Plot errors

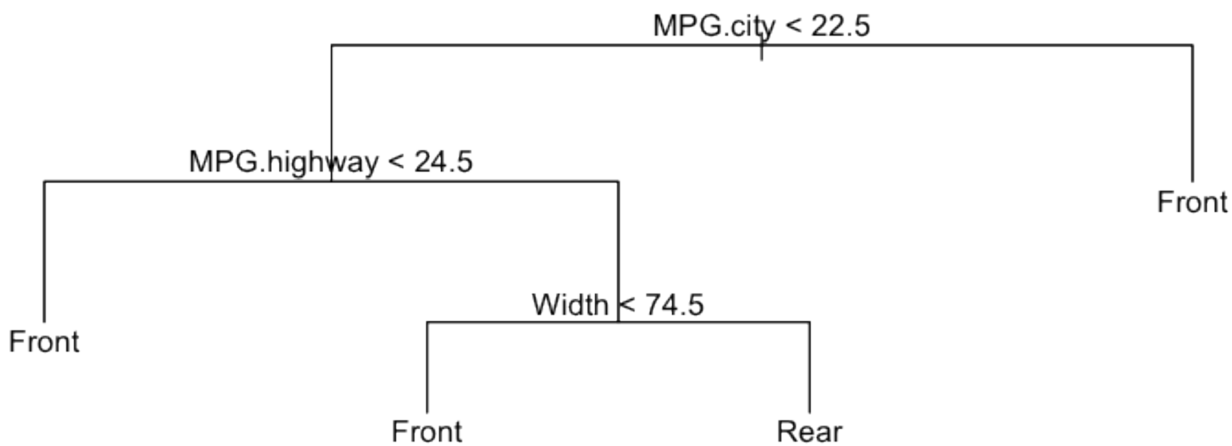
```
par(mfrow=c(1,2))
plot(cv.tree(treeCars,FUN=prune.tree,method="misclass"))
plot(cv.tree(treeCars))
```




```
pruneTree <- prune.tree(treeCars,best=4)
```

Prune the tree

```
pruneTree <- prune.tree(treeCars,best=4)
plot(pruneTree)
text(pruneTree)
```



Show resubstitution error *

```
table(Cars93$DriveTrain,predict(pruneTree,type="class"))
```

```
        4WD Front Rear
  4WD     5     5    0
  Front   1    66    0
  Rear    1    10    5
```

```
table(Cars93$DriveTrain,predict(treeCars,type="class"))
```

```
        4WD Front Rear
  4WD     5     5    0
  Front   2    61    4
  Rear    0     3   13
```


- Note that cross validation error is a better measure of test set accuracy

Notes and further resources

- Hector Corrada Bravo's Notes, code
- Cosma Shalizi's notes
- Elements of Statistical Learning
- Classification and regression trees
- Random forests