How do we represent data?

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How do we write about data?

- Each data point is usually represented by a capital letter.
  - H for height, W for weight.
- If there are more than one data point of the same type we use subscripts.
  - H_1, H_2, H_3 for three different people's heights.
- Sometimes it is more compact to write X_1 for height and X_2 for weight.
- Then we need another subscript for the individual data point
  - X_{11} for the height of the first person.
- Y representes general outcomes and X general covariates.
- In this course we will try to use informative letters when possible.

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Randomness

- Variables like X and Y are called *random variables* because we expect them to be *random* in some way.
- In general, randomness is a hard thing to define
- In this class a variable may be random because
    - It represents an incompletely measured variable
    - It represents a sample drawn from a population using a random mechanism.
- Once we are talking about a specific value of a variable we have observed it isn't random anymore, we write these values with lower case letters x, y, etc.
- We write $X = x$ or $X = 1$ to indicate we have observed a specific value x or 1.

Randomness and measurement

- A coin flip is commonly considered random
- But it can be modeled by deterministic equations
  - Dynamical bias in the coin toss (Diaconis, Holmes and Montgomery SIAM Review 2007)
  - Modeled the tossing as a dynamical system
  - Showed that a coin is more likely to land on the side it started
  - Did experiments that demonstrated it was a 51% chance
- Some have taken it a bit farther making predictable coin flipping machines based on physical properties.

Distributions

- In statistical modeling, random variables like X are assumed to be samples from a *distribution*
- A distribution tells us the possible values of X and the probabilities for each value.
- Probability is the chance something will happen and is abbreviated Pr
- The probabilities must all be between 0 and 1.
- The probabilities must add up to 1.
- An example:
  - Let's flip a coin and allow X to represent whether it is heads or tails
  - $X = 1$ if it is heads and $X = 0$ if it is tails
  - We expect that about 50% of the time it will be heads.
  - The distribution can then be written Pr($X = 1$)=0.5 and Pr($X = 0$)=0.5

Continuous versus discrete distributions

- *discrete* distributions specify probabilities for discrete values
  - Qualitative variables are discrete
  - So are variables that take on all values 0,1,2,3...
- *continuous* distributions specify probabilities for ranges of values
  - Quantitative variables are often assumed to be continuous
  - But we might only see specific values

Parameters

- Distributions are defined by a set of fixed values called *parameters*.
- *parameters* are sometimes represented by Greek letters like μ, σ, τ.
- Distributions are written as letters with the parameters in parentheses like $N(\mu, \sigma)$ or $Poisson(\lambda)$.
- $X \sim N(\mu, \sigma)$ means that X has the $N(\mu, \sigma)$ distribution.

The three most important parameters

- If X is a random variable, the mean of that random variable is written $E[X]$
  - Stands for expected value
  - Measures the "center" of a distribution
- The variance of that random variable is written $Var[X]$
  - Measures how "spread out" a distribution is
  - Measurement is in (units of X)^2
- The standard deviation is written $SD[X] = \sqrt{Var[X]}$
  - Also measures how "spread out" a distribution is
  - Measurement is in units of X

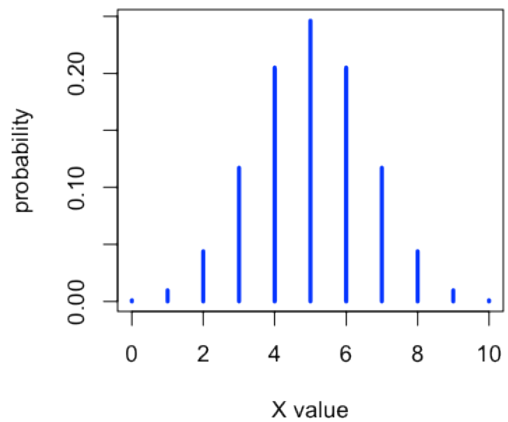
Conditioning

- The variables X are considered to be random
- The parameters are considered to be fixed values
- Sometimes we want to talk about a case where one of the random variables is fixed
- To indicate what is fixed, we *condition* using the symbol "|""
  - $X|\mu$ means that X is a random variable with fixed parameter μ
  - $Y|X = 2$ means Y is the random variable Y when X is fixed at 2.

Example distribution: Binomial

Binomial distribution: $Bin(n, p)$

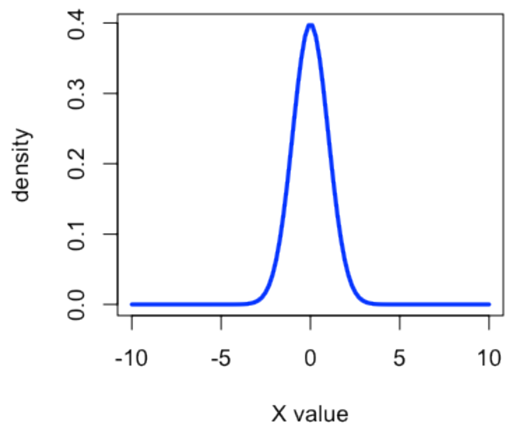
- $X \sim Bin(10, 0.5)$



Example distribution: Normal

Normal Distribution: $N(\mu, \sigma)$

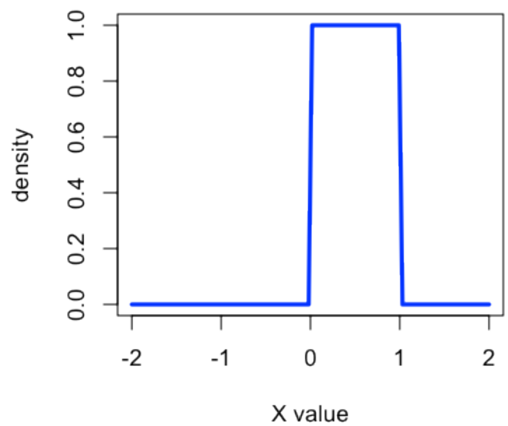
- $X \sim N(0, 1)$



Example distribution: Uniform

Uniform distribution: $U(\alpha, \beta)$

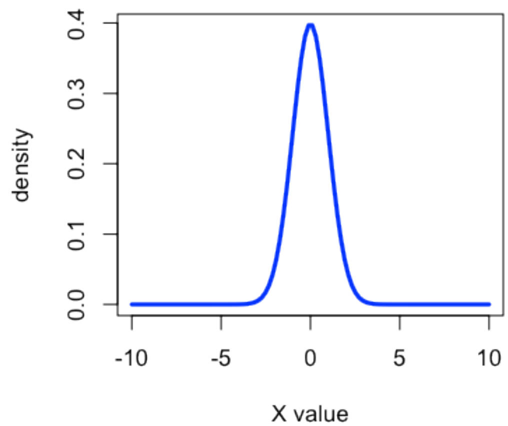
- $X \sim U(0, 1)$



Changing parameters

Normal Distribution: $N(\mu, \sigma)$

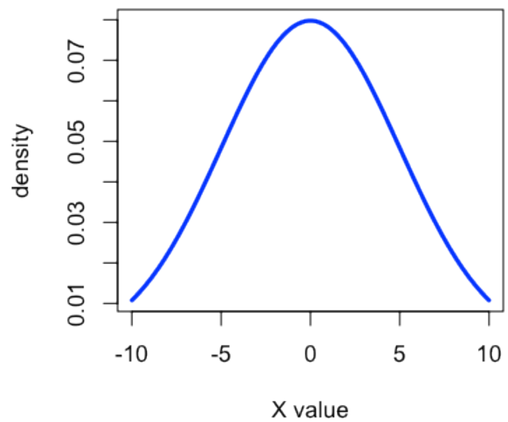
- $X \sim N(0, 1)$, $E[X] = \mu = 0$, $Var[X] = \sigma^2 = 1$



Changing parameters: the variance

Normal Distribution: $N(\mu, \sigma)$

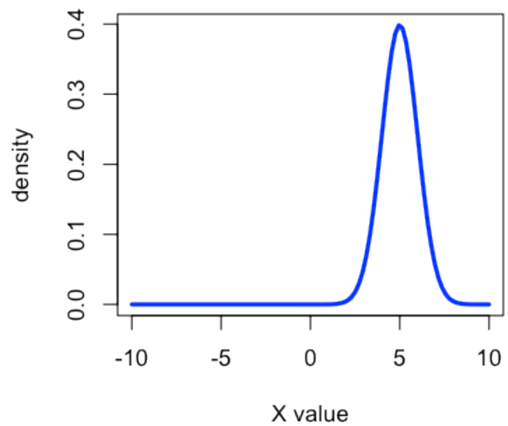
- $X \sim N(0, 5)$, $E[X] = \mu = 0$, $Var[X] = \sigma^2 = 25$



Changing parameters: the mean

Normal Distribution: $N(\mu, \sigma)$

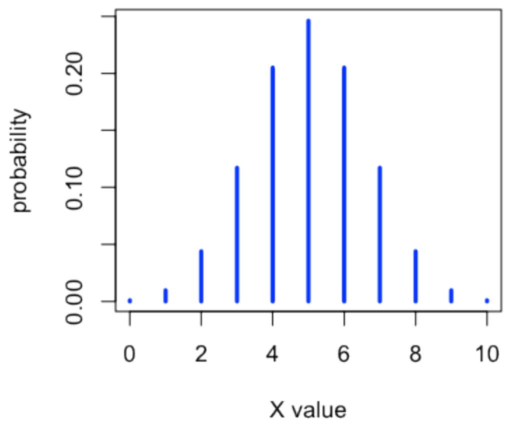
- $X \sim N(5, 1)$, $E[X] = \mu = 5$, $Var[X] = \sigma^2 = 1$



Example distribution: Binomial

Binomial distribution: $Bin(n, p)$

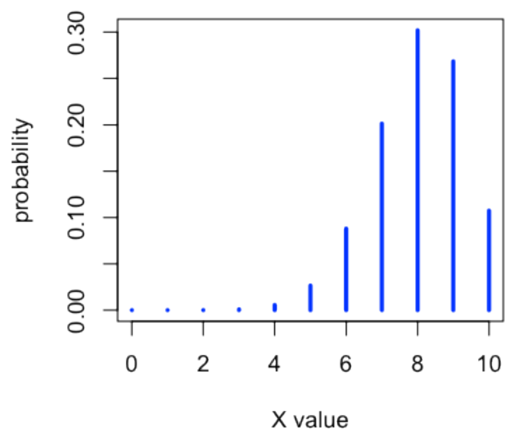
- $X \sim Bin(10, 0.5)$, $E[X] = n \times p = 5$, $Var[X] = n \times p \times (1 - p) = 2.5$



Changing parameters: both mean and variance

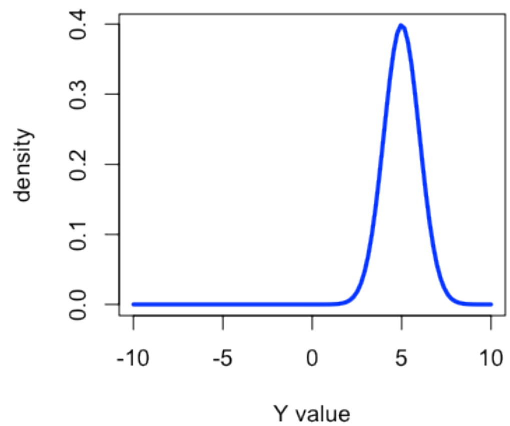
Binomial distribution: $Bin(n, p)$

- $X \sim Bin(10, 0.8)$, $E[X] = n \times p = 8$, $Var[X] = n \times p \times (1 - p) = 1.6$



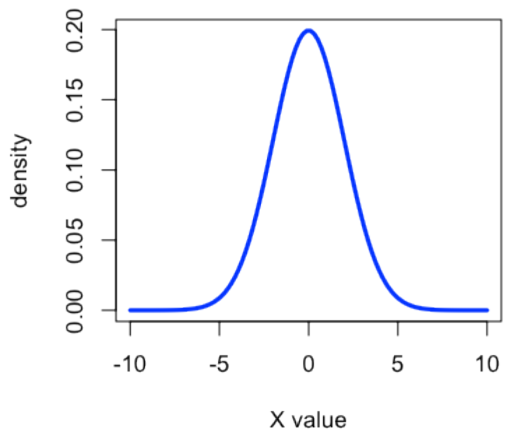
Conditioning

- Suppose $Y \sim N(X, 1)$ and $X \sim N(0, 1)$, then the distribution of $Y|X = 5$ is



Conditioning

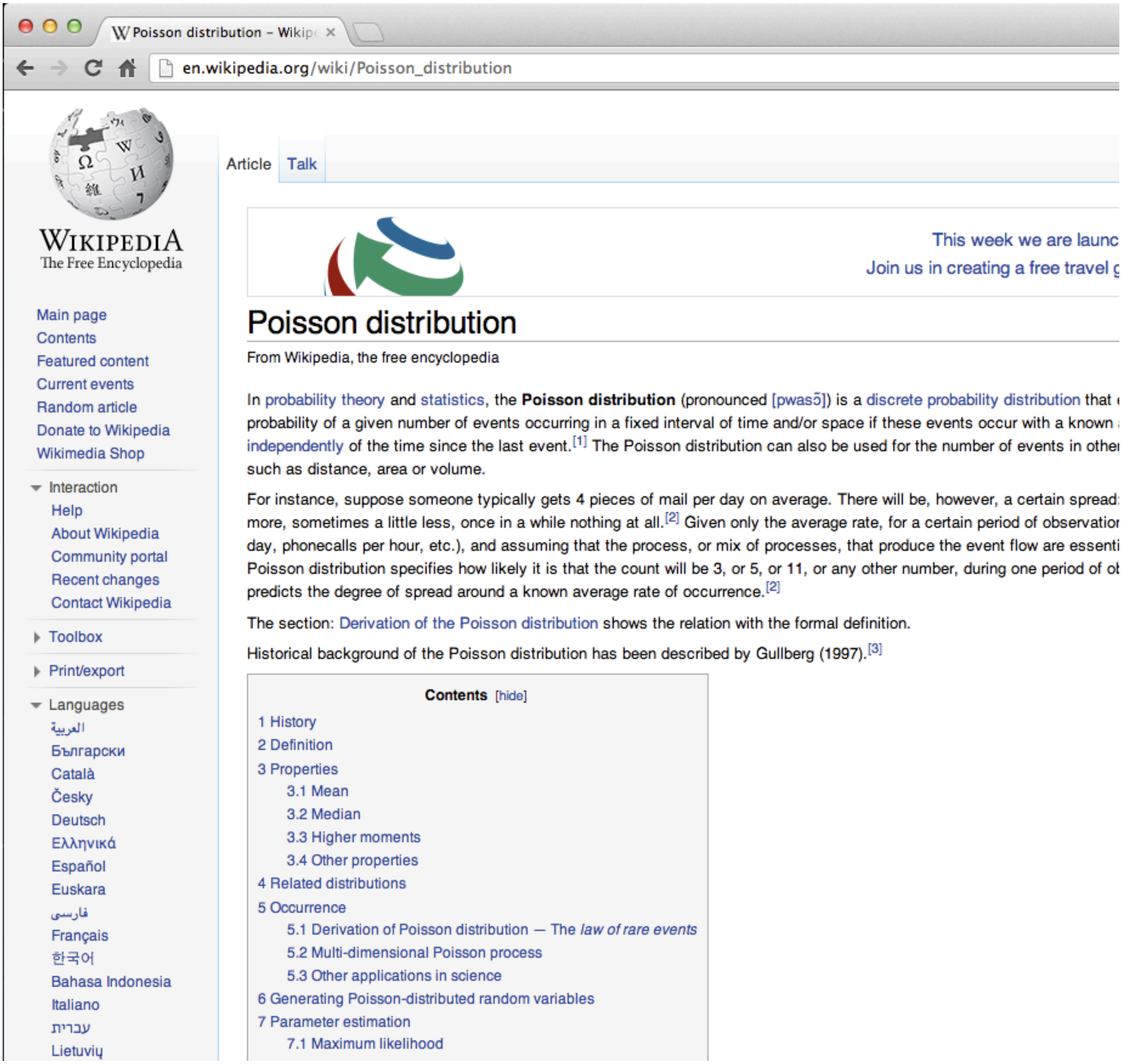
- Suppose $Y \sim N(X, 1)$ and $X \sim N(0, 1)$, then the distribution of Y is



http://en.wikipedia.org/wiki/Law_of_total_variance

http://en.wikipedia.org/wiki/Law_of_total_expectation

Learning more about a specific distribution



Poisson distribution – Wikip

en.wikipedia.org/wiki/Poisson_distribution

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Poisson distribution

From Wikipedia, the free encyclopedia

In probability theory and statistics, the **Poisson distribution** (pronounced [pwasɔ̃]) is a discrete probability distribution that
probability of a given number of events occurring in a fixed interval of time and/or space if these events occur with a known
independently of the time since the last event.[1] The Poisson distribution can also be used for the number of events in other
such as distance, area or volume.

For instance, suppose someone typically gets 4 pieces of mail per day on average. There will be, however, a certain spread
more, sometimes a little less, once in a while nothing at all.[2] Given only the average rate, for a certain period of observation
day, phonecalls per hour, etc.), and assuming that the process, or mix of processes, that produce the event flow are essenti
Poisson distribution specifies how likely it is that the count will be 3, or 5, or 11, or any other number, during one period of ob
predicts the degree of spread around a known average rate of occurrence.[2]

The section: Derivation of the Poisson distribution shows the relation with the formal definition.

Historical background of the Poisson distribution has been described by Gullberg (1997).[3]

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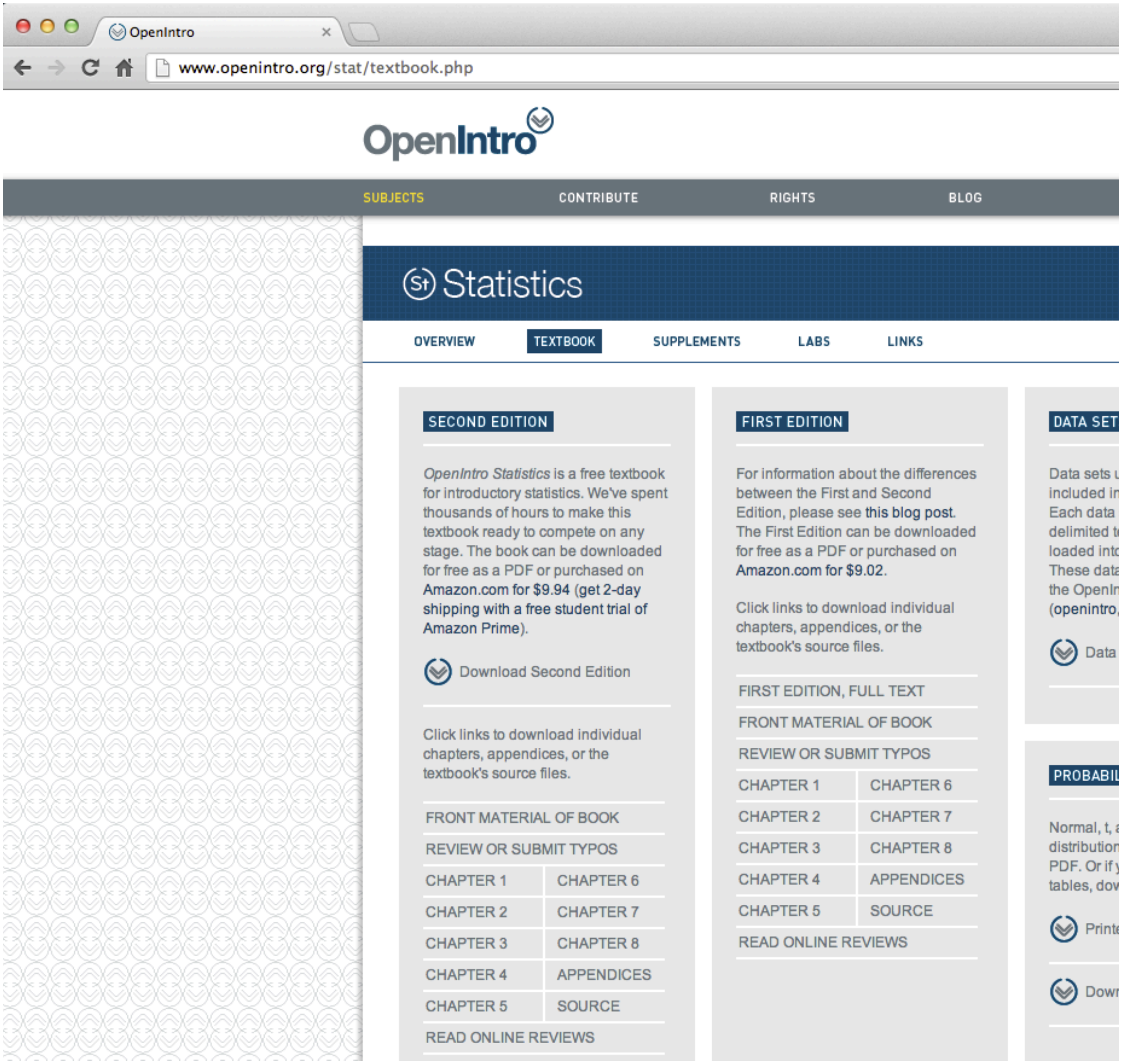
http://en.wikipedia.org/wiki/Poisson_distribution

Learning more about representing data



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