Basic least squares

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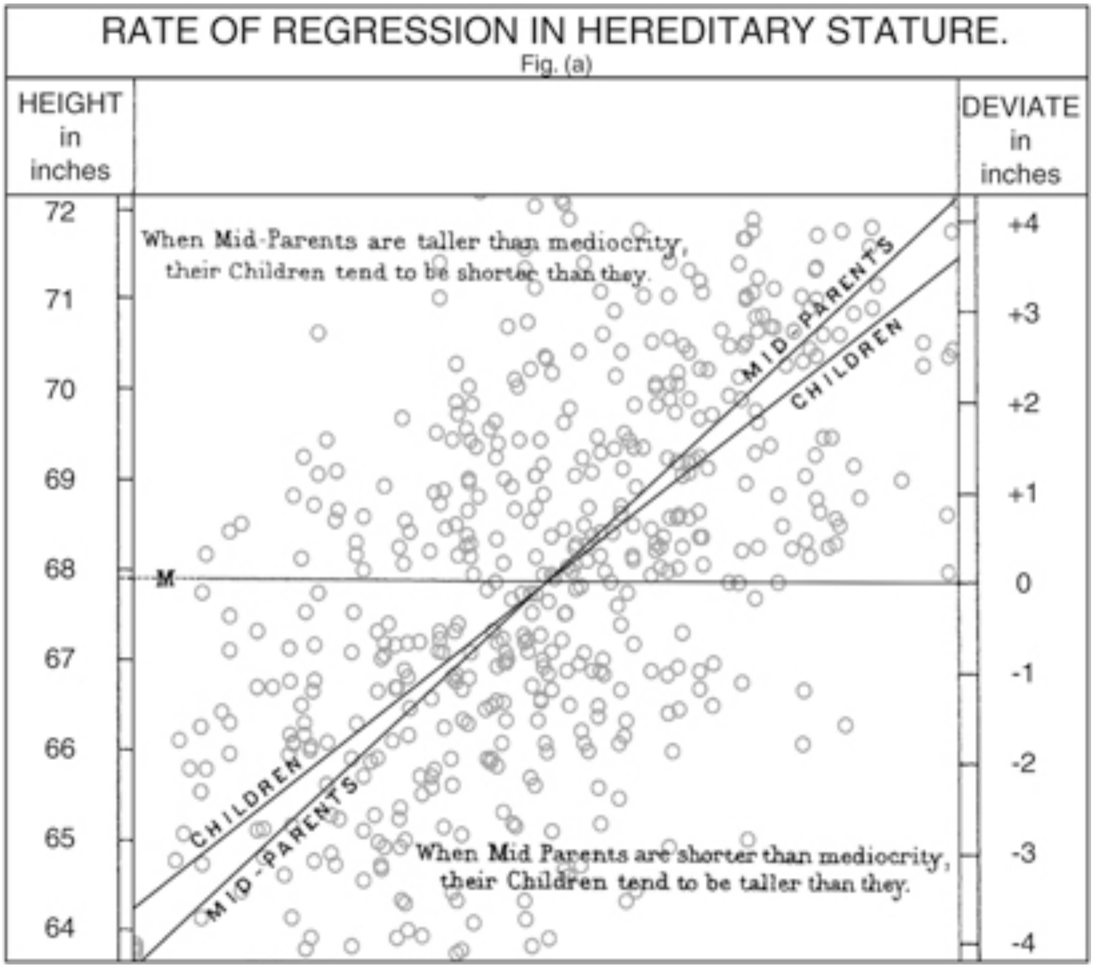
Goals of statistical modeling

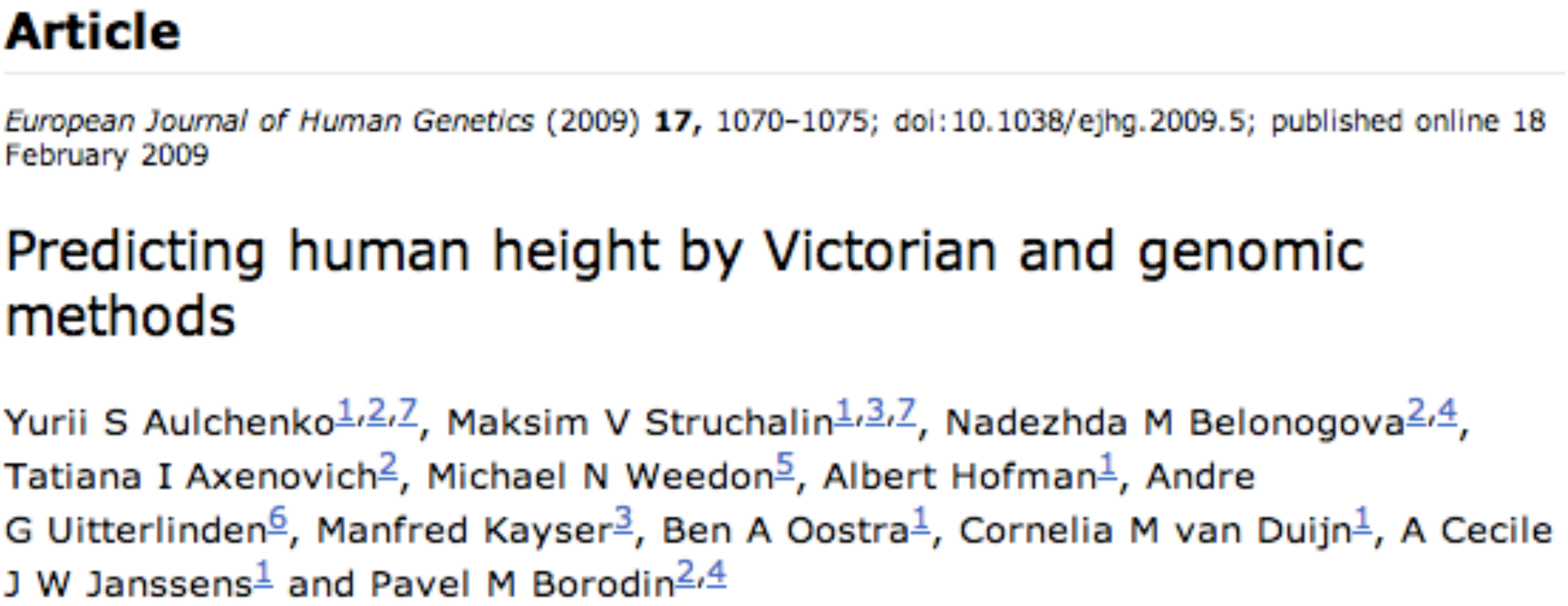
- Describe the distribution of variables
- Describe the relationship between variables
- Make inferences about distributions or relationships

Example: Average parent and child heights



http://www.nature.com/ejhg/journal/v17/n8/full/ejhg20095a.html

Still relevant



Article

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Predicting human height by Victorian and genomic methods

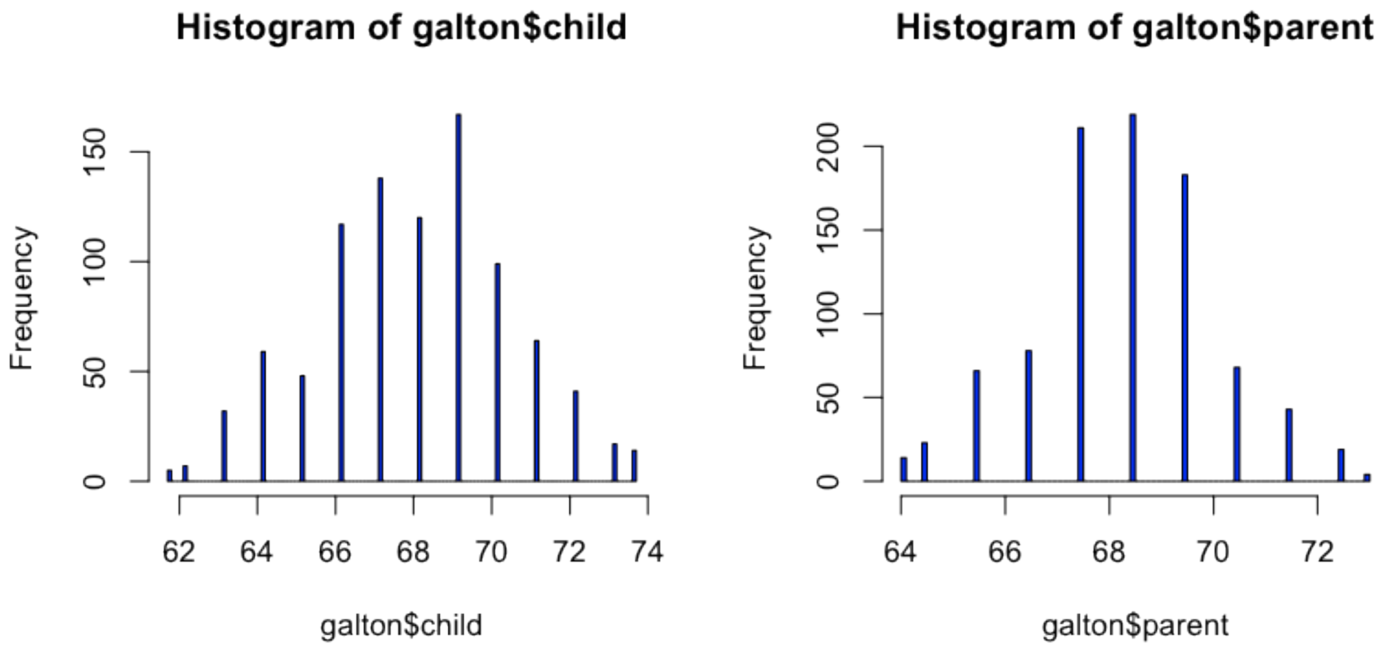
Yurii S Aulchenko[1,2,7], Maksim V Struchalin[1,3,7], Nadezhda M Belonogova[2,4], Tatiana I Axenovich[2], Michael N Weedon[5], Albert Hofman[1], Andre G Uitterlinden[6], Manfred Kayser[3], Ben A Oostra[1], Cornelia M van Duijn[1], A Cecile J W Janssens[1] and Pavel M Borodin[2,4]

http://www.nature.com/ejhg/journal/v17/n8/full/ejhg20095a.html

Predicting height: the Victorian approach beats modern genomics

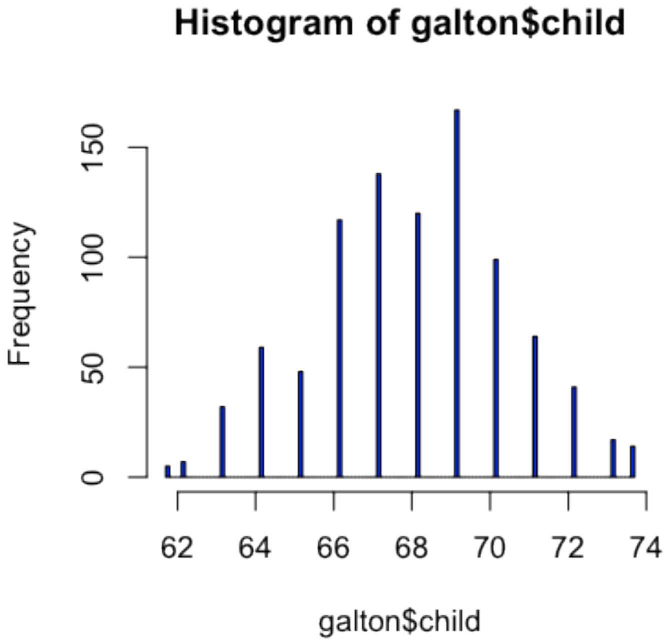
Load Galton Data

```
library(UsingR); data(galton)
par(mfrow=c(1,2))
hist(galton$child,col="blue",breaks=100)
hist(galton$parent,col="blue",breaks=100)
```



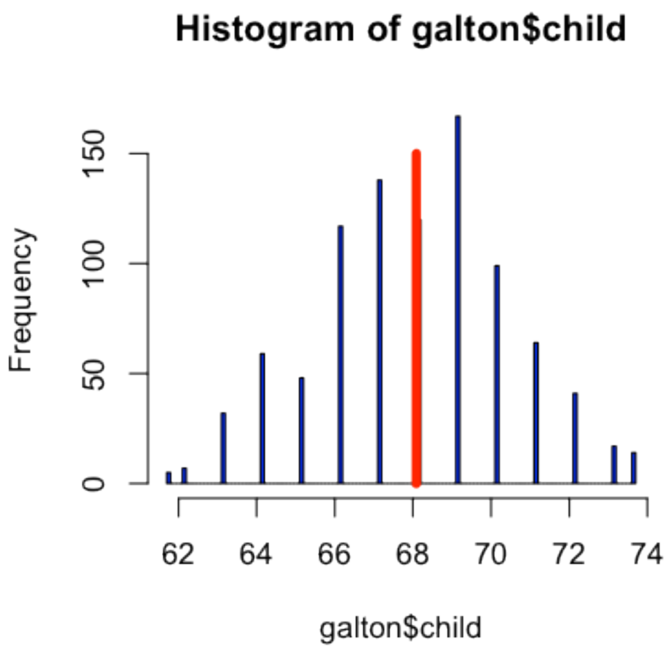
The distribution of child heights

```
hist(galton$child,col="blue",breaks=100)
```



Only know the child - average height

```
hist(galton$child,col="blue",breaks=100)
meanChild <- mean(galton$child)
lines(rep(meanChild,100),seq(0,150,length=100),col="red",lwd=5)
```



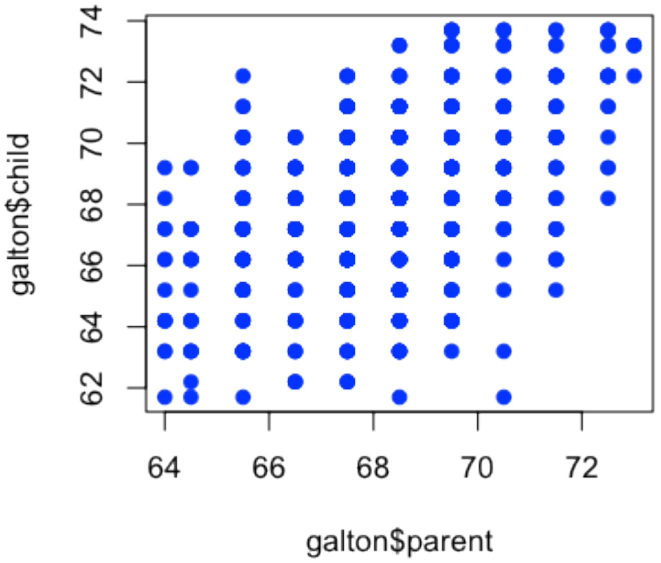
Only know the child - why average?

If C_i is the height of child i then the average is the value of μ that minimizes:

$$\sum_{i=1}^{928} (C_i - \mu)^2$$

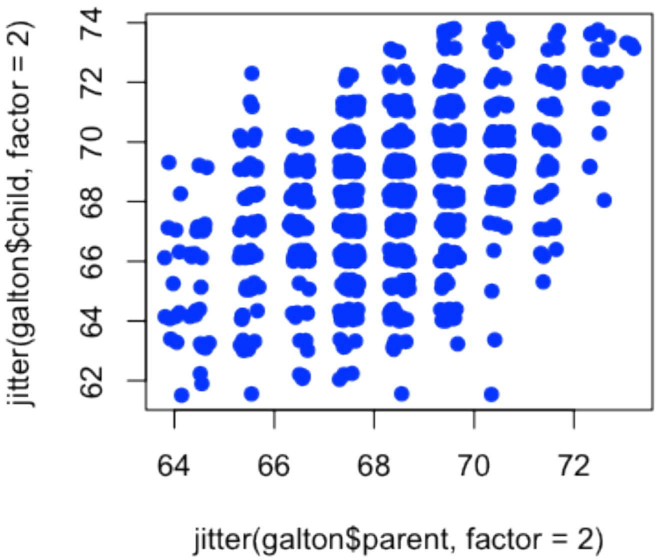
What if we plot child versus average parent

```
plot(galton$parent,galton$child,pch=19,col="blue")
```



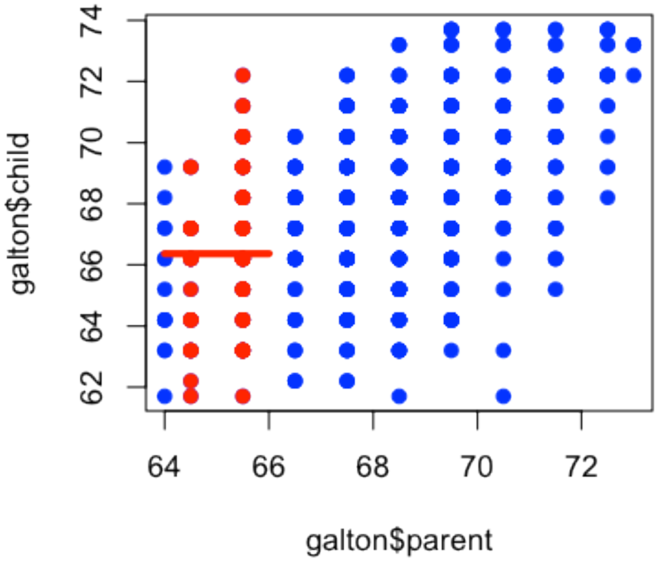
Jittered plot

```
set.seed(1234)
plot(jitter(galton$parent,factor=2),jitter(galton$child,factor=2),pch=19,col="blue")
```



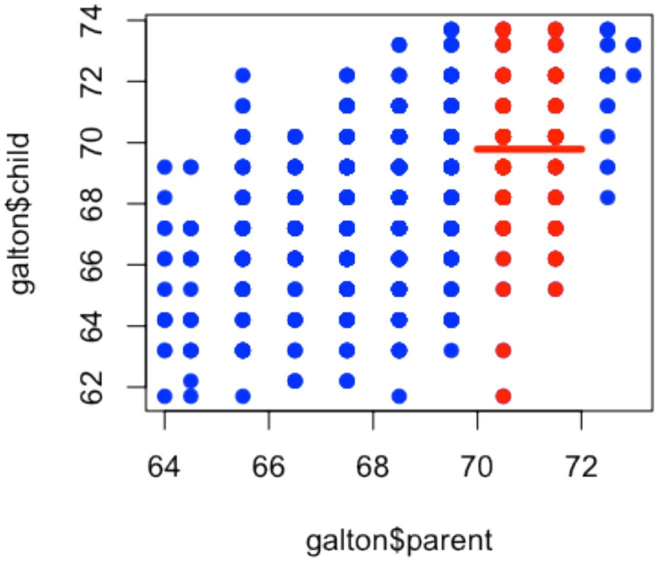
Average parent = 65 inches tall

```
plot(galton$parent,galton$child,pch=19,col="blue")
near65 <- galton[abs(galton$parent - 65)<1, ]
points(near65$parent,near65$child,pch=19,col="red")
lines(seq(64,66,length=100),rep(mean(near65$child),100),col="red",lwd=4)
```



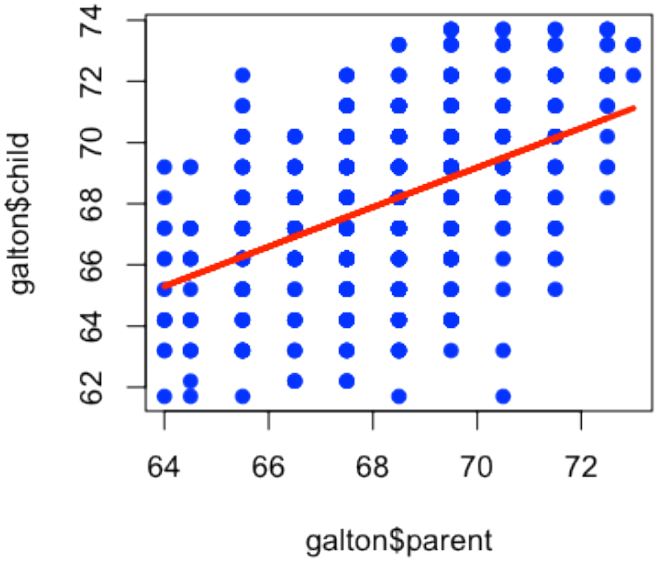
Average parent = 71 inches tall

```
plot(galton$parent,galton$child,pch=19,col="blue")
near71 <- galton[abs(galton$parent - 71)<1, ]
points(near71$parent,near71$child,pch=19,col="red")
lines(seq(70,72,length=100),rep(mean(near71$child),100),col="red",lwd=4)
```



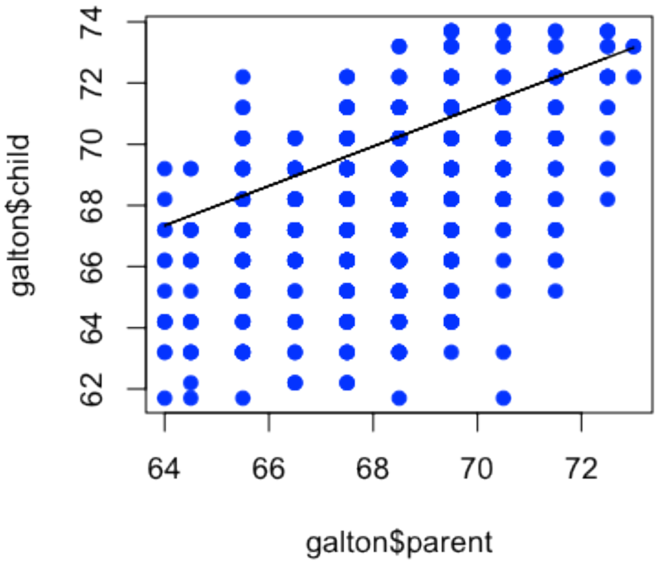
Fitting a line

```
plot(galton$parent,galton$child,pch=19,col="blue")
lm1 <- lm(galton$child ~ galton$parent)
lines(galton$parent,lm1$fitted,col="red",lwd=3)
```



Why not this line?

```
plot(galton$parent,galton$child,pch=19,col="blue")
lines(galton$parent, 26 + 0.646*galton$parent)
```



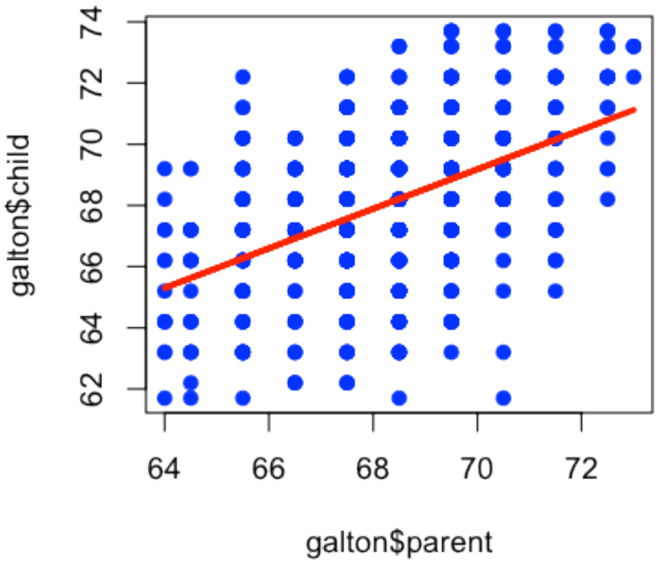
The equation for a line

If C_i is the height of child i and P_i is the height of the average parent, then we can imagine writing the equation for a line

$$C_i = b_0 + b_1 P_i$$

Not all points are on the line

```
plot(galton$parent,galton$child,pch=19,col="blue")
lines(galton$parent,lm1$fitted,col="red",lwd=3)
```



Allowing for variation

If C_i is the height of child i and P_i is the height of the average parent, then we can imagine writing the equation for a line

$$C_i = b_0 + b_1 P_i + e_i$$

e_i is everything we didn't measure (how much they eat, where they live, do they stretch in the morning...)

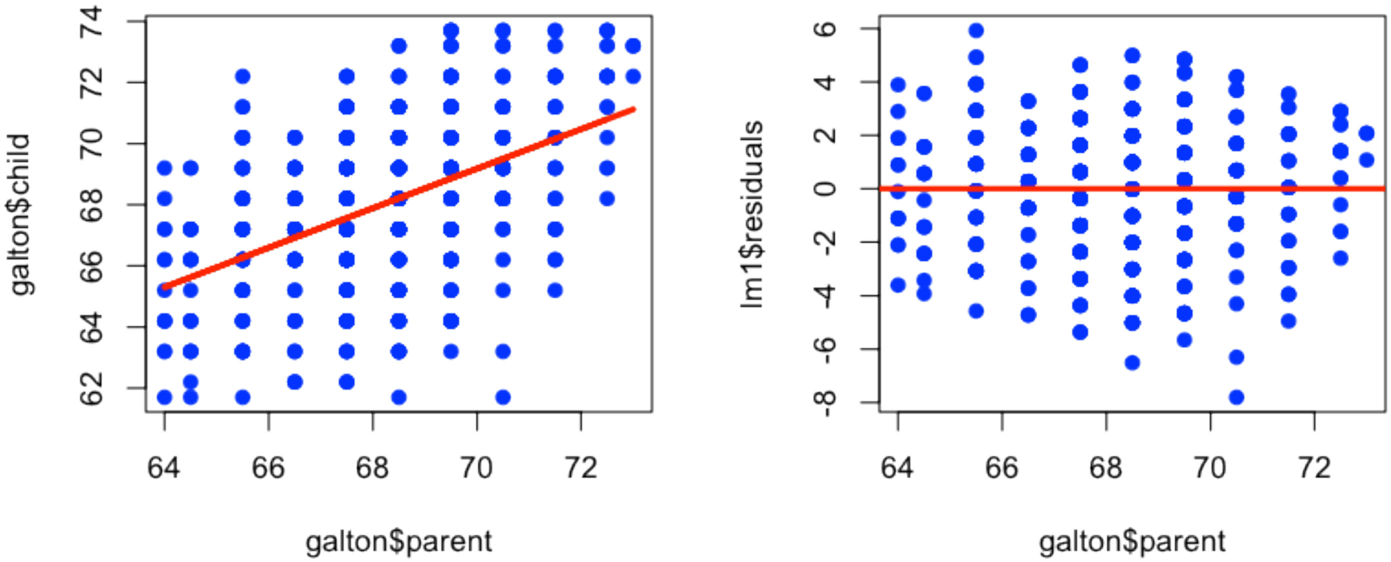
How do we pick best?

If C_i is the height of child i and P_i is the height of the average parent, pick the line that makes the child values C_i and our guesses

$$\sum_{i=1}^{928}(C_i - \{b_0 + b_1 P_i\})^2$$

Plot what is leftover

```
par(mfrow=c(1,2))
plot(galton$parent,galton$child,pch=19,col="blue")
lines(galton$parent,lm1$fitted,col="red",lwd=3)
plot(galton$parent,lm1$residuals,col="blue",pch=19)
abline(c(0,0),col="red",lwd=3)
```


galton$child
galton$parent
lm1$residuals
galton$parent