

Quiz 3

2) A) Log Likelihood:
$$\sum_{i=1}^n (\beta_0(1-x_i) + \beta_1 x_i) y_i - \sum_{i=1}^n \log(1 + e^{\beta_0(1-x_i) + \beta_1 x_i})$$

B)
$$\sum_{i \in G_0} (\beta_0 y_i) - \sum_{i \in G_0} \log(1 + e^{\beta_0}) + \sum_{i \in G_1} (\beta_1 y_i) - \sum_{i \in G_1} \log(1 + e^{\beta_1})$$

$$\star = n_0 \beta_0 \bar{y}_0 - n_0 \log(1 + e^{\beta_0}) + n_1 \beta_1 \bar{y}_1 - n_1 \log(1 + e^{\beta_1})$$

C)
$$\frac{\partial}{\partial \beta_0} = n_0 \bar{y}_0 - \frac{n_0 e^{\beta_0}}{1 + e^{\beta_0}} = 0$$

$$n_0 \bar{y}_0 = \frac{n_0 e^{\beta_0}}{1 + e^{\beta_0}}$$

$$\bar{y}_0 = \frac{e^{\beta_0}}{1 + e^{\beta_0}}$$

$$\bar{y}_0 + \bar{y}_0 e^{\beta_0} = e^{\beta_0}$$

$$\bar{y}_0 + \bar{y}_0 e^{\beta_0} - e^{\beta_0} = 0$$

$$e^{\beta_0} = \frac{\bar{y}_0}{\bar{y}_0 - 1}$$

$$e^{\beta_0} = \frac{\bar{y}_0}{1 - \bar{y}_0}$$

$$\hat{\beta}_0 = \log\left(\frac{\bar{y}_0}{1 - \bar{y}_0}\right)$$

$$\frac{\partial}{\partial \beta_1} = n_1 \bar{y}_1 - \frac{n_1 e^{\beta_1}}{1 + e^{\beta_1}} = 0$$

$$\Rightarrow \bar{y}_1 = \frac{e^{\beta_1}}{1 + e^{\beta_1}}$$

$$\Rightarrow \hat{\beta}_1 = \log\left(\frac{\bar{y}_1}{1 - \bar{y}_1}\right)$$