

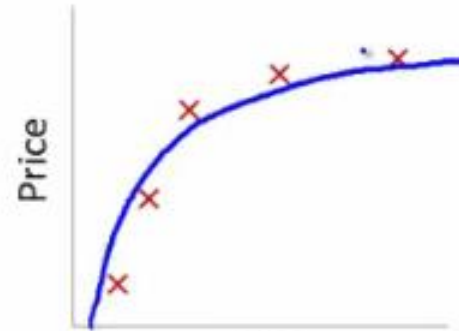
# Regularized Regression Models

# Linear regression fitting example



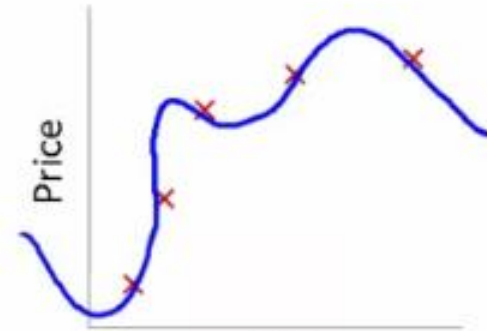
$$\theta_0 + \theta_1 x$$

High bias  
(underfit)



$$\theta_0 + \theta_1 x + \theta_2 x^2$$

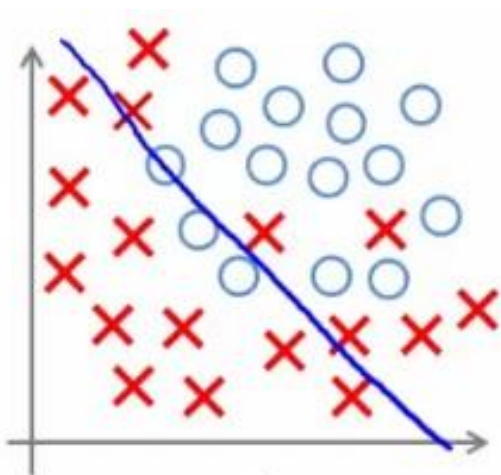
"Just right"



$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

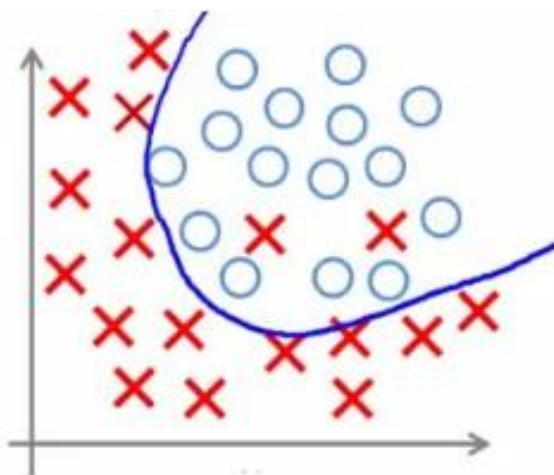
High variance  
(overfit)

# Logistic regression fitting example

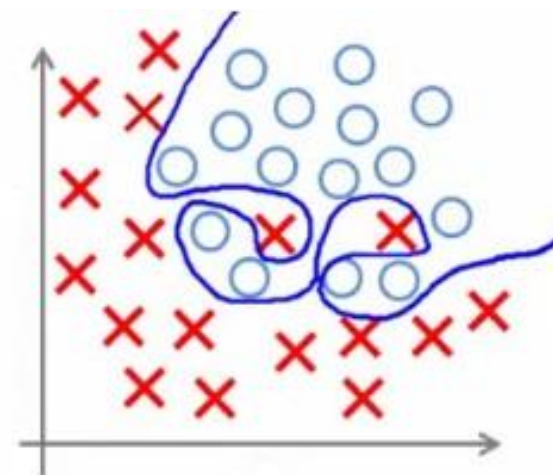


**Under-fitting**

(too simple to  
explain the  
variance)



**Appropriate-fitting**



**Over-fitting**

(forcefitting -- too  
good to be true)

# Overfitting

## Overfitting when.....

- Complex model, too many features, not enough training samples.

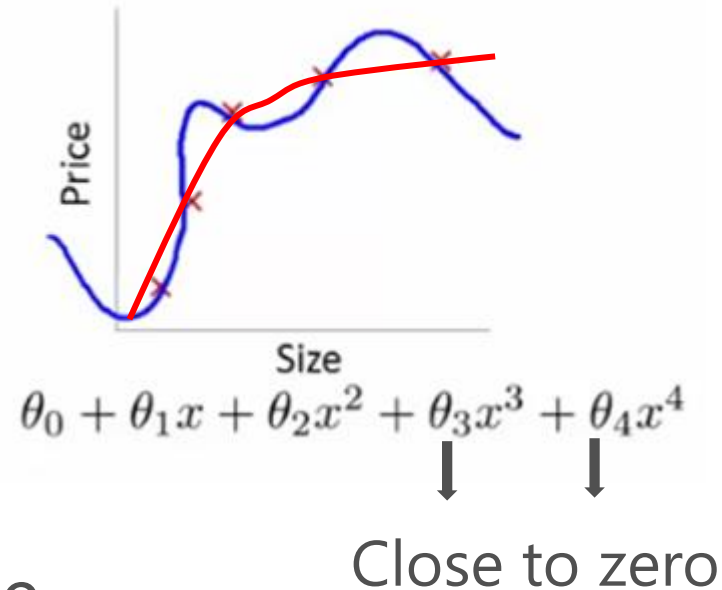
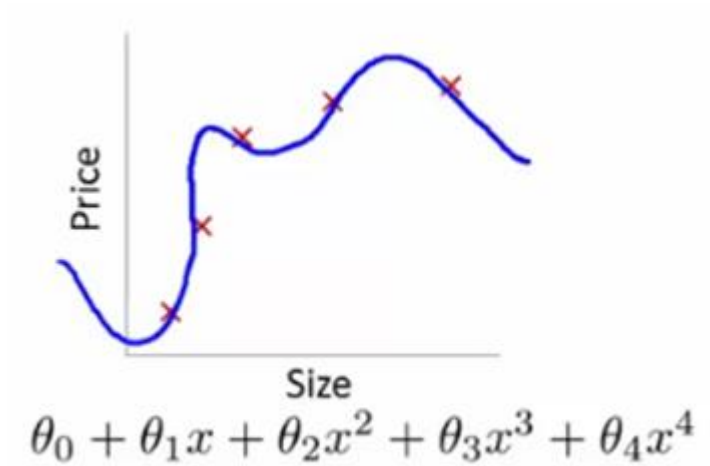
## How to address overfitting??

- Go through each features to decide which to keep.
- Use model selection algorithm to automatically choose features.

# Idea of Regularization

- Keep all the features, but reducing their magnitude of parameter effects in model.
- Shrink  $\theta_j$  parameters

# Regularized regression intuition



- Goal: To minimize cost function  $\theta_j$

$$\frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + 1000 \theta_3 + 1000 \theta_4$$

- Suppose we penalize and make  $\theta_3$  and  $\theta_4$  very small

# Regularization

- Two common types of regularization in linear regression
- L2 regularization (a.k.a ridge regression)

$$\sum_{j=1}^N (y_j - \sum_{i=0}^d \theta_i \cdot x_i)^2 + \lambda \sum_{i=1}^d \theta_i^2$$

- L1 regularization (a.k.a lasso regression)

$$\sum_{j=1}^N (y_j - \sum_{i=0}^d \theta_i \cdot x_i)^2 + \lambda \sum_{i=1}^d |\theta_i|$$

# Regularized-Ridge regression

Regularization by shrink  $\theta_j$  smaller values, as a result

- “less complex” hypothesis function without eliminating features
- More protection from overfitting.

L2: Ridge regression

$$J(\theta) = \frac{1}{2m} \left[ \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$

$\min_{\theta} J(\theta)$



# Regularized-Ridge regression

$$J(\theta) = \frac{1}{2m} \left[ \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$

$\min_{\theta} J(\theta)$

- Goal 1: find the best fit
- Goal 2: keep parameter  $\theta_j$  small
- $\lambda$  is regularization parameter to controls a trade off

# Regularized-Ridge regression

$$J(\theta) = \frac{1}{2m} \left[ \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$

$\min_{\theta} J(\theta)$

- If  $\lambda$  is too large,  $\theta_j$  become too small, as if features have no effect in predicting response.
- If  $\lambda$  is too small,  $\theta_j$  are not regularized.

# Questions?