Question of the Day

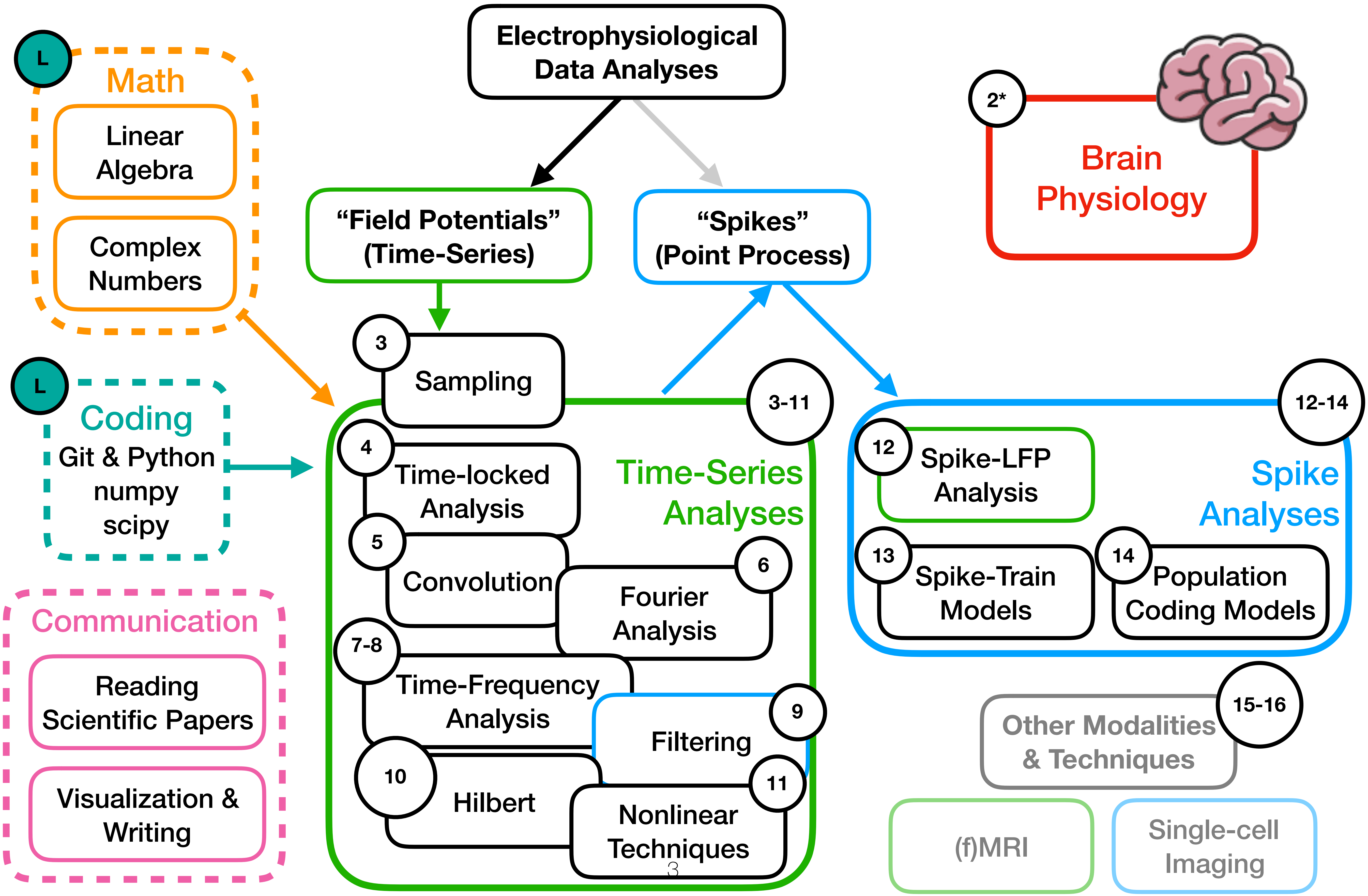
What happens if your Jennifer Aniston neuron died?

Spikes: Population Encoding Models

Lecture 14

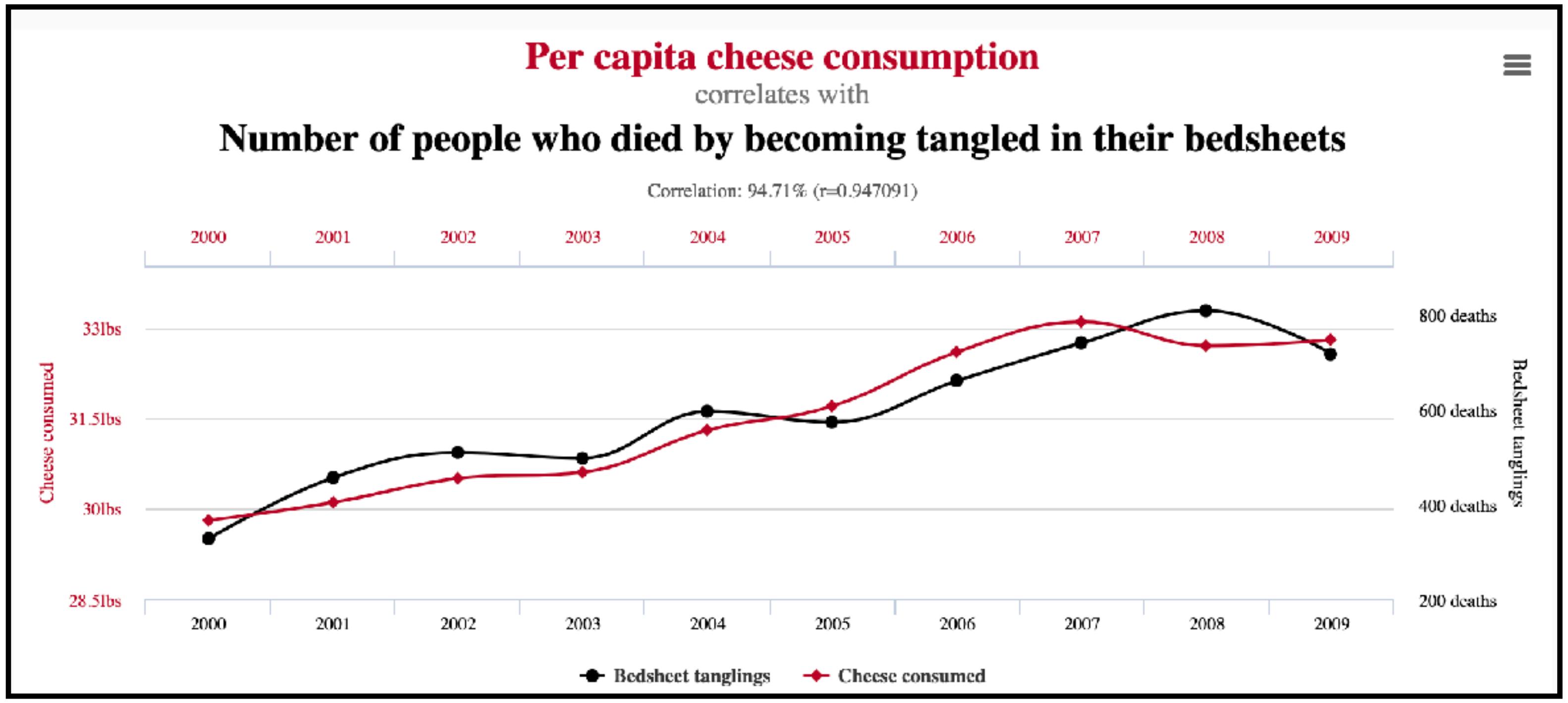
July 25, 2019

Course Outline: Road Map



1. Understand correlation & lagged correlation
2. Motivate population encoding models
3. Introduce population decoding methods

Correlation



$$r_{xy} \stackrel{\text{def}}{=} \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{(n-1)s_x s_y} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2 \sum_{i=1}^{n}(y_i-\bar{y})^2}},$$

x = [6, -1, 2, 6, 5, -9, 2, -3]

y = [-4, 8, 3, -2, 0, 6, 1, 4]

$\bar{x} =$ $\sigma_x =$

$\bar{y} =$ $\sigma_y =$

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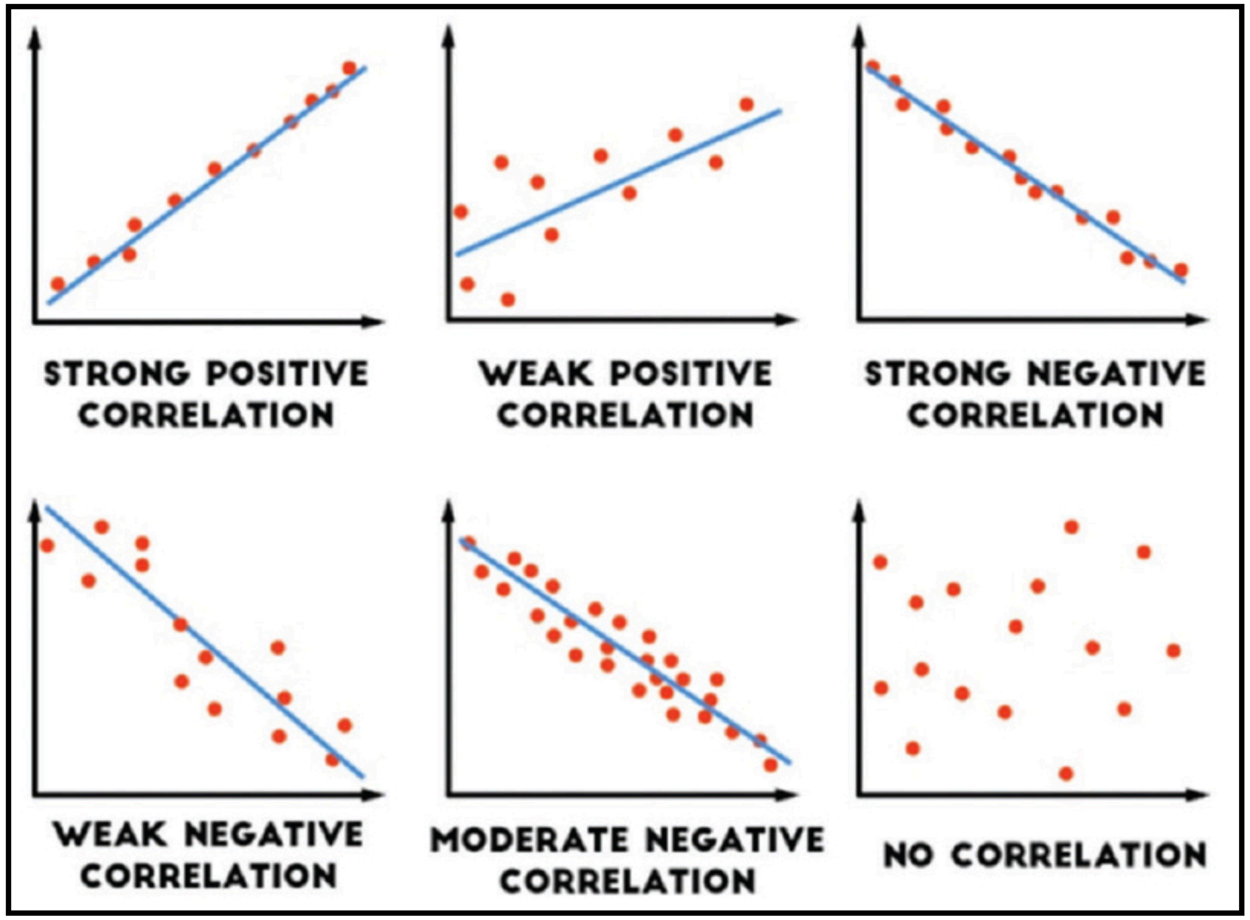
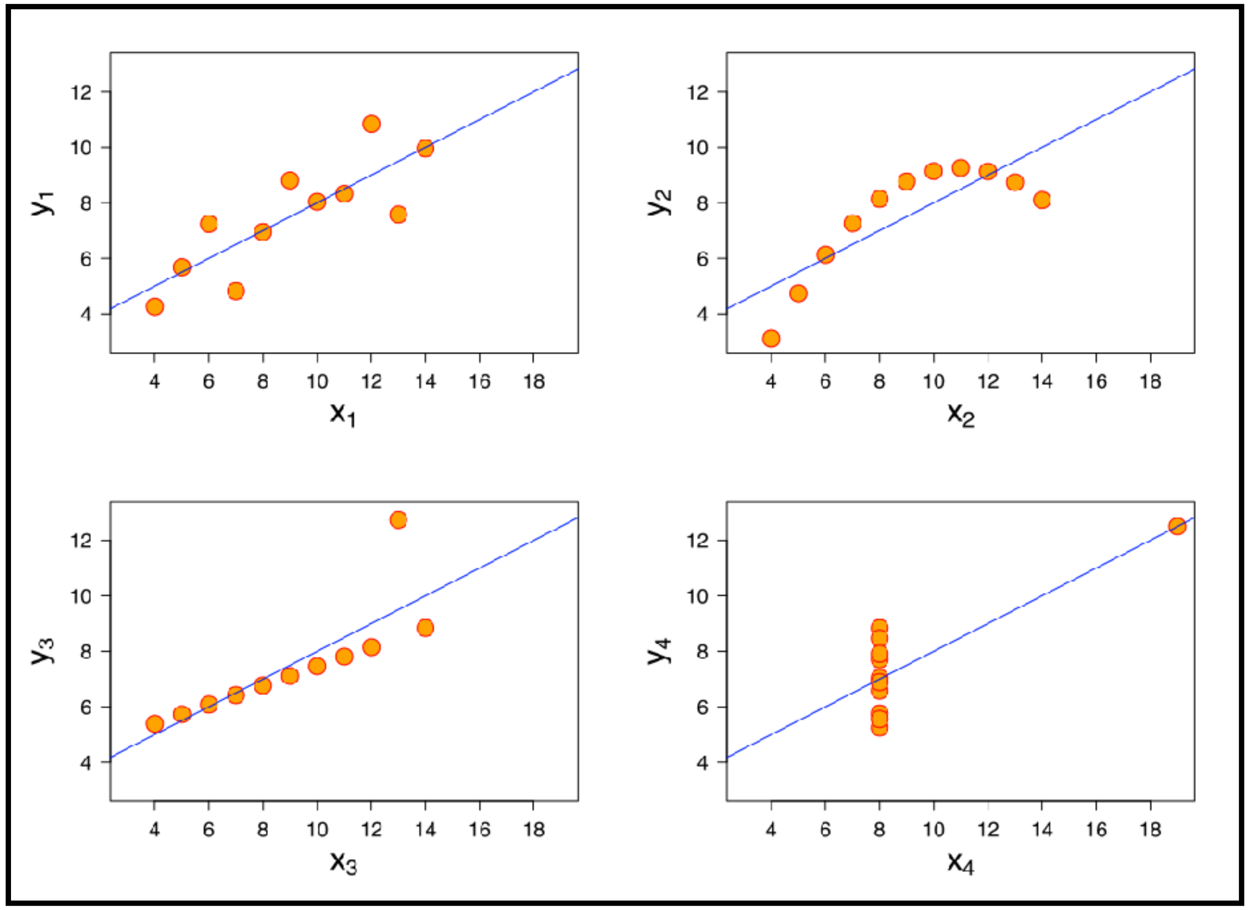
Correlation



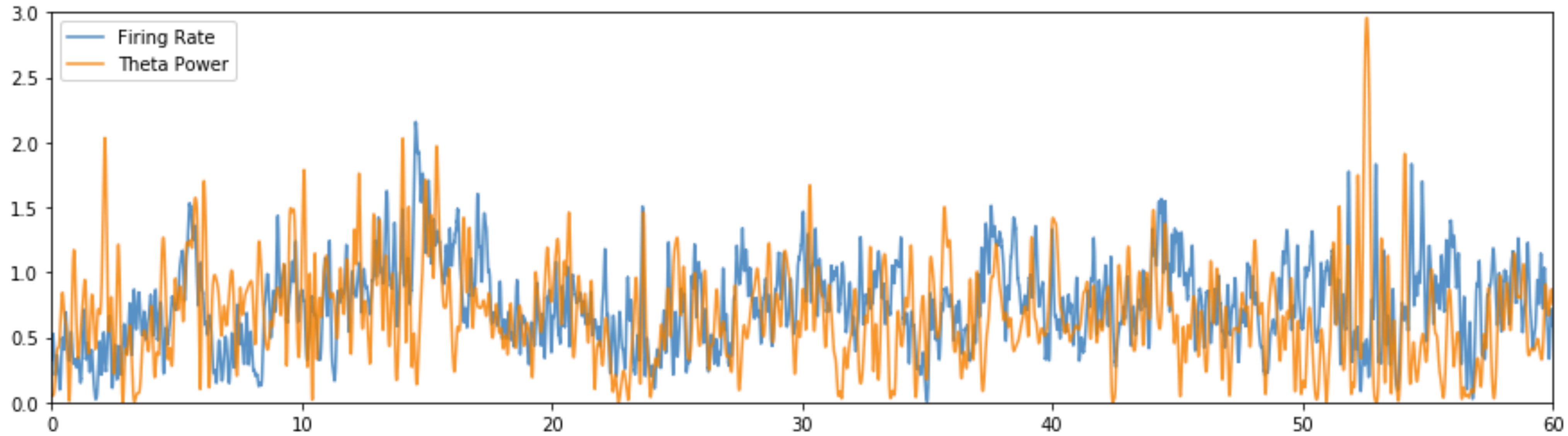
Anscombe's Quartet



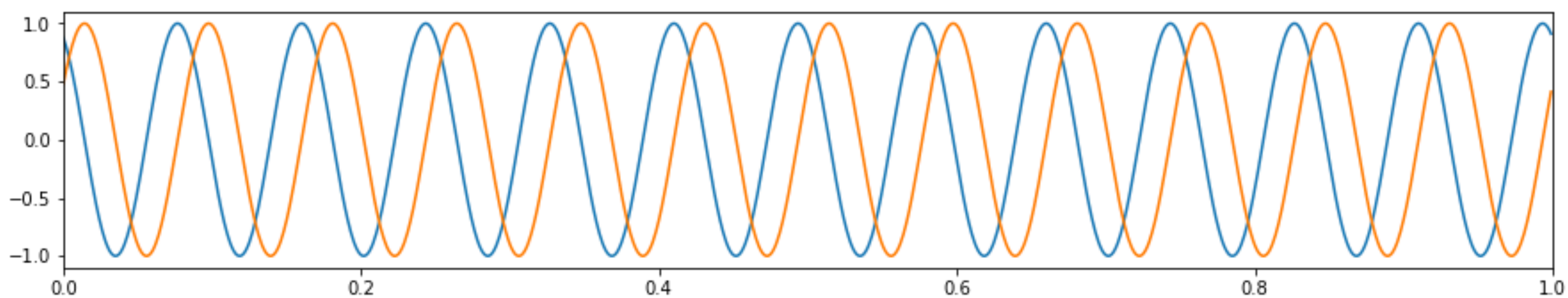
Correlation & other summary statistics may be misleading, plot your data!

Pairwise Correlation

Two correlated variables



What if it takes time for X to affect Y?

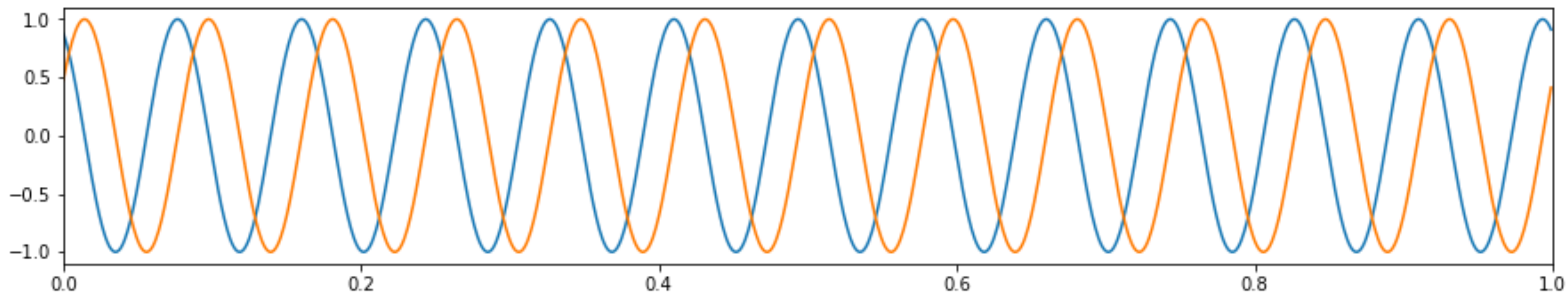


What's the correlation between these 2 signals?

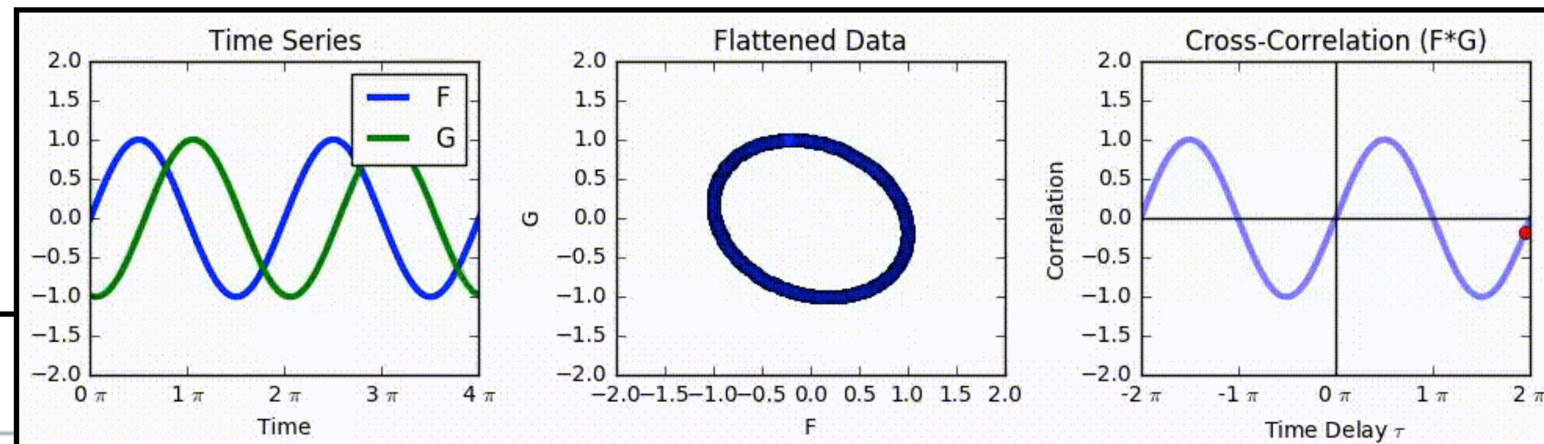
We want to capture relationships with delay as well.

Cross-Correlation (Lagged Correlation)

Solution: we compute correlation coefficient at different time lags.



slide the orange one along



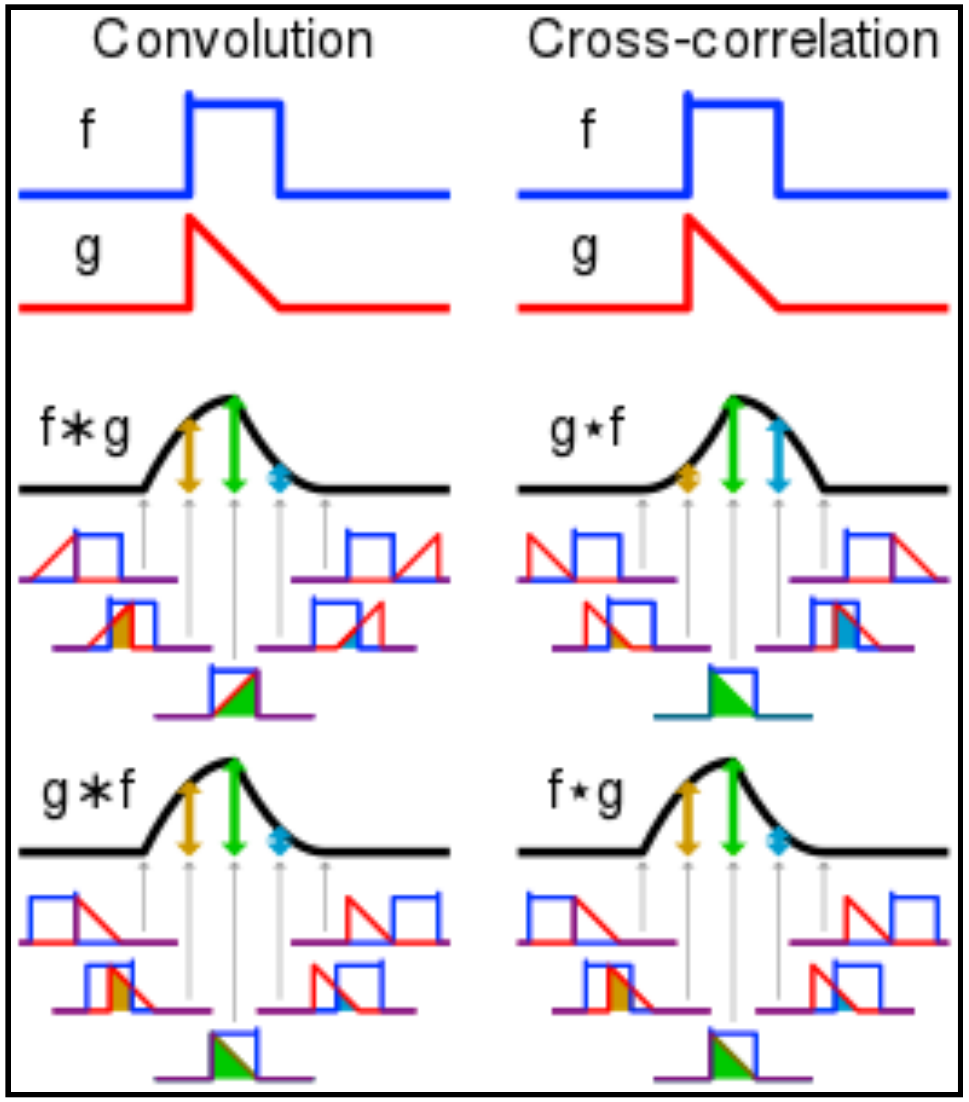
Cross-correlation

From Wikipedia, the free encyclopedia

In signal processing, **cross-correlation** is a measure of similarity of two series as a function of the displacement of one relative to the other. This is also known as a *sliding dot product* or *sliding inner-product*. It is

Want to know what lag produces the highest correlation, and the pattern.

Cross-Correlation vs. Convolution



Convolution: flip and slide (all the way through)

Cross-Correlation: just slide (centered at 0 lag, typically with a max lag parameter)

Mathematically (almost) equivalent, only different in practice.

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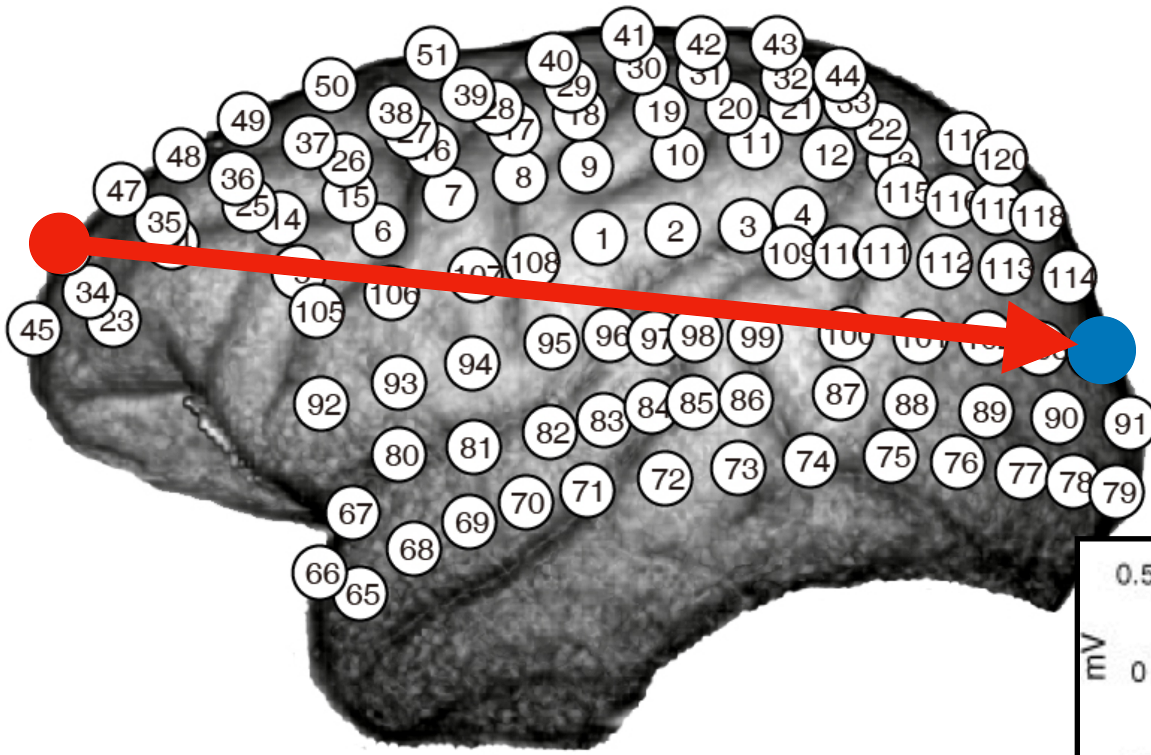
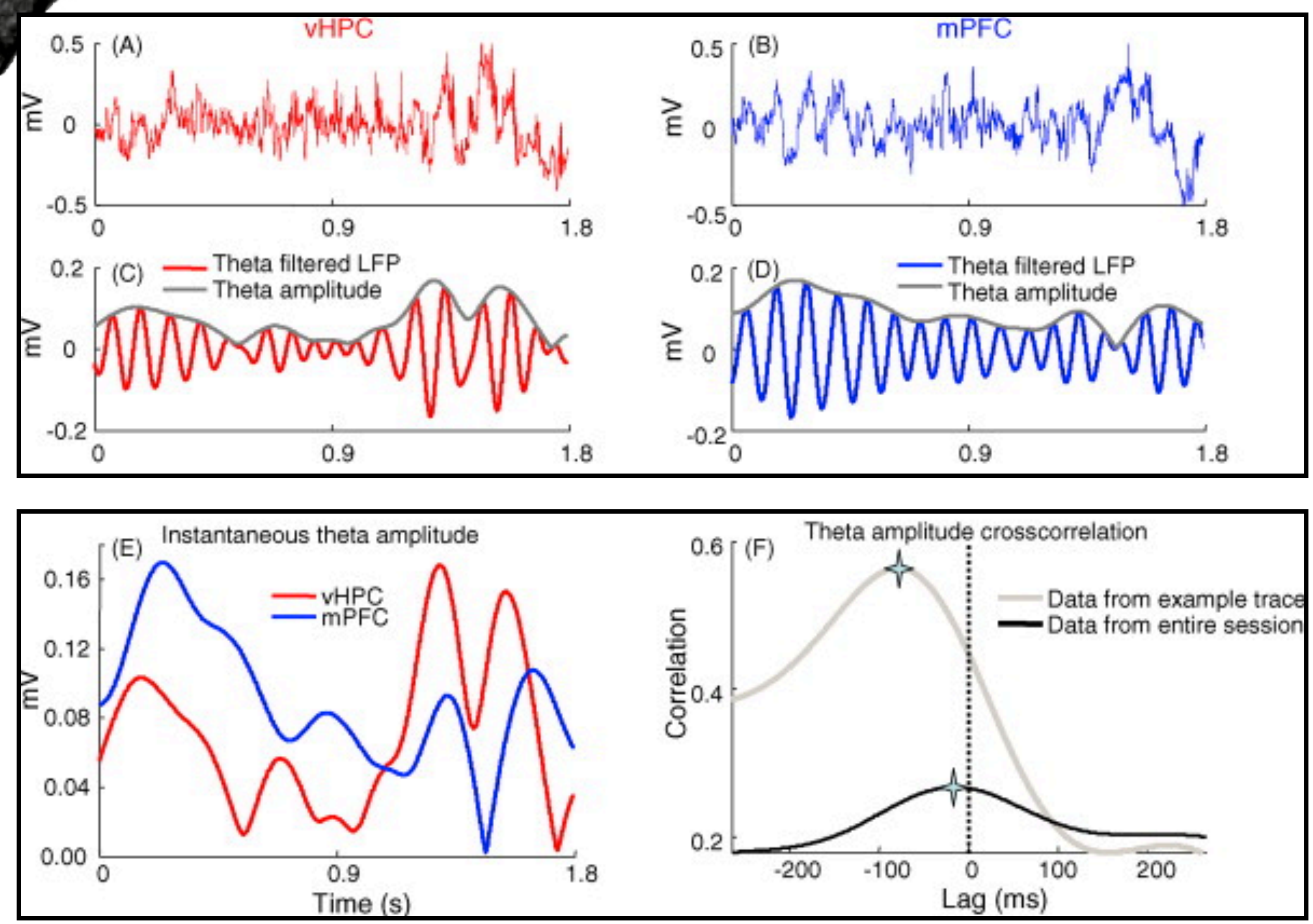
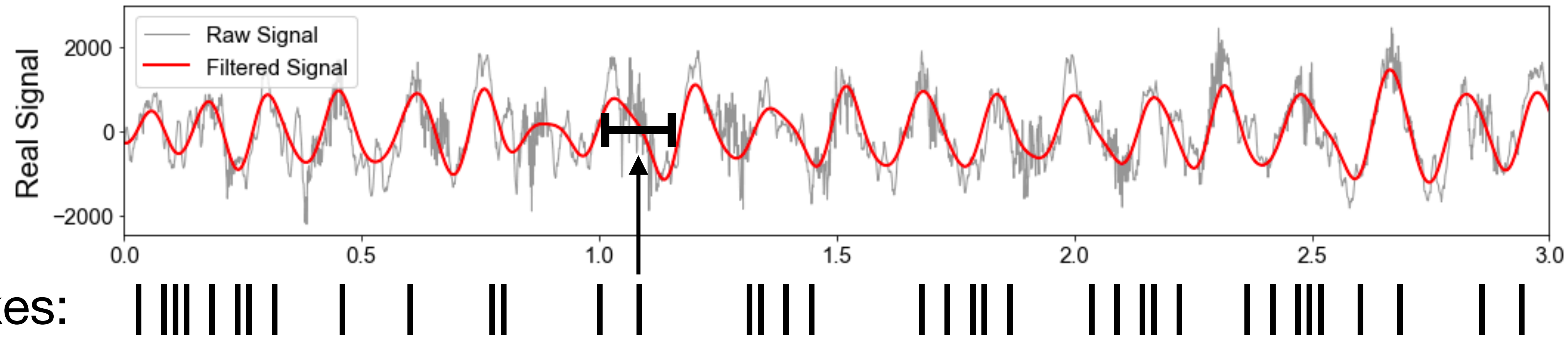
In Neural Signals



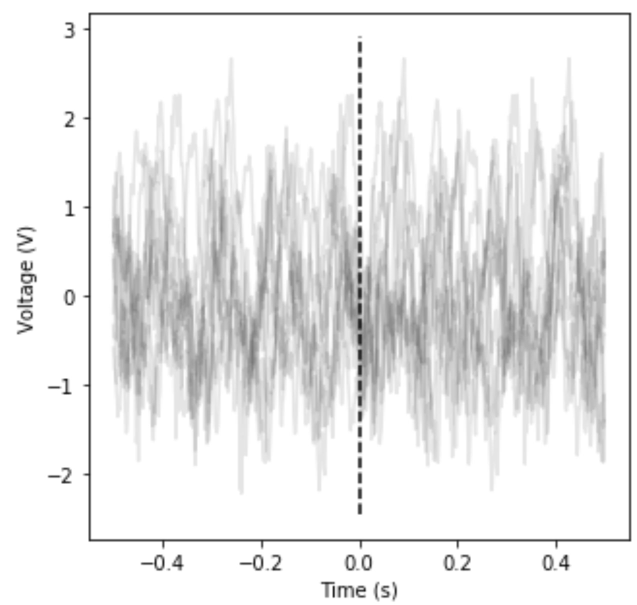
Non-trivial delay between brain regions



Spike Triggered Average

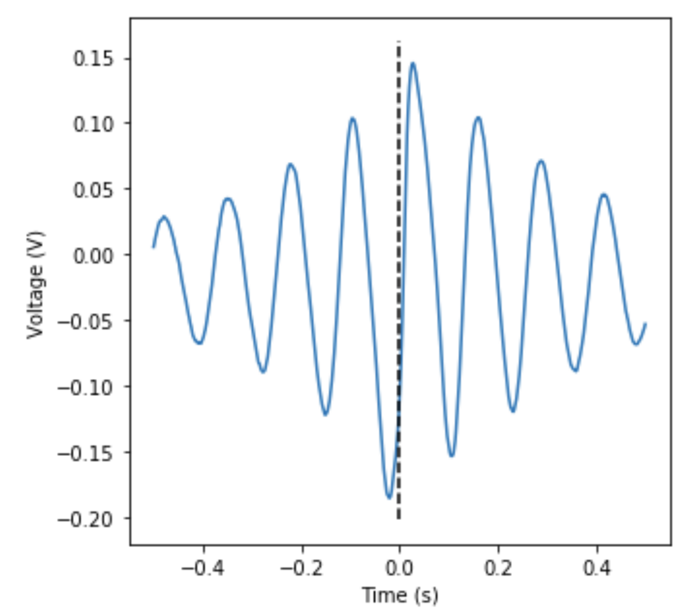


spikes:



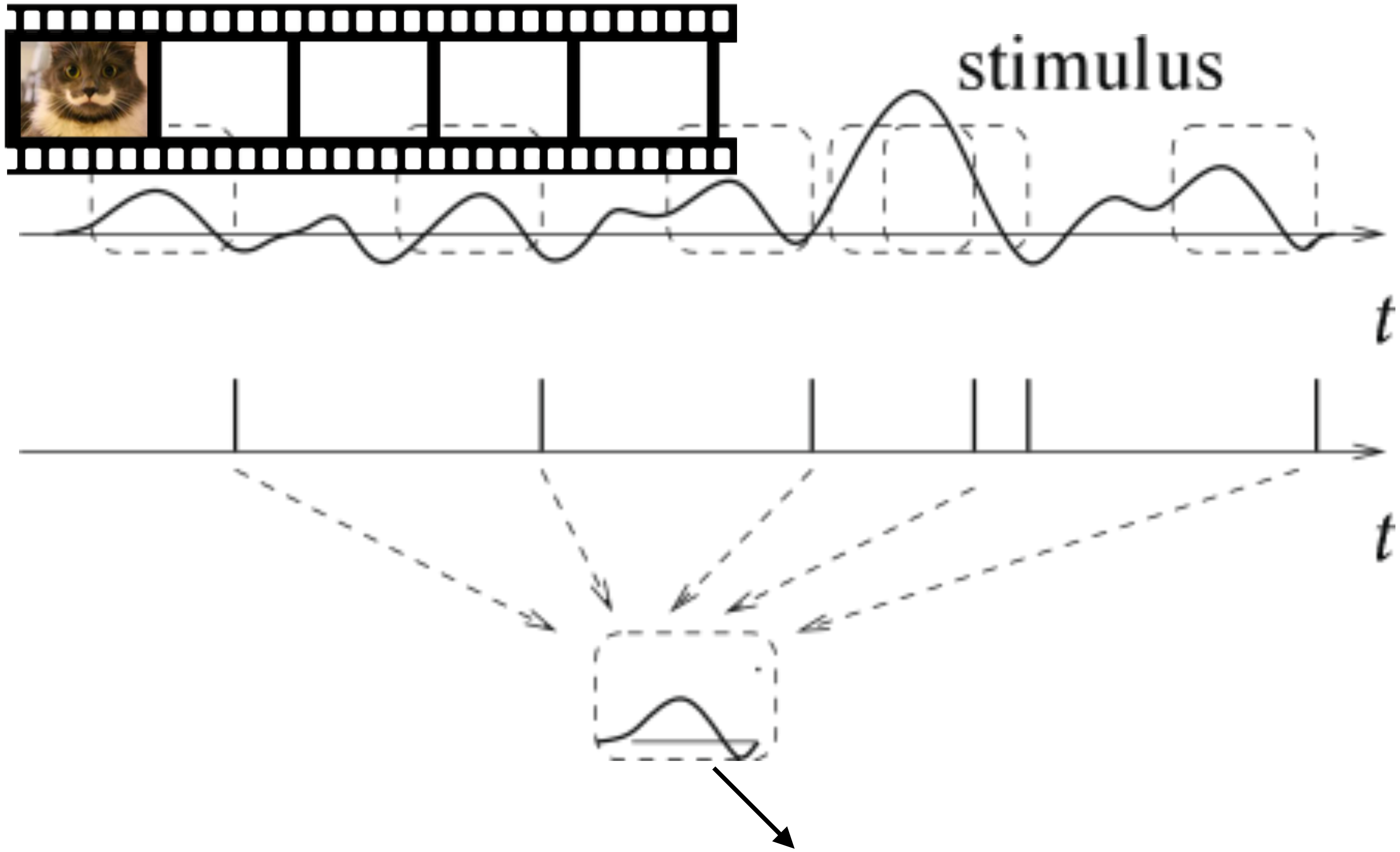
average →

STA

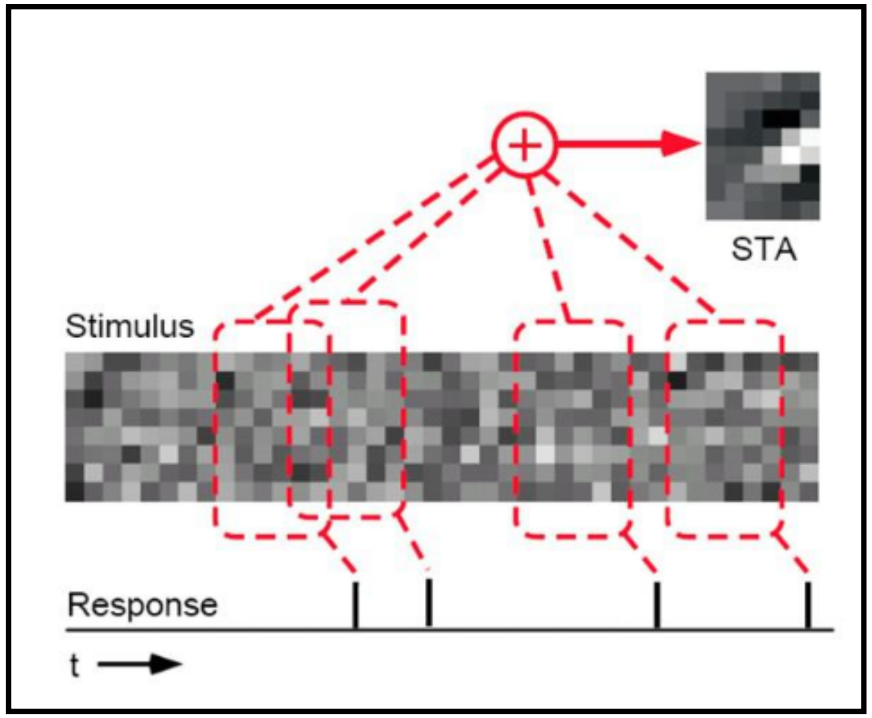


STA is the cross-correlation between binned spike count vector (delta-like train) with another signal.

STA for Stimulus Reconstruction



Average stimulus that triggers a spike.



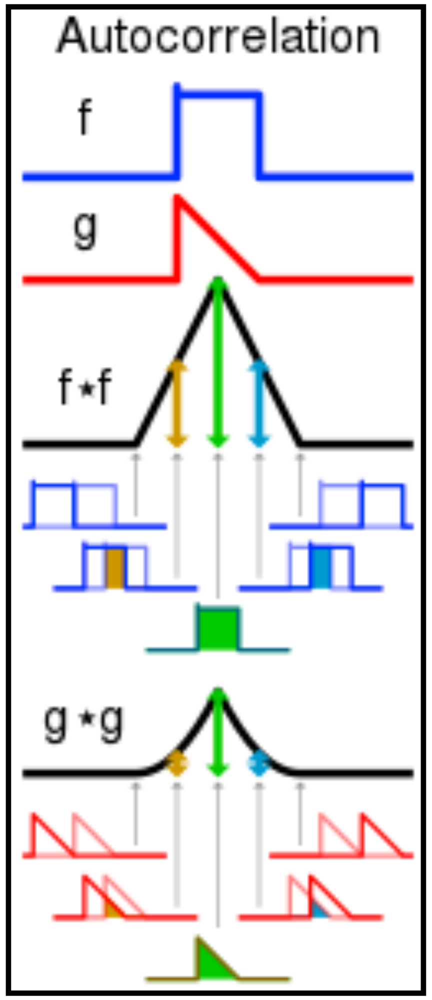
Want to see what the stimulus looks like, on average, just around a spike.

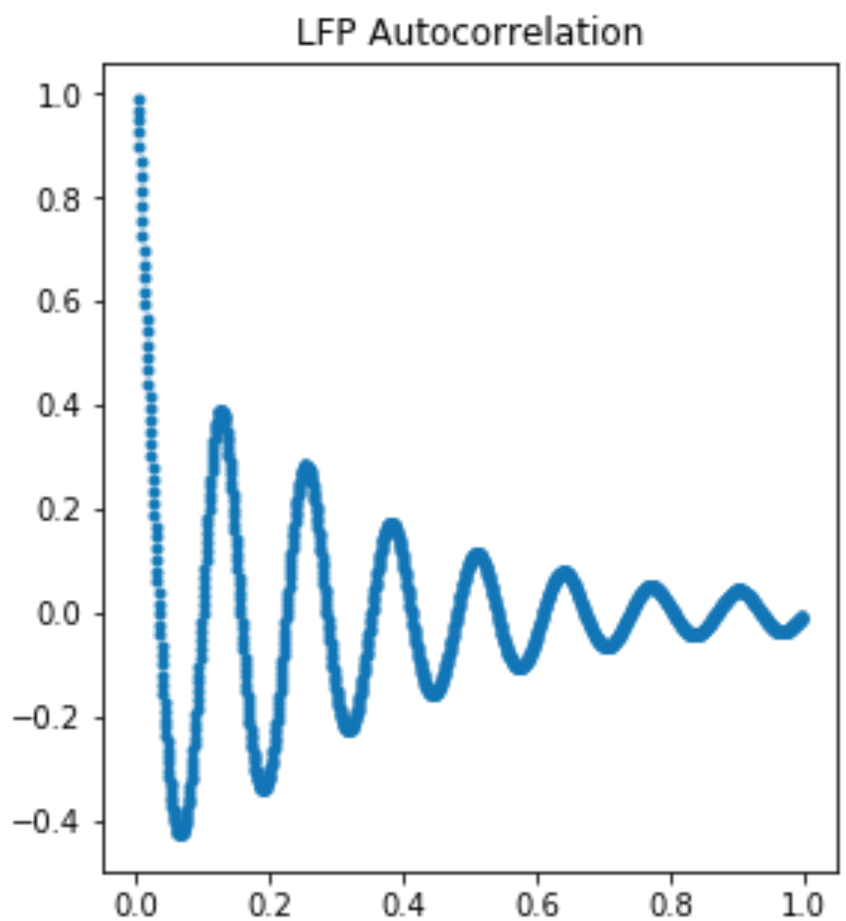
Inverse of ERP analysis (what does brain response look like around a stimulus)

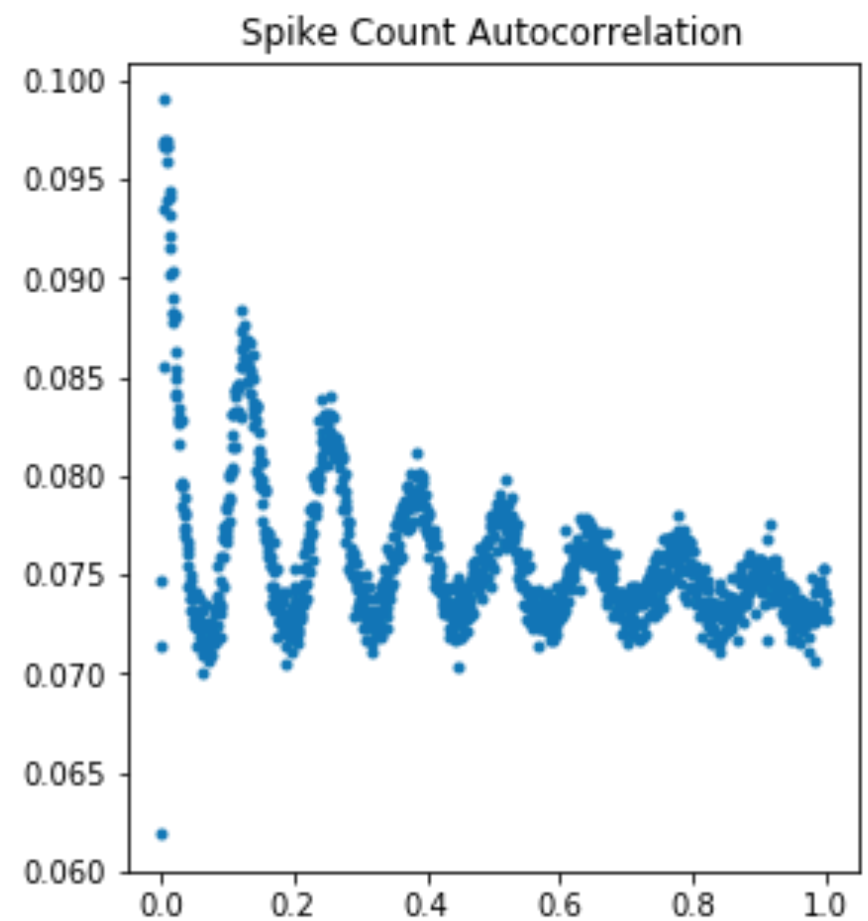

Presented clip
Clip reconstructed from brain activity

Autocorrelation

What happens if you cross-correlate a signal with itself?





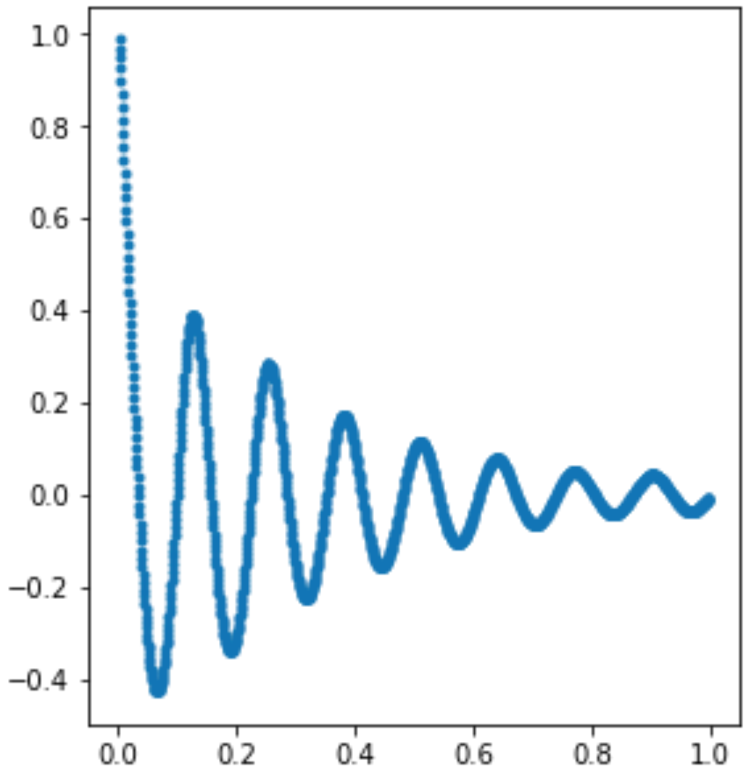


Always 1 at 0 lag, and symmetrical.

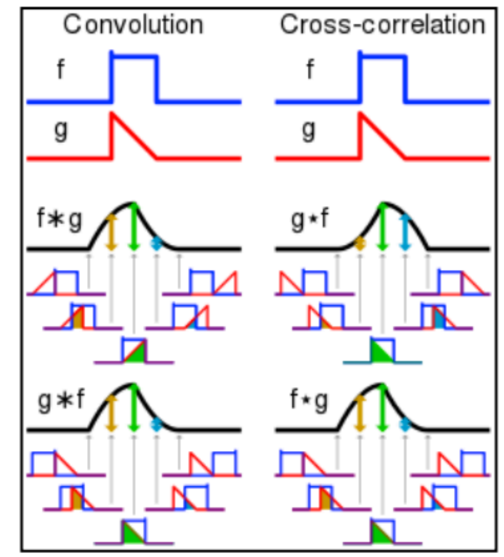
Finds temporal structures in the signal (decay, periodicity, etc.).
Very common to look at spike train autocorrelation.

Wiener-Khinchin Theorem

Autocorrelation



$$F_R(f) = \mathrm{FFT}[X(t)]$$
$$S(f) = F_R(f)F_R^*(f)$$
$$R(\tau) = \mathrm{IFFT}[S(f)]$$

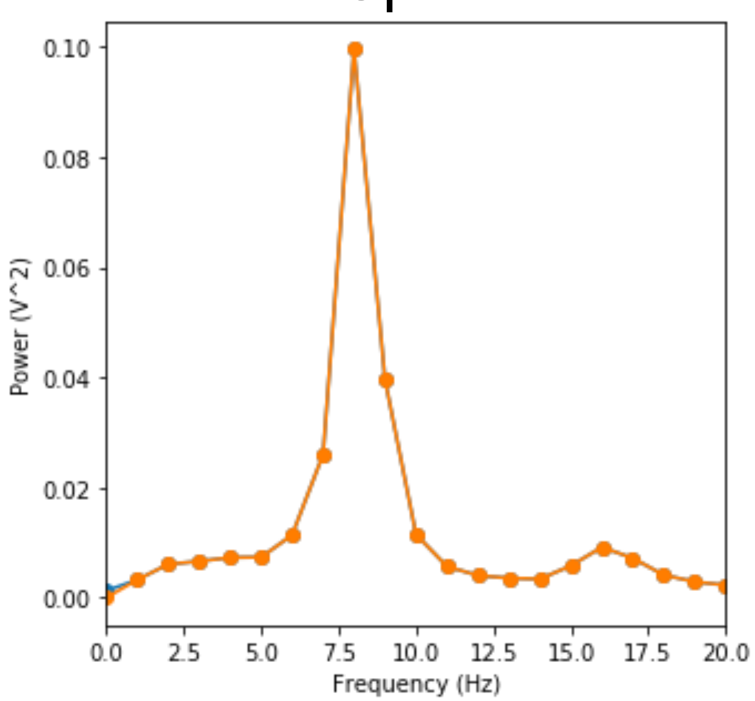


Convolution: flip and slide (all the way through)

Cross-Correlation: just slide (centered at 0 lag, typically with a max lag parameter)

Autocorrelation is convolution of a signal with itself, but time-reversed.

Power Spectrum



Convolution in time domain equals multiplication (of complex Fourier coefficients) in frequency domain.

I'm not really sure how to you go from time domain to frequency domain so easily

Autocorrelation is the same as multiplication between FT(x(t)) and FT(x(-t))

FT of autocorrelation is the power spectrum.

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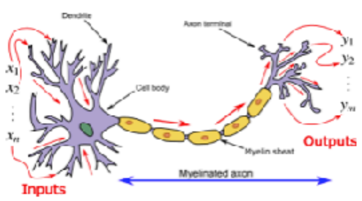
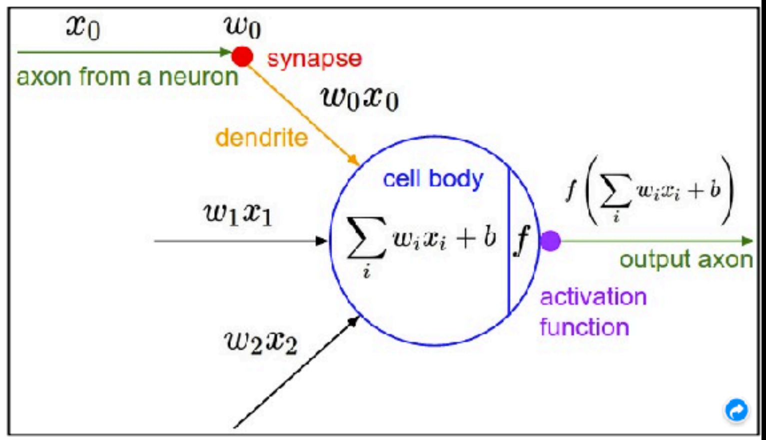
2. Motivate population encoding models

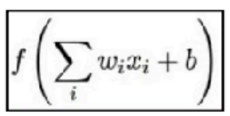
3. Introduce population decoding methods

Single-Neuron Representations

Single Neuron as a Computational Device



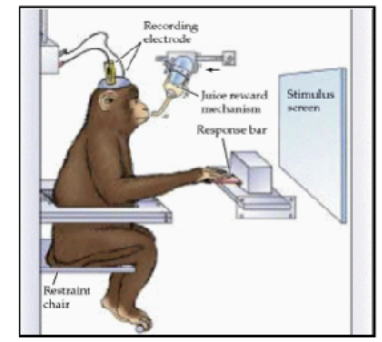




dot product
w: "synaptic" weights
x: inputs
b: bias
f( ): nonlinearity

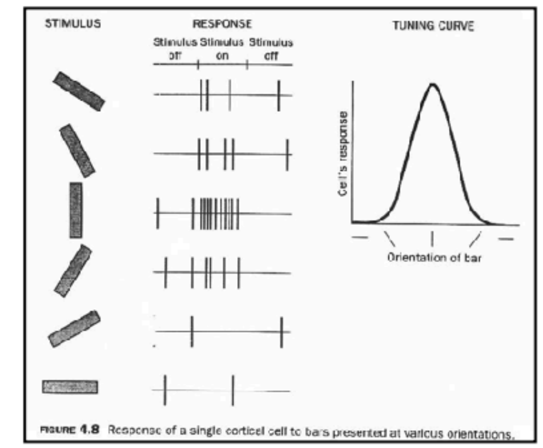
But what is x, physically?

Receptive Fields & Stimulus-Tuned Neurons



Stimulus with **varying aspects** are presented, e.g.:

- location
- orientation (angle)
- color
- sound frequency
- etc...



"This neuron has an orientation preference."

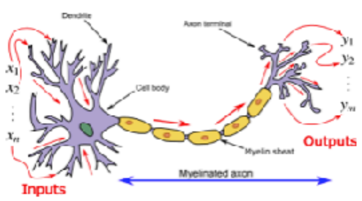
If a single neuron reliably responded to a stimulus, that's sometimes thought to be a "representation" in the brain, e.g.,

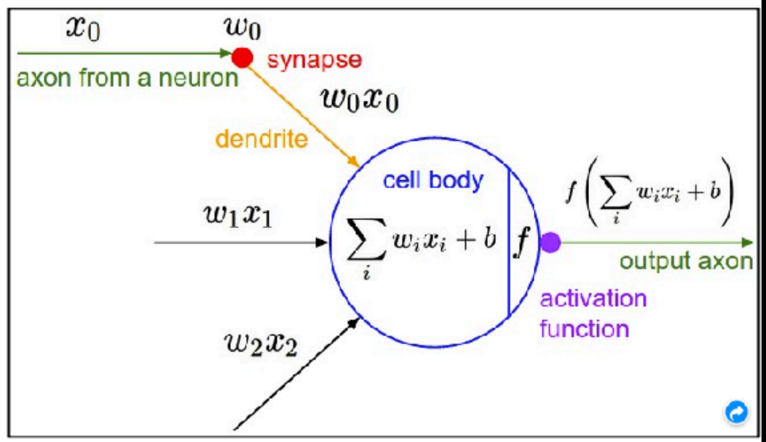
of place by your place cells,
of orientation by your orientation cells,
of Jennifer Aniston by your Jennifer Aniston cell.

Sparse representation (coding)

Not Single-Neuron Representations

Single Neuron as a Computational Device





$$f\left(\sum_i w_i x_i + b\right)$$

dot product
w: "synaptic" weights
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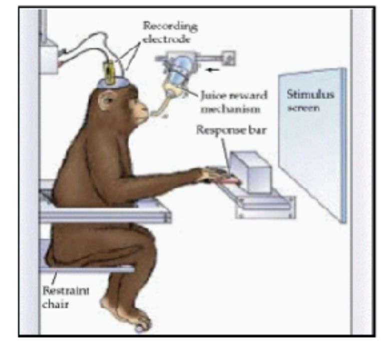
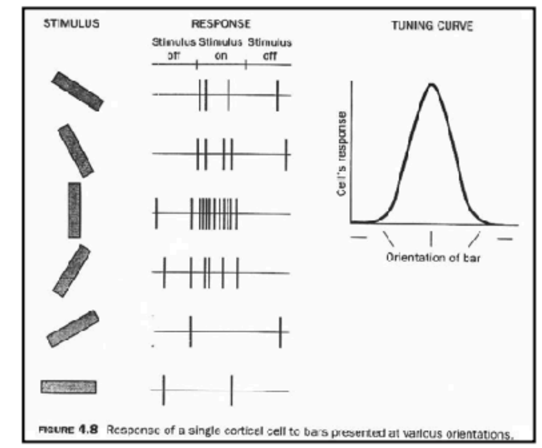
f( ): nonlinearity

But what is x, physically?

Receptive Fields & Stimulus-Tuned Neurons





Stimulus with **varying aspects** are presented, e.g.:

- location
- orientation (angle)
- color
- sound frequency
- etc...

"This neuron has an orientation preference."

Historical context: technology wasn't that great 50 years ago, so you could only record (more or less) one cell at a time, so (typically), they found the most active cells for a task.

Sparse coding is not very robust to biology and noise: what happens if your JA cell (or grandmother cell) died?

CogSci Tangent: theory of how the brain functioned depended on available methods

Population Codes

For a single neuron to be a stimulus representation, it has to be:
1) always active when the stimulus is present
2) not be active when the stimulus is not present

Turns out, most recorded neurons **don't satisfy** these completely…

…which makes sense.

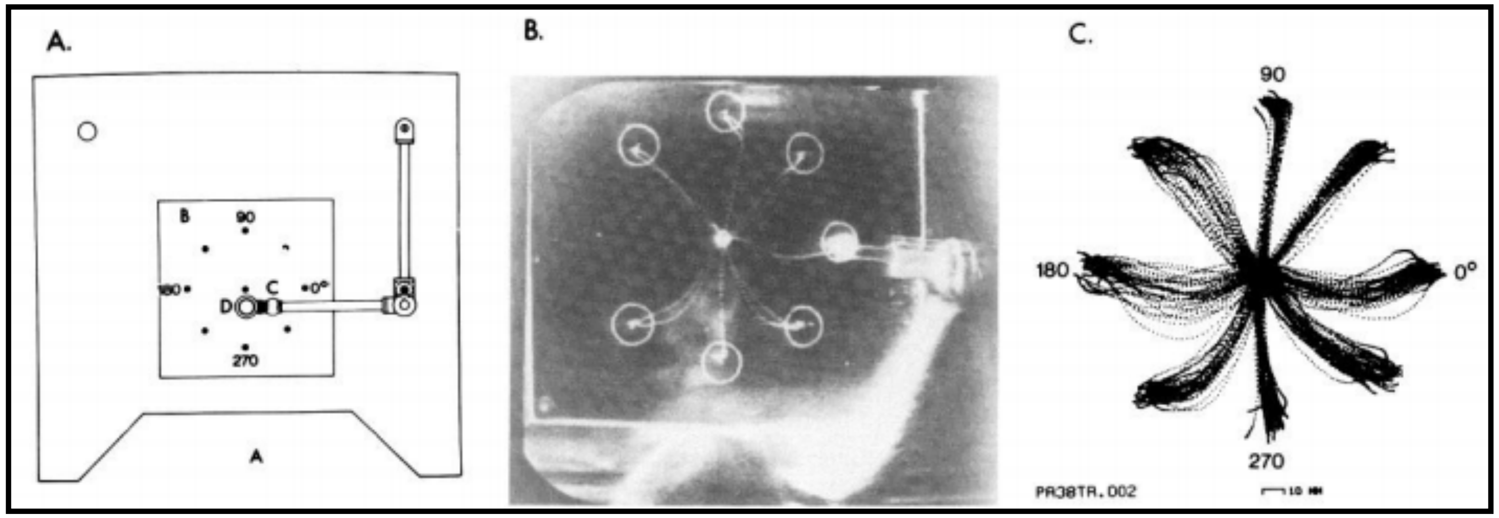
(but some do)

Neuronal Population Coding of Movement Direction

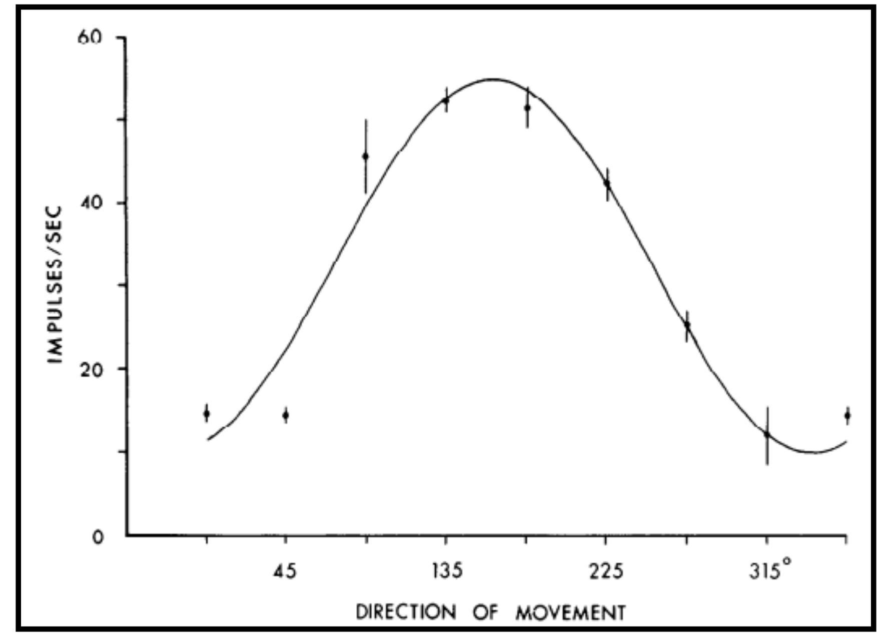
Apostolos P. Georgopoulos, Andrew B. Schwartz, Ronald E. Kettner

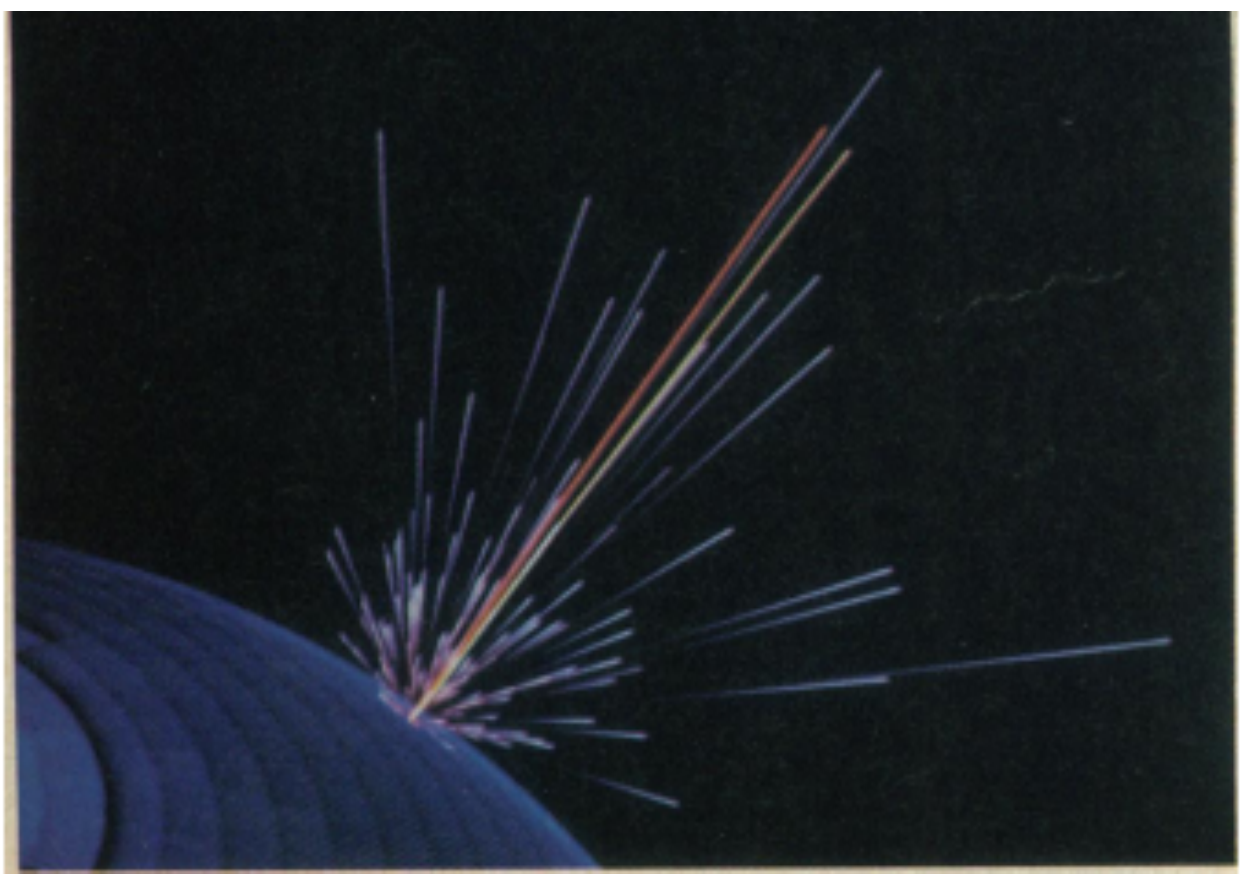
Population Codes

Movement Task



Wide Tuning Curve!





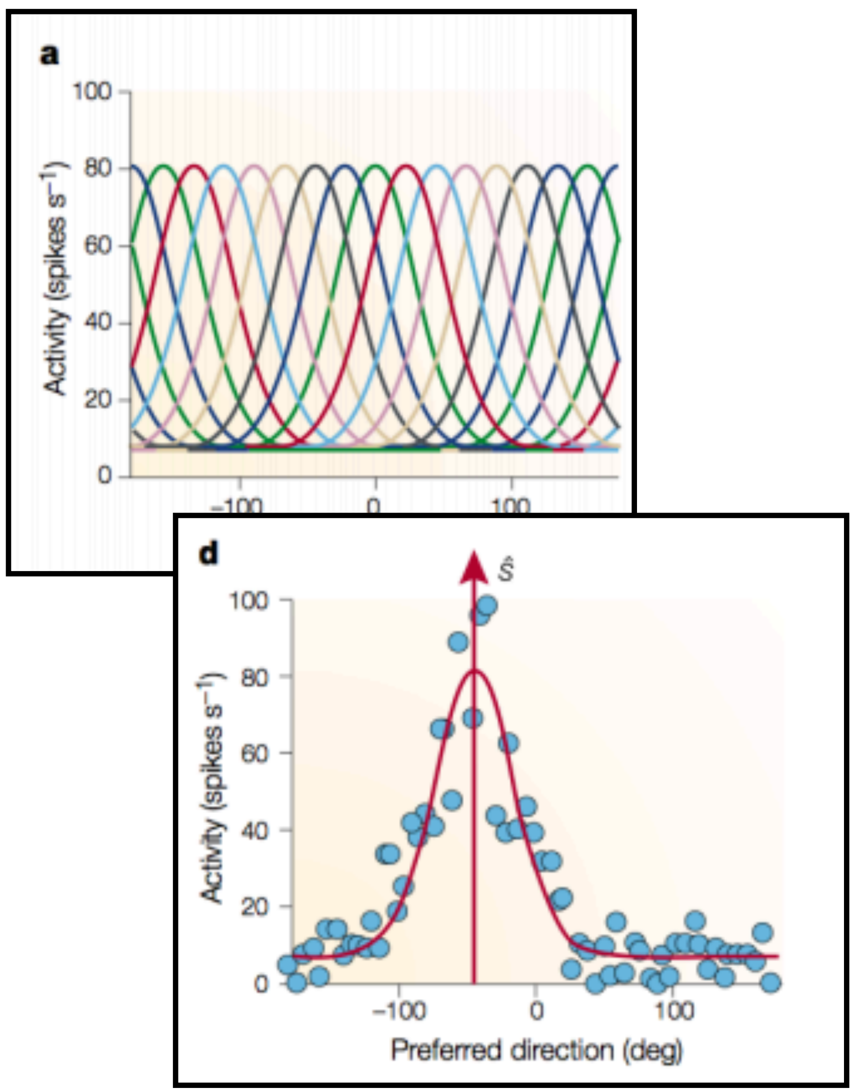
Average the vector response of all the neurons (weighed by their firing rate) to decode movement direction.

(Simple & outdated model)

Distributed Representation

Neuronal Population Coding of Movement Direction

Apostolos P. Georgopoulos, Andrew B. Schwartz, Ronald E. Kettner



Pouget et al., 2000

In contrast to sparse (local) representation is **distributed (& parallel) representation**

Parallel Distributed Processing (PDP, or connectionism) was the first name given to artificial neural networks.

Each neuron contributes a tiny amount of information about the stimulus or action (correlation structure).

Robust to noise and can have more efficient representations.

Goals for Today

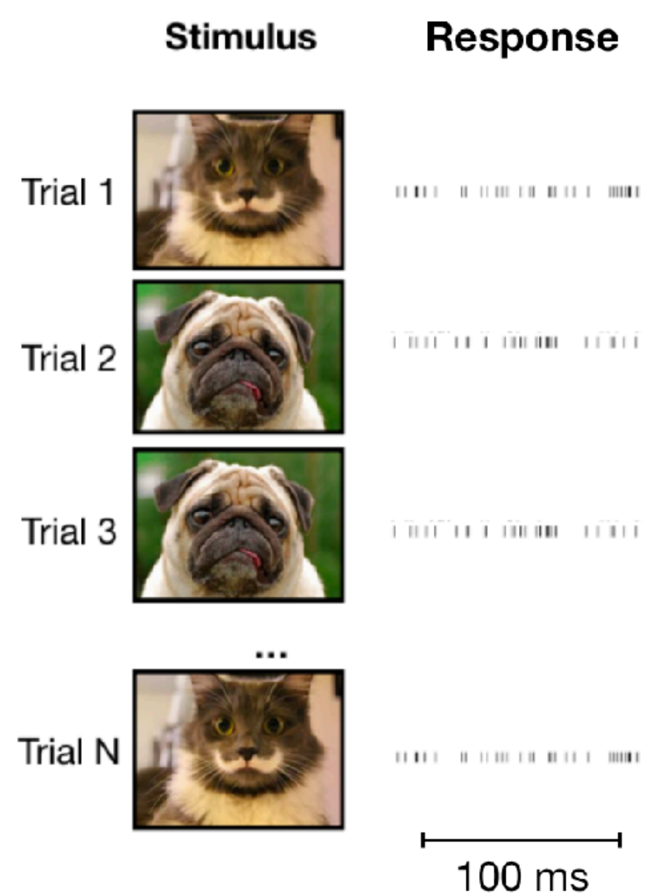
1. Understand correlation & lagged correlation

2. Motivate population encoding models

3. Introduce population decoding methods

Sparse vs. Distributed Coding

Let’s say you record 2 neurons



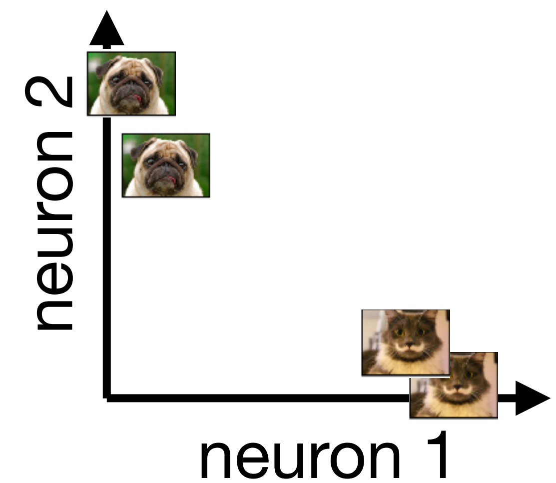
spikes

[10, 0]

[1, 8]

[0, 11]

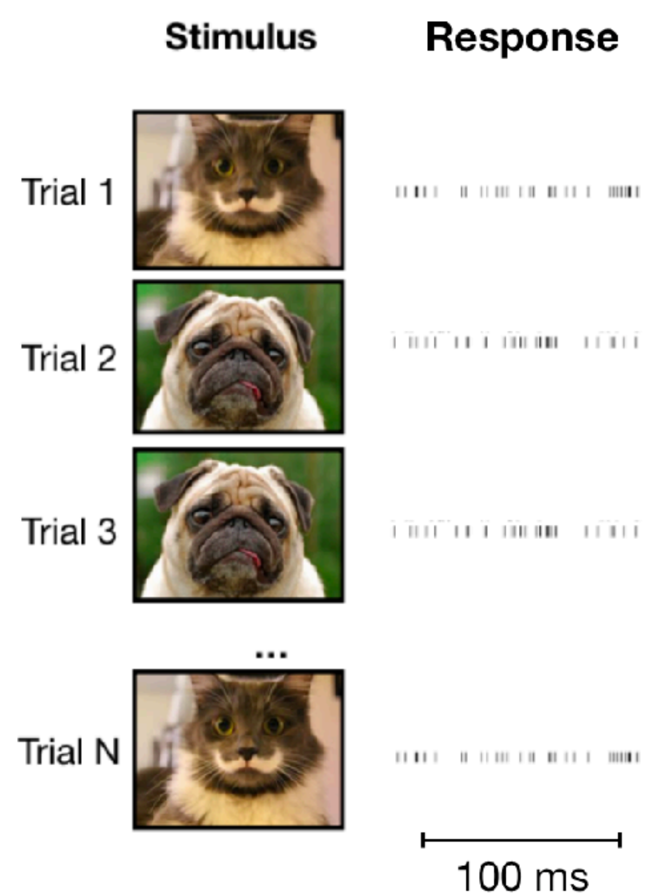
[9, 2]



Sparse code: dog neuron and cat neuron

Sparse vs. Distributed Coding

Let's say you record 2 neurons



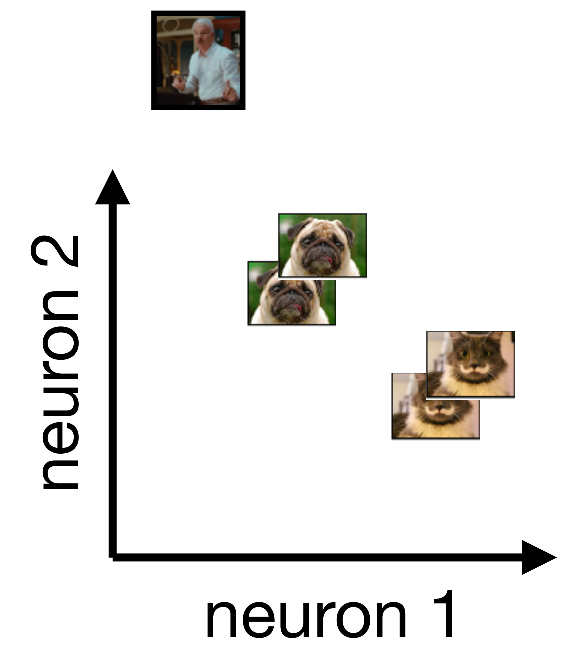
spikes

[6, 4]

[3, 9]

[4, 10]

[8, 3]



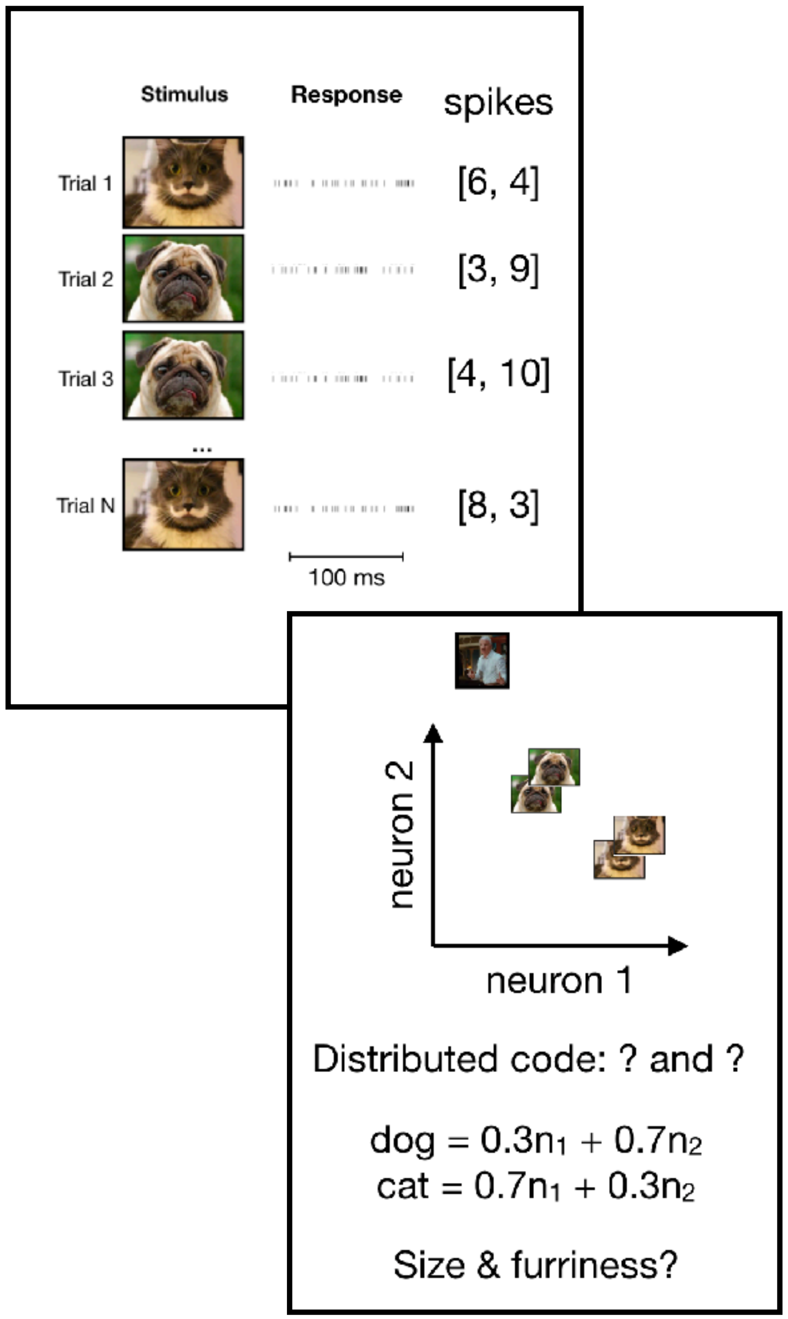
Distributed code: ? and ?

$$dog = 0.3n_1 + 0.7n_2$$
$$cat = 0.7n_1 + 0.3n_2$$

Size & furriness?

Analyzing Population Response

Regression, Classification, Correlation Decomposition

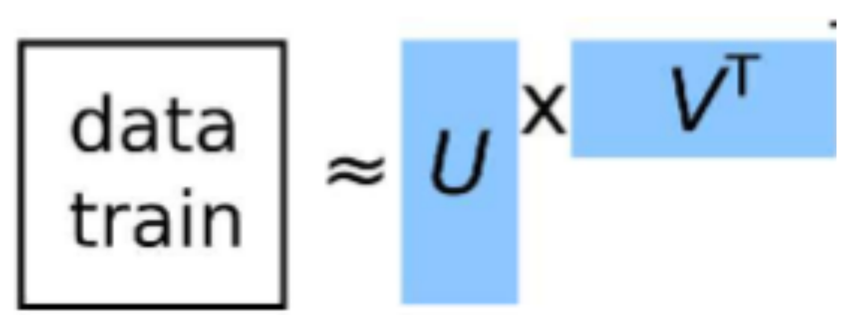


Regression / Classification

Find weights to decode stimulus from activity

Correlation Decomposition

Understand correlation structure between neurons



Summary

1. Understand correlation & lagged correlation

2. Motivate population encoding models

3. Introduce population decoding methods

https://tinyurl.com/cogs118c-att