K-Maps

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Karnaugh Maps Definitions

- Boolean algebra is prone to errors, requires memorization of rules and theorems
- Karnaugh Maps are a graphical method for simplifying boolean equations
- Invented in 1953 by Maurice Karnaugh

Karnaugh Maps Definitions

- Logic minimization involves combining terms (Minterms, SOP)
- Two terms containing an implicant P and the true and complementary forms of some variable A are combined to eliminate A:

$$PA + P\bar{A} = P$$

Eliminating redundant terms:
Ex: Minterms 3, 7 = ~abc, abc

If the function is true in both cases (F = ~abc + abc) with “~a” or “a”, so it doesn't matter, we can eliminate the literal “a” and keep only “bc”.

Karnaugh Maps Definitions

- Two dimensional truth table for 2, 3, and 4 variable functions
- There also 5 and 6 variable K-maps. Not covered.

a	b	x
0	0	0
0	1	0
1	0	0
1	1	1

A AND B

A \ B	0	1
0	0	0
1	0	1

a	b	f
0	0	0
0	1	1
1	0	1
1	1	1

A OR B

A \ B	0	1
0	0	1
1	1	1

a	b	x
0	0	0
0	1	1
1	0	1
1	1	0

A XOR B

A \ B	0	1
0	0	1
1	1	0

Karnaugh Maps Definitions

$$\overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

- 3-variable K-map
- K-maps are arranged such that adjacent cell differ by only one change
- BC -> 00 | 01 | 11 | 10
- 01 -> 10 (both digits change)
- 01 -> 11 (only first digit change)
- Numbering the cells helps

SOP	A	B	C	x
$\overline{A}\cdot\overline{B}\cdot\overline{C}$	0	0	0	0
$\overline{A}\cdot\overline{B}\cdot C$	0	0	1	0
$\overline{A}\cdot B\cdot\overline{C}$	0	1	0	0
$\overline{A}\cdot B\cdot C$	0	1	1	1
$A\cdot\overline{B}\cdot\overline{C}$	1	0	0	0
$A\cdot\overline{B}\cdot C$	1	0	1	1
$A\cdot B\cdot\overline{C}$	1	1	0	1
$A\cdot B\cdot C$	1	1	1	1

A \ BC	00	01	11	10
0	0	0	1	0
1	0	1	1	1

Karnaugh Maps Definitions

- 4 variable K-Map
- Any adjacent cell also only be different by a change in only one variable
- AB -> 00 | 01 | 11 | 10
- CD -> 00 | 01 | 11 | 10

ab \ cd	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

Karnaugh Maps For Simplification

$$a \cdot b + a \cdot \overline{b} = a$$

Karnaugh Maps For Simplification

$$a \cdot b + a \cdot \overline{b} = a$$

SOP	a	b	x
$\overline{a} \cdot \overline{b}$	0	0	0
$\overline{a} \cdot b$	0	1	0
$a \cdot \overline{b}$	1	0	1
$a \cdot b$	1	1	1

a \ b	0	1
0	0	0
1	1	1

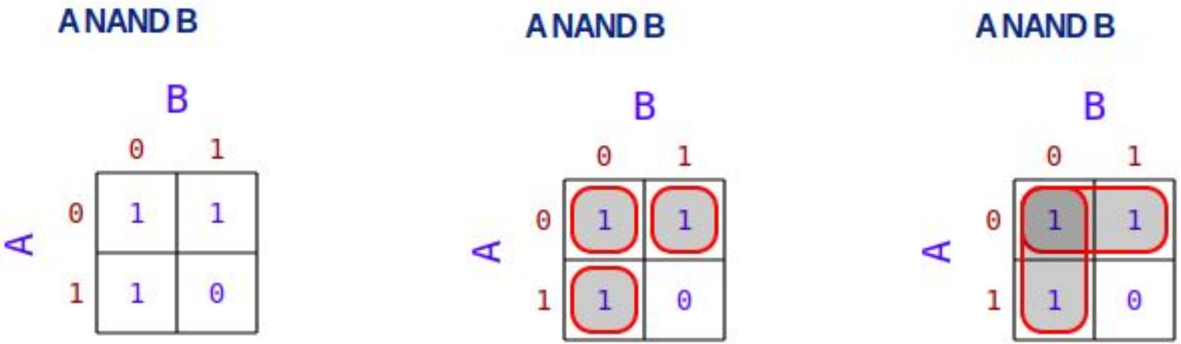
a \ b	0	1
0	0	0
1	1	1

Two Variable K-map Examples

- Grouping of powers of two implicants
- 1, 2 or 4 implicants
- What is the biggest group I can form with this minterm?

NAND $\overline{A}\cdot\overline{B} + \overline{A}\cdot B + A\cdot\overline{B}$

a	b	x
0	0	1
0	1	1
1	0	1
1	1	0

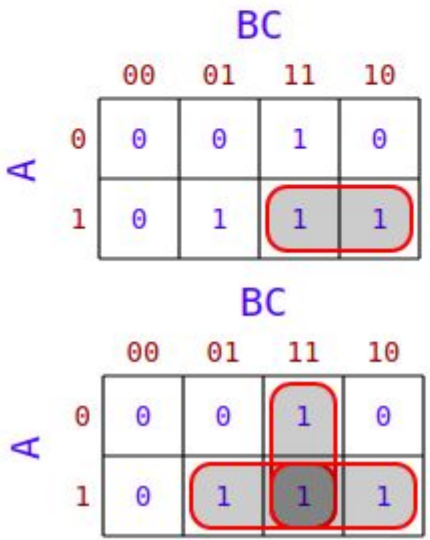


Karnaugh Maps For Simplification

$$\overline{A}BC + A\overline{B}C + AB\overline{C} + ABC =$$

Karnaugh Maps For Simplification

$\overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$ = A·B+A·C+B·C



K-map And Truth Table For Three Variables

- Grouping of powers of two implicants
- 1, 2, 4 or 8 implicants
- What is the biggest group I can form with this minterm?

A \ BC	00	01	11	10
0	m0	m1	m3	m2
1	m4	m5	m7	m6

- m0 is $\overline{A} \cdot \overline{B} \cdot \overline{C}$
- m1 is $\overline{A} \cdot \overline{B} \cdot C$
- m2 is $\overline{A} \cdot B \cdot \overline{C}$
- m3 is $\overline{A} \cdot B \cdot C$
- m4 is $A \cdot \overline{B} \cdot \overline{C}$
- m5 is $A \cdot \overline{B} \cdot C$
- m6 is $A \cdot B \cdot \overline{C}$
- m7 is $A \cdot B \cdot C$

A	B	C	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

A \ BC	00	01	11	10
0	0	1	1	1
1	0	0	0	1

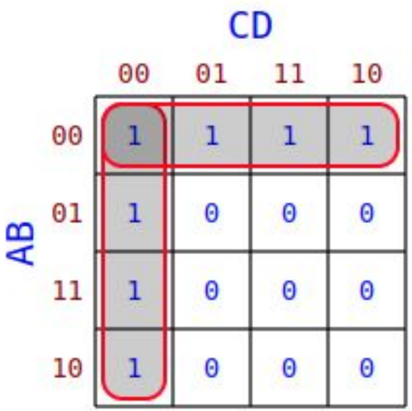
Karnaugh Maps For Simplification

~A~B~C~D + ~AB~C~D + AB~C~D + A~B~C~D +
~A~B~CD + ~A~BCD + ~A~BC~D =

Karnaugh Maps For Simplification

~A~B~C~D + ~AB~C~D + AB~C~D + A~B~C~D + ~A~B~CD + ~A~BCD + ~A~BC~D

= ~A~B + ~C~D



Four Variable K-maps

AB \ CD	00	01	11	10
00	m0	m1	m3	m2
01	m4	m5	m7	m6
11	m12	m13	m15	m14
10	m8	m9	m11	m10

- m0 is $\overline{A}\cdot\overline{B}\cdot\overline{C}\cdot\overline{D}$.
- m1 is $\overline{A}\cdot\overline{B}\cdot\overline{C}\cdot D$.
- m2 is $\overline{A}\cdot\overline{B}\cdot C\cdot\overline{D}$.
- m3 is $\overline{A}\cdot\overline{B}\cdot C\cdot D$.

- m4 is $\overline{A}\cdot B\cdot\overline{C}\cdot\overline{D}$.
- m5 is $\overline{A}\cdot B\cdot\overline{C}\cdot D$.
- m6 is $\overline{A}\cdot B\cdot C\cdot\overline{D}$.
- m7 is $\overline{A}\cdot B\cdot C\cdot D$.

- m8 is $A\cdot\overline{B}\cdot\overline{C}\cdot\overline{D}$.
- m9 is $A\cdot\overline{B}\cdot\overline{C}\cdot D$.
- m10 is $A\cdot\overline{B}\cdot C\cdot\overline{D}$.
- m11 is $A\cdot\overline{B}\cdot C\cdot D$.

- m12 is $A\cdot B\cdot\overline{C}\cdot\overline{D}$.
- m13 is $A\cdot B\cdot\overline{C}\cdot D$.
- m14 is $A\cdot B\cdot C\cdot\overline{D}$.
- m15 is $A\cdot B\cdot C\cdot D$.

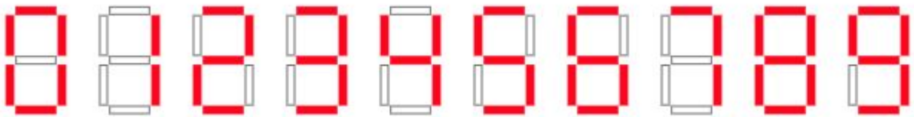
Don't cares

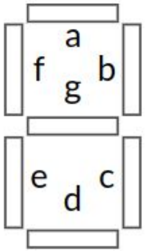
Sometimes your function has terms that could be 1 or 0 and that doesn't affect your systems output.

Example: Representing digits “0-9”

- Useful for 7-segment displays
- We need 4 inputs (ABCD), segment “a” in the example
- We can represent 0-15 with 4 inputs
- But we don't care from 10 (1010) to 15 (1111)

The digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 can be displayed with seven segments. One possibility is:





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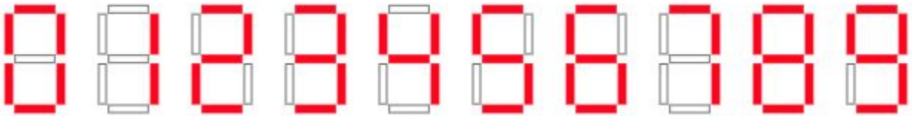
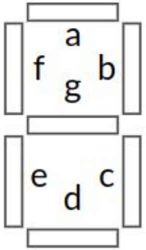
AB \ CD	00	01	11	10
00	1	0	1	1
01	0	1	1	1
11	X	X	X	X
10	1	1	X	X

The digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 can be displayed with seven segments. One possibility is:





Term Summary

- An implicant is a Boolean function term that makes the function true.
  - product/minterm term in Sum of Products (SOP)
  - sum/maxterm term in Product of Sums (POS)
- In an prime implicant, a variable in the expression cannot be removed.
  - A group of square or rectangle made up of bunch of adjacent minterms
  - What is the biggest group can form, 2, 4, 8, 16.
- An essential prime implicant contains a minterm not covered by any other implicant.
  - Always appear in final solution

K-Maps simplification examples

- Extra video lecture
  - ~A~B~C + A~B~C + A~BC
  - AB~C~D + A~B~C~D + ABC~D + A~BC~D
  - A~B~C~D + A~B~CD + A~BCD + A~BC~D + ~A~B~C~D + ~A~B~CD + ~A~BC~D + ~A~BCD

Questions?

- Next: Combinational building blocks
- Multiplex and decoders