Homework #2

Due Apr. 4, 2018 at 09:00 A.M.:

Academic Honesty

Academic honesty is taken seriously in this class during the semester. For homework problems or programming assignments, you are allowed to discuss the problems or assignments verbally with other class members, but under no circumstances can you look at or copy anyone else's written solutions or code relating to homework problems or programming assignments. All problem solutions and code submitted must be material you have personally written during this semester, except for any library or utility functions to be supplied. Failure to adhere to this guideline can result in a student receiving a failing grade in the class. It is the responsibility of each student to follow the course guideline.

1. [16 points] Prove the z-transforms and their ROCs for the discrete-time sequences in the table below:

		Signal, $x(n)$	z-Transform, $X(z)$	ROC
(1 point)	1	$\delta(n)$	1	All z
(1 point)	2	$u(n)$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
(1 point)	3	$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
(1 point)	4	$na^n u(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
(1 point)	5	$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
(1 point)	6	$-na^n u(-n-1)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
(2 points)	7	$(\cos\omega_0 n)u(n)$	$\frac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\lvert z\rvert > 1$
(2 points)	8	$(\sin\omega_0 n)u(n)$	$\frac{z^{-1}\sin\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\lvert z\rvert > 1$
(3 points)	9	$(a^n\cos\omega_0 n)u(n)$	$\frac{1-az^{-1}\cos\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
(3 points)	10	$(a^n\sin\omega_0 n)u(n)$	$\frac{az^{-1}\sin\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$

2. [20 points] For the LTI system described by the difference equation

$$y(n+2)+5y(n+1)+6y(n)=x(n+1)+4x(n)$$

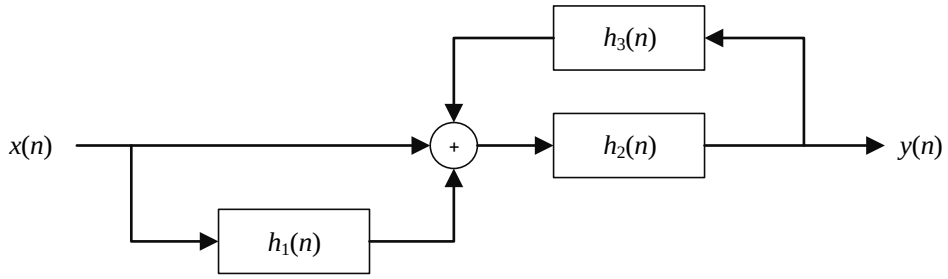
(a) [5 points] Find the transfer function $H(z)$ and plot the pole-zero diagram using MATLAB. You also need to show your MATLAB code to plot the pole-zero diagram.

(b) [5 points] Find the impulse response $h(n)$ when $H(z)$ is causal.

(c) [5 points] Find the output $y(n)$ using the inverse z-transform of $Y(z)$ to the input $x(n)=u(n)-u(n-3)$.

(d) [5 points] Find the output $y(n)$ using the convolution of $x(n)$ and $h(n)$ as $y(n)=h(n)*x(n)$. Show that your answer is the same as the one obtained in (c).

3. [24 points] Consider the following system below:



where $h_1(n) = \beta\delta(n\text{-}1)$ and $h_2(n) = \alpha^n u(n\text{-}1)$ with $|\alpha| < 1$. Assume that $h_3(n) = 0$ for 2-(a), -(b) and -(c).

(a) [4 points] Find the impulse response $h(n)$ of the overall system, i.e., such that $y(n) = h(n)*x(n)$.

(b) [4 points] Find the z-transform $H(z)$ and specify the ROC of $H(z)$.

(c) [4 points] Find the difference equation between the input $x(n)$ and the output $y(n)$.

(d) [4 points] Is the system causal? Why? or Why not?

(e) [4 points] Is the system stable? Why or Why not?

(f) [4 points] If $h_1(n) = 2\delta(n\text{-}1)$, $h_2(n) = 1/2\delta(n\text{-}1)$ and $h_2(n) = 1/3\delta(n\text{-}1)$, find $h(n)$.

4. [20 points] An LTI system is characterized by the system function below

$$H(z) = \frac{3 - 4z^{-1}}{1 - 3.5z^{-1} + 1.5z^{-2}}$$

(a) [5 points] Find the zeros of poles of $H(z)$ by your hand calculation. Find the zeros of poles of $H(z)$ using MATLAB and compared them to the ones found by your hand calculation. Give a pole-zero plot using MATLAB.

(b) [5 points] Find the partial fraction decomposition of $H(z)$, i.e., rewrite $H(z)$ as $H(z) = \sum_k \frac{A_k}{1 - p_k z^{-1}}$.

(c) [5 points] Specify the ROC of $H(z)$ and determine $h_i(n)$ for $i = 1, 2$ in the following two cases:

(i) $h_1(n)$: the system is stable.

(ii) $h_2(n)$: the system is anti-causal.

(d) [5 points] For which if the sequences above can you compute the DTFT? Justify your answer.

5. [20 points] Solve the following problems.

(a) [5 points] Determine the z-transform with its ROC for $x_1(n) = \left(\frac{1}{2}\right)^n u(n-3)$.

(b) [5 points] Determine the z-transform with its ROC for $x_2(n) = (1+n)\left(\frac{1}{3}\right)^n u(n)$.

(c) [5 points] Determine the z-transform with its ROC for $x_3(n) = n\left(\frac{1}{2}\right)^{|n|}$.

(d) [5 points] Determine the inverse z-transform of $X(z) = \frac{z^3 + 2z}{(z+1)^2(z+0.8)}$ where $X(z)$ is causal.