Korea Advanced Institute of Science and Technology

School of Electrical Engineering

EE432 Digital Signal Processing Spring 2018

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**Homework 3**

1. **Problem 1**

When the input to an LTI system is

1. Determine the system function of the system. Plot the poles and zeros of , and indicate the ROC

has one pole and one zero, and with .

The pole and zero of are presented in Figure 1.1. The ROC of is the gray area.



**Figure 1.1.** Poles and zeros of

1. Determine the impulse response of the system for all values of

Inverse z-Transform :

1. Write the difference equation that characterizes the system

Inverse z-Transform for both sides of the above equation:

So, the difference equation, characterizing the system is:

1. Is the system stable? Is it causal?

The system is stable because the ROC of includes the unit circle in z-plane.

The system is causal because the output only depends on the delayed sequences and does not depend on the advanced sequences.

1. **Problem 2**
2. Let and . Find such that
3. Use Matlab to plot the magnitude response and the phase response for the range . Plot the poles and zeros of using Matlab

* Plot the magnitude and phase response of the system

clear; clc;

w = (-1000:1:1000)\*pi/1000;

b0 = -37/48;

w0 = pi/3;

r = 0.75;

w1 = w0 - w;

w2 = w0 + w;

H = b0\*(1-exp(1j\*w1)).\*(1-exp(-1j\*w2))./((1-r\*exp(1j\*w1)).\*(1-r\*exp(-1j\*w2)));

magnitudeH = abs(H); phaseH = angle(H);

subplot(2,1,1); plot(w/pi,magnitudeH); grid on;

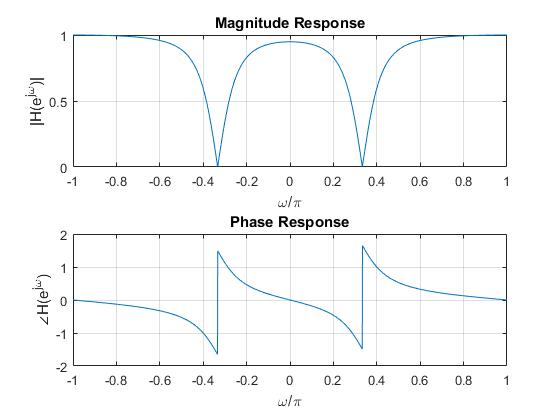
ylabel('|H(e^j^\omega)|'); title('Magnitude Response');

xlabel('\omega/\pi');

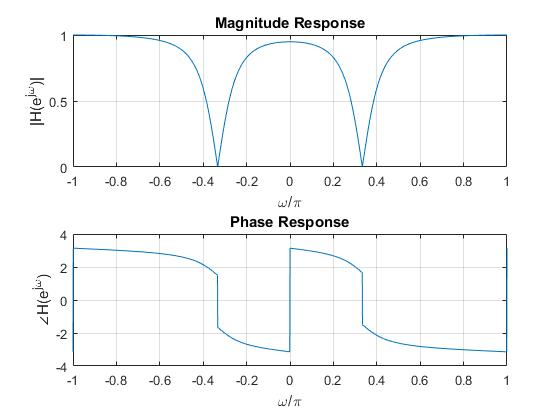
subplot(2,1,2); plot(w/pi,phaseH); grid on;

ylabel('\angleH(e^j^\omega)'); title('Phase Response');

xlabel('\omega/\pi');



**Figure 2.1.** The magnitude and phase response of the system with and



**Figure 2.2.** The magnitude and phase response of the system with and

* Plot the poles and zeros of

The Figure 2.3 depicts the poles and zeros of where .

clear; clc;

% Parameter

b0 = 37/48;

w0 = pi/3;

r = 0.75;

% Polynomial parameter

b = b0\*[1 -2\*cos(w0) 1];

a = [1 -2\*r\*cos(w0) r^2];

% Calculating pole

[R, p, C] = residuez(b, a);

% Plot zero-pole

[H1, H2, H3] = zplane(b, a);

set(H1, 'markersize', 10, 'color', 'r');

set(H2, 'markersize', 10, 'color', 'b');

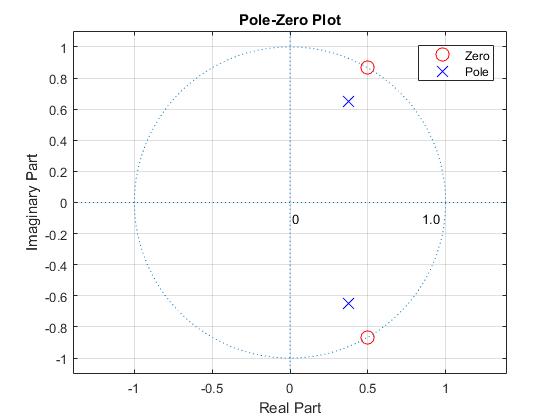
title('Pole-Zero Plot');

text(0.85, -0.1, '1.0');

text(0.01, -0.1, '0');

legend('Zero', 'Pole');

grid on;



**Figure 2.3.** The poles and zeros of for and

1. Now, let and . Find such that

For and :

1. Use Matlab to plot the magnitude response and the phase response for the range . Plot the poles and zeros of using Matlab

* Plot the magnitude and phase response of the system

clear; clc;

w = (-1000:1:1000)\*pi/1000;

b0 = 1141/1200;

w0 = pi/3;

r = 0.95;

w1 = w0 - w;

w2 = w0 + w;

H = b0\*(1-exp(1j\*w1)).\*(1-exp(-1j\*w2))./((1-r\*exp(1j\*w1)).\*(1-r\*exp(-1j\*w2)));

magnitudeH = abs(H);

phaseH = angle(H);

subplot(2,1,1); plot(w/pi,magnitudeH); grid on;

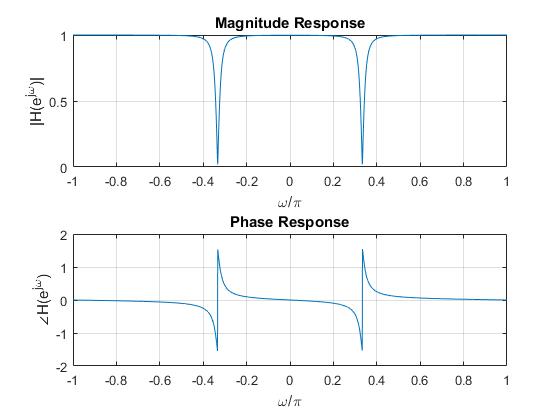
ylabel('|H(e^j^\omega)|'); title('Magnitude Response');

xlabel('\omega/\pi');

subplot(2,1,2); plot(w/pi,phaseH); grid on;

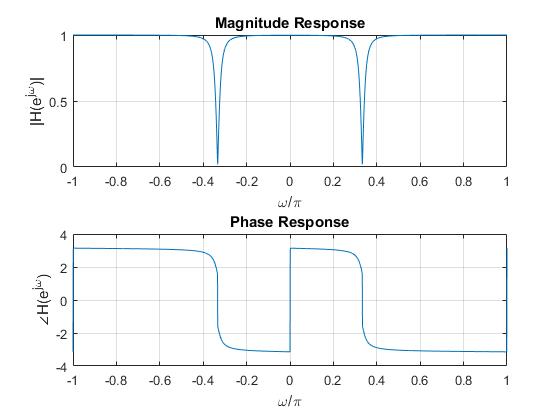
ylabel('\angleH(e^j^\omega)'); title('Phase Response');

xlabel('\omega/\pi');



**Figure 2.4.** The magnitude and phase response of the system

with and



**Figure 2.5.** The magnitude and phase response of the system

with and

* Plot the poles and zeros of

clear; clc;

% Parameter

b0 = 1141/1200;

w0 = pi/3;

r = 0.95;

% Polynomial parameter

b = b0\*[1 -2\*cos(w0) 1];

a = [1 -2\*r\*cos(w0) r^2];

% Calculating pole

[R, p, C] = residuez(b, a);

% Plot zero-pole

[H1, H2, H3] = zplane(b, a);

set(H1, 'markersize', 10, 'color', 'r');

set(H2, 'markersize', 10, 'color', 'b');

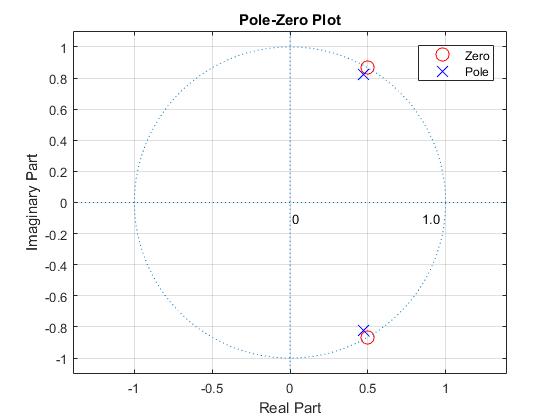
title('Pole-Zero Plot');

text(0.85, -0.1, '1.0');

text(0.01, -0.1, '0');

legend('Zero', 'Pole');

grid on;



**Figure 2.6.** The poles and zeros of for and

1. **Problem 3**

Consider a causal linear time-invariant system with system function

Where is real

1. Write the difference equation that relates the input and the output of this system

Inverse z-Transform for both sides of the above equation:

1. For what range of values of is the system stable?

The system always has 3 pole-zero pairs locating in difference sides of the unit circle in z-plane:

As long as , the system is stable.

1. For , plot the pole-zero diagram using Matlab and shade the region of convergence

clear; clc;

% Parameter

a = 0.5;

r1 = abs(a); r2 = abs(1/a);

r\_pole = min(r1, r2);

axis\_lim = max(r1, r2) + 0.5;

% Plot the poles

ReP = [a, 0, 0];

ImP = [0, a, -a];

plot(ReP, ImP, 'xr', 'MarkerSize', 7, 'DisplayName', 'Poles');

axis([-axis\_lim, axis\_lim, -axis\_lim, axis\_lim]);

axis equal; hold on; grid on;

% Plot the zeros

ReZ = [1/a, 0, 0];

ImZ = [0, 1/a, -1/a];

plot(ReZ, ImZ, 'ob', 'MarkerSize', 7, 'DisplayName', 'Zeros');

hold on;

% Plot the unit circle

[x\_unit, y\_unit] = circle(0, 0, 1);

plot(x\_unit, y\_unit, 'k--', 'LineWidth', 1, 'DisplayName', 'Unit circle');

% Plot the pole circle

[x\_pole, y\_pole] = circle(0, 0, r\_pole);

plot(x\_pole, y\_pole, 'r', 'LineWidth', 1, 'DisplayName', '|z| = 0.5');

[x, y] = circle(0, 0, 2\*axis\_lim);

h = patch([x,x\_pole], [y,y\_pole], [0.5 0.5 0.5], 'LineStyle', 'none', 'FaceAlpha', 0.25, 'DisplayName', 'ROC');

legend('show');

xlabel('Re(z)'); ylabel('Im(z)');

title('Problem 3c');

hold off;

%% Inner function

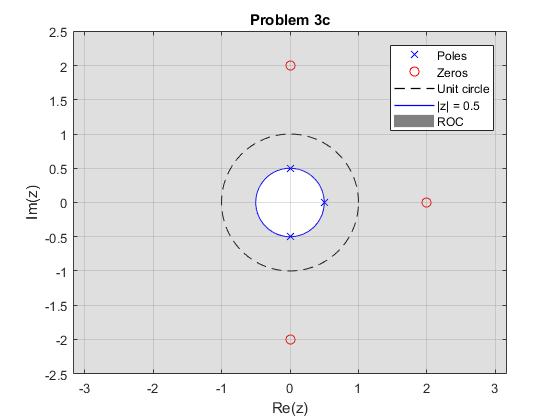
function [xc, yc] = circle(x, y, r)

theta = 0:pi/1000:2\*pi;

xc = x + r \* cos(theta);

yc = y + r \* sin(theta);

end



**Figure 3.1.** The pole-zero and ROC graph

1. Find the impulse response for the system

Applying the below inverse z-Transform property:

Finally, there are 2 cases:

Where:

1. Show that the system is an all-pass system, i.e., that the magnitude of the frequency response is a constant. Also, specify the value of the constant.

And the system is an all-pass system.

1. Plot the magnitude and phase responses of with for using Matlab

clear; clc;

w = (-1000:1:1000)\*pi/1000;

a = 0.5;

H = (1-1/a\*exp(-1j\*w)).\*(1+1/a^2\*exp(-2j\*w))./((1-a\*exp(-1j\*w)).\*(1+a^2\*exp(-2j\*w)));

magnitudeH = abs(H);

phaseH = angle(H);

subplot(2,1,1); plot(w/pi,magnitudeH); grid on;

axis([-1 1 7 9]);

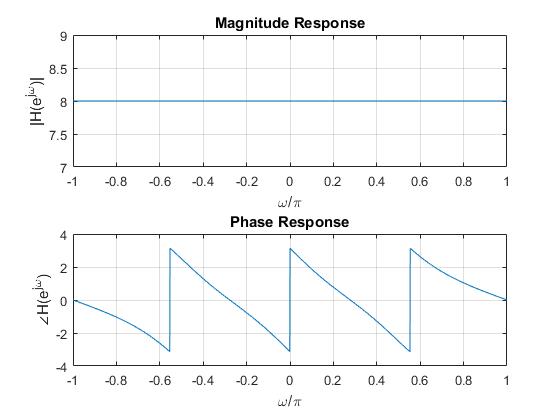
ylabel('|H(e^j^\omega)|'); title('Magnitude Response');

xlabel('\omega/\pi');

subplot(2,1,2); plot(w/pi,phaseH); grid on;

ylabel('\angleH(e^j^\omega)'); title('Phase Response');

xlabel('\omega/\pi');



**Figure 3.2.** The magnitude and phase response of the system where

1. **Problem 4**

Consider a stable LTI system given by

1. can be expressed as a cascade of a minimum-phase system and a unity-gain all-pass system . Determine a choice for and , and specify whether or not they are unique up to a scale factor

is a minimum-phase system because its poles do not exist but its zeros are inside of the unit circle:

is a all-pass system because there are 2 pole-zero pairs, the poles are inside but the zeros are outside of the unit circle.

1. Is the minimum-phase system a FIR system? Explain

Obviously, is a FIR.

1. Is the minimum-phase system a generalized linear-phase system? If not, can be represented as a cascade of a generalized linear-phase system and an all-pass system ? If your answer is yes, determine and . If your answer is no, explain why such representation does not exist.

is not linear

Therefore, the minimum-phase system is not a generalized linear-phase system.

can be represented as a cascade of a generalized linear-phase system and an all-pass system as below:

1. **Problem 5**

Consider the LTI system whose system function is given by

1. Determine all causal system functions that results in the same frequency-response magnitude as and for which the impulse responses are real value and of the same length as the impulse response associated with . There are four different such systems. Identify which system function is minimum phase and which to within a time shift, is maximum phase.



**Figure 5.1.** The zeros of 4 system (a) , (b) , (c) and (d)

* System 1:
* System 2:
* System 3:

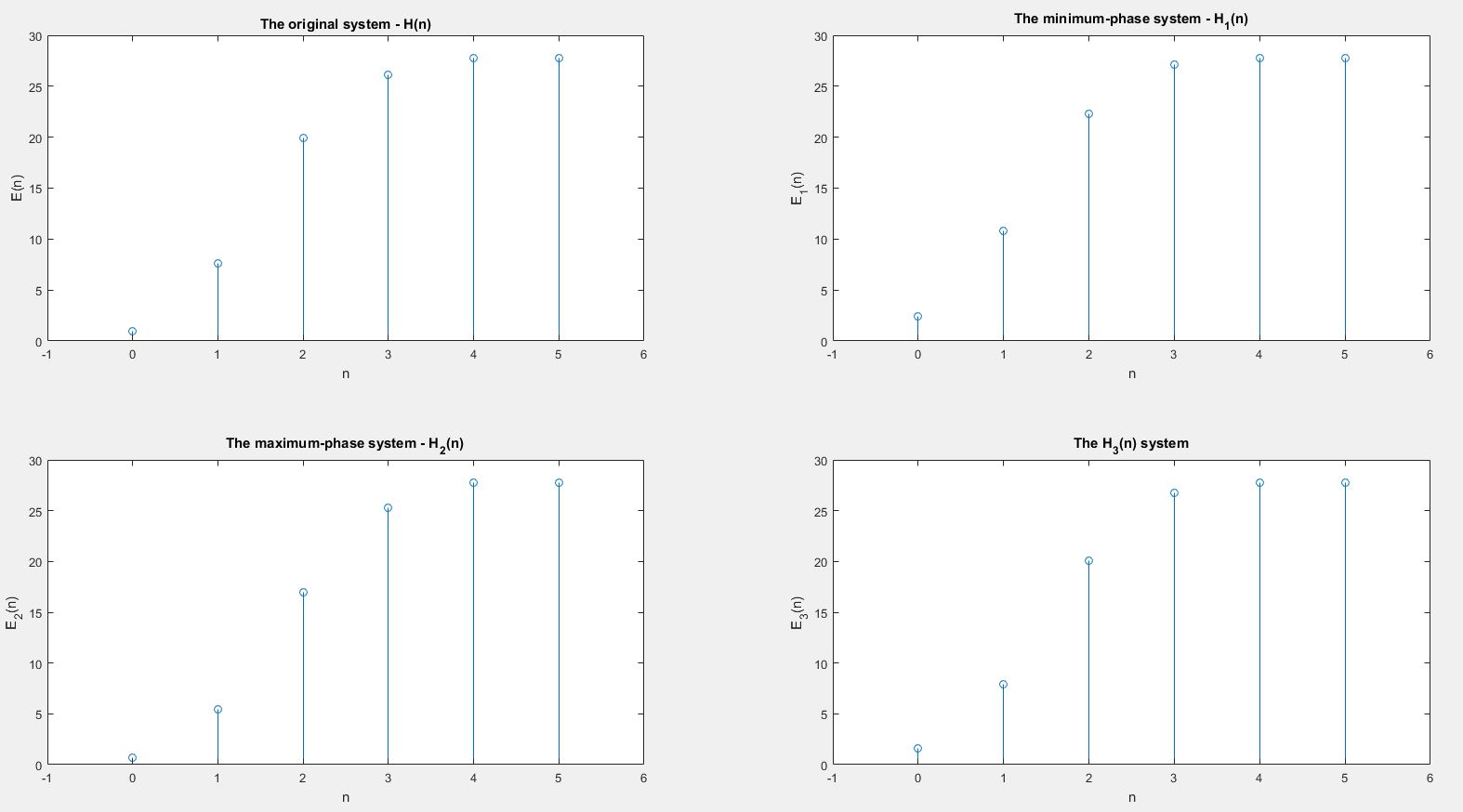
is a minimum-phase system because all its zeros are inside the unit circle.

is a maximum-phase system because all its zeros are outside the unit circle.

1. Determine the impulse response for the system function in part (a)
2. For each of the sequences in part (b), compute and plot the quantity for . Indicate explicitly which plot corresponds to the minimum-phase system.

**Table 5.1.** The energy of each sequence

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| 0 | 1 | 2.44 | 0.66 | 1.6 |
| 1 | 7.65 | 10.8 | 5.47 | 7.89 |
| 2 | 19.88 | 22.3 | 16.97 | 20.12 |
| 3 | 26.17 | 27.11 | 25.33 | 26.77 |
| 4 | 27.77 | 27.77 | 27.77 | 27.77 |
| 5 | 27.77 | 27.77 | 27.77 | 27.77 |



**Figure 5.2.** The quantity of energy in time domain

The plot of corresponds to the minimum-phase system.