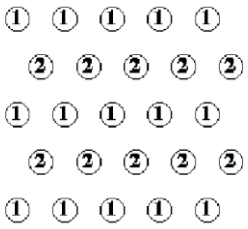
Homework #3

Due date: OCT 2 (Tue), 2018

Submission: Hard Copy (N1, 418)

1. Following figure illustrates Field-quincunx sampling lattice:



$$\mathbf{v} = \begin{bmatrix} \Delta x_1 & 0 & \Delta x_1/2 \\ 0 & 2\Delta x_2 & \Delta x_2 \\ 0 & 0 & \Delta t/2 \end{bmatrix}$$

Fig. Field-quincunx Sampling Lattice.

What is sampling matrix and its periodicity matrix?

2. Suppose that the basis vectors of two sampling lattices are

$$\text{Lattice A: } [\mathbf{v}_1] = [1 \quad 0]^{\mathrm{T}}, [\mathbf{v}_2] = [0 \quad 1]^{\mathrm{T}}$$

$$\text{Lattice B: } [\mathbf{v}_1] = [1 \quad 1]^{\mathrm{T}}, [\mathbf{v}_2] = [0 \quad 2]^{\mathrm{T}}$$

For each of these sampling lattices:

(a) Sketch the basis vectors and the sample points. Illustrate the Voronoi unit cell. Illustrate how the entire spatial domain is tiled by the shifted version of the unit cell.

(b) Determine the basis vectors of the reciprocal lattice. Repeat part (a) for this lattice.

(c) The basis vectors for a given lattice are not unique. Can you find another set of basis vectors for lattice B?

3. Let us consider the up-conversion from the sampling lattice $\mathbf{\Lambda}_1$ to $\mathbf{\Lambda}_2$. Their respective sampling matrices are

$$[\mathbf{V}_1] = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, [\mathbf{V}_2] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

- Assume the spectrum of the original continuous signal has a circular support. Let v_1^* and v_2^* denote Voronoi cell of lattice $\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$, respectively.

(a) Find the corresponding sampling matrices for the reciprocal lattices.

(b) How is low pass filter for the up-conversion

(c) Illustrate the up-conversion process from lattice $\mathbf{\Lambda}_1$ to lattice $\mathbf{\Lambda}_2$ in the given conditions (sketch the sampling lattice and its reciprocal before and after up-conversion in detail).