Report of 3-Point and 5-Point Algorithm

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1. Three-Point Perspective Algorithm

1.1. The problem

In 3D space, given the perspective projection of three points of a known triangle, the position of each of the vertices can be obtained.

(a, b, c) is known, (α, β, γ) can be determined from the unit vectors (j_1, j_2, j_3).

The length of S_1, S_2, S_3 and position of P_1, P_2, P_3 must be estimated.

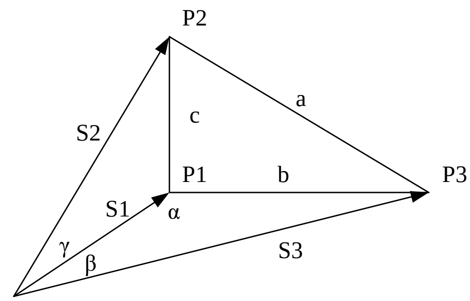

Figure 1.1. Illustrate the geometry of the three point space resection problem

1.2. The solutions

There are six solutions presents by Grunert (1841), Finsterwalder (1937), Merritt (1949), Fischler and Bolles (1981), Linnainmaa (1988) and Grafarend (1989).

$S_2^2 + S_3^2 - 2S_2S_3\cos\alpha = 0$ (1), $S_1^2 + S_3^2 - 2S_1S_3\cos\beta = 0$ (2)
$S_1^2 + S_2^2 - 2S_1S_2\cos\gamma = 0$ (3)

Table 1.1. Summarize of six solutions

Solutions	Change of Variables	Pairs of Equations	Variable Reduction	Solve New Variables
Grunert	$S_2 = uS_1$ $S_3 = vS_1$	(1, 2), (2, 3)	Substitution	
Finsterwalder	$S_2 = uS_1$ $S_3 = vS_1$	(1, 2), (2, 3)	Introduce a new variable	Set a perfect square into zero
Merritt	$S_2 = uS_1$ $S_3 = vS_1$	(1, 2), (1, 3)	Sustitution	
Fischler and Bolles	$S_2 = uS_1$ $S_3 = vS_1$	(1, 2), (1, 3)	Elimination	
Linnainmaa	$S_2 = u + \cos\gamma\, S_1$ $S_3 = v + \cos\beta\, S_1$		Elimination	
Grafarend	$S_2 = uS_1$ $S_3 = vS_1$	(1, 3), (2, 3)	Introduce a new variable	Set an eigensystem into zero

2. Five-Point Algorithm

2.1. The problem

Given the normalized image coordinates of five matching points, q and q', in two images, the camera matrix P must be estimated from them. Before jump to algorithm, three conditions must be considered.

$$q'Eq = 0 \ (2.1), \qquad \det(F) = 0 \ (2.2), \qquad EE^TE - \frac{1}{2}trace(EE^T)E = 0 \ (2.3)$$

Where: E and F is the essential and fundamental matrix, respectively.

2.2. The solutions

If 5 points are stacked, (2.1) become: $\tilde{q}^T\tilde{E} = 0$. Where:

$$\tilde{q} = [q_1q_1', q_2q_1', q_3q_1', q_1q_2', q_2q_2', q_3q_2', q_1q_3', q_2q_3', q_3q_3']^T$$

$$\tilde{E} = [E_{11}, E_{12}, E_{13}, E_{21}, E_{22}, E_{23}, E_{31}, E_{32}, E_{33}]^T$$

Therefore: $E = xX + yY + zZ + wW$

- ❖ Step 1: Extraction the nullspace of a 5×9 matrix $[\tilde{q}_1, \tilde{q}_2, \tilde{q}_3, \tilde{q}_4, \tilde{q}_5|I]^T$
- ❖ Step 2: Expansion of the cubic constraints (2.2) and (2.3)

$$|E| = o_2(o_1(E_{12}, E_{23}) - o_1(E_{13}, E_{22}), E_{31}) + o_2(o_1(E_{13}, E_{21}) - o_1(E_{11}, E_{23}), E_{32}) + o_2(o_1(E_{11}, E_{22}) - o_1(E_{12}, E_{21}), E_{33})$$

- ❖ Step 3: Gauss-Jordan elimination with partial pivoting on the 10×20 matrix
- ❖ Step 4: Expansion of the determinant polynomial of the 3×3 polynomial matrix to obtain the tenth degree polynomial

$$p_1 = B_{12}B_{23} - B_{13}B_{22}, \qquad p_2 = B_{13}B_{21} - B_{11}B_{23}, \qquad p_3 = B_{11}B_{22} - B_{12}B_{21}$$

$$\langle n \rangle = p_1B_{31} + p_2B_{32} + p_3B_{33}, \qquad x = \frac{p_1(z)}{p_3(z)}, \qquad y = \frac{p_2(z)}{p_3(z)}$$

- ❖ Step 5: Extraction of roots from the tenth degree polynomial
- ❖ Step 6: Recovery of R and t corresponding to each real root and point triangulation for disambiguation

$E \sim Udiag(1,1,0)V^T$ where $det(U) > 0$ and $det(V) > 0$. Then $t \sim [u_{13}, u_{23}, u_{33}]^T$ and $R_a = UDV^T$, $R_b = UD^TV^T$

There are 4 possible solutions for camera matrix:

$$P_1 = [R_a|t], \qquad P_2 = [R_a|-t], \qquad P_3 = [R_b|t], \qquad P_4 = [R_b|-t]$$