

Macro III: Problem Set 4

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September 2024

Deadline: Friday, 4 Oct 2024

1. **Aiyagari Model.** Time is discrete and indexed by $t = 0, 1, 2, \dots$. Let $\beta \in (0, 1)$ be the subjective discount factor, $c_t \geq 0$ be consumption at period t . Agents are *ex-ante* identical and have the following preferences:

Preferences:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} \right) \right],$$

where $\sigma > 0$. Expectations are taken over an idiosyncratic shock, z_t , on labor productivity, where

$$\ln(z_{t+1}) = \rho \ln(z_t) + \epsilon_{t+1}, \quad \rho \in [0, 1].$$

Variable ϵ_{t+1} is an iid shock with zero mean and variance σ_ϵ^2 . Markets are incomplete as in Huggett (1993) and Aiyagari (1994). There are no state contingent assets and agents trade a risk-free bond, a_{t+1} , which pays interest rate r_t at period t . In order to avoid a Ponzi game assume that $a \geq -\phi$ with $\phi > 0$.

Technology: There is no aggregate uncertainty and the technology is represented by $Y_t = K_t^\alpha N_t^{1-\alpha}$. Let I_t be investment at period t . Capital evolves according to:

$$K_{t+1} = (1 - \delta)K_t + I_t.$$

Let $\delta = 0.08$, $\beta = 0.96$, $\alpha = 0.4$ and $\sigma = 2$. Start with $\phi = 0$.

- (a) Use a finite approximation for the autoregressive process

$$\ln(z') = \rho \ln(z) + \epsilon.$$

where ϵ' is normal iid with zero mean and variance σ_ϵ^2 . Use a 7 state Markov process spanning 3 standard deviations of the log wage. Let ρ be equal to 0.98 and assume that $\sigma_z^2 = \frac{\sigma_\epsilon^2}{1-\rho^2} = 0.621$. Simulate this shock and report results.

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- (b) State the households' problem.
- (c) State the representative firm's problem.
- (d) Define the recursive competitive equilibrium for this economy.
- (e) For $r = 2\%$ (notice that w can also be found from the firm's problem), write down a code with VFI to solve the households problem. Find the policy functions for a' and c . Plot these functions and comment your results. How long does your code take? Suppose $\beta = 0.975$, how long does it take?
- (f) For $r = 2\%$ (notice that w can also be found from the firm's problem), write down a code with EGM to solve the households problem. Find the policy functions for a' and c . Plot these functions and comment your results. Compare the two solutions. How long does your code take? Suppose $\beta = 0.975$, how long does it take?
- (g) Solve out for the equilibrium allocations (r and w are determined in equilibrium) and compute statistics for this economy (stationary equilibrium). Report basic statistics about this economy, such as: investment rate, the capital-to-output ratio, cumulative distribution of income (e.g., bottom 1%, 5%, 10%, 50%, top 1%, top 5%, top 10%), cumulative distribution of wealth (e.g., bottom 1%, 5%, 10%, 50%, top 1%, top 5%, top 10%), Gini of income, Gini of Wealth.
- outer bisection
inner fsolve (h) Keeping all other parameters the same, could you find a value for β such that the equilibrium interest rate is equal to 4% per year? Report this value and explain how you found it.
- (i) Keeping all other parameters the same, could you find a value for σ_z^2 such that the p_{90}/p_{10} of wealth is equal to 7? Report this value and explain how you found it.
- (j) Keeping all other parameters the same, could you find joint value for β and σ_z^2 such that the equilibrium interest rate is equal to 4% per year and the p_{90}/p_{10} of wealth is equal to 7? Report these values and explain how you found them. If possible use the simulated methods of moments explained in the lectures.
- (k) Could you find a value for ϕ such that the debt over income is equal to 10%? Report this value and explain how you found it.
- (l) Could you find a value for $(\beta, \sigma_z^2, \phi)$ such that the equilibrium interest rate is equal to 4%, the p_{90}/p_{10} of wealth is equal to 7, and the debt over income is equal to 10%? Report the values and explain how you found them. If possible use the simulated methods of moments explained in the lectures.

2. **Hopenhayn model.** On paying a fixed operating cost $\kappa > 0$, an incumbent firm that hires n workers produces flow output $y = zn^\alpha$ with $0 < \alpha < 1$ where $z > 0$ is a firm-level productivity level. The productivity of an incumbent firm evolves according to an AR(1) in logs

$$\ln(z_{t+1}) = (1 - \rho) \ln(\bar{z}) + \rho \ln(z_t) + \sigma \epsilon_{t+1}, \quad \rho \in (0, 1), \quad \sigma > 0$$

where $\epsilon_{t+1} \sim N(0, 1)$. Firms discount flow profits according to a constant discount factor $0 < \beta < 1$. There is an unlimited number of potential entrants. On paying a sunk entry cost $\kappa_e > 0$, an entrant receives an initial productivity draw $z_0 > 0$ and then starts operating the next period as an incumbent firm. For simplicity, assume that initial productivity z_0 is drawn from the stationary productivity distribution implied by the AR(1) above.

Individual firms take the price p of their output as given. Industry-wide demand is given by the $D(p) = \bar{D}/p$ for some constant $\bar{D} > 0$. Let labor be the numeraire, so that the wage is $w = 1$. Let $\pi(z)$ and $v(z)$ denote respectively the profit function and value function of a firm with productivity z . Let v_e denote the corresponding expected value of an entering firm. Let $\mu(z)$ denote the (stationary) distribution of firms and let m denote the associated measure of entering firms.

- (a) Derive an expression for the profit function.
- (b) Set the parameter values $\alpha = 2/3$, $\beta = 0.8$, $\kappa = 20$, $\kappa_e = 40$, $\ln(\bar{z}) = 1.4$, $\sigma = 0.20$, $\rho = 0.9$ and $\bar{D} = 100$. Discretize the AR(1) process to a Markov chain on 33 nodes. Solve the model on this grid of productivity levels. Calculate the equilibrium price p^* and measure of entrants m^* . Let z^* denote the cutoff level of productivity below which a firm exits. Calculate the equilibrium z^* . Plot the stationary distribution of firms and the implied distribution of employment across firms. Explain how these compare to the stationary distribution of productivity levels implied by the AR(1).
- (c) Now suppose the demand curve shifts, with \bar{D} increasing to 120. How does this change the equilibrium price and measure of entrants? How does this change the stationary distributions of firms and employment? Give intuition for your results.
- (d) Now, as in Hopenhayn and Rogerson (1993), introduce a firing cost such that firms have to pay $g(n, n_{-1}) = \tau(n_{-1} - n)$ if $n < n_{-1}$ with $\tau > 0$, where n_{-1} corresponds to employment of the firm in the end of the previous period. In addition, $g(n, n_{-1}) = 0$ if $n \geq n_{-1}$. Repeat (a), (b) and (c) with different values for τ (e.g., $\tau = 0.1$, $\tau = 0.2$) and study the implications for aggregate output.