Computational Methods in Economics

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Problem Set 2

Due on March 18th, 2025 at 23h59

Please remember:

- *Late responses are only accepted within the first 24 hours and with a 20% penalty*
- *You are allowed to complete this problem in any programming language.*
- *You should submit the answers* ***along with a fully reproducible code****.*
- *Remember our good coding practices: code with documentation, folder structure, relative paths only, use of git, defensive programming, etc.*

Question 1. We want to find the root of the following function:

$$f(x) = x^3 - 14x^2 + 55x - 10$$

Start with the points $(4, f(4))$ and $(6, f(6))$.

a) Bracket the root.

b) Find the root using the bisection method. How many iterations are required to reach the tolerance of 10^{-8}?

c) Now use the secant method. How does it compare?

d) Since this is a polynomial, we can easily compute its derivative and use the Newton-Raphson method. How does it compare to the other methods?

Question 2. We want to minimize the function:

$$f(x) = x \times sin\left(\frac{x^2}{3}\right),$$

over the interval [0,10].

a) Choose 3 optimization methods and optimize this function using relative, absolute tolerance for $f(x)$ and x of 10^{-4}. If the method requires a starting point, use 0.

b) How does your answer depend on the starting point? And on the algorithm?

c) What happens if we change the tolerance to 10^{-8}?

Question 3. We have the following function with four unknowns:

$$f(x_1, x_2; \theta) = \theta_1 x_1 + \frac{\theta_2}{1 + exp(-\theta_3 x_2)} + \theta_4 x_1^{x_2}.$$

We are given four values:

$$\begin{aligned}
f(1, 1; \theta_0) &= y_1 = 35.8 \\
f(2, 4; \theta_0) &= y_2 = 547.6 \\
f(-1, 2; \theta_0) &= y_3 = 32.2 \\
f(2, -2; \theta_0) &= y_4 = 14.5
\end{aligned}$$

Our goal is to find the true θ_0. We know that $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)$ and that $\theta \in \mathbb{R}^4$

a) Create a function $g(\theta)$ that for each value of θ, it computes the sum of the quadratic difference between the function for that θ and the four known points. That is:

$$g(\theta) = (f(1, 1, \theta) - y_1)^2 + (f(2, 4, \theta) - y_2)^2 + (f(-1, 2, \theta) - y_3)^2 + (f(2, -2, \theta) - y_4)^2$$

b) What is the value of $g(\theta_1)$, when $\theta_1 = (0, 0, 0, 0)$?

c) Get your best estimate of θ_0 minimizing the function $g(\theta)$. You can choose your preferred algorithm.

d) For each iteration i, record the trying point (θ_i) and the value $g(\theta_i)$. Plot in a graph the value of the iteration and $g(\theta_i)$. Plot in 4 different graphs the iteration i and the trying point θ_i.

e) What is your estimate $\hat{\theta}$? How many iterations were necessary?

Question 4. We want to use optimization methods to compute the Pareto Frontier. That is, what is the allocation of goods that maximizes the social utility from the planners perspective. There are N individuals and $k = 1, 2, \ldots, K$ goods. Individual of individual i is given by:

$$u_i(x) = \sum_{k=1}^{K} \alpha_k^i \frac{(x_k^i)^{1-\sigma_k^i}}{1 - \sigma_k^i},$$

where $\alpha_k^i > 0$, $\sigma_k^i > 1$, and $x_k^i \geq 0$ is the amount of good k that individual i consumes.

The social planner problem is given by:

$$\max_{\{x^1,x^2,\ldots,x^N\}} \sum_{i=1}^{N} \lambda_i u_i(x^i) \;\; \text{s.t.} \;\; \sum_{i=1}^{N} x_k^i \leq \sum_{i=1}^{N} e_k^i, \forall k,$$

where $\lambda_i > 0$ is the weight of individual i and e_k^i is the endowment of good k of individual i.

a) Create a function that has inputs the vectors: $\lambda = (\lambda_1, \ldots, \lambda_N)$, $e = (e_1^1, \ldots, e_K^1, \ldots, e_1^N, \ldots, e_K^N)$, $\alpha = (\alpha_1^1, \ldots, \alpha_K^1, \ldots, \alpha_1^N, \ldots, \alpha_K^N)$, $\sigma = (\sigma_1^1, \ldots, \sigma_K^1, \ldots, \sigma_1^N, \ldots, \sigma_K^N)$ Respecting the imposed constraints. The function should give the optimal allocation $x = (x_1^1, \ldots, x_K^1, \ldots, x_1^N, \ldots, x_K^N)$. You can choose your optimization algorithm.

b) Consider $K = 2$ and $N = 2$. Compute the optimal allocation for $\lambda = (0, 1)$, $\lambda = (0.25, 0.75)$, $\lambda = (0.50, 0.50)$, $\lambda = (0.75, 0.25)$, $\lambda = (1, 0)$. For given α, σ, e of your choice.

c) How does your algorithm perform with $N = 6$ and $K = 3$?