Computational Methods in Economics

Sao Paulo School of Economics - FGV

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Problem Set 1

Due on March 11th, 2025 at 23h59

Please remember:

- *Late responses are only accepted within the first 24 hours and with a 20% penalty*
- *You are allowed to complete this problem in any programming language.*
- *You should submit the answers* ***along with a fully reproducible code****.*
- *Remember our good coding practices: code with documentation, folder structure, relative paths only, use of git, defensive programming, etc.*

Question 1. The table below shows the life expectancy at birth between 1940 and 2010, according to IBGE.[1]

Year	1940	1950	1960	1970	1980	1991	2000	2010
Life Expectancy	45.5	48.0	52.5	57.6	62.5	66.9	71.1	74.4

a) What is your best guess for the life expectancy at birth for someone born in 1996?

b) Using any interpolation method of your choice, plot the life expectancy for all years between 1940 and 2010.

Question 2. We are interested in interpolating the following function:

$$f(x) = \frac{(x+\varepsilon)^{(1-\sigma)}}{1-\sigma},$$

for $\sigma = 3$ and $\varepsilon = 10^{-8}$.

a) Create a vector of trying points, x. Draw a sample with 2500 numbers using a uniform distribution over the interval [0,10]. You should keep this fixed for the entire exercise.

[1] `https://agenciadenoticias.ibge.gov.br/agencia-noticias/2012-agencia-de-noticias/noticias/41984-em-2023-expectativa-de-vida-chega-aos-76-4-anos-e-supera-patamar-pre-pandemia`

b) Let our grid be from 0 to 10, we will use $n = 10$ points, equally spaced. First create the grid t. Then, compute the function for all the 10 grid points, generating a y vector.

Now we will approximate the function f for all the trying points x, using only the vectors t and y and some interpolation methods. That is, you should pretend you do not know the true function f.

c) For each interpolation method (linear, spline, and pchip/akima), compute the approximation $\hat{f}(x)$ for all trying points and the error $e(x) = f(x) - \hat{f}(x)$. Summarize the results showing in a table the average quadratic error for each method and the average time of execution (for all 2500 points).

d) What happens when we increase our grid to $n = 15$? And to $n = 20$? And to $n = 30$? And to $n = 50$?

e) Now let's repeat items b)–d) using a log-spaced grid.

f) What conclusions can you draw from this exercise?

Question 3. Repeat question 2 for this function:

$$g(x) = \frac{1}{1 + e^{k(-x+\xi)}},$$

with $k = 2.5$, $\xi = 5$, and $x \in [1, 10]$.

a) What conclusions are different in this case?

b) Given the shape of the function, what would be the best grid?

Question 4. Maria is finishing high school and is trying to decide whether to enroll in college or not. If she enrolls in college her human capital will grow and she will receive higher wages. However she needs to pay upfront costs to study. Assume that Maria observes four older friends, their decisions and wages:

Friend	Human Capital in HS	College	Wage
A	1.0	0	1.00
B	1.5	0	1.50
C	3.1	1	4.22
D	4.5	1	25.50

Maria has an $h = 2.8$. She will use exclusively the information from her four friends.

a) What would be her decision if she used nearest-neighbor interpolation to decide her optimal choice?

b) What would be her estimate of her wage using linear interpolation?

Now she learned more information. She also knows the counterfactual wages of each of her friends.

Friend	Human Capital in HS	College	Wage if HS	Wage if College
A	1.0	0	1.00	-13.00
B	1.5	0	1.50	-10.50
C	3.1	1	3.10	4.22
D	4.5	1	4.50	25.50

b) What is her best estimate now (you can choose your interpolation method) for her wage if she stays in high school? And if she goes to college?

c) Plot the two wage functions for any level of human capital in the interval $[1, 4.5]$.

d) Based exclusively on the college decisions and nearest-neighbor interpolation, what is the lowest level of human capital that would make Maria go to college?

e) Based on separate linear interpolations of the wage functions, what is the lowest level of human capital that would make Maria go to college?