CHE 116 Design Project 2024

Group: Drew Suzukawa, Vansh Nagpal, Roger Cruz

University of California, Riverside

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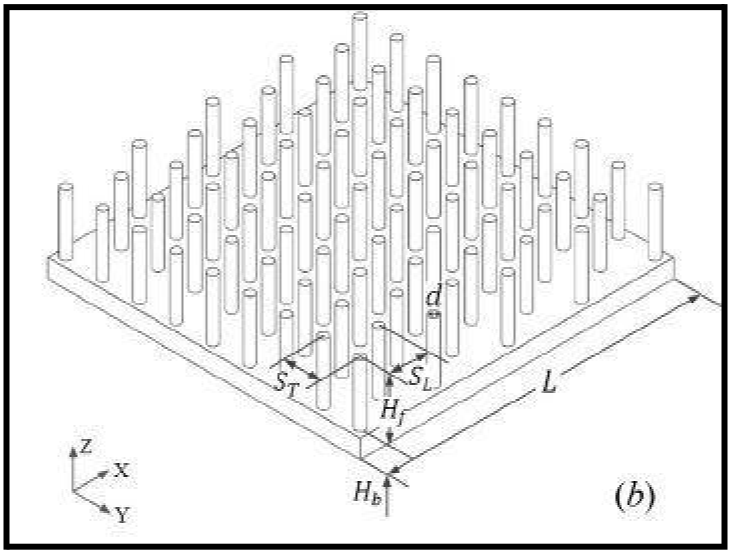

Table of Contents

Abstract

In order to develop a better understanding about convective heat transfer in extended surfaces, also referred to as fins, the purpose of this project was to design a the most cost efficient square array of aluminum fins capable of dissipating 50 W of heat from a $100mm \times 50mm$ rectangular device. To accomplish this, first the number of fins were obtained for 40 different numbers of fins using boxes in which each box represents one fin. Once the number of fins were determined the corresponding diameters and spacing were determined using equations (11), (12), (13); there were 5 spacing factors used to determine the area of the fins. There were 10 different lengths used for each case resulting in a total of 2000 configurations for the fins on the plate.

The key formula was equation (2), it was used for the total heat dissipation by an individual fin using the material's thermal conductivity, heat transfer coefficient, fin's perimeter, cross sectional area, length of fin, and change in temperature. It is followed by equation (1) to calculate the heat dissipation of n number of fins. In order to perform the computations, the problem parameters were defined in Python and ultimately reduced down to three variables, fin length, the number of fins, and fin diameter. After analyzing the data, the optimal configuration for the number of fins was determined to be 1600 fins, with a length of 9 mm, and a diameter of 1.33 mm; the total cost for this arrangement was determined using equation (17) around \$0.50. These values produce the highest efficiency and cost effectiveness based on the selected parameters of the report.

Introduction and Background

One of the most important design considerations in many engineering projects is the dissipation of heat to maintain a certain temperature for sensitive systems such as semiconductors, which must stay within a certain range of temperatures to operate properly. The primary ways of increasing the heat transfer out of these systems is to increase their surface area, increase the heat transfer coefficient, h, by providing fluid flow, or by decreasing the temperature of the environment in order to increase convective heat transfer. The latter two methods are often prohibitively expensive and impractical so increasing surface area is usually the simplest and cheapest method of maintaining temperature. An efficient way of increasing surface area is through the use of fins, surfaces that extend from an object in order to maximize surface area while using less volume than simply enlarging the object itself would.

There are two types of fins that are predominantly used, straight fins and annular fins. Straight fins are shapes that extrude from a flat surface while annular fins are fins that extrude radially from a surface, typically a cylinder. Typically, annular fins are better at dissipating heat than similar volume of straight fins but can be impractical to add to systems. Theoretically, the most efficient design of fins to maximize surface area while minimizing volume is an infinite amount of infinitely small fins, but design and manufacturing considerations limit how many fins can be used and how small they can be. Regardless, the most efficient fin designs utilize as large a number of short and thin fins as can be practically manufactured to meet a certain heat transfer requirement.

The goal of the project was to design a square array of aluminum straight cylindrical pin fins capable of dissipating 50 W of heat from a $100mm \times 50mm$ rectangular device at 85 ºC in a 30 ºC environment. The design was optimized to find the lowest possible cost of materials while dissipating the required heat by varying the number of fins, fin length, diameter, and spacing.

Theory and Methods

The calculations for the heat transfer of the device were based on the convection heat transfer of a square array of finite straight cylindrical fins attached to a thin $100mm \times 50mm$ rectangular plate. It was assumed that the device was at a uniform temperature of 85 ºC surrounded by a constant uniform environment of 30 ºC, with absence of heat conduction. Additionally, the thickness of the plate was not specified in the design problem statement, therefore, the value was taken to be 10 mm. The fins must be able to dissipate 50 W of heat from the device while maintaining a temperature of 85 ºC. To determine the number of fins as well as the spacing between the fins, each fin was assumed to occupy a single square within an $n \times n$ array, implying that the spacing between adjacent fins would be equivalent throughout the plate.

The overall heat transfer coefficient, h, was assumed to be uniform for both the fins and the plate, where the value was taken to be 15 $\frac{W}{m \cdot C}$. The properties of aluminum at 20 ºC were taken from Table 3 of Appendix A of Holman's Heat Transfer 10th Edition, the values for the thermal conductivity, k, and density, ρ, were found to be 204 $\frac{W}{m \cdot C}$ and 2707 $\frac{kg}{m^3}$, respectively. The cost of aluminum was assumed to be at a market value of \$2.63 per kg, adopted from Business Insider, as of the 21st of May, 2024.

All calculations and graphical representations of the data were completed using Python (version 3.12.1). The optimization of the dissipation of 50W of heat for minimum cost was accomplished by calculating the fin array heat transfer for different values of the fin length, L, fin diameter, D, the number of fins, n, and the spacing of fins, X. Based on these dimensions, the total mass of the fin array, M_F, and the mass of the bare plate, M_B, were used to determine the total mass of the device, M_T, which was used to find the total cost, C_T. After calculating all of the values, the designs incapable of meeting the heat transfer requirements were discarded and the configuration of minimal cost was selected. The control parameters for design optimization included arrangements ranging from a 1×1 to a 40×40 square array of fins, fin length ranging from 1 mm to 10 mm (at intervals of 1 mm), varying fin spacing as a % of the side length of a single square, and fin diameter. The parameters were determined as a function of the number of fins through a process described by equations (7), (11), (12), and (13). The total fin heat transfer, Q_F, was determined by multiplying the heat transfer of a single fin, Q_{fin}, by the number of fins in the array, n, given by equation (1).

$$Q_F = nQ_{fin} \tag{1}$$

The value for the heat transfer of a single fin was calculated using the formula for convective heat transfer of a finite cylindrical straight fin given by equations 2-6. This allowed for the calculation of total fin heat transfer by multiplying by number of fins to obtain equation (1).

$$Q_{fin} = M\frac{sinh(mL)+\frac{h}{mL}cosh(mL)}{cosh(mL)+\frac{h}{mL}sinh(mL)} \tag{2}$$

In the below equations, adapted from Holman's *Heat Transfer 10th Edition*[1], A_c is the cross-sectional area, P is the perimeter, and k is thermal conductivity of aluminum.

$$M = \sqrt{hPkA_c}\,\Delta T \tag{3}$$

$$m = \sqrt{\frac{hP}{kA_c}} \tag{4}$$

$$P = \pi D \tag{5}$$

The temperature difference between the surface and surrounding environment can be calculated using equation (6), where T_0 is 85 ºC and T_∞ is 30 ºC:

$$\Delta T = (T_0 - T_\infty) \tag{6}$$

[1] Holman, Jack P. Heat Transfer. 10th ed., McGraw-Hill Education, 2009.

The area of the plate is calculated using the giving length, *L*, and width, *W*, of 100 mm and 50 mm, respectively:

$$A_{plate} = L \times W = 5000mm^2 \quad (7)$$

The convective heat transfer of the plate left uncovered by the fins, $Q_{bare\ plate}$ was calculated by using the area of the aluminum plate left uncovered, $A_{bare\ plate}$.

$$Q_{bare\ plate} = hA_{bare\ plate}\Delta T \quad (8)$$

$$A_c = \frac{\pi D^2}{4} \quad (9)$$

The bare plate area was found by subtracting the total plate area by fin cross sectional area multiplied by the number of fins, given in equation (10).

$$A_{bare\ plate} = A_{plate} - nA_c \quad (10)$$

The area of each of the boxes for the number of fins on the plate is given by dividing the area of the plate by the number of boxes.

$$A_{square} = \frac{A_{plate}}{n} \quad (11)$$

The method for calculating the spacing, and diameter with respect to the number of fins corresponds to the number of boxes. The fins are centered at each box to have equal spacing to form a square array. The number of boxes are directly proportional to the number of fins. Using the area of the boxes, the max diameter of each fin can be calculated by finding the side length of the box, *X*, by square rooting the area of one plate.

$$X = \sqrt{A_{square}} \quad (12)$$

The max diameter of the fins may be equal to *X* but this configuration would not be ideal as it would introduce conductive heat transfer through the extruded surface. To offset this, a space factor was used to determine the diameter with respect to the number of fins, and the value of *D* that each fin can be without being the same value as *X*.

$$D = X \cdot S \quad (13)$$

The mass of the fins was determined by multiplying the number of fins by the volume and the density of aluminum, ρ, taken to be 2707 $\frac{kg}{m^3}$.

$$M_F = nA_cL\rho \quad (14)$$

The thickness of the plate, Z, was taken to be 10 mm, when calculating the mass of the plate.

$$M_B = A_{plate} Z \rho \tag{15}$$

Total mass was found by calculating the volume of the plate and fins then multiplying by the density of aluminum.

$$M_T = M_F + M_B \tag{16}$$

The cost, C_T, was calculated by multiplying the total mass of the device by the cost of aluminum per kilogram, \$2.63/kg.

$$C_T = M_T \cdot C_{Al} \tag{17}$$

Results and Discussion

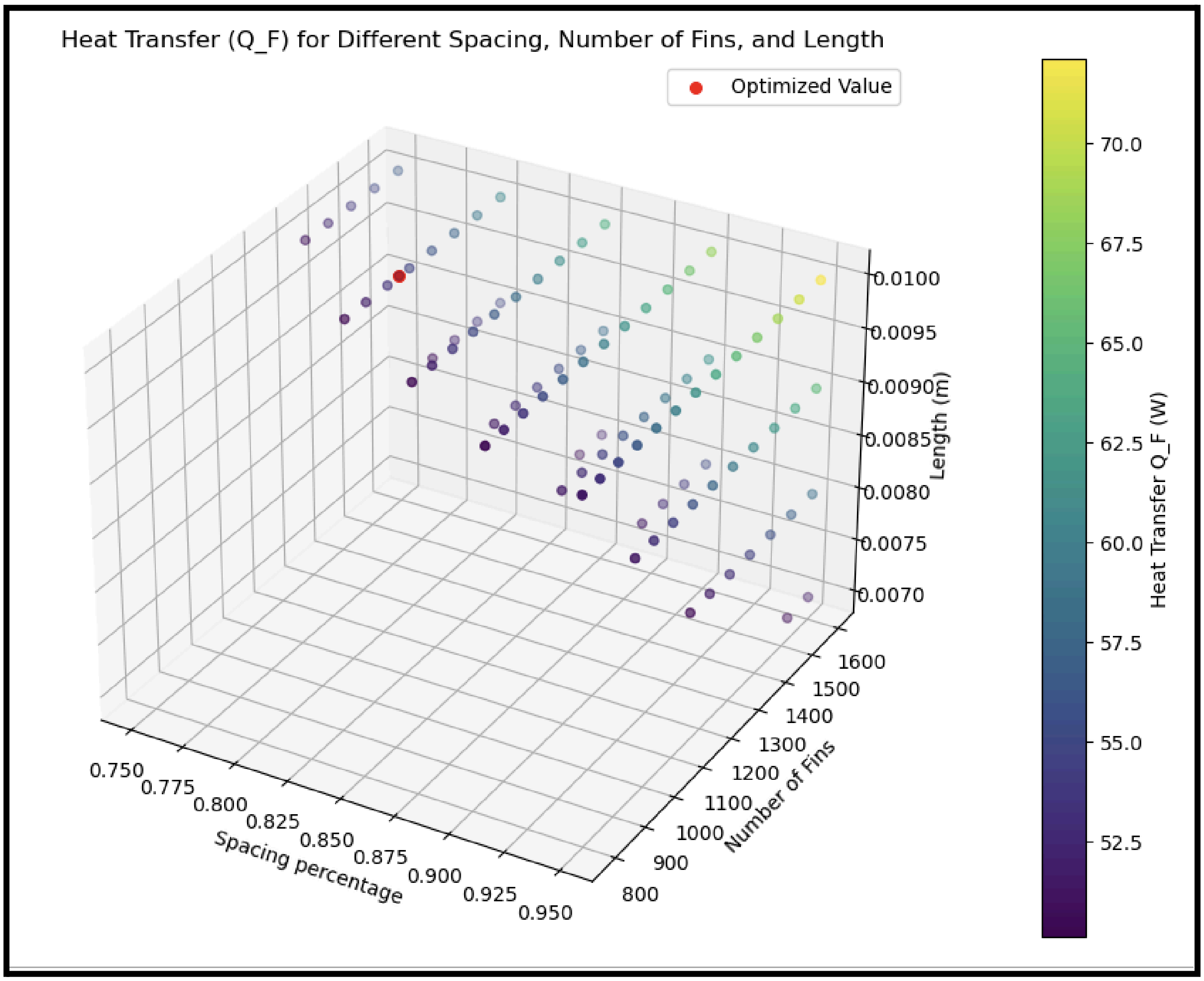

Figure 1.1: *Plot of spacing, number of fins, and length for the optimization process, where only values for heat transfer greater than or equal to 50 W are considered.*

Figure 1.1 shows that as could be predicted intuitively fin heat transfer increases significantly with length, increasing diameter from decreased spacing, and number of fins. The minimum cost was found at 1600 fins, 77.5% of square side length diameter, and 9 mm length. This indicates that the most cost effective way to increase heat transfer is to utilize a larger amount of fins as opposed to increasing dimensions, which comes with the tradeoff of making manufacturing more complicated and expensive in turn.

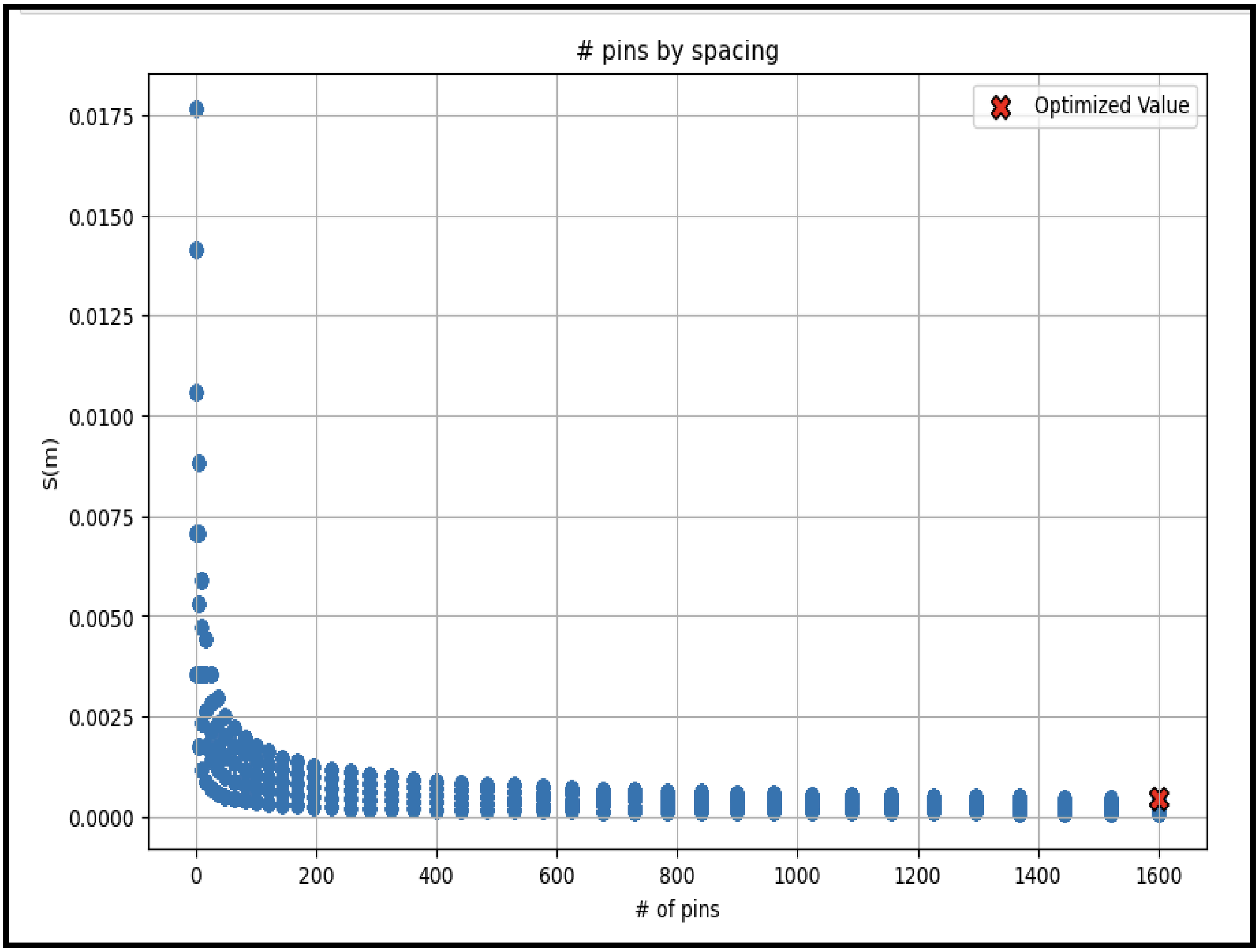

Figure 1.2 : *Plot of spacing (distance) between adjacent fins by the number of fins*

Figure 1.2 shows that relationship in the spacing relative to the number of fins. The more fins the less spacing and less fins the more spacing. The optimized value is at the end, it is one of the most fins for the data set but is the best value when considering the other variables.

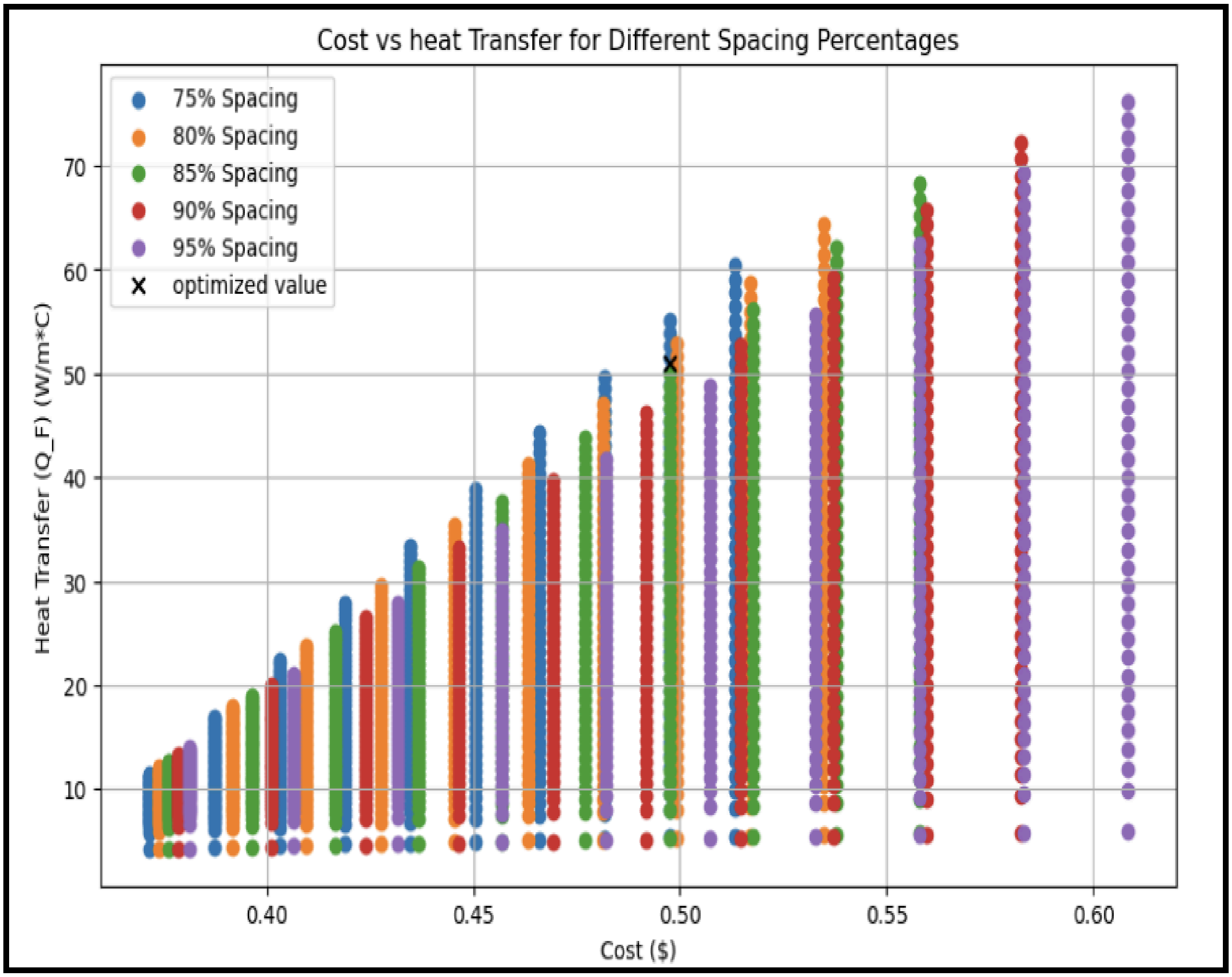

Figure 1.3 : *Cost of the configuration vs fin heat transfer*

The graph shows the difference between the cost vs heat transfer of the fins at different spacing. It appears that the higher the spacing factor the higher the heat transfer of the fin would be. For the optimized value, it is about in the middle. The graph shows that the optimized value appears to the first point where it crosses the 50 for the heat transfer so it would be the cheapest option at around $0.50.

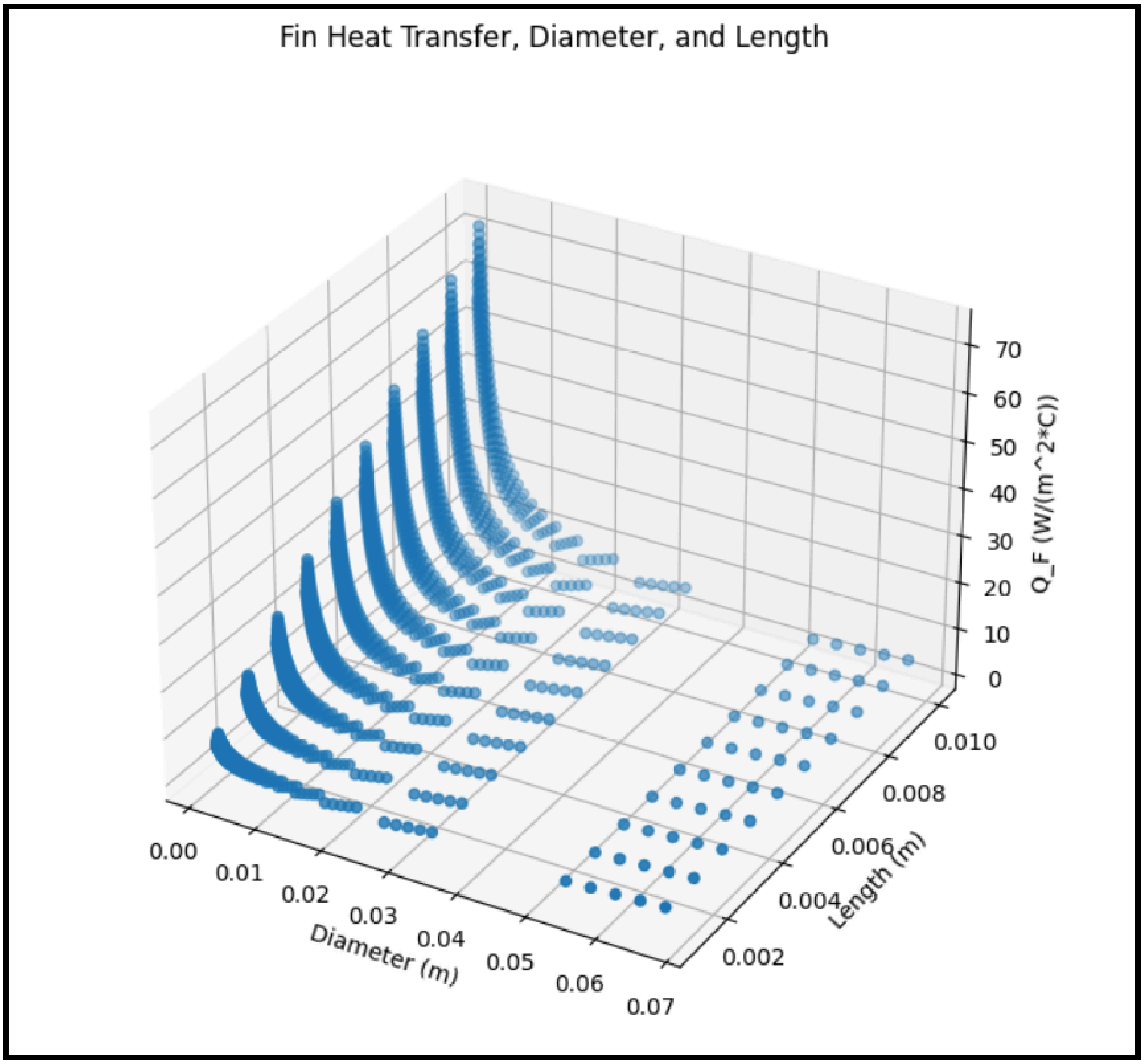

Figure 1.4 : *3D Plot of Fin Heat Transfer, Diameter, and Length*

The figure shows all data points, or configurations, that were produced by varying fin diameter and length. A noticeable trend is visualized with higher rates of heat transfer in fins with a smaller diameter and longer length compared to fins with larger diameters and shorter lengths. This indicates that smaller diameter and longer length will result in an optimized value for the rate of heat transfer through the fin.

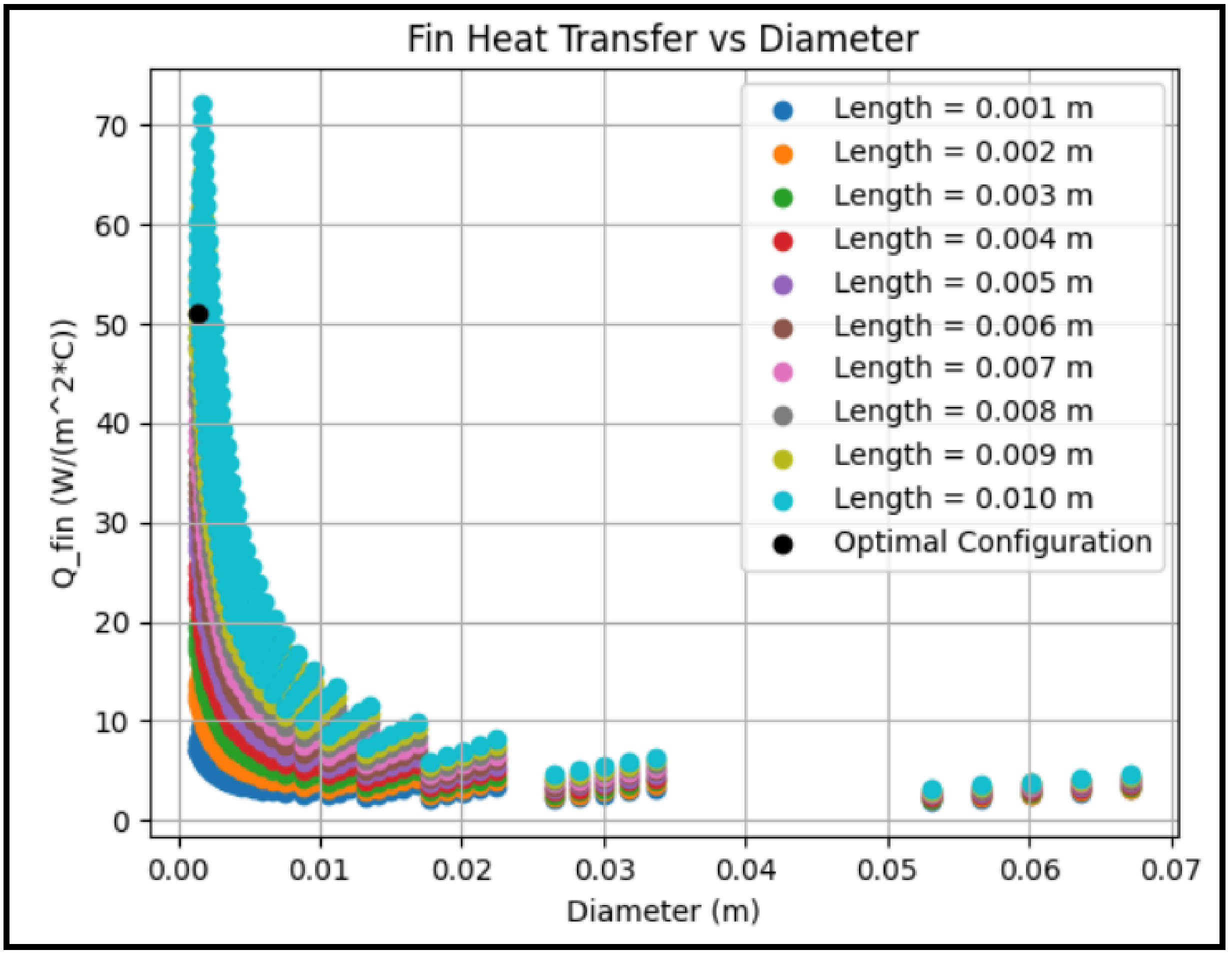

Figure 1.5 : *Heat transfer through the fins by diameter at varying lengths (1 mm - 10 mm)*

The graph shows the fin diameters by the heat transfer at different lengths. It appears that the majority of the higher heat transfer values are toward the beginning of the graph, where the smaller diameters are. Toward the end, the diameters are higher by the heat transfer is lower. This could mean smaller diameters with more fins dissipate more than a few fins with larger diameters. The optimal configuration is toward the front at 1.33 mm diameter and 9 mm length.

Conclusion

After attempting to optimize the various parameters of the configurations, it is evident that there are many different configurations that dissipate greater than or equal to 50 W through the fins. For this project, it was 10 different lengths, with 5 different spacing, with 40 different numbers of fins. With the data collected, out of the 2000 data points, the optimal quantities for the three variables were 1600 fins. 9mm length and 1.33mm diameter, yielding a cost of $0.50.

In the case of increased heat accumulation within the plate, it may be advantageous to consider an over-design to account for additional heat dissipation through the finned surfaces. In doing so, an alteration of the optimal configuration would provide 1600 pins fins, at a length of 9mm, and diameter of 1.5mm. This alteration would yield an additional 8 W of heat transfer at the cost of $0.04, resulting in a total of 62.2 W of total heat transfer, through the surface of the bare plate and fins, at a total cost of $0.54.

References and Appendix

Holman, Jack P. Heat Transfer. 10th ed., McGraw-Hill Education, 2009.

Kumar, Ganesh. “Design, Fabrication and Comparison of Heat Transfer Analysis in Annular Disc Fin and Annular Stepped Fins by Experimental and Numerical Methods.” International Journal of Engineering Research & Technology , vol. 5, no. 7, 2017.

“Price Today.” Markets Insider, Business Insider, markets.businessinsider.com/commodities/aluminum-price. Accessed 30 May 2024.

Project Code: From python

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

# FILM HEAT TRANSF. COEFF.
h = 15 # W / (m^2 * C)

# ALUMINUM PROPERTIES
k = 204 # W / (m * C) @ 20 C
DENSITY_AL = 2707 # kg/m^3
AL_PRICE_PER_KG = 2.63 # $ / kg (based on MarketInsider)

# SURROUNDING AND SURFACE TEMPS
T_INFINITY = 30 # C
T_ZERO = 85 # C
THETA = (T_ZERO - T_INFINITY)
```

```
# PLATE DIMENSIONS
LENGTH = 0.100 # m
WIDTH = 0.050 # m
# ASSUMED THICKNESS
Z = 0.010 # m

AREA_RECT = LENGTH * WIDTH # m^2

# CREATE AN ARRAY OF POSSIBLE ARRANGEMENTS (from 1x1 to 40x40 square array)
boxArray = np.arange(1, 41, 1)
# SQRT OF SQUARE ARRAY GIVES NUM SQUARES (AKA BOXES)
numBoxes = np.square(boxArray)
# AREA OF EACH SQUARE GIVEN BY RECT AREA OVER NUM SQUARES
areaBoxes = AREA_RECT / numBoxes
# SQRT OF BOX AREA GIVES LENGTH OF ONE SIDE OF SQUARES
boxLength = np.sqrt(areaBoxes)

# SPACINGS FROM 75% TO 95% OF SQUARE SIDE LENGTH
percentS = np.arange(0.75, 1, 0.05)
# VARYING PIN LENGTHS FROM 1MM to 10MM
lengthArray = np.arange(0.001, 0.011, 0.001)

# COMBINE THE ARRAYS INTO A SINGLE DATA ARRAY
data = np.column_stack((boxArray, numBoxes, areaBoxes, boxLength))

# REPLICATE VALUES TO ACCOUNT FOR EACH SPACING CONFIGURATION
data = np.tile(data, (np.size(percentS), 1))
spacing_rep = np.repeat(percentS, np.size(boxArray))
data = np.column_stack((data, spacing_rep))

# REPLICATE VALUES TO ACCOUNT FOR EACH LENGTH CONFIGURATION
data = np.tile(data, (np.size(lengthArray), 1))
```

```
length_rep = np.repeat(lengthArray, np.size(spacing_rep))
data = np.column_stack((data, length_rep))

# CREATE A TABLE WITH NECESSARY PARAMETERS
df = pd.DataFrame(data=data, columns=["Square Array", "# Pins", "A (m^2)",
"X (m)", "% Spacing", "L (m)"])

# CALCULATE FIN AND BARE PLATE DIMENSIONS USING SPECIFIED FORMULAS
df["D (m)"] = df["X (m)"] * df["% Spacing"]
df["P (m)"] = np.pi * df["D (m)"]
df["A_F (m^2)"] = np.pi * (df["D (m)"] / 2)**2
df["V (m^3)"] = df["A_F (m^2)"] * df["L (m)"]
df["S (m)"] = df["X (m)"] - df["D (m)"]
df["A_BARE (m^2)"] = AREA_RECT - df["A_F (m^2)"]
df["m"] = np.sqrt((h * df["P (m)"]) / (k * df["A_F (m^2)"]))
df["M"] = np.sqrt(h * df["P (m)"] * k * df["A_F (m^2)"]) * THETA
df["NUMER."] = np.sinh(df["m"] * df["L (m)"]) + (h / (df["m"] * k)) *
np.cosh(df["m"] * df["L (m)"])
df["DENOM."] = np.cosh(df["m"] * df["L (m)"]) + (h / (df["m"] * k)) *
np.sinh(df["m"] * df["L (m)"])

# CALCULATE FIN, BARE PLATE, AND TOTAL HEAT TRANSFER
df["Q_B"] = h * df["A_BARE (m^2)"] * THETA
df["Q_F"] = df["# Pins"] * df["M"] * (df["NUMER."] / df["DENOM."])
df["Q_T (W/m*C)"] = df["Q_B"] + df["Q_F"]

# CALCULATE FIN, BARE PLATE, AND TOTAL MASS
df["M_F"] = df["# Pins"] * (df["V (m^3)"] * DENSITY_AL)
M_B = (AREA_RECT * Z) * DENSITY_AL
df["M_T (kg)"] = M_B + df["M_F"]

# CALCULATE COST BASED ON TOTAL MASS OF CONFIGURATION
```

```
df["$"] = df["M_T (kg)"] * AL_PRICE_PER_KG

table = df[["Square Array", "# Pins", "% Spacing", "L (m)", "D (m)", "S
(m)", "Q_B", "Q_F", "Q_T (W/m*C)", "M_T (kg)", "$"]]

# FIND OPTIMAL CONFIGURATION WHERE FIN HEAT TRANSFER >= 50
table_f = table.loc[(table["Q_F"] >= 50) & (table["Q_T (W/m*C)"] <= 62.5)]
table_f = table_f.sort_values(by=["$", "Q_F", "Q_T (W/m*C)"],
ascending=[True, True, False])
optimal_config = table_f.iloc[0]

# GET OPTIMAL L, D, AND Q
optimal_L = optimal_config[3]
optimal_D = optimal_config[4]
optimal_Q = optimal_config[7]

# LOOP THROUGH VARYING LENGTH CONFIGS TO PLOT D VS Q WITH VARYING L
for length, group in table.groupby("L (m)"):
    plt.scatter(group["D (m)"], group["Q_F"], label=f"Length = {length:.3f}
m")
# PLOT OPTIMAL POINT ON D VS Q PLOT
plt.scatter(optimal_D, optimal_Q, label="Optimal Configuration", c='k')
plt.title("Fin Heat Transfer vs Diameter")
plt.xlabel("Diameter (m)")panel

plt.ylabel("Q_fin (W/(m^2*C))")
plt.xlim()
plt.legend()
plt.grid()

# PLOT DIAMETER VS LENGTH VS HEAT TRANSFER
fig = plt.figure(figsize=(12, 8))
```

```
ax = fig.add_subplot(111, projection='3d')
ax.scatter(table["D (m)"], table["L (m)"], table["Q_F"])
ax.scatter(optimal_D, optimal_L, optimal_Q, label="Optimal Configuration",
c='r')
ax.set_xlabel('Diameter (m)')
ax.set_ylabel('Length (m)')
ax.set_zlabel('Q_F (W/(m^2*C))')
ax.set_box_aspect(aspect=None, zoom=0.9)
ax.legend()

plt.title('Fin Heat Transfer, Diameter, and Length')
plt.show()
fig = plt.figure(figsize=(12, 8))
ax = fig.add_subplot(111, projection='3d')

# Extracting relevant columns for plotting
x = filtered_table['% Spacing']
y = filtered_table['# Pins']
z = filtered_table['L (m)']

# Creating the scatter plot
sc = ax.scatter(x, y, z, c=filtered_table['Q_F'], cmap='viridis')

# Adding color bar
cbar = plt.colorbar(sc, ax=ax, pad=0.1)
cbar.set_label('Heat Transfer Q_F (W)')

highlight_point = [0.75, 1600, 0.009]
ax.scatter(*highlight_point, color='red', s=30, label='Optimized Value')
# Labeling the axes
ax.set_xlabel('Spacing percentage')
ax.set_ylabel('Number of Fins')
```

```
ax.set_zlabel('Length (m)')

# Setting the title
plt.title('Heat Transfer (Q_F) for Different Spacing, Number of Fins, and
Length')

ax.legend()

plt.show()
plt.figure(figsize=(10,6))
plt.scatter(df["# Pins"], df["S (m)"])
highlighted_point = (1600,0.0004419417)
plt.scatter(*highlighted_point, color='red', s=100, edgecolor='black',
marker='X', label='Optimized Value')
plt.xlabel('# of pins')
plt.ylabel('S(m)')
plt.title("# pins by spacing")
plt.grid(True)
plt.legend()
plt.show()
plt.figure(figsize=(10, 6))
for percent in percentS:
subset = table[table["% Spacing"] == percent]
plt.scatter(subset["$"], subset["Q_T (W/m*C)"], label=f'{percent * 100:.0f}%
Spacing')

# Adding the specific configuration point from the screenshot
specific_config = {
"Square Array": 40,
"# Pins": 1600,
"% Spacing": 0.75,
"L (m)": 0.009,
```

```
"D (m)": 0.0013258252,
"S (m)": 0.0004419417,
"Q_B": 4.1238610193,
"Q_F": 50.9771769438,
"Q_T (W/m*C)": 55.1010379631,
"M_T (kg)": 0.1891662185,
"$": 0.4975071546
}
plt.scatter(specific_config["$"], specific_config["Q_F"], color='k', marker
= 'x', s = 40,label='optimized value')

plt.xlabel('Cost ($)')
plt.ylabel('Heat Transfer (Q_F) (W/m*C)')
plt.title('Cost vs heat Transfer for Different Spacing Percentages')
plt.legend()
plt.grid(True)
plt.show()
```

Q_F	Total heat transfer from n fins
n	Number of fins
Q_f	Heat transfer from single fin
Q_{plate}	Heat transfer from the plate
h	Film heat transfer coefficient
k	Thermal conductivity
ϱ	Density
T_∞	Fluid/Surrounding temperature
T_0	Surface temperature
ΔT	Temperature difference
A_c	Fin cross-sectional area
A_{plate}	Plate cross-sectional area
A_{square}	Area of single fin square
P	Fin perimeter
D	Fin diameter
L	Fin length
S	Spacing
M_F	Total mass of fins
M_B	Mass of the plate
M_T	Total mass
C_{Al}	Cost of aluminum per unit mass
X	Square side length
Q_F	Rate of heat transfer from fin surface
C_T	Total cost of material