Hydrodynamics of Pairs of Inclusions in Thin Liquid Crystal Films

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In this experiment we study the hydrodynamic interactions of pairs of circular inclusions in thin, freely suspended, fluid smectic membranes. By observing the velocity of pairs of islands subject to a force, we determined that the mobility of said islands is significantly increased. This falls in line with a model outlined by Kuriabova [1] that combines the Hughes, Pailthorpe, and White [2] theory for the mobility of islands with the Levine-Mackintosh [3] theory for point-force response functions. Kuriabova's theory describes an equal increase in the mobility of both islands as some function of their radius, a, the distance between them, l, and the angle made by the applied force and the line connecting the two islands. The effect becomes more pronounced with large islands and small separation distances.

I. INTRODUCTION

As a thin, freely suspended film; liquid crystal films provide a basis for studying quasi-2D hydrodynamic processes. These processes occur naturally among nature as biological membranes (e.g. lipid vesicles, cell membrane). Biological membranes have an assortment of proteins that are often densely packed together creating a complex system between the proteins, the surrounding fluid, and the membrane itself. This relationship was first described in detail by Saffman and Delbruck [4, 5]. Their analysis treated the proteins embedded in the film as cylinders and as they moved around in the film they move a portion of the surrounding film with them. The study of the movement of proteins in the cell membrane will give insight into many of the processes critical to life occurring on the cellular level.

The study of such membranes are generally difficult, as proteins in the membrane are difficult to manipulate. However, inclusions in a liquid crystal film are a good analogy to the hydrodynamics of proteins in a film. Liquid crystal islands, are especially similar in this regard. They are thicker portions of the membrane that protrude out as cylinders, interacting with the film and the surrounding fluid. Liquid crystal films can be readily made in a laboratory setting, and the islands can be manipulated into position using airjets or laser tweezers.[6]

Previous studies have already proven that liquid crystal islands also couple with the air, furthering their similarity to biological membranes [6–8] . While work has already done on individual inclusions on a liquid crystal film, this experiment seeks to create a greater understanding by determining the hydrodynamic interactions of a pair of inclusions. Although modeling the hydrodynamics of one or two inclusions is much more simple than the complex interactions of proteins in a cell membrane, it is a necessary step in order to completely understand the quasi-2D hydrodynamics present in liquid crystals and cell membranes.

II. BACKGROUND AND THEORY

Before the analysis of a pair of islands, it is important to understand the hydrodynamics of the film itself and of an isolated island. We will first give a brief overview of the hydrodynamics of the film. These membranes, which are only several molecular layers thick, are called *quasi*-2D because they interact with the surrounding fluid and contain some thickness to them. Any analysis of such a system must take its surroundings into account and as such, the hydrodynamics of a viscoelastic membrane separating two viscous fluids have been extensively studied by Levine and Mackintosh. Levine and Mackintosh have determined that the membrane's response at a distance χ to a force applied by a particle at the origin to be

$$\alpha_{||}(\chi) = \frac{1}{4\pi\eta_m}\left[\frac{\pi}{\beta}\mathbf{H}_1(\beta) - \frac{2}{\beta^2} - \frac{\pi}{2}[Y_0(\beta) + Y_2(\beta)]\right] \tag{1}$$

$$\alpha_{\perp} = \frac{1}{4\pi\eta_m}\left[\frac{\pi}{\beta}\mathbf{H}_0(\beta) - \frac{\pi}{\beta}\mathbf{H}_1(\beta) + \frac{2}{\beta^2} - \frac{\pi}{2}\left[Y_0(\beta) - Y_2(\beta)\right]\right] \tag{2}$$

Where $\beta = \dfrac{\chi}{l_s}$, l_s is the Saffman length, $\mathbf{H}_v$ are the Struve functions, and Y_v are the Bessel functions of the second kind. Levine and Mackintosh surmised that their is no distinction between the application of a force over a particular area or the application of a force on a particle occupying such an area. Because there is no distinction between the two scenarios, the equations are applicable to the study of an island applying a force to another island through the flow field. [3]

The case of a single particle interacting with the film must also be well understood before approaching the interaction of a pair of particles. If a particle is undergoing translational motion, it will follow the model developed by Saffman and Delbruck [4]

$$b = \frac{1}{4\pi\eta h}\left[ln\frac{2l_s}{a} - \gamma\right] \quad (3)$$

where μ is the mobility of the particle, a is the radius, and γ is the Euler constant. This model is valid given that the size of the membrane is large enough that the forces from the coupling to the surrounding fluid dominate and the radius of the particle is much less than one Saffman length.

The model has been extended by Hughes, Pailthorpe, and White [2] which predicted that a particle in a membrane would behave in a 3D regime if $a >> l_s$ and would transition to the 2D regime constructed by SD, as $a << l_s$. This model has since been simplified by Petrov and Schwille [9] into the SD-HPW-PS mobility expression

$$\mu_{HPW} = \frac{1}{4\pi\eta h}\left[\frac{ln(\frac{2}{\epsilon}) - \gamma + \frac{4\epsilon}{\pi} - \frac{\epsilon^2}{2}ln(\frac{2}{\epsilon})}{1 - \frac{\epsilon^3}{\pi}ln(\frac{2}{\epsilon} + \frac{c_1\epsilon^{b_1}}{1+c_2\epsilon^{b_2}})}\right] \quad (4)$$

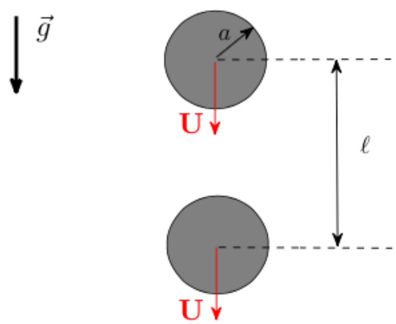

FIG. 1. Diagram of the system to be observed. A pair of particles a distance l apart and radius a are experiencing a force, in this case g, that cause them to move with velocity U

Now that the interactions between the film, the island and a force on the film itself are defined, we can consider the specific case where two particles are in line with a force, F. The derivation of the mobility of a pair of particles acting on each other is complex and is detailed in Tatiana Kuriabova's *Notes on Hydrodynamics of Smectic Islands* . However, given that the particles are arranged as in Fig. 1(i.e. Both particles and F fall along a line), the complex equation is simplified to the addition of the HPW Mobility(4) and the Response Functions(2).

$$U_F = \mu_{HPW}(a) + \alpha_{\parallel}(l) \quad (5)$$

The drag force on either island is the same in magnitude. This means that a particle A and a particle B will move as a pair, giving each other increased velocity. Particle A is pushing particle B and at the same time particle B is pulling on particle A. [1] This is much different than the expected everyday experience of "drafting" where the object in the front lowers the drag force felt by the object behind it.

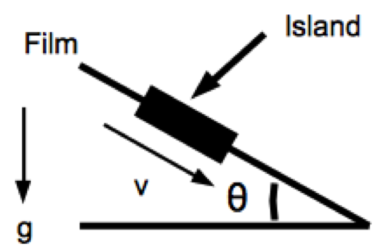

FIG. 2. Diagram of the methodology used to obtain the mobility of an island.

This experiment will gather data by collecting the mobility of pairs of islands on a liquid crystal film to experimentally confirm the validity of equation (5). Experimentally, the mobility can be obtained by applying a force to the pair of islands and measuring their velocity. In our case we will be using gravity as the applied force for its ease of application.

$$\mu = \frac{v}{mgsin\theta} \quad (6)$$

III. EXPERIMENTAL SETUP

To make our thin film we chose 8CB a liquid crystal that exhibits the Smectic A phase at room temperature. The Smectic A phase was chosen because of its ability to create very discrete and measurable molecular layers. The phase is also known to create a very stable and strong film that can easily create islands. The liquid crystal film and the islands have a very specific number of smectic layers(i.e. layers of molecules). [10] The number of layers can be easily determined by their reflectivity which we have calibrated to black glass in previous experiments. As the film becomes thicker the reflectivity increases and the films measured in this experiment are generally 6 molecular layers thick with islands ranging from 10-20 molecular layers thick.

The film itself is created by drawing the edge of a thin piece of glass, wetted with 8CB, across a 1cm diameter gap in a slide of glass and then placed in an enclosure to prevent erratic airflows from disturbing the film. In order to get accurate data the islands to be observed must lie near the center of the film, else they are limited by the boundary effects of the meniscus of the film. Thus, we chose a 1cm diameter film for its fairly thin film thus emphasizing the two-dimensional affects as well as its sizable region away from the edge of the film.

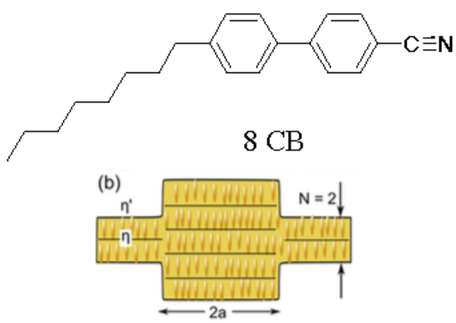

FIG. 3. Top: Molecular structure of 8CB, a well-studied liquid crystal used to create the film. Bottom: Structure of a liquid crystal film and island. Molecules are aligned forming a 2-layer thick film with a 5-layer thick island of radius a.

Islands are created by blowing a puff of air onto the film to agitate the thicker portions of the film at the meniscus. The air pushes portions of the thicker film into the center and spreads them out into dozens of small islands, mostly ranging anywhere from 20μm to 250μm. Pairs of islands of varying ranges were chosen to be studied.

To be able to take data we must get our islands into position as in Fig. 1. To do this we need to have a rotation stage to bring the islands in line as well as a method to tilt the entire microscope to apply gravity. To do this the film is placed on a rotation stage which is in turn attached to a light microscope and then attached to another rotation stage.

Once an isolated pair of islands are found the instrument was tilted to 4^o, chosen for giving a satisfactory length of time to view the islands before they move off-screen and a low brownian noise ratio. The islands are then allowed to reach their terminal velocity, before capturing images with a Watec-221S Camera, at 30 frames per second. The islands can then be tilted to -4^o to take multiple data points on the same set of islands.

The recorded images are analyzed using a particle tracking program based on the Canny method, an edge-finding algorithm. After finding the edges of the images the islands centers are then found by taking their center of masses(average position of the pixels). The velocity is then extracted by fitting a line to the X,Y coordinates of the island centers and using the slope as the velocity. The mobility is then found by using equation (6).

IV. RESULTS AND ANALYSIS

The collected data is represented in Figure 5 and the appendix. All data points collected have increased mobility compared to the HPW mobility. This shows that there is definite coupling between the two islands, and that this coupling causes an increase in mobility. Most

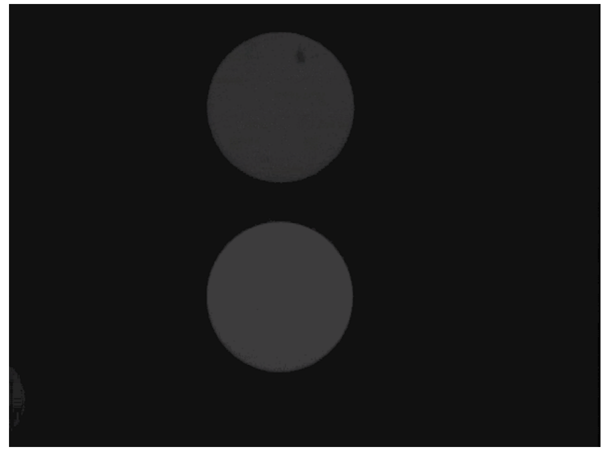

FIG. 4. Example of an observed pair of islands. The islands are on an 8CB film, rotated to be in line with the applied force. The dark, background region are 6 layers, the top island is 11 layers, and the bottom island is 13 layers thick. The islands were found, isolated from other islands.

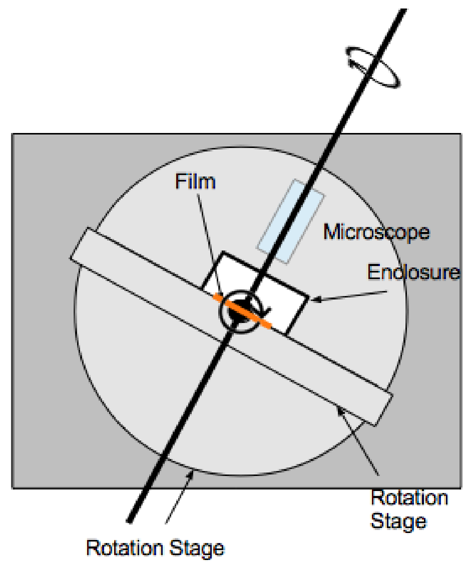

FIG. 5. Diagram of Experimental Apparatus. The entire apparatus is mounted onto a rotation stage and another stage connected to the sample. Utilizing two rotation stages allow the sample to be rotated into alignment as well as applying gravity to the sample.

points also fall fairly close to the curve predicted by theory, giving confidence that equation (5) accurately models the behavior.

However, despite most of the points falling within 10 percent relative error, several of the data points go up to 25 percent error. Furthermore, the data is a function of two variables, obfuscating our results. Because of this, it is inconclusive whether or not the model given by equation (5) is correct. It is apparent the apparatus needs to be improved or otherwise a large amount of data must be taken in order for there to be confidence in the data.

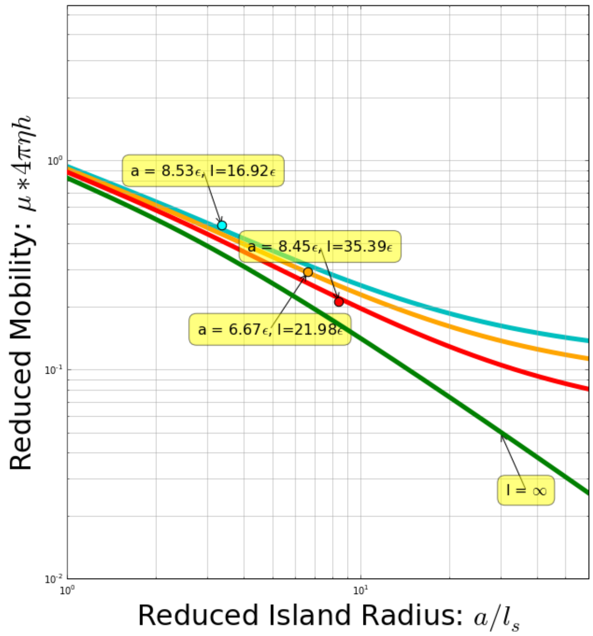

FIG. 6. Plotted data and curves for three points of data, including the data point with the largest, the smallest, and an intermediate separation distance, l.

V. CONCLUSION AND FUTURE WORK

Although the data does not match the curve exactly the results look promising. By modifying the experiment we can more accurately determine whether the proposed theory is correct. The accuracy of the data is obscured by the fact that it is a function of two variables, so in the future we will set the island radius to a constant, measuring how the mobility changes as a function of distance between the islands. To achieve this we can move from using islands to polystyrene beads of a set size, or we can sieve the islands into groupings of similar size. By changing the method of experimentation we can remove variables and more clearly see the coupling of inclusions.

To summarize, we have examined the hydrodynamic interactions of pairs of inclusions in a thin smectic membrane revealing a relationship between the two islands. When the pairs of islands are in line with the applied force, their velocities are directly proportional to the sum of the HPW mobility and the Response Function of the film at that point. These studies on the basic hydrodynamic interactions in thin membranes are inherently useful to the development of larger models of 2D fluids, including biological membranes such as lipid vesicles and plasma membranes.

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Island Radius	Separation Distance	Observed Mobility	Theory Mobility	Relative Error
(ε)	(μm)	-	-	-
6.32E+00	5.95E-04	2.60E-01	2.99E-01	-1.32E+01
7.17E+00	4.66E-04	2.85E-01	3.00E-01	-5.01E+00
5.38E+00	5.95E-04	4.24E-01	3.30E-01	2.84E+01
5.71E+00	6.57E-04	3.24E-01	3.11E-01	4.17E+00
5.74E+00	5.95E-04	3.94E-01	3.17E-01	2.43E+01
5.73E+00	5.95E-04	3.25E-01	3.18E-01	2.24E+00
6.32E+00	5.95E-04	2.60E-01	2.99E-01	-1.32E+01
6.55E+00	6.57E-04	2.08E-01	2.85E-01	-2.69E+01
6.61E+00	5.95E-04	2.91E-01	2.91E-01	-7.05E-02
6.66E+00	5.95E-04	2.12E-01	2.90E-01	-2.69E+01
3.37E+00	4.58E-04	4.86E-01	4.68E-01	3.87E+00
7.14E+00	3.85E-04	2.51E-01	3.22E-01	-2.21E+01
7.42E+00	3.80E-04	3.28E-01	3.17E-01	3.49E+00
8.45E+00	9.58E-04	2.11E-01	2.19E-01	-3.85E+00
5.66E+00	4.25E-04	4.51E-01	3.53E-01	2.78E+01
5.77E+00	4.00E-04	4.63E-01	3.56E-01	3.00E+01
6.33E+00	3.28E-04	3.97E-01	3.64E-01	8.89E+00
6.20E+00	3.35E-04	4.33E-01	3.65E-01	1.86E+01
3.57E+00	5.50E-04	4.04E-01	4.35E-01	-7.07E+00
5.51E+00	4.38E-04	2.77E-01	3.55E-01	-2.19E+01
5.08E+00	5.60E-04	2.92E-01	3.48E-01	-1.58E+01
4.61E+00	6.57E-04	2.98E-01	3.56E-01	-1.64E+01
6.32E+00	5.95E-04	2.60E-01	2.99E-01	-1.32E+01
7.17E+00	4.66E-04	2.85E-01	3.00E-01	-5.01E+00

FIG. 7. All data obtained throughout the experiment is catalogued in this table