CS 132 at Boston University
Assignment6*
Due 30th November @ 11:59pm using gSubmit†only.

1. (3 points) Let $\mathbf{A} = \begin{bmatrix} .4 & -.3 \\ .4 & 1.2 \end{bmatrix}$. Explain why $\mathbf{A}^k$ approaches $\begin{bmatrix} -.5 & -.75 \\ 1.0 & 1.5 \end{bmatrix}$ as $k \to \infty$

2. (5 points) Let $\mathbf{A} = \begin{bmatrix} -6 & 28 & 21 \\ 4 & -15 & -12 \\ -8 & a & 25 \end{bmatrix}$. For each value of a in the set $\{32, 31.9, 31.8, 32.1, 32.2\}$, compute the characteristic polynomial of $\mathbf{A}$ and the eigenvalues. In each case, create a graph of the characteristic polynomial $p(t) = det(\mathbf{A} - t\mathbf{I})$ for $0 \leq t \leq 3$. If possible, construct all graphs on one coordinate system. Describe how the graphs reveal the changes in the eigenvalues as a changes.

3. (5 points) Let $\beta = \{\mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}\}$ be a basis for the vector space V. Find $T(3\mathbf{b_1} - 4\mathbf{b_2})$ when T is a linear transformation from V to V whose matrix relative to β is: $[T]_\beta = \begin{bmatrix} 0 & -6 & 1 \\ 0 & 5 & -1 \\ 1 & -2 & 7 \end{bmatrix}$

4. (9 points) Define T: $\mathbb{P}^2 \to \mathbb{R}^3$ by $T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(-1) \\ \mathbf{p}(0) \\ \mathbf{p}(1) \end{bmatrix}$
   (a) Find the image under T of $\mathbf{p}(t) = 5 + 3t$
   (b) Show that T is a linear transformation.
   (c) Find the matrix T relative to the basis $\{\mathbf{1}, \mathbf{t}, \mathbf{t^2}\}$ for $\mathbb{P}^2$ and the standard basis for $\mathbb{R}^3$.

5. (6 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(\mathbf{x}) = \mathbf{A}\mathbf{x}$. Find a basis for β for $\mathbb{R}^2$ with the property that $[T]_\beta$ is diagonal.
   (a) $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -3 & 4 \end{bmatrix}$.
   (b) $\mathbf{A} = \begin{bmatrix} 5 & -3 \\ -7 & 1 \end{bmatrix}$.

6. Load the BU.jpg file provided on Blackboard, and run the following script on Matlab[1]

```
img_in=double((rgb2gray(imread('BU.jpg'))));
image(img_in)
[V,D]=eig(cov(img_in))
image((V(:,530:640)*V(:,530:640)'*img_in')')
```

*All matrices are in capital letters and bold. All vectors are in lower case and bold. All scalars are lower case and not bolded.

†if you are not familiar with gsubmit, come to my office hours and I am happy to show you how it works. No email submissions will be accepted

[1]You can run `help cov` for example on Matlab command line to find more information about the function cov

(a) (1 point) What does the function *cov* in Matlab do?

(b) (3 points) What does $cov(img\_in)$ do? Why is this required? What does the value of each cell in this matrix indicate in context of img_in?

(c) (1 point) What does the *eig* function in Matlab do?

(d) (3 points) What does the $\mathbf{V}$ and the $\mathbf{D}$ matrix in the above script contain? i.e. what are the columns of $\mathbf{V}$? And what are the diagonal entries of $\mathbf{D}$ contain?

(e) (3 points) Why does the $\mathbf{V}^{-1} == \mathbf{V}^T$. What is special about the columns of the matrix V? Hint: $\mathbf{V}(:, i)^T\mathbf{V}(:, j) = 0$ where $i \neq j$ and i and j are columns of the matrix $\mathbf{V}$.

(f) (3 points) What does `V(:,530:640)'*img_in'` mean? You must use your knowledge of change of basis matrix when answering this. Is the image in img_in getting compressed with the above code?

(g) (3 points) What does the number from 530 to 640 indicate? Why are these numbers sufficient to recreate the original image? What happens if you change the 530 to 640? How would you explain the image that gets generated in Matlab? Make sure to provide a screenshot of the image from MATLAB.

(h) (3 points) What happens when you take the smallest 400 eigenvectors and perform compression and then uncompression. What image gets produced? Why does this image is not closer to the true image? You must use your knowledge of variance when answering this question. Make sure to provide a screenshot of the image from MATLAB.

(i) (3 points) What does `V(:,530:640)*V(:,530:640)'*img_in'` mean? You must use your knowledge of change of basis matrix when answering this.