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CS 132
Assignment1
Due 22nd September @11:59pm using gSubmit*

1. Determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

   (a)
   $$\begin{bmatrix} 2 & 3 & h \\ 4 & 6 & 7 \end{bmatrix} \quad \begin{matrix}[4\ 6\ 7] \\ [2\ 3\ h]\end{matrix} \quad \begin{matrix}[4 & 6 & 7\ ] \\ [0 & 0 & h\text{-}3.5]\end{matrix}$$

   h = 3.5

   (b)
   $$\begin{bmatrix} 1 & -3 & -2 \\ 5 & h & -7 \end{bmatrix} \quad \begin{matrix}[5\ \ h\ \text{-}7] \\ [1\ \text{-}3\ \text{-}2]\end{matrix} \quad \begin{matrix}[5 & h & \text{-}7\ ] \\ [0 & \text{-}3\text{-}0.2h & \text{-}0.6]\end{matrix}$$

   h != -15

2. List five vectors in Span $\{\mathbf{v_1}, \mathbf{v_2}\}$ For each vector, show the weights on $\mathbf{v_1}$ and $\mathbf{v_2}$ used to generate the
3 vector and list the three entries of the vector. Do not make a sketch.

   (a)
   $$\mathbf{v_1} = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} \quad \mathbf{v_2} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$$

   $\mathbf{v}_3 = 1\mathbf{v}_1 + 2\mathbf{v}_2 = \begin{matrix}[\ \text{-}1\ ] \\ [\ 0\ ] \\ [\ 8\ ]\end{matrix}$ Entries = -1, 0, 8

   $\mathbf{v}_4 = 2\mathbf{v}_1 + 1\mathbf{v}_2 = \begin{matrix}[\ 4\ ] \\ [\ 0\ ] \\ [\ 7\ ]\end{matrix}$ Entries = 4, 0, 7

   $\mathbf{v}_5 = 1\mathbf{v}_1 + 4\mathbf{v}_2 = \begin{matrix}[\ \text{-}5\ ] \\ [\ 0\ ] \\ [\ 14\ ]\end{matrix}$ Entries = -5, 0, 14

   $\mathbf{v}_6 = 3\mathbf{v}_1 + 2\mathbf{v}_2 = \begin{matrix}[\ 5\ ] \\ [\ 0\ ] \\ [\ 12\ ]\end{matrix}$ Entries = 5, 0, 12

   $\mathbf{v}_7 = 2\mathbf{v}_1 + 2\mathbf{v}_2 = \begin{matrix}[\ 2\ ] \\ [\ 0\ ] \\ [\ 10\ ]\end{matrix}$ Entries = 2, 0, 10

   (b)
   $$\mathbf{v_1} = \begin{bmatrix} 7 \\ 1 \\ -6 \end{bmatrix} \quad \mathbf{v_2} = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$$

   $\mathbf{v}_3 = 1\mathbf{v}_1 + 2\mathbf{v}_2 = \begin{matrix}[\ \text{-}3\ ] \\ [\ 7\ ] \\ [\ \text{-}6\ ]\end{matrix}$ Entries = -3, 7, -6

   $\mathbf{v}_4 = 1\mathbf{v}_1 + 3\mathbf{v}_2 = \begin{matrix}[\ \text{-}8\ ] \\ [\ 10\ ] \\ [\ \text{-}6\ ]\end{matrix}$ Entries = -8, 10, -6

   $\mathbf{v}_5 = 1\mathbf{v}_1 + 4\mathbf{v}_2 = \begin{matrix}[\ \text{-}13\ ] \\ [\ 13\ ] \\ [\ \text{-}6\ ]\end{matrix}$ Entries = -13, -13, -6

   $\mathbf{v}_6 = 2\mathbf{v}_1 + 1\mathbf{v}_2 = \begin{matrix}[\ 9\ ] \\ [\ 5\ ] \\ [\ \text{-}12\ ]\end{matrix}$ Entries = 9, 5, -12

   $\mathbf{v}_7 = 3\mathbf{v}_1 + 1\mathbf{v}_2 = \begin{matrix}[\ 16\ ] \\ [\ 6\ ] \\ [\ \text{-}18\ ]\end{matrix}$ Entries = 16, 6, -18

3. Determine if b is a linear combination of the vectors formed from the columns of the matrix $\mathbf{A}$.

   (a)
   $$\mathbf{A} = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$$

   No (Inconsistent)

   $$\begin{matrix}[\ 1\ \text{-}4\ 2\,|\,3\ ] \\ [\ 0\ 3\ 5\,|\,\text{-}7\ ] \\ [\ \text{-}2\ 8\ \text{-}4\,|\,\text{-}3\ ]\end{matrix} \sim \begin{matrix}[\ 1\ \text{-}4\ 2\,|\,3\ ] \\ [\ \text{-}2\ 8\ \text{-}4\,|\,\text{-}3\ ] \\ [\ 0\ 3\ 5\,|\,\text{-}7\ ]\end{matrix} \sim \begin{matrix}[\ 1\ \text{-}4\ 2\,|\,3\ ] \\ [\ 0\ 0\ 0\,|\,3\ ] \\ [\ 0\ 3\ 5\,|\,\text{-}7\ ]\end{matrix}$$

   $$\begin{matrix}[\ 1\ \text{-}4\ 2\,|\,3\ ] \\ [\ 0\ 3\ 5\,|\,\text{-}7\ ] \\ [\ 0\ 0\ 0\,|\,3\ ]\end{matrix}$$

   (b)
   $$\mathbf{A} = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$$

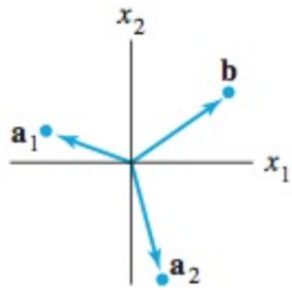
   Yes

   $$\begin{matrix}[\ 1\ \text{-}2\ \text{-}6\,|\,11\ ] \\ [\ 0\ 3\ 7\,|\,\text{-}5\ ] \\ [\ 1\ \text{-}2\ 5\,|\,9\ ]\end{matrix} \sim \begin{matrix}[\ 1\ \text{-}2\ \text{-}6\,|\,11\ ] \\ [\ 0\ 3\ 7\,|\,\text{-}5\ ] \\ [\ 0\ 0\ 11\,|\,\text{-}2\ ]\end{matrix} \sim \begin{matrix}[\ 1\ \text{-}2\ \text{-}6\,|\,11\ ] \\ [\ 0\ 3\ 7\,|\,\text{-}5\ ] \\ [\ 0\ 0\ 1\,|\,\text{-}2/11\ ]\end{matrix}$$

   $$\begin{matrix}[\ 1\ \text{-}2\ \text{-}6\,|\,11\ ] \\ [\ 0\ 3\ 0\,|\,\text{-}41/11\ ] \\ [\ 0\ 0\ 1\,|\,\text{-}2/11\ ]\end{matrix} \sim \begin{matrix}[\ 1\ \text{-}2\ \text{-}6\,|\,11\ ] \\ [\ 0\ 1\ 0\,|\,\text{-}41/33\ ] \\ [\ 0\ 0\ 1\,|\,\text{-}2/11\ ]\end{matrix} \sim \begin{matrix}[\ 1\ \text{-}2\ 0\,|\,109/11\ ] \\ [\ 0\ 1\ 0\,|\,\text{-}41/33\ ] \\ [\ 0\ 0\ 1\,|\,\text{-}2/11\ ]\end{matrix} \sim \begin{matrix}[\ 1\ 0\ 0\,|\,245/33\ ] \\ [\ 0\ 1\ 0\,|\,\text{-}41/33\ ] \\ [\ 0\ 0\ 1\,|\,\text{-}2/11\ ]\end{matrix}$$

*All matrices are in capital letters and bold. All vectors are in lower case and bold. All scalars are lower case and not bolded.If you are not familiar with gSubmit, come to my office hours and I am happy to show you how it works. No email submissions will be accepted

4. Let $\mathbf{a_1}, \mathbf{a_2}$, and $\mathbf{b}$ be the vectors in $\mathbb{R}^2$ shown in the figure, and let $\mathbf{A} = [\mathbf{a1}, \mathbf{a2}]$. Does the equation $\mathbf{A}x = \mathbf{b}$ have a solution? If so, is the solution unique? Explain.



Yes, this equation has an unique solution because both a1 and a2 are non colinear, so they can span R2

Figure 1:

5. Suppose $\mathbf{A}$ is a $3X3$ matrix and $\mathbf{b}$ is a vector in $\mathbb{R}^3$ with the property that $\mathbf{A}x = \mathbf{b}$ has a unique solution. Explain why the columns of $\mathbf{A}$ must span $\mathbb{R}^3$. According to the theorem 4, if Ax = b has a solution, the columns of A span IR3.

6. Could a set of three vectors in $\mathbb{R}^4$ span all of $\mathbb{R}^4$? Explain. What about n vectors in $\mathbb{R}^m$ when n is less than m? No. Suppose A = {v1 v2 v3}, A can have only 3 pivots columns at most, which can span up to R3 but not R4.

No. The reason is the same an the previous one.

7. Determine if the columns of the matrix span $\mathbb{R}^4$.

(a)

$$\begin{bmatrix} 7 & 2 & -5 & 8 \\ -5 & -3 & 4 & -9 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$$

No

$$\begin{bmatrix} 7 & 2 & -5 & 8 \\ -5 & -3 & 4 & -9 \\ 6 & 10 & -2 & 7 \\ 0 & 11 & -3 & 23 \end{bmatrix} \sim \begin{bmatrix} 7 & 2 & -5 & 8 \\ -5 & -3 & 4 & -9 \\ 0 & 32/5 & 14/5 & -19/5 \\ 0 & 11 & -3 & 23 \end{bmatrix} \sim \begin{bmatrix} 7 & 2 & -5 & 8 \\ -5 & -3 & 4 & -9 \\ 0 & 32 & 14 & -19 \\ 0 & 11 & -3 & 23 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 2 & -5 & 8 \\ -5 & -3 & 4 & -9 \\ 0 & 32 & 14 & -19 \\ 0 & 0 & -125/16 & 945/32 \end{bmatrix} \sim \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/3 & 3/7 & -23/7 \\ 0 & 32 & 14 & -19 \\ 0 & 0 & -125/16 & 945/32 \end{bmatrix} \sim \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11 & 3 & -23 \\ 0 & 32 & 14 & -19 \\ 0 & 0 & -125/16 & 945/32 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11 & 3 & -23 \\ 0 & 0 & 250/11 & 945/11 \\ 0 & 0 & -125/16 & 945/32 \end{bmatrix} \sim \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11 & 3 & -23 \\ 0 & 0 & -250 & 945 \\ 0 & 0 & -250 & 945 \end{bmatrix} \sim \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11 & 3 & -23 \\ 0 & 0 & 1 & -3.78 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11 & 0 & -11.56 \\ 0 & 0 & 1 & -3.78 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 1 & 0 & 1.06 \\ 0 & 0 & 1 & -3.78 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 7 & 2 & 0 & -10.9 \\ 0 & 1 & 0 & 1.06 \\ 0 & 0 & 1 & -3.78 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 7 & 0 & 0 & -13.02 \\ 0 & 1 & 0 & 1.06 \\ 0 & 0 & 1 & -3.78 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -1.86 \\ 0 & 1 & 0 & 1.06 \\ 0 & 0 & 1 & -3.78 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ -7 & -8 & 5 & 6 & -9 \\ 11 & 7 & -7 & -9 & -6 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix}$$

Yes

$$\begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ -7 & -8 & 5 & 6 & -9 \\ 11 & 7 & -7 & -9 & -6 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix} \sim \begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ -7 & -8 & 5 & 6 & -9 \\ 11 & 7 & -7 & -9 & -6 \\ 0 & 52/7 & -8/7 & -38/7 & 76/7 \end{bmatrix} \begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ -7 & -8 & 5 & 6 & -9 \\ 11 & 7 & -7 & -9 & -6 \\ 0 & 26 & -4 & -19 & 38 \end{bmatrix} \sim \begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ -7 & -8 & 5 & 6 & -9 \\ 0 & -39/7 & 6/7 & 3/7 & -141/7 \\ 0 & 26 & -4 & -19 & 38 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ -7 & -8 & 5 & 6 & -9 \\ 0 & -39 & 6 & 3 & -141 \\ 0 & 26 & -4 & -19 & 38 \end{bmatrix} \sim \begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ -7 & -8 & 5 & 6 & -9 \\ 0 & -39 & 6 & 3 & -141 \\ 0 & 0 & 0 & -17 & -84/19 \end{bmatrix} \sim \begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ -7 & -8 & 5 & 6 & -9 \\ 0 & -39 & 6 & 3 & -141 \\ 0 & 0 & 0 & 1 & 0.26 \end{bmatrix} \sim \begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ 0 & 13 & -2 & -1 & 19 \\ 0 & -39 & 6 & 3 & -141 \\ 0 & 0 & 0 & 1 & 0.26 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ 0 & 13 & -2 & -1 & 19 \\ 0 & 0 & 0 & 0 & -84 \\ 0 & 0 & 0 & 1 & 0.26 \end{bmatrix} \sim \begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ 0 & 13 & -2 & -1 & 19 \\ 0 & 0 & 0 & 1 & 0.26 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ 0 & 13 & -2 & -1 & 19 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ 0 & 13 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 8 & 11 & -6 & 0 & 0 \\ 0 & 13 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 8 & -28 & 0 & 0 & 0 \\ 0 & 13 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 8 & -28 & 0 & 0 & 0 \\ 0 & 1 & -0.1538 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -0.5385 & 0 & 0 \\ 0 & 1 & -0.1538 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$