CS 132 at Boston University
Assignment5*
Due 11th November @ 11:59pm using gSubmit†only.

1. Let $\mathbf{P} = \begin{bmatrix} 1 & 2 & -1 \\ -3 & 5 & 0 \\ 4 & 6 & 1 \end{bmatrix}$ and let

   $\mathbf{v_1} = \begin{bmatrix} -2 \\ 2 \\ 3 \end{bmatrix}$ $\mathbf{v_2} = \begin{bmatrix} -8 \\ 5 \\ 2 \end{bmatrix}$ $\mathbf{v_3} = \begin{bmatrix} -7 \\ 2 \\ 6 \end{bmatrix}$

   (a) (5 points) Find a basis $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ for $\mathbb{R}^3$, such that P is the change of coordinates matrix from $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ to the $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$. Hint: What do the columns of $\underset{C \leftarrow B}{P}$ represent?

   (b) (5 points) Find a basis $\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}$ for $\mathbb{R}^3$, such that P is the change of coordinates matrix from $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ to $\{\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}\}$.

2. Let $\beta = \{\mathbf{b_1}, \mathbf{b_2}\}$ and $C = \{\mathbf{c_1}, \mathbf{c_2}\}$ be bases for $\mathbb{R}^2$. In the following subparts find the change of coordinates matrix from β to C. Also find the change of coordinates matrix from C to β.

   (a) (3 points) $\mathbf{b_1} = \begin{bmatrix} 7 \\ 5 \end{bmatrix}, \mathbf{b_2} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ $\mathbf{c_1} = \begin{bmatrix} 1 \\ -5 \end{bmatrix}, \mathbf{c_2} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

   (b) (3 points) $\mathbf{b_1} = \begin{bmatrix} -1 \\ 8 \end{bmatrix}, \mathbf{b_2} = \begin{bmatrix} -1 \\ -5 \end{bmatrix}$ $\mathbf{c_1} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \mathbf{c_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (c) (3 points) $\mathbf{b_1} = \begin{bmatrix} -6 \\ 1 \end{bmatrix}, \mathbf{b_2} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $\mathbf{c_1} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \mathbf{c_2} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$

3. (5 points) In $\mathbb{P}_2$ find the change of coordinate matrix from the basis $\beta = \{1-2t+t^2, 3-5t+4t^2, 2t+3t^2\}$ to the standard basis $C = \{1, t, t^2\}$. Then find the β coordinate vector for $-1+2t$.

4. (5 points) Determine whether $\mathbf{w}$ is in the column space of $\mathbf{A}$, the null space of $\mathbf{A}$ or both, where:

   $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -3 \end{bmatrix}, \mathbf{A} = \begin{bmatrix} 7 & 6 & -4 & 1 \\ -5 & -1 & 0 & -2 \\ 9 & -11 & 7 & -3 \\ 19 & -9 & 7 & 1 \end{bmatrix}$

5. Determine whether the sets of polynomials form a basis for $\mathbb{P}_3$. Justify your conclusions:

   (a) (3 points) $3+7t, 5+t-2t^3, t-2t^2, 1+16t-6t^2+2t^3$

   (b) (3 points) $5-3t+4t^2+2t^3, 9+t+8t^2-6t^3, 6-2t+5t^2, t^3$

*All matrices are in capital letters and bold. All vectors are in lower case and bold. All scalars are lower case and not bolded.

†if you are not familiar with gsubmit, come to my office hours and I am happy to show you how it works. No email submissions will be accepted

6. (5 points) Let S be a finite set in a vector space V with the property that every $\mathbf{x}$ in V has a unique representation as a linear combination of elements of S. Show that S is a basis of V.

7. (5 points) Let $\beta = \{\mathbf{b_1}, \ldots, \mathbf{b_n}\}$ be a basis for the vector space V. Explain why the coordinates of β coordinate vectors $\mathbf{b_1}, \ldots, \mathbf{b_n}$ are the columns $\mathbf{e_1}, \ldots, \mathbf{e_n}$ of the nxn identity matrix. Note that $\mathbf{e_1}, \ldots, \mathbf{e_n}$ are the standard basis.

8. (5 points) Compute determinant of $\mathbf{B^4}$, where $\mathbf{B} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

9. Let $\mathbf{A}$ and $\mathbf{B}$ be a 3 x 3 matrices with det$\mathbf{A} = -3$ and det$\mathbf{B} = 4$. Use properties of determinant and find the following:
   (a) (2 points) det $\mathbf{AB}$
   (b) (2 points) det $5\mathbf{A}$
   (c) (2 points) det $\mathbf{B^T}$
   (d) (2 points) det $\mathbf{A^{-1}}$
   (e) (2 points) det $\mathbf{A^3}$

10. Find the determinant of the following when you know that: $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 7$
    (a) (3 points) $\begin{bmatrix} a & b & c \\ d & e & f \\ 3g & 3h & 3i \end{bmatrix}$
    (b) (3 points) $\begin{bmatrix} a & b & c \\ 5d & 5e & 5f \\ g & h & i \end{bmatrix}$
    (c) (3 points) $\begin{bmatrix} a & b & c \\ d+3g & e+3h & f+3i \\ g & h & i \end{bmatrix}$

11. Use cofactor expansion to find the determinant of the following matrices. Make sure to clearly tell us which row/column you have chosen for the expansion.
    (a) (3 points) $\begin{bmatrix} 3 & 4 & -3 & 1 \\ 3 & 0 & 1 & -3 \\ -6 & 0 & -4 & 3 \\ 6 & 8 & -4 & -1 \end{bmatrix}$
    (b) (3 points) $\begin{bmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 11 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{bmatrix}$

12. Suppose memory or size restrictions prevent your matrix program from working with matrices having more than 32 rows and 32 columns and suppose some project involves 50x50 matrices A and B.
    (a) (5 points) Solve $\mathbf{Ax} = \mathbf{b}$ for some vector $\mathbf{b}$ in $\mathbb{R}^{50}$, assuming that A can be partitioned into a 2x2 block matrix A_{ij}, with A_{11} an invertible 20 x 20 matrix, A_{22} an invertible 30 x 30 matrix, and A_{12} a zero matrix. Hint: Describe appropriate smaller systems to solve, without using any matrix inverse.