CS 132 at Boston University
Assignment7*
Due 11th December @ 11:59pm using gSubmit†only.

1. Let $\mathbf{A}$ be a mxn matrix. Use the steps below to show that a vector $\mathbf{x}$ in $\mathbb{R}^n$ satisfies $\mathbf{Ax} = \mathbf{0}$ if and only if $\mathbf{A^T Ax} = \mathbf{0}$. This will show that $Nul\mathbf{A} = Nul\mathbf{A^T A}$.
   (a) (2 points) Show that if $\mathbf{Ax} = \mathbf{0}$ then $\mathbf{A^T Ax} = \mathbf{0}$
   (b) (3 points) Suppose that $\mathbf{A^T Ax} = \mathbf{0}$. Explain why $\mathbf{x^T A^T Ax} = \mathbf{0}$ and use this to show that $\mathbf{Ax} = \mathbf{0}$.
2. Let $\mathbf{A}$ be a mxn matrix whose columns are linearly independent. [Careful: $\mathbf{A}$ need not be square].
   (a) (5 points) Use the previous question to show that $\mathbf{A^T A}$ is an invertible matrix.
   (b) (2 points) Explain why $\mathbf{A}$ must have alteast as many rows as columns.
   (c) (3 points) Determine the rank of $\mathbf{A}$
3. (4 points) Find a formula for the least squares solution of Ax=b when the columns of A are orthonormal[1].
4. Describe all the least squares solution of the equation $\mathbf{Ax} = \mathbf{b}$ using MATLAB[2].
   (a) (3 points) $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ 8 \\ 2 \end{bmatrix}$
   (b) (3 points) $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 7 \\ 2 \\ 3 \\ 6 \\ 5 \\ 4 \end{bmatrix}$.
5. (5 points) If data points tend to follow a pattern of the equation of the form:

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$$

   Describe the linear model that gives a least-squares fit of this type to data $(x_1, y_1) \ldots (x_n, y_n)$. You can generate the data points with some noise added in these data points by running the following code in MATLAB. Make sure to provide us the screenshot of your observations plotted in blue with circles AND the line in RED with thickness of 3 which is the best fit line solved using regression. Also do not forget to provide us your MATLAB code in the PDF.

*All matrices are in capital letters and bold. All vectors are in lower case and bold. All scalars are lower case and not bolded.

†if you are not familiar with gsubmit, come to my office hours and I am happy to show you how it works. No email submissions will be accepted

[1] `https://en.wikipedia.org/wiki/Orthonormal_basis`

[2] You must attach a screenshot of the MATLAB code. Also clearly tell us how many unknows are present and what are the values of the unknowns after you solve them

```
x=linspace(0,10,1000)';
y=6+5*x+3*x.^2+3*x.^3+x.^3.*rand(size(x));
plot(x,y,'bo')
```

6. (5 points) When the monthly sales of a product are subject to a seasonal fluctuations, a curve that approximates the sales data might have the form $y = \beta_0 + \beta_1 x + \beta_2 sin(\frac{2\pi x}{4})$ where x is the time in months. The term $\beta_0 + \beta_1 x$ gives the basic sales trend, and the sine term reflects the seasonal changes in sales. Give the coefficient matrix and the parameter vector the linear model that leads to a least-squares fit of the equation above. Assume the data are $(x_1, y_1) \ldots (x_n, y_n)$. We have already provided you the data in a separate text file on Blackboard. The first column is all your x's and the 2nd column is all your y's. Make sure to provide us the screenshot of your observations plotted in blue with circles AND the line in RED with thickness of 3 which is the best fit line solved using regression. Also do not forget to provide us your MATLAB code in the PDF.