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Suppose $A = \begin{bmatrix} a1 \\ a2 \\ a3 \end{bmatrix}$ $B = \begin{bmatrix} b1 \\ b2 \\ b3 \end{bmatrix}$ $A + B = \begin{bmatrix} a1 + b1 \\ a2 + b2 \\ a3 + b3 \end{bmatrix}$

$T(A) = \begin{bmatrix} a1 \\ 0 \\ a3 \end{bmatrix}$ $T(B) = \begin{bmatrix} b1 \\ 0 \\ b3 \end{bmatrix}$

$T(A + B) = \begin{bmatrix} a1 + b1 \\ 0 \\ a3 + b3 \end{bmatrix}$

CS 132 at Boston University
Assignment3*
Due 18th October @ 11:59pm using gSubmit† only.

Suppose $A = \begin{bmatrix} a1 \\ a2 \\ a3 \end{bmatrix}$ $cA = \begin{bmatrix} c*a1 \\ c*a2 \\ c*a3 \end{bmatrix}$ while c is a scalar

$T(A) = \begin{bmatrix} a1 \\ 0 \\ a3 \end{bmatrix}$ and $T(cA) = \begin{bmatrix} c*a1 \\ 0 \\ c*a3 \end{bmatrix}$ $c*T(A) = \begin{bmatrix} c*a1 \\ 0 \\ c*a3 \end{bmatrix}$

Since T(A + B) = T(A) + T(B), this satisfies the first property of the linear transformation

Since the transformation satisfies both properties of the linear transformation, we can conclude that T is a linear transformation.

Since T(cA) = c*T(A), this satisfies the second property of the linear transformation

1. Let T: $\mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a transformation that projects each vector $\mathbf{x} = (x_1, x_2, x_3)$ onto the plane $x_2 = 0$ so $T(x) = (x_1, 0, x_3)$. Show that T is a linear transformation.

2. [1] The given matrix **A** (in part a) and b)) determines a linear transformation T:

   (a)

   (i) $x = \begin{bmatrix} 3.5 * a \\ 4.5 * a \\ 0 \\ a \end{bmatrix}$ such that a is a scalar in R

   $$\mathbf{A} = \begin{bmatrix} 4 & -2 & 5 & -5 \\ -9 & 7 & -8 & 0 \\ -6 & 4 & 5 & 3 \\ 5 & -3 & 8 & -4 \end{bmatrix}$$

   (ii) Yes. $x = \begin{bmatrix} (3.5 * a) + 4 \\ (4.5 * a) + 7 \\ 1 \\ a \end{bmatrix}$ such that a is a scalar in R

   i. Find all $\mathbf{x}$ such that $T(\mathbf{x}) = \mathbf{0}$.

   ii. Let $\mathbf{b} = [7, 5, 9, 7]$ and let A be the matrix from above. Is $\mathbf{b}$ in the range of of the transformation $x :\rightarrow Ax$. If so, find an x whose image under the transformation is b.

   (b)

   (i) $x = \begin{bmatrix} -0.75 * a \\ -1.25 * a \\ 1.75 * a \\ a \end{bmatrix}$ such that a is a scalar in R

   $$\mathbf{A} = \begin{bmatrix} -9 & -4 & -9 & 4 \\ 5 & -8 & -7 & 6 \\ 7 & 11 & 16 & -9 \\ 9 & -7 & -4 & 5 \end{bmatrix}$$

   (ii) Yes. $x = \begin{bmatrix} -(0.75 * a) - 1.25 \\ -(1.25 * a) - 2.75 \\ (1.75 * a) + 3.25 \\ a \end{bmatrix}$ such that a is a scalar in R

   i. Find all $\mathbf{x}$ such that $T(\mathbf{x}) = \mathbf{0}$.

   ii. Let $\mathbf{b} = [-7, -7, 13, -5]$ and let $\mathbf{A}$ be the matrix from above. Is $\mathbf{b}$ in the range of of the transformation $x :\rightarrow \mathbf{Ax}$. If so, find an $\mathbf{x}$ whose image under the transformation is $\mathbf{b}$.

3. The figure shows vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$, along with the images $T(\mathbf{u})$ and $T(\mathbf{v})$ under the action of a linear transformation T:$\mathbb{R}^2 \rightarrow \mathbb{R}^2$. Copy this figure carefully, and draw the image $T(\mathbf{w})$ as accurately as possible. [Hint: First, write $\mathbf{w}$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$.]

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*All matrices are in capital letters and bold. All vectors are in lower case and bold. All scalars are lower case and not bolded.

†if you are not familiar with gsubmit, come to my office hours and I am happy to show you how it works. No email submissions will be accepted

[1] You must use Matlab to answer this question

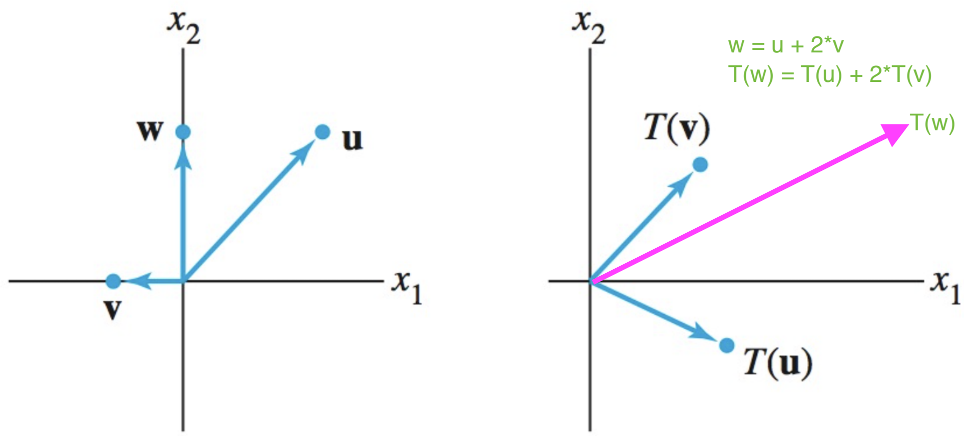

Figure 1: This figure is in relation to Question 3

$A = \begin{bmatrix} -2 & 7 \\ 5 & -3 \end{bmatrix}$

4. Let $\mathbf{x} = [x_1, x_2]$, $\mathbf{v_1} = [-2, 5]$ and $\mathbf{v_2} = [7, -3]$ and let Let T: $\mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\mathbf{x}$ onto $x_1\mathbf{v_1} + x_2\mathbf{v_2}$. Find a matrix $\mathbf{A}$ such that $T(\mathbf{x})$ is $\mathbf{Ax}$ for each $\mathbf{x}$.

5. [2] A useful way to test new ideas in matrix algebra, or to make conjectures, is to make calculations with matrices selected at random. Checking a property for a few matrices does not prove that the property holds in general, but it makes the property more believable. Also, if the property is actually false, you may discover this when you make a few calculations.

   (a) Describe in words what happens when you compute $\mathbf{A}^5$, $\mathbf{A}^{10}$; $\mathbf{A}^{20}$; and $\mathbf{A}^{30}$ for

$$\mathbf{A} = \begin{bmatrix} \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{5}{12} \end{bmatrix}$$

Each element of A is approaching to 1/3.

   (b) i. Construct a random[3] $4X4$ matrix $\mathbf{A}$ and test whether $(\mathbf{A}+\mathbf{I})(\mathbf{A}-\mathbf{I}) = \mathbf{A}^2 - \mathbf{I}$. The best way to do this is to compute $(\mathbf{A}+\mathbf{I})(\mathbf{A}-\mathbf{I}) - (\mathbf{A}^2 - \mathbf{I})$ is the zero matrix. Do this for three random matrices. Report your conclusions in words. Each element is approaching to zero. But the result is zero matrix if the random matrix is the integer matrix.

   ii. Now test $(\mathbf{A}+\mathbf{B})(\mathbf{A}-\mathbf{B}) = \mathbf{A}^2 - \mathbf{B}^2$ the same way for three pairs of random $4x4$ matrices. Report your conclusions in words. The equation is not true. The result of (A+B)(A-B) - (A^2 - B^2) goes larger when A and B is larger.

   iii. Let

$$\mathbf{S} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$S^2 = \begin{bmatrix} 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \end{bmatrix}$

$S^3 = \begin{bmatrix} 0&0&0&1&0 \\ 0&0&0&0&1 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \end{bmatrix}$

$S^4 = \begin{bmatrix} 0&0&0&0&1 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \end{bmatrix}$

$S^5 = \begin{bmatrix} 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \end{bmatrix}$

$S^6 = \begin{bmatrix} 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \end{bmatrix}$

   Compute $\mathbf{S}^k$ for $k = 2, \ldots, 6$

6. Let $\mathbf{A} = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}$. Construct a $2x2$ matrix $\mathbf{B}$ such that $\mathbf{AB}$ is the zero matrix. Use two different non zero columnns for $\mathbf{B}$. $B = \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$ or $\begin{bmatrix} 2a & 2b \\ a & b \end{bmatrix}$ for every a and b in R.

7. Let $\mathbf{r_1}, \ldots, \mathbf{r_p}$ be vectors in $\mathbb{R}^n$ and let $\mathbf{Q}$ be a mXn matrix. Write the matrix $[\mathbf{Qr_1}, \ldots, \mathbf{Qr_p}]$ as product of two matrices (neither of which is an identity matrix). Q x [r1 r2 r3 … rp]

[2] You must use Matlab to answer this question

[3] Refer to this https://www.mathworks.com/help/matlab/ref/rand.html when you are generating your random entries in the matrix