Fitting ODE-based models to hospitalization data

Deterministic ODE-based SIR

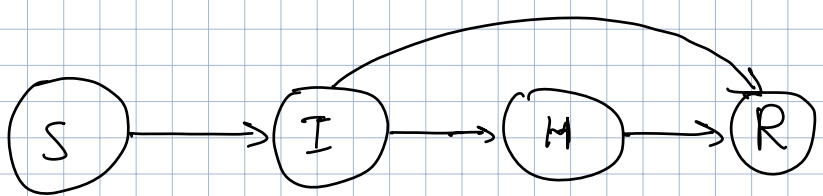
When population size $N \to \infty$, stochastic SIR approaches its deterministic limit, governed by the following system of ordinary differential equations:

$$\frac{dS(t)}{dt} = -\frac{\beta}{N} S(t) I(t)$$

$$\frac{dI(t)}{dt} = \frac{\beta}{N} S(t) I(t) - \gamma I(t)$$

$$\frac{dR(t)}{dt} = \gamma \cdot (1-q) I(t) + \alpha H(t)$$

$$\frac{dH(t)}{dt} = \gamma \cdot q I(t) - \alpha H(t)$$



β = transmission rate, γ - removal or hospitalization rate

q = infection-to-hospitalization ratio (probability) or IHR

$1/\alpha$ - average length of hospital stay

Hospitalization can be reported as incidence (new hospitalizations per day or week) or as prevalence (number of hospital beds occupied at a particular time). Prevalence reporting is common in US right now.

Suppose $y_1, \ldots, y_n$ are hospital beds reported at times $t_1, \ldots, t_n$. We can assume a negative binomial distribution as before

$$y_i \sim \text{Negative-binomial}\left(\mu = H(t_i), \sigma^2 = \mu\left(1 + \frac{\mu}{\varphi}\right)\right)$$

$\mu > 0$ mean of the negative binomial

$\varphi > 0$ - overdispersion parameter

Why negative binomial — Poisson could also work, but it is better to have a more flexible mean-variance relationship.

Note: $H(t_i) = H(t_i, \beta, \gamma, q, \alpha)$ - deterministic function of transmission model parameters

Since we now have a full data generating process described, we know what the likelihood function is

$Pr(y_1, \dots, y_n \mid \beta, \gamma, \rho, \alpha, \varphi) = \prod_{i=1}^{n} Pr(y_i \mid \beta, \gamma, \rho, \alpha, \varphi)$, where

$Pr(y \mid \beta, \gamma, \rho, \alpha, \varphi)$ is negative binomial probability mass function.

So the posterior distribution becomes

$$Pr(\beta, \gamma, \rho, \alpha, \varphi \mid y_1, \dots, y_n) \propto Pr(y_1, \dots, y_n \mid \beta, \gamma, \rho, \alpha, \varphi) \times \underbrace{Pr(\beta, \gamma, \rho, \alpha, \varphi)}_{\text{need to specify priors}}$$

Fitting ODE-based models to case data

$$\frac{dS(t)}{dt} = -\beta S(t) I(t)$$

$$\frac{dI(t)}{dt} = \beta S(t) I(t) - \gamma I(t)$$

$$\frac{dR(t)}{dt} = \gamma I(t)$$

$$\frac{dN_{SI}(t)}{dt} = \beta S(t) I(t)$$

But, we observe cases as incidence data

$y_1, \dots, y_n$, where y_i is number of new cases in interval $[t_i, t_{i+1}]$

$I(t_{i+1}) - I(t_i) \neq$ # of new infections

$N_{SI}(t)$ = cumulative incidence or the total number of infections up to time t.

$\Delta N_{SI}^{(i)} = N_{SI}(t_{i+1}) - N_{SI}(t_i)$ = # of new infections in time period $[t_i, t_{i+1})$

$y_i \sim$ Negative-binomial $\left(\mu = \rho \Delta N_{SI}^{(i)},\ \sigma^2 = \mu\left(1 + \frac{\mu}{\varphi}\right)\right)$

ρ – case detection probability

$Pr(y_1, \dots, y_n \mid \beta, \gamma, \rho, \varphi) = \prod_{i=1}^{n} Pr(y_i \mid \beta, \gamma, \rho, \varphi)$, where

$Pr(y_i \mid \beta, \gamma, \rho, \varphi)$ is negative binomial probability mass function.

As before, the posterior becomes

$Pr(\beta, \gamma, \rho, \varphi \mid y_1, \dots, y_n) \propto Pr(y_1, \dots, y_n \mid \beta, \gamma, \rho, \varphi) \times \underbrace{Pr(\beta, \gamma, \rho, \varphi)}_{\text{need to specify priors}}$

We can use R-H algorithm to approximate this posterior distribution