

CSIR NET.

Linear Algebra.

Topic: Nilpotent Matrix.

June-11, 12, 16.

J-11 Part-C:

Let N be 3×3 non-zero matrix with $N^3 = 0$.

Which of following is/are true?

1. ☒ N is not similar to a diagonal Matrix

2. ☒ N is similar to a diagonal Matrix.

3. ☒ N has one non-zero eigen vector.

4. ☒ N has three linearly Independent eigen vectors.

Solution:

\therefore 1st Analysis is N is a non-zero nilpotent matrix.

Fact: Non-zero nilpotent matrix is never diagonalizable.

1st option is correct and wrong.

Since N is a nilpotent matrix, '0' is the eigen value of matrix N .

Hence \exists at least one non-zero eigen vector corresponding to '0'. $\boxed{Ax = \lambda x, x \neq 0}$

for option (A).

Suppose if it has 3 linear independent eigen vectors.

Geometric Multiplicity of '0' = Algebraic Multiplicity of '0'

$$(ii) \text{ GM of '0' = AM of '0' = 3}$$

\Rightarrow It is diagonalizable.

Which is not true.

Hence (4)th option is not true.

J-12 P-c

N be a non-zero 3x3 matrix with $N^2 = 0$. which of following is/are true?

Same options as prev sum.

J-16 P-c

Let T be a nxn matrix with $T^n = 0$. which of following is/are true?

1. \times T has n distinct eigenvalues

2. \checkmark T has one eigenvalue of multiplicity n

3. \checkmark 0 is an eigenvalue of T.

4. \times T is similar to a diagonal matrix.

Solution:

T is nilpotent matrix.

T may be zero matrix or it may be non-zero matrix.

If T is non-zero nilpotent matrix, then it can't be diagonalizable.

so 4th is not always true.

Since T is nilpotent matrix. '0' is the eigen value with multiplicity 'n'.

So. 2 and 3 are correct.

1 is wrong.

Thank

you.