

IEEE 754 (bits)				
Tamanho	Sinal	expoente	polarização	mantissa
32	1	8	127	32
64	1	11	1023	52
Formato		Single	Double	
E_{min}		-126	-1022	
E_{max}		127	1023	
menor \aleph		$\approx 1,2 \times 10^{-38}$	$\approx 2,2 \times 10^{-308}$	
maior \aleph		$\approx 3,4 \times 10^{38}$	$\approx 1,8 \times 10^{308}$	
ϵ		$2^{-23} \approx 1,2 \times 10^{-7}$	$2^{-52} \approx 2,2 \times 10^{-16}$	
Precisão		≈ 7 dígitos decimais	≈ 16 dígitos decimais	
$F(\beta, t, m, M) \implies x = s \times \beta^e \quad e \in [-m, M]$				
$\beta^{-1}(1 - \frac{1}{2}\beta - t) \leq s \leq 1 - \frac{1}{2}\beta - t$				
MMQ				
$f(x) \approx g(x) \quad r(x_i) = \sum_{i=1}^n [f(x_i) - g(x_i)]^2$				
$r(x) = \int_c^d [f(x) - g(x)]^2 dx$				
$\begin{bmatrix} \langle g_1, g_1 \rangle & \langle g_1, g_2 \rangle & \cdots & \langle g_1, g_m \rangle \\ \langle g_2, g_1 \rangle & \langle g_2, g_2 \rangle & \cdots & \langle g_2, g_m \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle g_m, g_1 \rangle & \langle g_m, g_2 \rangle & \cdots & \langle g_m, g_m \rangle \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} = \begin{bmatrix} \langle g_1, f \rangle \\ \langle g_2, f \rangle \\ \vdots \\ \langle g_m, f \rangle \end{bmatrix}$				
Linearizações				
$g(x) = ae^{bx} \implies \ln(g(x)) = \ln(a) + bx$				
$g(x) = a10^{bx} \implies \log(g(x)) = \log(a) + bx$				
$g(x) = ax^b \implies \ln(g(x)) = \ln(a) + b \ln(x)$				
$g(x) = \frac{1}{a+bx} \implies \frac{1}{g(x)} = a + bx$				
$g(x) = \frac{ax}{b+x} \implies \frac{1}{g(x)} = \frac{b}{a} \frac{1}{x} + \frac{1}{a}$				
Resíduos MMQ				
$r(x_i) = \sum_{i=1}^n [f(x_i) - g(x_i)]^2$				
$R^2 = 1 - \frac{SQ_{res}}{SQ_{tot}}$				
$\bar{y} = \frac{1}{n} \sum_{i=1}^n f(x_i)$ (ponto médio)				
$SQ_{tot} = \sum_{i=1}^n [g(x_i) - \bar{y}]^2$ (erro total)				
$SQ_{res} = \sum_{i=1}^n [f(x_i) - \bar{y}]^2$ (erro residual)				
Serie Fourier				
$s_N(x) = \frac{a_0}{2} + \sum_{n=1}^N \left(a_n \cos(\frac{2\pi nx}{P}) + b_n \sin(\frac{2\pi nx}{P}) \right)$				
$= \sum_{n=-N}^N c_n e^{i \frac{2\pi nx}{P}}$				
$a_n = \frac{2}{P} \int_{x_0}^{x_0+P} s(x) \cos(\frac{2\pi nx}{P}) dx$				
$b_n = \frac{2}{P} \int_{x_0}^{x_0+P} s(x) \sin(\frac{2\pi nx}{P}) dx$				
$c_n = \frac{1}{P} \int_{x_0}^{x_0+P} s(x) e^{-i \frac{2\pi nx}{P}} dx$				

Forma interpoladora de Lagrange	
$f(x) \approx P_1(x) = a_0(x - x_1) + a_1(x - x_0)$	
$a_0 = \frac{f(x_0)}{x_0 - x_1}; a_1 = \frac{f(x_1)}{x_1 - x_0}$	
$f(x) \approx P_2(x) = (x - x_1)(x - x_2) \frac{f(x_0)}{(x_0 - x_1)(x_0 - x_2)}$	
$+ (x - x_0)(x - x_2) \frac{f(x_1)}{(x_1 - x_0)(x_1 - x_2)}$	
$+ (x - x_0)(x - x_1) \frac{f(x_2)}{(x_2 - x_0)(x_2 - x_1)}$	
$L_i(x) = \prod_{k=0, k \neq i}^n \frac{x - x_k}{x_i - x_k}, i = 0 \dots n$	
$f(x) \approx P_n(x) = \sum_{k=0}^n f(x_k) L_k(x)$	
Forma interpoladora de Newton	
$f(x) \approx P_n(x) = a_1$	
$+ a_2(x - x_0) + a_3(x - x_0)(x - x_1) + \dots$	
$+ a_n(x - x_0)(x - x_1) \dots (x - x_{n-1})$	
$a_1 = f(x_0)$	
$a_2 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$	
$a_3 = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$	
Forma Interpoladora de Newton - Diferenças divididas	
$f[x_i] = f(x_i)$	
$f[x_i, x_{i+1}] = \frac{f[x_{i+1}] - f[x_i]}{x_{i+1} - x_i}$	
$f[x_i, x_{i+1}, x_{i+2}] = \frac{f[x_{i+1}, x_{i+2}] - f[x_i, x_{i+1}]}{x_{i+2} - x_i}$	
$f[x_i, x_{i+1} \dots x_{i+k}] = \frac{f[x_{i+1} \dots x_{i+k}] - f[x_i, x_{i+k-1}]}{x_{i+k} - x_i}$	
$f(x) \approx P_n(x) = f[x_0] + f[x_0, x_1](x - x_0)$	
$+ f[x_0, x_1, x_2](x - x_0)(x - x_1) + \dots$	
$+ f[x_0, x_1 \dots x_n](x - x_0)(x - x_1) \dots (x - x_{n-1})$	
Erro na interpolação	
$f(x) = P_n(x) + R_n(x)$	
$R_n(x) = \frac{(x-x_0)(x-x_1)\dots(x-x_n)}{(n+1)!} f^{(n+1)}(\xi(x)), x_0 < \xi < x_n$	
$M > 0 \text{ e } \max_{t \in [x_0, x_n]} f^{(n+1)}(t) < M$	
$ R_n(x) \leq \frac{ x-x_0 x-x_1 \dots x-x_n }{(n+1)!} M$	
$ R_n(x) \leq \frac{ x-x_0 x-x_1 \dots x-x_n }{(n+1)!} f[x_0, x_1 \dots x_n, x_{n+1}] \iff \exists x_{n+1}$	

Métodos de Integração Numérica

Ponto Central	$\int_a^b f(x)dx \approx h \sum_{i=1}^N f\left(\frac{x_i + x_{i+1}}{2}\right)$
Trapézio	$\int_a^b f(x)dx \approx \frac{h}{2} [f(x_0) + 2(f(x_1) + \dots + f(x_{N-1})) + f(x_N)]$
Simpson 1/3	$\int_a^b f(x)dx \approx \frac{h}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{2N-2}) + 4f(x_{2N-1}) + f(x_{2N})]$
Simpson 3/8	$\int_a^b f(x)dx \approx \frac{3h}{8} [f(x_0) + 3f(x_1) + 3f(x_2) + 2f(x_3) + \dots + 2f(x_{3N-3}) + 3f(x_{3N-2}) + 3f(x_{3N-1}) + f(x_{3N})]$

Quadratura de Gauss

Geral	$\int_a^b w(x)f(x)dx \approx \sum_{i=1}^n nC_i f(x_i) \quad (C_i \text{ são os pesos e } x_i \text{ são os pontos tabelados})$
Gauss-Legendre	$w(x) = 1, a = -1, b = 1, n = 2 \implies C_1 = C_2 = 1, x_1 = -0,57735027, x_2 = 0,57735027$ $n = 2 \implies \int_{-1}^1 f(x)dx \approx f(-0,57735027) + f(0,57735027)$
Gauss-Chebyshev	$w(x) = \frac{1}{\sqrt{1-x^2}}, a = -1, b = 1$
Gauss-Laguerre	$w(x) = e^{-x}, a = 0, b = \infty$
Gauss-Hermite	$w(x) = e^{-x^2}, a = -\infty, b = \infty$

Erro na Integração Numérica

Trapézio	$R_1(f) = -\frac{(b-a)}{12} h^2 f''(\xi), \xi \in (x_0, x_1)$
Simpson 1/3	$R_2(f) = -\frac{(b-a)}{180} h^4 f^{(4)}(\xi), \xi \in (x_0, x_{2N})$
Simpson 3/8	$R_3(f) = -\frac{(b-a)}{80} h^4 f^{(4)}(\xi), \xi \in (x_0, x_{3N})$

Lista de Exercícios

$$y_n = \int_0^1 \frac{x^n}{x+a} dx \implies \int_0^1 \frac{x^1}{1+a} dx < y_n < \int_0^1 \frac{x^n}{a} dx$$

$$\frac{1}{(n+1)(1+a)} < y_n < \frac{1}{(n+1)a}$$

$$x^n = [(x+a) - a]^n = \sum_{k=0}^n (-1)^k \binom{n}{k} (x+a)^{n-k} a^k$$

$$y_n = \int_0^1 \sum_{k=0}^n (-1)^k \binom{n}{k} (x+a)^{n-k-1} a^k dx$$

$$= \sum_{k=0}^n (-1)^k a^k \binom{n}{k} \int_0^1 (x+a)^{n-k-1} dx$$

$$= \sum_{k=0-1}^n (-1)^k a^k \binom{n}{k} \int_0^1 (x+a)^{n-k-1} dx$$

$$+ (-1)^n a^n \binom{n}{n} \int_0^1 (x+a)^{-1} dx$$

Livro Neide

$$I_n = e^{-1} \int_0^1 x^n e^x dx \implies I_n < e^{-1} \max_{0 \leq x \leq 1} (e^x) \int_0^1 x^n dx < \frac{1}{n+1}$$

$$I_n = e^{-1} \left\{ [x^n e^x]_0^1 - \int_0^1 n x^{n-1} e^x dx \right\}$$

Formulas Gerais

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$f(x) \approx \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\sum_{k=0}^{n-1} ar^k = a \left(\frac{1-r^n}{1-r} \right)$$
