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| IEEE 754 (bits) | | | | |
|---|---|---|---|---|
| Tamanho | Sinal | expoente | polarização | mantissa |
| 32 | 1 | 8 | 127 | 32 |
| 64 | 1 | 11 | 1023 | 52 |

| Formato | Single | Double |
|---|---|---|
| $E_{min}$ | $-126$ | $-1022$ |
| $E_{max}$ | $127$ | $1023$ |
| menor № | $\approx 1,2 \times 10^{-38}$ | $\approx 2,2 \times 10^{-308}$ |
| maior № | $\approx 3,4 \times 10^{38}$ | $\approx 1,8 \times 10^{308}$ |
| $\epsilon$ | $2^{-23} \approx 1,2 \times 10^{-7}$ | $2^{-52} \approx 2,2 \times 10^{-16}$ |
| Precisão | $\approx$ 7 digitos decimais | $\approx$ 16 digitos decimais |

$$F(\beta, t, m, M) \implies x = s \times \beta^e \qquad e \in [-m, M]$$
$$\beta^{-1}(1 - \tfrac{1}{2}\beta{-t}) \leq s \leq 1 - \tfrac{1}{2}\beta{-t}$$

## MMQ

$$f(x) \approx g(x) \ r(x_i) = \textstyle\sum_{i=1}^{n}[f(x_i) - g(x_i)]^2$$
$$r(x) = \textstyle\int_c^d [f(x) - g(x)]^2 dx$$

$$\begin{bmatrix} \langle g_1, g_1 \rangle & \langle g_1, g_2 \rangle & \dots & \langle g_1, g_m \rangle \\ \langle g_2, g_1 \rangle & \langle g_2, g_2 \rangle & \dots & \langle g_2, g_m \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle g_m, g_1 \rangle & \langle g_m, g_2 \rangle & \dots & \langle g_m, g_m \rangle \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} = \begin{bmatrix} \langle g_1, f \rangle \\ \langle g_2, f \rangle \\ \vdots \\ \langle g_m, f \rangle \end{bmatrix}$$

## Linearizações

$$g(x) = ae^{bx} \implies \ln(g(x)) = \ln(a) + bx$$
$$g(x) = a10^{bx} \implies \log(g(x)) = \log(a) + bx$$
$$g(x) = ax^b \implies \ln(g(x)) = \ln(a) + b\ln(x)$$
$$g(x) = \tfrac{1}{a+bx} \implies \tfrac{1}{g(x)} = a + bx$$
$$g(x) = \tfrac{ax}{b+x} \implies \tfrac{1}{g(x)} = \tfrac{b}{a}\tfrac{1}{x} + \tfrac{1}{a}$$

## Resíduos MMQ

$$r(x_i) = \sum_{i=1}^{n}[f(x_i) - g(x_i)]^2$$
$$R^2 = 1 - \tfrac{SQ_{res}}{SQ_{tot}}$$
$$\bar{y} = \tfrac{1}{n}\sum_{i=1}^{n} f(x_i) \text{ (ponto médio)}$$
$$SQ_{tot} = \sum_{i=1}^{n}[g(x_i) - \bar{y}]^2 \text{ (erro total)}$$
$$SQ_{res} = \sum_{i=1}^{n}[f(x_i) - \bar{y}]^2 \text{ (erro residual)}$$

## Serie Fourier

$$s_N(x) = \frac{a_0}{2} + \sum_{n=1}^{N}\left(a_n \cos(\frac{2\pi nx}{P}) + b_n \sin(\frac{2\pi nx}{P})\right)$$
$$= \sum_{n=-N}^{N} c_n e^{i\frac{2\pi nx}{P}}$$
$$a_n = \tfrac{2}{P}\int_{x_0}^{x_0+P} s(x)\cos(\frac{2\pi nx}{P})dx$$
$$b_n = \tfrac{2}{P}\int_{x_0}^{x_0+P} s(x)\sin(\frac{2\pi nx}{P})dx$$
$$c_n = \tfrac{1}{P}\int_{x_0}^{x_0+P} s(x)e^{-i\frac{2\pi nx}{P}}dx$$

## Forma interpoladora de Lagrange

$$f(x) \approx P_1(x) = a_0(x - x_1) + a_1(x - x_0)$$
$$a_0 = \tfrac{f(x_0)}{x_0 - x_1}; a_1 = \tfrac{f(x_1)}{x_1 - x_0}$$
$$f(x) \approx P_2(x) = (x - x_1)(x - x_2)\frac{f(x_0)}{(x_0 - x_1)(x_0 - x_2)}$$
$$+ (x - x_0)(x - x_2)\frac{f(x_1)}{(x_1 - x_0)(x_1 - x_2)}$$
$$+ (x - x_0)(x - x_1)\frac{f(x_2)}{(x_2 - x_0)(x_2 - x_1)}$$

$$L_i(x) = \prod_{k=0, k\neq i}^{n}\frac{x - x_k}{x_i - x_k}, i = 0 \dots n$$
$$f(x) \approx P_n(x) = \sum_{k=0}^{n} f(x_k)L_k(x)$$

## Forma interpoladora de Newton

$$f(x) \approx P_n(x) = a_1$$
$$+ a_2(x - x_0) + a_3(x - x_0)(x - x_1) + \dots$$
$$+ a_n(x - x_0)(x - x_1)\dots(x - x_{x-1})$$
$$a_1 = f(x_0)$$
$$a_2 = \tfrac{f(x_1) - f(x_0)}{x_1 - x_0}$$
$$a_3 = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

## Forma Interpoladora de Newton - Diferenças divididas

$$f[x_i] = f(x_i)$$
$$f[x_i, x_{i+1}] = \tfrac{f[x_{i+1}] - f[x_i]}{x_{i+1} - x_i}$$
$$f[x_i, x_{i+1}, x_{i+2}] = \tfrac{f[x_{i+1}, x_{i+2}] - f[x_i, x_{i+1}]}{x_{i+2} - x_i}$$
$$f[x_i, x_{i+1} \dots x_{i+k}] = \tfrac{f[x_{i+1} \dots x_{i+k}] - f[x_i, x_{i+k-1}]}{x_{i+k} - x_i}$$
$$f(x) \approx P_n(x) = f[x_0] + f[x_0, x_1](x - x_0)$$
$$+ f[x_0, x_1, x_2](x - x_0)(x - x_1) + \dots$$
$$+ f[x_0, x_1 \dots x_n](x - x_0)(x - x_1)\dots(x - x_{n-1})$$

## Erro na interpolação

$$f(x) = P_n(x) + R_n(x)$$
$$R_n(x) = \tfrac{(x - x_0)(x - x_1)\dots(x - x_n)}{(n+1)!}f^{(n+1)}(\xi(x)), x_0 < \xi < x_n$$
$$M > 0 \text{ e } \max_{t \in [x_0, x_n]} |f^{(n+1)}(t)| < M$$
$$|R_n(x)| \leq \tfrac{|x - x_0||x - x_1|\dots|x - x_n|}{(n+1)!}M$$
$$|R_n(x)| \leq \tfrac{|x - x_0||x - x_1|\dots|x - x_n|}{(n+1)!}f[x_0, x_1 \dots x_n, x_{n+1}] \iff \exists x_{n+1}$$

| Métodos de Integração Numérica | |
|---|---|
| Ponto Central | $\int_a^b f(x)dx \approx h\sum_{i+1} Nf\left(\frac{x_i+x_{i+1}}{2}\right)$ |
| Trapézio | $\int_a^b f(x)dx \approx \frac{h}{2}\left[f(x_0)+2\left(f(x_1)+\cdots+f(x_{N-1})\right)+f(x_N)\right]$ |
| Simpson 1/3 | $\int_a^b f(x)dx \approx \frac{h}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\cdots+2f(x_{2N-2})+4f(x_{2N-1})+f(x_{2N})\right]$ |
| Simpson 3/8 | $\int_a^b f(x)dx \approx \frac{3h}{8}[f(x_0)+3f(x_1)+3f(x_2)+2f(x_3)+\ldots$ $+2f(x_{3N-3})+3f(x_{3N-2})+3f(x_{3N-1})+f(x_{3N})]$ |

| Quadratura de Gauss | |
|---|---|
| Geral | $\int_a^b w(x)f(x)dx \approx \sum_{i=1} nC_i f(x_i)$ ($C_i$ são os pesos e $x_i$ são os pontos tabelados |
| Gauss-Legendre | $w(x)=1, a=-1, b=1,\ n=2 \implies C_1=C_2=1, x_1=-0,57735027ex_2=-0,57735027$ |
| | $n=2 \implies \int_{-1}^1 f(x)dx \approx f(-0,57735027)+f(0,57735027)$ |
| Gauss-Chebyshev | $w(x)=\frac{1}{\sqrt{1-x^2}}, a=-1, b=1$ |
| Gauss-Laguerre | $w(x)=e^{-x}, a=0, b=\infty$ |
| Gauss-Hermite | $w(x)=e^{-x^2}, a=-\infty, b=\infty$ |

| Erro na Integração Numérica | |
|---|---|
| Trapézio | $R_1(f)=-\frac{(b-a)}{12}h^2f''(\xi), \xi\in(x_0,x_1)$ |
| Simpson 1/3 | $R_2(f)=-\frac{(b-a)}{180}h^4f^{(4)}(\xi), \xi\in(x_0,x_{2N})$ |
| Simpson 3/8 | $R_3(f)=-\frac{(b-a)}{80}h^4f^{(4)}(\xi), \xi\in(x_0,x_{3N})$ |

## Lista de Exercícios

$$y_n=\int_0^1 \tfrac{x^n}{x+a}dx \implies \int_0^1 \tfrac{x^1}{1+a}dx < y_n < \int_0^1 \tfrac{x^n}{a}dx$$

$$\tfrac{1}{(n+1)(1+a)} < y_n < \tfrac{1}{(n+1)a}$$

$$x^n=[(x+a)-a)]^n=\sum_{k=0}^{n}(-1)^k\begin{pmatrix} n\\ k\end{pmatrix}(x+a)^{n-k}a^k$$

$$y_n=\int_0^1\sum_{k=0}^{n}(-1)^k\begin{pmatrix} n\\ k\end{pmatrix}(x+a)^{n-k-1}a^kdx$$

$$=\sum_{k=0}^{n}(-1)^ka^k\begin{pmatrix} n\\ k\end{pmatrix}\int_0^1(x+a)^{n-k-1}dx$$

$$=\sum_{k=0}^{n-1}(-1)^ka^k\begin{pmatrix} n\\ k\end{pmatrix}\int_0^1(x+a)^{n-k-1}dx$$

$$+(-1)^na^n\begin{pmatrix} n\\ n\end{pmatrix}\int_0^1(x+a)^{-1}dx$$

## Livro Neide

$$I_n=e^{-1}\int_0^1 x^ne^xdx \implies I_n<e^{-1}\max_{0\le x\le 1}(e^x)\int_0^1 x^ndx<\frac{1}{n+1}$$

$$I_n=e^{-1}\left\{\left[x^ne^x\right]_0^1-\int_0^1 nx^{n-1}e^xdx\right\}$$

$$c_r=\tfrac{|P'(x)}{P(x)} \qquad c_r\le 1 \implies \text{bem condicionado}$$

## Formulas Gerais

$$\begin{pmatrix} n\\ k\end{pmatrix}=\tfrac{n!}{k!(n-k)!}$$

$$f(x)\approx\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$e^x=\textstyle\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$$

$$\sum_{k=0}^{n-1}ar^k=a\left(\frac{1-r^n}{1-r}\right)$$