Vinicius Pavanelli Vianna - NUSP 3155408

IEEE 754 (bits)				
Tamanho	Sinal	expoente	polarização	mantissa
32	1	8	127	32
64	1	11	1023	52

Formato	Single	Double
E_{min}	-126	-1022
E_{max}	127	1023
menor №	$\approx 1,2 \times 10^{-38}$	$\approx 2,2 \times 10^{-308}$
maior №	$\approx 3,4 \times 10^{38}$	$\approx 1,8 \times 10^{308}$
ϵ	$2^{-23} \approx 1,2 \times 10^{-7}$	$2^{-52} \approx 2,2 \times 10^{-16}$
Precisão	$\approx 7 digitos decimais$	$\approx 16 digitos decimais$

MMQ

$$f(x) \approx g(x) \; r(x_i) = \sum_{i=1}^{n}[f(x_i) - g(x_i)]^2$$

$$r(x) = \int_c^d [f(x) - g(x)]^2 dx$$

$$\begin{bmatrix} \langle g_1, g_1 \rangle & \langle g_1, g_2 \rangle & \dots & \langle g_1, g_m \rangle \\ \langle g_2, g_1 \rangle & \langle g_2, g_2 \rangle & \dots & \langle g_2, g_m \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle g_m, g_1 \rangle & \langle g_m, g_2 \rangle & \dots & \langle g_m, g_m \rangle \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} = \begin{bmatrix} \langle g_1, f \rangle \\ \langle g_2, f \rangle \\ \vdots \\ \langle g_m, f \rangle \end{bmatrix}$$

Linearizações

$$g(x) = ae^{bx} \implies \ln(g(x)) = \ln(a) + bx$$

$$g(x) = a10^{bx} \implies \log(g(x)) = \log(a) + bx$$

$$g(x) = ax^b \implies \ln(g(x)) = \ln(a) + b\ln(x)$$

$$g(x) = \frac{1}{a+bx} \implies \frac{1}{g(x)} = a + bx$$

$$g(x) = \frac{ax}{b+x} \implies \frac{1}{g(x)} = \frac{b}{a}\frac{1}{x} + \frac{1}{a}$$

Resíduos MMQ

$$r(x_i) = \sum_{i=1}^{n}[f(x_i) - g(x_i)]^2$$

$$R^2 = 1 - \frac{SQ_{res}}{SQ_{tot}}$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} f(x_i) \text{ (ponto médio)}$$

$$SQ_{tot} = \sum_{i=1}^{n}[g(x_i) - \bar{y}]^2 \text{ (erro total)}$$

$$SQ_{res} = \sum_{i=1}^{n}[f(x_i) - \bar{y}]^2 \text{ (erro residual)}$$

Forma interpoladora de Lagrange

$$f(x) \approx P_1(x) = a_0(x - x_1) + a_1(x - x_0), a_0 = \frac{f(x_0)}{x_0 - x_1}; a_1 = \frac{f(x_1)}{x_1 - x_0}$$

$$\begin{aligned} f(x) \approx P_2(x) = &(x - x_1)(x - x_2)\frac{f(x_0)}{(x_0 - x_1)(x_0 - x_2)} \\ &+ (x - x_0)(x - x_2)\frac{f(x_1)}{(x_1 - x_0)(x_1 - x_2)} \\ &+ (x - x_0)(x - x_1)\frac{f(x_2)}{(x_2 - x_0)(x_2 - x_1)} \end{aligned}$$

$$L_i(x) = \prod_{k=0, k\neq i}^{n} \frac{x - x_k}{x_i - x_k}, i = 0 \dots n$$

$$f(x) \approx P_n(x) = \sum_{k=0}^{n} f(x_k)L_k(x)$$

Forma interpoladora de Newton

$$\begin{aligned} f(x) \approx P_n(x) = a_1 & \\ &+ a_2(x - x_0) + a_3(x - x_0)(x - x_1) + \dots \\ &+ a_n(x - x_0)(x - x_1)\dots(x - x_{x-1}) \end{aligned}$$

$$a_1 = f(x_0)$$

$$a_2 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$a_3 = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

Forma Interpoladora de Newton - Diferenças divididas

$$f[x_i] = f(x_i)$$

$$f[x_i, x_{i+1}] = \frac{f[x_{i+1}] - f[x_i]}{x_{i+1} - x_i}$$

$$f[x_i, x_{i+1}, x_{i+2}] = \frac{f[x_{i+1}, x_{i+2}] - f[x_i, x_{i+1}]}{x_{i+2} - x_i}$$

$$f[x_i, x_{i+1} \dots x_{i+k}] = \frac{f[x_{i+1} \dots x_{i+k}] - f[x_i, x_{i+k-1}]}{x_{i+k} - x_i}$$

$$\begin{aligned} f(x) \approx P_n(x) = &f[x_0] + f[x_0, x_1](x - x_0) \\ &+ f[x_0, x_1, x_2](x - x_0)(x - x_1) + \dots \\ &+ f[x_0, x_1 \dots x_n](x - x_0)(x - x_1)\dots(x - x_{n-1}) \end{aligned}$$

Erro na interpolação

$$f(x) = P_n(x) + R_n(x)$$

$$R_n(x) = \frac{(x - x_0)(x - x_1)\dots(x - x_n)}{(n+1)!} f^{(n+1)}(\xi(x)), x_0 < \xi < x_n$$

$$M > 0 \text{ e } \max_{t\in[x_0, x_n]} |f^{(n+1)}(t)| < M$$

$$|R_n(x)| \leq \frac{|x - x_0||x - x_1|\dots|x - x_n|}{(n+1)!} M$$

$$|R_n(x)| \leq \frac{|x - x_0||x - x_1|\dots|x - x_n|}{(n+1)!} f[x_0, x_1 \dots x_n, x_{n+1}] \iff \exists x_{n+1}$$

Métodos de Integração Numérica

Ponto Central
$$\int_a^b f(x)dx \approx h\sum_{i+1} Nf\left(\frac{x_i+x_{i+1}}{2}\right)$$

Trapézio
$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(x_0)+2\left(f(x_1)+\cdots+f(x_{N-1})\right)+f(x_N)\right]$$

Simpson 1/3
$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\cdots+2f(x_{2N-2})+4f(x_{2N-1})+f(x_{2N})\right]$$

Simpson 3/8
$$\int_a^b f(x)dx \approx \frac{3h}{8}\left[f(x_0)+3f(x_1)+3f(x_2)+2f(x_3)+\cdots+2f(x_{3N-3})+3f(x_{3N-2})+3f(x_{3N-1})+f(x_{3N})\right]$$

Erro na Integração Numérica

Trapézio	$R_1(f) = -\frac{(b-a)}{12}h^2 f''(\xi), \xi \in (x_0, x_1)$
Simpson 1/3	$R_2(f) = -\frac{(b-a)}{180}h^4 f^{(4)}(\xi), \xi \in (x_0, x_{2N})$
Simpson 3/8	$R_3(f) = -\frac{(b-a)}{80}h^4 f^{(4)}(\xi), \xi \in (x_0, x_{3N})$

Quadratura de Gauss

$$\int_a^b w(x)f(x)dx \approx \sum_{i=1} nC_i f(x_i)$$ (C_i são os pesos e x_i são os pontos tabelados

Gauss-Legendre: $w(x)=1, a=-1, b=1,\ n=2 \implies C_1=C_2=1, x_1=-0,57735027 e x_2 = -0,57735027$

$$n=2 \implies \int_{-1}^1 f(x)dx \approx f(-0,57735027)+f(0,57735027)$$

Gauss-Chebyshev: $w(x)=\frac{1}{\sqrt{1-x^2}}, a=-1, b=1$

Gauss-Laguerre: $w(x)=e^{-x}, a=0, b=\infty$

Gauss-Hermite: $w(x)=e^{-x^2}, a=-\infty, b=\infty$

Lista de Exercícios

$$y_n = \int_0^1 \frac{x^n}{x+a}dx \implies \int_0^1 \frac{x^1}{1+a}dx < y_n < \int_0^1 \frac{x^n}{a}dx$$

$$\frac{1}{(n+1)(1+a)} < y_n < \frac{1}{(n+1)a}$$

$$x^n = [(x+a)-a]^n = \sum_{k=0}^n (-1)^k \binom{n}{k}(x+a)^{n-k}a^k$$

$$y_n = \int_0^1 \sum_{k=0}^n (-1)^k \binom{n}{k}(x+a)^{n-k-1}a^k dx$$

$$= \sum_{k=0}^n (-1)^k a^k \binom{n}{k}\int_0^1 (x+a)^{n-k-1}dx$$

$$= \sum_{k=0-1}^n (-1)^k a^k \binom{n}{k}\int_0^1 (x+a)^{n-k-1}dx$$

$$+ (-1)^n a^n \binom{n}{n}\int_0^1 (x+a)^{-1}dx$$

Formulas Gerais

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$f(x) \approx \sum_{n=0}^\infty \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$e^x = \sum_{n=0}^\infty \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$a + \cdots + ar^{n-1} = \sum_{k=0}^{n-1} ar^k = a\left(\frac{1-r^n}{1-r}\right)$$

Livro Neide

$$I_n = e^{-1}\int_0^1 x^n e^x dx \implies I_n < e^{-1}\max_{0\le x\le 1}(e^x)\int_0^1 x^n dx < \frac{1}{n+1}$$

$$I_n = e^{-1}\left\{[x^n e^x]_0^1 - \int_0^1 n x^{n-1}e^x dx\right\}$$