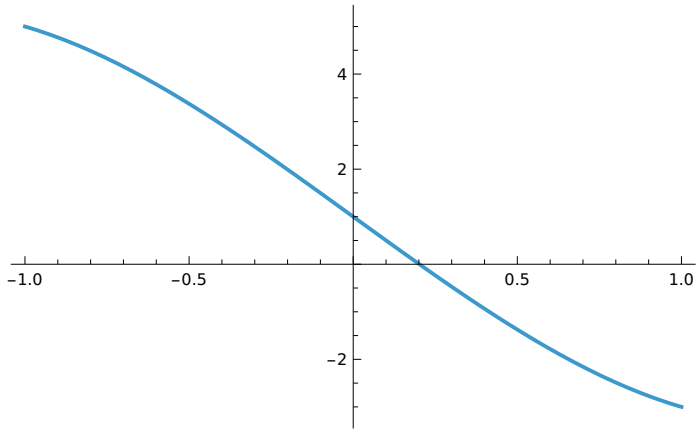
Made by *human* on Earth for Aliens

Bisection Method

Code 1

In[1]:=
```
f[x_] := x^3 - 5 x + 1;
a = 0;
b = 1;
nmax = 15;
Plot[f[x], {x, -1, 1}]
For[i = 1, i ≤ nmax, i++, c = (a + b) / 2;
Print["Root after iteration ", i, " is c = ", N[c, 6], " f(c) = ", N[f[c], 6], "\n"];
If[f[a] * f[c] < 0, b = c, a = c];
]
```

Out[5]=

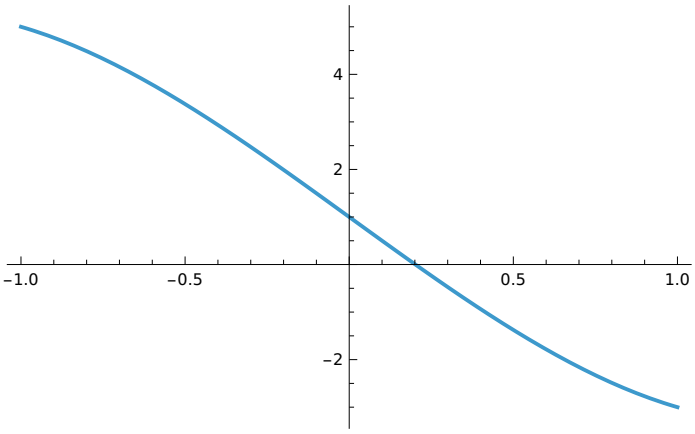


```
Root after iteration 1 is c = 0.500000 f(c) = -1.37500

Root after iteration 2 is c = 0.250000 f(c) = -0.234375

Root after iteration 3 is c = 0.125000 f(c) = 0.376953

Root after iteration 4 is c = 0.187500 f(c) = 0.0690918

Root after iteration 5 is c = 0.218750 f(c) = -0.0832825

Root after iteration 6 is c = 0.203125 f(c) = -0.00724411

Root after iteration 7 is c = 0.195313 f(c) = 0.0308881

Root after iteration 8 is c = 0.199219 f(c) = 0.0118129

Root after iteration 9 is c = 0.201172 f(c) = 0.00228208

Root after iteration 10 is c = 0.202148 f(c) = -0.00248160

Root after iteration 11 is c = 0.201660 f(c) = -0.0000999043

Root after iteration 12 is c = 0.201416 f(c) = 0.00109105

Root after iteration 13 is c = 0.201538 f(c) = 0.000495564

Root after iteration 14 is c = 0.201599 f(c) = 0.000197827

Root after iteration 15 is c = 0.201630 f(c) = 0.0000489610
```

Code 2

In[13]:=
```
f[x_] := x^3 - 5 x + 1;
a = 0;
b = 1;
nmax = 15;
Plot[f[x], {x, -1, 1}]
For[i = 1, i ≤ nmax, i++, c = (a + b) / 2;
Print[TableForm[{{i, N[c, 4], N[f[c], 4]}}]];
If[f[a] * f[c] < 0, b = c, a = c];
]
```

Out[17]=

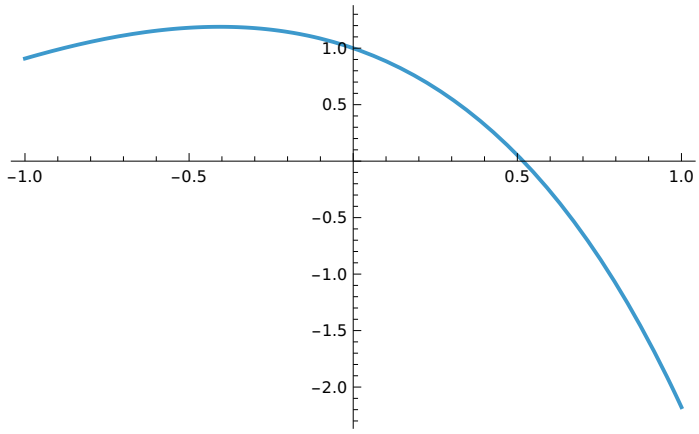


1	0.5000	-1.375
2	0.2500	-0.2344
3	0.1250	0.3770
4	0.1875	0.06909
5	0.2188	-0.08328
6	0.2031	-0.007244
7	0.1953	0.03089
8	0.1992	0.01181
9	0.2012	0.002282
10	0.2021	-0.002482
11	0.2017	-0.00009990
12	0.2014	0.001091
13	0.2015	0.0004956
14	0.2016	0.0001978
15	0.2016	0.00004896

Code 3

In[19]:=
```
f[x_] := Cos[x] - x * E ^ x;
a = 0;
b = 1;
nmax = 10;
Plot[f[x], {x, -1, 1}]
For[i = 1, i ≤ nmax, i++, c = (a + b) / 2;
Print[TableForm[{{i, N[c, 4], N[f[c], 4]}}]];
If[f[a] * f[c] < 0, b = c, a = c];
]
```

Out[23]=



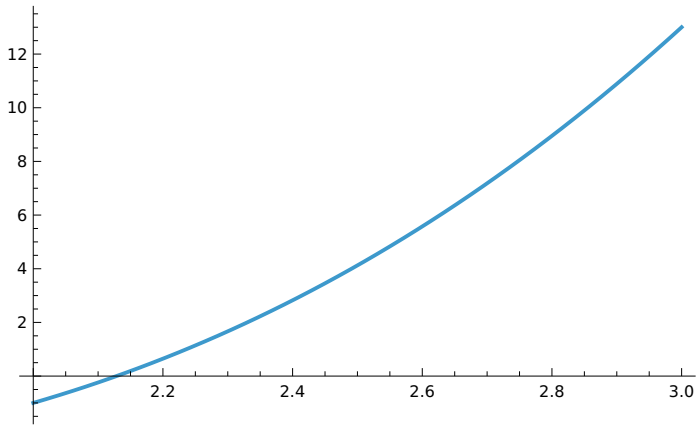

```
1    0.5000    0.05322
2    0.7500    -0.8561
3    0.6250    -0.3567
4    0.5625    -0.1413
5    0.5313    -0.04151
6    0.5156    0.006475
7    0.5234    -0.01736
8    0.5195    -0.005404
9    0.5176    0.0005452
10    0.5186    -0.002427
```

Secant Method

Code 1

In[7]:=
```
f[x_] := x^3 - 5 x + 1;
x0 = 2;
x1 = 3;
nmax = 5;
Plot[f[x], {x, x0, x1}]
For[ i = 1, i ≤ nmax, i++, x2 = N[x1 - ((x1 - x0) / (f[x1] - f[x0])) f[x1]];
Print["Root after iteration ", i, " is ", x2]; x0 = x1; x1 = x2;
]
```

Out[11]=

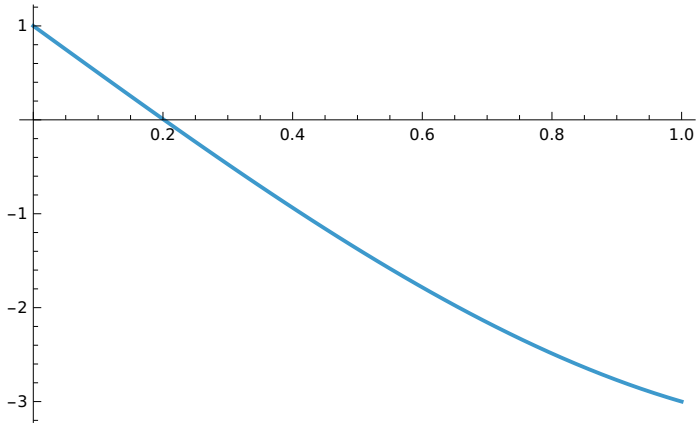



```
Root after iteration 1 is 2.07143
Root after iteration 2 is 2.10376
Root after iteration 3 is 2.12952
Root after iteration 4 is 2.1284
Root after iteration 5 is 2.12842
```

Code 2

In[31]:=
```
f[x_] := x^3 - 5 x + 1;
x0 = 0;
x1 = 1;
nmax = 8;
Plot[f[x], {x, x0, x1}]
For[ i = 1, i ≤ nmax, i++, x2 = N[x1 - ((x1 - x0) / (f[x1] - f[x0])) f[x1]];
Print[TableForm[{{i, N[x2], N[f[x2]]}}]]; x0 = x1; x1 = x2;
]
```

Out[35]=



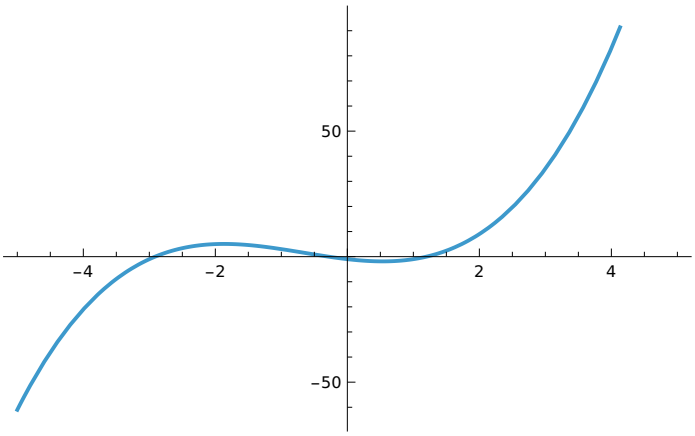
1	0.25	-0.234375
2	0.186441	0.0742773
3	0.201736	-0.000471116
4	0.20164	-8.64229×10^{-7}
5	0.20164	1.03527×10^{-11}
6	0.20164	-2.22045×10^{-16}
7	0.20164	1.11022×10^{-16}
8	0.20164	1.11022×10^{-16}

Regula Falsi Method

Code 1

In[67]:=
```
f[x_] := x^3 + 2 x^2 - 3 x - 1;
x0 = 1;
x1 = 2;
nmax = 10;
Plot[f[x], {x, -5, 5}]
For[ i = 1, i ≤ nmax, i++, x2 = N[x1 - ((x1 - x0) / (f[x1] - f[x0])) f[x1]];
Print["Root after iteration ", i, " is ", x2];
If[f[x0] * f[x2] < 0, x1 = x2, x0 = x2];
]
```

Out[71]=

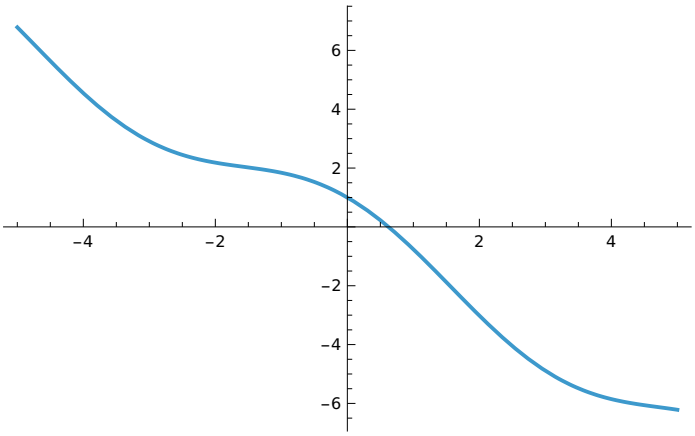



```
Root after iteration 1 is 1.1
Root after iteration 2 is 1.15174
Root after iteration 3 is 1.17684
Root after iteration 4 is 1.18863
Root after iteration 5 is 1.19408
Root after iteration 6 is 1.19658
Root after iteration 7 is 1.19773
Root after iteration 8 is 1.19825
Root after iteration 9 is 1.19849
Root after iteration 10 is 1.1986
```

Code 2

In[73]:=
```
f[x_] := Cos[x] - 1.3 x;
x0 = 1;
x1 = 2;
nmax = 10;
Plot[f[x], {x, -5, 5}]
For[ i = 1, i ≤ nmax, i++, x2 = N[x1 - ((x1 - x0) / (f[x1] - f[x0])) f[x1]];
Print["Root after iteration ", i, " is ", x2];
If[f[x0] * f[x2] < 0, x1 = x2, x0 = x2];
]
```

Out[77]=



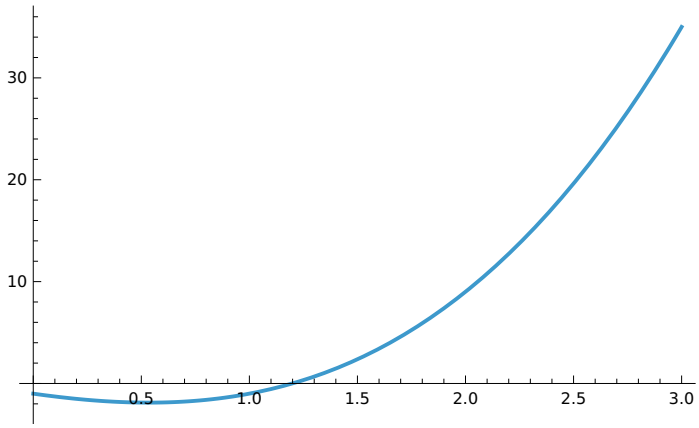

```
Root after iteration 1 is 0.663322
Root after iteration 2 is 0.629531
Root after iteration 3 is 0.624933
Root after iteration 4 is 0.62429
Root after iteration 5 is 0.624199
Root after iteration 6 is 0.624187
Root after iteration 7 is 0.624185
Root after iteration 8 is 0.624185
Root after iteration 9 is 0.624185
Root after iteration 10 is 0.624185
```

Newton Raphson Method

Code 1

In[79]:=
```
f[x_] := x^3 + 2 x^2 - 3 x - 1;
a = 1;
b = 2;
nmax = 6;
Plot[f[x], {x, a - 1, b + 1}]
For[i = 1, i ≤ nmax, i++, c = N[b - (f[b] / f'[b])];
Print["The ", i, " th iteration is: ", c];
a = b; b = c;
]
```

Out[83]=

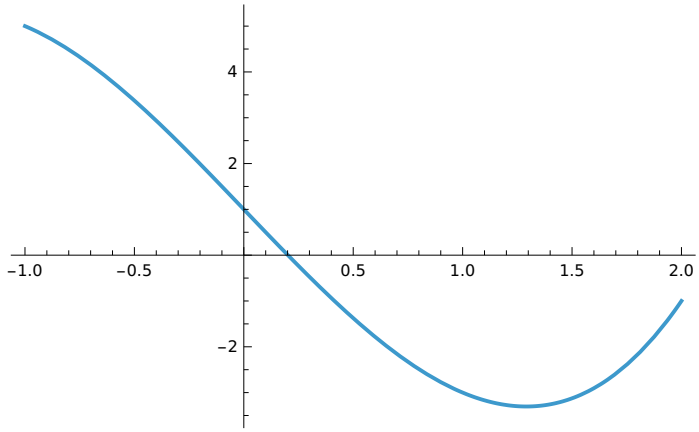



```
The 1 th iteration is: 1.47059
The 2 th iteration is: 1.24713
The 3 th iteration is: 1.2007
The 4 th iteration is: 1.19869
The 5 th iteration is: 1.19869
The 6 th iteration is: 1.19869
```

Code 2

In[85]:=
```
f[x_] := x^3 - 5 x + 1;
a = 0;
b = 1;
nmax = 6;
l = List[];
Plot[f[x], {x, a - 1, b + 1}]
For[i = 1, i ≤ nmax, i++, c = N[b - (f[b] / f'[b])];
a = b; b = c;
seq = AppendTo[l, {N[c], N[f[c]]}]
TableForm[seq, TableHeadings → {{"a", "b", "c", "d", "e", "f"}, {"i", "c", "f(c)"}},
 TableAlignments → Center]
```

Out[90]=



Out[92]//TableForm=

	i	c
a	-0.5	3.375
b	0.294118	-0.445146
c	0.200215	0.00695237
d	0.201639	1.22213×10^{-6}
e	0.20164	3.79696×10^{-14}
f	0.20164	1.11022×10^{-16}

Gauss Jacobi

Q. Solve the system of equations
8x+3y+4z = 15;
-2x+5y-2z = 1;
x+y+3z = 5;
using gauss jacobi iteration method. Use initial approximation as [x,y,z] = [0,0,0];

Code and Output

In[1]:=
```
x = 0;
y = 0;
z = 0;
nmax = 10;
For[i = 1, i ≤ nmax, i++, xnew = N[1/8 (15 - 3 y - 4 z), 5];
 ynew = N[1/5 (1 + 2 x + 2 z), 5]; znew = N[1/3 (5 - x - y), 5];
Print["The ", i, " th iteration is xnew = ", xnew, " ynew = ", ynew, " znew = ", znew];
x = xnew;
y = ynew;
z = znew
]
```

```
The 1 th iteration is xnew = 1.8750 ynew = 0.20000 znew = 1.6667
The 2 th iteration is xnew = 0.96667 ynew = 1.6167 znew = 0.97500
The 3 th iteration is xnew = 0.7813 ynew = 0.97667 znew = 0.80556
The 4 th iteration is xnew = 1.1060 ynew = 0.83472 znew = 1.0807
The 5 th iteration is xnew = 1.0216 ynew = 1.0747 znew = 1.0198
The 6 th iteration is xnew = 0.96212 ynew = 1.0166 znew = 0.96790
The 7 th iteration is xnew = 1.0098 ynew = 0.97201 znew = 1.0071
The 8 th iteration is xnew = 1.0069 ynew = 1.0068 znew = 1.0061
The 9 th iteration is xnew = 0.99443 ynew = 1.0052 znew = 0.99543
The 10 th iteration is xnew = 1.0003 ynew = 0.99594 znew = 1.0001
```

Gauss Seidel

Q. Solve the system of equations
8x+3y+4z = 15;
-2x+5y-2z = 1;
x+y+3z = 5;
using gauss seidel iteration method. Use initial approximation as [x,y,z] = [0,0,0];

Code and Output

In[11]:=
```
x = 0;
y = 0;
z = 0;
nmax = 10;
For[i = 1, i ≤ nmax, i++, xnew = N[1/8 (15 - 3 y - 4 z), 6];
 ynew = N[1/5 (1 + 2 xnew + 2 z), 5]; znew = N[1/3 (5 - xnew - ynew), 5];
Print["The ", i, " th iteration is xnew = ", xnew, " ynew = ", ynew, " znew = ", znew];
x = xnew;
y = ynew;
z = znew;
]
```

```
The 1 th iteration is xnew = 1.87500 ynew = 0.95000 znew = 0.72500
The 2 th iteration is xnew = 1.15625 ynew = 0.95250 znew = 0.96375
The 3 th iteration is xnew = 1.03594 ynew = 0.99988 znew = 0.98806
The 4 th iteration is xnew = 1.00602 ynew = 0.99763 znew = 0.99878
The 5 th iteration is xnew = 1.00150 ynew = 1.0001 znew = 0.99946
The 6 th iteration is xnew = 1.00023 ynew = 0.99988 znew = 0.99997
The 7 th iteration is xnew = 1.00006 ynew = 1.0000 znew = 0.99997
The 8 th iteration is xnew = 1.00001 ynew = 0.99999 znew = 1.0000
The 9 th iteration is xnew = 1.00000 ynew = 1.0000 znew = 1.0000
The 10 th iteration is xnew = 1.00000 ynew = 1.0000 znew = 1.0000
```

Module Command

In[16]:=
```
abs[x0_] := Module[{x = x0}, If[x > 0, x = x0, x = -x0]]
abs[-16]
```

Out[17]=
```
16
```

In[18]:=
```
arearect[x0_, y0_] := Module[{x = x0, y = y0, Area, Per}, Area = x*y; Per = 2*(x + y);
Print["Area = ", N[Area]];
Print["Per = ", N[Per]]
]
arearect[2.4, 4]
```

```
Area = 9.6
Per = 12.8
```

In[24]:=
```
fact[x0_] := Module[{n = x0}, factn = 1;
For[i = 1, i ≤ n, i++, factn = factn * i];
Print["Factorial of ", n, " is = ", factn];
]
fact[5]
```

```
Factorial of 5 is = 120
```

Gauss Jacobi Method using Matrix Method

In[30]:=
```
{{8, 3, 4}, {-2, 5, -2}, {1, 1, 3}} // MatrixForm
```

Out[30]//MatrixForm=

$$\begin{pmatrix} 8 & 3 & 4 \\ -2 & 5 & -2 \\ 1 & 1 & 3 \end{pmatrix}$$

In[29]:=
```
B = {{15}, {1}, {5}} // MatrixForm
```

Out[29]//MatrixForm=

$$\begin{pmatrix} 15 \\ 1 \\ 5 \end{pmatrix}$$

Code and Output

In[61]:=
```
A = {{8, 3, 4}, {-2, 5, -2}, {1, 1, 3}};
B = {{15}, {1}, {5}};
nmax = 10;

jacobi[x0_, y0_, z0_, iterations_] :=
 Module[{x, y, z, list, i},
  x[0] = x0; y[0] = y0; z[0] = z0;
  list = {{"Iteration", "x", "y", "z"}, {0, x[0], y[0], z[0]}};
  For[i = 0, i < iterations, i++,
   x[i + 1] = N[(B[[1, 1]] - A[[1, 2]] * y[i] - A[[1, 3]] * z[i]) / A[[1, 1]]];
   y[i + 1] = N[(B[[2, 1]] - A[[2, 1]] * x[i] - A[[2, 3]] * z[i]) / A[[2, 2]]];
   z[i + 1] = N[(B[[3, 1]] - A[[3, 2]] * y[i] - A[[3, 1]] * x[i]) / A[[3, 3]]];
   AppendTo[list, {i + 1, x[i + 1], y[i + 1], z[i + 1]}];
  ];
  TableForm[list]
 ]
jacobi[0, 0, 0, nmax]
```

Out[65]//TableForm=

Iteration	x	y	z
0	0	0	0
1	1.875	0.2	1.66667
2	0.966667	1.61667	0.975
3	0.78125	0.976667	0.805556
4	1.10597	0.834722	1.08069
5	1.02163	1.07467	1.01977
6	0.962116	1.01656	0.9679
7	1.00984	0.972006	1.00711
8	1.00694	1.00678	1.00605
9	0.994432	1.0052	0.995426
10	1.00034	0.995943	1.00012

Indexing of a Number

In[93]:=
```
X = {1, 5, 7}
X[[2]]
```

Out[93]=
```
{1, 5, 7}
```

Out[94]=
```
5
```

In[95]:=
```
Sum[k, {k, 1, n}]
```

Out[95]=
$$\frac{1}{2} n (1 + n)$$

In[96]:=
```
X = {1, 5, 7}
Product[x - X[[j]], {j, 1, 3}]
```

Out[96]=
```
{1, 5, 7}
```

Out[97]=
$$(-7 + x)\,(-5 + x)\,(-1 + x)$$

Lagrange Interpolation

Q. *Find the Lagrange interpolating polynomial for the following data (0,1) = (x0,f(x0)), (1,3) = (x1,f(x1)), (3,55) = (x2,f(x2))*

Code 1

In[98]:=
```
X = {0, 1, 3};
Y = {1, 3, 55};
n = 3;
For[k = 1, k ≤ n, k++,
L[n, k, x_] := Product[(x - X[[j]]) / (X[[k]] - X[[j]]), {j, 1, k - 1}] *
Product[(x - X[[j]]) / (X[[k]] - X[[j]]), {j, k + 1, n}]];
a = Sum[Y[[k]] × L[n, k, x], {k, 1, n}];
Print[a]
Simplify[a]
```

$$\frac{1}{3}(1 - x)(3 - x) + \frac{3}{2}(3 - x)\,x + \frac{55}{6}(-1 + x)\,x$$

Out[104]=
$$1 - 6x + 8x^2$$

Code 2

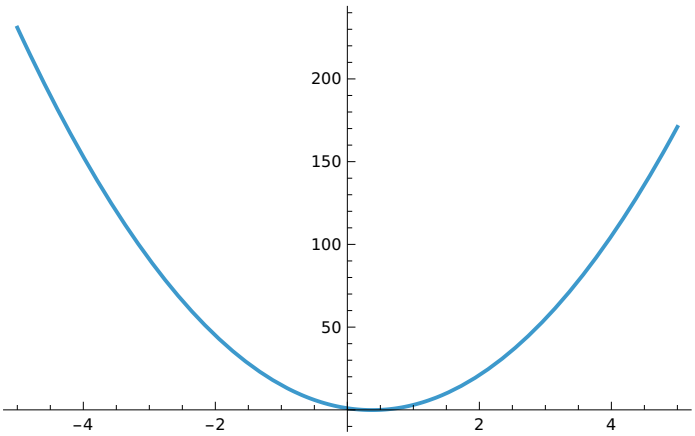
In[105]:=
```
X = {0, 1, 3};
Y = {1, 3, 55};
n = 3;
For[k = 1, k ≤ n, k++,
L[n, k, x_] := Product[(x - X[[j]]) / (X[[k]] - X[[j]]), {j, 1, k - 1}] *
Product[(x - X[[j]]) / (X[[k]] - X[[j]]), {j, k + 1, n}]];
Simplify[Sum[Y[[k]] × L[n, k, x], {k, 1, n}]]
```

Out[109]=
$$1 - 6x + 8x^2$$

In[110]:=

```
Plot[%, {x, -5, 5}]
```

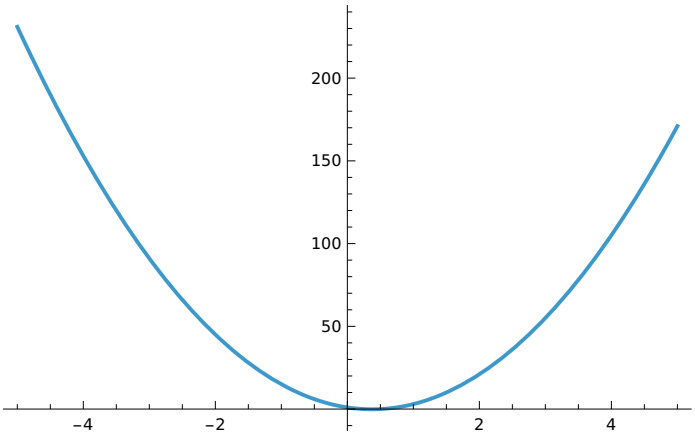
Out[110]=



In[111]:=

```
Plot[a, {x, -5, 5}]
```

Out[111]=



Code 3

In[143]:=

```
X = {-2, -1, 0, 1, 3, 4};
Y = {9, 16, 17, 18, 44, 81};
n = 6;
For[k = 1, k ≤ n, k++,
L[n, k, x_] := Product[(x - X[[j]]) / (X[[k]] - X[[j]]), {j, 1, k - 1}] *
Product[(x - X[[j]]) / (X[[k]] - X[[j]]), {j, k + 1, n}]];
a = Sum[Y[[k]] * L[n, k, x], {k, 1, n}];
Print[a]
Simplify[a]
```

$$-\frac{1}{20}(-1-x)(1-x)(3-x)(4-x)x-\frac{2}{5}(1-x)(3-x)(4-x)x(2+x)+\frac{17}{24}(1-x)(3-x)(4-x)(1+x)(2+x)+\frac{1}{2}(3-x)(4-x)x(1+x)(2+x)+\frac{11}{30}(4-x)(-1+x)x(1+x)(2+x)+\frac{9}{40}(-3+x)(-1+x)x(1+x)(2+x)$$

Out[149]=

$$17+x^3$$

Newton's Interpolation Method

Code and Output

In[119]:=

```
X = {0, 1, 3};
Y = {1, 3, 55};
n = 2;
d = Table["", {n + 1}, {n + 1}];
d[[All, 1]] = Y[[All]];
For[j = 1, j ≤ n, j++,
For[k = j, k ≤ n, k++,
d[[k + 1, j + 1]] = (d[[k + 1, j]] - d[[k, j]]) / (X[[k + 1]] - X[[k + 1 - j]])
];
];
For[k = 0, k ≤ n, k++,
p[k + 1, x_] := Product[(x - X[[j]]), {j, 1, k}]
];
a = Sum[d[[k + 1, k + 1]] * p[k + 1, x], {k, 0, n}];
Print[a]
Simplify[a]
```

$$1+2x+8(-1+x)x$$

Out[128]=

$$1-6x+8x^2$$

Trapezoidal Rule

Q. Find the approximate value of ∫(x+1) dx from 0 to 1.

Code and Output

In[1]:=
```
f[x_] := x + 1;
lowerlimit = a = 0;
upperlimit = b = 1;
fa = f[a];
fb = f[b];
h = b - a;
c = (h (fa + fb)) / 2;
Print["Integral = ", c]
```

Integral = $\frac{3}{2}$

Q. Find the approximate value of the integral ∫(x^2-x-1) dx from 3 to 5 and compare it with exact value of the integral also.

Code and Output

In[19]:=
```
f[x_] := x^2 - x - 1;
lowerlimit = a = 3;
upperlimit = b = 5;
fa = f[a];
fb = f[b];
h = b - a;
c = (h (fa + fb)) / 2;
Print["The area approximate by the trapezoidal rule is ", c]
int = Integrate[f[x], {x, a, b}];
Print["Original value of the integral is ", N[int], " and the error is ", N[c] - N[int]]
```

```
The area approximate by the trapezoidal rule is 24

Original value of the integral is 22.6667 and the error is 1.33333
```

Q. *Find the approximate value of the integral ∫(x^3+4x-13) dx from 3 to 4 and compare it with exact value of the integral also.*

Code and Output

In[29]:=
```
f[x_] := x^3 + 4 x - 13;
lowerlimit = a = 3;
upperlimit = b = 4;
fa = f[a];
fb = f[b];
h = b - a;
c = (h (fa + fb)) / 2;
Print["The area approximate by the trapezoidal rule is ", c]
int = Integrate[f[x], {x, a, b}];
Print["Original value of the integral is ", N[int], " and the error is ", N[c] - N[int]]
```

```
The area approximate by the trapezoidal rule is 93/2
Original value of the integral is 44.75 and the error is 1.75
```

Composite Trapezoidal Rule

Q. Find the approximate value of ∫(x+1) dx from 0 to 1 by using composite trapezoidal rule for n=10.
(/. x -> means replace x by a+i h)

Code and Output

In[66]:=
```
f[x_] := x + 1;
lowerlimit = a = 0;
upperlimit = b = 1;
n = 10;
fa = f[a];
fb = f[b];
h = (b - a) / n;
sum = 0;
For[i = 1, i < n, i++, sum = sum + N[f[x] /. x → (a + i h)]]
sum = (h / 2) (fa + 2 sum + fb);
Print["Integral = ", sum]
```

```
Integral = 1.5
```

Simpson's Rule

Q. *Find the approximate value of the integral ∫(x+1) dx from 0 to 1 by using simpson's rule.*

Code and Output

In[85]:=
```
f[x_] := x + 1;
lowerlimit = a = 0;
upperlimit = b = 1;
fa = f[a];
fb = f[b];
h = (b - a) / 2;
c = (h (fa + 4 f[a + h] + fb)) / 3;
Print["Integral = ", N[c]]
```

```
Integral = 1.5
```

Q. *Find the approximate value of the integral ∫(x^2+2x+7) dx from 1 to 2 by using simpson's rule and compare it with exact value of the integral also.*

Code and Output

In[104]:=
```
f[x_] := x^2 + 2 x + 7;
lowerlimit = a = 1;
upperlimit = b = 2;
fa = f[a];
fb = f[b];
h = (b - a) / 2;
c = (h (fa + 4 f[a + h] + fb)) / 3;
Print["The area approximate by the simpson's rule is ", N[c]]
int = Integrate[f[x], {x, a, b}];
Print["Original value of the integral is ", N[int], " and the error is ", N[c] - N[int]]
byTrap = N[(b - a) / 2 * (fa + fb)];
Print["The area approximate by the trapezoidal rule is ", N[byTrap]]
```

```
The area approximate by the simpson's rule is 12.3333
Original value of the integral is 12.3333 and the error is 0.
The area approximate by the trapezoidal rule is 12.5
```