L2(Binary Operations)

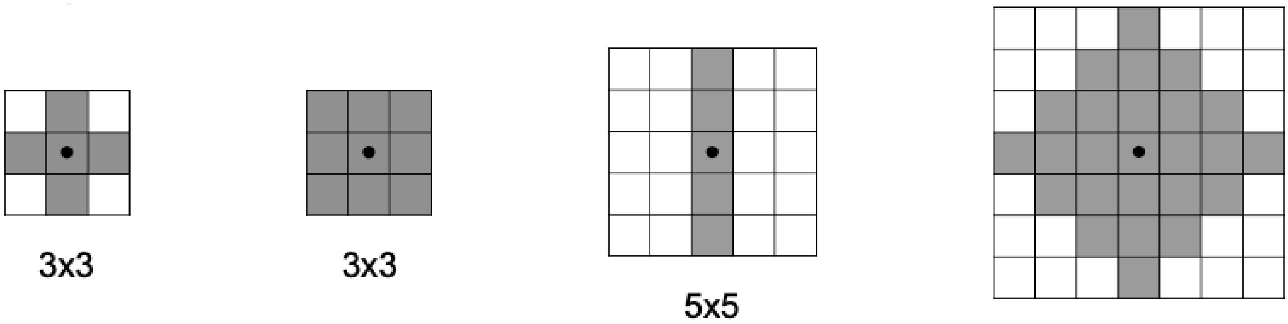
Morphological Filters

- Operations where an object is "hit" with a structuring element and thereby reduced/expanded to a more revealing shape.
- originally for binary signals, but extended for non-binary signals (grayscale images)

Dilation is "+" operator while erosion is "-" operator.

Binary Structuring Element

$b[m-k, n-l]$ denotes translation of $b[m, n]$ so that the origin is centered at position $[k, l]$.
Remember the origin is usually at $[0, 0]$, which corresponds to top left corner.
Examples for binary structuring elements:



3x3 3x3 5x5 7x7

Dilation

It expands or "grows" the white regions (foreground) of a binary image based on a small shape called the structuring element.
The structuring element b is moved across the image $s[m, n]$. At each position, the structuring element **hits** (overlaps) the image if any of its "1" values overlap with a "1" value in the image.
It's about **intersection** of structuring element and the image region.

$$s \oplus b = \{(k, l) \mid b[m-k, n-l] \cap s[m, n] \neq \emptyset\}$$

Intersection of structuring element and the image is not empty!

Effects

- expands the size of 1-valued objects (foreground)

- think of the structuring element as a stamp: wherever it touches white, it stamps more white in the output.
- closes holes and gaps
- smoothes object boundaries

Original image

Dilation with 3x3 structuring element

Dilation with 9x9 structuring element

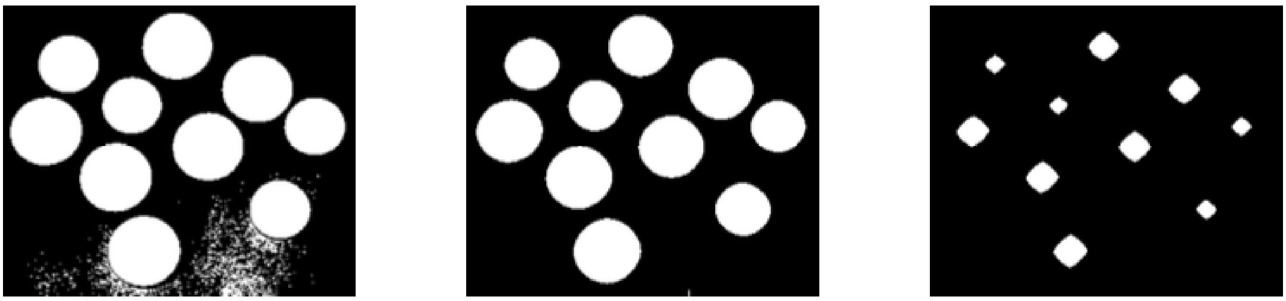
Erosion

Structuring element is **fully included** in the image region:

$$s \ominus b \; := \; \{(k,l) | b[m-k, n-l] \subset s[m,n]\}$$

Effects

- shrinks the size of 1-valued objects (foreground/white pixels)
- smooths object boundaries
- removes peninsulas, fingers, and small objects



Original image

Erosion with 5x5 structuring element

Erosion with 27x27 structuring element

Note that both in dilation and erosion, the original pixels get altered.

Properties of Erosion and Dilation

Both dilation and erosion have **translational invariance** and are generally irreversible operations.

Distributivity

Consecutive dilation or erosion by different structuring elements:

$$s \oplus (b \cup b') = (s \oplus b) \cup (s \oplus b')$$
$$s \ominus (b \cup b) = (s \ominus b) \cap (s \ominus b')$$

- If you dilate an image s using the union of two structuring elements b and b' , it's equivalent to dilating s with b and b' separately and then taking the union of the results.
- If you erode an image s using the union of two structuring elements b and b', it's equivalent to eroding s with b and b' separately and then taking the intersection of the results.

Duality

- The erosion of s by b is equivalent to the complement of the dilation of the background $\bar{s}$ by the reflection of b ($\hat{b}$)

$$\overline{s \ominus b} = \bar{s} \oplus \hat{b}$$
$$\overline{s \oplus b} = \bar{s} \ominus \hat{b}$$

- The dilation of s by b is equivalent to the complement of the erosion of the background $\bar{s}$ by the reflection of b ($\hat{b}$).
  For point symmetric structuring elements $b = \hat{b}$

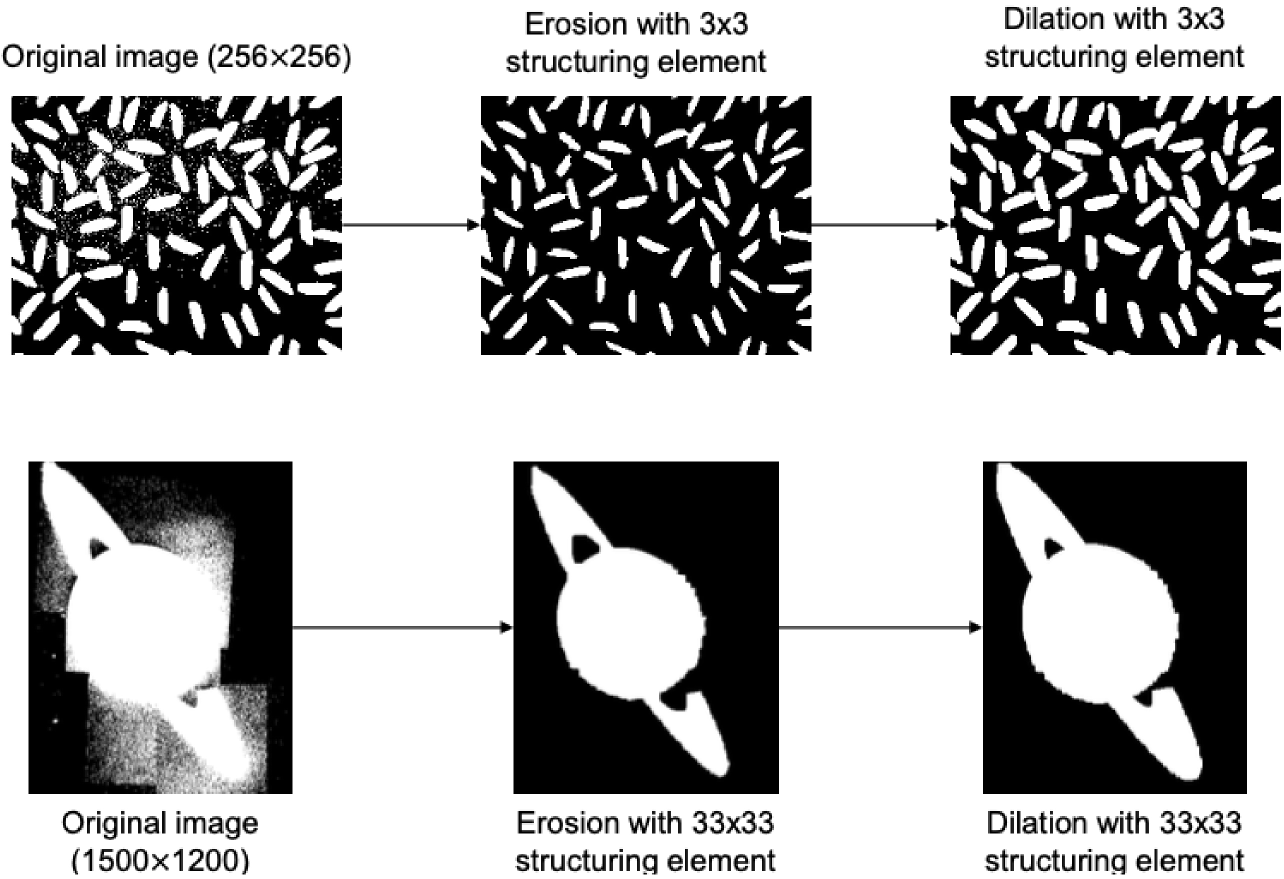
Opening and Closing

The goal here is to smooth without size change.

Opening

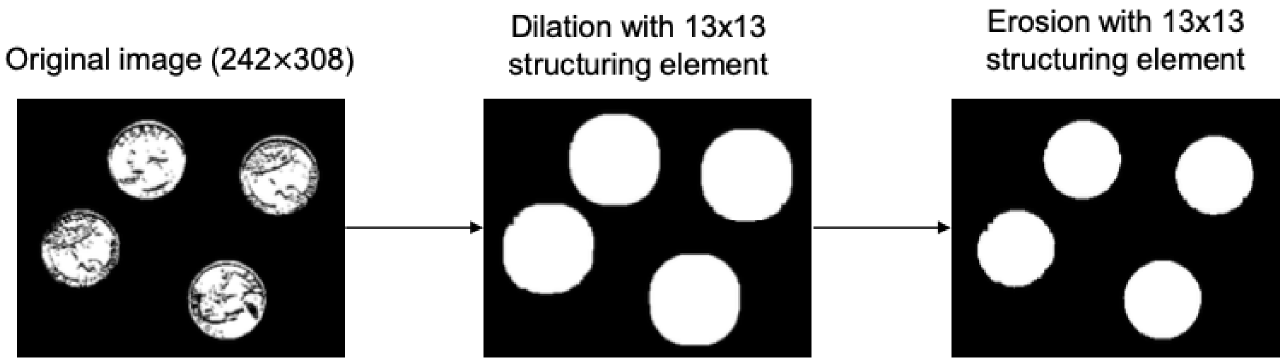
Opening is erosion combined by dilation:
Opening removes small regions(noise) in the image. Essentially, remove the noise with erosion which also shrinks the objects that we are interested. But this will be undone by a

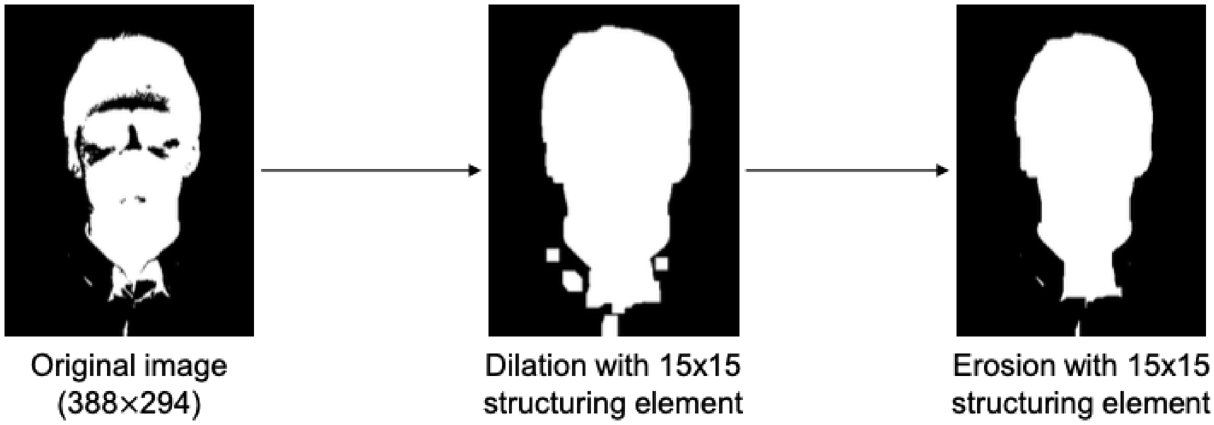
subsequent application of dilation.



Closing

Closing is dilation followed by erosion. First, it fills up the holes with dilation. Then, shrinks the objects of interest size back to normal with erosion.





Morphological Edge Detectors

White pixels represent object. We want to find out its boundary (edges)
It seems like the best filter would be the difference of dilated image and eroded image:

$$(s \oplus b) - (s \ominus b)$$

- Dilation expands the boundaries
- Erosion shrinks the boundaries
- The subtraction gives the boundary region by highlighting the difference between the expanded and shrunk versions

Iterative Hole Filling

Let r denote an 8-connected boundary of an object. Iterative **hole filling**:

$$s_k = (s_{k-1} \oplus b) \cap \bar{r} \qquad k = 1, 2, 3, \ldots$$

a seed point s_0, which is a known location inside the hole. (This is the starting point for filling). After each dilation constrain the result to remain inside the object by taking the intersection with s.

Grayscale images

Output would be the minimum value (erosion) or maximum value (dilation) of gray-scale image within structuring element