L1 (Exercise)

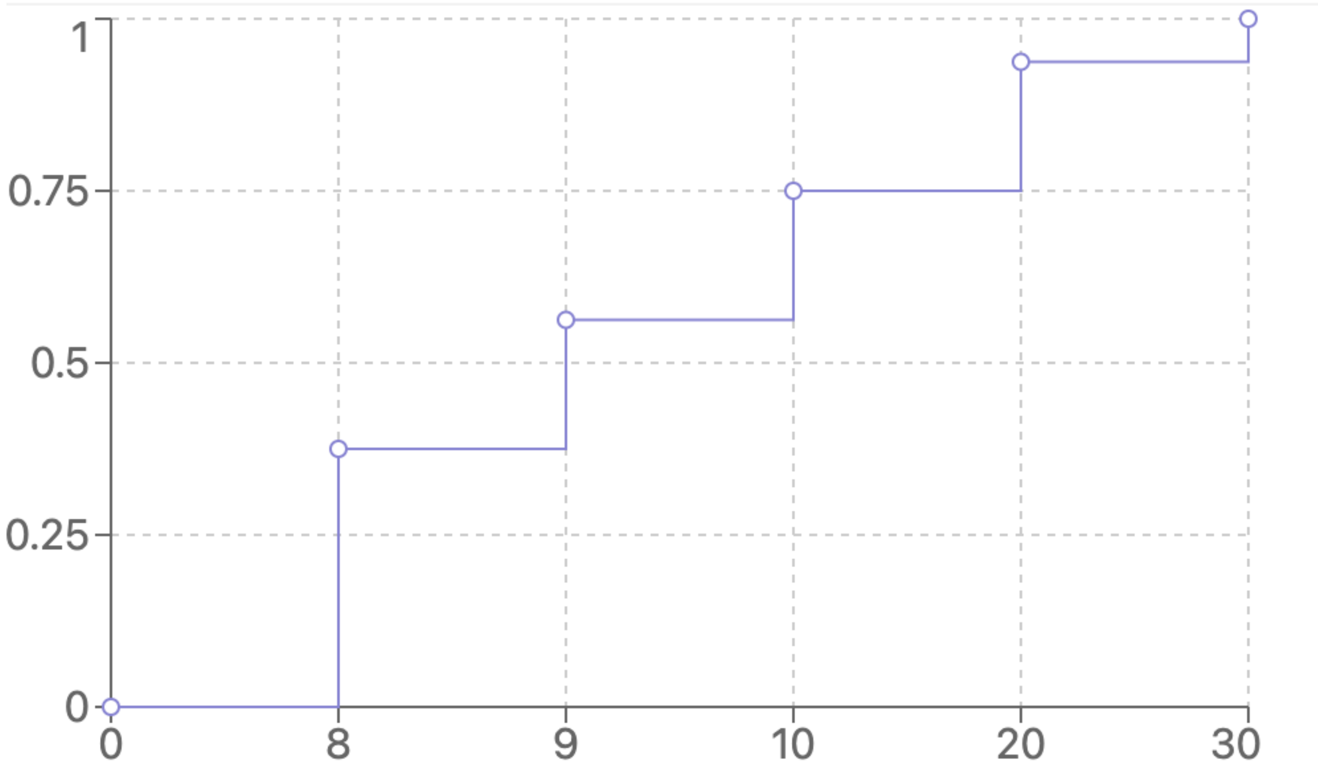
Gray levels L is the minimum bit depth required to fully represent all pixel values in an image.
Histogram Equalization:

- Input: Image $x[m,n]$ with L gray levels
- Compute:
  - Histogram $h[r]$
  - Cumulative distribution

$$P_r[r] = \frac{\text{cumulative count at r}}{\text{total count of pixels}}$$

- Histogram equalized image: $T[r] = floor((L-1) * P_r[r])$

Bin size here refers to how many values each bin covers from center to right(left to right).
CDF plot is like a staircase, with CDF value staying constant when there are no pixel values encountered:



L1 (Point Operations)

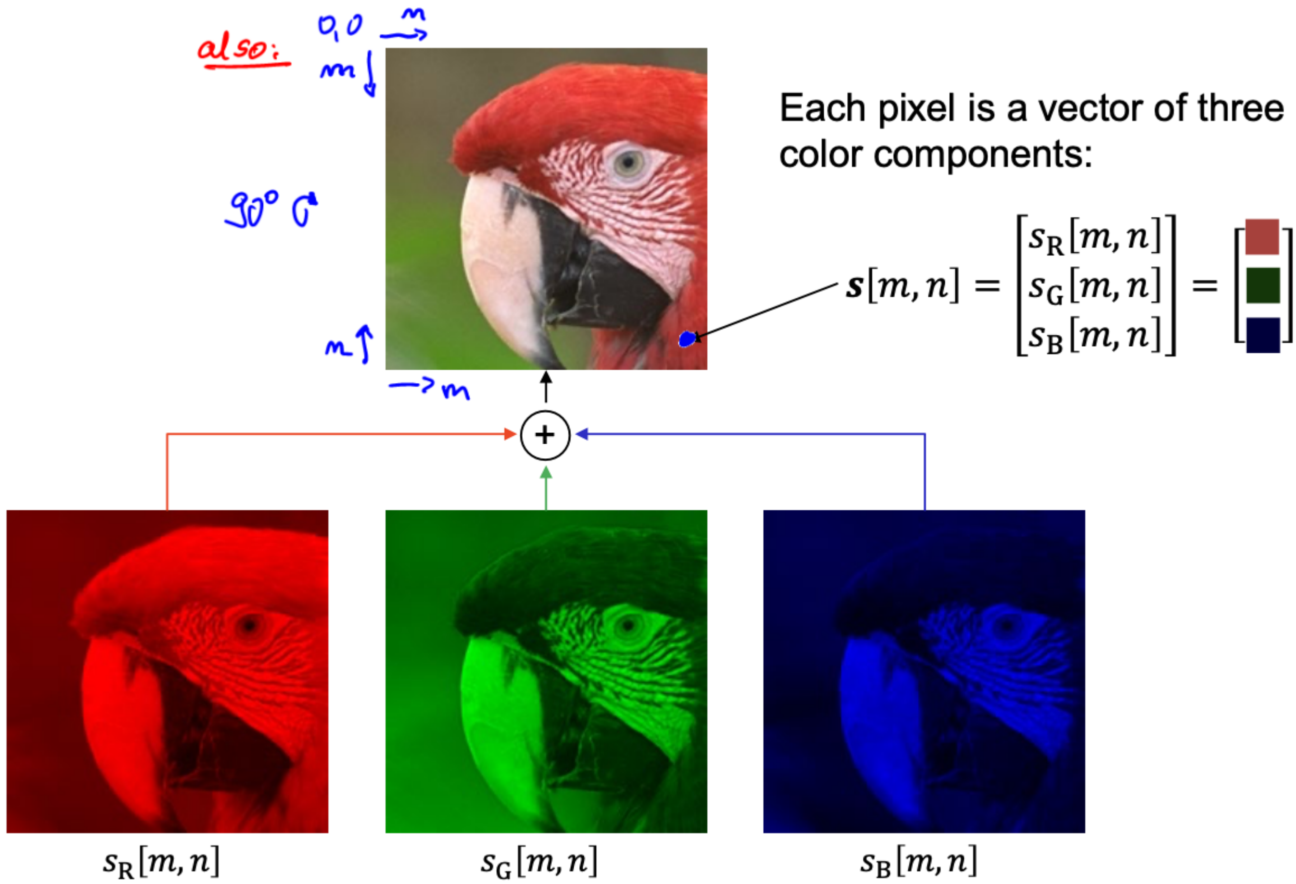
TLDR

- single frame point operations: **histogram correction** and **γ correction

- multi frame point operations: **image averaging, image subtraction, high dynamic range(HDR) imaging**

  Image matrix notation : $S_{MxN} = (s[m,n]|m = 0, \dots M-1, n = 0, \dots, N-1)$

  The starting point for **m** and **n** are at the top left corner:



Examples of Point Processing:

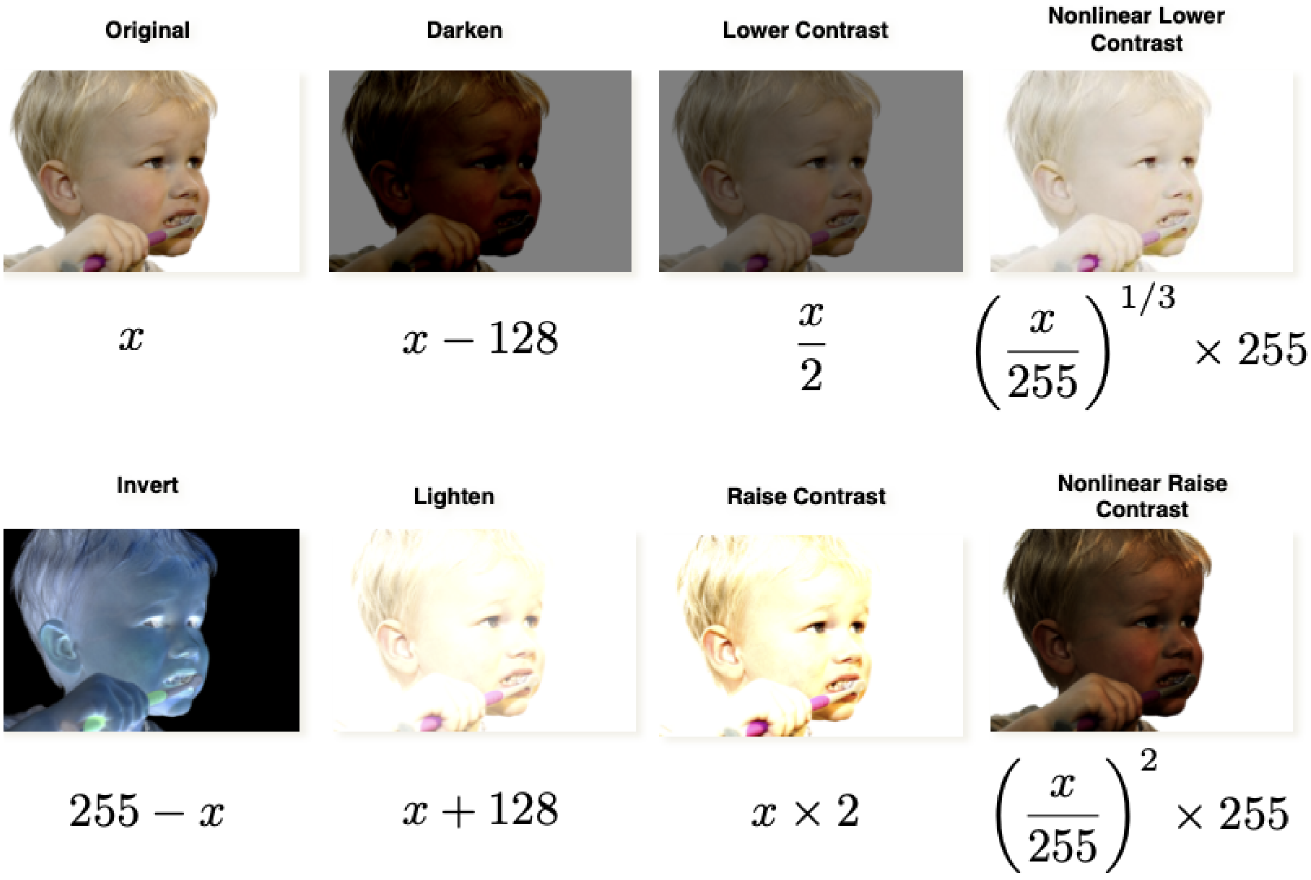


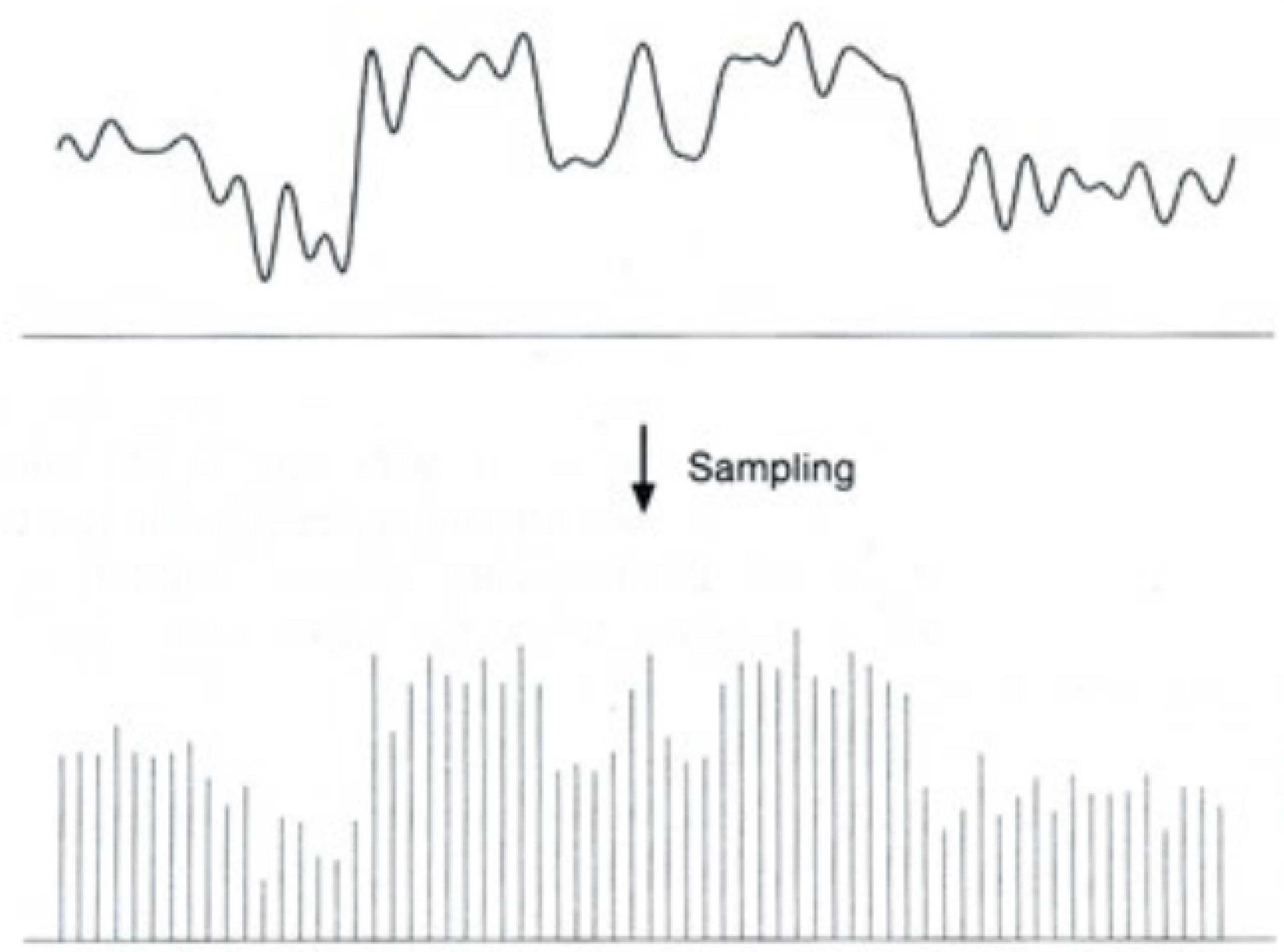
Image Resolution

Image resolution (number of pixels) can be increased by simple **pixel repetition** using nearest neighbor approach, but this can create blurry images.
There are better ways to achieve this:

- pre-filtering to avoid aliasing (what is aliasing?)
- smooth interpolation

Image Formation and Reconstruction

There are two processes: encoding(sampling) and decoding (reconstruction)
Sampling
We are given a continuous function. Our goal is to store this in our discrete representation space (histograms). The common way to do this is just sampling as many function values as possible at many points:



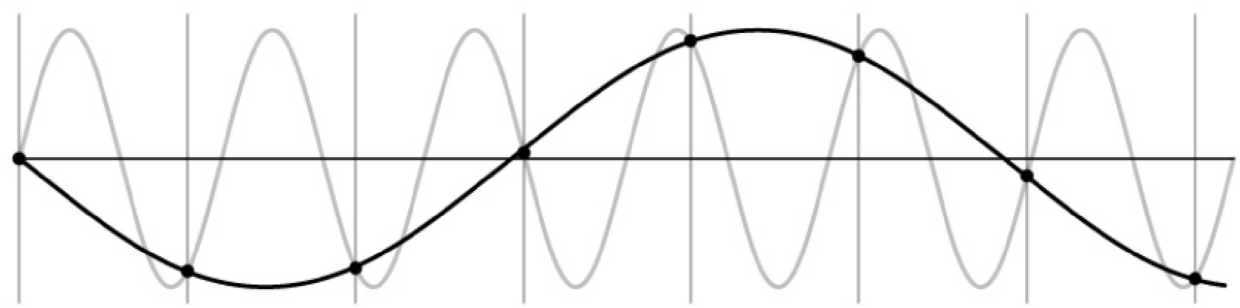
Reconstruction

How do we make samples back into a continuous function when displaying the signal (for output, analysis, etc)?

Example: a sine wave

Undersampling (Subsampling): what if we missed things between the samples? Obviously, some information is lost, but also the undersampled signal could misrepresent the original signal ("underfitting"),

Instead of a high-frequency sine wave, you get lower frequency sine wave:



This is a classic example of **aliasing** caused by **subsampling or undersampling**

The solution to aliasing effects is smoothing, effectively reducing the maximum frequency of image features. In other words smoothing removes t**he fast changes that sub-sampling would miss.**

Aliasing

Distortion or artifacts that occur when a signal is sampled at a rate that is **insufficient** to capture the detail in the signal (not enough samples). For example, aliasing happens because high-frequency components of the image (fine details or sharp edges) are misrepresented when sampled below **Nyquist rate**

To avoid this, one can do two things:

- **Pre-filtering**: use a low-pass filter(Gaussian blur) to remove high-frequency components **before** sampling or resizing. Get rid of some high frequencies. We will lose some information but it's better than aliasing.
- **Smooth interpolation**: bilinear or bicubic interpolation reduce visible artifacts compared to nearest-neighbor interpolation

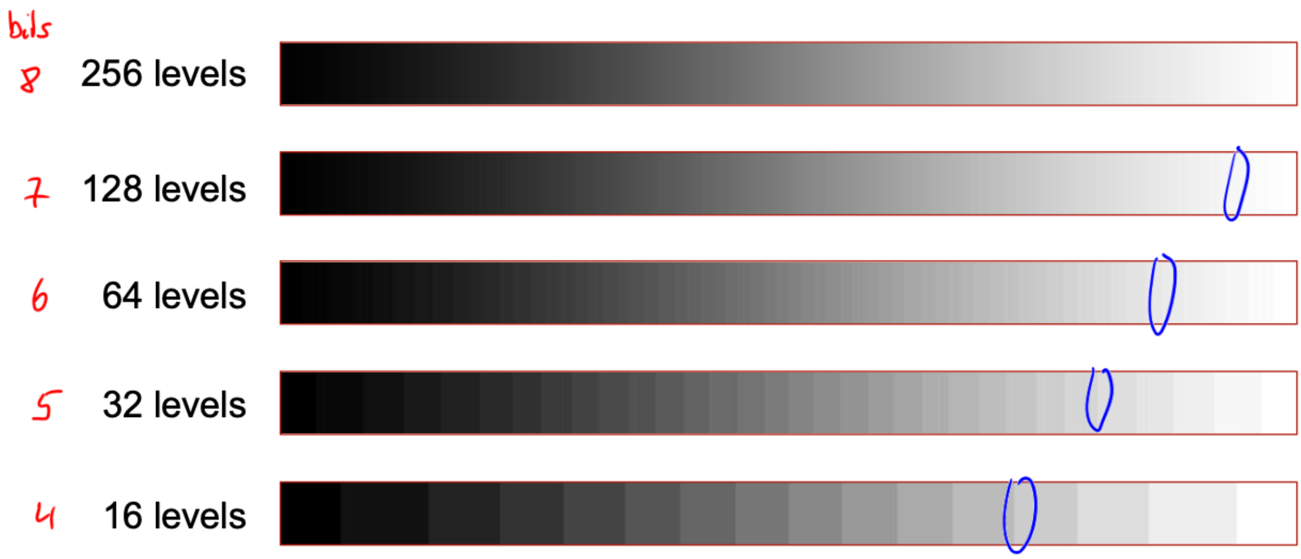
Monochromatic images

$$s = \frac{1}{3}(s_R + s_G + s_B)$$

s is created by averaging three RGB signal intensities. **Gray levels** refer to the number of distinct shades that can be represented in a grayscale image.
For example:

- **8 bits/pixel** can represent $2^8 = 256$ gray levels (from black to white)
- **4 bits/pixel** can represent $2^4 = 16$ gray levels

Higher bit depths allow for finer intensity resolution with smooth transition between shades of gray. Lower bits reduce memory/storage requirements but lead to coarser intensity resolution.
At lower levels, **contouring** becomes visible:



Single Point Operations

Histogram Equalization

Low contrast image may result from improper lighting or suboptimal quantization. Histogram equalization is introduced as a corrective method to make the intensity values more uniform and improve the overall visual appeal. It's all about **enhancing contrast**.

Goal: adjust the histogram of the image such that PDF of intensities are uniform.

It applies a non-linear transformation $s = T(r)$, where:

- r: original pixel intensity
- s: transformed pixel intensity

Properties:

- preserves dynamic range
- preserves black-white order

The transformation $s = T(r)$ is based on the **CDF** of the image's intensity histogram:

$$T(r) = \int_0^r p_r(w)dw$$

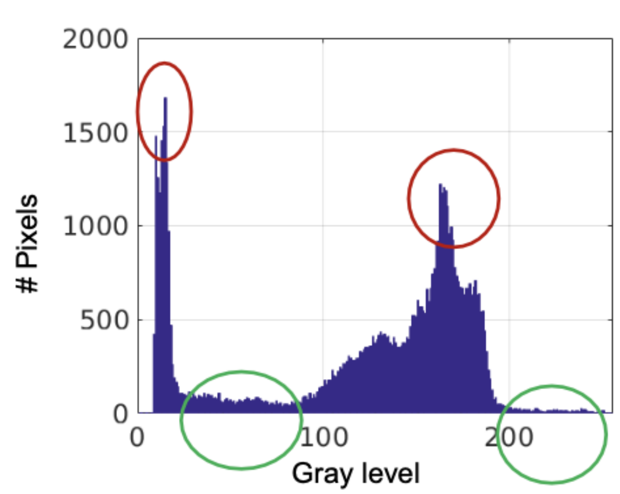
where :

- r: original intensity level (normalized between 0 and 1)
- $T(r)$: transformed intensity level
- $p_R(w)$: PDF of intensity levels
  In discrete terms:

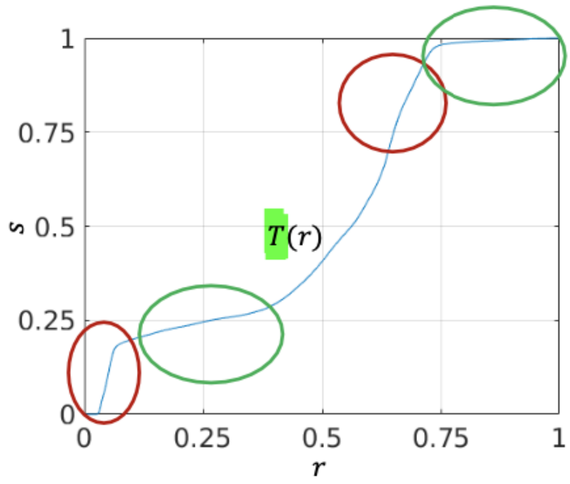
$$s = T(r) = \frac{\text{cumulative count at r}}{\text{total number of pixels}}$$

Larger accumulation of similar colors are "stretched out". Less frequent colors are "compacted":

Histogram



Equalization function



Discrete mapping function used in practice is CDF scaled by # of gray levels
After histogram equalization, CDF is approximately linear, that's ok. There might be empty bins since histogram equalization is still an approximation to the PDF.

PROBLEM

There is one big problem with global histogram equalization: global mapping leads to unnatural local equalization results.

Original image

…after histogram equalization

Adaptive Histogram Equalization

AHE applies histogram equalization locally using a **sliding window approach**, where the image is divided into smaller patches, and histograms are computed for each patch.
This improves **local contrast**, making features in specific regions more visible.
Pros:

- performs well in areas with **high local contrast**, enhancing details effectively.

Cons:

- tends to amplify **noise** in regions with **low local contrast**.

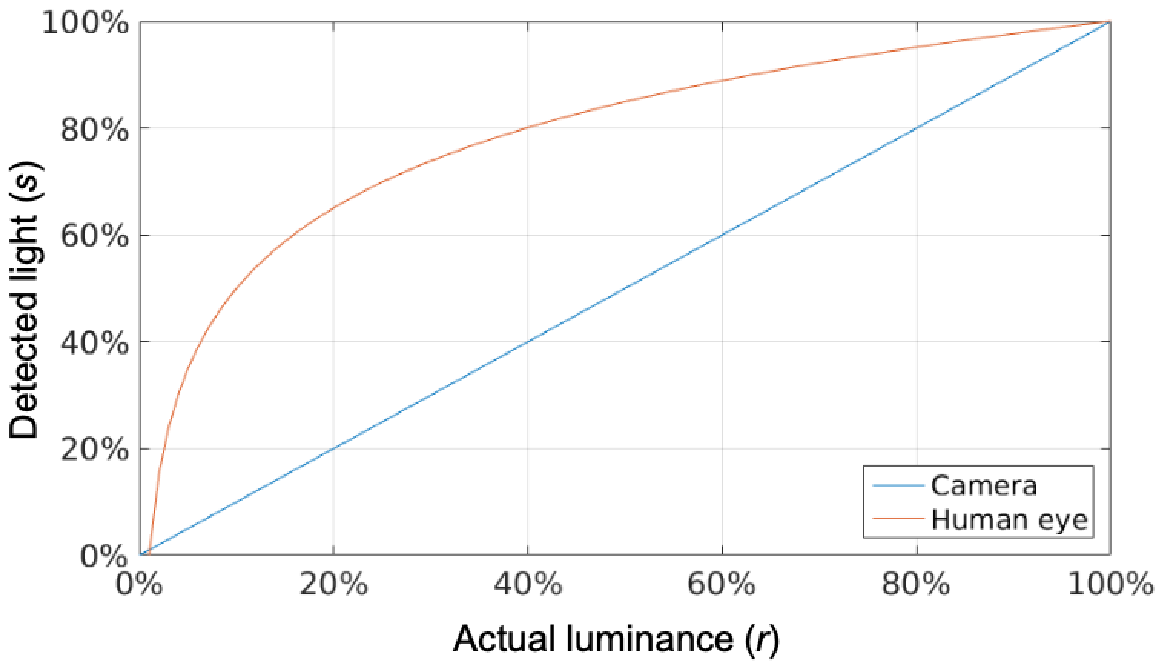
CLAHE (Contrast Limited AHE)

CLAHE addresses AHE's noise amplification issue by **limiting the contrast**.
This is done through **histogram clipping**, which caps the maximum slope of the cumulative distribution function (CDF), reducing the effect of large intensity variations in any given region.
Benefit:

- Provides improved contrast without the excessive noise amplification seen in AHE.

γ- adjustment

Non-linear point operation applied to images to encode or decode **luminance levels** to align with how humans perceive brightness.
Human vision perceives brightness logarithmically, not linearly.



Weber's Law

$$\frac{\Delta I}{I} = const$$

Fechner's Law

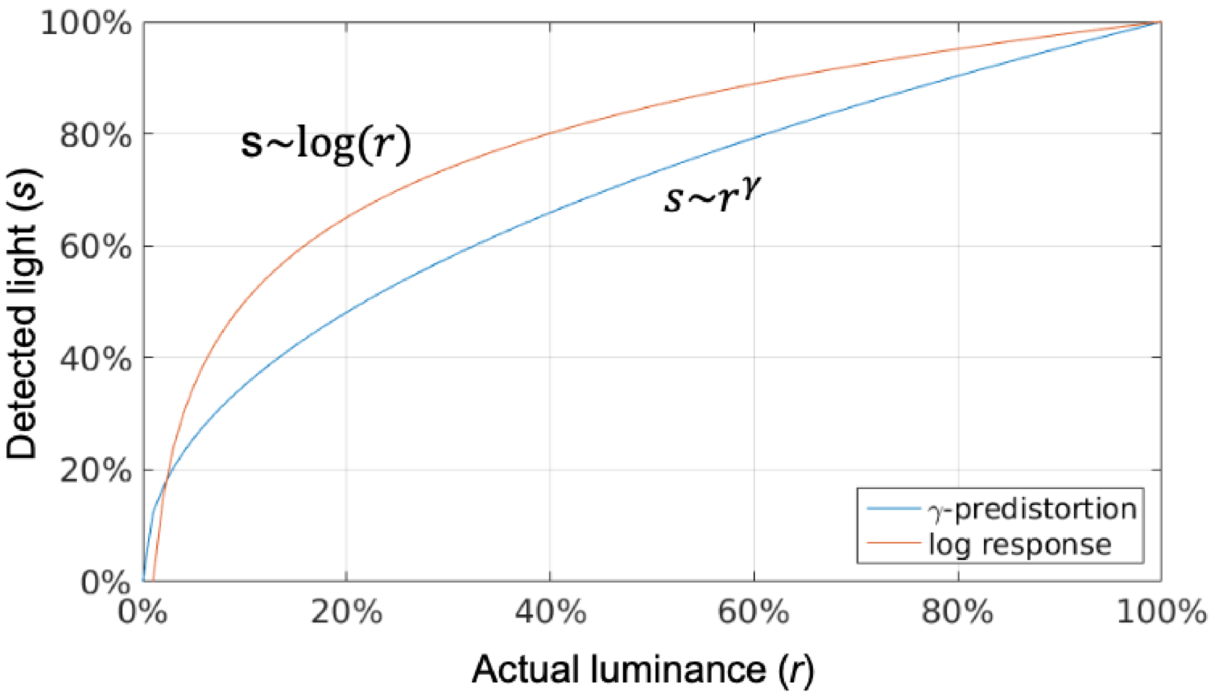
$$s \sim log(r)$$

Builds upon Weber's Law, proposing a logarithmic relationship between the perceived

intensity **of a stimulus (S) and its** physical intensity(I)*:
$$S=k\log(I)$$
Perception grow logarithmically. A large increase in stimulus is needed for the same perceived change at higher intensity levels.

Gamma Predistortion

Cameras contain γ - predistortion circuit to **mimic** human eye behavior:



Typically, values of $\gamma = \frac{1}{2.2}$ approximates human eye behavior pretty well.

Without gamma predistortion, we might have an excess of bits for brighter tones(camera more sensitive) and shortage of bits for darker tones(human eye more sensitive).

Gamma Adjustment Workflow:
γ-predistortion must be **compensated** before viewing. This is done by the video card to avoid unrealistically brightened images

Multi Frame Point Operations

For multi frame operations, the most important thing is the accurate alignment of frames.

Image Averaging

Temporal image averaging is used for noise reduction. The idea is to take N noisy images of the same scene. Suppose error-free image is given by $g[m,n]$ and the captured image is then:

$$s_i[m,n] = g[m,n] + v_i[m,n]$$

where $v_i[m, n]$ is an additive noise(uncorrelated)
Average image then would be:

$$\bar{s}[m, n] = \frac{1}{N} \sum_{i=1}^{N} s_i[m, n]$$

Noise gets reduced by N.

Image Subtraction

Find differences (errors, anomalies) between 2 mostly identical images. This is used in digital angiography to visualize blood vessels in brain by removing overlying structures like bone and soft tissues. Further contrast enhancement is applied

HDR Imaging

The goal is to extent the tonal range by combining images with multiple exposures by varying shutter speed, as different aperture affects depth of field.