

三角函数公式 (by lailai0916)

1. 两角和差公式

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

2. 诱导公式

第一组

$$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos \alpha$$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$$

$$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin \alpha$$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin \alpha$$

第二组

$$\sin(\pi + \alpha) = -\sin \alpha$$

$$\sin(\pi - \alpha) = \sin \alpha$$

$$\cos(\pi + \alpha) = -\cos \alpha$$

$$\cos(\pi - \alpha) = -\cos \alpha$$

第三组

$$\sin\left(\frac{3\pi}{2} + \alpha\right) = -\cos \alpha$$

$$\sin\left(\frac{3\pi}{2} - \alpha\right) = -\cos \alpha$$

$$\cos\left(\frac{3\pi}{2} + \alpha\right) = \sin \alpha$$

$$\cos\left(\frac{3\pi}{2} - \alpha\right) = \sin \alpha$$

第四组

$$\sin(2\pi + \alpha) = \sin \alpha$$

$$\sin(2\pi - \alpha) = -\sin \alpha$$

$$\cos(2\pi + \alpha) = \cos \alpha$$

$$\cos(2\pi - \alpha) = \cos \alpha$$

3. 二倍角公式

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 1 - 2 \sin^2 \alpha = 2 \cos^2 \alpha - 1$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

4. 三倍角公式

$$\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

$$\cos 3\alpha = -3 \cos \alpha + 4 \cos^3 \alpha$$

$$\tan 3\alpha = \tan \alpha \tan \left(\frac{\pi}{3} - \alpha \right) \tan \left(\frac{\pi}{3} + \alpha \right)$$

5. 半角公式

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} = \frac{\sin \alpha}{1 + \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha}$$

6. 积化和差公式

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin (\alpha + \beta) + \sin (\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2} [\sin (\alpha + \beta) - \sin (\alpha - \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos (\alpha + \beta) + \cos (\alpha - \beta)]$$

$$\sin \alpha \sin \beta = -\frac{1}{2} [\cos (\alpha + \beta) - \cos (\alpha - \beta)]$$

7. 和差化积公式

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

8. 辅助角公式

$$a \sin \theta + b \cos \theta = \sqrt{a^2 + b^2} \sin (\theta + \varphi), \tan \varphi = \frac{b}{a}$$

$$a \sin \theta + b \cos \theta = \sqrt{a^2 + b^2} \cos (\theta - \varphi), \tan \varphi = \frac{a}{b}$$

9. 万能公式

$$\sin \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

$$\cos \alpha = \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

$$\tan \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 - \tan^2 \frac{\alpha}{2}}$$