

1. Sketch the autocorrelation functions (ACF) for the following MA(2) models with parameters as specified:

(a) $\theta_1 = 0.5, \theta_2 = 0.4$.

(b) $\theta_1 = 1.2, \theta_2 = -0.7$.

Hint: You can use either the ACF formula for MA models or a useful R function like *ARMAacf* (as always first try *? ARMAacf* in R and find out the details).

(c) Simulate each the models in (a)-(b) with 150 observations, and plot their series, and their sample ACF.

2. Suppose that $\{Y_t\}$ is an AR(1) process with $-1 < \phi < +1$.

(a) Find the autocovariance function for $W_t = \nabla Y_t = Y_t - Y_{t-1}$ in terms of ϕ and σ_e^2 .

(b) In particular, show that $Var(W_t) = 2\sigma_e^2/(1 + \phi)$

3. Use the recursive formula to calculate and then sketch the autocorrelation functions for the following AR(2) models with parameters as specified. In each case specify whether the roots of the characteristic equation are real or complex. If the roots are complex, find the damping factor, R , and frequency, Θ for the corresponding autocorrelation function.

a) $\phi_1 = 0.6, \phi_2 = 0.3$.

(b) $\phi_1 = -.4, \phi_2 = .5$.

(c) $\phi_1 = 1.2, \phi_2 = -.7$.

Hint: You may use *polyroot* function in R to find roots of a polynomial.

(d) Simulate each the models in (a)-(c) with 150 observations, and plot their series, and their sample ACF.

4. Sketch the autocorrelation functions for each of the following ARMA models:

(a) ARMA(1,1), with $\phi = .7$ and $\theta = 0.4$

(b) ARMA(1,1) with $\phi = .7$ and $\theta = -0.4$

Hint: Like question 1., you can use *ARMAacf* function. (c) Simulate each the models in (a)-(b) with 150 observations, and plot their series, and their sample ACF.

5. For the ARMA(1,2) model $Y_t = 0.8Y_{t-1} + e_t + 0.7e_{t-1} + 0.6e_{t-2}$, show that

(a) $\rho_k = 0.8\rho_{k-1}$ for $k > 2$.

(b) $\rho_2 = 0.8\rho_1 + 0.6\sigma_e^2/\gamma_0$

6. (a) Find the autocovariance function of the time series $Y_t = e_t + 5/2e_{t-1} - 3/2e_{t-2}$, where $\{e_t\} \sim WN(0, 1)$
- (b) Find the autocovariance function of the time series $Y_t = e_t - 1/6e_{t-1} - 1/6e_{t-2}$, where $\{e_t\} \sim WN(0, 9)$
- (c)) Which of the MA models in 6. (a) and 6. (b) is invertible?