Aaron Vo MA 641 HW2 2/4/2024

1) Let $Y_1, \dots, Y_n$ be iid with $E(Y_i) = \mu$ and $V(Y_i) = \sigma^2$. Recall that $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$

↳ Find $E[(\bar{Y} - \mu)^2] = E[(\bar{Y} - E(\bar{Y}))^2]$

↳ $= E[(\frac{1}{n} \sum (Y_i - \mu))^2]$

↳ $= \sum E[(Y_i - \mu)^2] = \sum Var(Y_i)$

↳ $= n\sigma^2$

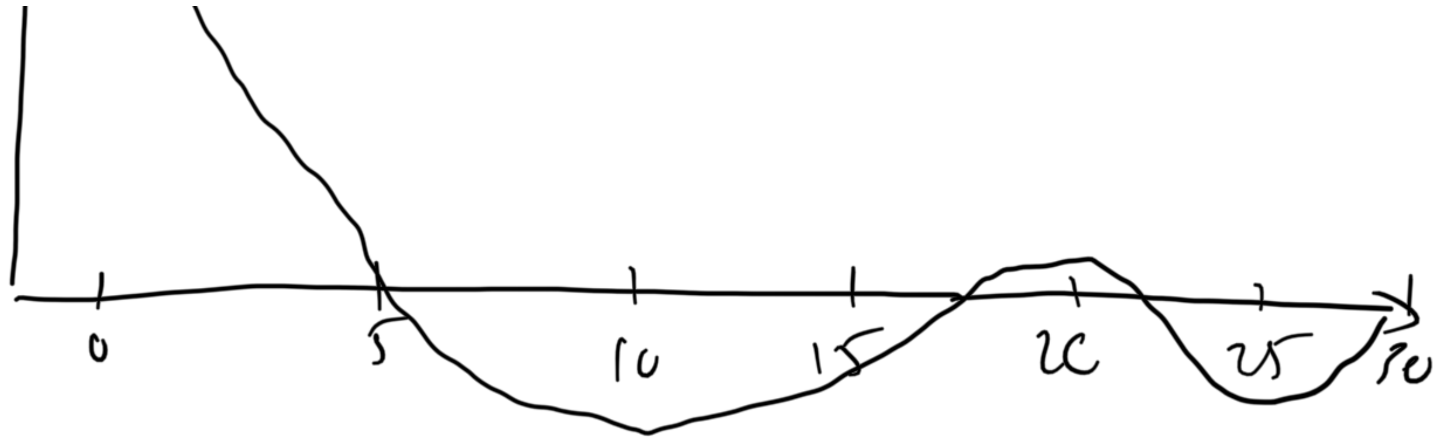
↳ $E[(\frac{1}{n} \sum (Y_i - \mu))^2] = \frac{1}{n^2} E[(\sum (Y_i - \mu))^2]$

↳ $= \frac{1}{n^2} \cdot n\sigma^2$
$= \frac{\sigma^2}{n}$ ■

2) $\rho_1 = .8$, $\rho_2 = .6$, $\rho_3 = .4$, $\rho_4 = .2$

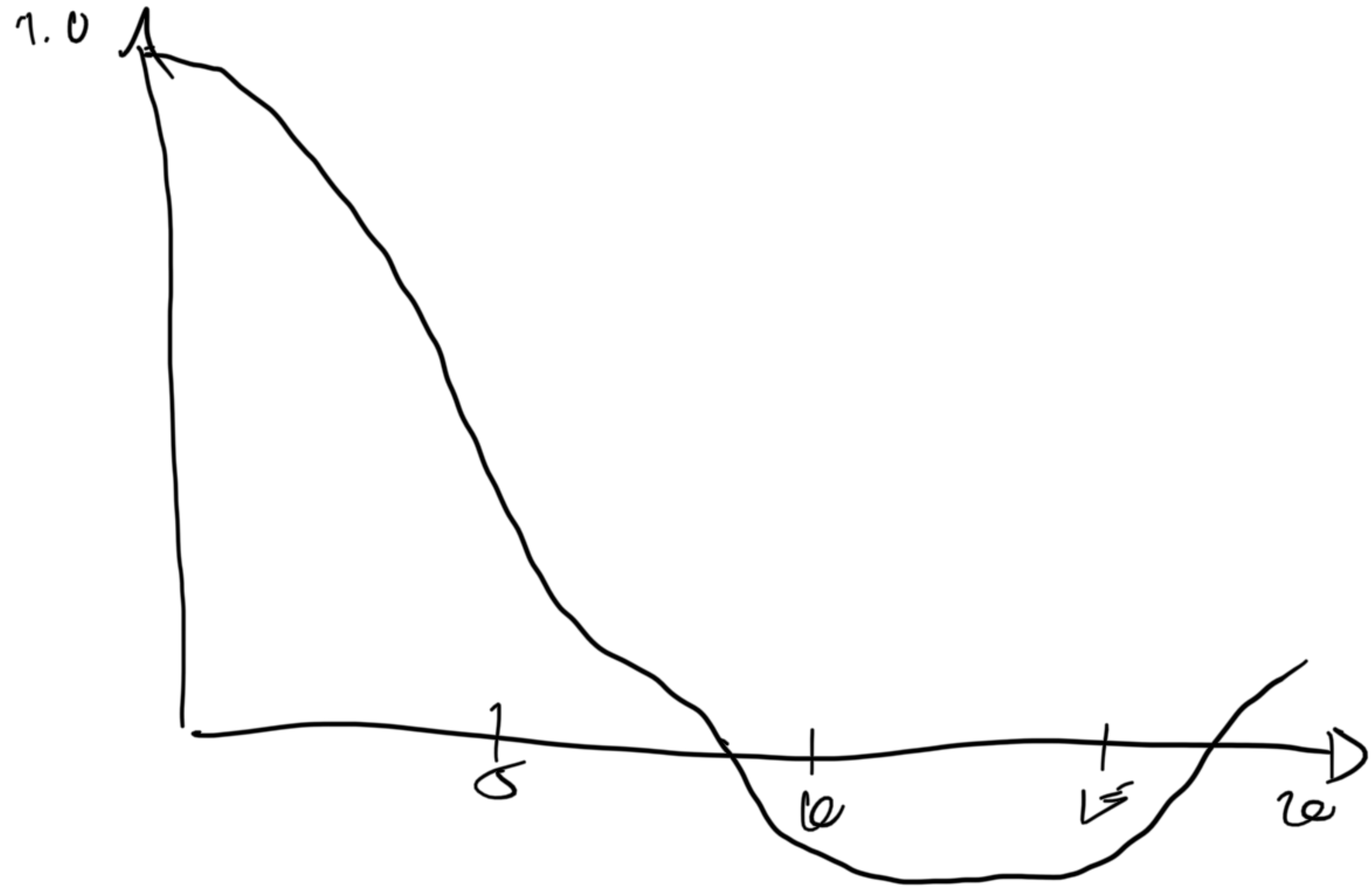
a. Sketch ACF Plot

↳ For 1000 samples




b.

↳ For 150 samples




4) $P(W_t = 0) = P(W_t = 1) = 1/2$ and
$P(Z_t = -1) = P(Z_t = 1) = 1/2$

- Show that $\{X_t\}$ is white noise but not i.i.d

↳ Show that $\{X\}$ is white noise

↳ $Var(X_t) = 1/4$

↳ $Cov(X_t, X_{t-1}) = E(X_t \cdot X_{t-1}) - E(X_t) \cdot E(X_{t-1})$
$= 0$

- The autocovariance function of $\{X_t\}$ is zero for all lags except 0, which means that $\{X_t\}$ is white noise

- $\{Y_t\}$ is not i.i.d because it is not the same for all points

3) Consider MA(1) model $Y_t = \mu - \theta e_{t-1} + e_t$

a. Find $E(Y_t)$

↳ $E(\mu - \theta e_{t-1} + e_t) = \mu - \theta E(e_{t-1}) + E(e_t)$
↳ $E(Y_t) = \mu$

b. $V(Y_t) = Cov(Y_t, Y_t)$

↳ $Var(Y_t) = Cov[(\mu - \theta e_{t-1} + e_t), (\mu - \theta e_{t-1} + e_t)]$
↳ $= \theta^2 Cov(e_{t-1}, e_{t-1}) + Cov(e_t, e_t)$
↳ $Var(Y_t) = \theta^2(1 + \theta^2)$

c. Find $Cov(Y_t, Y_{t-1})$

↳ $= -\theta\, Cov(e_{t-1}, e_{t-1}) = -\theta\sigma^2$

d. $\{Y_t\}$ is stationary because $E(Y_t)$ is constant and $Var(Y_t)$

is independent